The time-varying risk of Italian GDP

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THE TIME-VARYING RISK OF ITALIAN GDP

by Fabio Busetti*, Michele Caivano*, Davide Delle Monache* and Claudia Pacella*

Abstract

The uncertainty surrounding economic forecasts is generally related to multiple sources of risks, of both domestic and foreign origin. This paper studies the predictive distribution of Italian GDP growth as a function of selected risk indicators, relating to both financial and real economic developments. The conditional distribution is characterized by expectile regressions. Expectiles are closely related to the Expected Shortfall, a well-known measure of risk with desirable properties. Here a decomposition of Expected Shortfall in terms of the contributions of different indicators is proposed, which allows the main drivers of risk to be tracked over time. Our analysis of the predictive distribution of GDP confirms that financial conditions are relevant for the left tail of the distribution but it also highlights that indicators of global trade and uncertainty have strong explanatory power for both the left and the right tail. Their usefulness is also supported in a pseudo real-time predictive context. Overall, our findings suggest that Italian GDP risks have been driven mostly by foreign developments throughout the Great Recession, by the domestic financial conditions at the time of the sovereign debt crisis and by economic policy uncertainty in more recent years.

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Keywords: asymmetric least squares, expectiles, density forecasts, GDP growth, risks.
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Contents

1. Introduction ......................................................................................................................... 5
2. Methodology ........................................................................................................................ 7
   2.1 Expectile regression ...................................................................................................... 9
   2.2 Matching a parametric distribution ............................................................................ 10
3. Econometric estimates ....................................................................................................... 10
4. Out-of-sample properties ................................................................................................... 17
5. Disentangling the drivers of risk over time ....................................................................... 22
6. Concluding remarks ........................................................................................................... 24
References .............................................................................................................................. 26
Appendix A. Data description ................................................................................................. 29
Appendix B. Matching of skew-t distribution over 2008-14 ................................................. 30
Appendix C. Robustness check: properties of non-parametric distribution ......................... 32
Appendix D. Robustness check: drivers of the ES ................................................................. 33

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1 Introduction

Quantile methods have become increasingly popular in the analysis of macroeconomic time series. Indeed they provide a simple tool to characterize the distribution of the data allowing for nonlinear dynamics, such as time-varying volatility and skewness. For example, in a recent influential paper Adrian et al. (2019) have utilized quantile regressions to study the distribution of US GDP growth as a function of financial conditions, finding significant effects on the left tail dynamics of output but not on the right tail.

Probability distributions around point economic forecasts are of great importance to policymakers for devising and communicating the most appropriate policy actions. In the words of Greenspan (2003), ‘the conduct of monetary policy . . . requires an understanding of the many sources of risk and uncertainty that policymakers face and the quantifying of those risks when possible.’ Hence most central banks communicate their probabilistic assessment of current and future economic conditions by publishing conditional quantile forecasts, in the form of so-called ‘fan charts’; see e.g. Britton et al. (1998) for the case of the Bank of England. In those charts a skewness of the underlying distribution is used to reveal the presence of downside or upside risks around the forecast. In assessing the direction of risks policymakers mostly rely on qualitative arguments (based on the observation of the evolving economic landscape and of the possible sources of tail events) that are somehow mapped into subjective probability distributions.

As a complementary analysis, quantile methods can provide a kind of objective appraisal of risk, by relating in a mathematical way the tail dynamics of GDP and inflation to properly selected indicators. Indeed the ‘Growth-at-Risk’ framework of Adrian et al. (2019) is already regularly published by several institutions to detect vulnerabilities of macroeconomic conditions; see e.g. Prasad et al. (2019). In a similar vein, Giglio et al. (2016) examine the predictive power of several systemic risks measures for the distribution of industrial production growth and use these findings to obtain an aggregate index of risk. The conditional distribution of inflation

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1 We wish to thank Piergiorgio Alessandri, Michele Leonardo Bianchi, Arianna Miglietta and Mario Pietrunti, and seminar participants at the Bank of Italy, Bank of Spain, Federal Reserve Board of Governors for useful comments and suggestions.

2 See Pinheiro and Esteves (2012) for the methodology underlying a risk assessment exercise regularly conducted within the Eurosystem, Miani and Siviero (2010) for the case of the Bank of Italy.

3 Applying the same methodology to Italian data, Alessandri et al. (2019) confirm the link between left tail risk and financial conditions but they also find that the relationship may be unstable over time and may provide noisy signals.
has also been analyzed through quantile methods.\footnote{For example, Manzan and Zerom (2013) argue that macroeconomic indicators are useful for forecasting the distribution of US inflation. Similar results, but for the euro area, are given in Busetti et al. (2015), Busetti (2017), Béreau et al. (2018), Busetti et al. (2019), Tagliabracci (2020).}

In this paper we analyse the predictive distribution of GDP growth conditional on selected risk factors, focusing on Italian data. Differently from Adrian et al. (2019) we consider multiple sources of risks, related to both the financial and the real side of the economy. In particular, we argue that risks related to foreign developments may be very relevant for a small open economy like Italy and these can influence both the lower and the upper tail of the distribution of GDP growth. A further novelty is the use of expectile regression, as opposed to quantile regression, to study the conditional distribution of output growth. Expectiles are measures of location similar to quantiles (into which they can be easily mapped), but they are simpler to characterize in terms of minimization of a loss function (Efron, 1991; Newey and Powell, 1987). Furthermore, as shown in Taylor (2008), expectiles are closely linked to the Expected Shortfall, a widely used measure of risk with desirable properties. Here we propose a decomposition of the Expected Shortfall of Italian GDP in terms of contributions of various risk factors, which allows to track over time the main drivers of risk.

Overall, our analysis confirms that financial conditions are relevant for the left tail of the distribution of GDP growth but it also highlights that other risk factors have strong explanatory power for both the left and the right tail. In addition to a synthetic index of financial conditions, survey indicators of export orders at the global level and a measure of economic policy uncertainty appear closely linked to the predictive distribution of Italian GDP. Furthermore, in line with recent empirical studies (Alessandri et al., 2019; Reichlin et al., 2019), a pseudo real-time analysis shows some deterioration of the predictive content of financial indicators, particularly at longer horizon and for the US index of financial conditions. The trade and uncertainty indicators on the other hand retain strong explanatory power in pseudo real-time. Overall, our estimates suggest that downside risks of GDP were mostly driven by foreign developments around the Great Recession, by domestic financial conditions at the time of the Sovereign Debt Crisis and by economic policy uncertainty in more recent years.

