Is inflation targeting a strategy past its sell-by date?

by Alberto Locarno and Alessandra Locarno
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IS INFLATION TARGETING A STRATEGY PAST ITS SELL-BY DATE?

by Alberto Locarno* and Alessandra Locarno#

Abstract

In this paper we compare alternative monetary policy strategies to assess which one is best suited (1) to reduce output and inflation volatility and at the same time (2) minimise the frequency and costs of ZLB episodes. We consider only targeting rules, i.e. rules that minimise the loss function assigned by the Government to the monetary policymaker, who is assumed to set the policy rate under discretion. We run a horse race among eight different strategies. Our analysis confirms the theoretical findings by Svensson (1999) and Vestin (2006) that price-level targeting can guarantee a better performance than inflation targeting in terms of both of the criteria described above. These findings are valid regardless of whether interest-rate variability is included in the loss function or not and are robust to changes in model parameters. Nominal GDP-level targeting also performs well: though it is not uniformly superior to inflation targeting or average inflation targeting, it succeeds in ensuring better outcomes over a large range of model parameters and social preferences.

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Keywords: effective lower bound, inflation targeting, price-level targeting.
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1 Introduction

The global financial crisis (GFC) led central bankers to rethink how monetary policy should be conducted. The design of a “New Normal” needs to take into account not only the lessons learnt from the crisis but also changes in the structure of the economy, in particular the steady decline in the natural interest rates and the worsening of the output-inflation trade-off (Brainard 2017).

The natural real rate of interest has declined over the past two decades and across all the major advanced economies, reaching historical lows in the aftermath of the global financial crisis: \(^1\) because of the existence of the zero lower bound (ZLB), \(^2\) the lower the level of the natural rate, the narrower the space available to the monetary authority for cutting the policy rate to stabilise aggregate demand, and the less effective standard interest-rate policies.

The existence of the ZLB prevents the central bank from accommodating as much as necessary in response to strong deflationary shocks. Absent mechanisms to compensate for protracted periods of below-target inflation when the ZLB is not binding, actual and expected inflation are on average lower and the probability of hitting the ZLB increases even more. \(^3\)

The ability of a central bank to reach its inflation target is further hampered by the flattening of the Phillips curve observed in many advanced economies since the mid 1990s. On the one hand it allows inflation to remain subdued even for historically low levels of the unemployment rate; on the other hand it makes much more costly to counteract inflationary pressures. \(^4\)

\(^1\) According to the estimates of Laubach and Williams (2015), the natural rate of interest, which was about 2% before the crisis in the US, has become slightly negative in 2017. For the euro area Brand et al. (2018) estimate that the natural rate fell to or below zero in the second half of the current decade. According to Holston, Laubach and Williams (2016) Canada, the Euro Area and the United Kingdom have experienced a "moderate secular decline" in the natural rate in the period 1990-2007 and a stronger reduction over the last decade. For the analysis of the possible causes of the decline in the natural rate of interest, see Bernanke (2005); Caballero et al. (2017); Del Negro et al. (2017); Summers (2013); Gordon (2015) and (2016); Carvalho et al. (2016); Bielecki et al. (2018); Papetti (2019); Dynan et al. (2004); Cynamon and Fazzari (2014); Piketty (2014); Rachel and Smith (2015); Rannenberg (2018)

\(^2\) The terms zero lower bound (ZLB) and effective lower bound (ELB) are used here interchangeably.

\(^3\) According to Gust et al. (2017), policymakers who seek to minimize a (symmetric) quadratic loss function involving deviations of inflation and output from targets will achieve an average inflation rate below target due to the contractionary effects associated with hitting the ZLB. See also Fisher (2016), Constâncio (2016), Blanchard, Dell’Ariccia, and Mauro (2010) and Summers (2014)

\(^4\) There is no consensus about the factors that have determined this flattening. According to some scholars it is due to the greater effectiveness of monetary policy in anchoring of inflation expectations (Laxton and N'Diaye, 2002; Kiley, 2015 and Boivin, Kiley and Mishkin, 2010), or to the high degree of nominal price stickiness (Ball and Mazumder, 2011). Other academics
its origin, the flattening of the Phillips curve weakens the transmission of policy impulses, making inflation and output more volatile.

The fall in the natural rate of interest and the global financial crisis have exposed the major weaknesses of the inflation targeting (IT) regime currently adopted in most advanced economies. Although inflation targeting has proved to be very successful since its introduction in the 1990s, when inflation was high and volatile, nowadays it faces a wave of criticism, which brings into question its ability to fight severe recessions. In contrast to the situation in the early 1990s, the problem is now that inflation is too low, as many countries are undershooting, rather than overshooting, their targets.

Central banks can affect output and inflation by adjusting the policy rate or by managing expectations: under discretion and when the probability of hitting the ZLB is non-trivial, inflation targeting is ineffective in doing the former and ill-suited to achieve the latter.

The causes of this impairment may be easily detected by focusing on the channels through which the monetary policymaker can affect the output gap. Solving forward the IS curve, one obtains:

\[
x_t = -\frac{1}{\sigma} E_t \sum_{n=0}^{\infty} \left( i_{t+j} - \pi_{t+j} - r^n_{t+j} \right)
= -\frac{1}{\sigma} i_t - \frac{1}{\sigma} E_t \sum_{n=1}^{\infty} i_{t+j} + \frac{1}{\sigma} E_t \sum_{n=0}^{\infty} \pi_{t+j} + \frac{1}{\sigma} E_t \sum_{n=0}^{\infty} r^n_{t+j}
\]

(1)

where \(i_t\) is the policy rate, \(x_t\) is the output gap, \(\pi_t\) inflation and \(r^n_t\) the (real) natural rate of interest. As shown by the equation above, absent a commitment technology in normal times an inflation targeting central bank can affect the output gap only through the current policy rate \((-\frac{1}{\sigma} i_t)\); the remaining variables are either exogenous \((r^n_t)\) or outside the reach of the central bank (all terms representing expectations). When large enough shocks push \(i_t\) at the ZLB, the current policy rate becomes useless and the central bank loses the power to stabilise aggregate demand and to keep inflation on target. Agents will take into account the non-zero probability of hitting the ZLB and revise their expectations accordingly. At the ZLB, monetary policy is less accommodative that it would be otherwise and inflation and the output gap are accordingly lower; as inflation targeting lacks history dependence, the shortfall in price dynamics and economic activity during ZLB episodes is not compensated in other periods and hence average inflation is below the central bank’s target.
and average output gap is negative. Accordingly, inflation expectations adjusts downwards, raising the probability of hitting the ZLB and triggering a vicious circle with inflation expectations.\footnote{For a more thorough treatment of the limits of discretionary policy under inflation targeting at the ZLB, see Walsh (2017).}

The shortcomings plaguing inflation targeting are not shared by other monetary policy strategies. In the case of price-level targeting (PLT) the inability to accommodate the monetary stance when at the ZLB is matched by extra loosening when the economy is out of the liquidity trap, implying that episodes of below target inflation are followed by periods when the target is overshot. In terms of equation (1), this implies that, if credible, a price-level targeting central bank is able to affect both the anticipation of future interest rates (i.e. the term $\frac{1}{\sigma}E_t \sum_{n=1}^{\infty} i_{t+j}$) and the expectations of future inflation (i.e. the term $\frac{1}{\sigma}E_t \sum_{n=0}^{\infty} \pi_{t+j}$). Average inflation targeting (AIT) and nominal GDP (level) targeting (GDPT) exhibit similar properties and are therefore other candidates that can legitimately challenge the strategy currently adopted by most central banks.

Underperformance at the ZLB however is not enough to dismiss inflation targeting, as the frequency of ZLB episodes is quite low. What happens when the policy rate is well above the ZLB? For quite a long time the common view was that strategies like price level targeting were sub-optimal in normal times, as they tied the central bank to past actions and prevented it from acting with the boldness and swiftness required to stabilise the economy. This view is no longer dominant. Vestin (2006) showed that PLT can outperform IT even in normal times,\footnote{Nessén and Vestin (2005) reach similar conclusions for average inflation targeting.} though his analysis is confined to simple New-Keynesian models lacking endogenous inertia.

This paper tries and answers the question whether there exist alternative monetary policy strategies that can overcome at least in part the limits of an inflation targeting framework, which best represents the current monetary regime in most advanced economies. The main tenet of the paper is that the central bank acts under discretion. In order to have an unbiased ranking, all comparisons are made among strategies that are optimal for their own loss function, though their effectiveness is evaluated on the basis of the social welfare function, which is common to all of them. The ranking is based on the stabilisation properties of each strategy both in normal times and when the effective lower bound is binding; in the latter case, what matters is reducing the probability of hitting...
the ZLB and shortening the average and maximum duration of ZLB episodes. Sensitivity analyses are run to assess which structural parameters may affect the relative performance of the competing strategies and which is the impact of including interest rate volatility in the loss function.

The main results of the paper are the following. Our analysis confirms the theoretical findings by Svensson (1999) and Vestin (2006) that price-level targeting can guarantee a better performance than inflation targeting, as it is better suited to (1) reduce output and inflation volatility in normal times and (2) minimise the frequency and costs of ZLB episodes. The evidence for other history-dependent strategies is somewhat mixed. Average inflation targeting has no edge over inflation targeting, while nominal GDP-level targeting seems to perform well both in relative and in absolute terms. It is uniformly superior to neither inflation targeting nor average inflation targeting, but succeeds in ensuring better outcomes over a large range of model parameters and social preferences. These findings are valid regardless of whether interest-rate variability is included in the loss function or not and are robust to changes in model parameters, e.g. the slope of the Phillips curve and the degree of forward lookingness in either the Phillips curve or in the IS schedule.

The paper tries and provides an original contribution to the literature on PLT in a few ways. It generalises the findings documented in Vestin (2000) and Nessén and Vestin (2005) by (i) considering a model featuring endogenous inertia in both the Phillips and IS curve; (ii) allowing interest-rate variability to enter the loss function; and (iii) including nominal GDP level targeting among the scrutinised strategies. It extends the results in Busetti et al. (2020) by considering targeting rules rather than instrument rules. Finally, it presents results of several sensitivity analyses showing how the ranking of monetary policy strategies is affected by changes in the structural parameters of the model.

The paper is organised as follows. The next section provides a brief overview of the literature on the pros and cons of a few monetary policy strategies. Section 3 presents the model, the estimates obtained using euro area data and the strategies considered in the analysis. Section 4 presents analytic and simulation results on the effectiveness of each alternative monetary regime. The strategies are ranked in terms of their performance in minimising inflation and output-gap volatilities and of their ability to reduced the probability and the length of ZLB episodes. Section 5 presents a number of sensitivity analyses aimed at assessing which structural features of the economy affect the ranking of the competing monetary policy frameworks. Section 6 concludes.
2 Literature review

There has been much discussion lately on whether inflation targeting (IT) is a monetary framework that is already past its sell-by date. There seem to be two different schools of thought on the matter: evolution or revolution. The first suggests that IT could be rescued by either raising the inflation objective or by targeting average inflation over an extended period of time,\(^7\) while the second advocates a change in regime and calls for a price-level target or variants of it.

Raising the inflation target may appear like the simplest solution to prevent a higher incidence of the ZLB, as it would not require a radical change of the monetary policy framework, it would not jeopardise central bank’s credibility and it would be easy to communicate to the public. Those who advocate such a solution claim that there has never been a clearly optimal inflation target.

