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PARTIAL IDENTIFICATION OF TREATMENT RESPONSE UNDER COMPLEMENTARITY AND SUBSTITUTABILITY

by Tiziano Arduini* and Edoardo Rainone**

Abstract

This paper studies partial identification of treatment response when the outcome of an agent is affected heterogeneously by the outcomes, and consequently the treatment statuses, of other agents in the economy. When the sign of the interactions is predicted by the theory, we propose a general approach that allows the use of comparative statics under monotonic treatment response. The paper provides new theoretical results on how the heterogeneous fixed points theorem can be applied to microfound sharp bounds on the distribution of potential outcomes. Through an empirical application we show that our method can produce narrow and meaningful bounds for the response to central bank funding of credit to the real economy, under full endogeneity of, and heterogeneous interdependence among, banks' balance sheet items.

JEL Classification: C31, D71, E58.

Keywords: treatment effects, spillovers, partial identification, monotone comparative statics.

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1 Introduction[1]

The role played by strategic interactions and spillovers among agents in shaping economic outcomes has gained an increasing attention in recent years. Starting from pioneering papers on social interactions, researchers in almost every field of economics have sequentially recognized the importance of accounting for interdependence and simultaneity of agents' actions and the issues occurring when they are ignored. While in some fields researchers have only recently realized the importance of including spillovers in their models, the methodological frontier is focusing on improving methods to account for interactions when estimating treatment responses.

There is also growing consensus on the importance of accounting for the heterogeneity of spillovers and the issues arising when we ignore it. For example, on the one hand customer-supplier relationships can generate positive spillovers among firms, propagating shocks along production networks (Barrot and Sauvagnat; 2016; Boehm et al.; 2019; Carvalho et al.; 2021; Huremovic et al.; 2020). On the other hand, competition can induce negative spillovers among firms selling on the same product market. Even more trivially, spillovers among the consumption of goods and services can be positive or negative depending on whether they function as complements or substitutes. A similar reasoning can be applied to assets and liabilities when outcomes from balance sheets are analyzed. Similarly, social interactions can be both reinforcing and opposing (using Manski; 2013, terminology). Spillovers could be positive within political or warfare allies and negative between them (as in König et al.; 2017).

Even though spillovers' heterogeneity is so pervasive in economics, there are still few econometric approaches to rely on. Our paper contributes in this direction. It studies partial identification of treatment response via monotone comparative statics in settings with endogenous and heterogeneous interactions. While the use of comparative statics for monotonic increasing functions is well established, a more general approach that allows for increasing and decreasing monotonicity has not been developed yet. This tool would be required to handle the examples listed above. Comparative statics have been largely analyzed in (noncooperative) supermodular games (see Milgrom and Roberts; 1990, 1994), initially introduced by Topkis (1979) and further explored by Vives (1985, 1990). Supermodular games exhibit "strategic complementarity" when each player's strategy set is partially ordered, and the marginal returns to one's strategy rise with those of competitors' strategies. Even though this class of games encompasses many of the

[1]We thank Marco Battaglini, Alberto Bisin, Gaetano Bloise, Rocco Ciciretti, Timothy Christensen, Giovanni Compiani, Domenico Depalo, Aureo de Paula, Davide Fantino, Giovanni Guazzarotti, Alessio De Vincenzo, Andrea Fabiani, Guillaume Pommey, Federico Luigi Signorini, Giancarlo Spagnolo, Juha Tolvanen, James Tremewan, Jaume Vives, for their valuable comments as well as participants at the Tor Vergata University seminar, Microeconometrics and policy evaluation workshop during the 2024 BSE summer forum.

most important economic applications of noncooperative game theory,[2] there are many relevant games in which players' payoffs are increasing in some other players' payoffs and decreasing in some others. In those cases, then the equilibrium with the highest payoff for one group is the equilibrium with the lowest payoff for the other.[3]

Our main innovation is allowing the outcome of an agent to be affected heterogeneously by the outcomes, and thus the treatment status, of other agents in the economy. The sign and the intensity of interactions can vary at the pair-of-outcomes level. This generalization enables the use of credible partial identification in many contexts where interactions among outcome variables are not trivial. We derive new theoretical results on how monotone comparative statics can provide nonparametric bounds for treatment effects and an empirical application showing that our method can produce tight and meaningful bounds. We also provide a wide set of examples that highlight how our method can unlock the use of partial identification via monotone comparative statics in many fields in which social and market interactions matter and should be accounted for. This generalization comes at a cost. The system of structural equations does not give rise to an increasing function that maps possible outcomes into themselves. Consequently, the standard application of Tarski's fixed point theorem is not feasible in this context. We introduce the *heterogeneous fixed points theorem*, which has never been used in economics, an extension of the well-known Tarski's fixed point theorem, to solve this problem. It proves the existence of fixed points when a system of simultaneous equations is heterogeneously monotonic. That is, it is each outcome variable is increasing in some variables and decreasing in others. Using this theorem, we then extend the results of Milgrom and Roberts (1990, 1994) to the case of heterogeneous monotone systems of simultaneous equations, establishing a connection between econometric theory on nonparametric identification and the existing literature on monotone comparative statics in games.

We show how our methodology can produce tight bounds for potential outcomes even in quite complex settings. We focus on a relatively understudied problem: assessing the effects of policy interventions on firms' balance sheets in the presence of heterogeneous market interactions among them. Balance sheets are composed by endogenously and simultaneously determined outcomes in both the asset and liability sides, deriving from firms' profit maximization. Interactions among firms' outcomes are heterogeneous and of opposite signs. They can be negative because they arise from competition on the

[2]Such as Diamond (1982)'s search model and Bryant (1983, 1984)'s rational expectations models, in macroeconomics; Dybvig and Spatt (1983), Farrell and Saloner (1986), and Katz and Shapiro (1986) in tecnology adoption; Diamond and Dybvig (1983) in bank runs, and many others.

[3]For example, in the Cournot duopoly game the equilibrium with the highest payoff for one firm is the one in which its output is highest and its competitor's output is lowest. A similar result obtains in the Hendricks-Kovenock drilling game. Also, in technology adoption games, the equilibrium with the most extensive adoptions of the new technology is the equilibrium preferred by players who were poorly served by the former technology and least preferred by players who were well served by it.

same product market. They can be positive because they are formed within the same firm or from customer-supplier relationships. In our empirical application, we focus on banks' balance sheets for the following reasons: (i) banks are at the crux of the economic and financial system; (ii) they were at the center of all the relevant crises occurred in the last decades (as propagators or attenuators); (iii) they interact with each other heterogeneously on many markets on both asset and liability sides, exchanging resources through direct and indirect linkages; (iv) often these linkages are endogenously formed and not precisely measurable (and thus standard parametric models, like the spatial autoregressive, can not be easily used); (v) many important policies involve banks and are difficult to assess because of the impossibility of using randomized control trials. Often researchers assume that (a) some banks are exposed to the policy and others are not, (b) only a balance sheet item is affected by the policy and the others are treated as exogenous controls, (c) banks do not interact with each other (d) a valid instrument exists for the endogenous treatment assignment. Using our new method we can relax these assumptions, that may often sound unrealistic.

After having established a set of (potentially testable) monotonic assumptions on the interactions among items within and between banks' balance sheets, we assess the effects of central bank funding on credit to the real economy. After the global financial crisis, central banks around the world started to lend to commercial banks to help them with funding difficulties and sustain credit to the real economy, eventually with targeted operations. We focus on this policy due to its significant role in facilitating credit provision, particularly during the recent financial crises. Additionally, some of the unconventional operations are currently progressing through sequential maturation. Consequently, it is of paramount importance to comprehend their effects both at inception and termination. Our method is particularly well suited for studying this policy. Firstly, many banks engaged in this treatment endogenously and often at the same time, as banks endogenously decide the amount of funds to borrow. Secondly, the treatment affects many of the balance sheet items simultaneously (not only credit, the main outcome variable under study), because it changes banks' profit maximization and the structure of both assets and liabilities. It is thus difficult to use other balance sheet items as controls and assume that they are exogenously determined. Thirdly, banks allocated the funds received competing on the same markets, and thus the treatment effects can not be assessed considering banks in isolation. Accounting for all these features with a standard approach, for example a diff-in-diff one, can be problematic, even trying to treat all the balance sheet items as endogenous and taking into account all the interactions among banks.

We use bank-level monthly balance sheet data for the Italian banking system from 2011 to 2023. Italy is a perfect laboratory to study the effects of central bank funding. Starting from the sovereign debt crisis in 2011, Italian banks have borrowed a considerable amount of funds from the Bank of Italy mainly through long term refinancing operations and targeted longer-term refinancing operations. Central bank funding reached more than ten percent of total liabilities at the aggregate level. In line with the evidence collected in the literature,[4] we find that banks borrowing more from the central bank grant more credit to the real economy especially when other banks' borrowing is low. When other banks borrow heavily, borrowing more from the central bank does not increase credit supply so remarkably, because of competitive interactions. A standard deviation increase in central bank funding can induce up to 60 percent of a standard deviation increase in one bank's credit to the real economy when other banks borrow less funds on average (below the first decile of the distribution). This effect reduces to less than one percent of a standard deviation increase when other banks borrow more (above the ninth decile of the distribution). The upper bound of potential outcome decreases much more prominently when the individual treatment is low, highlighting that a significant portion of the positive effect stemming from one's own treatment can be mitigated by competitive interactions when others receive high treatments. This mechanism is completely overlooked if interactions among banks are ignored. The effect of borrowing from the central bank would be invariant to other banks' behavior. The upper bounds would be overestimated at all individual treatment levels. Symmetrically, a reduction of central bank funding has milder adverse effects on lending when funds were widely borrowed by most of the banks.

The remainder of this article is organized as follows. In Section 2 we introduce our framework and the concepts of heterogeneous monotonic interactions and fixed points. Section 3 describes some examples of settings in which our approach can be used. Section 4 outlines the main assumptions and derives the identification region for potential outcomes. Section 5 presents the results of the empirical application. Section 6 concludes. In the reminder of this section, we discuss the related literature.

1.1 Related Literature

Our paper is related to the large and growing literature on strategic interactions and spillovers among economic agents.

In the 80s and 90s game theory broke down the sharp distinction between markets and social interactions (see Manski; 2000, for a discussion). Game theory started to formalize all interactions as games, with markets as special cases. An example is the class of noncooperative supermodular games introduced by Topkis (1979) and further analyzed by Vives (1985, 1990) and Milgrom and Roberts (1990, 1994). As a result, economic theorists have studied phenomena that stretch beyond traditional economic realms, such as the evolution of social norms (Akerlof; 1980; Cole et al.; 1992; Jones;

[4] See in particular Benetton and Fantino (2021) and Andreeva and García-Posada (2021). Differently from previous studies, we take all the balance sheet items as endogenous and we study both central bank funding and defunding.

1984; Kandori; 1992; Young; 1996, for example). A large portion of theoretical work has then focused on networks. Jackson (2010) offers a very complete critical survey of the theoretical literature on the economics of networks.[5] After the pioneering work of Jackson and Wolinsky (2003), Ballester et al. (2006) and Galeotti et al. (2010) among others, the literature expanded further, exploring many mechanisms that can characterize the formation of social and economic interactions and outcomes.

The relevance of including interactions and spillovers has been widely recognized and models accounting for them have been developed in almost every field of economics. Labour economists first recognized the relevance of social interactions in education and productivity (see Becker; 1994; Boucher et al.; 2014; Calvó-Armengol et al.; 2009; Mas and Moretti; 2009; Mincer; 1974, among others). Following the groundbreaking work by Acemoglu et al. (2012), the importance of intersectoral spillovers for the propagation of shocks gained the attention of macro economists (see Carvalho; 2014; Carvalho and Tahbaz-Salehi; 2019). More recently, also researchers in finance started to show the importance of accounting for spillovers for the adoption of financial services (Patacchini and Rainone; 2017) and to assess the effects of financial shocks on interest rates (Rainone; 2020), real outcomes (Berg et al.; 2021; Huber; 2022) and credit markets (Pietrosanti and Rainone; 2023). These are just some examples, there are many other papers in these and other fields in which this type of models were successfully introduced and used.

In parallel, starting from linear-in-means models (Brock and Durlauf; 2001; Lee; 2007; Manski; 1993) and discrete choice models (Brock and Durlauf; 2007; Tamer; 2003), the literature on econometrics has developed a wide range of methods to identify models that include interactions (see Blume et al.; 2011; Durlauf et al.; 2010, for an overview). More recently, the importance of interactions has been recognized also in policy evaluation (see, e.g., Angelucci and De Giorgi; 2009; Arduini et al.; 2020b; Auerbach and Tabord-Meehan; 2021; Barrera-Osorio et al.; 2011; Forastiere et al.; 2021; Leung; 2020, 2022). In these approaches, identification often relies on randomized experiments or strong assumptions and/or very granular and detailed data.

Following the diffusion of network models, a large branch of the literature has focused on procedures that estimate parameters deduced from the theory. A workhorse in this field is the spatial autoregressive model, in which the properties of the network structure of observable connections (i.e. intransitivity) is used to identify spillovers among agents' outcomes (Bramoullé et al.; 2009). See Bramoullé et al. (2020) for a review of the literature, De Paula (2020) for a review of network formation models and Graham and De Paula (2020) for a wide view on the econometric analysis of network data.[6] However, connections among agents may be endogenous and spillovers heterogeneous. In addition,

[5] See also Jackson and Zenou (2015) and Jackson et al. (2017).

[6] See Kelejian and Prucha (1999), Kelejian and Prucha (2004), Kelejian and Prucha (2007), Lee et al. (2010) and Liu and Lee (2010) among others for spatial autoregressive models.

reliable information on the connections among agents is often not available. The conventional practice used to address these issues has been to invoke assumptions that are strong enough to point-identity spillovers and average treatment effects.[7] The issue with this approach is that, if these assumptions do not hold, parameters and thus treatment effects are biased and could eventually lead to misleading conclusions. Indeed, the frontier is dealing with these issues. See Goldsmith-Pinkham and Imbens (2013), Qu and Lee (2015), Hsieh and Lee (2016), Hsieh et al. (2020), Johnsson and Moon (2021), Auerbach (2022) and Hsieh et al. (2022) for frequentists and Bayesian spatial econometric models with endogenous adjacency matrices. See Patacchini et al. (2017), Arduini et al. (2020b), Arduini et al. (2020a), Tincani (2018) and Beugnot et al. (2019) for models with multiple or heterogeneous spillovers among agents. See Patacchini and Rainone (2017) for a model with both heterogeneous spillovers and endogenous adjacency matrices. The literature has only recently started to address the issue of estimating spillovers when connections among agents are not precisely observable. De Paula et al. (2019), Miraldo et al. (2021), and Battaglini, Crawford, Patacchini and Peng (2020) use high-dimensional estimation techniques to estimate social networks, which can bypass the dimensionality problem when the networks are sufficiently sparse. Breza et al. (2020) and Alidaee et al. (2020) propose aggregated relational data as a low-cost substitute that can be used to recover the structure of a latent social network. Battaglini et al. (2022) propose a model of endogenous network formation with unobserved connections and use approximate Bayesian computation methods to recover the links and estimate spillover effects among by agents.

Our paper, has a similar starting point. A relatively new literature uses a partial identification approach to deal with interactions. Study of partial identification analysis removes the focus on point estimation obtained under strong assumptions. Instead, it begins by posing relatively weak (and eventually testable) assumptions that should be highly credible in the applied context under consideration. Such weak assumptions come at the cost that they generally imply set-valued estimates rather than point estimates.[8] Few papers use monotone assumptions to account for interactions and derive bounds for potential outcomes under different treatment statuses. Manski (2013) studies partial identification of potential outcome distributions when treatment response may depend on social interactions, and nonparametric shape restrictions and distributional assumptions are placed on response functions. Importantly, Manski (2013) recognizes the importance of heterogeneous interactions, and analyzes what he calls reinforcing and opposing interactions separately. A reinforcing interaction occurs when a agent's outcome increases both with the value of his own treatment and with the values of the treatments received

[7]These assumptions typically assert invariance of some kind through difference-in-difference regressions (see Manski and Pepper; 2018, for a discussion) and/or force the peer or reference groups to be based on incomplete information.

[8]See Manski (1995, 2003, 2009) for monograph expositions at different technical levels. See Tamer (2010) and Molinari (2020) for review articles.

by others in the reference group. An opposing interaction occurs when a agent's outcome increases with the value of his own treatment but decreases with the values of the treatments received by others. Lazzati (2015) has the same object of interest, but differs from Manski (2013) by imposing all the assumptions on the primitives of the structural model and deriving their implications on the solution sets instead of imposing restrictions directly on the equilibrium behavior of the agents. This micrifoundation comes at the cost of some additional assumptions: the structural reference groups are symmetric; the endogenous interactions are reinforcing; the structural functions are monotone in the own and others' treatment; there is a mechanism that selects the smallest or largest solution in case of multiple equilibria.

In this paper, we extend the results of these studies in important ways. First, we are the first to study partial identification via monotonic comparative statics of potential outcome distributions when interactions can be both reinforcing and opposing (using Manski; 2013, terminology). This innovation allows the use of partial identification in settings with complementarity and substitutability, which are pervasive in economics as we discuss in Section 3. Second, we show a set of assumptions on the structural functions that can be used to bound the solution sets. By doing so, we introduce new concepts, such as the *heterogeneous fixed points*, which are new to economics and contribute to the literature on games. Third, we show how this approach can be used for policy evaluation when firms compete on the same product markets and are embedded in customer-supplier relationships. We provide a first application to real data of partial identification under positive and negative spillovers, and show how it can produce very tight and meaningful bounds for the response of credit to the real economy to central bank funding.

Additionally, we also provide a new tool and contribute to the vast literature studying the effects of policies and shocks on banks' (and more generally firms') balance sheets.[9] A far from complete list of influential works assessing the effects of different bank shocks includes Peek and Rosengren (2000), Khwaja and Mian (2008), Jimenéz et al. (2012), Schnabl (2012), Jiménez et al. (2014), Behn et al. (2016) and Jiménez et al. (2017). Moreover, our work relates to the growing strand of papers in macro-finance and banking focusing on spillovers' implications for the identification of shocks' effects. Mian et al. (2022) proposes a method to recover general equilibrium multipliers from differences in the regional impact of credit supply shocks. Berg et al. (2021) studies how spillovers from firms' interactions may affect the measurement of shocks' effect on firm level outcomes. Huber (2022) provides overall guidance to empirical researchers on how to deal

[9] There is a large literature studying the effects of central bank funding on credit provision and other items of banks balance sheets, see Benetton and Fantino (2021), Andreeva and García-Posada (2021), Carpinelli and Crosignani (2021), Garcia-Posada and Marchetti (2016), Crosignani, e Castro and Fonseca (2020), Afonso and Sousa-Leite (2020), Laine (2019), Balfoussia and Gibson (2016), Jasova et al. (2018), Acharya and Steffen (2015), Van der Kwaak (2015), Corbisiero (2022), Andrade et al. (2019), Daetz et al. (2018), Darracq-Paries and De Santis (2015), and Alves et al. (2021) among others.

with multiple contemporaneous spillovers (spatial, competitive, agglomeration, general equilibrium and so on).

2 Monotonic Interactions and Heterogeneous Fixed Points

This section begins with basic definitions from lattice theory, reports some known concepts in the literature, and concludes with our new results. We first introduce the heterogeneous fixed points (HFP) theorem, an extension of the well-known Tarski's fixed point theorem. This theorem was introduced in computer science by Kanade et al. (2005) and never used in economics. It proves fixed points exist when a system of simultaneous equations is heterogeneously monotonic. i.e., each outcome variable increases in some variables and decreases in others. Using HPF theorem, we extend the results of Milgrom and Roberts (1990, 1994) on monotone comparative statics for the extremal fixed points to the case of heterogeneous monotone systems of simultaneous equations.[10] The monotonicity properties of the structural functions w.r.t. some exogenous variables are preserved by the least and the greatest elements of the system's solution set.

Let S_n be a system of $n > 1$ simultaneous equations in n variables

$$
\begin{aligned}
y_1 &= f_1(y_2, y_3, \ldots, y_n), \\
y_2 &= f_2(y_1, y_3, \ldots, y_n), \\
&\vdots \\
y_n &= f_n(y_1, y_2 \ldots, y_{n-1}).
\end{aligned}
\tag{1}
$$

The functions $f_1, \ldots, f_n$ are called *structural functions* of S_n. Let $\mathbf{F}$ be a function defined as $\mathbf{F}(\mathbf{y}) = (f_1(\mathbf{y}), \ldots, f_n(\mathbf{y}))$, where $\mathbf{y} = (y_1, \ldots, y_n)$.

