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DEMOGRAPHICS AND THE NATURAL REAL INTEREST RATE: HISTORICAL AND PROJECTED PATHS FOR THE EURO AREA

by Andrea Papetti[*]

Abstract

This paper employs a large-scale overlapping generation (OLG) model quantifying that demographics account for a decrease in the natural real interest rate of about 1.4 percentage points in the euro area compared with the average for the 1980s to 2030 (roughly at its trough), under the baseline calibration. Two channels prevail in providing the downward impact: the increasing scarcity of effective labor input and the growing willingness of individuals to save due to longer life expectancy. Mitigating factors are: greater substitutability between labor and capital, higher intertemporal elasticity of substitution in consumption, higher productivity and participation by older individuals and, to a lesser extent, a higher retirement age. Absent pay-as-you-go pension systems, the natural rate would stand at a lower level of about 0.5 percentage points by 2030. The simulated paths of the natural real interest rate and output growth are consistent with econometric estimates.

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We have this unusual degree of knowledge concerning the future because of the long but definite time-lag in the effects of vital statistics. Nevertheless the idea of the future being different from the present is so repugnant to our conventional modes of thought and behaviour that we, most of us, offer a great resistance to acting on it in practice. (Keynes (1937), Some Economic Consequences of a Declining Population)

A temporary period of policy rates being close to zero or even negative in real terms is not unprecedented by any means. Over the past decades, however, we have seen long-term yields trending down in real terms as well, independent of the cyclical stance of monetary policy. (Draghi (2016), Addressing the Causes of Low Interest Rates)

1 Introduction[1]

Advanced economies are undergoing a demographic transition, the ageing process by which "populations move from initially high fertility and mortality with young age distributions to low fertilty and mortality with old age distributions" (Lee, 2016). As the number of people entering the world is shrinking and mortality rates are decreasing (i.e. the survival probabilities are increasing), the relative number of the elderly is dramatically increasing. While before the 1980s the ratio of the elderly (aged 65 and over) to working age (aged 15-64) has been less than 2 to 10, the United Nations (UN, 2017) project this proportion to rise above 5 to 10 by year 2050 in Europe.[2] Based on definite time-lags, demographic projections offer a relatively reliable knowledge of the future. It is therefore appealing to use demographic data as exogenous variation to explain macroeconomic dynamics.

Questioning the influence of demographic change on the real interest rate is certainly not new in economic research. What is new in recent years is that the topic is on the agenda of central bankers. The fact that real interest rates have been on a downward trend since the late 1980s across many countries leads to ask whether the natural interest rate has decreased as well and whether it will remain low in the years ahead, potentially hampering the effectiveness of monetary policy.

The definition of natural or neutral interest rate dates back to Wicksell (1898)[3] and in modern macroeconomics can be thought as the rate of interest that brings output in line with its potential or natural level in the absence of transitory shocks or nominal adjustment frictions (cf. Woodford (2003), Brand et al. (2018)).[4] It will be identified as the real rate of return on capital (net of

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[2] See Figure A.1 in the Appendix. The aging process is very similar across different European geographic areas, no matter if one considers the euro area composed by 19 countries, the big 5, the core 12, or the wide European Union composed by 28 countries.

[3] *"There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them. This is necessarily the same as the rate of interest which would be determined by supply and demand if no use were made of money and all lending were effected in the form of real capital goods. It comes to much the same thing to describe it as the current value of the natural rate of interest on capital". (Wicksell, 1898)*

[4] "*That is, the interest rate must at all times equal the Wicksellian natural rate of interest, which may be defined as the equilibrium real rate of return in the case of fully flexible prices. Under this definition, one observes a direct correspondence with the previously introduced concept of the natural rate of output. Indeed, the natural rate of interest is just the real rate of interest required to keep aggregate demand equal at all times to the natural rate of output.*" (Woodford, 2003).

depreciation) that allows the saving supply (by households) to meet the capital demand (by firms) in absence of any allocational friction or arbitrage. The *real* macro model employed in the paper will abstract from frictions and shocks that could capture business cycle variations. The natural rate of return on capital will be shortly called real interest rate.

Researchers have been recently looking for potential "slow-moving secular forces" as explanatory factors behind the downward trend in real interest rates (cf. Eggertsson et al. (2019)). Demographic change is one of such forces and how it can affect the natural real interest rate, an *unobservable* variable, is the exclusive focus of this paper.

In a standard Solow (1956)'s model with homogeneous population and a constant saving rate, as population growth decreases capital per worker rises dampening the marginal product of capital, so that in equilibrium the more abundant factor, capital, receives a lower remuneration than the other factor, labor: the real interest rate falls while the real wage rises. Models embedded in the neoclassical framework, like general equilibrium overlapping generations (OLG) models, can rarely escape the prediction that ageing leads to a lower real interest rate.

Since the seminal contribution by Auerbach and Kotlikoff (1987), OLG models are considered the most reliable tool to evaluate the macroeconomic effect of demographic change as they allow to use the full empirical age distribution, in a context of a flexible life-cycle behavior. Recent contributions employing fully-fledged OLG models all predict a downward trend of the real interest rate due to population aging, no matter if the model encompasses the whole world economy with different countries/areas (cf. e.g. Domeij and Flodén (2006), Krueger and Ludwig (2007), Attanasio et al. (2007)) or a single country/area modeled as closed-economy (cf. e.g. Gagnon et al. (2016) for US, Bielecki et al. (2018, 2020) for Europe, Sudo and Takizuka (2019) for Japan).

This paper employs a large-scale OLG model, similar to those existing in the literature, to provide a quantification of the impact of the demographic transition on the aggregates for the euro area, with particular focus on the channels through which the real interest rate is affected. There are four channels (cf. Krueger and Ludwig (2007), and Carvalho et al. (2016)).

(1) *Downward impact from increasing scarcity of labor supply*. A decrease of the growth rate of the effective labor supply relative to the total population in the economy is akin to a slowdown in total factor productivity for output per capita growth, which leads firms to demand less capital, reducing the marginal product of capital and so the real interest rate, everything else being equal. This effect is stronger the more the age-distribution shifts towards older cohorts that are parametrized to be less productive and participative in the labor market, and the lower the degree of substitutability in production between capital and labor.

(2) *Downward impact from higher life expectancy*. With the goal of smoothing consumption into the future, households anticipate their higher survival probabilities with a willingness of consuming less and saving more, i.e. becoming more patient, thus decreasing the real interest rate, all else being equal. This effect depends on the way households are insured against mortality risk as well as on the pension scheme in place, and is stronger the lower the intertemporal elasticity of

substitution in consumption.

(3) *Upward impact from a rising proportion of dissavers*. According to the life-cycle theory embedded in the model, households build up their wealth during the most productive working ages and dissave in later ages. The shift of the population distribution towards relatively more dissavers along the transition dynamics can decrease the aggregate saving rate thus increasing the real interest rate by making capital scarcer, all else being equal.

(4) *Upward impact from pension system crowding-out effect*. A pay-as-you-go (PAYGO) pension system with defined-benefit, as often run in European countries in spite of its different hybrid reformulations, requires an increase of labor taxes for the government budget to be balanced as the relative pool of retirees increases. This tends to crowd out productive capital in the economy and, as capital becomes less abundant, to increase the real interest rate, all else being equal.

The model is calibrated at the annual frequency for the euro area using demographic data and projections by UN (2017) as exogenous variations to study a perfect-foresight transition where the demographic change is perfectly anticipated by the agents in the economy. The quantitative exercise shows that the real interest rate path guided by demographic change exhibits a slight rise throughout the 1970s and 1980s, and then a prolonged and marked fall at least until 2030 (when it roughly reaches its trough according to the demographic projections). The shape is consistent with low-frequency econometric estimates (see Holston et al. (2017), Fiorentini et al. (2018), and Brand et al. (2018)).

In the baseline analysis the annual real interest rate declines about 1.4 percentage points going to 2030 compared to the average in the 1980s. Under the various sensitivity exercises the real interest rate is never found to reach a higher level compared to its initial steady state value in 1950. These exercises show that the dampening effect of aging could be mitigated not only by higher substitutability between labor and capital and higher elasticity of intertemporal substitution in consumption, but also by reforms aiming particularly at increasing the relative productivity of older cohorts and the participation rate. An increase of the retirement age with no other supporting reform leads to a limited increase in output per capita growth and to essentially no change in the real interest rate.

A decomposition of the main drivers in the model shows that the presence of the PAYGO pension system with defined-benefits is associated with an upward deviation of the real interest rate from the initial steady state of about 0.5 percentage point by 2030. By the same year the increase of the survival probabilities is associated with a negative deviation of more than 0.9 percentage points. Variations in the number of newborns in the model would account for the residual deviation of the real interest rate, about 0.5 percentage point, going to 2030 compared to the average in the 1980s.

Through the lens of this model, it does not seem to be really the case that demographics can meaningfully "reverse three multi-decade global trends" on real interest rates via the pension system channel as argued by Goodhart and Pradhan (2017). At least not until 2030. But then the system is predicted to stay at a permanent lower level of about 1 percentage point compared to the one prevailed in the 1980s. Coherently with what recently found by Rachel and Summers (2019a,b), this paper supports the view that in absence of offsetting policies such as PAYGO pension benefits the natural real interest rate would have been at lower level and that, going forward, bigger policy shifts are needed if one wants to overturn the downward impact of aging.

Interestingly, the model predicts that before 2007 one could easily interpret the impact of demographic change on output growth as a type of scaling factor, producing if anything mild variations around a constant, close to zero for the case of output per capita. However, after 2007 – which, coincidentally, is when the "Global Financial Crisis" started – the model predicts a dramatic decrease of output growth due to demographic change.[5] For the real interest rate too, the most dramatic phase is after 2007 with a variation of almost -1 percentage point between 2007 and 2030. Hence, what noticed by Gagnon et al. (2018) for the United States, employing a similar OLG model, seems also true for Europe: "the largest effects of demographics on interest rates and GDP growth coincide to some degree with the Global Financial Crisis". So that "downward pressures on interest rates and GDP growth due to demographics could be easily misinterpreted as persistent but ultimately temporary influences of the global financial crisis" (Gagnon et al. (2016), p. 3.). Instead, what OLG models seem to predict is a "new normal".

The quantitative results are not significantly affected when the model is augmented with exogenous time-varying total factor productivity (TFP) growth (inferred from a measure of the Solow residual). The associated re-calibration allows to be more consistent with the *level* of the econometric estimates of natural outcomes (real interest rate and output) as provided e.g. by Holston et al. (2017) while the *variation* that can attributed to aging remains of the same order of magnitude. Specifically, in the baseline with constant TFP growth in each period the real interest rate decreases by about 1.1 percentage points between 1990 and 2030, going from a value of about 2.24% in 1990 to 1.13% in 2030. Over the same period, the decrease is about 1.65 percentage points (2.22% to 0.57%) with the Hodrick-Prescott (HP) filtered TFP growth rate. Thus, the results from the OLG model allow to qualify the determinants of agnostic econometric estimates and provide a forecast relying on demographic projections. Compared to the average in the 1980s, the real interest rate decreases about 1.8 percentage points going to 2030. A decomposition shows that the progressive decline of TFP growth rates accounts for about 0.55 of those percentage points so that demographics account about two-thirds of the model's generated decline of the real interest rate.

This paper builds on a vast literature on large-scale OLG models, rapidly growing since the onset of the debate on "secular stagnation" (see Eggertsson et al. (2019)) where the closest match with the framework employed here can be probably found in Domeij and Flodén (2006), and Krueger and Ludwig (2007). The former focuses on capital flows in a multi-country setting, the latter is a seminal paper on the impact of aging on rates of return to capital providing quantitative estimates

[5] While in 2007 demographics was contributing to increase total output by 0.4% per annum, by 2015 the figure is -0.2%. Going to 2026 the model predicts a further drag on total output with an annual growth rate of about -0.6%. A similar persistent decrease is predicted for output per capita annual growth rate: it goes from -0.2% in 2007 to -0.4% in 2015, to -0.7% by 2026.

only from 2000. The purpose here is not to further refine their theoretical structure. Rather, to offer a quantification (for the euro area) of the total effect of aging as well as of the different channels in isolation in a model where the full age-structure of the population is allowed. Carvalho et al. (2016) study the channels above using the analytically-tractable set-up *à la* Gertler (1999) (applied to a representative OECD economy). They recognize that "tractability comes at the cost of not endowing the model with any flexibility to match the empirical age distribution", and "leave for future research an extensive comparison of the results obtained in this framework with those from a large-scale OLG model, disciplined by a richer set of moments from demographics data" (Carvalho et al. (2016), p. 212). Indeed, this paper tires to accomplish that need of future research.

Quantitative estimates for the euro area are provided by Kara and von Thadden (2016) using a Gertler (1999)'s type of model and by Bielecki et al. (2018, 2020) using a richer OLG model. Their results on the impact of demographic change on the real interest rate are of the same order of magnitude of those provided in this paper, even though there the focus is more on the interaction of demographic change with monetary policy in a New-Keynesian framework and with global factors.

The rest of the paper is organized as follows. Section 2 describes the OLG model environment. Section 3 studies the simplified two-period version of the model to understand and isolate analytically the channels. Section 4 provides the full set of quantitative analyses including calibration, description of the experiment to study the transition dynamics, and a sensitivity analysis of the results to different parameter values. Section 5 introduces non-zero total-factor-productivity (TFP) growth in the model and makes a comparison with econometric estimates. Section 6 concludes.

2 OLG model

Consider a model in the spirit of Domeij and Flodén (2006), Krueger and Ludwig (2007) for a closed-economy populated by overlapping generations (OLG) of households that solve a standard life-cycle consumption problem. The demographic development is exogenous.

Households. Each household consists of a single individual. Households within each cohort j are identical and their exogenous mass $N_{t,j}$ for time-period t evolves recursively according to:

$$N_{t,j} = N_{t-1,j-1} s_{t,j} \tag{2.1}$$

where $s_{t,j}$ is the conditional survival probability.[6]

A representative j–aged household entering the world at time t maximizes the utility function choosing consumption and the amount of assets to hold the following period for each life period,

[6]Given that an individual is aged $j-1$ at time $t-1$, $s_{t,j}$ is the probability to be alive at age j at time t. Following Domeij and Flodén (2006), data are taken for $N_{t,j}$ for all available t, j to get the implied survival probabilities $s_{t,j}$ which therefore can exceed 1 due to migration flows. The underlying assumption is that immigrants enter the economy without assets and are adopted by domestic households: assets are carried over between periods by a domestic cohort and then split among its survivors and the asset-less immigrants in the same age class.

$c_{t+j,j}, a_{t+j+1,j+1}$. Assuming a CRRA utility function, the maximization problem is:[7]

$$\max_{c_{t+j,j},a_{t+j+1,j+1}} \sum_{j=0}^{J} \beta^j \pi_{t+j,j} \frac{(c_{t+j,j})^{1-\sigma}}{1-\sigma} \tag{2.2}$$

subject to

$$\begin{aligned}
a_{t+j+1,j+1} &= \frac{a_{t+j,j}(1+r_{t+j})}{s_{t+j,j}} - c_{t+j,j} + y_{t+j,j} && (2.3)\\
y_{t+j,j} &= (1-\tau_{t+j})w_{t+j}h_{t+j,j}I(j<j_r) + d_{t+j,j}I(j \geq j_r) && (2.4)\\
a_{t+J+1,J+1} &= 0 && (2.5)\\
a_{t,0} &= 0 && (2.6)
\end{aligned}$$

where $\pi_{t+j,j} = \prod_{k=0}^{j} s_{t+k,k}$ is the unconditional survival probability with $s_{t,0} = 1$; β is the discount factor; σ is the coefficient of risk-aversion (here the inverse of the intertemporal elasticity of substitution in consumption); r_{t+j} is the real interest rate; w_{t+j} is the real wage; τ_{t+j} is the tax rate on labor income; $I(\cdot)$ is an indicator function; j_r denotes the retirement age, exogenously imposed; $h_{t+j,j} = h_j$ for all t is an exogenously given amount of hours to work depending on age, constant over time (cf. Figure A.3); $d_{t+j,j}$ is the pension transfer from the government. Each household is born with zero wealth and is not allowed to die with either positive or negative wealth. It is assumed that there exists a "perfect annuity market".[8]

Firms. Households have direct ownership of physical capital and rents it to the firms. Output (Y_t) is produced under perfect competition with constant elasticity of substitution (CES) technol-

[7]Notice that it is assumed that labor supply is exogenous. Of course, this a limitation of the analysis. However, similar OLG settings generally find small effects of introducing endogenous labour supply. For example, in a robustness analysis (section 6) of their seminal contribution, Attanasio et al. (2007) have: "The main finding is [that] endogenizing labor supply has little effect on the equilibrium". Therefore, it was opted to keep the analysis easier, not allowing for endogenous labor supply.

[8]The assumption of "perfect annuity market" means that the agents within each age group j agree to share the assets of the dying members of their age group among the surviving members, equally. Using the notation just introduced, consider those that at time t are aged j. The total amount of assets of the dying members is: $a_{t,j}(1-s_{t,j})N_{t-1,j-1}$, while the number of surviving members is: $N_{t,j} = N_{t-1,j-1}s_{t,j}$. Hence, in the budget constraint the asset holding in period $t+1$ will depend on what as been accumulated plus this sort of 'equal gift' from the dying members given the real interest rate (r_t) at which these assets can be invested (minus consumption plus income):

$$\begin{aligned}
a_{t+1,j+1} &= a_{t,j}(1+r_t) + \frac{a_{t,j}(1+r_t)(1-s_{t,j})N_{t-1,j-1}}{N_{t-1,j-1}s_{t,j}} - c_{t,j} + y_{t,j}\\
&= \frac{a_{t,j}(1+r_t)}{s_{t,j}} - c_{t,j} + y_{t,j}
\end{aligned}$$

which is the budget constraint written in the main text. Appendix H provides a different model structure and results where the assumption of "perfect annuity market" is relaxed. Instead, it is assumed that accidental bequests resulting from premature death are taxed by the government at a confiscatory rate and used for otherwise neutral government consumption.

ogy:[9]

$$Y_t = \left[\psi(K_t)^{\frac{\rho-1}{\rho}} + (1-\psi)(A_t L_t)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{2.7}$$

where $0 < \psi < 1$ is the capital share parameter, $0 < \rho < 1$ is the elasticity of substitution between capital and labor, A_t is the labor-augmenting technology assumed to grow exogenously: $A_t = g_t^A A_{t-1}$. The factor markets are also perfectly competitive. Therefore, one can consider a representative firm hiring (efficiency units of) labor L_t at a given hourly real wage w_t and renting capital K_t at the rental rate $r + \delta$, subject to the yearly depreciation rate δ. The representative firm maximizes per-period profits:

$$\max_{L_t, K_t} \{Y_t - w_t L_t - (r_t + \delta) K_t\} \tag{2.8}$$

Government. Given a certain level of generosity of the pay-as-you-go (PAYGO) pension system, i.e. the replacement rate $\bar{d}$ defined as the pension benefit d_t received by each household per unit of the average labor income $w_t(1-\tau_t)\bar{h}$, the government sets a tax rate τ_t such that its budget is balanced in each period:[10]

$$d_t = \bar{d} w_t (1-\tau_t)\bar{h} \tag{2.9}$$

$$\tau_t w_t L_t = d_t \sum_{j=j_r}^{J} N_{t,j} \tag{2.10}$$

Clearing. The factor markets and the goods market clear:

$$L_t = \sum_{j=0}^{J} h_j N_{t,j} \tag{2.11}$$

$$K_{t+1} = \sum_{j=0}^{J} a_{t+1,j+1} N_{t,j} \tag{2.12}$$

$$C_t + I_t = Y_t \tag{2.13}$$

where $C_t = \sum_{j=0}^{J} c_{t,j}$ is aggregate consumption, while investment I_t satisfies the low of motion of capital: $K_{t+1} = (1-\delta)K_t + I_t$.

Equilibrium. Given the exogenous demographic development (fully characterized by the incoming cohort size $N_{t,0}$ and the conditional survival probabilities $s_{t,j}$ according to (2.1)) and the labor-augmenting technology (A_t) in all periods $t = 0, 1, ..., \infty$ for all cohorts $j = 0, 1, ..., J$, the equilibrium for this (closed, perfectly competitive) economy is a sequence of prices $\{w_t, r_t\}_{t=0}^{\infty}$, policies $\{\tau_t\}_{t=0}^{\infty}$, transfers $\{d_t\}_{t=0}^{\infty}$ and quantities $\left\{\{c_{t,j}, a_{t,j}\}_{j=0}^{J}, K_t, L_t, Y_t, C_t, I_t\right\}_{t=0}^{\infty}$ such that:

[9] For $\rho \to 1$ the function is of the Cobb-Douglas form.
[10] Have: $\bar{h} = \sum_{j=0}^{j_r-1} h_j / j_r$.

1. Households solve the optimization problem (2.2) subject to constraints (2.3)–(2.6);
2. firms maximize profits solving problem (2.8) given the production function (2.7);
3. the fiscal authority sets a tax rate (2.10) such that its budget is balanced in each period given the individual pension transfer (2.9);
4. factor markets (2.11), (2.12) and the goods market (2.13) clear satisfying the low of motion of capital: $K_{t+1} = (1-\delta)K_t + I_t$, with aggregate consumption $C_t = \sum_{j=0}^{J} N_{t,j} c_{t,j}$.

3 Understanding the channels: two-period version of the model

Consider the special case of the model above in which the representative households lives only for two periods, at $j = 0$ as a worker and at $j = 1$ as a retiree. Under this simple setting it is possible to characterize the equilibria in the model analytically, revealing the main channels through which demographic change affects the real interest rate, thus providing some intuition for the quantitative results from the full model simulation provided in the next sections.[11]

Environment. Assume perfect foresight. Given the number of people in the working age in period t, $N_{t,0}$, and the probability to survive the next period (becoming a retiree), $s_{t+1,1}$, the number of old the subsequent period is given by: $N_{t+1,1} = s_{t+1,1} N_{t,0}$. A household born in period t has preferences over consumption $c_{t,0}, c_{t+1,1}$ representable by the utility function (consider the special case of log-preferences for simplicity, i.e. $\sigma = 1$):

$$\log(c_{t,0}) + \beta s_{t+1,1} \log(c_{t+1,1})$$

Households supply labor inelastically in the first period receiving a wage w_t. They retire in the second period receiving social security benefits d_{t+1} financed by payroll taxes on labor income. The holding of capital $(a_{t+1,1})$ gives a return r_{t+1} while the unintentional bequest is equally split among the survivors. Hence, the budget constraints are:

$$\begin{aligned} c_{t,0} + a_{t+1,1} &= (1-\tau_t) w_t \\ c_{t+1,1} &= \frac{a_{t+1,1}(1+r_{t+1})}{s_{t+1,1}} + d_{t+1} \end{aligned}$$

The aggregate labor supply is equal to the number of young people in the economy: $L_t = N_{t,0}$. Assuming a Cobb-Douglas production function ($\rho = 1$), $Y_t = K_t^{\psi} (A_t N_{t,0})^{1-\psi}$ with $A_t = (g^A)^t A_0$ and depreciation of capital $0 < \delta < 1$, profit maximization of firms implies:

$$\begin{aligned} r_t + \delta &= \psi k_t^{\psi-1} \\ w_t &= (1-\psi) A_t k_t \end{aligned}$$

[11] This section follows closely section 2 of Krueger and Ludwig (2007).

where $k_t = K_t/(A_t N_{t,0})$ denotes the stock of capital per efficiency unit of labor.

