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THE ECONOMIC DRIVERS OF VOLATILITY AND UNCERTAINTY

by Andrea Carriero*, Francesco Corsello** and Massimiliano Marcellino***

Abstract

We introduce a time-series model for a large set of variables in which the structural shocks identified are employed to simultaneously explain the evolution of both the level (conditional mean) and the volatility (conditional variance) of the variables. Specifically, the total volatility of macroeconomic variables is first decomposed into two separate components: an idiosyncratic component, and a component common to all of the variables. Then, the common volatility component, often interpreted as a measure of uncertainty, is further decomposed into three parts, respectively driven by the volatilities of the demand, supply and monetary/financial shocks. From a methodological point of view, the model is an extension of the homoscedastic Multivariate Autoregressive Index (MAI) model (Reinsel, 1983) to the case of time-varying volatility. We derive the conditional posterior distribution of the coefficients needed to perform estimations via Gibbs sampling. By estimating the model with US data, we find that the common component of volatility is substantial, and it explains at least 50 per cent of the overall volatility for most variables. The relative contribution of the demand, supply and financial volatilities to the common volatility component is variable specific and often time-varying, and some interesting patterns emerge.

JEL Classification: C15, C32, C38, C51, E30.
Keywords: Multivariate Autoregressive Index models, stochastic volatility, reduced rank regressions, Bayesian VARs, factor models, structural analysis.
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1 Introduction[1]

The seminal paper by Engle (1982) emphasized the importance of changing conditional volatilities in economic and financial variables. The ensuing literature on this topic is extensive, starting with time-series contributions but then entering also the mainstream economic literature, with the debate on the Great Moderation's origin, see, e.g., Primiceri (2005), Cogley & Sargent (2005), Sims & Zha (2006), and Fernández-Villaverde & Rubio-Ramírez (2013).[2] After the 2007 financial crisis there seems to be growing consensus on the importance of allowing for economic and financial shocks with particularly large variance during crisis periods, see for example Abbate, Eickmeier, Lemke & Marcellino (2016) and references therein.

While there is by now a large literature modeling volatilities as stochastic and time-varying, the economic identification of the shocks driving this variation in conditional variances has not been investigated. This is in stark contrast with the mainstream approach to study the effects of identified structural shocks on the conditional mean of macroeconomic variables as laid out in Sims (1980).

This paper introduces a model in which the identified structural shocks are employed to simultaneously explain the evolution of *both* the level (conditional mean) *and* the volatility (conditional variance) of macroeconomic variables. In particular, we provide a strategy to decompose the total volatility of each macroeconomic variable, into two separate components: an idiosyncratic component, and a component common to all of the variables, which is often interpreted as a measure of uncertainty (see e.g. Jurado, Ludvigson & Ng, 2015 and Carriero, Clark & Marcellino, 2018). The common volatility component is further decomposed into three parts, respectively driven by structural demand, supply and monetary / financial shocks. All these components are orthogonal to each other, so the proposed decomposition provides a measure of the contribution of each source to the total volatility.

Methodologically, the model is an extension of the homoschedastic Multivariate Autoregressive Index (MAI) Model of Reinsel (1983) and Carriero, Kapetanios & Marcellino (2016) [CKM16 henceforth] to the case of time varying volatility (MAI-SV). We derive the conditional posterior distributions of the model parameters, which are then used for the development of a Gibbs sampler algorithm. A key feature of our approach is the use

[1]We would like to thank Marco Del Negro, Barbara Rossi and participants at the GSE Summer Forum on Time Series Econometrics for useful comments on a previous draft. Marcellino is grateful to the Baffi-Carefin center for partially supporting this research.

[2]From a reduced form perspective, Clark & Ravazzolo (2015) reports evidence that introducing stochastic volatilities improves the forecasting accuracy for macro-financial variables, strengthening the evidence already gathered by D'Agostino, Gambetti & Giannone (2013) and Carriero, Clark & Marcellino (2015).

of a large information set, which is essential to avoid non-fundamentalness of the shocks and omitted variable bias, but requires efficient strategies to reduce the computational burden resulting from the large number of unobservable state variables.

Carriero, Clark & Marcellino (2016, 2018, 2019) also analyze small and large VAR models with a factor volatility structure, where each volatility is decomposed into common and idiosyncratic components. Yet, in those papers the factor volatility structure is imposed a priori, while in this paper it emerges as a consequence of the reduced rank restrictions. Moreover, while in those papers the common volatility component is unexplained, here it is linked to the volatilities of the structural shocks driving the indexes. Further, in those papers only the variances of the errors are time varying, while here the covariances can also change over time. Finally, the estimation algorithm is different, as here we need to take the nonlinearity of the conditional mean into account.

We estimate the MAI-SV model using a dataset of 20 macroeconomic and financial variables for the U.S.[3]. We use a common Choleski ordering to identify the demand, supply and monetary / financial shocks in the VAR for the indexes. While simple, in our application this identification scheme produces results in line with economic theory. In particular, a (positive) demand shock boosts the output factor, the financial factor and, to a lesser extent, the price factor; a (negative) supply shock increases prices, reduces the real output factor, and overall has non significant effects on the monetary / financial factor; and a (negative) monetary / financial shock lowers the real output factor and has negative, though barely significant, effects on the price factor.[4]

The demand, supply and financial structural shocks appear as important drivers not only of the level of macroeconomic and financial variables but also of their changing conditional volatility. The volatility of the three structural shocks explains, in general, more than 50% of the overall volatility, in line with the results in Carriero et al. (2018) obtained with a different model for the conditional means and variances. The relative importance of the estimated volatilities for the structural shocks varies substantially across different variables, and in some cases it changes over time.

The common component of volatility seems to have broadly diminished its importance over time for real variables, while it is rather stable and substantial for nominal variables (much more so for CPI and the PCE deflator than for PPI). A similar pattern emerges also for the financial variables, with the common component explaining about half of the variation in the volatility of the Fed Funds rate and the 10 year T-Bond. Finally, the

[3]The series included coincide with those in Bańbura, Giannone & Reichlin (2010) and Carriero et al. (2016), but for an extended period that covers 1964-2016.

[4]Our methodology could be also applied with different identification schemes, such as those discussed in Kilian & Lütkepohl (2017).

role of the common volatility component in explaining the volatility of the S&P500 is remarkably high.

In terms of the role of the various structural shocks, we find that demand shocks are clearly dominant in explaining the volatility of industrial production and capacity utilization. The same applies to employment, but only at the beginning of the sample: for this variable the role of demand shocks has progressively declined in favor of supply and monetary/financial shocks. Supply shocks are dominant in explaining the volatility of the nominal variables, in particular for CPI and the PCE deflator. With regards to the common volatility of financial variables, the contribution of different structural shocks is overall comparable and rather stable over time. The monetary / financial shock is particularly important to explain the common component of the S&P500 volatility, but even for this variable it only explains about 50%, with the other 50% close to equally split between demand and supply shocks.

From an economic point of view, the volatility of the structural demand shock could be related to changes in the inventory mechanism (e.g., McConnell & Perez-Quiros, 2000) or to the globalization process (e.g., Bianchi & Civelli, 2015), while that of the supply shock can depend on the behavior of the oil market (e.g., Lee, Ni & Ratti, 1995) or on firms' productivity (e.g., Christiano, Motto & Rostagno, 2010), and the volatility of the financial/monetary shock can be influenced by financial innovation (e.g., Dynan, Elmendorf & Sichel, 2006) or by changes in the conduct of monetary policy (e.g., Clarida, Gali & Gertler, 2000). Justiniano & Primiceri (2008) introduce a DSGE model with stochastic volatility in several structural shocks. Specifically, they find that a reduction in the variance of investment shocks, interpreted also as a proxy for unmodeled financial frictions, is the main driver of the US Great Moderation, with a limited role for changes in monetary policy. Their evidence is somewhat in line with our common volatility decomposition that shows the gradually decreasing role of the monetary/financial shock contribution from the 1980s to the Great Financial Crisis. However, our structural decomposition is based on the common part of the volatility which is only a fraction of the total volatility. Moreover, Benati & Surico (2009) show that it can be difficult to map changes in the variance of structural VAR shocks with those in DSGE models.

The remainder of the paper is structured as follows. Section 2 describes the model. Section 3 introduces the volatility decomposition. Section 4 presents the choice of prior distributions and develops the MCMC algorithm for estimation, with additional details provided in the appendix. Section 5 discusses the empirical results. Section 6 concludes.

2 Description of the model

This section presents the econometric model used in the paper. The model is an extension of the homoschedastic Multivariate Autoregressive Index (MAI) Model of Reinsel (1983) to the case of time varying volatility.

Let y_t be a n-dimensional vector containing the variables of interest, and consider the following model:

$$y_t = \sum_{\ell=1}^{p} A_\ell B_0 \; y_{t-\ell} + u_t, \tag{1}$$

where A_ℓ are $n \times r$ matrices, $\ell = 1, \ldots, p$, B_0 a is $r \times n$ matrix, and $u_t \overset{iid}{\sim} \mathcal{MN}(0, \Omega)$ is a vector of i.i.d. disturbances. The product $A_\ell B_0$ is then a sequence of n-dimensional reduced rank (r) matrices, $\ell = 1, \ldots, p$, and defining $\Phi_\ell = A_\ell B_0$ makes clear that the model above is a constrained VAR. One advantage of the rank-reduction is that as the dimension of the cross-section n increases, the model remains relatively parsimonious, as long as $r \ll n$. A second important advantage is that the terms $B_0 \; y_{t-\ell}$ and A_ℓ have a straightforward interpretation as a set of r indeces (or factors) and their loadings.[5] This model was originally named Multivariate Autoregressive Index model (Reinsel (1983)). If we define the observable factors as:

$$F_t \equiv B_0 y_t, \tag{2}$$

the MAI model in (1) can be written as:

$$y_t = \sum_{\ell=1}^{p} A_\ell \; F_{t-\ell} + u_t. \tag{3}$$

As pointed out by Carriero et al. (2016) [CKM16 henceforth], by pre-multiplying (3) by B_0 it is apparent that the MAI model implies a $VAR(p)$ representation for the factors:

$$F_t = \sum_{\ell=1}^{p} C_\ell F_{t-\ell} + \omega_t, \tag{4}$$

[5]Of course, as it happens also in factor models and in cointegrated VARs, it is the case that one can rotate these matrices arbitrarily, e.g. $A_\ell \cdot B_0 = A_\ell Q' \cdot Q B_0$, where Q is an orthogonal matrix, and hence identification restrictions are needed to pin down only one of these possible rotations. Identification can be straightforwardly achieved e.g. as in Reinsel (1983) by assuming that the first r rows and columns of B are an identity matrix, $B_0 = \left[\begin{array}{cc} I_r & \widetilde{B}_0 \end{array} \right]$. Here we follow a similar approach in which the only difference is the position of the columns of the matrix I_r. In addition, due to the grouping of the variables, we also impose some additional zero restrictions, see Section 4 for details.

where $C_\ell = B_0 A_\ell$, $\ell = 1, ..., p$, and $\omega_t \overset{i}{\sim} \mathcal{MN}(0,\ \Xi)$, with $\Xi \equiv B_0 \Omega B_0'$. Grouping (3) and (4), we obtain a factor augmented VAR (FAVAR) - type model, with the notable difference that the factors of the MAI model are observable, while the factors of a FAVAR model are considered as unobservable. In addition, the VAR model for the factors is imposed in FAVARs, while in our framework it is a consequence of the MAI model for the variables. Furthermore, factors in FAVAR are in general orthogonal, while here they can be correlated.

In this paper we generalize the MAI model by allowing the disturbances to have time varying volatility:

$$u_t = G_t^{-1} \Sigma_t \varepsilon_t, \tag{5}$$

where $\varepsilon_t \overset{iid}{\sim} \mathcal{MN}(0, I_n)$, and:

$$G_t = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ g_{1,t} & 1 & \ddots & & \vdots \\ g_{2,t} & g_{3,t} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ g_{m-n+2,t} & g_{m-n+3,t} & \dots & g_{m,t} & 1 \end{bmatrix}, \qquad \Sigma_t = \begin{bmatrix} \sigma_{1,t} & 0 & \dots & 0 \\ 0 & \sigma_{2,t} & \ddots & \vdots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \sigma_{n,t} \end{bmatrix}.$$

Hence, the reduced form MAI errors become $u_t \overset{i}{\sim} \mathcal{MN}(0, \Omega_t)$, with $\Omega_t = G_t^{-1} \Sigma_t \Sigma_t' G_t^{-1\prime}$, so that both the variances and the covariances are time-varying.[6]

Following Primiceri (2005), we specify the following law of motions for the n-dimensional vector of (log) standard deviations σ_t and the m-dimensional vector $(m = n(n-1)/2)$ of off-diagonal elements $g_t = (g_{1t}, g_{2t}, ..., g_{mt})'$:

$$\log \sigma_t = \log \sigma_{t-1} + \nu_{\sigma,t},$$

$$g_t = g_{t-1} + \nu_{g,t},$$

with $\nu_{\sigma,t} \overset{iid}{\sim} \mathcal{MN}(0,\ Q_\sigma)$, $\nu_{g,t} \overset{iid}{\sim} \mathcal{MN}(0, Q_g)$, and $\forall (t,h), \nu_{\sigma,t} \perp \nu_{g,t+h}$.

Note that the errors in the $VAR(p)$ for the observable factors F_t also feature time variation in their volatilities. Specifically, the disturbances to the factor VAR in (4) have the following distribution:

$$\omega_t \overset{i}{\sim} \mathcal{MN}(0, \Xi_t), \tag{6}$$

where $\Xi_t \equiv B_0 \Omega_t B_0'$. The equations in (3) and (4), augmented with the error specifications

[6]Any positive definite matrix Ω_t can be decomposed as $\Omega_t = G_t^{-1} \Sigma_t \Sigma_t' G_t^{-1'}$, while of course other decompositions are possible.

in (5) and (6), are similar to a FAVAR model with stochastic volatility (SV) in both the (observable) factors and idiosyncratic errors.[7]

When the model has a large number of variables, the off-diagonal elements contained in G_t can create a computational bottleneck. For example, in a model with 20 variables as in our empirical application, g_t becomes an 190-dimensional stochastic process. For this reason, in our empirical application, we consider two versions of the model. The first version only features time variation in the volatilities, while the covariances g_t are assumed to remain constant over time. We will refer to this model as MAI-SV. The second version of the model features time variation also in the covariances. We will refer to this model as MAI-SVCV.

3 Decomposing volatility

In this section we present an approach to decompose the total volatility of the variables y_t into orthogonal components. This is achieved in two stages. In the first stage, we decompose total volatility into a part which is common to all variables, which can be interpreted as a measure of uncertainty, and an idiosyncratic component. In the second stage we further decompose the common component into parts, each corresponding to a different structural shock.

3.1 Common and idiosyncratic volatilities

We start with decomposing the total volatility into two parts, a part which is common to all variables, and an idiosyncratic component. CKM16 introduce the following identity:[8]

$$I_n = \Omega B_0' \Xi^{-1} B_0 + B_{0\perp}' \left(B_{0\perp} \Omega^{-1} B_{0\perp}' \right)^{-1} B_{0\perp} \Omega^{-1}, \tag{7}$$

where $B_{0\perp}$ is a $(n-r) \times n$ matrix, such that $B_0 B_{0\perp}' = \mathbf{0}_{r\times(n-r)}$. Using this decomposition in the context of the MAI-SV or MAI-SVCV models, we can then split the residuals u_t in their projection over the subspace generated by the factors' residuals ω_t, and their projection over the subspace generated by the $n - r$ idiosyncratic errors ψ_t (orthogonal to ω_t):

$$y_t = \sum_{\ell=1}^{p} A_\ell \cdot F_{t-\ell} + \Omega_t B_0' \Xi_t^{-1} \underbrace{B_0 u_t}_{\omega_t} + B_{0\perp}' \left(B_{0\perp} \Omega_t^{-1} B_{0\perp}' \right)^{-1} \underbrace{B_{0\perp} \Omega_t^{-1} u_t}_{\psi_t}. \tag{8}$$

[7]A small factor model with common stochastic volatility in the innovations to both factors and idiosyncratic errors is studied by Marcellino et al. (2016).

[8]See also Johansen (1995) and Centoni & Cubadda (2003).

We can then further decompose the total error volatility Ω_t into the volatility of the common component and that of the idiosyncratic component:[9]

$$\Omega_t = \Omega_t^{com} + \Omega_t^{idio}, \tag{9}$$

with $\Omega_t^{com} = \Omega_t B_0' \Xi_t^{-1} B_0 \Omega_t$ and $\Omega_t^{idio} = B_{0\perp}' \left(B_{0\perp}\Omega_t^{-1}B_{0\perp}'\right)^{-1} B_{0\perp}$.

The matrix Ω_t can be interpreted as the variance covariance matrix of the 1-step ahead forecast errors. We can apply a similar decomposition also to the variance of the h-step ahead forecast errors, to understand whether the common component contributes differently across horizons.

Finally, as mentioned in the introduction, in the MAI-SV model the variance decomposition in (9) into common and idiosyncratic components emerges from the model structure, while in Carriero et al. (2016, 2018, 2019) it is imposed a priori (in a linear rather than nonlinear VAR model).

3.2 Decomposition of the common volatility

In this section we further decompose the common volatility component Ω_t^{com} into r parts, each corresponding to a different structural shock. Starting from the shocks to the observed factors ω_t we can recover the structural shocks to the factors ε_t^{ω} using a Choleski decomposition:

$$\omega_t = S_t^{\omega} \cdot \varepsilon_t^{\omega},$$

where S_t^{ω} is the lower-triangular cholesky factor of Ξ_t (i.e. the variance of ω_t), and ε_t^{ω} is an i.i.d. standard Normal variable, which represents the structural shocks. We can now insert the structural shocks ε_t^{ω} into the decomposition in (8), obtaining:

$$y_t = \sum_{\ell=1}^{p} A_\ell \cdot F_{t-\ell} + \sum_{j=1}^{r} \Omega_t B_0' \Xi_t^{-1} S_t^{\omega} \varepsilon_{t,j}^{\omega} \cdot e_j + B_{0\perp}' \left(B_{0\perp}\Omega_t^{-1}B_{0\perp}'\right)^{-1} \psi_t$$

where e_j is the $r \times 1$ column vector of the r-dimensional canonical base.

