Temi di discussione

Protectionism and the effective lower bound in the euro area

by Pietro Cova, Alessandro Notarpietro and Massimiliano Pisani

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PROTECTIONISM AND THE EFFECTIVE LOWER BOUND IN THE EURO AREA

ONLINE-ONLY APPENDIX

by Pietro Cova[*], Alessandro Notarpietro[*] and Massimiliano Pisani[*]

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A Model overview

In what follows we report the main equations of the simulated DSGE model of the world economy. The model has four regions: Home (EA), US (USA), CH (China), and RW (rest of the world). The size of the world economy is normalized to 1. EA, US, CH, and RW have sizes equal to s^{EA}, s^{US}, s^{CH}, and $\left(1 - s^{EA} - s^{CH} - s^{US}\right)$, respectively, with $s^{EA}, s^{US}, s^{CH} > 0$ and $s^{EA} + s^{US} + s^{CH} < 1$. For each region, the size refers to the overall households' population and to the number of firms operating in each sector (intermediate tradable, intermediate non-tradable, final non-tradable private consumption, final non-tradable public consumption, final non-tradable investment). Each region has a central bank that sets the nominal interest rate according to a standard Taylor rule, and reacts to domestic consumer prices and GDP growth.[1]

Households consume a final good, which is a composite of fuel and non-fuel intermediate non-tradable goods and intermediate tradable goods. Intermediate non-fuel tradables are domestically produced or imported. All households supply differentiated labor services to domestic firms and act as wage setters in monopolistically competitive labor markets, as they charge a wage mark-up over their marginal rate of substitution between consumption and leisure.

Households trade two bonds. One is traded domestically, and is denominated in the domestic currency. The other is internationally traded, and is denominated in US dollars. The related first–order conditions imply that, in each region other than the US, an uncovered interest parity condition holds, linking the differential between domestic and US monetary policy rates to the expected depreciation of the nominal exchange rate of the domestic currency vis-à-vis the US dollar.

On the production side there are firms that, under perfect competition, produce three final manufacturing goods (private consumption, public consumption, and investment goods) and firms that, under monopolistic competition, produce intermediate (internationally) tradable and non-tradable goods.

The final manufacturing goods are sold domestically and are produced combining intermediate goods using a constant-elasticity-of-substitution (CES) production function. The resulting investment and consumption bundles can have different composition. In particular, the investment basket, different from consumption, does not have a fuel component and the consumption basket of the public sector is composed by intermediate non-tradable goods only.

Intermediate tradable and non-tradable goods are produced combining, in each sector-specific production function, capital, labor, and fuel. Capital and labor are supplied by the domestic

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[1] We make the assumption of a cashless economy. This Appendix follows Pesenti (2008), that reports an exhaustive description of a multi-country DSGE model of the world economy.

households and are assumed to be mobile across the two intermediate sectors.

In each region there is a distribution sector. Firms in the distribution sector act under perfect competition. They produce distribution services using intermediate non-tradables according to a Leontief technology. The distribution services allow non-oil consumption goods to be distributed to households. The distribution sector introduces a wedge between the wholesale and the retail price of the consumption goods. It implies that the exchange rate pass-through to retail prices is lower than the pass-through at the border (wholesale price), consistent with empirical evidence.

Given the assumption of differentiated intermediate goods, non-oil firms have market power, are price-setters, and restrict output to create excess profits. Intermediate tradable goods can be sold domestically and abroad. It is assumed that markets for tradable goods are segmented, so that firms can set a different price in each of the three regions.

Oil supply is, by assumption, a worldwide constant endowment, distributed among the four regions. The oil market operates under perfect competition. The law of one price holds, so that the price of oil is the same everywhere when evaluated in the same currency. In each region both households and firms demand oil, for consumption and production purposes, respectively.

In line with other dynamic general equilibrium models (see, among the others, Warne et al. 2008 and Gomes et al. 2010), we include adjustment costs on real and nominal variables, ensuring that consumption, production, wages, and prices react in a gradual way to a given shock. On the real side, habits and quadratic costs prolong the adjustment of consumption and investment, respectively. On the nominal side, quadratic costs make wages and prices sticky.[2] Wages and price are also indexed to a weighted average of previous-period inflation and central bank inflation target.

In what follows, we report the main equations for the Home country. Similar equations hold in the other regions (if not so, we report the differences).

B Firms

We initially show the final goods' sectors (private consumption, investment good, and public sector good). Thereafter, the intermediate goods' sectors (intermediate non-tradable goods, and intermediate tradable goods) and the fuel sector. We report only equations of the Home (EA) economy. Similar equations hold for the other regions. We explicitly state when this is not the case.[3]

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[2]See Rotemberg (1982).

[3]For a detailed description ot the main features of the model see also Pesenti (2008), which provides a description of the GEM (the International Monetary Fund Global Economy Model).

B.1 Final private consumption good

The private consumption bundle is produced by generix firm x according to a CES function of non-fuel $Y_{CV,t}$ and fuel $FU_{C,t}$ bundles:

$$Y_{C,t}(x) = \left[(1 - a_{FU_C})^{\frac{1}{\rho}} C_{V,t}(x)^{\frac{\rho-1}{\rho}} + a_{FU_C}^{\frac{1}{\rho}} FU_{C,t}(x)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \tag{A.1}$$

where a_{FU_C} $(0 < a_{FU_C} < 1)$ is the share of fuel in the bundle and $\rho > 0$ measures the elasticity of substitution between non-fuel consumption, $C_{V,t}$, and fuel, $FU_{C,t}$.

The private non-fuel consumption bundle, $C_{V,t}$, is produced according to a CES function of intermediate, tradable and non-tradable goods ($C_{TC,t}$ and $C_{N,t}$, respectively):

$$C_{V,t}(x) = \left[a_{TC}(x)^{\frac{1}{\eta}} C_{TC,t}^{\frac{\eta-1}{\eta}} + (1 - a_{TC})^{\frac{1}{\eta}} C_{N,t}(x)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{A.2}$$

where the parameter a_{TC} $(0 < a_{TC} < 1)$ is the weight of tradable goods in the consumption bundle and $\eta > 0$ is the elasticity of substitution between tradable and non-tradable goods.

The basket of tradable goods $C_{TC,t}$ is

$$C_{TC,t}(x)^{\frac{\eta_T-1}{\eta_T}} = a_{EA,C}^{\frac{1}{\eta_T}} C_{EA,t}(x)^{\frac{\eta_T-1}{\eta_T}} + a_{US,C}^{\frac{1}{\eta_T}} C_{US,t}(x)^{\frac{\eta_T-1}{\eta_T}}$$
$$+ a_{CH,C}(x)^{\frac{1}{\eta_T}} C_{CH,t}(x)^{\frac{\eta_T-1}{\eta_T}} + (1 - a_{EA,C} - a_{US,C} - a_{CH,C})^{\frac{1}{\eta_T}} C_{RW,t}(x)^{\frac{\eta_T-1}{\eta_T}}, \tag{A.3}$$

where the parameters $a_{EA,C}, a_{US,C}, a_{CH,C}$ $(0 < a_{EA,C}, a_{US,C}, a_{CH,C} < 1$, $a_{EA,C} + a_{US,C} + a_{CH,C} < 1$) are respectively the weights of EA, US, and CH goods in the bundle ($C_{EA,t}$, $C_{US,t}$, and $C_{CH,t}$, respectively), while $\eta_T > 0$ is the elasticity of substitution among tradable goods.