The budget balance of the government running the PAYGO pension system implies the following social security transfers:

$$d_{t+1} = \tau_{t+1} w_{t+1} \frac{g^N_{t+1}}{s_{t+1,1}}$$

where $g^N_{t+1} = N_{t+1,0}/N_{t,0}$ denotes the growth rate of the young generation. Assuming a fixed replacement rate $\bar{d}$ the pension transfers must also satisfy:

$$d_t = \bar{d} w_t (1 - \tau_t)$$

Finally, clearing of the capital market and the law-of-motion of capital require: $K_{t+1} = N_{t,0} a_{t+1,1}$, $K_{t+1} = (1-\delta)K_t + I_t$. Rewritten in terms of efficiency units of labor:

$$\begin{aligned} g^N_{t+1} A_{t+1} k_{t+1} &= a_{t+1,1} \\ g^A g^N_{t+1} k_{t+1} &= (1-\delta) k_t + i_t \end{aligned}$$

BGP characterization. As shown in Appendix D, the equilibrium on a *balanced-growth path* (BGP) with constant effective capital stock ($k_t = k$ for all t) can be fully characterized by the following capital demand (by firms) and supply (by households):

$$\iota^D \quad : \quad r = \frac{g^A g^N - (1-\delta)}{\iota} - \delta \tag{3.1}$$

$$\iota^S \quad : \quad r = \frac{\tau}{(1-\tau)\frac{\beta s_1}{g^A g^N} - \iota \frac{1+\beta s_1}{(1-\psi)[g^A g^N - (1-\delta)]}} - 1 \tag{3.2}$$

where ι denotes the investment-output ratio which equals the aggregate saving rate (given the assumption of closed economy). Assuming a PAYGO pension system with fixed replacement rate $\bar{d}$ the tax rate is given by:

$$\tau = \frac{\bar{d}}{\bar{d} + \frac{g^N}{s_1}} \tag{3.3}$$

Aging shock channels. Aging is characterized by decreasing fertility and increasing survival probabilities. In this simple model the former is represented by decreasing g^N_{t+1}, the latter by increasing $s_{t+1,1}$, both constant on a BGP. The BGP characterization above allows to determine the impact of aging in terms of movements of the capital demand (ι^D) and supply (ι^S). Figure 1 depicts such movements assuming that the economy moves from an initial stationary equilibrium (BGP$_0$) to a final one (BGP$_f$) facing a reduction of $g^A g^N$ from 1.025 per annum to 1 and an increase of s_1 from 0.146 to 0.561 (according to the empirical old-dependency ratio, see Figure A.1).[12] It allows to detect four channels through which aging impacts the equilibrium real interest rate.[13]

[12]Notice that in the two-period model the BGP survival probability $s_1 = N_1/N_0$ is the old-dependency ratio, i.e. the number of elderly (aged 65+) over the number of workers (aged 15–64).

[13]Cf. Carvalho et al. (2016) in the context of a model *à la* Gertler (1999).

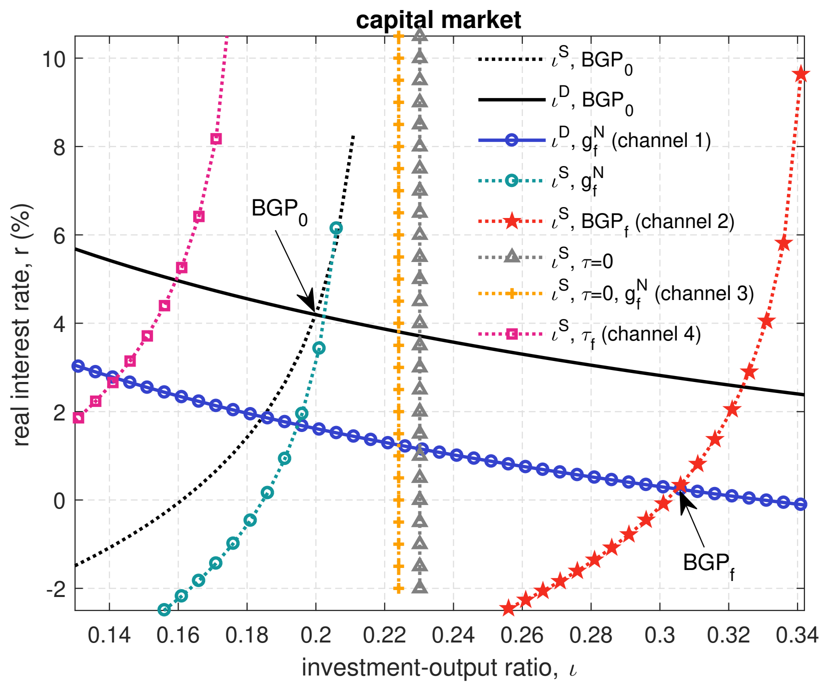

Figure 1: Two-period model: capital market equilibrium

Note. Movements of capital demand (ι^D) and supply (ι^S) to aging (i.e. decrease of g^N, increase of s_1). See equations (3.1), (3.2) and Appendix D for derivation. BGP$_0$ and BGP$_f$ denote the initial and final balanced-growth path (BGP), respectively. It is assumed that one period corresponds to $p = 30$ years. The term g_f^N indicates that the unique demographic shock is in the labor growth, taking its assumed value in the final BGP ($g^A g^N = 1$ compared to a value in the initial BGP of 1.025 per annum, i.e. $g^A g^N = 1.025^p$). From the initial to the final BGP s_1 goes from 0.146 to 0.561 (according to the old-dependency ratio, see Figure A.1). With an annual depreciation rate of $\delta_a = 0.0952$ (as in the subsequent section), the depreciation is $\delta = 1 - (1 - \delta_a)^p \approx 0.95$. The capital bias in production has the standard value of $\psi = 0.33$. In the initial BGP the target is an investment-output ratio $\iota = 0.2$ which implies an annual real interest rate of about $0.0419 = r = \{\psi[g^A g^N - (1-\delta)]/\iota + (1-\delta)\}^{1/p} - 1$. The replacement rate is set to $\bar{d} = 0.45$, implying a tax rate τ of about 6% in the initial BGP, 20% in the final BGP. These values imply an individual discount factor $\beta = \{\iota/[g^A g^N - (1-\delta)] + \tau(1-\psi)/(1+r)\}/\{(1-\tau)(1-\psi)s_1/(g^A g^N) - \iota s_1/[g^A g^N - (1-\delta)]\}$. The annualized effective discount factor: $(s_1\beta)^{1/p} = 0.9798$ in the initial BGP. The term τ_f in the legend indicates that the unique demographic shock is in the tax rate, taking its final BGP value.

1. *Labor supply*. A decrease of g^N induces firms to demand less capital (downward shift of ι^D) as it is optimal to reduce the capital stock in support of relatively scarcer labor input. At fixed capital supply, this decreases the real interest rate by making capital relatively more abundant than the labor employed in production, thus depressing the marginal product of capital all else being equal.

2. *Life expectancy*. An increase of the survival probability s_1, all else being equal, makes households more patient (notice that it always multiplies the discount factor β) making them will-

ing to save more in the expectation of financing consumption for a longer life, for given retirement age, thus contributing to depress the real interest rate (shift to the right of ι^S).

3. *Savers/dissavers composition.* While a higher survival probability s_1 pushes up the saving rate, a decrease of the fertility growth g^N *per se* (in absence of a pension system) tends to decrease it, all else being equal. The reason is that at a lower g^N there are relatively less young people who are the only savers in the economy as compared to retirees. A lower saving rate makes capital scarcer, thus increasing the real interest rate (shift to the left of ι^S).[14]

4. *Pension system.* In presence of a PAYGO pension system with fixed replacement rate, aging increases the tax rate (see equation (3.3)). With less income-generating workers (lower g^N) in support of more old people (higher s_1) the fiscal authority needs to set a higher tax rate to have the budget balanced in each period and provide the agreed social security transfers. As the tax rate increases there is a crowding-out effect of private capital (shift to the left of ι^S). As capital becomes less abundant the equilibrium real interest rate increases when the tax rate increases, all else being equal. If the tax rate was always zero the savings by households would be higher.

Without any further assumption it is not possible to find a unique closed-form solution for the equilibrium real interest rate by equating the capital demand to the capital supply in (3.1) and (3.2). However, assuming full depreciation of capital ($\delta = 1$) is sufficient to have the following closed form solution:[15]

$$\begin{aligned} \iota &= \frac{s_1\beta\psi(1-\psi)(1-\tau)}{(1+s_1\beta)\psi+\tau(1-\psi)} \\ r &= \frac{\psi}{\iota}g^A g^N - 1 \end{aligned}$$

In sum, this section reveals that in a stationary equilibrium such as a BGP with constant effective capital stock, aging tends to reduce the real interest rate via two main channels. On the one hand, aging makes the available labor in production relatively scarcer (decrease in g^N, equivalent to a decrease of labor-augmenting productivity growth g^A) inducing firms to demand less capital (channel 1); on the other hand, aging makes individuals more willing to supply capital (increase in s_1) thus increasing the investment-output ratio (ι) – which in a closed economy equals the gross

[14] Notice that in absence of a pension system, $\tau = 0$, the savings supply becomes inelastic. From equation (3.2), for $1 + r > 0$, it results:

$$\iota = \frac{\beta s_1}{1+\beta s_1}(1-\psi)\left(1-\frac{1-\delta}{g^A g^N}\right)$$

As g^N decreases, ι decreases (i.e. ι^S shifts to the left) via channel 3. However, this impact tends to be small as δ is not far from 1 over a period (corresponding to 30 years) in the model. In case of $\delta = 1$, channel 3 is nil in this setting.

[15] Notice that since a period here corresponds to 30 years, it is reasonable to assume full depreciation.

saving rate (channel 2). Both channels independently decrease the real interest rate while the latter can be dampened by the crowding out effect on savings associated with the increase of the tax rate implied by a PAYGO pension system with defined benefits (channel 4). Therefore, a stationary equilibrium with a higher survival probability is unambiguously characterized by a higher investment-output ratio. The potential downward impact on the saving rate (hence upward impact on the real interest rate) stemming from the increasing number of dissavers does not materialize in a stationary equilibrium given the current setting. This of course does not exclude that along the transition dynamics from an initial to a final stationary equilibrium the saving rate can decrease while the number of dissavers increases (channel 3).[16]

4 Quantitative analysis

The goal of the quantitative analysis is to study the transition dynamics of the macroeconomic system of section 2 from an initial to a final steady state, where the unique perfectly-anticipated exogenous driving process is the time-varying demographic structure. The focus is on the euro area composed by 12 countries (EA12 henceforth) modeled as a closed economy.[17]

4.1 Experiment and baseline calibration

One period of the model corresponds to one year. Following Domeij and Flodén (2006), equation (2.1) is directly used to retrieve the conditional survival probabilities using data on the empirical number of people, $N_{t,j}$, by single age-group j for each year t in the time-range 1950-2100. Data are taken from the United Nations (UN, 2017) *World Population Prospects: The 2017 Revision* including the medium variant projections until year 2100.[18] Hence, the demographic evolution in the model is fully captured by the incoming cohort size ($N_{t,0}$) and the implied unconditional survival probabilities $\pi_{t+j,j}$ for all t and j via the recursive formulation: $N_{t+j,j} = \pi_{t+j,j} N_{t,0}$.[19]

It is assumed that in the initial steady state the system has the demographics prevailing in year 1951 in the data, i.e. for all $j = 0, 1, ..., J-1$ the demographic structure is given by: $N_{1951,j+1} = \pi_{1951,j+1} N_{1951,0}$. The experiment is such that while in 1950 (as well as in all previous periods) the

[16] Only one curve shift in Figure 1 has not been discussed above. That pertains to the shift of ι^S with $\tau > 0$ when g^N decreases ("ι^S, g_f^N" line). The saving supply shifts slightly to the right. The reason is that with a lower g^N for given τ the pension transfers decrease. Hence, with the goal of smoothing consumption, individuals want to save more. The quantitative importance of this channel will be discussed later when comparing a PAYGO pension system with fixed vs endogenous replacement rate.

[17] EA12 is composed by the following countries: Austria (AT), Belgium (BE), Finland (FI), France (FR), Germany (DE), Greece (EL), Ireland (IE), Italy (IT), Luxembourg (LU), Netherlands (NL), Portugal (PT), Spain (ES).

[18] Before year 1990, the number of people aged more than 80 are grouped together in the set 80+ for all countries. As a strategy to identify the number of people in each single age group after age 80 for years 1950-1989, the implied survival probabilities of 1990 for those aged more than 80 have been applied backwards. The shape of the number of people of different ages in each year has been slightly smoothed via the Matlab "smooth" function with quadratic fit and a span of 0.08.

[19] Figure A.2 plots both $N_{t,0}$ and $\pi_{t+j,j}$ for all years t from 1951 to 2100 and all ages in the model.

system is assumed to be in the initial steady state with the 1951 demographics, in 1951 there is the information shock: agents learn about the new demographic development for all subsequent years, and what this implies for macroeconomic variables in a perfect-foresight environment. In year 2100 the conditional survival probabilities and the incoming cohort size start remaining fixed forever. This implies an evolution of the demographic structure that eventually gets stationary again so that around year 2200 the system reaches a new final steady state.[20]

Given the demographics, the structural model parameters and the implied solution values for the initial and final steady state, the dynamic equilibrium is solved using a standard deterministic simulation set-up where the numerical problem of solving a nonlinear system of simultaneous equations is managed by means of a Newton-type method.[21]

The baseline calibration employs standard parameter values summarized in Table 1. The role of most parameter values will be explored in subsequent sections via sensitivity analyses, including an exercise where the model is fed with a series measuring total-factor-productivity growth in the data to capture technology growth. Here, to isolate the effect of demographics, the labor-augmenting technological parameter A_t is simply set to 1 (without loss of generality) in all periods t. Therefore, the model in the baseline features no technology growth.[22]

In the baseline calibration the representative firm employs a Cobb-Douglas production function which corresponds to the case when the elasticity of substitution between the factors of production tends to 1. Consistently with a widely used value in the literature the bias towards capital in the production function ψ is set to about 1/3.

The constant relative risk aversion (CRRA) parameter σ, which is the inverse of the intertemporal elasticity of substitution, is set to the standard value of 1, a widely used assumption in the literature (see e.g. Krueger and Ludwig (2007)), implying that the representative households have logarithmic preferences over consumption.

The fixed net replacement rate $\bar{d}$ of the PAYGO pension system is set 0.45, in line with what assumed by Kara and von Thadden (2016) in a model for the euro area.[23]

[20] The dynamic simulations of the model are run until year 2351.

[21] This method is employed under the "perfect foresight solver" available in *Dynare*. Specifically, the Jacobian of the system has dimension 177×400 given that there are 177 endogenous variables over 400 periods (years). On the households side there are 85 endogenous variables for consumption and as many minus 1 for assets holdings (the initial asset is exogenously given, equal to zero) for a total of 169. The remaining endogenous variables are: real interest rate, wage rate, tax rate, pension transfers, and aggregate capital, assets, income, consumption. In the first 150 periods (from 1951 to 2100) the system has a different demographic structure in each period as in the data by UN (2017). In the remaining periods, a time-span more than sufficient for the system to reach the final steady state, the conditional survival probabilities and incoming cohort sizes take the values of 2100.

[22] This is standard practice with the argument of isolating the effect of demographic factors. See e.g. De la Croix et al. (2013), Gagnon et al. (2016).

[23] This number is roughly the average across the EA12 countries of the replacement rates reported by Barany et al. (2018) who use the same type of PAYGO pension system adopted in this paper to compare most of the world countries. Their measure is based on an upgrade of the official replacement rates to take into account concerns about measurement errors or potential biases in the percentages of retirees receiving benefits and working-age population contributing to pensions.

Table 1: Baseline calibration: parameter values

Parameter	Value	Note
Structural:		
A_t	$1\ \forall t$	labor-augmenting technology, no technology growth
ρ	1	capital-labor substitutability, Cobb-Douglas production function
	0.33	bias towards capital in the production function
δ	0.0952	depreciation rate of capital
σ	1	constant relative risk aversion (CRRA), log preferences
$\bar{d}$	0.45	net replacement rate of PAYGO pension system
β	0.9598	individual discount factor, endogenous in the initial steady state: $K/Y = 2.1$
Demographic:		
J	84	terminal life-age (99)
j_r	50	retirement age (65)
h_j	Figure A.3	individual life-cycle effective labor supply. Source: Domeij and Flodén (2006)
$N_{t,0}$	Figure A.2	incoming cohort size (age 15). Source: UN (2017)
$\pi_{t+j,j}$	Figure A.2	unconditional survival probabilities, retrieved from UN (2017) applying (2.1)

For what concerns demographics, individual labor supply in efficiency units, h_j, is interpolated using the data-points provided by Domeij and Flodén (2006), obtained by interacting the profiles of both productivity and the participation rates.[24] It is assumed that households enter the world as workers at age 15, all retiring at age 65 which correspond to $j_r = 50$ (this is why h_j drops abruptly to zero after age 64). A standard assumption, see Kara and von Thadden (2016), Bielecki et al. (2018) for the euro area, in line with what reported for most OECD economies (see Carvalho et al. (2016), Table 2). Households live at most until age 99 corresponding to $J = 84$ so that in each year there are 85 overlapping generations.

The initial steady state is calibrated to a capital-output ratio of 2.1 and an investment-output ratio of 0.2. These values are such that the capital-output ratio and the investment-output ratio generated by the simulation are in line with what can be found in the data using series for the gross fixed capital formation,[25] and in line with values normally found in the literature.[26] Furthermore,

[24]See Figure A.3. A similar profile is employed for European countries by Cooley et al. (2019), see their Figure 7. Following a common assumption in OLG modeling (that can be found in most of the literature mentioned in this paper), labor force participation rates are assumed to be constant over time. Hence, the model is not tailored to directly capture structural changes such as the increase in female labor force participation. Attanasio et al. (2007) explicitly take into account the time-varying nature of female participation rates and find that the "participation transition is virtually exhausted in 2005 in the North" (where the North includes: North America, Europe, Japan, Australia, and New Zealand). Hence, the impact of participation on factor prices should be limited after 2005. This is reassuring given that, as shown later, most of the downward impact of demographic change on the real interest rate is found to occur after the mid-2000s. Furthermore, extreme variations in h_j (such as those studied in section 4.4.3) are not found to have such a comparably sizable effect on the real interest rate.

[25] To have empirical estimates for the capital-output ratio, the series used here are: "Gross fixed capital formation (constant LCU)" and "GDP (constant LCU)", data source: World Development Indicators (WDI) by the World Bank (update: January 2018). The capital stock is estimated by applying the perpetual inventory method (cf. e.g. Caselli (2005)). The initial capital stock (the base year is 1970, the first year data are available for all EA12 countries) is computed using the formula: $K_{1970} = I_{1970}/(g_I + \delta_K)$ where I_{1970} corresponds to the gross capital formation in 1970, g_I is the average growth rate, while δ_K is set to 7.5%. The capital stock is obtained via the law-of-motion: $K_{t+1} = (1-\delta)K_t + I_t$ for all years t from 1970 to 2016. The series for EA12 are obtained by weighting each country with its real GDP share in year 2000.

[26]For example, De la Croix et al. (2013) target a capital-output ratio of 2.4 in 2010 for France; Attanasio et al. (2007)

they give an annual depreciation rate of capital $\delta = 0.0952$ which is consistent with what set e.g. by Gomes et al. (2012) in a model for the euro area.[27] Hence in the initial steady state the real interest rate is about 6.19% per annum.[28] While the reader might find this value relatively high, especially if compared with recent work on the topic, it is in line with seminal contributions in the literature.[29]

Given these targets and structural parameter values the annual individual discount factor β is solved endogenously (by means of a solver for non-linear systems of equations) in the initial steady state. The resulting value is 0.9598.

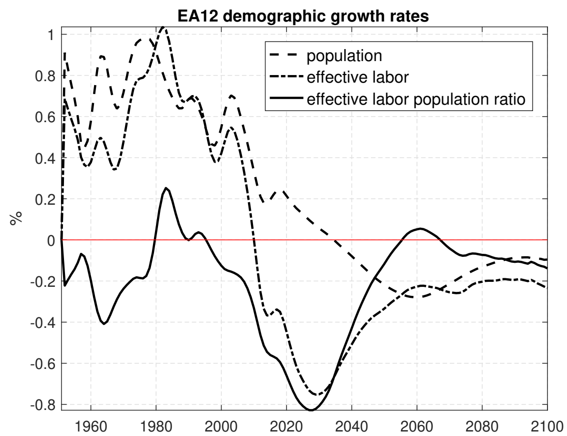

Figure 2: Demographic growth rates in EA12

Note. The series plotted are the annual growth rates of population $N_t = \sum_j N_{t,j}$, effective labor supply $L_t = \sum_j h_j N_{t,j}$ (see Figure A.3 for the age-dependent units of efficiency h_j), and their ratio, where the ages j considered involve people aged between 15 and 100, consistently with the model. The number of people $N_{t,j}$ is taken from data provided by the United Nations (UN, 2017) *World Population Prospects: The 2017 Revision* for year $t \in [1950, 2100]$, medium variant after year 2016 (see footnote 18).

Figure 2 plots the annual growth rates of population $N_t = \sum_j N_{t,j}$, aggregate effective labor supply $L_t = \sum_j h_j N_{t,j}$, and their ratio. The growth rate of the effective labor to population ratio

have a value of 2.5 in 2005 for a composite of advanced economies representing the "North"; Domeij and Flodén (2006) have a slightly higher number of 2.74 for the world in 1990. Compare these numbers with Figure 4 below.

[27]Notice that targeting a capital-output ratio $K/Y = 2.1$ with an investment-output ratio $I/Y = 0.2$ gives, by the law-of-motion of capital in steady state a depreciation rate $\delta = (I/Y)/(K/Y) = 0.2/2.1 \approx 0.0952$. Gomes et al. (2012) set the depreciation rate at the quarterly frequency to 0.025. At the annual frequency this implies a depreciation rate $1 - (1 - .025)^4 \approx 0.0963$.

[28]From the representative firm first order condition with respect to capital the real interest rate is $\psi/(K/Y) - \delta = 0.33/2.1 - 0.0952 \approx 0.0619$.

[29]See e.g. Domeij and Flodén (2006) (world), Attanasio et al. (2007) ("North" = "more developed regions" according to UN), Krueger and Ludwig (2007) (world), De la Croix et al. (2013) (France). They all target a real interest rate in the range of 6–7% per annum in a period spanning from 1950 to 2010. Among the recent contributions with a lower real interest rate target see e.g. Eggertsson et al. (2019) (US), Bielecki et al. (2018) (EU), Gagnon et al. (2016) (US). Section 5 will explore a different calibration in the attempt to match data and econometric estimates for the euro area from different sources. Controversy on the point arises because the model, as well as the models in the literature mentioned above, features a unique asset; and because the natural real interest rate is an unobserved variable.

is a key determinant of the aggregate dynamics of the system along the transition path. Between 1985 and 2030 it decreases by about 1 percentage point going from a growth rate of 0.2% to -0.8% per year. While the growth rate of both population and effective labor decrease over time until 2030, the latter decreases more turning negative after 2010 which is approximately the year when the baby boom generations start to retire (at age 65). The retirement of those generations determine a widening of the gap between the population and labor force growth rates until 2030. After that there is a re-bouncing of the growth rate of the effective labor to population ratio as the increasing scarcity of effective labor becomes less strong after the baby boom generations have ceased to exist. But the growth rate of population and effective labor are both negative in the projected horizon. In the period between 1990 to 2030 the scarcity of effective labor is exacerbated by the fact that the demographic distribution is tilted towards relatively older people who are calibrated to provide relatively less effective labor.

Once the medium variant projections from UN (2017) are used until 2100, the model predicts that the real interest rate decreases in the long-run (while the real wage increases) going from 6.19% in the initial steady state to 4.56% per annum in the final steady state. It results that individual labor (pension) income is slightly higher (lower) in the long-run. Furthermore, as factor prices adjust in the long-run saving becomes less attractive (and borrowing during the early stages of life more attractive) so that the individual age-profile of asset holdings in the final steady state lies always below its equivalent in the initial steady state. However, as the population distribution shifts in the long-run in favor of relatively older people, the distribution of both aggregate savings and aggregate consumption in the long run has relatively more mass on older ages.[30]

Figure 3 compares the life-cycle profiles generated by the simulated model for a selection of generations (continuous black lines) along the transition dynamics with the data for the European countries available in the *National Transfer Accounts* (see note under Figure 3). The model finds visual correspondence with the data. Few features are worth emphasizing.

