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“GREEN” FISCAL POLICY MEASURES AND NON-STANDARD MONETARY POLICY IN THE EURO AREA

by Anna Bartocci*, Alessandro Notarpietro* and Massimiliano Pisani*

Abstract

This paper evaluates the macroeconomic effects of increasing taxes on fossil fuels (“carbon tax”) and subsidies for renewable energy and reducing labor income tax in the euro area, and the interaction of these effects with domestic monetary policy. The tax increase is announced, gradually implemented and fully anticipated by agents (thus it is conceptually different from a sudden and unexpected positive shock affecting the international prices of fossil fuels). The analysis makes use of a New Keynesian two-country model with an energy sector, calibrated to the euro area and the rest of the world. The main results are the following. First, an increase in the carbon tax generates recessionary effects. Second, higher subsidies for green energy and a lower labor tax can limit the macroeconomic cost of increasing the carbon tax. Third, if the monetary policy rate is at its effective lower bound, the fiscal policy mix generates short-run recessionary effects, which can be offset if the central bank, for monetary policy purposes, purchases long-term sovereign bonds in the secondary market, thus keeping long-term interest rates low.

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1 Introduction

The European Union has set over time several targets for cutting greenhouse gas emissions. The ensuing and longstanding debate has focused on the policy measures to achieve these targets. Carbon taxes i.e., charges on the carbon content of fossil (“brown”) fuels, and subsidies for renewable (“green”) sources of energy, levied by fiscal authorities, have always been considered as key measures to reduce carbon dioxide (CO\textsubscript{2}) emissions. One question to address is if and to which extent these policy measures generate macroeconomic costs in terms of lower economic activity. The issue is particularly relevant for the euro-area (EA) countries. Over the past years, the EA has been characterized by the effective lower bound (ELB) on the policy rate. Hence, when facing recessionary shocks, the EA central bank cannot reduce the policy rate as much as needed to stabilize inflation and macroeconomic conditions. It has to rely on the so-called non-standard measures, e.g. asset purchases, for monetary policy purposes.

In this paper, we assess the macroeconomic effects on the EA economy of a mix of (i) higher carbon tax and subsidies for renewable sources of energy, and (ii) a lower labour income tax, and their interaction with central bank purchases of long-term sovereign bonds in the secondary markets for monetary policy reasons.

The analysis is based on a two-country model calibrated to the EA and the rest of the world (RW). The model is New Keynesian, allowing for nominal price and wage rigidities and, thus, for a non-trivial stabilization role of the monetary policy.

The main features of the model are the following ones.

First, there are several ways to produce energy, according to the used source. The oil-based energy sector uses capital, labour, and oil in the production function. A similar production function holds in the coal-, gas-, nuclear-, and green source-based energy sectors, such that capital and labour are combined with the sector-specific source of energy. The energy outputs obtained from the different sectors are aggregated in an en-

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1 The views expressed in this paper are those of the authors alone and should not be attributed to the Bank of Italy or the Eurosystem. We thank two anonymous referees, Fabio Busetti, Michele Caivano, Pietro Cova, and participants at the Working Group on Econometric Modelling for useful comments. All remaining errors are ours.

2 Current European Union-wide emission reduction targets and policy objectives for the period 2021-30 are based on the 2030 Climate and Energy Framework adopted by the European Council in 2014. The framework sets binding targets for cutting greenhouse gas emissions to below 1990 levels, namely a reduction in emissions of 20% by 2020 and at least 40% by 2030. With the 2030 Climate Target Plan, the Commission proposes to reduce greenhouse gas emissions to at least 55% below 1990 levels by 2030. Policies to reduce carbon emissions in the European Union comprise the Emissions Trading System (ETS) and national measures in sectors that are not covered by the ETS, such as transport, heating, and agriculture.
ergy bundle and used, joint with capital and labour, by firms that produce intermediate manufacturing goods. Moreover, the different types of energy are also assembled into a basket that enters the households’ final consumption joint with (non-energy) consumption goods.

Second, in each country there is a representative fiscal authority that levies taxes on fossil sources of energy (i.e., oil, coal, and gas) and on households’ labour income, provides subsidies for the physical capital used to produce renewable energy, and issues short- and long-term sovereign bonds.

Third, in some simulations, the EA central bank can implement, for monetary policy purposes, discretionary purchases of long-term sovereign bonds in the secondary market. In such case it is assumed, consistent with the current low levels of the interest rates in the EA, that the ELB does not allow the central bank to reduce the policy rate as much as needed to stabilize macroeconomic conditions in response to recessionary or disinflationary shocks. In the model, central bank purchases have non-trivial effects on economic activity and inflation because of financial market segmentation. Some households, labeled “restricted”, have access only to the market for domestic long-term sovereign bonds (thus, their access to financial markets is “restricted”). Moreover, they own, jointly with domestic “Ricardian” households (that have access to all financial markets), the domestic producers of physical capital. Central bank asset purchases reduce long-term interest rates, which induces restricted households to substitute consumption and investment in physical capital for long-term sovereign bonds.³

Our analysis is based on counterfactual simulations. We simulate the implementation of the following measures in the EA. First, a gradual increase in the carbon tax, taking place over nine years.⁴ Second, a fiscal policy mix based on simultaneously and gradually (i) increasing the carbon tax and the subsidy to physical capital used to produce green energy, and (ii) reducing the labour income taxes paid by households. Third, the fiscal policy mix is simulated also under the assumption that the ELB holds and that the central bank implements purchases of the sovereign bonds in the secondary market for monetary policy purposes. Finally, we run a sensitivity analysis to evaluate the extent to which some key parameters of the model affect the results of the fiscal policy mix. The tax increase – as well as the other fiscal measures – is announced, gradually implemented

³See Chen et al. (2012).
⁴All our scenarios for carbon taxation calculate the economic effects relative to the current situation, with some form of carbon taxation already existing in many industry sectors. Thus, this tax increase is imposed on top of the existing taxes on energy and the carbon tax. We do not make any claim about our simulated fiscal measures being expected to achieve a full switch to a carbon-free economy in the longer term.
and fully anticipated by agents (thus it is conceptually different from a sudden and unexpected positive shock affecting the international prices of fossil fuels).

The main results are as follows. First, an increase in the carbon tax generates recessionary effects and exerts a moderate downward pressure on consumer price inflation. The latter reflects the large negative effect of the carbon tax on aggregate demand, which offsets the positive effect of the tax on fossil fuels and, thus, on the energy component of consumer price inflation. Second, higher subsidies for green energy and lower labour income tax can limit the macroeconomic cost of increasing the carbon tax. Third, if the monetary policy rate is at the ELB, the fiscal policy mix generates short-run recessionary effects, which can be offset if the central bank, for monetary policy purposes, purchases long-term sovereign bonds in the secondary market, thus keeping long-term interest rates low.

Our paper contributes to the literature on the macroeconomic effects of fiscal measures aiming at reducing CO$_2$ emissions. Golosov et al. (2014) simulate a dynamic general equilibrium model with an externality through climate change from using fossil energy. They find that the marginal externality damage of emissions (or, equivalently, the optimal carbon tax) is proportional to current GDP, with the proportion depending on discounting, the expected damage elasticity (how much output flow is lost from an extra unit of carbon in the atmosphere), and the structure of carbon depreciation in the atmosphere. Heutel (2012) analyzes the optimal environmental policy response to economic fluctuations caused by persistent productivity shocks within a DSGE model and finds that optimal policy allows carbon tax to be procyclical (increasing during expansions and decreasing during recessions). Annicchiarico et al. (2017) simulate a dynamic general equilibrium model calibrated to the Italian economy and find that tax incentives encouraging the use of clean energy sources, by discouraging the use of fossil fuel, produce a sizeable reallocation of emissions across sectors and are expansionary. Bartocci and Pisani (2013), by simulating a real business cycle model enriched with an energy sector, show that taxing motor vehicle fuels for private transportation, so as to reduce taxes on electricity consumption and increase subsidies for renewable sources of electricity generation, help reduce CO$_2$ emissions and favor the development of electricity generation from renewable sources, without jeopardizing economic activity in the

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$^5$It is important to stress that our model-based results are not forecasts nor projections, but illustrative counterfactual simulations. The results are inevitably subject to considerable uncertainty, since specific assumptions have to be made regarding the values of the parameters and the specification of the model, especially for what concerns the production of energy from different sources.

$^6$We do not evaluate whether the simulated policy measures allow to achieve the environmental targets set at international levels. Our analysis should be intended as positive and not normative.
main EA economies. Ferrari and Nispi Landi (2022a) use a DSGE model to analyze the environmental impact of a program of green-asset purchases by the central bank in presence of a carbon tax. Their model is characterized by two kinds of firms: one polluting (producing “brown” goods) and one using green technologies. They find that purchases help reduce CO$_2$ emissions when green and brown goods are substitutes, while the emissions increase when the two are complement. They also find that the earlier the central bank purchases green bonds, the larger the CO$_2$ reduction. Varga et al. (2021) analyze the transitional costs of moving towards a net zero emissions economy using a DSGE model with energy sectors, calibrated on the European Union. They find that the costs of moving towards a net zero emissions economy can be significantly reduced when carbon taxes are used and are recycled to reduce other distortionary taxes, or for subsidizing clean energy. Different from these contributions, we use a New Keynesian model and assess the interaction between fiscal and monetary policy in limiting possible macroeconomic costs of environmental fiscal measures.

The paper is organized as follows. The next section describes model setup and calibration. Section 3 illustrates the simulated scenarios. Section 4 reports the results. Section 5 concludes.

2 Model

We first provide an overview of the model (Section 2.1), then describe the production of energy (Section 2.2), the production of intermediate goods (Section 2.3), the households’ consumption basket (Section 2.4), the fiscal sector (Section 2.5), the monetary policy (Section 2.6), and discuss the calibration (Section 2.7).

2.1 Overview

The model is New Keynesian (nominal wage and price rigidities hold) and represents the whole EA economy and the RW. The size of the world economy is normalized to 1. EA and RW have sizes equal to $s^{EA}$, and $1 - s^{EA}$, respectively (with $0 < s^{EA} < 1$). In each country, the central bank sets the policy rate and, in the case of the EA, can purchase long-term sovereign bonds on the secondary market for monetary policy purposes. Long-term sovereign bonds are issued by the domestic fiscal authority, which also raises taxes on domestic capital, labour, and consumption. Crucially, the authority also raises taxes

\footnote{For each country, size refers to the overall population and to the number of firms operating in each sector.}
on “brown” sources of energy and subsidies on “green” sources of energy, where brown sources are those that generate CO$_2$ emissions, while green sources are the renewable ones (more later).

The model has two key features. First, it includes energy generated from oil, coal, gas, nuclear, and a green source. The latter is representative of solar, wind, hydro, and bio sources of energy. Sources other than those of green energy and nuclear energy generate CO$_2$ emissions (so-called “brown” sources). The second key model feature is financial segmentation as in Chen et al. (2012), that allows central bank asset purchases to have real effects in our model.\textsuperscript{8}

Figure 1 shows the links between intermediate manufacturing (non-energy) sector, energy sector, and final consumption and investment goods sectors. There are firms that produce (i) energy, (ii) intermediate tradable goods, (iii) three final non-tradable goods (private consumption, public consumption, and investment goods), and (iv) physical capital.

Each type of energy is produced by firms under perfect competition using a constant-elasticity-of-substitution (CES) production function whose inputs are domestic capital, labour, and the related source of energy. It is assumed that capital, labour, and the energy services are not traded across the two countries. Within each country, capital and labour are mobile across sectors. Moreover, EA imports oil, coal, and gas from the RW. The nuclear source is assumed to be nontradable.

The different types of energy enter a CES energy bundle. The latter enters, joint with value added, the CES production function of intermediate goods. The value added is produced according to a Cobb-Douglas production function whose inputs are domestic capital and labour. Intermediate goods are traded across countries. Each firm in the intermediate manufacturing sector acts under monopolistic competition, thus it has market power and sets domestic and foreign prices of its intermediate good taking into account the demand by firms in the local (i.e., country-specific) final sector and subject to quadratic price adjustment costs, which introduce short-run nominal price rigidities in the model.

Intermediate goods are assembled in private consumption, public consumption, and investment goods by firms in the domestic final good sector, under perfect competition according to a CES production function.

Energy also enters the households’ consumption basket. A CES bundle of energy services is part of the overall private consumption basket, jointly with the CES bun-

\textsuperscript{8}See also Bartocci et al. (2017).
dle of intermediate goods. The latter bundle is composed of domestic and imported intermediate tradables.

“Capital producers” (i.e., firms producing physical capital) optimally choose investment in physical capital to maximize profits under perfect competition, subject to the law of capital accumulation and quadratic adjustment costs on investment, taking prices as given. They rent capital to domestic firms producing intermediate goods and rebate profits to domestic “restricted” and “Ricardian” households (see below).

In each country there are three types of households, labeled restricted, Ricardian, and “Rule-Of-Thumb” (ROT).