The consumption good C_{EA} is a composite basket of a continuum of differentiated intermediate goods, each supplied by a different EA firm h. It is produced according to the following function:

$$C_{EA,t}(x) = \left[\left(\frac{1}{s^{EA}} \right) \int_0^{s^{EA}} C_{EA,t}(h,x)^{\frac{\theta_T-1}{\theta_T}} dh \right]^{\frac{\theta_T}{\theta_T-1}}, \tag{A.4}$$

where $1 < \theta_T < \infty$ is the elasticity of substitution among EA brands. Similar bundles hold for other EA tradable (imported) and non-tradable goods.

Consumption deflators. The implied overall consumption deflator is

$$P_{C,t} = \left[(1 - a_{FU_C}) P_{V,t}^{1-\rho} + a_{FU_C} P_{FU,t}^{1-\rho} \right]^{\frac{1}{1-\rho}}, \tag{A.5}$$

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where $P_{FU,t}$ is the fuel price deflator and $P_{V,t}$ the deflator of the non-fuel component, equal to

$$P_{V,t} = \left[a_{TC} P_{TC,t}^{1-\eta} + (1 - a_{TC}) P_{N,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{A.6}$$

where $P_{N,t}$ is the price of the bundle of intermediate non-tradables.

The deflator of the non-fuel tradable consumption basket, $P_{TC,t}$, is

$$P_{TC,t} = \left[a_{EA,C} P_{EA,t}^{1-\eta_T} + a_{US,C} P_{US,t}^{1-\eta_T} + a_{CH,C} P_{CH,t}^{1-\eta_T} + (1 - a_{EA,C} - a_{US,C} - a_{CH,C}) P_{RW,t}^{1-\eta_T} \right]^{\frac{1}{1-\eta_T}}, \tag{A.7}$$

where $P_{EA,t}$, $P_{US,t}$, $P_{CH,t}$, and $P_{RW,t}$ are the EA consumption prices of EA, US, CH, and RW tradable goods, respectively.

The EA consumer price of the generic US good is

$$P_{US,t} = \bar{P}_{US,t} \left(1 + \tau_{EA,US,t} \right) + \eta^{distr} P_{N,t}, \tag{A.8}$$

where $\tau_{EA,US,t} > 0$ is the *ad valorem* tariff the EA applies on the generic imported US good and $\bar{P}_{US,t}$ is the border (ex-tariff) price of the imported euro-invoiced good. The term $\eta^{distr} > 0$ is a parameter and is due to the presence of distribution services intensive in local intermediate non-tradable goods. The distribution services introduce a wedge between the border (i.e., wholesale) price and the consumer (i.e., retail) price of the tradable good. Similar equations hold in other regions and for other tradable goods (by assumption, intermediate non-tradable goods do not need a distribution sector to be distributed to local households).

B.2 Final investment good

The production of investment goods I is isomorphic to that of consumption, Eq. (A.2). Specifically, the private investment bundle, I_t, is produced according to a CES function of intermediate, tradable and non-tradable goods ($I_{T,t}$ and $I_{N,t}$, respectively):

$$Y_{I,t}(y) = \left[a_{TI}^{\frac{1}{\eta}} I_{T,t}(y)^{\frac{\eta-1}{\eta}} + (1 - a_{TI})^{\frac{1}{\eta}} I_{N,t}(y)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{A.9}$$

where the parameter a_{TI} ($0 < a_{TI} < 1$) is the weight of tradable goods in the investment bundle and $\eta > 0$ is the elasticity of substitution between tradable and non-tradable goods.

The basket of tradable goods $I_{T,t}$ is

$$I_{T,t}(y)^{\frac{\eta_T-1}{\eta_T}} = a_{EA,I}^{\frac{1}{\eta_T}} I_{EA,t}(y)^{\frac{\eta_T-1}{\eta_T}} + a_{US,I}^{\frac{1}{\eta_T}} I_{US,t}(y)^{\frac{\eta_T-1}{\eta_{TI}}} + a_{CH,I}^{\frac{1}{\eta_T}} I_{CH,t}(y)^{\frac{\eta_T-1}{\eta_T}}$$
$$+ (1 - a_{EA,I} - a_{US,I} - a_{CH,I})^{\frac{1}{\eta_T}} I_{RW,t}^{\frac{\eta_T-1}{\eta_T}}, \tag{A.10}$$

where the parameters $a_{EA,I}, a_{US,I}, a_{CH,I}$ $(0 < a_{EA,I}, a_{US,I}, a_{CH,I} < 1, a_{EA,I} + a_{US,I} + a_{CH,I} < 1$) are respectively the weights of EA, US, and CH goods in the bundle ($I_{EA,t}$, $I_{US,t}$, and $I_{CH,t}$, respectively), while $\eta_T > 0$ is the elasticity of substitution among tradable goods.

The investment good I_{EA} is a composite basket of a continuum of differentiated intermediate goods, each supplied by a different EA firm h. It is produced according to the following function:

$$I_{EA,t}(y) = \left[\left(\frac{1}{s^{EA}} \right) \int_0^{s^{EA}} I_{EA,t}(h,y)^{\frac{\theta_T-1}{\theta_T}} dh \right]^{\frac{\theta_T}{\theta_T-1}}, \tag{A.11}$$

where $1 < \theta_T < \infty$ is the elasticity of substitution among EA brands.

Investment deflators. The implied investment deflator is

$$P_{I,t} = \left[a_{TI} P_{TI,t}^{1-\eta} + (1 - a_{TI}) P_{N,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}. \tag{A.12}$$

The deflator of the tradable investment basket, $P_{TI,t}$, is

$$P_{TI,t} = \left[a_{EA,I} P_{EA,t}^{1-\eta_T} + a_{US,I} P_{US,t}^{1-\eta_T} + a_{CH,I} P_{CH,t}^{1-\eta_T} + (1 - a_{EA,I} - a_{US,I} - a_{CH,I}) P_{RW,t}^{1-\eta_T} \right]^{\frac{1}{1-\eta_T}}. \tag{A.13}$$

B.3 Final public consumption good

The public consumption good $C_{N,t}^g$ is fully biased towards the intermediate non-tradable good

$$C_{N,t}^g(p) = \left[\left(\frac{1}{s^{EA}} \right)^{\theta_N} \int_0^{s^{EA}} C_{N,t}^g(n,p)^{\frac{\theta_N-1}{\theta_N}} dn \right]^{\frac{\theta_N}{\theta_N-1}}, \tag{A.14}$$

where $\theta_N > 1$ is the elasticity of substitution among brands in the non-tradable sector.

B.4 Distribution sector

It is assumed that there is a competitive local distribution sector. Bringing one unit of traded goods to consumers requires η^d units of a basket of differentiated non-traded goods. Each firm

distr in the distribution sector has the following technology

$$\eta^d\left(distr\right) = \left[\left(\frac{1}{s^{EA}}\right)^{\theta_N} \int_0^{s^{EA}} \eta^d\left(n, distr\right)^{\frac{\theta_N-1}{\theta_N}} dn\right]^{\frac{\theta_N}{\theta_N-1}}, \tag{A.15}$$

where $\theta_N > 1$.

