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SUBSIDIZING BUSINESS ENTRY IN COMPETITIVE CREDIT MARKETS

by Vincenzo Cuciniello*, Claudio Michelacci§ and Luigi Paciello§

Abstract

Business creation subsidies are a means for reducing firm debt and bankruptcy risk. Do they work? To answer the question, we consider a general equilibrium model where firms are financially constrained at entry and borrow in a competitive market by issuing long-term debt. A subsidy stimulates entry and market competition, which increases the bankruptcy rate of incumbent firms. If the subsidy is paid out ex ante to finance start-up expenditures, the subsidy reduces the debt and the bankruptcy rate of start-ups; if paid out ex post as a refund for start-up expenditures, the subsidy crowds out the equity rather than the debt of start-ups and their bankruptcy rate also increases. The model is calibrated to match North-South differences across Italian provinces. The optimal subsidy in the South is paid out entirely ex ante and yields an increase in welfare equivalent to almost one percent of consumption. When the same subsidy is paid out ex post as a proportion of 60 per cent, it results in a welfare loss of a similar amount. We discuss the implications for the 'I Stay in the South' policy recently introduced in Italy.

JEL Classification: E44, E62, G32, G33.
Keywords: firm dynamics, overborrowing, ratchet effect.
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Contents

* Bank of Italy, Directorate General for Economics, Statistics and Research.
§ EIEF and CEPR.

1 Introduction[1]

Business creation subsidies are often proposed as a means to stimulate business entry, make start-ups less indebted and reduce the risk that they might inefficiently go bankrupt, see for example OECD (1997, 2022). We show that under perfect competition in credit markets—possibly due to the abundant credit of the last two decades—the effects of the subsidy on the debt of start-ups and aggregate bankruptcies depend on how the subsidy affects the trade-off between debt and equity financing, with important welfare consequences.

We consider a model where firms borrow in a perfectly competitive market using long-term non-state contingent debt, as in Aguiar and Amador (2020) and DeMarzo and He (2021). At entry, firms are financially constrained and need to finance part of the start-up investment through debt. After entry, they are subject to idiosyncratic risk. Greater debt increases the risk of bankruptcy and bankruptcy is inefficient: it occurs even if the present value of firm cash-flows is positive, due to the limited liability of debt. Firms cannot commit to future funding choices and have incentives to dilute the value of past debt which yields a *leverage ratchet effect* (Admati, Demarzo, Hellwig, and Pfleiderer 2018): the firm never buys back or cancels any of its debt before it reaches maturity because the cost of the debt reduction would be borne by equity holders while the increase in firm value (due to the lower bankruptcy risk) is appropriated by existing creditors. Conditional on survival, the firm leverage ratio (debt over value added) gradually converges toward a reference target.

A business creation subsidy promotes business entry, stimulates market competition, and reduces firm profitability, which increases the bankruptcy rate of incumbent firms. The design of the subsidy also affects the capital structure of start-ups, through a novel effect of government *reimbursements* on the firm debt-equity trade-off. Since the firm is initially financially constrained, the firm uses all available liquidity to finance the start-up investment. Then, if the subsidy is paid out ex-ante, it is used to reduce firm leverage, which makes start-ups less likely to go bankrupt. If the subsidy is paid-out ex-post as a refund of start-up expenditures, the firm still needs to rely on debt to finance the initial start-up investment. Since the firm never cancels debt before it reaches maturity due to the leverage ratchet effect, the ex-post payment of the subsidy increases dividend pay-outs and crowds out firm equity rather than debt. As a consequence, the debt of start-ups remains unchanged amid decreased profitability, which increases their leverage ratio, resulting in a higher bankruptcy rate of start-ups.

To study the quantitative relevance of the mechanism we incorporate our model of firm dynamics into a fully integrated economy with a large number of provinces. In each province there is an endogenous number of firms with market power that hire workers in a competitive local labor market. Firm goods are freely tradable and there are aggregate demand complementarities à la Dixit-Stiglitz. Workers can slowly migrate across provinces and thereby, in the long run, the utility of living in a province is equalized as in Rosen (1979) and Roback (1982). Living costs in a province are increasing in the local labor force, a congestion externality that restricts the number of workers in the province. Provinces differ in firm productivity, business idiosyncratic risk, start-up costs, initial financial conditions, and debt guarantees. The risk adjusted cost

[1]We thank Manuel Amador, Paola Di Casola, Alessandro Ferrari, Francesco Lippi, Francesca Lotti, and Enrico Sette for useful comments, as well as seminar participants at Bank of Italy, EIEF, Goethe University, University of Michigan, ESCB Research Cluster on Monetary Economics, the Mannheim Workshop on Firm Heterogeneity and Macroeconomics, ESSIM 2023, SED in Cartagena, and BSE Summer Forum 2023. The views expressed in the paper are those of the authors and do not necessarily reflect those of the Bank of Italy.

of credit is constant and the transitional dynamics of the model remains tractable because the (properly standardized) firm optimal policy function is invariant to changes in the cross-sectional distribution of firms within provinces and the allocation of workers across provinces.

We study the optimal subsidy to business creation in the least productive provinces of the economy, financed through non distortionary taxes. Welfare is the present discounted value of aggregate consumption net of living costs, starting from a steady state without subsidies. The subsidy affects welfare through the leverage ratio of start-ups and incumbent firms, which determines the number of inefficient bankruptcies, and through the entry rate, which influences the number of firms and their productivity. Since lowering the debt of start-ups reduces the risk of inefficient bankruptcies, an optimally positive subsidy is entirely paid out ex-ante. For a given firm leverage, entry could be inefficient because of (i) aggregate demand complementarities, (ii) spatial misallocation of labor, or (iii) firm overborrowing.[2] (i) and (ii) cause too low entry and call for setting a positive subsidy.[3] (iii) causes excessive entry and calls for a tax on business creation. In the short run, a subsidy also increases the leverage ratio and thereby the bankruptcy rate of incumbent firms: a bust in business creation makes the local labor market tighter which reduces firm profitability. The optimal subsidy efficiently trades off the welfare costs of this firms' shake-out and of the excessive entry due to overborrowing with the welfare gains from reducing start-ups' leverage and correcting the insufficient entry due to (i) and (ii).

We use data from the 2000s and calibrate the model to match differences in firm dynamics between the North and South of Italy, a country with sizable regional disparities in productivity, firm leverage ratios, business creation and exit rates. The risk-adjusted cost of credit is similar across provinces, consistent with a perfectly integrated Italian credit market, which reflects the widespread expansion of large banks throughout the entire national territory since the early 00's (Accetturo et al. 2022). Business exit rates and leverage ratios are both higher in the South than in the North. These differences materialize only after firms mature (roughly after the first 8 years of life of the business), consistent with the possibility that Southern firms have a stronger appetite for credit, which increases their bankruptcy rate. The age profiles of firm employment size and labor productivity are relatively similar in the North and the South, with a North-South productivity gap of roughly 30 percent, stable over the firm life cycle. We also show that (i) the standard deviation of idiosyncratic shocks is twenty percent higher in the South than in the North and (ii) mature firms in the South are more likely to exit due to their excessive debt. We construct a novel measure of idiosyncratic shocks by examining a representative panel of mature firms (INVIND) and use projection methods to evaluate the elasticity of business exit to idiosyncratic shocks, separately for Northern and Southern firms. The elasticity is significantly larger in the South than in the North, largely due to differences in firm leverage.

The calibrated model matches well North-South differences in productivity, leverage and bankruptcy rates and explains why Southern firms demand more debt and become more likely to go bankrupt as they age. Quantitatively, differences in business risk accounts for two thirds of the North-South variation in business exit rates. Starting from the calibrated steady-state

[2]The misallocation of labor is due partly to congestion externalities and partly to the overinvestment caused by overborrowing, whose marginal costs are not equalized across provinces. In practice we find that this latter effect is quantitatively unimportant.

[3]In our model, as in Rivera-Batiz and Romer (1991) (see also Chapter 6 in Barro and Sala-i-Martin (2003)), the start-up investment is in output units and labor is used as an input in the production of varieties. Under these assumptions the Dixit-Stiglitz aggregate demand externality causes an inefficiently low business creation rate. This differs from the original formulation in Dixit and Stiglitz (1977) where labor is the only input in both production of varieties and business creation: in the latter case the business creation rate is efficient.

equilibrium, we calculate the business creation subsidy in the South that maximizes the welfare of Italians. The optimal subsidy is entirely financed ex-ante and is roughly equal to 105,000€, which represents about one quarter of the average investment of a start-up in the South. At the optimal subsidy, per capita aggregate welfare, measured as a consumption equivalent, increases by almost 1 percent. If all provinces contribute to the financing of the subsidy in proportion to their per-capita consumption, both Northern and Southern provinces gain. The design of the subsidy matters for the results. If more than 40 percent of the optimal subsidy is paid-out ex-post, aggregate welfare falls rather than increases, with potentially sizeable losses, equivalent to a 2 percent fall in per-capita consumption when the subsidy is entirely paid-out ex-post.

In 2017, the Italian government introduced a business creation subsidy for start-ups in the South, dubbed "I Stay in the South" (hereafter ISS). In the subsequent years, the ISS subsidy has become more generous and is now close to the optimal size of our calibrated economy. Around 50 percent of ISS is paid ex-post. Taking into account that roughly 20 percent of start-ups in the South are subsidized, we find that, in our model, ISS yields small welfare gains equivalent to an increase in consumption of just 5 basis points. The gains would be 4 times larger if the ISS subsidy were paid entirely ex-ante. Under the current policy, the ISS subsidy increases the average leverage ratio of incumbent firms and unsubsidized start-ups as well as the aggregate bankruptcy rate. Southern start-ups receiving the ISS subsidy benefit in terms of lower leverage ratios and bankruptcy probabilities. Some difference-in-differences evidence is consistent with the claim that provinces exposed to the ISS policy have experienced an increase in leverage ratios and bankruptcy rates.

Our model is a general equilibrium version of DeMarzo and He (2021), which builds on the leverage ratchet effect discussed by Admati et al. (2018). Crouzet and Tourre (2021) use the model to study credit market interventions during the 2020 recession. DeMarzo, He, and Tourre (2023) focus on sovereign default. Perla, Pflueger, and Szkup (2022) study the effects of equity payout restrictions. We emphasize the general equilibrium properties of the model under free entry of firms, studying welfare and the optimal design of business creation subsidies.

Regional disparities are widespread (Austin, Glaeser, and Summers 2018 and Lagakos 2020). Several papers (Fajgelbaum and Gaubert 2020, Bilal 2023, Lagakos, Mobarak, and Waugh 2023 and Ferrari and Ossa 2023) have considered spatial models to analyze the welfare properties of place-based policies. These papers typically abstract from firm financial conditions and neglect geographical variation in firm leverage and bankruptcy. We underscore the implications of firm finance for the optimal design of entry subsidies, a commonly used place based policy. We show that cheap credit could lead to excessive entry, leverage and bankruptcy and show that a model with debt dilution fits well the substantial regional disparities in leverage and bankruptcy observed in Italy, which build up as firms age.

Since Cooley and Quadrini (2001) an extensive body of literature has underscored the importance of firm age in identifying the effects of firms' financial constraints. Here, similarly to Kochen (2022), we rely on the age profile of firms to identify whether regional differences in leverage and bankruptcy stem from the supply or demand of firm credit.

Itskhoki and Moll (2019) study the Ramsey dynamic optimal policy of an emerging economy where pro-business interventions can help in relaxing firms' financial constraints. We do not study a full Ramsey problem: we focus just on a once-and-for-all business creation subsidy targeted to the least productive areas of an economy, highlighting the implications of the subsidy for the debt-equity trade-off. We note that, even if entrepreneurs are initially financially constrained, business entry could be excessive when credit is cheap as in recent decades.

There is an extensive literature on the effects of wealth transfers to entrepreneurs. Evans and Jovanovic (1989) first provided evidence that greater entrepreneur wealth stimulates business creation, a claim challenged by Hurst and Lusardi (2004) but later confirmed by Schmalz, Sraer, and Thesmar (2017); see Quadrini (2009) for a literature review and Buera (2009) for further analysis of the relation between wealth and business entry. Holtz-Eakin, Joulfaian, and Rosen (1994), Schmalz et al. (2017), and Cingano et al. (2022) also document the effects of entrepreneur wealth on firm survival. We are the first to note that, under a leverage ratchet effect, the welfare effects of transfers also depend on their timing.

Section 2 describes North-South differences in Italy and uses difference-in-differences methods to study the ISS policy. Section 3 introduces the model of firm dynamics. Section 4 analyzes spatial equilibrium. Section 5 discusses calibration and the fit of the model. Section 6 studies the optimal subsidy in the South. Section 7 analyzes the ISS policy. Section 8 concludes.

2 Preliminary evidence on North-South differences

We describe data sources, define empirical variables and document differences across Italian provinces. We show that (i) on average, Southern firms are more indebted and more likely to go bankrupt, (ii) North-South differences in leverage and bankruptcy build up as firms age, and (iii) there is evidence that mature Southern firms fail more because of their excessive debt. We conclude by running difference-in-differences regressions for the effects of the ISS subsidy.

2.1 Data and definitions

We focus on the universe of limited liability companies. Firm total financial debt (sum of bank debt plus other financial debt), and value added are from the Company Accounts Data Service (CADS); firm employment is from "Universo Imprese INPS" (UNINPS); information on bad debts, debt guarantees and bank debt are from the Central Credit Register; firm interest rates are from TAXIA; information on bankruptcy procedures is from the Business Register by the Chambers of Commerce. We match firms in CADS with UNINPS, the Credit Register, the Business Register, and TAXIA using the fiscal code of the company. Aggregate data (GDP, working age population, total employment, and CPI inflation) are from the Italian Statistical Institute (ISTAT). Appendix A further describes the data.

We exploit the long time coverage of UNINPS, available since 1990, to accurately identify business age, entry and exit. A firm is *new* if it employs workers for the first time. A firm *exits* if its employment drops forever to zero—which requires information for some years after exit. Firm age is the number of years since the firm has first employed some workers, calculated for all firms born after 1990. A firm exits with *bankruptcy* if the firm exits leaving behind some bad loans or after a formal bankruptcy procedure. We track firms over time to characterize their life cycle. Time averages (unless otherwise specified) are calculated over the years 2007-2015. We pool in the same age group (labelled age 20) all firms with more than 16 years of age.[4]

The exit rate in province i in a year t, f_{it}, is the fraction of firms in the province at the beginning of year t that exit by the end of year t. The business creation rate $\tilde{m}_{it}$ is the number of start-ups in year t divided by total employment in the province. The risk-adjusted real cost

[4]2007 is the first year when we observe firms with at least 17 years of age: those already employing workers in 1990 and still active in 2007. We stop in 2015 to leave 5 additional years of data to identify exit.

of credit r_{ic} in a province is the difference between the average real interest rate on term loans and the average bankruptcy rate of businesses in the province multiplied by the fraction of unguaranteed debt.[5] The leverage ratio is equal to the firm's total financial debt (bank debt plus other financial debt excluding trade payables) over value added. We check robustness using only (total) bank debt to calculate firm leverage.

To measure idiosyncratic shocks we use INVIND: a representative sample of relatively mature firms over the years 1995-2019 (Bank of Italy 2014).[6] INVIND has information on: (i) expected and realized changes in prices and sales; and (ii) the elasticity of demand expected by firms. We use (i)-(ii) to identify (non-parametrically) unexpected shifts to firm demand by assuming—as in the model below—, that a firm faces the following iso-elastic log-linear demand for its goods:

$$\ln q_{jt} = \ln Y_{ist} + z_{jt} - \nu_j \ln p_{jt} \tag{1}$$

where q_{jt} is firm j's output, Y_{ist} is an aggregate demand shifter for province i in year t possibly varying according to the sector s where the firm operates, p_{jt} is the price set by firm j, z_{jt} is an idiosyncratic shifter to firm demand and $\nu_j > 1$ is the price elasticity of firm j's demand. Firm j's revenue is equal to $r_{jt} \equiv p_{jt}q_{jt}$. For each firm present in two consecutive waves of INVIND we calculate the following Wold innovations (expectation errors) for revenue ϵ^r_{jt}, and price ϵ^p_{jt}:

$$\epsilon^r_{jt} = \frac{r_{jt} - E_{jt-1}(r_{jt})}{r_{jt-1}} \qquad \text{and} \qquad \epsilon^p_{jt} = \frac{p_{jt} - E_{jt-1}(p_{jt})}{p_{jt-1}}.$$

Given (1), and the approximation $(x_{jt} - x_{jt-1})/x_{jt-1} \simeq \ln x_{jt} - \ln x_{jt-1}$, the Wold innovation on the demand shifter of firm j, ϵ^z_t, can be expressed as equal to

$$\epsilon^z_t \equiv \frac{z_{jt} - E_{jt-1}(z_{jt})}{z_{jt-1}} = \epsilon^r_{jt} + (\nu_j - 1)\,\epsilon^p_{jt} - \epsilon^A_{ist}, \tag{2}$$

where $\epsilon^A_{ist} = \ln Y_{ist} - E_{t-1}(\ln Y_{ist})$ is a shock common to all firms in the same province and sector.

We calculate the idiosyncratic shock ϵ^z_{jt} as the residual of the following regression

$$\epsilon^r_{jt} + (\nu_j - 1)\,\epsilon^p_{jt} = d_{st} + d_{it} + \epsilon^z_{jt}, \tag{3}$$

where d_{st}, and d_{it} are a full set of sector-time and province-time dummies, which control for the aggregate shock ϵ^A_{ist}. The elasticity of firm demand ν_j needed to evaluate the left-hand side of (3) is recovered using a unique feature of INVIND. Both in 1996 and in 2007, firm managers in INVIND were asked about the value of $(1 - \nu_j) \times 0.1$ through the following question: "*If your firm were to increase the selling prices by 10%, what percentage change in your nominal sales would be obtained, provided that all your competitors were to keep their prices unchanged and you were to leave all the other terms unchanged?*". We take the average self-reported sector-specific elasticity ν_j as an estimate of the demand elasticity faced by firms in the sector.[7] We

[5]Formally $r_{ic} = r_i - f^m_i \times (1 - \varphi_i)$ where r_i is the average interest rate on term loans of duration greater than 5 years over the period 2009-2015 minus realized inflation over the next 5 years, f^m_i is the bankruptcy rate of firms older than 10 years of age in the province, φ_i are the contractual guarantees on long term loans multiplied by 60 percent, which is the average debt recovery rate for guaranteed debt (Fischetto et al. 2018).

[6]We match firms in INVIND with CADS and UNINPS which allows us (i) to measure firm employment size, leverage ratio and return on assets and (ii) to follow firms even after they are no longer present in INVIND.

[7]As discussed in Pozzi and Schivardi (2016) the implied reported elasticities ν_j's range between 1.2 and 5.5 and are in the order of magnitude estimated by the literature.

check that the residuals ϵ^z_{jt}'s in (3) are serially uncorrelated over time, which is a key property of expectation errors.[8] For each province and year we calculate the province level standard deviation of ϵ^z_{jt} using the sample weights provided by INVIND. The standard deviation of idiosyncratic risk in a province σ_i is the resulting time average.

2.2 Descriptive evidence

Figure 1 shows the variation in GDP per capita across the 102 provinces in our sample. GDP per capita is in logs demeaned by the cross-sectional average. Milan in Lombardy has the highest GDP per capita (70 percent above the national average). Agrigento in Sicily and Cosenza in Calabria have the lowest (50 percent below average). Generally, the lowest GDP per capita provinces are concentrated in the South: the correlation between a province's latitude and its GDP per capita is around 85%. Thereafter, we use the GDP per capita of a province to measure how further in the (economic) North the province is located. For expositional simplicity, the North-South difference corresponds to provinces that differ by 0.6 in terms of logged GDP per capita, roughly equal to twice its cross-sectional standard deviation.

Figure 1: Geographical variation in GDP per capita

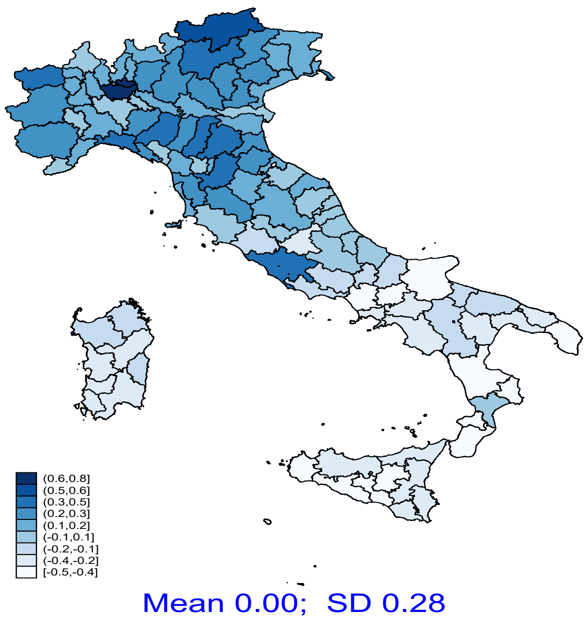


Table 1 reports the cross-sectional average and standard deviation of productivity (logged GDP over total employment), the business exit rate f_i, the business creation rate $\tilde{m}_i$, the firm leverage ratio, the risk-adjusted real cost of credit r_{ic}, and the standard deviation of idiosyncratic shocks σ_i. The third row of Table 1 also reports the correlation with GDP per capita; see Appendix B for the corresponding scatter plots. GDP per capita is strongly positively correlated with labour productivity (correlation of 0.8) and negatively correlated with both the business creation rate and the business exit rate. The average exit rate in a province f_i is 1.5 percentage points higher in the South than in the North. GDP per capita is mildly negatively correlated with the firm leverage ratio (correlation of minus 0.25). The risk adjusted real interest rate on

[8] Our structural shock to firm idiosyncratic demand are therefore immune to the serial correlation of the Wold innovations on sales ϵ^r_{jt} documented by Ma, Ropele, Sraer, and Thesmar (2022), roughly equal to 10 percent.

debt is constant across provinces, roughly equal to 1 percent. The fact that provinces in the South have a relatively high leverage ratio with similar cost of credit is prime facie evidence that Southern firms do not face a tighter supply of credit. Finally, GDP is negatively correlated with the standard deviation of firm level shocks, σ_i (correlation of -0.40).

Table 1: Cross-sectional average, dispersion and correlation with GDP per capita

	Latitude	Aggregate labor productivity	Business exit rate	Business creation rate, %	N. of firms per capita	Firm leverage ratio	Risk adjusted interest rate	SD of shocks
Mean	42.73	0.00	0.08	0.76	0.08	1.84	0.98	0.26
Standard Deviation	2.65	13.86	0.01	0.39	0.03	0.38	0.44	0.08
Correlation with GDP	0.82	0.81	-0.63	-0.84	-0.81	-0.26	0.01	-0.36

Notes: Cross-sectional mean, standard deviation and correlation with GDP per capita (in log with mean normalized to zero). The sample period is 2007-2015 except for GDP and labor productivity whose averages are over the period 2010-2015. Labor productivity is GDP divided by aggregate employment in logs with mean normalized to zero. The business exit rate is the fraction of the beginning of the year number of limited liability companies that exit during the year. The leverage ratio is equal to the sum of the total financial debt (bank debt plus other financial debt excluding trade payables) of companies in the province over the sum of their value added. Value added is the value of production plus all provisions minus the sum of the expenditures in raw material and intermediate inputs. The business creation rate is the ratio between the number of start-ups in a year divided by total employment in the province multiplied by 100. Number of firms per capita is the number of companies over total employment. The risk-adjusted real cost of credit in a province $r_{ic} = r_i - f_i^m \times (1 - \varphi_i)$ is the difference between the average real interest rate on term of loans of firms (nominal interest rates minus CPI inflation) r_i and the average bankruptcy rate of firms older than 10 years in the province f_i^m multiplied by the fraction of unguaranteed debt $1 - \varphi_i$, in %.

To measure North-South differences in the age profile of businesses we use the cross-sectional averages of the 102 provinces and run regressions of the type:

$$X_{ia} = cte_a^X + \beta_a^X GDP_i + \text{error} \tag{4}$$

where X_{ia} corresponds to either the business exit rate (with or without bankruptcy) or the leverage ratio in province i for firms of age a, GDP_i is the average logged GDP per capita of province i over the period, cte_a^X is a constant and β_a^X measures how variable X_{ia} varies across provinces according to its GDP per capita.[9] The North-South difference is measured by $0.6 \times \beta_a^X$.

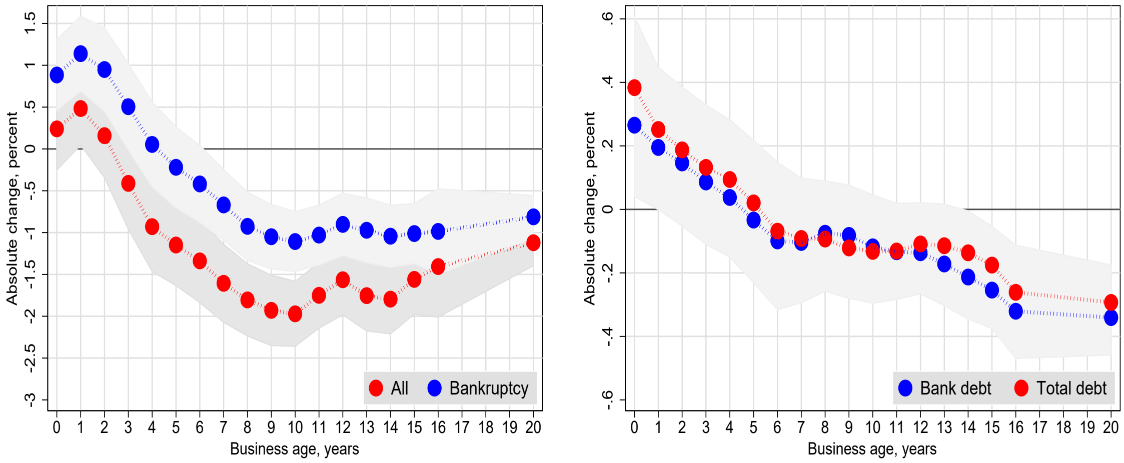
The red line in panel (a) of Figure 2 plots $0.6 \times \beta_a^X$ as a function of age a when X_{ia} is the overall exit rate; the blue line is analogous but for the bankruptcy rate. Grey areas correspond to 95 percent confidence intervals. The business exit rate during the first 2 years of life is not larger in the South than in North. It is only after 5 years of age that Southern firms fail with higher probability than Northern firms. After 10 years of age, the difference in the exit probability reaches a plateau at an absolute value slightly above one percentage point, for exit with or without bankruptcy.[10]

Panel (b) plots $0.6 \times \beta_a^X$ when X_{ia} is the leverage ratio, calculated using total debt (red line) or bank debt (blue line). The leverage ratio starts from a value of 2 at birth and progressively falls to roughly 1.6 after 16 years of life (see Appendix B). Northern firms at birth have a leverage ratio 40 percentage points higher than Southern firms. After more than 15 years of life,

[9]Results changes little once using the latitude of the province rather than its GDP per capita.

[10]In Appendix B we show that North-South differences in the age profile of exit rates are roughly unchanged when focusing on all legal entities rather than on limited liability companies.

Figure 2: North-South differences in the age profile of exit rates and leverage ratios



(a) N-S difference in business exit rates **(b) N-S difference in leverage ratio**

Notes: Data are for the universe of limited liability companies. Panel (a) plots the North-South difference in the average exit rate of companies of a given age. Panel (b) plots the difference in average leverage ratios. Averages are calculated over the 2007-2015 period. The average leverage ratio is weighted across firms using firm value added. In panel (a) the red line corresponds to exit rates; the blue line to exit rates with bankruptcy—i.e. leaving some bad loans to banks or with a formal bankruptcy procedure. In panel (b) the blue and red line corresponds to leverage measured with bank and total debt, respectively.

the leverage ratio of Northern firms becomes 20 percentage points lower than in the South.[11] Differences are similar with total or bank debt.

In the appendices we perform several additional exercises. In Appendix C we use CADS to run firm level regressions for the age profile of business exit rates and leverage ratios controlling for a full set of sector and province dummies as well as for firm employment size and in some specifications for firm dummies. We confirm that North-South differences in business exit rates and leverage ratios do indeed decrease with firm age (as in Figure 2) and that the pattern emerged more strongly during the period of cheap and abundant credit in the 2000's.

In Appendix D we also show that mature firms in the South are more likely to fail because of their excessive debt which implies a causation from the higher demand for credit of Southern firm in panel (b) of Figure 2 to their higher exit rate in panel (a). To show this, we use projection methods to estimate the response of the business exit probability of firm j to the shock ϵ^z_{jt} in (3) using firm level data from INVIND. Responses are consistent with the predictions of a canonical demand shock: firm prices and quantities increase while the exit probability falls. We find that highly indebted firms have a higher elasticity of business exit to the shock and that part of the North-South variation in the exit rate of mature firms is indeed due to the fact that in the South there is a larger mass of firms that have accumulated excessive debt.

In Appendix B we also characterize North-South differences in the age profile of firm employment size and firm labor productivity. Employment size and productivity increase as firms age.

[11]Kochen (2022) examines differences in the age profile of firm exit and leverage between high GDP per capita European countries and low GDP per capita European countries. His study reveals that exit rates and leverage decline with age (consistent with our findings), and that European firms in high GDP per capita countries exhibit lower exit rates and higher leverage at all ages (which contrasts with the evidence presented in Figure 2 and Appendix C). These patterns indicate that the credit markets across regions within Italy are substantially more integrated than the credit markets across countries in Europe.

Northern firms are larger and more productive than Southern firms. North-South differences remain stable as firms age, with a gap of around 40 percent in employment size and 30 percent in productivity.

2.3 Effects of the I Stay in the South subsidy

To promote business creation in Southern provinces Law n. 123 of 03/08/2017 introduced a subsidy for Southern start-ups, dubbed "I Stay in the South" (ISS), whose generosity and coverage have been expanded by 5 subsequent laws.[12] As of 2021, all Southern entrepreneurs under the age of 55 are entitled to the ISS subsidy, which covers nearly all expenditures to start up a business. We consider all firms in which at least one of the founding shareholders (those present at the time of creation) is less than 55 years old, which is the start-up population targeted by the ISS policy. This group represent around 75 percent of the population of new companies in Italy. We track these start-ups over time and for each province i, year t and age a we calculate firm-level averages. Since we can identity the shareholders of companies only starting from 2010, the sample period covers the years 2010-2020.[13] We rely on this pseudo panel of firm averages to run difference-in-differences regressions of the type

$$X_{iat} = d_{ia} + d_{ta} + \beta_{SR} \times \textit{Eligible-to-Subsidy}_{iat} + \beta_{SI} \times \textit{South-Incumbent}_{iat} + \epsilon_{it} \tag{5}$$

where X_{iat} could be either the logged number of new businesses created in province i in year t (in this case $a = 0$), or the exit rate of businesses of age a in province i in year t, or the average leverage ratio, or the logged average labor productivity (value added divided by firm employment) or the logged average firm employment size of businesses of age a in province i in year t. d_{ia} is a full set of provinces times firm-age dummies. d_{ta} is another full set of year times firm-age dummies.[14] Only firms created after 21 June 2017 were entitled to the subsidy. *Eligible-to-Subsidy* identifies the group of firms in the South that in principle were eligible for the ISS subsidy. It is a dummy which is equal to one for the group of firms in the South that: in 2017 are new ($a = 0$); in 2018 have less than 1 year of age ($a \leq 1$); in 2019 have less than 2 years of age ($a \leq 2$); in 2020 have less than 3 years of age ($a \leq 3$). Otherwise the dummy *Eligible-to-Subsidy* is equal to zero. The coefficient β_{SR} in (5) measures the effect of being eligible for the subsidy relative to other firms located in provinces of the North where no firms in the province are entitled to the ISS subsidy. There is another group of Southern firms that are directly affected by the policy: incumbent firms not entitled to the ISS subsidy that compete in the local market with other subsidized firms. These firms are identified by the dummy *South-Incumbent* which is equal to one for the group of firms in the South that: in 2017 have at least 1 year of age ($a > 0$); in 2018 have at least 2 years of age ($a > 1$); in 2019 have at least 3 years of age ($a > 2$); in 2020 at least 4 years of age ($a > 3$). Otherwise, the dummy *South-Incumbent* is equal to zero. The coefficient β_{SI} in (5) measures the effect of being a firm born without

[12]Law n. 123 of 03/08/2017 covered only startups by young entrepreneurs (with less than 35 years of age) in specific sectors. Law n. 145 of 12/30/2018 extended the age limit to 46 years of age. Law decree n. 34 of 19/05/2020 expanded the amount of the non-repayable grant. Law n. 77 of 17/07/2020 increased the amount of the maximum bank loan guaranteed by the government. Law n. 178 of 30/12/2020 extended the age limit up to 56 years of age. Law n. 156 of 9/11/2021 extended the subsidy to the commerce sector.

[13]We exclude firms in 7 partially treated provinces (Aquila, Teramo, Rieti, Macerata, Ascoli, Perugia and Terni), where only start-ups in a subset of seismic counties within the province were entitled to the ISS subsidy.

[14]For the regression where the dependent variable is the business creation rate, a is always equal to zero.

receiving the ISS subsidy in a market where some other firms got subsidized. Again the effect is relative to other firms in the North where none is entitled to the subsidy.

Table 2 shows the results from estimating the regression in (5).[15] Provinces exposed to the ISS subsidy experience an increase in the business creation rate of around 10 percentage points (column 1). The exit rate of start-ups entitled to the subsidy and of other incumbent firms both increase by 54 and 22 basis points, respectively (column 2). In treated provinces, the average leverage ratio of firms also increases both for start-ups entitled to the subsidy and unsubsidized incumbent firms (column 3). The average productivity of firms increases both for new and incumbent firms by around 3 percentage points (column 4). The employment size of start-ups is little affected by the subsidy, while there is evidence that incumbents firms not entitled to the subsidy scale down their size by 3 percentage points.

Table 2: Difference-in-differences effects of ISS subsidy

VARIABLES	(1) Creation	(2) Exit rate	(3) Leverage	(4) Productivity	(5) Size
Eligible-to-Subsidy	0.10***	0.54***	0.14***	0.04***	0.01
	(0.01)	(0.21)	(0.04)	(0.02)	(0.02)
South-Incumbent		0.22	0.08**	0.02*	-0.03*
		(0.19)	(0.04)	(0.01)	(0.02)
Observations	1,045	6,175	6,175	6,175	6,175
R^2	0.99	0.40	0.36	0.60	0.70
Province dummy	Y	Y	Y	Y	Y
Year dummy	Y	Y	Y	Y	Y
Age dummy	N	Y	Y	Y	Y
Province × Age dummy	N	Y	Y	Y	Y
Year × Age dummy	N	Y	Y	Y	Y

Notes: Estimates from running the regression in (5) on the population of companies in which at least one of the founding shareholders is younger than 55 years of age. Business creation, productivity and employment size are the log of firm averages. The business exit rate is in percentage terms (i.e. multiplied by 100). The leverage ratio is in level. Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.10.

3 Firm dynamics and free entry

We model firm dynamics in a province under free entry of firms. First, we focus on the problem of incumbent firms, then on start-ups. The analysis clarifies the leverage ratchet effect and the difference between a business creation subsidy paid-out ex-ante and one paid-out ex-post.

3.1 Assumptions

Time is continuous. There is a measure one of provinces $i \in [0, 1]$. We focus on a province, dropping reference to its identify. We also drop reference to time for all quantities and functions

[15] In Appendix E, we consider an event study for the β_{SR}'s coefficients in (5) and show that the parallel trend assumption holds for the variables in Table 2 .

that will remain time invariant in the general equilibrium analysis of Section 4.

Firm profits The profits of firm j are $\mathcal{R}Z_{jt}$ where $\mathcal{R}$ characterizes market *profitability* and Z_{jt} is an idiosyncratic shifter which evolves according to the geometric Brownian motion

$$dZ_{jt} = \sigma Z_{jt} d\omega_{jt}, \tag{6}$$

where ω_{jt} is a standard Brownian motion (zero mean and unit variance) idiosyncratic across firms, σ measures idiosyncratic risk; Z_{jt} is firm j's *technology.*

Debt Firms are owned by risk neutral entrepreneurs who have a time discount rate r and maximize the expected present value of consumption. After entry, a firm can borrow in a competitive market which charges a risk-free interest rate $r_c \leq r$, set by a government authority using lending subsidies, financed through lump sum taxes on entrepreneurs.[16] The net profits of the credit sector are rebated as lump-sum payments to entrepreneurs. Firm debt is modelled as a bond with coupon $\varkappa$ that reaches maturity at Poisson arrival rate ρ. A fraction φ of the debt is guaranteed.