Ball (2013) estimated the risk of zero-bound episodes in the US by analysing the behaviour of interest rates in past recessions and showed that a higher inflation target could be beneficial as it would lower the probability of incurring a liquidity trap. Had the 4% inflation target proposed by Ball been in place in the past decades, the ZLB constraint would have been binding only in 2 recessions out of 8, instead of the 4 cases that resulted from the current 2% target.

Another argument in favour of raising the inflation target is given by downward wage rigidity, which may be particularly relevant in the current low inflation environment, as it prevents real wages to adjust as much as needed to keep unemployment low (Krugman 2014). A higher inflation target would allow employers to cut real wages without affecting nominal ones, thus reducing involuntary unemployment.

However, inflation has costs that should be thoroughly assessed before deciding to adopt a higher target. Some of the main concerns associated with a higher inflation target regard inflation variability and price dispersion. Inflation variability seems to be positively correlated with the level of inflation and so does price dispersion, which lowers welfare through an inefficient allocation of resources.\(^8\) Another major risk in raising the target is jeopardizing the credibility of central banks, since one of the greatest achievements in monetary policy is

---

\(^7\)See in particular the proposal put forth by Nessén and Vestin (2005), who advocate average inflation targeting.

\(^8\)If uncoordinated and adopted in a single country, an increase in the inflation target would also affect the volatility of the exchange rate. A steady depreciation of the domestic currency would be needed to offset the impact on competitiveness of higher trend inflation: the volatility of the latter would be therefore transmitted to the former.
the anchoring of inflation expectations around 2%. Ascari and Sbordone (2014) provide a detailed analysis of the problems generated by higher trend inflation, which results in a lower level of steady-state output (and thus welfare), a flatter Phillips curve and a less effective monetary policy.\footnote{See Ascari and Ropele (2009) on the impact of different level of the inflation target on indeterminacy.}

Unfortunately, neither the benefits nor the costs of a higher inflation target have been clearly and unambiguously quantified in the literature, making it hard to assess whether a target of 4\% would be beneficial or detrimental to society’s welfare. It is noteworthy to point out that the costs of a higher target are permanent, so that even if they are small in any given year, they add up. Bernanke (2015), for example, while acknowledging that this proposal has some merits, adds that it is not the most effective way to deal with the ZLB problem. Nonetheless, this suggestion has gained a foothold in the aftermath of the crisis (Yellen 2017).

Rather than fixing inflation targeting, some authors have proposed to do without it. A departure from inflation targeting was proposed prior to the crisis: Woodford (2003) argued that the optimal policy under commitment, which exhibits history-dependence, can be implemented by targeting the price level or, somewhat less effectively, nominal GDP.

Until the late 1990s, PLT was viewed with a lot of skepticism by academics and policymakers. The common wisdom at the time was that there was a trade-off between long-term price-level variability and short-term inflation volatility, due to the history-dependence of price level targeting: history-dependence implies that if the price target has been overshot in the past, it must be undershot in the future (and the other way around) in order to bring back the price level to its desired value. This generates higher inflation and output variability than under IT, in particular if nominal rigidities are present.

The first author who questioned the superiority of IT over PLT was Svensson in 1999. Svensson proved that in a backward-looking New Classical model with high output persistence, a central bank unable to commit can indeed improve the variability of inflation without worsening that of output by targeting the price level. PLT is superior even when judged on the basis of a loss function that depends on inflation, not the price level. This result appears to be a sort of "free lunch", as suggested by the title of Svensson’s paper.

Vestin (2000) showed that these findings hold in a forward-looking New Keynesian model as well. In Vestin’s paper the output gap is driven by expected
future inflation (the forward-looking aspect of monetary policy) and not by inflation surprises, as in Svensson (1999). The central bank uses the output gap as an instrument and is, once again, unable to commit. Vestin found that the equilibrium under PLT is in general very close to the commitment solution, the more so when the persistence of the cost-push (i.e. supply) shock is low. The reason why PLT is more effective in reducing the trade-off between inflation and output-gap variability is precisely that it exhibits history dependence, which allows the central bank to affect expectations.

It shouldn’t come as a surprise that price level targeting is similar to and, under certain conditions, coincides with the commitment solution. Woodford states that policy under commitment is optimal because it entails history dependence: a history-dependent policy can tame inflationary pressure with less contraction of output. Eggertsson and Woodford (2003) identified a PLT regime, either with a time-varying or a fixed target, as an optimal policy. Price-level targeting, by committing to undo any deflation with subsequent inflation, has a built-in automatic stabiliser that an inflation targeting regime does not possess.\(^\text{10}\) This feature is particularly useful when the natural rate of interest is low and the probability of hitting the ZLB is non trivial. It is of course important that the strategy is well understood by the public, which requires that the central bank is very careful in communicating its objectives and targets.

With the Great Recession, the ZLB stopped being only a theoretical concern and became a potential threat. John Williams, President of the New York Fed, was one of the early supporters of price level targeting. In Mertens and Williams (2019) he claims that in order to be effective, a monetary strategy should entail the promise to keep rates lower for longer after a ZLB episode, thus stimulating the economy precisely at a time when the central bank is constrained. PLT exhibits such a feature and so do policies allowing an otherwise standard interest rate rule to make up for the sum of past shortfalls in interest rate cuts, as in Reifschneider and Williams (2000).

Recently, a milder version of price-level targeting has been proposed by Bernanke (2017), who suggests resorting to the implementation of a temporary PLT strategy only in periods when conventional policies are constrained by the zero lower bound: the standard practice of targeting inflation would be maintained in normal times, while a makeup policy would kick in in periods

\(^{10}\) A similar claim is in Svensson (2019), who claims that price-level targeting implies some – or even substantial – “automatic” stabilisation, which makes monetary policy more effective especially in situations when the ZLB is binding.
when inflation is persistently below target. According to Bernanke, the adoption of a temporary price-level target would improve economic performance and would require only a relatively modest shift in central banks’ current framework, avoiding the communication challenges vexing the move to a strategy of full PLT.

In conclusion, a large strand of the literature identifies price-level targeting - or variants of it - as the optimal policy at all times, but particularly when a liquidity trap prevents interest rate policies to provide the degree of accommodation needed to stabilise the economy.

In 2005, Nessén and Vestin proposed an alternative strategy, named average inflation targeting (AIT from here on), which lies in-between price-level targeting and inflation targeting: AIT exhibits some degree of history dependence and is sufficiently similar to the current policy framework. They found that such a strategy may outperform both price-level and inflation targeting when price setters’ behaviour is relatively backward-looking.

Nessén and Vestin define average inflation targeting as “a policy where the central bank’s objective is to keep average inflation measured over several years stable”. The main difference from inflation targeting is that the central bank does not need to reach the inflation target in one period, but instead aims at keeping on target inflation averaged over a given horizon. The reference to past price developments makes AIT history dependent, though less than PLT, and enhances its effectiveness in reducing output and inflation variability.

Vestin (2000) proved that in a forward-looking model PLT is superior to inflation targeting (IT). Nessén and Vestin (2005) extended his analysis to include average inflation targeting and sought to assess how it measures up to the other two regimes. They found that under discretion PLT is superior to both inflation targeting and average inflation targeting, but AIT dominates IT.

Busetti et al. (2020) use a medium-size DSGE model for the euro area to assess the stabilisation properties of a number of monetary policy strategies. They find that price-level targeting outperforms other frameworks in mitigating output and inflation variability in normal times and is the most effective in reducing the probability of hitting the ELB. Inflation targeting ranks low in terms of both criteria.

A dissenting view is outlined in Walsh (2019), who shows that in a model with sticky wages and shocks to productivity (but without endogenous persistence in inflation or the output gap) the ranking of monetary policy strategies radically changes, with PLT falling at the bottom. With sticky prices and wages,
a shift in productivity requires a persistent change in real wages: what worsen
the performance of PLT is the attempt to force too much of the adjustment to
fall on wages. Walsh therefore concludes that it is too early to count IT out in
the competition over policy design.

3 Model estimates and simulation design

The model used for assessing the merits of alternative monetary policy strategies
is a three-equation New-Keynesian model; its parameters are estimated with
Bayesian methods on euro-area quarterly data. Inflation ($\pi_t$) and the output
gap ($x_t$) exhibit endogenous inertia and the two structural shocks – a supply
($u_t$) and a demand ($z_t$) shock – are first-order autoregressive processes. The
formulation of the interest rate ($i_t$) equation is such that is encompasses the
optimal rule for all strategies.

The specification of the model is the following:

$$
\begin{align*}
\pi_t &= \varphi E_t \pi_{t+1} + (1 - \varphi) \pi_{t-1} + \kappa x_t + u_t \\
x_t &= \psi E_t x_{t+1} + (1 - \psi) x_{t-1} - \sigma (i_t - E_t \pi_{t+1}) + z_t \\
i_t &= \gamma_1 \pi_{t-1} + \gamma_2 x_{t-1} + \delta p_{t-1} + \sum_{j=2}^k \zeta_j \pi_{t-j} + \gamma_3 u_t + \gamma_4 z_t \\
u_t &= \rho^s u_{t-1} + \varepsilon_t \\
z_t &= \rho^s z_{t-1} + \eta_t
\end{align*}
$$

where each variable is expressed in deviation from its steady-state value.\textsuperscript{11} The
interest rate equation is written so as to encompass the optimal rule under all
strategies.\textsuperscript{12}

The monetary policy strategies whose effectiveness is compared with that of
inflation targeting (IT) are: (1) price-level targeting (PLT); (2) average inflation
targeting (AIT); (3) nominal GDP level targeting (GDPT). AIT can be defined
in several ways, depending on the time interval over which average inflation is
computed: five alternatives are considered, corresponding to 2-, 4-, 8-, 12-, and

\textsuperscript{11}The term $z_t$ in the IS curve is usually viewed as generated by a technology shock. It is a
linear function of the deviation of the natural rate of interest from its steady-state value. A
positive technology shock pushes inflation and output in opposite direction, but the sign of
the correlation switches for inflation and the output gap, as the increase in demand falls short
of the rise in the natural level of output. Since the technology shock moves $\pi_t$ and $\pi_t$ in the
same direction, $z_t$ is dubbed a demand shock, in line with the common usage of distinguishing
supply and demand shocks on the basis of the co-movement of inflation and the output gap.
See Gali (2008) for more detailed information.

\textsuperscript{12}In particular, $\delta$ is non-zero for price level targeting and nominal GDP targeting, while the
$\zeta_i$’s only for AIT.
3.1 Optimal policy under alternative strategies

In order to ensure a level-playing field across all strategies, only optimal policies are considered. Optimality is judged on the basis of a loss function that can differ from the one that reflects social preferences. The assumption is that the government delegates monetary policy to a central bank that is assigned a particular loss function; as in practice, the central bank retains discretion in setting the policy rate.

Each strategy is assessed on the basis of its performance both in normal times and when the ZLB is binding: in the first case what matters are the volatilities of inflation and the output gap, whose relative weight is determined by social preferences, as represented by the loss function $L^S = E_t \sum_{n=0}^{\infty} \beta^n L^S_{t+n}$, where $L^S_t = \pi^2_t + \lambda^S x^2_t$; in the second the ranking is inversely related to the probability of hitting the ZLB and to the average and maximum duration of ZLB episodes.