Restricted households have access only to the domestic long-term sovereign bond market and, joint with domestic Ricardian, own shares of domestic capital producers. This setup allows asset purchases by the central bank to have non-trivial effects on economic activity and inflation. Purchases of long-term sovereign bonds by the central bank for monetary policy purposes reduce the long-term sovereign interest rates, inducing restricted households to substitute consumption and investment in physical capital for long-term sovereign bonds.9

Ricardian households have multiple investment choices, because they invest in domestic short-term sovereign bonds, long-term sovereign bonds, and international short-term bonds. The latter are traded with Ricardian households of the other country, and are denominated in Foreign currency, implying that an uncovered interest parity condition holds in the model that links the differential between Home and Foreign monetary policy rates to the expected nominal exchange rate depreciation of the Home currency. Ricardian households own domestic firms operating in the final and intermediate sectors and hold shares of the domestic capital producers.

ROT households do not have access to financial markets and consume all available labour income in each period.

All households supply differentiated labour services to domestic firms (other than capital producers) and act as nominal wage setters in monopolistically competitive labour markets, as they charge a wage markup over their marginal rate of substitution between consumption and leisure. Nominal wage and labour decisions are taken by Ricardian households for all households taking into account labour demand by firms and subject to quadratic adjustment costs on nominal wages. The overall wage income is equally distributed across all households.

The fiscal bloc includes, in each country, the government budget constraint and a

9See Chen et al. (2012).
fiscal rule. The government issues short- and long-term bonds in the domestic financial market. A fiscal rule, if activated, commands changes in lump-sum transfers (or lump-sum taxes) to stabilize public debt. The government buys consumption goods and raises taxes on local households and firms (labour income, consumption, and capital income). Importantly, the government levies a tax on the brown sources of energy (a “carbon tax”) and a subsidy on the production of energy from green sources.

In each country, the central bank sets the policy rate according to a Taylor-type rule to stabilize inflation and economic activity. Moreover, in the case of the EA, it can implement discretionary purchases of long-term sovereign bonds for monetary policy purposes when the policy rate is constrained by the ELB.

We consider a symmetric equilibrium, where in each country there is a representative household for each type of households, and a representative firm for each sector. In what follows, we report key equations for the EA. Similar equations hold for the RW.

2.2 Production of energy

Energy from oil, denoted \( EN_{oil,t} \), is produced by a representative firm under perfect competition according to the production function

\[
\begin{align*}
EN_{oil,t}^{-\rho_{EN_{oil}}^{-1}} &= \gamma_{oil,source}^{\rho_{EN_{oil}}^{-1}} OIL_t^{\rho_{EN_{oil}}^{-1}} + \gamma_{oil,k}^{\rho_{EN_{oil}}^{-1}} K_{oil,t}^{\rho_{EN_{oil}}^{-1}} \\
&
+ (1 - \gamma_{oil,k} - \gamma_{oil,source})^{\rho_{EN_{oil}}^{-1}} L_{oil,t}^{\rho_{EN_{oil}}^{-1}},
\end{align*}
\]

where \( OIL_t, K_{oil,t}, \) and \( L_{oil,t} \) are oil, physical capital, and labour, respectively. The parameters \( \gamma_{oil,source} \) and \( \gamma_{oil,k} \) (\( 0 < \gamma_{oil,k}, \gamma_{oil,source} < 1, \gamma_{oil,k} + \gamma_{oil,source} < 1 \)) are the weights of oil source and capital, respectively. The parameter \( \rho_{EN_{oil}} > 0 \) is the elasticity of substitution among the different inputs.

The representative firm producing energy from the oil source optimally chooses capital, labour, and the oil source to maximize profits subject to the technology constraint (Eq. 1) and taking prices as given.

The first order condition with respect to the oil source \( OIL_t \) is

\[
OIL_t = \gamma_{oil,source} \left( \frac{p_{oil,t} + tax_{oil,t}}{rmc_{oil,t}} \right)^{-\rho_{EN_{oil}}} EN_{oil,t},
\]

\[\text{Eq. 2}\]

\[\text{10}\text{The other first order conditions are standard and we do not report them to save on space. They are reported in the Appendix.}\]
where $tax_{oil,t} > 0$ is the carbon tax on oil paid by the firms in the oil-based energy sector, $rmc_{oil,t}$ is the real marginal cost (in domestic consumption units) to produce oil-based energy and $p_{oil,t}$ is the relative price of oil (in domestic consumption units). Oil demand for energy production is lower, the higher the value of the carbon tax. In our simulations, we assume that the before-tax relative price of oil, i.e., priced in units of domestic consumption, in each period $t$ is constant at its baseline level, to have a clean evaluation of the macroeconomic impacts of the carbon tax. This amounts to assuming that the RW has its own endowment of oil that exogenously adjusts to satisfy global demand, in particular demand originating in the EA. The EA imports oil, coal, and gas from the RW, while nuclear and renewable sources of energy are held domestically (and thus not internationally traded). For simplicity, we do not allow for cross-country trade in energy.\(^{11}\)

Similar assumption holds for coal-, gas-, nuclear-, and renewable source-based energy sectors. Each sector has its own production function, similar to the one of the oil sector, in which capital and labour are combined with the sector-specific source of energy. In particular, the representative firms in the gas- and coal-based energy sectors pay a sector-specific carbon tax levied on gas and coal, respectively. Instead, the representative firms in the nuclear and renewable energy sector do not pay the carbon tax because they do not generate CO\(_2\) emissions.

In the case of the firm producing energy from renewable (green) sources, profit maximization implies the following first-order condition with respect to capital $K_{res,t}$:

$$K_{res,t} = \gamma_{res,k} \left( \frac{r_{k,t}(1 - subs_{res,t})}{rmc_{res,t}} \right)^{-\rho_{ENres}} EN_{res,t},$$

where $\gamma_{res,k}$ ($0 < \gamma_{res,k} < 1$) is the weight of capital in the production function of energy, $subs_{res,t} > 0$ is the subsidy to physical capital, $\rho_{ENres} > 0$ is the elasticity of substitution among inputs used to produce the green energy, $r_{k,t}$ is the real rental rate of capital, $rmc_{res,t}$ is the real marginal cost to produce energy, $EN_{res,t}$ is the amount of green energy produced. Ceteris paribus, the demand for capital to produce energy from renewable sources is higher, the higher the subsidy.

### 2.3 Production of intermediate goods

There is a representative profit-maximizing firm that operates under perfect competition and produces the intermediate (non-energy) good $Y_t$ according to a technology that

\(^{11}\)We leave the extension to cross-border trade in energy for future research.
combines valued added \( VA_{y,t} \) and the energy bundle \( EN_{Y,t} \):
\[
Y_t^{\rho_Y - 1} = \gamma_{pr_{va,va}} VA_{y,t}^{\rho_Y - 1} + (1 - \gamma_{pr_{va,va}}) \rho_Y EN_{Y,t}^{\rho_Y - 1},
\]
where the parameter \( \gamma_{pr_{va,va}} \) (0 < \( \gamma_{pr_{va,va}} < 1 \)) measures the weight of value added. The parameter \( \rho_Y > 0 \) is the elasticity of substitution among inputs. The value added is
\[
VA_{y,t} = K_{Y,t}^{\gamma_{pr_{va,k}}} L_{Y,t}^{1 - \gamma_{pr_{va,k}}},
\]
where \( K_{Y,t} \) is physical capital and \( L_{Y,t} \) is labour. The parameter \( \gamma_{pr_{va,k}} \) (0 < \( \gamma_{pr_{va,k}} < 1 \)) is the elasticity of value added with respect to capital.

The energy bundle is
\[
EN_{Y,t}^{\rho_{EN} - 1} = \gamma_{pr_{oil}} EN_{oil,t}^{\rho_{EN} - 1} + \gamma_{pr_{coal}} EN_{coal,t}^{\rho_{EN} - 1} + \gamma_{pr_{gas}} EN_{gas,t}^{\rho_{EN} - 1} + \gamma_{pr_{nuc}} EN_{nuc,t}^{\rho_{EN} - 1} + (1 - \gamma_{pr_{oil}} - \gamma_{pr_{coal}} - \gamma_{pr_{gas}} - \gamma_{pr_{nuc}}) EN_{resy,t}^{\rho_{EN} - 1},
\]
where the parameters \( \gamma_{pr_{oil}}, \gamma_{pr_{coal}}, \gamma_{pr_{gas}}, \gamma_{pr_{nuc}} \) measure the weights of energy obtained from oil \( EN_{oil,t} \), coal \( EN_{coal,t} \), gas \( EN_{gas,t} \), and nuclear \( EN_{nucy,t} \) in the energy bundle, respectively.\(^\text{12}\) The term \( EN_{resy,t} \) represents energy obtained from the renewable source. The parameter \( \rho_{EN} > 0 \) is the elasticity of substitution among the different types of energy.

**2.4 Final consumption basket**

The final consumption basket \( C_t \) is a CES bundle of (non-energy) intermediate manufacturing consumption basket \( C_{manu,t} \) and of energy basket \( C_{EN,t} \):
\[
C_t^{\rho_{C} - 1} = \gamma_{pr_{manu}} C_{manu,t}^{\rho_{C} - 1} + (1 - \gamma_{pr_{manu}}) \frac{1}{\rho_{C}} C_{EN,t}^{\rho_{C} - 1},
\]
where the parameter \( \gamma_{pr_{manu}} \) (0 < \( \gamma_{pr_{manu}} < 1 \)) measures the weight of non-energy final consumption basket in the overall consumption bundle. The parameter \( \rho_{C} > 0 \) is the elasticity of substitution between non-energy and energy consumption. The non-energy consumption basket is a CES of domestic and foreign consumption goods. The

\(^{12}\)0 < \( \gamma_{pr_{oil}}, \gamma_{pr_{coal}}, \gamma_{pr_{gas}}, \gamma_{pr_{nuc}} < 1, \gamma_{pr_{oil}} + \gamma_{pr_{gas}} + \gamma_{pr_{coal}} + \gamma_{pr_{nuc}} < 1.\)
latter are bundles of goods produced by firms in the domestic and foreign intermediate sectors, respectively.\footnote{The final investment good is a CES bundle of domestic and foreign investment goods. Differently from the consumption good, the investment good does not have an energy component.}

The energy consumption bundle $C^t_{EN}$ is

\[
C_{EN,t} = \frac{\rho_{EN}}{\rho_{EN}^t} - 1\rho_{EN}^t \left[ \gamma_{prc,oil}^{\rho_{EN}} EN_{oilc,t} + \gamma_{prc,gas}^{\rho_{EN}} EN_{gasc,t} + \gamma_{prc,coal}^{\rho_{EN}} EN_{coalc,t} + \gamma_{prc,nuc}^{\rho_{EN}} EN_{nucc,t} \right] + (1 - \gamma_{prc,oil}^{\rho_{EN}} - \gamma_{prc,gas}^{\rho_{EN}} - \gamma_{prc,coal}^{\rho_{EN}} - \gamma_{prc,nuc}^{\rho_{EN}}) \frac{1}{\rho_{EN}} EN_{resc,t},
\]

(8)

where the parameters $\gamma_{prc,oil}^{\rho_{EN}}, \gamma_{prc,gas}^{\rho_{EN}}, \gamma_{prc,coal}^{\rho_{EN}}, \gamma_{prc,nuc}^{\rho_{EN}}$ measure the weights of energy obtained from oil ($EN_{oilc,t}$), gas ($EN_{gasc,t}$), coal ($EN_{coalc,t}$), and nuclear ($EN_{nucc,t}$) in the energy consumption bundle, respectively.\footnote{0 < $\gamma_{prc,oil}^{\rho_{EN}}, \gamma_{prc,gas}^{\rho_{EN}}, \gamma_{prc,coal}^{\rho_{EN}}, \gamma_{prc,nuc}^{\rho_{EN}}$ < 1, $\gamma_{prc,oil}^{\rho_{EN}} + \gamma_{prc,gas}^{\rho_{EN}} + \gamma_{prc,coal}^{\rho_{EN}} + \gamma_{prc,nuc}^{\rho_{EN}}$ < 1.} The term $EN_{resc,t}$ represents energy obtained from the renewable source. The parameter $\rho_{EN} > 0$ is the elasticity of substitution among the different types of energy.