First, the calibration of the individual labor supply in efficiency units, h_j, with the assumption of exogenous retirement at age 65, allows to match the hump-shaped labor income profile found in the data, roughly corresponding to the median across the labor income profiles for the countries observed.

Second, the hump-shaped profile of labor income coupled with the consumption-smoothing motive entails that households start to build up positive savings only after the first 20-25 years of working life (when they are 35-40 years old). This is a standard life-cycle profile captured by OLG models (cf. e.g. Cooley and Henriksen (2018)) and finds some correspondence in the data.[31]

[30]These patterns are confirmed in Figure A.4 showing not only the life-cycle profiles for the initial and the final steady state but also counter-factual (partial equilibrium) profiles for the final steady state where the real interest rate is fixed at the initial steady state value (see dotted lines). It confirms that had the system not incurred any general equilibrium adjustment in factor prices, households – facing higher survival probabilities in the long-run – would have saved more in all life-cycle periods in the long-run.

[31]Compared to data for consumption and labor income, data for total savings shown in Figure 3 are available for less countries, seem to be measured with more noise and suggest more variability across countries.

Third, the individual consumption profile tends to be upward sloping both in the model and in the data. In the model this feature is due to the assumption of a "perfect annuity market" which ensures that there exists a perfect insurance against individual lifetime uncertainty as the end-of-period assets of the households prematurely dying are equally redistributed among the surviving members of the same generation (see footnote 8). Practically, this assumption ensures that the conditional survival probabilities do not enter the inter-temporal conditions (see (B.1) in Appendix B) preventing a hump-shaped profile of consumption in steady state.[32] As endogenous forces lead the real interest rate to decrease progressively over time (as shown in the next sections) the consumption profile of subsequent generations tends to be flatter.[33]

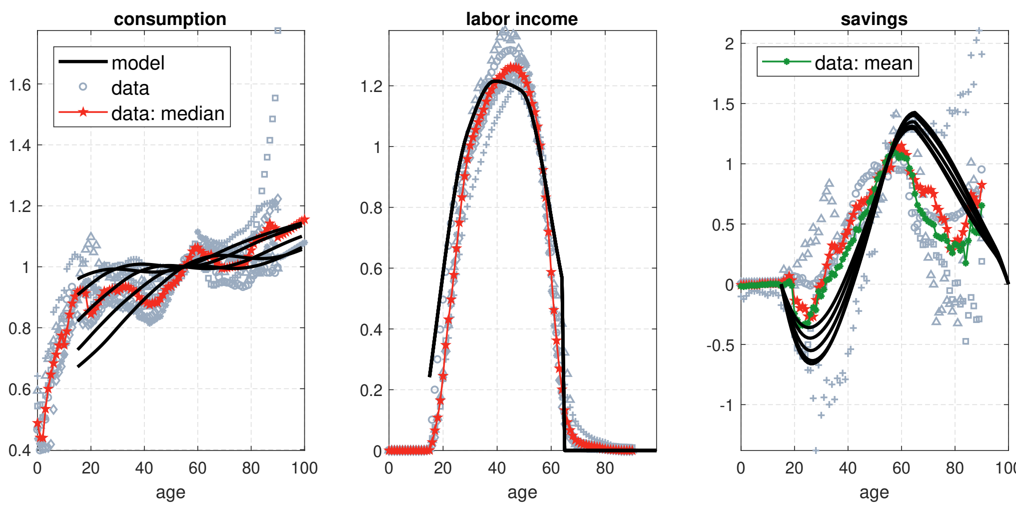

Figure 3: Profiles of individual consumption, labor income and savings: model vs data, normalized on mean for persons 50-60 years old.

Note. The *grey markers* with no line represent data from the *National Transfers Accounts* `www.ntaccounts.org` (NTA), a global project whose main reference is Lee and Mason (2011). The European countries for which data is available are (year used in parenthesis - often only one year is available, results are not sensitive to the specific year used): Austria (2005), Finland (2004), France (2005), Germany (2003), Italy (2008), Spain (2000). For each age of the life-cycle until 90+ (or 100 in few cases) the dataset includes data for the per capita ("smooth mean") components of the following identity: `consumption = labor_income + asset_income - saving + transfers` which finds correspondence in the model. The NTA report the flows for a particular year for a cross-section of age groups. Hence the patterns in the data reflect the effects of age as well as cohort differences. To facilitate comparison both the data and the model series are normalized to the respective mean for persons 50–60 years old. Data on saving available only for Austria, Finland, Italy, Spain. The *continuous black line* represent the life-cycle profiles in the model along the transition dynamics for a selection of generations, those born (aged 15) in: 1971, 1981, 1991, 2001.

[32] See Appendix H for a relaxation of the perfect annuity market assumption.

[33] See Lee and Mason (2011) on how the empirical results based on the *National Transfer Accounts* might differ from those based on e.g. the US *Consumer Expenditure Survey* where the life-cycle profile of consumption is generally found to be more hump-shaped (cf. Fernndez-Villaverde and Krueger (2007)).

4.2 Baseline results

Figure 4 shows the resulting transition dynamics under the baseline calibration (see Table 4.1) for the main macroeconomic variables with (solid line) and without (dashed line) PAYGO pension system. Demographic change is the only exogenous driver in the model.

A decrease of the growth rate of the effective labor to population ratio is akin to a negative shock in the growth rate of total factor productivity (TFP) for output per capita growth. This is why the annual growth rate of output per capita follows closely the growth rate of the effective labor to population ratio.[34] Similarly, the growth rate of total output follows closely the growth rate of effective labor. Compare the first panel of Figure 4 with Figure 2. While the presence of the PAYGO pension system affects the level of output by crowding-out capital accumulation, the growth rate of output is essentially unaffected even though along the transition it stands always at a slightly higher level when the PAYGO pension system is not in place. Instead, by assumption, the growth rate of output in both the initial and final steady state is zero.

As emphasized above, aging means lower fertility rates and higher survival probabilities. As (effective) labor supply becomes increasingly scarcer over time and households are willing to save more in the expectation of a longer life (for given retirement age), capital becomes relatively more abundant than labor in the economy. Hence, the capital-labor ratio (and so the capital-output ratio) increases over time, depressing the marginal product of capital and so the equilibrium real interest rate. This is the most intuitive impact of aging, encompassing channel 1 and 2 above (section 3), and seems immediately clear in the acutest phase of the aging process, between 1990 and 2030.

Notice, however, that in this period the aggregate saving rate (equal to the investment rate by the assumption of closed-economy) decreases. In principle, as shown in the simple model of section 3, there can exist a negative relationship between the saving rate and the real interest. But here they both decrease. What is the intuition? The saving rate starts decreasing few years before 2010, roughly when the first wave of baby boomers start retiring, and keeps doing so until about 2040, when most of the baby boomers have most likely chased to exist. It goes from about 23% of output in 2010 to 21% in 2040. It is conceivable that part of the decrease of the saving rate is due to the flow of dissaving that originates from the retiring baby boomers (channel 3 above). Additionally, one needs to consider not only that with labor becoming relatively scarcer it is optimal to reduce the capital stock to exploit the complementarity between factors in production, but also other general equilibrium effects that connect the decrease of the real interest rate to the saving rate.

Nonetheless, the capital stock in the economy decreases slowly over time and so does the holding of wealth over the course of retirement. Since it is the capital stock rather than the flow of saving that determines the real interest rate, the general equilibrium analysis under the baseline calibration

[34] To see this, consider the Cobb-Douglas production function: $Y_t = K_t^{\psi} L_t^{1-\psi}$. Divide by the total number of people in the economy, N_t, to have variables per capita, denoted by an over-line. Then, $\overline{Y}_t = \overline{K}_t^{\psi} \overline{L}_t^{1-\psi}$. Hence the growth rate of output per capita: $\overline{Y}_{t+1}/\overline{Y}_t = (\overline{L}_{t+1}/\overline{L}_t)^{1-\psi}(\overline{K}_{t+1}/\overline{K}_t)^{\psi}$. That is, a decrease of the growth rate of effective labor to population ratio, $\overline{L}_{t+1}/\overline{L}_t = (L_{t+1}/N_{t+1})/(L_t/N_t)$, is akin to a negative shock to TFP growth.

shows that ultimately the main driver in the period between 1990 and 2030 is the relative scarcity of labor (channel 1) coupled with an increased willingness to save by households discounting higher survival probabilities (channel 2) in spite of general equilibrium forces that lead the saving rate to decrease. As a matter of fact, the real interest rate starts re-bouncing around year 2030, when the growth rate of the effective labor to population ratio start re-bouncing too (see Figure 2). However, in spite of the re-bouncing, in the very long-run the real interest rate is permanently at a lower level compared to its initial steady state value as households discount permanently higher survival probabilities.

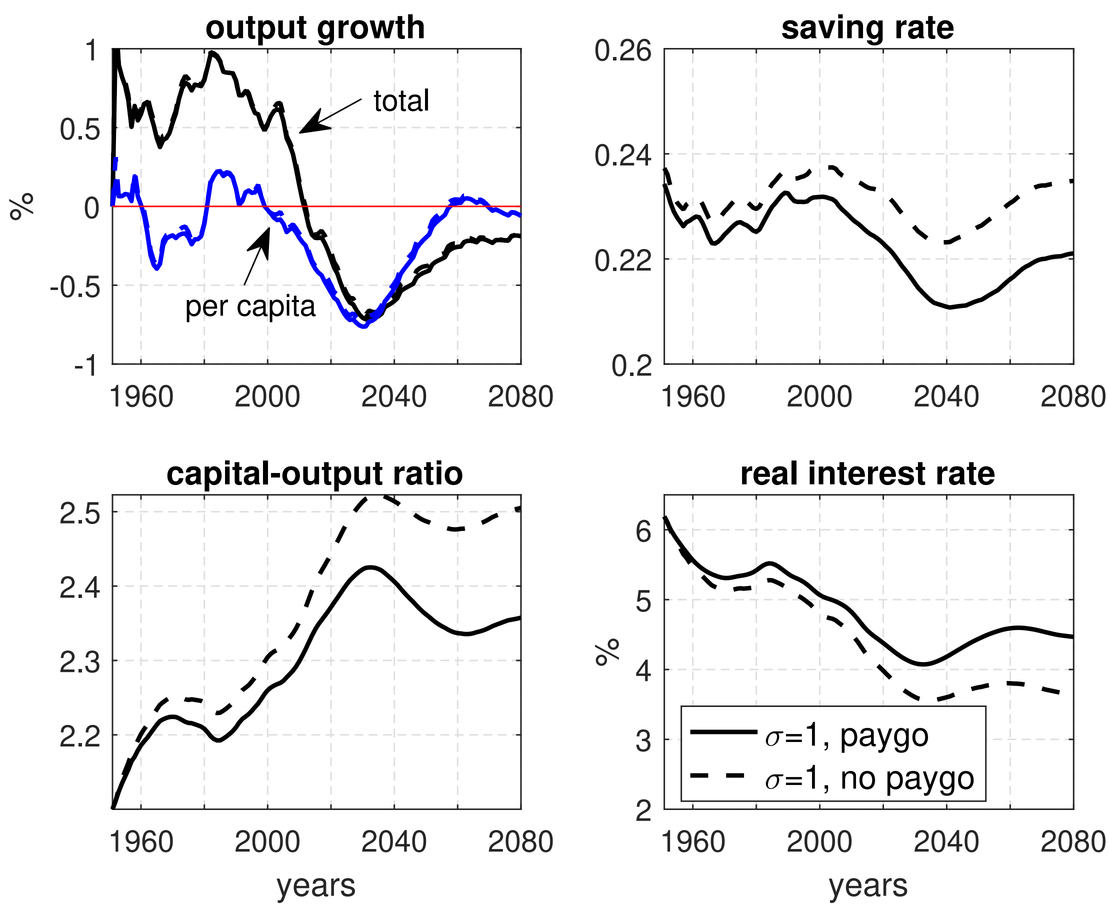

Figure 4: Baseline transition dynamics in EA12

Note. Calibration of Table 1. In the "no paygo" scenario the tax rate is $\tau_t = 0$ for all years t and the individual discount factor β is recalibrated to match the target capital-output ratio in the initial steady state. The aggregate saving rate in the figure is: $I_t/Y_t = [K_{t+1} - (1-\delta)K_t]/Y_t$.

The dashed lines in Figure 4 show what happens when the PAYGO pension system is not in place, i.e. when households need to finance the retirement period by means of private savings only. It is immediately clear that without social security benefits there is no crowding-out effect on capital so that both the saving rate and the capital-output ratio are always at a higher level, going hand in hand with a lower real interest rate along all the transition (channel 4 above). There is not only a level effect but also a relative magnitude effect: between 1990 and 2030 the real interest rate decreases by 1.3 (1.56) percentage points with (without) the PAYGO pension system in the economy. If the economy did not run any pension system, the model predicts that the real interest rate by 2030 would stand at a lower level of about 0.5 percentage points (the real interest rate in 2030 is about 4.1% with PAYGO system, 3.6% without).

Therefore, while it is true that the presence of a PAYGO pension system mitigates the downward impact of aging on the real interest rate (by crowding-out capital investment, a well established result in the literature), under the baseline calibration it does not seem the case that demographics can meaningfully "reverse three multi-decade global trends" on real interest rates via this channel as argued by Goodhart and Pradhan (2017). The next sections will explore whether this might or might not happen studying the transition dynamics under different calibrations and assumptions.

Finally, it is interesting to notice that before 2007 one could easily interpret the impact of demographic change on output growth as a type of scaling factor, producing if anything mild variations around a constant, close to zero for the case of output per capita. However, after 2007 – which, coincidentally, is when the "global financial crisis" started – the model predicts a dramatic decrease of output growth due to demographic change.[35] While in 2007 demographics was contributing to increase total output by 0.4% per year, by 2015 the figure is -0.2%. Going to 2026 the model predicts a further drag on total output with an annual growth rate of about -0.6%. A similar persistent decreases is predicted for output per capita annual growth rate: it goes from -0.2% in 2007 to -0.4% in 2015, to -0.7% by 2026. For the real interest rate too, the most dramatic phase is after 2007 with a variation of about -1 percentage point between 2007 and 2030 (compared to -1.3 percentage points between 1990 and 2030). Hence, what noticed by Gagnon et al. (2018) for the United States seems also true for Europe: "the largest effects of demographics on interest rates and GDP growth coincide to some degree with the Global Financial Crisis". So that "downward pressures on interest rates and GDP growth due to demographics could be easily misinterpreted as persistent but ultimately temporary influences of the global financial crisis" (Gagnon et al., 2016). Instead, what OLG models seem to predict is a "new normal".[36]

[35]This is made clearer in Figure A.5 (that also reports the results of different simulations varying the intertemporal elasticity of substitution σ, see section 4.4.1).

[36]For completeness, Figure A.6 reports the transition dynamics for output, consumption and capital per capita as well as for the real interest rates until year 2200 when the system has essentially reached the final steady state after the demographic evolution has become stationary again once the incoming cohort size and the conditional survival probabilities have been fixed at the 2100 level forever. For the way the experiment is designed, consumption decreases upon impact in 1951 while both capital and output are fixed (and so the real interest rate) as the former is a pre-determined variable and the latter depends both on capital and labor which, by assumption, has the same value of 1950. This downward jump of aggregate consumption determines an upward jump of the aggregate saving rate that goes from 20% of the initial steady state to slightly more than 23% in 1951. See Figure 4. Figure A.7 explores the jumps of consumption in 1951 on the age-profiles for different cohorts. In 1951 households have to re-optimize given the new set of information about the future. Most of the generations respond to higher survival probabilities in the future by saving more (i.e. with a consumption profile that tends to jump downward upon impact). However, younger generations (roughly younger than 30 in 1951) have *ex-post* saved too much during the first years of working

4.3 Isolating the drivers

Figure 5 shows the results of running different counterfactual simulations where the conditional survival probabilities are fixed at their level in the first period of the transition dynamics;[37] and where the economy is assumed not to be endowed with a PAYGO pension system (in this case the individual discount factor β is recalibrated to match the target capital-output ratio in the initial steady state, as in Figure 4). The reference year is chosen to be 1987 as in this year the baseline simulated series takes a value, normalized to zero in Figure 5, approximately equal to its mean over the 1980s.

Notice that (see equation (2.1)) the evolution of the population distribution in the model depends on three elements: the initial population distribution, the incoming cohort size (the newborns in the model, aged 15) and the conditional survival probabilities. When the conditional survival probabilities are fixed at the first period level, the model counterfactually captures not only the impact on savings decisions (channel 2) but also the shift of the population towards older cohorts (which tend to be dissavers, channel 3) due to higher survival probabilities. Furthermore, the labor supply in this case follows an implied evolution that does not only depend on the incoming cohort size and the initial population distribution but also on the fixed survival probabilities themselves. In this sense, fixing the survival probabilities isolates only imperfectly the life expectancy, the labor supply and the composition channels above. The residual difference between the "fixed survival probabilities" line and the zero line can be attributed to forces other than those due to the increase in the survival probabilities. In particular, from equation (2.1), the only exogenous variation in this case is given by the incoming cohort size in each period, thus capturing the effects of variations in fertility rates over time.

Through the lens of this decomposition, the only force that provides upward pressure to the real interest rate is the presence of the PAYGO pension system (channel 4) but, as already seen in the previous section, it is not strong enough to counteract the other forces. Compared to the mean in the 1980s the real interest rate decreases about 1.4 percentage points until 2030. If no PAYGO pension system was ever run the real interest rate would be about 0.5 percentage points lower by 2030. The increase of the survival probabilities over time has an important role in explaining the decrease of the real interest rate in the model. Alone they are associated with a negative deviation of the real interest rate from the mean in the 1980s of about 0.9 percentage points by 2030 (cf. the "Mortality" shaded area in Figure 5). The residual decrease of about 0.5 percentage points by 2030 can be attributed to forces related to the decrease of the newborns in the model (cf. the "Fertility"

(when they were planning for a steeply upward-sloping consumption profile based on higher real interest rate). Now, learning not only about higher survival probabilities but also lower real interest rate in the future, they re-optimize tending to save less for the future (correcting in turn the 'excessive' capital accumulation in the past).

[37] The first period of the transition was preferred to the initial steady state to fix the survival probabilities at. The reason is that the initial steady state does not use actual survival probabilities, for the way the model is built. It uses instead the ones implied by the demographic structure prevailing in the chosen year (1951). Consequently, the counterfactual results so obtained are better interpreted in terms of a reference year standing some time later than the initial steady state, to avoid also potential front-loading effects, as done in Figure 5.

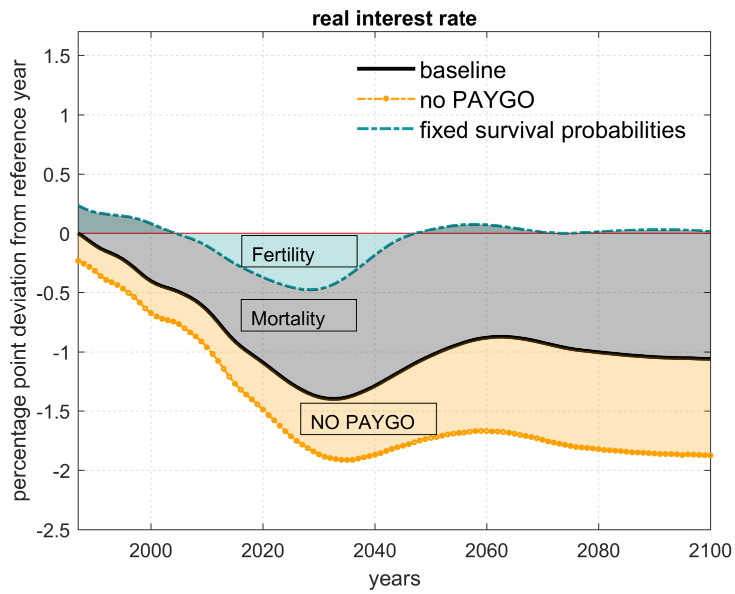

Figure 5: Transition dynamics in EA12: different channels

Note. Calibration of Table 1. The results are obtained by running the simulations fixing the conditional survival probabilities at the first period of the transition dynamics and assuming that the economy is not endowed with a PAYGO pension system (cf. dashed line in Figure 4). The reference year is 1987 (when the baseline series takes approximately its mean value over the 1980s, normalized to zero).

shaded area in Figure 5). These findings are in line with the independent work by Bielecki et al. (2020) where, in a similar OLG model applied to the euro area, they find: "Of the two demographic factors, mortality is the more important one. Fertility also plays a significant role, but only since about 2000, and its contribution is even slightly positive in the 1990s because of the echo effect of the postwar baby boom".[38]

Interestingly, the fertility headwind fades away progressively after 2030 (at its trough) until it is essentially reabsorbed by 2050. The reason is that in the projected horizon the growth rate of the newborns in the model gets progressively less negative until reaching again zero, as in the initial steady state. This is the main driver of the re-bouncing after the 2030. Hence all the permanent decrease of the real interest rate in the very long run, about 1 percentage point compared to the average in the 1980s, is due to the permanent increase in the survival probabilities. This should not be surprising in light of the discussion in section 3. As the growth rates of the incoming cohort size and technology are set to zero in the (initial and final) stationary equilibria, all the demographic effects on the real interest rate in the long run depend on the survival probabilities. Section 5,

[38]The relative importance of the mortality channel is generally found in similar OLG models even for other countries. For example, Krueger and Ludwig (2007) report: "[...] the main sources of changes in factor prices and other aggregates between 2005 and 2080, at least in the U.S., come from changes in mortality rates". Also the results in Carvalho et al. (2016) and Lisack et al. (2017), for a representative OECD economy, point to the relative importance of the life expectancy channel. Gagnon et al. (2016) instead, in a OLG model for the US, find a more limited role for mortality changes, at least from 1980 to 2015.

particularly subsection 5.3, will explore these results in presence of non-zero technology growth.

4.4 Sensitivity analysis

The goal of the sensitivity analysis is to understand how the results change when the model departs from the baseline calibration. This sheds light not only on the robustness of the quantitative estimates but also on the economic significance of the main parameters in the model.

4.4.1 Intertemporal elasticity of substitution

In the current setting, the intertemporal elasticity of substitution in consumption is represented by the inverse of the constant relative risk aversion (CRRA) coefficient, σ. By changing the intertemporal condition for consumption, a variation of σ modifies the reaction of households to the higher survival probabilities brought about by aging, i.e. it modifies the transmission of channel 2 above. In particular, a higher σ, by making the consumption-smoothing motive stronger, would tend to amplify channel 2, thus tending to reduce the real interest rate even more than in the baseline.

Eichenbaum et al. (1988) suggest that an appropriate range is $\sigma \in [0.5, 3]$, a common range in the business-cycle literature. Rarely $\sigma < 1$. Most often σ is simply set to 1, which corresponds to the case of logarithmic preferences. In a systematic analysis of the literature on the estimates of the intertemporal elasticity of substitution, Havranek (2013) documents that "the typical range of calibrations lies between 0.2 and 2", corresponding to $\sigma \in [0.5, 5]$. However, the value of σ remains controversial.[39]

In the literature on OLG models it is quite common to set σ either equal to 1 or to 2. Notable examples of quantitative OLG models calibrated for both Europe and the US, as well as other OECD economies, include with $\sigma = 1$: Krueger and Ludwig (2007), Gagnon et al. (2016), with $\sigma = 2$: Domeij and Flodén (2006), Attanasio et al. (2007).

Figure 6 shows results of the transition dynamics under three different values of σ: 1 (baseline), 2, 2.5 with and without the PAYGO pension system. In each simulation the value of the individual discount factor β is recalibrated in order to match the target capital-output ratio of 2.1 in the initial steady state. Table 2 reports, for each experiment, the values of β as well as the percentage points change of the real interest rate between 1990 and 2030.