The paper is organized as follows. Section 2 briefly describes the method of expectile regression and the related analyses employed in the rest of the paper. Section 3 presents the empirical specifications adopted for analyzing the predictive distribution of Italian GDP, shows the in-sample properties of our estimates and discusses the predictive content of various risk indicators. The out-of-sample properties of
our empirical models are examined in section 4, where the forecast accuracy of the distributions is compared across specifications. Section 5 introduces the Expected shortfall and its decomposition in terms of various indicators that allows to describe the evolution over time of the drivers of risk. Section 6 concludes.

2 Methodology

Expectiles are measures of location similar to quantiles, but they are determined by tail expectations rather than tail probabilities. For a random variable $y$ with distribution function $F(.)$ and finite mean, the expectile of order $\tau \in (0, 1)$, denoted as $m(\tau)$, is defined by the following equation:

$$\tau = \frac{\int_{-\infty}^{m(\tau)} |y - m(\tau)| dF(y)}{\int_{-\infty}^{\infty} |y - m(\tau)| dF(y)}$$  \hspace{1cm} (1)$$

In words, $m(\tau)$ defines the point in the distribution such that the average distance of the data below that point is the fraction $\tau$ of the distance between $m(\tau)$ and all data points. While quantiles are not sensitive to values in the tails but only to the ordering of data, expectiles depend on all admissible points of the distribution; see e.g. Kuan et al. (2009), Bellini and Di Bernardino (2017) for further details.

For the quantile of order $\alpha \in (0, 1)$, denoted as $q(\alpha)$, the analogous of equation 1 is:

$$\alpha = \int_{-\infty}^{q(\alpha)} dF(y)$$

Quantiles and expectiles can be easily mapped into each other. As showed in Yao and Tong (1996), for a given quantile $q(\alpha)$ there is a corresponding expectile of order $\tau(\alpha)$ given by:

$$\tau(\alpha) = \frac{\alpha q(\alpha) + \int_{q(\alpha)}^{\infty} ydF(y)}{2 \int_{q(\alpha)}^{\infty} ydF(y) - (1 - 2\alpha) q(\alpha)}$$  \hspace{1cm} (2)$$

In practice, given the estimate of an expectile the corresponding quantile order can be obtained by counting the numbers of observations below that value (Efron, 1991). Furthermore, as we will see in Section 5 expectiles are closely linked to the Expected Shortfall, a widely used measure of risk with desirable properties such as dependence on extreme values.

Given a set of observations $\{y_1, \ldots, y_n\}$, the sample expectile is obtained by
minimizing the following loss function:

\[ L(\tau) = \sum_{t=1}^{n} \rho_{\tau}(y_t - m(\tau)) \]  

(3)

where \( \rho_{\tau}(u) = u^2 \mid \tau - 1(u < 0) \mid \) (Newey and Powell, 1987).  

The sample quantile on the other hand minimizes:

\[ L(\alpha) = \sum_{t=1}^{n} \delta_{\alpha}(y_t - q(\alpha)) \]

where \( \delta_{\alpha}(u) = u(\alpha - 1(u < 0)) \). Figure 1 shows the loss function for expectiles (quantiles) at the different expectile (quantile) order. The expectile loss function is quadratic and it is also called ‘asymmetric least squares’ as the estimate minimizes the squared residuals giving them different weight according to whether they are positive or negative. Note that \( \tau = 0.5 \) corresponds to OLS and hence the estimate is the sample mean.

---

**Figure 1:** Loss functions for quantile (red) and expectile (blue) regression

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5Busetti and Harvey (2010) construct tests of stability of a distribution function based on partial sums of the first derivative of \( \rho_{\tau}(y_t - \hat{m}(\tau)) \), where \( \hat{m}(\tau) \) is the sample expectile; similar tests are obtained for quantiles. De Rossi and Harvey (2009) extend a standard unobserved component framework to track time variation in quantiles and expectiles.
2.1 Expectile regression

Expectiles may depend on covariates. Assuming a linear relation, \( m_t(\tau) = \beta(\tau)'x_t \), the vector of parameter \( \beta(\tau) \) is estimated by the expectile regression:

\[
\hat{\beta}(\tau) = \arg \min_{\beta} \sum_{t=1}^{n} \rho_{\tau}(y_t - \beta(\tau)'x_t)
\]

which is the obvious extension of (3). As shown in Newey and Powell (1987), \( \hat{\beta}(\tau) \) can be expressed as 'weighted least square estimator':

\[
\hat{\beta}(\tau) = \left[ \sum_{t=1}^{n} w(\tau)x_t'x_t \right]^{-1} \sum_{t=1}^{n} w(\tau)x_t y_t
\]

where the weights are \( w(\tau) = |\tau - 1 (y_t - x_t'\hat{\beta}(\tau) < 0)| \). Since the weights themselves depend on the estimated coefficients, \( \hat{\beta}(\tau) \) can be computed by iterating formula (5) starting from an initial guess. For a given expectile order \( \tau_k \), a good initial value is \( \hat{\beta}(\tau_m) \), where \( \tau_m \) is close to \( \tau_k \), starting with \( \hat{\beta}(0.5) = \hat{\beta}_{OLS} \). Newey and Powell (1987) show that, under regularity conditions, the expectile regression estimator (5) is asymptotically Gaussian. The limiting variance can also be easily computed. As seen from the previous formulae, expectile regression are based on a smooth loss function that facilitates estimation and makes it suitable to generalisations such as time-varying parameters (cf. (Busetti et al., 2019)). Moreover, although sample quantiles and expectiles may violate the theoretical property of being non-decreasing functions of their order (i.e. they may cross), this issue occurs much less frequently for expectiles; see Waltrup et al. (2015) where a detailed discussion on the relative merits of quantile and expectile regressions is provided. A measure of goodness of fit is the analogous of the pseudo-\( R^2 \) proposed by Koenker and Machado (1999) in the context of quantile regression:

\[
R^2(\tau) = 1 - \frac{\sum_{t=1}^{n} \rho_{\tau}(y_t - \hat{m}_t(\tau))}{\sum_{t=1}^{n} \rho_{\tau}(y_t - \hat{m}_{unc}(\tau))}
\]

where \( \hat{m}_t(\tau) \) is the fitted value of the expectile regression and \( \hat{m}_{unc}(\tau) \) is the unconditional sample expectile (the fitted value of an expectile regression including only the intercept term). The version of this coefficient adjusted for the degrees of freedom is:

\[
R^2_{adj}(\tau) = 1 - \frac{n-1}{n-k} (1 - R^2(\tau)),
\]

where \( k \) is the number of covariates in the regression.
2.2 Matching a parametric distribution