### 2.5 Fiscal sector

The EA (representative) government budget constraint is

\[
B^G_t - B^G_{t-1} R^B_t + P^long_t B^G,_{long,t} - R^long_t^t P^long_t B^G,_{long,t-1} = P_{EA,t} G_t - TAX_t - T_t - s_{EA} \times (tax_{oil,t} OIL_t + tax_{gas,t} GAS_t + tax_{coal,t} COAL_t) + s_{EA} \times subs_{res,t} \times r^K_t \times K_{res,t},
\]

(9)

where $B^G_t$ is the short-term (one-period) bond which pays the gross interest rate $R^B_t$, $B^G,_{long,t}$ is the long-term bond and $P^long_t$ its price. Following Woodford (2001), the bond is formalized as a perpetuity paying an exponentially decaying coupon $\kappa_{long} \in (0, 1]$. The gross interest rate $R^long_t$ on the long-term bond is equal to

\[
R^long_t = \frac{1}{P^long_t} + \kappa_{long}.
\]

(10)

The variable $G_t$ is government purchases of goods and services (i.e. public spending for consumption). Consistent with the empirical evidence, $G_t$ is fully biased towards the domestic intermediate manufacturing good. Therefore, it is multiplied by the corre-
sponding price index $P_{EA,t}$.\footnote{See Corsetti and Müller (2006).}

The variable $TAX_t > 0$ denotes lump-sum taxes imposed on Ricardian households. The variables $tax_{oil,t}$, $tax_{gas,t}$, $tax_{coal,t}$ are the carbon taxes on oil, gas, coal, respectively, and $subs_{res,t}$ is the subsidy to physical capital used as input to produce energy form renewable sources. Tax rates on labour income, capital income, and consumption are $\tau^w_t$, $\tau^k_t$, $\tau^c_t$, respectively ($0 \leq \tau^w_t$, $\tau^k_t$, $\tau^c_t \leq 1$). The same tax rates apply to the domestic representative Ricardian, restricted, and ROT households. Total government revenues from distortionary taxation $T_t$ other than the carbon tax are given by the identity

$$T_t \equiv \tau^w_t W_t s^{EA} L_t + \tau^k_t R_t s^{EA} K_{t-1} + \tau^c_t P_t s^{EA} (s_{ric,t} C_{ric,t} + s_{restr,t} C_{restr,t} + (1 - s_{ric} - s_{res}) C_{rot,t}), \quad (11)$$

where $0 < s_{ric}, s_{restr} < 1$ and $1 - s_{ric} - s_{res} (s_{ric} + s_{res} < 1)$ are the population shares of Ricardian, restricted, and ROT households, respectively (the sum of the three shares is equal to 1). Variables $C_{ric,t}$, $C_{restr,t}$, and $C_{rot,t}$ are consumption of representative Ricardian, restricted and ROT households, respectively. The government follows a fiscal rule defined on lump-sum taxes $TAX_t$ to bring the short-term public debt as a percentage of domestic GDP, $b^G_t > 0$, in line with its long-run (steady-state) target $\bar{b}^G$.

The fiscal rule is

$$\frac{tax_t}{TAX_t} = \left( \frac{b^G_t}{\bar{b}^G} \right)^{\phi_{bG}}, \quad (12)$$

where the parameter $\phi_{bG} > 0$ calls for an increase (reduction) in lump-sum taxes as a ratio to GDP, $tax_t$, relative to its steady-state value $\overline{tax}$, whenever the current-period short-term public debt as a ratio to GDP, $b^G_t$, is above (below) the steady-state target, $\bar{b}^G$. We choose lump-sum taxes to stabilize public finance as they are non-distortionary and, thus, allow for a “clean” evaluation of the macroeconomic effects of environmental taxes and subsidies. For simplicity, it is assumed that the changes in issued long-term sovereign bonds are proportional to the changes in issued short-term sovereign bonds dictated by Eq. (12).
2.6 Monetary policy

We assume the following specification for the monetary policy rule:

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_r} \left( \frac{\pi_{EA,t}}{\bar{\pi}_{EA}} \right)^{(1-\rho_r)\rho_\pi} \left( \frac{y_{EA,t}}{y_{EA,t-1}} \right)^{(1-\rho_r)\rho_y}.
\]

The rule describes how the central bank conducts its monetary policy. The variable \(R_t\) is the gross policy rate and \(\bar{R}\) its steady-state value. The parameters \(0 \leq \rho_r \leq 1\), \(\rho_\pi > 0\), and \(\rho_y\) measure the sensitivity of the policy rate to its lagged value, to (quarterly) gross inflation rate (in deviation from the target \(\bar{\pi}_{EA}\)), and to the quarterly gross growth rate of output \(y_{EA} (y_{EA,t}/y_{EA,t-1})\), respectively.\(^{16}\)

Moreover, consistent with the currently observed low level of the short-term interest rates, in some simulations it is assumed that the EA policy rate is constrained by the ELB and that the EA central bank, for monetary policy purposes, discretionally purchases on the secondary market domestic long-term sovereign bonds issued by the EA fiscal authority. This type of non-standard measure contributes to sustain inflation dynamics in a macroeconomic context characterized by low inflation, low policy rate, and low natural rate of interest.

2.7 Calibration

The model is calibrated at quarterly frequency. The chosen calibration allows our model to adequately capture the dynamics of the main EA variables. For simplicity and consistent with the focus on EA, it is assumed that the RW calibration is rather symmetric to that of the EA but for size, degree of openness and imports (the EA imports fossil fuels from the RW).

Table 1 reports the (flexible-price) steady-state equilibrium. We calibrate the model so that the net natural (real) interest rate is equal to 0.1 in steady state, while the steady-state net annualized inflation rate is 2%. The (net) policy rate is set to 2.1%.\(^{17}\)

Private consumption, public consumption, investment, and imports are set to 60%, 21%, 19%, and 20% of GDP, respectively. Long-term public debt amounts to 100% of GDP, half of which is held by Ricardian and half by restricted households.

The energy shares of oil-, gas-, coal-, nuclear-, and green-based energy are set to

\(^{16}\)The lagged interest rate ensures that the policy rate is adjusted smoothly and captures the idea that the central bank prefers to avoid large changes and reversals in its policy instrument.

\(^{17}\)For evidence of a low level of the natural rate in the EA, see Neri and Gerali (2017).
34.4, 22.4, 14.4, 13.4, and 15.4, respectively (see European Commission 2020). Energy shares in EA firms’ production costs and households’ consumption are set to 9.7% and 10.2%, respectively

Tables 2 and 3 show parameters regulating preferences and technology. As reported in Table 2 the elasticity of intertemporal substitution is set to 1 (i.e., log preferences in consumption). The discount factors of Ricardian and restricted households are set to 0.9998 and 0.999, respectively. The consumption habit parameter is set to 0.7. The Frisch labour elasticity is set to 0.5. The shares of Ricardian, restricted, and ROT households in the population are set to 0.55, 0.2, 0.25. Ricardian households hold a share of capital producers equal to 40%, restricted households equal to 60%.

The production functions and the consumption baskets are calibrated in line with the literature and according to the following criteria. First, the elasticity of substitution between non-energy and energy inputs in the production function of the intermediate manufacturing good and in the (final) consumption basket is relatively low (lower than one). Second, as reported in Table 1, the weights of energy services are set to match data on the shares of oil, gas, coal, nuclear, and green energy in energy consumption. Third, the responses of main macroeconomic variables to an oil shock are in line with empirical evidence for the EA.

The elasticity of substitution among energy types is 0.45. As this is a central parameter in assessing the necessary conditions for a green transition, we perform a sensitivity analysis on it. For final goods, the elasticity of substitution between domestic and imported intermediate goods is 1.5.

Table 4 reports the markups and the elasticities of substitution among intermediate brands and among labour varieties. They are set to 6 and 4.3, respectively, which correspond to steady-state mark-ups of 1.2 and 1.3.

Table 5 contains the adjustment costs. The investment adjustment cost is equal to 6. Concerning nominal rigidities, the parameter measuring the cost for adjusting the price of goods is set to 380. The one for adjusting nominal wages is set to 400. The parameter that measures the degree of indexation to previous-period inflation is set to 0.7 for both prices and wages.

Table 6 reports the parameters of the monetary policy and fiscal rule. For monetary policy, the response to inflation, $\rho_\pi$, is relatively large and equal to 1.7, consistent with

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18 For a review of the empirical literature on the elasticity of substitution among energy sources see Papageorgiou et al. (2017).

19 The corresponding Calvo (1983) probabilities of not adjusting prices and wages are 0.89 and 0.81, respectively.
the estimated value reported by Warne et al. (2008). The policy rate is adjusted slowly, given that the coefficient measuring the inertia in interest rate setting, $\rho_r$, is set to 0.87. The response to output growth, $\rho_y$, is set to 0.1. For fiscal policy, lump-sum taxes respond to public debt according to a coefficient set to 0.6.

3 Simulated scenarios

We initially simulate a gradual carbon tax increase in the EA starting from the first quarter of 2022 (starting period of the simulations). The carbon tax raises by $75 in 2030 on each source of energy per metric ton of CO$_2$ emissions, and is calibrated as follows.\textsuperscript{20} Relative to the baseline, in 2030 the price of oil increases by $31.1$ (from a baseline level of $73.7$), the price of an oil-barrel equivalent of gas increases by $22.7$ (from a baseline value of $52.5$), and the price of an oil-barrel equivalent of coal increases by $46.9$ (from a baseline value of $21.7$).

Second, we consider the case of an ex ante budget-neutral fiscal policy mix based on the gradual increase in both the carbon tax and the subsidy to physical capital used to produce green energy, and a gradual reduction in labour income taxes paid by households.\textsuperscript{21} The subsidy rate is in 2030 equal to 95% of the return of physical capital in the green energy sector (it is equal to zero at the beginning of the simulation). For the labour income tax, in the benchmark simulation the tax rate paid by households in 2030 is 36.64%, i.e., 3.36 percentage points below the steady-state level (40%), consistent with the ex ante budget-neutrality of the fiscal mix.

Third, we newly run the fiscal policy mix scenario under the assumptions that the ELB endogenously binds the EA policy rate and that the EA central bank enacts domestic long-term sovereign bond purchases for monetary policy purposes.

Finally, we run a sensitivity analysis by simulating the fiscal policy mix under alternative values of the elasticity of substitution among the different types of energy, the weight of sources in the energy generation, and speed of the fiscal policy mix implementation.

\textsuperscript{20}According to International Monetary Fund (2019), limiting global warming to 2C or less requires policy measures such as an immediate global carbon tax that will rise rapidly to $75 a ton of CO$_2$ in 2030.

\textsuperscript{21}The composition of such fiscal policy mix is suggestive and does not reflect any considerations about the optimal use of revenues from the higher carbon tax. Other measures may include, e.g., short-term targeted transfers to those households, such as ROT consumers, which are likely to be more severely hit by the recessionary effects of the green transition. At the same time, well-designed incentive schemes may favour investment in green and sustainable production, which expands long-run production.
The fiscal rule, Eq. 12, is not active during the first nine years, i.e., the lump-sum taxes do not change as a ratio to GDP during the first nine years. The rule is newly active starting from the tenth year, so as to stabilize public debt.

All simulations are run under perfect foresight. Households and firms perfectly anticipate the path of policy shocks and are surprised only in the initial period of the simulations.

4 Results

4.1 Carbon tax increase

Figure 2 reports the macroeconomic effects of an increase in EA carbon tax. The higher carbon tax has recessionary effects that gradually increase over time, consistent with the nine-year gradual increase in the tax. Firms producing intermediate goods face a gradual increase in the cost of brown energy inputs, which are directly hit by the carbon tax. Thus, they decrease their demand for brown energy sources and, given the low elasticity of substitution between value added and energy in the production process, also their demand for other inputs (i.e., capital and labour). Green energy production increases only to a small extent, because green and brown energies are not perfect substitutes. Thus, capital and labour used in the green energy sector increase only mildly. Demand for labour and, in the long run, for capital decreases, with negative effects on labour and capital income. Given the higher energy costs and the low substitutability between energy and non-energy products, consumption and investment decrease as well. The latter initially mildly increases, reflecting the lower rental rate of capital, but eventually falls, because of the lower demand for capital. EA exports decrease because of the appreciation of the EA nominal (and real) exchange rate, due to the contraction of the EA goods supply. EA imports of fossil fuels decrease over time because of the increase in the carbon tax. EA imports of manufacturing goods mildly increase consistent with the appreciation of the EA exchange rate. Overall, EA economic activity and output decrease.

EA inflation falls, consistent with the lower aggregate demand, the higher after-tax energy price notwithstanding. Specifically, the energy component of consumer price inflation immediately and persistently increases, reflecting the higher production costs

\footnote{Also Vermeulen et al. (2018) and Kara (2019), using the NiGEM model, find the carbon tax has recessionary effects on GDP.}

\footnote{Spillovers to the RW economic activity are small and negative in all simulations.}
of energy due to the carbon tax. To the opposite, the price of the manufacturing good falls, driven by the gradual decrease in aggregate demand. The decrease in inflation induces the central bank to reduce the policy rate, consistent with the Taylor rule (see Eq. 13).  

Long-term sovereign bonds issued by the governments decrease (not reported), given the higher revenues from carbon taxes. The price of long-term debt increases and, thus, the long-term interest rate decreases (see Eq. 10), by around 0.6 percentage points at the end of the transition.

Figure 3 reports the corresponding effects on the use of energy sources. There is a drop in the use of brown sources of energy (especially coal), which have become more expensive because of the higher carbon tax, while green energy increases because it is not taxed and it is a substitute (although an imperfect one) to brown sources. The increase in the use of green energy is contained, because the two types of energy, green and brown, are not perfect substitute and, crucially, because overall economic activity drops, which reduces the demand for all energy sources.

Overall, the increase in carbon tax has recessionary effects.

A few remarks are in order to further clarify the results.

First, the carbon tax increase –as well as the other fiscal measures as well – is announced, gradually implemented and fully anticipated by agents. Thus, the carbon tax is conceptually different from a sudden and unexpected positive shock affecting the international prices of fossil fuels, which would have stagflationary effects on the EA economy.

Second, a sudden increase in the carbon tax, as opposed to the gradual implementation assumed in our simulations, would generate a positive (temporary) impact effect on consumer price inflation, followed by a reduction over time.