B.5   Demand for intermediate goods

Final consumption goods are composed by CES bundles of differentiated intermediate goods, each produced by a single firm under conditions of monopolistic competition,

$$C_{EA,t}\left(x\right) = \left[\left(\frac{1}{s^{EA}}\right)^{\theta_T} \int_0^{s^{EA}} C_t\left(h, x\right)^{\frac{\theta_T-1}{\theta_T}} dh\right]^{\frac{\theta_T}{\theta_T-1}}, \tag{A.16}$$

$$C_{US,t}\left(x\right) = \left[\left(\frac{1}{s^{US}}\right)^{\theta_T} \int_{s^{EA}}^{s^{EA}+s^{US}} C_t\left(f, x\right)^{\frac{\theta_T-1}{\theta_T}} df\right]^{\frac{\theta_T}{\theta_T-1}}, \tag{A.17}$$

$$C_{CH,t}\left(x\right) = \left[\left(\frac{1}{s^{CH}}\right)^{\theta_T} \int_{s^{EA}+s^{US}}^{s^{EA}+s^{US}+s^{CH}} C_t\left(g, x\right)^{\frac{\theta_T-1}{\theta_T}} dg\right]^{\frac{\theta_T}{\theta_T-1}}, \tag{A.18}$$

$$C_{RW,t}\left(x\right) = \left[\left(\frac{1}{1-s^{EA}-s^{US}-s^{CH}}\right)^{\theta_T} \int_{s^{EA}+s^{US}+s^{CH}}^{1} C_t\left(z, x\right)^{\frac{\theta_T-1}{\theta_T}} dz\right]^{\frac{\theta_T}{\theta_T-1}}, \tag{A.19}$$

$$C_{N,t}\left(x\right) = \left[\left(\frac{1}{s^{EA}}\right)^{\theta_N} \int_0^{s^{EA}} C_{N,t}\left(n, x\right)^{\frac{\theta_N-1}{\theta_N}} dn\right]^{\frac{\theta_N}{\theta_N-1}}, \tag{A.20}$$

where firms in the Home intermediate tradable and non-tradable sectors are respectively indexed by $h \in (0, s^{EA}]$ and $n \in (0, s^{EA}]$, firms in US by $g \in (s^{EA}, s^{EA} + s^{US}]$, firms in CH by $f \in (s^{EA} + s^{US}, s^{EA} + s^{US} + s^{CH}]$, and firms in the RW by $r \in (s^{EA} + s^{US} + s^{CH}, 1]$. Parameters θ_T, $\theta_N > 1$ are respectively the elasticity of substitution among brands in the tradable and non-tradable sector. The prices of the intermediate non-tradable goods are denoted $P(i)$. The representative firm producing the final consumption good takes these prices as given when minimizing production costs. The resulting demand for the generic intermediate non-tradable input n is

$$C_{N,t}\left(n, x\right) = \left(\frac{1}{s^{EA}}\right) \left(\frac{P_t\left(n\right)}{P_{N,t}}\right)^{-\theta_N} C_{N,t}\left(x\right), \tag{A.21}$$

where $P_{N,t}$ is the cost-minimizing price of one basket of local non-tradable intermediates,

$$P_{N,t} = \left[\left(\frac{1}{s^{EA}}\right) \int_0^{s^{EA}} P_t\left(n\right)^{1-\theta_N} di\right]^{\frac{1}{1-\theta_N}}. \tag{A.22}$$

Firms producing the final investment goods have similar demand curves. Demand for generic intermediate non-tradable good n by the generic firm in the final consumption, investment, and distribution sectors is

$$\left(\frac{P_t(n)}{P_{N,t}} \right)^{-\theta_N} \left(C_{N,t}(x) + I_{N,t}(y) + C^g_{N,t}(p) \right)$$

$$+\eta^d(d) \int_0^{s^{EA}} \left(C_{EA}(x) + C_{US}(x) + C_{CH}(x) + C_{RW}(x) \right) dx. \tag{A.23}$$

Home demands for (intermediate) domestic and imported tradable goods and the cost-minimizing prices of the corresponding baskets can be derived in a similar way.

B.6 Supply of intermediate goods

Non-tradable goods

The production function for the generic EA intermediate non-tradable good n is:

$$Y_{N,t}(n) = \left[\ (1 - a_{FU_N})^{\frac{1}{\xi_{Y,N}}} V_{N,t}(n)^{\frac{\xi_Y-1}{\xi_Y}} + a_{FU_N}^{\frac{1}{\xi_{Y,N}}} \left(FU_{N,t}(n) \right)^{\frac{\xi_Y-1}{\xi_Y}} \ \right]^{\frac{\xi_Y}{\xi_Y-1}}, \tag{A.24}$$

where the variable $FU_{N,t}(n)$ represents fuel, $V_{N,t}(n)$ is the value added input, the parameter a_{FU_N} $(0 < a_{FU_N} < 1)$ is the weight of fuel in the production, and the parameter $\xi_Y > 0$ measures the elasticity of substitution between value added and fuel.

The value added input of the generic EA intermediate non-tradable good h is:

$$V_{N,t}(n) = \left[(1 - a_{NL})^{\frac{1}{\xi_N}} K_{N,t}(n)^{\frac{\xi_N-1}{\xi_N}} + a_{NL}^{\frac{1}{\xi_N}} L_{N,t}(n)^{\frac{\xi_N-1}{\xi_N}} \right]^{\frac{\xi_N}{\xi_N-1}}, \tag{A.25}$$

where the variable $K_{N,t}(n)$ is the end-of-period physical capital, rented from domestic households in a competitive market, and $L_{N,t}(n)$ is labor, supplied by domestic households. The parameter $\xi_N > 0$ measures the elasticity of substitution between capital and labor. The parameter a_{NL} $(0 < a_{NL} < 1)$ is the weight of labor in the production.

The variable $L_N(n)$ is a composite of a continuum of differentiated labor inputs, , each supplied by a different domestic household j under monopolistic competition:

$$L_{N,t}(n) = \left[\left(\frac{1}{s^{EA}} \right) \int_0^{s^{EA}} L_{N,t}(j)^{\frac{\theta_L-1}{\theta_L}} dj \right]^{\frac{\theta_L}{\theta_L-1}}, \tag{A.26}$$

where $1 < \theta_L < \infty$ is the elasticity of substitution among labor varieties.

FOCs: demand of inputs Denoting W_t the nominal wage index and R_t^K the nominal rental price of capital, cost minimization implies that

$$L_{N,t}(n) = a_{NL} \left(\frac{W_t}{MC_{N,t}(n)} \right)^{-\xi_N} V_{N,t}(n) \tag{A.27}$$

and

$$K_{N,t}(n) = (1 - a_{NL}) \left(\frac{R_t^K}{MC_{N,t}(n)} \right)^{-\xi_N} V_{N,t}(n), \tag{A.28}$$

where $MC_{N^s,t}(h)$ is the nominal marginal cost:

$$MC_{N,t}(n) = \left(a_{NL} W_t^{1-\xi_N} + (1 - a_{NL}) \left(R_t^K \right)^{1-\xi_N} \right)^{\frac{1}{1-\xi_N}}. \tag{A.29}$$

Tradable goods

The production function of the generic EA intermediate tradable good h is:

$$Y_{H,t}(h) = \left[\; (1 - a_{FU_H})^{\frac{1}{\xi_Y}} V_{H,t}(h)^{\frac{\xi_Y - 1}{\xi_Y}} + a_{FU_H}^{\frac{1}{\xi_Y}} \left(FU_{H,t}(h) \right)^{\frac{\xi_Y - 1}{\xi_Y}} \; \right]^{\frac{\xi_Y}{\xi_Y - 1}}, \tag{A.30}$$

where the variable $FU_{H,t}(h)$ represents fuel, bought from the domestic fuel sector, the variable $V_{H,t}(h)$ is value added input, the parameter a_{FU_H} $(0 < a_{FU_H} < 1)$ is the weight of fuel in the production, and the parameter $\xi_Y > 0$ measures the elasticity of substitution between value added and fuel.