We call $\mathbf{F}$ the function vector of S_n. We assume that the outcome y_i with $i = 1, \ldots, n$, takes value from a finite complete lattice $\mathbb{P}_i = (\mathbb{P}_i, \leq_i, \sqcap_i, \sqcup_i)$ where $\leq_i$ is the partial order over the set $\mathbb{P}_i$, and $\sqcap_i$ and $\sqcup_i$, are respectively the meet and the join of the lattice. The set $\mathbb{P}_i$ is a lattice if for each two-element subset $\{x, z\} \subseteq \mathbb{P}_i$ there is a supremum and an infimum denoted join and meet, respectively.[11] The lattice is complete if for all subset $\mathrm{T} \subset \mathbb{P}_i$, $inf(T)$ and $sup(T) \in \mathbb{P}_i$. Let us define $\mathbb{P} = (\mathbb{P}, \leq, \sqcap, \sqcup) = \mathbb{P}_1 \times \ldots \times \mathbb{P}_n$, a function $f_i : \mathbb{P} \to \mathbb{P}_i$ and the function vector $\mathbf{F} : \mathbb{P} \to \mathbb{P}$.

Definition 1. *The outcome y_i* depends *on y_j where $1 \leq i \leq n$ and $1 \leq j \leq n, j \neq i$, if and only if there exists at least two distinct values a_j, a'_j of y_j, such that for any evaluation of y_k, $1 \leq k \leq n, k \neq j$ we have that $f_i(y_1, \ldots, a_j \ldots, y_n) \neq_i f_i(y_1, \ldots, a'_j \ldots, y_n)$.*

[10] In particular, Theorem 2.3 extends the results in Theorem 3 of Milgrom and Roberts (1994).

[11] Given a subset S of $\mathbb{P}_i$, $\bar{b}$ is called an upper bound for S if $\bar{b} \geq x$ for all $x \in S$. It is the supremum of S, denoted $\sup(S)$, if $b \leq \bar{b}$ for all upper bounds b of S.

Let R be a relation, called *dependence relation*, such that $< y_i, y_j >$ are in R if and only if y_i depends on y_j. Let us define $D = (V, C)$ as the *dependence graph* of S_n, where V is the set of variables and C is the set of relations R. We say that the system is *closed* if its dependence graph is strongly connected.[12] Without losing generality, we consider only strongly connected D in what follows.

Definition 2. *A structural function* $f_i : \mathbb{P} \to \mathbb{P}_i$ *is* monotonically increasing *in a variable* y_j *where* $1 \leq i \leq n$ *and* $1 \leq j \leq n, j \neq i$*, if and only if for any evaluation of* y_k*,* $1 \leq k \leq n, k \neq j$*, and for all* $a_j, a'_j \in \mathbb{P}_j$*,*

$$a_j \leq_j a'_j \implies f_i(y_1, \ldots, a_j \ldots, y_n) \leq_i f_i(y_1, \ldots, a'_j \ldots, y_n)$$

The definition of monotonic decreasing follows the same argument. Observe that if a variable y_i does not depend on y_j, the function is both monotonically increasing and decreasing in y_j.

Definition 3. *A vector function* $\mathbf{F} : \mathbb{P} \to \mathbb{P}$ *is* monotonic *if and only if for all* $C, D \in \mathbb{P}$

$$C \leq D \implies \boldsymbol{F}(C) \leq \boldsymbol{F}(D),$$

In other words, a function vector $\mathbf{F}$ is monotonic if and only if each of its component functions, f_i, $1 \leq i \leq n$, is monotonically increasing in all variables.

2.1 Identification of a Valid Partition

Let us define $\leq$ as the usual coordinate-wise order. Consider a partition (A, B) of the set $\{1, \ldots, n\}$. Let us call A and B the blocks of the partition. $A \cup B = \{1, \ldots, n\}$ and $A \cap B = \varnothing$. We say a vector function is *heterogeneous monotonic* or simply *h-monotonic* w.r.t. a partition (A, B) if and only if the functions whose subscripts belong to A are monotonically decreasing (increasing) in the endogenous variables with the same subscripts and monotonically increasing (decreasing) in the variables whose subscripts belong to B. At the same time, the functions with B-subscripts are monotonically decreasing (increasing) in the variables whosesubscripts belong to B and monotonically increasing (decreasing) in the variables with subscripts in A. The following results can be generalized when blocks are more than two.

Definition 4. *A vector function* $\mathbf{F} : \mathbb{P} \to \mathbb{P}$ *is* h-monotonic *w.r.t. a partition* (A, B)*, if and only if,*

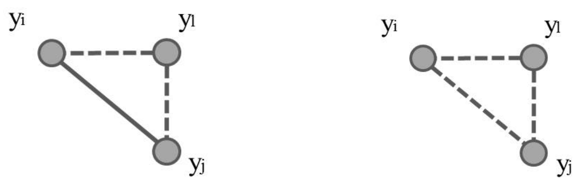
- *for any* $i \in A$*,* f_i *monotonically increases in* y_j*, if* $j \in A, j \neq i$ *and monotonically decreases in* y_j*, if* $j \in B$*.*

[12] A graph is said to be strongly connected if every vertex is reachable from every other vertex.

- *for any $i \in B$, f_i monotonically increases in y_j, if $j \in B, j \neq i$ and monotonically decreases in y_j, if $j \in A$.*

If a function vector $\mathbf{F}$ of a system S_n is h-monotonic w.r.t. a partition (A, B) of the set $\{1, \dots, n\}$, then (A, B) is said to be a *valid* partition of S_n. Let us provide the conditions under which a given system has valid partitions. If in the dependence graph of S_n, $D = (V, E)$, y_i depends positively on y_j (i.e., f_i is increasing in y_j), then we have that in the set E this is represented by a solid edge. On the contrary if y_i depends negatively on y_j (i.e. f_i is decreasing in y_j), then there is a dashed edge from y_i to y_j. Let a *path* in a graph be a sequence of edges that joins two vertices. The *parity* of a path in D is even if it has an even number of dashed edges. Otherwise, its parity is odd. If all paths between every pair of nodes have the same parity, then the dependencies between those variables are consistent (a visual example is provided in Figure 1). If the parity is even, then the two variables are monotonically increasing. If the parity is odd, then the two variables are monotonically decreasing. The univocal monotonic relationship between two variables cannot be determined if paths connect the same nodes with different parities.

Figure 1: Consistent and Inconsistent Dependences



(a) Consistent (b) Inconsistent

Notes. Grey nodes are the variables, y_i, y_l, and y_j. Dashed edges represent negative relationships, and solid ones represent positive ones.

Definition 5. *Dependences in a system of simultaneous equations are* consistent *if for every pair of nodes (y_i, y_j) contained in a strongly connected component of the dependence graph, all paths between y_i and y_j have the same parity.*

Having *consistent* dependencies is particularly important to establish *h-monotonicity* of a system w.r.t. a partition, which can be used to model heterogeneity in strategic interactions. We can now state conditions of a valid partition's existence (and uniqueness) in a system of simultaneous equations.

Lemma Kanade et al., 2005 (Existence of a Valid Partition). *A valid partition for a system S_n iff the dependencies in S_n are consistent. If (A, B) is a valid partition for the system S_n, then there is no other valid partition except (B, A).*

The proof is in Lemma 1 and 2 of Kanade et al. (2005). For example, consistent dependencies can be assumed among outcomes of interacting units embedded in a network of relationships belonging to different blocks. Using Manski (2013)'s terminology, the partition would be valid as long as interactions are *reinforcing* within the same block and *opposing* between blocks. The following theorem provides formal conditions for such a structure of interactions to generate a valid partition.

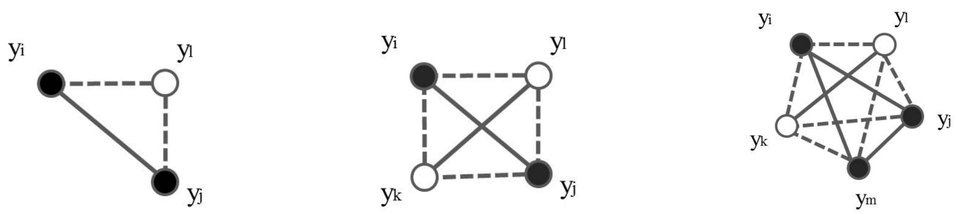
Theorem 2.1. *In a partition (A, B) of the set $\{1, \dots, n\}$ in which interactions are* reinforcing *within the same block and* opposing *between blocks, the dependencies are* consistent *and the partition is valid.*

See the Appendix for all the proofs. Theorem 2.1 tells us that when interactions are reinforcing within the same block and opposing between blocks, the partition's validity is easily implied. Figure 2 provides simple visual examples of consistent dependencies among 3, 4, and 5 nodes belonging to two blocks (in black or white). Solid edges represent reinforcing interactions among nodes of the same block, and dashed edges represent opposing ones between nodes of different blocks. The partition validity for these types of interactions is related to the extensive literature in social psychology, particularly the theory of structural balance (Cartwright and Harary; 1956; Harary; 1953; Heider; 1946). This influential concept explores how relationships form among individuals and their perceptions of shared objects. In a P-O-X triad, where P represents a focal individual, O is a second individual, and X can be either another individual or a common object, Heider proposed that a triad is balanced if P likes O, O likes X, and P also likes X. Conversely, an imbalance occurs if any of these conditions are violated. Heider's primary claim is that balance represents a state of psychological equilibrium, and individuals in networks strive to achieve and maintain this balance. Harary (1953), and Cartwright and Harary (1956) extend the theory to signed graphs, where relationships between pairs of nodes can be represented as either positive or negative. Overall, structural balance theory sheds light on how individuals seek cognitive harmony in social interactions, influencing sentiments and stability within networks. Figure 1 can be interpreted as follows from this perspective. In Figure 1 (a), nodes i and j are friends who share a common enemy, l. This configuration represents a stable relationship between social actors or groups. Conversely, in Figure 1 (b), node i is the enemy of both j and k, who are enemies as well -Such configuration results in an unstable graph. Therefore, a signed graph is deemed stable if and only if it can be partitioned into two sets: one consisting of positive relationships (plus-set) and the other of negative relationships. The stability of the signed graph is the same as having consistent dependencies in S_n. This concept is formally articulated as follows.

Theorem 2.2 (Harary (1953))**.** *A signed graph is balanced, iff the set of vertices, V, can be partitioned into two subsets, called plus-sets, such that all positive edges are between vertices within a plus-set and all negative edges are between vertices in different plus-sets.*

Theorem 2.2 establishes that possessing this bipartition structure is also necessary for ensuring the consistency of S_n. Consequently, regardless of the observed partition in the data, the partition is valid if the analyst can categorize the nodes into two positive within-interaction blocks and a set of negative links between these blocks.

Figure 2: Heterogeneous Interactions and Consistent Dependences



(a) 3 units (b) 4 units (c) 5 units

Notes. Black and white nodes belong to different blocks. Dashed edges represent cross-block connections, and solid ones represent within-block connections.

2.2 Heterogeneous Fixed Points and Monotone Comparative Statics

Since the component lattice $\mathbb{P}_i$ is a complete lattice, any subset of $\mathbb{P}_i$ has a *supremum* and an *infimum*. Let $FP(\mathbf{F})$ be the set of fixed points of $\mathbf{F}$. Let us define $FP_i(\mathbf{F})$ as the set of elements from $\mathbb{P}_i$ that belong to some fixed points of $\mathbf{F}$.

If $\mathbf{F}$ is monotonic (not h-monotonic) -e.g. $f_i(\cdot)$ is increasing in y_j, $\forall i, j$-, then $FP(\mathbf{F})$ has two subsets $LFP(\mathbf{F})$ and $GFP(\mathbf{F})$ containing respectively the least and greatest fixed points, which in turn are the greatest and the least (element-wise) in each dimension ($\mathbb{P}_i$). When $\mathbf{F}$ is h-monotonic, it no longer holds. Thus, a fixed point can be the greatest in one dimension and the least in another. We now define a subset of fixed points $FP(\mathbf{F})$, which are the greatest and least in different dimensions, depending on which partition the specific dimension belongs to.

Let us define $HFP^{(A,B)}(\mathbf{F})$ as the set of HPF of the system S_n w.r.t. a partition (A, B) such that its element $HFP_i^{(A,B)}(\mathbf{F})$ is sup of $FP_i(\mathbf{F})$ if $i \in A$ and is inf of $FP_i(\mathbf{F})$ if $i \in B$.

$$HFP_i^{(A,B)}(\mathbf{F}) = \begin{cases} \sqcap_i FP_i(\mathbf{F}) & if\ i \in A \\ \sqcup_i FP_i(\mathbf{F}) & if\ i \in B \end{cases}$$

At the same time, let us define $HFP^{(B,A)}(\mathbf{F})$ as the set of HPF of the system S_n w.r.t. a partition (A, B) such that its element $HFP_i^{(B,A)}(\mathbf{F})$ is *infimum* of $FP_i(\mathbf{F})$ if $i \in A$ and is *supremum* of $FP_i(\mathbf{F})$ if $i \in B$.

$$HFP_i^{(B,A)}(\mathbf{F}) = \begin{cases} \sqcup_i FP_i(\mathbf{F}) & if\ i \in A \\ \sqcap_i FP_i(\mathbf{F}) & if\ i \in B \end{cases}$$

Let us define $HFP(\mathbf{F}) \equiv HFP^{(B,A)}(\mathbf{F}) or HFP^{(A,B)}(\mathbf{F})$ and introduce its existence theorem provided by Kanade et al. (2005).

HFP Theorem (Kanade et al., 2005). *If the function vector* $\mathbf{F} : \mathbb{P} \to \mathbb{P}$ *of a system* S_n *is h-monotonic w.r.t. a valid partition* (A, B)*, then* $HFP^{(A,B)}(\mathbf{F}), HFP^{(B,A)}(\mathbf{F}) \in FP(\mathbf{F})$*.*

- $HFP^{(A,B)}(\mathbf{F})$ *has the least possible values from the component lattice with a subscript in* A *and the greatest possible values from the component lattice whose subscripts belong to* B*;*
- $HFP^{(B,A)}(\mathbf{F})$ *has the least possible values from the component lattice with a subscript in* B *and the greatest possible values from the component lattice whose subscripts belong to* A*;*

Let us now extend the result of Milgrom and Roberts (1994) for the case of HFP. In doing so, we allow our system S_n to contain $\mathbf{t} = (t_1, \ldots, t_n)$ defined on any partially ordered set T as additional exogenous variables. Observe that the concept of *h-monotonicity* easily extends to the presence of exogenous variables, we thus assume that $\mathbf{F}$ is *h-monotonic* in t later on, when not specified. Let us define $\mathbf{y_a^L} \in \sqcap_i FP_i(\mathbf{F})$, and $\mathbf{y_a^U} \in \sqcup_i FP_i(\mathbf{F})$ $if\, i \in A$; $\mathbf{y_b^L} \in \sqcup_i FP_i(\mathbf{F})$, and $\mathbf{y_b^U} \in \sqcap_i FP_i(\mathbf{F})$ $if\, i \in B$.

Theorem 2.3. *Let* $\mathbb{P}$ *be a complete lattice and* $\mathbf{F} : \mathbb{P} \times T \to \mathbb{P}$*. Suppose* $\mathbf{F}$ *is* h-monotonic *with respect to a valid partition* (A, B)*, then also* $\mathbf{y_a^L}$ $\mathbf{y_a^U}$*,* $\mathbf{y_b^L}$ *and* $\mathbf{y_b^U}$ *are* h-monotonic *in* $\mathbf{t}$.

3 Possible interpretations

As mentioned above, our framework is amenable to many possible interpretations and applications in which there are heterogeneous interactions. In all these contexts, the proposed method can be useful especially if connections among agents are not observable with precision, assuming specific functional forms may be restrictive, economic theory provides straightforward monotone assumptions, treatments and networks are potentially endogenous. Here, we discuss some of these possible applications.

3.1 Social interactions

This framework is useful whenever reinforcing and opposing interactions are possibly at play (Manski; 2013). A reinforcing interaction occurs when a person's outcome increases both with the value of his own treatment and with the values of the treatments received by others in the reference group. An opposing interaction occurs when a person's outcome increases with the value of his own treatment but decreases with the values of the treatments received by others.

3.1.1 Legislative Activity

There is an important body of work in political science that has stressed the importance of taking into account interpersonal relations and social connections when studying how legislatures work. For example, the history of the U.S. Congress is indeed rich in examples where voting coalitions are shaped by social connections formed inside and outside the legislative chambers. We can imagine that interactions could be reinforcing within a party and opposing between parties that compete for voters, or follow other and finer criteria, for example considering committees or geographical location as well.

While scholars in political science have recognized the relevance of social connections between lawmakers for quite some time, only recently data availability and advances in network analysis has allowed to move beyond descriptive analyses.[13] Peoples (2008), Masket (2008), Rogowski and Sinclair (2012), Cohen and Malloy (2014) and Harmon et al. (2019) have studied whether social links affect voting behavior. Battaglini and Patacchini (2018) have studied the impact of the legislators' social connection on PAC's contributions. Fowler (2006), Kirkland (2011) and Battaglini, Sciabolazza and Patacchini (2020) have studied the relationship between legislators effectiveness in the U.S. Congress and their social connections.

The main challenges are that social networks are endogenous and not directly observed in legislatures (as, indeed, in most other social groups). With few exceptions, the standard approach in the empirical literature is to assume that there is an observable network; and that it can be used to estimate key parameters (for example the effect of covariates, the spillover effect, the cost of link formation and other parameters that matter for the network). A remarkable exception is Battaglini et al. (2022), which develop a model of endogenous network formation to recover unobserved social networks using only observable outcomes and approximate Bayesian computation methods.

3.1.2 Conflicts and Warfare

In many episodes, like civil conflicts or wars, there are links among alliances and enmities. Military alliances can also be influenced by trading relationships (Jackson and Nei; 2015). König et al. (2017) construct a stylized theory of conflict that captures the effect of informal networks of alliances and enmities, and apply it to the empirical study of the

[13]Earlier work include Rice (1927), Rice (1938), Routt (1938), Eulau (1962). See Victor et al. (2017) and Battaglini and Patacchini (2019) for recent surveys.

Second Congo War. In their model, the fighting effort of group i's allies increases group i's operational performance, whereas the fighting effort of its enemies decreases it. Since the cost of fighting is borne individually by each group, a motive for strategic behavior arises among both enemies and allies. The complex externality web affects the optimal fighting effort of all groups.[14] A similar approach can be used to study conflicts between or interactions within criminal groups (Dell; 2015; Glaeser et al.; 1996; Lee et al.; 2021; Patacchini and Zenou; 2008) or ethnic groups (Arbatl et al.; 2020; Bisin and Verdier; 2000).

3.2 Market Interactions

When outcomes are formed as an expression of trade and monetary exchanges, negative (positive) spillovers can be generated between substitutes (complements). For example, the outcome of a market participant can affect negatively the outcomes of competitors on the same side of the market.

3.2.1 Financial Services and Markets

Financial networks, the web of business links that banks and other financial operators establish among themselves, have long been recognized as important factors in financial crises, for interbank liquidity, and the diffusion of investment choices (see Allen and Babus; 2009, for a survey of this literature). Financial networks are often endogenous, typically unobservable, and embedding positive and negative spillovers. For example, direct financial links between banks are formed through credit exposures in the interbank market (see Denbee et al.; 2021; Drehmann and Tarashev; 2011). These exposures, however, are often not perfectly observed and results are often sensitive to how they are measured (Upper; 2006). Exposures are generally recovered from the banks' balance sheets which only provide information on the aggregate exposure of a bank.[15]

As banks exchange reserves on the interbank market, increased supply of a bank can affect positively the borrowing behavior of a bank on the other side of the market.

In addition, banks interact with each other on other markets simultaneously. They compete in the markets for deposits and loans to the real economy, thus increased supply of a bank can affect negatively the supply of other banks (Pietrosanti and Rainone; 2023). At the same time, outcomes on these markets are not independent within the

[14] In their study, the network of interactions is identified using information from a variety of expert and data sources. The identification strategy exploits the exogenous variation in the average weather conditions facing, respectively, the set of allies and of enemies of each group.