As σ increases the impact of aging on the real interest rate becomes more negative. For example, when $\sigma = 2$ the real interest rate decreases by 2.32 percentage points between 1990 and 2030 compared to a baseline decrease of 1.3 percentage points. Looking at Figure 6, to higher values of σ correspond paths of the real interest rate that lie at lower and lower levels in all periods of the transition. While the path of the real interest rate for the cases of no PAYGO pension system seems unrealistically low, it might serve as a useful counterfactual (for cases of σ not believed to

[39] Havranek (2013) finds "strong publication bias: researchers report negative and insignificant estimates less often than they should, which pulls the mean estimate [of the inverse of σ] up by about 0.5", pointing that the "corrected mean of micro estimates ... is around 0.3–0.4", i.e. a mean value of σ between 2.5 and 3.34.

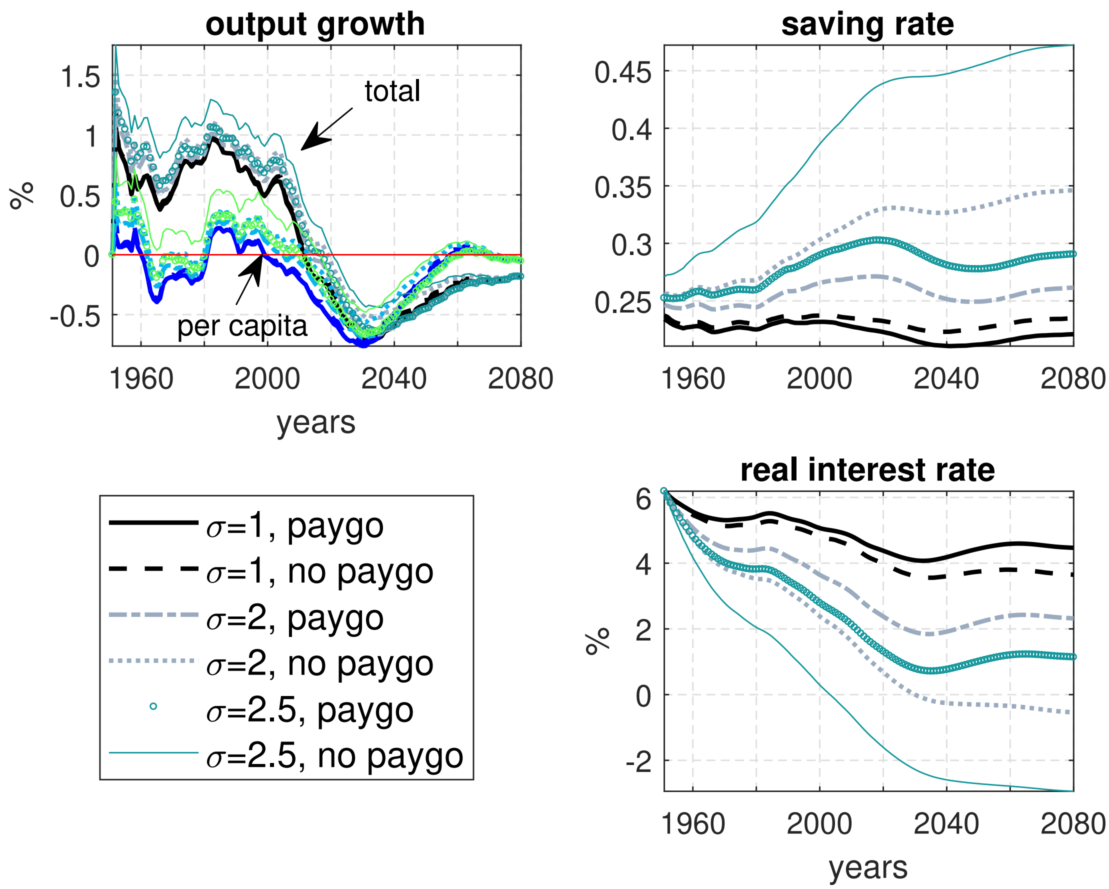

Figure 6: Transition dynamics in EA12: different values of σ

Note. Calibration of Table 1 with different values of σ. In each simulation β is recalibrated to be consistent with the target capital-output ratio of 2.1 in the initial steady state.

be unrealistically high) supporting the view that absent policy interventions (such as the PAYGO system) real interest rates would be at even lower levels than the those observed in the data.[40]

It is interesting to note that for the cases of $\sigma = 2, 2.5$, especially for the no PAYGO scenario, the income effect seems to prevail over the substitution effect in general equilibrium so that the saving rate increases as the real interest rate decreases. With more accumulation of capital, a higher σ is also associated with a significantly higher growth rate of output along the transition.[41]

Overall, the big (probably unrealistic) impact that the model predicts for the values of σ higher than in the baseline hints to stay on the conservative side when calibrating the intertemporal elasticity of substitution in a frictionless modeling context where intertemporal considerations linked to demographic change are the only drivers of savings.

[40] Cf. Rachel and Summers (2019a): "[...] *neutral real interest rates would have declined by far more than what has been observed in the industrial world and would in all likelihood be significantly negative but for offsetting fiscal policies over the last generation*".

[41] This is made clearer by Figure A.5.

Table 2: Different values of σ: impact on the real interest rate, 1990-2030 percentage points change

	σ	β	$\Delta r_{1990-2030}$
	1	0.9598	-1.30
PAYGO	2	0.9783	-2.32
	2.5	0.9877	-2.71
	1	0.9566	-1.56
NO PAYGO	2	0.9717	-3.16
	2.5	0.9794	-3.63

4.4.2 Capital-labor substitutability

As seen above, a driver of the declining real interest rate over the demographic transition after 1990 is the scarcity of labor as production input. As labor becomes scarcer, the demand for capital by firms is reduced thus leading to a decrease of the real interest rate everything else being equal (see channel 1 in section 3). A criticism to the quantitative estimates above based on this channel is that the demand for capital comes from the assumption of a Cobb-Douglas production function, i.e. a unitary elasticity of substitution between labor and capital ρ.

This channel might not matter at all if one admits that capital and labor are sufficiently substitutable between each other. In the limit, when capital and labor are perfect substitutes, changes in relative factor quantities have no impact on relative factor prices. To what extent can a higher substitutability between labor and capital mitigate the negative impact on the real interest rate induced by the demographic transition? Consider the CES production function in expression (2.7). When $\rho \to 1$, the production function is a Cobb-Douglas. When $\rho > 1$, capital and labor are said to be *gross substitutes*. In this case, a lower supply of one input leads to added demand for the other input. The opposite occurs when $\rho \leq 1$, in which case capital and labor are said to be *gross complements*. Therefore, when the labor supply decreases in the process of population aging with labor and capital as gross substitutes in production, the capital demand is affected differently compared to the case of a Cobb-Douglas production function.

In his summary of the empirical literature, Chirinko (2008) concludes that "the weight of the evidence suggests that ρ lies in the range between 0.40 and 0.60".[42] Nonetheless, it might be interesting to consider values of ρ grater than unity. One could speculate that ongoing economic processes are changing the nature of capital, so that estimates based on historical data might not be reliable any longer. For example, automation can be thought as a process that by making labor increasingly superfluous in production is leading to an increasing degree of substitutability between labor and capital.[43] Furthermore, there might be reasons to believe that $\rho > 1$. Karabarbounis and

[42] Of the 31 sources listed, 26 shows a ρ strictly less than one with a median of 0.52. The maximum value, reported only by one source, points to a value of 2. The remaining sources point to $1 < \rho < 1.5$ with one exception where ρ is slightly bigger than 1.5.

[43] Acemoglu and Restrepo (2018) "argue theoretically and document empirically that aging leads to greater (industrial) automation, and in particular, to more intensive use and development of robots".

Neiman (2014) need ρ greater than one to explain the simultaneous decrease of the relative price of investment goods and the labor share of income. They estimate $\rho = 1.25$. Piketty (2014) too needs ρ greater than one to explain the fact that historically the capital share of income was lower when the capital-output ratio was lower, even though he does not provide estimates of ρ.[44]

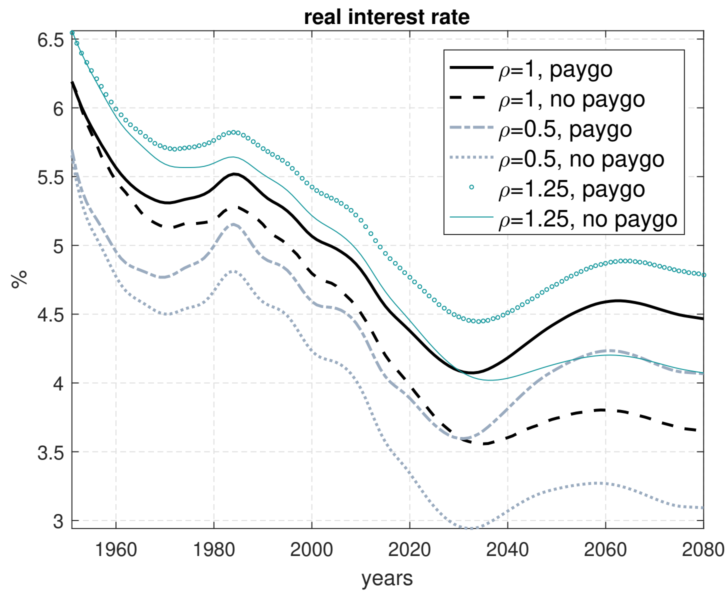

Figure 7: Transition dynamics in EA12: different values of ρ

Note. Calibration of Table 1 with different values of capital-labor elasticity of substitution ρ. For $\rho > 1 (< 1)$ capital and labor are gross substitutes (complements). In each simulation the capital-output ratio in the initial steady state is allowed to differ from the baseline target value.

Figure 7 plots the real interest rate resulting from the demographic transition for three values of ρ: 1 (baseline), 0.5, 1.25 with and without the PAYGO pension system. For each simulation, given the parameter values of Table 1 (including β), the capital-output ratio in the initial steady state is allowed to differ from the baseline value and so also the initial real interest rate value.[45]

As expected, compared to the baseline, the demographic impact on the real interest rate is more negative for values of ρ smaller than 1, less negative for values bigger than 1. The numbers underlying these series are reported in Table 3 as percentage points change between 1990 and 2030 (the table also reports the initial steady state value of the real interest rate, r_0, in the different simulations). When ρ takes the mid-point value in the range suggested by the empirical literature, $\rho \in [0.4, 0.6]$, the demographic impact on the real interest rate between 1990 and 2030 is -1.34 (-1.65) percentage points for the case with (without) PAYGO pension system. Values very close to the ones found under the baseline scenarios. However the real interest rate stands systematically at lower levels. Across comparable scenarios, there is a gap between the simulated series of about 0.5

[44]Cf. Rognlie (2014) for an assessment of Piketty (2014) in relation to ρ.

[45]Notice that the steady state real interest rate is: $r = \psi(K/Y)^{-\rho} - \delta$, so that targeting the same capital-output ratio of the baseline would give a different value of the real interest rate when ρ is different from the baseline value.

percentage points that widens after 2030. Overall, despite higher substitutability between labor and capital dampens the baseline negative impact of the demographic transition on the real interest rate, the sign is not reversed for any sensible value of ρ.[46] In fact, for values of ρ in the range reported by the empirical literature the demographic impact on the real interest rate is even more negative than what estimated in the baseline specification above, particularly in the period between 1990 and 2030.

Table 3: Different values of ρ: impact on the real interest rate, 1990-2030 percentage points change

	ρ	r_0	$\Delta r_{1990-2030}$
PAYGO	0.5	5.70	-1.34
	1	6.19	-1.30
	1.25	6.55	-1.23
NO PAYGO	0.5	5.66	-1.65
	1	6.19	-1.56
	1.25	6.56	-1.43

4.4.3 Age-dependent labor efficiency

The exogenous aggregate labor supply, $L_t = \sum_j h_j N_{t,j}$, is composed by two elements: (a) the number of people in the labor force, in the baseline assumed to be those that are between 15 and 64 years old; (b) the age-dependent labor supply in units of efficiency, h_j, obtained by interacting the profiles for productivity and participation rate (see Figure A.3). The final profile h_j is such that each person on average across all the ages is able to provide one unit of labor input. For example, between age 30 and 50 individuals provide more than twice as much labor as in the remaining periods of life, because they are either more productive or more participative in the labor market providing more worked hours, or both. Therefore, as the demographic distribution varies over time also the aggregate efficiency of labor varies over time. To isolate the impact of this time-varying aggregate efficiency, Figure 8 plots the series for output growth and the real interest rate under the hypothetical case that for each person in the working age the labor efficiency is equal to the average, namely to 1, so that in this case the aggregate labor supply is simply captured by the total number of people in the working age: $L_t = \sum_{j=0}^{j_r-1} N_{t,j}$.

Looking at the output growth rates in Figure 8, it is apparent that when one does not adjust for the age-dependent labor efficiency the impact of aging is more negative in the periods before 2013, while it is attenuated afterwords. The reason is that before the turning point the demographic distribution is tilted towards relatively younger cohorts, more productive and participative according to the profile h_j. The opposite occurs afterwards when a greater mass of those in the labor force

[46] Simulations for values of $\rho > 1.25$ up to value 2, not reported here, confirm that aging exerts a downward pressure on the real interest rate in all cases of comparable magnitude across the simulations, while the starting level can be significantly higher.

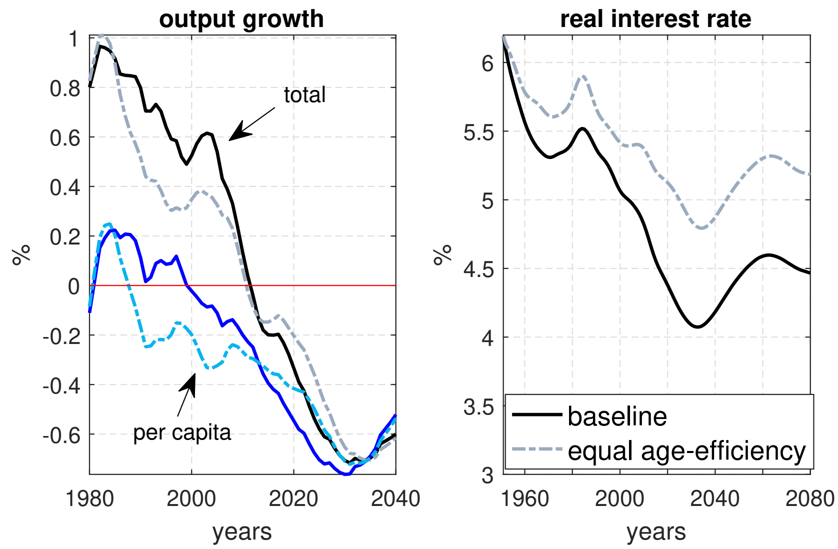

Figure 8: Transition dynamics in EA12: equal age-dependent efficiency units of labor h_j

Note. Calibration of Table 1 with $h_j = 1$ for all $j = 0, 1, ...j_r$. The discount factor β is recalibrated to be consistent with the target capital-output ratio of 2.1 in the initial steady state. The resulting β has value 0.9489.

is relatively older. So the growth rate of the aggregate labor input is relatively smaller (higher) before (after) 2013 when it is not evaluated with the age-dependent labor efficiency. This translated directly into lower (higher) output growth.

When the individual labor supply is flat at the value of 1, there is less scope for savings over the life cycle (even though in this case they do not borrow in the first part of their life). The saving rate tends to be systematically lower than in the baseline. This is reflected in a systematically higher real interest rate. Furthermore, as the aggregate labor supply varies by less along the transition, the real interest rate also varies by less. Between 1990 and 2030 the real interest rate decreases by only -0.84 percentage points (compared to -1.3 in the baseline).[47]

4.4.4 Type of pension system

The type of pension system might significantly matter for the results. In the baseline analysis above, it is assumed that the PAYGO pension system has a replacement rate fixed at 45% in all periods of the transition while the tax rate adjusts in order for the government to have the budget

[47] Figure A.8 shows the demeaned series over the period 1960-2020 decomposing the impact of demographic change into "labor quantity", stemming from the simulation where the aggregate labor supply is simply given by the number of people in the working age, and a residual component that captures the evolving "labor efficiency", i.e. how the efficiency profile h_j interacts with the number of people in the economy making the aggregate labor input more or less scarce. It shows that, compared to the mean over 1960 to 2020, before the 2000s both "labor quantity" and "labor efficiency" exert an upward pressure on the real interest rate; afterwards they both turn negative. It is remarkable how the results from the OLG model align with the results from the econometric model by Fiorentini et al. (2018)

balanced in each period (see equation (2.10)). Figure 9 reproduces the baseline path of the real interest rate and shows the associated increase in the tax rate under such a pension scheme. The tax rate increases by about 14 percentage points in the long-run, going from about 6% in 1950 to more than 20% in 2080. The path of the tax rate along the transition follows closely the evolution of the old-dependency ratio, the number of retirees over the number of effective workers in the economy.

The dash-dotted lines in Figure 9 show what happens when the government runs a different pension system. Instead of fixing the replacement rate and varying the tax rate over time, the government fixes the tax rate at the value of 6.21% (the initial steady state value corresponding to a replacement rate of 45%) and allows the replacement rate to varies endogenously in such a way, again, that its budget is balanced in each period. It results that the replacement rate decreases significantly going to slightly less than 12% in the long-run. Also the evolution of the replacement rate is directly connected to the evolution of the old-dependency ratio. The resulting path of the real interest rate when the replacement rate adjusts endogenously is almost identical to the case in which there is no pension system in the economy.[48]

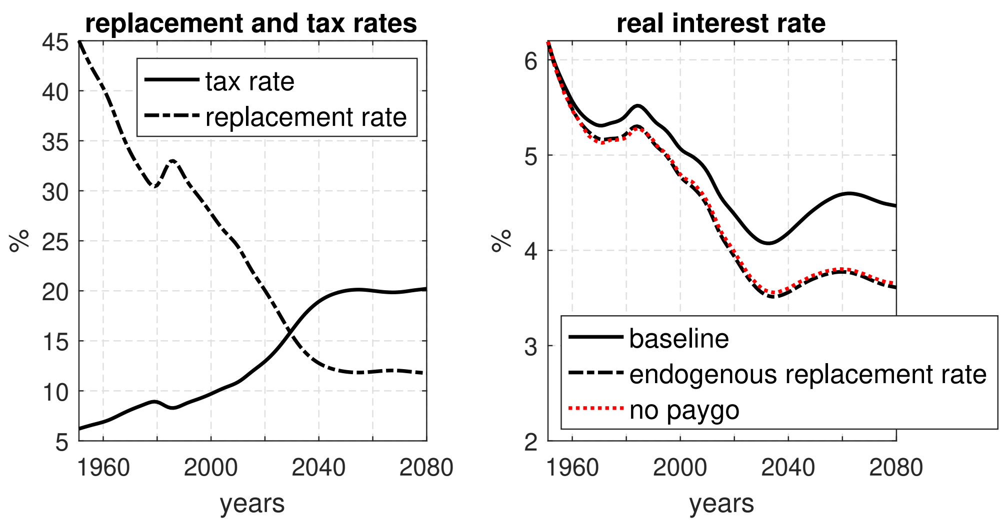

Figure 9: Transition dynamics in EA12: different pension schemes

Note. Calibration of Table 1 with two different pension schemes: (a) in the baseline (continuous lines) the replacement rate ρ is fixed at 45% while the tax rate τ_t varies over time such that the government budget is balanced in each period (see equation (2.10)); (b) in the second scheme (dashed-dotted lines) the tax rate is fixed at the initial steady state value of 6.21% while the replacement rate is allowed to vary over time such that the government budget is balanced in each period (see equation (2.10)).

4.4.5 Retirement age

The relative scarcity of the labor supply in the economy is determined not only by the age-dependent efficiency of labor but also by the retirement age (here exogenously imposed) which in turn influ-

[48]These results are in line with the literature, cf. e.g. section 5.5. of Krueger and Ludwig (2007).

ences the overall willingness to save and, in particular, the pervasiveness of the pension system effects. What happens when individuals are allowed to retire later or earlier compared to the baseline (age 65)? Figure 10 offers an answer.

When the retirement age is increased by 10 years, so that all individuals retire at age 75 – a big policy shift – general equilibrium forces lead the path of the real interest rate to be essentially the same as in the baseline. The same occurs when the retirement age is lowered to age 60. Given the parametrization of the efficiency age-profile h_j, the associated increase (decrease) of aggregate labor efficiency when the retirement age is increased (decreased) is not material enough to determine a significant modification of the consumption-savings decisions.[49] On the one hand, an increase of the retirement age makes labor less scarce making firms demanding more capital. This should put upward pressure on the real interest rate. On the other hand, an increase of the retirement age reduces the total amount of pension benefits to be financed via the PAYGO system so that the tax rate reduces, implying smaller crowding-out effects on private savings, thus a smaller real interest rate. These two effects seem to compensate each other (with the flipped side when the retirement age is decreased) so that the impact on the real of a policy shift on the retirement age is nil.

Output per capita growth tends to be higher (lower) when the retirement age is increased (decreased) in the acutest phase of the demographic transition. If in 2006 the output per capita growth is basically the same in the three scenarios (retirement age at 65, 75, 60) standing at about -0.15% on a annual basis, in 2020 the model predicts that demographic change leads to a growth rate of -0.55% in the baseline (retirement age at 65), -0.45% when the retirement age is at 75, -0.75% when it is at 60. A magnitude that, arguably, is insufficient to justify a huge policy shift such as an increase of the retirement age by 10 years.

The dashed lines show what happens when not only the retirement age is increased by 10 years but also individuals are made equally efficient in production. In this way the baby boomers not only retire 10 years later but they are not a drag on the economy when they approach retirement as they are assumed to be equally efficient in production of e.g. a 30 years old. As a consequence in 2020 the impact of demographic change on output per capita growth is about the same as the one in 2006. Nonetheless, the model predicts that no matter which scenario considered by 2040 demographic change leads the growth rate of output per capita at about -0.5%.

The real interest rate, on the contrary, stands persistently at a higher level when the individuals in the economy are equally efficient across the ages. More so the higher the retirement age. To strengthen the point, the thin lines in Figure 10 show the results from the extreme case in which individuals work continuously until they reach the death age and are equally efficient irrespective of age. In this case the growth rate of the aggregate labor supply is equal to the population growth rate. As discussed also in section 4.2, in this case there is really no shock induced by aging to the output

[49] This is confirmed when one allows the efficiency profile not to be re-normalized such that the average efficiency along the life-cycle is 1 when the retirement age is varied. The thin dashed line in Figure 10 shows that the increase in average efficiency associated with individuals staying ten more years in the labor force (it goes from 1 to about 1.0752) does not lead to any meaningful variation in the simulation series.