Default and exit The firm declares bankruptcy when its equity value falls below the expected value of default. Upon default, with probability $\phi \in [0, 1)$ the firm renegotiates its debt B, obtains an haircut $1 - \alpha$ to B, and thereafter restarts production with new debt αB; with probability $1 - \phi$, the renegotiation fails, the firm pays the debt guarantees φB and exits forever from the market. The firm also exits if the entrepreneur exogenously dies (instantaneous probability δ). Upon death, debt guarantees are void. The bond price of a firm with debt B, technology Z and market profitability $\mathcal{R}$ is equal to the expected present value of debt payments discounted at the financiers rate r_c, equal to

$$X(\mathbf{S}) = E\left[\int_0^{\xi} e^{-(r_c+\delta+\rho)s}(\varkappa + \rho)\, ds + e^{-r_c\xi}\left[(1-\phi)\varphi + \phi\alpha X(\alpha B_{t+\xi}, Z_{t+\xi}, \mathcal{R})\right]\right], \tag{7}$$

where $\mathbf{S} = (B, Z, \mathcal{R})$ and ξ is the stopping time of bankruptcy. $B_{t+\xi}$, $Z_{t+\xi}$, and $\mathcal{R}$ denote debt, technology, and profitability at the time of bankruptcy (time $t + \xi$).

Firm creation A large mass $\delta\epsilon$ of immobile entrepreneurs is born in the province with wealth ϖ. Newborn entrepreneurs can start-up a business by making the investment k. After the investment, the business starts producing with initial technology $Z_0 = e^z$ where z is a discrete random variable that assigns probability g_z to $z \in \mathcal{Z}$, with $\sum_{z\in\mathcal{Z}} g_z = 1$.

The government subsidizes business creation in the province with (up to) two non-repayable grants: λk is paid ex-ante and can be used to finance the start-up investment; τk is paid ex-post as a reimbursement for the start-up investment. $\lambda + \tau$ measures the *size* of the subsidy. The ratio $\tilde{\tau} \equiv \tau/(\lambda + \tau)$ measures its *timing*: the fraction of subsidy paid ex-post. Subsidies are financed through lump-sum taxes on entrepreneurs.

Debt of start-ups The newborn entrepreneur is liquidity constrained and finances the start-up cost k partly by using her initial wealth ϖ, partly by using the ex-ante subsidy λk and partly by pledging B_0 bonds of the firm once productive. In equilibrium, given the convexity of the

[16] The tax advantage of debt over equity generally implies that $r_c < r$. In practice r_c can be controlled by a monetary or a fiscal authority: by the monetary authority through a discount window lending facility, by the fiscal authority through lending subsidies or taxes. The assumption that the lump-sum taxes are on entrepreneurs is without loss of generality.

firm value function with respect to B_0 (see below), the entrepreneur wants to minimize B_0, so that

$$B_0 = \frac{1}{x_0} \cdot \max\left\{(1-\lambda)\,k - \varpi, 0\right\}, \tag{8}$$

where x_0 is the bond price of a start-up in the province, $(1-\lambda)k$ is the initial start-up expenditure to be financed ex-ante and ϖ is the internal liquidity available to the entrepreneur.

Due to the leverage ratchet effect (more below), the ex-post subsidy is never used to buy back debt before maturity. Hence x_0 is equal to the expected bond value of the firm $X(B_0, e^z, \mathcal{R})$, which depends on the pledged bonds B_0, the firm technology e^z and market profitability $\mathcal{R}$:

$$x_0 = \sum_{z \in \mathcal{Z}} X(B_0, e^z, \mathcal{R}) \cdot g_z. \tag{9}$$

In addition to (possible) business income, an entrepreneur obtains per-period income ς.[17]

3.2 The problem of an incumbent firm

The equity value of a firm in the province with outstanding debt B, technology Z, and market profitability $\mathcal{R}$, is denoted by $V(\mathbf{S})$ satisfying the Hamilton-Jacobi-Bellman (HJB) equation

$$(r+\delta)V(\mathbf{S}) = \max_L \mathcal{R}Z - (\varkappa + \rho)\,B + X(\mathbf{S})\,L + L\frac{\partial V(\mathbf{S})}{\partial B} + \mathbb{L}V \tag{10}$$

where $\mathbb{L}V$ is the following differential operator that characterizes how exogenous changes in $\mathbf{S}=(B, Z, \mathcal{R})$ affect the firm equity value:

$$\mathbb{L}V \equiv -\rho B\frac{\partial V(\mathbf{S})}{\partial B} + \frac{\sigma^2 Z^2}{2} \cdot \frac{\partial^2 V(\mathbf{S})}{\partial Z^2}.$$

The first term in (10) is firm profits. The second term is the payments for serving current debt B; the third term is the cash flow from issuing new debt L, optimally chosen by the firm. The last two terms in (10) are the capital gains due to L and $\mathbb{L}V$.

The problem in (10) is further characterized by a bankruptcy boundary $\bar{B}(Z, \mathcal{R})$: a firm with technology Z under market profitability $\mathcal{R}$ whose debt B is greater or equal than $\bar{B}(Z, \mathcal{R})$ declares bankruptcy. After bankruptcy, with probability $1-\phi$ the firm pays the debt guarantees φB and exits, while with probability ϕ the firm restarts production with debt $\alpha\bar{B}(Z, \mathcal{R})$. Then at $\bar{B}(Z, \mathcal{R})$, the following value matching condition holds:

$$V\left(\bar{B}(Z,\mathcal{R}), Z, \mathcal{R}\right) = -\left(1-\phi\right)\varphi\bar{B}(Z,\mathcal{R}) + \phi V\left(\alpha\bar{B}(Z,\mathcal{R}), Z, \mathcal{R}\right). \tag{11}$$

At $\bar{B}(Z, \mathcal{R})$ we also have the two following smooth pasting conditions:

$$\left.\frac{\partial V}{\partial Z}\right|_{B=\bar{B}(Z,\mathcal{R})} = 0, \quad \text{and} \quad \left.\frac{\partial V}{\partial B}\right|_{B=\bar{B}(Z,\mathcal{R})} = -\left(1-\phi\right)\varphi + \phi\alpha\left.\frac{\partial V}{\partial B}\right|_{B=\alpha\bar{B}(Z,\mathcal{R})}. \tag{12}$$

By maximizing with respect to L in (10) we obtain that

$$X(\mathbf{S}) = -\frac{\partial V(\mathbf{S})}{\partial B}, \tag{13}$$

which says that the firm issues bonds until its equity value V is unaffected at the margin by L.

[17] ς is assumed to be large enough to guarantee that an entrepreneur running a business can finance equity injections (negative dividend payments) and honour debt guarantees.

Scaled value function To simplify the problem in (10), we define the firm *debt-value ratio*:

$$b \equiv \frac{B}{\mathcal{R}Z}.$$

In Appendix F, we guess and then verify that the value function $V(\mathbf{S})$ can be written as

$$V(B, Z, \mathcal{R}) = v(b)\,\mathcal{R}Z.$$

Let $\bar{b} \equiv \bar{B}(Z,\mathcal{R})/(\mathcal{R}Z)$ denote the threshold for the debt-value ratio, which triggers firm bankruptcy. For $b \in [0, \bar{b})$, we show that

$$v(b) = \frac{1}{r+\delta} - \bar{\varphi}b + \frac{(1-\phi\alpha)\,\bar{\varphi} - (1-\phi)\,\varphi}{(1+\gamma)\,(1-\phi\alpha^{1+\gamma})}\left(\frac{b}{\bar{b}}\right)^{\gamma} b, \tag{14}$$

where $\bar{\varphi} = \frac{\varkappa+\rho}{r+\delta+\rho}$ is the cost of a bond to the entrepreneur in the absence of default and γ is a positive constant equal to

$$\gamma = \frac{\rho}{\sigma^2} - \frac{1}{2} + \sqrt{\left(\frac{\rho}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2\,(r+\delta+\rho)}{\sigma^2}}.$$

For $b \geq \bar{b}$, the value function $v(b)$ can be evaluated recursively.[18]

The boundary conditions (11) and (12) determine the threshold $\bar{b}$ which is equal to

$$\bar{b} = \frac{1}{r+\delta} \cdot \frac{\left(1+\frac{1}{\gamma}\right)(1-\phi)}{(1-\phi\alpha)\,\bar{\varphi} - (1-\phi)\,\varphi}. \tag{15}$$

Given (13) and (14), the equilibrium market price of firm debt is equal to

$$x(b) = \bar{\varphi} - \frac{(1-\phi\alpha)\,\bar{\varphi} - (1-\phi)\,\varphi}{1-\phi\alpha^{1+\gamma}}\left(\frac{b}{\bar{b}}\right)^{\gamma}, \tag{16}$$

provided that $b \in [0, \bar{b}]$.[19] In Appendix G, we also show that a firm with debt-value ratio b issues the following amount of new debt per value of $\mathcal{R}Z$:

$$l(b) \equiv \frac{L}{\mathcal{R}Z} = (r - r_c)\,\frac{-v'(b)}{v''(b)}. \tag{17}$$

The condition (17) arises from equating the financial gains of a new bond to its marginal cost: the marginal gain is equal to the cash flow from a new bond, $x(b) = -v'(b)$, times the difference

[18] Notice that if $b \in \left[\alpha^{1-n}\bar{b}, \alpha^{-n}\bar{b}\right)$, $n = 1, 2, ...$, the firm restarts production only after n (successful) renegotiations, otherwise it pays the debt guarantees and exits, so that

$$v(b) = -(1-\phi^n)\,\varphi b + \phi^n v(\alpha^n b), \qquad \forall b \in \left[\alpha^{-n+1}\bar{b}, \alpha^{-n}\bar{b}\right).$$

[19] For $b \geq \bar{b}$, the price of debt $x(b)$ is again obtained recursively: if $b \in \left[\alpha^{1-n}\bar{b}, \alpha^{-n}\bar{b}\right)$, $n = 1, 2, ...$, we have

$$x(b) = (1-\phi^n)\,\varphi + \phi^n x(\alpha^n b), \qquad \forall b \in \left[\alpha^{-n+1}\bar{b}, \alpha^{-n}\bar{b}\right).$$

in discount rates between the entrepreneur and creditors $r - r_c$; the marginal cost is equal to the fall in the bond value $x(b)$ due to a marginal increase in debt, equal to $l(b)\, v''(b)$—which is positive due to the convexity of the value function, see (18).

After using (14) to substitute $v'(b)$ and $v''(b)$ in (17), we obtain that

$$l(b) = \frac{r - r_c}{\gamma} \left[\bar{l} \left(\frac{b}{\bar{b}} \right)^{-\gamma} - 1 \right] b. \tag{18}$$

Notice that $l(b)$ is strictly positive $\forall b \in [0, \bar{b}]$ because

$$\bar{l} \equiv \frac{\left(1 - \phi\alpha^{1+\gamma}\right) \bar{\varphi}}{\left(1 - \phi\alpha\right) \bar{\varphi} - \left(1 - \phi\right) \varphi} > 1.$$

Using the Ito's lemma and (6), we obtain that $\hat{b} \equiv \ln b$ evolves according to

$$d\hat{b} = \left[\frac{l(b)}{b} + \frac{\sigma^2}{2} - \rho \right] dt - \sigma d\omega, \tag{19}$$

which says that, conditional on survival, the firm gradually adjusts its debt value ratio toward a target level b^* identified by the condition $E(d\hat{b}) = \frac{l(b^*)}{b^*} + \frac{\sigma^2}{2} - \rho = 0$.

The Ito's lemma and (6) also imply that $\hat{z} \equiv \ln(\mathcal{R}Z)$ evolves as

$$d\hat{z} = -\frac{1}{2}\sigma^2 dt + \sigma\, d\omega. \tag{20}$$

Leverage ratchet effect The model exhibits a leverage ratchet effect: the firm never buys back and cancels its debt before it reaches maturity. To see this notice that the optimal policy of the firm in (17) implies that $l(b)$ is strictly positive for all debt value ratios in the relevant range, $\forall b \in [0, \bar{b}]$. Moreover, the firm never finds optimal to discretely adjust its debt value ratio. A firm that considers reducing its debt value from b to b' would face the problem

$$\max_{b' \in [0, \bar{b}]} \left[v(b') - x(b')\, (b - b') \right],$$

which given the convexity of the value function, $v'' > 0$, and (16) is solved at $b' = b$. This says that the firm only adjust its debt smoothly over time constantly issuing new debt, $l(b) > 0$. This also implies that, upon entry (when the debt value ratio is initially equal to zero), the entrepreneur wants to minimize the debt of the start-up.

3.3 Start-ups and free-entry

After using (16), (9) implies that the bond price of a start-up x_0 satisfies

$$x_0 = \sum_{z \in \mathcal{Z}} x \left(\frac{B_0}{\mathcal{R}e^z} \right) \cdot g_z. \tag{21}$$

The expected equity value of a start-up in the province, excluding the ex-post subsidy, is

$$\mathcal{V} = \sum_{z \in \mathcal{Z}} v \left(\frac{B_0}{\mathcal{R}e^z} \right) \cdot \mathcal{R}e^z \cdot g_z. \tag{22}$$

Under free entry with strictly positive business creation, the wealth forgone by the entrepreneur should be equal to the expected equity value of the start-up plus the ex post subsidy:

$$\varpi = \mathcal{V} + \tau k. \tag{23}$$

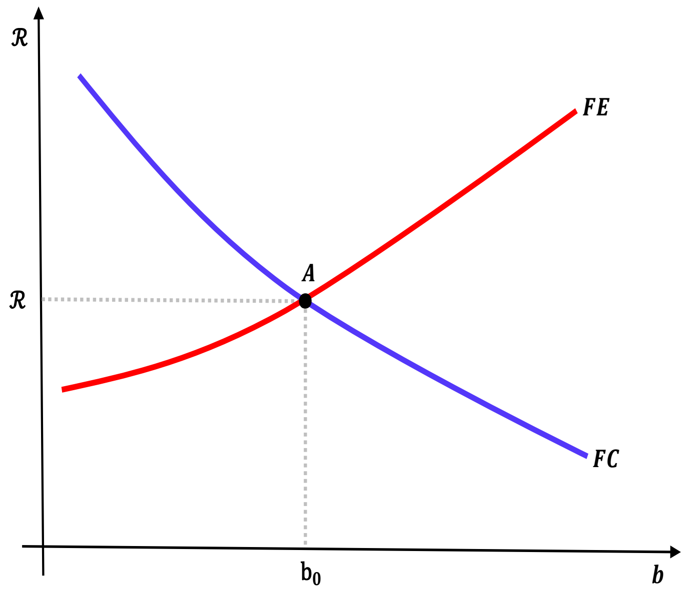
The free entry condition in (23) again uses the leverage ratchet effect: the firm never finds optimal to buy back debt and uses the ex-post subsidy to increase dividend payments.

Insulating property We use (21) to substitute x_0 in (8). We call the resulting equation the **FC**-condition. It establishes a relation between the initial debt of a start-up B_0 and market profitability, as measured by $\mathcal{R}$. Generally, along the **FC**-condition a greater $\mathcal{R}$ leads to a lower B_0: a more profitable market (greater $\mathcal{R}$) makes debt safer (greater $x(b_0)$), which implies that the firm can finance the start-up investment k with lower debt (lower B_0). When all start-ups in the province enter with the same technology Z_0, the **FC**-condition reads as follows

$$k = \mathcal{R}\, Z_0\, b_0\, x(b_0) + \varpi + \lambda k, \tag{\textbf{FC}}$$

which we plot in the b_0-$\mathcal{R}$ space. It corresponds to the negatively sloped blue line in Figure 3.

Figure 3: Insulating equilibrium: constant $\mathcal{R}$ in each province



After using (22) to substitute $\mathcal{V}$ in (23), we obtain a second relation between the initial debt of the start-up B_0 and market profitability $\mathcal{R}$, that we call the **FE**-condition: generally greater $\mathcal{R}$ implies that the free entry condition can be sustained with a higher start-up debt (higher B_0). When start-ups have all the same technology Z_0, the **FE**-condition reads as follows

$$\varpi = v(b_0)\, \mathcal{R}\, Z_0 + \tau k, \tag{\textbf{FE}}$$

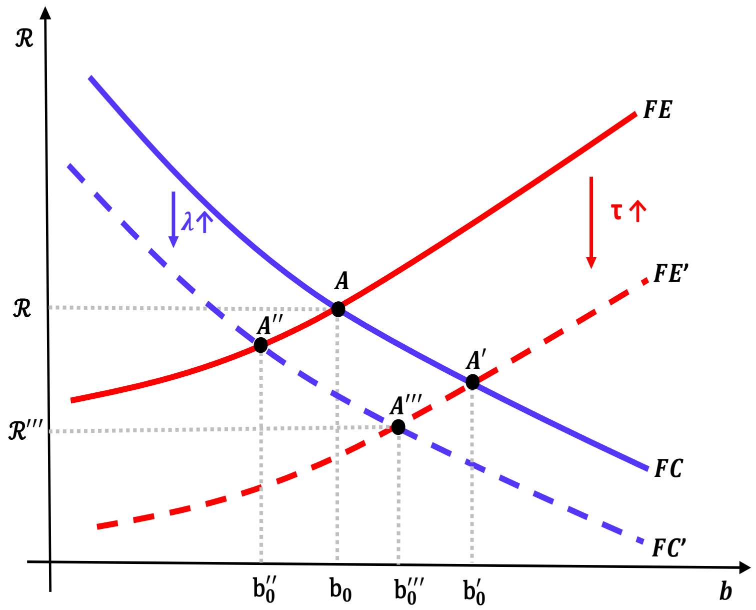
which in Figure 3 corresponds to the positively sloped red line.

In every province, **FE** and **FC** represent a system of two equations in the two unknown b_0 and $\mathcal{R}$, whose solution (represented by point A in Figure 3) allows to sustain an equilibrium where firms in every province are insulated from aggregate dynamics. The assumption that the entry cost k is in output units is key for constructing an insulating equilibrium.[20]

Reimbursement effect We can now use **FC** and **FE** to discuss why a subsidy paid ex-ante λ and one paid ex-post τ have different effects on the bankruptcy rate of start-ups. Notice that, since the threshold $\bar{b}$ in (15) is unaffected by market profitability $\mathcal{R}$, the bankruptcy rate of start-ups increases whenever their debt-value ratio at entry b_0 increases.

An increase in the ex-post subsidy τ shifts down the **FE**-condition, leaving the **FC**-condition unchanged. In Figure 4, the equilibrium moves from point A to point A', with a lower market profitability $\mathcal{R}$ and a higher debt value ratio at entry b_0. The ex-post subsidy stimulates entry pushing down market profitability $\mathcal{R}$. The fall in $\mathcal{R}$ reduces the market value of debt and start-ups should demand more debt B_0 to finance the same initial investment $(1-\lambda)k$. Since B_0 increases and $\mathcal{R}$ falls, the debt value ratio at entry b_0 increases, making firms more likely to go bankrupt during their first years of life.

Figure 4: Effects of an ex-post vs ex-ante subsidy



An increase in the ex-ante subsidy λ shifts down the **FC**-condition, leaving the **FE**-condition unchanged. In Figure 4, the equilibrium moves from A to A'', with a lower debt value ratio at entry b_0 and a lower market profitability $\mathcal{R}$. b_0 falls, because the ex-ante subsidy reduces the financial needs to start-up a business.

Business creation subsidies might have both an ex-ante and an ex-post component, and in Figure 4 the equilibrium can move from A to A'''. To evaluate the effects of the subsidy on the bankruptcy rates of start-ups, the two components of the subsidy should be analyzed separately.

[20] There could be multiple solution. This is because $b_0 x(b_0)$ in **FC** could be hump-shaped in b_0: the revenue from issuing new debt could be low either because the firm issues little debt (b_0 is low) or because the debt issued is so high that its market value is very low ($x(b_0)$ is low). Since welfare is decreasing in firm debt, in case of multiplicity we always select the solution with the highest value of x_0—i.e. with the lowest debt level B_0. This amounts to choosing a debt value ratio b_0 on the positively sloped arm of the $b_0 x(b_0)$ relation. This is why we plot the **FE**-condition as generally positively sloped.

3.4 Excessive entry and exit

Financial frictions generate welfare losses because of excessive entry as well as excessive exit.

Excessive entry We combine (8) with (23) to show that the free-entry condition implies that the start-up investment (net of entry subsidies) should be equal to the expected private value of a start-up (the sum of its equity and debt value):

$$(1 - \lambda - \tau)\, k = \mathcal{V} + B_0 x_0.$$

The private value of a firm with debt value ratio b, technology Z and market profitability $\mathcal{R}$ can be written as $s(b)Z\mathcal{R}$ where

$$s(b) = v(b) + x(b)b. \tag{24}$$

In Appendix H we combine the Bellman equation for $v(b)$ and $x(b)$ and show that $s(b)$ evolves according to the following HJB equation:

$$(r + \delta)s(b) = 1 - \rho b s'(b) + \frac{\sigma^2}{2} b^2 s''(b). \tag{25}$$

For given firm debt policy in (19) and bankruptcy threshold in (15), the social value of a firm is the present value of its net-output. This firm social value, divided by $\mathcal{R}Z$, is denoted by $s_*(b)$, which (as shown in Appendix H) satisfies the following HJB equation:

$$(r + \delta)s_*(b) = 1 + \left[\frac{l(b)}{b} - \rho\right] b s_*'(b) + \frac{\sigma^2}{2} b^2 s_*''(b). \tag{26}$$

Differently from (25), the HJB equation for $s_*(b)$ takes into account that the firm constantly issues new debt according to $l(b)$ in (18).

The dashed red line of Figure 5 plots $s(b)$. The solid blue line plots $s_*(b)$. $s(b)$ and $s_*(b)$ are expressed as a function of the log difference between the debt-value ratio b and the bankruptcy threshold $\bar{b}$ in (15). Panel (a) and (b) correspond to the two parameter configurations (for the North and the South) discussed in Section 5. Since $l\,(b) > 0$, $\forall b \in [0, \bar{b}]$ (a manifestation of the leverage ratchet effect) and $s_*'(b) < 0$, we have that $s(b) < s_*(b)\ \forall b$, as in Figure 5.

Equity owners issue debt to appropriate the private gains of cheap credit $r_c < r$. In equilibrium the private benefits of credit exactly compensate for the fall in firm value due to increased bankruptcy risk, see (13). In practice, cheap credit has no social value: all profits of the financial sector are rebated back to private agents, so debt payments are wealth transfers with no direct welfare effects. As a result, the private value of firms fails to internalize that the issuance of new debt $\frac{l(b)}{b}$ increases the risk of bankruptcy, which reduces the social value of the firm. As a result, for given initial debt value ratio b_0, entry is excessive causing an over-investment problem.[21]

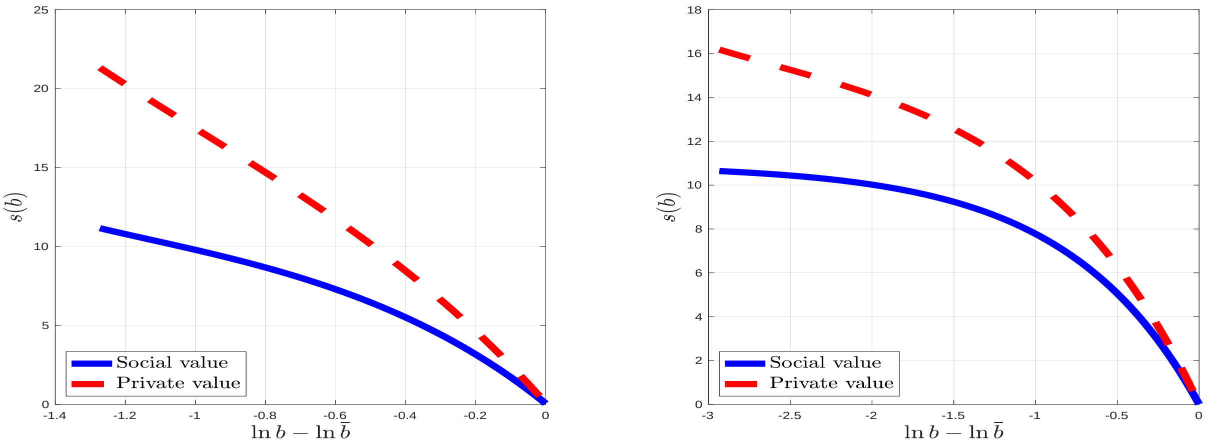
Excessive exit All exits due to bankruptcy are socially inefficient. The optimal social value of a firm, scaled by $\mathcal{R}Z$, under efficient exit would be equal to

$$s_{**} = \frac{1}{r + \delta}.$$

When a firm exits because of excessive debt, private agents are destroying a socially efficient production unit of value s_{**}. Then, reducing the bankruptcy rate is welfare improving.

[21] The result bears similarities with Aguiar, Amador, and Fourakis (2020). They consider a model where credit is too cheap because an impatient government has access to international bond markets. As a result the government overconsumes and over-borrows inefficiently increasing the bankruptcy risk of the country.

Figure 5: Private and social value of a firm for given debt policy



(a) Firm value, $s(b)$ & $s_*(b)$, in the North **(b) Firm value, $s(b)$ & $s_*(b)$, in the South**

4 Spatial equilibrium

We now describe the production technology and the economic geography of the model introducing explicit reference to the identity of the province i. Then we define the equilibrium and discuss additional sources of welfare inefficiencies.

4.1 Assumptions

Workers Workers are infinitely lived with discount rate r. They inelastically supply one unit of labor in the province i of current residence. The labor market is perfectly competitive with wage w_{it}. In province i at t there are ℓ_{it} workers and the aggregate supply is normalized to one:

$$\int_0^1 \ell_{it} di = 1. \tag{27}$$

With instantaneous probability ψ_t, workers have the option to choose the province of residence, so in the long run worker utility gets equalized across provinces. As in Rosen (1979) and Roback (1982), living costs $h_i(\ell_{it})$ are increasing in the province workforce, due to (un-modelled) congestions in the use of housing, infrastructure or amenities. Workers instantaneous utility in province i at t, u_{it}, is increasing in consumption and decreasing in living costs:

$$u_{it} = w_{it} - h_i(\ell_{it}), \tag{28}$$

where $h_i(\ell) = \bar{h}_i \ell^\eta$, with $\bar{h}_i, \eta > 0$.

Output Final output is produced by a representative firm with CES production function

$$Y_t = \left[\int_0^{M_t} (Z_{jt})^{\frac{1}{\nu}} (q_{jt})^{\frac{\nu-1}{\nu}} dj \right]^{\frac{\nu}{\nu-1}} \tag{29}$$

with $\nu > 2$.[22] M_t is the aggregate mass of firms in the economy which satisfies

$$M_t = \int_0^1 m_{it} di, \tag{30}$$

where m_{it} is the number of firms in province i at t. In (29), q_{jt} is firm j's output and Z_{jt} evolves as in (6), where σ_i varies by province. Firm j has access to production function a linear in labor, $q_{jt} = n_{jt}$, where n_{jt} is firm j's employment. Running the firm involves a leisure cost $\chi_i Z_{jt}$ to the entrepreneur. The output of firm $j \in [0, M_t]$ is freely tradable and there is monopolistic competition with flexible pricing.

Market profitability Final output is the numeraire so that

$$1 = \left[\int_0^{M_t} Z_j \left(p_{jt}\right)^{1-\nu} dj \right]^{\frac{1}{1-\nu}} \tag{31}$$

where p_{jt} is firm j price at t. From (29), it follows that firm j faces the demand

$$q_{jt} = Z_{jt} \left(p_{jt}\right)^{-\nu} Y_t. \tag{32}$$

Given the production technology $q_{jt} = n_{jt}$ and the demand in (32), firm j optimally chooses

$$n_{jt} = \left(1 - \frac{1}{\nu}\right)^{\nu} w_{it}^{-\nu} Y_t Z_{jt}. \tag{33}$$

Firm j's revenue net of labour and the leisure costs is $\mathcal{R}_i Z_{jt}$ where

$$\mathcal{R}_i \equiv \frac{\mathcal{A}_i}{\nu} - \chi_i, \tag{34}$$

is a sufficient statistic for market profitability in province i. $\mathcal{A}_i$ in (34) is the market *value added* per technology unit in province i equal to

$$\mathcal{A}_i = \left(\frac{\nu - 1}{\nu\ w_{it}}\right)^{\nu-1} Y_t, \tag{35}$$

which is decreasing in the local wage w_{it} and increasing in aggregate output Y_t.

4.2 Equilibrium conditions

We focus on an equilibrium where there is positive business creation $\tilde{m}_{it} > 0$ $\forall t$ and $\forall i \in [0, 1]$. This means that the free entry condition (23) holds $\forall i$ and $\forall t$ and that the $\mathcal{R}_i$'s and $\mathcal{A}_i$'s are constant through time, which implies that firms are insulated from aggregate dynamics.

[22]The requirement $\nu > 2$ guarantees the existence of the equilibrium.

Kolmogorov forward equation Remember that $\hat{b} \equiv \ln b$ and $\hat{z} \equiv \ln(\mathcal{R}Z)$. The inflow in province i at $\hat{\mathbf{s}} = (\hat{b}, \hat{z})$ due to business creation is

$$f_{it}^{0}(\hat{\mathbf{s}}) = \tilde{m}_{it} \times \sum_{z \in \mathcal{Z}_i} g_{iz} \cdot \mathbf{\Delta}\left(\hat{b}, \ln B_{i0} - z - \ln \mathcal{R}_i\right) \times \mathbf{\Delta}(\hat{z}, z + \ln \mathcal{R}_i), \tag{36}$$

where $\mathbf{\Delta}(x, y)$ denotes the Dirac delta function, infinite at $x = y$ and zero elsewhere.

Let $f_{it}(\hat{\mathbf{s}})$ denote the mass of firms in province i at t with state $\hat{\mathbf{s}} = (\hat{b}, \hat{z})$. For $\hat{b} < \ln \bar{b}_i$ with $\hat{b} \neq \ln(\alpha_i \bar{b}_i)$, $f_{it}(\hat{\mathbf{s}})$ solves the Kolmogorov forward equation

$$\frac{\partial f_{it}(\hat{\mathbf{s}})}{\partial t} = f_{it}^{0}(\hat{\mathbf{s}}) - \delta_i f_{it}(\hat{\mathbf{s}}) - \frac{\partial\left[\mathrm{B}_i(\hat{b}) f_i(\hat{\mathbf{s}})\right]}{\partial \hat{b}} + \frac{\sigma_i^2}{2} \cdot \frac{\partial f_{it}(\hat{\mathbf{s}})}{\partial \hat{z}} + \frac{\sigma_i^2}{2}\left[\frac{\partial^2 f_{it}(\hat{\mathbf{s}})}{\partial \hat{b}^2} - 2\frac{\partial^2 f_{it}(\hat{\mathbf{s}})}{\partial \hat{b} \partial \hat{z}} + \frac{\partial^2 f_{it}(\hat{\mathbf{s}})}{\partial \hat{z}^2}\right], \tag{37}$$

where $\mathrm{B}_i(\hat{b})$ is a function of $\hat{b}$ obtained using the (province specific) policy function $l_i(b)$ in (18):

$$\mathrm{B}_i(\hat{b}) = e^{-\hat{b}} \cdot l_i\left(e^{\hat{b}}\right) + \frac{\sigma_i^2}{2} - \rho_i.$$

The left-hand side is the change over time of $f_{it}(\hat{\mathbf{s}})$. The first term in the right-hand side of (37) is the instantaneous inflow of firms into state $\hat{\mathbf{s}}$ due to business creation $f_{it}^{0}(\hat{\mathbf{s}})$. The second term is the fall in $f_{it}(\hat{\mathbf{s}})$ due to entrepreneur death (arrival rate δ_i). The third term is the change in $f_{it}(\hat{\mathbf{s}})$ due to the mean change in the debt-value ratio $\hat{b}$ in (19). The fourth term is analogous but for the mean change of $\hat{z}$ in (20). The last term is the (standard) second order effect for the two dimensional diffusion processes in (19)-(20).

Equilibrium output The number of firms in province i at t is equal to

$$m_{it} = \int_{R^2} f_{it}(\hat{\mathbf{s}}) d\hat{\mathbf{s}}. \tag{38}$$

We use (35) to write

$$w_{it} = \frac{\nu - 1}{\nu}\left(\frac{Y_t}{\mathcal{A}_i}\right)^{\frac{1}{\nu-1}}, \tag{39}$$

which we substitute into (33). Clearing of the labor market in province i at t implies that the labor force in the province ℓ_{it} should be equal to labor demand, obtained by aggregating (33) across firms in the province. By imposing this labor market clearing condition we obtain that

$$\ell_{it}\left(Y_t\right)^{\frac{1}{\nu-1}}\left(\frac{1}{\mathcal{A}_i}\right)^{\frac{\nu}{\nu-1}} = \int_{R^2} \frac{\exp(\hat{z})}{\mathcal{R}_i} f_{it}(\hat{\mathbf{s}}) d\hat{\mathbf{s}}. \tag{40}$$

The value added produced in province i at time t is equal to

$$y_{it} = \mathcal{A}_i \int_{R^2} \frac{\exp(\hat{z})}{\mathcal{R}_i} f_{it}(\hat{\mathbf{s}}) d\hat{\mathbf{s}} = \ell_{it}\left(\frac{Y_t}{\mathcal{A}_i}\right)^{\frac{1}{\nu-1}}. \tag{41}$$

where the second equality uses (40).

Aggregate output at t is $Y_t = \int_0^1 y_{it} di$, which after using (41) yields

$$Y_t = \left[\int_0^1 \ell_{it}\left(\frac{1}{\mathcal{A}_i}\right)^{\frac{1}{\nu-1}} di\right]^{\frac{\nu-1}{\nu-2}}. \tag{42}$$

Y_t is a weighted average across provinces of the inverse of the firm value added per technology units $\mathcal{A}_i$, with weights equal to the province workforce ℓ_{it}: maximizing output requires allocating the workforce to the provinces with the lowest $\mathcal{A}_i$'s, i.e. those with the highest wage.

Worker mobility The value to the worker from moving to province i at t is equal to

$$U_{it} = \int_0^\infty e^{-rs} u_{it+s} ds, \tag{43}$$

where u_{it} is the instantaneous utility in (28).[23] We denote by

$$U_t^* \equiv \max_{i \in [0,1]} U_{it} \tag{44}$$

the maximum worker utility across provinces at t. The proportion ψ_t of workers who can choose the province of residence exert the option if $U_{it} < U_t^*$ so that

$$\frac{\dot{\ell}_{it}}{\ell_{it}} = -\psi_t \cdot \mathbb{I}\left(U_t^* - U_{it} > 0\right), \tag{45}$$

where $\dot{\ell}_{it} = \frac{d\ell_{it}}{dt}$ denotes the time derivative of ℓ_{it}.

Welfare We measure aggregate welfare $\mathbb{W}_t$ by the present value of the sum across provinces of the instantaneous utility flow of workers and entrepreneurs equal to

$$\mathbb{W}_t = \int_0^\infty e^{-rs} \left(C_{t+s} - H_{t+s}\right) ds, \tag{46}$$

where H_t measures aggregate total living costs equal to

$$H_t = \int_0^1 h_i\left(\ell_{it}\right) \ell_{it} di, \tag{47}$$

and C_t is aggregate consumption net of leisure costs equal to

$$C_t = \int_0^1 \left(y_{it} - k_i \tilde{m}_{it} - \mathfrak{c}_{it}\right) di, \tag{48}$$

where y_{it} is firm output in province i at t as in (41), $k_i \tilde{m}_{it}$ is business creation investment in the province, $\mathfrak{c}_{it} = \frac{\chi_i}{\mathcal{R}_i} \int_{R^2} \exp\left(\hat{z}\right) f_{it}(\hat{\mathbf{s}}) d\hat{\mathbf{s}}$ is leisure costs in the province.[24]

4.3 Definition

Let $\mathbb{X}_{it} = \left[\mathbb{X}_{it}^1, \mathbb{X}_{it}^2\right]$, $i \in [0,1]$, be the province-i specific tuple obtained by combining the time invariant tuple

$$\mathbb{X}_i^1 = \left[\mathcal{R}_i, \mathcal{A}_i, l_i\left(b\right), \bar{b}_i, x_i\left(b\right), x_{i0}, B_{i0}\right],$$

with the time varying tuple

$$\mathbb{X}_{it}^2 = \left[\tilde{m}_{it}, \ell_{it}, \dot{\ell}_{it}, U_{it}, f_{it}(\hat{\mathbf{s}}), f_{it}^0(\hat{\mathbf{s}}), m_{it}, w_{it}\right].$$

Let $\mathbf{X}_t = \left(U_t^*, Y_t, C_t, H_t\right)$ characterize the integrated economy. An equilibrium is a combination of $\mathbb{X}_{it}$'s, $i \in [0,1]$ and $\mathbf{X}_t$ that satisfy the following conditions:

[23] In equilibrium no worker finds optimal to switch more than once, so (43) is written as if the worker could not move again after first moving to province i.