It follows that one can decompose the common volatility Ω_t^{com} into orthogonal com-

[9] The complete derivation is the following:

$$\Omega_t^{com} = \Omega_t B_0' \Xi_t^{-1} \mathbb{E}\left(\omega_t \omega_t'\right) \Xi_t^{-1} B_0 \Omega_t \;=\; \Omega_t B_0' \Xi_t^{-1} \Xi_t \Xi_t^{-1} B_0 \Omega_t \;=\; \Omega_t B_0' \Xi_t^{-1} B_0 \Omega_t.$$

$$\Omega_t^{idio} = B_{0\perp}' \left(B_{0\perp}\Omega_t^{-1}B_{0\perp}'\right)^{-1} B_{0\perp}\Omega_t^{-1} \mathbb{E}\left(u_t u_t'\right) \Omega_t^{-1} B_{0\perp}' \left(B_{0\perp}\Omega_t^{-1}B_{0\perp}'\right)^{-1} B_{0\perp} = B_{0\perp}' \left(B_{0\perp}\Omega_t^{-1}B_{0\perp}'\right)^{-1} B_{0\perp}.$$

ponents, each associated with a structural shock:

$$\Omega_t^{com} = \sum_{j=1}^{r} \Omega_t B_0' \Xi_t^{-1} \left(S_t^{\omega} e_j \cdot e_j' S_t^{\omega\prime} \right) \Xi_t^{-1} B_0 \Omega_t. \tag{10}$$

A similar decomposition can be applied to the common component of the h-step ahead covariance matrix, which is informative to understand whether the contribution of various structural shocks changes with the forecast horizon.

A comparable decomposition can be also applied with different shock identification methods, such as sign restrictions or external instruments, see Kilian & Lütkepohl (2017) for an exhaustive review and analysis on shock identification. The choice of the proper structural identification method crucially depends on the empirical application of interest.

In the empirical section we will apply these formulae to decompose the estimated conditional volatilities of US macro and financial variables into their common and idiosyncratic components, further studying the contribution of various structural shocks when decomposing the common volatility component.

4 Estimation

In this section we discuss, in turn, the identification restrictions, the priors on model parameters, and the Gibbs Sampler to draw from the joint posterior, which is not a known distribution.

4.1 Identification restrictions

In order to identify the model, we need to restrict at least r^2 elements in the $r \times n$ matrix B_0. Similarly to Carriero et al. (2016), we divide the n variables in r subgroups or blocks, so to have as many blocks as factors. Each block may have in principle its own number of variables, i.e. $y_t = (y_t^{1\prime}, y_t^{2\prime}, \ldots, y_t^{r\prime}),\ n = \sum_{j=1}^{r} n_j$. The proposed identification for the observable factors $F_t = B_0 y_t$ assumes that each factor is a linear combination of the variables in its associated block. Moreover, we normalize to 1 the weight of the first variable of each block. This is equivalent to the following representation:

$$B_0 = \begin{bmatrix} 1 & \widetilde{B}_{0,1} & 0 & \mathbf{0}_{1\times(n_2-1)} & \cdots & 0 & \mathbf{0}_{1\times(n_r-1)} \\ 0 & \mathbf{0}_{1\times(n_1-1)} & 1 & \widetilde{B}_{0,2} & \cdots & 0 & \mathbf{0}_{1\times(n_r-1)} \\ \vdots & \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \mathbf{0}_{1\times(n_1-1)} & 0 & \mathbf{0}_{1\times(n_2-1)} & \cdots & 1 & \widetilde{B}_{0,r}, \end{bmatrix},$$

which implies that the unrestricted elements are contained in each of the $1 \times (n_j - 1)$ vectors $\widetilde{B}_{0,j}$, for a total of $n - r$ unrestricted elements in the matrix B_0.

4.2 Specification of the prior distributions

The set of prior distributions is constructed using a training sample. Prior knowledge for the unrestricted elements of B_0 is elicited with a Normal distribution. To calibrate these prior distributions, for each block $j = 1, \ldots, r$, we compute the largest eigenvalue score via principal components analysis, so to obtain a set of r score series $\{S_t^j\}_{t=1,\ldots,T}^{j=1,\ldots,r}$. Using these scores, we consider the following $n - r$ univariate regressions:

$$S_t^j = b_{0,j,k} \cdot y_{t,k}^j + u_{j,k,t}, \quad u_{j,k,t} \overset{iid}{\sim} \mathcal{N}(0, \sigma_{j,k}^2); \quad j = 1, \ldots, r, \quad k = 2, \ldots, n_j.$$

The OLS estimates of these regressions, and their standard errors, are used to calibrate mean and variances of the prior distributions. Since each $b_{0,j,1}$ is set to 1 (as shown in the previous subsection), each prior mean of the elements in $\widetilde{B}_{0,1}$ is divided by $\hat{b}_{0,j,1}$. Prior covariances among elements are set to zero.

Defining $A \equiv [A_1|...|A_p]$, the prior on $a = vec\,(A')$ is multivariate Normal, centered on $\mathbf{0}$, and with diagonal variance V_a resembling a Minnesota prior. In particular, it holds that

$$V_a = Diag\left(\begin{bmatrix} \widehat{\sigma}_{y,1}^2 \\ \vdots \\ \widehat{\sigma}_{y,n}^2 \end{bmatrix}\right) \otimes \begin{bmatrix} \Psi_1 & 0 & \ldots & 0 \\ 0 & \Psi_2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \ldots & 0 & \Psi_p \end{bmatrix}, \forall \ell, \quad \Psi_\ell = \frac{\lambda_a}{\ell^d} \cdot Diag\left(\begin{bmatrix} \widehat{\sigma}_{F,1}^{-2} \\ \vdots \\ \widehat{\sigma}_{F,r}^{-2} \end{bmatrix}\right),$$

where $\widehat{\sigma}_{y,j}^2$ and $\widehat{\sigma}_{F,s}^2$ are the residual variances of a univariate AR(1) for, respectively, each variable j and each factor s (computed using the prior mean of B_0). The parameter λ_a is a tightness parameter, and d is a decay parameter. We chose standard calibration borrowing from the VAR literature, i.e. $\lambda_a = 0.2$ and $d = 2$.

In the MAI-SV case, the prior for the elements of G is a multivariate Normal distribution centered at zero, with large diagonal covariance matrix. In the MAI-SVCV case, the prior of the initial vector g_0 is a multivariate normal centered at zero with identity matrix. The prior for σ_0 is a multivariate normal, centered at $[\widehat{\sigma}_{y,1}^2|...|\widehat{\sigma}_{y,n}^{2\prime}]'$, with identity covariance matrix, as in Primiceri (2005). Prior distributions for innovation covariance matrices Q_g and Q_σ are calibrated as in de Wind & Gambetti (2014) and Primiceri (2005).

4.3 Gibbs Sampler

This subsection describes the main steps of the Gibbs Sampler used to simulate from the joint posterior distribution of both parameters and unobservable states of the MAI-SV and MAI-SVCV models. When drawing the volatility states, the algorithm makes use of the approximation of Omori et al. (2007), which requires drawing a set of mixture states $\{S_t\}_{t=1}^T$.

The algorithm is more general than the one presented in Carriero et al. (2016) since it introduces the step to draw the time-varying volatilities, following Primiceri (2005) and Del Negro & Primiceri (2015). A Markov Chain of 10 thousand draws (with additional 30% initial draws discarded as burn-in) is simulated to obtain the posterior distributions of the coefficients and unobservable states. Convergence statistics confirm that convergence has been achieved and mixing is good.

The Gibbs Sampler steps for the MAI-SV model are:

1. Draw a history of volatilities $\{\sigma_t\}_{t=1}^T \Big| \theta, \{S_t\}_{t=1}^T$,
2. Draw $\theta, \{S_t\}_{t=1}^T \,|\, \{\sigma_t\}_{t=1}^T$. This second step is further split as follows:
   (a) Draw the elements in $\theta | \{\sigma_t\}_{t=1}^T$
      i. Draw the covariance of volatilities' innovations $Q_\sigma \Big| A, B_0, G, \{\sigma_t\}_{t=1}^T$
      ii. Draw the loadings $A \Big| B_0, G, Q_\sigma, \{\sigma_t\}_{t=1}^T$,
      iii. Draw the factor weights $B_0 \Big| A, G, Q_\sigma, \{\sigma_t\}_{t=1}^T$,
      iv. Draw the off-diagonal elements in $G \Big| A, B_0, Q_\sigma, \{\sigma_t\}_{t=1}^T$
   (b) Draw a history of indexes of the mixture in $\{S_t\}_{t=1}^T | \theta, \{\sigma_t\}_{t=1}^T$

In the MAI-SVCV model, instead of drawing the time invariant off-diagonal elements in G, we need to draw the unobservable states $\{g_t\}_{t=1}^T$ and the covariance matrix of their innovations Q_g. The augmented sampler encompasses the following steps:

1. Draw a history of volatilities $\{\sigma_t\}_{t=1}^T \Big| \theta, \{S_t\}_{t=1}^T$
2. Draw $\theta, \{S_t\}_{t=1}^T \,|\, \{\sigma_t\}_{t=1}^T$. This second step is further split as follows:
   (a) Draw the elements in $\theta | \{\sigma_t\}_{t=1}^T$
      i. Draw the covariance of volatilities' innovations $Q_\sigma \Big| A, B_0, \{g_t\}_{t=1}^T, Q_g, \{\sigma_t\}_{t=1}^T$
      ii. Draw the loadings $A \Big| B_0, \{g_t\}_{t=1}^T, Q_g, Q_\sigma, \{\sigma_t\}_{t=1}^T$,

iii. Draw the factor weights $B_0 \left| A, \{g_t\}_{t=1}^T, Q_g, Q_\sigma, \{\sigma_t\}_{t=1}^T \right.$,

iv. Draw the TV off-diagonal elements $\{g_t\}_{t=1}^T \left| A, B_0, Q_g, Q_\sigma, \{\sigma_t\}_{t=1}^T \right.$,

v. Draw the covariance of TV off-diagonal elements $Q_g \left| A, B_0, \{g_t\}_{t=1}^T, Q_\sigma, \{\sigma_t\}_{t=1}^T \right.$,

(b) Draw a history of indexes of the mixture in $\{S_t\}_{t=1}^T | \theta, \{\sigma_t\}_{t=1}^T$

The Gibbs Sampler steps for the MAI-SV and the MAI-SVCV are described in detail in sections A and B of the Appendix.

5 Empirical Results

5.1 Data and model specification

We collected from the FRED database 20 monthly macroeconomic variables for the period from January 1964 to December 2016. The selected series coincide with those in Bańbura, Giannone & Reichlin (2010) and CKM16. The first 84 observations, from January 1964 to December 1970, are used as training sample to calibrate the prior distributions, while the rest of the sample is used in estimation. The variables are used in differences, standardized and demeaned. We set the number of lags to 13, following common practice for Bayesian VARs at monthly frequency.

Following CKM16, we select a specification using 3 factors. The first factor is a real factor, gathering information from the real economic activity variables; the second factor is a nominal factor, representing changes in price dynamics; the third factor is a monetary/financial factor. Table 1 lists the variables and their factor grouping[10].

5.2 Identifying the structural shocks and their effects

In this section we discuss the identification of demand, supply and monetary/financial shocks, and their effects on the variables under analysis.

The estimated factors are reported in Figure 1, along with NBER recessions (as shaded areas) and the uncertainty events highlighted in Bloom (2009) (as black lines). Due to the

[10] Carriero et al. (2019) estimate a BVAR with independent stochastic volatilities for 125 variables (including macroeconomic indicators, an array of interest rates, some stock return measures, and exchange rates). A factor analysis of the volatilities indicates two components to account for the vast majority of innovations to volatilities. Here, we use three rather than two factors, even if the number of variables is smaller, to get robustness to potential omission of a third factor, as the third factor is significant in the conditional means, and the three factors can be given a meaningful economic interpretation as real, nominal and financial factors. Computing the first three principal components of the median volatility of residuals, we are able to explain more than 90% of time variation across the sample, in line with Carriero et al. (2019).

definition of factors and estimated coefficients, improved economic conditions are associated with higher demand factor and lower supply and monetary/financial factors. The factors are reaching extreme values during the Oil crisis in the 1970s and in correspondence of the Great Financial Crisis, while from the mid 1980s to the early 2000s they appear less volatile (Great Moderation), even in case of recessions and rare events. It is interesting to notice how the demand and supply factors seem negatively correlated in case of supply-driven recessions, as the Oil crises, and positively correlated in other recessions as the recent Great Financial Crisis.

Structural analysis can be performed similarly as in FAVARs applications. Indeed, as shown before, the factors' law of motion is given by (4) where the variance of the shocks ω_t is given by $\Xi_t \equiv B_0 \Omega_t B_0'$, and where the factors are ordered as real, nominal and monetary / financial. Differently from FAVAR models, this model features observable factors (since $F_t = B_0 y_t$) and heteroskedasticity in the factor innovations ω_t.

From an economic perspective, it makes sense to expect that real activity variables take some time to react to exogenous changes in prices, while prices are likely to react contemporaneously to changes in real activity. Financial variables are likely to react immediately to changes in prices and real activity. These considerations support the use of a simple Cholesky scheme for the factor VAR in which the real activity factor is ordered first, the nominal factor second, and the financial factor is ordered last.

This identification scheme provides three orthogonal structural shocks, each featuring time-varying volatility. The posterior median of the volatilities of the structural shocks to the factors, and their posterior 68% credible bounds are reported in Figure 2, along with NBER recessions and Bloom (2009) uncertainty events. The Great Moderation is evident for all shocks, but the volatility of supply and financial shocks seems to increase again from the mid-90s, and all volatilities peak during the recent Global Financial Crisis.

From an economic point of view, the volatility of the structural demand shock could be related to changes in the inventory mechanism (e.g., McConnell & Perez-Quiros, 2000) or to the globalization process (e.g., Bianchi & Civelli, 2015), while that of the supply shock can depend on the behavior of the oil market (e.g., Lee et al., 1995) or on firms' productivity (e.g., Christiano et al., 2010), and the volatility of the financial/monetary shock can be influenced by financial innovation (e.g., Dynan et al., 2006) or by changes in the conduct of monetary policy (e.g., Clarida et al., 2000). Justiniano & Primiceri (2008) introduce a DSGE model with stochastic volatility in several structural shocks. Specifically, they find that a reduction in the variance of investment shocks, interpreted also as a proxy for unmodeled financial frictions, is the main driver of the US Great Moderation, with a limited role for changes in monetary policy. Their evidence is somewhat in line with the gradually decreasing role of the monetary/financial shock contribution from the 1980s to the Great Financial Crisis in our common volatility decomposition. However, our structural decomposition is based on the common part of the volatility which is only a fraction of the total volatility. Moreover, Benati & Surico (2009) show that it can be difficult to map changes in the variance of structural VAR shocks with those in DSGE models.

Fernández-Villaverde et al. (2015) estimate a DSGE model with stochastic volatilities similar to that of Justiniano & Primiceri (2008), but using a second order approximation of the equilibrium dynamics of the economy. This approach allows changes in shock volatilities to directly affect the equilibrium solution along with the shocks, contrarily to the case of a first-order approximation of the solution. The model is estimated using a non-Bayesian methodology. In light of these methodological differences, the estimated stochastic volatilities of structural shocks show some discrepancies to analogous measures in Justiniano & Primiceri (2008). Moreover, they don't report the contribution of stochastic volatilities in explaining unconditional variances of the observable variables. In general, they find that "*the 1970s and the 1980s were more volatile than the 1960s and the 1990s, creating a tougher environment for monetary policy*", which is in line with our results.

We can also compare these volatilities with the uncertainty estimates of Carriero et al. (2018). Our demand factor volatility appears much correlated with their macro uncertainty estimate, even though the only large volatility spike after 1970s is observed in correspondence of the last Great Financial Crisis, and our measure of demand factor volatility reaches record lows in the end of 2016. The supply factor volatility shows a smaller degree of time variation than the demand factor during the Great Moderation, but it has been changing more since the spike of 2008, probably because of large fluctuations in commodity prices in the last years. As for the financial/monetary factor volatility, the largest spike throughout the sample is reasonably observed in coincidence of the years 2008-2009, differently from Carriero et al. (2018) financial uncertainty estimate, for which there is a comparable spike also in the early 2000s. It is interesting to observe how volatility of the monetary/financial factor is increasing in the years 2013-2016, most likely because of increased volatilities of bond yields and exchange rates.

Figure 3 reports the dynamic response of each factor after each structural shock (posterior median), measured at the NBER troughs, to assess whether there are differences across the various recessions. A (positive) demand shock boosts the output factor, the financial factor and, to a lesser extent, the price factor; a (negative) supply shock increases prices, reduces the real output factor, and overall has non significant effects on the financial factor; finally, a (negative) financial shock lowers the real output factor and has non-significant effects on the price factor. The fact that the estimated responses are in line with economic theory provides support for the use of our Choleski identification scheme for this application.

The temporal heterogeneity in the responses is due to different shocks' sizes (i.e. their standard deviations) and also to the time-varying simultaneous relations across variables. The price factor displays the larger degree of time variation in median posterior responses, especially at larger horizon, even though these differences among NBER troughs are barely significant when estimation uncertainty is taken into account.

Using the factor representation, it is possible to compute the response of variables y_t to the structural shocks, again as in FAVAR models. Figures 4,5 and 6 report the responses of all variables to, respectively, the identified demand, supply and financial shocks, at the NBER troughs. The responses are broadly in line with those in CKM16: a demand shock increases output variables and decreases unemployment, and triggers an increase of both prices and Fed Funds rate, along with an appreciation of the real exchange rate and a reduction in monetary indicators. The negative supply shock increases prices and worsens the real economy indicators. Interestingly, the response of the Fed Funds rate has an opposite sign in the last two recessions with respect to the previous recessions. Instead, in almost all cases there is a depreciation of the real exchange rate. Responses to the financial/monetary shock are also in line with economic theory: real variables decrease along with monetary aggregates, while the effects on prices are not significant. In terms of time variation, there are some differences in the responses at different NBER troughs, because of different shocks' size and changing simultaneous relationships. A demand shock in November 2009 has a larger effect on Industrial Production, Capacity Utilization and Housing Starts, while a monetary/financial shock in November 2001 has a large effect on stocks and macro variables. However, when considering estimation uncertainty, such time variation is not much significant.

To provide a different measure of time variation of the responses to the structural shocks, we have also computed the temporal evolution of the responses at fixed horizons of 1, 12 and 48 months. The estimated median responses, together with 68% bands, reported in Figure 11 to Figure 19 in section C of the Appendix are broadly in line with those commented above.

5.3 Decomposing the Stochastic Volatilities

In this section we use the decompositions discussed in Section 3 to assess the contributions of different sources of volatlity to the total volatility of the macroeconomic series.