Third, in our simulations it is assumed that the gradual carbon tax increase is fully anticipated by households and firms. Additional simulations, not reported to save on space, show that the very same gradual carbon tax increase would have mild inflationary effects and almost zero effects on output growth during the first year, if households and firms do not anticipate the planned future carbon tax increase and, instead, are surprised

\(^{24}\) Konradt and di Mauro (2021) find that carbon taxes do not have to be inflationary and may even have deflationary effects. Their evidence, based on the experience of CO\(_2\) taxes introduced in Europe and Canada over the last thirty years, suggests that the increase in energy prices was more than offset by a fall in the prices of services and other non-tradables. Moessner (2022), using data for 35 OECD economies, shows that an increase in carbon taxes has no significant effects on headline CPI inflation.

\(^{25}\) In the ninth year, revenues from the (increased) carbon tax are 1.9% of GDP, broadly in line with figures for G20 economies reported in International Monetary Fund (2019).
in each of the first four quarters by the carbon tax rise. From the second year onwards, when agents are assumed to fully anticipate the planned carbon tax increase, effects would be disinflationary. This scenario could be interpreted as a case of a carbon tax rise plan which is not announced or, if announced, not fully credible and/or not clearly communicated. For qualitative similar results see Ferrari and Nispi Landi (2022b).

4.2 Fiscal policy mix

Figure 4 reports the macroeconomic impact of the EA implementing a fiscal policy mix, based on gradually increasing the carbon tax and the subsidy to renewable energy, and gradually reducing labour taxes. Different from the case of higher carbon tax, the effects on economic activity are only mildly recessionary and only at the end of the transition. Output is very close to its baseline value along the overall nine-year transition and it is mildly above the baseline in the initial periods. The higher carbon tax has negative effects on economic activity, because it depresses aggregate demand, as illustrated in the previous subsection. However, the lower labour income tax stimulates aggregate demand for consumption, while the subsidy to the green sector stimulates sector-specific demand for labour and capital. Overall, the carbon tax-induced recessionary effect and the expansionary effect associated with higher subsidies and lower labour tax broadly offset one another. Inflation mildly and persistently decreases below its baseline level, consistent with the weak aggregate demand, and eventually increases, as the large increase in the energy component tends to prevail, over time, on the reduction in the non-energy one. The central bank persistently reduces the policy rate below the baseline, consistently with consumer price inflation dynamics.

There is a reduction in the use of brown sources of energy (Fig. 5), because the higher carbon tax makes them more expensive. The use of green sources of energy increases more than in the scenario of carbon tax increase, because they are more convenient thanks to the subsidies.²⁶

Overall, a fiscal mix of (i) carbon tax, (ii) subsidies for renewable sources of energy and (iii) lower labour income tax can greatly reduce the macroeconomic costs associated

²⁶Such scenario is quite conservative if compared to existing studies. Vermeulen et al. (2018) report that in 2015, the share of renewable energy in the Dutch economy’s energy mix was 19% and, according to most energy experts, such share will be more than 50% by 2050. Thus, Vermeulen et al. (2018) design a technological breakthrough scenario that allows the share of renewable energy to double in five years. Kara (2019) considers a similar scenario for Netherlands, in which the amount of fossil fuel used to produce a unit of energy falls by 25% over five years, which amounts to doubling the share of renewable energy over the same period. If we considered such technological change in our scenarios and its interaction with higher subsidies, the increase in the share of green energy would be larger.
with the imposition of the carbon tax and favour the additional increase in the use of renewable sources of energy.\footnote{A mix without carbon tax and based only on higher subsidies for renewable sources and lower labour income tax would have expansionary effects, reduce the use of brown sources and favour the use of green sources of energy.}

### 4.3 Fiscal policy mix, ELB, and central bank asset purchases

We newly simulate the fiscal mix previously illustrated, under the assumption that in the EA the ELB can endogenously bind and, thus, the central bank cannot reduce the policy rate as much as needed to stabilize inflation and economic activity.\footnote{Technically, the endogenous ELB is implemented by imposing the max operator on the Taylor rule, Eq. (13), as follows:}

\[
\frac{R_t}{R} = \max \left( \frac{1}{R^*} \left( \frac{R_{t-1}}{R^*} \right)^{\rho_r} \left( \frac{\pi_{E_A,t}}{\bar{\pi}_{E_A}} \right)^{(1-\rho_r)\rho_r} \left( \frac{y_{E_A,t}}{y_{E_A,t-1}} \right)^{(1-\rho_r)\rho_y} \right),
\]

\footnote{where \(\max\) is the max operator (note that \(R\) is the monetary policy rate in gross terms, so the term \(\frac{1}{R^*}\) implies that the ELB is zero in net terms). In the simulations, we set the ELB at approximately 20 annualized basis points below the steady-state level of the nominal interest rate, for computational reasons. In general, the smaller is the room for manoeuvre for the central bank to decrease the policy rate to counteract disinflationary shocks, the larger are the associated recessionary and disinflationary effects we obtain.}

We also simulate a scenario in which, on top of the fiscal mix and the endogenous ELB, the central bank buys EA long-term sovereign bonds in the secondary market for monetary policy purposes. Net purchases are gradually implemented during the first four years. The stock of overall purchases amounts to 3.75\% of the initial steady-state annualized GDP.\footnote{The chosen size of the overall purchases is illustrative. It is relatively small compared to historical evidence. The first wave of APP purchases announced by the European Central Bank in January 2015 amounted to about 10\% of EA annualized GDP (see Altavilla et al., 2015).} The central bank keeps the overall stock of purchased long-term sovereign bonds in its balance sheet for around seven years.

Fig. 6 contains the results. When the ELB is binding (red dashed line), the EA central bank cannot reduce the policy rate by the amount needed to stabilize inflation, which decreases following the recessionary effects associated with the gradual increase in the carbon tax. Thus, the (ex ante) real interest rate increases more than in the no-ELB case, inducing households and firms to further reduce aggregate demand. The additional deterioration in aggregate demand generates further disinflationary pressures that increase the real interest rate. In equilibrium, relative to the no-ELB case, inflation and economic activity decrease to a larger extent in the first four years.

The stronger disinflationary pressures in the ELB-case can be offset by the stimu-
lating effects of the central bank asset purchases (green line). The latter, by reducing the long-term interest rate, favor aggregate demand and, thus, economic activity and inflation dynamics. The mild medium-term fall in output observed under the green fiscal policy mix is more than offset and output actually increases along the transition. Inflation only very mildly falls, showing values close to those observed in the no-ELB case. Consistent with the relative improvement in macroeconomic conditions, the policy rate does not achieve the ELB, because the relative improvement in inflation and economic activity generated by the asset purchases allows the central bank to reduce the policy rate to a lower extent.

Overall, a gradual increase in the carbon tax can have disinflationary effects, even if accompanied by other expansionary fiscal measures that broadly offset the recessionary effects on economic activity. If the ELB on the policy rate binds, such disinflationary effects are amplified. Central bank purchases of long-term sovereign bonds for monetary policy purposes, by reducing long-term interest rates, can sustain aggregate demand and, thus, economic activity and inflation dynamics.

4.4 Sensitivity analysis

We newly run the ex ante budget-neutral fiscal policy mix (higher carbon tax, higher subsidies for renewable energy sector, and lower labour income tax) under the assumption that the elasticity of substitution among the different types of energy, for both households and firms, has a smaller (0.2) or, alternatively, higher value (0.9) than the benchmark one (0.45).\(^{30}\) Fig. 7 reports the results. The macroeconomic effects are virtually identical across the three scenario. In the long run, GDP decreases to a lower extent under high elasticity than under low elasticity, as households and firms substitute green energy for brown energy more easily under high elasticity. Inflation mildly decreases across the three cases.

We also simulate the fiscal policy mix assuming that the weights of fossil fuels in the corresponding energy generation technologies are larger than in the benchmark calibration (we correspondingly reduce the weight of capital in each production function). Specifically, we set the weights of oil, gas, and coal to 0.5 (instead of 0.4 as in the benchmark calibration), 0.45 (0.35) and 0.4 (0.3), respectively. Results in Fig. 9 show that the carbon tax impact has slightly larger recessionary effects on economic activity than in the benchmark case.\(^{31}\)

\(^{30}\)In this Section we assume that the ELB is not binding.

\(^{31}\)We have also simulated the increase in carbon tax under the assumption of lower weights of energy

23
Finally, we simulate a faster implementation of the fiscal mix, which is now assumed to last five years instead of nine as in the benchmark simulations. As reported in Fig. 8, the short-run recessionary effects would be more pronounced than in the benchmark case, because households would face a more rapid increase in the carbon tax, which would exert its negative effects on aggregate demand more rapidly than in the benchmark case. The inflation rate would initially decrease and then increase, in a more rapid way than in the benchmark case, consistent with the more rapid increase in the carbon tax.

Overall, the sensitivity analysis suggests that, to minimize the possible macroeconomic costs along the transition, the design of the carbon tax should take into account the ability of the economic system to adapt and react to the implied changes in relative prices of energy sources.

5 Conclusions

In this paper we have studied the macroeconomic effects of fiscal policy measures implemented in the EA to reduce CO$_2$ emissions. The carbon tax rise is gradually implemented and announced by the fiscal authority. Thus, it is conceptually different from a sudden and unexpected positive shock to the international prices of fossil fuels, which would have stagflationary effects on the EA economy.

According to our results, a policy mix based on higher carbon tax, subsidies for green energy, and a lower labour income tax can greatly limit the macroeconomic cost of the higher carbon tax. Moreover, while under the ELB the policy mix can have short-run macroeconomic costs, the latter can be offset if the central bank, for monetary policy purposes, keeps long-term interest rates low by purchasing long-term sovereign bonds.

Our paper can be extended along several dimensions. First, a different composition of the policy mix than the one suggested in the paper may induce positive effects on output in the medium-to-long term. For example, our model does not feature long-term endogenous growth, driven for instance by investment in R&D to develop green technology, which, with appropriate incentive, could have expansionary effects on economic activity and output growth in the short and long run. Second, green transition

\[24\]

\[\text{DArcangelo et al. (2022) highlights the need for developing decarbonisation strategies based on a wide policy mix consisting of three main components: 1) emission pricing policy instruments; 2) standards and regulations; 3) complementary policies to facilitate the reallocation of capital, labour, and innovation towards low-carbon activities and to offset the adverse distributional effects of reducing emissions.}\]
policies could be implemented also in the RW, to assess the domestic and international macroeconomic and environmental effects of simultaneous measures implemented by the EA and other countries, from both positive and normative (optimal policy) perspectives. We leave these issues for future research.
References


Bartocci, Anna and Massimiliano Pisani, “Green fuel tax on private transportation services and subsidies to electric energy. A model-based assessment for the main European countries,” Energy Economics, 2013, 40 (S1), 32–57.