[15] Bilateral credit positions vis-a-vis the other banks are generally reconstructed using ad hoc algorithms such as the maximum entropy method (which is based on the idea that banks spread their position uniformly across other banks). Among others, the maximum entropy algorithm has been used by Drehmann and Tarashev (2011), Anand et al. (2015) and Peltonen et al. (2019). See Mistrulli (2011) for a survey and discussion of these approaches.

single bank's balance sheet. Increasing funding through deposits for example allows granting more loans. These interactions between and within banks' outcomes, creates a complex system of interdependence embedding both positive and negative interactions. As shown by Allen and Gale (2000), among others, contagion is very sensitive to the shape of the financial network. For regulations on financial stability, monetary policy, and policy intervention more generally, it is therefore important to incorporate the effects of underlying links between and within financial operators to get unbiased estimates of policy measures and external shocks. Indeed, there is a growing strand of papers in macro-finance and banking focusing on spillovers' implications for the identification of shocks' effects (Berg et al.; 2021; Huber; 2022; Mian et al.; 2022). More discussion and details on this topic are provided in our empirical analysis in Section 5.

4 Identification of Treatment Response with Heterogeneous Interactions

This section studies the identification of treatment response with heterogeneous interactions. The results in Section 2 are used to construct identification regions for potential outcome distributions via monotone comparative statics under conditions motivated by economic theory.

4.1 Model Specification

Let the population $\mathcal{P}$ be partitioned into two blocks, A and B. The vector of outcomes for units in the k_{th} block is denoted as $\mathbf{y}^k$. Each unit has a reference group, G, composed of some units from each block. Reference groups are *mutually exclusive* and *symmetric*, with unit l being a member of j's group if and only if j belongs to l's group. The structural functions are assumed to be block-specific. A potential treatment $t \in \mathcal{T}_i$ is administered to population units that are not necessarily randomly selected. Additionally, we define the exposure mapping $h^0 : \mathcal{T} \to \mathcal{D} \subset \mathbb{R}^2$, as a function that maps the treatment vector into an *effective treatment* for each unit (Manski; 2013). For example, a widely used exposure function is the fraction of treated units in group G, i.e. $h^0(\mathbf{t}) = (t_i, \frac{1}{n^A}\sum_{j\in,A/i} t_j, \frac{1}{n^B}\sum_{j\in B/i} t) = \{d_i^{0I}, d_i^{0A}, d_i^{0B}\} = \mathbf{d}_i^0 \in \mathcal{D}$ (see, e.g., Leung; 2020; Vazquez-Bare; 2023). Let us define $f_i(\cdot)$ as the *structural function*. We introduce a constant treatment response assumption (CTR, Manski; 2013) that only restricts interactions within reference groups and uses the exposure mapping function to describe the interference mechanism.

Assumption CTR. *For all $i \in \mathcal{P}$, and for all $\boldsymbol{t}$, $\boldsymbol{t}' \in \mathcal{T}$ such that $h^0(\boldsymbol{t}) = h^0(\boldsymbol{t}')$, the* *following holds:*[16]

$$f_i^k(h^0(\boldsymbol{t}), \cdot) = f_i^k(h^0(\boldsymbol{t}'), \cdot).$$

Formally, the structural equation for the generic i_{th} unit in block $k \in \mathcal{K} \equiv \{A, B\}$ and group G is as follows:

$$y_i^k = f_i^k(\mathbf{d}^0, S_i^0(\mathbf{y}^{\mathbf{k}\setminus\mathbf{i}}(\mathbf{t})), S_i^0(\mathbf{y}^{\mathbf{l}}(\mathbf{t}))), \;\; i \in G, k, l \in \mathcal{K}. \tag{2}$$

here, $S_i^0(\cdot)$ is any function of the outcome distribution that respects stochastic dominance, such as a quantile or the mean of an increasing function of the outcome (Manski; 2013). In equation (2), the outcome of unit i depends on the group vector of assignments through the exposure mapping $h^0(\cdot)$, the outcomes of other units in the same block (excluding her), and the outcomes of units in the other block.[17] Observe that thanks to the CTR assumption, the structural function can be indexed by the effective treatment, $\mathbf{d}^0$. In what follows, we implicitly assume that the CTR assumption holds.

Following Lazzati (2015), we characterize groups indexed by r based on the structural functions of the group members in a given block. Two units are of the same type if they react similarly to different treatment configurations and the behavior of other units in the same block and group. Let $\mathcal{S}_y^k$ and $\mathcal{Y}^k$ denote the set of all possible values assumed by $S(\cdot)$ on $\mathbf{y}_r^k$ and $\mathbf{y}_r^k$ itself. We drop the superscript k to denote the set of all possible values for all units in the group r. If $\mathcal{Y}^k$ and $\mathcal{T}_i$ are countable, then there are countably many different systems of structural equations that can describe a group type. Let p_r^k be the fraction of group-types r for a given block k. The group is characterized by the structural functions of its members: $\mathbf{f}_r^k(\cdot) = [f_{ir}^k(\cdot), i \in k, r]$ with $f_{ir}^k : \mathcal{T}_i \times \mathcal{S}_y \to \mathcal{Y}^k$. According to this probabilistic framework, $[(\mathbf{f}_r^k, p_r^k)]$ describes the universe of group types in the population for a given block.

For example, in Table 1, we describe the structural functions of a population with two blocks and three groups. Outcomes are assumed to be binary, i.e., y_{ir}^k equals one if units exert some effort and zero otherwise. The numbers in the first row and column represent the structural function y_{11}^A when the outcome of y_{21}^A is equal to one, and at least one unit in block B exerts some effort. Interactions are assumed to be reinforcing within a block and opposing between blocks. While the structural functions of units in block A and B are the same in groups 1 and 2, they differ from the third group. Thus, in this population, there are two group types $r = 1, 2$ with $\mathbf{p}^A = \mathbf{p}^B = \{2/3, 1/3\}$. Observe that

[16] The CTR assumption is also referred to as *neighborhood interference* in Forastiere et al. (2021). See also the exposure mapping definition in Aronow and Samii (2017). Additionally, we implicitly assume what is called *consistency* in causal inference, which ensures that potential outcomes are well-defined by excluding the possibility of multiple versions of the treatment. Consistency and CTR constitute the components of the so-called Stable Unit Treatment Value Assumption (SUTVA, Rubin; 1986).

[17] In our model, the primitives do not explicitly incorporate covariate information; however, in principle, they can. In such instances, the primitives should be interpreted as conditional on specific covariate values (Lazzati; 2015).

p_r^k can also depend on the relative size of the block.

We denote the vectors of potential outcomes, which solves the system of structural equations, as $\mathbf{y}_\mathbf{r}^\mathbf{k}(\mathbf{d}_r^0) : \mathcal{D} \to Y^k$. Each group has an observable realized treatment collected by the vector $\mathbf{t}_\mathbf{r}^\mathbf{0} \in T$, observed effective treatment $\mathbf{z}_r^0 \in \mathcal{D}$, and realized outcomes $\mathbf{y}_\mathbf{r}^\mathbf{k} \equiv \mathbf{y}_\mathbf{r}^\mathbf{k}(\mathbf{z}_\mathbf{r}^\mathbf{0})$. The available data are (t_{ir}^{0k}, y_{ir}^k), $i \in \mathcal{P}$, $k \in \mathcal{K}$, and $r \in \mathcal{R}$. The main assumptions about the shape of the structural functions are described below.

Table 1: Structural functions in the population

Group 1

Block A t=1,0	$y^{A/i} = 1, S(\mathbf{y}^B) = 1$	$y_1^{A/i} = 1, S(\mathbf{y}^B) = 0$	$y^{A/i} = 0, S(\mathbf{y}^B) = 1$	$y_1^{A/i} = 0, S(\mathbf{y}^B) = 0$
y_{11}	1,1	1,1	0,0	1,0
y_{21}	1,1	1,1	1,0	1,1

Block B t=1,0	$y^{B/i} = 1, S(\mathbf{y}^A) = 1$	$y^{B/i} = 1, S(\mathbf{y}^A) = 0$	$y^{B/i} = 0, S(\mathbf{y}^A) = 1$	$y^{B/i} = 0, S(\mathbf{y}^A) = 0$
y_{31}	1,1	1,1	0,0	1,0
y_{41}	1,0	1,1	1,0	1,1

Group 2

Block A t=1,0	$y^{A/i} = 1, S(\mathbf{y}^B) = 1$	$y^{A/i} = 1, S(\mathbf{y}^B) = 0$	$y^{A/i} = 0, S(\mathbf{y}^B) = 1$	$y^{A/i} = 0, S(\mathbf{y}^B) = 0$
y_{12}	1,1	1,1	0,0	1,0
y_{22}	1,1	1,1	1,0	1,1

Block B t=1,0	$y^{B/i} = 1, S(\mathbf{y}^A) = 1$	$y^{B/i} = 1, S(\mathbf{y}^A) = 0$	$y^{B/i} = 0, S(\mathbf{y}^A) = 1$	$y^{B/i} = 0, S(\mathbf{y}^A) = 0$
y_{32}	1,1	1,1	0,0	1,0
y_{42}	1,0	1,1	1,0	1,1

Group 3

Block A t=1,0	$y^{A/i} = 1, S(\mathbf{y}^B) = 1$	$y^{A/i} = 1, S(\mathbf{y}^B) = 0$	$y^{A/i} = 0, S(\mathbf{y}^B) = 1$	$y^{A/i} = 0, S(\mathbf{y}^B) = 0$
y_{13}	1,0	1,1	0,0	1,0
y_{23}	1,1	1,1	1,0	1,1

Block B t=1,0	$y^{B/i} = 1, S(\mathbf{y}^A) = 1$	$y^{B/i} = 1, S(\mathbf{y}^A) = 0$	$y^{B/i} = 0, S(\mathbf{y}^A) = 1$	$y^{B/i} = 0, S(\mathbf{y}^A) = 0$
y_{33}	1,1	1,1	0,0	1,0
y_{43}	1,0	1,0	1,0	1,1

4.2 Heterogeneous Monotonic Structural Functions

In this section, we impose restrictions on the structural equations and derive their implications in terms of equilibrium behavior justified by the results in the previous sections. We then use the result of the main Lemma to derive the identifying region of the potential outcome distribution when *h-monotonicity* is assumed.

Without further restrictions, the system of equations (2) might have no solution. In order to avoid this case, we provide a set of assumptions that precludes this possibility. To make the derivations easier, we assume that $S^0_{ir}(\cdot) = E(\cdot)$.

Assumption HMI (H-Monotonic Interactions)**.** *For each $i \in \mathcal{P}$, $k, l \in \mathcal{K}$, $r \in \mathcal{R}$, and $\mathbf{d}^0_{ir} \in \mathcal{D}$, $f^k_{ir}(\mathbf{d}^0_{ir}, E(\mathbf{y}^{\mathbf{k/i}}_{\mathbf{r}}), E(\mathbf{y}^{\mathbf{l}}_{\mathbf{r}}))$ are increasing in $E(\mathbf{y}^{\mathbf{k/i}}_{\mathbf{r}})$ and decreasing in $E(\mathbf{y}^{\mathbf{l}}_{\mathbf{r}})$.*

Assumption **HMI** requires reinforcing interactions between units in the same block and opposing between different blocks. This implies the partition is valid, and we can apply the **HFP Theorem**. Among the models that satisfy Assumption **HMI**, we have games that exhibit both strategic complements and strategic substitutes; see Section 3 for some examples.

Let $\phi^k(\mathbf{d}_r, r)$ be the solution set of the system of structural equations for a given k, r, $\mathbf{d}_r$. The following result states the existence of extrema equilibria. To make the notation lighter we drop the subscript r from the vector of potential and realized effective treatment.

Lemma 1. *If assumption **HMI** holds, then $\phi^A(\mathbf{d}^0, r)$ always has a least (or greatest) solution and $\phi^B(\mathbf{d}^0, r)$ has a greatest (or least) solution for all $\mathbf{d}^0 \in \mathcal{D}$.*

Let $\mathbf{y}^k_r(\mathbf{d}^0)$ denote the element of $\phi^k(\mathbf{d}^0, r)$ that is selected by the units that get effective treatment $\mathbf{d}^0$. The expectation of the potential outcome vector over the population type distribution is

$$E(\mathbf{y}^k(\mathbf{d}^0)) = \sum_{r \in \mathcal{R}} \mathbf{y}^k_r(\mathbf{d}^0) p^k_r. \tag{3}$$

Researchers typically do not know the true exposure function and must select a candidate that minimizes the likelihood of model misspecification. The choice of the exposure function is subject to two constraints. Firstly, the exposure function must satisfy stochastic dominance, which characterizes endogenous interactions in the structural form through $S^0_{ir}(\cdot)$ and allows the prediction of the direction of variation in $\mathbf{d}^0$. Secondly, the exposure function must be no *finer* than the true one. Observe that while $S^0_{ir}(\cdot)$ can theoretically take any sign-preserving form, $h^0(\cdot)$, which is unknown, must be "almost" correctly specified. This is crucial for the (partial) identification of the average treatment effects. We adopt the definition of *coarseness* introduced in Vazquez-Bare (2023) to formalize this constraint:

Definition 6 (Coarseness.)**.** *Given two exposure functions* $h(\cdot) : \mathcal{T} \to \mathcal{D}$ *and* $\tilde{h}(\cdot) : \mathcal{T} \to \tilde{\mathcal{D}}$*, we say* $h(\cdot)$ *is coarser than* $\tilde{h}(\cdot)$ *if there exist a funtion* $f(\cdot) : \tilde{\mathcal{D}} \to \mathcal{D}$ *such that* $h(\mathbf{t}) = f(\tilde{h}(\mathbf{t}))$ *for all* $\mathbf{t} \in \mathcal{T}$*.*

The concept of *coarseness* implies that the exposure function, $h(\cdot)$, imposes more restrictions on the potential outcome than $\tilde{h}(\cdot)$. For instance, a constant exposure function, which is coarser than the fraction of treated units in a group, is coarser than the entire treatment vector. We assume that the true exposure function, $h^0(\cdot)$, satisfies the stochastic dominance constraints, and it is coarser than the exposure function selected by the researcher, $h(\cdot)$:

Assumption EF (Exposure function)**.** $h^0(\cdot)$ *is coarser than or equal to* $h(\cdot)$ *and it is a function which respects stochastic dominance.*

Assumption **EF** allows to condition only on $h(\mathbf{t}) = \mathbf{z}$ given that uniquely determine the value of the true exposure $h^0(\mathbf{t}) = \mathbf{z}^0$. Let $p^k_{r|z}$ be the proportion of units that belong to a group r conditional on the realized effective treatment vector $\mathbf{z}$. Then,

$$
\begin{aligned}
E(\mathbf{y}^k|\mathbf{z}=\mathbf{s}) &= \sum_{\mathbf{s}^0\in\mathcal{D}} E(\mathbf{y}^k(\mathbf{s}^0)|\mathbf{z}=\mathbf{s},\mathbf{z}^0=\mathbf{s}^0)P^k(\mathbf{z}^0=\mathbf{s}^0|\mathbf{z}=\mathbf{s}) \\
&= \sum_{\mathbf{s}\in\mathcal{D}} E(\mathbf{y}^k(\mathbf{s})|\mathbf{z}^0=\mathbf{s})P^k(\mathbf{z}^0=\mathbf{s})
\end{aligned} \tag{4}
$$

where the third equality is given by coarseness, $E(\mathbf{y}^k(\mathbf{s})|\mathbf{z}=\mathbf{s}) = \sum_{r\in\mathcal{R}} \mathbf{y}^k_r(\mathbf{s})p^k_{r|\mathbf{z}=s}$, and $P^k(\mathbf{z})$ denotes the distribution of realized effective treatments across group-types for a given block. See also Lemma 1 in Vazquez-Bare (2023). Observe that under **EF**, we can use $\mathbf{d}$ instead of $\mathbf{d}^0$, thus allowing for partial misspecification of the exposure function and the resulting random variables. The next assumption formalizes that the structural equations are *h-monotonic* in potential treatments.

Assumption 1 (HMTR)**.** *For each* $i \in \mathcal{P}$*,* $r \in \mathcal{R}$*,* $E(\mathbf{y_r}^{\mathbf{A/i}}) \in \Delta^{A/i}_y$*, and* $E(\mathbf{y^B}_r) \in \Delta^B_y$*,*
$f^A_{ir}(d^I_{ir},\cdot) \geq f^A_{ir}(d^{I'}_{ir},\cdot)$ *and*
$f^A_{ir}(\mathbf{d}^A_{ir},\cdot) \geq f^A_{ir}(\mathbf{d}^{A'}_{ir},\cdot)$*, while*
$f^A_{ir}(\mathbf{d}^B_{ir},\cdot) \leq f^A_{ir}(\mathbf{d}^{B'}_{ir},\cdot)$
for all $\mathbf{d}^k_{ir} \geq \mathbf{d}^{k'}_{ir}$*, and* $d^I_{ir} \geq d^{I'}_{ir}$*.*

As showed in Lazzati (2015) without further restrictions, Assumption **HMTR** allows counterfactual predictions only at the extremal solutions of $\phi^k(\mathbf{d}, r)$. Under Assumptions **HMI** and **HMTR**, which guarantee that the greatest and least elements (w.r.t. the partitions A, B) are h-monotonic in $\mathbf{d}$, we can use the HFP Theorem to motivate the following additional assumption.

Assumption ES1 (Equilibrium Selection)**.** *Within each group units in block A select either the largest (the smallest) element of $\phi^A(\mathbf{d}, r)$ and units in block B select the smallest (the largest) element of $\phi^B(\mathbf{d}, r)$ for each $\mathbf{d} \in \mathcal{D}$.*

Assumption ES2. *Within each group, units in block A select the largest (the smallest) element of $\phi^A(\mathbf{d}, r)$, and units in block B select the smallest (the largest) element of $\phi^B(\mathbf{d}, r)$ and the selection rule is the same for each $r \in \mathcal{R}$.*

Assumption **ES1** is useful because it guides us always to choose either the smallest or the largest equilibrium when predicting the effect of $\mathbf{d}$ on the expectation of potential outcomes. Observe, however, that the credibility of this assumption would be difficult to assess in practice (Manski; 2013) and depends on the context. If the population is partitioned by A and B, the **HFP Theorem** states that the largest equilibrium in A-block corresponds to the smallest in the B-block so that Assumption **ES1** can be justified using the concept of Pareto dominance. For example, it holds if the utility in block A increases for outcomes of units in the same partition but decreases for units in block B. The opposite is true for the utility of units in block B. The condition holds if the block-wise structural functions correspond to the best replies of a game, the largest equilibrium dominates the others for A, and the smallest dominates the others for B if for all $\mathbf{y}_r^A \geq \mathbf{y}_r'^A$ and $\mathbf{y}_r^B \geq \mathbf{y}_r'^B$ we have

$$U_{ir}^A(\mathbf{y}_r^A, \mathbf{y}_r'^B, \mathbf{t}_r) \geq U_{ir}^A(\mathbf{y}_r'^A, \mathbf{y}_r'^B, \mathbf{t}_r) \geq U_{ir}^A(\mathbf{y}_r'^A, \mathbf{y}_r^B, \mathbf{t}_r).$$

Assumption **ES2** is stronger and requires the same selection rule across groups. Alternatively, we can apply different selection rules, such as the second rule used in Lazzati (2015), based on making the solution set strongly increase with the potential treatment.

We next state that under all previous assumptions, we can apply Theorem 2.3, and hence, the equilibrium outcome is *h-monotonic* in t.

Lemma 2. *If Assumptions **HMI**, **HMTR**, **EF**, and **ES1** hold, then $E(\mathbf{y}_r^A(\mathbf{d}))$ and $E(\mathbf{y}_r^B(\mathbf{d}))$ are* h-monotonic *in $\mathbf{d}$.*

Lemma 2 uses two different shape restrictions on the outcome function of units in A and B. First, *h-monotonicity* of unit choices -i.e., **HMI**-. Second, *h-monotonicity* with respect to the treatment -i.e. **HMTR**-. These two types of assumptions play different roles in Lemma 2. The first guarantees the existence of extremal equilibria through the **HFP Theorem**. The second allows us to obtain the new *h-monotone* comparative statics results that we exploit to construct the identification region for potential outcomes functional.

We now introduce heterogeneous monotone treatment selection. Let $\mathbf{f}^k$ denote a random vector of function with support on $(\mathbf{f}_r^k, r \in \mathcal{R})$, and let us define $P(\mathbf{f}^k = \mathbf{f}_{r|\mathbf{z}}^k) \equiv p_{r|\mathbf{z}}^k$. We assume that the probability of having a type-r structural function (given the type of the members), conditional on the realized treatment, is statistically h-monotonic in z. We can interpret this condition as follows: units in group r and block k that self-select into higher treatment vectors exhibit stochastically higher structural functions than those with lower ones.