Figure 7 reports the stochastic volatilities for the 20 variables, estimated using the MAI-SVCV model. Stochastic volatilities appear to be significantly changing over time for all variables, highlighting the importance of allowing for a time-varying conditional variance in macro-financial applications. The Great Moderation is evident in variables such as employment, earnings, consumption, CPI and the PCE deflator. The recent financial crisis is associated with volatility peaks in many real and nominal variables, including employment, earning, consumption, industrial production, housing starts, CPI, PPI. Also for monetary/financial variables, volatility peaks during the last global crisis are particularly evident for the Fed Funds rate, stock prices, money measures and reserves.

We start applying the decomposition in (9) and (10) and partition each of the reduced form volatilities into the common component, driven by innovations to the three factors and interpretable as an uncertainty measure, and the idiosyncratic components, driven by the idiosyncratic innovations, ψ_t.

Figure 7 plots also the decomposition of the total volatility for each variable into the two orthogonal components. From a graphical inspection, the common component of volatility is dominant for most variables, but a non-negligible part of variation is also driven by the idiosyncratic component. In order to obtain a more precise measure of the contribution of each component, Figure 8 reports the time-varying percentage shares of explained volatility by the common and idiosyncratic components.

In the real activity variables, the common component of volatility seems to have diminished its importance over time for employment and, less so, for earnings, personal income, and consumption, with values well below 50% at the end of the sample. The fraction of volatility explained by the common component is much higher for industrial production and capacity utilization, but it drops substantially for both variables after the financial crisis, from values around 80-90% to slightly above 50%. For the unemployment rate and housing starts, the fraction is instead stable at about 50% over the entire sample.

In the price variables, the fraction of their conditional time-varying volatility explained by the common shocks is rather stable and high, much more so for CPI and the PCE deflator than for PPI, but still well above 50% for the latter.

A similar pattern emerges also for the financial variables, with values around 50% for the Fed Funds rate and the 10 year T-Bond yield but, interestingly, much higher for the S&P500, whose time-varying volatility seems to be substantially affected by economic shocks.

We further decompose the common component share of volatility into three orthogonal sub-components, each driven by the volatility of structural demand, supply and monetary/financial shocks identified in the previous subsection. Figure 9 reports the time-varying percentage contribution of each structural shock in determining the common component volatility.

In the real activity variables, demand shocks are clearly dominant for industrial production and capacity utilization. The same was true for employment at the beginning of the sample, but then the role of demand shocks has progressively declined in favor of supply and monetary/financial shocks, that combined explained more than 50% of the common volatility component of employment at the end of the sample. A declining role for demand shocks is evident, though milder, also for housing starts. Instead, for earning, income, consumption and unemployment the share of demand shocks is rather stable over time, below 50% for the three variables and close to 50% for unemployment. Supply and monetary/financial shocks explain a comparable fraction of the remaning common volatility component of these four variables.

In the price variables, supply shocks are dominant, in particular for CPI and the PCE deflator, with demand shocks ranked second and monetary/financial shocks third, though slightly more important for PPI.

In the financial variables, the contribution of the three types of structural shocks is overall comparable and rather stable over time. The volatility of the monetary / financial shock is particularly important to explain the common component of the S&P500 volatility, but even for this variable it only explains about 50%, with the other 50% close to equally split between demand and supply shocks.

Overall, the structural demand, supply and financial shocks appear as important drivers not only of the level of macroeconomic and financial variables but also of their changing conditional volatility. In fact, the volatility of demand, supply and monetary / financial shocks explains, in general, more than 50% of the overall volatility, with their relative importance being variable dependent and, sometimes, changing over time.

5.4 Results for longer forecast horizons

In Figures 10 and 11 we report the decomposition of the 12- and 36-steps ahead forecast error covariance matrices into common and idiosyncratic components (results for longer horizons are very similar to those for $h = 36$ and are available upon request). It is interesting that for virtually all variables and periods the role of the common component of volatility becomes even larger at longer horizons, suggesting that common shocks are even more important than idiosyncratic shocks at long than at short horizons. Figures 12 and 13 further decompose the common volatility component into the contribution of the supply, demand and monetary / financial shocks' volatilities. The relative contribution of each shock is rather stable across forecast horizons, but it is worth mentioning the larger importance at longer horizon of supply and monetary shocks for industrial production, capacity utilization and unemployment.

6 Conclusions

Many economic variables feature changes in their conditional volatility, and stochastic volatility specifications are commonly used in macroeconomic applications to model this feature. In models with stochastic volatility, different shocks drive the levels and volatilities of the variables, with volatility shocks left unexplained. In this paper we decompose the shocks driving the volatility processes into the volatility of structural economic shocks, in order to understand the relative importance over time of demand, supply and monetary / financial shocks as drivers of volatility.

The structural shocks are obtained from a Multivariate Autoregressive Index (MAI) model, a particular reduced rank VAR that can be also interpreted as a factor model, featuring stochastic volatility, over a large set of real, nominal and financial indicators.

We have developed a Gibbs Sampling algorithm for (Bayesian) inference, introducing efficient strategies to reduce the computational burden.

Using the model with US data for the period 1964-2016, we have found that the common component of volatility is substantial, it explains at least 50% of the overall volatility for most variables, though the share is declining over time for some real variables. Moreover, a large fraction of the common volatility component is driven by the volatility of structural demand, supply and financial shocks, in general more than 50%, with their relative importance being variable dependent and, sometimes, changing over time.

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Table 1: List of variables and composition of Factors

Variable	**F1**	**F2**	**F3**
Employees (Total nonfarm)	1	0	0
Average hourly earnings	$B_0^{(1,2)}$	0	0
Personal Income	$B_0^{(1,3)}$	0	0
Real Consumption	$B_0^{(1,4)}$	0	0
Industrial Production	$B_0^{(1,5)}$	0	0
Capacity Utilization	$B_0^{(1,6)}$	0	0
Unemployment rate	$B_0^{(1,7)}$	0	0
Housing starts	$B_0^{(1,8)}$	0	0
CPI all items	0	1	0
Producer Price Index (Farm & Foods)	0	$B_0^{(2,10)}$	0
Implicit price deflator for personal cons. exp.	0	$B_0^{(2,11)}$	0
Producer Price Index (Industrials)	0	$B_0^{(2,12)}$	0
Federal Funds, effective	0	0	1
M1 money stock	0	0	$B_0^{(3,14)}$
M2 money stock	0	0	$B_0^{(3,15)}$
Total reserves of depository institutions	0	0	$B_0^{(3,16)}$
Nonborrowed reserves of depository institutions	0	0	$B_0^{(3,17)}$
S&P's common stock price index	0	0	$B_0^{(3,18)}$
Interest rate on treasury bills, 10 year constant maturity	0	0	$B_0^{(3,19)}$
Effective Exchange rate	0	0	$B_0^{(3,20)}$

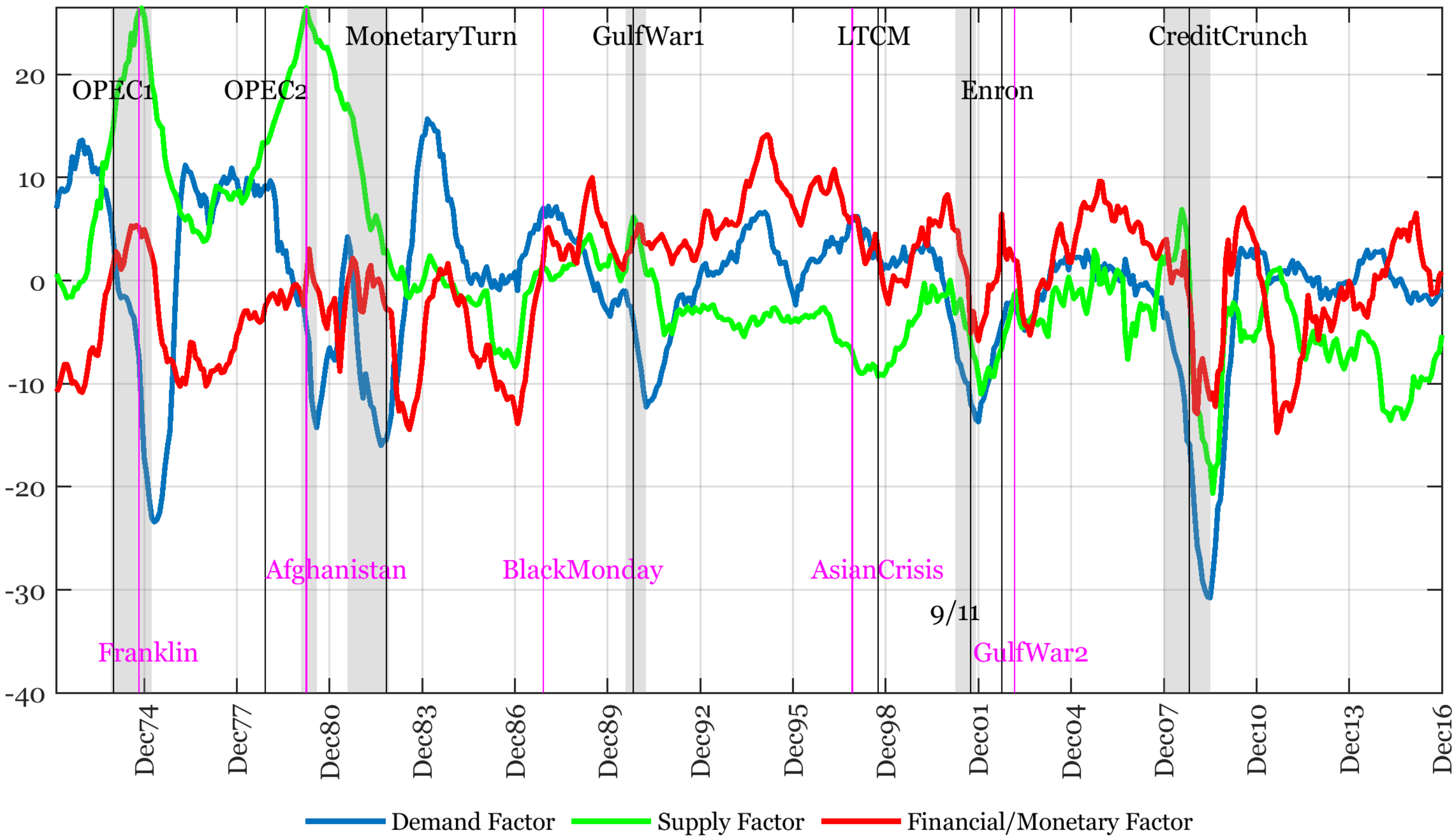

Figure 1: Estimated observable factors, with NBER recessions (shaded areas) and uncertainty events (black vertical lines).

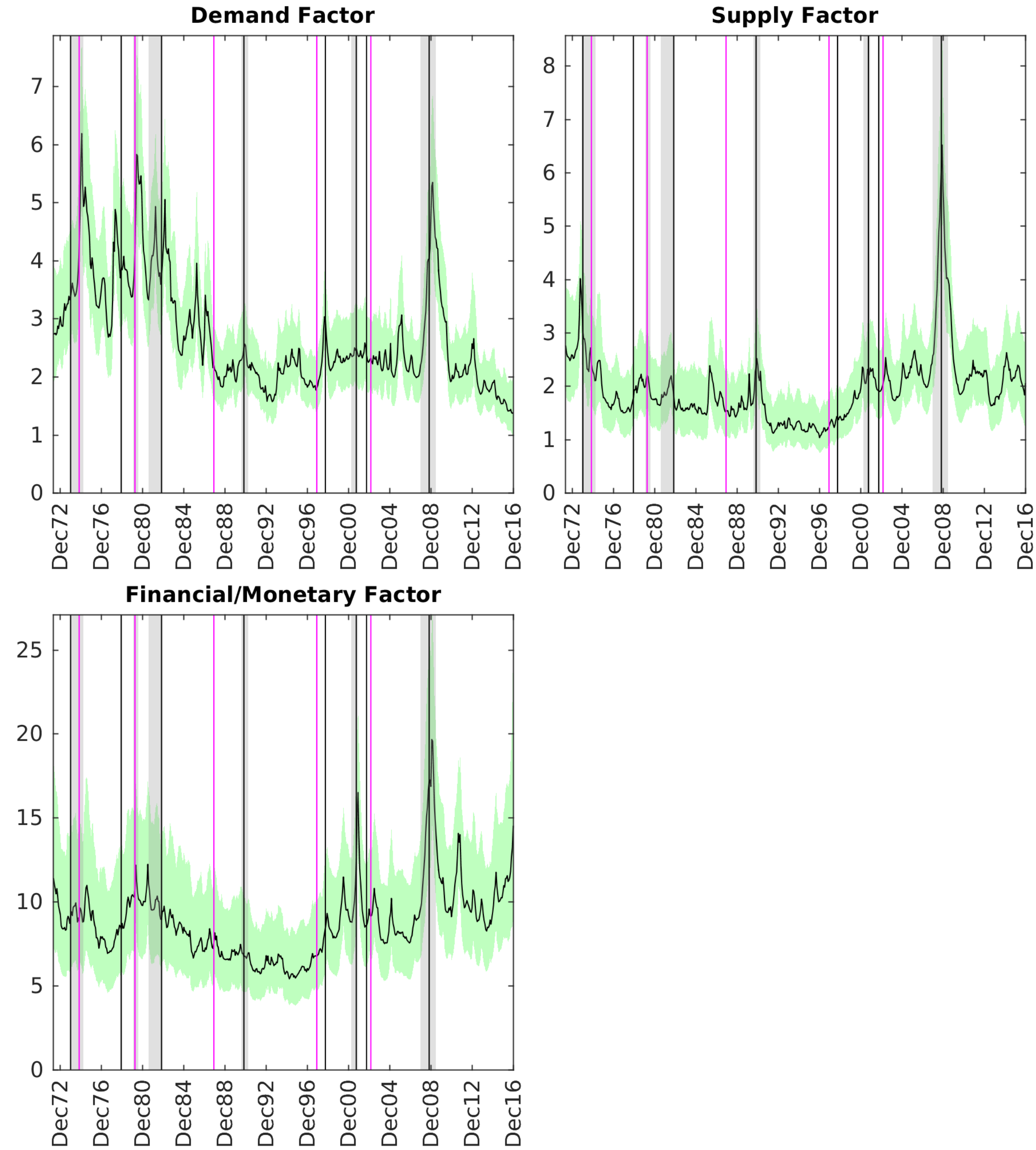

Figure 2: Volatilities of the structural shocks. Green Bands correspond to the 68% credible regions of the posterior distribution.

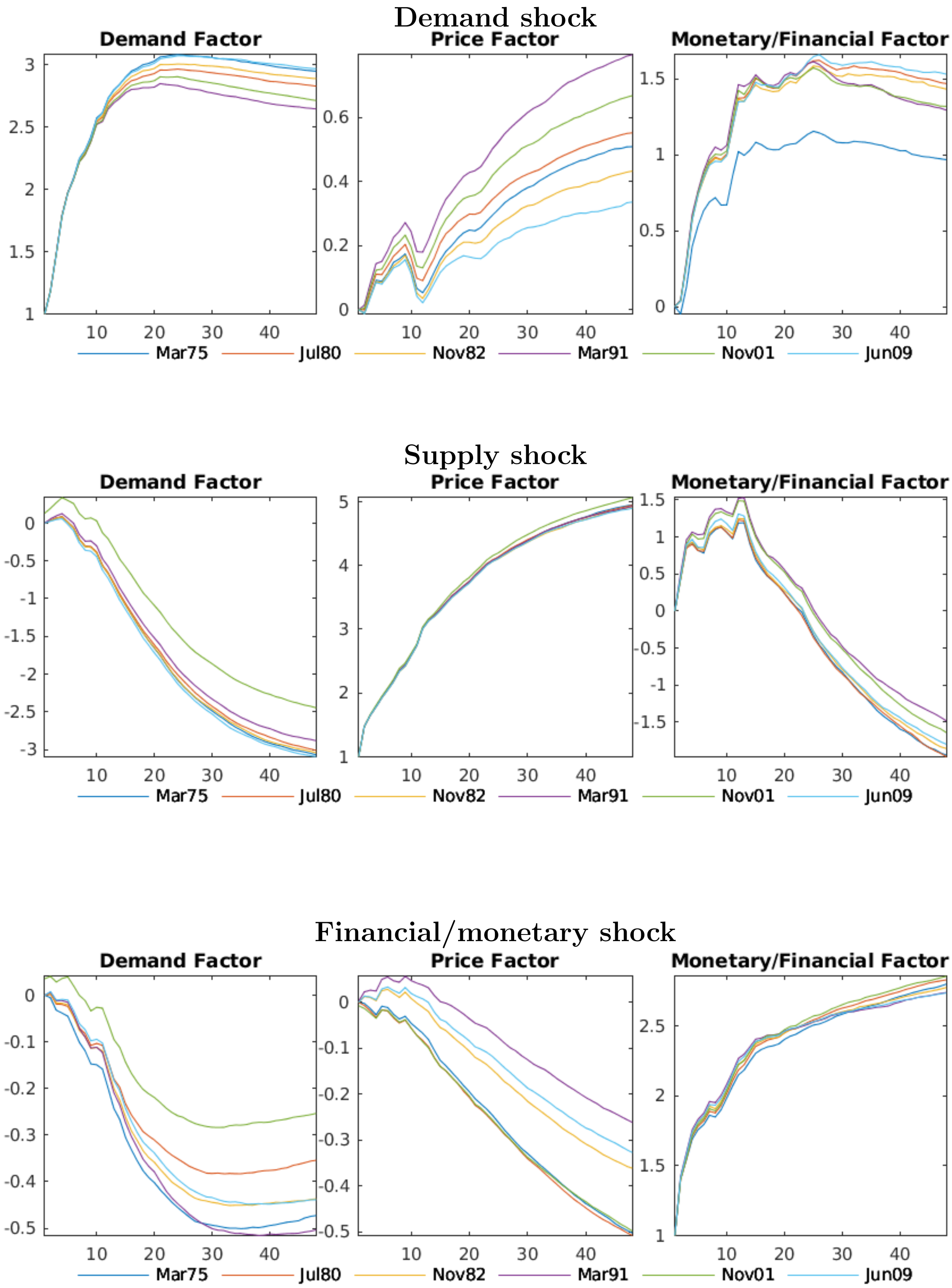

Figure 3: Response of each factor to all shocks at NBER troughs. Means of the posterior distribution.