Strategies are distinguished on the basis of the loss function that the government assigns to the central bank, whose generic time-$t+n$ element is:

$$
L^I_{t+n} = \pi^2_{t+n} + \lambda^I x^2_{t+n}
$$

$$
L^{PLT}_{t+n} = p^2_{t+n} + \lambda^{PLT} x^2_{t+n}
$$

$$
L^{AIT_k}_{t+n} = \left( \frac{1}{k} \sum_{j=0}^{k-1} \pi_{t+n-j} \right)^2 + \lambda^{AIT_k} x^2_{t+n}
$$

$$
L^{GDPT}_{t+n} = \left( p_{t+n} + x_{t+n} \right)^2
$$

with $k = 2, 4, 8, 12$ or 16.

Optimal policies are computed subject to the constraint represented by the Phillips curve and under the assumption that no lower bound for the policy rate exists. The output gap is the central bank’s instrument and the value of the policy rate is obtained by inverting the IS curve. Strategies are compared also under the assumption that the loss function assigned to the monetary authority includes the volatility of the short-term (i.e. policy) rate as an additional argument\(^\text{13}\): in that case the IS curve becomes the second constraint. The optimisation algorithm is the one proposed in Dennis (2007) and described in Appendix A.

In all cases it is supposed that the central bank acts under discretion and that its commitment to follow transparent rules and policy goals is fully credible.

\(^{13}\)In all simulations, the weight attached to interest-rate volatility in the loss function is the one suggested in Giannoni (2010)
Policy is the outcome of the game between the monetary authority and the private sector and is thus time consistent in equilibrium.

In what follows, the parameter $\lambda^S$ in the social loss function is set equal to 0.25, which is the value used in Mertens and Williams (2019). Though such a value is presumably higher than that characterising social preferences in the euro area, at least as coded in the ECB mandate, there is a valid reason to pick out a fairly high value for $\lambda^S$. The main objective of this paper is to assess whether inflation targeting is the best strategy a central bank can adopt in an environment of very low level of the real natural rate of interest. As shown by Vestin (2006) and Nessén and Vestin (2005), price level targeting (PLT) and average inflation targeting (AIT) tend to outperform IT because they reduce inflation volatility at the cost of a moderate increase in the standard deviation of the output-gap. As we rank alternative strategies by means of society’s loss function, a high value of $\lambda^S$ makes more difficult for PLT and AIT to outperform IT. Setting a value for $\lambda^S$ that is larger (and probably much larger) than the appropriate value for the euro-area is a way to treat IT more than fairly when comparing its merits against those of other strategies, making it more difficult to dismiss IT as a still viable option for a central bank.

3.2 Model estimates

The model parameters are estimated with Bayesian methods on euro-area quarterly data for inflation, the output gap and the three-month nominal interest rate. For the most recent period, when money-market interest rates are constrained by the effective lower bound, the shadow rate estimated by Kortela (2016) is used. Inflation is measured by the HICP net of the most volatile components, i.e. food and energy goods. The output gap is an internal, non-public measure of economic slack and coincides with the one featuring in the Eurosystem’s projection exercises.

The estimated model is the following:

\[
\begin{align*}
\pi_t &= \varphi E_t \pi_{t+1} + (1 - \varphi) \pi_{t-1} + \kappa x_t + u_t \\
x_t &= \psi E_t x_{t+1} + (1 - \psi) x_{t-1} - \sigma (i_t - E_t \pi_{t+1} - \bar{r}) + z_t \\
i_t &= \delta i_{t-1} + (1 - \delta) (\bar{r} + \bar{\pi}) + \alpha_\pi (\pi_t - \bar{\pi}) + \alpha_x x_t + \theta_t
\end{align*}
\]

\[^{14}\text{The value } \lambda^S = 0.25 \text{ captures the balanced approach to the dual mandate adopted by the Federal Reserve. Assuming that for the United States the Okun’s law is } x_t = 2(u^*_t - u_t), \text{ a loss function assigning the same weight to the variance of inflation and that of the unemployment gap is equivalent to one where the relative weight of the output gap is 0.25.}

[^15]\text{Model estimates have been provided by Andrea Gerali.}\]
where all shocks are AR(1) processes. The interest-rate equation is a standard Taylor-type feedback rule, modeling the policy rate as a function of its own lag, inflation and the output gap. The policy rule is used only for closing the three-equation system and estimating the structural parameters of the model: in the next sections it is substituted by the optimal interest-rate rule of each of the strategies considered in the paper.

The target inflation rate, which is set at 1.9% annualised, is consistent with the ECB monetary policy strategy; the natural real rate is not estimated either, but calibrated at 1.1% annualised, which seems an acceptable average value for the sample period. Data spans a 23-year time frame, starting in 1995Q1 and ending in 2017Q4.

Table 1: Prior and Posterior Moments of Model Parameters

<table>
<thead>
<tr>
<th>Model param.</th>
<th>prior mean</th>
<th>posterior mean</th>
<th>90% HPD interval</th>
<th>prior distribution</th>
<th>prior standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varphi$</td>
<td>0.800</td>
<td>0.304</td>
<td>0.184 0.422</td>
<td>Beta</td>
<td>0.100</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.100</td>
<td>0.056</td>
<td>0.032 0.080</td>
<td>Gamma</td>
<td>0.050</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.600</td>
<td>0.744</td>
<td>0.601 0.880</td>
<td>Beta</td>
<td>0.200</td>
</tr>
<tr>
<td>$\delta$</td>
<td>1.500</td>
<td>0.530</td>
<td>0.275 0.777</td>
<td>Gamma</td>
<td>0.500</td>
</tr>
<tr>
<td>$\rho_\pi$</td>
<td>0.700</td>
<td>0.684</td>
<td>0.619 0.744</td>
<td>Beta</td>
<td>0.040</td>
</tr>
<tr>
<td>$\rho_x$</td>
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<td>1.157</td>
<td>1.004 1.315</td>
<td>Normal</td>
<td>0.100</td>
</tr>
<tr>
<td>$\alpha_{\pi}$</td>
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<td>0.065</td>
<td>0.033 0.099</td>
<td>Normal</td>
<td>0.100</td>
</tr>
<tr>
<td>$\rho_{\pi}$</td>
<td>0.950</td>
<td>0.965</td>
<td>0.929 0.999</td>
<td>Beta</td>
<td>0.040</td>
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<tr>
<td>$\rho_x$</td>
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<td>0.924</td>
<td>0.863 0.986</td>
<td>Beta</td>
<td>0.040</td>
</tr>
<tr>
<td>$\rho_i$</td>
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<td>0.692</td>
<td>0.593 0.786</td>
<td>Beta</td>
<td>0.100</td>
</tr>
<tr>
<td>$\sigma_{\pi}$</td>
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<td>0.044</td>
<td>0.032 0.056</td>
<td>Inv.Gamma</td>
<td>5.000</td>
</tr>
<tr>
<td>$\sigma_x$</td>
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<td>0.128</td>
<td>0.085 0.168</td>
<td>Inv.Gamma</td>
<td>5.000</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>0.200</td>
<td>0.105</td>
<td>0.088 0.122</td>
<td>Inv.Gamma</td>
<td>5.000</td>
</tr>
</tbody>
</table>

Table 1 presents the prior and posterior distributions of the estimated parameters. The choice of the prior distribution is related to the nature of the coefficients: parameters bounded between 0 and 1 (the weight of the forward looking components in the IS and Phillips curve, $\varphi$ and $\psi$ respectively; $\delta$, the implicit assumption is that the steady state of the model is efficient, thanks to a government subsidy that offsets the distortion induced by the monopoly power of firms. This assumption however is not likely to affect the assessment of the effectiveness of alternative monetary policy strategies. As proved in Benigno and Woodford (2005) and neatly shown in Galí (2008), provided that the steady-state distortion is small, under the optimal policy the presence of a distorted steady state affects the response to shocks of neither the output gap nor inflation and hence does not alter the ranking of alternative monetary policy strategies. It however affects the average levels of inflation and the output gap.
parameter of lagged interest rate in the monetary policy rule; the autocorrelation coefficients of the structural shocks: $\rho_\pi$, $\rho_x$ and $\rho_i$ are assumed to follow a beta distribution; positive parameters not constrained to be lower than 1 ($\kappa$, the output gap-inflation trade-off in the Phillips curve; $\sigma$, the elasticity of intertemporal substitution) have a gamma distribution; finally, unbounded parameters ($\alpha_\pi$, the interest rate response to inflation; and $\alpha_x$, the coefficient of the output gap in the Taylor rule) are assumed to follow the Gaussian distribution.

The prior mean for the coefficient of expected inflation in the Phillips curve is set at 0.8, while $\kappa$ is assumed to be 0.1. Regarding the parameters of the IS curve, the prior mean of the coefficient of next-period output gap is set at 0.6, while that of $\sigma$ is equal to 1.5. For the monetary policy rule, the prior mean for the lagged interest rate coefficient is set at 0.7, while those measuring the response to inflation and the output gap are centred around 1.1 and 0.2 respectively.

Finally, the prior means of the parameters of the structural shocks are chosen based on the assumption that all shocks — save the monetary policy shock — are highly autocorrelated.

The standard deviation of all prior distributions is sufficiently loose to allow the likelihood function to play a meaningful role in determining the posterior mean, but at the same time sufficiently tight to be informative and reduce the probability of having flat or multimodal posteriors.

The posterior distribution of the parameters is estimated using the Metropolis–Hastings algorithm. The joint posterior distribution of all estimated parameters is obtained in two steps. First, the posterior mode and an approximate covariance matrix, based on the inverse Hessian matrix evaluated at the mode, is obtained by numerical optimization on the log posterior density. Then, the posterior distribution is explored by generating draws using the Metropolis–Hastings algorithm. The distribution is taken to be the multivariate normal density centered at the previous draw with a covariance matrix proportional to the inverse Hessian at the posterior mode.

The results are reported in Table 1, which shows for all the parameters the mean as well as the 5th and 95th percentiles of the posterior distribution. According to the estimates, the inertia in the Phillips curve is quite elevated (0.6972) and much higher than implied by the prior mean, presumably due to the protracted period of stubbornly low inflation experienced in the final decade of the sample. The output gap coefficient is small (0.0560), in line with the value suggested in Woodford (2003). More relevant is the forward-looking
component of the IS curve (0.7437), which features a fairly high value (0.5298)
of the coefficient measuring the impact of the ex-ante real interest rate on the
output gap. Fairly standard are the posterior means of the parameters of the
interest rate rule, which however do not play any role in the analysis presented
in the following sections.

A comparison with other papers is not easy. Estimates of a three-equation
New-Keynesian model tend to be very different across countries and heavily de-
pendent on the selection of the sample period and observable variables. While
in this paper the sample covers basically the first 20 years of the monetary
union and includes the output gap among the observables, Smets (2003) uses
data over the two pre-EMU decades, Gali et al. (2001) rely upon real marginal
costs and Dennis (2005) focuses on the United States. Other papers resort to
calibrated parameters (e.g. Söderström et al. (2003)) or uses a different estima-
tion approach (e.g. Juselius (2008)). With these caveats in mind, a comparison
with the above-mentioned papers shows that our estimates are quite in line with
those available in the literature. The Phillips curve inertia in our paper is higher
than in Smets (2003) and Dennis, but lower than in Söderström et al. (2003);
the opposite occurs for inertia in the IS curve, which is instead somewhat lower,
though the estimates of other papers fall within the 90% HPD interval. The out-
put gap coefficient in the Phillips curve is nearly identical to the one in Dennis
(2005), but about half the size of Smets’ and Söderström’s estimate/calibration.
The only exception is the coefficient measuring the interest-rate sensitivity of
the output gap in the IS curve, which is substantially higher, though half the
size of the calibrated value in Gali (2008).