Assumption HMTS (H-Monotone Treatment Selection). $P(\mathbf{f}^k|\mathbf{z} = \mathbf{s})$ *is stochastically* h-monotonic *in* $\mathbf{s}$.

HMTS is a h-monotonic condition applied on the conditional probability that groups are of a given type. Thus, it compares potential outcomes across different types and blocks. Lemma 3 in Appendix B shows that if $\mathbf{f}^A_r$ is (point-wise) greater or equal to $\mathbf{f}^B_{r'}$, then also the corresponding elements in the solution set are larger than the solution set in the other block-group. Importantly, this also holds when units belong to the same block. If we add **HMTS** to **HMI** and **ES2**, we have that $\mathbf{y}^A_r(\mathbf{d}) \geq \mathbf{y}^B_{r'}(\mathbf{d})$ for all groups such that $\mathbf{f}^A_r \geq \mathbf{f}^B_{r'}$. **HMTS** states that the structural functions are statistically increase in realized treatments. Following Manski and Pepper (2018), we can interpret the condition as group types that self-select into higher treatments within a block because they have stochastically larger structural functions than those that self-select into lower ones. The following lemma uses the selection rule to move from selection into treatment vectors of the structural functions to the solutions sets.

Lemma 3. *If Assumptions **HMI**, **HMTS**, **EF**, and **ES2** hold, then for each* $\mathbf{d} \in \mathcal{D}$, $E(\mathbf{y}^A(\mathbf{d})|\mathbf{z} = \mathbf{s})$ *and* $E(\mathbf{y}^B(\mathbf{d})|\mathbf{z} = \mathbf{s})$ *are* h-monotonic *in* $\mathbf{s}$.

In practice, let us assume that A and B are the blocks which partition $\mathcal{P}$, Lemma 3 shows that if **HMI**, **HMTS**, **EF**, and **ES2** hold, and $\mathbf{s}^A \geq \mathbf{s}^{A'}, \mathbf{s}^B \geq \mathbf{s}^{B'}$ then $E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^{B'}) \geq E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^B) \geq E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A = \mathbf{s}^{A'}, \mathbf{z}^B = \mathbf{s}^B)$ and $\mathrm{E}(\mathrm{y}^B(\mathbf{d})|\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^{B'}) \leq E(\mathbf{y}^B(\mathbf{d})|\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^B) \leq E(\mathbf{y}^B(\mathbf{d})|\mathbf{z}^A = \mathbf{s}^{A'}, \mathbf{z}^B = \mathbf{s}^B)$, where $\mathbf{y}^k(\mathbf{d}) = (\mathbf{y}^k_r(\mathbf{d}), r \in \mathcal{R})$.

4.3 Identification Region for Potential Outcome Distributions

In this section, we translate the monotone comparative statics results in the previous sections into sharp distributional bounds for the potential outcomes. Given the existence of the equilibrium and the results about of heterogeneous monotone comparative statics discussed above, all the propositions in this section are built on Manski and Pepper (2000). Using the Law of Total Probability, we can formulate the expected potential outcomes as follows.

$$E(\mathbf{y}(\mathbf{d})) = E(\mathbf{y}(\mathbf{d})|\mathbf{z} = \mathbf{d})P(\mathbf{z} = \mathbf{d}) + E(\mathbf{y}(\mathbf{t})|\mathbf{z} \neq \mathbf{d})P(\mathbf{z} \neq \mathbf{d}). \tag{5}$$

Under **HMI** and given equations (3) and (4), the empirical evidence reveals $E(\mathbf{y}(\mathbf{d})|\mathbf{z} = \mathbf{d}) = E(\mathbf{y}|\mathbf{z} = \mathbf{d})$, $P(\mathbf{z} = \mathbf{d})$ and $P(\mathbf{z} \neq \mathbf{d})$. The observation of treatments and outcomes alone remains silent about the potential outcome distribution for those units that have realized treatments different from the potential treatment, i.e., $E(\mathbf{y}(\mathbf{d})|\mathbf{z} \neq \mathbf{d})$. If we assume that the vectors of outcomes have lower and upper bounds equal to $\underline{y}$, $\overline{y}$, then $E(\mathbf{y}) \in [\underline{y}, \overline{y}]$ and the resulting bounds are sharp. Let Δ_{y^A} and Δ_{y^B} be the set of (conditional) expectations of $\mathcal{Y}^A$ and $\mathcal{Y}^B$, for blocks A and B respectively and let H be the set of expectations which characterize the sharp identification region for $E(\mathbf{y}^A(\mathbf{d}))$. We present the result only for block A, but by reversing the inequality, we obtain the identification region for $H\{E(\mathbf{y}^B(\mathbf{d}))\}$, thanks to the symmetry of the problem.

Proposition 1. *If Assumption* **HMI** *holds, then for all* $\mathbf{d} \in \mathcal{D}$*,*

$$H\{E(\mathbf{y}^A(\mathbf{d}))\} = \left\{ \delta_A \in \Delta_{y^A} : \begin{array}{c} E(\mathbf{y}^A|\mathbf{z}=\mathbf{d})P(\mathbf{z}=\mathbf{d}) + \overline{y}^A P(\mathbf{z} \neq \mathbf{d}) \\ \\ \geq \delta_A \geq \\ \\ E(\mathbf{y}^A|\mathbf{z}=\mathbf{d})P(\mathbf{z}=\mathbf{d}) + \underline{y}^A P(\mathbf{z} \neq \mathbf{d}) \end{array} \right\}.$$

The size of $H\{E(\mathbf{y}^A(\mathbf{d}))\}$ varies inversely with $P(\mathbf{z}=\mathbf{d})$. The set of expectations is the singleton $E(\mathbf{y}^A(\mathbf{d}))$ when $P(\mathbf{z}=\mathbf{d}) = 1$. It is informative when $P(\mathbf{z}=\mathbf{d}) = 0$. The result is standard and follows by Manski (1990, 2013). If $P(\mathbf{z} \neq \mathbf{d}) = 1$, these bounds are sharp but completely uninformative. We next show that the identification region can be substantially tightened by adding either heterogeneous monotone treatment response or monotone treatment selection assumptions.

If **HMI**, **HMTR**, **EF** and **ES1** hold, then by Lemma 2 $E(\mathbf{y}^A(\mathbf{d}))$ and $E(\mathbf{y}^B(\mathbf{d}))$ are *h-monotonic* in $\mathbf{d}$. H-monotonicity assumption denotes that response functions in block A, $\mathbf{y}^A(\mathbf{d}^A)$ are weakly increasing in $\mathbf{d}^A$, while response functions in the partition B are weakly decreasing in $\mathbf{d}^A$. For example, consider a specific group r, a realized policy $\mathbf{z}$, and a realized outcome $\mathbf{y}_r^A$. If $\mathbf{d}^A \geq \mathbf{z}^A$, and $\mathbf{d}^B \leq \mathbf{z}^B$, then $\mathbf{y}_r^A$ is a sharp lower bound for $\mathbf{y}_r^A(\mathbf{d})$. Otherwise, the empirical evidence is uninformative and $\underline{y}_r^A$ is a sharp lower bound. If $\mathbf{d}^A \leq \mathbf{z}^A$, and $\mathbf{d}^B \geq \mathbf{z}^B$, then $\mathbf{y}_r^A$ is a sharp upper bound for $\mathbf{y}_r^A(\mathbf{d})$. Otherwise, the empirical evidence is uninformative, and $\overline{y}_r^A$ is a sharp upper bound. Since the group r is arbitrarily chosen, this analysis extends to all group types and justifies the next result.

Proposition 2. *If Assumptions* **HMI***,* **HMTR***, **EF**, and* **ES1** *hold, then for all* $\mathbf{d} \in \mathcal{D}$*,*

$$H\{E(\mathbf{y}^A(\mathbf{d}))\} = \left\{ \delta_A \in \Delta_{y^A} : \begin{array}{c} \overline{y}^A P(\mathbf{d}^A > \mathbf{z}^A \text{ or/and } \mathbf{d}^B < \mathbf{z}^B) \\ +E(\mathbf{y}^A|\mathbf{d}^A \leq \mathbf{z}^A, \mathbf{d}^B \geq \mathbf{z}^B)P(\mathbf{d}^A \leq \mathbf{z}^A, \mathbf{d}^B \geq \mathbf{z}^B) \\ \\ \geq \delta_A \geq \\ \\ E(\mathbf{y}^A|\mathbf{d}^A \geq \mathbf{z}^A, \mathbf{d}^B \leq \mathbf{z}^B)P(\mathbf{d}^A \geq \mathbf{z}^A, \mathbf{d}^B \leq \mathbf{z}^B) \\ +\underline{y}^A P(\mathbf{d}^A < \mathbf{z}^A \text{ or/and } \mathbf{d}^B > \mathbf{z}^B) \end{array} \right\}.$$

We now use the heterogenous monotone treatment selection assumption **HMTS** . If **HMI**, **HMTS**, **EF**, and **ES2** hold, then $E(\mathbf{y}^A|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B)$ is a sharp lower bound for $E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)$ and a sharp upper bound for $E(y^A(\mathbf{d})|\mathbf{z}^A \leq \mathbf{d}^A, \mathbf{z}^B \geq \mathbf{d}^B)$, while $E(\mathbf{y}^B|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B)$ is a sharp lower bound for $E(\mathbf{y}^B(\mathbf{d})|\mathbf{z}^A \leq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)$ and a sharp upper bound for $E(\mathbf{y}^B(\mathbf{d})|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)$. Hence, the identification region is the following.

Proposition 3. *If Assumptions* **HMI***,* **HMTS***,* ***EF****, and* **ES2** *hold, then for all* $\mathbf{d} \in \mathcal{D}$*,*

$$H\{E(\mathbf{y}^A(\mathbf{d}))\} = \left\{ \delta_A \in \Delta_{y^A} : \begin{array}{c} E(\mathbf{y}^A|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B)P(\mathbf{z}^A \leq \mathbf{d}^A, \mathbf{z}^B \geq \mathbf{d}^B) \\ +\overline{y}^A P(\mathbf{z}^A > \mathbf{d}^A \text{ or, and } \mathbf{z}^B < \mathbf{d}^B) \\ \\ \geq \delta_A \geq \\ \\ E(\mathbf{y}^A|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B)P(\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) \\ +\underline{y}^A P(\mathbf{z}^A < \mathbf{d}^A \text{ or, and } \mathbf{z}^B > \mathbf{d}^B) \end{array} \right\}.$$

It is worth noting that the **HTMS** assumption can be tested for each block using the approach outlined in Lee et al. (2018) andHsu et al. (2019).

We now combine Assumptions **HMTR** and **HMTS** as in Manski and Pepper (2000). In this case, the bounds are informative even if the outcome is assumed to be unbounded. If Assumptions **HMI**, **HMTR**, **HMTS**, **EF**, and **ES2** hold, then for all $\mathbf{s}^{A'} \leq \mathbf{s}^A$, and $\mathbf{s}^{B'} \geq \mathbf{s}^B$ we have that

$$E(\mathbf{y}^A|\mathbf{z}^A = \mathbf{s}^{A'}, \mathbf{z}^B = \mathbf{s}^{B'}) = E(\mathbf{y}^A(\mathbf{s}')|\mathbf{z}^A = \mathbf{s}^{A'}, \mathbf{z}^B = \mathbf{s}^{B'}) \quad (6)$$
$$\leq E(\mathbf{y}^A(\mathbf{s})|\mathbf{z}^A = \mathbf{s}^{A'}, \mathbf{z}^B = \mathbf{s}^{B'}) \leq E(\mathbf{y}^A(\mathbf{s})|\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^B) = E(\mathbf{y}^A|\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^B),$$

$$E(\mathbf{y}^B|\mathbf{z}^A = \mathbf{s}^{A'}, \mathbf{z}^B = \mathbf{s}^{B'}) = E(\mathbf{y}^B(\mathbf{s}')|\mathbf{z}^A = \mathbf{s}^{A'}, \mathbf{z}^B = \mathbf{s}^{B'})$$
$$\geq E(\mathbf{y}^B(\mathbf{s})|\mathbf{z}^A = \mathbf{s}^{A'}, \mathbf{z}^B = \mathbf{s}^{B'}) \geq E(\mathbf{y}^B(\mathbf{s})|\mathbf{z}^A = \mathbf{s}^{A}, \mathbf{z}^B = \mathbf{s}^{B}) = E(\mathbf{y}^B|\mathbf{z}^A = \mathbf{s}^{A}, \mathbf{z}^B = \mathbf{s}^{B}).$$

Observe that the data reveal $E(\mathbf{y}^A|\mathbf{z}^A = \mathbf{s}^{A'}, \mathbf{z}^B = \mathbf{s}^{B'})$ and $E(\mathbf{y}^A|\mathbf{z}^A = \mathbf{s}^{A}, \mathbf{z}^B = \mathbf{s}^{B})$ so that we can test the two joint hypotheses (see footnote 9 in Manski and Pepper; 2000). Thus, the identification region is

Proposition 4. *If Assumptions* **HMI**, **HMTR**, **HMTS**, ***EF***, *and* **ES2** *hold, then for all* $\mathbf{d} \in \mathcal{D}$,

$$H\{E(\mathbf{y}^A(\mathbf{d}))\} = \left\{ \delta_A \in \Delta_{y^A} : \begin{array}{c} E(\mathbf{y}^A|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B)P(\mathbf{z}^A \leq \mathbf{d}^A, \mathbf{z}^B \geq \mathbf{d}^B) + \\ \sum_{\substack{\mathbf{d}^{A'} > \mathbf{d}^A \\ \textit{or, and } \mathbf{d}^{B'} < \mathbf{d}^B}} E(\mathbf{y}^A|\mathbf{z}^A = \mathbf{d}^{A'}, \mathbf{z}^B = \mathbf{d}^B)P(\mathbf{z}^A = \mathbf{d}^{A'}, \mathbf{z}^B = \mathbf{d}^B) \\ \\ \geq \delta_A \geq \\ \\ E(\mathbf{y}^A|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B)P(\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) + \\ \sum_{\substack{\mathbf{d}^{A'} < \mathbf{d}^A \\ \textit{or, and } \mathbf{d}^{B'} > \mathbf{d}^B}} E(\mathbf{y}^A|\mathbf{z}^A = \mathbf{d}^{A'}, \mathbf{z}^B = \mathbf{d}^B)P(\mathbf{z}^A = \mathbf{d}^{A'}, \mathbf{z}^B = \mathbf{d}^B) \end{array} \right\}.$$

If two particular values of the potential treatment are specified we can construct bounds for the average treatment effects based on the previous results. For example, let $\mathbf{d}$ and $\mathbf{s}$ be two potential effective treatments with $\mathbf{d}^A > \mathbf{s}^A$, and $\mathbf{d}^B < \mathbf{s}^B$. Then let us define $ATE(\mathbf{t}, \mathbf{s}) = E(\mathbf{y}^A(\mathbf{d})) - E(\mathbf{y}^A(\mathbf{s}))$. Following Manski and Pepper (2000) the sharp upper bound of $ATE(\mathbf{t}, \mathbf{s})$ will be the upper bound of $E(\mathbf{y}^A(\mathbf{d}))$ minus the lower bound of $E(\mathbf{y}^A(\mathbf{s}))$. The lower bound can be computed as the difference between the upper bound of $E(\mathbf{y}^A(\mathbf{s}))$ and the lower bound of $E(\mathbf{y}^A(\mathbf{d}))$. Under Assumption **HMTR**, however, it is not feasible to compute the lower bound in this way because the result is generally negative. In this case, Manski and Pepper (2000) showed that the lower bound of $ATE(\mathbf{d}, \mathbf{s})$ must be no less than zero.

The next section provides an empirical application of our new method. We will focus on market interactions where all participating firms are observed and part of the reference group. Consequently, the imprecision in our results is attributable solely to the fact that specific potential outcomes cannot be observed rather than being caused by any random variability resulting from the sampling process (Manski and Pepper; 2018). However, suppose groups are randomly sampled. In that case, the researcher can estimate the proposed identification regions by the sample analog of our bounds and provide a confidence set based on the literature on inference for settings with partial identification (see e.g., Lazzati; 2015; Manski; 2013, and references therein). Note that conditioning on other characteristics is permissible. Classical estimation is feasible in cases where these characteristics assume finitely many values, and the researcher can estimate bounds by calculating the corresponding sample averages.. However, when the characteristics involve continuous components, nonparametric methods such as Local Polynomial Regression or Nadaraya-Watson Estimator can be employed for estimation.

5 Empirical Application

As discussed in Section 3, our method can be used in a wide range of settings characterized by social and market interactions. In the context of market interactions, firms maximize their profits competing on the same product market or exchanging inputs and outputs. This creates interactions between and within firms' outcomes, such as income reports and balance sheets items. It follows that when we analyze balance sheet items, we deal with simultaneously determined outcomes on both the asset and liability sides.

5.1 Banks' Balance Sheet Items

We consider the following outcomes of banks' balance sheets. On the asset side, a_i is the value of securities held (like bonds, treasuries and equity), c_i is the value of credit provided to the non financial sector (i.e. households and firms), l_i is the value of lending in the interbank market, r_i is the value of reserves held at the central bank, by bank i. On the liability side, b_i is the value of borrowing in the interbank market, s_i is the value of bonds issued by bank i, d_i is the value of deposits held at bank i. Suppose the banking system is composed of $\mathcal{N} = \{1, .., n\}$ banks. Let $A \equiv \{a_i, c_i, l_i\}_{i\in\mathcal{N}}$ and $B = \{b_i, s_i, d_i\}_{i\in\mathcal{N}}$.[18]

In what follows we refer to the log of gross nominal monetary value of the stock of balance sheet items, the approach can be applied also to flows, or other transformations of both them. It can also accommodate different or eventually multiple characteristics of each balance sheet item, like interest rates.

5.2 Dependencies among Asset and Liability Outcomes

In the majority of empirical studies, these variables are not considered as simultaneous and endogenous outcomes. Often one of these outcomes, say c_i, is regressed on a treatment variable and the other items are used as exogenous controls or treatments themselves, for each bank in isolation, thus ignoring the endogeneity arising from interactions within and between banks' balance sheet items.[19]

[18] For simplicity, we abstract from reserves on the asset side.

[19] Remarkable exceptions include Benetton and Fantino (2021), Andreeva and García-Posada (2021) and Berg et al. (2021), in which competitive interactions between banks are considered.

This popular approach bypasses the interactions among the balance items generated by the simultaneous optimization of banks interconnected through markets (like the markets for retail deposits, credit to non financial corporations and households and the interbank market). These interactions can be classified based on both the markets in which banks operate and the side of the balance sheet to which the relative outcome belongs. For example, c_i and c_j are on the asset side respectively of bank i and bank j and present opposing interactions generated by the competition between bank i and j for lending to non financial corporations and households. In what follows, we propose a set of general and simple monotonicity assumptions for between-banks and within-bank interactions among balance sheet outcomes. For simplicity, we assume that there are two banks i and j in the system and they operate in the same markets, the results easily extend to a banking system with any number N of participating banks when we aggregate the other market participants and use $-i$ instead of j in the notation.

5.2.1 Between Banks Dependencies

Banks operate on the same side of many markets. For example, bank i and bank j offer loans to non financial corporations. With respect to the outcomes listed before, we have the following interactions. In credit markets, loans supplied by bank i (c_i) affect negatively the supply of credit by bank j (c_j), being substitutes for borrowers in the non financial sectors.[20] In deposit markets, deposits collected by bank i (d_i) affect negatively the retail funding of bank j (d_j), being substitute as well for depositors. In interbank markets, banks compete on both the supply and the demand side, thus l_i (b_i) affects negatively l_j (b_j).[21] In financial markets, banks demand securities (a_i and a_j), and supply them, issuing bonds (s_i and s_j). We call these same-side-of-the-market interactions (M).

[20] In principle, there could also be complementarities among banks' lending for syndicated loans (one bank's participation is conditional on, or at least encouraged by, other banks' participation), because of customer-supplier relationships in the production network among their clients, or induced by the information available in credit registers (used by one bank to supply credit to some borrower provided the same borrower has a good standing with other banks). The implicit assumption here is that these mechanisms are on average overruled by competition among banks. This is supported by the findings of Pietrosanti and Rainone (2023), which shows that substitutability of credit across different relations of the same firm prevails on average. However, if information on credit registers inquiries, composition of syndicated loans and production networks is available and allows to identify the pair of banks for which credit complementarity (instead of substitutability) should be more prominent, the sign of monotonicity could be reverted and (if the system of dependencies still produces a valid partition) bounds can be estimated as well. Importantly, this could be done only because of the main innovation we propose in the paper, i.e. allowing for both substitutability and complementarities among the endogenous variables in the system.