The value added input of the generic EA intermediate tradable good h is:

$$V_{H,t}(h) = \left[(1 - a_{HL})^{\frac{1}{\xi}} K_{H,t}(h)^{\frac{\xi - 1}{\xi}} + a_{HL}^{\frac{1}{\xi}} L_{H,t}(h)^{\frac{\xi - 1}{\xi}} \right]^{\frac{\xi}{\xi - 1}}, \tag{A.31}$$

where the variable $K_{H,t}(h)$ is the end-of-period physical capital, rented from domestic households in a competitive market, and $L_{H,t}(h)$ is labor, supplied by domestic households. The parameter $\xi > 0$ measures the elasticity of substitution between capital and labor. The parameter a_{HL} $(0 < a_{HL} < 1)$ is the weight of labor in the production.

The variable $L_H(h)$ is a composite of a continuum of differentiated labor inputs, each supplied by a different domestic household j under monopolistic competition:

$$L_{H,t}(h) = \left[\left(\frac{1}{s^{EA}} \right) \int_0^{s^{EA}} L_{H,t}(j)^{\frac{\theta_L - 1}{\theta_L}} \, dj \right]^{\frac{\theta_L}{\theta_L - 1}}, \tag{A.32}$$

where $1 < \theta_L < \infty$ is the elasticity of substitution among labor varieties. The assumption of monopolistic supply allows us to have nominal wage rigidity in the model.

The generic firm in the intermediate tradable sector minimizes its production costs by optimally choosing the amount of inputs given the above technology constraints and the corresponding prices (the gross nominal rental rate of capital R_t^K, the nominal wage rate W_t, the price of fuel $P_{FU,t}$). The corresponding first order conditions are analogous to equations (A.27) and (A.28):

$$L_{H,t}(h) = a_{HL}\left(\frac{W_t}{MC_{T,t}(h)}\right)^{-\xi} V_{H,t}(h) \tag{A.33}$$

and

$$K_{H,t}(h) = (1 - a_{HL})\left(\frac{R_t^K}{MC_{T,t}(h)}\right)^{-\xi} V_{H,t}(h), \tag{A.34}$$

where $MC_{T,t}(h)$ is the nominal marginal cost:

$$MC_{T,t}(h) = \left(a_{HL}W_t^{1-\xi} + (1 - a_{HL})\left(R_t^K\right)^{1-\xi}\right)^{\frac{1}{1-\xi}}. \tag{A.35}$$

Moreover, the firm sets prices in each (domestic and foreign) destination market in local currency, taking into account local demand conditions and the presence of a local distribution sector. We introduce nominal price rigidities by assuming that the firm pays market-specific quadratic costs for adjusting ex-tariff nominal prices.[4]

Thus, when setting the ex-tariff optimal price, the firm takes into account that tariffs could affect relative prices and thus demand for the produced good. Specifically, and consistent with the definitions of the bundles and deflators reported above, the following consumption demand equation for the generic US brand f holds in the EA:

$$C_{US,t}(f,x) = a_{US,TC}a_{TC}(1 - a_{FU_C})\left(\frac{1}{s^{US}}\right)\left(\frac{P_{US,t}(f)}{P_{US,t}}\right)^{-\theta_T}\left(\frac{P_{US,t}}{P_{TC,t}}\right)^{-\eta_T}$$
$$\left(\frac{P_{TC,t}}{P_{V,t}}\right)^{-\eta}\left(\frac{P_{V,t}}{P_{C,t}}\right)^{-\rho} C_t(x), \tag{A.36}$$

where prices are inclusive of tariff (a similar equation holds for the investment demand). An increase in the tariff on the US good would raise its price inclusive of tariff, according to Eq. (A.8), inducing consumers to substitute other goods for the US ones.

FOCs: supply of intermediate tradables We assume that there is cross-country market segmentation because nominal prices are invoiced and sticky in the currency of the destination market (local currency pricing) and because of local distribution costs intensive in non-tradable

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[4]See Rotemberg (1982).

services, that introduce a wedge between the price of the tradable at the border (wholesale price) and at consumer (retail price) level.

The (generic) Home firm producing the brand h chooses the optimal wholesale prices $\bar{P}_t(h)$ in the Home market, $\bar{P}_t^*(h)$ in the US, $\bar{P}_t^{**}(h)$ in CH, and $\bar{P}_t^{***}(h)$ in the RW to maximize the expected flow of profits (in terms of domestic consumption units),

$$
E_t \sum_{\tau=t}^{\infty} \beta^{\tau} \lambda_{t,\tau} \left[
\begin{array}{c}
\frac{\bar{P}_{\tau}(h)}{P_{\tau}} Y_{\tau}(h) + \frac{S_{\tau} \bar{P}_{\tau}^*(h)}{P_{\tau}} Y_{\tau}^*(h) \\
+ \frac{S_{\tau} \bar{P}_{\tau}^{**}(h)}{S_{\tau}^{**} P_{\tau}} Y_{\tau}^{**}(h) + \frac{S_{\tau} \bar{P}_{\tau}^{***}(h)}{S_{\tau}^{***} P_{\tau}} Y_{\tau}^{***}(h) \\
- \frac{MC_{H,\tau}(h)}{P_{\tau}} \left(Y_{\tau}(h) + Y_{\tau}^*(h) + Y_{\tau}^{**}(h) + Y_{\tau}^{***}(h) \right)
\end{array}
\right], \qquad \text{(A.37)}
$$

where the term E_t denotes the expectation operator conditional on the information set at time t, $0 < \beta < 1$ is the domestic household's discount factor, $\lambda_{t,\tau}$ is the domestic household's stochastic discount factor, and $MC_{T,t}(h)$ is the nominal marginal cost, S, S^{**}, S^{***} are the nominal exchange rates of the Home, CH, and RW currency vis-à-vis the US dollar, respectively.

The maximization is subject to the demand of the destination market and (destination-specific) quadratic price adjustment costs.

The demand curves for the generic Home intermediate good h depend on local demand for all Home goods and on relative prices. Thus, the demand curves in Home country, US, CH, JP, and RW are

$$
Y_{\tau}(h) = \left(\frac{1}{s^{EA}} \right) \left(\frac{P_{H,\tau}(h)}{P_{H,\tau}} \right)^{-\theta_H} \int_0^{s^{EA}} C_{H,\tau}(h,x) \, dx + \int_0^{s^{EA}} I_{H,\tau}(h,y) \, dy, \qquad \text{(A.38)}
$$

$$
Y_{\tau}^*(h) = \left(\frac{1}{s^{EA}} \right) \left(\frac{P_{H,\tau}^*(h)}{P_{H,\tau}^*} \right)^{-\theta_H} \int_{s^{EA}}^{s^{EA}+s^{US}} C_{H,\tau}(h,x^*) \, dx^* + \int_{s^{EA}}^{s^{EA}+s^{US}} I_{H,\tau}(h,y^*) \, dy^*, \qquad \text{(A.39)}
$$

$$
Y_{\tau}^{**}(h) = \left(\frac{1}{s^{EA}} \right) \left(\frac{P_{H,\tau}^{**}(h)}{P_{H,\tau}^{**}} \right)^{-\theta_H} \int_{s^{EA}+s^{US}}^{s^{EA}+s^{US}+s^{CH}} C_{H,\tau}(h,x^{**}) \, dx^{**} + \int_{s^{EA}+s^{US}}^{s^{EA}+s^{US}+s^{CH}} I_{H,\tau}(h,y^{**}) \, dy^{**}, \qquad \text{(A.40)}
$$

$$
Y_{\tau}^{***}(h) = \left(\frac{1}{s^{EA}} \right) \left(\frac{P_{H,\tau}^{***}(h)}{P_{H,\tau}^{***}} \right)^{-\theta_H} \int_{s^{EA}+s^{US}+s^{CH}}^{1} C_{H,\tau}(h,x^{***}) \, dx^{***} + \int_{s^{EA}+s^{US}+s^{CH}}^{1} I_{H,\tau}(h,y^{***}) \, dy^{***}, \qquad \text{(A.41)}
$$

respectively.