4 Results of the horse race

4.1 The ranking of strategies in good times

To assess whether there exist strategies that are able to outperform inflation
targeting, it seems appropriate to consider for each option the one maximising
social welfare, i.e. the one that is enforced by choosing a central banker as
conservative as needed to remove the stabilisation bias. This implies to choose
for each strategy the value of $\lambda$ that solves the following condition:

$$\lambda^j = \arg \min_{\lambda^S} L^S \left[ \pi \left( \lambda^j \right), x \left( \lambda^j \right); \lambda^S \right]$$  \hspace{1cm} (5)
for \( j = IT, PLT, AIT_k, GDPT \). The steps needed to find the optimal policy are two: in the first, the optimal interest-rate rule \( i(\lambda^j) \) and the MSV solutions for \( \pi(\lambda^j) \) and \( x(\lambda^j) \) are computed for a given \( \lambda^j \); in the second, the range of value of \( \lambda^j \) is scanned to find the one maximising social welfare \(-L^S[\pi(\lambda^j), x(\lambda^j); \lambda^S]\).

The second step is needed because, as shown in Clarida et al. (1999), in discretionary policymaking there is a bias towards under-stabilisation of inflation, which does not depend on the desire to maintain output above potential, as in Rogoff (1985). For the central bank there is always an incentive to accommodate inflation shocks when they occur, in spite of the optimal policy requiring a tougher response. If price-setting depends on expectations of future conditions, a monetary policymaker that can credibly commit to a rule faces a more favourable output-inflation trade-off. Absent a commitment technology, an improved equilibrium can be achieved under discretion if the government appoints a conservative central banker, i.e. a policymaker that assigns to output stabilisation less importance than society does.\(^{17}\)

Unfortunately, what is good in theory does not always work in practice, as it will be shown shortly.

Fig.1 plots on the y-axis the social loss function of each alternative strategy relative to that of IT; on the x-axis there are the values of \( \lambda \), i.e. the relative weight of the variance of the output-gap in the loss function, which can be interpreted as a label representing the type of central banker - the more conservative the lower \( \lambda \) - that the government can appoint.

Four features of the chart clearly stand out: first, for \( \lambda = 0 \), IT, PLT and AIT provide exactly the same results; second, there seems to be no advantage in moving from an IT strategy to AIT, regardless of the order of the averaging process; third, PLT is uniformly superior to IT; fourth, nominal GDP targeting is better than IT for most values of \( \lambda \), including the one assumed for the social loss function.

The first finding seems quite unexpected: how can strategies that are so different provide the same outcome for the variance of both inflation and the output-gap? Actually, such a result is not surprising: if the central bank is not concerned with output stabilisation (i.e. if \( \lambda = 0 \)), it can keep inflation on target in every period (i.e. at zero, since each variable is measured in deviation

\(^{17}\)The same conclusion is reached in Nakata and Schmidt (2018). The authors consider an economy with an occasionally binding ZLB and show that, because of anticipation of future ZLB episodes, inflation systematically falls below target even when the policy rate is above zero. Nakata and Schmidt prove that this bias can be mitigated if a conservative central banker is appointed.
from the steady-state value).\footnote{The current value of the structural shocks is in the central bank’s information set and hence the inflation objective can be achieved with absolute precision.} Moreover, if inflation is always zero, the Phillips curve becomes:

\[ 0 = \kappa x_t + u_t \]  

(6)

implying that \( x_t = -\frac{1}{\kappa} u_t \) and \( \text{Var}(x_t) = \frac{1}{\kappa^2} \text{Var}(u_t) \), which is the same for all strategies, save GDPT.

A possible explanation of the second finding, which is somewhat at odds with most of the literature, is that under AIT, regardless of the length of the window over which average inflation is computed, it is more difficult for the private sector to anticipate the reaction of the central bank to shocks. Apparently, the benefits of adopting a history-dependent strategy are offset by the disadvantage of blurring the role of the target as an attractor for actual and expected inflation.\footnote{See Neri and Ropele (2019) on the costs of not having a clearly defined inflation target.}

The third result is consistent with predictions from economic theory: PLT is uniformly superior to inflation targeting\footnote{This claim is somewhat overstated: for values of \( \lambda \) very close to zero, IT guarantees the best performance overall in terms of social welfare. However, the improvement with respect to PLT is tiny, disappears for value of \( \lambda \) slightly above 0.05 and is not robust to changes in parameters of the Phillips or IS curve.} because it is more effective in keeping inflation anchored and in approximating the equilibrium outcome that can be achieved under commitment.

Finally, the good performance of GDPT can be justified on the basis of Woodford’s (2012) considerations: “A simple nominal GDP target path would not achieve quite the full welfare gains associated with a credible commitment to the gap-adjusted price level target. […] Nonetheless, such a proposal would retain several of the desirable characteristics of the gap-adjusted price level target […]. Essentially, the nominal GDP target path represents a compromise between the aspiration to choose a target that would achieve an ideal equilibrium if correctly understood and the need to pick a target that can be widely understood and can be implemented in a way that allows for verification of the central banks pursuit of its alleged target, in the spirit of Milton Friedman’s celebrated proposal of a constant growth rate for a monetary aggregate.”

There are also practical advantages – not fully captured by the overly simple 3-equation model used in this paper – that allow nominal GDP-level targeting to gain the upper hand over IT and AIT: as stressed by Beckworth and Hendrickson (2016), nominal GDP targeting (i) allows central bankers to focus only
on one variable rather than two; (ii) does not require to respond to real variable beyond the policymaker’s control; (iii) makes unnecessary for the central bank to distinguish, in real time, between shocks to aggregate supply and shocks to aggregate demand.

Similar evidence is shown in Fig.2, which replicates Fig.1 for the case where the variance of the interest rate enters the loss function. This assumption may be justified on the grounds that excessive volatility of the policy (i.e. short-term) rate may jeopardise the stability of the financial system. A similar provision applies to the Federal Reserve of the United States, whose mandate is to “promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates.”

There are two main results that are highlighted by Fig.2. First, contrary to what has been for a long time the conventional wisdom, the adoption of price-level targeting does not involve higher short-run inflation and interest rate volatility: PLT remains the most effective strategy even when excessive central bank’s activism is penalised. Second, the relative performance of Nominal GDP targeting increases: it remains inferior to PLT, but becomes superior to both IT and AIT even when the weight assigned to the variance of the output gap is zero or extremely small.

Overall, the message that Fig.1 and Fig.2 convey is that there exists a clear ranking among the strategies: PLT is the most effective in stabilising the economy, regardless of the weight attached to the variance of the output gap; nominal GDP targeting ranks second when interest-rate volatility is a matter of concern for the policymaker, and even when it is not, it outperforms IT and AIT for most values of $\lambda$. Finally, $AIT_k$ does not improve upon IT even when inflation is averaged over 4 years, a fairly long period of time.

4.2 What makes a good strategy?

Fig.1 shows that when the variance of the policy rate is not a matter of concern for the central bank, the optimal value of $\lambda$ for PLT, IT and $AIT_k$ is zero and the performance of IT and $AIT_k$ rapidly deteriorates for higher values of this parameter. Since it seems unlikely that the government can choose with high precision the ‘type’ of central banker to appoint, it seems inappropriate

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21 The relative weight in the loss function of the variance of the short-term interest rate is 0.5, as in Mertens and Williams (2019).
22 $\lambda$ does not enter the loss function for nominal GDP targeting and hence its value is irrelevant for this strategy.
to compare the competing monetary policy strategies at such a value of \( \lambda \); moreover, an environment where inflation is always on target and the output gap fluctuates widely does not seem the most realistic benchmark. Luckily, a viable and satisfactory alternative is available.

A sensible choice for \( \lambda^{IT} \) and \( \lambda^{AIT} \) is \( \lambda^{S} \). For PLT the pick is less obvious, given that the argument of the loss function is the square of the price level and not of inflation. However, the performance of price-level targeting is quite insensitive to the value of \( \lambda^{PLT} \) and accordingly \( \lambda^{S} \) may be a neutral choice for PLT as well. For nominal GDP targeting the issue is irrelevant, as the loss function does not depend on the value of \( \lambda \).

To understand what makes one strategy outperform the others, we can look at the behaviour of the variables in equilibrium. Table 2 to 4 show for inflation, the output gap and the policy rate the coefficients of the MSV solution (in the first six columns), the variance and the first-order autocorrelation coefficient (in the following two).

A few results are worth stressing. First, under PLT the interest rate responds more to past inflation and less to the current supply shock, i.e. it exhibits more history dependence. The response to the lagged output gap is the same as for the other strategies and is equal to \( 1 - \psi \frac{1}{\sigma} \). The variance of the policy instrument is much smaller than in the other cases: PLT is more effective in steering expectations and hence the interest rate need not move much to control aggregate demand. This feature of the policy rule is what makes inflation less inertial and less volatile, though at the cost of a somewhat higher variance of the output gap. The trade-off is however extremely favourable, as inflation volatility is two order of magnitudes lower than under alternative strategies, while the variance of the output gap is less than 25% higher. Second, the ineffectiveness of average inflation targeting is due to very low coefficients on the lags of inflation, which makes AIT of any order barely distinguishable from IT. Third, nominal GDP targeting is unlike any other strategy, but shares some features with PLT: it responds weakly to the supply shock; it exhibits low inflation and interest-rate volatility; it trade off some degree of output-gap stabilisation for achieving a firmer control of inflation.

22When the interest rate is not an argument of the loss function, the output gap is the central bank’s instrument and hence does not enter the MSV solution of inflation and the output gap. This is true for all strategies. The solution for the policy rate is obtained by inverting the IS curve and is equal to \( i_t = \frac{1}{\sigma} \left( 1 - \psi \right) x_{t-1} - x_t + \psi E_t x_{t+1} + z_t \). Since \( \pi_t \) and \( x_t \) do not depend on the lagged output gap, the coefficient of \( x_{t-1} \) in the MSV solution of \( i_t \) is \( \frac{1}{\sigma} \) for all strategies.
Table 5 to 7 present the same evidence as table 2 to 4 for the case where interest rate volatility enters the loss function. The results confirm the main finding outlined for the benchmark case. The only noticeable difference is that under price-level and nominal GDP targeting both inflation and the interest-rate are more inertial, which is not surprising as now changes in the policy rate are more costly.

4.3 Effectiveness at the ELB

In order to be effective, a monetary policy strategy should perform well also in periods when the ELB is binding. The existence of an ELB introduces a non-linearity, which complicates the solution of the model. The standard procedure adopted in the literature is to solve the system under perfect foresight, assuming that agents set to zero future shocks and are surprised every period, when shocks materialise. To ensure convergence to an equilibrium, it is in addition posited that ELB episodes have a maximum length and/or an emergency fiscal stimulus package is enacted.\(^{24}\) I instead follow an approach akin to Walsh (2019), where linearity is justified by the use of a shadow rate instead of a policy rate. The former differs from the latter in the sense that it is allowed to be negative, i.e. it coincides with the policy rate that the central bank would set were the ELB nonexistent.