### Table 1: Main variables

<table>
<thead>
<tr>
<th></th>
<th>EA</th>
<th>RW</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Macroeconomic variables</strong></td>
<td></td>
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<tr>
<td>Private consumption</td>
<td>60.4</td>
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<tr>
<td>Public consumption</td>
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<td>21</td>
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<tr>
<td>Investment</td>
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<tr>
<td>Imports</td>
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<td>Imports of consumption goods</td>
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<td>Imports of investment goods</td>
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<tr>
<td>Imports of oil, coal, and gas</td>
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<td>Share of world GDP</td>
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<td>83.9</td>
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<tr>
<td><strong>Financial variables</strong></td>
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<tr>
<td>Nominal short-term rate</td>
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</tr>
<tr>
<td>Nominal long-term rate</td>
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<td>2.4</td>
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<tr>
<td>Long-term public debt</td>
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<td>100</td>
</tr>
<tr>
<td>Held by Ricardian households</td>
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<tr>
<td>Held by restricted households</td>
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<tr>
<td>Short-term public debt</td>
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<td>Net foreign asset position</td>
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<td><strong>Energy shares in total energy production</strong></td>
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<tr>
<td>Share of oil-based energy</td>
<td>34.4</td>
<td>34.4</td>
</tr>
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<td>Share of gas-based energy</td>
<td>22.4</td>
<td>22.4</td>
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<td>Share of coal-based energy</td>
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<td>Share of nuclear-based energy</td>
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<td>Share of renewable-based energy</td>
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<td>15.4</td>
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<tr>
<td><strong>Energy shares</strong></td>
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<tr>
<td>Energy share in firms’ production costs</td>
<td>9.7</td>
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<tr>
<td>Energy share in households’ consumption</td>
<td>10.2</td>
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Note: EA = euro area. RW = rest of the world. Macroeconomic variables are as % of GDP. Inflation and interest rates are in %, annualized term. Public debt as % of annualized GDP. Energy shares as %.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>EA</th>
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<tr>
<td><strong>Ricardian households discount factor $\beta_{\text{ric}}$</strong></td>
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<td><strong>Restricted discount factor $\beta_{\text{res}}$</strong></td>
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<td><strong>Intertemporal elasticity of substitution $1/\sigma$</strong></td>
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<td><strong>Habit $bb$</strong></td>
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<td><strong>Inverse of Frisch elasticity of labour supply $\tau$</strong></td>
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<table>
<thead>
<tr>
<th><strong>Share of households in population</strong></th>
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<th></th>
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<td><strong>Ricardian households $\lambda_{\text{ric}}$</strong></td>
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<tr>
<td><strong>Restricted households $\lambda_{\text{res}}$</strong></td>
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<table>
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<td><strong>Depreciation rate of capital $\delta$</strong></td>
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<td>0.025</td>
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<td>1.0</td>
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<tr>
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<td>0.29</td>
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<td><strong>Bias towards value added $\gamma_{\text{pr},va}$</strong></td>
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<tr>
<td><strong>Bias towards coal $\gamma_{\text{pr},coal}$</strong></td>
<td>0.12</td>
<td>0.12</td>
</tr>
<tr>
<td><strong>Bias towards nuclear energy $\gamma_{\text{pr},nuc}$</strong></td>
<td>0.13</td>
<td>0.13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Final consumption goods</strong></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Elasticity subst. btw. manufacturing good and energy $\rho$</strong></td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td><strong>Bias towards manufacturing goods $a_{\text{manu}}$</strong></td>
<td>0.76</td>
<td>0.76</td>
</tr>
<tr>
<td><strong>Elasticity subst. btw. dom. and imported manuf. goods $\phi$</strong></td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td><strong>Bias towards domestic tradable goods $a_{\text{EA}}$</strong></td>
<td>0.85</td>
<td>0.90</td>
</tr>
<tr>
<td><strong>Elasticity subst. among energy types $\rho$</strong></td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td><strong>Bias towards oil energy $a_{\text{oil}}$</strong></td>
<td>0.30</td>
<td>0.34</td>
</tr>
<tr>
<td><strong>Bias towards gas energy $a_{\text{gas}}$</strong></td>
<td>0.20</td>
<td>0.23</td>
</tr>
<tr>
<td><strong>Bias towards coal energy $a_{\text{coal}}$</strong></td>
<td>0.14</td>
<td>0.14</td>
</tr>
<tr>
<td><strong>Bias towards nuclear energy $a_{\text{nuc}}$</strong></td>
<td>0.14</td>
<td>0.12</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Final investment goods</strong></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Elasticity subst. btw. dom. and imported goods $\phi$</strong></td>
<td>1.50</td>
<td>1.50</td>
</tr>
<tr>
<td><strong>Bias towards domestic tradable goods $a_{\text{HI}}$</strong></td>
<td>0.85</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Note: EA = euro area. RW = rest of the world.
Table 3: Energy production technology

<table>
<thead>
<tr>
<th>Parameter</th>
<th>EA</th>
<th>RW</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Oil-based energy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elasticity subst. btw. factors of production</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Bias towards capital $\alpha_{oil,k}$</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Bias towards oil $\alpha_{oil,source}$</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td><strong>Gas-based energy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elasticity subst. btw. factors of production</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Bias towards capital $\alpha_{gas,k}$</td>
<td>0.55</td>
<td>0.55</td>
</tr>
<tr>
<td>Bias towards gas $\alpha_{gas,source}$</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td><strong>Coal-based energy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elasticity subst. btw. factors of production</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Bias towards capital $\alpha_{coal,k}$</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td>Bias towards coal $\alpha_{coal,source}$</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>Renewable-based energy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elasticity subst. btw. factors of production</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Bias towards capital $\alpha_{res,k}$</td>
<td>0.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Bias towards renewable source $\alpha_{res,source}$</td>
<td>0.01</td>
<td>0.01</td>
</tr>
<tr>
<td><strong>Nuclear-based energy</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elasticity subst. btw. factors of production</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td>Bias towards capital $\alpha_{nuc,k}$</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Bias towards nuclear $\alpha_{nuc,source}$</td>
<td>0.2</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Note: EA = euro area. RW = rest of the world.
Table 4: Gross markups (elasticities of substitution)

<table>
<thead>
<tr>
<th></th>
<th>EA</th>
<th>RW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intermediate goods</td>
<td>1.2 ($\theta_T = 6.0$)</td>
<td>1.3 ($\psi = 4.3$)</td>
</tr>
<tr>
<td>labour</td>
<td>1.2 ($\theta_T = 6.0$)</td>
<td>1.3 ($\psi = 4.3$)</td>
</tr>
</tbody>
</table>

Note: EA = euro area. RW = rest of the world.

Table 5: Adjustment costs

<table>
<thead>
<tr>
<th>Parameter</th>
<th>EA</th>
<th>RW</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ricardian households</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long-term sovereign bond $\phi_{ric,long}^I$</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>International bond $\phi_{B1}$</td>
<td>0.05</td>
<td>–</td>
</tr>
<tr>
<td>International bond $\phi_{B2}$</td>
<td>0.05</td>
<td>–</td>
</tr>
<tr>
<td><strong>Restricted households</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long-term sovereign bond $\phi_{res,long}^I$</td>
<td>0.001</td>
<td>0.001</td>
</tr>
<tr>
<td><strong>Firms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Physical capital $\phi_{I}$</td>
<td>6.0</td>
<td>6.0</td>
</tr>
<tr>
<td>Nominal wages $\kappa_W$</td>
<td>400</td>
<td>400</td>
</tr>
<tr>
<td>EA intermediate tradable goods $\kappa_{EA}$</td>
<td>380</td>
<td>380</td>
</tr>
<tr>
<td>RW intermediate tradable goods $\kappa_{RW}$</td>
<td>380</td>
<td>380</td>
</tr>
<tr>
<td>EA price indexation to past inflation $\alpha_{EA}$</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>RW price indexation to past inflation $\alpha_{RW}$</td>
<td>0.7</td>
<td>0.7</td>
</tr>
<tr>
<td>Wage indexation to past inflation $\alpha_W$</td>
<td>0.7</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Note: EA = euro area. RW = rest of the world.
Table 6: Monetary policy rules, fiscal policy rules and steady-state taxes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>EA</th>
<th>RW</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Monetary policy rule</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lagged interest rate $\rho_r$</td>
<td>0.87</td>
<td>0.87</td>
</tr>
<tr>
<td>Inflation $\rho_\pi$</td>
<td>1.70</td>
<td>1.70</td>
</tr>
<tr>
<td>Output growth $\rho_y$</td>
<td>0.10</td>
<td>0.10</td>
</tr>
<tr>
<td><strong>Fiscal policy rule</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lump-sum taxes sensitivity to public debt $\phi_{G^C}, \phi_{G^C}^*$</td>
<td>0.6</td>
<td>0.6</td>
</tr>
<tr>
<td><strong>Taxes</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Consumption tax rate $\tau_c, \tau_c^*$</td>
<td>20.0</td>
<td>20.0</td>
</tr>
<tr>
<td>Labour income tax rate $\tau_w, \tau_w^*$</td>
<td>40.0</td>
<td>40.0</td>
</tr>
<tr>
<td>Capital income tax rate $\tau_k, \tau_k^*$</td>
<td>30.0</td>
<td>30.0</td>
</tr>
<tr>
<td>Carbon tax</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>Subsidy rate to capital (renewables’ sector) $subs_{res,t}, subs_{res,t}^*$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
</tbody>
</table>

*Note: EA = euro area. RW = rest of the world. "*" refers to RW. Tax ans subsidy rates are in %.*
Figure 1: The role of energy in production and consumption
Figure 2: EA carbon tax: main macroeconomic variables

Notes: quarters on the horizontal axis; on the vertical axis, % deviations from the baseline; inflation and interest rates: annualized pp deviations; public debt as % of annualized GDP; for real exchange rate, + is a depreciation.
Figure 3: Carbon tax: energy variables

Notes: quarters on the horizontal axis; on the vertical axis, % deviations from the baseline.
Figure 4: Fiscal policy mix: main macroeconomic variables

Notes: quarters on the horizontal axis; on the vertical axis, % deviations from the baseline; inflation and interest rate: annualized pp deviations; for real exchange rate, + is a depreciation.
Figure 5: Fiscal policy mix: energy variables

Notes: quarters on the horizontal axis; on the vertical axis, % deviations from the baseline.
Figure 6: Fiscal policy mix: ELB and central bank asset purchases

Notes: quarters on the horizontal axis; on the vertical axis, % deviations from the baseline; inflation and interest rates: annualized pp deviations; for real exchange rate, + is a depreciation.
Figure 7: Sensitivity: fiscal policy mix and elasticity of substitution

Notes: quarters on the horizontal axis; on the vertical axis, % deviations from the baseline; inflation and interest rates: annualized pp deviations.
Figure 8: Sensitivity: fiscal policy mix and weight of fossil fuels

Notes: quarters on the horizontal axis; on the vertical axis, % deviations from the baseline; inflation and interest rates: annualized pp deviations.
Figure 9: Sensitivity: speed of fiscal policy mix implementation

Notes: quarters on the horizontal axis; on the vertical axis, % deviations from the baseline; inflation and interest rates: annualized pp deviations.
Appendix: the model

A.1 Energy sector

A.1.1 Production of energy $EN_{oil,t}(o)$ from oil

- Production function

The generic firm $o$ produces energy $EN_{oil,t}(o)$ using oil under perfect competition. It maximizes profits taking all prices and the technology constraint as given. The CES production function is

$$EN_{oil,t}(o) = \frac{\rho_{EN_{oil}}}{\rho_{EN_{oil}} - 1} \left( \gamma_{oil,source} \frac{\rho_{EN_{oil}}}{\rho_{EN_{oil}} - 1} OIL_t(o) + \gamma_{oil,k} \frac{\rho_{EN_{oil}}}{\rho_{EN_{oil}} - 1} K_{oil,t}(o) \right. + \left(1 - \gamma_{oil,k} - \gamma_{oil,source}\right) \frac{\rho_{EN_{oil}}}{\rho_{EN_{oil}} - 1} L_{oil,t}(o),$$

(1)

where $OIL_t(o)$, $K_{oil,t}(o)$, and $L_{oil,t}(o)$ are oil, capital, and labour, respectively. The parameter $\rho_{EN_{oil}} > 0$ is the elasticity of intratemporal substitution among inputs. The parameters $0 < \gamma_{oil,source}, \gamma_{oil,k} < 1$, $\gamma_{oil,source} + \gamma_{oil,k} < 1$ are the weights of oil and capital in the production process, respectively.

The implied demands for oil, capital, and labour are reported in what follows.

- Demand for oil

$$OIL_t(o) = \gamma_{oil,source} \left( \frac{p_{oil,t} + tax_{oil,t}}{r_{mc_{enoil,t}}} \right)^{-\rho_{EN_{oil}}} EN_{oil,t}(o)$$

(2)

where $p_{oil,t}$, $tax_{oil,t}$, and $r_{mc_{enoil,t}}$ are the relative price of oil (in units of domestic consumption), the carbon tax, and the real marginal cost of producing energy from oil, respectively.

- Demand for physical capital

$$K_{oil,t}(o) = \gamma_{oil,k} \left( \frac{r_{k,t}}{r_{mc_{enoil,t}}} \right)^{-\rho_{EN_{oil}}} EN_{oil,t}(o)$$

(3)

where $r_{k,t}$ is the rental rate on capital.
• Demand for labour

\[ L_{\text{oil},t}(o) = (1 - \gamma_{\text{oil};k} - \gamma_{\text{oil};source}) \left( \frac{w_t}{r m c_{\text{enoil},t}} \right)^{-\rho_{\text{ENoil}}} EN_{\text{oil},t}(o) \]  

where \( w_t \) is the real wage.

Similar equations hold for energy production from gas \((GAS_t(g))\) and coal \((COAL_t(co))\) produced by the generic firms \(g\) and \(co\), respectively.

### A.1.2 Production of energy \( EN_{\text{res},t}(r) \) from renewable sources

The generic firm \(r\) produces energy \( EN_{\text{res},t}(r)\) using a generic renewable source of energy \( RES_t(r)\), capital, and labour.

• Production function

\[
EN_{\text{res},t}(r) \left( \frac{\rho_{\text{ENres}} - 1}{\rho_{\text{ENres}}} \right) = \frac{1}{\gamma_{\text{res;source}}} RES_t(r) \left( \frac{\rho_{\text{ENres}} - 1}{\rho_{\text{ENres}}} \right) + \gamma_{\text{res;k}}/\rho_{\text{ENres}} K_{\text{res},t}(r) \left( \frac{\rho_{\text{ENres}} - 1}{\rho_{\text{ENres}}} \right) \\
+ (1 - \gamma_{\text{res;k}} - \gamma_{\text{res;source}}) \frac{1}{\rho_{\text{ENres}}} L_{\text{res},t}(r) \left( \frac{\rho_{\text{ENres}} - 1}{\rho_{\text{ENres}}} \right) \tag{5}
\]

• Demand for capital

\[
K_{\text{res},t}(r) = \gamma_{\text{res;k}} \left( \frac{r_{k,t}(1 - \text{subs}_{\text{res},t})}{r m c_{\text{enres},t}} \right)^{-\rho_{\text{ENres}}} EN_{\text{res},t}(r) \tag{6}
\]

where \(0 < \text{subs}_{\text{res},t} < 1\) is the subsidy rate to the capital used to produce energy from renewable sources, and \( r m c_{\text{enres},t} \) is the real marginal cost.