[21] Since after the global financial crisis money markets shifted to centralized secured venues, we abstract from the case in which bank i lend (l_i) directly to bank j (b_j). We discuss possible extensions below.

5.2.2 Within Banks Dependencies

Outcomes are not only linked between different banks interacting in the market layers described above, they are also linked within the same bank. For example, an euro of extra retail funding (d_i) can be employed in lending to the non financial sector (c_i) (or the other way around) or in other assets, like buying securities a_i. Basically balance sheet expansion or reduction from one side of the balance sheet reflects, mechanically or not, on the other side. We call these between-sides-complementarity interactions (C).

It follows that in this context the following monotonic assumption are credibly weak. M interactions generate negative monotone effects, because of competition between banks. C interactions generate positive monotone effects, because of banks' balance sheet expansion on both asset and liability sides. Let us define the set of same-side-of-the-market (M) and between-sides-complementarity (C) as MC interactions. Table 2 reports the upper triangular matrix that summarizes the signs of MC interactions among banks' outcomes, considering a banking system with two banks, i and j. The lower triangular has the same signs, as the full matrix is symmetric. We can see that even with quite simple and straightforward monotonic assumptions the interdependences within and between banks generate a quite complex system of interactions among balance sheet outcomes. From this perspective, it is easy to see that the effect of a shock or a policy on even only one single item of one single bank, can potentially reverberate to all the items of all the other banks through internal optimization and market equilibrium adjustments.

In general terms, more complexity could be embedded in the system of dependencies outlined. We provide an as simple as possible setting, grounding our assumptions on the evidence collected and consolidated in the existing literature, which focused mainly on competing interactions (see Andreeva and García-Posada; 2021; Benetton and Fantino; 2021, for example). It follows that we refrain from setting an application that relies on assumptions not fully tested (or departs from what has been identified as the main source of spillovers among banks' lending) by the existing literature and lever the data mentioned above, because it is often not available (even to us).

Proposition 5. *Under MC interactions, dependencies among banks' A-L outcomes are* consistent *and it always exists a valid partition, which implies the existence of heterogeneous fixed points.*

In Appendix C we also provide the conditions for a valid partition under substitutability within the same balance sheet side, assets or liabilities.

Table 2: Monotonic Assumptions - Interactions among Banks' Assets and Liabilities -

		A						L					
		c_i	c_j	l_i	l_j	a_i	a_j	d_i	d_j	b_i	b_j	s_i	s_j
	c_i		$-$ [M]					+ [C]		+ [C]		+ [C]	
	c_j								+ [C]		+ [C]		+ [C]
A	l_i				$-$ [M]			+ [C]		+ [C]		+ [C]	
	l_j								+ [C]		+ [C]		+ [C]
	a_i						$-$ [M]	+ [C]		+ [C]		+ [C]	
	a_j								+ [C]		+ [C]		+ [C]
	d_i								$-$ [M]				
	d_j												
L	b_i										$-$ [M]		
	b_j												
	s_i												$-$ [M]
	s_j												

Notes. A and L stand respectively for assets and liabilities. i and j are two banks. l_i and b_i are respectively the amount of lending and borrowing in the interbank market. c_i is the amount of credit provided by bank i. d_i is the amount of deposits held at bank i. a_i is the amount of securities held by bank i. s_i is the amount of bonds issued by bank i. $+$ and $-$ indicate respectively positive and negative interactions. [M]: same side of the market. [C]: balance sheet expansion complementarity.

5.3 Selected Treatment: Central Bank Lending

In our empirical exercise, we focus on an important and recently widely adopted policy: central bank lending to commercial banks. The central bank can lend funds to banks for several purposes and under different conditions, usually through open market operations (OMO). Before the global financial crisis, central bank funding was mainly used to provide banks with the adequate amount of reserves to settle interbank transactions, maintain the desired target rate, and meet the regulatory requirements under competitive auctions. After the global financial and sovereign debt crisis, and subsequent stress in unsecured interbank markets (see Afonso et al.; 2011; Angelini et al.; 2011; Rainone; 2017, among others), this tool started to be used by central banks to help banks with funding difficulties and sustain credit to the real economy, eventually with targeted operations.[22]

Money injections by the central bank must increase mechanically the consolidated balance sheet of the banking sector. However, depending of the specific policy considered, the configuration of the balance sheet, in terms of shares of assets and liabilities, can vary. Moreover, this configuration is not solely determined by the policy itself but also hinges on

[22]Given the relanvance of the topic, there is a large literature studying the effects of these operations on credit provision and other items of banks balance sheets, see Benetton and Fantino (2021), Andreeva and García-Posada (2021), Carpinelli and Crosignani (2021), Garcia-Posada and Marchetti (2016), Crosignani, e Castro and Fonseca (2020), Afonso and Sousa-Leite (2020), Laine (2019), Balfoussia and Gibson (2016), Jasova et al. (2018), Acharya and Steffen (2015), Van der Kwaak (2015), Corbisiero (2022), Crosignani, e Castro and Fonseca (2020), Andrade et al. (2019), Daetz et al. (2018), Darracq-Paries and De Santis (2015), and Alves et al. (2021) among others. Generally, the literature agreeds on the positive effects of these operations on credit, but potential externalities and competition are mentioned as factors that can curb the direct effects of this policy.

the optimal responses of banks, which are influenced by their interactions in the markets they participate in. It means that under different configurations of optimal choices of and interactions among banks operating in the system, the same amount of money injected could have a different effect on aggregate credit to the non financial sector (NFS). For example, suppose there are two banks of equal size, i and j, in the system. If bank i intends to borrow funds from the central bank and allocate a portion to lending to the NFS, the aggregate NFS credit can vary significantly depending on whether bank j utilizes the same funds for lending or for purchasing securities.

We assume positive monotonicity between central bank funding and credit to the NFS at the individual bank-level and negative monotonicity (substitutability) between banks' credit to the NFS.

This framework enables the expansion of the consolidated banking sector (and the central bank's) balance sheet, as the positive effect of central bank funding can outweigh the negative effect of inter-bank substitution. This alignment with balance-sheet constraints at the macro-level can be achieved solely through the assumption of monotonicity, without the imposition of additional constraints, thereby maintaining a light and general set of assumptions.[23]Let us make a real-life example that may highlight the advantages of our methodology. Assume bank i and bank j have a true (but unobserved) response of credit to the NFS of 0.3 for each euro of central bank funding and that an additional euro of lending from bank j to the NFS decreases the lending of bank i of 0.2. For simplicity, assume that the remaining amount of funds not allocated in the credit market is held in reserves at each bank and credit is deposited outside the banking system, thus the balance sheet constraint holds for all entities. An aggregate injection of 200 of central bank funding, 100 to bank i and 100 to bank j, would then increase aggregate credit to the NFS by 54, with a macro-level effect of 0.27. In this example, the bank-level effect of central bank funding is 0.3, the macro-level effect is ten percent lower because of substitutability of (competitive interactions among) banks' lending. Assume that there exists another bank k that does not borrow any funds from the central bank, and thus work as a control group for micro estimates, and whose lending decreases (as for bank i) by 0.2 for each euro of lending by bank j. Any variation of lending is counterbalanced by purchases or sales of a security issued outside the system, and thus balance sheet constraint holds. Diff-in-diff estimates, usually employed because the effect of aggregate central bank funding is indistinguishable from other correlated forces operating at the macro-level, would suggest a macro-level effect of 0.33, twenty percent higher than the true. Our partial identification approach provides an upper bound of 0.3 and a lower bound of 0.24 for such effect. Even if not point identified, the true effect is in the identified region. The width and the position of the region reflect the uncertainty about the intensity and the

[23]It's worth noting that while we do not impose such constraints initially, they can be incorporated if necessary.

sign of the interactions at work.

5.4 Data Description

This section describes our data and helps to gauge the correlations in the data at aggregate and individual level. Our analysis is based on a unique, proprietary data set of balance sheet items (BSI) at bank level (Individual Balance-Sheet Items or IBSI), which is regularly updated by the Bank of Italy. In the context of European regulations, national central banks regularly collect and disseminate monthly data on banks' balance sheet items (BSI).[24] BSI data represent a crucial source for the production of both national and euro-area monetary aggregates (M1, M2 and M3) and their counterparts, which have a major role for the ECB assessment of the risks to price stability and stability. See Altavilla et al. (2017) for a description of the unique representativeness of this data. The IBSI statistics encompass information on the balance sheet of banks, both on the assets and liabilities side, which makes it an ideal dataset for the application of our methodology to banks' outcomes. The asset side indicators include loans to households and NFCs, securities, and funds lent in money markets. On the liabilities side, time series are collected for deposits included and not included in the broad money aggregate M3, debt securities issued, and funds borrowed in money markets.[25] As a result, the Bank of Italy monthly collects and elaborates a huge amount of individual balance sheet data reported by the entire population of Italian banks (385 reporting banks at the end of 2023) in order to compile aggregate statistics to be provided to the ECB and the public.

5.5 Descriptive Statistics

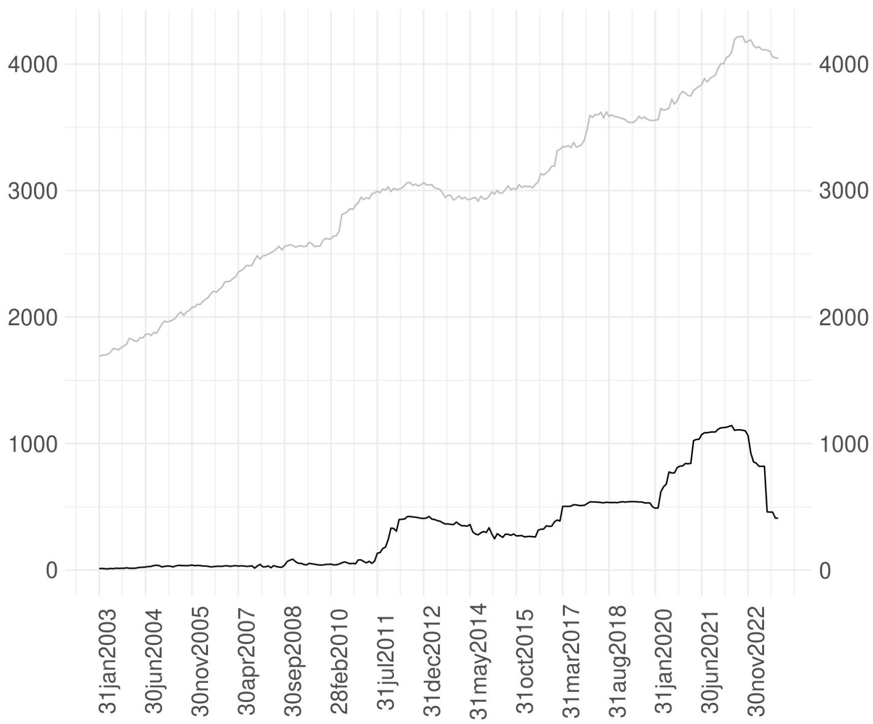
In Table 5, we report end of the year aggregate statistics for the balance sheet items under study in our application, c, a, l, b, s, d and our selected treatment t, from 2011 to 2023 the years we focus on in the empirical analysis. Credit to the real economy, c, is probably the most studied item of banks' assets, given its relevance for the transmission of monetary

[24]See the ECB Regulation ECB/2013/33 on the balance sheet of the monetary financial institutions sector, and the ECB Guideline of 4 April 2014 on monetary and financial statistics (ECB/2014/15). At the European level, the reporting obligations are specified in different legal frameworks, including the ECB statistical regulations on balance sheet items (BSI) and interest rates (MIR) of monetary financial institutions (MFIs), the sectoral module of Securities Holdings Statistics (SHS-S), granular credit and credit risk (AnaCredit), as well as the European Banking Authority's Implementing Technical Standards on supervisory reporting and resolution reporting and the ECB Regulation on reporting of supervisory financial information. See the https://www.ecb.europa.eu/stats/ecb$_s tatistics/co - operation_a nd_s tandards/reporting/html/index.en.htmlESCBlong - termstrategyforbanks' datareporting$

[25]National central banks of the Eurosystem collect outstanding amounts and, for some of them, the corresponding adjustment series, covering information on revaluations for changes in prices and exchanges rates, reclassifications or loan write-offs/write-downs. For loans, additional data on loan transfers, linked for instance to securitisation, are also covered. See Morandi et al. (2016) for more details on the content of BSI data and the sample of banks. See Marinelli et al. (2021) for a discussion of quality of the data and methods to improve it.

policy. Indeed, the relationship between our main treatment variable, the central bank funding, and loans supply to firms and households has been widely investigated.[26] In the period under analysis, Italy has been hit by multiple macro and idiosyncratic shocks, and central bank interventions were often aimed at restoring the soundness of banks' funding and let them continuing providing credit to the real economy. In Figure 3, we report more frequent, monthly, time series of the average stock of central bank funding and loans to the NFS for the Italian banking system for a wider time period, from January 2003 to October 2023.

Figure 3: Aggregate time series
- Stock of loans to the NFS and central bank funding -



Notes. Monthly data of average stock of loans to the NFS and central bank funding for the IBSI sample of the Italian banking system in million euro. The stock of loans to the NFS is reported in grey. The stock of central bank funding is reported in black.

We can see from this figure that the stock of loans steadily increased before the global financial crisis, even in absence of significant central bank funding. Since then its growth rate declined, also reflecting the adverse effects of the sovereign debt crisis. Central bank funding started to flow intensively in banks' balance sheets after mid 2011, mainly through long term refinancing operations (LTRO) following the second peak of the sovereign debt

[26]See Afonso and Sousa-Leite (2020); Andrade et al. (2019); Andreeva and García-Posada (2021); Benetton and Fantino (2021); Carpinelli and Crosignani (2021); Esposito et al. (2020); Garcia-Posada and Marchetti (2016) among others. Observe that our method can be used to assess also other conventional and unconventional policies, see Abbassi et al. (2016); Acharya and Steffen (2015); Crosignani, Faria-e Castro and Fonseca (2020); Krishnamurthy et al. (2018).

crisis, in which Italy was at the epicenter. From that time period, we can see a remarkable correlation in the behavior of the two time series, which supports the hypothesis that central bank funding supported credit to the real economy. The introduction of the targeted longer-term refinancing operations (TLTROs) strengthened such linkage even more, given that the amount of funds that banks can borrow is linked to their loans to the NFS. More loans to NFS (except loans to households for house purchases) imply more attractive interest rates.[27] Central bank funding increased significantly again in 2020 with the pandemic crisis when also a new series of non-targeted pandemic emergency longer-term refinancing operations (PELTROs) flanked TLTROs. Recently the outstanding amount of central bank funding started to decline quite sharply, given the expiration of the main long term operations. Even if other policies affect the stock of loans to the NFC, like the level of interest rates or the public guarantee schemes during the pandemic crisis, the influence of central bank funding is quite evident from this plot, and generally supported by the empirical studies mentioned above. Given the diffuse interest on this relationship, we will mainly focus on the effect of central bank funding (the treatment variable) on the credit to the NFS (the outcome variable) from mid 2011 in the remainder of the paper.

5.6 Main Results

5.6.1 Empirical Bounds

We are interested in estimating bounds for the credit to the NFS (c_i) that a bank i gives when it gets a certain treatment (central bank funding, t_i). Let $h(\mathbf{t}_{-i})$ be the average treatment received by the rest of the banking system.

Below we report the empirical upper and lower bounds for the potential outcome for the individualistic treatment response (ITR),[28] which assumes that there are no interactions among balance sheet items and under the different assumptions outlined before: (i) the *HMI* bounds from Proposition 1; (ii) the *HMIR* bounds form Proposition 2; (iii) the *HMIS* bounds from Proposition 3; (iv) the *HMIRS* bounds from Proposition 4.
The individualistic treatment response without interactions considers only the individual treatment status:

$$\begin{aligned} LB_{ITR}[c_i(t_i)] &= E(c_i(s)|t_i = s)P(t_i = s) \\ &+ \underline{c}_i P(t_i \neq s), \end{aligned} \qquad (7)$$

[27]A first series of TLTROs was announced on 5 June 2014, a second series (TLTRO II) on 10 March 2016 and a third series (TLTRO III) on 7 March 2019.

[28]We follow the definition of Manski (2013) to mark the assumption that one unit's outcome may vary only with its own treatment, not with those of other members of the population. Cox (1958) called this 'no interference between units'. Rubin (1978) called it the 'stable unit treatment value assumption' (SUTVA).

$$UB_{ITR}[c_i(t_i)] = E(c_i(s)|t_i = s)P(t_i = s) + \bar{c}_i P(t_i \neq s), \tag{8}$$

where $\underline{c}_i$ and $\bar{c}_i$ are respectively the minimum and maximum observed credit granted by a bank in the sample.

The *HMI* bounds account for interactions and considers the whole treatment vector:

$$LB_{HMI}[c_i(t_i, h(\mathbf{t}_{-i}))] = E(c_i(s)|t_i = s, h(\mathbf{t}_{-i}) = k)P(t_i = s, h(\mathbf{t}_{-i}) = k) + \underline{c}_i P(t_i \neq s, or\, h(\mathbf{t}_{-i}) \neq k), \tag{9}$$

$$UB_{HMI}[c_i(t_i, h(\mathbf{t}_{-i}))] = E(c_i(s)|t_i = s, h(\mathbf{t}_{-i}) = k)P(t_i = s, h(\mathbf{t}_{-i}) = k) + \bar{c}_i P(t_i \neq s, or\, h(\mathbf{t}_{-i}) \neq k). \tag{10}$$

The *HMI* bounds are very conservative as they use the observed outcomes at $t_i, h(\mathbf{t}_{-i})$ and the extreme values $\bar{c}_i$ and $\underline{c}_i$ for the other observations.
The *HMIR* bounds are:

$$LB_{HMIR}[c_i(t_i, h(\mathbf{t}_{-i}))] = E(c_i(s)|t_i \geq s, h(\mathbf{t}_{-i}) \leq k)P(t_i \geq s, h(\mathbf{t}_{-i}) \leq k) + \underline{c}_i P(t_i < s, or\, h(\mathbf{t}_{-i}) > k), \tag{11}$$

$$UB_{HMIR}[c_i(t_i, h(\mathbf{t}_{-i}))] = E(c_i(s)|t_i \leq s, h(\mathbf{t}_{-i}) \geq k)P(t_i \leq s, h(\mathbf{t}_{-i}) \geq k) + \bar{c}_i P(t_i > s, or\, h(\mathbf{t}_{-i}) < k). \tag{12}$$

The *HMIR* bounds are generally tighter, as they give smaller weights to the extreme values. This is because in the *HMIR* bounds outcomes of units having smaller (larger) own treatments, and larger (smaller) others' treatments are used to increase (decrease) the lower (upper) bound, exploiting the monotone treatment response assumption.
The *HMIS* bounds are:

$$LB_{HMIS}[c_i(t_i, h(\mathbf{t}_{-i}))] = E(c_i(s)|t_i = s, h(\mathbf{t}_{-i}) = k)P(t_i \leq s, or\, h(\mathbf{t}_{-i}) \geq k) + \underline{c}_i P(t_i > s, or\, h(\mathbf{t}_{-i}) < k), \tag{13}$$

$$UB_{HMIS}[c_i(t_i, h(\mathbf{t}_{-i}))] = E(c_i(s)|t_i = s, h(\mathbf{t}_{-i}) = k)P(t_i \geq s, or\, h(\mathbf{t}_{-i}) \leq k) + \bar{c}_i P(t_i < s, or h(\mathbf{t}_{-i}) > k). \tag{14}$$

Also the *HMIS* bounds are generally tighter than the *HMI*, because they exploit the monotone treatment selection assumption. In *HMIS* bounds, outcomes of units with a certain realized own treatment are used to set an upper (lower) bound for the expected value of the outcome of units with smaller (larger) realized own treatments. Units with a certain realized others' treatment are used to construct a lower (upper) bound for the expected value of the outcome of units with larger (smaller) realized own treatments.