We use our estimated expectiles to match a flexible distribution, the skewed $t_{a,b}$ of Jones and Faddy (2003) that has the following probability density function: \(^6\)

$$f(y|\mu, \sigma, a, b) = \frac{C(a, b)}{\sigma} \left( 1 + \frac{z}{\sqrt{a + b + z^2}} \right)^{a+1/2} \left( 1 - \frac{z}{\sqrt{a + b + z^2}} \right)^{b+1/2}$$  \(6\)

where $C(a, b) = \frac{2^{1-a-b}}{B(a, b)\sqrt{a+b}}$, $B(.)$ is the Beta function and $z = \frac{y-\mu}{\sigma}$.

The distribution depends upon four parameters: location $\mu$, scale $\sigma$, and two shape parameters $a$ and $b$ which are real positive numbers. When $a = b$, $t_{a,b}$ reduces to the Student–t distribution with $2a$ degrees of freedom; when $a < b$ ($a > b$), it is negatively (positively) skewed. A closed form expression for the distribution function is available. The moments $E[y^r]$ are defined for $a, b > r/2$:

$$E[y^r] = \frac{(a+b)^{r/2}}{2^r B(a, b)} \sum_{i=0}^{r} \binom{r}{i} (-1)^i B \left( a + \frac{r}{2} - i, b - \frac{r}{2} + 1 \right)$$  \(7\)

Let $\theta = (\mu, \sigma, a, b)$ be the vector of parameters of the $t_{a,b}$ distribution. Given $\theta$ and $\tau$, the theoretical expectile of the distribution, say $m_\theta(\tau)$, is derived by rearranging equation (1) and solving for $m(\tau)$. Note that some of the integrals inside the equation need to be evaluated numerically. The estimation of $\theta$ is obtained by minimizing the distance between some set $\Upsilon$ of theoretical and fitted expectiles:

$$\hat{\theta} = \min_{\theta} \sum_{\tau \in \Upsilon} \{ \hat{m}(\tau) - m_\theta(\tau) \}^2,$$  \(8\)

where $\hat{m}(\tau)$ are the fitted values of expectile regressions. In the computations below we use four expectiles.

3 Econometric estimates

The results of Adrian et al. (2019) has opened the way to the calculation of ‘Growth-at-Risk’ statistic that links the likelihood of an economic downturn to the current state of financial markets. Here, the aim is to investigate which indicators may systematically anticipates risks for economic growth. To do so we characterize the

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\(^6\)The approach is similar to Adrian et al. (2019) who use estimated quantiles to match the skewed t-distribution of Azzalini and Capitanio (2003).
predictive distribution of the Italian GDP growth by means of the expectile regression:

\[
m^h_{t+h}(\tau) = \beta_0(\tau) + \beta_1(\tau)y_t + \delta(\tau)'x_t + \epsilon_{t+h}(\tau) \tag{9}
\]

where \( m^h_{t+h}(\tau) \) is the expectile of order \( \tau \) of the target variable, \( y^h_{t+h} = \frac{400}{h} \Delta^h \log Y_{t+h} \), that is the (annualised) average growth rate of real GDP between \( t \) and \( t + h \). As regressors we have, \( y_t = 400 \Delta \log Y_t \), that is the annualised quarter-on-quarter growth rate, and \( x_t \) is a vector of selected indicators related to relevant risk factors affecting the evolution of GDP over and beyond what could be inferred by its lagged dynamics. The predicted value of the regression, obtained using the loss function (3) is the estimated expectile of the target variable \( y^h_{t+h} \), conditional on the information set up to \( t \) (denoted by \( I_t \)):

\[
\hat{m}^h_{t+h|t}(\tau) = \hat{\beta}_0(\tau) + \hat{\beta}_1(\tau)y_t + \hat{\delta}(\tau)'x_t \tag{10}
\]

The approach is similar to that of Adrian et al. (2019) except that we replace quantiles with expectiles and we allow for additional indicators of risk other than financial conditions. While it is known that indicators of financial distress may anticipate future recessions they do not appear much related to the dynamics of GDP in the upper tail of the distribution.\(^7\) In order to extend the set of variables to be included in our analysis, we conjecture that: risks related to foreign developments may be very relevant for a small open economy like Italy and these can influence both the lower and the upper tail of the distribution; shocks to the real economic activity may be as relevant as the financial shocks; the uncertainty over economic policies may be a significant driver of GDP growth, as suggested in several empirical studies (see e.g. Baker et al. (2016)).