• Demand for renewable sources

\[
RES_t(r) = \gamma_{\text{res;source}} \left( \frac{p_{\text{res},t}}{r m c_{\text{enres},t}} \right)^{-\rho_{\text{ENres}}} EN_{\text{res},t}(r) \tag{7}
\]

where \(p_{\text{res},t}\) is the relative price (in units of domestic consumption) of the renewable source of energy.

• Demand for labour

\[
L_{\text{res},t}(r) = (1 - \gamma_{\text{res};k} - \gamma_{\text{res;source}}) \left( \frac{w_t}{r m c_{\text{enres},t}} \right)^{-\rho_{\text{ENres}}} EN_{\text{res},t}(r) \tag{8}
\]
A.1.3 Production of energy $EN_{nuc,t}(n)$ from nuclear source

The generic firm $n$ produces energy $EN_{nuc,t}(n)$ using the nuclear source of energy $NUC_t(n)$, capital, and labour. Equations are similar to those reported for the cases of oil-based and renewable-source-based productions of energy. Nuclear energy production is not subject to either tax or subsidy.

- Production function

$$EN_{nuc,t}(n) = \gamma_{nuc,k} \rho_{EN_{nuc}}^{-1} NUC_t(n) + \gamma_{nuc,source} K_{nuc,t}(n) + (1 - \gamma_{nuc,k} - \gamma_{nuc,source}) L_{nuc,t}(n)$$

- Demand for capital

$$K_{nuc,t}(n) = \gamma_{nuc,k} \left( \frac{\rho_{k,t}}{rmc_{enuc,t}} \right)^{-\rho_{EN_{nuc}}} E N_{nuc,t}(n)$$

- Demand for nuclear source

$$NUC_t(n) = \gamma_{nuc,source} \left( \frac{p_{nuc,t}}{rmc_{enuc,t}} \right)^{-\rho_{EN_{nuc}}} E N_{nuc,t}(n)$$

- Demand for labour

$$L_{nuc,t}(n) = (1 - \gamma_{nuc,k} - \gamma_{nuc,source}) \left( \frac{\rho_{l}}{rmc_{enuc,t}} \right)^{-\rho_{EN_{nuc}}} E N_{nuc,t}(n)$$

A.2 Intermediate goods sector

The generic firm $h$ produces an intermediate good $Y_t(h)$ under monopolistic competition. It chooses inputs, i.e., labour, capital, and energy, to minimize the production costs taking as given the technology constraint and the input prices.
Production function

\[ Y_t(h^{\rho_Y-1}) = \frac{1}{\rho_Y} VA_{y,t}(h^{\rho_Y-1}) + (1 - \gamma_{pr_{y,vu}}) \frac{1}{\rho_Y} EN_{y,t}(h^{\rho_Y-1}), \] (13)

where \( VA_{y,t}(h) \) and \( EN_{y,t}(h) \) are the value added and the energy bundle, respectively. The parameter \( \rho_Y > 0 \) is the elasticity of substitution among inputs and the parameter \( 0 < \gamma_{pr_{y,vu}} < 1 \) is the weight of value added in the production process.

Value added

The value added is a combination, according to a Cobb-Douglas technology, of physical capital and labour

\[ VA_{y,t}(h) = K_{y,t}(h)^{\gamma_{pr_{v,u,k}}} L_{y,t}(h)^{1-\gamma_{pr_{v,u,k}}}, \] (14)

where \( K_{y,t}(h) \) and \( L_{y,t}(h) \) are capital and labour, respectively, while the parameter \( 0 < \gamma_{pr_{v,u,k}} < 1 \) is the elasticity of value added with respect to capital.

Energy bundle

The energy bundle combines, according to a CES technology, the different types of energy obtained from oil \( (EN_{oily,t}(h)) \), coal \( (EN_{coaly,t}(h)) \), gas \( (EN_{gasy,t}(h)) \), nuclear source \( (EN_{nucy,t}(h)) \), and renewable source \( (EN_{resy,t}(h)) \):

\[ EN_{y,t}(h) = \frac{1}{\rho_{EN}} \left( \gamma_{pr_{y,oil}} EN_{oily,t}(h) \gamma_{pr_{y,gas}} EN_{gasy,t}(h) \right) \gamma_{pr_{y,nuc}} EN_{nucy,t}(h) + (1 - \gamma_{pr_{y,oil}} - \gamma_{pr_{y,gas}} - \gamma_{pr_{y,nuc}}) \right)^{\frac{1}{\rho_{EN}}} EN_{resy,t}(h), \] (15)

where \( \rho_{EN} > 0 \) is the elasticity of substitution among the different types of energy, \( 0 < \gamma_{pr_{y,oil}} < 1, 0 < \gamma_{pr_{y,gas}} < 1, 0 < \gamma_{pr_{y,nuc}} < 1 \) are the weights of energy obtained from oil, coal, gas, and nuclear source, respectively.

The implied demands for value added, energy bundle, capital, labour, and different types of energy, obtained by the cost minimization problem, are reported in what follows.
• Demand for value added

\[ V_{A_y,t}(h) = \gamma_{pr,va} \left( \frac{p_{va,t}}{rmc_{y,t}} \right)^{-\rho_Y} Y_t(h) \]  

(16)

where \( p_{va,t} \) is the relative price of the value added and \( rmc_{y,t} \) is the real marginal production cost of the intermediate good.

• Demand for energy bundle

\[ EN_{y,t}(h) = (1 - \gamma_{pr,va}) \left( \frac{p_{en,t}}{rmc_{y,t}} \right)^{-\rho_Y} Y_t(h) \]  

(17)

where \( p_{en,t} \) is the relative price of the energy bundle.

• Demand for capital

\[ K_{y,t}(h) = \gamma_{pr,va,k} \left( \frac{r_k,t}{p_{va,t}} \right)^{-1} V_{A_y,t}(h) \]  

(18)

• Demand for labour

\[ L_{y,t}(h) = (1 - \gamma_{pr,va,k}) \left( \frac{w_t}{p_{va,t}} \right)^{-1} V_{A_y,t}(h) \]  

(19)

• Demand for energy obtained from oil

\[ EN_{oily,t}(h) = \gamma_{pr,y,ool} \left( \frac{rmc_{enOil,t}}{p_{en,t}} \right)^{-\rho_{EN}} EN_{y,t}(h) \]  

(20)

Similar equations hold for demands for energy obtained from coal, gas, nuclear, and renewable sources.

• Optimal price of the EA intermediate good in the EA market

The generic firm \( h \) chooses the price of its good to maximize profits subject to the demand constraint and the quadratic costs to adjust the nominal price. Thus, the firm faces (short-term) nominal rigidities. We assume that EA and RW markets are exogenously segmented and the generic firm \( h \) price-discriminates across markets.
The implied first-order condition (i.e., optimal price of brand $h$ in the EA market) is

\[(1 - \theta_T)p_{EA,t}(h) + \theta_T rmc_{EA,t}(h) = \kappa_{EA} \left( \frac{P_{EA,t}(h)/P_{EA,t-1}(h) - 1}{\pi_{EA,t-1}^{indEA} \pi_{EA,t-1}^{target}} \right) \frac{P_{EA,t}/P_{EA,t-1}}{\pi_{EA,t}^{indEA} \pi_{EA,t}^{target}} \]

where $\theta_T > 1$ is the elasticity of substitution among different brands produced by firms belonging to the same sector, $\kappa_{EA} > 0$ is a parameter measuring the cost of adjusting the nominal price, $P_{EA,t}(h)$ is the nominal price of the intermediate good $h$, $\pi_{EA,t}$ is the sector-specific gross inflation rate, and $\pi_{target}$ is the central bank (gross) inflation target. The parameter $0 < ind_{EA} < 1$ measures indexation of current prices to previous-period inflation. Correspondingly, $1 - ind_{EA}$ measures indexation to the central bank target. Thus, the optimal price setting scheme is subject to a double indexation, i.e., to past inflation and to the central bank inflation target.

A similar equation holds for the price of good $h$ in the Foreign market.

### A.3 Final-good sectors

There are three sectors producing final goods: consumption goods for households, investment goods, and public sector consumption goods. Firms act under perfect competition. They choose inputs to maximize profits subject to the technology constraint and taking all prices as given.

#### A.3.1 Private consumption good

- **Overall basket**

The generic firm $x$ produces the consumption good $C_t(x)$ according to the CES production function

\[C_t(x) \sim_{EC}^{-1} = \frac{1}{\gamma_{pc,manu}^C} C_{manu,t}(x) \sim_{EC}^{-1} + (1 - \gamma_{pc,manu}) \sim_{EC}^{-1} C_{EN,t}(x) \sim_{EC}^{-1} \]

(22)
where \( \rho_C > 0 \) is the elasticity of substitution among inputs, \( C_{\text{manu},t}(x) \) is the bundle of non-energy intermediate goods, \( C_{\text{EN},t}(x) \) is the energy bundle. The parameter \( 0 < \gamma_{\text{pr},\text{manu}} < 1 \) is the weight of the non-energy consumption bundle.

- **Basket of the manufacturing goods**

  The bundle of manufacturing goods is a CES function of domestic and imported consumption goods, \( C_{E\text{A},t}(x) \) and \( C_{R\text{W},t}(x) \), respectively:

  \[
  C_{\text{manu},t}(x)^{\frac{\theta_T-1}{\theta_T}} = a_{E\text{A},C}^{E\text{A},t}(x)^{\frac{\theta_T-1}{\theta_T}} + (1 - a_{E\text{A},C})^{R\text{W},t}(x)^{\frac{\theta_T-1}{\theta_T}}
  \]

  where the parameters \( a_{E\text{A},C} \), and \( (1 - a_{E\text{A},C}) \) (\( 0 < a_{E\text{A},C} < 1 \)) are the weights of EA and RW goods in the bundle \( C_{E\text{A},t} \) and \( C_{R\text{W},t} \), respectively, while \( \eta_T > 0 \) is the elasticity of substitution among tradable goods.

- **Basket of domestically-produced manufacturing good for consumption purposes \( C_{E\text{A}}(x) \)**

  The domestically-produced manufacturing good for consumption purposes \( C_{E\text{A}} \) is a composite basket of a continuum of differentiated intermediate goods, each supplied by a different EA firm \( h \) operating in the intermediate sector. It is produced according to the following function:

  \[
  C_{E\text{A},t}(x) = \left[ \left( \frac{1}{s_{E\text{A}}} \right)^{\theta_T} \int_0^{s_{E\text{A}}} C_{E\text{A},t}(h,x)^{\frac{\theta_T-1}{\theta_T}} dh \right]^{\frac{\theta_T}{\theta_T-1}}
  \]

  where \( \theta_T > 1 \) is the elasticity of substitution among EA intermediate brands \( h \) used as inputs by the firms \( x \), \( C_{E\text{A},t}(h,x) \), and \( s_{E\text{A}} \) is the size of the EA.\(^{33}\)

- **Basket of manufacturing goods \( C_{R\text{W}}(x) \)**

  The basket of imported RW goods has a structure similar to that of EA goods, i.e.,

  \[
  C_{R\text{W},t}(x) = \left[ \left( \frac{1}{1 - s_{E\text{A}}} \right)^{\theta_T} \int_{s_{E\text{A}}}^{1} C_{R\text{W},t}(f,x)^{\frac{\theta_T-1}{\theta_T}} df \right]^{\frac{\theta_T}{\theta_T-1}}
  \]

  where \( (1 - s_{E\text{A}}) \) is the size of RW.

\(^{33}\)For each country, size refers to the overall population and to the number of firms operating in each sector.
Energy consumption bundle $C_{EN,t}(x)$

The energy consumption bundle is a CES aggregator of the different types of energy produced using oil, gas, coal, nuclear source, and renewable source ($EN_{oilc,t}(x)$, $EN_{gasc,t}(x)$, $EN_{coalc,t}(x)$, $EN_{nucc,t}(x)$, and $EN_{resc,t}(x)$, respectively):

$$C_{EN,t}(x) = \frac{\rho_{EN}^{-1}}{\rho_{EN}} = \frac{1}{\gamma_{pr_{c,coal}}} EN_{coalc,t}(x) + \frac{1}{\gamma_{pr_{c,gas}}} EN_{gasc,t}(x) + \frac{1}{\gamma_{pr_{c,coal}}} EN_{coalc,t}(x) + (1 - \gamma_{pr_{c,oil}} - \gamma_{pr_{c,coal}} - \gamma_{pr_{c,gas}} - \gamma_{pr_{c,nuc}}) \frac{1}{\rho_{EN}} EN_{resc,t}(x) \frac{\rho_{EN}^{-1}}{\rho_{EN}} \tag{26}$$

where $\rho_{EN} > 0$ is the elasticity of substitution among the different types of energy, $\gamma_{pr_{c,oil}}$, $\gamma_{pr_{c,gas}}$, $\gamma_{pr_{c,coal}}$, $\gamma_{pr_{c,nuc}}$ ($0 < \gamma_{pr_{c,oil}}, \gamma_{pr_{c,gas}}, \gamma_{pr_{c,coal}} < 1, \gamma_{pr_{c,oil}} + \gamma_{pr_{c,gas}} + \gamma_{pr_{c,coal}} + \gamma_{pr_{c,nuc}} < 1$) are the weights of the energy produced using oil, gas, coal, and nuclear source, respectively.