[24] Notice that the present value of aggregate exogenous entrepreneurs income $\mathcal{E}_i = \frac{1}{r} \int_0^1 \left(\delta_i \varpi_i + \varsigma_i\right) \rceil \epsilon_i di$ is constant, irrelevant for welfare comparisons and thereby for expositionally simplicity it is excluded from (46).

1. *Firm maximization* Given $\mathcal{R}_i$ that solves (23) and $\mathcal{A}_i$ in (34), firms declare bankruptcy when the debt value ratio is above $\bar{b}_i$ in (15) and issue debt according to $l_i(b)$ in (18).

2. *Province equilibrium* Given the distribution $f_{it}(\hat{\mathbf{s}})$ which satisfies (37) with $f_{it}^0(\hat{\mathbf{s}})$ given in (36), clearing of the province-i's labor market implies that the wage w_{it} satisfies (39); free entry implies that business creation $\tilde{m}_{it}$ clears the labor market so that (40) holds; clearing of financial markets requires that the bond price $x_i(b)$ is equal to (16), that the debt of a start-up B_{i0} satisfies (8) with x_{i0} that solves (21).

3. *Worker maximization* The emigration rate $-\dot{\ell}_{it}/\ell_{it}$ satisfies (45) with U_{it} and U_t^* in (43) and (44), respectively.

4. *Aggregate market clearing* Clearing of the integrated labor market requires that (27) holds. Clearing of the goods market implies that aggregate output Y_t, living costs H_t, and consumption C_t satisfy (42), (47), and (48), respectively.

Since $\mathcal{R}_i$ and $\mathcal{A}_i$ are time-invariant, aggregate output Y_t, the labor force in each province ℓ_{it}, and the wage w_{it} are all predetermined at every point in time t. The equilibrium is sustained through a strictly positive level of business creation $\tilde{m}_{it}$ that adjusts to clear the labor market and make (40) satisfied at these predetermined values. In each province, the firm value function $v_i(b)$, the debt policy $l_i(b)$, the bankruptcy threshold $\bar{b}_i$, the debt value of start-ups x_{i0}, and their initial debt B_{i0} remain insulated from aggregate dynamics. The variables in the vectors $\mathbb{X}_{it}^2$ and $\mathbf{X}_t$ adjust over time: the wages w_{it}'s and the labor forces ℓ_{it}'s in provinces slowly adjust, which determine the dynamics of aggregate output Y_t and aggregate welfare $\mathbb{W}_t$.

4.4 Additional welfare inefficiencies

In the spatial economy, we introduced two additional sources of welfare inefficiencies, due to spatial misallocation of labor and aggregate demand externalities.

Spatial misallocation There are two reasons why labor may be misallocated across provinces. The first is due to the congestion externality that affects living costs. The second is due to the marginal cost of overinvestment which might differ across provinces. When migrating to a location, workers do not internalize that their choices affect living costs in the provinces of origin and destination. This might justify subsidizing workers to stay in the provinces with lower wages. To see this, suppose that one half of provinces are in the North, that pays wage w_N with labor force ℓ_N in each province, and the remaining one half of provinces are in the South that pays lower wages $w_S < w_N$ with labor force ℓ_S in each province. In the long run, it must be that

$$w_N - h_N(\ell_N) = w_S - h_S(\ell_S), \tag{49}$$

which implies that living costs are higher in provinces with higher wages: a common feature of any spatial equilibrium. Now consider the effect on aggregate living costs H in (47) of reallocating one worker from the high-wage provinces of the North to provinces in the South. H falls by

$$\frac{\partial\left[\ell_S h_S(\ell_S)\right]}{\partial \ell_S} - \frac{\partial\left[\ell_N h_N(\ell_N)\right]}{\partial \ell_N} = -(1+\eta)(w_N - w_S) < 0,$$

where the second equality makes use of the spatial equilibrium condition in (49). By subsidizing workers to stay in the South, aggregate living costs H in (47) fall.

Workers fail also to internalize the overinvestment implications of their migration decisions. If there is cross-sectional variation in overinvestment, reallocating workers across provinces can increase aggregate consumption. To see this effect, notice that (33) implies that a start-up in province i at t employs an expected number of workers equal to

$$\bar{n}_{it}^{0} = \sum_{z \in \mathcal{Z}_i} \left(\frac{\nu - 1}{\nu} \right)^{\nu} w_{it}^{-\nu} \, Y_t \, e^z \cdot g_{iz}.$$

If a worker moves to province i, the number of start-ups in the province increases by $1/\bar{n}_{it}^{0}$.[25] Overinvestment in the province increases by the product of the number of new start-ups, $1/\bar{n}_{it}^{0}$, times the difference between the social cost k_i and the social value s_{*i} of the start-up:

$$\Omega_{it} = \frac{k_i}{\bar{n}_{it}^{0}} \left[1 - \sum_{z \in \mathcal{Z}_i} s_{*i} \left(\frac{B_{i0}}{\mathcal{R}_i e^z} \right) \cdot \frac{\mathcal{R}_i}{k_i} e^z \cdot g_{iz} \right].$$

Ω_{it} measures the amount of overinvestment caused by a (marginal) worker in province i at t. Reallocating workers from provinces with high Ω_{it} to provinces with low Ω_{it} increases aggregate consumption, inducing a welfare gain. In our calibration the cross-sectional differences in Ω_{it}'s are small, and this source of misallocation turns out to be quantitatively unimportant.

Demand externality The demand of each single firm in (32) depends on the aggregate demand shifter Y that satisfies (29) and thereby is function of the aggregate number of firms M in the economy. This well-known Dixit-Stiglitz aggregate demand externality might lead to inefficient entry. It is known that when the start-up investment is in output units and labor is used as an input in the production of varieties (as in our model), the Dixit-Stiglitz aggregate demand externality causes an inefficiently low business creation rate, see Rivera-Batiz and Romer (1991) and Chapter 6 in Barro and Sala-i-Martin (2003).[26]

5 Calibration

There are two types of provinces, one half in the North, one half in the South. They represent Italian provinces with GDP per capita 30% above and 30% below the national average, respectively. The economy is in steady state, provinces have the same labor force $\ell_i = 1$ with no subsidies, $\lambda_i = \tau_i = 0 \; \forall i = N, S$. In the model, the leverage ratio of a firm with debt value ratio b in province i is equal to $b\mathcal{R}_i/\mathcal{A}_i$. We target the age profiles of the leverage ratio, total business exit rate and exit rate with bankruptcy, which correspond to the red dashed lines in Figure 6 for the cross-province average (left column) and the North-South difference (right column). Table 3 reports the parameter values in the calibration.

The yearly discount rate of firms matches the long-run real return on Italian wealth net of GDP growth, around 3 percent (Jordà et al. 2019). The parameter governing the elasticity of

[25] The wage in a province satisfies (39), so unaffected by the workforce in the province. An increase in the labor supply of a province is accommodated entirely through an increase in the labor demand by start-ups: $\tilde{m}_{it}$ increases to guarantee that the labor market clearing condition in (40) remains satisfied.

[26] This differs from the original formulation in Dixit and Stiglitz (1977) where labor is the only input in both production of varieties and business creation: in this latter case the business creation rate is efficient.

Table 3: Parameter values

Parameter	Value	Targeted Moment	Data value	Model value
Firm discount rate, r	.030	Yearly wealth return	.030	.030
Elasticity of substitution, ν	5	Average INVIND	5	5
Financiers discount rate, r_c	.011	Risk adjusted interest rate on debt	.011	.011
Debt repayment arrival rate, ρ	1/30	Leverage ratio at 4 years of age relative to age 0	.83	.79
Congestions elasticity to workforce, η	2.00	Long-run elasticity of workforce to wages	2.7	2.7
Debt coupon, $\varkappa_i$	$r+\delta_i$	Bond cost to entrepreneur absent default	1.00	1.00
Scale of congestions, $(\bar{h}_N;\bar{h}_S)$	(0.34;0.10)	North workforce & living costs/labor income	(1.00; .33)	(1.00; .33)
Idiosyncratic risk, $(\sigma_N;\sigma_S)$	(.27;.31)	Standard deviation of shocks, North & South	(.27; .31)	(.27; .31)
Debt guarantees, $(\varphi_N;\varphi_S)$	(.28;.32)	% of guaranteed debt, North & South	(.28;.32)	(.28;.32)
Entry investment cost, $(k_N;k_S)$	(29.7; 11.5)	Average firm size, ℓ_i/m_i, North & South	(18.17; 10.00)	(18.17; 10.00)
Entrepreneur death rate, $(\delta_N;\delta_S)$	(.025;.033)	Average exit rate without bankruptcy, North & South	(.025; .033)	(.025; .033)
Min technology at entry, $(z_N^l;z_S^l)$	(9.70; 0.84)	Average labor productivity, y_i/ℓ_i, North & South	(1; .7)	(1; .7)
Max technology at entry, $(z_N^h;z_S^h)$	(20.65; 1.79)	Average exit rate, age 0, North & South	(.105; .102)	(.104; .101)
Probability of z^h at entry, $(q_N;q_S)$	(.41; .41)	Average exit rate, age 1, North & South	(.112; .107)	(.111; .105)
Renegotiation probability, $(\phi_N;\phi_S)$	(.47;.36)	Exit rate with bankruptcy, age 12-14 yrs, North & South	(.038;.050)	(.040;.051)
Recovery rate at renegotiation, $(\alpha_N;\alpha_S)$	(.43; .40)	Recovery rate upon bankruptcy, North & South	(.35; .35)	(.35; .35)
Initial entrepreneur's wealth, $(\varpi_N;\varpi_S)$	(.48; .0.26)	Firm leverage ratio at entry, North & South	(2.77;2.39)	(2.77;2.39)
Entrepreneur leisure cost, $(\chi_N;\chi_S)$	(.025;.035)	Leverage ratio, age 12-14 yrs, North & South	(1.71;1.83)	(1.67;1.82)
Worker mobility rate $\tilde{\psi}$	.069	Half-life duration of aggregate shocks	10yrs	10yrs

substitution across varieties is set to $\nu = 5$, roughly the average demand elasticity self-reported by firms in INVIND (Pozzi and Schivardi 2016).

The risk-adjusted cost of credit r_c is 1.1 percent, equal across provinces, in line with the evidence of Table 1. The debt maturity arrival rate is $\rho_i = 1/30$, to match a fall of 16 percent in the leverage ratio of firms during their first 5 years of life, see panel (a) of Figure 6.

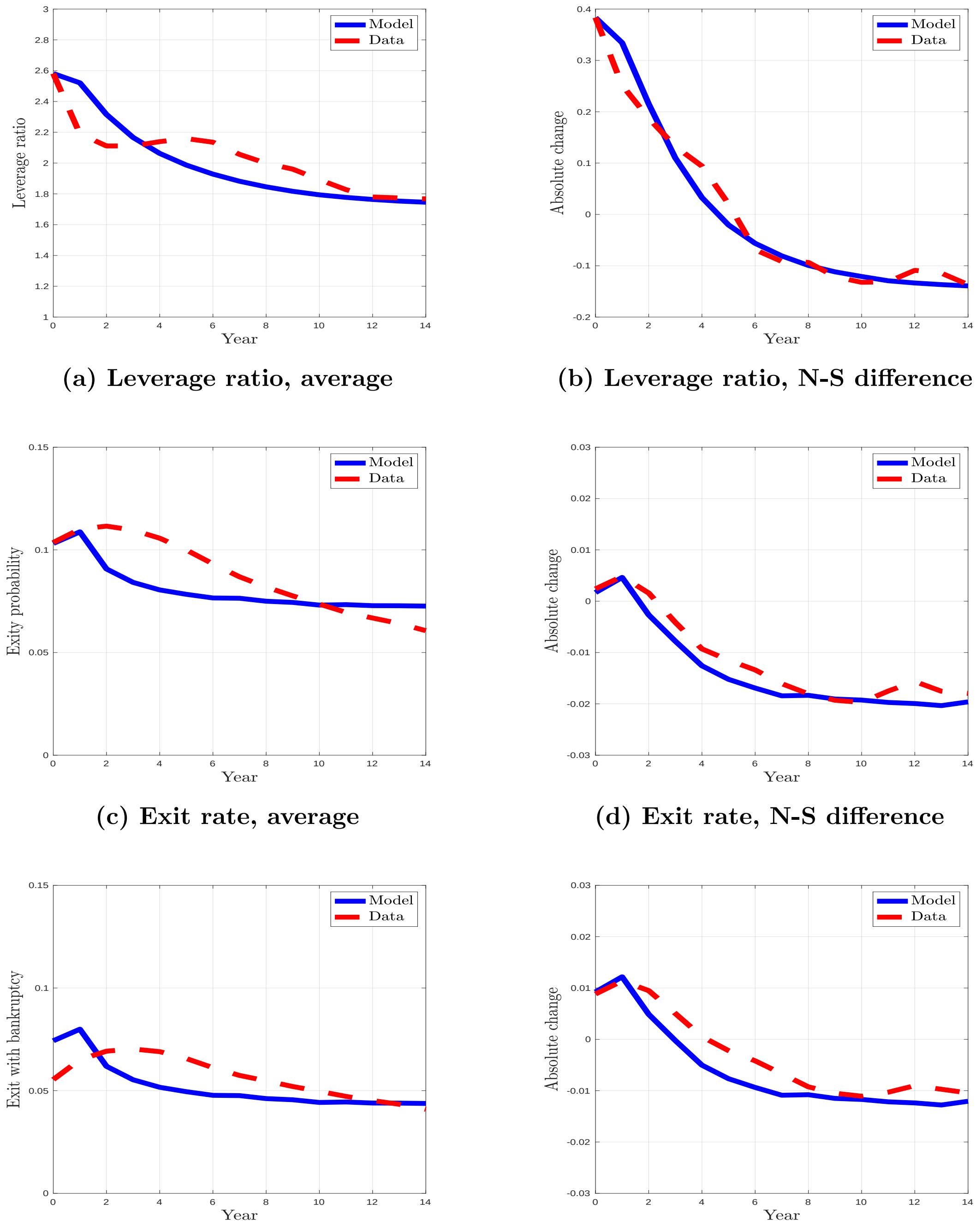
The technology at birth, $\exp(z_{i0})$ is a discrete two points random variable $\exp(z_{i0}) \in \{z_i^l, z_i^h\}$ with probability q_i and $1-q_i$, respectively. z_N^l and z_S^l target an aggregate labor productivity normalized to 1 in the North, $y_N/\ell_N = 1$ and one 30 percent smaller in the South, $y_S/\ell_S = 0.7$, see Table 1. z_i^h and q_i are set to match the average exit rate of firms during their first and second year of life, see panel (c) and (d) of Figure 6.

Combining (39) with (41), we obtain that $w_{it} = \frac{\nu-1}{\nu} \cdot \frac{y_{it}}{\ell_{it}}$, which, given the targeted labor productivity y_i/ℓ_i, pins down the wages w_N and w_S. The scale parameters for the living costs function in North and South, $\bar{h}_i$, $i = N, S$ match (i) the spatial equilibrium condition in (49) at the steady state labor force of 1, which implies that

$$h_N - h_S = \frac{\nu - 1}{\nu} \times 0.3,$$

and (ii) a ratio between aggregate living costs to aggregate labor income of 1/3, roughly equal

Figure 6: Age profiles of business failure and leverage: model vs data



(a) **Leverage ratio, average** (b) **Leverage ratio, N-S difference**

(c) **Exit rate, average** (d) **Exit rate, N-S difference**

(e) **Bankruptcy exit rate, average** (f) **Bankruptcy exit rate, N-S difference**

Notes: The red dashed line corresponds to data for all companies from CADS. Leverage is total firm debt divided firm value added. The bankruptcy exit rate is the exit rate of companies with bankruptcy. The left column plots cross-province averages, the right column North-South differences. North-South differences correspond to the difference between two provinces that differ in logged GDP per capita by 60 percent. Blue lines correspond to model simulated data using 100,000 firms for 15 years.

to the incidence of housing costs on labor income (ISTAT 2020), so that $\bar{h}_N + \bar{h}_S = \frac{1.7}{3} \cdot \frac{\nu-1}{\nu}$.

Given (49), the long-run elasticity of the local labor force to a permanent wage change is

$$\hat{\beta} \equiv \frac{\partial \ln \ell_i}{\partial \ln w_i} = \frac{w_i}{\bar{h}_i} \cdot \frac{1}{\eta},$$

which, given the already calibrated values for w_i and $\bar{h}_i$, fully identifies the elasticity of living costs to the local labor force η. Since the pioneering work by Blanchard and Katz (1992), there is a large literature estimating $\hat{\beta}$. Basso, D'Amuri, and Peri (2018) document that estimates are similar in the US and Europe. We target a value for $\hat{\beta}$ in the South equal to 2.7, in line with the evidence in Notowidigdo (2020).

The standard deviations of idiosyncratic shocks, $\sigma_N = 0.27$ and $\sigma_S = 0.31$, are calculated using our measure of idiosyncratic shocks in provinces with GDP per capita 30% above and 30% below the national average, respectively. We use the Credit Registry and proceed analogously for the fraction of guaranteed debt, concluding that $\varphi_N = 0.28$ and $\varphi_S = 0.32$.

The debt recovery rate upon bankruptcy is $(1 - \phi_i)\,\varphi_i + \phi_i \alpha_i$. We set α_N and α_S to match a recovery rate of 33 percent in North and South, its average value over the 2015-2017 period (Fischetto et al. 2018). We normalize the cost of debt to the firm in the hypothetical case of no bankruptcy to one, $\bar{\varphi}_i = 1$, which pins down the debt coupon $\varkappa_i$. The entry cost κ_i implicitly targets the average firm employment size in North and South, equal to $\ell_N/m_N = 18.17$ and $\ell_S/m_S = 10.00$ (from CADS matched with UNIMPS); see also Table 1. We set the entrepreneur death rate, δ_i, as equal to the difference between the total business exit rate and the exit rate with bankruptcy, which yields $\delta_N = 2.5\%$ in the North and $\delta_S = 3.3\%$ in the South.

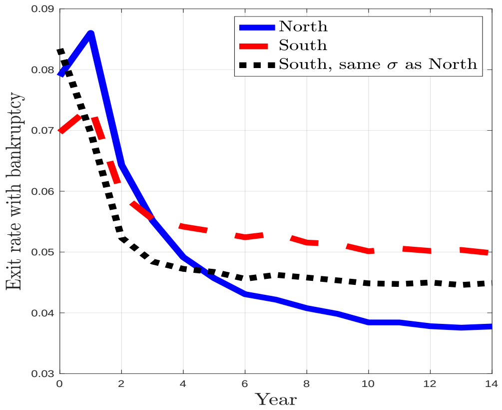
The probability of renegotiating debt upon bankruptcy, ϕ_i, matches the exit rate with bankruptcy of firms with 13-15 years of age, equal to 3.8% in the North and 5% in the South. We use entrepreneur's wealth at birth ϖ_i and the entrepreneurs' cost of running a firm χ_i to target the leverage ratio of firms at entry (in North and South) and the leverage ratio of firms with 13-15 years of age, respectively.

In steady state worker mobility is irrelevant. For any (permanent) shock $\mathcal{T}$, ψ_t is set to yield a half-life of the shock of 10 years, in line with the evidence by Michael and Manning (2018) and Monras (2018). Let $\ell_S(\mathcal{T})$ denote the new steady state work force in the South after the shock $\mathcal{T}$. Let ℓ_{St} denote the labor force at t. We set $\psi_t = \tilde{\psi}\left|\frac{\ell_S(\mathcal{T})}{\ell_{St}} - 1\right|$, with $\tilde{\psi} = -\ln(0.5)/10$.

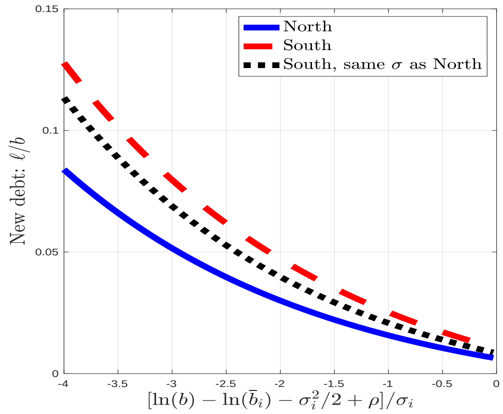
We simulate the history of 100,000 firms for 15 years separately in the North and the South under our calibration. The blue solid lines in Figure 6 are the resulting age-profile of the leverage ratio (panels a and b), total exit rate (panels c and d) and exit rate with bankruptcy (panels e and f). The empirical counterparts from the data correspond to the red dashed lines. Our relatively parsimonious model of firm dynamics fits reasonably well the slope of the average age profile observed in the data and the magnitude of the North-South differences.

Panel (a) of Figure 7 shows the age profiles of the business exit rate, in the North (blue solid line), in the South (red dashed line), and in a counterfactual economy where Southern firms face the same idiosyncratic shocks σ_i as in the North (black dotted line). Panel (b) shows the debt issuance policy $l_i(b)/b$ in (18) in the three economies. In panel (b), the x-axis is standardized so that, at the same point on the x-axis, if firms stop issuing new debt forever $l_i(b) = 0$, the bankruptcy probability is the same in the three economies. Panel (a) shows that around half of the North-South differences in the exit rate of mature firms are due to differences in idiosyncratic risk. Panel (b) shows that this happens partly because the higher risk in the South gives Southern firms stronger incentives to issue more debt when close to the bankruptcy threshold.

Figure 7: Age profiles of business failure rates and leverage: the role of risk



(a) Age profile of exit rate



(b) New debt issuance policy, $l_i(b)/b$

6 A subsidy in the South

The economy is initially in a steady state without subsidies. We study an unexpected once-and-for-all permanent change at $t = 0$ in the entry subsidy for all Southern provinces to $\mathcal{T} \equiv \{\lambda_S + \tau_S, \tilde{\tau}_S\}$, parameterized in terms of size $\lambda_S + \tau_S$ and timing $\tilde{\tau}_S = \tau_S/(\lambda_S + \tau_S)$. In the North, subsidies remain equal to zero. The government budget is balanced and the subsidy is financed through (non distortionary) lump-sum taxes on entrepreneurs (an immobile factor of production). We characterize the equilibrium response to $\mathcal{T}$, $\forall t \geq 0$, see Appendix I for the computational details.

Then we study the optimal subsidy $\mathcal{T}^*$: the $\mathcal{T}$ that maximizes welfare in (46). Welfare gains are measured as the equivalent per-period permanent consumption percentage increase of the status quo consumption,

$$\mathbb{W}_0(\mathcal{T}) \equiv \int_0^\infty re^{-rt} \left[\left(\frac{Y_t - Y}{C} \right) - \left(\frac{I_t - I}{C} \right) - \left(\frac{\mathfrak{C}_t - \mathfrak{C}}{C} \right) - \left(\frac{H_t - H}{C} \right) \right] dt \tag{50}$$

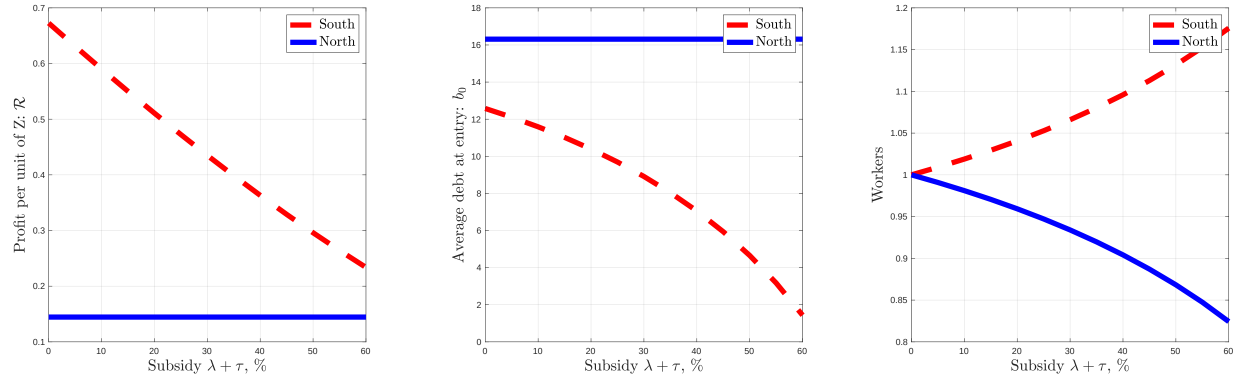
where a variable without time subindex indicates its initial steady state value. $\mathbb{W}_0(\mathcal{T})$ is the sum of the increase in (the present value of) output Y_t net of the increase in the sum of investment $I_t = \frac{1}{2}(k_N \tilde{m}_{Nt} + k_S \tilde{m}_{St})$, leisure costs of entrepreneurs $\mathfrak{C}_t = \frac{1}{2}(\mathfrak{c}_{Nt} + \mathfrak{c}_{St})$, and living costs H_t.

6.1 Equilibrium response

In any province $i = N, S$, the subsidy $\mathcal{T}$ leaves unaffected the bankruptcy threshold $\bar{b}_i$ in (15) and the debt issuance policy $l_i(b)$ in (18). Panels (a) and (b) of Figure 8 show the response to $\lambda_S + \tau_S$ (with $\tilde{\tau}_S = 0$) of market profitability $\mathcal{R}_i$ and the average debt value ratio at entry b_{i0}, respectively. The solid blue line is for the North, the red dashed line for the South. In the South, market profitability falls from $\mathcal{R}_S$ to $\mathcal{R}_S(\mathcal{T})$ while the debt of start-ups changes to $B_{S0}(\mathcal{T})$, which determines the debt-value ratio $b_{S0}(z; \mathcal{T})$ (up or down depending on $\tilde{\tau}_S$), see (21)-(23). In the North, market profitability is unchanged, $\mathcal{R}_N = \mathcal{R}_N(\mathcal{T})$ and so is the debt-value ratio of start-ups, $b_{N0}(z)$.

At $t = 0$, given the $\mathcal{A}_i(\mathcal{T})$'s and the predetermined labor forces ℓ_{i0}'s, output Y_0 and wages w_{i0}'s are determined by (42) and (39). The business creation rate instantaneously jumps up to

Figure 8: Long-run effects of subsidy size $\lambda_S + \tau_S$ with $\tilde{\tau}_S = 0$



(a) **Market profitability** $\mathcal{R}_i$ (b) **Debt value at entry,** $E(b_{i0})$ (c) **Labor force,** ℓ_i

$\tilde{m}^+_{i0} > 0$, to guarantee that the labor market clears so that (40) holds. In the South at $t = 0$, there is a shake-out in the market: the distribution of the (logged) debt value ratio $\hat{b}$ instantaneously shifts to the right by $\mathcal{D} = \ln \mathcal{R}_S - \ln \mathcal{R}_S(\mathcal{T})$, so some firms cross the threshold $\bar{b}_S$ and declare bankruptcy. Some bankrupt firms exit. Others renegotiate debt down and remain active. Since $\mathcal{R}_N$ is unchanged, no shake-out happens in the North.

The steady state labor force in a province of the South increases to $\ell_S(\mathcal{T})$, see red dashed line in panel (c) of Figure 8. Starting from the initial steady state with $\ell_S = 1$, Northern workers gradually move to the South so that

$$\ell_{St} = \ell_S e^{-\tilde{\psi}t} + \ell_S(\mathcal{T})\left(1 - e^{-\tilde{\psi}t}\right). \tag{51}$$

The labor force in a province of the North is $\ell_{Nt} = 2 - \ell_{St}$. Again, given the $\mathcal{A}_i(\mathcal{T})$'s and the predetermined ℓ_{it}'s, (42) determines output Y_t and (39) determines wages w_{it} $\forall t > 0$. The business creation rate $\tilde{m}_{it}$ guarantees that (40) holds $\forall t > 0$. The Kolmogorov forward equation in (37) dictates the evolution of the firm distribution $f_{it}(\hat{\mathbf{s}}; \mathcal{T})$.

Figure 9 plots the responses over time to a subsidy of size $\lambda_S + \tau_S = 0.25$, entirely paid out ex ante, $\tilde{\tau}_S = 0$. The labor force gradually moves from the South to the North according to (51) (panel a), value added per worker increases, more in the South than in the North (panel b). This is due to the greater business creation investment in the South (panel c), directly caused by the subsidy that stimulates entry more in the South than in the North (panel e). The exit rate spikes up in the South on impact due to the shake-out resulting from the fall in $\mathcal{R}_S$ (panel d). This also contributes to the higher business creation of the South.

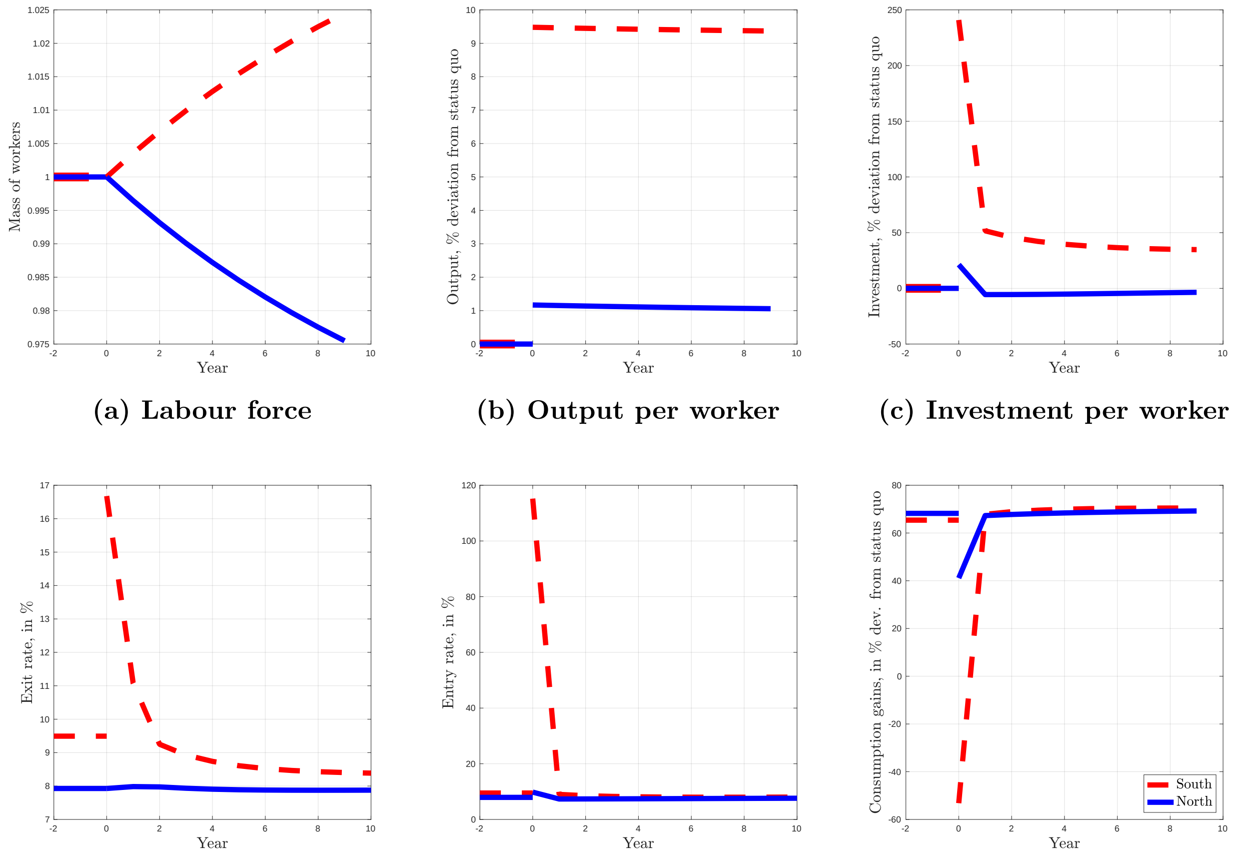
We assume that a province of type $i = N, S$, finances the cost of the subsidy

$$\vartheta_t = (\lambda_S + \tau_S) k_S \frac{\tilde{m}_{St}}{2}$$

in proportion to its initial steady state output: a province in the South pays the (lump sum) tax $\frac{2y_S}{y_N + y_S} \cdot \vartheta_t$; one in the North pays $\frac{2y_N}{y_N + y_S} \cdot \vartheta_t$, where y_i is the initial steady state output in province i as defined in (41). Then, the welfare gains per worker in a province of type i are

$$\mathbb{W}^i_0(\mathcal{T}) = \int_0^\infty \frac{re^{-rt}}{c_i}\left[\left(\frac{y_{it}}{\ell_{it}} - y_i\right) - k_i\left(\frac{\tilde{m}_{it}}{\ell_{it}} - \tilde{m}_i\right) - \left(h_i(\ell_{it}) - \bar{h}_i\right) - \left(\frac{\mathfrak{C}_{it}}{\ell_{it}} - \mathfrak{C}_i\right) + \frac{\mathbb{I}_i 2\vartheta_t y_N}{(y_N + y_S)\ell_{it}}\right] dt \tag{52}$$

Figure 9: Impulse responses at the optimal subsidy



(a) Labour force **(b) Output per worker** **(c) Investment per worker**

(d) Exit rate **(e) Entry rate** **(f) Welfare flow per worker**

where $\mathbb{I}_i$ is an indicator function, equal to one if province i is in the South and minus one otherwise. Again, variables without time subindex refer to their initial steady state value. $\mathbb{W}_0^i(\mathcal{T})$ is the sum of the increase in (the present value of) output in province i minus costs due to local investment, leisure and living net of tax transfers, which are positive (negative) in the South (North). Panel (f) plots the response of the welfare-flow-gains per worker (the integrand of (52)) in a province of the North and the South. In the first years after $\mathcal{T}$, welfare per worker falls both in the North and the South due to the increase in start-up investment and the initial shake-out in the South, but then it recovers and eventually turns positive.

6.2 Optimal subsidy

The subsidy optimal size $\lambda_S + \tau_S$ and timing $\tilde{\tau}_S = \tau_S/(\tau_S + \lambda_S)$ solve

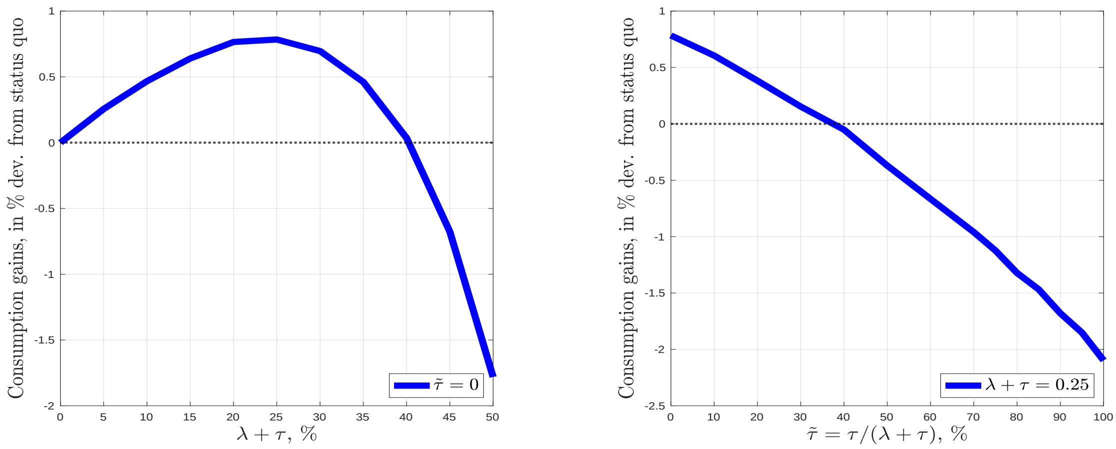
$$\mathbb{W}^* = \max_{\lambda_S+\tau_S,\, \tilde{\tau}_S} \mathbb{W}_0(\mathcal{T}).$$

Both $\lambda_S + \tau_S$ and $\tilde{\tau}_S$ have support on the unit interval. Numerically, the unit interval is discretized uniformly with 21 points (step size 0.05).