Specifically, we assume that the main channels driving the Italian growth are: the financial conditions (both at national and global level) that are meant to capture the financial shocks, the world real economic activity that pins down the real shocks, and the uncertainty that should capture a forward looking component over the firms and households spending attitudes. To summarize, the vector \( x_t \) contains the following four covariates: (i) a domestic financial condition index (IT FCI), introduced in Miglietta and Venditti (2019); (ii) the National Financial Conditions Index for the US economy (US FCI) of the Chicago FED (Brave and Butters, 2011) as proxy of global financial conditions; (iii) the global Purchasing Managers’ Index on new export orders (PMI) as a leading indicator of world demand and trade developments,

\(^7\)For European economies even an ‘inversion of the yield curve’, the traditional leading indicator of recession for the US, does not have a significant explanatory power (Estrella and Mishkin, 1997; Moneta, 2005).
so it summarizes the risks related to the real economic activity from the global perspective;\(^8\) (iv) the world Economic Policy Uncertainty (EPU) index defined in Baker et al. (2016) in order to detect the impact of uncertainty over firms and households spending attitudes. Although those variables result to be correlated, such correlation does not show to be very high, our specification does not suffer of collinearity, and we can argue that the selected variables are able to capture different drivers of risk.\(^9\)

Someone may argue that the FCI is a composite index summarizing different indicators and therefore it can mix up different risk factors. However, Adrian et al. (2019) find that the conditional quantile function is more sensitive to the overall index rather than to specific subcategories (e.g. risk, credit, leverage). Therefore, bearing in mind the issue we use the aggregate FCI index. This said, it would be of interest for future research to investigate whether specific subcategories may have more leading properties for different expectiles of the distribution.\(^10\)

Tables 1, 2, 3 report the regression estimates over the sample period 1993Q1-2018Q4 for one and four steps ahead predictive horizon \((h = 1, 4)\) and for selected expectile orders \(\tau = .05, .10, .50, .90, .95,\) the IT FCI and US FCI enter the regression in levels, while the PMI and EPU enter in differences (quarterly changes).\(^11\)

<table>
<thead>
<tr>
<th>(\tau)</th>
<th>(h = 1)</th>
<th>(h = 4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>IT FCI</td>
<td>US FCI</td>
</tr>
<tr>
<td>0.05</td>
<td>0.64</td>
<td>-11.47</td>
</tr>
<tr>
<td></td>
<td>(10.4)</td>
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</tr>
<tr>
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<td>-9.92</td>
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<tr>
<td></td>
<td>(8.84)</td>
<td>(-2.54)</td>
</tr>
<tr>
<td>0.50</td>
<td>0.57</td>
<td>-7.69</td>
</tr>
<tr>
<td></td>
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<td>(-2.29)</td>
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<tr>
<td>0.90</td>
<td>0.53</td>
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</tr>
<tr>
<td></td>
<td>(7.16)</td>
<td>(-2.48)</td>
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<tr>
<td>0.95</td>
<td>0.56</td>
<td>-10.73</td>
</tr>
<tr>
<td></td>
<td>(8.87)</td>
<td>(-2.73)</td>
</tr>
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</table>

\(^8\)The PMI index turns out to have good leading properties for the GDP growth: the contemporaneous correlation is about 0.2, it increases to roughly 0.4 for lag 1,...,4, and vanishes at leads.

\(^9\)See Appendix A for more details on data descriptions and pairwise correlations.

\(^{10}\)A similar argument may apply to the EPU indicator.

\(^{11}\)The results for two and three steps ahead predictive horizons and for other expectile orders are available upon request.
### Table 2: Estimation results: the full model

<table>
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<tr>
<th>$\tau$</th>
<th>GDP</th>
<th>IT FCI</th>
<th>US FCI</th>
<th>PMI</th>
<th>EPU</th>
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<th>IT FCI</th>
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<th>PMI</th>
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<td>(3.38)</td>
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<td>(-4.73)</td>
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<td>(2.89)</td>
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### Table 3: Estimation results: the baseline model

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<th>GDP</th>
<th>IT FCI</th>
<th>PMI</th>
<th>EPU</th>
<th>GDP</th>
<th>IT FCI</th>
<th>PMI</th>
<th>EPU</th>
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<tr>
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<td>(-1.63)</td>
<td>(2.92)</td>
<td>(3.02)</td>
<td>(1.8)</td>
<td>(-1.78)</td>
<td>(2.04)</td>
<td>(-1.45)</td>
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<tr>
<td>0.90</td>
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<td>(4.57)</td>
<td>(-2.28)</td>
<td>(1.96)</td>
<td>(4.06)</td>
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<td>-7.60</td>
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</tr>
<tr>
<td></td>
<td>(5.15)</td>
<td>(-2.27)</td>
<td>(1.92)</td>
<td>(4.80)</td>
<td>(1.94)</td>
<td>(-0.76)</td>
<td>(2.92)</td>
<td>(-2.2)</td>
</tr>
</tbody>
</table>

Notes: For each horizon and expectile order the coefficients and the corresponding t–statistic (below in parentheses) are reported.

We first consider a predictive model that includes only one risk indicator at a time (or no indicators at all), in addition to the intercept and current GDP growth. The results, in terms of coefficients values and t-statistics, are displayed in Table 1. For $h = 1$ IT FCI, PMI and EPU are statistically significant, with the sign one would expect, over all regions of the predictive distribution. On the other hand, the global financial conditions (US FCI) is significant, and strongly so, only for the left tail of the distribution. Interestingly, this latter result mirrors the findings of Adrian et al. (2019) for the US. Qualitatively similar results hold for $h = 4$, where however the link with GDP growth tends to become weaker for all indicators (implying that at some expectile order IT FCI is no longer statistically significant), except for the US FCI for which the coefficients retain their magnitude and significance. Intermediate results would hold for $h = 2$ and $h = 3$. Finally note that, as expected, the impact
of current GDP conditions (displayed in the first column for the model without risk indicators), becomes lower from one to four step ahead predictions.

Table 2 reports the results for the full model where all four risk indicators and current GDP are included as covariates. It is interesting to see that each indicator tends to retain its statistical significance, especially for the case \( h = 1 \). Generally, financial conditions appear significant drivers of GDP risk in the left tail while trade developments (PMI) and uncertainty (EPU) are significant both in the left and the right tail (although the magnitude of the coefficients is smaller in the right tail). For \( h = 4 \) current GDP is no longer a relevant driver, while economic policy uncertainty loses significance in the lower tail of the distribution.

Table 3 shows similar results but for a model where the US FCI is excluded from the covariates, i.e. where there is a single indicator of financial condition (IT FCI). As we will see later, this specification (denoted as ‘baseline model’) appears to work relatively better than the previous one (denoted as ‘full model’) in an out-of-sample context, so it is worthwhile to examine the in-sample fit. The figures are to a large extent in line with those in the previous table, except that the magnitude of the coefficient of IT FCI tend to be larger, in absolute values, since it partly captures also the impact of global financial conditions.