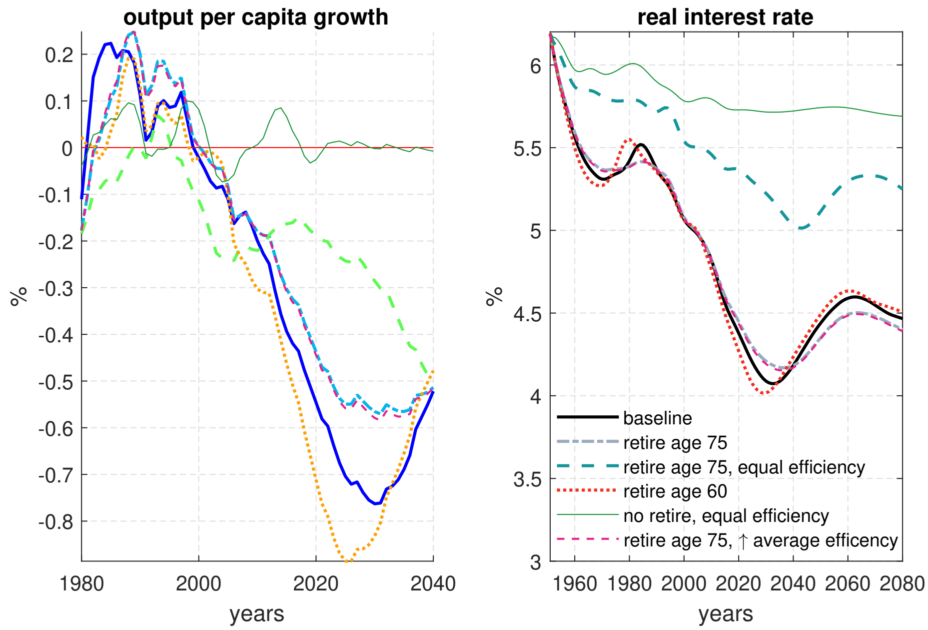

Figure 10: Transition dynamics in EA12: different retirement age j_r

Note. Calibration of Table 1 with different retirement ages j_r. In each simulation the discount factor β is recalibrated to be consistent with the target capital-output ratio of 2.1 in the initial steady state.

per capita growth which mildly fluctuates around zero. The decrease of the real interest rate is immaterial, about -0.18 percentage points between 1990 and 2030, associated with the lengthening of the survival rates.

These results suggest that, to mitigate the negative impact of aging on both the growth rate of output per capita and the real interest rate, increasing the retirement age *per se* does not do a satisfactory job. The main reason resides in the hump-shaped nature of the productivity-participation profile across the ages so that increasing the number of people in the labor force at the margin does not sufficiently reduce the effective scarcity of labor in the economy. A policy that not only increased the retirement age but also bent upwards such a hump-shaped profile by making older individuals more productive would provide a more satisfactory outcome.

5 Comparing the model with the data: the role of TFP growth

In the attempt to compare the model with the data at least three orders of considerations arise.

First, to have non-zero long-run economic growth (and to match the historical levels of trend growth) one needs to introduce at least one source of growth other than demographics. Following a standard view, that one can find in most macroeconomic textbooks, in this section the approach is to allow for non-zero exogenous *total-factor-productivity* (TFP) growth.

Second, the natural level of both the real interest rate and output are *unobservable* variables. As such, one needs to employ econometric estimates to compare the model's results with.

Third, the concept of natural interest rate revolves around a *single* measure, with no difference across asset classes. In this regard, the OLG model employed here seems appropriate in allowing savings in a single asset whose return in equilibrium, coherently with Wicksellian theory, does not differ from the marginal product of capital which features as the natural rate (given the absence of nominal variables and frictions in the model).[50] Hence, this paper embraces the approach by Rachel and Summers (2019b) who find empirically a common downward trend across different types of assets[51] and thus assert: " [...] for analyzing long-term trend movements in neutral real rates, it is appropriate to focus on factors relating to saving and investment propensities rather than issues of liquidity or risk".

This section introduces a new calibration with non-zero TFP growth which allows to be consistent with recent econometric estimates of natural outcomes (real interest rate and output) for the euro area. The simulation results from this new calibration are compared not only with econometric estimates of natural outcomes but also with actual data pertaining to estimates of the marginal product of capital.[52]

The comparison suggests a broad consistency between the OLG model's simulation results, econometric estimates and data from different sources, providing evidence in support of a prominent role of demographics to explain low-frequency movements of real interest rates and output. Nonetheless, a certain degree of arbitrariness seems thus far inevitable when it comes to measuring the marginal product of capital in the data so that its level is likely to remain debatable.[53] Arguably, the results from the model's simulation on the real interest rate are better interpreted in terms of *variation* over time associated with demographic change (and TFP growth).[54] Empirical measures of the marginal product of capital reported below shall be interpreted accordingly.

[50] See Anderson (2005) on the interpretation of Wicksell's natural rate.

[51] Specifically, by means of a principal components analysis they find that in a set of various US real yields (spanning government debt, corporate bond and equity markets) "the first principal component, which picks up the downward trend visible in all returns, explains 94% of the total variance in the underlying series" (Rachel and Summers, 2019b).

[52] Appendix G compares the OLG model's simulations also with measures of real yields for a set of different assets.

[53] Gomme et al. (2011), Gomme et al. (2015), Caballero et al. (2017), Marx et al. (2018) all argue in favor of an increasing discrepancy between real interest rates and real returns to productive capital. On the contrary, most contributions in the OLG model literature do not take this potential discrepancy into consideration by targeting either a measure of the marginal product of capital (like in section 2, cf. e.g. Domeij and Flodén (2006), Krueger and Ludwig (2007), Attanasio et al. (2007), Ludwig et al. (2012), De la Croix et al. (2013)) or a comparatively lower measure coming from data on real government bonds yields (cf. e.g. Carvalho et al. (2016), Gagnon et al. (2016), Bielecki et al. (2018), Jones (2018), Aksoy et al. (2019)). Eggertsson et al. (2019) also enter in this second set of contributions but allow for markups that can drive a wedge between the marginal product of capital and the interest rate. Ludwig et al. (2016) and Sudo and Takizuka (2019) represent exceptions allowing households savings in more than one asset.

[54] This view is reinforced in OLG models such as those of Ludwig et al. (2016) and Sudo and Takizuka (2019) where it is generally found that the impact of demographic change along the transition dynamics does not differ much across different types of assets, while the initial level of the associated returns can differ markedly. Sudo and Takizuka (2019) report: "While the returns on capital r_t^K and the return on government bonds r_t^B are sizably different in level, [...], the difference is negligible when comparing the effect of demographic changes on the two rates".

5.1 Experiment and calibration with non-zero TFP growth

To compare the model's results with the data one needs to introduce non-zero long-run growth in the model. To do so, the approach here adopted is to have non-zero exogenous long-run total-factor-productivity (TFP) growth. Consider the following Cobb-Douglas production function (a special case of the function in equation (2.7)):

$$Y_t \;=\; Z_t K_t^{\psi} L_t^{1-\psi} \tag{5.1}$$

where $Z_t = g_t^Z Z_{t-1}$ is the TFP level assumed to grow exogenously at gross rate g_t^Z. The rest of the model is the same one described in section 2.

The experiment is the same perfect-foresight exercise described in section 4.1. Here, however, in addition to demographics also TFP growth is exogenously time-varying. The transition dynamics occurs between an initial and a final stationary equilibrium characterized by constant non-zero TFP growth. Such an equilibrium is generally referred to as *balanced-growth path* (BGP) as variables grow at a constant rate over time.[55] In the simulations TFP growth is assumed either to be constant along the whole transition or to come from the (filtered) Solow residual obtained via a standard growth regression.[56]

Table 4 summarizes the parameter values in the calibration that allows to compare the model with the data. The calibration is done in order to be consistent with the levels of the real interest rate and output that find correspondence in the econometric estimates of the "natural rate of interest" and the "trend growth rate of the natural rate of output" in Holston et al. (2017) (HLW, henceforth) for the euro area.[57] To this end, in the baseline the TFP growth rate is set to $g_t^Z = g^Z = (1.017)^{1-\psi}$ for all periods t, namely to the long-run growth rate of total output of 1.7% per annum.[58]

The annualized estimates of the natural interest rate for the euro area by HLW range from a maximum of about 3.2% in 1973 to a minimum of -0.2% in 2014. To be consistent with this range of values in those years the initial level of the real interest rate is targeted to be about 3%. Furthermore, to be consistent with data the initial investment-output ratio is targeted to be 0.2 (equally to the calibration in section 4.1). The bias towards capital in the production function is set to the relatively low level of 0.22 to have an implied depreciation rate of capital which does not deviate much from what assumed in the literature (generally about 10% per annum, see section

[55] On a BGP the variables $\{c_t, a_t, d_t, w_t, K_t, C_t, Y_t\}_{\forall t}$ grow at rate $(g^Z)^{1/(1-\psi)}$. See Appendix C.

[56] Furthermore, it is assumed that TFP growth reverts in the long-run to its unconditional mean determined by the estimation of an autoregressive process of order 1, AR(1). See Appendix E.

[57] The latest estimates are updated online, see https://www.newyorkfed.org/research/policy/rstar. The law of motion for the natural rate of interest in HLW is assumed to be: $r_t^* = g_t + z_t$, where g_t is the trend growth rate of the natural rate of output and z_t captures other determinants of r^*.

[58] Across different simulations it is found that $g^Z = (1.017)^{1-\psi}$ allows to broadly match the dynamics of trend growth reported by HLW for the euro area in the period 1972–2018, with little discrepancies depending on the range of sensible values chosen for the other parameters in the model. Notice that to impose long-run growth $(g^Z)^{1/(1-\psi)} = 1.017$ one needs to impose a TFP growth rate $g^Z = (1.017)^{1-\psi}$, $0 < \psi < 1$.

4.1).[59] It results $\delta = 0.11$.[60]

The value of the individual discount factor consistent with these parameter values and the exogenous demographics in the initial BGP is found numerically (by means of a solver for non-linear systems) and results: $\beta = 0.994878$.

Table 4: Calibration with non-zero TFP growth: parameter values

Parameter	Value	Note
Structural:		
ρ	1	capital-labor substitutability, Cobb-Douglas production function
	0.22	bias towards capital in the production function
δ	0.11	depreciation rate of capital
σ	1	constant relative risk aversion (CRRA), log preferences
$\bar{d}$	0.45	net replacement rate of PAYGO pension system
β	0.994878	individual discount factor, endogenous in the initial BGP: $\bar{r} = 0.0297, \bar{\iota} = 0.2$
Demographic:		
J	84	terminal life-age (99)
j_r	50	retirement age (65)
h_j	Figure A.3	individual life-cycle effective labor supply. Source: Domeij and Flodén (2006)
$N_{t,0}$	Figure A.2	incoming cohort size (age 15). Source: UN (2017)
$\pi_{t+j,j}$	Figure A.2	unconditional survival probabilities, retrieved from UN (2017) applying (2.1)
Technology:		
g_t^Z	Figure E.1	TFP growth, $(1.017)^{1-\psi}$ $\forall t$ in baseline experiment

Before solving for the transition dynamics the final stationary equilibrium is solved numerically for the endogenous real interest rate prevailing with the long-run demographics, imposing the parameter values in Table 4. Two main final stationary equilibria are studied: one where g_t^Z is simply set to its initial value of $(1.017)^{1-\psi}$ – the same value imposed throughout the whole transition; another one where the final value of g_t^Z is the unconditional mean obtained by estimating an autoregressive process of order 1 on the Solow residual (see Appendix E). In the former case the long-run real interest rate is about 1.63%, in the latter (with g^Z such that the long-run output growth is about 1.2%) about 1.18%.[61]

[59] Bielecki et al. (2018) calibrate a similarly low value, 0.25, for the bias towards capital in production. They target the average real interest rate of 1.2% for the euro area in the period 1999-2008.

[60] Denote with an upper bar the targeted values, $\bar{r} = 0.0297, \bar{\iota} = 0.2$. On a BGP the real interest rate reads: $\bar{r} = \psi/\kappa - \delta$, where κ denotes the capital-output ratio: $\kappa = \bar{\iota}/[\delta + (g^Z)^{1/(1-\psi)} - 1]$ with $\bar{\iota}$ denoting the targeted investment-output ratio. Given $\psi = 0.22, g^Z = 1.017$, from these two equations it results that the implied depreciation rate of capital is: $\delta = \{\bar{r} - (\psi/\bar{\iota})[(g^Z)^{1/(1-\psi)} - 1]\}/(\psi/\bar{\iota} - 1) = 0.11$.

[61] One can easily compute the value of TFP growth that would allow to restore a level of 3% for the real interest rate once the demographic transition is ended. One needs to solve numerically the final stationay equilibrium for g^Z imposing a value of the final real interest rate $\bar{r}$ equal to 0.03. It is found that the value of g^Z consistent with a target $\bar{r}$ tends to be such that the growth rate of output equals the targeted real interest rate. The output growth rate in the long-run is given by $(g^Z)^{1/(1-\psi)} = (\bar{r} + \delta)(\bar{\iota}/\psi) - \delta + 1$. Across different values for $\bar{r}$ the implied investment-output ratio in the long-run $\bar{\iota}$ tends to be at a value very close to 0.22, which is the value assumed for ψ. Hence $(g^Z)^{1/(1-\psi)} \approx 1 + \bar{r}$. Which also means that absent technology growth the final real interest rate implied by demographics only would be close to zero given the current calibration.

5.2 Results: model vs data

Figure 13 compares data and model's simulations on total output growth and investment-output ratio under different scenarios for the exogenous TFP growth rate.

Output growth. When the TFP growth rate is the "actual" Solow residual growth rate (see Appendix E) the correspondence between the model's simulations and data from the Area Wide Model (AMW) dataset on output growth is quite close because of the very way the Solow residual is constructed.[62] In this case it is assumed that in the long-run the TFP growth series reverts to its unconditional mean (estimated via an AR(1) process in the period 1999-2018) which implies a long-run output growth of about 1.2%. The simulated series is at a minimum around year 2030 with a value of about 0.5% growth per annum.

Assuming that the TFP growth is constant throughout the whole transition (at a value which implies a long-run output growth of 1.7%) allows to match quite closely the estimate of trend growth provided by HLW, with discrepancies between the two series in the range of about $\pm$ 0.3 percentage points. Before the 1970s, however, it generates values that seem too low if compared with the actual data (shaded area). Also in this case the series is at a minimum around year 2030 when it has a value of about 0.9% growth per annum.

Using the low frequency component of the Solow residual (as implied by the Hodrick-Prescott filter, see Appendix E) allows to have more sensible values before the 1970s. It returns a bleaker picture in all years since the late 1990s, also reaching a minimum around year 2030 at a value of about 0.3% growth per annum.

These results show that once a certain level of exogenous technology growth is accounted for, demographic change seems to be a key determinant of the low-frequency movements of total output growth which find close correspondence with econometric estimates such as those provided by HLW. Thus, the results from the OLG model allow to qualify the determinants of those agnostic econometric estimates and provide a forecast relying on demographic projections. As explained in the previous sections, aging in the model exerts a persistent downward pressure on output growth due to the increasing scarcity of effective labor at least until 2030. Holding fixed the long-run TFP growth, between 1990 and 2030 the output growth decreases by about 1.45 percentage point. When one takes into account also the decline in TFP growth (measured by the filtered Solow residual) the figure is about 2.04 percentage points decline over the same period.

Investment-output ratio. The investment-output ratio is quite stable throughout the transition if compared with the range of values in the data. It is remarkable how the model with both the actual and the filtered TFP growth allows to match quite closely the main variations in the data. The model predicts that in the projected horizon the investment output ratio goes from a value of

[62] Since using the "actual" TFP growth leads the model's simulation series on output growth to be very close to the actual series, the model in this case is not capturing the potential or natural output growth. Nonetheless, it is a useful exercise to see the overall direction of the real interest rate so captured, especially given that in this way the OLG model provides the closest results with the estimates of the natural rate by HLW (see Figure 11).

about 22% in 2016 to about 21% in 2040 with a decline that has no precedents in the past according to the model, no matter which simulation's specification is used.

The close correspondence between data and model for output growth and investment-output ratio lays the basis to the model's results on the real interest rate whose counterpart in the data is more debatable.

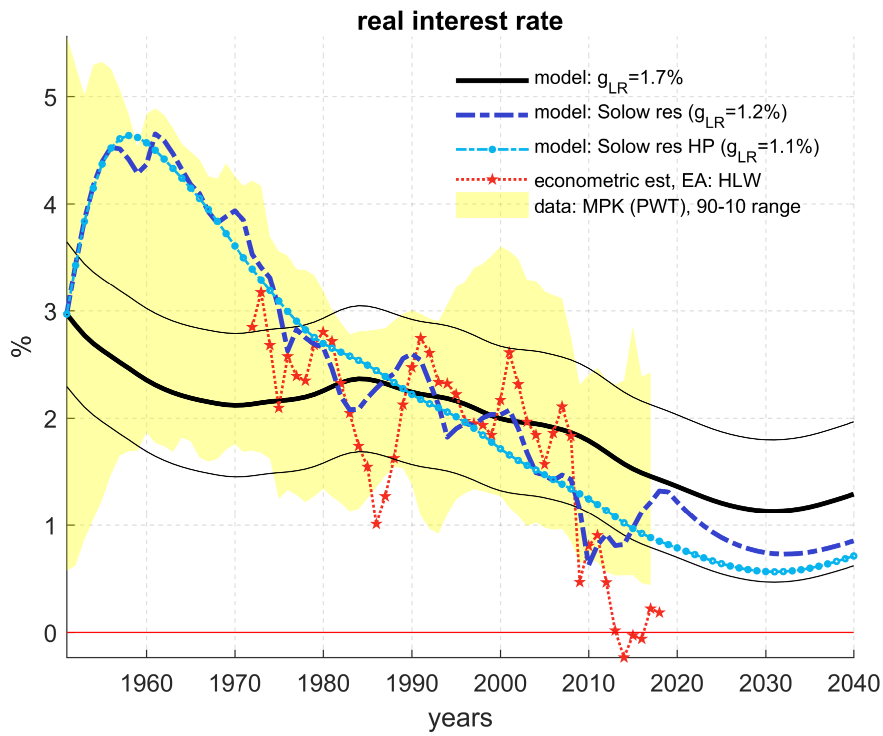

Figure 11: Model vs data: real interest rate

Note. The long-run growth imposed in the final stationary equilibrium is denoted by $g_{LR} = (g^Z)^{1/(1-\psi)}$. See Appendix E for the Solow residual used in the simulations: actual and Hodrick-Prescott (HP) filtered. Econometric estimate for the euro area (EA) corresponding to the "natural rate of interest" in Holston et al. (2017) (HLW). The bright (yellow) shaded area shows the 90-10 percentile range of EA12 countries on a measure of the marginal product of capital (MPK) computed using data from the Penn World Table (PWT) with time varying depreciation rates. Details in Appendix F. The thin black lines mark a band corresponding to $g_{LR} = 1.7\% \pm 0.6\%$ in the OLG model's simulations.

Real interest rate. Figure 11 compares the model's simulation results with the econometric estimate of the natural interest rate by HLW for the euro area.In addition, it shows the 90-10 percentile range fora measure of the marginal product of capital net of depreciation (MPK) derived from the Penn World Table (PWT).[63] These results deserve few comments.

[63]Details on measures and series constructions are provided in Appendix F. Appendix G makes a further comparison of the model's results with data showing a set of measures of the real interest rate that span: *(i)* yields on short- and long-term government securities; *(ii)* a measure of the return of capital which in addition to government securities yields incorporate equity dividend yields and housing rental returns with data provided in the Jordá et al. (2019) (JST)

First, the impact of demographic change on the real interest rate is largely unaffected compared with the calibration in section 4.1. When TFP growth is fixed in each period of the transition (at a value implying a long-run output growth of 1.7%) the real interest rate decreases by about 1.1 percentage points between 1990 and 2030, going from a value of about 2.24% in 1990 to 1.13% in 2030. Over the same period the decrease is about 1.86 percentage points (2.6% to 0.74%) if the model is augmented with the actual TFP growth rate, 1.65 percentage points (2.22% to 0.57%) with the HP filtered TFP growth rate.

Second, there is quite a good correspondence between the OLG model's simulations and the econometric estimate of HLW. This correspondence deteriorates after 2008 suggesting that forces other than demographics and TFP might be at play in this period (perhaps due to factors linked to liquidity and risk not captured by the OLG model), determining the low-frequency movements of the real interest rate in the econometric estimate. But this discrepancy might be also due to the relative inaccuracy with which the HLW methodology leads to estimate the natural rate compared to trend growth (see Box 3 in Brand et al. (2018)) and which might magnify its downward trend (see Buncic (2019)).

Third, the model augmented with the actual TFP growth produces a simulated series which mimic very closely the econometric estimate by HLW. But even without the cumbersome utilization of the actual TFP growth series, the overall trend generated by the OLG model captures the dynamics implied by the econometric estimate.[64] This provides support for the prominent role of demographics in explaining the downward trend in the natural rate generally found by econometric estimates, but left unexplained due to the very structure of the econometrics employed. In particular, no matter the long-run value of technological growth, the OLG model predicts a downward impact of aging on the real interest rate at least until 2030.[65]

Finally, in the early part of the simulations, before the 1970s, the high TFP growth rate (which allows to match the observed growth rate of output, see Figure 13) translates inevitably in a relatively high level of the real interest rate. This level is still within the 90-10 percentile range for the MPK and consistent with the levels of the real yields in the data. But it is probably on the high side (especially if compared with the econometric estimate at the world level by DGGT, see Appendix G). Overall, one shall consider with caution the simulation results in the early part of the sample also because of possible front-loading effects that might be linked to the assumption that the system is at the initial stationary equilibrium in 1950.[66] Still, it is remarkable that the model's simulation

"return on everything" dataset. It further plots the econometric estimates of the natural real interest rate of Del Negro et al. (2019) (DGGT) for the world economy.

[64] The Pearson's correlation coefficient between the two series is about 0.8 in the 1972-2018 period, when the model has the actual TFP growth. This coefficient is about 0.75 and 0.76 for the model with the HP filtered TFP growth and with fixed TFP growth, respectively.

[65] When the model assumes a long-run TFP growth such that in the final stationary equilibrium output growth is 1.7% (1.2%, 1.1%) and the implied real interest rate is about 1.63% (1.18%, 1.05%), the real interest rate in 2030 is 1.13% (0.74%, 0.57%). That is, irrespective to the long-run technology growth assumed, the real interest rate in 2030 is lower with respect to its long-run value of about 0.5 percentage points or slightly less.

[66] Also, consider that due to lack of data for all years before 1955 in the transition the TFP growth rate has the same

series capture the overall trend found in the data for the MPK, in the early part of the sample too.

5.3 Isolating the drivers

To isolate the quantitative contribution of the main exogenous drivers in the model, Figure 12 reports the results from three simulations:[67] (i) using the trend TFP growth (which implies a long-run output growth of 1.1%); (ii) fixing the TFP growth rate at the initial level throughout the whole transition (which implies a long-run output growth of 1.7%); (iii) in addition to (ii), fixing the survival probabilities at their first period level throughout the whole transition. To facilitate comparison with section 4.3 (compare Figure 12 with Figure 5) the initial reference year is 1987, which is when the baseline series takes approximately its mean value over the 1980s (normalized to zero in the figure). The series from the first two simulations correspond to the lightblue and black lines in Figure 11, respectively, reported as percentage point deviation from the reference year in Figure 12. Similarly to what discussed in section 4.3, the third exercise allows to isolate the contributions hinging upon changes in the mortality and fertility rates over time.

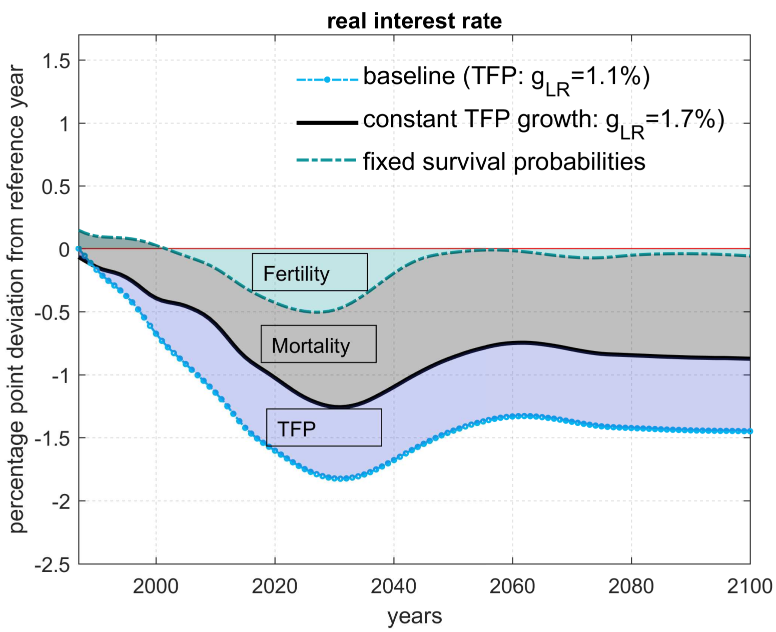

Figure 12: Real interest rate in EA12: drivers

Note. Calibration of Table 4. The results are obtained by running the simulations fixing the conditional survival probabilities at the their first period values and assuming that the economy has the TFP growth of the initial steady state throughout the whole transition (cf. black continuous line in Figure 11). The reference year is 1987 (when the baseline series takes approximately its mean value over the 1980s, normalized to zero).