Panel (a) of Figure 10 sets $\tilde{\tau}_S = 0$ and plots the welfare gains $\mathbb{W}_0(\mathcal{T})$ in (50) as a function of $\tau_S + \lambda_S$ (in percentage). Panel (b) sets $\lambda_S + \tau_S = 0.25$ and plots $\mathbb{W}_0(\mathcal{T})$ as a function of

$\tilde{\tau}_S$ (in percentage). Welfare is hump shaped in $\tau_S + \lambda_S$ and is monotonically decreasing in $\tilde{\tau}_S$, implying that it is optimal setting $\tilde{\tau}_S = 0$. Welfare is maximized at $\mathcal{T}^* = \{0.25, 0\}$, which yields

Figure 10: Welfare effects of subsidy



(a) Size of subsidy, $\lambda_S + \tau_S$ **(b) Timing of subsidy, $\tilde{\tau}_S$**

a consumption equivalent increase in welfare of two-thirds of a percentage point. The optimal subsidy $(\tau_S + \lambda_S)k_S$ amounts to roughly 105,000€, which follows from multiplying 0.25 to the start-up investment cost measured in 2020 Euros in the South into 2020, k_S^{20}, obtained using

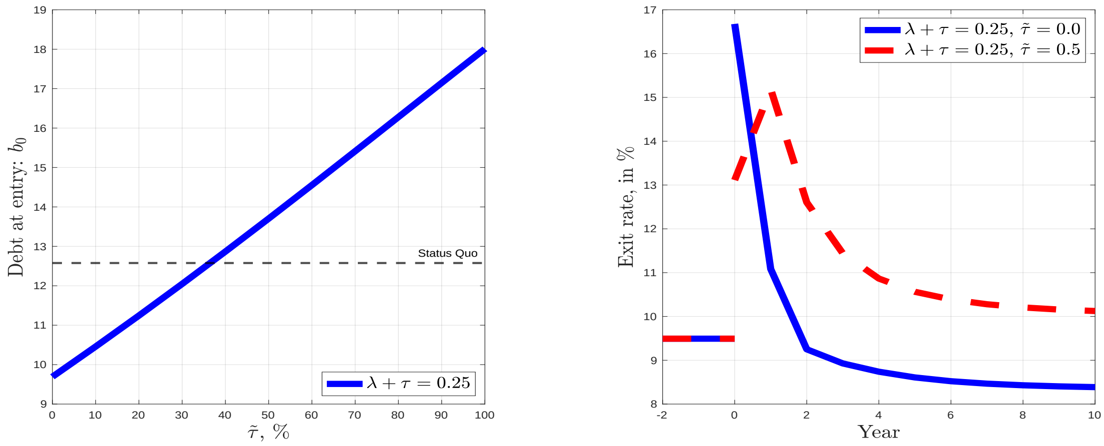
$$k_S^{20} = \frac{y_S^{20}/\ell_S^{20}}{y_S/\ell_S} \times k_S = 417,900€ \tag{53}$$

where y_S^{20}/ℓ_S^{20} denotes GDP per capita in the South in 2020 equal to 25,400€, while y_S/ℓ_S and k_S are as in Table 3.

For $\tilde{\tau}_S$ greater than 0.4, a subsidy of optimal size, $\tau_S + \lambda_S = 0.25$, yields welfare losses, see panel (b) of Figure 10. Welfare losses could be sizeable: $\tilde{\tau}_S = 1$ yields a consumption equivalent losses of 2 percentage points. Figure 11 explains why this happens. We set the subsidy at its optimal size $\tau_S + \lambda_S = 0.25$. Panel (a) shows that the average debt value ratio of start-ups in the South b_{S0} is increasing in $\tilde{\tau}_S$: a manifestation of the reimbursement effect discussed in Section 4. As shown in panel (b) this makes new firms in the South more likely to go bankrupt, which leads to welfare losses.

Table 4 studies welfare gains under the optimal subsidy $\mathcal{T}^*$. The first column focuses on aggregate welfare: it reports the welfare gains $\mathbb{W}_0(\mathcal{T})$ (row 1), the steady welfare gains neglecting transitional dynamics (row 2), and the 4 components of $\mathbb{W}_0(\mathcal{T})$ in (50), due to greater output Y_t (row 3), investment I_t (row 4), leisure costs $\mathfrak{C}_t$ (row 5) and living costs H_t (row 6). Column 2 and 3 are analogous but focus on the welfare gains per worker in the South $\mathbb{W}_0^S(\mathcal{T})$ and the North $\mathbb{W}_0^N(\mathcal{T})$, respectively. We use (52) and decompose the welfare gains per worker in province $i = N, S$ into 5 terms: the province level counterpart of the 4 components in column 1 (due to output, investment, leisure and livings costs) plus the contribution of net transfers, which contributes positively to welfare in the South and negatively in the North. Column 4 of Table 4 is equal to the difference between column 1 and half the sum of columns 2-3. It arises because the labor force, that was initially balanced between North and South, progressively moves to the South.

Figure 11: Leverage, exit and timing in the South



(a) Debt value ratio at entry

(b) Impulse response of exit rate

Table 4: Consumption equivalent gains at optimal subsidy, $\lambda + \tau = 0.25$ & $\tilde{\tau} = 0$

	Aggregate $\mathbb{W}_0$	South $\mathbb{W}_0^S$	North $\mathbb{W}_0^N$	Reallocation $\mathbb{W}_0 - \frac{\mathbb{W}_0^S+\mathbb{W}_0^N}{2}$
Total welfare	0.78	0.81	0.78	-0.80
SS welfare only	3.25	4.12	2.44	-3.31
Output	4.90	9.62	1.42	-6.14
Investment	-5.58	-11.10	0.10	5.42
Congestions	1.70	-1.10	3.46	-0.66
Entrepreneur leisure	-0.23	-0.52	-0.04	0.33
Net tax transfer	0.00	3.91	-4.16	0.25

The steady state welfare gains are between three and four times larger than the true welfare gains $\mathbb{W}_0(\mathcal{T})$: steady state welfare neglects the initial costs in business creation and the initial spike in bankruptcies. At $\mathcal{T}^*$, $\mathbb{W}_0(\mathcal{T})$ is positive because the increase in output and the reduction in living costs more than compensate for the increase in investment and leisure costs: output yields a 4.9 percentage points increases in consumption and the fall in living costs amounts to a 1.70 percentage points increase in consumption compared to the fall in consumption of 5.81 percentage points due to the combined effects of investment and leisure costs. Welfare gains are distributed evenly between the North and the South: the percentage increase in consumption in the South is 0.81 compared with 0.78 in the North. The South gains more in terms of output and tax transfers, but lose more in terms of greater investment and experience an increase in livings costs. The North gains less in output, it has to pay for the tax transfers, but gains in terms of living costs because its labor force progressively move to the South.

Table 5 replicates Table 4 with a suboptimal timing of $\tilde{\tau}_S = 0.30$. Welfare gains become negative both in the South and the North. Relative to the case $\tilde{\tau}_S = 0$, the output gains fall, while investment costs are little affected, due to the increased bankruptcy rate of firms. Neglecting the transitional dynamics would mistakenly lead to the conclusion that the subsidy yields sizeable welfare gains, greater than 1 percentage point.

Table 5: **Gains at optimal size and suboptimal timing,** $\lambda + \tau = 0.25$ **&** $\tilde{\tau} = 0.50$

	Aggregate $\mathbb{W}_0$	South $\mathbb{W}_0^S$	North $\mathbb{W}_0^N$	Reallocation $\mathbb{W}_0 - \frac{\mathbb{W}_0^S + \mathbb{W}_0^N}{2}$
Total welfare	-0.37	-0.34	-0.37	0.34
SS welfare only	1.42	2.05	0.84	-1.48
Output	3.45	6.84	1.00	-4.39
Investment	-4.90	-9.76	0.07	4.79
Congestions	1.24	-0.77	2.47	-0.46
Entrepreneur leisure	-0.15	-0.35	-0.03	0.23
Net tax transfer	0.00	3.71	-3.88	0.17

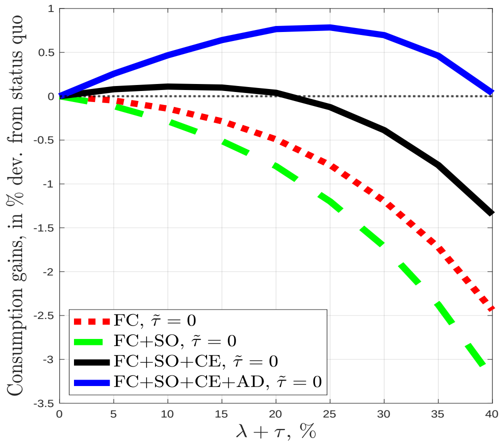
6.3 Determinants of welfare gains

The subsidy affects welfare through the entry rate and the exit rate margin. Entry in the South is inefficient because of aggregate demand externalities (**AD**), a spatial allocation of labor distorted by congestion externalities (**CE**), and firm financial conditions (**FC**). **AD** and **CE** cause too low entry. **FC** causes excessive entry. The subsidy affects the exit rate margin through **FC** which determines the debt value ratio of start-ups and the probability that they may go bankrupt. In the short run, the subsidy also causes a shake-out (**SO**): the bankruptcy rate instantaneously increases because local wages increase, which reduces firm profitability. To quantify how **AD**, **CE**, **FC**, and **SO** contribute to the welfare gains $\mathbb{W}_0(\mathcal{T})$, we solve for three counterfactual economies (see Appendix J for the details). In the first we isolate the contribution of **FC**, by solving a version of the model where the labor forces ℓ_i's and the aggregate demand shifter Y remain at their initial steady-state value. Additionally, we remove the shake-out effect by imposing that on impact the distribution of incumbent firms is unaffected by the instantaneous change in $\mathcal{R}_S$ due to $\mathcal{T}$. To isolate the contribution of **SO**, we solve for a second counterfactual economy with labor forces and demand shifter at steady state value but where now, on impact, the density of the debt-value ratio of firms in the South shifts to the right by $\mathcal{D} = \ln \mathcal{R}_S - \ln \mathcal{R}_S(\mathcal{T})$. To measure the contribution of **CE,** we solve for a third counterfactual economy where also the labor force responds to $\mathcal{T}$, still maintaining the demand shifter at the steady state value Y. Finally we allow the demand shifter $\mathbf{Y}(\mathcal{T})$ to respond and measure the contribution of **AD** as a residual.

Figure 12 shows the welfare gains for different sizes of the subsidy $\lambda_S + \tau_S$ when $\tilde{\tau}_S = 0$. The red dotted line corresponds to the welfare gains in the first counterfactual economy, the green dashed line to the second, the black solid line to the third and the blue solid line to our economy. The red dotted line shows that with **FC** only, welfare falls with $\lambda_S + \tau_S$. This is the result of the excess entry caused by cheap credit. The difference between the green dashed line and the red dotted line measures the contribution to welfare of **SO**, which is sizeable and negative. At the optimal subsidy $\mathcal{T}^* = \{0.25, 0\}$, **FC** together with **SO** yield a welfare loss equivalent to almost a 1 percentage point fall in status quo consumption. The difference between the black solid line and the green dashed line measures the contribution to welfare of **CE**, which is sizeable and positive. Finally the difference between the blue solid line and the black solid line is the contribution of **AD.** At the optimal subsidy $\mathcal{T}^*$, **CE** and **AD** account almost equally for the difference in welfare gains between the baseline (blue solid line) and the combined effects of **FC** and **SO** (green dashed line).

To further analyze the effects of financial frictions, we also compare our economy with a

Figure 12: Welfare gains decomposition at optimal subsidy



Notes: **FC** refers to an economy where the labor forces ℓ_i's and the demand shifter Y are at their initial steady-state value and on impact the distribution of incumbent firms is unaffected by the change in $\mathcal{R}_S$. **FC+SO** refers to an economy where the labor forces ℓ_i's and the demand shifter Y are at their initial steady-state value and on impact the distribution of debt value ratios $\hat{b}$ in the South instantaneously shifts to the right by $\mathcal{D} = \ln \mathcal{R}_S - \ln \mathcal{R}_S(\mathcal{T})$ due to the fall in market profitability. **FC+SO+CE** refers to an economy where only the demand shifter Y is at its initial steady-state value. **FC+SO+CE+AD** is the baseline economy.

more canonical *Dixit-Stiglitz* economy without debt and bankruptcy. We assume that there are no frictions in financing start-ups and that firms do not dilute past debt. As a result, firms in province i exit exogenously at rate δ_i. Due to (6), the value of a firm with technology Z_{jt} in province i is simply equal to

$$V_i^{DS}(Z_{jt}) = \frac{\mathcal{R}_i Z_{jt}}{r + \delta_i},$$

and the free-entry condition in province i reads as

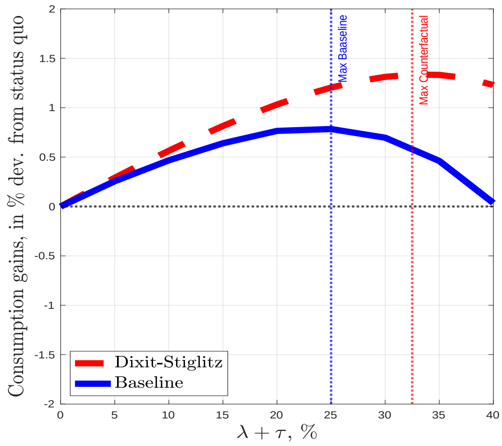
$$(1 - \lambda_i - \tau_i) k_i = \frac{\mathcal{R}_i E_i[Z_{j0}]}{r + \delta_i}, \tag{54}$$

where $E_i[Z_{j0}]$ is the expected technology upon entry in province i and $\lambda_i + \tau_i$ is the size of the business creation subsidy in the province. Notice that now the timing of the subsidy is irrelevant for the free entry condition. (54) pins down $\mathcal{R}_i$ in our counterfactual Dixit-Stiglitz economy. All the other assumptions of the model (in terms of geography, worker mobility, tax financing, etc.) are as in the baseline economy, see Appendix K for the details.

In solving the Dixit-Stiglitz economy, we recalibrate δ_i to match the average exit rate of the data, and set k_i to guarantee that market profitability $\mathcal{R}_i$ is initially unchanged, so that the baseline and the Dixit-Stiglitz economy with zero subsidies share the same output Y, wages, and labor allocation across provinces.[27] All the other parameters are as in Table 3. The red dashed line of Figure 13 plots welfare gains in the Dixit-Stiglitz economy as a function of the size of the subsidy $\lambda_S + \tau_S$. The blue solid line plots welfare gains in the baseline economy with $\tilde{\tau}_S = 0$. The optimal subsidy in the Dixit-Stiglitz economy is 0.33 compared with 0.25 in the baseline. The welfare gains at the optimal subsidy are twice as large. Financial market conditions not

[27] The recalibrated values are $\delta_S = 0.097$, $\delta_N = 0.087$, $k_S = 10.6024$, and $k_N = 26.99$.

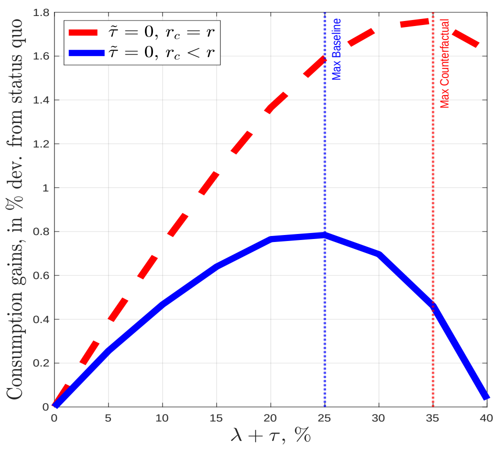
Figure 13: Comparison with a Dixit-Stiglitz economy



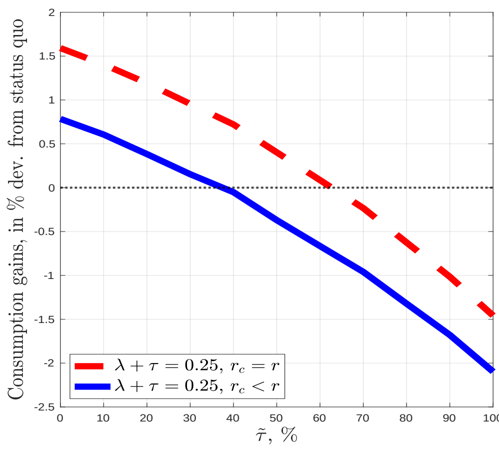
only matter for the optimal design of the subsidy. They also tend to dampen both the size of and the welfare gains at the optimal subsidy.

Firms' incentives to overborrow are stronger when credit is cheap as in recent years: $l_i(b)$ in (18) is decreasing in r_c and equal to zero when $r = r_c$. To analyze the contribution of cheap credit to the welfare results, we study an economy where the cost of credit is $r_c = r$, keeping all the other parameters unchanged. Panel (a) in Figure 14 plots welfare gains as a function of the size of the subsidy when $\tilde{\tau} = 0$ in the baseline economy (blue solid line) and in an economy with $r_c = r$ (red dashed line). The optimal subsidy is approximately 10 percentage points larger than in the baseline calibration. With $r_c = r$, welfare gains are twice as large: firms accumulate less debt, the number of inefficient bankruptcies fall and thereby subsidizing entry is more valuable. Panel (b) studies the effects on welfare of changing the timing of the subsidy $\tilde{\tau}$ at the optimal subsidy size (different for the two economies). Because of the ratchet effect, it remains optimal setting $\tilde{\tau} = 0$ also when $r_c = r$. When $r_c = r$, at the optimal size of the subsidy, $\mathcal{T}$ yields welfare losses when $\tilde{\tau}_S$ is greater than 0.6, compared with 0.4 in the baseline.

Figure 14: Optimal subsidy and the cost of credit r_c



(a) The role of $r_c < r$: optimal size



(b) The role of $r_c < r$: optimal timing

7 I stay in the South

We now use the model to study the effects of the ISS policy introduced in Italy in 2017. First we calibrate the ISS subsidy, then we incorporate the ISS policy into the model and study its welfare properties. Finally we run difference-in-differences regressions on model simulated data and compares results with the empirical estimates reported in Table 2.

Calibration On average in 2022, an ISS beneficiary received a non-repayable grant of about ζ_N =70,000€ and an eight-years bank-loan of 60,000€.[28] The loan is fully insured against the risk of default by a government agency (Fondo di Garanzia) and interest rates are paid by another agency (Invitalia). To quantify the value to the firm of the government guarantee ζ_G and of the exemption from interest rate payments ζ_I, we price a bond that pays a per-period coupon $\varkappa_S$, matures at Poisson arrival rate 1/8 (8 years maturity), goes bankrupt at Poisson arrival rate $\tilde{\delta}_S$ and upon bankruptcy pays a debt guarantee φ_S. Financiers would price the bond at

$$x_B = \frac{\varkappa_S + \tilde{\delta}_S \varphi_S + 1/8}{r_c + \delta_S + \tilde{\delta}_S + 1/8}.$$

With the government guarantee, financiers get fully reimbursed upon bankruptcy and the value of the bond increases from x_B to $x_G = \frac{\varkappa_S + \tilde{\delta}_S + 1/8}{r_c + \delta_S + \tilde{\delta}_S + 1/8}$. To obtain 60,000€, the firm issues B_G =60,000/x_G fully guaranteed bonds. The parameters r_c, r, δ_S, φ_S and $\varkappa_S$ are as in Table 3 and we set $\tilde{\delta}_S = 0.075$, equal to the average bankruptcy rate of firms during their first 8 years of life. The value of the debt guarantee to the firm, ζ_G, is equal to the induced increase in the market value of bonds:

$$\zeta_G = (x_G - x_B) \times B_G = \frac{\tilde{\delta}_S (1 - \varphi_S)}{\varkappa_S + \tilde{\delta}_S + 1/8} \times 60,000 = 11,635€.$$

The value to the firm of the interest rate exemption, ζ_I, is equal to the expected present value of interest rate payments discounted at the entrepreneur's discount rate:

$$\zeta_I = \frac{\varkappa_S}{r + \delta_S + \tilde{\delta}_S + 1/8} \times \frac{60,000}{x_G} = 13,280€.$$

Using k_S^{20} in (53), we conclude that the size of the ISS subsidy is

$$\lambda_S^{ISS} + \tau_S^{ISS} = \frac{\zeta_N + \zeta_G + \zeta_I}{k_S^{20}} = 0.2028 \simeq 0.23.$$

The non repayable grant ζ_N is paid in two equal tranches: the first upon completing half of the start-up investment; the second upon full completion, within two months after submitting all invoices to Invitalia. Therefore, the first tranche of ζ_N and the debt guarantee ζ_G are paid ex-ante while the second tranche of ζ_N and the interest rate exemption ζ_I are received ex-post. Then, the fraction of ISS subsidy paid ex-post is equal to

$$\tilde{\tau}_S^{ISS} = \frac{\frac{\zeta_N}{2} + \zeta_I}{\zeta_N + \zeta_G + \zeta_I} = 0.5087 \simeq 0.5.$$

[28]The numbers are obtained from `https://opencoesione.gov.it` as downloaded on 24 April 2023.

Modelling Only a proportion ι of Southern start-ups get subsidized. We refer to ι as the *incidence* of the ISS subsidy, roughly equal to 20% in 2020. Entrepreneurs invest their wealth in the start-up and get to know whether they receive the subsidy at the time of the start-up investment k_S. Firms entitled to the subsidy issue debt B^Y_{S0}, those without subsidy issue B^N_{S0} equal to

$$\begin{aligned} B^Y_{S0} &= \frac{1}{x_{S0}} \cdot \left[\left(1-\lambda^{ISS}_S\right) k_S - \varpi_S\right] \\ B^N_{S0} &= \frac{1}{x_{S0}} \cdot \left(k_S - \varpi_S\right) \end{aligned}$$

The value of a bond used to finance the start-up investment x_{S0} satisfies

$$x_{S0} = \sum_{z\in\mathcal{Z}_i} \left[\iota x_S\left(\frac{B^Y_{S0}}{e^z\mathcal{R}_S}\right) + (1-\iota)\, x_S\left(\frac{B^N_{S0}}{e^z\mathcal{R}_S}\right)\right] \cdot g_{Sz},$$

which is analogous to (21) after incorporating the probability $1-\iota$ that the start-up might not obtain the subsidy. The assumption that the bonds of subsidized and unsubsidized start-ups are priced equally is conservative: it makes the subsidy more beneficial for welfare because it increases the value of the bonds sold by unsubsidized start-ups which reduces the (negative) impact of the subsidy on their leverage ratio and bankruptcy risk.

The expected equity value of a Southern start-up (excluding the ex-post subsidy) becomes

$$\mathcal{V}_S = \sum_{z\in\mathcal{Z}_S} \left[\iota\, v_S\left(\frac{B^Y_{S0}}{\mathcal{R}_S e^z}\right) + (1-\iota)\, v_S\left(\frac{B^N_{S0}}{\mathcal{R}_S e^z}\right)\right] \cdot \mathcal{R}_S e^z \cdot g_{Sz}.$$

Given the incidence of the subsidy ι, the free entry condition analogous to (23) is now

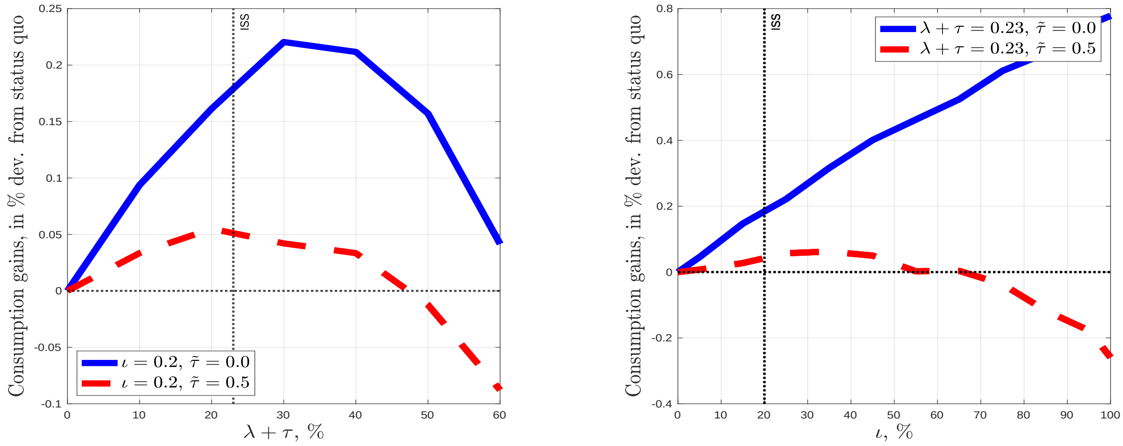
$$\varpi_S = \mathcal{V}_S + \iota\tau_S k_S.$$

All other equilibrium conditions are as in Section (4).

Welfare effects Panel (a) of Figure 15 illustrates the consumption equivalent welfare gains (relative to the status quo), $\mathbb{W}_0(\mathcal{T})$, as a function of the size of the subsidy $\lambda_S + \tau_S$. The blue solid line represents the welfare gains with incidence $\iota = 0.2$ and (optimal) timing $\tilde{\tau} = 0$. The red dashed line represents the welfare gains with incidence $\iota = 0.2$ and timing $\tilde{\tau} = 0.5$. The welfare gains under the ISS policy correspond to the evaluation of the red dashed line at the subsidy size $\lambda_S + \tau_S = 0.23$. It shows small consumption gains of 5 basis points. Welfare gains exhibit a hump-shaped pattern in response to the size of the subsidy. The peak of welfare is 4 times larger with $\tilde{\tau} = 0$ than with the (suboptimal) ISS timing of $\tilde{\tau} = 0.5$.

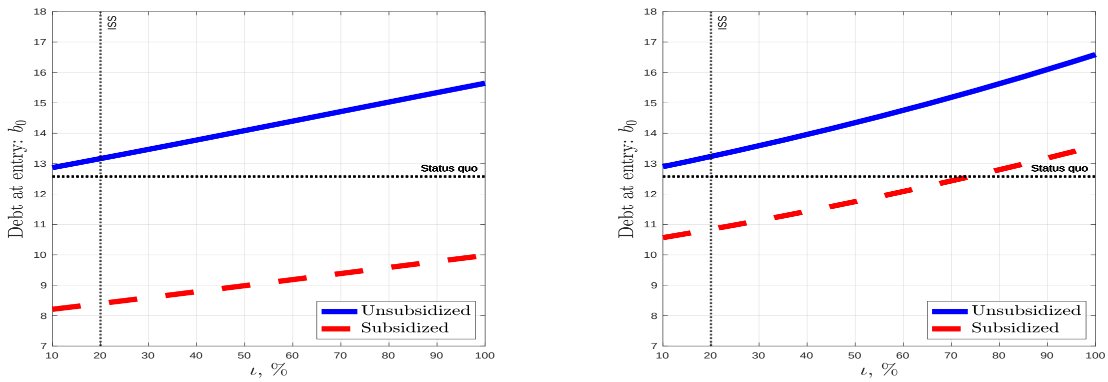
Panel (b) of Figure 15 illustrates the effects on welfare of increasing the incidence of the subsidy ι. The subsidy is set at the ISS size $\lambda_S + \tau_S = 0.23$. The blue solid line represents welfare gains $\mathbb{W}_0(\mathcal{T})$ as a function of ι under optimal timing $\tilde{\tau} = 0$. The red dashed line represents welfare gains under the suboptimal ISS timing $\tilde{\tau} = 0.5$. At the optimal timing $\tilde{\tau} = 0$, welfare increases as ι increases. Under the suboptimal ISS timing $\tilde{\tau} = 0.5$, welfare first increases and then decreases: with an incidence greater than 50 percent the policy yields welfare losses. Expanding the incidence of the subsidy to 100% under the ISS timing leads to a fall in status quo consumption of 0.4%. This indicates that while the size of the ISS subsidy is close to optimal, its timing is suboptimal.

Figure 15: Welfare effects of I-Stay-in-the-South subsidy

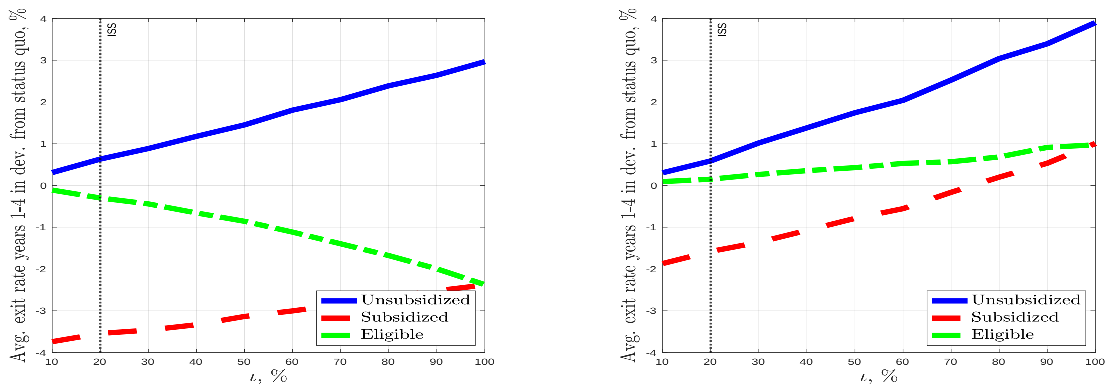


(a) Size **(b) Incidence**

Figure 16: Effects in the South of changing the incidence of ISS



(a) Initial debt to value ratio, $\tilde{\tau} = 0$ **(b) Initial debt to value ratio, $\tilde{\tau} = 0.5$**



(c) Exit rate in first 4 years, $\tilde{\tau} = 0$ **(d) Exit rate in first 4 years, $\tilde{\tau} = 0.5$**

Notes: For all lines the size of the subsidy is $\lambda_S + \tau_S = 0.23$, which is the calibrated size under the ISS policy. Under the ISS policy the incidence of the subsidy is $\iota = 0.2$ and the timing is $\tilde{\tau} = 0.5$.

We further investigate why the ISS policy yields small welfare gains and why it would yield welfare losses if the incidence ι were increased. In Figure 16, we set the subsidy at the ISS size

$\lambda_S + \tau_S = 0.23$, and vary the incidence of the subsidy ι. The left panels (a) and (c) depict the results under the optimal timing $\tilde{\tau} = 0$. The right panels (b) and (d) show the results under the ISS timing $\tilde{\tau} = 0.5$. Panels (a) and (b) display the variations in the average debt value ratio of Southern start-ups $E(b_{S0})$ for subsidized start-ups (red dashed line) and unsubsidized start-ups (blue solid line). The horizontal black dotted line represents the value of $E(b_{S0})$ under the baseline calibration without subsidies. Panels (c) and (d) focus on the average exit rate response of Southern firms in the 4 years after the introduction of the subsidy (corresponding to the years 2017-2020), in deviation from their value in the absence of subsidy. The red dashed line represents the response for subsidized firms, the green dashed line represents the response for all ex-ante eligible start-ups, and the solid blue line represents all unsubsidized firms (all incumbent firms plus the proportion $1 - \iota$ of ex-ante eligible start-ups).

With $\tilde{\tau} = 0.5$, the debt value ratio of subsidized start-ups falls less than under the optimal timing $\tilde{\tau} = 0$ and eventually when the ISS incidence becomes bigger than 70 percent, the debt value ratio of subsidized start-ups even increases relative to its status quo value. Unsubsidized start-ups always experience an increase in their debt value ratio, both when $\tilde{\tau} = 0$ and when $\tilde{\tau} = 0.5$. The increase is more pronounced when $\tilde{\tau} = 0.5$, particularly so when the incidence of the subsidy becomes large: a higher incidence reduces market profitability $\mathcal{R}_S$ more, which amplifies the negative externality of the subsidy on the debt value ratio of unsubsidized firms (either start-ups or incumbents). With the suboptimal ISS timing ($\tilde{\tau} = 0.5$) the bankruptcy rate of ex-ante eligible start-ups is always above its status-quo value and keeps increasing with the incidence of the subsidy ι. With an optimal timing ($\tilde{\tau} = 0$), the bankruptcy rate of ex-ante eligible start-ups is smaller than its status quo value and it keeps falling with ι. Under the current ISS policy ($\lambda + \tau = 0.23$, $\tilde{\tau} = 0.5$ and $\iota = 0.2$), recipients of the ISS subsidy experience a large reduction in their bankruptcy rate, close to 2 percentage points, in line with the policy evaluation report commissioned by Invitalia (Italiacamp 2022), which reports low bankruptcy rates among ISS recipients.

To evaluate the performance of the model in reproducing the sign and magnitude of the difference-in-differences estimates reported in Table 2, we estimate the regression (5) on model simulated data. Table 6 shows the results. We mimic the sample of provinces in the data over 11 years, 7 before the introduction of the ISS subsidy (2010-2016) and 4 after its introduction (2017-2020). For province $i = N, S$ and $t = 2010, ...2020$ we calculate business creation, $\tilde{m}_{it}$. In 2010-2016 $\tilde{m}_{it}$ is equal to its steady state value in the economy without subsidies. In 2017-2020 $\tilde{m}_{Nt}$ and $\tilde{m}_{St}$ are equal to the response in the first 4 years after the introduction of the ISS subsidy. We define firm groups $\mathbf{G} =(t, i, a)$ by year t, province type i and firm age a. For each group $\mathbf{G} =(t, i, a)$ we calculate the corresponding (i) business exit rate in percentage terms (i.e. multiplied by 100), (ii) average leverage ratio, (iii) logged average labor productivity and (iv) the logged average firm employment size. We also construct the dummies for *Eligible-to-Subsidy* and *South-Incumbent* constructed as in the data and then run the regression (5) on model simulated data, see Appendix L for further details. The coefficient on *Eligible-to-Subsidy* measures the effect of being eligible for the subsidy relative to other firms located in provinces of the North. The coefficient on *South-Incumbent* measures the effect (again relative to the North) of being a firm born without receiving the ISS subsidy in a market where other firms got subsidized.

Provinces exposed to the ISS subsidy experience an increase in the business creation rate of around 16 percentage points compared with 10 percentage points in the data (column 1). The exit rate of start-ups entitled to the subsidy and of other incumbent firms both increase by 21 and 37 basis points compared with 54 and 22 basis points in the data (column 2). In treated provinces, the average leverage ratio of firms also increases both for start-ups eligible for the subsidy and unsubsidized incumbent firms (column 3). The increase in the leverage ratio is less than in the data possibly because bad loans remain in the balance sheets of CADS firms longer than in the model. The average productivity of firms increase both for new and incumbent firms by around 4 percentage points, roughly in line with the data (column 4). Incumbent firms scale down their employment size by around 6 percentage points compared with 3 pecerntage points in the data (column 5). The model accurately reflects the direction of the effects reported in Table 2 and the differences in magnitudes are not statistically significant given the standard errors of the empirical estimates.

Table 6: Difference-in-differences effects of ISS subsidy on model simulated data

	(1)	(2)	(3)	(4)	(5)
VARIABLES	Creation	Exit rate	Leverage	Productivity	Size
Eligible-to-Subsidy	0.16***	0.25***	0.01***	0.05***	-0.09***
	(0.01)	(0.01)	(0.00)	(0.00)	(0.00)
South-Incumbent		0.40***	0.03***	0.05***	-0.07***
		(0.01)	(0.00)	(0.00)	(0.00)
Observations	1,045	6,175	6,175	6,175	6,175
R^2	0.78	0.99	1.00	1.00	1.00
Province dummy	Y	Y	Y	Y	Y
Year dummy	Y	Y	Y	Y	Y
Age dummy	N	Y	Y	Y	Y
Province × Age dummy	N	Y	Y	Y	Y
Year × Age dummy	N	Y	Y	Y	Y

Notes: Estimates from running the regression in (5) on model simulated data. Business creation, productivity and employment size are the log of firm averages. The business exit rate is in percentage terms (i.e. multiplied by 100). The leverage ratio is in level. Standard errors in parentheses *** p<0.01, ** p<0.05, * p<0.10.