Table 4 provides goodness of fit measures for the model specifications considered in the previous tables, for \( h = 1, 4 \) step ahead predictive horizons. Along the lines of the quantile weighted predictive score by Gneiting and Ranjan (2011), we also obtain a goodness of fit indicator for the whole distribution by taking an average across expectile orders of the \( \hat{R}_{adj}^2(\tau) \) measure defined in the previous section:

\[
\hat{R}_{adj}^2 = \sum_\tau \omega^*(\tau)\hat{R}_{adj}^2(\tau)
\]

where \( \omega^*(\tau) = \frac{\omega(\tau)}{\sum_\tau \omega(\tau)} \) are normalized weights under three cases: (a) equal weights, \( \omega(\tau) = 1 \) for all \( \tau \), (b) relatively higher weights in the left tail, \( \omega(\tau) = (1 - \tau)^2 \), (c) relatively higher weights in the right tail, \( \omega(\tau) = \tau^2 \).

The following main results emerge. First, goodness of fit clearly decreases for longer predictive horizons. Second, models including at least one risk indicator show a better fit than a model that considers only current GDP. Third, the left tail of the distribution is better captured than the right tail for most indicators and predictive horizons. Fourth, models that jointly consider several indicators can fit the predictive distribution of GDP distinctively better than considering a single indicator, either of the real or of the financial type. There is a small advantage of the full model with respect to the baseline model (where US FCI is excluded). This ranking will however be reversed in the out-of-sample exercise presented in the next section.
Table 4: Goodness of fit: adjusted pseudo R square

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>GDP</th>
<th>IT FCI</th>
<th>US FCI</th>
<th>PMI</th>
<th>EPU</th>
<th>Full</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
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<td>0.05</td>
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<td>0.64</td>
<td>0.60</td>
</tr>
<tr>
<td>0.50</td>
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<td>0.35</td>
<td>0.32</td>
<td>0.41</td>
<td>0.42</td>
<td>0.49</td>
<td>0.47</td>
</tr>
<tr>
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<td>0.24</td>
<td>0.28</td>
<td>0.23</td>
<td>0.29</td>
<td>0.36</td>
<td>0.39</td>
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</tr>
<tr>
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<td>0.28</td>
<td>0.23</td>
<td>0.27</td>
<td>0.36</td>
<td>0.38</td>
<td>0.39</td>
</tr>
<tr>
<td>left tail</td>
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<td>0.41</td>
<td>0.44</td>
<td>0.50</td>
<td>0.51</td>
<td>0.59</td>
<td>0.56</td>
</tr>
<tr>
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<td>0.36</td>
<td>0.35</td>
<td>0.42</td>
<td>0.44</td>
<td>0.50</td>
<td>0.49</td>
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<tr>
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<td>0.27</td>
<td>0.34</td>
<td>0.38</td>
<td>0.43</td>
<td>0.42</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\tau$</th>
<th>GDP</th>
<th>IT FCI</th>
<th>US FCI</th>
<th>PMI</th>
<th>EPU</th>
<th>Full</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
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<td>0.12</td>
<td>0.29</td>
<td>0.18</td>
<td>0.14</td>
<td>0.38</td>
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<td>0.10</td>
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<td>0.14</td>
<td>0.25</td>
<td>0.20</td>
<td>0.17</td>
<td>0.34</td>
<td>0.21</td>
</tr>
<tr>
<td>0.50</td>
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</tr>
<tr>
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<td>0.12</td>
<td>0.11</td>
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<td>0.18</td>
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</tr>
<tr>
<td>0.95</td>
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<td>0.12</td>
<td>0.11</td>
<td>0.20</td>
<td>0.19</td>
<td>0.24</td>
<td>0.24</td>
</tr>
<tr>
<td>left tail</td>
<td>0.12</td>
<td>0.15</td>
<td>0.21</td>
<td>0.18</td>
<td>0.16</td>
<td>0.30</td>
<td>0.21</td>
</tr>
<tr>
<td>equal</td>
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<td>0.14</td>
<td>0.15</td>
<td>0.18</td>
<td>0.17</td>
<td>0.26</td>
<td>0.21</td>
</tr>
<tr>
<td>right tail</td>
<td>0.11</td>
<td>0.13</td>
<td>0.11</td>
<td>0.18</td>
<td>0.17</td>
<td>0.23</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Finally, we use the fitted values of the expectile regressions to match a flexible parameter distribution, the skewed $t_{a,b}$ distribution, through the method described in the previous section. The graphs in Figure 2 show the results for $h = 1$ and $h = 4$ over the entire period 1994Q1-2018Q4 for the full model. It can be seen how the conditional distribution evolves over time, with changing dispersion and skewness. In more detail, Appendix B provides the graphs of the conditional density functions (for two alternative model specifications) for all periods between 2008Q1 and 2014Q4 for $h = 1$ and $h = 4$ (figures B.1 and B.2).

Figures 3 and 4 show the evolution over time of the second and third moments of the matched distribution for 1-step ahead and 4-step ahead for the full model and the model including only current GDP. Both measures are expressed as centered moving average of 3 terms to get a smoother picture. The figure shows that the distribution of Italian GDP growth is not constant over time and that there are several periods when the predictive distribution is far from symmetric.
Figure 2: Conditional distribution of GDP growth.

Figure 3: Variance for the GDP model (red) and the Full model (blue).

Figure 4: Skewness for the GDP model (red) and the Full model (blue).
The variance of the full model hovers around 4 for both horizons showing two peaks in the years corresponding to the two most recent recessions. Conversely, the difference in variance in the two phases of the business cycles for the GDP model is much lower. For most of the sample the GDP model tends to overestimate the uncertainty, in particular during tranquil periods. On the other hand, the full model is more suitable to capture the spikes in uncertainty around the crises periods.

Regarding the third moment, the distributions are negatively skewed for $h = 4$ in most periods, while for $h = 1$ skewness is mostly related to recessionary periods. More generally Table 5 shows a significant correlation between skewness (but also dispersion) and future GDP growth. Overall, recessionary periods appear to be characterized by higher variance and more left skewness. This feature is confirmed when we compute the skewness using a non-parametric indicator based on the fitted expectiles (Appendix C).