Walsh draws from Wu and Zhang (2017), who propose a modified version of the three-equation New Keynesian model, whose main feature is to include in a highly-stylised way non-standard monetary policy measures. They substitute in the IS curve the policy (i.e. short-term) rate with a longer-term one, that equals the former plus a term premium. The term premium, which is allowed to become negative, can be reduced by the central bank via outright asset purchases, the size of which is set so that the impact on the aggregate demand is the same that would be obtained by means of the shadow rate.\(^{25}\) In this setup the frequency of negative interest rates is interpreted as the recurrence of central banks engaging in quantitative easing (QE), without imposing an ELB-induced structural break.

The main disadvantage of the approach suggested by Walsh is that it relies

\(^{24}\)See for instance Kiley and Roberts (2017), who avoid indeterminacy by imposing an emergency fiscal package and assuming that agents never expects the ZLB to bind for more than 15 years. A similar approach is adopted by Williams (2009).

\(^{25}\)Wu and Zhang show that the shadow rate estimated for the US is highly correlated (-0.94) with the Fed’s balance sheet in the post-GFC period.
on the strong assumption that non-standard measures, namely QE, can fully substitute for interest-rate policies at the ELB, which therefore would be no longer a constraint. According to Debortoli et al. (2019) the empirical relevance of the ELB is indeed dubious. Their working assumption is that the performance of the economy should be different when the ELB binds from when it does not. They focus on two dimensions of the performance that are likely to be affected by a binding ELB: (i) the volatility of macro variables and (ii) their response to shocks. A rise in volatility could be expected as a result of the central bank’s hands being tied due to the policy rate having hit the ZLB, since this prevents the "usual" stabilizing policy response to aggregate shocks. Similar considerations apply to the behaviour for instance of the long-term interest rate, which in a liquidity trap should respond differently to shocks, lacking the guidance provided by the policy rate. Using US data, Debortoli et al. (2019) find little evidence against the irrelevance hypothesis, with their estimates suggesting that the responses of output, inflation and the long-term interest rate were hardly affected by the binding ELB constraint, possibly as a result of the adoption and fine-tuning of unconventional monetary policies.

The main advantage of the Walsh approach is instead that determinacy is achieved without resorting to fiscal policy or some ad-hoc assumption about the maximum length of ELB episodes.

Tables 8-10 report for all policy strategies three statistics: (i) the probability of hitting the ELB, measured by the relative frequency of periods when the policy rate is below the ELB; (ii) the average length of a liquidity trap episode; (iii) the maximum duration of an ELB occurrence. Results are obtained by stochastic simulations (one thousand 400-quarter long replications), assuming that the ELB is 100, 200 and 300bp below the steady-state value of the policy rate. As unconventional policies are unlikely to be as effective as standard ones, it is possible that in the real world it will take longer for the economy to exit a liquidity trap: the numbers in Tables 8-10 are accordingly better viewed as lower bounds rather than unbiased estimates of the probability and severity of ELB episodes.

As in the previous sections, two cases are considered: (i) social welfare is unaffected by the volatility of the short-term rate; (ii) $Var(i_t)$ enters the loss function. As the frequency and duration of ELB episodes depend on the level of

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24 Another drawback of the approach is that inflation and output-gap volatilities are the same both when the policy rate is above the ELB and when it is stuck at it, since we are assuming that unconventional measures are effective in removing the lower bound constraint.
the natural (nominal) rate of interest, the tables correspond to three different values of $i^*$.\footnote{Assuming a 2\% inflation target, the three cases correspond to a natural real rate of interest of 1\%, 0\% and -1\%.}

Let’s consider the benchmark case, where $i^* = 2\%$. If the criterion used to rank strategies is the minimisation of the frequency of ELB episodes, once again simulation results confirm that price-level targeting is the best-performing option: (i) the lower bound of the policy rate is hit once every four-five quarters; (ii) the average length of ELB episodes is three quarters; (iii) the maximum duration of ELB occurrences is 67 periods. Inflation targeting is inferior in all respects: the lower bound for $i_t$ is binding more frequently and the economy ends up being stuck at the ELB for a much longer period of time. The longest ELB episode covers nearly half of the simulation horizon, suggesting that under IT it may be extremely difficult for the monetary policymaker to offset a sequence of large recessionary shocks. Average inflation targeting does not perform better, even when the average is computed over a 4-year time frame. According to Table 9, nominal GDP targeting performs approximately as well as PLT.

The ranking of the strategies does not change substantially if the volatility of the policy rate is included in the loss function. Two findings are however worth mentioning: first, the effectiveness gap between PLT and GDPT on the one hand and IT or AIT on the other hand increases; second, nominal GDP targeting now outperforms PLT by all three criteria reported in Table 9.

The estimated probability of hitting the ELB resulting from Tables 8-10 is in the ballpark of other empirical studies: for the US, Kiley and Roberts (2017) find that, when the steady-state nominal interest rate is 3\% the short-term interest rate could be at zero (that is, the ZLB could be binding) as much as 30\% of the time, which compares with 23.5\% in our paper; for the euro area, Andrade et al. (2020) estimate a slightly lower frequency of hitting the ELB, which is however set at -0.5\%, not at zero. It should be noted however that the model used in this paper does not allow for a stabilising role of fiscal policy, unlike for instance the FRB/US model employed in Kiley and Roberts (2017), and gives the central bank a narrower policy space to offset deflationary shocks, as the interest rate can fall from the steady-state value only, respectively, 100bp, 200bp or 300bp.
5 Sensitivity analysis

In order to understand to what extent the ranking of monetary policy strategies is affected by the specific values of the parameters of the model, sensitivity analyses have been conducted.

5.1 Forward-lookingness in the Phillips curve

One crucial parameter is $\phi$, the degree of forward-lookingness of the Phillips curve, whose estimated value for the euro area model $\hat{\phi}$ is 0.304. What makes history-dependent strategies more effective is their ability to steer expectations, providing additional accommodation even at the ELB. It is a fair guess that, for very low values of $\phi$, price-level targeting should no longer outperform the other strategies and this is indeed what happens.

Fig. 3 shows the loss function of all strategies relative to that of inflation targeting for all values of $\phi$ included in the [0, 1] interval, while the grey bars measure the welfare loss under IT. For very low values of $\phi$ both PLT and GDPT are less effective than IT or AIT: in the first case, the difference is negligible, at most 6%; in the second, it is substantial, reaching almost 80%. The edge of IT (and AIT) with respect to PLT disappears when $\phi > 0.15$ and the same occurs with respect to GDPT when $\phi > \hat{\phi}$: from that point on, the higher $\phi$ the poorer the performance of IT (and AIT).

Finally, when the degree of backward-lookingness of the Phillips curve approaches zero, nominal GDP targeting becomes as effective as PLT, if not slightly better.

Similar results obtain when the central bank cares for the variance of the policy rate.

5.2 Forward-lookingness in the IS curve

Expectations may matter even when the Phillips curve is entirely backward looking, as they affect how the output gap responds to monetary policy actions. This is the case when interest-rate variability is an argument of the loss function, because otherwise the output gap is the central bank’s instrument.

Fig. 4 shows the loss function – which includes $Var(i_t)$ – of all strategies relative to that of inflation targeting for all values of $\psi$ included in the [0, 1] range. Once again for very low values of $\psi$ both PLT and GDPT turn out to be

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28This set of results, which is not shown, is available upon request.
less effective than IT or AIT. The efficiency loss for PLT is however negligible and disappears when $\psi > 0.17$; it is instead substantial and persists for most values of $\psi$ for GDPT. For values close to $\hat{\psi} = 0.744$, which is the estimate for the euro area, the evidence suggests that PLT and GDPT are the most effective strategies.

As shown by the grey bars, the loss function under IT is barely sensitive to the degree of forward lookingness of the IS curve.

5.3 The slope of the Phillips curve

The flatter the Phillips curve, the less responsive inflation to changes in the output gap. According to estimates reported in Section 3.1, a sensible value for the euro area is $\hat{\kappa} = 0.056$. A priori it is not clear which strategy is most affected by a lower (or higher) value of $\kappa$, but the evidence presented in Fig.5 seems to suggest that the relative performance of PLT deteriorates more when the slope of the Phillips curve becomes steeper and that of GDPT when it is flatter. The ranking seems to be a non-linear function of the slope of the Phillips curve. This result is consistent with the working of the transmission channel in the model: monetary policy, through changes in the policy rate and/or by steering expectations, affects the output gap, which in turn affects inflation. When $\kappa$ is close to zero, inflation hardly moves in response to central bank’s stimuli, all strategies perform poorly and the ranking depends to a large extent on how they manage to stabilise the output gap. History dependent policies like PLT and GDPT are less effective in this respect and are accordingly penalised relatively more: PLT is still the best strategy, but the effectiveness gap with respect to IT or AIT narrows. On the contrary, for high values of $\kappa$ the interest-rate channel works pretty well, which reduces the benefits from a tighter control of expectations, and all strategies are similarly effective. In this case as well the performance of PLT and GDP does not differ much from that of IT or AIT. Price-level targeting remains nonetheless the most powerful strategies, while GDPT turns out to be less effective than IT.

As shown by the grey histograms, which represent the absolute loss under inflation targeting, the effectiveness of monetary policy is positively related to the value of $\kappa$: when the slope of the Phillips curve is close to zero, the effectiveness of monetary policy in mitigating output and inflation fluctuations is extremely low, regardless of the strategy adopted.
5.4 The slope of the IS curve

The parameter $\sigma$ in the IS curve measures how strong the output gap reacts to changes in the (short-term) real rate of interest. Since inflation is driven by current and expected output gaps, *ceteris paribus* the higher $\sigma$ the stronger the impact of a monetary policy stimulus. It is therefore presumable that when the IS curve is flat (i.e. $\sigma$ is high), the interest rate channel is working at full strength and the gains that can be achieved from a tighter control of expectations are limited; on the contrary, when the IS curve is steep, changes in the policy rate are ineffective and the only way to steer economic activity and inflation is to manage expectations. Fig.6 shows that this is indeed the case: price level targeting and, to a lesser extent, nominal GDP targeting do much better than inflation targeting – average or not – for very low values of $\sigma$, while the performance gap reduces (or disappears in the case of GDPT) when $\sigma$ is high. For the value estimated for the euro area – i.e. $\hat{\sigma} = 0.529$ – the two history-dependent strategies outperform the others.

Fig.6 shows also that the absolute effectiveness of monetary policy is a function of the slope of the IS curve: as the grey histograms suggest, social welfare is a few orders of magnitude lower when $\sigma$ is close to zero, regardless of the strategy adopted.

5.5 Shocks materialising after the policy rate is set

The optimisation process described in Appendix A is based on the assumption that the central bank observes the current value of the supply and demand shocks before deciding on the monetary policy stance. This premise is what makes all strategies except nominal GDP targeting able to perfectly control inflation when the central bank’s loss function attaches a zero weight to output-gap variability (i.e. when the parameter $\lambda$ in the loss function is zero), which makes the loss function of all strategies the same. It might be informative to check what happens in a more realistic environment, where current supply and demand shocks are not in the central bank’s information set. The changes in the numerical procedure suggested by Dennis (2007) needed to compute the optimal targeting rules are described in Appendix B.