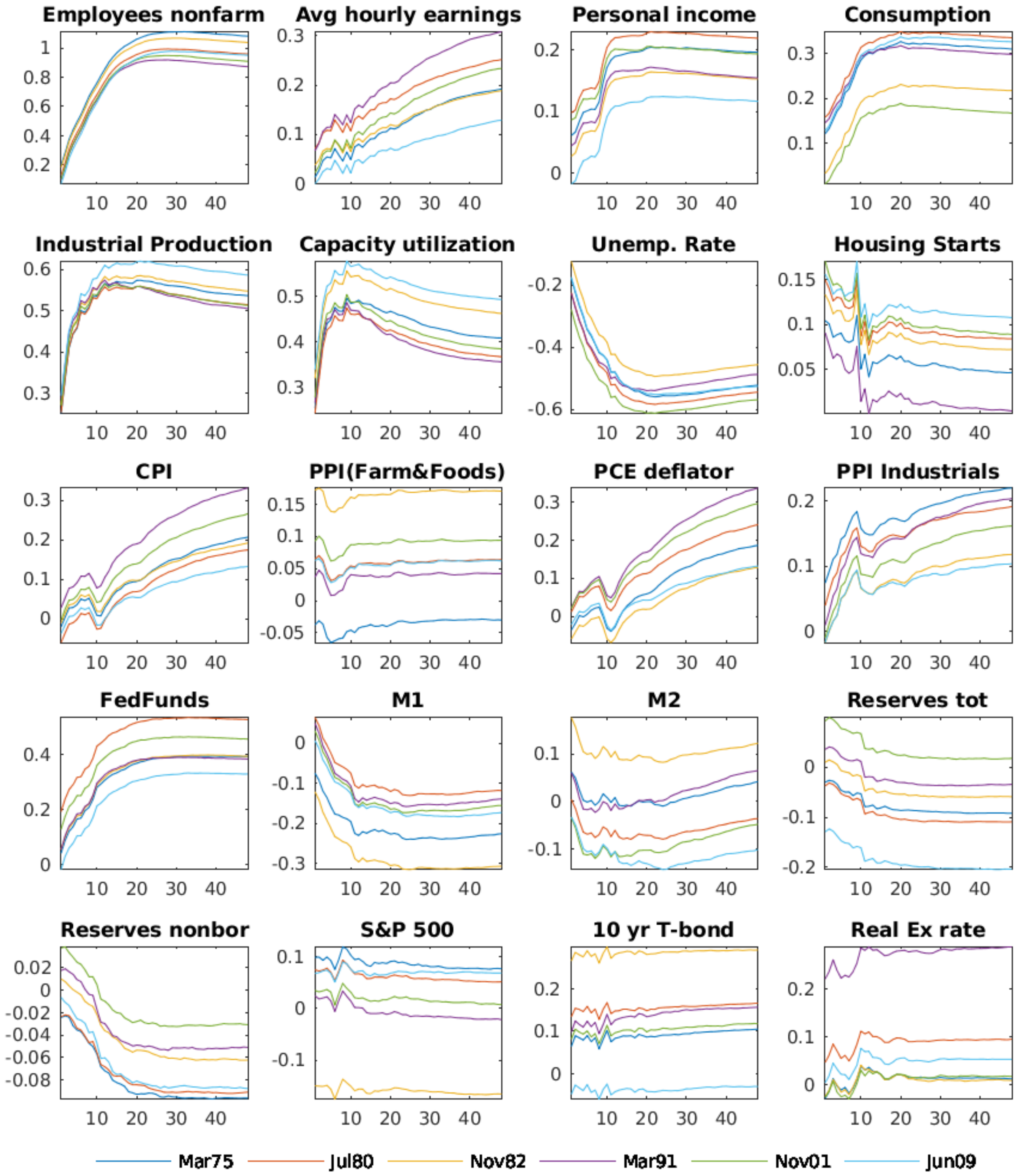

Figure 4: Response to a permanent Demand shock at NBER troughs. Means of the posterior distribution.

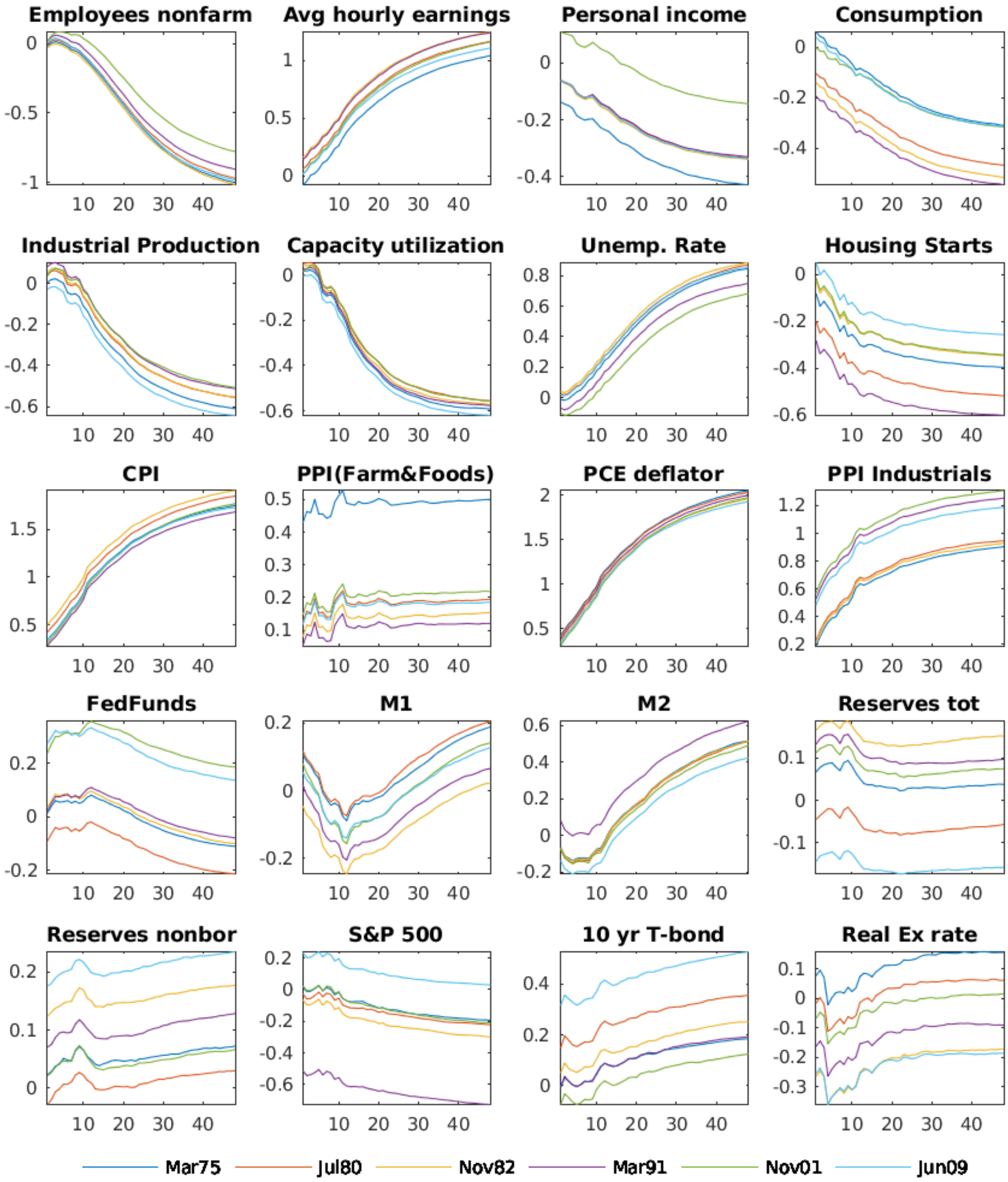

Figure 5: Response to a permanent Supply shock at NBER troughs. Means of the posterior distribution.

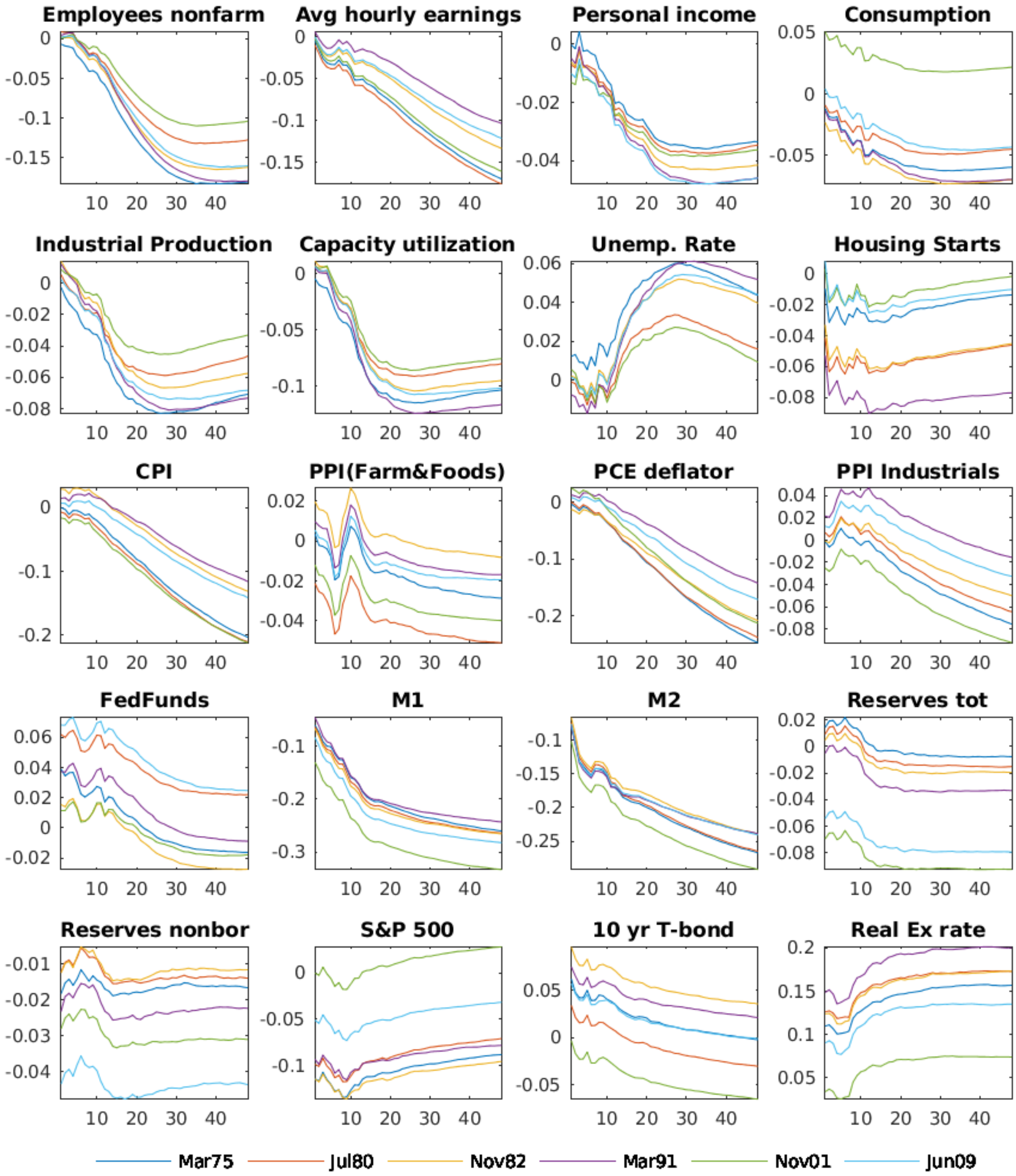

Figure 6: Response to a permanent Financial shock at NBER troughs. Means of the posterior distribution.

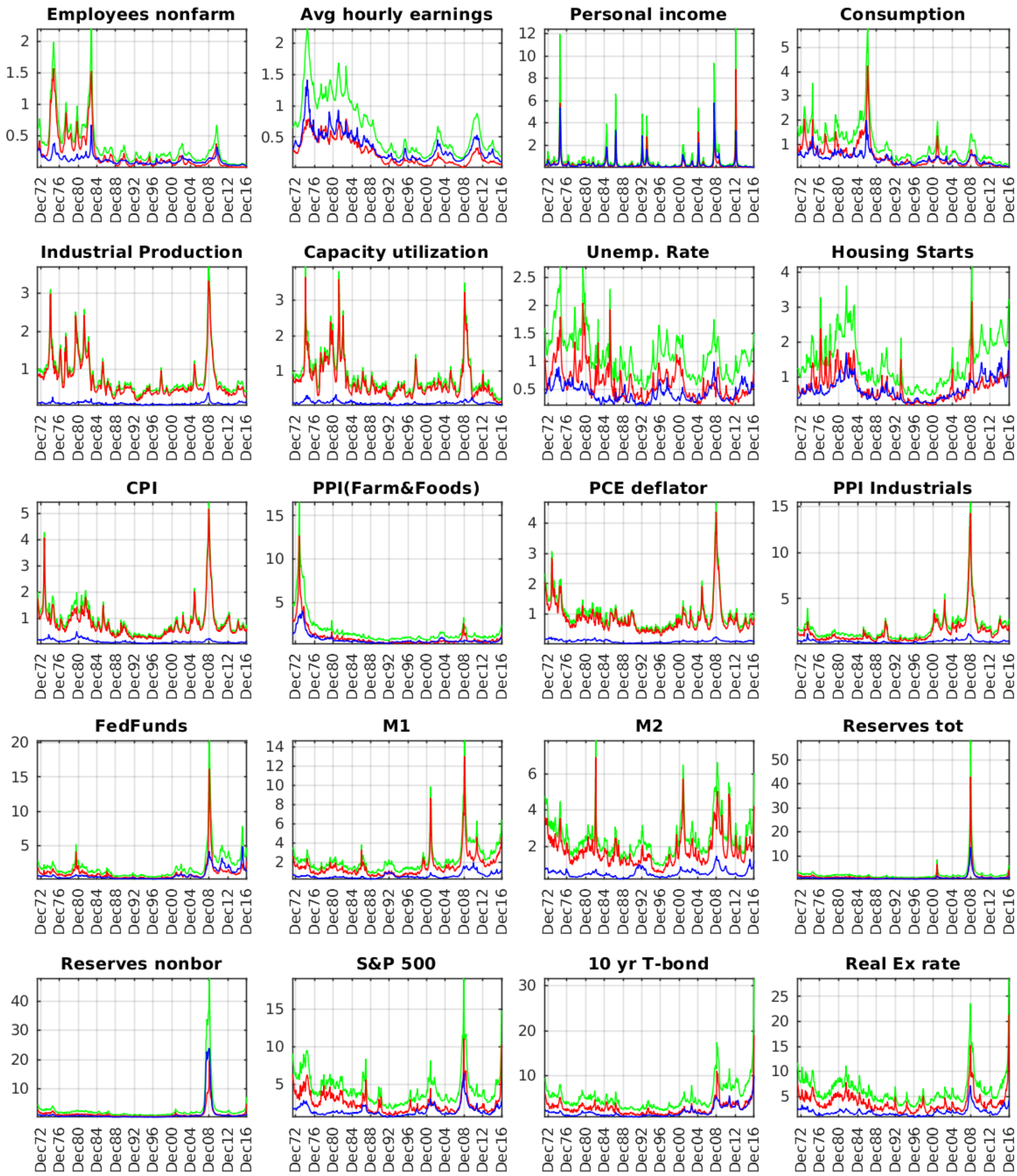

Figure 7: Residual volatility and its decomposition: total (green), common factors (red), Idiosyncratic (blue)

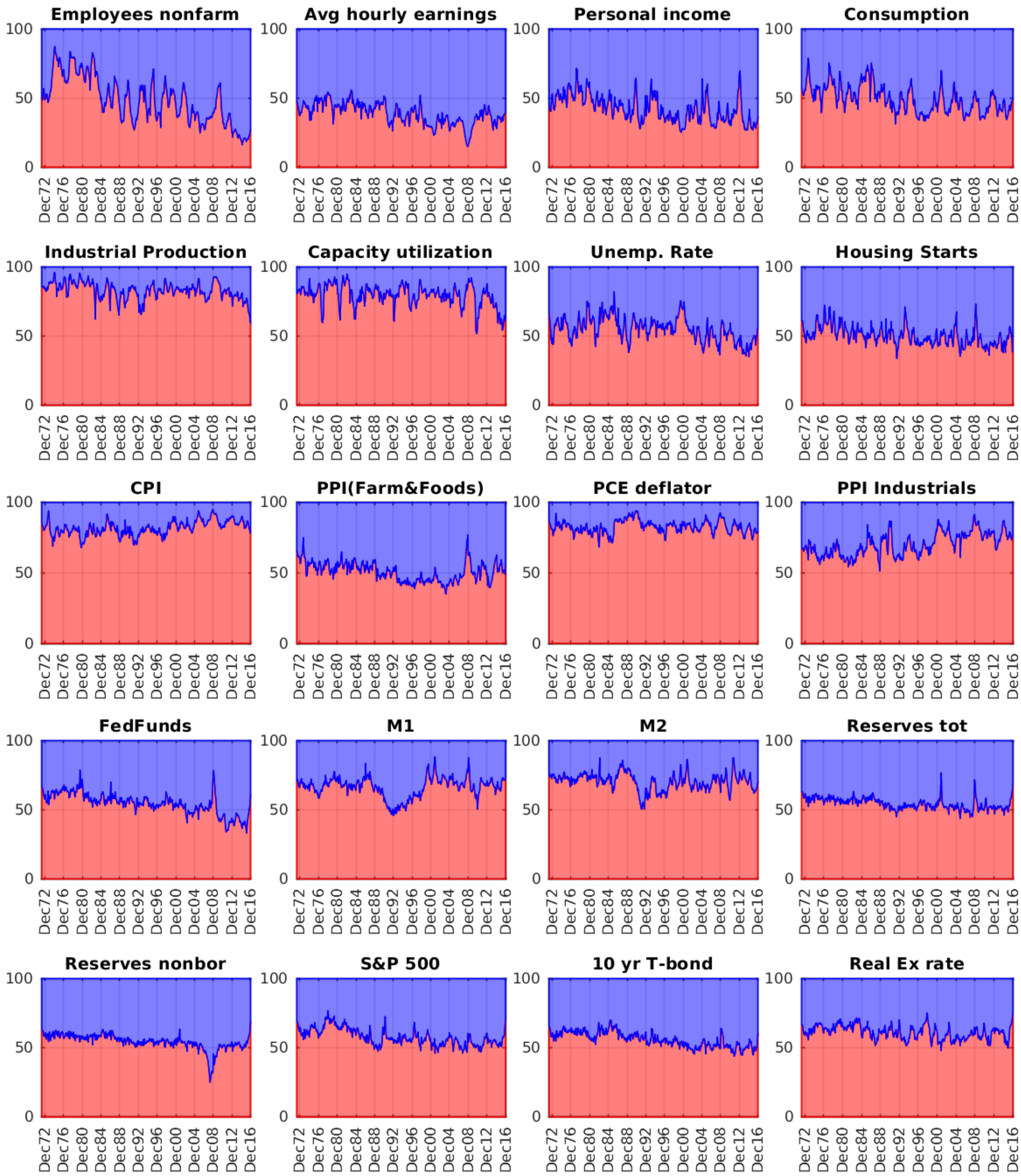

Figure 8: Residual volatility shares (%): common factors (red), Idiosyncratic (blue)