The *HMIRS* bounds are:

$$\begin{aligned} LB_{HMIRS}[c_i(t_i, h(\mathbf{t}_{-i}))] &= E(c_i(s)|t_i = s, h(\mathbf{t}_{-i}) = k)P(t_i \leq s, h(\mathbf{t}_{-i}) \geq k) \\ &+ E(c_i(s)|t_i > s, or\, h(\mathbf{t}_{-i}) < k)P(t_i > s, or\, h(\mathbf{t}_{-i}) < k) \end{aligned} \quad (15)$$

$$\begin{aligned} UB_{HMIRS}[c_i(t_i, h(\mathbf{t}_{-i}))] &= E(c_i(s)|t_i = s, h(\mathbf{t}_{-i}) = k)P(t_i \geq s, h(\mathbf{t}_{-i}) \leq k) \\ &+ E(c_i(s)|t_i < s, or\, h(\mathbf{t}_{-i}) < k)P(t_i < s, or\, h(\mathbf{t}_{-i}) < k) \end{aligned} \quad (16)$$

The *HMIRS* bounds can be even tighter as they combine the h-monotone treatment response and selection assumptions, thus less informative extreme values of c_i are not used to compute upper and lower bounds.

5.6.2 Estimated Bounds

In Table 3, we report the estimated upper and lower bounds for the potential outcome when the individual treatment varies, under the different assumptions outlined before: *HMI*, *HMIR*, *HMIS* and *HMIRS*. We consider ten intervals for the individual treatment and the average treatment status of others, corresponding to the deciles of the distributions. Both the treatment and the outcome are expressed in logs.

The first panel reports the *ITR* bounds for the response to the individual treatment. The bounds are increasing in the treatment. Interestingly, the lower bound is negative, suggesting that even with high treatment values the outcome is not necessarily positive, which could be explained by negative spillovers from others' treatments.

In the other panels, the response of the potential outcome to the individual treatment is combined with the treatment status of others: a low and high level of others' treatment, respectively below the first and above the last decile of its distribution.

The second panel reports the *HMI* bounds for the response to the individual treatment, under h-monotonic interactions. The introduction of the **HMI** assumption, slightly widens the bounds. This is due to the fact that accounting also for monotonic interactions increases the uncertainty about the potential outcome.

The third panel reports the *HMIR* bounds for the response to the individual treatment, under h-monotonic interactions and h-monotonic response to the treatment vector. The introduction of the **HMTR** assumption, h-monotonicity in the treatment response, narrows the bounds, more prominently the lower bounds. In particular, the lower bounds are much higher when the treatment of others is low, substantially narrowing the potential outcome region. The lower bounds decrease remarkably if the treatment of others is high.

The fourth panel reports the *HMIS* bounds for the response to the individual treatment, under h-monotonic interactions and endogenous selection into the treatment vector. The introduction of **HMTS**, the monotone treatment selection, decreases sensibly the upper bounds w.r.t. the **HMI**, much more when the treatment of others is low, less remarkably when it is high. This means that accounting for self-selection into the treatment makes the highest potential outcome lower especially if others' treatment status is low.

The fifth panel reports the *HMIRS* bounds for the response to the individual treatment, under h-monotonic interactions and h-monotonic treatment response and selection. When both **HMTR** and **HMTS** are imposed the bounds are remarkably tighter. A high treatment of others substantially decreases both the lower and the upper bounds. When the treatment of others is low, bounds are strictly higher than when the treatment of others is high. Their lower bounds are regularly greater than the upper bounds when the treatment of others is high.

In Table 4, we report the estimated upper and lower bounds for the potential outcome when others' treatment status varies, under the HMIR, HMIS and HMIRS. The response of the potential outcome to others' treatment status is combined with a low and high level of own treatment, respectively below the first and above the last deciles of its distribution. From the first panel, we can see that the **HMTR** produces bounds quite stable across different levels of others' and own treatments, inline with the findings of Table 3. The second panel shows that the **HMTS** instead generates lower bounds remarkably decreasing in the treatment of others. The upper bound decreases much more prominently when the individual treatment is low, highlighting that a significant portion of the positive effect of own treatment can be offset by competitive interactions if the others receive high treatment. The third panel highlights that when both **HMTR** and **HMTS** are assumed the upper bounds decrease much more sharply with others' treatment status.

Observe that nearly all of the bounds (except *HMIR* lower bounds when others' and and own' treatment status is low) include zero. This means that under **HMI**, **HMTR**, and **HMTS** assumptions, we cannot generally identify the sign of the treatment effect. Thus, in Figure 4, we focus on the tightest identified set and provide a more detailed and graphical representation of the *HMIRS* bounds. The left (right) panel represents the *HMIRS* bounds when own (others') treatment vary, under low, medium and high others' (own) treatment status, respectively in light gray, dark gray and black (black, medium and light gray). From panel (a) we can see that upper and lower bounds increase remarkably with the treatment when the treatment of others is low and, to a lower extent, when it is

Table 3: Estimated Upper and Lower Bounds under Different Monotonic and Selection Assumptions - Own Treatment Variation -

Bounds type						Monotonic assumptions			
						HMI	**HMTR**	**HMTS**	**ES**
ITR						N	N	N	N
		Interval	Own treatment level	Lower bound	Upper bound				
		(0.001,3.58]	1	-6.589	12.820				
		(3.58,4.33]	2	-6.627	12.869				
		(4.33,4.8]	3	-6.588	12.863				
		(4.8,5.23]	4	-6.594	12.874				
		(5.23,5.58]	5	-6.586	12.880				
		(5.58,6.02]	6	-6.574	12.885				
		(6.02,6.5]	7	-6.565	12.893				
		(6.5,7.13]	8	-6.553	12.906				
		(7.13,8.34]	9	-6.523	12.931				
	Own treatment	Low others' treatment level		High others' treatment level					
	level	Lower bound	Upper bound	Lower bound	Upper bound				
HMI						Y	N	N	Y
	1	-6.892	13.012	-6.873	12.997				
	2	-6.887	13.011	-6.889	13.009				
	3	-6.885	13.010	-6.879	13.005				
	4	-6.891	13.014	-6.875	13.004				
	5	-6.888	13.013	-6.862	13.000				
	6	-6.887	13.013	-6.869	13.004				
	7	-6.886	13.013	-6.868	13.005				
	8	-6.891	13.015	-6.866	13.006				
	9	-6.878	13.014	-6.866	13.010				
HMIR						Y	Y	N	Y
	1	2.746	12.954	-4.857	11.799				
	2	3.027	12.962	-4.819	11.981				
	3	3.347	12.970	-4.775	12.120				
	4	3.661	12.980	-4.722	12.268				
	5	3.982	12.986	-4.659	12.405				
	6	4.316	12.993	-4.604	12.535				
	7	4.659	12.999	-4.544	12.662				
	8	5.014	13.006	-4.481	12.781				
	9	5.399	13.011	-4.418	12.887				
HMIS						Y	N	Y	Y
	1	-6.731	7.577	-4.247	11.717				
	2	-6.737	7.973	-4.469	11.747				
	3	-6.754	7.961	-4.578	11.857				
	4	-6.771	8.250	-4.841	11.872				
	5	-6.785	8.406	-5.080	11.910				
	6	-6.802	8.379	-5.340	11.952				
	7	-6.821	8.290	-5.628	11.982				
	8	-6.838	8.904	-5.909	12.067				
	9	-6.851	9.776	-6.196	12.232				
HMIRS						Y	Y	Y	Y
	1	3.902	4.846	1.622	2.303				
	2	4.043	5.525	1.577	2.252				
	3	4.164	5.825	1.616	2.265				
	4	4.310	6.414	1.519	2.194				
	5	4.459	6.872	1.439	2.154				
	6	4.611	7.152	1.340	2.094				
	7	4.771	7.370	1.215	2.022				
	8	4.943	8.291	1.099	1.994				
	9	5.132	9.472	0.976	2.015				

Notes. Levels and log million euro. The nine levels for the individual treatment correspond to the deciles of the distribution of strictly positive values. The first (last) level are not considered in the comparison because the HMIRS lower (upper) bound can not be computed. The low others' treatment level corresponds to first decile of the mean of others' treatment distribution, the first is not considered as the log function has values equal or close to zero. The high others' treatment level corresponds to the last decile of the mean of others' treatment distribution. ITR stands for individual teratment response. HMI stands for the h-monotonic interactions assumptions from Proposition 1, HMIR stands for the h-monotonic treatment response assumptions from Proposition 2, HMIS stands for the h-monotonic treatment selection assumptions from Proposition 3, HMIRS stands for the h-monotonic treatment selection and responseassumptions from Proposition 4. The bounds are constructed with the formulas in Section 5.6.1.

Table 4: Estimated Upper and Lower Bounds under Different Monotonic and Selection Assumptions - Others' Treatment Variation -

Bound type	Interval	Others' treatment level	Low own treatment level Lower bound	Low own treatment level Upper bound	High own treatment level Lower bound	High own treatment level Upper bound	Monotonic assumptions **HMI**	**HMTR**	**HMTS**	**ES**
HMI							Y	Y	N	Y
	(0.508,1.05]	1	-5.795	12.307	-6.876	13.016				
	(1.05,1.16]	2	-5.875	12.372	-6.852	13.012				
	(1.16,1.25]	3	-5.902	12.388	-6.856	13.013				
	(1.25,1.33]	4	-5.919	12.373	-6.874	13.016				
	(1.33,1.37]	5	-5.963	12.405	-6.873	13.016				
	(1.37,1.53]	6	-5.899	12.364	-6.873	13.016				
	(1.53,1.58]	7	-6.019	12.481	-6.831	13.011				
	(1.58,1.6]	8	-6.103	12.516	-6.842	13.013				
	(1.6,1.62]	9	-5.892	12.391	-6.874	13.016				
HMIR							Y	Y	N	Y
		1	2.427	12.281	5.812	13.016				
		2	1.378	11.543	4.558	13.009				
		3	0.403	10.842	3.249	13.002				
		4	-0.546	10.104	2.012	12.998				
		5	-1.479	9.371	0.757	12.994				
		6	-2.371	8.620	-0.503	12.990				
		7	-3.323	7.948	-1.774	12.982				
		8	-4.161	7.259	-3.065	12.974				
		9	-4.921	6.528	-4.368	12.970				
HMIS							Y	N	Y	Y
		1	-5.693	7.071	-6.877	10.703				
		2	-4.425	7.813	-6.825	10.787				
		3	-3.237	8.418	-6.776	11.053				
		4	-2.090	8.916	-6.743	11.393				
		5	-0.876	9.535	-6.708	11.755				
		6	0.352	10.119	-6.673	12.046				
		7	1.777	10.836	-6.604	12.173				
		8	2.897	11.311	-6.540	12.401				
		9	4.183	11.794	-6.512	12.575				
HMIRS							Y	Y	Y	Y
		1	3.902	4.846	5.132	9.472				
		2	3.561	3.806	4.598	8.204				
		3	3.236	3.639	4.109	7.314				
		4	2.944	3.405	3.591	6.479				
		5	2.663	3.034	3.064	5.781				
		6	2.349	2.804	2.527	4.861				
		7	2.111	2.508	2.028	3.672				
		8	1.940	2.358	1.526	2.879				
		9	1.622	2.303	0.976	2.015				

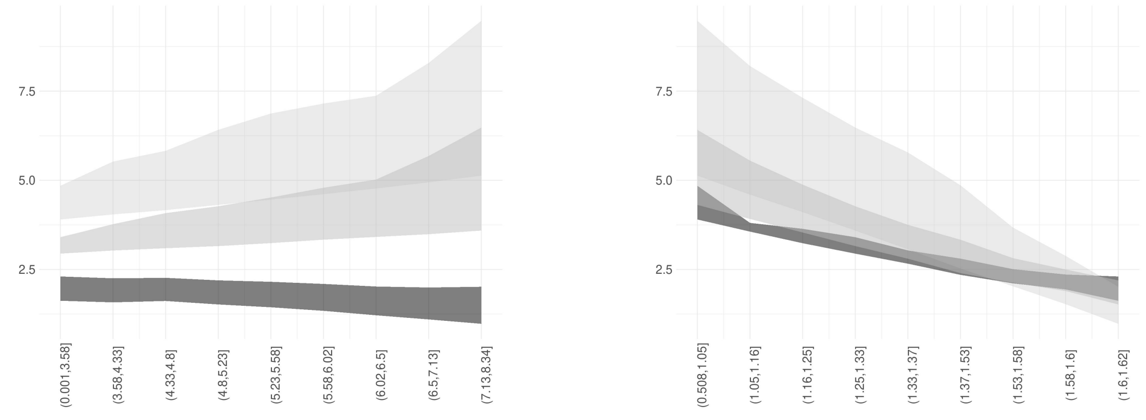
Notes. Levels and log million euro. The nine levels for the individual treatment correspond to the deciles of the distribution of strictly positive values. The low others' treatment level corresponds to first decile of the mean of others' treatment distribution, the first is not considered as the log function has values equal or close to zero. The high others' treatment level corresponds to the last decile of the mean of others' treatment distribution. HMI stands for the h-monotonic interactions assumptions from Proposition 1, HMIR stands for the h-monotonic treatment response assumptions from Proposition 2, HMIS stands for the h-monotonic treatment selection assumptions from Proposition 3, HMIRS stands for the h-monotonic treatment selection and responseassumptions from Proposition 4. The bounds are constructed with the formulas in Section 5.6.1.

medium. When others' treatment is high, the bounds do not even increase with the own treatment status and remain strictly inferior to those when others' treatment status is low or medium, pushed down by the highest competitive externalities. In addition, the lower the treatment status of competitors the higher the increase of the potential outcome following an increase of the own treatment. From panel (b) we can appreciate how a gradual increase of the treatment status of others reduces the own potential outcome bounds, sharply converging to a quite narrow area much closer to zero. A high own treatment status attenuates the negative effect induced by higher others' treatments.

When the own treatment status is the lowest (the darkest area in the plot) the upper bound is remarkably lower than when it is medium or high and this holds true the lower is others' treatment status. This means that when competitors treatment status is high, the own treatment has very small effect on the outcome almost certainly.

We find that banks borrowing more from the central bank grant more credit to the real economy when other banks' borrowing is low. When competitors in the market borrow heavily, the effect of central bank funding on credit attenuates remarkably. A standard deviation increase in central bank funding can induce up to 60 percent of a standard deviation point increase in a single bank credit provision to the real economy when other banks get limited funding from the central bank on average (about the first decile of the distribution). This upper bound reduces to less than one percent of a standard deviation point when other banks borrow more funds (about the ninth decile of the distribution). Symmetrically, a reduction of central bank funding, which is included in our data (especially in the last periods), has milder adverse effects on lending when funds were widely borrowed by most of the banks.

Figure 4: Partial Identification under HMIRS
- Estimated Upper and Lower Bounds when Own and Others' Treatment Varies -



(a) Own treatment (b) Others' treatment

Notes. panel (a) x-axis: levels of own treatment status. panel (b) x-axis: levels of others' treatment status. Levels are expressed in log million euro. The levels correspond to the deciles of the empirical distribution. y-axis: estimated upper and lower bounds for the own potential outcome. In panel (a), light grey: low others' treatment status; dark grey: medium others' treatment status; black: high others' treatment status. The low others' treatment level corresponds to first decile of the mean of others' treatments distribution, the medium others' treatment level corresponds to fifth decile of the mean of others' treatments distribution, the high others' treatment level corresponds to last decile of the mean of others' treatments distribution. In panel (b), light grey: low own treatment status; dark grey: medium own treatment status; black: high own treatment status. The low own treatment level corresponds to first decile of the own treatment distribution, the medium own treatment level corresponds to fifth decile of the own treatment distribution, the high own treatment level corresponds to last decile of the own treatment distribution. HMIRS stands for the h-monotonic treatment selection and response assumptions from Proposition 4. The bounds are constructed with the formulas in 5.6.1.

6 Concluding Remarks

This paper provides identification results for treatment response in contexts with endogenous and heterogeneous interactions among agents by means of monotone comparative statics. Our method enables an agent's treatment response to be a function of the entire vector of treatments received by the population, and to react differently according to heterogeneous interactions, which can arise within or between different agents' types or outcomes in the population. Compared to previous work, this generalization allows the use of credible partial identification via comparative statics in many contexts where the relationships between outcome variables are not trivial, for example under complementarity and substitutability. It relies neither on random treatment assignment nor on random assignment of agents to types or interactions. In doing so, we bridge the theory of identification of treatment effects that exploits monotone restrictions with that on games featuring strategic complementarity and substitutability. In particular, we introduce the heterogeneous fixed points theorem in economics to show that our bounds are coherent with equilibrium behavior arising from the assumptions made on the primitives of the structural model. We derive identification regions for the true distribution of outcome variables and show that such bounds can be highly informative under relatively weak monotonic assumptions.

While the mathematical foundation of our method may look complex, it produces bounds that are very easy to compute, it just requires few expected values and frequencies. In practice, the primary effort involves constructing a system of coherent monotonic assumptions based on economic theory. This exercise, often necessary and required in any case, can be facilitated by ancillary statistical tests if needed. The empirical application illustrates the usefulness of our method to inform monetary policy and financial stability. Allowing for full endogeneity of and heterogeneous interdependence among banks' balance sheet items, we show how the method can produce tight and meaningful bounds for the response to central bank funding of credit to the real economy.

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APPENDIX

Appendix A: Useful Results and Extensions

Let us include some useful results to make this paper self-contained.

Tarski's Fixed Point Theorem (TFP). *If L is a complete lattice and $f : L \to L$ is an increasing function, then f has a fixed point. Furthermore, the set of fixed points of f has a least and a greatest element. (Tarski, 1955 ADD REF)*

Milgrom and Roberts (1990, 1994) Theorem. *If L is a complete lattice, and T is a partially ordered set, and $f : L \times T \to L$ is an increasing function, then the least and the greatest fixed points of f are increasing in t on T. (Milgrom and Roberts; 1990)*

Appendix B: Proofs

Monotonic Interactions and Heterogeneous Fixed Points

Proof of Theorem 2.1. Suppose $\mathbf{y} = (y_1, \ldots, y_n)$ is the outcome vector of n units as in system (1), and the population is split into two separate groups, A and B. If interactions are reinforcing within the same group and opposing between groups, the direct effect of y_i on y_j is positive if iI and j belong to the same group, it ; it is negative otherwise. In this case, the indirect effects follow the sign of the direct effects. The indirect effect of y_i on y_j trough y_k is positive if i and j belong to the same group, negative otherwise, whatever group k belongs to. If k is in the same group, the direct effects of y_i on y_k and y_k on y_j are positive, then the indirect effect of y_i on y_j is positive too. If k is in the other group the direct effects of y_i on y_k and y_k on y_j are negative, then the indirect effect of y_i on y_j is positive. The same holds for longer paths of influence. We formalize this propriety in what follows. Let $D_k = \{< y_i, y_j, k >\}_{i,j \in N, k \in \mathbb{N}^+}$ be a $N \times N$ matrix with $d_{ij,k} = 1$ if the impact of y_j on y_i is positive and $d_{ij,k} = -1$ if the impact of y_j on y_i is negative passing through k other variables (in a chain of influences).

Set $D_0 = I_N$. By construction we have that $D_k = D_{k-1} \cdot D_1$. Let $R = \{< y_i, y_j >\}_{i,j \in N}$ be the reduced form matrix whose generic element r_{ij} represents the sign of the total impact of y_j on y_i (the total derivative through all the possible chains). Let $r_{ij} = 1$ if the final impact of y_j on y_i is positive and $r_{ij} = -1$ if the impact of j on i is negative.

R embeds all the direct and indirect effects, providing a final sign. According to Kanade et al. (2005), the system of equations is consistent if $R = D_1 = D_2 = \cdots D_{\mathbb{N}^+}$. Suppose that each variable y_i represents a specific unit i's outcome and units are connected through a network G of connections, which keeps track of who influences whom. g_{ij} is equal to 1 is j influences i, 0 otherwise. Let S be a matrix such that $s_{ij} = 1$ if $g_{ij} \neq 0$ and $i, j \in A$ or $i, j \in B$, $s_{ij} = -1$ if $g_{ij} \neq 0$ and $i \in A, j \in B$ or $i \in A, j \in B$ and $s_{ij} = 0$ if $g_{ij} = 0$. W.l.o.g. assume G is strongly connected. It implies that $S = D_1$,

$sign(S^2) = D_2$, ... , $sign(S^k) = D_k$ and $R = sign(H)$ where $H = (I_N - \theta S)^{-1} = Z^{-1}$, $\theta \in \Theta^+$ where Θ^+ is the positive subset of the parameter space for inversion of Z. W.l.o.g. Suppose units are sorted, and the interaction matrix is split accordingly $G = \begin{bmatrix} G_A & G_{A,B} \\ G_{B,A} & G_B \end{bmatrix}$. It is easy to see that it implies $s = \begin{bmatrix} P & M \\ M & P \end{bmatrix}$., $s^2 = \begin{bmatrix} PP+MM & PM+MP \\ MP+PM & PP+MM \end{bmatrix}$., $s^3 = \begin{bmatrix} PPP+MMP+PMM+MPM & PPM+MMMPMP+MPP \\ MPP+PMP+MPP+MMM & MPM+PMM+PPP+MMP \end{bmatrix}$., ... , $s^k = \begin{bmatrix} \rho_k P & \mu_k M \\ \mu_k M & \rho_k P \end{bmatrix}$, where P is a square matrix of ones, M is a square matrix of minus ones, μ_k and ρ_k are integers parameters strictly positive. It implies that $sign(S^k) = \begin{bmatrix} P & M \\ M & P \end{bmatrix}$, $\forall k$. It thus follows that for any G and any (A, B) we have that $D_1 = D_2 = \cdots = D_{\mathbb{N}^+} = R$.