Table 5: Correlation between moments and actual future GDP growth for the Full model.

<table>
<thead>
<tr>
<th></th>
<th>$h = 1$</th>
<th>$h = 4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>variance</td>
<td>-0.62</td>
<td>-0.55</td>
</tr>
<tr>
<td></td>
<td>(-7.83)</td>
<td>(-6.60)</td>
</tr>
<tr>
<td>skewness</td>
<td>0.54</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>(6.36)</td>
<td>(8.60)</td>
</tr>
</tbody>
</table>

4 Out-of-sample properties

Here we consider the out-of-sample properties of the empirical models estimated in the previous section. A natural way to evaluate the forecast performance at a given expectile order $\tau$ is through the ‘out-of-sample loss’ $\rho_\tau(y_{t+h} - \hat{m}^h_{t+h|t}(\tau))$, where $\hat{m}^h_{t+h|t}(\tau)$ is the expectile forecast of $y_{t+h}$ at horizon $h$, computed estimating the model with data up to time $t$. In order to obtain an overall measure for the forecast distribution, the loss can be averaged across expectile orders:

$$L_{t+h|t} = \int_0^1 \omega^*(\tau) \rho_\tau(y_{t+h} - \hat{m}^h_{t+h|t}(\tau))d\tau$$

where $\omega^*(\tau)$ are weights as for the in-sample goodness of fit measure used in section 3. A statistic to measure the forecast performance is obtained by aggregating the
loss over the evaluation period:

\[ L_h = \frac{1}{T-h} \sum_{t=1}^{T-h} L_{t+h|t} \]

Table 6 reports the loss statistics associated to the full and the baseline models, as well as to the more restricted model specifications analyzed in section 3. The predictive loss is computed over the period 2007Q1-2018Q4 and it is reported in relative terms with respect to the loss of a regression with only the intercept term, which can be interpreted as a model for the unconditional distribution. Lower values of the loss are associated to a better predictive performance: a value higher than 1 means that the unconditional distribution is better than that conditional of risk factors. The results are reported using three alternative weighting schemes, in order to appraise the forecast performance over the whole distribution (equal weights) and over the right and left tails.

**Table 6:** Predictive loss over the evaluation period 2007Q1-2018Q4.

<table>
<thead>
<tr>
<th>( \tau )</th>
<th>GDP</th>
<th>IT FCI</th>
<th>US FCI</th>
<th>PMI</th>
<th>EPU</th>
<th>Full</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.61</td>
<td>0.53</td>
<td>0.40</td>
<td>0.45</td>
<td>0.36</td>
<td>0.30</td>
<td>0.40</td>
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<tr>
<td>0.10</td>
<td>0.60</td>
<td>0.53</td>
<td>0.46</td>
<td>0.44</td>
<td>0.41</td>
<td>0.35</td>
<td>0.42</td>
</tr>
<tr>
<td>0.50</td>
<td>0.63</td>
<td>0.63</td>
<td>0.72</td>
<td>0.51</td>
<td>0.52</td>
<td>0.51</td>
<td>0.47</td>
</tr>
<tr>
<td>0.90</td>
<td>0.70</td>
<td>0.73</td>
<td>1.02</td>
<td>0.64</td>
<td>0.57</td>
<td>0.57</td>
<td>0.55</td>
</tr>
<tr>
<td>0.95</td>
<td>0.69</td>
<td>0.74</td>
<td>1.08</td>
<td>0.65</td>
<td>0.57</td>
<td>0.57</td>
<td>0.57</td>
</tr>
<tr>
<td>left tail</td>
<td>0.61</td>
<td>0.57</td>
<td>0.56</td>
<td>0.47</td>
<td>0.44</td>
<td>0.41</td>
<td>0.43</td>
</tr>
<tr>
<td>equal</td>
<td>0.63</td>
<td>0.63</td>
<td>0.73</td>
<td>0.52</td>
<td>0.50</td>
<td>0.49</td>
<td>0.48</td>
</tr>
<tr>
<td>right tail</td>
<td>0.66</td>
<td>0.69</td>
<td>0.90</td>
<td>0.58</td>
<td>0.55</td>
<td>0.56</td>
<td>0.53</td>
</tr>
</tbody>
</table>

For \( h = 4 \) the financial conditions (US FCI and IT FCI) have significant predictive power for the left tail of the distribution of Italian GDP, but US FCI does not appear
to be a useful indicator for the right tail. On the other hand, EPU and PMI have very good predictive ability for both tails of the distribution that appears overall superior to that of financial indicators. The full and the baseline models, which include several indicators jointly, are generally preferable.

Figure 5 shows that the predictive content of each indicator deteriorates as the forecast horizon $h$ becomes larger. This is particularly visible for financial indicators while PMI and EPU continue to deliver a better forecast distribution than the unconditional one. The US FCI is a very poor indicator for the right tail of the distribution, as seen in the right hand side of the figure (case of $\tau = 0.95$). For $h = 4$ the detailed results at several expectile orders are presented in the lower part of Table 6.

The poor out of sample properties of US FCI are to some extent transferred to the full model which includes that indicator. Hence the baseline model (constructed using only IT FCI, PMI and EPU) seems a better choice in practice.

![Figure 5: Predictive loss over the evaluation period 2007Q1-2018Q4.](image)

(a) $\tau = 0.05$

(b) $\tau = 0.95$

Overall our results show the usefulness of indicators of global trade developments and uncertainty for capturing changes in the distribution of Italian GDP growth. In an out-of-sample perspective, PMI and EPU appear even more relevant than the
financial indicators that were adopted in previous studies.\textsuperscript{12}

To look into the calibration properties of the out-of-sample forecast distribution, we match the expectiles of a parametric skewed $t_{a,b}$ distribution. Figure 6 shows the Probability Integral Transform (PIT) of the forecast distribution implied by the baseline model and the model which contains only current GDP, for $h = 1$ and $h = 4$ step ahead forecasts. As highlighted in Diebold et al. (1998), a correctly calibrated density forecast produces PITs that are uniformly distributed. The baseline model appears better calibrated than the model that includes only current GDP for both $h = 1$ and $h = 4$. Figure 7 confirms this evidence. Following Rossi and Sekhposyan (2019) the cumulative distribution function of the PIT is plotted together with the critical values. For a well calibrated distribution the cdf should stay close to the 45 degree line. It appears that the model with only current GDP cannot capture correctly the right tail.