8 Conclusions

The age profile of business exit rates and leverage ratios vary across Italian provinces. As they age, Southern firms accumulate more debt (relative to productivity) and become more prone to failure due to excessive debt. To explain these differences, we used a geographical model of firm dynamics where financial frictions arise because firms inefficiently accumulate new debt to dilute the value of past debt. The incentive to overaccumulate debt is stronger when idiosyncratic risk is high, as in the South, and credit is cheap, as in recent years.

We studied the optimal business creation subsidy in the least productive provinces of the economy (the South). The dynamics of leverage and bankruptcy are important for the optimal design of the subsidy because of general equilibrium effects. The subsidy always stimulates business creation and reduces firm profitability due to increased competition in the local market. If paid out before business formation (ex-ante), the subsidy reduces the debt of start-ups and their bankruptcy rate; if paid out after business formation (ex-post) as a refund of start-up expenditures, the subsidy crowds out the equity rather than the debt of start-ups leading to higher leverage ratios for all firms (exhisting and new) and an increase in aggregate bankruptcies. Under our calibration, the optimal subsidy in the South yields a consumption equivalent increase in welfare of almost one percent. A subsidy of optimal size, paid out ex-post in a proportion of 60 percent, yields a similar reduction in consumption. In the calibrated model, the I Stay in the South business creation subsidy, available since 2017 to Southern start-ups, yields only marginal welfare gains due to a suboptimal timing in the payment of subsidies. The analysis shows that general equilibrium forces are important for the welfare effect of policies, through their impact on leverage ratios and bankruptcy rates.

For simplicity, we did not consider agglomeration externalities, which the field of economic geography has extensively documented as highly relevant for welfare (see Duranton and Puga 2004; and Venables 2010 for discussions on their significance). Although it would be relatively straightforward to incorporate certain agglomeration externalities into our model, specifying their form and accurately quantifying their magnitude in the North and South of Italy would pose greater challenges. Even in the presence of agglomeration externalities, the welfare effects of business creation subsidies would still depend on their impact on firms' capital structure, the trade-off between debt and equity financing, and the overall response of the bankruptcy rate. Our findings indicate that these effects, which are often overlooked in policy evaluation studies, have important implications for welfare. These considerations remain valid regardless of the presence of agglomeration externalities.

Moreover, the model does not assume asymmetric information regarding project quality and default probability. An ex-post subsidy, such as a tax credit, would provide two distinct advantages. First, tax credits incentivize projects with a higher likelihood of generating future profits (or having a low probability of default), as they are only beneficial in the presence of taxable profits. Additionally, in the face of potential tax evasion, tax credits promote profit declaration and consequently the reporting of other items like invoices to suppliers, employment contracts, etc., which contribute to the tax base. We contend that the absence of asymmetric information appears particularly plausible in the case of new firms, where the entrepreneur must also learn to assess the true quality of the project. Furthermore, in the context of start-ups, profits often materialize after several years; as a result, measures like tax credits may have a diminished incentivizing effect.

We focused the analysis on subsidies to business entry but the reimbursement effect emphasized in this paper is more general and could be relevant for any business subsidy aimed at stimulating investment, innovation, hiring, or the adoption of new technologies.

In practice, assessing the quality of business projects in advance can be challenging, and there is always a risk that unscrupulous entrepreneurs exploit subsidies opportunistically. This explains why policymakers are often cautious about providing business subsidies entirely upfront. We have found that, in a world with perfectly competitive credit markets featuring cheap credit, providing the subsidy as a refund of expenditures can result in significant welfare losses. All this suggests that policy makers should formally require entrepreneurs to allocate the reimbursed funds toward debt repayment rather than granting entrepreneurs full discretion on funds usage.

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Appendix
"Subsidizing Business Entry in Competitive Credit Markets"

Appendix A further describes the data. Appendix B contains additional empirical results. Appendix C reports firm level regressions for the age profile of business exit rates and leverage ratio and shows that the evidence in Figure 2 is robust. Appendix D investigates how the response of business exit to idiosyncratic demand shock is affected by firm leverage. Appendix E checks the parallel trend assumption for the results in Table 2. Appendix F verifies the guess for the standardized value function. Appendix G derives the optimal policy for firm leverage. Appendix H derives the Hamilton-Jacobi-Bellman equation for the private and social value of a firm in (25) and (26). Appendix I characterizes equilibrium over the transitional dynamics and discusses numerical procedures to solve the model. Appendix J solves for the equilibrium of the counterfactual economies discussed in Section 6.3 and decomposes the source of welfare gains. Appendix K compares welfare gains in our model with those in a Dixit-Stiglitz version of our economy without debt. Appendix L describes how we constructed the sample of firms used to estimate the regression (5) on model simulated data as reported in Table 6.

A Data appendix

We combine *six* sources of data for Italy: i) the Credit Register managed by the Bank of Italy; ii) the Business Register by the Chambers of Commerce (InfoCamere); iii) Universo imprese INPS (UNIMPS) from the Italian National Social Security Institute (INPS); iv) the Company Accounts Data Service (CADS) assembled by CERVED Group; v) the Survey of Industrial and Service Firms (INVIND) run by the Bank of Italy; and vi) the Labor Force Survey and other data from the Italian National Institute of Statistics (ISTAT). i-iv are administrative data, v) and vi) are surveys. Access to i-vi is subject to strict confidentiality requirements lifted only to Bank of Italy employees.

Central Credit Registry and TAXIA Bank loans to businesses are from the Central Credit Registry (CCR), operated by the Bank of Italy. CCR provides monthly information on loans by banks and other financial companies to borrowers. It reports (i) all loans to borrowers whose overall exposure to a single intermediary exceeds a threshold (equal to €75,000 before January 2009 and to €30,000 thereafter) and (ii) all loans of borrowers with some non-performing loans (NPLs) classified as bad loans ("Sofferenze"). CCR specifies the loan amount, its insolvency status, the presence of collateral or guarantees, the origination date of the loan and its maturity date. Banks and other financial intermediaries are legally obliged to submit information to CCR. Loans in CCR are classified into the following three categories: (i) credit lines, (ii) fixed-term loans, and (iii) loans backed by account receivables. Data on interest rates charged by financial intermediaries are from TAXIA which is part of CCR. TAXIA provides information at the quarterly frequency on the interest rates paid by clients with a total bank exposure exceeding €75,000, excluding all NPLs. TAXIA covers more than 80% of all loans granted by the banking system. We measure interest rates as the sum of gross annual interest rate payments plus fees, and other commissions.

InfoCamere InfoCamere is managed by the Italian Chambers of Commerce. It is available at `https://accessoallebanchedati.registroimprese.it/abdo/?lang=it`). It is constructed using the Business Register digitized since 1993. InfoCamere provides information on the name of all businesses, their legal form, type of business activity, identity of board members and directors, tax code of the business, and in case of business exit, its reason. We exclude sole proprietorships from the analysis and focus on corporations— and in some robustness exercises also on partnerships. To identify exit of a business with bankruptcy, we use information on insolvency proceedings which is recorded at the start of a bankruptcy procedure. Insolvency proceedings involve the participation of courts and/or other public authorities and are formally codified by the Italian bankruptcy legislation. They happen at the beginning of a bankruptcy procedure which is typically concluded only after a lengthy process, which lasts on average 7.5 years according to a report by the Ministry of Justice in 2019. InfoCamere also reports information on the geographic location of businesses and since 2010 on the identity of all business shareholders and their corresponding share-holdings.

UNINPS For welfare related reasons, INPS routinely collects data from all private (non-agricultural) employers. Since 1990 UNINPS reports the number of employees and wage payments of all businesses with at least one employee. We use UNINPS to calculate the average number of payroll employees in the year of each business. We use this information to identify the year of birth, the year of exit and the age (in years) of the business. A firm is *new*, if it employs workers for the first time. A firm *exits*, if its employment drops forever to zero—which requires information for some years after exit. Firm age is the number of years since the firm has first employed some workers, calculated for all firms born established 1990. A firm exits with "bankruptcy" if the firm exits leaving some bad loans, as recovered from the Credit Register, or with a formal bankruptcy procedure, as recovered from the insolvency proceedings by InfoCamere. We follow firms over time to identify how they evolve as they age. We calculate time averages over the years 2007-2015. We pool in the same age group (labelled age 20) all firms with more than 17 years of age. Note that 2007 is the first year when we observe firms of age 17.

CADS CADS is a proprietary database owned by Cerved Group, a European credit rating agency. CADS reports the balance sheet and the income statement of all Italian limited liability companies since 1993. Firms are required by law to submit this information to the local Chambers of Commerce. Cerved uses this information to construct CADS which is updated annually. We use CADS to calculate total financial debt, value added, earnings, EBIT and assets of firms. Total financial debt is the sum of bank debt and other financial debt, excluding trade payables. Value added is the sum of the value of production and all provisions minus the sum of expenditures in raw materials and intermediate inputs. The leverage ratio is equal to total financial debt over firm's value added. For robustness, we also calculate the leverage ratio as equal to total bank debt over firm's value added. Return on Assets (ROA) is equal to Earnings Before Interest and Taxes (EBIT) over total assets.

INVIND INVIND, accessible through the interface at `https://www.bancaditalia.it/statistiche/tematiche/indagini-famiglie-imprese/imprese-industriali/distribuzione-microdati/index.html`, is a survey run annually by the Bank of Italy on a representative sample of firms. INVIND is available since 1995. INVIND is representative of the universe of companies with at least 20 employees operating in industrial sectors (manufacturing, energy, and extractive industries) and in non-financial private services, with administrative headquarters located in Italy. The universe of companies targeted by INVIND represents around 70% of total sales in Italy. INVIND uses a one-stage stratified sample design based on (i) 11-sectors, (ii) the number of employees of the company, and (iii) the region of the firm's headquarter; see (Bank of Italy 2014) for further description of INVIND. In the most recent waves, INVIND contains about 4,000 firms, 3,000 in the industrial sector and 1,000 in the service sector. Table 7 reports the list of the 20 NACE codes available in INVIND, that we use to construct sector dummies. We dropped from the sample firms operating in mineral extraction or construction and firms involved in extraordinary company operations (incorporations, mergers, spin-offs, and extraordinary assets transfers).

Table 7: List of NACE codes in INVIND

Codes	Section/Subsection
CB	Mining and quarrying except energy producing materials
DA	Manufacture of food products; beverages and tobacco
DB	Manufacture of textiles and textile products
DC	Manufacture of leather and leather products
DD	Manufacture of wood and wood products
DE	Manufacture of pulp, paper & paper product; publishing & printing
DF	Manufacture of coke, refined petroleum products & nuclear fuel
DG	Manufacture of chemicals, chemical products and man-made fibres
DH	Manufacture of rubber and plastic products
DI	Manufacture of other non-metallic mineral products
DJ	Manufacture of basic metals and fabricated metal products
DK	Manufacture of machinery and equipment n.e.c.
DL	Manufacture of electrical and optical equipment
DM	Manufacture of transport equipment
DN	Manufacturing n.e.c.
E	Electricity, gas and water supply
G	Wholesale & retail trade; repair of motor vehicles
H	Hotels and restaurants
I	Transport, storage and communication
K	Real estate, renting and business activities

The elasticity of firm demand ν_j in (3) is recovered using a specific question of INVIND. Both in 1996 and in 2007, firm managers in INVIND were asked about the value of $(1-\nu_j)\times 0.1$ through the following question: "*If your firm were to increase the selling prices by 10%, what percentage change in your nominal sales would be obtained, provided that all your competitors were to keep their prices unchanged and you were to leave all the other terms unchanged?*". We take the average self-reported sector-specific elasticity ν_j as an estimate of the demand elasticity faced by firms in the sector, see Pozzi and Schivardi (2016) for the implied reported elasticities ν_j. For each firm present in two consecutive waves of INVIND we calculate the following Wold innovations for revenue ϵ^r_{jt}, and prices ϵ^p_{jt}:

$$\epsilon^r_{jt} = \frac{r_{jt} - E_{jt-1}(r_{jt})}{r_{jt-1}} \qquad \text{and} \qquad \epsilon^p_{jt} = \frac{p_{jt} - E_{jt-1}(p_{jt})}{p_{jt-1}}.$$

The Wold innovation on revenue ϵ^r_{jt} is computed using the following threequestions in INVIND: (i) Revenue from the sale of goods and services in year t, r_{jt} (Mnemonic v210); (ii) Revenue from the sale of goods and services in year $t-1$, r_{jt-1} (Mnemonic v209); and (iii) Expected revenue from the sale of goods and services in year t, $E_{jt-1}(r_{jt})$ (Mnemonic v437).
The Wold innovation on price changes ϵ^p_{jt} is calculated using the following two questions in INVIND: (i) The mean percentage change in invoiced prices between year t and $t-1$, $\frac{p_{jt}-p_{jt-1}}{p_{jt-1}}$ (Mnemonic v220a); (ii) The expected mean percentage change in invoiced prices between year

t and $t-1$, $E_{jt-1}\left(\frac{p_{jt}-p_{jt-1}}{p_{jt-1}}\right)$ (Mnemonic v440). Table 8 reports descriptive statistics for our INVIND sample.

Table 8: INVIND sample, 1995-2019

	Mean	St.Dev.	Median	p25	p75	Obs.
Exit at 1yr	0.02	0.12	0.00	0.00	0.00	36,165
Exit at 2yr	0.03	0.18	0.00	0.00	0.00	34,871
Exit at 3yr	0.06	0.23	0.00	0.00	0.00	32,782
Leverage$_{jt-1}$	1.33	1.37	0.89	0.27	1.91	33,901
ROA$_{jt-1}$	0.05	0.05	0.04	0.02	0.07	34,167
Size$_{jt-1}$	4.55	1.04	4.40	3.70	5.27	35,431
Year	2,009.75	5.88	2,010.00	2,006.00	2,015.00	36,169

Notes: *Size* is the logarithm of firm employment size and is from INVIND. *ROA* is the ratio of firm earnings before interest and taxes over total assets. *Leverage* is the ratio of firm total financial debt over firm value added. Both *ROA* and *Leverage* are from CADS.

Labor Force Survey The Italian Labour Force Survey (ILFS) is a cross-sectional household survey conducted by ISTAT. A restricted version of ILFS without province level information can be accessed at `https://esploradati.istat.it/databrowser/#/it`. In each year, ILFS reports labor market information for approximately 250,000 households and 600,000 individuals. ILFS is part of the European Labor Force Survey and is the official source of labor market statistics in Italy. We use ILFS to calculate the total population and employment for each province.

Table 9: List of variables at the firm level

Variable	Source	Description
Financial Debt	CADS	Bank debt plus other financial debt, sum of Mnemonics [01.24], [01.27], [01.47] and [01.48]
Return on Assets (ROA)	CADS	EBIT (Mnemonic 01.68) over Total assets (Mnemonic 01.15)
Value Added	CADS	Value of production - Overhead expenses - Raw material costs + Provisions (Mnemonic 01.58)
Leverage Ratio	CADS	Financial debt over Value added.
Leverage Ratio (alternative)	CADS	Bank debt (sum of Mnemonics 01.24 and 01.27) over Value added
Employment	UNINPS	Average number of payroll employees throughout the year
Bad Loans	CCR	Bank bad loans ("sofferenze"); banks are obliged to report NPLs that exceeds €250
Loans	CCR	Bank loans; banks are obliged to report credit relationships that exceed a €30,000 (€75,000) threshold as of (before) January 2009
Demand elasticity, ν_j in (3)	INVIND	Revenue elasticity to price changes. Question: "*If your firm were to increase the selling prices by 10%, what percentage change in your nominal sales would be obtained, provided that all your competitors were to keep their prices unchanged and you were to leave all the other terms unchanged?*" (Mnemonic v558) asked in wave 1996 and 2007.
Wold innovation for revenue, ϵ^r_{jt} in (3)	INVIND	Wold innovation for revenue. Questions: (i) Revenue from the sale of goods and services in year t (Mnemonic v210); (ii) Revenue from the sale of goods and services in year $t-1$ (Mnemonic v209); Expected revenue from the sale of goods and services in year t (Mnemonic v437).
Wold innovation for prices, ϵ^p_{jt} in (3)	INVIND	Wold innovation for prices. Questions: (i) Average annual change in invoiced prices between year t and $t-1$ (Mnemonic v220a0; (ii) Expected annual change in invoiced prices between year t and $t-1$ (Mnemonic v440).

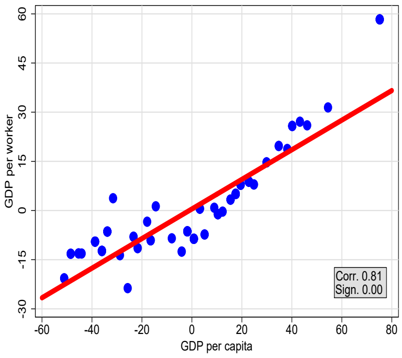
B Additional empirical evidence

First, we present additional scatter plots and age profile. Next, we show impulse responses to idiosyncratic shocks. Then, we report results for the North-South differences in the age profile of exit and leverage ratios using micro level regressions with firm level data. Finally, we show that the parallel trend assumption holds reasonably well in the regressions reported in Table 2.

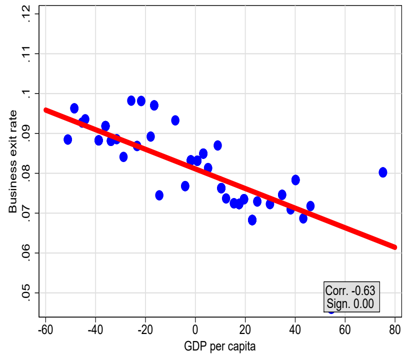
B.1 Additional figures

Figure 17 shows bin-scatter plots for some of the variables in Table 1. Averages are calculated

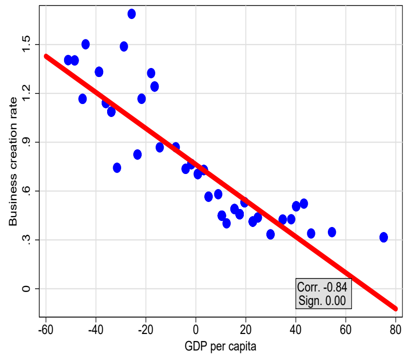
Figure 17: Variation across Italian provinces



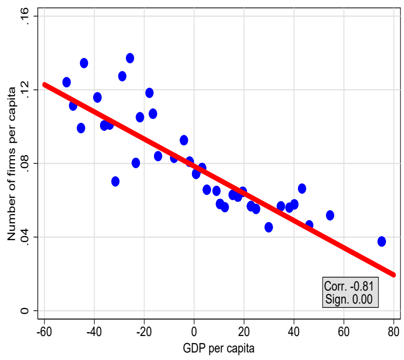
(a) Labor productivity



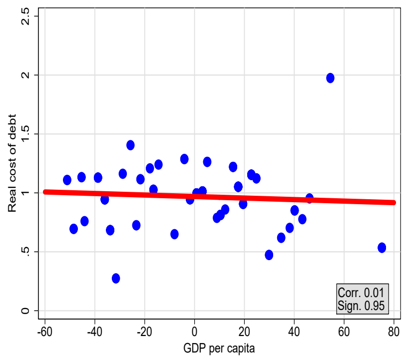
(b) Business exit rate



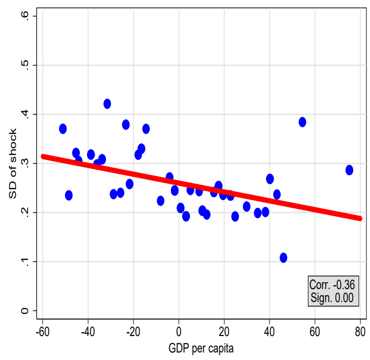
(c) Business creation rate



(d) Number of firms per worker



(e) Risk adjusted cost of credit



(f) SD of shocks

Notes: Bin scatter plots of average cross-sectional differences across Italian provinces. On the x-axis there is the average province specific GDP per capita in logs with mean normalized to zero. On the y-axis we have: in panel (a) aggregate labor productivity equal to aggregate GDP divided by aggregate employment in logs with mean normalized to zero; in panel (b) the business exit rate equals the ratio of limited liability companies from CADS that exit divided by the fraction of companies in the previous year; ; in panel (c) the business creation rate equal to the ratio between the number of start-ups in a year divided by total employment in the province in percentage terms (multiplied by 100); in panel (d) the number of firms per capita equal to the number of companies over total employment; in panel (e) the risk-adjusted real cost of credit $r_{ic} = r_i - f_i^m \times (1 - \varphi_i)$ in a province equal to the difference between the average real interest rate on term of loans of firms r_i and the average bankruptcy rate of firms older than 10 years in the province f_i^m multiplied by the fraction of unguaranteed debt $1 - \varphi_i$, in %; and in panel (f) an estimate of the Standard Deviation of shocks calculated using the idiosyncratic shock from INVIND. The sample period is 2007-2015 except for labor productivity and GDP per capita which is 2010-2015.

over the sample period 2007-2015 except for labor productivity and GDP per capita which are calculated over the years 2010-2015. On the x-axis there is the average province specific GDP per capita in logs with the mean normalized to zero. On the y-axis we have: in panel (a) aggregate labor productivity equal to aggregate GDP divided by aggregate employment in logs with the mean normalized to zero; in panel (b) the business exit rate calculated as the ratio of limited liability companies from CADS that exit divided by the fraction of companies at the beginning of the year; in panel (c) the business creation rate equal to the ratio between the number of start-ups in a year divided by total employment in the province in percentage terms (multiplied by 100); in panel (d) the number of firms per capita equal to the number of companies over total employment; in panel (e) the risk-adjusted real cost of credit $r_{ic} = r_i - f_i^m \times (1 - \varphi_i)$ in a province equal to the difference between the average real interest rate on term of loans of firms r_i and the average bankruptcy rate of firms older than 10 years in the province f_i^m multiplied by the fraction of unguaranteed debt $1 - \varphi_i$, in %; and in panel (f) an estimate of the Standard Deviation of shocks calculated using the idiosyncratic shock from INVIND.

Figure 18 shows the average age profile of the leverage ratio across provinces: it plots the coefficient cte_a^X (as a function of age) of the regression 4 where X_{ia} is the average firm leverage ratio of firms of age a in province i over the years 2007-2015. The red line represents the results obtained using the leverage ratio calculated using total debt, the blue line corresponds to the results obtained using the leverage ratio calculated using bank debt.

Figure 18: Average profile of leverage

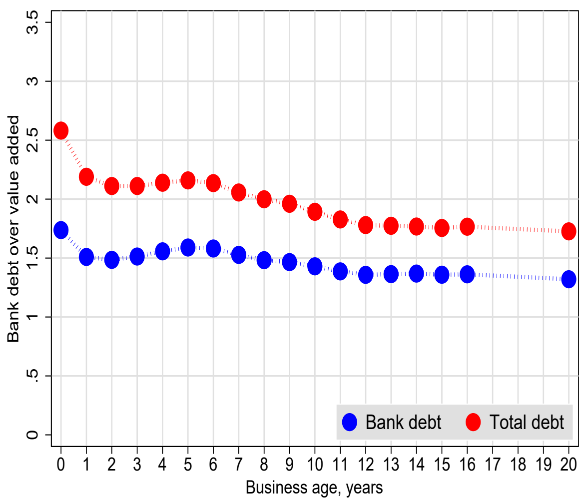
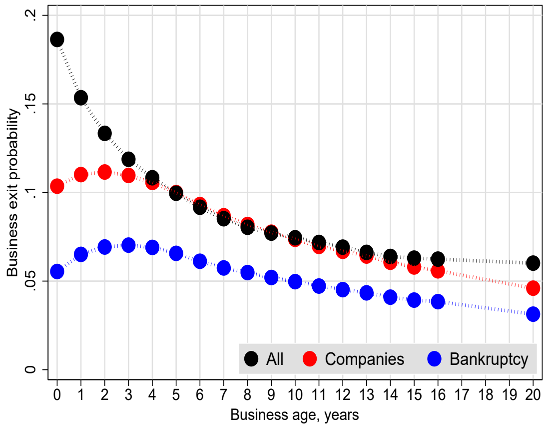
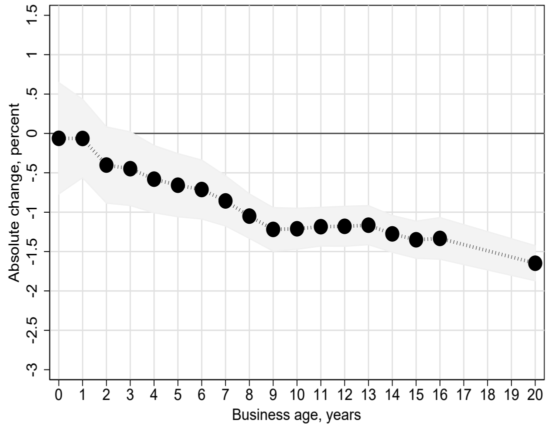


Figure 19 shows the results from running regression (4) using different definition of business exit rates (different X_{ia}'s in (4)): the black lines correspond to the business exit rates of all businesses either legal entities or sole proprietors; the red lines correspond to the exit rate of all limited liability companies; the blue lines correspond to the business exit rate of all limited liability companies with bankruptcy—i.e. leaving some bad loans to banks or exit after a formal bankruptcy procedure. Panel (a) plots the value of the constants cte_a^X in (4) as a function of age. Panel c-d plots $0.6 \times \beta_a^X$ as a function of age a for the three different exit rates, X_{ia}'s.

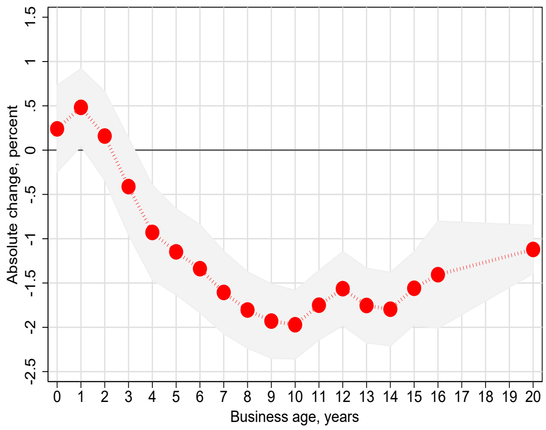
Figure 19: Exit and bankruptcy rates of Italian businesses



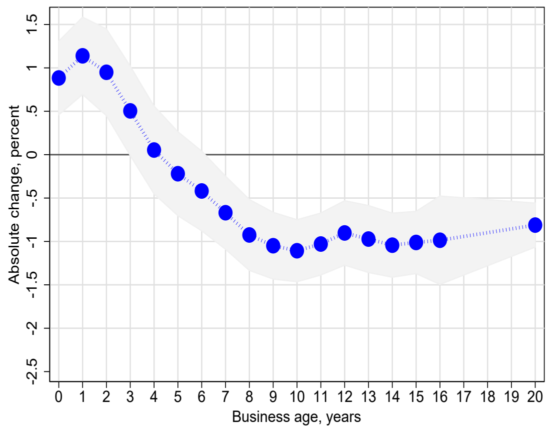
(a) Average profile of business exit rate



(b) N-S difference exit rate, all businesses



(c) N-S difference exit rate, CADS

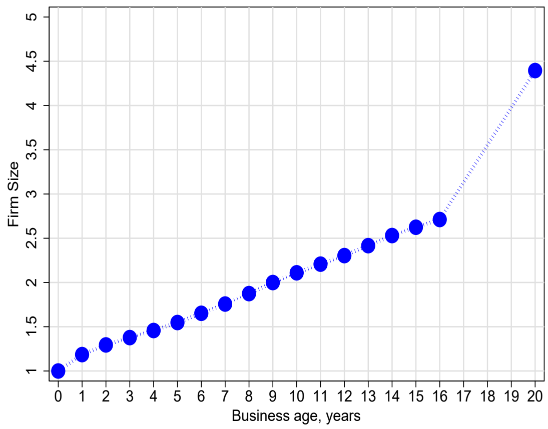


(d) N-S difference bankruptcy rate,CADS

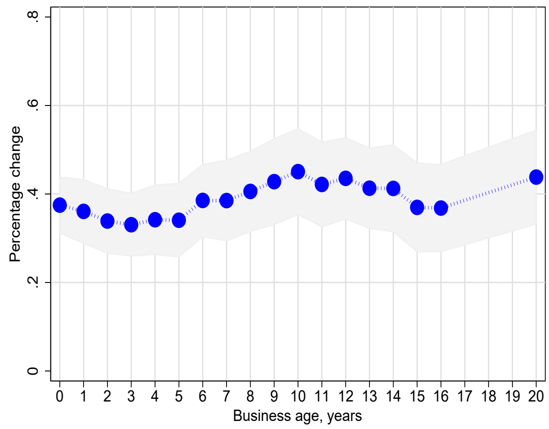
Notes: Black lines correspond to the business exit rates of all businesses (legal entities or sole proprietors). Red lines correspond to the exit rate of all limited liability companies. Blue lines correspond to the business exit rate of limited liability companies with bankruptcy—i.e. leaving some bad loans to banks or exit after a formal bankruptcy procedure. The source of data is UNINPS matched with CADS, Credit Register and Business Register.

Figure 20 shows the results from running the regression (4) with the dependent variable being firm employment size in panel (a) and (b) and firm labor productivity (valued added over employment) in panel (c) and (d). Employment is normalized to one at entry. Panels (a) and (c) show the average age profile of employment and labor productivity by plotting the constants cte_a^X's in (4) as a function of age. Panels (b) and (d) plot the value of $0.6 \times \beta_a^X$ as a function of age a for employment and labor productivity, respectively.

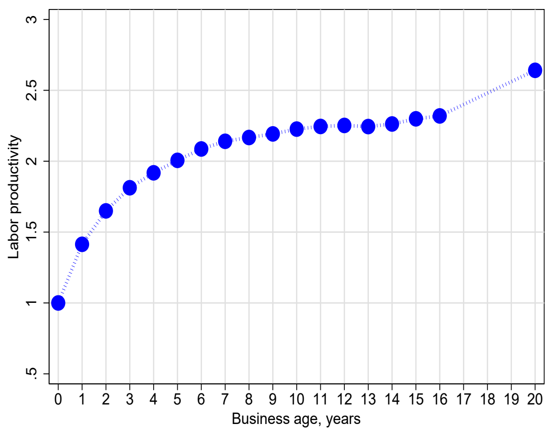
Figure 20: Firm life cycle of employment size and productivity



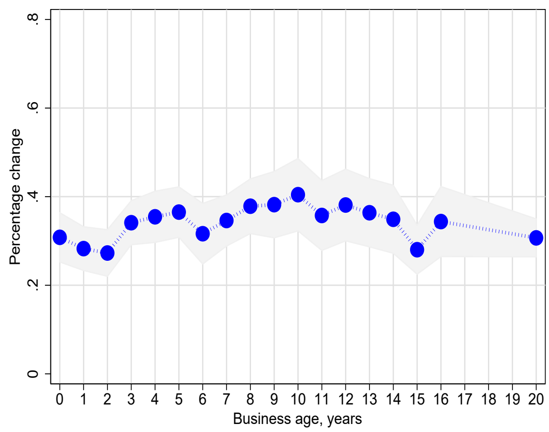
(a) Firm employment size



(b) Firm size and GDP



(c) Labor Productivity



(d) Productivity and GDP

Notes: Universe of limited liability companies. Employment is in logs. In Panel (a) and (c) the constant cte_a^X is normalized to one at entry.

C Firm level regressions for age profiles

Figure 2 shows that North-South differences in exit rates and leverage ratio increases with the age of the firm: Southern firms become relatively more likely to exit and more indebted as they grow older. We now check robustness of results by running regressions on firm-level data. We also show that the geographical differences documented in Figure 2 have become more pronounced in recent decades, coinciding with a period of ample credit availability and heightened competition in the Southern credit market, largely due to the diffusion of large Italian banks over the entire national territory (Accetturo et al. 2022). In running regressions, we control for the sector, the year and size of the firm and in some specifications also for individual firm dummies. We also check that results are robust to identifying firms in the South based on their geographical location (firms based in the eight Southern regions of Abruzzo, Basilicata, Calabria, Campania, Molise, Puglia, Sardegna or Sicilia), rather than using the (normalized) GDP per capita of the province, as in Figure 2. To use the full sample of companies from CADS over the entire sample period 1995-2019, the *Age* of a firm is measured as the number of years since its foundation which is reported by CADS.

Exit rate To check robustness for North-South differences in exit rates we run the following regression on firm-level data from CADS

$$Exit_{jt} = \beta_A Age_{jt} + \beta_E Size_{jt-1} + \beta_S South_j + \beta_I South_j \times Age_{jt} + Industry_j + Year_t + \varepsilon_{jt}, \quad (55)$$

where $Exit_{jt}$ is a dummy variable equals to 1 if firm j exits in year t and 0 otherwise, $Size_{jt-1}$ is the log of firm j employment at the end of year $t-1$ (obtained form UNIMPS), Age_{jt} is the log of firm j age in year t, $South_j$ is a dummy variable equals to 1 if firm j operates in the geographical South of Italy and 0 otherwise. In some specification we use GDP_j instead of the $South_j$ dummy. Finally, the variables $Industry_j$ and $Year_t$ denote industry and time fixed effects, respectively.

We are interested in the sign of the coefficient β_I on the interaction between the *South* dummy and firm *Age*. Figure 2 indicates that β_I is positive: Southern firms becomes relatively more likely to exit as they age, compared with Northern firms. Additionally, Figure 2 indicates that the coefficient β_S on the *South* dummy is negative when the regression includes the interaction between the *South* dummy and firm *Age*: at birth Southern firms have a lower exit probability than Northern firms.

We run the regression in (55) on the full sample period (1995-2019) from CADS. Table 10 shows some descriptive statistics over the sample.

Table 10: Summary statistics, exit rate: 1995-2019

	Mean	St.Dev.	Median	p25	p75	Obs.
Exit	0.06	0.25	0.00	0.00	0.00	10,726,776
Size(t-1)	1.74	1.26	1.61	0.69	2.48	10,726,776
Age	2.49	0.74	2.48	1.95	3.04	10,726,776
Year	2,008.60	6.89	2,009.00	2,003.00	2,015.00	10,726,776
South	0.25	0.43	0.00	0.00	1.00	10,726,776

Table 11 reports the regression results. Column (1) reports the coefficient β_A on *Age* in a regression that controls only for a full set of sector and year dummies. Column (2) adds firm size as a control *Size*, column (3) adds the *South* dummy, column (4) also includes the interaction between the *South* dummy and firm *Age*. In column (5) we interact firm *Age* with the *GDP* of the province where the firm operates rather with the *South* geographical dummy. Table 11 shows that the probability of firm exit decreases with firm age and firm employment size: the coefficients β_A and β_E in (55) are both negative. The exit probability is generally higher in the South than in the North, as indicated by the positive coefficient β_S on the *South* dummy in column 3, which excludes the interaction between *South* and *Age*. The coefficient β_I on the interaction between the *South* dummy and firm age *Age* in column (4) is negative which means that Southern firms becomes relatively more likely to exit than Northern firms as firms age. The effect is present both when we focus on the *South* dummy and when we measure geographical location using *GDP* per capita. There is also evidence that young Southern firms have lower exit rates than Northern young firms: the coefficient β_S on the *South* dummy is negative and the coefficient on *GDP* in level is positive.

We show that these results emerged more strongly during the cheap and abundant credit of the 2000's. Table 12 reports the results when we run the regression in (55) using data over the 1995-2005 period. We find that the coefficient on the *South*-*Age* interaction in column (4) or on the *South*-*GDP* interaction in column 5 falls by roughly one half: the interaction coefficient in column (4) falls from 1.1 percent in the full sample to 0.5 percent in the 1995-2005 sample; the interaction coefficient in column (5) falls in absolute value from 1 percent in the full sample to 0.5 percent the 1995-2005 sample.