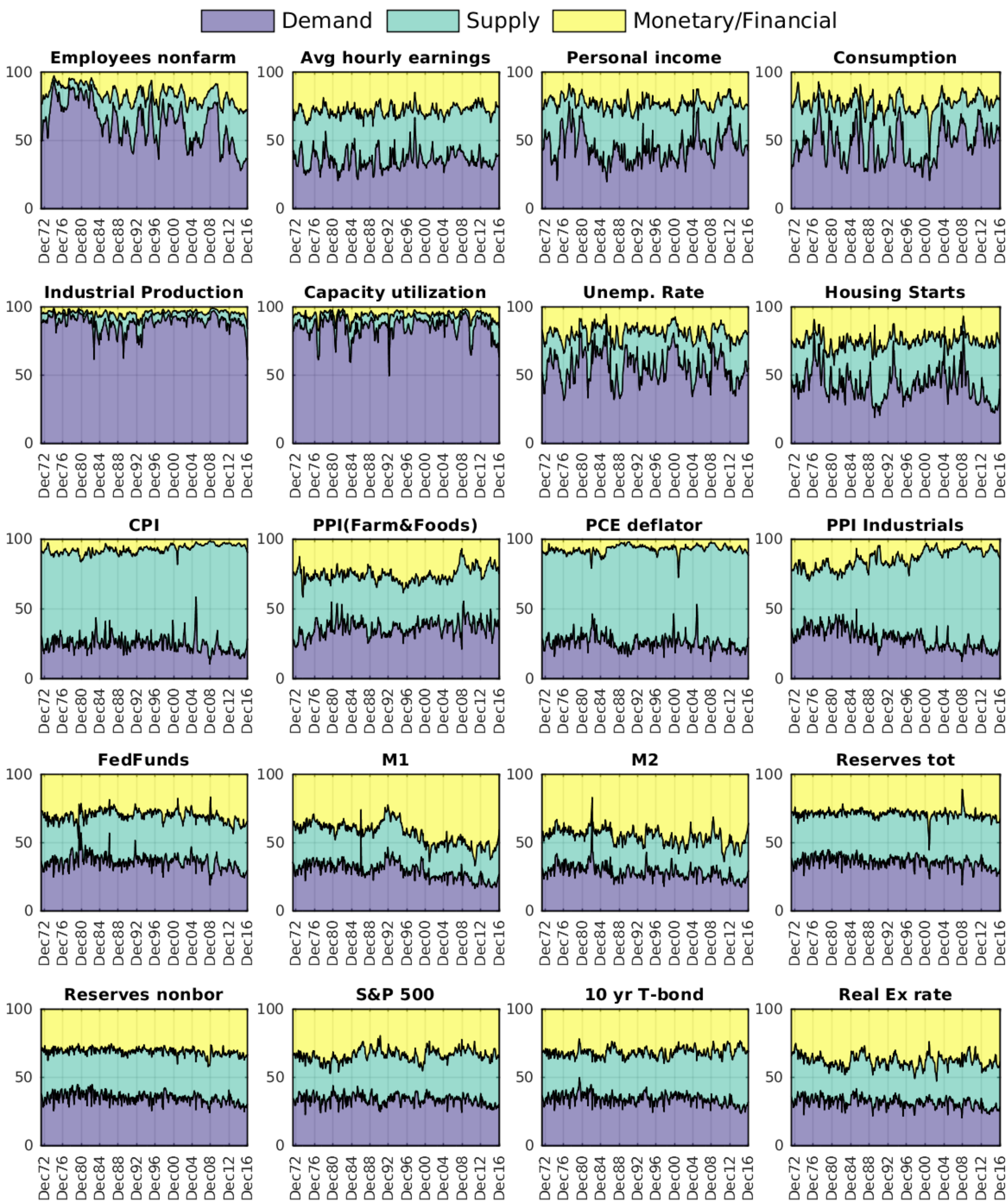

Figure 9: Contribution (%) of the identified shocks over the common factor component of residual volatility

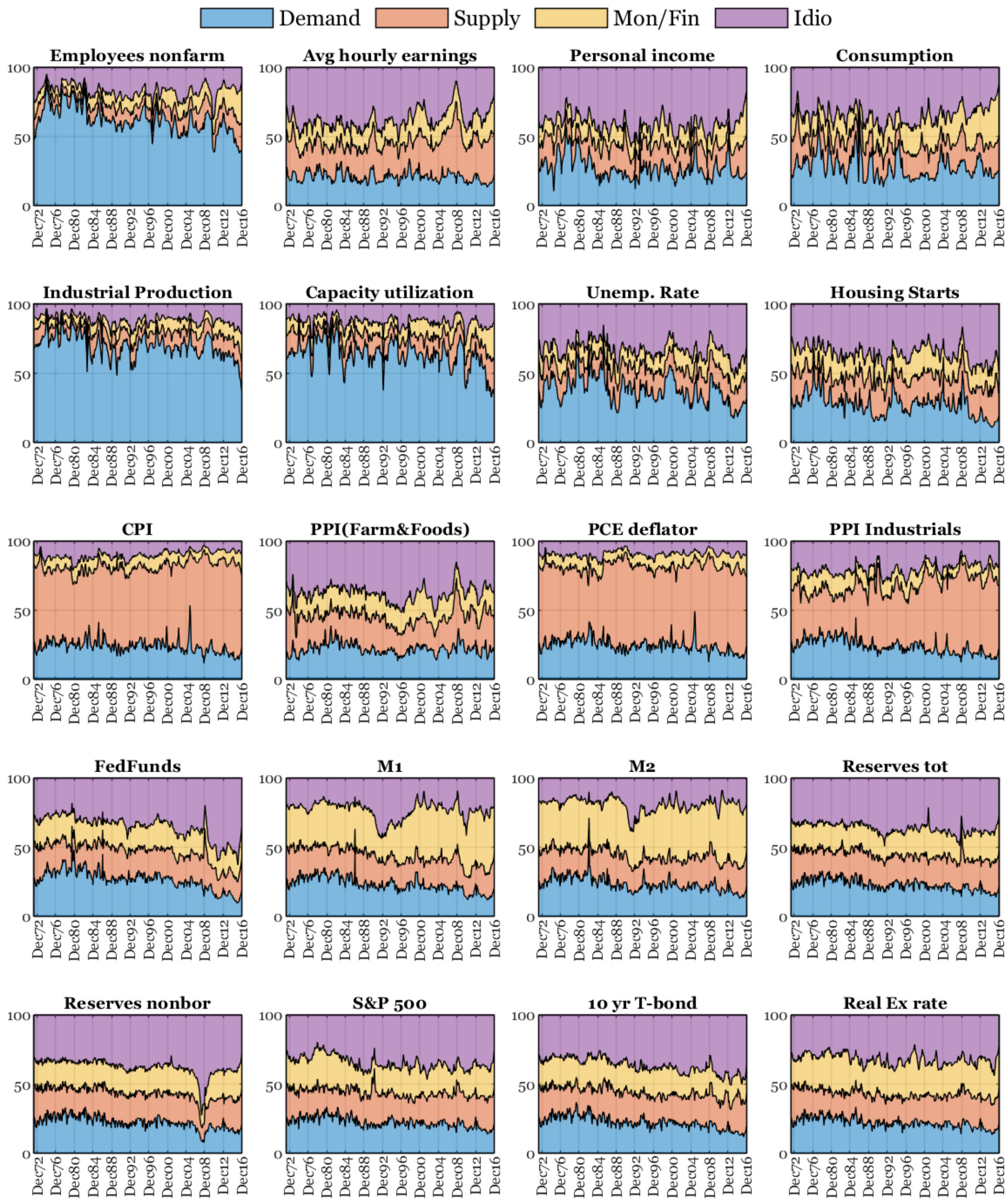

Figure 10: Contribution (%) of the common and idiosyncratic components to the 12-steps ahead forecast error variance

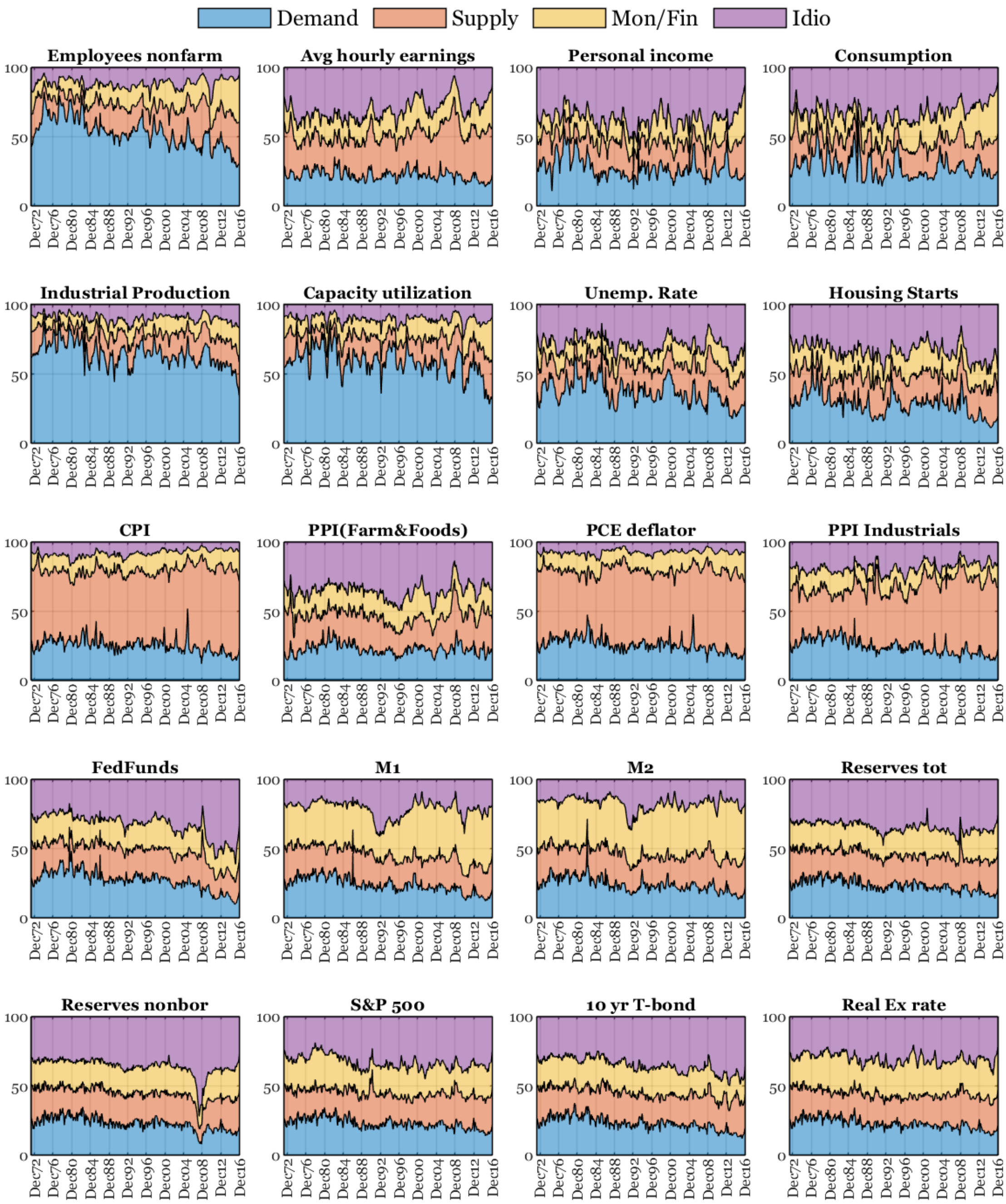

Figure 11: Contribution (%) of the common and idiosyncratic components to the 36-steps ahead forecast error variance

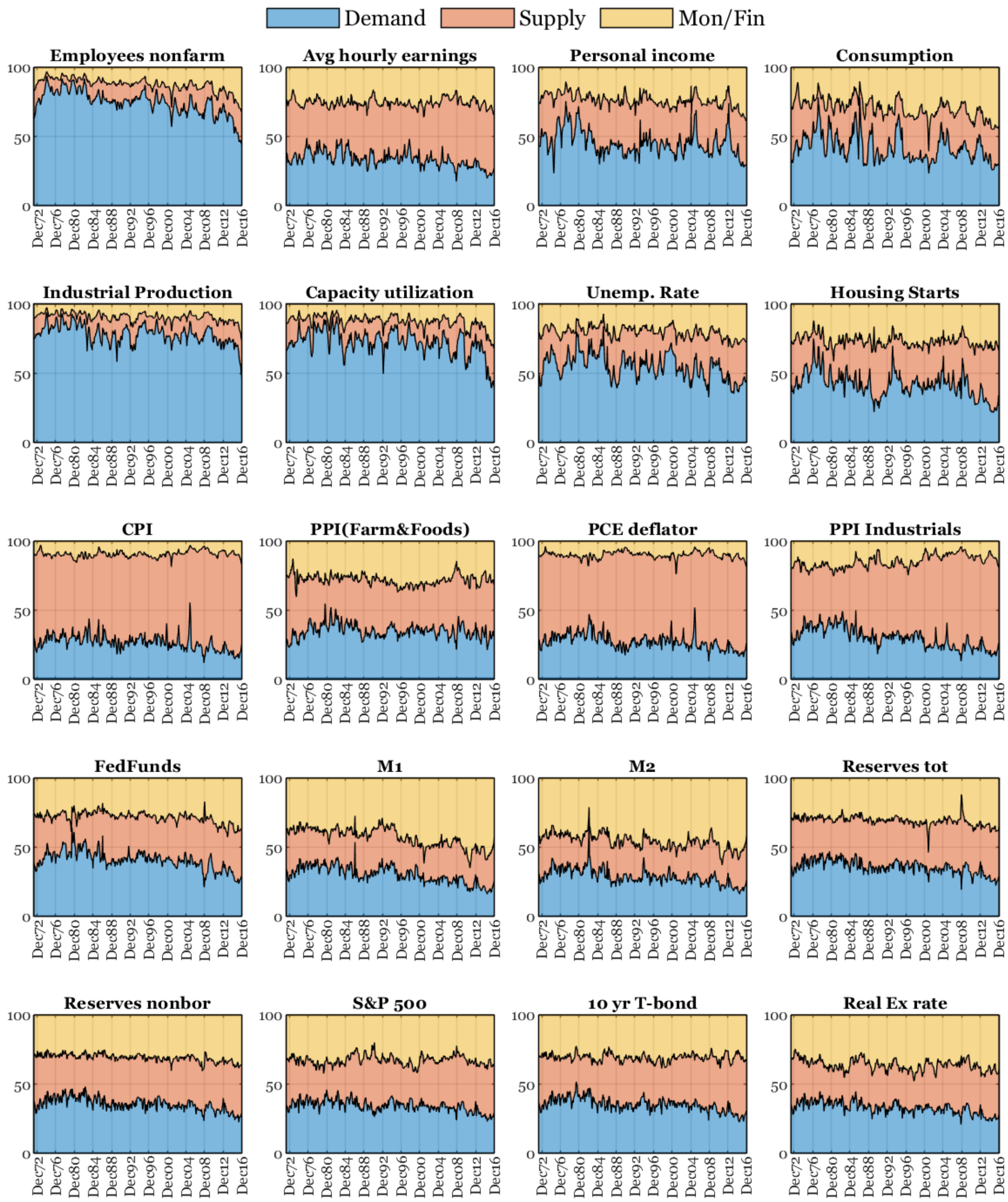

Figure 12: Contribution (%) of the identified shocks over the common factor component to the 12-steps ahead forecast error variance

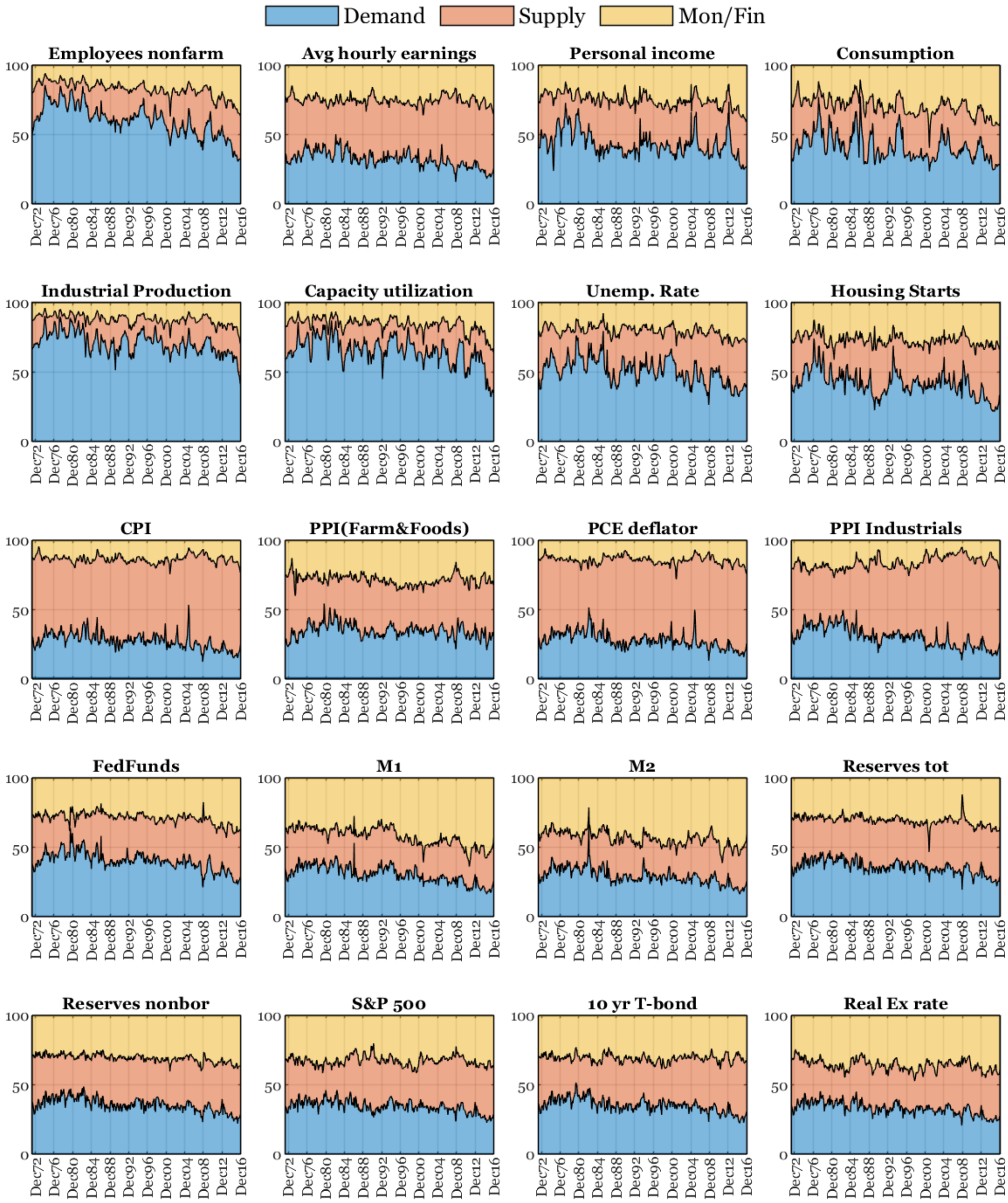

Figure 13: Contribution (%) of the identified shocks over the common factor component to the 36-steps ahead forecast error variance

Online Appendix for "The Economic Drivers of Volatility and Uncertainty"

The four sections of this online Appendix provide additional details on Gibbs Sampler algorithm for the MAI-SV and MAI-SVCV models (Appendix A, B and C), and some additional figures for the empirical application (Appendix D).