$\square$

Proof of HFP Theorem (Kanade et al.; 2005). See proof Theorem 1 in Kanade et al. (2005). $\square$

Proof of Theorem 2.3. Let us define the vector functions $\mathbf{F_a} = (f_i(\mathbf{y}, t), i \in A)$, with $\mathbf{F_a} : \mathbb{P} \times T \to \mathbb{P}_a$ and $\mathbf{F_b} = (f_i(\mathbf{y}, t), i \in B)$ with $\mathbf{F_b} : \mathbb{P} \times T \to \mathbb{P}_b$. Let us define $\mathbf{y_a^L} = inf\{\mathbf{y_a} | \mathbf{F_a}(\mathbf{y}, \mathbf{t}) \leq \mathbf{y_a}\}$, $\mathbf{y_a^U} = sup\{\mathbf{y_a} | \mathbf{F_a}(\mathbf{y}, \mathbf{t}) \geq \mathbf{y_a}\}$, $\mathbf{y_b^L} = inf\{\mathbf{y_b} | \mathbf{F_b}(\mathbf{y}, \mathbf{t}) \leq \mathbf{y_b}\}$ and $\mathbf{y_b^U} = sup\{\mathbf{y_b} | \mathbf{F_b}(\mathbf{y}, \mathbf{t}) \geq \mathbf{y_b}\}$. Define $S_A(\mathbf{t}) = \{\mathbf{y_a}(\mathbf{t}) | \mathbf{F_a}(\mathbf{Y}(\mathbf{t})), \mathbf{t}) \leq \mathbf{y_a}\}$ so that $\mathbf{y_a^L}(\mathbf{t}) \equiv inf(S_A(\mathbf{t}))$. Since $\mathbf{F_a}(\mathbf{Y}(\mathbf{t}), \mathbf{t})$ is *h-monotonic*, for all $\mathbf{y_a}(\mathbf{t}) \in S_A$, $\mathbf{y_a^L}(\mathbf{t}) \in HFP^{(A,B)} \in FP(\mathbf{F})$ by the heterogeneous fixed point theorem. Furthermore, we have also that $\mathbf{y_b^U}(\mathbf{t}) \in HFP^{(A,B)} \in FP(\mathbf{F})$. Since $\mathbf{y_a^L}(\mathbf{t})$ is a lower bound of S_A we have $\mathbf{y_a^L}(\mathbf{t}) \leq \mathbf{F_a}(\mathbf{y_a^L}(\mathbf{t}), \mathbf{t})$ and given that $\mathbf{F}$ is *h-monotonic* in $\mathbf{t}$ w.r.t. (A, B) we have that the set $S_A(\mathbf{t})$ becomes smaller as $\mathbf{t}$ increases for the elements of the vector $i \in A$ and it becomes larger for elements of the vector $i \in B$. Hence, we have that $\mathbf{y_a^L} \equiv inf(S_A(\mathbf{t}))$ is a *h-monotonic* function of $\mathbf{t}$. The same argument holds for the set $S_B(\mathbf{t}) = \{\mathbf{y_b} | \mathbf{f_b}(\mathbf{Y}, \mathbf{t}) \geq \mathbf{y_b}\}$ and is supremum $\mathbf{y_b^U}$. Given the fact that $\mathbf{F}$ is *h-monotonic* function, then there are no $\mathbf{y_a}$ and $\mathbf{y_b}$, which satisfy the two equations $\mathbf{y_a} = \mathbf{F_a}(\mathbf{y_a}, \mathbf{t})$ and $\mathbf{y_b} = \mathbf{F_b}(\mathbf{y_b}, \mathbf{t})$ for two different values of $\mathbf{t}$. So $\mathbf{y_a^L}$ and $\mathbf{y_b^U}$ must be *h-monotonic* as well. The same way of reasoning can be applied if $\mathbf{F}(\mathbf{y}, \mathbf{t})$ is an increasing (or decreasing) function. In that case, the solutions of the system will preserve the sign of the monotonicity of the structural vector function. The result follows from Milgrom and Roberts (1990, 1994). $\square$

Heterogeneous Monotonic Structural Functions

The strategy of the following proofs is similar in spirit to Lazzati (2015). However, they are applied to different blocks/outcomes, use heterogenous monotonic shape functions, and use conditional expectations rather than the outcome's probability. W.l.o.g., we focus on two blocks A and B, which partitions the population $\mathcal{P}$.

Proof of Lemma 1. Consider the mapping

$$N^A_{\mathbf{d},r} : \Delta_y \to \Delta^A_y,$$
$$E(Y) \to E(Y^A)$$

where $E(Y^A) = (1/|A|)\sum_i f^A_{ir}(t_{ir}, E(\mathbf{y_r^{A/i}}(t_{ir})), E(\mathbf{y_r^B}(t_{ir})))$ for block A. By construction, the elements of the solution set $\phi^A(\mathbf{d}, r)$ coincide with the extremal fixed points of $N^A_{\mathbf{d},r}$. Then, the proof of Lemma 1 reduces to show that the set of fixed points of $N^A_{\mathbf{d},r}$ has a least and greatest element. By Lazzati (2015), $(\Delta^A_y, \geq)$ is a complete lattice for the usual (coordinatewise) order. By Assumption **HMI**, $f^A_{ir}(\cdot)$, is *h-monotonic* in $E(\mathbf{y_r^{A/i}}(t))$ and $E(\mathbf{y_r^B}(t))$. Thus, given the linearity of the expectation operator , also $N^A_{\mathbf{d},r}$ are *h-monotonic* in the same arguments, and the result follows by the HFP Theorem. By symmetry, the same strategy can be applied for block B.

□

Proof of Lemma 2. If **HMI** holds, then Lemma 1 shows that the solution set has at least two heterogeneous solutions: the greatest, and the least for the two blocks we consider.. Let **HMR** holds. Then, fixing r we have that $N^A_{\mathbf{d},r}$ as defined in (17) is h-monotonic in $\mathbf{d}$, given Assumption **EF**. The fact that the extremal elements of $\phi^k(\mathbf{d}, r)$ are *h-monotonic* in $\mathbf{d}$ follows by Theorem 2.3. Then, condition **ES1** is sufficient to prove our claim. □

Proof of Lemma 3. The first part of the proof will show that if $\mathbf{f}^k_r \geq \mathbf{f}^{k'}_r$, then $\mathbf{y}^k_r(\mathbf{d}) \geq \mathbf{y}^{k'}_r(\mathbf{d})$ for all $\mathbf{d} \in \mathcal{D}$. Assume that **HMI** hold, then Lemma 1 shows that the solutions $\phi^k(\mathbf{d}, r)$ and $\phi^{k'}(\mathbf{d}, r)$ has at least two heterogeneous solutions: one is the greatest and the other is the smallest. Observe that $\mathbf{f}^k_r \geq \mathbf{f}^{k'}_r$ implies that $N^k_{\mathbf{d},r}$ is larger than $N^{k'}_{\mathbf{d},r}$. Then since $\phi^k(\mathbf{d}, r)$ and $\phi^{k'}(\mathbf{d}, r)$ are the sets of heterogeneous fixed points of $N^k_{\mathbf{d},r}$ and $N^{k'}_{\mathbf{d},r}$ this implies that the largest (smallest) element of $\phi^k(\mathbf{d}, r)$ is larger than the largest (smallest) element of $\phi^{k'}(\mathbf{d}, r)$ for each $\mathbf{d} \in \mathcal{D}$ by Theorem 2.3. Thus, $\mathbf{y}^k_r(\mathbf{d}) \geq \mathbf{y}^{k'}_r(\mathbf{d})$ for each $d \in \mathcal{D}$ by **ES2** and **EF**. Using similar argument we can show that if $\mathbf{f}^k_r \geq \mathbf{f}^k_{r'}$, then $\mathbf{y}^k_r(\mathbf{d}) \geq \mathbf{y}^k_{r'}(\mathbf{d})$ for all $\mathbf{d} \in \mathcal{D}$ given **ES2** and **EF**.

For the second part of the proof, we have by definition that

$$E(\mathbf{y}^k|\mathbf{z} = \mathbf{s}) = \sum_{r\in\mathcal{R}}(\mathbf{y}^k_r(\mathbf{s}))p^k_{r|\mathbf{z}=s}.$$

Thus, if $\mathbf{y}^k_r(\mathbf{d}) \geq \mathbf{y}^k_{r'}(\mathbf{d})$ for each $\mathbf{d} \in \mathcal{D}$, then $P(\mathbf{f}^k = \mathbf{f}^k_{r|\mathbf{z}}) = P^k_{r|\mathbf{z}}$ we have that $P(\mathbf{f}^k|\mathbf{z}^k = \mathbf{s}^k, \mathbf{z}^l = \mathbf{s}^{l\prime}) \geq_{st} P(\mathbf{f}^k|\mathbf{z}^k = \mathbf{s}^k, \mathbf{z}^l = \mathbf{s}^l) \geq_{st} P(\mathbf{f}^k|\mathbf{z}^k = \mathbf{s}^k, \mathbf{z}^l = \mathbf{s}^{l\prime})$ and $P(\mathbf{f}^l|\mathbf{z}^k = \mathbf{s}^k, \mathbf{z}^l = \mathbf{s}^{l\prime}) \leq_{st} P(\mathbf{f}^l|\mathbf{z}^k = \mathbf{s}^k, \mathbf{z}^l = \mathbf{s}^l) \leq_{st} P(\mathbf{f}^l|\mathbf{z}^k = \mathbf{s}^k, \mathbf{z}^l = \mathbf{s}^{l\prime})$ if $\mathbf{s}^k > \mathbf{s}^{k\prime}$, and $\mathbf{s}^l > \mathbf{s}^{l\prime}$, by **HMTS** and **EF**. Hence, $E(\mathbf{y}(\mathbf{d})|\mathbf{z} = \mathbf{s})$ is h-monotonic in $\mathbf{s}$ and the result follows.

□

Identification Region for Potential Outcome Distributions

Proof of Proposition 1. See Manski and Pepper (2000) and Manski (2013). □

The proof of Proposition 2 requires an additional Lemma.

Lemma F4. *Assume **HMI**, **HMR**, **EF**, and **ES1** hold. Then if* $\mathbf{z}^A \geq \mathbf{d}^A$, *and* $\mathbf{z}^B \leq \mathbf{d}^B$, *we have (i)* $E(\mathbf{y}^A|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) \geq E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)$; *(ii)* $E(\mathbf{y}^B|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) \leq E(\mathbf{y}^B(\mathbf{d})|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)$.

Proof. (i)

$$E(\mathbf{y}^A|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) = \sum_{\mathbf{s}\in\mathcal{D}} \left\{ \sum_{r\in\mathcal{R}} y_r^A(\mathbf{s}) p^A_{r|\mathbf{z}=\mathbf{s}} \right\} 1(\mathbf{s}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) P^A(\mathbf{z}=\mathbf{s}|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) \geq \tag{17}$$
$$\sum_{\mathbf{s}\in\mathcal{D}} \left\{ \sum_{r\in\mathcal{R}} y_r^A(\mathbf{d}) p^A_{r|\mathbf{z}=\mathbf{s}} \right\} 1(\mathbf{s}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) P^A(\mathbf{z}=\mathbf{s}|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) =$$
$$E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B).$$

Under **HMI**, **HMR**, **EF** and **ES1**, the inequality follows by Lemma 2, as it implies that $E(\mathbf{y}^A(\mathbf{s})) \geq E(\mathbf{y}^A(\mathbf{d}))$ for all $\mathbf{s}^A \geq \mathbf{d}^A, \mathbf{s}^B \leq \mathbf{d}^B$,, and for every $r \in \mathcal{R}$.

(ii)

$$E(\mathbf{y}^B|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) = \sum_{\mathbf{s}\in\mathcal{D}} \left\{ \sum_{r\in\mathcal{R}} y_r^B(\mathbf{s}) p^A_{r|\mathbf{z}=\mathbf{s}} \right\} 1(\mathbf{s}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) P^B(\mathbf{z}=\mathbf{s}|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) \leq \tag{18}$$
$$\sum_{\mathbf{s}\in\mathcal{D}} \left\{ \sum_{r\in\mathcal{R}} y_r^B(\mathbf{d}) p^B_{r|\mathbf{z}=\mathbf{s}} \right\} 1(\mathbf{s}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) P^B(\mathbf{z}=\mathbf{s}|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) =$$
$$E(\mathbf{y}^B(\mathbf{d})|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B).$$

Under **HMI**, **HMR**, **EF** and **ES1**, the inequality follows by Lemma 2, as it implies that $E(\mathbf{y}^B(\mathbf{s})) \leq E(\mathbf{y}^B(\mathbf{d}))$ for all $\mathbf{s}^A \geq \mathbf{d}^A, \mathbf{s}^B \leq \mathbf{d}^B$, and for every $r \in \mathcal{R}$.

□

Proof of Proposition 2. We have to prove that

$$E(\mathbf{y}^A(\mathbf{d})) \leq E(\mathbf{y}^A|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) P(\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) + \overline{y}^A P(\mathbf{z}^A < \mathbf{d}^A \text{ or/and } \mathbf{z}^B > \mathbf{d}^B)$$

$$
\begin{aligned}
& E(\mathbf{y}^A(\mathbf{d})) && (19)\\
= \; & E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)P(\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) \\
+ \; & E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A < \mathbf{d}^A \text{ or/and })P(\mathbf{z}^A < \mathbf{d}^A \text{ or/and } \mathbf{z}^B > \mathbf{d}^B) \\
\leq \; & E(\mathbf{y}^A|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)P(\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) \\
+ \; & E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A < \mathbf{d}^A, \mathbf{z}^B > \mathbf{d}^B)P(\mathbf{z}^A < \mathbf{d}^A \text{ or/and } \mathbf{z}^B > \mathbf{d}^B) \\
\leq \; & E(\mathbf{y}^A|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)P(\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) + \overline{y}^A P(\mathbf{z}^A < \mathbf{d}^A \text{ or/and } \mathbf{z}^B > \mathbf{d}^B).
\end{aligned}
$$

$$
E(\mathbf{y}^B(\mathbf{d})) \geq E(\mathbf{y}^B|\mathbf{z}^A \leq \mathbf{d}^A, \mathbf{z}^B \geq \mathbf{d}^B)P(\mathbf{z}^A \leq \mathbf{d}^A, \mathbf{z}^B \geq \mathbf{d}^B) + \underline{y}^B P(\mathbf{z}^A > \mathbf{d}^A \text{ or/and } \mathbf{z}^B < \mathbf{d}^B)
$$

$$
\begin{aligned}
& E(\mathbf{y}^B(\mathbf{d})) && (20)\\
= \; & E(\mathbf{y}^B(\mathbf{d})|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)P(\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) \\
+ \; & E(\mathbf{y}^B(\mathbf{d})|\mathbf{z}^A < \mathbf{d}^A, \text{ or/and } \mathbf{z}^B > \mathbf{d}^B)P(\mathbf{z}^A < \mathbf{d}^A \text{ or/and } \mathbf{z}^B > \mathbf{d}^B)) \\
\geq \; & E(\mathbf{y}^B|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)P(\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) \\
+ \; & E(\mathbf{y}^B(\mathbf{d})|\mathbf{z}^A < \mathbf{d}^A, \text{ or/and } \mathbf{z}^B > \mathbf{d}^B)P(\mathbf{z}^A < \mathbf{d}^A \text{ or/and } \mathbf{z}^B > \mathbf{d}^B) \\
\geq \; & E(\mathbf{y}^B|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)P(\mathbf{z}^A \geq \mathbf{d}^A \text{ or/and } \mathbf{z}^B \leq \mathbf{d}^B) + \underline{y}^B P(\mathbf{z}^A < \mathbf{d}^A \text{ or/and } \mathbf{z}^B > \mathbf{d}^B).
\end{aligned}
$$

Under **HMI**, **HMR**, **EF** and **ES1**, the first inequality follows by Lemma F4. The second one is true as $\overline{y}^A$ is an upper bound, and $\underline{y}^B$ is a lower bound for any conditional expectation of potential outcomes. The proof for the remaining bounds follows the same argument. To show that the bounds are sharp, first observe that we restrict $E(\mathbf{y}^A(\mathbf{d}))$ to the set of all possible expectations that are consistent with the nature of the outcomes, i.e., $E(\mathbf{y}^A(\mathbf{d})) \in \Delta_{y^A}$. Furthermore, given the data, our assumptions are consistent with both $E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A < \mathbf{d}^A \text{ or/and } \mathbf{z}^B > \mathbf{d}^B) = \underline{y}^A = \overline{y}^A$. Then, $E(\mathbf{y}^A(\mathbf{d}))$ can coincide with any element within Δ_{y^A} that lies between the considered lower and upper bounds. A similar argument applies to $E(\mathbf{y}^B(\mathbf{d}))$.

□

The proof of Proposition 3 requires an additional Lemma.

Lemma F5. *Assume **HMI**, **HMTS**, **EF** and **ES2**, hold. we have (i)* $E(y^A(\mathbf{d})|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) \geq E(y^A|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B)$ *and* $E(y^A|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B) \geq E(y^A(\mathbf{d})|\mathbf{z}^A \leq \mathbf{d}^A, \mathbf{z}^B \geq \mathbf{d}^B)$ *; (ii)* $E(y^B(\mathbf{d})|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) \leq E(y^B|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B)$ *and*

$E(y^B|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B) \leq E(y^B(\mathbf{d})|\mathbf{z}^A \leq \mathbf{d}^A, \mathbf{z}^B \geq \mathbf{d}^B)$.

Proof. (i)

$$\begin{aligned}
& E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) \quad (21)\\
= & \sum_{\mathbf{s}\in\mathcal{D}} E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^B)1(\mathbf{s}^A \geq \mathbf{d}^A, \mathbf{s}^B \leq \mathbf{d}^B)P^A(\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^B|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)\\
\geq & \sum_{\mathbf{s}\in\mathcal{D}} E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B)1(\mathbf{s}^A \geq \mathbf{d}^A, \mathbf{s}^B \leq \mathbf{d}^B)P^A(\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^B|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)\\
= & E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B)\sum_{\mathbf{s}\in\mathcal{D}} 1(\mathbf{s}^A \geq \mathbf{d}^A, \mathbf{s}^B \leq \mathbf{d}^B)P^A(\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^B|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)\\
= & E(\mathbf{y}^A|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B).
\end{aligned}$$

Under **HMI**, **HMTS**, **EF** and **ES2**, the inequality follows by Lemma 3. The third line holds as $E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B)$ is independent of $\mathbf{s}$. The last line holds as the conditioning event is on block A and $\sum_{\mathbf{s}\in\mathcal{D}} 1(\mathbf{s}^A \geq \mathbf{d}^A, \mathbf{s}^B \leq \mathbf{d}^B)P^A(\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^B|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) = 1$. The proof of the second claim is similar; thus, we omit it.