Table 11: All companies, exit rate: 1995-2019

	(1) Exit	(2) Exit	(3) Exit	(4) Exit	(5) Exit
Age	-0.019***	-0.011***	-0.011***	-0.013***	-0.009***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Size(t-1)		-0.020***	-0.020***	-0.020***	-0.020***
		(0.000)	(0.000)	(0.000)	(0.000)
South			0.001***	-0.023***	
			(0.000)	(0.001)	
Age*South				0.010***	
				(0.000)	
GDP					0.027***
					(0.001)
Age*GDP					-0.011***
					(0.000)
Sector Fixed Effect	Y	Y	Y	Y	Y
Year Fixed Effect	Y	Y	Y	Y	Y
Observations	10,726,776	10,726,776	10,726,776	10,726,776	10,726,776
R^2	0.010	0.019	0.019	0.019	0.019

Notes: Robust standard errors in parentheses (clustered at the firm-level).
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The regressions include sector fixed effect defined using a NACE 2-digit industry classification and time dummies. Age is in years since the date of incorporation.

Table 12: All companies, exit rate: 1995-2005

	(1) Exit	(2) Exit	(3) Exit	(4) Exit	(5) Exit
Age	-0.017***	-0.009***	-0.009***	-0.010***	-0.009***
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Size(t-1)		-0.015***	-0.015***	-0.015***	-0.015***
		(0.000)	(0.000)	(0.000)	(0.000)
South			0.002***	-0.010***	
			(0.000)	(0.001)	
Age*South				0.005***	
				(0.000)	
GDP					0.012***
					(0.001)
Age*GDP					-0.005***
					(0.001)
Sector Fixed Effect	Y	Y	Y	Y	Y
Year Fixed Effect	Y	Y	Y	Y	Y
Observations	3,639,260	3,639,260	3,639,260	3,639,260	3,639,260
R^2	0.007	0.013	0.014	0.014	0.014

Notes: Robust standard errors in parentheses (clustered at the firm-level).
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The regressions include sector fixed effect defined using a NACE 2-digit industry classification and time dummies. Age in years since the date of incorporation.

Leverage ratio To check robustness for the age profile of the leverage ratio we run the following regression analogous to (55) but where the dependent variable is the leverage ratio of the firm denoted by $Leverage_{jt}$:

$$Leverage_{jt} = \gamma_A Age_{jt} + \gamma_E Size_{jt-1} + \gamma_S South_j + \gamma_I South_j \times Age_{jt} + Industry_j + Year_t + Firm_j + \varepsilon_{jt}, \quad (56)$$

In this specification we can also include a full set of firm fixed effects $Firm_j$. We are interested in the sign of the coefficient γ_I on the interaction between the *South* dummy and firm *Age*. We run regression (56) on the full sample of companies in CADS. Table 13 shows some descriptive statistics for the sample used.

Table 13: Summary statistics, leverage: 1995-2019

	Mean	St.Dev.	Median	p25	p75	Obs.
Leverage	1.25	3.65	0.14	0.00	1.08	8,733,643
Size(t-1)	1.82	1.27	1.79	0.69	2.56	8,733,643
Age	2.50	0.74	2.48	1.95	3.04	8,733,643
Year	2,008.76	6.89	2,009.00	2,003.00	2,015.00	8,733,643
South	0.25	0.43	0.00	0.00	0.00	8,733,643

Table 14 shows the regression results. Column (1) reports the coefficient γ_E on *Size*, γ_S on *South* and γ_A on the *Age*, in a regression that controls only for a full set of sector and year dummies. Column (2) also includes the interaction between the *South* dummy and firm *Age*. The specification in column (3) is analogous to the specification in column (2) but now it also

Table 14: All companies, leverage ratio, 1995-2019 period

	(1) Leverage	(2) Leverage	(3) Leverage	(4) Leverage	(5) Leverage
Size(t-1)	-0.17***	-0.17***	-0.43***	-0.17***	-0.43***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
South	0.01*	-0.79***			
	(0.01)	(0.02)			
Age	0.33***	0.25***	0.07***	0.40***	0.19***
	(0.00)	(0.00)	(0.01)	(0.00)	(0.01)
Age*South		0.34***	0.28***		
		(0.01)	(0.01)		
GDP				0.98***	
				(0.02)	
Age*GDP				-0.42***	-0.40***
				(0.01)	(0.01)
Sector Fixed Effect	Y	Y	Y	Y	Y
Year Fixed Effect	Y	Y	Y	Y	Y
Firm Fixed Effect	N	N	Y	N	Y
Observations	8,733,643	8,733,643	8,539,195	8,733,643	8,539,195
R^2	0.040	0.041	0.486	0.041	0.486

Notes: Robust standard errors in parentheses (clustered at the firm-level).
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The regressions include sector fixed effect defined using a NACE 2-digit industry classification and time dummies. Age in years since the date of incorporation.

includes *Firm* fixed effects. In column (4) we interact firm *Age* with the *GDP* of the province where the firm operates rather with the *South* geographical dummy. The specification in column (5) is analogous to the specification in column (4) but now it also includes *Firm* fixed effects. Figure 2 indicates that δ is positive: Southern firms becomes relatively more indebted as they age, compared with Northern firms. This pattern is robust and full confirmed by Table 14 even after controlling for a full set of *Firm* fixed effect. Figure 2 also indicates that the coefficient γ_S on the *South* dummy is negative when the regression includes the interaction between the *South* dummy and firm *Age*: at birth Southern firms are less indebted than Northern firms. This is confirmed by column 2 in Table 14.

Again these North-South differences in the age profile of leverage ratios emerged more strongly during the cheap and abundant credit of the 2000's. Table 15 reports the results when we run the regression in (56) using data over the 1995-2005 period. We find that the coefficient on the *South-Age* interaction in columns (2) and (3) or on the *South-GDP* interaction in columns (4) and (5) fall significantly especially after controlling for *Firm* fixed effects. For example the interaction coefficient in column (4) falls in absolute value from .42 in the full sample to .31 in the 1995-2005 sample; when including *Firm* fixed effects the fall is from .40 in the full sample to .16 in the 1995-2005 sample (see column 5).

Table 15: All companies, leverage ratio, 1995-2005 period

	(1) Leverage	(2) Leverage	(3) Leverage	(4) Leverage	(5) Leverage
Size(t-1)	-0.05***	-0.05***	-0.30***	-0.05***	-0.30***
	(0.00)	(0.00)	(0.01)	(0.00)	(0.01)
South	0.06***	-0.59***			
	(0.01)	(0.03)			
Age	0.16***	0.11***	0.09***	0.22***	0.12***
	(0.00)	(0.01)	(0.02)	(0.01)	(0.02)
Age*South		0.28***	0.03		
		(0.01)	(0.02)		
GDP				0.69***	
				(0.03)	
Age*GDP				-0.31***	-0.16***
				(0.01)	(0.03)
Sector Fixed Effect	Y	Y	Y	Y	Y
Year Fixed Effect	Y	Y	Y	Y	Y
Firm Fixed Effect	N	N	Y	N	Y
Observations	2,889,184	2,889,184	2,791,270	2,889,184	2,791,270
R^2	0.031	0.032	0.513	0.032	0.513

Notes: Robust standard errors in parentheses (clustered at the firm-level).
*** $p < 0.01$, ** $p < 0.05$, * $p < 0.1$. The regressions include sector fixed effect defined using a NACE 2-digit industry classification and time dummies. Age in years since the date of incorporation.

D Impulse responses to idiosyncratic shocks

We use projection methods to estimate the response of the business exit probability of firm j to the shock ϵ^z_{jt} in (3) using firm level data. First we show that responses are consistent with the predictions of a canonical demand shock: both prices and quantities increase while the exit probability falls. Secondly we show that the elasticity of business exit to shock is higher in the South than in the North. Thirdly we show that highly indebted have a higher elasticity of business exit to shock and that differences in firm leverage accounts for some of the North-South variation in business exit elasticities.

D.1 Average aggregate response

We calculate the average response of firm j in terms of prices p_{jt}, quantities q_{jt}, business exit F_{jt} and demand shifter $\tilde{z}_{jt}$ to the idiosyncratic shock ϵ^z_{jt} in (3). Business exit F_{jt} is a dummy equal to one if firm j exits at t and zero otherwise. Given (1), the change in the demand shifter between $t+n-1$ and $t+n$ can be calculated as

$$\Delta \tilde{z}_{jt+n} = \Delta r_{jt+n} + (\nu_j - 1)\, \Delta p_{jt+n}$$

where Δ is the growth rate operator: $\Delta x_{jt} = \frac{x_{jt} - x_{jt-1}}{x_{jt-1}}$. To estimate the response of $x = \tilde{z}, p, q, F$ to the shock ϵ^z_{jt} at $n = 0, 1, 2, 3$ we run the following regression:

$$\mathbf{R}^{\mathbf{x}}_{jt+n} = \beta^n_x \epsilon^z_{jt} + d_{st} + d_{it} + d_j + \gamma_x X_{jt-1} + \text{error}_{jt} \tag{57}$$

where $\mathbf{R}^{\mathbf{x}}_{jt+n}$ is the response of variable x, n periods after the shock, d_{st} and d_{it} are a full set of sector-time dummies and province-time dummies to control for the aggregate shocks ϵ^A_{ist}'s , and X_{jt-1} is a set of controls that contain only information available at time $t-1$and therefore are orthogonal to ϵ^z_{jt}. The dependent variable $\mathbf{R}^{\mathbf{x}}_{jt+n}$ is equal to $\sum_{k=0}^{n} \Delta x_{jt+k}$ when $x = \tilde{z}, p, q$. For this set of x's, the set of controls X_{t-1} includes the growth rate of x at $t-1$ Δx_{jt-1}, and the expected (at $t-1$) growth rate at t $E_{jt-1}(\Delta x_{jt})$, which is information available from INVIND. For failure probabilities, $\mathbf{R}^{\mathbf{F}}_{jt+n}$ is equal to one if the firm has failed by year $t+n$ and zero otherwise (exiting is an absorbing state), so that

$$\mathbf{R}^{\mathbf{F}}_{jt+n} = \max_{k=1,\ldots n} F_{jt+n}. \tag{58}$$

The coefficient β^n_x measures the response of variable x, n periods after a doubling of demand—a shock of 100 percent.

Figure 21 shows the average response of firms to the shock as characterized by the coefficients β^n_x's in (57) as function of n on the x-axis. Responses are consistent with the predictions of a canonical demand shock: both prices and quantities increase while the probability of going out of business falls. The shocks are highly persistent. A doubling of demand is associated with an increase in prices of roughly 10 percent, an increase in quantities of 17 percent, and a fall in the failure probability of roughly 2 percentage points.

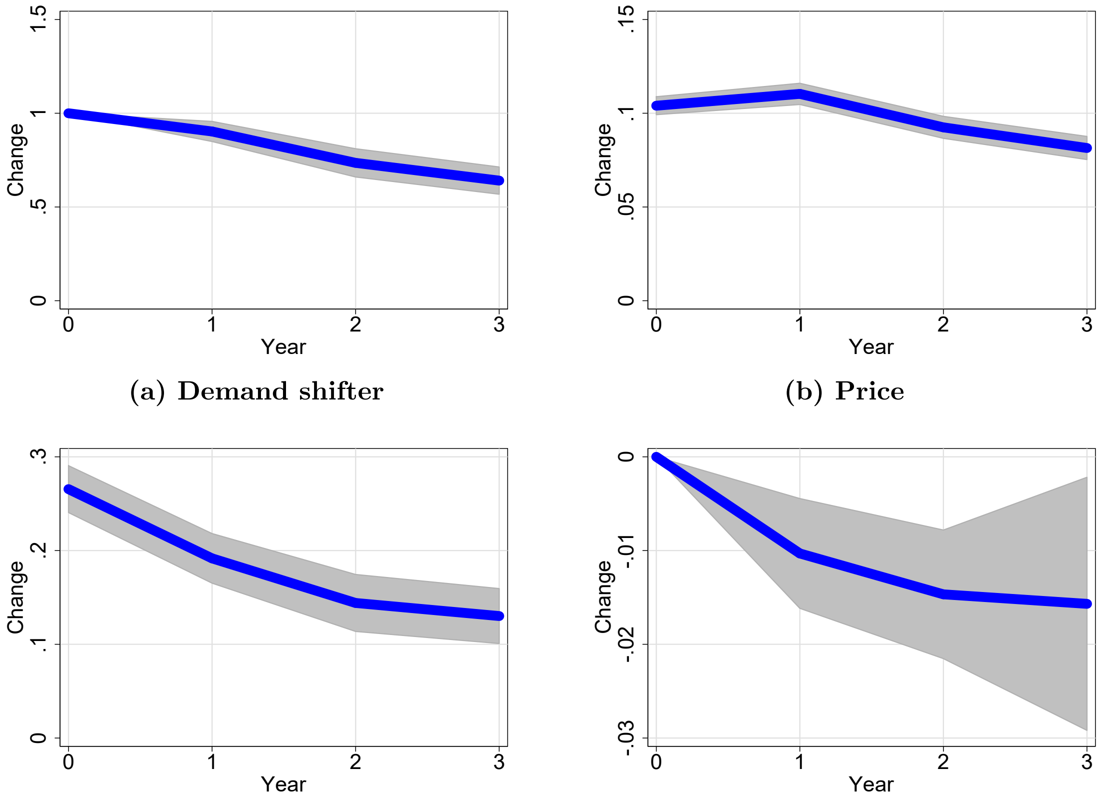
D.2 Geographical variation

Let $\mathbf{R}^{\mathbf{F}}_{jt+n}$ be equal to one if firm j exits between year t and year $t+n$, $n = 1, 2, 3$, and zero otherwise. To measure the response of $\mathbf{R}^{\mathbf{F}}_{jt+n}$ in province i to ϵ^z_{jt}, n years after the shock, we estimate the following regression with $n = 1, 2, 3$

$$\mathbf{R}^{\mathbf{F}}_{jt+n} = (\beta_{Fs} + \beta_{Fi})\, \epsilon^z_{jt} + d_{st} + d_{it} + \text{error}_{jt}, \tag{59}$$

where d_{st} and d_{it} are a full set of dummies for sector and province interacted with time. The coefficient β_{Fs} measures the average (national) response of the business exit probability in the sector s, where firm j operates. To identify the full set of province dummies we set to zero the value of β_{Fs} for the reference sector (sector NACE3). β_{Fi} is the coefficient of interest estimated

Figure 21: Average impulse responses



(a) Demand shifter

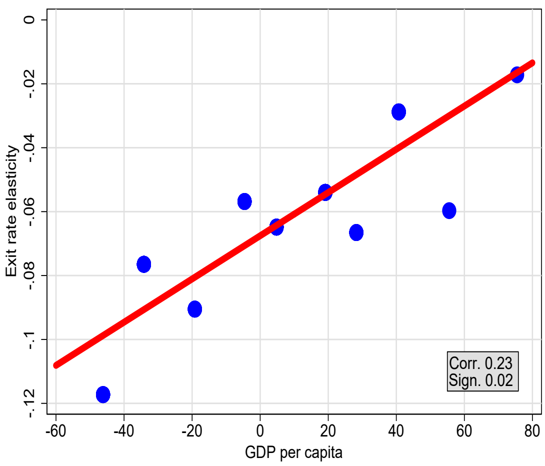
(b) Price

(c) Quantity

(d) Failure probability

by pooling all years $n = 1, 2, 3$: it measures the average response to a doubling in demand (a shock of 100 percent) of the business failure probability in province i, after controlling for the sector of the firm. Figure 22 shows a binscatter plot for the value of β_{Fi} in the province as a function of its GDP per capita. In response to a doubling in firm demand, the failure probability falls substantially more for Southern firms: it responds little for firms in the provinces with the highest GDP per capita, while it falls by more than 10 percentage points for firms in provinces with GDP per capita lower than 50 percent of the national average.

Figure 22: Variation in exit rate elasticity to shock, 3 years average

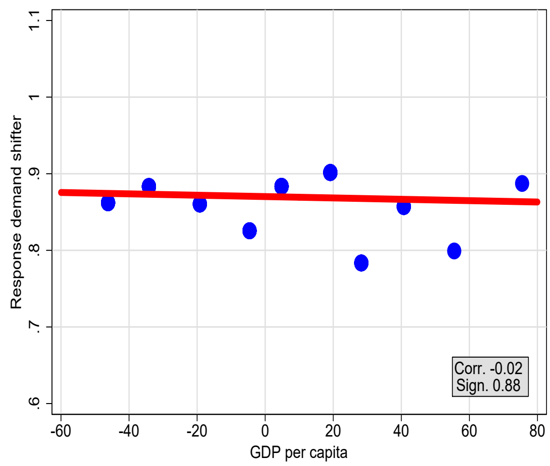


We show that the differences in the exit rate elasticity to shock in Figure 22 are not due to differences across provinces in the response of the firm demand shifter z_{jt}. To show this, we estimate the following regressions where the impulse response coefficient β_z^n are allowed to vary by sector s and province i

$$\sum_{k=0}^{n} \Delta\tilde{z}_{jt+k} = (\beta_{zs} + \beta_{zi})\,\epsilon^z_{jt} + d_{st} + d_{it} + \gamma_x X_{xt-1} + \text{error}_{jt}, \quad n = 1, 2, 3 \tag{60}$$

As in regression 59, we pool together the responses for all n+1,2,3. The coefficient β_{zs} measures the average response of the demand shifter z over the 3 periods after the shock in the sector s where firm j operates. To identify the full set of province dummies we set to zero the value of β_{xs} for the reference sector (sector NACE3). Figure 23 shows a binscatter plot for how the response of the demand shifter β_{zi} varies across provinces according to the GDP per capita of the province. The demand shock is highly persistent with small differences across provinces.

Figure 23: Variation in demand shock: 3 years average after demand shock



D.3 Elasticity of firm exit to shocks and firm leverage

We test whether highly indebted have a higher elasticity of business exit to shocks. We also check whether differences in firm leverage account for the observed difference in the response of business exit across provinces. For the purpose we run the following regression analogous to (59):

$$\mathbf{R}^{\mathbf{F}}_{jt+n} = \beta^n_y \times GDP_i \times \epsilon^z_{jt} + \beta^n_l Leverage_{jt-1} \times \epsilon^z_{jt} + \beta^n_s \times \epsilon^z_{jt} + d_{st} + d_{it} + d_j + \gamma_x X_{jt-1} + \text{error}_{jt}, \tag{61}$$

We interact the shock ϵ^z_{jt} with (i) the GDP per capita in the province GDP_i (in logs with mean normalized to zero), (ii) the firm leverage ratio before the shock $Leverage_{jt-1}$, and (iii) a full set of sector dummies.[29] The regression also includes a full set of sector-time dummies, province-time dummies and the controls X_{jt-1} evaluated at $t-1$ (before the shock) which contain the firm leverage ratio *Leverage*, log-assets *Size*, and the return on assets *ROA* (earnings before

[29]Results are almost unchanged when we do not interact the shock with the full set of sector dummies—i.e. if we drop the term $\beta^n_s \times \epsilon^z_{jt}$ from the regression in (61).

interest and taxes over total assets) trimmed at 5 percent. To detect early exit, a firm exit if either its employment drops forever to zero, or it has accumulated bad loans or have started a formal bankruptcy procedure. The leverage ratio is calculated only for firms with positive value added and is expressed as a difference with respect to the average leverage ratio in the province over the full sample: this is consistent with the model below where firms fail when their debt is excessive relative to their long-run average target, which is province specific. Standard errors are clustered at the province level.

The coefficient β_y^n in (61) measures how the response of the exit probability of a firm n years after the shock ϵ_{jt}^z varies according to the GDP per capita of the province where the firm operates. Table 16 shows the results from estimating (61) one year after the shock $n = 1$ (columns 1-3), two years after the shock $n = 2$ (columns 4-6), and three years after the shock $n = 3$ (columns 7-9), with and without controlling for firm leverage, employment size or ROA. β_y^n is positive consistent with Figure 22.

We test whether the value of β_y^n falls after controlling for firm leverage: if firm leverage explains differences in the response of firms across provinces, the effect of GDP should drop after controlling for firm leverage. To check that the difference in responses is indeed due to firm leverage rather than firm size or firm profitability, in columns 3, 6, and 9 we also interact ϵ_{jt}^z with firm employment size, and ROA. After controlling for firm leverage, the effect of GDP per capita on business exit is halved and it becomes statistically insignificant for $n = 1$. For $n = 2$ and $n = 3$ the effect of GDP per capita on business exit falls by roughly 30 percent. This indicates that on average Southern firms fails more in response to negative shocks because in the South there is a larger mass of firms that have accumulated excessive debt. Results change little after controlling for firm size and profitability, which is again consistent with the hypothesis that in the South there is a larger proportion of over-indebted firms.

Table 16: Response of business exit to idiosyncratic shocks

	Exit at								
	1 year	1 year	1 year	2 year	2 year	2 year	3 year	3 year	3 year
$GDP_i \times \epsilon_{jt}^z$	0.026*	0.013	0.008	0.054***	0.037**	0.030*	0.068***	0.052***	0.035*
	(0.014)	(0.009)	(0.009)	(0.019)	(0.015)	(0.016)	(0.018)	(0.016)	(0.018)
$Leverage_{jt-1} \times \epsilon_{jt}^z$		-0.008***	-0.006***		-0.013***	-0.009***		-0.013***	-0.008**
		(0.003)	(0.002)		(0.003)	(0.003)		(0.004)	(0.004)
$ROA_{jt-1} \times \epsilon_{jt}^z$			0.162***			0.387***			0.504***
			(0.052)			(0.067)			(0.100)
$Size_{jt-1} \times \epsilon_{jt}^z$			0.002			0.001			0.006
			(0.002)			(0.004)			(0.005)
Province×Year FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Sector×Year FE	Y	Y	Y	Y	Y	Y	Y	Y	Y
Observations	36,153	33,867	33,810	34,859	32,639	32,582	32,769	30,651	30,594
R^2	0.082	0.093	0.097	0.087	0.098	0.104	0.094	0.109	0.115

Notes: Interaction coefficient of shock and province level GDP per capita. Firm data from INVIND matched with CADS over the period 1995-2019. GDP_i is the average GDP per capita in the province (in logs with mean across provinces normalized to zero), $Leverage_{jt-1}$ is the firm leverage ratio, ROA_{jt-1} is the ratio of firm earnings before interest and taxes over total assets and $Size_{jt-1}$ is employment size in logs. $Leverage_{jt-1}$ is demeaned by the province average over the sample. Descriptive statistics for the sample are in Table 8. The leverage ratio is calculated only for firms with positive value added and we drop observations in the top and bottom 5 percent of the distribution of leverage ratio and ROA. Standard errors robust to heteroscedasticity and clustered at the province level are in parentheses with p-value denoted by *** if p<.01, ** if p<.05, and * if p<.1.

E Event study for the I-Stay-in-the-South subsidy

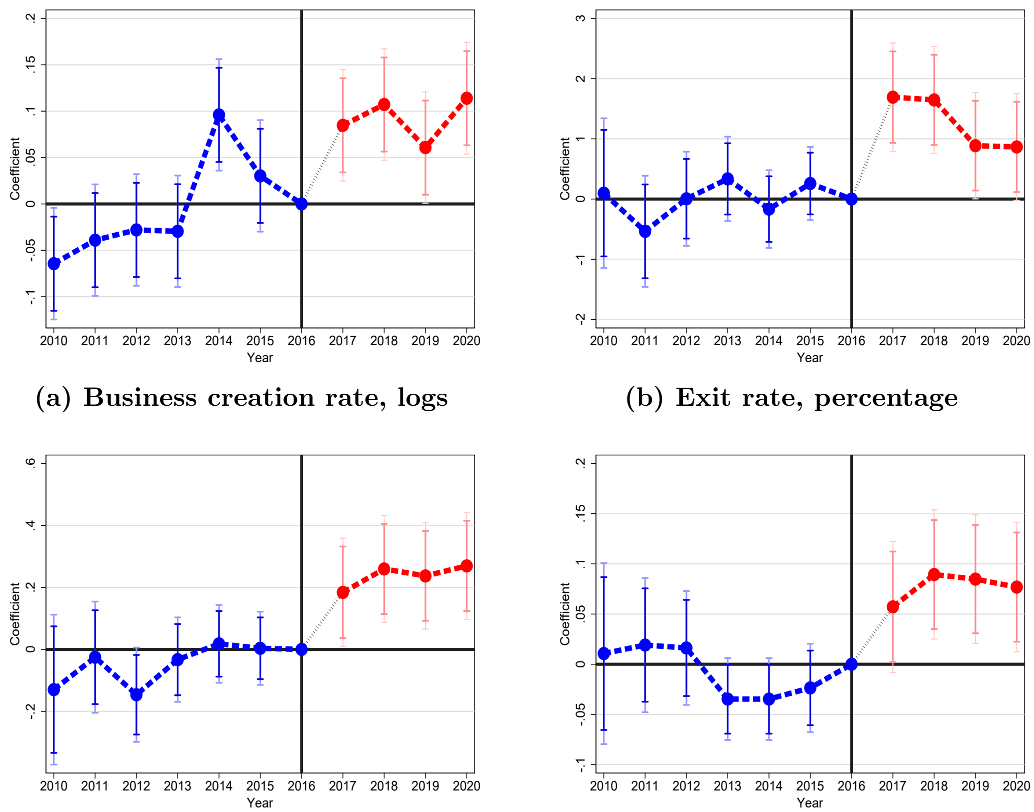
To check that the parallel trend assumption holds in the regressions reported in Table 2, we run regressions of the type

$$X_{iat} = d_{ia} + d_{ta} + \sum_{\tau=0}^{11} \beta_{SR}^{2010+\tau} \times \textit{Eligible-to-Subsidy2}_{it} + \beta_{SI} \times \textit{South-Incumbent}_{iat} + \epsilon_{it} \quad (62)$$

where the coefficient $\beta_{SR}^{2010+\tau}$ is allowed to vary in each the year of the sample period 2010-2020. The value of the coefficient in 2016 (the year before the introduction of the ISS subsidy) is normalized to one. We construct a dummy variable *Eligible-to-Subsidy2* which is equal to one if the firm is created in year t in one of the provinces of the South exposed to the ISS policy. Otherwise the dummy *Eligible-to-Subsidy2* is equal to zero. We drop from the sample some firms of age greater than zero that might have received the subsidy. In particular we drop firms that in 2018 have 1 year of age; firms that in 2019 have 1 or 2 years of age; firms that in 2020 have 1, 2 or 3 years of age. As in (5) d_{ia} is a full set of provinces times age dummies and d_{ta} is another full set of year times age dummies. As in (5) *South-Incumbent* is a dummy which is equal to one for the group of firms in the South that: in 2017 have at least 1 year of age; in 2018 have at least 2 years of age; in 2019 have at least 3 years of age; in 2020 at least 4 years of age. Otherwise, the dummy *South-Incumbent* is equal to zero.

Figure 24 shows the event study for the time profile of the β_{SR}'s coefficients. As in Table 2

Figure 24: Parallel trend, firm entitled to subsidy



(a) Business creation rate, logs

(b) Exit rate, percentage

(c) Leverage ratio

(d) Labor productivity, logs

X_{iat} could be either the logged number of new businesses created in province i in year t (in this case $a = 0$) (panel a), or the exit rate of businesses of age a in province i in year t (panel b), the average leverage ratio (panel c), or the logged average labor productivity of businesses of age a

in province i in year t (panel d). The parallel trend hypothesis seems to hold reasonably well in the data. There is only a one year spike in the business creation rate in 2014 that predates the introduction of the policy.

F Verifying the guess for the standardized value function

For simplicity we dropped reference to the province where the firm operates. The HJB equation of the firm problem is given in (10). We define

$$V(B,Z,\mathcal{R}) = v(b)\mathcal{R}Z \tag{63}$$

with $b = B/(\mathcal{R}Z)$ and

$$\begin{aligned}
\frac{\partial V(B,Z,\mathcal{R})}{\partial B} &= v'(b) \\
\frac{\partial^2 V(B,Z,\mathcal{R})}{\partial B^2} &= v''(b)\frac{1}{\mathcal{R}Z} \\
\frac{\partial V(B,Z,\mathcal{R})}{\partial Z} &= v(b)\mathcal{R} - v'(b)b\mathcal{R} \\
\frac{\partial^2 V(B,Z,\mathcal{R})}{\partial Z^2} &= -v'(b)b\frac{\mathcal{R}}{Z} + v'(b)b\frac{\mathcal{R}}{Z} + v''(b)b^2\frac{\mathcal{R}}{Z} = v''(b)b^2\frac{\mathcal{R}}{Z} \\
\frac{\partial V(B,Z,\mathcal{R})}{\partial \mathcal{R}} &= v(b)Z - v'(b)bZ
\end{aligned}$$

Under the guess (63) and (13) we have that (10) implies that

$$(r+\delta)v(b) = 1 - (\varkappa + \rho)\,b - \rho v'(b)b + \frac{\sigma^2}{2}v''(b)b^2. \tag{64}$$

We guess that

$$v(b) = \bar{v}_0 - \bar{v}_1 b + \frac{\bar{v}_2}{1+\gamma}\left(\frac{b}{\bar{\bar{b}}}\right)^{\gamma} b. \tag{65}$$

Under the guess (65) we have that

$$v'(b) = \bar{v}_2\left(\frac{b}{\bar{\bar{b}}}\right)^{\gamma} - \bar{v}_1 \tag{66}$$

$$v''(b) = \bar{v}_2\gamma\left(\frac{b}{\bar{\bar{b}}}\right)^{\gamma} \times \frac{1}{b} \tag{67}$$

By substituting the guess for $v(b)$ in (65) into the HJB in (64) we obtain that the guess (65) is verified if

$$\begin{aligned}
(r+\delta)\left[\bar{v}_0 - \bar{v}_1 b + \frac{\bar{v}_2}{1+\gamma}\left(\frac{b}{\bar{\bar{b}}}\right)^{\gamma} b\right] &= 1 - (\varkappa+\rho)\,b \\
+\rho\left[\bar{v}_1 b - \bar{v}_2\left(\frac{b}{\bar{\bar{b}}}\right)^{\gamma} b\right] &+ \frac{\sigma^2}{2}\bar{v}_2\gamma\left(\frac{b}{\bar{\bar{b}}}\right)^{\gamma} b,
\end{aligned} \tag{68}$$

Since the value matching has to be satisfied it must also be that

$$\bar{v}_0 - \bar{v}_1\bar{b} + \frac{\bar{v}_2}{1+\gamma}\bar{b} = -(1-\phi)\,\varphi\,\bar{b} + \phi\left[\bar{v}_0 - \bar{v}_1\alpha\bar{b} + \frac{\bar{v}_2\alpha^{1+\gamma}}{1+\gamma}\bar{b}\right] \tag{69}$$

Finally, from the smooth pasting condition it follows that that

$$\bar{v}_2 - \bar{v}_1 = -(1-\phi)\varphi + \phi\alpha\left(\bar{v}_2\alpha^{\gamma} - \bar{v}_1\right). \tag{70}$$

By using (68), we obtain that under the guess (65) $\bar{v}_0$ should be equal to

$$\bar{v}_0 = \frac{1}{r+\delta} \tag{71}$$

and $\bar{v}_1$ should be equal to

$$-(r+\delta)\,\bar{v}_1 = -(\varkappa+\rho) + \rho\bar{v}_1$$

which implies that

$$\bar{v}_1 = \overline{\varphi} = \frac{\varkappa+\rho}{r+\delta+\rho} \tag{72}$$

Moreover it has to be the case that

$$(r+\delta) = -\rho(1+\gamma) + \frac{\sigma^2}{2}\gamma(1+\gamma),$$

which requires that γ should satisfy the equation

$$\frac{\sigma^2}{2}\gamma^2 - \left(\rho - \frac{\sigma^2}{2}\right)\gamma - (r+\delta+\rho) = 0. \tag{73}$$

The solution to $ay^2 + by + c = 0$ has the form

$$y_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Then there are two solutions for γ in (73), but we are only interested in the solution $\gamma > 0$, which guarantees that $\bar{b} > 0$. We conclude that

$$\gamma = \frac{\rho - \frac{\sigma^2}{2} + \sqrt{\left(\rho - \frac{\sigma^2}{2}\right)^2 + 2\sigma^2(r+\delta+\rho)}}{\sigma^2} > 0 \tag{74}$$

The smooth pasting condition in (70) together with (72) implies that $\bar{v}_2$ in (65) should be equal to

$$\bar{v}_2 = \frac{-(1-\phi)\varphi + (1-\phi\alpha)\bar{v}_1}{1-\phi\alpha^{1+\gamma}} = \frac{(1-\phi\alpha)\overline{\varphi} - (1-\phi)\varphi}{1-\phi\alpha^{1+\gamma}} \tag{75}$$

Given (71), (72) and (75), $\bar{b}$ is determined by making the value matching condition in (69) satisfied, which requires that

$$(1-\phi)\,v_0 + \frac{\bar{v}_2}{1+\gamma}\bar{b} = \left[\frac{\phi\bar{v}_2\alpha^{1+\gamma}}{1+\gamma} - (1-\phi)\varphi + (1-\phi\alpha)\bar{v}_1\right]\bar{b} \tag{76}$$

After using the fact that (70) implies that

$$(1-\phi\alpha)\,\bar{v}_1 = (1-\phi)\varphi + \left(1-\phi\alpha^{1+\gamma}\right)\bar{v}_2$$

the condition in (76) reads as

$$(1-\phi)\,\bar{v}_0 = \left[\left(1-\phi\alpha^{1+\gamma}\right) - \frac{\left(1-\phi\alpha^{1+\gamma}\right)}{1+\gamma}\right]\bar{v}_2\bar{b}$$

which implies that

$$\begin{aligned}
\bar{b} &= \frac{\left(1+\frac{1}{\gamma}\right)(1-\phi)\,\bar{v}_0}{\left(1-\phi\alpha^{1+\gamma}\right)\bar{v}_2} \\
&= \frac{1}{r+\delta}\cdot\frac{\left(1+\frac{1}{\gamma}\right)(1-\phi)}{(1-\phi\alpha)\overline{\varphi}-(1-\phi)\,\varphi}
\end{aligned} \tag{77}$$

which concludes the derivation of the standardized value function in the main text.

G Derivation of the optimal leverage policy in (17)

For simplicity we drop reference to the province where the firm operates. Notice that

$$\begin{aligned}
V_B\left(B,Z,\mathcal{R}\right) &= v'(b) \\
V_{BB}\left(B,Z,\mathcal{R}\right) &= v''(b)\frac{1}{Z} \\
V_Z\left(B,Z,\mathcal{R}\right) &= v(b)-v'(b)b \\
V_{ZZ}\left(B,Z,\mathcal{R}\right) &= -v'(b)b\frac{1}{Z}+v'(b)b\frac{1}{Z}+v''(b)b^2\frac{1}{Z}=v''(b)b^2\frac{1}{Z}
\end{aligned}$$

We have argued that that the optimal choice of L (in absence of debt renegotiation) requires that

$$X(B,Z,\mathcal{R})=-V_B\left(B,Z,\mathcal{R}\right)$$

holds. The HJB of (7) implies that

$$\begin{aligned}
\left[r_c+\delta+\rho\right]X(B,Z,\mathcal{R}) &= (\varkappa+\rho)+\left[L\left(B,Z\right)-\rho B\right]\frac{\partial X(B,Z,\mathcal{R})}{\partial B} \\
&\quad +\frac{\sigma^2Z^2}{2}\frac{\partial^2 X(B,Z,\mathcal{R})}{\partial Z^2}+\frac{\partial X(B,Z,\mathcal{R})}{\partial t}.
\end{aligned} \tag{78}$$

By taking the partial derivative of (10) with respect to B and after using the fact that $X(B,Z,\mathcal{R})=-V_B(B,Z,\mathcal{R})$ and that $V(Bg,Zg)=gV(B,Z,\mathcal{R})$ we obtain that

$$\begin{aligned}
-(r+\delta)X(B,Z,\mathcal{R}) &= -\left(\varkappa+\rho\right)+\rho X(B,Z,\mathcal{R}) \\
&\quad +\rho B\frac{\partial X(B,Z,\mathcal{R})}{\partial B}-\frac{\sigma^2Z^2}{2}\frac{\partial^2 X(B,Z,\mathcal{R})}{\partial Z^2}-\frac{\partial X(B,Z,\mathcal{R})}{\partial t}
\end{aligned} \tag{79}$$

After adding side by side (79) to (78) we obtain

$$\left(r_c-r\right)\,X(B,Z,\mathcal{R})=L\left(B,Z,\mathcal{R}\right)\frac{\partial X(B,Z,\mathcal{R})}{\partial B}$$

which can be rewritten as

$$\left(r-r_c\right)\,\frac{\partial V(B,Z,\mathcal{R})}{\partial B}=-L\left(B,Z,\mathcal{R}\right)\frac{\partial^2 V(B,Z,\mathcal{R})}{\partial B^2}$$

which yields

$$-L\left(B,Z,\mathcal{R}\right)\frac{\partial^2 V(B,Z,\mathcal{R})}{\partial B^2}=\left(r-r_c\right)\,\frac{\partial V(B,Z,\mathcal{R})}{\partial B}$$

which can be finally be written as

$$L\left(B,Z,\mathcal{R}\right)=\frac{-\left(r-r_c\right)V_B}{V_{BB}} \tag{80}$$

This implies that

$$l\left(b\right)\equiv\frac{L}{Z}=\left(r-r_c\right)\frac{-v'(b)}{v''(b)} \tag{81}$$

H Derivation of (25) and (26)

Derivation of (25) By following the same steps as in Section F, we conclude that the HJB for $v(b)$ is as follows

$$(r+\delta)v(b) = 1 - (\varkappa+\rho)\,b - \rho b v'(b) + \frac{\sigma^2}{2} b^2\, v''(b) \tag{82}$$

We use the fact that $x(b) = -v'(b)$ to write the following the HJB for $x(b)$:

$$(r+\delta)x(b) = (\varkappa+\rho) - \rho x(b) - \rho b x'(b) + \frac{\sigma^2}{2}\left[2bx'(b) + b^2 x''(b)\right] \tag{83}$$

Adding side by side (82) and (83), and after using the definition of $s(b)$ in (24) we conclude that

$$(r+\delta)s(b) = 1 - \rho b s'(b) + \frac{\sigma^2}{2} b^2 s''(b)$$

which coincides with (25) in the main text.