Demand for energy services produced from oil source

$$EN_{oilc,t}(x) = \gamma_{pr_{c,oil}} \left( \frac{rmc_{enoi,t}}{pen,t} \right)^{-\rho_{EN}} C_{EN,t}(x) \tag{27}$$

Similar demand equations hold for energy services produced from coal, gas, nuclear and renewable sources.

Demand for the generic brand $h$

Firm $x$ demand for the generic brand $h$ is

$$C_{EA,t}(h,x) = \frac{1}{s_{EA}} a_{EA,C} \gamma_{pr_{c,manu}} \left( \frac{P_{EA,t}(h)}{P_{EA,t}} \right)^{-\theta_{T}} \left( \frac{P_{EA,t}}{P_{manu,t}} \right)^{-\eta_{T}} \left( \frac{P_{manu,t}}{P_{t}} \right)^{-\rho_{C}} C_{t}(x) \tag{28}$$

where

$$P_{EA,t} = \left[ \int_{0}^{s_{EA}} P_{EA,t}(h)^{1-\theta_{T}} dh \right] \frac{1}{1-\theta_{T}} \tag{29}$$

$$P_{manu,t} = \left[ a_{EA,C} P_{EA,t}^{1-\eta_{T}} + (1 - a_{EA,C}) P_{RW,t}^{1-\eta_{T}} \right] \frac{1}{1-\eta_{T}} \tag{30}$$
\[ P_{C,t} = \left[ \gamma_{pr,\text{manu}} P_{\text{manu},t}^{1-\rho_c} + (1 - \gamma_{pr,\text{manu}}) P_{EN,t}^{1-\rho_c} \right]^{\frac{1}{1-\rho_c}} \]  

are the price deflators of EA goods’ consumption bundle, non-energy consumption bundle, overall consumption bundle, respectively. An equation similar to the price deflator of the EA goods’ consumption bundle holds for the price deflator of the imported (i.e., RW) goods.

### A.4 Investment good

The sector producing final investment goods has a structure similar to one of the consumption goods’ sector. The only difference is that the energy bundle does not enter the overall investment basket. Only non-energy (domestic and imported) intermediate goods do.

#### Overall basket

The generic firm \( i \) produces a basket (CES aggregator) of bundles of EA and imported (RW) goods, \( I_{EA,t} \) and \( I_{RW,t} \), respectively:

\[
I_{T,t}(i) = \left[ a_{EA,I} \left( \frac{1}{\eta_T} \right)^{\eta_T-1} + (1 - a_{EA,I}) \left( \frac{1}{\eta_T} \right)^{\eta_T-1} \right]^{\frac{1}{\eta_T-1}}
\]

where the parameter \( 0 < a_{EA,I} < 1 \) is the weight of EA goods in the bundle, while \( \eta_T > 0 \) is the elasticity of substitution among tradable goods.

#### Basket of domestic goods \( I_{EA}(i) \)

The investment good \( I_{EA} \) is a composite basket of a continuum of differentiated domestic intermediate goods, each supplied by a different EA firm \( h \). It is produced according to the following function:

\[
I_{EA,t}(i) = \left[ \left( \frac{1}{s_{EA}} \right) \int_{0}^{s_{EA}} I_{EA,t}(h,i) \left( \frac{\eta_T-1}{\eta_T} \right) dh \right]^{\frac{1}{\eta_T-1}}
\]
• Basket of imported goods $I_{RW}(i)$

The investment good $I_{RW}$ is a composite basket of a continuum of differentiated domestic intermediate goods, each supplied by a different RW firm $f$. It is produced according to the following function:

$$I_{RW,t}(i) = \left[ \left( \frac{1}{1 - s^{EA}} \right) \int_{s^{EA}}^{1} I_{RW,t}(f,i) \frac{\theta^{-1}}{\theta^{-1} - 1} \, df \right]^{\frac{\theta^{-1}}{\theta^{-1} - 1}} \tag{34}$$

Implied demand equations for generic brands and implied deflators are similar to corresponding equations for private consumption goods.

A.5 Public consumption good

• Overall basket

The public consumption good $C_{g,EA,t}^g$, produced by the generic firm $g$ under perfect competition, is fully biased towards the intermediate domestic brands, i.e.,

$$C_{g,EA,t}^g(g) = \left[ \left( \frac{1}{s^{EA}} \right)^{\theta^{-1}} \int_{0}^{s^{EA}} C_{g,EA,t}^g(h,g) \frac{\theta^{-1}}{\theta^{-1} - 1} \, dg \right]^{\frac{\theta^{-1}}{\theta^{-1} - 1}} \tag{35}$$

Implied demand equations for generic brands and implied deflators are similar to the corresponding equations for private consumption goods.

A.6 Households

A.6.1 Ricardian Household

In each country there is a continuum of Ricardian households $j$ of mass $s^{ric}$ ($0 < s^{ric} < 1$).

• Preferences

Each household $j$ maximizes its lifetime expected utility subject to the budget constraint. The lifetime expected utility, in consumption of goods $C_{ric}$, and labour $L_{ric}$ is

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta_{ric}^t \left[ \frac{(C_{ric,t}(j) - hab_{ric,t-1})^{1-\sigma}}{(1 - \sigma)} - \frac{L_{ric,t}(j)^{1+\tau}}{1 + \tau} \right] \right\}, \tag{36}$$
where $E_0$ is period-0 expectation term, $0 < \beta_{ric} < 1$ the discount factor, $0 < hab < 1$ the (external) consumptions’ habit parameter, $\sigma > 0$ the reciprocal of the intertemporal elasticity of substitution, and $\tau > 0$ the Frish elasticity of labour supply.

- **Budget constraint**

The budget constraint is

$$
B^G_t(j) - B^G_{t-1}(j) R^G_{t-1} + S_t B^P_t(j) - S_t B^P_{t-1}(j) R^*_{t-1} (1 - \Gamma_{BP,t-1}) + P^L_t B^L_{ric,t}(j) - R^L_t P^L_{ric,t-1}(j) = (1 - \tau^L_t) W_t(j) L_{ric,t}(j) + \Pi^P_t(j) + \Pi^{prof}_t(j) - P_t (1 + \tau^L_t) C^L_{ric,t}(j) + TR_t(j) - \kappa_W \frac{t}{2} \left( \frac{W_t(j)/W_{t-1}(j)}{\pi^{indw}_{t-1}/\pi^{indw}_{target}} - 1 \right)^2 W_t L_{ric,t} - \frac{\phi B}{2} (P^L_{ric,t}(j) - \bar{B}^L_{ric})^2,
$$

where: $B^G_t$ is the end-of-period holdings of short-term (one-period) bonds issued by the domestic government, which pays the (gross) interest rate $R^G$; $B^P_t$ is the bond exchanged with other domestic and RW Ricardian households, which pays the (gross) RW monetary policy rate $R^*$ and denominated in RW currency ($S_t$ is nominal exchange rate expressed as number of euro per unit of RW currency); the function $\Gamma_{BP,t}$ captures the costs of undertaking positions in the international bond market; $B^L_{ric,t}$ is the long-term sovereign bond issued by the domestic government; $W_t$ is the nominal wage; $0 < \tau^L_t < 1$ is the labour income tax rate; $\Pi^P_t$ are profits from ownership of domestic capital producers, rebated to Ricardian households in a lump-sum way; $\Pi^{prof}_t$ are profits from ownership of domestic firms (other than capital producers), rebated to Ricardian households in a lump-sum way; $P_t$ is the consumer price deflator; $0 < \tau^L_t < 1$ is the consumption tax rate; $TR_t$ are lump-sum transfers from the government ($TR_t < 0$ are lump-sum taxes paid to the government); the last two terms are quadratic costs paid to adjust the nominal

---

34The adjustment cost in the bond markets has the following functional form:

$$
\Gamma_{BP,t} \equiv \exp \left( \phi_b \left( \frac{S_t B^P_t}{P_t} - b^P \right) \right)
$$

The parameter $\phi_b > 0$ controls the speed of convergence to the non-stochastic steady state and $b^P$ is the steady-state position. The adjustment cost is imposed to ensure the stationarity of the net foreign asset position.
wage and the position in the long term sovereign bond, respectively. Specifically, $\kappa_W > 0$ is a parameter measuring the nominal wage stickiness; $0 < \text{ind}_W < 1$ is a parameter measuring the degree of indexation to previous-period inflation, $\pi_{t-1}$ and, correspondingly, $1 - \text{ind}_W$ measures the indexation to the central bank target; in the bond adjustment cost, the term $\phi_B > 0$ is a parameter, while $B_{ric}^L$ is the Ricardian household’s steady-state position in the bond.

The long-term interest rate is

$$R_{t}^{\text{long}} = \frac{1}{P_{t}^{\text{long}}} + \kappa_{t}^{\text{long}} \tag{38}$$

In what follows we report the first order conditions implied by the household’s utility maximization subject to the budget constraint.

- FOC with respect to consumption $C_{ric,t}(j)$

$$\lambda_{ric,t}(j) (1 + \tau_t^c) = \left(C_{ric,t}(j) - h a b C_{ric,t-1}(j)\right)^{-\sigma} \tag{39}$$

where $\lambda_{ric,t}$ is the consumption marginal utility.

- FOC with respect to domestic bond $B_{t}^G(j)$

$$\lambda_{ric,t}(j) = \beta_{ric} E_t \left(R_t^{G} \pi_{t+1}^{-1} \lambda_{ric,t+1}(j)\right) \tag{40}$$

where

$$\pi_t \equiv \frac{P_t}{P_{t-1}} \tag{41}$$

is the gross consumer price inflation rate.

- FOC with respect to foreign bond $B_{t}^P(j)$

$$\lambda_{ric,t}(j) = \beta_{ric} E_t \left(R_t(1 - \Gamma_{B_{t}^P}) \pi_{t+1}^{-1} S_{t+1} \lambda_{ric,t+1}(j)\right) \tag{42}$$

- FOC with respect to long-term sovereign bond $B_{ric,t}^L(j)$

$$\lambda_{ric,t}(j) P_t^{L} \left(1 + \phi_B \left(P_t^{L} B_{ric,t}^L(j) - B_{ric}^L\right)\right) = \beta_{ric} E_t \left(\left(1 + \kappa P_{t+1}^{L}\right) \pi_{t+1}^{-1} \lambda_{ric,t+1}(j)\right) \tag{43}$$
• FOC with respect to nominal wage $W_t(j)$

The household supplies its labour variety under monopolistic competition; she sets the nominal wage taking into account of demand by domestic firms and subject to quadratic adjustment costs of setting nominal wages. The implied optimal wage setting equation is

$$\theta L \frac{W_t(j)^{-\theta_L(1+\tau)-1}}{W_t^{-\theta_L(1+\tau)}} L_{ric,t}^\tau + (1 - \theta L) \frac{W_t(j)^{-\theta_L}}{W_t^{-\theta_L}} = \lambda_{ric,t}(j) \kappa W \left( \frac{W_t(j)/W_{t-1}(j)}{\pi_{t-1}^{\text{ind} - \text{ind}_w} - 1} - 1 \right) \frac{W_t/W_{t-1}(j)}{\pi_t^{\text{ind} - \text{ind}_w} \pi_{t-1}^{\text{ind}} \pi_{t-1}^{\text{ind}_w} \pi_{t-1}^{\text{target}}}$$

where the parameter $\theta_L > 1$ measures the elasticity of substitution among different labour varieties supplied by households and $\pi_{t-1}$ is the previous-period gross inflation rate. The parameter $0 < \text{ind}_W < 1$ measures indexation of current-period wage to previous-period inflation. Correspondingly, $1 - \text{ind}_W$ measures indexation to the central bank target.

### A.6.2 Restricted Households

There is a continuum of restricted households, with mass $0 < s^{\text{restr}} < 1$.

• Preferences

The generic restricted households $j'$ chooses consumption $C_{\text{restr},t}$ to maximize her utility function, while she supplies an amount of labour $L_{\text{restr},t}$ equal to that chosen by the Ricardian household. The intertemporal utility function is

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta_{\text{restr}}^t \left[ \frac{(C_{\text{restr},t}(j') - \text{hab}C_{\text{restr},t-1})^{1-\sigma}}{(1 - \sigma)} - \frac{L_{\text{restr},t}(j')^{1+\tau}}{1 + \tau} \right] \right\},$$

where $0 < \beta_{\text{restr}} < 1$ is the household’s discount factor, $0 < \text{hab} < 1$ the parameter measuring (external) consumption habit, $\sigma > 0$ the reciprocal of the intertemporal elasticity of substitution, $\tau > 0$ the Frish elasticity of labour supply.
• Budget constraint

\[
P_t^L B_{restr,t}^L (j') - R_t^L P_t^L B_{restr,t-1}^L (j') = \Pi_t^{prof} (j') + (1 - \tau_t^f) W_t (j') L_{restr,t} (j') - P_t (1 + \tau_c^t) C_{restr,t}^L (j') - \frac{\phi}{2} (P_t^L B_{restr,t}^L (j') - B_{restr}^L)^2. \tag{46}
\]

The household invests in domestic long-term sovereign bonds \(B_{restr,t}^L\); she gets profits, in a lump-sum way, from domestic capital producers \(\Pi_t^{prof}\) and the nominal wage \(W_t\), equal to the one chosen by the Ricardian household; she pays a quadratic adjustment cost to change her position in the long-term sovereign bond with respect to the steady-state value \(B_{restr}^L\). In what follows we report the first order conditions.