Appendix A Detailed steps of the Gibbs Sampler for the estimation of the MAI-SV model

A.1 Stacking of the model

The observation equation of the MAI-SV model written as:

$$\underbrace{y_t}_{n\times 1} = \sum_{\ell=1}^{p} \underbrace{A_\ell}_{n\times r} \cdot \underbrace{B_0}_{r\times n} y_{t-\ell} + u_t \tag{11}$$

establishes that the observables in y_t depend on p lags of the r common components $B_0 \cdot y_{t-\ell}$, plus an error term. The equation in 11 can be stated in a more compact formulation.

Defining the $n\times rp$ matrix $A \equiv \begin{bmatrix} A_1 & \dots & A_p \end{bmatrix}$ and the $n\times p$ matrix $x_t^\bullet \equiv \begin{bmatrix} y_{t-1} & \dots & y_{t-p} \end{bmatrix}$, we can define Z_t as

$$Z_t \equiv \begin{bmatrix} B_0 y_{t-1} \\ \vdots \\ B_0 y_{t-p} \end{bmatrix} = vec\left(B_0 \cdot x_t^\bullet\right),$$

and since

$$\sum_{\ell=1}^{p} A_\ell B_0 \cdot y_{t-\ell} = \begin{bmatrix} A_1 & \dots & A_p \end{bmatrix} \begin{bmatrix} B_0 y_{t-1} \\ \vdots \\ B_0 y_{t-p} \end{bmatrix},$$

we can finally restate the model as

$$y_t = A \cdot Z_t + u_t. \tag{12}$$

A.2 Step 1: Draw a history of volatilities $\{\sigma_t\}_{t=1}^T$

This step concerns the draw of the (unobservable) stochastic volatilities conditional on $\theta = \{A, B_0, G, Q_\sigma\}$ and the indexes of Normal components of the mixture $\{S_t\}_{t=1}^T$. The order of these steps is in line with Del Negro & Primiceri (2015).

In order to draw the volatilities, we apply the triangular reduction of the errors $u_t = G^{-1}\Sigma_t\varepsilon_t$ to transform the model in the following way:

$$y_t = A \cdot Z_t + G^{-1}\Sigma_t\varepsilon_t,$$

$$\underbrace{G\left(y_t - A \cdot Z_t\right)}_{\widetilde{y}_t} = \Sigma_t\varepsilon_t,$$

$$\widetilde{y}_t = \Sigma_t\varepsilon_t.$$

Moreover, using the Hadamard product operator, we can first write

$$\Sigma_t\varepsilon_t = \Sigma_t \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \\ \vdots \\ \varepsilon_{n,t} \end{bmatrix} = \begin{bmatrix} \sigma_{1,t}\ \varepsilon_{1,t} \\ \sigma_{2,t}\ \varepsilon_{2,t} \\ \vdots \\ \sigma_{n,t}\ \varepsilon_{n,t} \end{bmatrix} = \sigma_t \odot \varepsilon_t,$$

and taking the square element by element on both sides of $\widetilde{y}_t = \Sigma_t\varepsilon_t$, we obtain:

$$(\widetilde{y}_t)^{\cdot 2} = (\Sigma_t\varepsilon_t)^{\cdot 2} = \sigma_t^{\cdot 2} \odot \varepsilon_t^{\cdot 2} \quad \Longleftrightarrow \quad \begin{bmatrix} \widetilde{y}_{1,t}^2 \\ \widetilde{y}_{2,t}^2 \\ \vdots \\ \widetilde{y}_{n,t}^2 \end{bmatrix} = \begin{bmatrix} \sigma_{1,t}^2\ \varepsilon_{1,t}^2 \\ \sigma_{2,t}^2\ \varepsilon_{2,t}^2 \\ \vdots \\ \sigma_{n,t}^2\ \varepsilon_{n,t}^2 \end{bmatrix}.$$

We add on the left hand-side a small constant[11] $\bar{c} = 10^{-3}$ and apply the logarithm on

[11] The addition of a small constant term has numerical stability purposes, as explained in Fuller (2009) and Primiceri (2005).

both sides[12] in order to break the non-linearity, to obtain:

$$\underbrace{\begin{bmatrix} \log\ \left(\widetilde{y}_{1,t}^2 + \bar{c}\right) \\ \log\ \left(\widetilde{y}_{2,t}^2 + \bar{c}\right) \\ \vdots \\ \log\ \left(\widetilde{y}_{n,t}^2 + \bar{c}\right) \end{bmatrix}}_{\widetilde{y}_t^*} = 2 \begin{bmatrix} \log\ \sigma_{1,t} \\ \log\ \sigma_{2,t} \\ \vdots \\ \log\ \sigma_{n,t} \end{bmatrix} + \begin{bmatrix} \log\ \varepsilon_{1,t}^2 \\ \log\ \varepsilon_{2,t}^2 \\ \vdots \\ \log\ \varepsilon_{n,t}^2 \end{bmatrix},$$

$$\widetilde{y}_t^* = 2 \log \sigma_t + \log\ \left[(\varepsilon_t)^{\cdot 2}\right].$$

Since $\varepsilon_t \overset{iid}{\sim} \mathcal{MN}(\mathbf{0}, I_n)$, each term $\log \varepsilon_{j,t}^2$ has a $\log \chi_1^2$ distribution, and $\log\ \left[(\varepsilon_t)^{\cdot 2}\right]$ is a vector of independent $\log \chi_1^2$ random variables.

Then, conditioning on $\{S_t\}_{t=1}^T$, i.e. the sequence of $n \times 1$ vectors that specify the indexes of Normal components of the mixture, the vector $\log\ \left[(\varepsilon_t)^{\cdot 2}\right]$ has the following Gaussian distribution:

$$\log\ \left[(\varepsilon_t)^{\cdot 2}\right] \big| \{S_t\}_{t=1}^T \sim \mathcal{MN}\left(\underbrace{\begin{bmatrix} m_{s_{1,t}}^m \\ m_{s_{2,t}}^m \\ \vdots \\ m_{s_{n,t}}^m \end{bmatrix}}_{\varphi_t}, \underbrace{\begin{bmatrix} m_{s_{1,t}}^v & 0 & \dots & 0 \\ 0 & m_{s_{2,t}}^v & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \dots & 0 & m_{s_{n,t}}^v \end{bmatrix}}_{\Upsilon_t} \right)$$

Hence, given the Gaussian distribution of $\log\ \left[(\varepsilon_t)^{\cdot 2}\right] \big| \{S_t\}_{t=1}^T$, we can define the needed state space form as:

$$\widetilde{y}_t^* = \varphi_t + 2 \log \sigma_t + \zeta_t, \qquad \zeta_t \overset{i}{\sim} \mathcal{MN}\left(\ \mathbf{0},\ \Upsilon_t\ \right),$$

$$\log \sigma_t = \log \sigma_{t-1} + \nu_{\sigma,t}, \qquad \nu_{\sigma,t} \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0},\ \underbrace{Q_\sigma}_{n \times n} \right).$$

At this point, the Forward Filtering Backward Sampling (FFBS) procedure, introduced by Carter & Kohn (1994), can be implemented to draw a history of volatilities $\{\sigma_t\}_{t=1}^T$. The procedure is described below. For simplicity we define $\widetilde{\sigma}_t \equiv \log \sigma_t$, since the FFBS procedure is implemented on the log-volatilities.

[12]Noticing that $\log \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \log a \\ \log b \end{bmatrix}$, and $\begin{bmatrix} a \\ b \end{bmatrix}^{\cdot 2} = \begin{bmatrix} a^2 \\ b^2 \end{bmatrix}$.

The filter can be initialized at the following values:

$$\widetilde{\sigma}_{0|0} = \log(\bar{\sigma}), \qquad P_{\sigma,0|0} = \bar{P}_{\sigma},$$

where $\log(\bar{\sigma})$ and $\bar{P}_{\sigma}$ are respectively the mean and the covariance matrix of the prior distribution of $\log \sigma_0$.

Recursively, for each $\left(\widetilde{\sigma}_{t-1|t-1},\ P_{\sigma,t-1|t-1}\right)$, we compute the filter:

$$\begin{aligned}
P_{\sigma,t|t-1} &= P_{\sigma,t-1|t-1} + Q_{\sigma}, \\
K_{\sigma,t} &= 2P_{\sigma,t|t-1}\left(4P_{\sigma,t|t-1} + \Upsilon_t\right)^{-1}, \\
\widetilde{\sigma}_{t|t} &= \widetilde{\sigma}_{t-1|t-1} + K_{\sigma,t}\left(\widetilde{y}_t^* - 2\widetilde{\sigma}_{t-1|t-1} - \varphi_t\right), \\
P_{\sigma,t|t} &= P_{\sigma,t|t-1} - 2K_{\sigma,t}P_{\sigma,t|t-1}.
\end{aligned}$$

Having an entire set of updating and prediction steps $\left(\widetilde{\sigma}_{t|t},\ P_{\sigma,t|t},\ P_{\sigma,t|t-1}\right)_{t=1}^{T}$, we start to sample backward, beginning by sampling $\widetilde{\sigma}_T$ from $\mathcal{MN}\left(\widetilde{\sigma}_{T|T},\ P_{\sigma,T|T}\right)$, and then for each $t \in \{T-1, T-2, \ldots, 2, 1\}$ we sample recursively each $\widetilde{\sigma}_t$ from $\mathcal{MN}\left(\widetilde{\sigma}_{t|t+1},\ P_{\sigma,t|t+1}\right)$ where:

$$\begin{aligned}
\widetilde{\sigma}_{t|t+1} &= \widetilde{\sigma}_{t|t} + P_{\sigma,t|t}P_{\sigma,t+1|t}^{-1}\left(\widetilde{\sigma}_{t+1} - \widetilde{\sigma}_{t|t}\right), \\
P_{\sigma,t|t+1} &= P_{\sigma,t|t} - P_{\sigma,t|t}P_{\sigma,t+1|t}^{-1}P_{\sigma,t|t}.
\end{aligned}$$

A.3 Step 2(a): Draw $\theta| \{\sigma_t\}_{t=1}^T$

A.3.1 Substep 2(a).i: Draw the covariance of volatilities' innovations Q_σ

Conditioning on $\{\sigma_t\}_{t=0}^T$, we can draw the covariance matrix Q_σ. Indeed, recall that:

$$\log \sigma_t = \log \sigma_{t-1} + \nu_{\sigma,t}, \qquad \nu_{\sigma,t} \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0},\ \underbrace{Q_\sigma}_{n\times n}\right).$$

But then, having a complete history of the sigmas, given the random walk law of motion, is equivalent to having a complete histories of innovations $\nu_{\sigma,t}$. Stacking the $\nu_{\sigma,t}$ across time, we get:

$$\underbrace{\nu_\sigma^*}_{n\times T} = \begin{bmatrix}\nu_{\sigma,1} & \nu_{\sigma,2} & \ldots & \nu_{\sigma,T}\end{bmatrix},$$

and we can easily compute the innovations sum of squares matrix:

$$\underbrace{S_\sigma}_{n\times n} = \underbrace{\nu^*_\sigma}_{n\times T}\underbrace{\nu^{*\prime}_\sigma}_{T\times n}.$$

If the prior on the matrix Q_σ is a $n \times n$ Inverse Wishart with scale matrix $\bar{Q}_\sigma$ and degrees of freedom $\tau_{\sigma,0}$:

$$Q_\sigma \sim \mathcal{IW}_n\left(\bar{Q}_\sigma,\ \tau_{\sigma,0}\right),$$

then the posterior is conjugate and given by:

$$Q_\sigma \big| \left\{\sigma^i_t\right\}_{t=0}^T \sim \mathcal{IW}_n\left(S_\sigma + \bar{Q}_\sigma,\ \tau_{\sigma,0} + T\right).$$

A.3.2 Substep 2(a).ii: Draw the loadings A

To draw the loadings contained in A, we rewrite the model in (12) in the following stacked form:

$$\begin{bmatrix} y_1' \\ y_2' \\ \vdots \\ y_T' \end{bmatrix} = \begin{bmatrix} Z_1' \\ Z_2' \\ \vdots \\ Z_T' \end{bmatrix} A' + \begin{bmatrix} u_1' \\ u_2' \\ \vdots \\ u_T' \end{bmatrix},$$

$$\underbrace{y}_{T\times n} = \underbrace{Z}_{T\times rp} \cdot \underbrace{A'}_{rp\times n} + u.$$

Defining $a \equiv vec\,(A')$, and exploiting the Kronecker properties, the stacked form can be vectorized and transformed into:

$$vec\,(y) = vec\,(Z \cdot A' \cdot I_n) + vec\,(u)\,,$$

$$\underbrace{Y}_{nT\times 1} = \underbrace{(I_n \otimes Z)}_{n\times nrp} \cdot \underbrace{a}_{nrp\times 1} + U,$$

where U has the following distribution:

$$\underbrace{U}_{nT\times 1} \sim \mathcal{MN}\left(\mathbf{0},\ \underbrace{V_u}_{nT\times nT}\right)$$

and

$$
V_u \equiv \begin{bmatrix}
\Omega_1^{(1,1)} & 0 & \cdots & 0 & \cdots & \cdots & \Omega_1^{(1,n)} & 0 & \cdots & 0 \\
0 & \Omega_2^{(1,1)} & \ddots & \vdots & \cdots & \cdots & 0 & \Omega_2^{(1,n)} & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 & \cdots & \cdots & \vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & \Omega_T^{(1,1)} & \cdots & \cdots & 0 & \cdots & 0 & \Omega_T^{(1,n)} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
\Omega_1^{(n,1)} & 0 & \cdots & 0 & \cdots & \cdots & \Omega_1^{(n,n)} & 0 & \cdots & 0 \\
0 & \Omega_2^{(n,1)} & \ddots & \vdots & \cdots & \cdots & 0 & \Omega_2^{(n,n)} & \ddots & \vdots \\
\vdots & \ddots & \ddots & 0 & \cdots & \cdots & \vdots & \ddots & \ddots & 0 \\
0 & \cdots & 0 & \Omega_T^{(n,1)} & \cdots & \cdots & 0 & \cdots & 0 & \Omega_T^{(nn)}
\end{bmatrix}
$$

$$
= \sum_{t=1}^{T} \left[\Omega_t \otimes (e_t \cdot e_t') \right].
$$

To use an informative prior on a, we follow the approach in Gelman et al. (2014). The strategy incorporates the prior as observations. Considering a multivariate normal prior with the following moments:

$$
a \sim \mathcal{MN}\left(\, \bar{a},\ V_a \,\right),
$$

it is possible to augment the model with nrp observations that express the prior information:

$$
\begin{bmatrix} Y \\ \bar{a} \end{bmatrix} = \begin{bmatrix} I_n \otimes Z \\ I_{nrp} \end{bmatrix} a + \begin{bmatrix} U \\ U_a \end{bmatrix},
$$

$$
Y^{\diamond} = Z^{\diamond} a + U^{\diamond}, \quad U^{\diamond} \sim \mathcal{MN}\left(\, \mathbf{0}_{nT+nrp},\ V^{\diamond} \,\right),
$$

$$
V^{\diamond} = \begin{bmatrix} V_u & \mathbf{0}_{nT \times nrp} \\ \mathbf{0}_{nrp \times nT} & V_a \end{bmatrix}.
$$

A draw for a then comes from the following posterior:

$$
a \sim \mathcal{MN}\left(\widetilde{a},\ \left(Z^{\diamond\prime} V^{\diamond-1} Z^{\diamond} \right)^{-1} \right),
$$

$$
\widetilde{a} = \left(Z^{\diamond\prime} V^{\diamond-1} Z^{\diamond} \right)^{-1} Z^{\diamond\prime} V^{\diamond-1} Y^{\diamond}.
$$

In order to decrease the computational burden of this step throughout the sampling, we adopt the strategy proposed by Carriero et al. (2019). In particular, the triangular representation of the system is exploited, and coefficients are drawn equation by equation. The approach proposed in Carriero et al. (2019) and its generalization to allow for time-varying covariances is analytically documented in the appendix (section C.2).

A.3.3 Substep 2(a).iii: Draw the factor weights elements in B_0

Given the restrictions and the non-linear role of B_0, a Random Walk Metropolis step on the posterior kernel of each element of B_0 is implemented, nested into the Gibbs Sampling algorithm. In order to do this, we first write the likelihood of the model.

Given the reduced form VAR written as:

$$y_t = A \cdot Z_t + u_t \ , \qquad u_t \overset{i}{\sim} \mathcal{MN}\left(\mathbf{0}, \ \Omega_t \right),$$

conditioning on all other elements and using the chain rule, we can write the likelihood kernel as:

$$f\left((y_t)_{t=1}^T \middle| A, (\Omega_t)_{t=1}^T , \ B_0 \right) \propto \left(\prod_{t=1}^T |\Omega_t|^{-\frac{1}{2}} \right) \exp\left\{ -\frac{1}{2} \sum_{t=1}^T (y_t - A \cdot Z_t)' \, \Omega_t^{-1} \, (y_t - A \cdot Z_t) \right\}.$$

Next, let us consider the $r^* \equiv n - r$ scalar unrestricted elements of B_0, i.e. $(b_{0,j})_{j=1}^{r^*}$. Then, $\forall j \in \{1, \dots, r^*\}$, we can define the set $b_{0,j-} \equiv (b_{0,s})_{s \neq j}$.