The region-specific adjustment costs paid by the generic firm h to adjust nominal wholesale prices (one in each region) are

$$AC^p_{H,\tau}(i) \equiv \frac{\kappa^p_H}{2} \left(\frac{\bar{P}_{H,\tau}(h)/\bar{P}_{H,\tau-1}(h)}{\bar{\pi}^{ind_H}_{H,\tau-1} \pi^{1-ind_H}_{target}} - 1 \right)^2 \frac{\bar{P}_{H,\tau}}{P_\tau} Y_{H,\tau}, \tag{A.42}$$

$$AC^{p*}_{H,\tau}(h) \equiv \frac{\kappa^{p*}_H}{2} \left(\frac{\bar{P}^*_{H,\tau}(h)/\bar{P}^*_{H,\tau-1}(h)}{\left(\bar{\pi}^*_{H,\tau-1}\right)^{ind^*_H} \pi^{1-ind^*_H}_{target^*}} - 1 \right)^2 \frac{S_\tau \bar{P}^*_{H,\tau}}{P^*_\tau} Y^*_{H,\tau}, \tag{A.43}$$

$$AC^{p**}_{H,\tau}(i) \equiv \frac{\kappa^{p**}_H}{2} \left(\frac{\bar{P}^{**}_{H,\tau}(h)/\bar{P}^{**}_{H,\tau-1}(h)}{\left(\bar{\pi}^{**}_{H,\tau-1}\right)^{ind^{**}_H} \pi^{1-ind^{**}_H}_{target^{**}}} - 1 \right)^2 \frac{S_\tau \bar{P}^{**}_{H,\tau}}{S^{**}_\tau P^{**}_\tau} Y^{**}_{H,\tau}, \tag{A.44}$$

$$AC^{p***}_{H,\tau}(i) \equiv \frac{\kappa^{p***}_H}{2} \left(\frac{\bar{P}^{***}_{H,\tau}(h)/\bar{P}^{***}_{H,\tau-1}(h)}{\left(\bar{\pi}^{***}_{H,\tau-1}\right)^{ind^{***}_H} \pi^{1-ind^{***}_H}_{target^{***}}} - 1 \right)^2 \frac{S_\tau \bar{P}^{**}_{H,\tau}}{S^{***}_\tau P^{***}_\tau} Y^{***}_{H,\tau}, \tag{A.45}$$

in the domestic, US, CH, and RW markets, respectively. The parameters κ^p_H, $\kappa^{p}_H{}^*$, $\kappa^{p}_H{}^{**}$, $\kappa^{p}_H{}^{***} > 0$ measure the degree of nominal rigidity in the Home country, US, CH, and RW, respectively, whereas ind_H, ind^*_H, ind^{**}_H, ind^{***}_H are the corresponding indexation parameters. Moreover, π_{target}, π_{target^*}, $\pi_{target^{**}}$, $\pi_{target^{***}}$ denote the long-run (consumer-price) inflation targets set by the central bank in EA, US, CH, and RW, respectively.

First order condition with respect to the domestic price of the Home tradable good.
After some algebraic manipulations, the first order condition with respect to the wholesale price of Home good set in the domestic market, $\bar{P}_t(h)$, under the assumption of symmetric firms (i.e., $\bar{P}_{H,t}(h) = \bar{P}_{H,t}$ for every h) is, in real terms (i.e., in units of domestic consumption, $\bar{p}_{H,t} \equiv \bar{P}_{H,t}/P_t$, similar expressions hold for other relative prices and for the real marginal cost):

$$p_{H,t} = \theta_H \left(\bar{p}_{H,t} - mc_{H,t} \right) + A_{H,t}, \tag{A.46}$$

where $p_{H,t} \equiv P_{H,t}/P_t$ is the retail price of the Home good and mc_t is the real marginal cost of producing the good and $A_{H,t}$ is defined as

$$A_{H,t} \equiv p_{H,t} \kappa_H^p \frac{\bar{\pi}_{H,t}}{\bar{\pi}_{H,t-1}^{ind_H} \pi_{target}^{1-ind_H}} \left(\frac{\bar{\pi}_{H,t}}{\bar{\pi}_{H,t-1}^{ind_H} \pi_{target}^{1-ind_H}} - 1 \right)$$
$$-p_{H,t} \beta \kappa_H^p E_t \left[\frac{1}{\pi_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \frac{\bar{\pi}_{H,t+1}^2}{\pi_{H,t}^{ind_H} \pi_{target}^{1-ind_H}} \left(\frac{\bar{\pi}_{H,t+1}}{\bar{\pi}_{H,t}^{ind_H} \pi_{target}^{1-ind_H}} - 1 \right) \frac{Y_{H,t+1}}{Y_{H,t}} \right], \qquad (A.47)$$

where λ is the marginal utility of consumption.

First order condition with respect to the price, invoiced in US dollars, of the Home tradable good in the US. After some algebraic manipulations, the first order condition with respect to the wholesale Home price set in the domestic market, $\bar{P}_t^*(h)$, under the assumption of symmetric firms (i.e., $\bar{P}^*_{H,t}(h) = \bar{P}^*_{H,t}$ for every h) is, in real terms (i.e., in units of domestic consumption):

$$p_{H,t}^* = \theta_H \left(\bar{p}^*_{H,t} - mc_{H,t} rer_t \right) \left(1 + tariff_t^{US} \right) + A_{H,t}^*, \qquad (A.48)$$

where $tariff^{US} > 0$ is the US tariff imposed on Home goods, rer is the real exchange rate between the Home currency and the US dollar ($rer_t \equiv S_t P_t^{US}/P_t^H$ and $p_{H,t} \equiv P_{H,t}/P_t$ (similar expressions hold for other relative prices), $A_{H,t}^*$ is defined as

$$A_{H,t}^* \equiv p_{H,t}^* \kappa_H^{p*} \frac{\bar{\pi}_{H,t}^*}{\left(\bar{\pi}_{H,t-1}^* \right)^{ind_H} \pi_{target*}^{1-ind_H}} \left(\frac{\bar{\pi}_{H,t}^*}{\left(\bar{\pi}_{H,t-1}^* \right)^{ind_{H*}} \pi_{target*}^{1-ind_{H*}}} - 1 \right)$$
$$-p_{H,t}^* \beta \kappa_H^{p*} E_t \left[\frac{1}{\pi_{t+1}} \frac{S_{t+1}}{S_t} \frac{\lambda_{t+1}}{\lambda_t} \frac{\bar{\pi}_{H,t+1}^{*2}}{\left(\bar{\pi}_{H,t}^* \right)^{ind_{H*}} \pi_{target*}^{1-ind_{H*}}} \left(\frac{\bar{\pi}_{H,t+1}^*}{\left(\bar{\pi}_{H,t}^* \right)^{ind_{H*}} \pi_{target*}^{1-ind_{H*}}} - 1 \right) \frac{Y_{H,t+1}^*}{Y_{H,t}^*} \right],$$
$$(A.49)$$

where $p_{H,t}^*$ is the retail price of the Home good in the US market.