The relative accuracy of alternative forecast distributions models can be analyzed by comparing their log-scores, whose difference can be tested using a Diebold-Mariano statistic (Amisano and Giacomini, 2007). In our case for $h = 1$ the log-score of the baseline model is significantly higher than that of the benchmark model that includes only current GDP (t-statistic = -4.11). For $h = 4$ the log-score of the baseline model remains higher but the difference between the two models is no longer significant (t-statistic = -0.61).

\textsuperscript{12}The lower forecasting performance of IT FCI might reflect, to some extent, the difficulty to correctly capture the causality between financial and macroeconomic conditions. Financial conditions are both a driver of GDP growth and they are affected by economic activity: the causality is bi-directional. On the other hand, PMI and EPU are referred to global developments that can be regarded as mostly exogenous for Italian GDP growth. The poor performance of the US FCI is instead mainly related to the large errors in predicting Italian GDP during the sovereign debt crisis of 2011-12 (a period when financial conditions were very favourable in the US but not in the euro area).
Figure 6: Probability density functions of the PITs (normalized).

The red dashed lines represent the 95% confidence intervals, constructed using a normal approximation to a binomial distribution, as in Diebold et al. (1998).

Figure 7: Cumulative distribution functions of the PITs with 95% critical values based on Rossi and Sekhposyan (2019).
5 Disentangling the drivers of risk over time

In the previous sections we investigated the predictive distribution of Italian GDP growth conditional to a set of indicators. In this section, we quantify the overall risk for the economic growth using the Expected Shortfall, a well-known measure of risk with desirable properties (Acerbi and Tasche, 2002a; 2002b; Taylor, 2008). Moreover, a decomposition of Expected Shortfall in terms of contributions of different indicators is proposed and this allows to track over time the main drivers of risk.

The tools we use are borrowed from financial risk management. The so-called ‘Growth-at-risk measure’ introduced by Adrian et al. (2019), which is based of the concept of Value-at-Risk (VaR), essentially aims at controlling adverse scenarios based on the worst episodes in history. These are the low-probability but high-cost events that are commonly known as ‘tail risks’. Simple measures of dispersion, like the standard deviation, often fail to account for the size of such extreme events. The left tail can get fatter (i.e. the probability of very bad events can rise) without materially raising the standard deviation. In other circumstances point forecasts may remain substantially the same while the probability in the left tail of the distribution (i.e. the chance of a very bad outcome) rises. In those cases, as the left tail gets fatter lower quantiles can drop sharply. When tail risks rise, policymakers (acting as risk managers) reasonably respond to their perception that Growth-at-risk has gone up.

In particular, the VaR is the upper bound of all potential losses that have probability less than a given confidence level, as such it only considers the probability of such losses and not the magnitude of the losses themselves. On the other hand, the Expected Shortfall (ES) considers the magnitude of such potential losses by computing the average of those with probability less than a given confidence level.

Specifically, the VaR of order \( \alpha \), VaR\((\alpha) \), is computed as the conditional quantile \( q(\alpha) \). Similarly to the financial VaR, the Growth-at-risk is computed as the quantile of the distribution of GDP growth conditional on some indicators and it indicates that future GDP growth will be less or equal such value with probability \( \alpha \). The ES of order \( \alpha \), ES\((\alpha) \), is instead the expected value of the future GDP growth in the left tail of the distribution delimited by the quantile \( q(\alpha) \), with \( \alpha < 0.5 \); it is therefore more sensitive to the shape of the tail of the distribution, unlike the VaR.\(^\text{13}\)

Formally, the ES\((\alpha) \) is defined as follows:

\[
ES(\alpha) = \mathbb{E}[Y|Y < q(\alpha)] = \frac{1}{\alpha} \int_0^\alpha F^{-1}(u)du,
\]

\(^\text{13}\)For a financial investment the ES is the conditional expectation of the loss given that such loss is beyond the VaR level.
while the upper tail counterpart ($\alpha > 0.5$) is known as Expected Longrise.

It is worth to stress that the computation of the ES from quantile regression requires first an approximation of the distribution and then numerical integration (see Adrian et al. (2019)). On the other hand, as shown in Taylor (2008), the expectiles regression allows us to directly obtain the ES as following:

$$ES(\alpha) = \left[ 1 + \frac{\tau(\alpha)}{(1-2\tau(\alpha))\alpha} \right] m(\tau(\alpha)) - \frac{\tau(\alpha)}{(1-2\tau(\alpha))\alpha} m(0.5)$$

with $\tau(\alpha)$ being the expectile order corresponding to the $\alpha$-quantile, $m(\tau(\alpha))$ is the expectile, and $m(0.5)$ is the mean.

As our empirical expectile model for Italian GDP growth contains multiple risk indicators (related to financial conditions, global trade developments and economic policy uncertainty, respectively), it seems interesting to try to disentangle the impact of each of these indicator on the overall ES measure.

Such decomposition of ES is reported in Figure 8 for three subperiods: the Global Financial Crisis (2007-10), the Sovereign Debt Crisis (2011-13) and the post-crisis recovery (2014-18). The figures are obtained from the fitted values of the expectile regression baseline model for 1- and 4-step ahead predictions (in the left and right panel, respectively) that are mapped into values of the ES at the 10% probability level. The colored bars in the figure represent the contributions of financial conditions (red), international trade (green), global economic policy uncertainty (blue); the residual term is attributable to the initial conditions (lagged GDP) and the unconditional mean (intercept term), that must be added to the risk drivers to obtain the ES (the thick black line).

According to this decomposition, GDP risks appear to have been mostly driven by foreign developments around the Global Financial Crisis, by domestic financial conditions at the time of the Sovereign Debt Crisis and by economic policy uncertainty in more recent years.

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14 Since we are interested in downside risks we use the 10% probability level as a proxy for the left tail. Findings are however robust to other lower quantiles (e.g. 5% and 1%).