Fig.7 shows that PLT still outperforms the other strategies, but its superiority is no longer so clear-cut: for low values of $\lambda$ IT achieves the better combination of inflation and output-gap volatility. This is no longer the case when $\lambda > 0.10$, with PLT returning to be the most effective option.
5.6 Variances of the shocks

Is the ranking of the strategies affected by the relative size of demand and supply shocks? Since the loss function is quadratic and the equations describing the working of the economy are linear, the solution of the optimisation problem does not depend on the covariance matrix of the shocks; the second moments of the endogenous variables and the policy rate however do and this in principle might alter the relative performance of the strategies.

The case of interest is the one where the variance of the interest rate is an argument of the loss function.\textsuperscript{29} Tables 11 to 13 show for all strategies the value of the loss function and of its arguments corresponding to different pairs of standard deviations of the two shocks. In Table 11 the variance of the supply shock is set to zero; in Table 13 it is ten times larger than that of the demand shock; in Table 12 it is equal to the estimated value for the euro area. In all three case the variance of the demand shocks is the same and corresponds to the value reported in Table 1. The results confirm that the ranking does not depend on the shocks hitting the economy: PLT is once again more effective than the other strategies, in particular IT and AIT; GDPT turns out to be the second best option.

Table 11 shows that when the variance of the supply shock is zero, monetary policy comes close to fully stabilise both inflation and the output gap: the so-called "divine coincidence" nearly materialises. Compared with the benchmark case, the loss function is ten times smaller; it would be zero if the central bank were allowed to disregard the volatility of the policy rate.

6 Conclusions

Before the Global Financial Crisis the common view was that shocks large enough to push the policy rate to the ELB were rare events and that in normal times central banks had enough space for adjusting the policy rate to manage aggregate demand and keep inflation under control. The experience of the past decade changed this perception: the probability of entering a liquidity trap is now viewed as high and represents the main threat to monetary policymaking.

\textsuperscript{29}When the volatility of the policy rate does not affect social welfare, shocks to the IS curve are fully offset by the central bank. As shown in Table 2 and 3, neither inflation nor the output gap depend on the demand shock: their variance and the value of the loss function are proportional to the variance of the supply shock only, whose changes affect all strategies in the same way and leave the ranking unchanged.
The marked and persistent reduction of the natural rate of interest, which has trimmed down the degree of accommodation that central banks are able to provide, has made the situation worse. The limited effectiveness of non-standard monetary measures has made imperative to limit as much as possible the frequency and duration of ELB episodes.

An additional lesson taught by the crisis is that an inflation targeting framework has limits: it fails to keep average inflation close to the desired level and is not the most powerful way to steer expectations.

In this paper we tried to compare alternative monetary strategies to assess which one is better suited (1) to reduce output and inflation volatility in normal times and at the same time is capable of (2) minimising the frequency and costs of ZLB episodes. The model used in our analysis is a simple three-equation New Keynesian model, where both inflation and output exhibit endogenous inertia. We considered only targeting rules, i.e. optimal rules that minimise the loss function assigned by the Government to the monetary policymaker, who is assumed to set the policy rate under discretion. We ran a horse race among eight strategies: inflation targeting; price level targeting; average inflation targeting (five variants); nominal GDP (level) targeting.

Our analysis confirms the theoretical findings by Svensson (1999) and Vestin (2006) that price-level targeting can guarantee a better performance than inflation targeting in terms of both criteria described above. PLT is uniformly superior to both nominal GDP targeting and AIT as well, regardless of how average inflation is defined. As suggested by Vestin (2005), what makes PLT a better strategy is that it delivers a more favourable trade-off between inflation and output gap variability and comes closer to implement the commitment equilibrium. The mechanism behind these results is a better control of expectations: the private sector realises that the central bank’s incentive to offset shocks increases with a price level target. Accordingly, reduced expectations about future inflation are beneficial for the central bank when the economy is hit by a cost-push shock.

These findings are valid regardless of whether interest-rate variability is included in the loss function or not.

The ranking among the strategies considered in the paper is robust to changes in model parameters. PLT remains the best-performing framework even if the degree of forward lookingness in either the Phillips curve or in the IS schedule is lowered, which is surprising, given that what makes PLT superior is its ability to steer expectations. The achievements of PLT are affected neither
by a change in the steepness of the Phillips or IS curves.

Nominal GDP-level targeting is another well-performing strategy. It is not uniformly superior to IT or AIT, but succeeds in ensuring better outcomes over a large range of model parameters and social preferences. For high values of the variance of the supply shocks, it outmatches PLT. As stressed by Beckworth and Hendrickson (2016), nominal GDP targeting has the advantage that (i) it allows central bankers to focus only on one variable rather than two; (ii) it does not require to respond to real variable beyond the policymaker’s control; (iii) it makes unnecessary for the central bank to distinguish, in real time, between shocks to aggregate supply and shocks to aggregate demand.

No matter how robust, the findings of the paper are not general, as they are based on two critical assumptions: (1) the limited size of the model, which includes only two types of shocks; (2) the model-consistent expectations formation mechanism.

What would happen if these assumptions were removed is not obvious. Walsh (2019) provides evidence that the performance of PLT deteriorates significantly relative to IT and AIT in the presence of wage rigidities and shocks to productivity; Busetti et al. (2019) find instead that PLT is the most effective strategy in stabilising inflation and output and reducing the frequency and duration of ELB episodes even using a medium-scale DSGE model. Concerning the expectations formation mechanism, the intuition would suggest that in order to minimise transition costs under adaptive learning it is safer to avoid big changes in the way the central bank operates, which should favour keeping IT; Aoki and Nikolov (2005), building on the literature of feedback control, show instead that PLT performs better, as it possesses elements of integral control. These are however questions that need further research to be answered.

Appendices

A Finding the optimal interest-rate rule

The solution algorithm used to find the optimal discretionary monetary policy in a rational expectations model is the one suggested by Dennis (2007). The method is more general than existing alternatives and is simpler to apply, as it does not require the optimization constraints to be written in state-space
In spite of its merits, the use of the state-space representation may at times be inconvenient, as it forces the distinction between predetermined and nonpredetermined variables and requires to manipulate the model accordingly. Moreover, Dennis’ method allows the constraints to be written in structural form rather than in state-space form and has the additional advantage that it can be applied to models whose optimization constraints contain the expectation of next period’s policy instrument(s). Finally, it supplies the Euler equation for the optimal discretionary policy, which makes it particularly convenient when a “targeting rule” rather than an “instrument rule” is sought.

The policymaker’s optimization problem have two components: the loss function that is to be minimised and the set of equations constraining the optimization process. Let \( y_t \) be the \( n \)-element vector of endogenous variables and \( x_t \) be the \( p \)-element vector of policy instruments, where the variables in both \( y_t \) and \( x_t \) represent deviations from nonstochastic steady-state values. The policymaker aims at minimising the loss function

\[
L = E_0 \sum_{t=0}^{\infty} \beta^t \left( y_t' W y_t + x_t' Q x_t \right)
\]  

(7)

where \( \beta \) is the discount factor, \( W \) and \( Q \) are symmetric, positive semidefinite matrices capturing policy preferences and \( E_0 \) represents the expectations operator conditional on information available at time \( t = 0 \). \( L \) does not necessarily coincide with the social loss function.\(^\text{30}\)

The constraints facing the policymaker are given by the equations describing the working of the economy, namely

\[
A_0 y_t = A_1 y_{t-1} + A_2 E_t y_{t+1} + A_3 x_t + A_4 E_t x_{t+1} + A_5 v_t
\]  

(8)

where \( v_t \sim iid (0, \Omega) \) is the \( s \)-element vector of innovations (with \( s \leq n \)) and the matrices \( A_j \) contain the model’s structural parameters. In particular, \( A_0 \) must be nonsingular; \( A_5 \) can be a null matrix, as \( v_t \) can be included into the vector \( y_t \); \( A_4 \) is in general zero, but becomes useful for models containing an interest rate term structure.

As the state variables of the model are \( y_{t-1} \) and \( v_t \), if a solution exists it is

\(^{30}\)The omission in the loss function of cross-products of elements of the vectors \( y_t \) and \( x_t \) is only apparent, as it is always possible to include \( x_t \) within \( y_t \), with off-diagonal coefficients in \( W \) representing the penalty terms. This ad-hoc definition of the \( y_t \) vector is possible because the constraints are expressed in structural form.
of the form:
\[ y_t = H_1 y_{t-1} + H_2 v_t \]
\[ x_t = F_1 y_{t-1} + F_2 v_t \] (9)

To find the values of the \( H_j \) and \( F_j \) matrices minimising the loss function, we start by substituting equation (9) into (8), which gives
\[ D y_t = A_1 y_{t-1} + A_3 x_t + A_5 v_t \] (10)

where the matrix \( D \) is defined as\(^{31}\)
\[ D \equiv A_0 - A_2 H_1 - A_4 F_1 \] (11)

The same substitution in the loss function allows to get rid of future values of \( y_t \) and \( x_t \), obtaining
\[ L = y_t' P y_t + x_t' Q x_t + \frac{\beta}{1 - \beta} \text{tr} \left( \left( F_2' Q F_2 + H_2' P H_2 \right) \Omega \right) \] (12)

where
\[ P \equiv W + \beta F_1' Q F_1 + \beta H_1' P H_1 \] (13)

The last term on the right-hand side of (12) does not depend on the policy instrument \( x_t \) and hence does not affect the optimisation problem.

The constrained optimisation problem can be transformed into an unconstrained one by inserting (10) into (12). The objective function then becomes
\[ L = (A_1 y_{t-1} + A_3 x_t + A_5 v_t)' D^{-1} P D^{-1} (A_1 y_{t-1} + A_3 x_t + A_5 v_t) + x_t' Q x_t + K \] (14)

where \( K \) collects terms independent from the instrument vector \( x_t \). The first order condition of the problem is:
\[ \frac{\partial L}{\partial x_t} = A_3' D^{-1} P D^{-1} (A_1 y_{t-1} + A_3 x_t + A_5 v_t) + Q x_t = 0 \]
\[ = A_3' D^{-1} P y_t + Q x_t = 0 \] (15)

\( ^{31} \)In setting \( x_t \), the policymaker considers the matrix \( D \) rather \( A_0 \) to gauge the response of \( y_t \) to her/his actions, which implies that future policymakers are followers with respect to her/him: the policymaker optimising today is the Stackelberg leader and private-sector agents and future policymakers are Stackelberg followers. The solution represents a Markov-perfect Stackelberg-Nash equilibrium.
Solving (15) for the policy instrument gives
\[ x_t = - \left( Q + A_3 D_3' D^{-1} PD_{3^{-1}} A_3 \right)^{-1} A_3' D_3' D^{-1} (A_1 y_{t-1} + A_5 v_t) \]
\[ = F_1 y_{t-1} + F_2 v_t \tag{16} \]

implying the following law of motion for the endogenous variables:
\[ y_t = D_{3^{-1}} (A_1 + A_3 F_1) y_{t-1} + D_{3^{-1}} (A_5 + A_3 F_2) v_t \]
\[ = H_1 y_{t-1} + H_2 v_t \tag{17} \]

\(P\) and \(D\) are implicit functions of the matrices \(F_1, F_2, H_1\) and \(H_2\), which in turn depend on \(P\) and \(D\) according to the following set of relationships:
\[
F_1 \equiv - \left( Q + A_3 D_3' D^{-1} PD_{3^{-1}} A_3 \right)^{-1} A_3' D_3' D^{-1} A_1 \\
F_2 \equiv - \left( Q + A_3 D_3' D^{-1} PD_{3^{-1}} A_3 \right)^{-1} A_3' D_3' D^{-1} A_5 \\
H_1 \equiv D_{3^{-1}} (A_1 + A_3 F_1) \\
H_2 \equiv D_{3^{-1}} (A_5 + A_3 F_2) \tag{18} \]

Accordingly, the solution to the minimisation problem requires the computation of a fixed point and can be achieved by means of a numerical procedure. The only complication in the computation of the solution matrices is related to the calculation of \(P\), which is the unknown of a Sylvester equation. A Sylvester equation is of the form
\[ M = R + SMT \tag{19} \]
where \(M\) is the square matrix to be computed and \(R, S\) and \(T\) are specified in advance. \(S\) and \(T\) have stable eigenvalues, which allows to express \(M\) as an infinite sum, namely \( M = \sum_{j=0}^{\infty} S^j T^j \).\(^{32}\) For the subject in question, \( M = P, R = W + \beta F_1' Q F_1, S = \beta H_1' \) and \( T = H_1 \).