First order conditions with respect to the price of the Home tradable good in CH and RW. Similar equations hold for the prices $P_t^{**}(h)$ and $P_t^{***}(h)$ set in the CH and RW market, respectively.

First order condition with respect to the price of the Home non-tradable good. Each firm n sets the price $P_{N,t}(n)$ in the Home intermediate non-tradable sector to maximize

the present discounted value of profits

$$E_t \sum_{\tau=t}^{\infty} \beta^t \lambda_{t,\tau} \left[\left(\frac{P_{N,\tau}(n)}{P_\tau} - \frac{MC_{N,\tau}(n)}{P_\tau} \right) Y_{N,\tau}(n) \right], \tag{A.50}$$

where the term E_t denotes the expectation operator conditional on the information set at time t, $0 < \beta < 1$ is the domestic household's discount factor, $\lambda_{t,\tau}$ is the domestic household's stochastic discount factor, and $MC_{N,t}(n)$ is the nominal marginal cost.

The maximization is subject to the demand constraint

$$Y_{N,\tau}(n) = \left(\frac{1}{s^{EA}} \right) \left(\frac{P_t(n)}{P_{N,t}} \right)^{-\theta_N} \left(\int_0^{s^{EA}} C_{N,t}(n,x)\,dx + \int_0^{s^{EA}} I_{N,t}(n,y)\,dy + \int_0^{s^{EA}} C_{N,t}^g(n,p)\,dp \right), \tag{A.51}$$

and the quadratic adjustment cost,

$$AC_{N,\tau}^p(n) \equiv \frac{\kappa_N^p}{2} \left(\frac{P_{N,\tau}(n)/P_{N,\tau-1}(n)}{\pi_{N,\tau-1}^{ind_N} \pi_{target}^{1-ind_N}} - 1 \right)^2 \frac{P_{N,\tau}}{P_\tau} Y_{N,\tau}. \tag{A.52}$$

The adjustment cost is paid in unit of sector-specific product $Y_{N,t}$, where $\kappa_N^p \geq 0$ is a parameter that measures the degree of price stickiness, $\pi_{N,t-1}$ is the previous-period gross inflation rate of non-tradable goods $(\pi_{N,t} \equiv P_{N,t}/P_{N,t-1})$, π_{target} is the long-run (consumer-price) inflation target set by the central bank, and $0 \leq ind_N \leq 1$ is a parameter that measures indexation to previous-period inflation.

The FOC with respect to $P_{N,t}(n)$ is

$$p_{N,t} = \frac{\theta_N}{\theta_N - 1} mc_{N,t} - \frac{A_{N,t}}{\theta_N - 1}, \tag{A.53}$$

where $A_{N,t}$ contains terms related to the presence of price adjustment costs

$$A_{N,t} \equiv p_{N,t} \kappa_N^p \frac{\pi_{N,t}}{\pi_{N,t-1}^{ind_N} \pi_{target}^{1-ind_N}} \left(\frac{\pi_{N,t}}{\pi_{N,t-1}^{ind_N} \pi_{target}^{1-ind_N}} - 1 \right)$$

$$-\beta \kappa_N^p E_t \left[p_{N,t} \frac{1}{\pi_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{N,t+1}^2}{\pi_{N,t}^{ind_N} \pi_{target}^{1-ind_N}} \left(\frac{\pi_{N,t+1}}{\pi_{N,t}^{ind_N} \pi_{target}^{1-ind_N}} - 1 \right) \frac{Y_{N,t+1}}{Y_{N,t}} \right]. \tag{A.54}$$

Labor bundle   In the case of the generic firm n operating in the intermediate non-tradable sector, the labor input $L_N(n)$ is a CES combination of differentiated labor inputs supplied by

domestic agents and defined over a continuum of mass equal to the country size ($j \in \left[0, s^{EA}\right]$):

$$L_{N,t}(i) \equiv \left(\frac{1}{s^{EA}}\right)^{\frac{1}{\sigma_L}} \left[\int_0^{s^{EA}} L_t(n,j)^{\frac{\sigma_L-1}{\sigma_L}} dj\right]^{\frac{\sigma_L}{\sigma_L-1}}, \tag{A.55}$$

where $L(n,j)$ is the demand of the labor input of type j by the producer of good n and $\sigma_L > 1$ is the elasticity of substitution among labor inputs. Cost minimization implies that

$$L_t(n,j) = \left(\frac{1}{s^{EA}}\right) \left(\frac{W_t(j)}{W_t}\right)^{-\sigma_L} L_{N,t}(j), \tag{A.56}$$

where $W(j)$ is the nominal wage of labor input j and the wage index W is

$$W_t = \left[\left(\frac{1}{s^{EA}}\right) \int_0^{s^{EA}} W_t(j)^{1-\sigma_L} dj\right]^{\frac{1}{1-\sigma_L}}. \tag{A.57}$$

Similar equations hold for firms producing intermediate tradable goods. Each household is the monopolistic supplier of a labor input j and sets the nominal wage facing a downward-sloping demand obtained by aggregating demand across domestic firms.

C   Households

In the Home country there is a countinuum of households of mass $j \in [0, n]$. Each household j maximizes its lifetime expected utility subject to the budget constraint. The lifetime utility, in consumption C and labor L, is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log\left(C_t(j) - b_c C_{t-1}\right) - \frac{1}{1+\zeta} L_t(j)^{1+\zeta}\right), \tag{A.58}$$

where $0 < \beta < 1$ is the discount factor, $b_c \in (0,1)$ is the external habit parameter, C_t is the private consumption bundle, defined in eq. (A.1), $\zeta > 0$ is the reciprocal of the Frisch elasticity of labor supply, and L_t is the demand of the household-specific labor type by domestic firms,

$$L_t(j) = \left(\frac{W_t(j)}{W_t}\right)^{-\theta_L} L_t, \tag{A.59}$$

where the parameter $\theta_L > 1$ measures the elasticity of substitution among different labor brands supplied by different households. The budget constraint is

$$B_{H,t}(j) + S_t B_{F,t}(j) \leq R_{t-1} B_{H,t-1}(j)$$

$$+ R_{t-1}^{US} \left[1 - \Gamma_{B_F,t-1}\right] S_t B_{F,t-1}(j) + R_t^k K_{t-1}(j) + W_t(j)L_t(j)$$

$$- \frac{\kappa_W}{2} \left(\frac{W_t(j)/W_{t-1}(j)}{\pi_{W,t}^{ind_W} \pi_t^{1-ind_W}} - 1 \right)^2 W_t L_t + \Pi_t^{prof}(j) - P_{C,t}C_t(j) - P_{I,t}I_t(j) - TAX_t(j), \quad \text{(A.60)}$$

where the parameter $\kappa_W > 0$ measures the degree of nominal wage rigidity and L_t is the total amount of labor in the Home economy. $B_{H,t}$ is the end-of-period t position in a nominal bond denominated in the Home currency, $B_{F,t}$ is the end-of period position in a nominal bond denominated in US dollars, and $TAX_t > 0$ (< 0) are lump-sum taxes (transfers). The two bonds respectively pay the domestic R_t and US R_t^{US} (gross nominal) policy rates at the beginning of period $t + 1$. The interest rates are known at time t (consistent with the riskless bond assumption). The variable S_t is the bilateral nominal exchange rate of the domestic currency vis-à-vis the US dollar, defined as number of Home currency units per unit of US dollar.