• FOC with respect to consumption \(C_{restr,t}(j')\)

\[
\lambda_{restr,t}(j') (1 + \tau_t^f) = (C_{restr,t}(j') - hC_{restr,t-1})^{-\sigma} \tag{47}
\]

where \(\lambda_{restr,t}\) is the marginal utility of consumption

• FOC with respect to long-term sovereign bond \(B_{restr,t}^L(j')\)

\[
\lambda_{restr,t}(j') P_t^L (1 + \phi B (P_t^L B_{restr,t}^L (j') - B_{restr}^L)) = \beta_{restr} E_t \left( (1 + \kappa P_{t+1}^L) \pi_{t+1}^{-1} \lambda_{restr,t+1}(j') \right) \tag{48}
\]

A.6.3 Rule-of-thumb households

There is a continuum of rule-of-thumb households \(j''\), with mass \(0 < s^{rot} < 1.\)\(^{35}\) In each period the generic household consumes all the available wage income. The nominal wage and the labour supply are the same as the Ricardian household’s corresponding variables.

• Budget constraint

\[
(1 + \tau_c,t) P_{C,t} C_{rot,t}(j'') = (1 - \tau_w,t) W_t L_{rot,t}(j''). \tag{49}
\]

\(^{35}\) \(s^{ric} + s^{restr} + s^{rot} = 1.\)
A.7 Capital producers

The generic capital goods producer $c$ produces private physical capital. It is owned by Ricardian and restricted households. Capital producers optimally choose the end-of-period capital $K$ and investment $I$ subject to the law of capital accumulation, the adjustment costs on investment, distortionary taxes on capital income levied by the domestic government, and taking all prices as given. Capital producers rent existing physical capital stock $K$ in a perfectly competitive market at the nominal rate $R^K$ to domestic firms producing intermediate goods.

• Capital accumulation law

$$K_t(c) = (1-\delta)K_{t-1}(c) + \left[1 - \frac{\psi}{2} \left(\frac{I_t(c)}{I_{t-1}(c)} - 1\right)^2\right]I_t(c) \quad (50)$$

where $0 < \delta < 1$ is the depreciation rate and investment is subject to a quadratic adjustment cost ($\psi > 0$ is a parameter);

• FOC with respect to the end-of-period capital $K_t(c)$

$$\begin{align*}
& (\text{share}_{ric}\lambda_{ric,t} + (1 - \text{share}_{ric})\lambda_{restr,t}) Q_t(c) \\
& = E_t \left(\text{share}_{ric}\beta_{ric}\lambda_{ric,t+1} + (1 - \text{share}_{ric})\beta_{restr}\lambda_{restr,t+1}\right) r^{K}_{t+1}(1 - \tau^k_{t+1}) \\
& + E_t \left(\text{share}_{ric}\beta_{ric}\lambda_{ric,t+1} + (1 - \text{share}_{ric})\beta_{restr}\lambda_{restr,t+1}\right) (1 - \delta)Q_{t+1}(c)
\end{align*} \quad (51)$$

where: $0 < \text{share}_{ric} < 1$ is the share of capital producers owned by Ricardian households, while $\lambda_{ric,t}$ is the generic Ricardian household marginal utility of consumption; correspondingly, $1 - \text{share}_{ric}$ is the share of capital producers owned by the restricted households and $\lambda_{restr,t}$ is the generic restricted household marginal utility of consumption. $Q(c)$ is the Tobin’s $Q$ (i.e., the multiplier of the capital accumulation law), $0 < \tau^k < 1$ is the tax rate on the return of capital, $r^K$;
FOC with respect to investment $I_t(c)$

$$(share_{ric} \lambda_{ric,t} + (1 - share_{ric}) \lambda_{restr,t}) p_{I,t} = Q_t(c) (share_{ric} \lambda_{ric,t} + (1 - share_{ric}) \lambda_{restr,t}) \times \left[1 - \frac{\psi}{2} \left(\frac{I_t(c)}{I_{t-1}(c)} - 1\right)^2 - \psi \left(\frac{I_t(c)}{I_{t-1}(c)} - 1\right) \frac{I_t(c)}{I_{t-1}(c)} \right] + (share_{ric}\beta_{ric}\lambda_{ric,t+1} + (1 - share_{ric})\beta_{restr}\lambda_{restr,t+1}) \psi \left[\left(\frac{I_{t+1}(c)}{I_t(c)} - 1\right) \frac{I_{t+1}(c)}{I_t^2(c)} \right]$$

(52)

A.8 Monetary policy

The central bank sets the (gross) quarterly policy rate $R_t$ according to the Taylor rule.

• Taylor rule

$$\left(\frac{R_t}{R}\right)^4 = \left(\frac{R_{t-1}}{R}\right)^{4\rho_R} \left(\frac{\pi_{t,t-3}}{\pi_{\text{target}}^4}\right)^{(1-\rho_R)\rho_\pi} \left(\frac{RGDP_t}{RGDP_{t-1}}\right)^{(1-\rho_R)\rho_{RGDP}}$$

(53)

where $\pi_{t,t-3}$ is the annual gross inflation rate, $RGDP_t$ is the quarterly real gdp, $0 < \rho_R < 1$ is a parameter capturing inertial setting of the policy rate, $\rho_\pi > 0$, $\rho_{RGDP}$ measure responsiveness of the policy rate to inflation deviation from the central bank target $\pi_{\text{target}}$ and to real gdp growth, respectively.

We also allow, in some simulations, the central bank to purchase long-term sovereign bonds for monetary policy purposes. The purchases are exogenously set.

A.9 Fiscal policy

In each country there is a fiscal authority.

• Budget constraint

$$B^G_t - B^G_{t-1}R^G_{t-1} + P^\text{long}_t B^G_{t,\text{long}} - R^\text{long}_t P^\text{long}_t B^G_{t-1,\text{long}} = P_{G,t} G_t - TAX_t - T_t - s^E A \times (\text{tax}_\text{oil,}t OIL_t + \text{tax}_\text{gas,}t GAS_t + \text{tax}_\text{coal,}t COAL_t) + subs_{res,t} r^K_t \times s^E A \times K_{res,t}$$

(54)
where: $B^G > 0$ is the short-term (one-period) sovereign bond, which pays the gross interest rate $R^G$; $B^{G, long}_t > 0$ the long-term sovereign bond, paying the rate $R^{long}_t$ and whose price is $P^{long}_t$; $G_t$ is purchases of domestic goods, at the price $P_{G,t}$; $TAX_t > 0$ are lump-sum taxes paid by the Ricardian households; $T_t$ are Total government revenues from distortionary taxation other than the carbon tax; $tax_{oil,t}, tax_{gas,t}, tax_{coal,t}$ are the carbon taxes on oil, gas, and coal, respectively; $subs_{res,t}$ are subsidies for the return on physical capital $K_{res,t}$ (whose rate of return is $r^K_t$) for generation of electricity from the renewable source.

- Total government revenues from distortionary taxation $T_t$ other than the carbon tax are equal to

$$T_t \equiv \tau^w_t W_t s^{EA} L_t + \tau^K_t s^{EA} R^K_t K_{t-1} + \tau^c_t P_t s^{EA} (s^{ric,t} C_{ric,t} + s^{restr,t} C_{restr,t} + (1 - s^{ric} - s^{restr}) C_{rot,t}) \quad (55)$$

where $0 < \tau^w_t, \tau^K_t, \tau^c_t < 1$ are tax rates on labour income, physical capital income, and consumption, respectively.

- Fiscal rule

The fiscal authority stabilizes the public debt as a ratio to GDP by setting the lump-sum taxes $tax_t$ according to the following fiscal rule:

$$\frac{tax_t}{\bar{tax}} = \left( \frac{b_{G,t}}{\bar{b}_G} \right)^{\phi_G} \quad (56)$$

where $\bar{tax}$ is the steady-state lump-sum-taxes-to-gdp ratio, $b_{G,t}$ the short-term public debt as a ratio to GDP, $\bar{b}_G$ its steady-state value, $\phi_G > 0$ a parameter measuring the responsiveness of taxes to the debt. Long-term sovereign debt is assumed to change proportionally to changes in the short-term debt. Thus the rule indirectly stabilizes also changes in the long-term sovereign debt.

A.10 Market clearing conditions

In what follows we report the market clearing conditions of goods and bonds holding in the EA. Similar equations hold for the RW.
• Labour market
\[
\int_{0}^{s_{EA}^{ric}} L_{ric,t}(j) \, dj + \int_{s_{EA}^{ric}}^{s_{EA}(s^{ric}+s^{extr})} L_{res,t}(j') \, dj' + \int_{s_{EA}(s^{ric}+s^{extr})}^{s_{EA}} L_{rot,t}(j'') \, dj'' = \\
\int_{0}^{s_{EA}} L_{y,t}(h) \, dh + \int_{0}^{s_{EA}} L_{oil,t}(o) \, do + \int_{0}^{s_{EA}} L_{coal,t}(co) \, dco + \\
\int_{0}^{s_{EA}} L_{gas,t}(g) \, dg + \int_{0}^{s_{EA}} L_{res,t}(r) \, dr + \int_{0}^{s_{EA}} L_{nuc,t}(n) \, dn
\]

(57)

• Capital
\[
\int_{0}^{s_{EA}} K_{t}(c) \, dc = \int_{0}^{s_{EA}} K_{y,t}(h) \, dh + \int_{0}^{s_{EA}} K_{oil,t}(o) \, do + \int_{0}^{s_{EA}} K_{coal,t}(co) \, dco + \\
\int_{0}^{s_{EA}} K_{gas,t}(ga) \, dga + \int_{0}^{s_{EA}} K_{res,t}(r) \, dr + \int_{0}^{s_{EA}} K_{nuc,t}(n) \, dn
\]

(58)

• Oil market
\[
OIL_{RW,t}^{*} = \int_{0}^{s_{EA}} OIL_{t}(o) \, do + \int_{s_{EA}}^{1} OIL_{t}(o^{*}) \, do^{*}
\]

(59)

• Coal market
\[
COAL_{RW,t}^{*} = \int_{0}^{s_{EA}} COAL_{t}(co) \, dco + \int_{s_{EA}}^{1} COAL_{t}(co^{*}) \, dco^{*}
\]

(60)

• Gas market
\[
GAS_{RW,t}^{*} = \int_{0}^{s_{EA}} GAS_{t}(ga) \, dga + \int_{s_{EA}}^{1} GAS_{t}(ga^{*}) \, ga^{*}
\]

(61)

• Nuclear source market
\[
NUC_{EA,t} = \int_{0}^{s_{EA}} NUC_{t}(n) \, dn
\]

(62)

• Renewable sources market
\[
RES_{EA,t} = \int_{0}^{s_{EA}} RES_{t}(r) \, dr
\]

(63)
• Market of oil-based energy (similar conditions hold for other types of energy)

\[
\int_{0}^{s_{EA}} E_{\text{oil},t}(o) \, do = \int_{0}^{s_{EA}} E_{Y,t}(x) \, dx + \int_{0}^{s_{EA}} C_{\text{EN},t}(x) \, dh \tag{64}
\]

• Short-term sovereign bond

\[
\int_{0}^{s_{EA,s^{ric}}} B_{t}^{G}(j) \, dj = B_{t}^{G} \tag{65}
\]

• Long-term sovereign bond

\[
\int_{0}^{s_{EA,s^{ric}}} B_{t}^{L}(j) \, dj + \int_{s_{EA,s^{ric}}}^{s_{EA}(s^{ric}+s^{restr})} B_{t}^{L}(j') \, dj' + B_{CB,t}^{G,long} = B_{t}^{G,long} \tag{66}
\]

• Internationally traded bond

\[
\int_{0}^{s_{EA,s^{ric}}} B_{t}^{P}(j) \, dj + \int_{s_{EA}}^{s_{EA}+(1-s_{EA})s^{ric}} B_{t}^{P}(j^*) \, dj^* = 0 \tag{67}
\]

• Generic EA intermediate tradable \( h \) sold in EA

\[
\int_{0}^{s_{EA}} Y_{EA,t}(h) \, dh = \int_{0}^{s_{EA}} C_{EA,t}(x) \, dx + \int_{0}^{s_{EA}} I_{EA,t}(i) \, di \tag{68}
\]

• Generic EA intermediate tradable \( h \) sold in RW

\[
\int_{0}^{s_{EA}} Y_{EA,t}(h) \, dh = \int_{0}^{1} C_{EA,t}(x^*) \, dx^* + \int_{0}^{1} I_{EA,t}(i^*) \, di^* \tag{69}
\]

• Generic intermediate tradable \( h \)

\[
\int_{0}^{s_{EA}} Y_{T,t}(h) \, dh = \int_{0}^{s_{EA}} Y_{EA,t}(h) \, dh + \int_{0}^{s_{EA}} Y_{EA,t}^*(h) \, dh \tag{70}
\]
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