Compared to the average in the 1980s, in the baseline – with time-varying TFP growth rate

(relatively high) value it has in 1955, see Appendix E.

[67] Compared to section 4.3 the only difference here is the presence of non-zero TFP growth.

(as captured by the trend component of the Solow residual) – the real interest rate is about 1.8 percentage points lower in 2030 when the series reaches its trough. Of these, about 0.55 percentage points are due to a lower TFP growth (cf. the "TFP" shaded area in Figure 12); the remaining 1.25 percentage points are due to demographics, of which 0.75 can be attributed to changes in the survival probabilities and the residual 0.5 to changes in the growth rate of the number of newborns in the model (cf. the "Mortality" and "Fertility" shaded areas in Figure 12, respectively).

Equally to what found in section 4.3, fertility is responsible for the rebouncing of the real interest rate after 2030 while the permanently higher survival probabilities and lower TFP growth rate lead the real interest rate in the very long-run to be permanently lower compared to the mean in the 1980s. In 2100 the real interest rate is about 1.5 percentage points lower than the mean in the 1980s, of which 0.55 are due to TFP growth and the remainder to the survival probabilities.

These results confirm the orders of magnitude reported in Figure 5, adding the contribution of TFP growth. If one admits that demographics and TFP growth are the main drivers of the real interest rate (given the closeness of the econometric and OLG model's results, cf. Figure 11), one can argue that demographics account for about two-thirds of the projected decline of the real interest rate in the euro area until 2030 compared to the 1980s average (cf. Bielecki et al. (2020)).

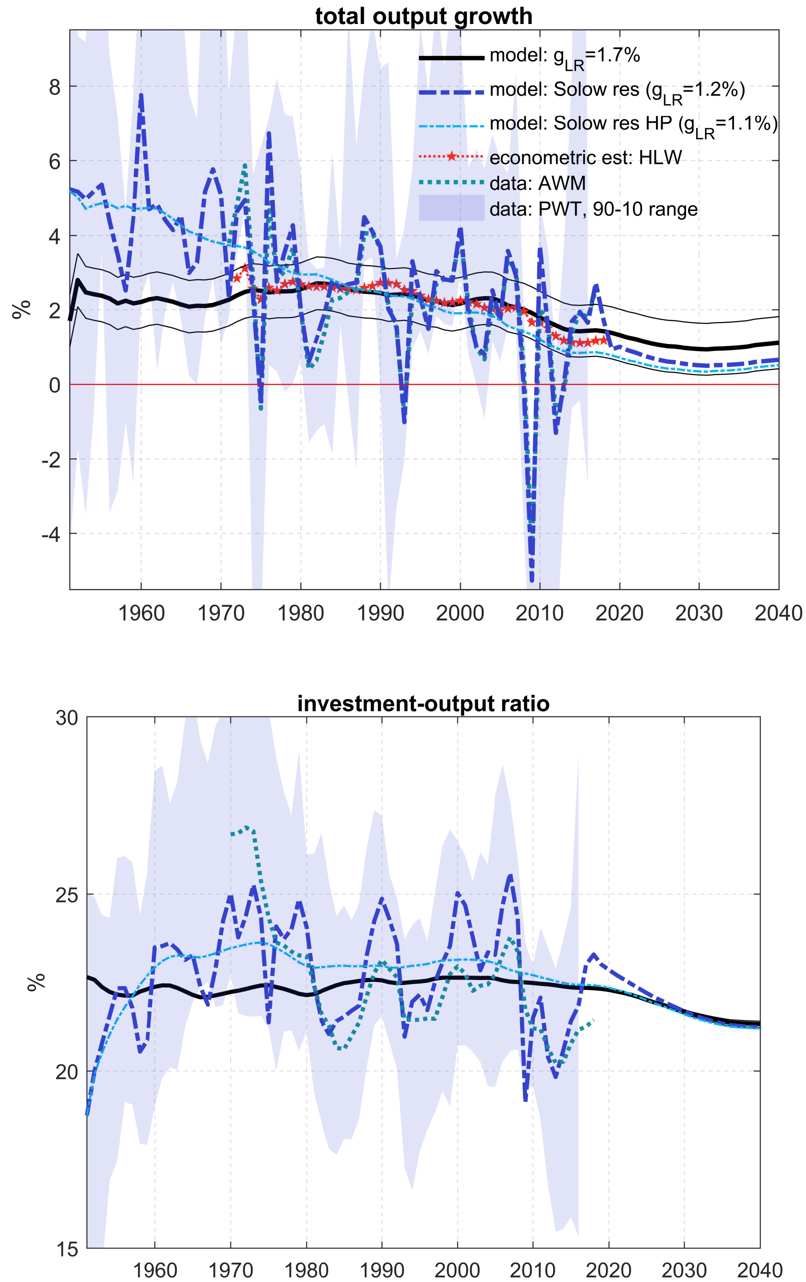

Figure 13: Model vs data: total output growth and investment-output ratio, euro area

Note. The long-run growth imposed in the final stationary equilibrium is denoted by $g_{LR} = (g^Z)^{1/(1-\psi)}$. See Appendix E for the Solow residual used in the simulations: actual and Hodrick-Prescott (HP) filtered. Total output growth econometric estimate corresponding to "trend growth rate of the natural rate of output" in Holston et al. (2017) (HLW). Data from the "Area Wide Model" (AMW) dataset refer to the annualized growth rate of the `YER` series (Gross Domestic Product (GDP) at market prices, Chain linked volume, Reference year 1995") and to the ratio of `IER` (Gross Fixed Capital Formation, Chain linked volume, Reference year 1995) over `YER`. The shaded area denotes the 90–10 percentile range for the twelve EA12 countries, where the series used are from the Penn World Table (PWT) national accounts: `q_gdp` ("GDP at constant national 2011 prices"), `q_i` ("Investment at constant national 2011 prices"). The thin black lines mark a band corresponding to $g_{LR} = 1.7\% \pm 0.6\%$.

6 Concluding remarks

By means of a large-scale overlapping generation (OLG) model, this paper finds that demographic change has a significant impact on the natural real interest rate for the euro area: a slight upward pressure in the 70s and 80s and a prolonged and marked downward pressure that extends at least until 2030 as the aging process unfolds (according to UN (2017) demographic projections). The model predicts in the baseline a decrease of the natural real interest rate of about 1.4 percentage point going to 2030 compared to the average in the 1980s. This estimate is never significantly dampened according to a set of sensitivity specifications which, if anything, suggest that it is a conservative number. The results indicate that the downward impact of aging could be mitigated not only by higher substitutability in production between labor and capital and higher intertemporal elasticity of substitution in consumption, but also by reforms aiming particularly at increasing the relative productivity of older cohorts and the participation rate. Increasing the retirement age per se has a very limited, if not null, mitigating effect.

Two drivers prevail explaining why aging has a downward total impact on the natural real interest rate: effective labor supply becomes scarcer and individuals increase their willingness to save in anticipation of higher survival probabilities. The latter driver is found to have relative more prominence in explaining the downward pressure of aging on the natural real interest rate especially in the very long-run. The presence of a pay-as-you-go pension system with defined benefits can in part offset this downward pressure by crowding-out productive capital but cannot significantly overturn it.

Importantly, the simulation results from the OLG model finds correspondence in econometric estimates of natural outcomes (real interest rate and output growth). Hence the results from the OLG model allow to qualify the determinants of agnostic econometric estimates and provide a forecast relying on demographic projections.

Finally, while the ongoing demographic transition seems to have definite time-lags, that allow to have a relatively clear picture of demographics in the future, its macroeconomic impact going forward might be altered by fundamental changes in the underlying structure of the economy. Such changes might comprise not only variations of parameters that the model in this paper allowed to control for (such as potential changes in the elasticity of substitution between labor and capital in production, intertemporal elasticity of substitution in consumption, productivity by age), but also technological changes that might alter the very nature of the production function (for example via the adoption of automation technologies) or processes of endogenous growth involving investment in human capital, as well as issues related to public debt sustainability and social tenability. All these potential changes lie outside the scope of the model adopted in this paper but offer interesting avenues for future research speculations.

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Appendix

A Additional figures

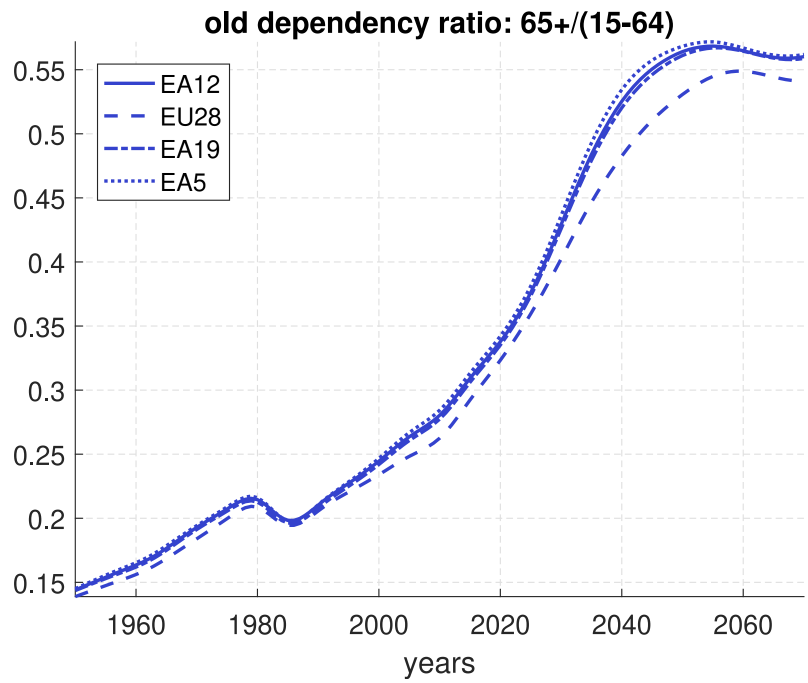


Figure A.1: Old dependency ratio in Europe

Note. The indicator in the figure is the the number of people aged more than 64 over the number of people aged between 15 and 64. Data from the United Nations (UN, 2017) *World Population Prospects: The 2017 Revision*, medium variant after year 2016 (cf. footnote 18). The following groups of countries hold. EA19: Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, Netherland, Portugal, Slovakia, Slovenia, Spain; EA12 is EA19 excluding Cyprus, Estonia, Latvia, Lithuania, Malta, Slovakia, Slovenia; EA5: France, Germany, Italy, Netherlands, Spain; EU28 comprises EA19 and the following non-EA members: Bulgaria, Croatia, Czech Republic, Hungary, Poland, Romania, Sweden, Denmark and United Kingdom.

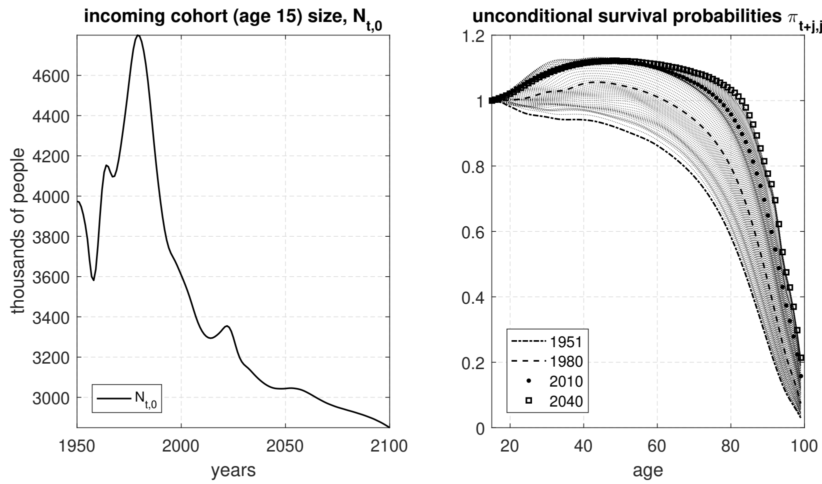


Figure A.2: Euro area (EA12) demographics in the model

Note. Data are from UN (2017) including medium variant projections until year 2100. The unconditional survival probabilities $\pi_{t+j,j}$ are retrieved applying the recursive formula $N_{t+j,j} = \pi_{t+j,j} N_{t,0}$ using data for the cohort size $N_{t,j}$ for each year t and age-bin j, with $N_{t,0}$ corresponding to the incoming cohort size, those aged 15. The marked lines correspond to $\pi_{\tau+j,j}$ for $\tau \in \{1951, 1980, 2010, 2040\}$ for all ages j in the model, the other lines capture the profiles in the remaining years.

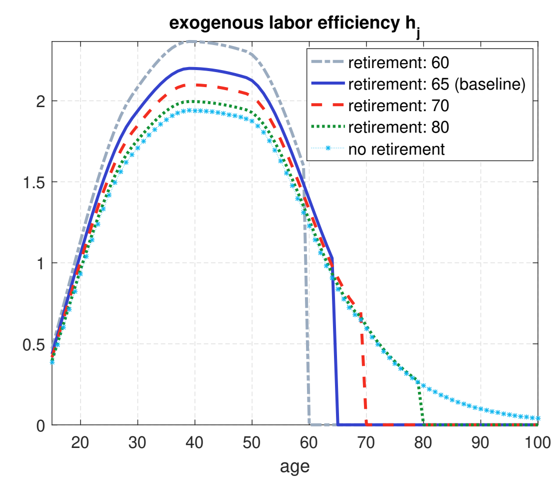


Figure A.3: Age dependent labor supply in efficiency units, h_j

Note. The profile is obtained with a cubic interpolation (for age 15 to 70) on the data points provided in Domeij and Flodén (2006). These data points are the product of participation rates provided by Fullerton (1999) and productivity provided by Hansen (1993). Lacking data, for $j \geq 70$ the profile is obtained from the following logistic function: $C/(1 + Ae^{-Bj})$, with $A = .49$, $C = 50$, $B = (1/70) \log [h_{70} A/(C - h_{70})]$. The blue continuous line denotes the baseline profile with exogenous retirement age at $jr + 1 = 65$. For each exogenous retirement age, the productivity profile h_j is normalized across the ages such that its mean is equal to 1.

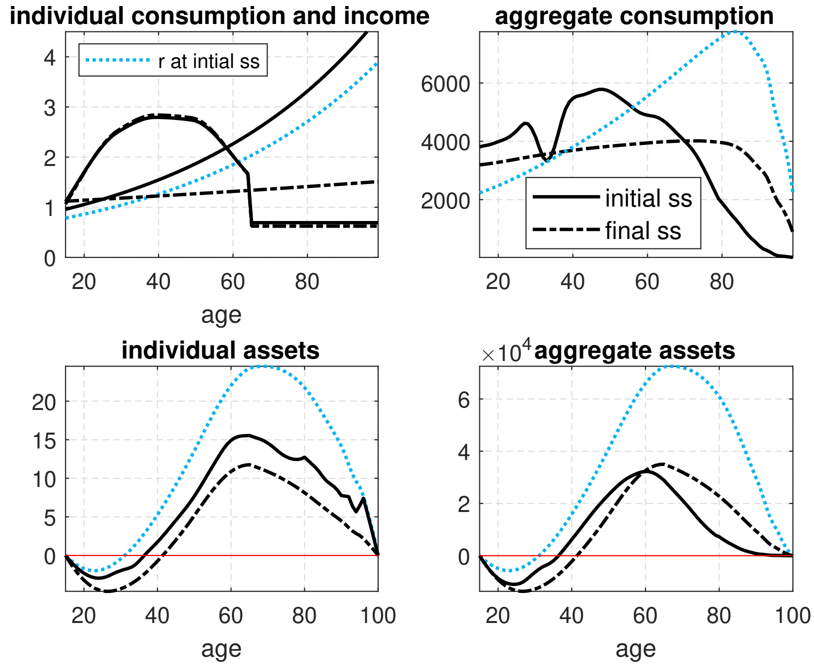


Figure A.4: Age profiles: initial versus final steady state

Note. Baseline calibration, see Table 1. The dotted lines show the hypothetical (partial equilibrium) final steady state scenario in which the real interest rate is fixed at the initial steady state value.

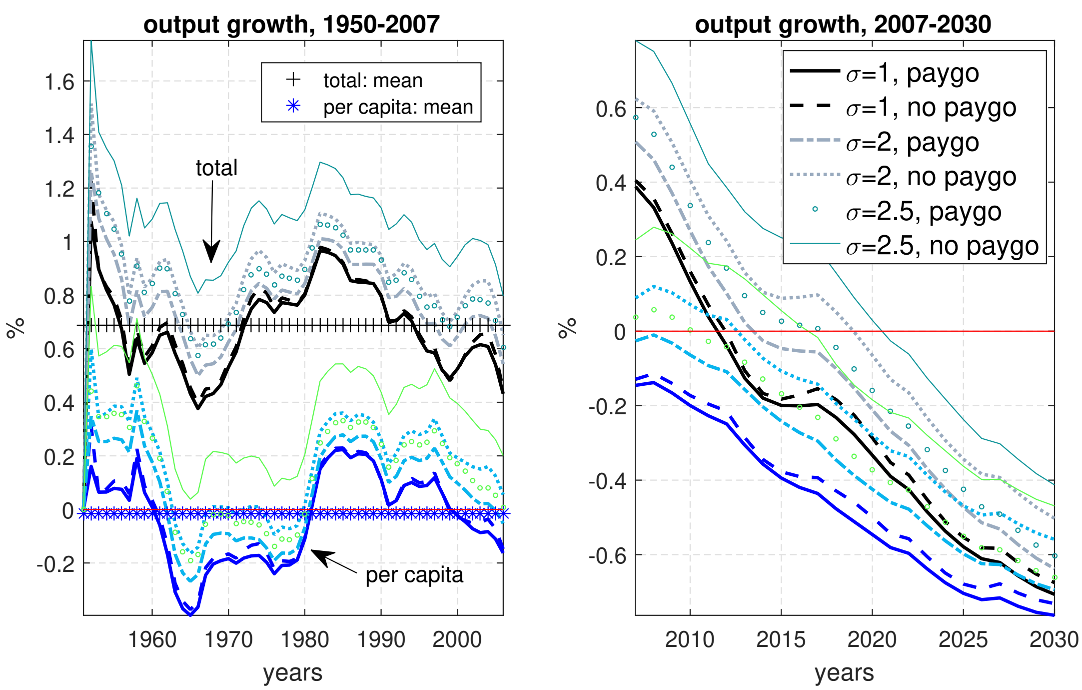


Figure A.5: Growth rate of output: different sample periods

Note. The sample means in the first panel refer to the baseline simulation: $\sigma = 1$ with PAYGO pension system.

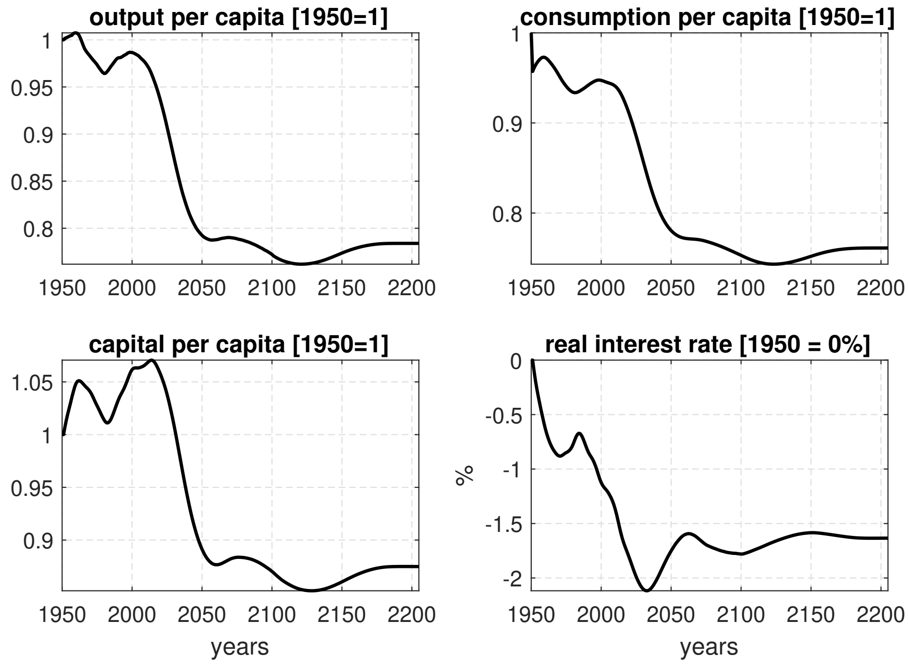


Figure A.6: EA12 transition dynamics: main aggregates per capita

Note. Baseline calibration, see Table 1

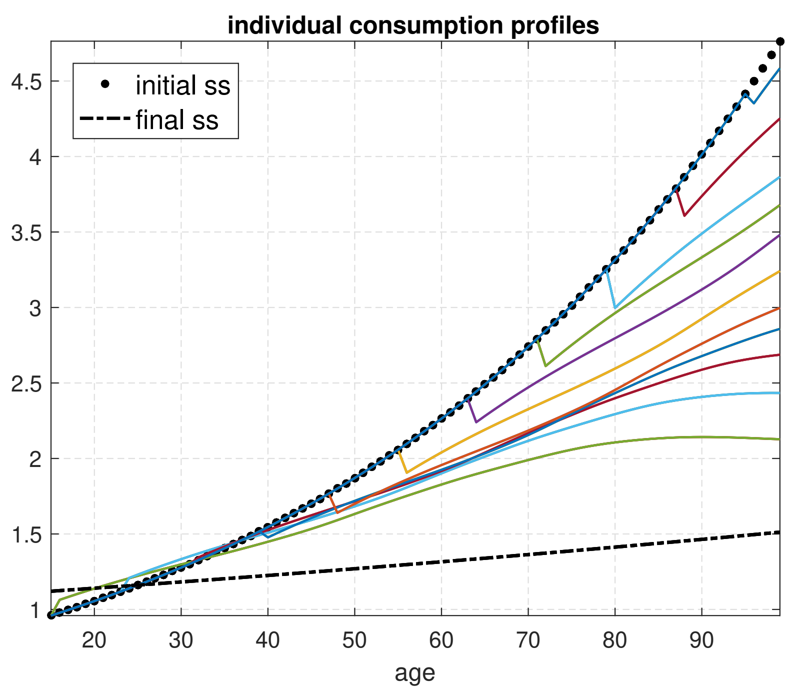
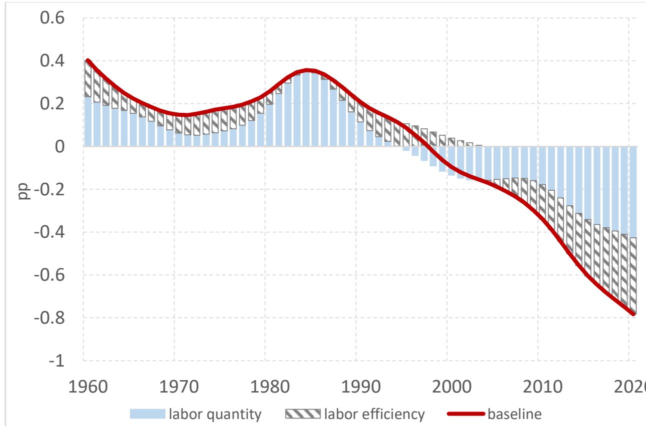
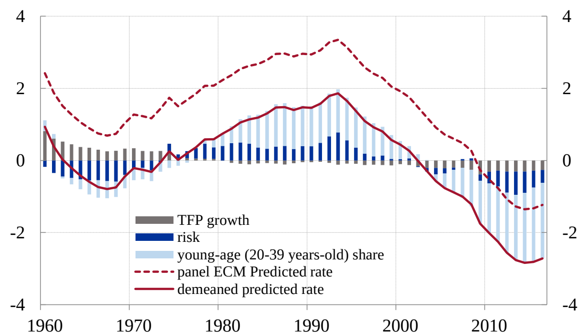


Figure A.7: Individual consumption profiles: selected cohorts versus initial and final steady state

Note. Baseline calibration, see Table 1.