For a given prior $f\left(b_{0,j}\right)$ on each element $b_{0,j}$, we can write the kernel of the conditional posterior of $b_{0,j}$ as:

$$f_{post}\left(b_{0,j} | \, (y_t, \Omega_t)_{t=1}^T , \ A, \ b_{0,j-} \right) \propto f\left((y_t)_{t=1}^T \middle| A, \ B_0, \ (\Omega_t)_{t=1}^T \right) \cdot f\left(b_{0,j}\right).$$

We are now ready to design the Metropolis step, separately for each j. Given the last step B_0^{i-1}, a random walk candidate is computed as:

$$b_{0,j}^* = b_{0,j}^{i-1} + c_j \cdot \eta_t,$$

where c_j is a scaling factor calibrated to have an acceptance rate of approximately 30%-35% and $\eta_t \overset{iid}{\sim} \mathcal{N}\left(0, v_j\right)$, where v_j is the variance of the prior $f\left(b_{0,j}\right)$. The candidate draw is accepted with probability:

$$\alpha_j = \min\left\{ 1, \frac{ f_{post}\left(b_{0,j}^* | \left(y_t, \Omega_t^{i-1} \right)_{t=1}^T , \ A, \ b_{0,j-}^{i-1} \right) }{ f_{post}\left(b_{0,j}^{i-1} | \left(y_t, \Omega_t^{i-1} \right)_{t=1}^T , \ A, \ b_{0,j-}^{i-1} \right) } \right\}.$$

When the candidate is accepted, then $b^i_{0,j-} = b^*_{0,j}$, otherwise $b^i_{0,j-} = b^{i-1}_{0,j-}$. Repeating this procedure for $\forall j \in \{1, \ldots, r^*\}$, we obtain a draw B^i_0 from the distribution of interest.

A.3.4 Substep 2(a).iv: draw the off-diagonal elements in G

To draw the off-diagonal elements in G, we restate the reduced form as:

$$\begin{aligned} y_t &= \underbrace{A}_{n\times rp} \cdot \underbrace{Z_t}_{rp\times 1} + u_t, \\ y_t - A \cdot Z_t &= G^{-1}\Sigma_t\varepsilon_t, \\ \widehat{y_t} &= G^{-1}\Sigma_t\varepsilon_t, \\ G \cdot \widehat{y_t} &= \Sigma_t\varepsilon_t. \end{aligned}$$

Removing ones from the diagonal of G, and bringing off-diagonal elements on the right hand side, produces:

$$G = I_n + G^*.$$

This can be combined in the model to obtain:

$$\begin{aligned} (I_n + G^*)\ \widehat{y_t} &= \Sigma_t\ \varepsilon_t, \\ \widehat{y_t} &= -G^*\ \widehat{y_t} + \Sigma_t\ \varepsilon_t. \end{aligned}$$

Exploiting the Kronecker properties, we then get:

$$-\ I_n \underbrace{G^*}_{n\times n} \underbrace{\widehat{y_t}}_{n\times 1} = -\underbrace{(I_n \otimes \widehat{y}_t')}_{n\times n^2} \underbrace{vec\,(G^{*\prime})}_{n^2\times 1},$$

where $vec\,(G^{*\prime})$ have zeros in positions $[(i-1)\,n + j]^{i\in\{1,\ldots,n\}}_{j\in\{1,\ldots,n\}}$. By removing the zeros, we obtain exactly the elements below the main diagonal of G, gathered in the m-dimensional vector g. Removing the corresponding columns in $-\,(I_n \otimes \widehat{y}_t')$, we construct the matrix

W_t, which has the following form:

$$\underbrace{W_t}_{n\times m} = -1 \cdot \begin{bmatrix} 0 & \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots & & & & & & & & & & 0 \\ \widehat{y}_{1,t} & 0 & \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots & & & & & & & & & \vdots \\ 0 & \widehat{y}_{1,t} & \widehat{y}_{2,t} & 0 & \dots\dots\dots\dots\dots\dots\dots\dots\dots & & & & & & & \vdots \\ 0 & 0 & 0 & \widehat{y}_{1,t} & \widehat{y}_{2,t} & \widehat{y}_{3,t} & 0 & \dots\dots\dots\dots\dots\dots & & & & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & 0 \\ 0 & \dots\dots\dots\dots\dots\dots\dots\dots & & & & & & 0 & \widehat{y}_{1,t} & \widehat{y}_{2,t} & \widehat{y}_{3,t} & \dots & \widehat{y}_{n-1,t} \end{bmatrix}.$$

We can then rewrite the model as:

$$\begin{aligned} \widehat{y}_t &= -G^*\widehat{y}_t + \Sigma_t\varepsilon_t, \\ \widehat{y}_t &= -\left(I_n \otimes \widehat{y}_t'\right) vec\left(G^{*\prime}\right) + \Sigma_t\varepsilon_t, \\ \widehat{y}_t &= W_t\ g + \varepsilon_t^*, \qquad \varepsilon_t^* \sim \mathcal{MN}\left(\ \mathbf{0}_{n\times 1},\ \Sigma_t^2\ \right). \end{aligned}$$

Next, we stack the model as:

$$\begin{bmatrix} \widehat{y}_1 \\ \widehat{y}_2 \\ \vdots \\ \widehat{y}_T \end{bmatrix} = \begin{bmatrix} W_1 \\ W_2 \\ \vdots \\ W_T \end{bmatrix} g + \begin{bmatrix} \varepsilon_1^* \\ \varepsilon_2^* \\ \vdots \\ \varepsilon_T^* \end{bmatrix},$$

$$\underbrace{\widehat{y}}_{nT\times 1} = \underbrace{W}_{Tn\times m} \cdot \underbrace{g}_{m\times 1} + \varepsilon^*, \qquad \varepsilon^* \sim \mathcal{MN}\left(\ \mathbf{0}_{nT\times 1},\ \Sigma^2\ \right),$$

where Σ is the diagonal matrix containing all the stacked stochastic volatilities vectors in the main diagonal:

$$\Sigma = Diag\left(\begin{bmatrix} \sigma_1' & \sigma_2' & \dots & \sigma_T' \end{bmatrix}'\right).$$

We can then use a similar approach as the one implemented for a, following Gelman et al. (2014). Specifically, given the prior :

$$g \sim \mathcal{MN}\left(\ \bar{g},\ V_g\ \right),$$

we augment the model with r observations that express the prior information:

$$\begin{bmatrix} \widehat{y} \\ \bar{g} \end{bmatrix} = \begin{bmatrix} W \\ I_m \end{bmatrix} g + \begin{bmatrix} \varepsilon^* \\ \varepsilon_g \end{bmatrix},$$

$$\widehat{Y}^\diamond = W^\diamond g + \varepsilon^\diamond, \quad \varepsilon^\diamond \sim \mathcal{MN}\left(\mathbf{0}_{nT+m},\ V_\varepsilon^\diamond \right),$$

$$V_\varepsilon^\diamond = \begin{bmatrix} \Sigma^2 & \mathbf{0}_{nT\times m} \\ \mathbf{0}_{m\times nT} & V_g \end{bmatrix}.$$

A draw for g then comes from the following posterior:

$$g \sim \mathcal{MN}\left(\widetilde{g},\ \left(W^{\diamond\prime} V_\varepsilon^{\diamond -1} W^\diamond\right)^{-1} \right),$$

$$\widetilde{g} = \left(W^{\diamond\prime} V_\varepsilon^{\diamond -1} W^\diamond\right)^{-1} W^{\diamond\prime} V_\varepsilon^{\diamond -1} \widehat{Y}^\diamond.$$

A.4 Step 2(b): Draw a history of indexes of the mixture $\{S_t\}_{t=1}^T |\theta, \{\sigma_t\}_{t=1}^T$

Starting from the following formulation seen in Step 1 of the algorithm

$$\widetilde{y}_t^* = 2\log\sigma_t + \log\ \left[(\varepsilon_t)^{\cdot 2}\right],$$

we can notice that, since $\varepsilon_t \overset{iid}{\sim} \mathcal{MN}(\mathbf{0}, I_n)$, each term $\log \varepsilon_{j,t}^2$ has a $\log \chi_1^2$ distribution, and $\log\ \left[(\varepsilon_t)^{\cdot 2}\right]$ is a vector of independent $\log \chi_1^2$ random variables.

Omori et al. (2007), improving upon Kim et al. (1998), show that the $\log \chi_1^2$ distribution is very well approximated by a mixture of ten Normal distributions:

$$f_{\log \chi_1^2}(x) \approx \sum_{j=1}^{10} m_j^p\ f_{\mathcal{N}}\left(x \middle| m_j^m, m_j^v\right),$$

where m_j^p, m_j^m and m_j^v are contained in the following table:

j	1	2	3	4	5	6	7	8	9	10
m_j^p	0.00609	0.04775	0.13057	0.20674	0.22715	0.18842	0.12047	0.05591	0.01575	0.00115
m_j^m	1.92677	1.34744	0.73504	0.02266	−0.85173	−1.97278	−3.46788	−5.55246	−8.68384	−14.65000
m_j^v	0.11265	0.17788	0.26768	0.40611	0.62699	0.98583	1.57469	2.54498	4.16591	7.33342

Therefore, in order to have a conditionally Gaussian measurement equation, we should condition each element of the vector $\log\ \left[(\varepsilon_t)^{\cdot 2}\right]$ on the index that specifies the Normal components of the mixture. Defining the $n \times 1$ vector S_t that contains the indexes of components in period t, we can write

$$S_t \equiv \begin{bmatrix} s_{1,t} \\ \vdots \\ s_{n,t} \end{bmatrix}, \quad \text{where} \quad \left[\log\ \varepsilon_{h,t}^2 \middle|\, s_{h,t} = j\right] \sim \mathcal{N}\left(m_j^m, m_j^v\right).$$

Conditioning on a history of volatilities $(\sigma_t)_{t=1}^T$, we can restate the model as

$$\log\ \left[(\varepsilon_t)^{\cdot 2}\right] = \widetilde{y}_t^* - 2\log\sigma_t,$$

Then, the element $s_{h,t}$ that indexes the specific component from which log $\varepsilon_{h,t}^2$ is drawn, has support $J = \{1, \ldots, 10\}$ and the following discrete probability distribution:

$$\forall j \in J, \qquad Pr\left[s_{h,t} = j \middle|\, \widetilde{y}_{h,t}^*, \sigma_t\right] = \frac{m_j^p\ f_{\mathcal{N}}\left(\widetilde{y}_{h,t}^* - 2\log\sigma_{h,t} \middle|\, m_j^m, m_j^v\right)}{\sum_{\iota=1}^{10} m_\iota^p\ f_{\mathcal{N}}\left(\widetilde{y}_{h,t}^* - 2\log\sigma_{h,t} \middle|\, m_\iota^m, m_\iota^v\right)}.$$

Independent draws for all variables $h \in \{1, \ldots, n\}$ at all periods $t \in \{1, \ldots, T\}$ from this distribution will form the new history of indexes of mixture's components $\{S_t\}_{t=1}^T |\theta, \{\sigma_t\}_{t=1}^T$.

Appendix B Detailed steps of the Gibbs Sampler for the estimation of the MAI-SVCV model

The MAI-SVCV model has time-varying off diagonal elements in the triangular reduction $u_t = G_t^{-1}\Sigma_t\varepsilon_t$. Therefore, we have to substitute SubStep 2(a).iv with two Substeps: the first one draws the elements $\{g_t\}_{t=1}^T$, and the second one the covariance matrix of their innovations Q_g. The Gibbs sampler in which the rank reduction of de Wind & Gambetti (2014) is not applied, and hence matrix Q_g is full rank, is similar to the one described in Primiceri (2005). If the rank reduction of de Wind & Gambetti (2014) is applied to attenuate the curse of dimensionality, the draw of $\{g_t\}_{t=1}^T$ is further split into two substeps. The following subsections will provide further details on the procedure needed in the case of reduced rank of Q_g, following de Wind & Gambetti (2014), which is more general than the full rank case.

B.1 Alternative SubStep 2(a).iv for the MAI-SVCV: Draw a history of TV off-diagonal elements $\{g_t\}_{t=1}^T$

Using the rank reduction strategy of de Wind & Gambetti (2014), the vector g_t is decomposed as:

$$g_t = \Lambda_g \widetilde{g}_t + M_g g_0,$$

with

$$\underbrace{\Lambda_g}_{m\times q_r} = \underbrace{V_g}_{m\times q_g} \underbrace{D_g}_{q_g\times q_g}, \qquad M_g = I_m - \Lambda_g \left(\Lambda_g' \Lambda_g\right)^{-1} \Lambda_g',$$

where D_g is the diagonal matrix containing the square roots of the q_g non-zero eigenvalues of Q_g, while V_g is the matrix whose columns are the associated eigenvectors (normalized to unit length). $M_g g_0$ represents the time invariant residual and $\widetilde{g}_t$ is the q_g-dimensional stochastic vector of TV components, having the following law of motion:

$$\widetilde{g}_t = \widetilde{g}_{t-1} + \widetilde{\nu}_{g,t}, \qquad \underbrace{\widetilde{\nu}_{g,t}}_{q_g\times 1} \overset{iid}{\sim} \mathcal{MN}\left(\ \mathbf{0},\ I_{q_g}\ \right).$$

As in de Wind & Gambetti (2014), the transformed time-varying components $\Lambda_g \widetilde{g}_t$ and the non-TV component $M_g g_0$ are drawn separately.

In order to build the state space form, we first transform the reduced form of the model in the following way:

$$y_t = A \cdot Z_t + G_t^{-1} \Sigma_t \varepsilon_t,$$

$$\underbrace{y_t - A \cdot Z_t}_{\widehat{y}_t} = G_t^{-1} \Sigma_t \varepsilon_t,$$

$$G_t\ \widehat{y}_t = \Sigma_t\ \varepsilon_t.$$

Removing ones from the diagonal of G_t, and bringing off-diagonal elements on the right hand side, produces:

$$G_t = I_n + G_t^*.$$

This can be inserted in the model to obtain:

$$(I_n + G_t^*)\ \widehat{y}_t = \Sigma_t\ \varepsilon_t,$$

$$\widehat{y}_t = -G_t^*\ \widehat{y}_t + \Sigma_t\ \varepsilon_t.$$

Exploiting the Kronecker properties, we get:

$$-\, I_n \underbrace{G_t^*}_{n\times n} \underbrace{\widehat{y}_t}_{n\times 1} = -\underbrace{(I_n \otimes \widehat{y}_t')}_{n\times n^2} \underbrace{vec\,(G_t^{*\prime})}_{n^2\times 1},$$

where $vec\,(G_t^{*\prime})$ have zeros in positions $[(i-1)\,n + j]_{j\in\{1,\ldots,n\}}^{i\in\{1,\ldots,n\}}$. By removing the zeros we obtain exactly the elements below the main diagonal of G_t, gathered in the vector g_t. Removing the corresponding columns in $-\,(I_n \otimes \widehat{y}_t')$, we construct the matrix W_t, which has the following form:

$$\underbrace{W_t}_{n\times m} = -1 \cdot \begin{bmatrix} 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ \widehat{y}_{1,t} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \widehat{y}_{1,t} & \widehat{y}_{2,t} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & 0 & 0 & \widehat{y}_{1,t} & \widehat{y}_{2,t} & \widehat{y}_{3,t} & 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & \ddots & \ddots & \vdots & \vdots & \vdots & \vdots & 0 \\ 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 & \widehat{y}_{1,t} & \widehat{y}_{2,t} & \widehat{y}_{3,t} & \cdots & \widehat{y}_{n-1,t} \end{bmatrix}.$$

Hence, we obtain a measurement equation with g_t as states:

$$\widehat{y}_t = -G_t^* \widehat{y}_t + \Sigma_t \varepsilon_t,$$

$$\widehat{y}_t = -\,(I_n \otimes \widehat{y}_t')\, vec\,(G_t^{*\prime}) + \Sigma_t \varepsilon_t,$$

$$\widehat{y}_t = W_t\ g_t + \Sigma_t \varepsilon_t.$$

B.1.1 SubSubStep: draw a history of TV components $\{\widetilde{g}_t^i\}_{t=1}^T$

Let us consider the de Wind & Gambetti (2014) decomposition on the m-dimensional process g_t:

$$g_t = \Lambda_g \widetilde{g}_t + M_g g_0, \qquad Q_g = \underbrace{\Lambda_g}_{m\times q_g} \Lambda_g',$$

and plug it into the measurement equation:

$$\widehat{y}_t = W_t g_t + \Sigma_t \varepsilon_t,$$

$$\widehat{y}_t - W_t M_g g_0 = W_t \Lambda_g \cdot \widetilde{g}_t + \Sigma_t \varepsilon_t,$$

$$\widehat{y}_t^* = W_t^*\ \widetilde{g}_t + \Sigma_t \varepsilon_t.$$

The resulting state space form is:

$$\widehat{y}_t^* = W_t^* \widetilde{g}_t + \Sigma_t \varepsilon_t, \qquad \varepsilon_t \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0},\ I_n \right),$$

$$\underbrace{\widetilde{g}_t}_{q_g \times 1} = \widetilde{g}_{t-1} + \widetilde{\nu}_{g,t}, \qquad \underbrace{\widetilde{\nu}_{g,t}}_{q_g \times 1} \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0},\ I_{q_g} \right).$$

The FFBS procedure can be implemented to draw an history $\{\widetilde{g}_t^i\}_{t=1}^T$. The filter, following de Wind & Gambetti (2014), can be initialized at:

$$\widetilde{g}_{0|0} = R_g \bar{g} \qquad P_{g,0|0} = R_g \bar{P}_g R_g'.$$

where

$$R_g \equiv \left(\Lambda_g' \Lambda_g\right)^{-1} \Lambda_g'.$$

B.1.2 SubSubStep: draw a vector of time invariant component $M_g g_0$ of g_t

We operate the following transformation

$$\widehat{y}_t = W_t g_t + \Sigma_t \varepsilon_t = W_t \left(\Lambda_g \widetilde{g}_t + M_g g_0 \right) + \Sigma_t \varepsilon_t,$$

$$\underbrace{\widehat{y}_t - W_t \Lambda_g \widetilde{g}_t}_{\widehat{y}_t^\bullet} = W_t \underbrace{M_g g_0}_{\mu_{g,0}} + \underbrace{\Sigma_t \varepsilon_t}_{\widehat{\varepsilon}_t},$$

to obtain the regression model of interest:

$$\widehat{y}_t^\bullet = W_t\ \mu_{g,0} + \widehat{\varepsilon}_t, \qquad \widehat{\varepsilon}_t \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0},\ \Sigma_t^{\cdot 2} \right).$$

We want to impose the $q_g < m$ restrictions given by:

$$\underbrace{R_g}_{q_g \times r} \underbrace{\mu_{g,0}}_{r \times 1} = \mathbf{0}_{q_g \times 1}.$$

Stacking the time dimensions in columns, we write the restricted regression model as:

$$\underbrace{\widehat{Y}^\bullet}_{nT \times 1} = \underbrace{W}_{nT \times m}\ \mu_{g,0} + \underbrace{\xi}_{nT \times 1}, \qquad \xi \sim \mathcal{MN}\left(\mathbf{0}_{nT \times 1},\ \Sigma^{\cdot 2} \right),$$

$$\underbrace{R_g}_{q_g \times m} \underbrace{\mu_{g,0}}_{r \times 1} = \mathbf{0}_{q_g \times 1}.$$

Then, given the prior for g_0 and the unrestricted Bayesian regression moments:

$$g_0 \sim \mathcal{MN}\left(\ \bar{g},\ \bar{P}_g\ \right),$$
$$\Psi_g = \left(W'\Sigma^{\cdot -2}W + \bar{P}_g^{-1}\right)^{-1},$$
$$\psi_g = \Psi_g\left(W'\Sigma^{\cdot -2}\ \widehat{Y}^{\bullet} + \bar{P}_g^{-1}\bar{g}\right),$$

the sampling posterior distribution for $\mu_{g,0}$ is given by:

$$\mu_{g,0} \sim \mathcal{MN}\left(\ \widetilde{\mu}_{g,0}\ ,\ P_{\mu_g}\ \right),$$
$$\widetilde{\mu}_{g,0} = \Psi_{\mu_g}\psi_g, \qquad P_{\mu_g} = \Psi_{\mu_g}\Psi_g,$$
$$\Psi_{\mu_g} = I_r - \Psi_g R_g'\left(R_g\Psi_g R_g'\right)^{-1} R_g.$$

B.1.3 SubSubStep: sum the TV and non-TV draws of g_t

We can now sum the draws obtained in the two previous substeps:

$$g_t = \Lambda_g\widetilde{g}_t + M_g g_0, \qquad \forall t \in \{1,\ldots,T\},$$

finally obtaining a history of time-varying off-diagonal elements $(g_t^i)_{t=1}^T$.