If \(M\) is low dimension, it can be computed by exploiting the identity: \( \text{vec}(SMT) = \left[ T' \otimes S \right] \text{vec}(M) \), which gives
\[ \text{vec}(M) = \left[ I - T' \otimes S \right]^{-1} \text{vec}(R) \]

\(^{32}\)If the eigenvalues of \(S\) and \(T\) are smaller than 1 in absolute value, it is possible to repeatedly substitute \(M\) with \(R + SMT\) on the right-hand side of (A13), eventually obtaining \( M = \sum_{j=0}^{\infty} S^j T^j \).
In general, the numerical solution of a Sylvester equation can be found by applying a doubling algorithm, which is an iterative procedure based on the following recursions:

\[
\begin{align*}
M^{(j)} &= M^{(j-1)} + S^{(j-1)}M^{(j-1)}T^{(j-1)} \\
S^{(j)} &= S^{(j-1)}S^{(j-1)} \\
T^{(j)} &= T^{(j-1)}T^{(j-1)}
\end{align*}
\]

The numerical procedure to find the solutions for \(F_1, F_2, H_1\) and \(H_2\) involves the following four steps:

**Step 1.** Initialise \(F_1, F_2, H_1\) and \(H_2\).

**Step 2.** Compute \(D\), according to equation (11) and solve the Sylvester equation (13) to find \(P\).

**Step 3.** Update \(F_1, F_2, H_1\) and \(H_2\) according to the system of equations (18).

**Step 4.** Iterate over step 2 and 3 until convergence.

Convergence is usually achieved after a small number of iterations. Using the computed values of \(F_1, F_2, H_1\) and \(H_2\) it is then possible to obtain the variance of \(y_t\) and \(x_t\).

### B Optimal policies when time-\(t\) shocks are not in the information set

When time-\(t\) shocks are not in the information set, the optimal interest rate policy can respond only to lagged variables. In this case, the constraints describing the working of the economy that the policymaker has to face are summarised in the following matrix equation:

\[
A_0y_t = A_1y_{t-1} + A_2E_{t-1}y_{t+1} + A_3x_t + A_4E_{t-1}x_{t+1} + A_5v_t \tag{20}
\]

where the symbols are the same as in the previous section. If they exist, the MSV solutions for the endogenous variables and for the policy instrument are of the following form:

\[
\begin{align*}
y_t &= H_1y_{t-1} + H_2v_t \\
x_t &= F_1y_{t-1}
\end{align*} \tag{21}
\]

Substituting equation (21) into (20) gives
\begin{align*}
A_0 y_t &= A_1 y_{t-1} + A_2 H_1 E_{t-1} y_t + A_3 x_t + A_4 F_1 E_{t-1} x_t + A_5 v_t \\
&= A_1 y_{t-1} + A_2 H_1 (y_t - H_2 v_t) + A_3 x_t + A_4 F_1 (y_t - H_2 v_t) + A_5 v_t
\end{align*}

which can be simplified to

\begin{align*}
(A_0 - A_2 H_1 - A_4 F_1) y_t &= A_1 y_{t-1} + A_3 x_t + (A_5 - A_2 H_1 H_2 - A_4 F_1 H_2) v_t \\
G y_t &= A_1 y_{t-1} + A_3 x_t + L v_t
\end{align*}

The numerical procedure to find the solutions for \( F_1, H_1 \) and \( H_2 \) involves the same steps as in the previous section, provided that the matrices \( G \) and \( L \) defined above are used instead of \( D \) and \( A_5 \) in Step 2 of the numerical procedure described in Appendix A.

\section*{C References}


Table 2: Inflation under alternative strategies

<table>
<thead>
<tr>
<th></th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \zeta )</th>
<th>( \xi )</th>
<th>( \text{Var}(\pi_t) )</th>
<th>( \rho )</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT</td>
<td>0.845</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.789</td>
<td>-</td>
<td>0.610</td>
</tr>
<tr>
<td>PLT</td>
<td>0.603</td>
<td>-</td>
<td>-0.118</td>
<td>-</td>
<td>0.359</td>
<td>-</td>
<td>0.003</td>
</tr>
<tr>
<td>AIT2</td>
<td>0.845</td>
<td>-</td>
<td>-</td>
<td>0.845</td>
<td>0.794</td>
<td>-</td>
<td>0.615</td>
</tr>
<tr>
<td>AIT4</td>
<td>0.850</td>
<td>-</td>
<td>-</td>
<td>0.844</td>
<td>0.804</td>
<td>-</td>
<td>0.626</td>
</tr>
<tr>
<td>AIT8</td>
<td>0.863</td>
<td>-</td>
<td>-</td>
<td>0.842</td>
<td>0.825</td>
<td>-</td>
<td>0.653</td>
</tr>
<tr>
<td>AIT12</td>
<td>0.874</td>
<td>-</td>
<td>-</td>
<td>0.841</td>
<td>0.844</td>
<td>-</td>
<td>0.685</td>
</tr>
<tr>
<td>AIT16</td>
<td>0.884</td>
<td>-</td>
<td>-</td>
<td>0.840</td>
<td>0.861</td>
<td>-</td>
<td>0.722</td>
</tr>
<tr>
<td>GDPT</td>
<td>0.834</td>
<td>-</td>
<td>-0.105</td>
<td>-</td>
<td>1.847</td>
<td>-</td>
<td>0.191</td>
</tr>
</tbody>
</table>

The table shows the coefficients of the MSV solution for inflation, the output gap and interest rate corresponding to the alternative monetary policy strategy. The specification encompassing all solutions is shown in the upper part of the table. IT stands for inflation targeting, PLT for price-level targeting, GDPT for nominal GDP targeting and AIT for average inflation targeting, with the number indicating the order of the average. \( \rho \) is the 1st-order autocorrelation coefficient of inflation and \( \text{Var}(\cdot) \) is the variance of, respectively, inflation, the output gap and the interest rate.
Table 5: Inflation under alternative strategies (Var(\(i_t\)) in the LF)

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\gamma)</th>
<th>(\Sigma \alpha_j)</th>
<th>(\zeta)</th>
<th>(\xi)</th>
<th>Var((\pi_t))</th>
<th>(\rho)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT</td>
<td>0.921</td>
<td>0.021</td>
<td>-</td>
<td>-</td>
<td>0.700</td>
<td>0.085</td>
<td>0.447</td>
</tr>
<tr>
<td>PLT</td>
<td>0.701</td>
<td>0.015</td>
<td>-0.081</td>
<td>-</td>
<td>0.472</td>
<td>0.110</td>
<td>0.006</td>
</tr>
<tr>
<td>AIT2</td>
<td>0.921</td>
<td>0.021</td>
<td>-</td>
<td>0.921</td>
<td>0.702</td>
<td>0.085</td>
<td>0.450</td>
</tr>
<tr>
<td>AIT4</td>
<td>0.922</td>
<td>0.021</td>
<td>-</td>
<td>0.920</td>
<td>0.708</td>
<td>0.086</td>
<td>0.458</td>
</tr>
<tr>
<td>AIT8</td>
<td>0.929</td>
<td>0.021</td>
<td>-</td>
<td>0.919</td>
<td>0.724</td>
<td>0.088</td>
<td>0.480</td>
</tr>
<tr>
<td>AIT12</td>
<td>0.939</td>
<td>0.021</td>
<td>-</td>
<td>0.918</td>
<td>0.737</td>
<td>0.088</td>
<td>0.502</td>
</tr>
<tr>
<td>AIT16</td>
<td>0.947</td>
<td>0.021</td>
<td>-</td>
<td>0.917</td>
<td>0.745</td>
<td>0.087</td>
<td>0.520</td>
</tr>
<tr>
<td>GDPT</td>
<td>0.842</td>
<td>0.012</td>
<td>-0.093</td>
<td>-</td>
<td>1.840</td>
<td>0.093</td>
<td>0.206</td>
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Table 6: Output gaps under alternative strategies (Var(\(i_t\)) in the LF)

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\gamma)</th>
<th>(\Sigma \alpha_j)</th>
<th>(\zeta)</th>
<th>(\xi)</th>
<th>Var((x_t))</th>
<th>(\rho)</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT</td>
<td>-0.531</td>
<td>0.239</td>
<td>-</td>
<td>-</td>
<td>-11.252</td>
<td>0.599</td>
<td>8.168</td>
</tr>
<tr>
<td>PLT</td>
<td>-2.095</td>
<td>0.207</td>
<td>-0.677</td>
<td>-</td>
<td>-12.439</td>
<td>0.958</td>
<td>8.960</td>
</tr>
<tr>
<td>AIT2</td>
<td>-0.531</td>
<td>0.240</td>
<td>-</td>
<td>-0.531</td>
<td>-11.237</td>
<td>0.601</td>
<td>8.166</td>
</tr>
<tr>
<td>AIT4</td>
<td>-0.515</td>
<td>0.241</td>
<td>-</td>
<td>-0.517</td>
<td>-11.197</td>
<td>0.607</td>
<td>8.159</td>
</tr>
<tr>
<td>AIT8</td>
<td>-0.448</td>
<td>0.242</td>
<td>-</td>
<td>-0.458</td>
<td>-11.102</td>
<td>0.617</td>
<td>8.141</td>
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<tr>
<td>AIT12</td>
<td>-0.375</td>
<td>0.244</td>
<td>-</td>
<td>-0.396</td>
<td>-11.030</td>
<td>0.619</td>
<td>8.122</td>
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<tr>
<td>AIT16</td>
<td>-0.322</td>
<td>0.245</td>
<td>-</td>
<td>-0.352</td>
<td>-10.996</td>
<td>0.598</td>
<td>8.106</td>
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<tr>
<td>GDPT</td>
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<td>0.153</td>
<td>-0.733</td>
<td>-</td>
<td>-1.976</td>
<td>0.771</td>
<td>9.904</td>
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</table>

Table 7: Interest rates under alternative strategies (Var(\(i_t\)) in the LF)