15 The contributions of each risk factor is obtained by recomputing the ES after setting to zero the other factors in the baseline expectile regression model. In Appendix D a robustness check is provided using fitted expectiles obtained by regressions that include only one indicator at a time.
Figure 8: Contributions to the $ES(0.1)$ in the three subsamples: the Global Financial Crisis (2007-10), the Sovereign Debt Crisis (2011-13) and the post-crisis recovery (2014-18).

6 Concluding remarks

We have studied the predictive conditional distribution of Italian GDP growth as a function of several risk factors, of domestic and foreign origin. We have considered multiple sources of risks, related to both financial and real economic developments.
The predictive distribution has been characterized through expectile regressions and a related measure of risk, the Expected Shortfall, has been computed.

Our empirical evidence confirms that financial conditions are relevant for the left tail of the distribution of GDP growth but other risk factors, such as survey indicators of export orders at the global level and a measure of economic policy uncertainty, appear to have strong explanatory power for both the left and the right tail. However, a pseudo real-time analysis shows some deterioration of the predictive content of financial indicators, particularly at longer horizon and for the US index of financial conditions, in line with other recent empirical works. The trade and uncertainty indicators on the other hand retain their statistical significance in pseudo real-time.

The evolution of Italian GDP risk has been tracked in terms of the contribution of different drivers, pointing to a marked heterogeneity over time. Our findings indicate that risks were mostly driven by foreign developments around the Great Recession, by domestic financial conditions at the time of the Sovereign Debt Crisis and by economic policy uncertainty in more recent years.
References


A Data description

- **GDP**: Real GDP for Italy. Source: ISTAT.
- **IT FCI**: Italian Financial Condition Index. Source: Estimates by Miglietta and Venditti (2019). Quarterly averages of weekly data, with the week between two months assigned to the second one.
- **PMI**: Global Purchasing Managers Index on new export orders. Source: IHS Markit. Quarterly averages of monthly data.

Table A.1: Contemporaneous correlation among the different variables

<table>
<thead>
<tr>
<th></th>
<th>GDP</th>
<th>US FCI</th>
<th>IT FCI</th>
<th>PMI</th>
<th>EPU</th>
</tr>
</thead>
<tbody>
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<td>1.0</td>
<td>-0.4</td>
<td>-0.6</td>
<td>0.2</td>
<td>0.1</td>
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</tr>
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<td>EPU</td>
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<td></td>
<td>1.0</td>
</tr>
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</table>

Notes: The standard error for each correlation coefficient is reported below in parentheses.
Figure B.1: In sample estimates of the distribution of GDP growth for $h = 1$.

Note: The black vertical line represents the realized average annualized GDP growth between $t$ and $t + h$, where $t + h$ refers to the quarter in the title.
Figure B.2: In sample estimates of the distribution of GDP growth for $h = 4$.

Note: The black vertical line represents the realized average annualized GDP growth between $t$ and $t + h$, where $t + h$ refers to the quarter in the title.
C Robustness check: properties of non-parametric distribution

The fitted expectiles implicitly define a conditional predictive distribution of Italian GDP growth which is varying over time according to the risk indicators. Time variation concerns location as well as other moments of the distribution, such as skewness. Regarding the latter, a simple non-parametric skewness indicator can be constructed taking the ratio of the left tail to the right tail of the distribution as follows:

\[
Sk_h(\tau) = \frac{\hat{m}_{t+h|t}(0.5) - \hat{m}_{t+h|t}(\tau)}{\hat{m}_{t+h|t}(1 - \tau) - \hat{m}_{t+h|t}(0.5)}
\]

where the tails are measured by the fitted expectiles of order \(\tau\) and \(1 - \tau\) with \(\tau < 0.5\).

A value of \(Sk\) greater (lower) than 1 implies negative (positive) skewness.

Figure C.1 shows the evolution of \(Sk_h\) at \(\tau = 0.1\) for the Full model and the model including only lagged GDP with \(h = 1, 4\). This measure has the opposite interpretation with respect to the classical indicator of skewness presented in Figure 4, because the latter indicates left (right) skewness if it is lower (greater) than 0, while the former is greater (lower) than 1. It follows that negative correlation of actual future GDP growth with \(Sk_h\) corresponds to positive correlation with classical skewness. Lower panels of figure C.1 match the evidence of table 5, because the downward slope of the dots in scatterplot can be summerized in significant negative correlations for both horizons (\(h = 1\): \(r = -0.6, t = -6.9\) and \(h = 4\), \(r = -0.3, t = -2.9\)). All in all, the finding of section 3 related to Figure 4 are confirmed.

16The indicator provides essentially the same information as the quantile-based Bowley coefficient, \(B = \frac{q(0.75) + q(0.25) - 2q(0.5)}{q(0.75) - q(0.25)}\), except that quantiles are replaced by expectiles. Note that for a symmetric distribution the Bowley coefficient is equal to 0 while \(Sk_h\) is equal to 1.
Figure C.1: Non-parametric skewness indicator.
Upper panels plot the $Sk_h(0.1)$ for the GDP model (red) and the Full model (blue) for $h = 1, 4$. The measure is represented as a centered moving average of 3 terms to get a smoother picture. Lower panels show the scatter plots of $Sk_h(0.1)$ against the GDP growth.

D Robustness check: drivers of the ES

In this appendix we address the issue of which driver was more relevant for the dynamics of the ES in a given quarter using an alternative methodology to the one applied in section 5. The decomposition in Figure D.1 is obtained by using the risk measures implied by the single indicator models. The marginal contribution of each risk driver is computed as the difference between the ES of the model with only one exogenous regressor and the one of the GDP model. A preliminar demeaning of the ESs is performed, using the average over the entire period.
Overall, results confirm that GDP downside risks have been mostly driven by foreign
developments around the Global Financial Crisis, by domestic financial conditions at the time of the Sovereign Debt Crisis and by economic policy uncertainty in more recent years.

**Figure D.1**: Main drivers of \( ES(0.1) \) in the three subsamples: the Global Financial Crisis (2007-10), the Sovereign Debt Crisis (2011-13) and the post-crisis recovery (2014-18).
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