(a) Theoretical model



(b) Econometric model (see Brand et al. (2018))

Figure A.8: Natural real interest rate, demeaned: theoretical vs econometric model: drivers

***Note* to A.8a**. The real interest rate path of the model is obtained with the baseline specification of Figure 4. Since labor in efficiency units (L_t) depends on two parameters, their different impact is considered in isolation: "labor quantity" denotes the impact of the mere number of people in the labor force (aged between 15 and 64); "labor efficiency" denotes the impact of the age-varying productivity (technically, it is obtained by running the model twice: first, when L_t is the actual one: $\sum_{j=0}^{jr} h_j N_{t,j}$; second, considering $L_t = \sum_{j=0}^{jr} N_{t,j}$, i.e. by setting $h_j = 1$ for all j (see section 4.4.3). Finally, the difference between the two implied curves is taken). All the implied series are demeaned over the period 1960–2020 in order to produce the standardized results in the figure.

***Note* to A.8b**. Estimates of the natural real interest rate provided by Fiorentini et al. (2018) for the Brand et al. (2018)'s report from a panel error correction model (ECM) at annual frequency over the period 1899-2016. The unbalanced panel of advanced economies includes the following 17 countries: Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, and US. The observed real rate serves as dependent variable in the ECM, while some indicators about total factor productivity, demographics and risk serve as regressors. Data on TFP growth comes from Penn World Tables and Total Economy Database by The Conference Board `https://www.conference-board.org/data/economydatabase/`; data on demographic composition come from the Human-Mortality-Database `http://mortality.org`. The spread between long-term and short-term interest rate is used as proxy for the term premium to measure the time-varying risk aversion of agents. Interest rates data come from the Jordá et al. (2019) Macro history Database and from the OECD Main Economic Indicators database.

B OLG model with zero technology growth

B.1 Set of equilibrium equations

In an equilibrium with perfect foresight, the following optimal conditions hold (labor augmenting technology $A_t = 1\ \forall t$).

Households:

$$\text{for } j = 0, 1, ..., J-1: \quad c_{t+1,j+1} = [\beta(1 + r_{t+1})]^{\frac{1}{\sigma}} c_{t,j} \tag{B.1}$$

$$\text{for } j = 0, 1, ..., J: \quad c_{t,j} + a_{t+1,j+1} = \frac{a_{t,j}(1 + r_t)}{s_{t,j}} + w_t h_j (1 - \tau_t) I(j < j_r) + d_t I(j \geq j_r) \tag{B.2}$$

with: $a_{t,0} = 0$

$a_{t,J+1} = 0$

Government:

$$\tau_t = \frac{\bar{d}\bar{h} \sum_{j=j_r}^{J} N_{t,j}}{L_t + \bar{d}\bar{h} \sum_{j=j_r}^{J} N_{t,j}} \tag{B.3}$$

$$d_t = \bar{d}\bar{h} w_t (1 - \tau_t) \tag{B.4}$$

Firms:

$$r_t + \delta = \left(\frac{Y_t}{K_t}\right)^{\frac{1}{\rho}} \tag{B.5}$$

$$w_t = (1 - \psi)\left(\frac{Y_t}{L_t}\right)^{\frac{1}{\rho}} \tag{B.6}$$

Clearing:[68]

$$K_{t+1} = \mathcal{A}_t, \quad \mathcal{A}_t = \sum_{j=0}^{J} a_{t+1,j+1} N_{t,j} \tag{B.7}$$

$$C_t + K_{t+1} - (1 - \delta)K_t = Y_t, \quad C_t = \sum_{j=0}^{J} c_{t,j} N_{t,j} \tag{B.8}$$

Check: 177 equations for 177 endogenous unknowns ($J = 84$):

$$\{c_t\}_{j=0}^{J}, \{a_t\}_{j=1}^{J}, r_t, w_t, \tau_t, d_t, K_t, \mathcal{A}_t, C_t, Y_t$$

[68] Notice that the current decision on savings determine the next-period level of capital. That is, as it is standard, capital is a predetermined variable.

B.2 Steady state

A steady state is defined as a stationary equilibrium where the exogenous demographic variables assume a fixed value forever. Economic growth is zero in such equilibrium with zero technology growth. Variables in steady state have no time subscript. The following system of optimal conditions hold.

$$
\begin{aligned}
L &= \sum_{j=0}^{J} h_j N_j \\
r &= \left(\frac{K}{Y}\right)^{-\rho} - \delta \\
w &= (1-\psi)\left(\frac{r+\delta}{}\right)\left(\frac{K}{L}\right)^{\frac{1}{\rho}} \\
\tau &= \frac{\overline{dh}\sum_{j=j_r}^{J} N_j}{L + \overline{dh}\sum_{j=j_r}^{J} N_j}, \quad \bar{h} = \sum_{j=0}^{J} h_j / j_r \\
d &= \overline{dh}(1-\tau)w \\
y_j &= (1-\tau) w h_j I(j < j_r) + d I(j \geq j_r), \quad \text{for } j = 0, 1, ..., J \\
c_0 &= \frac{\sum_{j=0}^{J}\left(\frac{1}{1+r}\right)^j \pi_j y_j}{\sum_{j=0}^{J}\left(\frac{1}{1+r}\right)^j \pi_j (\beta(1+r))^{\frac{j}{\sigma}}} \\
c_{j+1} &= [\beta(1+r)]^{\frac{1}{\sigma}} c_j, \quad \text{for } j = 0, 1, ..., J-1 \\
a_{j+1} &= \frac{a_j(1+r)}{s_j} + y_j - c_j, \quad \text{for } j = 0, 1, ..., J \\
a_0 &= 0 \\
a_{J+1} &= 0 \\
C &= \sum_{j=0}^{J} c_j N_j \\
K &= \sum_{j=0}^{J} a_j N_j \\
Y &= \left[\psi K^{\frac{\rho-1}{\rho}} + (1-\psi) L^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \\
C + \delta K &= Y
\end{aligned}
$$

Exogenous demographics: $\{N_j\}_{j=0}^{J}$, from which the survival probabilities are derived: $s_j = N_j/N_{j-1}$ with $N_{-1} = N_0$ (such that $s_0 = 1$), $\pi_j = \prod_{k=0}^{j} s_k$; parameters: $\{h_j\}_{j=0}^{J}, j_r, \psi, \delta, \bar{d}, \beta, \sigma, \rho$.

C OLG model with non-zero TFP growth

C.1 Set of scaled equilibrium equations

Consider the model with production function (5.1): $Y_t = Z_t K_t^{\psi} L_t^{1-\psi}$, $Z_t = g_t^Z Z_{t-1}$. To have stationarity in presence of technology (TFP) growth g_t^Z, each variable $x_t \in \{c_t, a_t, d_t, K_t, C_t, Y_t\}$ is scaled by the level of technology and effective labor:[69]

$$\widetilde{x}_t = x_t/(Z_t^{\frac{1}{1-\psi}} L_t)$$

Further, have $g_t^L = L_t/L_{t-1}$ and $\widetilde{w}_t = w_t/(Z_t^{\frac{1}{1-\psi}})$. Then the following optimal conditions hold:

Households:

$$\text{for } j = 0,1,...,J-1: \quad \widetilde{c}_{t+1,j+1} = [\beta(1+r_{t+1})]^{\frac{1}{\sigma}} \widetilde{c}_{t,j} \left[(g_{t+1}^Z)^{\frac{1}{1-\psi}} g_{t+1}^L \right]^{-1} \tag{C.1}$$

$$\text{for } j = 0,1,...,J: \quad \widetilde{c}_{t,j} + \widetilde{a}_{t+1,j+1} (g_{t+1}^Z)^{\frac{1}{1-\psi}} g_{t+1}^L = \widetilde{a}_{t,j}(1+r_t)/s_{t,j} + \tag{C.2}$$
$$+ \widetilde{w}_t (h_j/L_t)(1-\tau_t) I(j<j_r) + \widetilde{d}_t I(j \geq j_r)$$

$$\text{with:} \quad \widetilde{a}_{t,0} = 0, \quad \widetilde{a}_{t,J+1} = 0 \tag{C.3}$$

Government:

$$\tau_t = \frac{\bar{d}\bar{h} \sum_{j=j_r}^{J} N_{t,j}}{L_t + \bar{d}\bar{h} \sum_{j=j_r}^{J} N_{t,j}} \tag{C.4}$$

$$\widetilde{d}_t = \bar{d}(\bar{h}/L_t)\widetilde{w}_t(1-\tau_t) \tag{C.5}$$

Firms:

$$r_t + \delta = \frac{\widetilde{Y}_t}{\widetilde{K}_t} \tag{C.6}$$

$$\widetilde{w}_t = (1-\psi)\widetilde{Y}_t \tag{C.7}$$

Clearing (notice: $\widetilde{\mathcal{A}}_t = \mathcal{A}_t/(Z_{t+1}^{\frac{1}{1-\psi}} L_{t+1})$):

$$\widetilde{K}_{t+1} = \widetilde{\mathcal{A}}_t, \quad \widetilde{\mathcal{A}}_t = \sum_{j=0}^{J} \widetilde{a}_{t+1,j+1} N_{t,j} \tag{C.8}$$

$$\widetilde{C}_t + \widetilde{K}_{t+1}(g_{t+1}^Z)^{\frac{1}{1-\psi}} g_{t+1}^L - (1-\delta)\widetilde{K}_t = \widetilde{Y}_t, \quad \widetilde{C}_t = \sum_{j=0}^{J} \widetilde{c}_{t,j} N_{t,j} \tag{C.9}$$

[69] A balanced-growth-path (BGP) is characterized by constant effective capital. To characterize what is effective capital, and thus find the scaling factor in the model, consider that on a BGP the capital-output ratio needs to be constant. Its inverse reads:

$$\frac{Y_t}{K_t} = \frac{Z_t K_t^{\psi} L_t^{1-\psi}}{K_t} = \left(\frac{K_t}{Z_t^{\frac{1}{1-\psi}} L_t} \right)^{\psi-1}$$

which is constant if and only if $\widetilde{K}_t \equiv K_t/(Z_t^{\frac{1}{1-\psi}} L_t)$ is constant.

C.2 Balanced growth path

A balanced-growth-path (BGP) is defined as a stationary equilibrium where the exogenous demographic variables and the exogenous growth rate of technology assume a fixed value forever. Scaled variables are constant over time so that each variable grow at a constant rate over time determined by the exogenous growth rate of technology. That is, for each $x_t \in \{c_t, a_t, d_t, \{y_{t,j}\}_{j=0}^{J}, K_t, C_t, Y_t\}$ their respective constant growth rate on a BGP is given by:[70]

$$g_t^x = x_t/x_{t-1} = (g^Z)^{\frac{1}{1-\psi}} \quad \forall t$$

Variables on a BGP have no time subscript. The following system of optimal scaled conditions hold.

$$\begin{aligned}
L &= \sum_{j=0}^{J} h_j N_j \\
\widetilde{w} &= (1-\psi)\left(\frac{}{r+\delta}\right)^{\frac{}{1-\psi}} \\
\tau &= \frac{\bar{d}\bar{h}\sum_{j=j_r}^{J} N_j}{L + \bar{d}\bar{h}\sum_{j=j_r}^{J} N_j}, \quad \bar{h} = \sum_{j=0}^{J} h_j/j_r \\
\widetilde{d} &= \bar{d}(\bar{h}/L)(1-\tau)\widetilde{w} \\
\widetilde{y}_j &= (1-\tau)\widetilde{w}(h_j/L)I(j < j_r) + \widetilde{d}I(j \geq j_r), \quad \text{for } j = 0, 1, ..., J \\
\widetilde{c}_0 &= \frac{\sum_{j=0}^{J} \left(\frac{1}{1+r}\right)^j \pi_j \widetilde{y}_j (g^Z)^{\frac{j}{1-\psi}}}{\sum_{j=0}^{J} \left(\frac{1}{1+r}\right)^j \pi_j (\beta(1+r))^{\frac{j}{\sigma}}} \\
\widetilde{c}_{j+1} &= [\beta(1+r)]^{\frac{1}{\sigma}} \widetilde{c}_j (g^Z)^{-\frac{1}{1-\psi}}, \quad \text{for } j = 0, 1, ..., J-1 \\
\widetilde{a}_{j+1} &= \{\widetilde{a}_j(1+r)/s_j + \widetilde{y}_j - \widetilde{c}_j\}(g^Z)^{-\frac{1}{1-\psi}}, \quad \text{for } j = 0, 1, ..., J \\
\widetilde{a}_0 &= 0 \\
\widetilde{a}_{J+1} &= 0 \\
\widetilde{C} &= \sum_{j=0}^{J} \widetilde{c}_j N_j \\
\widetilde{K} &= \sum_{j=0}^{J} \widetilde{a}_j N_j \\
\widetilde{Y} &= \widetilde{K} \\
\widetilde{C} + [(g^Z)^{\frac{1}{1-\psi}} - 1 + \delta]\widetilde{K} &= \widetilde{Y}
\end{aligned}$$

Exogenous demographics $\{N_j\}_{j=0}^{J}$, from which the survival probabilities are derived: $s_j = N_j/N_{j-1}$ with $N_{-1} = N_0$ (such that $s_0 = 1$), $\pi_j = \prod_{k=0}^{j} s_k$; parameters: $\{h_j\}_{j=0}^{J}, j_r, \psi, \delta, \bar{d}, \beta, \sigma, g^Z$.

[70] Notice that the steady state studied in (B.2) represents simply a special case of BGP where the TFP growth rate is zero, i.e. $g^Z = 1$.

D Two-period version of the model

The two-period economy described in section 3 of the main text is fully characterized by the following equations.

Firms' first order conditions:

$$r_t + \delta = \psi k_t^{\psi-1} \tag{D.1}$$
$$w_t = (1-\psi) A_t k_t \tag{D.2}$$

Social security:

$$d_{t+1} = \tau_{t+1} w_{t+1} \frac{g_{t+1}^N}{s_{t+1,1}} \tag{D.3}$$
$$d_t = \bar{d} w_t (1-\tau_t) \tag{D.4}$$

Households' Euler equation and budget constraint:

$$c_{t+1,1} = \beta(1+r_{t+1}) c_{t,0} \tag{D.5}$$
$$c_{t,0} + a_{t+1,1} = (1-\tau_t) w_t \tag{D.6}$$
$$c_{t+1,1} = \frac{a_{t+1,1}(1+r_{t+1})}{s_{t+1,1}} + d_{t+1} \tag{D.7}$$

Capital market clearing and law-of-motion of capital:

$$g_{t+1}^N A_{t+1} k_{t+1} = a_{t+1,1} \tag{D.8}$$
$$g^A g_{t+1}^N k_{t+1} = (1-\delta) k_t + i_t \tag{D.9}$$

Analysis. To derive an expression for the households' savings supply, plug (D.6) into (D.7) and then use (D.5) to have:

$$\beta(1+r_{t+1}) c_{t,0} = \frac{[(1-\tau_t) w_t - c_{t,0}](1+r_{t+1})}{s_{t+1,1}} + d_{t+1}$$

Using (D.3) and rearranging, it results:

$$c_{t,0} = \frac{1}{1+\beta s_{t+1,1}} \left[(1-\tau_t) w_t + \tau_{t+1} w_{t+1} \frac{g_{t+1}^N}{1+r_{t+1}} \right]$$

Equation (D.3) needs to be used also into (D.4) to have the equilibrium tax rate:

$$\tau_{t+1} = \frac{\bar{d}}{\bar{d} + \frac{g_{t+1}^N}{s_{t+1,1}}} \tag{D.10}$$

Hence, the household's savings from (D.6) read:

$$a_{t+1,1} = (1-\tau_t) w_t \frac{\beta s_{t+1,1}}{1+\beta s_{t+1,1}} - \tau_{t+1} w_{t+1} \frac{g_{t+1}^N}{(1+r_{t+1})(1+\beta s_{t+1,1})}$$

First use (D.2):

$$a_{t+1,1} = (1-\tau_t)(1-\psi)A_t k_t \frac{\beta s_{t+1,1}}{1+\beta s_{t+1,1}} - \tau_{t+1}(1-\psi)A_{t+1}k_{t+1}^{\psi}\frac{g_{t+1}^N}{(1+r_{t+1})(1+\beta s_{t+1,1})}$$

then use (D.8) and simplify to have:

$$k_{t+1} = (1-\tau_t)(1-\psi)k_t \frac{\beta s_{t+1,1}}{g^A g_{t+1}^N(1+\beta s_{t+1,1})} - \frac{\tau_{t+1}(1-\psi)k_{t+1}^{\psi}}{(1+r_{t+1})(1+\beta s_{t+1,1})} \tag{D.11}$$

Only two equations have not been used to arrive to this result: (D.1) which identifies the capital demand by firms and the law-of-motion of capital (D.9). The goal now is to characterize the economy in terms of capital demand by firms and capital supply by households where the unique endogenous variables are the real interest rate r_t and the aggregate saving rate. The latter equals the investment-output ratio (due to the assumption of closed economy) which from (D.9) reads:

$$\iota_t \equiv \frac{I_t}{Y_t} = \frac{i_t}{y_t} = \frac{g^A g_{t+1}^N k_{t+1} - (1-\delta)k_t}{k_t} \tag{D.12}$$

Consider a *balanced-growth path* (BGP) characterized by a constant effective capital stock: $k_t = k$ for all t. From (D.12) the saving rate on a BGP reads:

$$\iota = [g^A g^N - (1-\delta)]k^{1-\psi} \tag{D.13}$$

where $k^{1-\psi}$ is the capital-output ratio. Plug (D.13) into (D.1) to have:

$$\iota^D : \boxed{r = \frac{g^A g^N - (1-\delta)}{\iota} - \delta} \tag{D.14}$$

which represents the *firms' capital demand* ι^D on the ι, r-Cartesian plane.

To have a comparable expression on the households' side, add and subtract $(1-\delta)k_t$ to (D.11) and evaluate on a BGP to have:

$$\frac{g^A g^N k - (1-\delta)k}{k^{\psi}} = (1-\tau)(1-\psi)\frac{\beta s_1}{(1+\beta s_1)} - \frac{g^A g^N \tau(1-\psi)}{(1+r)(1+\beta s_1)} - (1-\delta)\frac{k}{k^{\psi}}$$

Using (D.13), it reduces to:

$$\iota = \left(1 - \frac{1-\delta}{g^A g^N}\right)\left[(1-\tau)(1-\psi)\frac{\beta s_1}{(1+\beta s_1)} - \frac{g^A g^N \tau(1-\psi)}{(1+r)(1+\beta s_1)}\right]$$

To have $1+r$ on the left-hand-side, the expression can be rearranged to have:

$$\iota^S : \boxed{r = \frac{\tau}{(1-\tau)\frac{\beta s_1}{g^A g^N} - \iota\frac{1+\beta s_1}{(1-\psi)[g^A g^N - (1-\delta)]}} - 1} \tag{D.15}$$

Assuming a PAYGO pension system with fixed replacement rate, the tax rate is given by (D.10) on a BGP: $\tau = \bar{d}/(\bar{d} - g^N/s_1)$.

E Solow residual in the euro area

The Solow residual is identified by the TFP component in the Cobb-Douglas production function employed in the OLG model:

$$Z_t = \frac{Y_t}{K_t^{\psi} L_t^{1-\psi}} \tag{E.1}$$

using data on output (Y_t), capital (K_t), labor (L_t) and imposing a value for the capital bias ψ. Recall that for the sake of the OLG model and economic interpretation only its growth rate matters: $g_t^Z = Z_t/Z_{t-1}$.

The main series are computed using the "Area Wide Model" (AWM) dataset (`https://eabcn.org/page/area-wide-model`) which provides main macroeconomic series at the quarterly frequency for the euro area as a whole in the period 1970-2017.[71] To have a value for 2018 the same procedure used on AMW data is applied on data from the "World Development Indicators" (WDI, `wdi.worldbank.org`) summing the series for the 12 euro area countries forming EA12 which are available for the period 1970-2018.[72] The resulting Solow residual growth rates obtained employing the two dataset are essentially identical except for the 4 years before 1975 casting doubt on data reliability before that. To obviate this, for the period 1955-1975 the Solow residual growth rate comes from the the growth rate of "TFP at constant national prices (2011=1)" (`rtfpna`) provided by the "Penn World Table" (PWT, `https://www.rug.nl/ggdc/productivity/pwt/`). For the period 1975-2018 the series on TFP growth provided by PWT and the one obtained applying (E.1) with AMW (and WDI) data are very similar in level and co-move very strongly with a Pearson's correlation coefficient of about 0.996. The series obtained from the AWM (and WDI) dataset for the period 1975-2018 is preferred due to its closer theoretical correspondence with the OLG model.

The data used to measure the annual Solow residual in the euro area for the period 1970-2017 are the following:

- *Output*. Annualized series `YER` ("Gross Domestic Product (GDP) at market prices, Million Euro, Chain linked volume, Calendar and seasonally adjusted data, Reference year 1995") in the AMW dataset.

- *Capital*. The capital stock in the euro area is built on the basis of the annualized series `IER` ("Gross Fixed Capital Formation, Millions of euros, Chain linked volume, Calendar and seasonally adjusted data, Reference year 1995") in the AMW dataset. Following standard practice (cf. e.g. Caselli (2005)) the perpetual inventory method is applied. The initial capital stock (the base year is 1970, the first year data are available) is computed using the formula: $K_{1970} = I_{1970}/(g_I + \delta)$ where I_{1970} corresponds to the gross capital formation in 1970, g_I is the average growth rate (about 1.657% in the period 1971-2017), while δ_K is set to 11% (the same value set in the OLG model calibration with non-zero TFP growth). The capital stock is obtained via the law-of-motion: $K_{t+1} = (1-\delta)K_t + I_t$ for all years t from 1970 to 2017.

[71] For a description of this database, see Fagan et al. (2001).

[72] EA12 is composed by the following countries: Austria (AT), Belgium (BE), Finland (FI), France (FR), Germany (DE), Greece (EL), Ireland (IE), Italy (IT), Luxembourg (LU), Netherlands (NL), Portugal (PT), Spain (ES).

- *Labor*. To be consistent with the OLG model data on labor are the same ones employed in the OLG model's simulations. That is, $L_t = h_j N_{t,j}$ measures the *effective* labor input where the profile h_j (Figure A.3) takes into account productivity and participation rates by age with exogenous retirement at age 65, and $N_{t,j}$ is the respective number of people for each year t for all ages j between 15 and 65 provided by UN (2017).

Setting the parameter ψ to 0.22 (to be consistent with the OLG model calibration) the series for output, capital and labor are used to measure the Solow residual Z_t in (E).

Figure E.1 plots the annual growth rate of the resulting Solow residual in the euro area used in the OLG model's simulations (recall that before 1975 the series comes from the estimates of TFP growth provided in the PWT). For each year in the period 1950-1954 the growth rate is simply set to the value in 1955 (the first year data are available). In the projected horizon it is assumed that the series reverts to its unconditional mean implied by an autoregressive process of order 1, AR(1), estimated over the period 1999-2018 as detailed in the note under the figure.