Derivation of (26) The social value of a firm, $S_*\left(B, Z, \mathcal{R}\right)$ is the present value of output produced by the firm which evolves according to the following HJB equation given by

$$(r+\delta)S_*\left(B, Z, \mathcal{R}\right) = \mathcal{R}Z + \left[L(B, Z, \mathcal{R}) - \rho B\right] S_B^* + \frac{\sigma^2 Z^2}{2} \cdot S_{ZZ}^* \tag{84}$$

We guess and then verify that

$$S_*(B, Z, \mathcal{R}) = s_*(b)\mathcal{R}Z$$

with $b = B/\left(\mathcal{R}Z\right)$. By following the same steps as in Section F we obtain that

$$(r+\delta)s_*(b) = 1 + \left[\frac{l(b)}{b} - \rho\right] b s_*'(b) + \frac{\sigma^2}{2} b^2 s_*''(b), \tag{85}$$

which corresponds to (26) in the main text.

I Numerical solution

We first solve for the steady state equilibrium of the model. Then we analyze equilibrium after the shock $\mathcal{T}$ as the economy reaches the new steady state.

I.1 Steady state equilibrium

Step 1 (market profitability and initial debt) For each type of province $i = N, S$, the system of equations (21)-(23) determines $\mathcal{R}_i$ as well as the initial debt of start-ups B_i. The technology of a start-up in province i, Z_{0i} is a discrete two points random variable $Z_{0i} \in \{z_i^l, z_i^h\}$ with probability $1 - q_i$ and q_i, respectively. In the North the debt-value ratio of start-ups is $b_{N0}(z) = e^{-z} B_N / (Z_{0N} \mathcal{R}_N)$. In the South it is $b_{S0}(z) = B_{S0}/(Z_{0S}\mathcal{R}_S)$. Given $\mathcal{R}_i$, (34) determines $\mathcal{A}_i$ $i = N, S$.

Step 2 (equilibrium wages and labor force) The labor force in a province of the South is ℓ_S. Given (27), the labor force in a province of the North is $\ell_N = 2 - \ell_S$. Given the $\mathcal{A}_i$'s, (42) determines steady state output as equal to

$$Y = \left[\frac{\ell_S}{2} \left(\frac{1}{\mathcal{A}_S} \right)^{\frac{1}{\nu-1}} + \frac{2 - \ell_S}{2} \left(\frac{1}{\mathcal{A}_N} \right)^{\frac{1}{\nu-1}} \right]^{\frac{\nu-1}{\nu-2}}. \tag{86}$$

Given the $\mathcal{A}_i$'s and Y in (86), (39) implies that the wage in province $i = N, S$ is equal to

$$w_i = \frac{\nu - 1}{\nu} \left(\frac{1}{\mathcal{A}_i} \right)^{\frac{1}{\nu-1}} \cdot \left[\frac{\ell_S}{2} \left(\frac{1}{\mathcal{A}_S} \right)^{\frac{1}{\nu-1}} + \frac{2 - \ell_S}{2} \left(\frac{1}{\mathcal{A}_N} \right)^{\frac{1}{\nu-1}} \right]^{\frac{1}{\nu-2}} \tag{87}$$

By substituting the expression for w_i in (87) $i = N, S$ into the spatial equilibrium condition in (49) we obtain a non linear equation in ℓ_S, which has a unique solution. By solving the equation we determine the steady state value of ℓ_S, which in turn determines ℓ_N. Then using (86) and (87), we solve for Y, w_S, and w_N.

Step 3 (steady state distribution and entry) In each province i we take the labor force ℓ_i, the wage w_i and output Y as determined in step 2. Market profitability $\mathcal{R}_i$ and value added per technology unit $\mathcal{A}_i$ are determined as in step 1. We follow Moll (2020) and Achdou, Han, Lasry, Lions, and Moll (2022) to solve for the invariant distribution of firms in province i. We discretize the Kolmogorov Forward equation in (37) and write it in matrix form. For simplicity, we omit the sub-index i. The state of a firm is given by the pair $\hat{s}=(\hat{z}, \hat{b})$ where $\hat{b} = \ln(b)$ and $\hat{z} = \ln(Z\mathcal{R})$. We discretize the possible values of $\hat{z}$ and $\hat{b}$. The distance between two consecutive grid points of $\hat{z}$ and $\hat{b}$ is denoted by dz and db, respectively, and is invariant over the grid. We denote by G_z and G_b the number of grid points for $\hat{z}$ and $\hat{b}$, respectively. The corresponding vector of grid points is denoted by $\hat{\mathbf{z}}$ and $\hat{\mathbf{b}}$, respectively. The total number of grid points is $G = G_z \cdot G_b$. We denote by $\mathbf{S}$ the matrix of dimension $G_z \times G_b$ which collects all possible values of $\hat{\mathbf{s}}$ on the grid. The matrix is ordered so that in each row (column) of $\mathbf{S}$ the value of $\hat{z}$ ($\hat{b}$) remains unchanged. We denote by $\mathbf{f}$ the vector of dimension $G \times 1$ which collects the mass of firms at the corresponding grid points. $\mathbf{f}$ is ordered so that the mass of firms in row j corresponds to the state in row i of the vector, $\mathbf{vec}(\mathbf{S})$, obtained by vectorizing the matrix of states $\mathbf{S}$. We denote by j^* the row of $\hat{\mathbf{b}}$ associated with the maximum $\hat{b}$ smaller than $\ln \bar{b}$. In this section, we adopt the convention that objects (numbers, vectors or matrices) with a "*" denote the corresponding object without "*" obtained by dropping all elements of $\hat{\mathbf{b}}$ whose $\hat{b}$ are greater or equal than

$\ln \bar{b}$: for example $G^* = j^* \times G_z$ and $\hat{\mathbf{b}}^*$ is the vector of dimension $j^* \times 1$ containing all $\hat{b}$'s smaller than $\ln \bar{b}$.

Remember that $\text{B}(\hat{b}) = e^{-\hat{b}} l(e^{\hat{b}}) + \frac{\sigma^2}{2} - \rho$ with the function l given in (18). $\text{B}(\hat{b})$ governs the time evolution of $\hat{b}$ in (19). We construct the $G \times G$ matrix $\mathbf{T}_{\text{B}}$ such that the vector $\mathbf{f}_{\text{B}}$ which contains the derivative of $\text{B}(\hat{b})f(\hat{s})$ with respect to $\hat{b}$, $\frac{\partial[\text{B}(\hat{b})f(\hat{s})]}{\partial \hat{b}}$, at the corresponding grid point $\hat{s}$ can be written as

$$\mathbf{f}_{\text{B}} = \mathbf{T}_{\text{B}} \times \mathbf{f}.$$

The matrix $\mathbf{T}_{\text{B}}$ is such that the derivative is computed forward if at that corresponding value of $\hat{b}$ we have $\text{B}(\hat{b}) = e^{-\hat{b}} l(e^{\hat{b}}) + 0.5\sigma^2 - \rho < 0$, otherwise the derivative is calculated backward. Similarly, we construct the $G \times G$ matrix $\mathbf{T}_z$ such that the vector containing the derivatives of the distribution $\mathbf{f}$ with respect to $\hat{z}$ at the corresponding grid point $\hat{s}$ can be written as

$$\mathbf{f}_z = \mathbf{T}_z \times \mathbf{f}$$

Since $d\hat{z} = -\sigma^2/2\, dt + \sigma\, d\omega$ has a negative drift, the derivatives in the matrix $\mathbf{T}_z$ are always calculated forward. We proceed analogously for the matrices $\mathbf{T}_{zz}$, $\mathbf{T}_{bb}$, and $\mathbf{T}_{zb}$ that allow to express the second partial derivative of $\mathbf{f}$ with respect to $\hat{z}$ at all $\hat{s}$'s as $\mathbf{f}_{zz} = \mathbf{T}_{zz} \times \mathbf{f}$, the second partial derivative of $\mathbf{f}$ with respect to $\hat{b}$ at all $\hat{s}$'s as $\mathbf{f}_{bb} = \mathbf{T}_{bb} \times \mathbf{f}$ and the cross partial derivative with respect to $\hat{z}$ and $\hat{b}$ as equal to $\mathbf{f}_{zb} = \mathbf{T}_{zb} \times \mathbf{f}$.[30] Let $\mathbf{I}$ denote the identity matrix of dimension $G \times G$. We define the following matrix $\mathbf{A}$ of dimension $G \times G$ that characterizes the evolution of the mass of incumbent firms in the Kolmogorov forward equation in (37):

$$\mathbf{A} = -\delta \times \mathbf{I} - \mathbf{T}_{\text{B}} + \frac{\sigma^2}{2}\mathbf{T}_z + \frac{\sigma^2}{2}\mathbf{T}_{bb} - \sigma^2 \mathbf{T}_{zb} + \frac{\sigma^2}{2}\mathbf{T}_{zz}. \tag{88}$$

In the absence of bankruptcy, the matrix $\mathbf{A}$ has the property that the sum of its entries in each column j would be equal to minus δ (the instantaneous death probability of entrepreneurs). We follow our convention and denote by $\mathbf{f}^*$ the $G^* \times 1$ vector which collects the steady state mass of firms in the province for all states whose $\hat{b}$ is smaller than $\ln \bar{b}$. We also denote by $\mathbf{h}^*$ the $G^* \times 1$ vector which collects the probability that new firms in the province enter at the corresponding grid points: it is a vector of zeros except at the two grid points $(\ln(z_0^l \mathcal{R}), \ln(B_0/(z_0^l \mathcal{R})))$ and $(\ln(z_0^h \mathcal{R}), \ln(B_0/(z_0^h \mathcal{R})))$ which have probability $(1-q)$ and q respectively. Notice that $\mathbf{h}^*$ is a probability distribution: the sum of its entries adds up to one. Let $\tilde{m}$ denote the steady state number of start-ups in the province. We write the discretized Kolmogorov forward equation in (37) in steady state so that

$$0 = \frac{\mathbf{f}_t^* - \mathbf{f}_{t-dt}^*}{dt} = \mathbf{H} \times \mathbf{f}^* + \tilde{m} \cdot \mathbf{h}^*. \tag{89}$$

The matrix $\mathbf{H}$ of dimension $G^* \times G^*$ is constructed in two steps. First, we take the matrix $\mathbf{A}^*$ of dimension $G^* \times G^*$ obtained by dropping all columns and rows of $\mathbf{A}$ in (88) whose corresponding state (by row or column) has $\hat{b}$ greater or equal than $\bar{b}$. Secondly, for each column j of $\mathbf{A}^*$, with associated state $(\hat{z}_j, \hat{b}_j)$, we calculate the (instantaneous) probability $\tilde{\phi}_j$ that the firm declares bankruptcy—i.e. the probability that $\hat{b}_j$ becomes greater or equal than $\ln \bar{b}$— and then add $\phi\tilde{\phi}_j$ to the entry in the row of column j of $\mathbf{A}^*$ which corresponds to the re-injection point $(\hat{z}_j, \ln(\alpha\bar{b}))$. The bankruptcy probability for a firm in column j, $\tilde{\phi}_j$, is equal to minus the sum of all entries in

[30] All entries on the diagonal of $\mathbf{T}_z$, $\mathbf{T}_{zz}$, $\mathbf{T}_{bb}$, and $\mathbf{T}_{zb}$ are strictly negative; all entries on the diagonal of $\mathbf{T}_{\text{B}}$ are strictly negative if $\text{B}(\hat{b}) \leq 0$, and strictly positive otherwise. All rows of the matrices $\mathbf{T}$'s sum to 0.

column j of $\mathbf{A}^*$ minus δ. It is also equal to the sum of all entries in the column of $\mathbf{A}$ (associated with column j of $\mathbf{A}^*$) not present in $\mathbf{A}^*$.

Let m denote the steady state number of active firms in the province. Let $\hat{m}_e$ denote the overall number of firms that declares bankruptcy in a period. Since firms at the boundary are reinjected at some $\hat{\mathbf{s}}^*$ with probability ϕ it must be that

$$\tilde{m} = (1-\phi)\hat{m}_e + \delta m, \tag{90}$$

which says, that in steady state, the number of start-ups is equal to the number of firms that exit (either because of unsuccessful renegotiation upon bankruptcy or because the entrepreneur dies). We notice that the steady state condition in (89), $\mathbf{f}^*_t = \mathbf{f}^*_{t-dt}$, determines the steady state value of $\mathbf{f}^*$ up to a scaling factor. We normalize vectors by the number of firms in the province, m, and define $\hat{\mathbf{f}}^* = \frac{1}{m}\mathbf{f}^*$. With this normalization also the vector $\hat{\mathbf{f}}^*$ is a probability distribution: its entries add up to one and (90) can be rewritten as

$$\frac{\tilde{m}}{m} = (1-\phi)\frac{\hat{m}_e}{m} + \delta. \tag{91}$$

We find $\hat{\mathbf{f}}^*$ by solving the following fixed point problem

$$\hat{\mathbf{f}}^* = (-\mathbf{H})^{-1}\left(\frac{\tilde{m}}{m}\cdot\mathbf{h}^*\right). \tag{92}$$

where the right hand side depend on $\hat{\mathbf{f}}^*$ through the business creation rate $\frac{\tilde{m}}{m}$, which should satisfy (91) and guarantee that indeed the entries of $\hat{\mathbf{f}}^*$ add up to one. The fixed point in (92) is solved by iterating over the business creation rate $\frac{\tilde{m}}{m}$ as follows: (i) we guess that $\frac{\tilde{m}}{m}$ is equal to $\frac{\tilde{m}^0}{m^0}$; (ii) we calculate the vector $(\mathbf{H})^{-1}\left(\frac{\tilde{m}^0}{m^0}\mathbf{h}^*\right)$ and check whether its entries add up to one, if it does the algorithm has converged otherwise we update $\frac{\tilde{m}^0}{m^0}$ and go back to step (i) and keep iterating until we achieve convergence. Figure 25 shows the marginal CDF in the North and the South of logged technology $\ln Z$ (panel a and b) and of the debt value ratio $\ln b$ (panel c and d). The red dashed line is obtained using the Kolomogorof equation in steady state as in (92). The blue solid line is obtained by simulating the model using the firm policy function in (15) and (18). The blue and the red lines overlap almost perfectly.

The number of firms m solves the labor market clearing condition in (40) so that

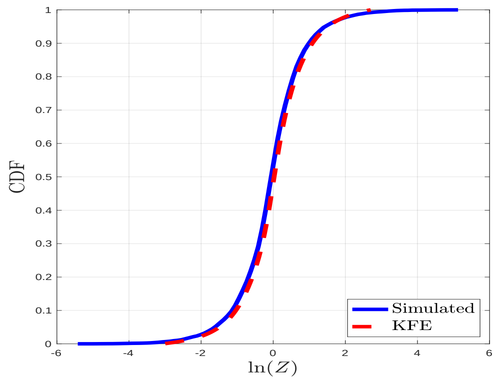
$$m = \frac{\mathcal{R}}{E_{\hat{\mathbf{f}}^*}\left[e^{\hat{z}}\right]}\cdot \ell Y^{\frac{1}{\nu-1}}\left(\frac{1}{\mathcal{A}}\right)^{\frac{\nu}{\nu-1}}$$

where $E_{\hat{\mathbf{f}}^*}\left[e^{\hat{z}}\right]$ is the mean value of $Z\mathcal{R}$ under the probability distribution $\hat{\mathbf{f}}^*$, ℓ is the labor supply in the province and Y is aggregate output both already determined (in step 2). Given m, business creation is equal to $\tilde{m} = m\cdot\frac{\tilde{m}}{m}$. The instantaneous number of firms that declare bankruptcy $\hat{m}_e$ is determined using (90). Given the probability distribution $\hat{\mathbf{f}}^*$, the steady state distribution of firms over all states $\hat{\mathbf{f}}$ is equal to $\hat{\mathbf{f}}^*$ for all $\hat{b}$'s smaller than $\ln\bar{b}$ and zero otherwise.

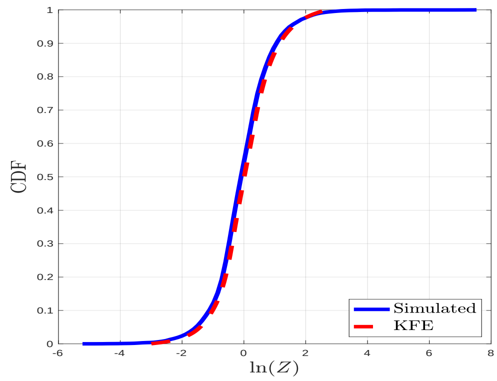
I.2 Equilibrium after the shock $\mathcal{T}$

The economy is initially in a steady state without subsidies, with a distribution of firms in provinces of type $i = N, S$ equal to $\mathbf{f}_i$ (determined as in the previous subsection). We solve for the equilibrium after an unexpected once-and-for-all permanent change at $t = 0$ in the entry subsidy for all provinces of the South to $\mathcal{T} \equiv \{\lambda_S + \tau_S, \tilde{\tau}_S\}$. The government budget is balanced and the subsidy is financed through lump-sum taxes on entrepreneurs. We construct the equilibrium after $\mathcal{T}$ recursively by first solving for the response at $t = 0$ and then at $t > 0$:

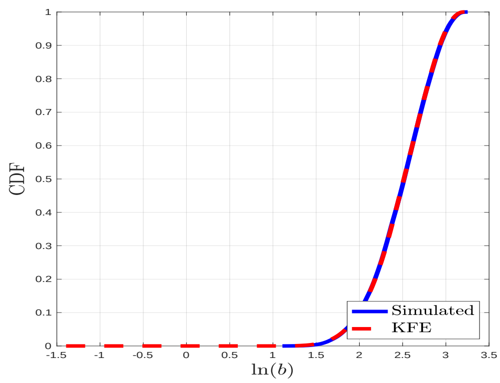
Figure 25: Marginal CDF of log technology and debt value ratio in logs



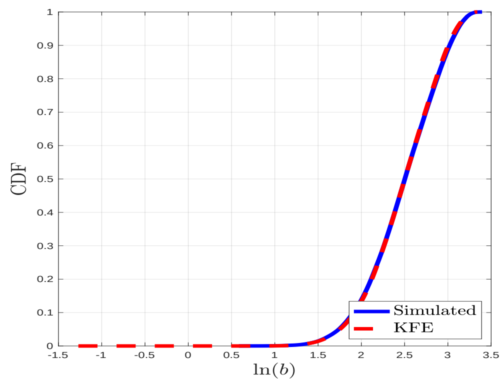
(a) Marginal CDF of $\ln \mathbf{Z}$ in the North



(b) Marginal CDF of $\ln \mathbf{Z}$ in the South



(c) Marginal CDF of $\ln \mathbf{b}$ in the North



(d) Marginal CDF of $\ln \mathbf{b}$ in the South

1. **FE & FC** The system of equations (21)-(23) implies that market profitability in the South falls from $\mathcal{R}_S$ to $\mathcal{R}_S(\mathcal{T})$ while the debt of start-ups changes to $B_{S0}(\mathcal{T})$, which determines the debt-value ratio $b_{S0}(z;\mathcal{T}) = e^{-z}B_{S0}(\mathcal{T})/\mathcal{R}_S(\mathcal{T})$. In the North, market profitability is unchanged, $\mathcal{R}_N = \mathcal{R}_N(\mathcal{T})$ and so is the debt-value ratio of start-ups, $b_{N0}(z)$. Using $\mathcal{R}_i(\mathcal{T})$, (34) determines $\mathcal{A}_i(\mathcal{T})$.
2. **Output and wages at t=0** Given the predetermined ℓ_{i0}'s (equal to their initial steady state value) and the $\mathcal{A}_i(\mathcal{T})$'s, (42) determines output Y_0. Then, using the $\mathcal{A}_i(\mathcal{T})$'s and Y_0, (39) determines wages w_{i0} $i = N, S$.
3. **Shake-out at t=0** Since $\mathcal{R}_S$ falls to $\mathcal{R}_S(\mathcal{T})$, in the South $\hat{b} \equiv \ln b$ instantaneously shifts to the right and $\hat{z} \equiv \ln(\mathcal{R}Z)$ to the left by

$$\mathcal{D} = \ln \mathcal{R}_S - \ln \mathcal{R}_S(\mathcal{T}).$$

Some firms cross the bankruptcy threshold $\bar{b}_i$ and declares bankruptcy. A fraction $1-\phi_S$ of the firms in bankruptcy exit. The remaining fraction ϕ_S renegotiates debt down to a proportion α_S of the pre-existing debt and remain active.[31] After the instantaneous shake-out, the firm density is $f^+_{S0}(\hat{\mathbf{s}};\mathcal{T})$. Its CDF corresponds to the red dashed line in panel (b) of Figure 26.

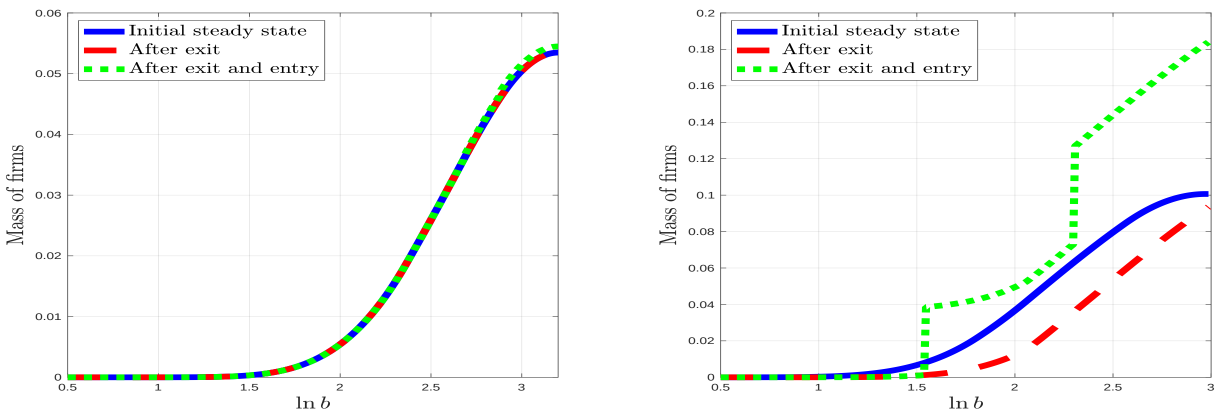
[31]This assumes that all firms in bankruptcy fall below the bankruptcy threshold after one successful round of debt renegotiation which requires that $\mathcal{D} + \ln \alpha_S < 0$. This assumption holds true in the quantitative analysis.

$f^+_{S0}(\hat{\mathbf{s}};\mathcal{T})$ is equal to

$$f^+_{S0}(\hat{\mathbf{s}};\mathcal{T}) = \begin{cases} f_S\left(\hat{b}-\mathcal{D}, \hat{z}+\mathcal{D}\right) & \text{if } \hat{b} \leq \ln \bar{b}_S \;\; \& \;\; \hat{b} \notin \left(\underline{g}, \overline{g}\right) \\ f_S\left(\hat{b}-\mathcal{D}, \hat{z}+\mathcal{D}\right) + \phi_S f_S\left(\hat{b}-\mathcal{D}-\ln\alpha_S, \hat{z}+\mathcal{D}\right) & \text{if } \hat{b} \in \left(\underline{g}, \overline{g}\right) \\ 0 & \text{if } \hat{b} > \ln \bar{b}_S \end{cases} \tag{93}$$

The thresholds $\underline{g} = \ln \bar{b}_S + \ln \alpha_S$ and $\overline{g} = \ln \bar{b}_S + \mathcal{D} + \ln \alpha_S$ incorporate that some firms declare bankruptcy and renegotiate debt down. Since $\mathcal{R}_N$ is unchanged, no shake-out happens in the North, $f^+_{N0}(\hat{\mathbf{s}};\mathcal{T}) = f_N(\hat{\mathbf{s}})$: the blue solid line and the red dashed line in panel (a) of Figure 26 overlap perfectly.

Figure 26: Impact effect of subsidy on firm distribution



(a) CDF in the North at $t = 0$ **(b) CDF in the South at $t = 0$**

4. **Entry at t=0** Given $f^+_{i0}(\hat{\mathbf{s}};\mathcal{T})$, the business creation rate instantaneously jumps up to $\tilde{m}^+_{i0} > 0$, $i = N, S$, to guarantee that the labor market clears and (40) holds at the Y_0 and w_{i0}'s determined in step 2.[32] The resulting end-of-period firm density in province $i = N, S$ is

$$f_{i0}(\hat{\mathbf{s}};\mathcal{T}) = f^+_{i0}(\hat{\mathbf{s}};\mathcal{T}) + \tilde{m}^+_{i0} \times \sum_{z\in\mathcal{Z}_i} g_{iz} \cdot \mathbf{\Delta}\left(\hat{b}, \ln B_{i0}(\mathcal{T}) - z - \ln \mathcal{R}_i(\mathcal{T})\right) \times \mathbf{\Delta}(\hat{z}, z + \ln \mathcal{R}_i(\mathcal{T})) \tag{94}$$

The CDF of $f_{i0}(\hat{\mathbf{s}};\mathcal{T})$ is plotted as a dotted green line in Figure 26.

5. **Dynamics at** $t > 0$ The steady state labor force in the South $\ell_S(\mathcal{T})$ (determined as in the previous subsection) increases. Starting from the initial steady state with $\ell_S = 1$, workers in the North gradually move to the South according to

$$\ell_{St} = \ell_S e^{-\tilde{\psi}t} + \ell_S(\mathcal{T})\left(1 - e^{-\tilde{\psi}t}\right)$$

[32]For the equilibrium to exist it has to be that $\tilde{m}^+_{i0}$ is strictly positive, which is necessarily the case. In the North after a positive subsidy, relative to the original steady state (with no subsidies), the left hand side of (40) has gone up—because of the greater Y_0 due to the lower $\mathcal{A}_S$ (see (42))—while the right hand side is unchanged. Since $f_{N0}(\hat{\mathbf{s}};\mathcal{T})$ in (94) is strictly increasing in $\tilde{m}^+_{N0}$, restoring (40) in $i = N$ at $t = 0$ requires $\tilde{m}^+_{N0} > 0$. Similarly, in the South, the left hand side of (40) is greater—because of both the higher Y_0 and the lower $\mathcal{A}_S$—while the right hand side is lower due to the shake out on impact. Then, restoring (40) in $i = S$ at $t = 0$ yields $\tilde{m}^+_{S0} > 0$.

which corresponds to (51) in the main text. The labor force in a province of the North is $\ell_{Nt} = 2 - \ell_{St}$. Using the predetermined ℓ_{it}'s and $\mathcal{A}_i(\mathcal{T})$'s, (42) determines output Y_t as equal to

$$Y_t = \left[\sum_{i \in S,N} \frac{\ell_{it}}{2} \left(\frac{1}{\mathcal{A}_i(\mathcal{T})} \right)^{\frac{1}{\nu-1}} \right]^{\frac{\nu-1}{\nu-2}}.$$

Given the $\mathcal{A}_i(\mathcal{T})$'s and Y_t, (39) determines wages w_{it} $\forall t > 0$. The business creation rate $\tilde{m}_{it}$ guarantees that (40) holds $\forall t > 0$. Then, given $f_{i0}(\hat{\mathbf{s}}; \mathcal{T})$ in (94), the Kolmogorov forward equation in (37) dictates the time evolution of $f_{it}(\hat{\mathbf{s}}; \mathcal{T})$. We now describe more in detail how $\tilde{m}_{it}$ and $f_{it}(\hat{\mathbf{s}}; \mathcal{T})$ are jointly determined. After impact and given f_{i0} from step 0, we discretize time in intervals of size dt corresponding to 1 year, $dt = 1$. At each $t = dt, 2dt, ...100$, we calculate $\mathbf{f}^*_{it}$ by discretizing the Kolmogorov forward equation in (37). Given the mass distribution $\mathbf{f}^*_{it}$ in province i at t, the overall mass distribution of firms in the province at t is equal to $\mathbf{f}^*_{it}$ for all $\hat{b}$'s smaller than $\ln \bar{b}$ and zero otherwise. In solving the Kolmogorov forward equation for $\mathbf{f}^*_{it}$ we use the implicit method, see Achdou et al. (2022) for a discussion why the implicit method guarantees that $\mathbf{f}^*_{it}$ converges to its steady state value. Then we write

$$\frac{\mathbf{f}^*_{it} - \mathbf{f}^*_{it-dt}}{dt} = \mathbf{H}_i \times \mathbf{f}^*_{it} + \tilde{m}_{it} \mathbf{h}^*_i(\mathcal{T}), \tag{95}$$

which implies that

$$\mathbf{f}^*_{it} = (\mathbf{I}_{G^*} - \mathbf{H}_i \, dt)^{-1} \times \left[\mathbf{f}^*_{it-dt} + \tilde{m}_{it} \mathbf{h}^*_i(\mathcal{T}) dt \right] \tag{96}$$

where $\mathbf{I}_{G^*}$ is the identity matrix of dimension $G^* \times G^*$. The transition matrix $\mathbf{H}_i$ is unchanged relative to the initial steady-state economy before the shock $\mathcal{T}$. The vector $\mathbf{h}^*_i(\mathcal{T})$ is a $G^* \times 1$ vector of zeros with the exception of $(\ln(B_{0i}(\mathcal{T})/(\mathcal{R}_i(\mathcal{T}) z^l_{0i})), \ln(\mathcal{R}_i(\mathcal{T}) z^l_{0i}))$ and $(\ln(B_{0i}(\mathcal{T})/(\mathcal{R}_i(\mathcal{T}) z^h_{0i})), \ln(\mathcal{R}_i(\mathcal{T}) z^h_{0i}))$, with mass $1 - q_i$ and q_i respectively. $\tilde{m}_{it}$ is the number of start-ups in province i at t. The firms $\tilde{m}_{it}$ enter at the end of period $t - dt$ (before the realization of shocks between $t - dt$ and t) and starts producing in period t. For given $\mathbf{f}^*_{it-dt}$, (96) pins down the current period distribution $\mathbf{f}^*_{it}$ provided that the business creation rate $\tilde{m}_{it}$ is known. For each i and t and given $\mathbf{f}^*_{it-dt}$, $\tilde{m}_{it}$ is set to make (40) satisfied so that the following condition holds

$$\ell_{it} \left(Y_t \right)^{\frac{1}{\nu-1}} \left[\frac{1}{\mathcal{A}_i(\mathcal{T})} \right]^{\frac{\nu}{\nu-1}} = \int_{\mathcal{R}^2} \frac{\exp\left(\hat{z}\right)}{\mathcal{R}_i} f_{it}(\hat{\mathbf{s}}) d\hat{\mathbf{s}}.$$

The integral in the right hand side is calculated over the grid points of the distribution $\mathbf{f}_{it}$.

I.3 Firm histories

To calculate age profile and other statistics, we simulate the history of 500,000 new firms for 20 years. A new firm starts with debt B_{0i} and initial technology Z_{0i} which is a discrete two points random variable $Z_{0i} \in \{z^l_i, z^h_i\}$ with probability $1 - q_i$ and q_i, respectively. The simulations are taken over discrete time intervals of size $dt = 1/100$. Given the initial debt value ratio $b_0 \equiv B_{0i}/(\mathcal{R}_i Z_0)$ and Z_0, we simulate the dynamics of the firm using the following two discretized differential equations:

$$\begin{aligned} \ln(b_{t+dt}) &= \ln(b_t) + \left[\frac{\ell_i(b_t)}{b_t} - \rho_i + \frac{\sigma_i^2}{2} \right] dt - \sqrt{dt}\, \sigma_i \, \epsilon \\ \ln(Z_{t+dt}) &= \ln(Z_t) - \frac{\sigma_i^2}{2} dt + \sqrt{dt}\, \sigma_i \, \epsilon \end{aligned}$$

where ϵ is a iid Gaussian shock from a standard normal. If b_{t+dt} is above the bankruptcy threshold $\bar{b}$, the firm declares bankruptcy. Upon bankruptcy, with probability $1 - \phi_i$ the firm exits; with probability ϕ_i it renegotiates debt down to $\alpha_i \bar{b}$. We store the value of the debt value ratio b, and technology Z as the firm ages. For each year of age $a = 1, 2, 3, ..$, we compute the fraction of firms that exits the market, the leverage ratio, and all other relevant statistics.

J Decomposition

A business creation in the South yields welfare gains (or losses) because of aggregate demand externalities (**AD**), spatial misallocation of labor (**CE**), firm financial conditions (**FC**) and the initial firm shake-out after the introduction of the subsidy $\mathcal{T}$ (**SO**). We first quantify the separate contribution of **AD**, **CE**, **SO**, and **FC** to the welfare gains under the optimal subsidy $\mathcal{T}^*$. To decompose welfare gains we solve for the three counterfactual economies, whose detailed equilibrium conditions are described in Subsections J.2-J.6 below.