The function $\Gamma_{B_F,t}$ captures the costs of undertaking positions in the international asset market and pins down a well-defined steady-state. It has the following functional form:

$$\Phi\left(\frac{S_t B_{F,t}}{P_t} - b\right) \equiv \exp\left(\phi_b\left(\frac{S_t B_{F,t}}{P_t} - b\right)\right) \qquad \phi_B \geq 0.$$

The parameter ϕ_B controls the speed of convergence to the non-stochastic steady state.[5]

The sources of the household income are physical capital $K_t(j)$, which is rented to domestic intermediate firms at the net rate R_t^k, labor $L_t(j)$, which is supplied to domestic intermediate firms and earns the nominal wage $W_t(j)$, and $\Pi_t^{prof}(j)$, which represents profits from ownership of domestic firms (the profits are rebated in a lump-sum way to households).

The variable $I_t(j)$ is investment in physical capital. The latter is accumulated according to the following law:

$$K_t(j) = (1-\delta)K_{t-1}(j) + \left[1 - \frac{\psi}{2}\left(\frac{I_t(j)}{I_{t-1}(j)} - 1\right)^2 I_t(j)\right], \qquad \text{(A.61)}$$

where $0 < \delta < 1$ is the depreciation rate and investment is subject to a quadratic adjustment

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[5] The function $\Phi(.)$ depends on real holdings of the foreign assets in the entire Home economy. Hence, domestic households take it as given when deciding on the optimal holding of the foreign bond. We require that $\Phi(0) = 1$ and that $\Phi(.) = 1$ only if $S_t B_{F,t}/P_t = b$, where b is the steady state real holdings of the foreign assets in the entire Home economy. The function $\Phi(.)$ is assumed to be differentiable and decreasing at least in the neighborhood of the steady state. The payment of this cost is rebated in a lump-sum fashion to foreign agents.

cost.

C.1 First-order conditions

The household maximizes the intertemporal utility with respect to consumption $C_t(j)$, $B_{H,t}(j)$, $B_{F,t}(j)$, $W_t(j)$, $K_t(j)$, $I_t(j)$, subject to the budget constraint, the capital accumulation law, the R&D accumulation law, and the adjustment costs.

The corresponding FOCs in the generic period t are:

- with respect to domestic bond $C_t(j)$

$$\lambda_t(j) = (C_t(j) - b_c C_{t-1})^{-1}, \tag{A.62}$$

- with respect to domestic euro-denominated bond $B_{H,t}(j)$

$$\lambda_t(j) = \beta E_t \lambda_{t+1}(j) R_t \pi_{t+1}^{-1}, \tag{A.63}$$

- with respect to US-dollar bonds $B_{F,t}(j)$

$$\lambda_t(j) = \beta E_t \lambda_{t+1}(j) R_t^{US}(1 - \Gamma_{B,t}) \frac{\Delta S_{t+1}}{\pi_{t+1}}, \tag{A.64}$$

- with respect to the end-of-period capital $K_t(j)$

$$Q_t(j) = \beta E_t \left[\lambda_{t+1} r_{t+1}^K + Q_{t+1}(j)(1 - \delta) \right], \tag{A.65}$$

where $Q(j)$ is the Tobin's Q (i.e., the multiplier of the capital accumulation law),

- with respect to investment $I_t(j)$

$$\lambda_t(j) p_{I,t} = Q_t(j) \left[1 - \frac{\psi}{2} \left(\frac{I_t(j)}{I_{t-1}(j)} - 1 \right)^2 - \psi \left(\frac{I_t(j)}{I_{t-1}(j)} - 1 \right) \frac{I_t(j)}{I_{t-1}(j)} \right]$$
$$+ \beta E_t Q_{t+1}(j) \psi \left[\left(\frac{I_{t+1}(j)}{I_t(j)} - 1 \right) \frac{I_{t+1}^2(j)}{I_t^2(j)} \right], \tag{A.66}$$

- with respect to nominal wage $W_t(j)$

$$\theta_L \frac{W_t(j)^{-\theta_L(1+\zeta)-1}}{W_t^{-\theta_L(1+\zeta)}} L_t^\zeta + (1 - \theta_L) \frac{W_t(j)^{-\theta_L}}{W_t^{-\theta_L}} = \lambda_t \kappa_W \left(\frac{W_t(j)/W_{t-1}(j)}{\pi_{W,t-1}^{ind_w} \pi_{target}^{1-ind_w}} - 1 \right) \frac{W_t/W_{t-1}(j)}{\pi_{W,t-1}^{ind_w} \pi_{target}^{1-ind_w}}$$
$$- \beta \lambda_{t+1} \kappa_W \left(\frac{W_{t+1}(j)/W_t(j)}{\pi_{W,t}^{ind_w} \pi_{target}^{1-ind_w}} - 1 \right) \frac{W_{t+1} W_{t+1}(j)/W_t(j)^2 L_{t+1}}{\pi_{W,t}^{ind_w} \pi_{target}^{1-ind_w} L_t}. \tag{A.67}$$

D   Oil market

There is a continuum of firms o that, under perfect competition, transform crude oil in fuel according to the following linear technology:

$$Y_{FU,t}(o) = Oil(o). \tag{A.68}$$

Fuel is sold to domestic households and firms. It is assumed that each county has a constant oil endowment, labelled $E_t^{EA} = E^{EA}$.

The US dollar price is set in the international crude oil market. The law of one price holds for oil, i.e., the price is the same in every region once expressed in the same currency. In the EA case, the oil price in EA currency is

$$P_{oil,t} = S_t P_{oil,t}^{US}, \tag{A.69}$$

where $P_{oil,t}^{US}$ is the US dollar price and is the nominal exchange rate between the euro and the US dollar (number of euros per dollar).

It is assumed that the price of crude oil is immediately and fully pass-troughed in the fuel price. Thus,

$$P_{FU,t} = Poil, t. \tag{A.70}$$

E   Monetary and fiscal policy

E.1   Monetary policy

In the case of the EA central bank, the gross monetary policy rate R^{EA} is set according to the rule:

$$\left(\frac{R_t^{EA}}{\overline{R}^{EA}}\right) = max\left(\frac{1}{\overline{R}^{EA}}, \left(\frac{R_{t-1}^{EA}}{\overline{R}^{EA}}\right)^{\phi_R^{EA}} \left(\frac{\Pi_{C,t}^{EA}}{\overline{\Pi}_C^{EA}}\right)^{\left(1-\phi_R^{EA}\right)\phi_\Pi^{EA}} \left(\frac{GDP_t^{EA}}{GDP_{t-1}^{EA}}\right)^{\left(1-\phi_R^{EA}\right)\phi_{gY}^{EA}}\right), \tag{A.71}$$

where $\overline{R}^{EA}$ is the steady-state gross monetary policy rate, $\overline{\Pi}_{C,t}^{EA} \equiv \overline{P}_{C,t}^{EA}/\overline{P}_{C,t-1}^{EA}$ is the ex-tariff gross headline inflation rate, $\overline{\Pi}_C^{EA}$ is the long-run steady-state inflation target, and GDP_t^{EA} the EA output in real terms (i.e., evaluated at constant prices), the terms ϕ_R^{EA}, ϕ_Π^{EA}, and ϕ_{gY}^{EA} are parameters $(0 < \phi_R^{EA} < 1)$.