(ii)

$$\begin{aligned}
& E(\mathbf{y}^B(\mathbf{d})|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) \quad (22)\\
= & \sum_{\mathbf{s}\in\mathcal{D}} E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^B)1(\mathbf{s}^A \geq \mathbf{d}^A, \mathbf{s}^B \leq \mathbf{d}^B)P^B(\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^B|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)\\
\leq & \sum_{\mathbf{s}\in\mathcal{D}} E(\mathbf{y}^B(\mathbf{d})|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B)1(\mathbf{s}^A \geq \mathbf{d}^A, \mathbf{s}^B \leq \mathbf{d}^B)P^B(\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^B|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)\\
= & E(\mathbf{y}^A(\mathbf{d})|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B)\sum_{\mathbf{s}\in\mathcal{D}} 1(\mathbf{s}^A \geq \mathbf{d}^A, \mathbf{s}^B \leq \mathbf{d}^B)P^B(\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^B|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B)\\
= & E(\mathbf{y}^B|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B).
\end{aligned}$$

Under **HMI**, **HMTS**, **EF** and **ES2**, the inequality follows by Lemma 3. The third line holds as $E(\mathbf{y}^B(\mathbf{d})|\mathbf{z}^A = \mathbf{d}^A, \mathbf{z}^B = \mathbf{d}^B)$ is independent of $\mathbf{s}$. The last line holds as the conditioning event is on block B and $\sum_{\mathbf{s}\in\mathcal{D}} 1(\mathbf{s}^A \geq \mathbf{d}^A, \mathbf{s}^B \leq \mathbf{d}^B)P^B(\mathbf{z}^A = \mathbf{s}^A, \mathbf{z}^B = \mathbf{s}^B|\mathbf{z}^A \geq \mathbf{d}^A, \mathbf{z}^B \leq \mathbf{d}^B) = 1$. The proof of the second claim is similar; thus, we omit it. □

Proof of Proposition 3. The proof follows using Lemma F5 and the same reasoning we did for Proposition 2. □

Proof of Proposition 4. The proof follows by applying Lemma F4, Lemma F5, and the same reasoning used for Proposition 2 and Proposition 3. □

Within Banks Dependencies

Proof of Proposition 5. Let $Y \equiv \{A, L\}$ with dimension N and $D_1 = \{< y_i, y_j, 1 >\}_{i,j \in N}$ be the $N \times N$ matrix with $d_{ij,1} = 1$ if the direct impact of y_j on y_i is positive and $d_{ij,k} = -1$ if it is negative, according to the signs in Table 2. Under assets-liabilities interactions D_1 can be decomposed into four submatrices: $D_1 = \begin{bmatrix} AA & AL \\ LA & AA \end{bmatrix}$, where AA (LL) is the partition of D_1, which represents the signs of interactions among asset (liabilities) items, and AL ($=LA'$) is the matrix that represents the signs of interactions between assets and liabilities. Let $D_k = \{< y_i, y_j, k >\}_{i,j \in N, k \in \mathbb{N}^+}$ be the $N \times N$ matrix with $d_{ij,k} = 1$ if the impact of y_j on y_i is positive and $d_{ij,k} = -1$ if the impact of y_j on y_i is negative passing through k other variables. Let $R = \{< y_i, y_j >\}_{i,j \in N}$ be the reduced form matrix whose generic element r_{ij} represents the sign of the total impact of y_j on y_i (the total derivative through all the possible chains). Let $r_{ij} = 1$ if the final impact of y_j on y_i is positive and $r_{ij} = -1$ if the impact of j on i is negative. Under assets-liabilities interactions D_1 can be decomposed in four submatrices: $R = \begin{bmatrix} \tilde{AA} & \tilde{AL} \\ \tilde{LA} & \tilde{AA} \end{bmatrix}$.

Let $D_0 = I_N$, D_1 be the $N \times N$ matrix with $d_{ij,1} = 1$ if the direct impact of y_j on y_i is positive and $d_{ij,k} = -1$ if it is negative, according to the signs for M, O and C in Table 2. To prove that A-L outcomes are *consistent*, we can show that $R = D_1 = D_2 = \cdots D_{\mathbb{N}^+}$, which is a sufficient condition for a system of equations to be consistent, see the proof of Theorem 2.1. Under MOC interactions, we have that $D_2 = \begin{bmatrix} AA & AL \\ LA & AA \end{bmatrix}^2 = \begin{bmatrix} AA^2 + AL(AL') & AA(AL) + AL(LL) \\ AL'(AA) + LL(AL') & AL'(AL) + LL^2 \end{bmatrix} = \begin{bmatrix} AA & AL \\ LA & AA \end{bmatrix} = D_1$, which implies $D_k = D_{k-1}D_1$ and $R = D_1 = D_2 = \cdots D_{\mathbb{N}^+}$. This proves that there exists a valid partition for the system of equations, which guarantees the existence of **HFP**.

□

Appendix C: Additional Tables, Figures, and Results for the Empirical Application

Table 5: Banks' Balance Sheet Items Descriptives

Variable	Description
Loans to NFS (c)	"Stock of loans to the non financial sector in milion euro. It includes loans to non financial firms, households, non-profit institutions, public administrations, insurance companies, mutual funds of the euro area and to the rest of the world. All maturities are considered."
Securities (a)	"Stock of securities held in milion euro. It includes equities, bonds and any debt issued by other financial institutions, NFS (non financial firms, non-profit institutions, households, public administrations, insurance companies) of the euro area and to the rest of the world. All maturities are considered. Intragroup exposures are not computed, as banking groups are considered. "
Interbank lending (l)	"Stock of loans to other financial institutions in milion euro. It includes secured and unsecured transactions agreed OTC, through a CCP, third party repos with counterparties of the euro area and to the rest of the world. All maturities are considered. Intragroup exposures are not computed, as banking groups are considered."
Interbank borrowing (b)	"Stock of loans from other financial institutions in milion euro. It includes secured and unsecured transactions agreed OTC, through a CCP, third party repos with counterparties of the euro area and to the rest of the world. All maturities are considered. Intragroup exposures are not computed, as banking groups are considered."
Bonds issued (s)	"Stock of bonds issued by the financial institution in milion euro. All maturities and seniorities are considered. "
Deposits (d)	"Stock of deposits at the financial institution in milion euro. Deposits included and not included in M3 are considered. Sight and term deposits of NFC (non financial firms, non-profit institutions, households, public administrations, insurance companies, mutual and pension funds) of the euro area and the rest of the world are considered."
Central bank funding (t)	"Stock of funds borrowed from the central bank in milion euro. All maturities and operation types (including OMO, LTRO, TLTRO, PELTRO) are considered."

	N	Mean	Std	Median	10^{th} pct	90^{th} pct
Time: 31dec2011						
Loans to NFS (c)	623	2,992	19,583	270	41	2,595
Securities (a)	623	1,226	9,750	51	1	497
Interbank lending (l)	623	752	8,047	19	4	151
Interbank borrowing (b)	623	1,048	9,202	26	1	533
Deposits (d)	623	2,014	13,067	170	31	1,624
Bonds and equity issued (s)	623	1,209	9,421	62	-	614
Central bank funding (t)	623	331	2,508	-	-	95
Time: 31dec2012						
Loans to NFS (c)	602	3,062	19,931	287	38	2,398
Securities (a)	602	1,297	8,521	81	0	799
Interbank lending (l)	602	773	7,195	25	4	163
Interbank borrowing (b)	602	1,042	8,405	42	1	495
Deposits (d)	602	2,160	13,019	197	31	1,695
Bonds and equity issued (s)	602	1,091	8,643	59	-	553
Central bank funding (t)	602	408	2,456	-	-	259
Time: 31dec2013						
Loans to NFS (c)	593	2,939	18,614	286	36	2,450
Securities (a)	593	1,370	8,606	113	0	945

Interbank lending (l)	593	771	7,075	24	4	185
Interbank borrowing (b)	593	1,067	8,504	50	1	674
Deposits (d)	593	2,166	12,629	219	33	2,045
Bonds and equity issued (s)	593	1,003	7,992	49	-	491
Central bank funding (t)	593	379	2,074	-	-	342
Time: 31dec2014						
Loans to NFS (c)	579	2,956	18,577	283	38	2,310
Securities (a)	579	1,382	8,750	129	0	1,078
Interbank lending (l)	579	399	2,680	29	5	196
Interbank borrowing (b)	579	683	4,061	46	1	505
Deposits (d)	579	2,250	12,875	240	41	1,986
Bonds and equity issued (s)	579	916	7,275	46	-	468
Central bank funding (t)	579	335	1,855	-	-	397
Time: 31dec2015						
Loans to NFS (c)	564	3,025	18,719	295	41	2,273
Securities (a)	564	1,393	8,832	130	1	1,079
Interbank lending (l)	564	395	2,883	27	5	184
Interbank borrowing (b)	564	724	4,160	40	0	621
Deposits (d)	564	2,402	13,833	268	46	2,361
Bonds and equity issued (s)	564	831	6,637	34	-	377
Central bank funding (t)	564	273	1,617	-	-	294
Time: 31dec2016						
Loans to NFS (c)	529	3,194	19,447	292	43	2,387
Securities (a)	529	1,427	9,170	141	0	1,265
Interbank lending (l)	529	410	2,875	27	4	196
Interbank borrowing (b)	529	720	4,237	44	0	576
Deposits (d)	529	2,682	15,557	290	49	2,552
Bonds and equity issued (s)	529	741	5,982	22	-	318
Central bank funding (t)	529	382	2,489	-	-	302
Time: 31dec2017						
Loans to NFS (c)	474	3,488	21,799	347	51	2,580
Securities (a)	474	1,493	9,864	148	-	1,234
Interbank lending (l)	474	469	3,335	30	5	201
Interbank borrowing (b)	474	714	4,138	48	0	574
Deposits (d)	474	3,076	18,681	357	54	2,694
Bonds and equity issued (s)	474	683	5,524	12	-	271
Central bank funding (t)	474	526	3,645	-	-	370
Time: 31dec2018						
Loans to NFS (c)	450	3,574	21,537	379	53	2,745
Securities (a)	450	1,642	10,356	182	-	1,359
Interbank lending (l)	450	533	3,782	26	4	255
Interbank borrowing (b)	450	855	4,704	52	1	713
Deposits (d)	450	3,317	19,126	404	68	3,208
Bonds and equity issued (s)	450	622	5,181	5	-	219
Central bank funding (t)	450	537	3,649	-	-	372
Time: 31dec2019						
Loans to NFS (c)	437	3,553	20,846	394	59	2,842
Securities (a)	437	1,780	10,931	193	-	1,558
Interbank lending (l)	437	558	4,446	8	0	274
Interbank borrowing (b)	437	814	4,556	33	-11	930
Deposits (d)	437	3,575	20,167	459	77	3,655
Bonds and equity issued (s)	437	643	5,238	3	-	172
Central bank funding (t)	437	499	3,229	-	-	352
Time: 31dec2020						
Loans to NFS (c)	422	3,771	25,118	400	62	3,082
Securities (a)	422	1,924	11,884	240	-	2,020
Interbank lending (l)	422	505	3,378	8	0	356
Interbank borrowing (b)	422	699	3,808	53	-9	673
Deposits (d)	422	4,084	26,240	527	87	3,718
Bonds and equity issued (s)	422	584	5,532	1	-	149
Central bank funding (t)	422	843	5,345	-	-	687
Time: 31dec2021						
Loans to NFS (c)	404	3,965	24,328	436	73	3,451

Securities (a)	404	2,077	12,662	278	-	1,878
Interbank lending (l)	404	487	3,260	7	0	317
Interbank borrowing (b)	404	664	3,377	60	-14	643
Deposits (d)	404	4,511	27,471	572	92	4,057
Bonds and equity issued (s)	404	593	5,557	-	-	134
Central bank funding (t)	404	1,113	7,625	-	-	777
Time: 31dec2022						
Loans to NFS (c)	385	4,167	23,871	480	83	3,985
Securities (a)	385	2,124	12,838	290	-	2,105
Interbank lending (l)	385	532	3,382	5	0	443
Interbank borrowing (b)	385	791	3,627	85	-	1,006
Deposits (d)	385	4,713	27,871	616	92	4,179
Bonds and equity issued (s)	385	623	5,735	0	-	130
Central bank funding (t)	385	915	6,091	-	-	499

Notes. End of year monthly data in million euro. Individual Balance sheet items (IBSI) data for the Italian banking system. Individual banks are aggregated in banking groups. Data is aggregated at the banking group level according to the group strucuture at each time observation. Descriptive statistics are computed across banking groups at the end of the year. Any zero values in the archives may refer either to a phenomenon that does not exists or to periods for which the data do not reach the significant figure of the minimum amount considered in the statistical reporting. The institutional sectors correspond to those of the European System of Accounts (ESA2010). Monetary financial institutions include: the Bank of Italy, banks, money market funds, electronic money institutions and Cassa Depositi e Prestiti spa. The latter is not included in the sample. Public administrations include central government and "other general government institutions", which in turn can be distinguished into local government and social security agencies. "Other residents" include insurance and pension funds, other financial institutions, including nonmonetary mutual funds monetary, non-financial corporations, households, and nonprofit institutions serving households. Stocks do not correct for differences in stocks to account for reclassifications, value adjustments, and any other changes (except those due to exchange rates) that do not originate from economic transactions. Statistical reclassifications are due, for example, to changes in the reporting population or re-attributions of balance sheet items; value adjustments are, for example, loan write-downs or changes in the price of securities. In addition, adjusted flows neutralize the increases in stocks attributable to any merger or incorporation transactions. In the case of loans, decreases due to disposals (net of acquisitions) are also adjusted. For further details on the content of the items, see the notes for Section 1 within the issue of the Statistics, Methods and Sources series of https://www.bancaditalia.it/pubblicazioni/metodi-e-fonti-note/metodi-note-2020/index.html?com.dotmarketing.htmlpage.language=1Banks and Money: National Data. In particular, within section 1.4 are highlighted all the statistical discontinuities that over time have been reflected in the consistencies.

Extensions of Monotonic Assumptions among Banks' Balance Sheet Items

In addition to the between-sides-complementarity interactions (C), which generate monotone positive effects among same bank's outcomes in opposed sides of the balance sheet, there could also be interactions among same bank's outcomes in the same side of the balance sheet. For example, there could be substitutability within the same balance sheet side, assets or liabilities. On the liabilities side, the bank decides its funding structure, given its desired balance sheet size. On the assets side, a bank decides how to allocate its resources, given the budget constraint. For example, an additional euro of loans (c_i) can be substituted by selling securities (a_i) or other assets, in absence of a balance sheet expansion. These within-side-substitution interactions (S) can generate monotone negative effects among same bank's outcomes. Furthermore, if we are studying a period in which unsecured bilateral interbank trading is relevant, like before the global financial crisis, we can assume that bank i can lend (l_i) directly to bank j (b_j). We call these opposite-side-of-the-market interactions (O). O interactions can generate positive monotone effects, because banks' jointly increase their outcomes. In addition, bank i, by lending to customers (c_i) that borrow to pay bank j's clients, can affect positively the retail funding of bank j. We define the system of previously described interactions MC plus these ones as $MOCS$. Table 6 reports the upper triangular matrix that summarizes the signs of the $MOCS$ interactions among banks' outcomes, considering a banking system with two banks, i and j.

Table 6: Monotonic Assumptions - MOCS -

		A						L					
		c_i	c_j	l_i	l_j	a_i	a_j	d_i	d_j	b_i	b_j	s_i	s_j
A	c_i		$-$ [M]	$-$ [S]		$-$ [S]		+ [C]	+ [C]	+ [C]		+ [C]	
	c_j				$-$ [S]		$-$ [S]	+ [C]	+ [C]		+ [C]		+ [C]
	l_i				$-$ [M]	$-$ [S]		+ [C]		+ [C]	+ [O]	+ [C]	
	l_j						$-$ [S]		+ [C]	+ [O]	+ [C]		+ [C]
	a_i						$-$ [M]	+ [C]		+ [C]		+ [C]	+ [O]
	a_j								+ [C]		+ [C]	+ [O]	+ [C]
L	d_i								$-$ [M]	$-$ [S]		$-$ [S]	
	d_j										$-$ [S]		$-$ [S]
	b_i										$-$ [M]	$-$ [S]	
	b_j												$-$ [S]
	s_i												$-$ [M]
	s_j												

Notes. A and L stand respectively for assets and liabilities. i and j are two banks. l_i and b_i are respectively the amount of lending and borrowing in the interbank market. c_i is the amount of credit provided by bank i. d_i is the amount of deposits held at bank i. a_i is the amount of securities held by bank i. s_i is the amount of bonds issued by bank i. + and $-$ indicate respectively positive and negative interactions. [M]: same side of the market. [O]: opposite side of the market. [S]: asset liability substitution. [C]: balance sheet expansion complementarity.

Let $D^{MOCS} = \begin{bmatrix} AA & AL \\ LA & LL \end{bmatrix}$, where AA (LL) is the partition of D_1 which represents the signs of interactions among asset (liability) items and AL ($=LA'$) is the matrix that represents the signs of interactions between assets and liabilities under $MOCS$. Let $D_K^{MOCS} = (D^{MOCS})^K$ $R^{MOCS} = \sum_{K=1}^{\infty}(D^{MOCS})^K$

Proposition 6. *Under $MOCS$ interactions, the **HMI** bounds provided by Proposition 1, the **HMI**, **HMTR**, and **ES** bounds provided by Proposition 2, the **HMI**, **HMTS**, and **ES** bounds provided by Proposition 3 and the **HMI**, **HMTR**, **HMTS**, and **ES** bounds provided by Proposition 4, derive from structural dependencies among banks' A-L outcomes iff $R^{MOCS} = D^{MOCS}$.*

Proof of Proposition 6. The proof follows from the proof of Proposition 5.

□

The condition $R^{MOCS} = D^{MOCS}$, basically constraints the indirect effects to not offset the first order effect of one outcome variable on the other, and provides immediately a valid partition. It is required as $MOCS$ interactions do not guarantee per se a valid partition, since O and S interactions may generate indirect effects with opposite sign w.r.t. the direct effects. While it could look a strong assumption, for A-L interdependencies it is quite plausible. Below we provide an example.

Example. Let us focus on the effect of an increase of one unit of d_i on c_i, i.e. the effect of one additional unit of deposits on credit granted to the NFS. According to MC and $MOCS$ interactions, the first order effect is positive. Under $MOCS$ there is also a substitution mechanism at play: d_i may have a negative effect for example on b_i, the interbank borrowing of bank i, which in turn could generate a negative effect on c_i. While bank i could increase its revenues more just expanding the asset side (this is why we presented MC interactions as our primary set of assumptions), it could also prefer to maintain the size of the balance sheet stable and substitute a unit of b_i with a unit of d_i. In this more complex case, there are also a series of indirect effects with potentially contrasting signs, as the substitution is among any item within both sides of the balance sheet. The condition $R^{MOCS} = D^{MOCS}$ states that the final effect of an increase in d_i on c_i is monotone positive. While in other environments it could be a strong assumption, for banks A-L interactions it is quite reasonable. In this example, consider three scenarios in which an expansion of a liability induce equally positive direct effects on the asset items (one third, in our stylized balance sheet). We abstract for the sake of simplicity from higher order effects, that tend to zero and do not change qualitatively the illustrative point of this example. In the first scenario, the increase of d_i is a pure balance sheet expansion, thus there is no reduction in other liabilities. In this case, dependencies are fully consistent and c_i, a_i and l_i increase by a third. In the second, the increase of d_i reduces b_i by one half. In this case, it only partially reduces others liabilities, thus still being a balance sheet expansion, ceteris paribus. In this case, dependencies are not fully consistent anymore but c_i, a_i and l_i increase by a sixth, with a final reduced form effect that is positive. In the third scenario, the increase of d_i is fully offset by a reduction of b_i, there is no balance sheet expansion and thus assets can not be expanded as well. In all these scenarios there is no decrease of c_i or other assets items.

Whether or not the Democrats are the incumbent party in a Congressional district is a deterministic function of their vote share in the prior election. Assuming that there are two parties, consider the following model of Congressional elections:

$v_{i2} = \alpha w_{i1} + \beta v_{i1} + \gamma d_{i2} + e_{i2}$

$d_{i2} = 1(v_{i1} \geq 1/2)$

v_{it} is the vote share for the Democratic candidate in Congressional district i in election year t. d is the indicator variable for whether the Democrats are the incumbent party during the electoral race in year 2. It is a deterministic function of whether the Democrats won election 1. w_{i1} refers to characteristics determined by agents' choices as of election day in year 1.

The first line in (3) is a standard regression model describing the causal impacts of w_{i1}, which is a vector of variables that reflect all relevant characteristics. These could represent the partisan make-up of the district, party resources, or the quality of potential nominees. v_{i1} is also permitted to impact v_{i2}. For example, a higher vote share may attract more campaign donors, which in turn, could boost the vote share in election year 2.

The potentially discontinuous jump in how v_{i1} impacts v_{i2} is driven by the relationship between w_{i1}, v_{i1}, and d_{i2}.

Possible model with HMI:
$v_{i2} = \alpha w_{i1} + \alpha_R w_{i1}^R + \alpha w_{-i1} + \alpha_R w_{-i1}^R + \beta v_{i1} + \gamma d_{i2} + \phi \frac{1}{n_j} \sum_j d_{j,2} + \phi_R \frac{1}{n_j}^R \sum_j d_{j,2}^R + e_{i2}$
$d_{i2} = 1(v_{i1} \geq 1/2)$

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