J.1 Welfare effects due to AD, CE, SO, and FC

We express $\mathbb{W}_0(\mathcal{T})$ in (50) as a function of the debt value ratio of start-ups in the South $b_{S0}(\mathcal{T})$, the firm density in the South after the shake-out $\boldsymbol{f}^+_{\boldsymbol{S0}}(\mathcal{T})$, the sequences of labor forces $\boldsymbol{\ell}(\mathcal{T})$ and aggregate output $\mathbf{Y}(\mathcal{T})$:

$$\mathbb{W}_0(\mathcal{T}) = \mathbb{W}(b_{S0}(\mathcal{T}), \boldsymbol{f}^+_{\boldsymbol{S0}}(\mathcal{T}), \boldsymbol{\ell}(\mathcal{T}), \mathbf{Y}(\mathcal{T})).$$

We solve for the three counterfactual economies. In the first we isolate the contribution of **FC**, by solving a version of the model where the labor force and the aggregate demand shifter remain at their initial steady-state value, with a labor force equal to one (in all provinces) and a shifter equal to Y as in the initial steady state economy without subsidies. Additionally, we remove the shake-out effect by imposing that the distribution of incumbent firms after $\mathcal{T}$ is unaffected by $\mathcal{R}_S$, so that $\boldsymbol{f}^+_{\boldsymbol{S0}}(\mathcal{T}) = \boldsymbol{f}_{\boldsymbol{S0}}$. This implies that $\mathcal{T}$ affects the initial debt value of startups, $\boldsymbol{b}_{\boldsymbol{S0}}(\mathcal{T})$, business entry and investment and that output in (41) responds only in provinces of the South through $\mathcal{A}_S$. The welfare gains relative to the status quo of this counterfactual economy measures the contribution of **FC** and is denoted by

$$\mathbb{W}_0^{\mathbf{FC}}(\mathcal{T}) = \mathbb{W}(b_{S0}(\mathcal{T}), \boldsymbol{f}_{\boldsymbol{S0}}, 1, Y).$$

To isolate the contribution of **SO**, we solve for a second counterfactual economy with labor forces ℓ and demand shifter Y at steady state value but where now, on impact, $\mathcal{R}_S$ does shift the density of firms in the South to $\boldsymbol{f}^+_{\boldsymbol{S0}}(\mathcal{T})$. The contribution of **SO** is measured by the difference between the welfare gains under this and the previous counterfactual economy:

$$\mathbb{W}_0^{\mathbf{SO}}(\mathcal{T}) = \mathbb{W}(b_{S0}(\mathcal{T}), \boldsymbol{f}^+_{\boldsymbol{S0}}(\mathcal{T}), 1, Y) - \mathbb{W}_0^{\mathbf{FC}}(\mathcal{T})$$

To measure the contribution of **CE,** we now do allow also the labor force to respond to $\mathcal{T}$, still maintaining the demand shifter at the steady state value Y. The contribution of **CE** is measured by the difference between the welfare gains under this and the second counterfactual economy:

$$\mathbb{W}_0^{\mathbf{CE}}(\mathcal{T}) = \mathbb{W}(b_{S0}(\mathcal{T}), \boldsymbol{f}^+_{\boldsymbol{S0}}(\mathcal{T}), \boldsymbol{\ell}(\mathcal{T}), Y) - \mathbb{W}(b_{S0}(\mathcal{T}), \boldsymbol{f}^+_{\boldsymbol{S0}}(\mathcal{T}), 1, Y)$$

Finally we allow the demand shifter $\mathbf{Y}(\mathcal{T})$ to respond and measure the contribution of **AD** as a residual:

$$\mathbb{W}_0^{\mathbf{AD}}(\mathcal{T}) = \mathbb{W}_0(\mathcal{T}) - \mathbb{W}(b_{S0}(\mathcal{T}), \boldsymbol{f}^+_{\boldsymbol{S0}}(\mathcal{T}), \boldsymbol{\ell}(\mathcal{T}), Y).$$

Obviously we have:

$$\mathbb{W}_0(\mathcal{T}) = \mathbb{W}_0^{\mathbf{FC}}(\mathcal{T}) + \mathbb{W}_0^{\mathbf{SO}}(\mathcal{T}) + \mathbb{W}_0^{\mathbf{CE}}(\mathcal{T}) + \mathbb{W}_0^{\mathbf{AD}}(\mathcal{T})$$

Figure 12 decomposes the welfare gains for different $\mathcal{T}$. The subsidy is paid optimally, $\tilde{\tau}_S = 0$ and we allow the size of the subsidy $\lambda_S + \tau_S$ to change. The red dotted line shows that with **FC** only, welfare falls with $\lambda_S + \tau_S$. This is the result of the excess entry caused by cheap credit.

The difference between the green dashed line and the red dotted line measures the contribution to welfare of **SO**, which is sizeable and negative. At the optimal subsidy $\mathcal{T}^* = \{0.25, 0\}$, **FC** together with **SO** yield a welfare loss equivalent to more than a 1% fall in status quo consumption. The difference between the black solid line and the green dashed line measures the contribution to welfare of **CE**, which is sizeable and positive. Finally the difference between the blue solid line and the black solid line is the contribution of **AD.** At the optimal subsidy $\mathcal{T}^*$, **CE** and **AD** account almost equally for the difference in welfare gains between the baseline (blue solid line) and the combined effects of **FC** and **SO** (green dashed line).

J.2 More notation

In every counterfactual economy, $\forall i = N, S$, $B_{i0}(\mathcal{T})$, $x_{i0}(\mathcal{T})$, and $\mathcal{R}_i(\mathcal{T})$ denote the values that solves the following system of equations

$$B_{i0}(\mathcal{T}) = \frac{1}{x_{i0}} \cdot \max\{(1-\lambda_i)\, k_i - \varpi_i, 0\} \tag{97}$$

$$x_{i0}(\mathcal{T}) = \sum_{z \in \mathcal{Z}_i} x_i \left(\frac{B_{i0}(\mathcal{T})}{\mathcal{R}_i(\mathcal{T}) e^z}\right) \cdot g_{iz} \tag{98}$$

$$\varpi_i = \sum_{z \in \mathcal{Z}_i} v_i \left(\frac{B_{i0}(\mathcal{T})}{\mathcal{R}_i(\mathcal{T}) e^z}\right) \cdot \mathcal{R}_i(\mathcal{T}) e^z \cdot g_{iz} + \tau_i k_i \tag{99}$$

$\mathcal{A}_i(\mathcal{T})$ always satisfies (34) and is equal to

$$\mathcal{A}_i(\mathcal{T}) = \nu \left[\mathcal{R}_i(\mathcal{T}) + \chi_i\right]. \tag{100}$$

In every counterfactual economy, there are three distributions of firms: the initial distribution before the shock denoted by f_{i0}^- (equal to the distribution in the steady state of the model without subsidies), the distribution immediately after the change in subsidy that incorporates the shake-out due to $\mathcal{R}_i(\mathcal{T})$, which is denoted by f_{i0}^+, and finally there is the distribution after the adjustment in business creation which is denoted by f_{i0}. At $t = 0$, the instantaneous jump in business creation guarantees that the insulating property holds $\forall t$.

J.3 Economy with all effects

The equilibrium with all the effects **AD**, **CE,** **FC**, and **SO** is a tuple

$$(f_{i0}^+, Y_t, \tilde{m}_{it}, \ell_{it}, f_{it}(\hat{\mathbf{s}}), \mathbb{W}_t, C_t, I_t, \mathfrak{C}_{it}, H_t)$$

that satisfies the following conditions

1. Instantaneous response of $\mathcal{R}_i$ and B_{i0} implied by the system of equations 97-99
2. In every province, the labor force ℓ_{it} moves to maximize worker utility so that (51) holds.
3. Aggregate output is equal

$$Y_t = \left[\int_0^1 \ell_{it} \left[\frac{1}{\mathcal{A}_i(\mathcal{T})}\right]^{\frac{1}{\nu-1}} di\right]^{\frac{\nu-1}{\nu-2}} \tag{101}$$

4. f_{i0}^{+} is constructed using $f_{i0}(0)$ (steady state distribution of economy without business creation subsidy) plus horizontal shift due to jump in $\ln \mathcal{R}_i(\mathcal{T}) - \ln \mathcal{R}_i(0)$, plus debt renegotiation as described by (93).

5. Given f_{i0}^{+}, the business creation rate jump at $t = 0$ to $\tilde{m}_{it}^{+}$ and determine $f_{i0}(\hat{\mathbf{s}}; \mathcal{T})$, so that (40) holds also on impact. More generally, the business creation rate $\tilde{m}_{it}$ makes the insulating property satisfied at all times so that (40) holds

$$\ell_{it} \left(Y_t\right)^{\frac{1}{\nu-1}} \left[\frac{1}{\mathcal{A}_i(\mathcal{T})}\right]^{\frac{\nu}{\nu-1}} = \int_{R^2} \frac{\exp\left(\hat{z}\right)}{\mathcal{R}_i(\mathcal{T})} f_{it}(\hat{\mathbf{s}}) d\hat{\mathbf{s}}$$

6. The distribution evolves according to the Kolmogorov forward equation in (37), given the initial distribution $f_{i0}(\hat{\mathbf{s}})$

7. Aggregate welfare $\mathbb{W}_t$ is given by (46).

$$\mathbb{W}_t = \int_0^{\infty} e^{-rs} \left(C_{t+s} - H_{t+s}\right) ds \tag{102}$$

where H_t, C_t, I_t and $\mathfrak{C}_{it}$ satisfies

$$\begin{aligned} H_t &= \int_0^1 h_i\left(\ell_{it}\right) \ell_{it} di \\ C_t &= Y_t - I_t - \mathfrak{C}_{it} \\ I_t &= \int_0^1 k_i \tilde{m}_{it} di \\ \mathfrak{C}_{it} &= \int_0^1 \mathfrak{c}_{it} di \\ \mathfrak{c}_{it} &= \frac{\chi_i}{\mathcal{R}_i(\mathcal{T})} \int_{\left[0, \bar{b}_i\right] \times \mathcal{R}} \exp\left(\hat{z}\right) f_{it}(\hat{\mathbf{s}}) d\hat{\mathbf{s}} \end{aligned}$$

J.4 Economy with only FC

The equilibrium of the economy used to identify only **FC** is a tuple

$$\left(f_{i0}^{+FC}, Y_t^{FC}, \tilde{m}_{it}^{FC}, \ell_{it}^{FC}, f_{it}^{FC}(\hat{\mathbf{s}}), \mathbb{W}_t^{FC}, C_t^{FC}, I_t^{FC}, \mathfrak{C}_{it}^{FC}, H_t^{FC}\right)$$

that satisfies the following conditions

1. There is no shake out in determining f_{i0}^{+FC} neither in the South nor in the North so that $\forall i = N, S$
$$f_{i0}^{+FC} = f^{SS}(0)$$

2. The labor force is at its steady state value $\ell^{SS} = 1 \ \forall i$.

3. Aggregate output Y_t^{FC} is equal

$$Y_t^{FC} = Y^{FC}(\mathcal{T}) = \int_0^1 \ell^{SS} \left(Y^{SS}\right)^{\frac{1}{\nu-1}} \left[\frac{1}{\mathcal{A}_i(\mathcal{T})}\right]^{\frac{1}{\nu-1}} di$$

4. In every province $i = N, S$, $\forall t$ the business creation rate $\tilde{m}_{it}^{FC}$ satisfies

$$\ell^{SS}\left(Y^{SS}\right)^{\frac{1}{\nu-1}}\left[\frac{1}{\mathcal{A}_i(\mathcal{T})}\right]^{\frac{\nu}{\nu-1}} = \int_{R^2}\frac{\exp\left(\hat{z}\right)}{\mathcal{R}_i(\mathcal{T})}f_{it}^{FC}(\hat{\mathbf{s}})d\hat{\mathbf{s}}$$

5. For every province $i = N, S$, the distribution $f_{it}^{FC}(\hat{\mathbf{s}})$ evolves according to

$$\begin{aligned}\frac{\partial f_{it}^{FC}(\hat{\mathbf{s}})}{\partial t} &= f_{it}^{0FC}(\hat{\mathbf{s}}) - \delta_i f_{it}^{FC}(\hat{\mathbf{s}}) - \frac{\partial\left[\mathrm{B}_i(\hat{b})f_i^{FC}(\hat{\mathbf{s}})\right]}{\partial\hat{b}} + \frac{\sigma_i^2}{2}\cdot\frac{\partial f_{it}^{FC}(\hat{\mathbf{s}})}{\partial\hat{z}} \\ &\quad + \frac{\sigma_i^2}{2}\left[\frac{\partial^2 f_{it}^{FC}(\hat{\mathbf{s}})}{\partial\hat{b}^2} - 2\frac{\partial^2 f_{it}^{FC}(\hat{\mathbf{s}})}{\partial\hat{b}\partial\hat{z}} + \frac{\partial^2 f_{it}^{FC}(\hat{\mathbf{s}})}{\partial\hat{z}^2}\right]\end{aligned}$$

with

$$f_{it}^{0FC}(\hat{\mathbf{s}}) = \tilde{m}_{it}^{FC} \times \sum_{z\in\mathcal{Z}_i} g_{iz}\cdot\mathbf{\Delta}\left(\hat{b}, \ln B_{i0}(\mathcal{T}) - z - \ln\mathcal{R}_i(\mathcal{T})\right)\times\mathbf{\Delta}(\hat{z}, z + \ln\mathcal{R}_i(\mathcal{T}))$$

6. Aggregate welfare $\mathbb{W}_t^{FC}$, C_t^{FC}, I_t^{FC}, H_t^{FC} and $\mathfrak{C}_{it}^{FC}$ satisfy

$$\mathbb{W}_t^{FC} = \int_0^\infty e^{-rs}\left(C_{t+s}^{FC} - H^{FC}\right)ds$$

where

$$\begin{aligned}H^{FC} &= H^{SS} = \int_0^1 h_i\left(\ell_i^{SS}\right)\ell_i^{SS}di \\ C_t^{FC} &= Y^{FC}(\mathcal{T}) - I_t^{FC} - \mathfrak{C}_{it}^{FC} \\ I_t^{FC} &= \int_0^1 k_i\tilde{m}_{it}^{FC}di \\ \mathfrak{C}_{it}^{FC} &= \int_0^1 \mathfrak{c}_{it}^{FC}di \\ \mathfrak{c}_{it}^{FC} &= \frac{\chi_i}{\mathcal{R}_i(\mathcal{T})}\int_{\left[0,\bar{b}_i\right]\times\mathcal{R}}\exp\left(\hat{z}\right)f_{it}^{SO}(\hat{\mathbf{s}})d\hat{\mathbf{s}}\end{aligned}$$

J.5 Economy with FC and SO

The equilibrium of the economy used to measure the effect of **FC** and **SO** is a tuple

$$(f_{i0}^{+SO}, Y_t^{SO}, \tilde{m}_{it}^{SO}, \ell_{it}^{SO}, f_{it}^{SO}(\hat{\mathbf{s}}), \mathbb{W}_t^{SO}, C_t^{SO}, I_t^{SO}, \mathfrak{C}_{it}^{SO}, H_t^{SO})$$

that satisfies the following conditions

1. There is a shake-out effect in the determination of f_{i0}^{+} in the South so that f_{S0}^{+} incorporates the change in $\mathcal{R}_i(\mathcal{T})$ as described by (93).

2. The labor force is at its steady state value $\ell^{SS} = 1\ \forall i$.

3. Aggregate output Y_t^{SO} is equal

$$Y_t^{SO} = Y^{SO}(\mathcal{T}) = Y^{FC}(\mathcal{T}) = \int_0^1 \ell^{SS} \left(Y^{SS}\right)^{\frac{1}{\nu-1}} \left[\frac{1}{\mathcal{A}_i(\mathcal{T})}\right]^{\frac{1}{\nu-1}} di$$

4. In every province $i = N, S$, $\forall t$ the business creation rate $\tilde{m}_{it}^{SO}$ satisfies

$$\ell^{SS} \left(Y^{SS}\right)^{\frac{1}{\nu-1}} \left[\frac{1}{\mathcal{A}_i(\mathcal{T})}\right]^{\frac{\nu}{\nu-1}} = \int_{R^2} \frac{\exp(\hat{z})}{\mathcal{R}_i(\mathcal{T})} f_{it}^{SO}(\hat{\mathbf{s}}) d\hat{\mathbf{s}}$$

5. For every province $i = N, S$, the distribution of firms $f_{it}^{SO}(\hat{\mathbf{s}})$ evolves according to

$$\begin{aligned} \frac{\partial f_{it}^{SO}(\hat{\mathbf{s}})}{\partial t} &= f_{it}^{0SO}(\hat{\mathbf{s}}) - \delta_i f_{it}^{SO}(\hat{\mathbf{s}}) - \frac{\partial\left[\mathrm{B}_i(\hat{b}) f_i^{SO}(\hat{\mathbf{s}})\right]}{\partial \hat{b}} + \frac{\sigma_i^2}{2} \cdot \frac{\partial f_{it}^{SO}(\hat{\mathbf{s}})}{\partial \hat{z}} \\ &\quad + \frac{\sigma_i^2}{2}\left[\frac{\partial^2 f_{it}^{SO}(\hat{\mathbf{s}})}{\partial \hat{b}^2} - 2\frac{\partial^2 f_{it}^{SO}(\hat{\mathbf{s}})}{\partial \hat{b}\partial \hat{z}} + \frac{\partial^2 f_{it}^{SO}(\hat{\mathbf{s}})}{\partial \hat{z}^2}\right] \end{aligned}$$

with

$$f_{it}^{0SO}(\hat{\mathbf{s}}) = \tilde{m}_{it}^{SO} \times \sum_{z \in \mathcal{Z}_i} g_{iz} \cdot \mathbf{\Delta}\left(\hat{b}, \ln B_{i0}(\mathcal{T}) - z - \ln \mathcal{R}_i(\mathcal{T})\right) \times \mathbf{\Delta}(\hat{z}, z + \ln \mathcal{R}_i(\mathcal{T}))$$

6. Aggregate welfare $\mathbb{W}_t^{SO}$, C_t^{SO}, I_t^{SO}, H_t^{SO} and $\mathfrak{C}_{it}^{SO}$ satisfy

$$\mathbb{W}_t^{SO} = \int_0^\infty e^{-rs}\left(C_{t+s}^{SO} - H^{SO}\right) ds$$

where

$$\begin{aligned} H^{SO} &= H^{FC} = H^{SS} = \int_0^1 h_i\left(\ell_i^{SS}\right) \ell_i^{SS} di \\ C_t^{SO} &= Y^{SO}(\mathcal{T}) - I_t^{SO} - \mathfrak{C}_{it}^{SO} \\ I_t^{SO} &= \int_0^1 k_i \tilde{m}_{it}^{SO} \\ \mathfrak{C}_{it}^{SO} &= \int_0^1 \mathfrak{c}_{it}^{SO} di \\ \mathfrak{c}_{it}^{SO} &= \frac{\chi_i}{\mathcal{R}_i(\mathcal{T})} \int_{[0,\bar{b}_i]\times\mathcal{R}} \exp(\hat{z})\, f_{it}^{SO}(\hat{\mathbf{s}}) d\hat{\mathbf{s}} \end{aligned}$$

J.6 Economy with FC, SO and CE

The equilibrium of the economy used to measure the effect of **FC**, **SO** and **CE** is a tuple

$$(f_{i0}^{+CE}, Y_t^{CE}, \tilde{m}_{it}^{CE}, \ell_{it}^{CE}, f_{it}^{CE}(\hat{\mathbf{s}}), \mathbb{W}_t^{CE}, C_t^{CE}, I_t^{CE}, \mathfrak{C}_{it}^{CE}, H_t^{CE})$$

that satisfies the following conditions

1. There is a shake-out effect in the determination of f_{i0}^{+} in the South so that f_{S0}^{+} incorporates the change in $\mathcal{R}_i(\mathcal{T})$ as described by (93).

2. In every province, the labor force ℓ_{it} moves to maximize worker utility so that ℓ_{it} evolves according to (51) using the corresponding long-run steady state value of the labor force.

3. Aggregate output Y_t^{CE} is equal

$$Y_t^{CE} = Y_t^{CE}(\mathcal{T}) = \int_0^1 \ell_t \cdot \left(Y^{SS}\right)^{\frac{1}{\nu-1}} \left[\frac{1}{\mathcal{A}_i(\mathcal{T})}\right]^{\frac{1}{\nu-1}} di$$

4. In every province $i = N, S$, and $\forall t$ the business creation rate $\tilde{m}_{it}^{CE}$ satisfies

$$\ell_{it} \left(Y^{SS}\right)^{\frac{1}{\nu-1}} \left[\frac{1}{\mathcal{A}_i(\mathcal{T})}\right]^{\frac{\nu}{\nu-1}} = \int_{R^2} \frac{\exp\left(\hat{z}\right)}{\mathcal{R}_i(\mathcal{T})} f_{it}^{CE}(\hat{\mathbf{s}}) d\hat{\mathbf{s}}$$

5. For every province $i = N, S$, the distribution of firms $f_{it}^{CE}(\hat{\mathbf{s}})$ evolves according to

$$\begin{aligned}\frac{\partial f_{it}^{CE}(\hat{\mathbf{s}})}{\partial t} &= f_{it}^{0CE}(\hat{\mathbf{s}}) - \delta_i f_{it}^{CE}(\hat{\mathbf{s}}) - \frac{\partial \left[\mathrm{B}_i(\hat{b}) f_i^{CE}(\hat{\mathbf{s}})\right]}{\partial \hat{b}} + \frac{\sigma_i^2}{2} \cdot \frac{\partial f_{it}^{CE}(\hat{\mathbf{s}})}{\partial \hat{z}} \\ &\quad + \frac{\sigma_i^2}{2}\left[\frac{\partial^2 f_{it}^{CE}(\hat{\mathbf{s}})}{\partial \hat{b}^2} - 2\frac{\partial^2 f_{it}^{CE}(\hat{\mathbf{s}})}{\partial \hat{b} \partial \hat{z}} + \frac{\partial^2 f_{it}^{CE}(\hat{\mathbf{s}})}{\partial \hat{z}^2}\right]\end{aligned}$$

   with

$$f_{it}^{0CE}(\hat{\mathbf{s}}) = \tilde{m}_{it}^{CE} \times \sum_{z \in \mathcal{Z}_i} g_{iz} \cdot \mathbf{\Delta}\left(\hat{b}, \ln B_{i0}(\mathcal{T}) - z - \ln \mathcal{R}_i(\mathcal{T})\right) \times \mathbf{\Delta}(\hat{z}, z + \ln \mathcal{R}_i(\mathcal{T}))$$

6. Aggregate welfare $\mathbb{W}_t^{CE}$, C_t^{CE}, I_t^{CE}, H_t^{CE} and $\mathfrak{C}_{it}^{CE}$ satisfy

$$\mathbb{W}_t^{CE} = \int_0^\infty e^{-rs}\left(C_{t+s}^{CE} - H_t^{CE}\right) ds$$

   where

$$\begin{aligned}H_t^{CE} &= \int_0^1 h_{it} \ell_{it} di \\ C_t^{CE} &= Y_t^{CE} - I_t^{CE} - \mathfrak{C}_{it}^{CE} \\ I_t^{CE} &= \int_0^1 k_i \tilde{m}_{it}^{CE} \\ \mathfrak{C}_{it}^{CE} &= \int_0^1 \mathfrak{c}_{it}^{CE} di \\ \mathfrak{c}_{it}^{CE} &= \frac{\chi_i}{\mathcal{R}_i(\mathcal{T})} \int_{R^2} \exp\left(\hat{z}\right) f_{it}^{CE}(\hat{\mathbf{s}}) d\hat{\mathbf{s}}\end{aligned}$$

K Dixit-Stiglitz economy

To measure the contribution of finance to the welfare gains under the optimal subsidy, we compare the effects of a subsidy in our economy to the effects that would arise in a Dixit-Stiglitz version of our economy without debt. First, we characterize the Dixit-Stiglitz economy, then we compare the two economies.

K.1 Characterization of Dixit-Stiglitz economy

We assume that there are no frictions in financing start-ups and that firms do not dilute past debt.

There is a measure one of provinces. One half of them are in the North, the remaining one half are in the South. In every province, the labor force ℓ_{it} moves to maximize worker utility so that ℓ_{it} evolves according to (51) using the steady state value of the labor force in the economy. The (endogenous) mass of firms in the economy is equal to the sum of firms in the provinces of the North and the South:

$$M_t = \frac{1}{2}\left(m_{Nt} + m_{St}\right) \tag{103}$$

where m_{St} and m_{St} is the number of firms the North and South, respectively. Aggregate output is

$$Y_t = \left[\frac{1}{2}\int_0^{m_{Nt}} \left(Z_{jt}\right)^{\frac{1}{\nu}} \left(q_{jt}\right)^{\frac{\nu-1}{\nu}} dj + \frac{1}{2}\int_0^{m_{St}} \left(Z_{jt}\right)^{\frac{1}{\nu}} \left(q_{jt}\right)^{\frac{\nu-1}{\nu}} dj\right]^{\frac{\nu}{\nu-1}} \tag{104}$$

Z_{jt} is an idiosyncratic demand shifter which evolves according to the geometric Brownian motion in (6) so that

$$dZ_j = \sigma_i \, Z_j d\omega_{jt}$$

which implies that

$$E_t[Z_{jt+s}] = Z_{jt} \tag{105}$$

which follows from the fact that $Z_{jt+s} = e^{\ln Z_{jt+s}}$ where $\ln Z_{jt+s} \sim N(\ln Z_{jt} - \frac{1}{2}\sigma^2 t, \sigma^2 t)$ and the formula $E[e^x] = e^{\mu + \frac{1}{2}\sigma^2}$ with$\sim N(\mu, \sigma^2)$. Firms in province i exit exogenously at Poisson arrival rate δ_i. Firm j faces the demand

$$q_{jt} = Z_{jt}\left(p_{jt}\right)^{-\nu} Y_t$$

Given the production technology $q_{jt} = n_{jt}$ and the demand in (32), firm j optimally chooses

$$q_{jt} = \left(\frac{\nu - 1}{\nu w_{it}}\right)^{\nu} Y_t \, Z_{jt}, \tag{106}$$

The revenue of firm j with technology Z_{jt} in province i can again be written as $\mathcal{A}_{it} Z_{jt}$. The revenue net of labor costs and leisure costs of entrepreneurs is $\mathcal{R}_{it} Z_{jt}$ where as in the baseline model

$$\begin{aligned} \mathcal{A}_{it} &= \left(\frac{\nu - 1}{\nu w_{it}}\right)^{\nu - 1} Y_t, \\ \mathcal{R}_{it} &\equiv \frac{\mathcal{A}_{it}}{\nu} - \chi_i, \end{aligned} \tag{107}$$

Given (105) and the exogenous exit, the value of a firm with technology Z_{jt} in province i is equal to

$$V_i^{DS}\left(Z_{jt}\right) = \frac{\mathcal{R}_i Z_{jt}}{r + \delta_i}$$

Free entry in province $i = N, S$ implies that

$$(1 - \lambda_i - \tau_i)\, k_i = \frac{\mathcal{R}_i E_i[Z_{j0}]}{r + \delta_i} \tag{108}$$

where $\lambda_i + \tau_i$ is the size of the business creation subsidy in province i. Notice that now the timing of the subsidy is irrelevant for firm dynamics. Notice that (108) pins down $\mathcal{R}_i$ and thereby

$$\mathcal{A}_i = \nu\,(\mathcal{R}_i + \chi_i) \tag{109}$$

Using the fact that $q_{jt} = n_{jt}$ we use (106) to aggregate the labor demand of all firms in the province. After using the fact that in province i there are m_{it} firms, that firms exit exogenously and that in expected value have all productivity Z_{i0} we obtain that clearing of the labor market implies that $\forall t$ and $\forall i = N, S$

$$w_{it} = \frac{\nu - 1}{\nu} \left[\frac{Y_t m_{it} Z_{i0}}{\ell_{it}}\right]^{\frac{1}{\nu}} \tag{110}$$

After using (107) to express wages as a function of $\mathcal{A}_i$ we obtain that the condition for the clearing of the labor market of province i reads as follows:

$$m_{it} Z_{i0} = \ell_{it} \left(\frac{1}{\mathcal{A}_i}\right)^{\frac{\nu}{\nu-1}} (Y_t)^{\frac{1}{\nu-1}} \tag{111}$$

By substituting (111) into the definition of Y_t in (29), we obtain that as in the main model aggregate output is equal to

$$Y_t = \left\{ \frac{\ell_{Nt}}{2} \left[\frac{1}{\mathcal{A}_N(\mathcal{T})}\right]^{\frac{1}{\nu-1}} + \frac{\ell_{St}}{2} \left[\frac{1}{\mathcal{A}_S(\mathcal{T})}\right]^{\frac{1}{\nu-1}} \right\}^{\frac{\nu-1}{\nu-2}} \tag{112}$$

Summing up: (108) pins down $\mathcal{R}_i$, which in turn determines $\mathcal{A}_i$ using (109); at any point in time ℓ_{it} is predetermined and evolves according to (51) using the steady state value of the labor force in the economy; given ℓ_{it}, Y_t is determined by (112); given $\mathcal{A}_i$ and Y_t wages w_{it} are determined using (39); given $\mathcal{A}_i$, Y_t and ℓ_{it} the number of firms in the province m_{it} is determined using (111); the business creation rate $\tilde{m}_{it}$ always adjust to guarantee that the number of firms in the province are exactly equal to the m_{it} dictated by the labor market condition in (111). At the time of the introduction of the subsidy this requires a jump in the business creation rate.

We measure aggregate welfare $\mathbb{W}_t^{DS}$ by the present value of the sum across provinces of the instantaneous utility flow of workers and entrepreneurs equal to

$$\mathbb{W}_t^{DS} = \int_0^{\infty} e^{-rs} \mathcal{F}_s ds$$

where the integrand $\mathcal{F}_t$ is the income welfare flow at time t equal to

$$\begin{aligned} \mathcal{F}_t \;=\; & Y_t - \frac{k_N \tilde{m}_{Nt} + k_S \tilde{m}_{St}}{2} - \frac{\chi_N}{2} \int_{-\infty}^{t} \tilde{m}_{Nj} e^{-\delta_N (t-j)} Z_{N0} dj - \frac{\chi_S}{2} \int_{-\infty}^{t} \tilde{m}_{Sj} e^{-\delta_S (t-j)} Z_{S0} dj \\ & - \frac{h_N(\ell_{Nt})\, \ell_{Nt} + h_S(\ell_{St})\, \ell_{St}}{2} \end{aligned}$$

L Regression (5) on model simulated data

We describe how we constructed the sample of firms used to estimate the regression (5) on model simulated data as reported in Table 6.

Set-up There are 33 provinces in the South and 62 in the North. All provinces in the North are the same. All provinces in the South are the same.

There are 11 years of data, 7 before the introduction of the ISS subsidy (2010-2016) and 4 after its introduction (2017-2020). Let (t, i, a) denote the triple for year $t = 2010, 2011....2020$, province $i = N, S$ and age group a. We will consider 130 groups: 65 for the North, 65 for the South.

Variables For each (t, i, a) we construct the following variables.

1. For province $i = N, S$ and $t = 2010, 2011....2020$ we calculate the business creation, $\tilde{m}_{it}$. In 2010-2016 $\tilde{m}_{Nt}$ and $\tilde{m}_{St}$ is equal to its steady state value $\tilde{m}_N$ and $\tilde{m}_S$ for the economy without subsidies. In 2017-2020 $\tilde{m}_{Nt}$ and $\tilde{m}_{St}$ are the response in the first 4 years after the introduction of the ISS subsidy—which we have already calculated.

2. For group $\mathbf{G} = (t, i, a)$, we calculate (i) the beginning of year number of firms in group $\mathbf{G}$, $\widehat{m}_{tia}^{-}$, (ii) the end of year number of firms in group $\mathbf{G}$, $\widehat{m}_{tia}$, (iii) the end of period total debt of firms in group $\mathbf{G}$, $\widehat{b}_{tia} = \int_{j\in\mathbf{G}} B_{jt} dj$, (iv) the end of period total value added produced by firms in group $\mathbf{G}$, $\widehat{y}_{tia} = \int_{j\in\mathbf{G}} \mathcal{A}_i Z_{jt} dj$, (v) the end of period total employment by firms in group $\mathbf{G}$ $\widehat{n}_{tia} = \int_{j\in\mathbf{G}} n_{jt} dj$ where $n_{jt} = \left(\frac{\nu-1}{\nu w_{it}}\right)^{\nu} Y_t Z_{jt}$—$j \in \mathbf{G}$ denotes integration over all firms j in group $\mathbf{G}$. We collect these 5 variables for group $\mathbf{G}$ in the vector $\mathbf{X}_{tia} = \left(\widehat{m}_{tia}^{-}, \widehat{m}_{tia}, \widehat{b}_{tia}, \widehat{y}_{tia}, \widehat{n}_{tia}\right)$.

Other variables We calculate the vector $\mathbf{X}_{tia} = \left(\widehat{m}_{tia}^{-}, \widehat{m}_{tia}, \widehat{b}_{tia}, \widehat{y}_{tia}, \widehat{n}_{tia}\right)$ for the relevant $\mathbf{G} = (t, i, a)$. We proceed as follows:

1. **Year t=2010** Consider a flow $\tilde{m}_i$ of firms born at the beginning of year t =2010 in province $i = N, S$. At the end of year 2010 we calculate the vector $\mathbf{X}_{tia}$ for $t = 2010$, province $i = N, S$ and age $a = 0$.

2. **Year t=2010+x, x=1,2,3,4,5,6** Consider a flow $\tilde{m}_i$ $i = N, S$ of firms born at the beginning of year t =2010+**x** (the same flow as in 2010). At the end of the year t =2010+**x** we calculate the vector $\mathbf{X}_{tia}$ for t = 2010+**x**, provinces $i = N, S$ and age $a = 0, 1...,$**x** (all firms born in all previous years).

3. **Year t=2017 (introduction of ISS)** Consider a flow $\tilde{m}_i$ of firms born at the beginning of year t =2017 in province $i = N, S$. For $i = N$ we proceed as for $t < 2017$. For $i = S$, we assume that $\tilde{m}_S/2$ of firms enter with the same initial debt value ratio b_{S0} as in the steady state of the model without subsidies (they correspond to the firms created before June 2017). The other remaining mass $\tilde{m}_S/2$ of firms enter with initial debt value ratio $b_{S0}\left(\mathcal{T}^{ISS}\right)$ equal to the value under the ISS subsidy: a fraction ι of firms receives the ISS subsidy, the remaining fraction $1 - \iota$ of firms do not receive the ISS subsidy. We assume that market profitability switches from $\mathcal{R}_S$ to $\mathcal{R}_S(\mathcal{T}^{ISS})$ after 6 months in 2017. Only the first half of firms (those born in the South before June 2017) experience the unexpected jump in $\mathcal{R}_S$ to $\mathcal{R}_S(\mathcal{T}^{ISS})$. All firms created in years before 2017 also experience the jump

in $\mathcal{R}_S$ to $\mathcal{R}_S(\mathcal{T}^{ISS})$. In the North there is no jump and $\mathcal{R}_N$ is unchanged. At the end of the year 2017 we calculate the vector $\mathbf{X}_{tia}$ for $t = 2017$, $i = N, S$ and age $a = 0, 1...7$ (due to firms born in previous years).

4. **Year t=2017+x, x=1,2,3 (xth year after ISS)** Consider a flow $\tilde{m}_i$ of firms born at the beginning of year t =2017+**x** in province $i = N, S$. Firms in the South have initial debt value ratio as after the introduction of ISS, $b_{S0}\left(\mathcal{T}^{ISS}\right)$: a fraction ι of firms receives the subsidy, the remaining fraction $1 - \iota$ does not receive the subsidy. In the North firms enter with b_{N0} as in 2010. In the South market profitability is $\mathcal{R}_S(\mathcal{T}^{ISS})$. In the North $\mathcal{R}_N$ is unchanged. At the end of the year t =2017+**x** we calculate the vector $\mathbf{X}_{tia}$ for $\mathbf{t}$ =2017+**x**, $i = N, S$ and age $a = 0, 1...7..., 7+$**x** (due to firms born in all previous years).

Regression specification We then run regression 4 in the paper:

$$X_{iat} = d_{ia} + d_{ta} + \beta_{SR} \times \textit{Eligible-to-Subsidy}_{iat} + \beta_{SI} \times \textit{South-Incumbent}_{iat} + \epsilon_{it}$$

The dependent variables will be (i) the logged business creation $\ln \tilde{m}_{it}$, (ii) the logged average (within group) leverage ratio $\ln\left(\frac{\widehat{b}_{tia}}{\widehat{y}_{tia}}\right)$, (iii) the logged average (within group) labor productivity $\ln\left(\frac{\widehat{y}_{tia}}{\widehat{n}_{tia}}\right)$, (iv) the logged average (within group) firm employment size $\ln\left(\frac{\widehat{n}_{tia}}{\widehat{m}_{tia}}\right)$ and (v) the business exit rate in percentage terms (i.e. multiplied by 100), $100\times\left(1 - \frac{\widehat{m}_{tia}}{\widehat{m}^{-}_{tia}}\right)$. The regression includes (i) age dummies, (ii) South $\times$ age dummies, (iii) South dummy, (iv) Year $\times$ age dummies and *Eligible-to-Subsidy* and *South-Incumbent* constructed as in the data. We then produce the equivalent of table 3 with the corersponding OLS standard errors. When running the regression, in 2010 we have data only for the group of age $a = 0$, in 2011 for age $a = 0, 1$, in 2011 per age $a = 0, 1, 2$, in 2012 for age $a = 0, 1, 2, 3$, ...in 2020 for age $a = 0, 1, 2, ...10$. In total there should be 6175 observations for the regression for variables ii-v and 1045 observations for the regression with business creation $\ln \tilde{m}_{it}$.

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