In the case of all the remaining regions, a similar rule holds, except that the max operator is

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not active.

E.2 Fiscal policy

The government budget constraint is

$$B_{G,t} - B_{G,t-1}R_{t-1} = P_{N,t}C_{N,t}^g - TAX_t, \tag{A.72}$$

where $B_{G,t} > 0$ is public debt, which is financed by a one-period nominal bond issued in the domestic bond market, paying the (gross) monetary policy interest rate R_t. The variable $C_{N,t}^g$ represents government purchases of goods and services, while $TAX_t > 0$ (< 0) are lump-sum taxes (transfers) to households. Consistent with the empirical evidence, $C_{G,t}$ is fully biased towards the non-tradable intermediate good. Therefore, it is multiplied by the corresponding price index $P_{N,t}$.[6]

The government follows a fiscal rule defined on lump-sum taxes to bring the public debt as a % of domestic GDP, $b_G > 0$, in line with its long-run (steady-state) target $\bar{b}_G$ and to stabilize its rate of change.[7]

The rule is

$$\frac{TAX_t}{TAX_{t-1}} = \left(\frac{b_{G,t}}{\bar{b}_G}\right)^{\phi_1} \left(\frac{b_{G,t}}{b_{G,t-1}}\right)^{\phi_2}, \tag{A.74}$$

where parameters ϕ_1, ϕ_2 are greater than zero, calling for a increase (reduction) in lump-sum taxes whenever the current-period public debt (as a ratio to GDP) is above (below) the target and the previous-period public debt, respectively. We choose lump-sum taxes to stabilize public finance as they are non-distortionary.

F   Market clearing conditions

In what follows we report the market clearing conditions for goods and bonds holding in EA (i.e., Home). Similar equations hold for other countries.

- EA bond

$$\int_0^{s^{EA}} B_{H,t}(j)\, dj = B_{G,t} \tag{A.75}$$

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[6]See Corsetti and Muller (2006).

[7]The definition of nominal GDP is

$$GDP_t = P_tC_t + P_t^I I_t + P_{N,t}C_{G,t} + P_t^{EXP}EXP_t - P_t^{IMP}IMP_t, \tag{A.73}$$

where P_t, is the price of private consumption, P_t^I, $P_{N,t}$, P_t^{EXP}, P_t^{IMP} are prices of private investment in physical capital, public consumption, exports, and imports, respectively.

- labor market

$$\int_0^{s^{EA}} L_t(j)\,dj = \int_0^{s^{EA}} L_{H,t}(h)\,dh + \int_0^{s^{EA}} L_{N,t}(n)\,dn \tag{A.76}$$

- internationally traded bond

$$\int_0^{s^{EA}} B_{F,t}(j)\,dj + \int_{s^{EA}}^{s^{EA}+s^{US}} B_{F,t}(j^*)\,dj^*$$
$$+ \int_{s^{EA}+s^{US}}^{s^{EA}+s^{US}+s^{CH}} B_{F,t}(j^{**})\,dj^{**} + \int_{s^{EA}+s^{US}+s^{CH}}^{1} B_{F,t}(j^{***})\,dj^{***} = 0 \tag{A.77}$$

- generic EA intermediate tradable h sold in EA

$$Y_{H,t}(h) = \int_0^{s^{EA}} C_{H,t}(h,x)\,dx + \int_0^{s^{EA}} I_{H,t}(h,y)\,dy \tag{A.78}$$

- generic EA intermediate tradable h sold in US

$$Y_{H,t}^*(h) = \int_{s^{EA}}^{s^{EA}+s^{US}} C_{H,t}(h,x^*)\,dx^* + \int_{s^{EA}}^{s^{EA}+s^{US}} I_{H,t}(h,y^*)\,dy^* \tag{A.79}$$

- generic EA intermediate tradable h sold in CH

$$Y_{H,t}^{**}(h) = \int_{s^{EA}+s^{US}}^{s^{EA}+s^{US}+s^{CH}} C_{H,t}(h,x^{**})\,dx^{**} + \int_{s^{EA}+s^{US}}^{s^{EA}+s^{US}+s^{CH}} I_{H,t}(h,y^{**})\,dy^{**} \tag{A.80}$$

- generic EA intermediate tradable h sold in RW

$$Y_{H,t}^{***}(h) = \int_{s^{EA}+s^{US}+s^{CH}}^{1} C_{H,t}(h,x^{***})\,dx^{***} + \int_{s^{EA}+s^{US}+s^{CH}}^{1} I_{H,t}(h,y^{***})\,dy^{***} \tag{A.81}$$

and, finally, supply of generic intermediate tradable h is equal to worldwide aggregate demand:

$$Y_{TH,t}(h) = Y_{H,t}(h) + Y_{H,t}^*(h) + Y_{H,t}^{**}(h) + Y_{H,t}^{***}(h) \tag{A.82}$$

- Home intermediate non-tradable good n

$$Y_{N,t}(n) = Y_{CN,t}(n,x) + Y_{IN,t}(n,y)$$
$$+ \int_0^{s^{EA}} \eta^d(n,distr) \int_0^{s^{EA}} (C_{EA}(x) + C_{US}(x) + C_{CH}(x) + C_{RW}(x))\,dx\,ddistr \tag{A.83}$$

- Home final consumption good

$$\int_0^{s^{EA}} Y_{C,t}(x)\,dx = \int_0^{s^{EA}} C(j)\,dj \tag{A.84}$$

- Home final investment good

$$\int_0^{s^{EA}} Y_{I,t}(x)\,dx = \int_0^{s^{EA}} I(j)\,dj \tag{A.85}$$

- Home fuel

$$\int_0^{s^{EA}} Y_{FU,t}(o)\,do = \int_0^{s^{EA}} FU_{C,t}(x)\,dx + \int_0^{s^{EA}} FU_{H,t}(h)\,dh + \int_0^{s^{EA}} FU_{N,t}(n)\,dn \tag{A.86}$$

- world oil market

$$E_t^{EA} + E_t^{US} + E_t^{CH} + E_t^{RW} =$$

$$\int_0^{s^{EA}} O_t(o)\,do + \int_{s^{EA}}^{s^{EA}+s^{US}} O_t\left(o^{US}\right)\,do^{US}$$

$$+ \int_{s^{EA}+s^{US}}^{s^{EA}+s^{US}+s^{CH}} O_t\left(o^{CH}\right)\,do^{CH} + \int_{s^{EA}+s^{US}+s^{CH}}^{1} O_t\left(o^{RW}\right)\,do^{RW} \tag{A.87}$$

G   Equilibrium

We define a symmetric equilibrium. In each country there is a representative household, and a representative firm in each sector. The equilibrium is a sequence of allocations and prices such that households maximize utility subject to their budget constraint, firms maximize profits subject to technology and demand constraints, the monetary policy and fiscal policy rules hold, the public sector budget constraint holds, and all market clearing conditions hold.

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