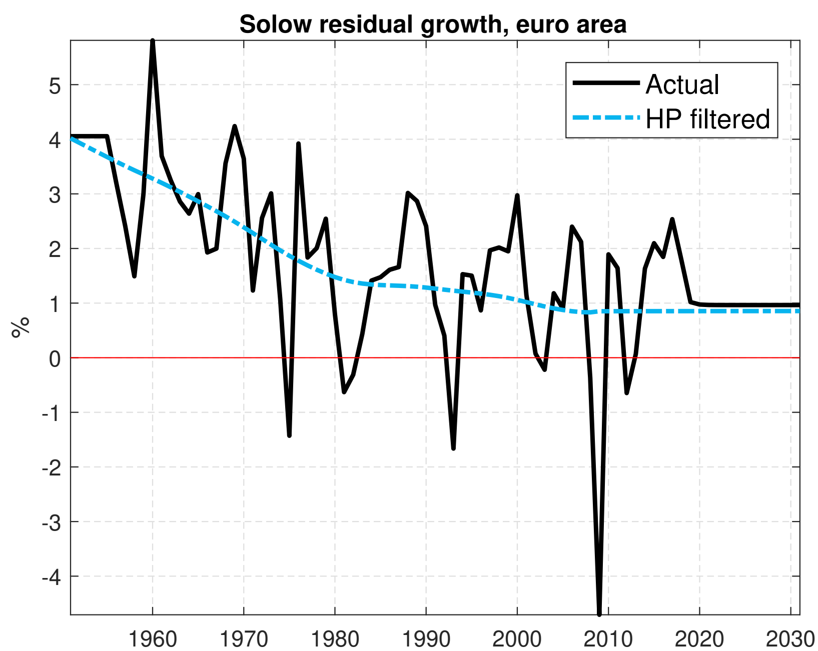


Figure E.1: Solow residual: annual growth rate, euro area

Note. "Actual" denotes the growth rate of the TFP component in (E) for the period 1975–2018, while for the period 1950–1974 the series is from the TFP growth provided by the PWT, as explained in the main text. It is assumed that in the projected horizon (after 2018) the series follows an AR(1) process $z_t = \theta_1 + \theta_2 z_{t-1}$ where θ_1, θ_2 are estimated over the 1999-2018 period. The estimation implies that in the long-run the series reverts to its unconditional mean: $\theta_1/(1-\theta_2) = 0.00964$. "HP filtered" denotes the series resulting from the application of the Hodrick–Prescott filter with a multiplier equal to 1000 (a similar high value is used by Gagnon et al. (2016) in the context of OLG simulations for the US). In this case the series is assumed to revert to its unconditional mean beginning with year 2007, where this mean results from the estimation of an AR(1) process over the entire sample period 1951–2018.

F Measures of real interest rate in the euro area

To compare the model's simulation results with the data a set of different measures of the real interest rate is used (see Figure 11) for each of the EA12 countries.

From the dataset built by Jordá et al. (2019) (JST), available at http://www.macrohistory.net/data, the following measures of the yield component of the total return are used:

- `bond_rate`: "long-term interest rate", 10-year government bonds
- `bill_rate`: "short-term interest rate", i.e. "yield on Treasury bills, i.e., short-term, fixed-income government securities" or money market rate or deposit rate when T-bill unavailable).
- `capital` measured as the unweighted average of `bond_rate`, `bill_rate`, `eq_dp` ("equity dividend yield") and `housing_rent_rtn` ("housing rental return").

Each nominal return series x_t above, for each year t, is measured in real terms dividing by the annual variation of consumer prices p_t (`cpi`, "consumer prices (index, 1990=100)") such that the respective real interest rate is: $(1 + x_t)/(1 + \pi_t) - 1$, with $\pi_t = p_t/p_{t-1} - 1$.

Figure F.1 shows the `capital` series for those EA12 countries where series from the JST dataset were available.

A measure of the marginal product of capital (MPK) net of depreciation is derived applying the representative firm's first order condition in the OLG model:

$$r = MPK - \delta = \psi/(K/Y) - \delta \tag{F.1}$$

using data from the Penn World Table (PWT) national accounts, available at https://www.rug.nl/ggdc/productivity/pwt/,[73] on the capital-output ratio (K/Y) and the time-varying depreciation rate of capital (δ).[74] The series used for Y, K and δ are respectively: `rgdpna` ("Real GDP at constant 2011 national prices (in mil. 2011US$)"), `rnna` ("Capital stock at constant 2011 national prices (in mil. 2011US$)") and `delta` ("Average depreciation rate of the capital stock"). Since this depreciation rate tends to be lower than what calibrated in the OLG model (the average across the 12 countries over time in the PWT is about 3.5%) the capital bias in production ψ is set to the value of 0.28, higher than what assumed in the OLG model (0.22) in order to be consistent with the overall level of the real interest as found in the econometric estimate of HLW which is the main comparison reference for the OLG model. It comes as a consequence that the level of the marginal product of capital net of depreciation must be treated with caution.

Figure F.2 shows the resulting series of the marginal product of capital net of depreciation for the EA12 countries obtained using the PWT data. There is a marked negative trend common to most countries. The figure compares these series with the series for the marginal product of capital obtained using the standard method to measure capital employed in section E from AWM data for the euro area.[75] There is a marked discrepancy in the early years of the sample which, as already noted in section E, casts doubt on the reliability of AWM data in that part of the sample.

[73] See Feenstra et al. (2015) for a description

[74] As explained in Feenstra et al. (2015), in contrast to studies such as Caselli (2005) or Caselli and Feyrer (2007) (representative for the broader literature) the depreciation rate in the PWT varies across countries and over time as countries differ in the asset composition of their capital stock and depreciation differs across assets.

[75] In this case, the real interest rate is still identified by (F.1) but the parameter values are those set in the OLG model ($\psi = 0.22$ and $\delta = 0.11$ for all periods).

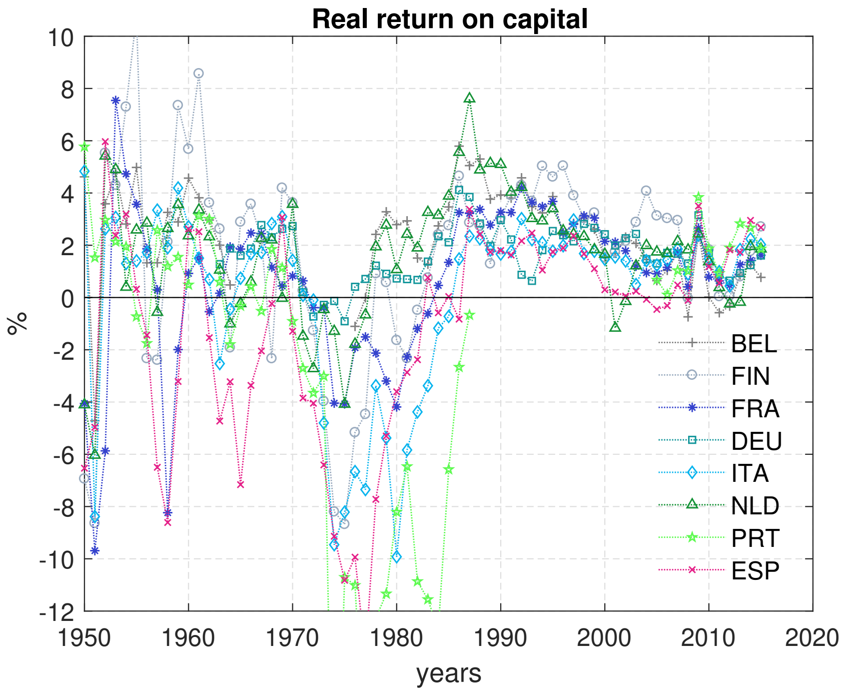


Figure F.1: Real return on capital in the euro area

Note. Data source: JST. The return on capital is measured as the unweighted average of the yield component of the four assets returns available in the JST dataset: bond, bill, equity, housing.

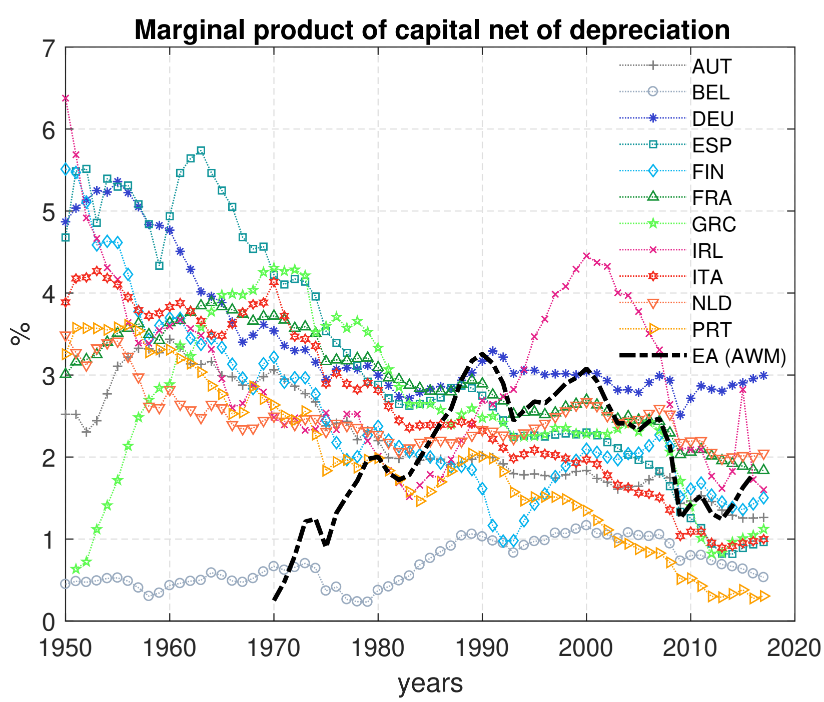


Figure F.2: Marginal product of capital net of depreciation in the euro area

Note. Data source: PWT. Luxembourg excluded from EA12 as evident outlier.

G Real interest rate: further comparison of the model results with data

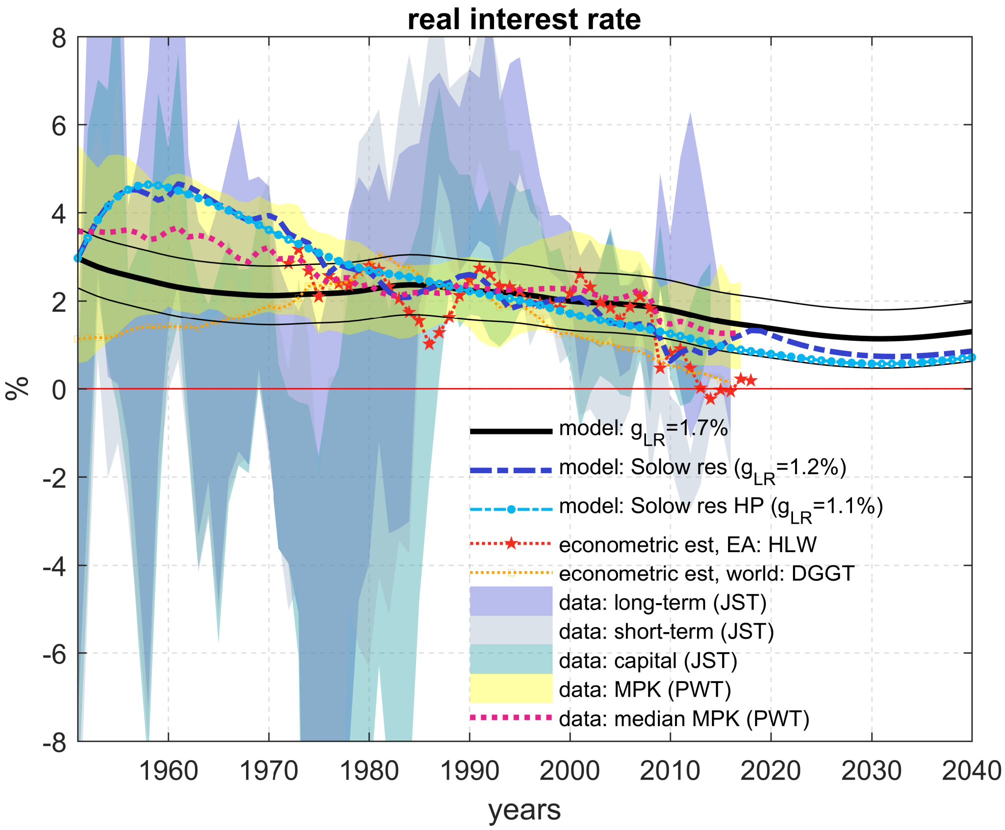


Figure G.1: Model vs data: real interest rate

Note. The long-run growth imposed in the final stationary equilibrium is denoted by $g_{LR} = (g^Z)^{1/(1-\psi)}$. See Appendix E for the Solow residual used in the simulations: actual and Hodrick-Prescott (HP) filtered. Econometric estimate for the euro area (EA) corresponding to the "natural rate of interest" in Holston et al. (2017) (HLW). The econometric estimate for the world economy is provided in Del Negro et al. (2019) (DGGT) on yield data provided by Jordá et al. (2019) (JST). Data from the same source are used for the darker shaded areas comparing the 90-10 percentile range of EA12 countries on real yields of different maturities and types (T-bill, T-bond, equity, housing). The brighter (yellow) shaded area shows the 90-10 percentile range of EA12 countries on a measure of the marginal product of capital (MPK) computed using data from the Penn World Table (PWT) with time varying depreciation rates. Details in Appendix F. The thin black lines mark a band corresponding to $g_{LR} = 1.7\% \pm 0.6\%$ in the OLG simulations.

H Robustness: alternative to perfect annuity market

If one assumes that there are no annuities, one needs to make an assumption about the allocation of the assets left by deceased households ("accidental bequest"). Following e.g. Börsch-Supan et al. (2014), here it is assumed that accidental bequests resulting from premature death are taxed by the government at a confiscatory rate and used for otherwise neutral government consumption. Under this assumption the representative household is not ensured against the risk of prematurely dying with positive wealth.[76] In this case, the budget constraint (2.3) in the main text needs to be substituted by:

$$a_{t+j+1,j+1} = a_{t+j,j}(1+r_{t+j}) - c_{t+j,j} + y_{t+j,j} \tag{H.1}$$

Among the clearing conditions one needs to add government consumption G_t:

$$G_t = \sum_{j=0}^{J} a_{t,j} N_{t-1,j-1}(1-s_{t,j})(1+r_t) \tag{H.2}$$

$$C_t + G_t + I_t = Y_t \tag{H.3}$$

Importantly, when one solves for the optimal conditions, in the Euler equations the conditional survival probabilities do not cancel out anymore. Hence, in addition to the new budget constraint, in steady sate the following new dynamics of consumption holds:

$$c_0 = \frac{\sum_{j=0}^{J}\left(\frac{1}{1+r}\right)^j y_j}{\sum_{j=0}^{J}\left(\frac{1}{1+r}\right)^j (\beta(1+r))^{\frac{j}{\sigma}} \pi_j^{\frac{1}{\sigma}}} \tag{H.4}$$

$$c_{j+1} = [\beta(1+r)s_{j+1}]^{\frac{1}{\sigma}} c_j, \quad \text{for } j = 0, 1, ..., J-1 \tag{H.5}$$

The analytic expressions for the rest of the model are unaffected by the new assumption on accidental bequest.

Figure H.1 plots the life-cycle profiles of consumption and assets holdings in the final steady state comparing the results obtained with (baseline) and without perfect annuity market. Since the conditional survival probabilities now enter the Euler equation, individual consumption is not increasing constantly over the life-cycle but exhibits a hump-shape pattern. Consistently with what found in Hansen and Imrohoroglu (2008), consumption declines at later stages of life because individuals are not compensated via annuities for their increasing effective rate of discount (due to survival probabilities falling as an individual ages). This decline starts happening when an individual's effective discount rate is larger than the interest rate.

The new simulation with no annuities is done assuming that the economy starts from the same value of the real interest rate in the initial steady state (0.0619). This requires to re-calibrate the discount factor β which results to be 0.9733 (instead of 0.9598 in the baseline). The remaining parameter values are the same of the baseline calibration (see Table 4.1). The real interest rate in the final steady state with no annuities is lower than in the baseline (about 3.15% per annum versus 4.55% in the baseline).

[76] As noted e.g. by Quadrini and Ríos-Rull (2015), an alternative with identical implications, except for the use of public revenues, would be to assume that any household is like a pharaoh and assets are buried with their owners. For sensitivity on different degrees of annuitization of household's wealth, see Hansen and Imrohoroglu (2008).

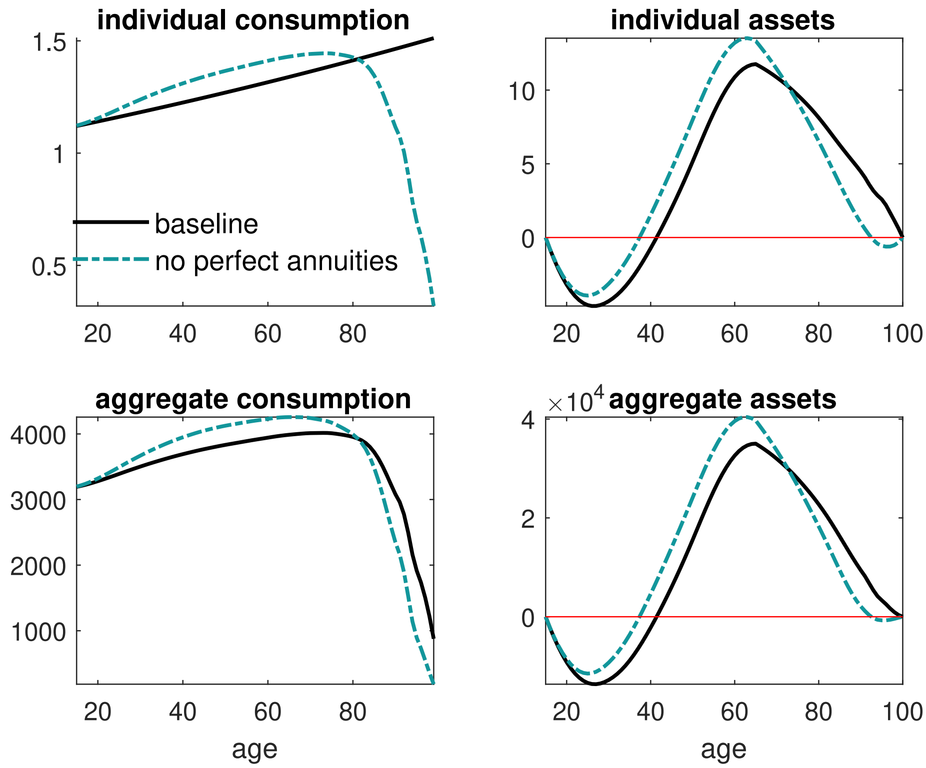


Figure H.1: Final steady state: profiles of consumption and asset holdings

Note. Baseline calibration (see Table 4.1). In the "no perfect annuity" case the individual discount factor β is recalibrated in order to have the same real interest rate value in the initial steady state of the baseline. It results $\beta = 0.9733$.

Figure H.2 shows the results of the transition dynamics in the model for the main macroeconomic variables with (baseline) and without perfect annuities. Compared to the baseline, the absence of annuities increases the the willingness to save of individuals in the face of aging. The saving rate stands at a higher level throughout the whole transition, associated with a higher capital-output ratio and a lower real interest rate. Output growth benefits slightly, tending to stay at a slighter higher level. Overall, the economy is more sensitive to aging. So, while the saving rate is always at a higher level it decreases more in the acutest phase of the transition: between 1990 and 2040 (roughly the peak to trough) it decreases by about 2.9 percentage points (in the baseline about 2.15 percentage points over the same period). Similarly, the real interest rate decrease by about 1.55 percentage points between 1990 and 2030 (compared with 1.3 percentage points in the baseline). If anything, this exercise offers evidence that the downward impact of aging on the real interest rate in the baseline simulation is a conservative estimate.

Figure H.3 strengthens the comparison with the baseline by showing the the dynamics of the main aggregates in terms of deviations from the initial steady state until year 2200. Clearly, the case of no perfect annuities entails a a bigger downward pressure of aging on the real interest rate.

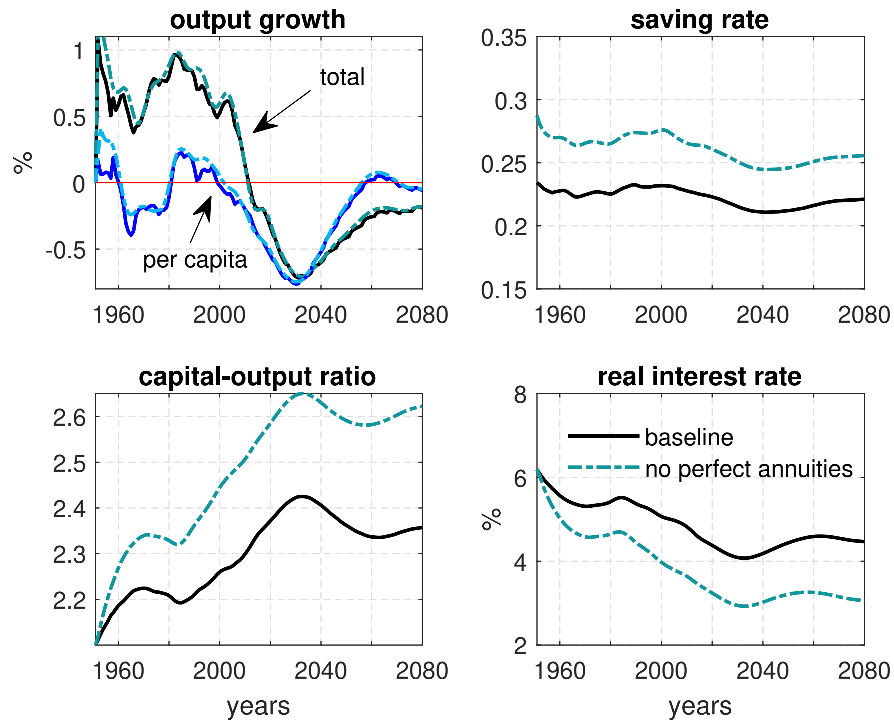


Figure H.2: EA12 transition dynamics: main macro variables

Note. For calibrations, see note of Figure H.1. Perfect foresight solution for the “no perfect annuities” case found via a homotopy method (see *Dynare* reference manual).

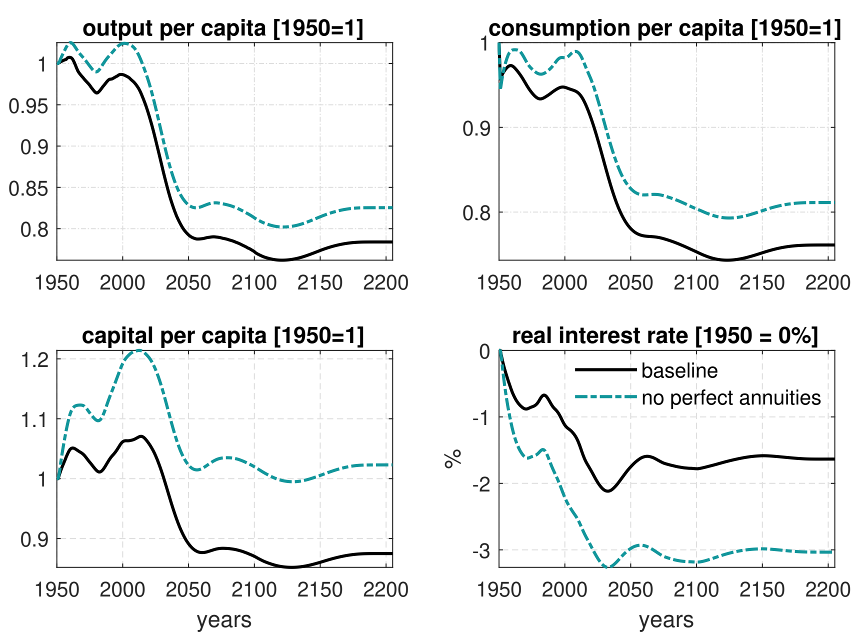


Figure H.3: EA12 transition dynamics: main aggregates per capita

Note. See note of Figure H.2.

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