<table>
<thead>
<tr>
<th>(\alpha)</th>
<th>(\beta)</th>
<th>(\gamma)</th>
<th>(\Sigma \alpha_j)</th>
<th>(\zeta)</th>
<th>(\xi)</th>
<th>Var((i_t))</th>
<th>(\rho)</th>
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<tbody>
<tr>
<td>IT</td>
<td>0.974</td>
<td>0.121</td>
<td>-</td>
<td>-</td>
<td>2.784</td>
<td>1.840</td>
<td>1.176</td>
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<tr>
<td>PLT</td>
<td>1.020</td>
<td>0.106</td>
<td>0.304</td>
<td>-</td>
<td>1.731</td>
<td>1.358</td>
<td>0.366</td>
</tr>
<tr>
<td>AIT2</td>
<td>0.974</td>
<td>0.121</td>
<td>-</td>
<td>0.974</td>
<td>2.777</td>
<td>1.840</td>
<td>1.179</td>
</tr>
<tr>
<td>AIT4</td>
<td>0.950</td>
<td>0.120</td>
<td>-</td>
<td>0.949</td>
<td>2.777</td>
<td>1.843</td>
<td>1.190</td>
</tr>
<tr>
<td>AIT8</td>
<td>0.909</td>
<td>0.120</td>
<td>-</td>
<td>0.899</td>
<td>2.819</td>
<td>1.854</td>
<td>1.219</td>
</tr>
<tr>
<td>AIT12</td>
<td>0.898</td>
<td>0.121</td>
<td>-</td>
<td>0.877</td>
<td>2.882</td>
<td>1.866</td>
<td>1.247</td>
</tr>
<tr>
<td>AIT16</td>
<td>0.905</td>
<td>0.122</td>
<td>-</td>
<td>0.875</td>
<td>2.935</td>
<td>1.875</td>
<td>1.268</td>
</tr>
<tr>
<td>GDPT</td>
<td>0.133</td>
<td>0.213</td>
<td>0.222</td>
<td>-</td>
<td>-0.125</td>
<td>1.566</td>
<td>0.361</td>
</tr>
</tbody>
</table>

The tables show the coefficients of the MSV solution for inflation, the output gap and interest rate corresponding to the alternative monetary policy strategy. The variance of the policy (i.e. short-term) rate enters the loss function. The specification encompassing all solutions is shown in the upper part of the table. IT stands for inflation targeting, PLT for price-level targeting, GDPT for nominal GDP targeting and AIT for average inflation targeting, with the number indicating the order of the average. \(\rho\) is the 1st-order autocorrelation coefficient of inflation and Var(\(\cdot\)) is the variance of, respectively, inflation, the output gap and the interest rate.
### Table 8: Frequency and features of ELB episodes when $i^* = 3\%$

<table>
<thead>
<tr>
<th>Var($i_t$) not in the Loss Function</th>
<th>Var($i_t$) in the Loss Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob($i_t &lt; ELB$)</td>
<td>mean</td>
</tr>
<tr>
<td><strong>IT</strong></td>
<td>23.54</td>
</tr>
<tr>
<td><strong>PLT</strong></td>
<td>13.18</td>
</tr>
<tr>
<td><strong>AIT</strong></td>
<td>18.91</td>
</tr>
<tr>
<td><strong>AIT4</strong></td>
<td>23.68</td>
</tr>
<tr>
<td><strong>AIT8</strong></td>
<td>23.93</td>
</tr>
<tr>
<td><strong>AIT12</strong></td>
<td>24.21</td>
</tr>
<tr>
<td><strong>AIT16</strong></td>
<td>24.53</td>
</tr>
<tr>
<td><strong>GDPT</strong></td>
<td>11.27</td>
</tr>
</tbody>
</table>

### Table 9: Frequency and features of ELB episodes when $i^* = 2\%$

<table>
<thead>
<tr>
<th>Var($i_t$) not in the Loss Function</th>
<th>Var($i_t$) in the Loss Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob($i_t &lt; ELB$)</td>
<td>mean</td>
</tr>
<tr>
<td><strong>IT</strong></td>
<td>31.27</td>
</tr>
<tr>
<td><strong>PLT</strong></td>
<td>22.66</td>
</tr>
<tr>
<td><strong>AIT</strong></td>
<td>27.51</td>
</tr>
<tr>
<td><strong>AIT4</strong></td>
<td>31.37</td>
</tr>
<tr>
<td><strong>AIT8</strong></td>
<td>31.56</td>
</tr>
<tr>
<td><strong>AIT12</strong></td>
<td>31.79</td>
</tr>
<tr>
<td><strong>AIT16</strong></td>
<td>32.05</td>
</tr>
<tr>
<td><strong>GDPT</strong></td>
<td>20.79</td>
</tr>
</tbody>
</table>

### Table 10: Frequency and features of ELB episodes when $i^* = 1\%$

<table>
<thead>
<tr>
<th>Var($i_t$) not in the Loss Function</th>
<th>Var($i_t$) in the Loss Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob($i_t &lt; ELB$)</td>
<td>mean</td>
</tr>
<tr>
<td><strong>IT</strong></td>
<td>40.03</td>
</tr>
<tr>
<td><strong>PLT</strong></td>
<td>35.10</td>
</tr>
<tr>
<td><strong>AIT</strong></td>
<td>38.02</td>
</tr>
<tr>
<td><strong>AIT4</strong></td>
<td>40.09</td>
</tr>
<tr>
<td><strong>AIT8</strong></td>
<td>40.20</td>
</tr>
<tr>
<td><strong>AIT12</strong></td>
<td>40.31</td>
</tr>
<tr>
<td><strong>AIT16</strong></td>
<td>40.47</td>
</tr>
<tr>
<td><strong>GDPT</strong></td>
<td>33.88</td>
</tr>
</tbody>
</table>

The tables show the frequency and features of ELB episodes for different values of the natural (nominal) rate of interest $i^*$. IT stands for inflation targeting, PLT for price-level targeting, GDPT for nominal GDP targeting and AIT for average inflation targeting, with the number indicating the order of the average.
The tables show for all strategies the values of the loss functions - which includes $\text{Var}(i)$ - and the variances of inflation, the output gap and the interest rate for different combinations of the standard deviation (Std) of the supply and demand shocks. IT stands for inflation targeting, PLT for price-level targeting, GDPT for nominal GDP targeting and AIT for average inflation targeting, with the number indicating the order of the average.

### Table 11: Loss function under different combinations of shock variances

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$\text{Var}(\pi)$</th>
<th>$\text{Var}(x)$</th>
<th>$\text{Var}(i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT</td>
<td>0.423</td>
<td>0.071</td>
<td>1.194</td>
</tr>
<tr>
<td>PLT</td>
<td>0.188</td>
<td>0.004</td>
<td>0.367</td>
</tr>
<tr>
<td>AIT</td>
<td>0.424</td>
<td>0.071</td>
<td>1.197</td>
</tr>
<tr>
<td>AIT4</td>
<td>0.427</td>
<td>0.072</td>
<td>1.208</td>
</tr>
<tr>
<td>AIT8</td>
<td>0.434</td>
<td>0.075</td>
<td>1.237</td>
</tr>
<tr>
<td>AIT12</td>
<td>0.439</td>
<td>0.076</td>
<td>1.266</td>
</tr>
<tr>
<td>AIT16</td>
<td>0.439</td>
<td>0.076</td>
<td>1.287</td>
</tr>
<tr>
<td>GDPT</td>
<td>0.194</td>
<td>0.005</td>
<td>0.361</td>
</tr>
</tbody>
</table>

### Table 12: Loss function under different combinations of shock variances

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$\text{Var}(\pi)$</th>
<th>$\text{Var}(x)$</th>
<th>$\text{Var}(i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT</td>
<td>3.100</td>
<td>0.466</td>
<td>0.688</td>
</tr>
<tr>
<td>PLT</td>
<td>2.429</td>
<td>0.006</td>
<td>0.350</td>
</tr>
<tr>
<td>AIT</td>
<td>3.104</td>
<td>0.468</td>
<td>0.689</td>
</tr>
<tr>
<td>AIT4</td>
<td>3.116</td>
<td>0.477</td>
<td>0.693</td>
</tr>
<tr>
<td>AIT8</td>
<td>3.148</td>
<td>0.499</td>
<td>0.701</td>
</tr>
<tr>
<td>AIT12</td>
<td>3.180</td>
<td>0.522</td>
<td>0.707</td>
</tr>
<tr>
<td>AIT16</td>
<td>3.205</td>
<td>0.540</td>
<td>0.708</td>
</tr>
<tr>
<td>GDPT</td>
<td>2.862</td>
<td>0.206</td>
<td>0.356</td>
</tr>
</tbody>
</table>

### Table 13: Loss function under different combinations of shock variances

<table>
<thead>
<tr>
<th>Strategy</th>
<th>$\text{Var}(\pi)$</th>
<th>$\text{Var}(x)$</th>
<th>$\text{Var}(i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>IT</td>
<td>2245.7</td>
<td>331.4</td>
<td>424.7</td>
</tr>
<tr>
<td>PLT</td>
<td>1879.6</td>
<td>2.0</td>
<td>14.7</td>
</tr>
<tr>
<td>AIT</td>
<td>2247.9</td>
<td>333.2</td>
<td>426.5</td>
</tr>
<tr>
<td>AIT4</td>
<td>2255.2</td>
<td>338.9</td>
<td>432.6</td>
</tr>
<tr>
<td>AIT8</td>
<td>2276.4</td>
<td>355.6</td>
<td>450.2</td>
</tr>
<tr>
<td>AIT12</td>
<td>2299.6</td>
<td>373.5</td>
<td>469.4</td>
</tr>
<tr>
<td>AIT16</td>
<td>2320.5</td>
<td>389.4</td>
<td>486.3</td>
</tr>
<tr>
<td>GDPT</td>
<td>2237.6</td>
<td>168.1</td>
<td>3.9</td>
</tr>
</tbody>
</table>
Fig. 1: Loss Functions (relative to IT) with varying output-gap volatility weights

Each curve plots the loss function relative to that of inflation targeting of price level targeting (PLT), average inflation targeting (AIT2, AIT4, AIT8, AIT12 and AIT16) and Nominal GDP targeting. IT (level) shows on the RHS scale the values of the loss functions for IT.
Each curve plots the loss function, relative to that of inflation targeting, of price level targeting (PLT), average inflation targeting (AIT2, AIT4, AIT8, AIT12 and AIT16) and nominal GDP targeting. In each case the loss function includes the variance of the policy interest rate. IT (level) shows on the RHS scale the values of the loss function for IT.

Fig.2: Loss Functions (relative to IT) with varying output-gap volatility weights

Fig.3: Loss Functions (relative to IT) corresponding to different values of the parameter $\phi$

Fig.4: Loss Functions (relative to IT) corresponding to different values of the parameter $\psi$

Fig.5: Loss Functions (relative to IT) corresponding to different values of the parameter $\kappa$

Fig.6: Loss Functions (relative to IT) corresponding to different values of the parameter $\sigma$

Fig.7: Loss Functions (relative to IT) when current shocks are observed after $i_t$ has been set
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ERCOLANI V. and J. VALLE E AZEVEDO, How can the government spending multiplier be small at the zero lower bound?, Macroeconomic Dynamics, v. 23, 8. pp. 3457-2482, WP 1174 (April 2018).


RIGGI M., Capital destruction, jobless recoveries, and the discipline device role of unemployment, Macroeconomic Dynamics, v. 23, 2, pp. 590-624, WP 871 (July 2012).

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