B.2 Additional SubStep 2(a).v for the MAI-SVCV: Draw a reduced rank covariance matrix Q_g

Conditioning on the drawn $\{g_t^i\}_{t=1}^T$, we can draw the reduced rank covariance matrix Q_g. Indeed, since:

$$g_t = g_{t-1} + \nu_{g,t}, \qquad \nu_{g,t} \overset{iid}{\sim} \mathcal{MN}\left(\mathbf{0},\ \underbrace{Q_g}_{r\times r}\right),$$

having a complete history $(g_t)_{t=1}^T$, given the random walk law of motion, is equivalent to having a complete history of innovations $\nu_{g,t}$. Stacking the $\nu_{g,t}$ across time, we get:

$$\underbrace{\nu_g^*}_{r\times T} = \begin{bmatrix}\nu_{g,1} & \nu_{g,2} & \ldots & \nu_{g,T}\end{bmatrix},$$

and we can easily compute the innovation sum of squares matrix:

$$\underbrace{S_g}_{r\times m} = \underbrace{\nu_g^*}_{m\times T}\underbrace{\nu_g^{*\prime}}_{T\times m}.$$

By construction, the rank of S_g would be $q_g < m$. Hence, if the prior on the matrix Q_g is an $m \times m$ Singular Inverse Wishart with rank $q_g < m$, scale matrix $\bar{Q}_g$ and degrees of freedom $\tau_{g,0}$:

$$Q_g \sim \mathcal{SIW}_m^{q_g}\left(\bar{Q}_g,\ \tau_{g,0}\right),$$

then, the posterior is conjugate and given by:

$$Q_g\big|\left(g_t^i\right)_{t=0}^T \sim \mathcal{SIW}_m^{q_g}\left(S_g + \bar{Q}_g,\ \tau_{g,0} + T\right).$$

Section C.1 in the appendix discusses how to compute efficiently a draw from a Singular Inverse Wishart distribution.

Appendix C Additional details for the GS

C.1 Procedure to draw from a Singular Inverse Wishart distribution

Consider the $m \times m$ matrix Ξ having the following $\mathcal{SIW}$ distribution:

$$\Xi \sim \mathcal{SIW}_m^q\left(\underbrace{S}_{m\times m},\ d\right),$$

where S is a scale matrix, d the degrees of freedom, and q is the rank of both Ξ and S.

Following Bodnar & Okhrin (2008) and Dıaz-Garcıa et al. (1997), and applying some modifications tailored at improving the algorithm efficiency, it is possible to draw from the distribution of Ξ through the following steps:

1. Construct the Moore–Penrose pseudoinverse S^+ of the scale matrix S. Moore Penrose computation follows the highly efficient Qrginv algorithm proposed by Ataei (2014).

2. Construct the diagonal matrix $\underbrace{\Lambda_S}_{q\times q}$ containing the non-zero eigenvalues of S^+ in the main diagonal, and the matrix $\underbrace{U_S}_{m\times q}$ whose columns are the associated eigenvectors.

3. Draw d independent vectors $\underbrace{\widetilde{z}}_{q\times 1}$ from the multivariate normal $\mathcal{MN}\left(\,\mathbf{0},\ \Lambda_S\,\right)$ and stack them as columns in the matrix $\underbrace{\widetilde{Z}}_{q\times d}$.

4. Construct the matrices $\underbrace{Z}_{m\times q} = U_S\widetilde{Z}$ and $\underbrace{W}_{m\times m} = ZZ'$. The matrix W is a draw from a Singular Wishart distribution with scale matrix S^+ and d degrees of freedom.

5. Compute the Moore–Penrose pseudoinverse W^+ of W. The matrix $\widetilde{\Xi} = W^+$ is a draw from the Singular Inverse Wishart of interest.

C.2 Procedure to draw variable-specific coefficients with triangular structure of heteroskedastic innovations

This approach generalizes the one proposed by Carriero et al. (2019). Consider the following multivariate regression, with variable-specific coefficients, common regressors across equations, and heteroskedastic errors, whose time variation can be decomposed through a standard triangular reduction.

We are considering the following model:

$$\forall t \in \{1,\ldots,T\}, \qquad \underbrace{Z_t}_{n\times 1} = \underbrace{\Theta}_{n\times k} \cdot \underbrace{F_t}_{k\times 1} + \underbrace{u_t}_{n\times 1}, \qquad u_t \overset{i}{\sim} \mathcal{MN}\left(\mathbf{0},\ \underbrace{\Omega_t}_{n\times n}\right),$$

$$\Omega_t = W_t\Sigma_t\Sigma_t W_t', \qquad u_t = W_t\Sigma_t\varepsilon_t, \qquad \varepsilon_t \overset{iid}{\sim} \mathcal{MN}\left(\,\mathbf{0},\ I_n\,\right),$$

where

$$\underbrace{W_t}_{n\times n} = \begin{bmatrix} 1 & 0 & \dots & \dots & 0 \\ w_{1,t} & 1 & \ddots & \ddots & \vdots \\ w_{2,t} & w_{3,t} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ w_{m-n+2,t} & w_{m-n+3,t} & \dots & w_{m,t} & 1 \end{bmatrix}, \qquad \underbrace{w_t}_{m\times 1} \equiv \begin{bmatrix} w_{1,t} \\ w_{2,t} \\ \vdots \\ w_{m,t} \end{bmatrix}, \qquad m \equiv \frac{n\,(n-1)}{2},$$

$$\underbrace{\Sigma_t}_{n\times n} = \begin{bmatrix} \sigma_{1,t} & 0 & \dots & 0 \\ 0 & \sigma_{2,t} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \sigma_{n,t} \end{bmatrix}, \qquad \underbrace{\sigma_t}_{n\times 1} \equiv \begin{bmatrix} \sigma_{1,t} \\ \sigma_{2,t} \\ \vdots \\ \sigma_{n,t} \end{bmatrix}.$$

When $n \cdot k$ gets large, it is convenient to split the previous system in n sets of univariate equations. Indeed, splitting Θ in row vectors it is possible to write the system as:

$$\Theta = \begin{bmatrix} \theta_{[1,:]} \\ \theta_{[2,:]} \\ \vdots \\ \theta_{[n,:]} \end{bmatrix} \quad \rightarrow \quad \begin{cases} z_{1,t} = \theta_{[1,:]} \cdot F_t + \sigma_{1,t}\varepsilon_{1,t} \\ z_{2,t} = \theta_{[2,:]} \cdot F_t + w_{1,t}\sigma_{1,t}\varepsilon_{1,t} + \sigma_{2,t}\varepsilon_{2,t} \\ \vdots \\ z_{n,t} = \theta_{[n,:]} \cdot F_t + w_{m-n+2,t}\sigma_{1,t}\varepsilon_{1,t} + \cdots + w_{m,t}\sigma_{n-1,t}\varepsilon_{n-1,t} + \sigma_{n,t}\varepsilon_{n,t} \end{cases}$$

that, defining

$$\underbrace{W_t^*}_{n\times n} = \begin{bmatrix} 0 & 0 & \dots & \dots & 0 \\ w_{1,t} & 0 & \ddots & \ddots & \vdots \\ w_{2,t} & w_{3,t} & 0 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ w_{m-n+2,t} & w_{m-n+3,t} & \dots & w_{m,t} & 0 \end{bmatrix} = W_t - I_n,$$

and

$$\xi_t = W_t^* \Sigma_t \varepsilon_t,$$

can be written as:

$$\begin{cases} z_{1,t} - \xi_{1,t} = \theta_{[1,:]} \cdot F_t + \sigma_{1,t}\varepsilon_{1,t} \\ z_{2,t} - \xi_{2,t} = \theta_{[2,:]} \cdot F_t + \sigma_{2,t}\varepsilon_{2,t} \\ \vdots \\ z_{n,t} - \xi_{n,t} = \theta_{[n,:]} \cdot F_t + \sigma_{n,t}\varepsilon_{n,t} \end{cases}$$

This system is constituted by the following set of n separate equations:

$$\forall j \in \{1, \dots, n\}, \qquad z_{j,t} - \xi_{j,t} = \theta_{[j,:]} \cdot F_t + \varepsilon_{j,t}^*, \quad \varepsilon_{j,t}^* \sim \mathcal{N}\left(0, \sigma_{j,t}^2\right)$$

Notice that for $j \geq 2$ each $\xi_{j,t}$ depends on $\left\{\varepsilon_{1,t}^*, \dots, \varepsilon_{j-1,t}^*\right\}$, since only the first $j-1$ elements of the j-th row of W_t^* are non-zero.

Then, conditioning on $(W_t^*, \sigma_t, F_t)_{t=1}^T$, and given the following priors on the elements of Θ:

$$\forall j \in \{1, \dots, n\}, \qquad vec\left(\theta_{[j,:]}\right) \sim \mathcal{MN}\left(\ \bar{\theta}_{[j,:]},\ V_{j,\theta}\ \right)$$

it is possible to design the following recursive procedure to draw from the posterior of Θ.

Starting from $j = 1$:

$$z_{1,t} - \xi_{1,t} = \theta_{[1,:]} \cdot F_t + \varepsilon^*_{1,t}, \quad \varepsilon^*_{1,t} \sim \mathcal{N}\left(0, \sigma^2_{1,t}\right)$$

and given that $\forall t, \xi_{1,t} = 0$, it is possible to draw from the posterior of $\theta_{[1,:]}$ using the following stacked regression:

$$\begin{bmatrix} z_{1,1} \\ \vdots \\ z_{1,T} \end{bmatrix} = \begin{bmatrix} F'_1 \\ \vdots \\ F'_T \end{bmatrix} \cdot vec\left(\theta_{[1,:]}\right) + \varepsilon^*_1, \quad \varepsilon^*_1 \sim \mathcal{MN}\left(\mathbf{0}, \begin{bmatrix} \sigma^2_{1,1} & 0 & \dots & 0 \\ 0 & \sigma^2_{1,2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \sigma^2_{1,T} \end{bmatrix}\right),$$

using the prior

$$vec\left(\theta_{[1,:]}\right) \sim \mathcal{MN}\left(\ \bar{\theta}_{[1,:]},\ V_{1,\theta}\ \right).$$

Then, setting the initial condition $\widetilde{\xi}_{j-1,t} = 0$ and the posterior draw $\widetilde{\theta}_{[1,:]}$, the recursive step for any $j \geq 2$ is:

1. The posterior draw $vec\left(\widetilde{\theta}_{[j-1,:]}\right)$ is used to compute the following $\forall t$:

$$\widetilde{\varepsilon}^*_{j-1,t} = z_{j-1,t} - \widetilde{\xi}_{j-1,t} - \widetilde{\theta}_{[j-1,:]} F_t,$$

$$\widetilde{\xi}_{j,t} = W^*_{t,[j,:]} \cdot \begin{bmatrix} \widetilde{\varepsilon}^*_{1,t} \\ \vdots \\ \widetilde{\varepsilon}^*_{j-1,t} \end{bmatrix}.$$

2. The following stacked regression is used to draw from the posterior of $\theta_{[j,:]}$:

$$\begin{bmatrix} z_{j,1} - \widetilde{\xi}_{j,1} \\ \vdots \\ z_{1,T} - \widetilde{\xi}_{j,T} \end{bmatrix} = \begin{bmatrix} F'_1 \\ \vdots \\ F'_T \end{bmatrix} \cdot vec\left(\theta_{[j,:]}\right) + \varepsilon^*_j, \quad \varepsilon^*_j \sim \mathcal{MN}\left(\mathbf{0}, \begin{bmatrix} \sigma^2_{j,1} & 0 & \dots & 0 \\ 0 & \sigma^2_{j,2} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \sigma^2_{j,T} \end{bmatrix}\right),$$

using the following prior distribution: $\quad vec\left(\theta_{[j,:]}\right) \sim \mathcal{MN}\left(\ \bar{\theta}_{[j,:]},\ V_{j,\theta}\ \right).$

Repeating this iteration until $j = n$, we have drawn from the conditional posterior:

$$\left.\begin{bmatrix} \theta'_{[1,:]} & \theta'_{[2,:]} & \dots & \theta'_{[n,:]} \end{bmatrix}'\right| \left(W^*_t, \sigma_t, F_t, Z_t\right)^T_{t=1}.$$

If W_t^* is non time-varying, the above procedure is implemented in the same way as in the TV case, simply by considering that $\forall t$ we have $W_t^* = W^*$.

Appendix D Additional Figures

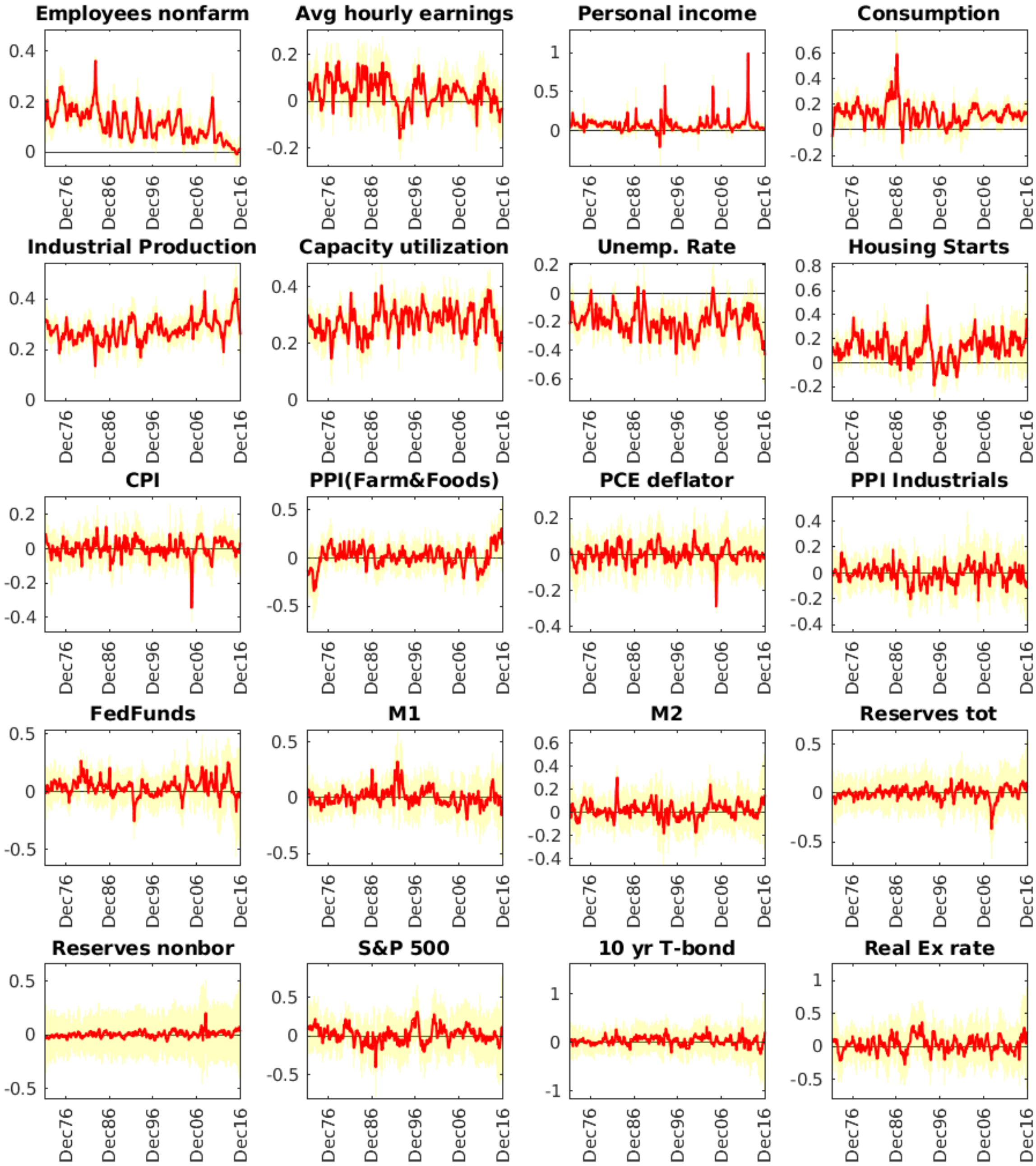

Figure 14: Response to a permanent Demand shock at horizon $h = 1$. Posterior bands.

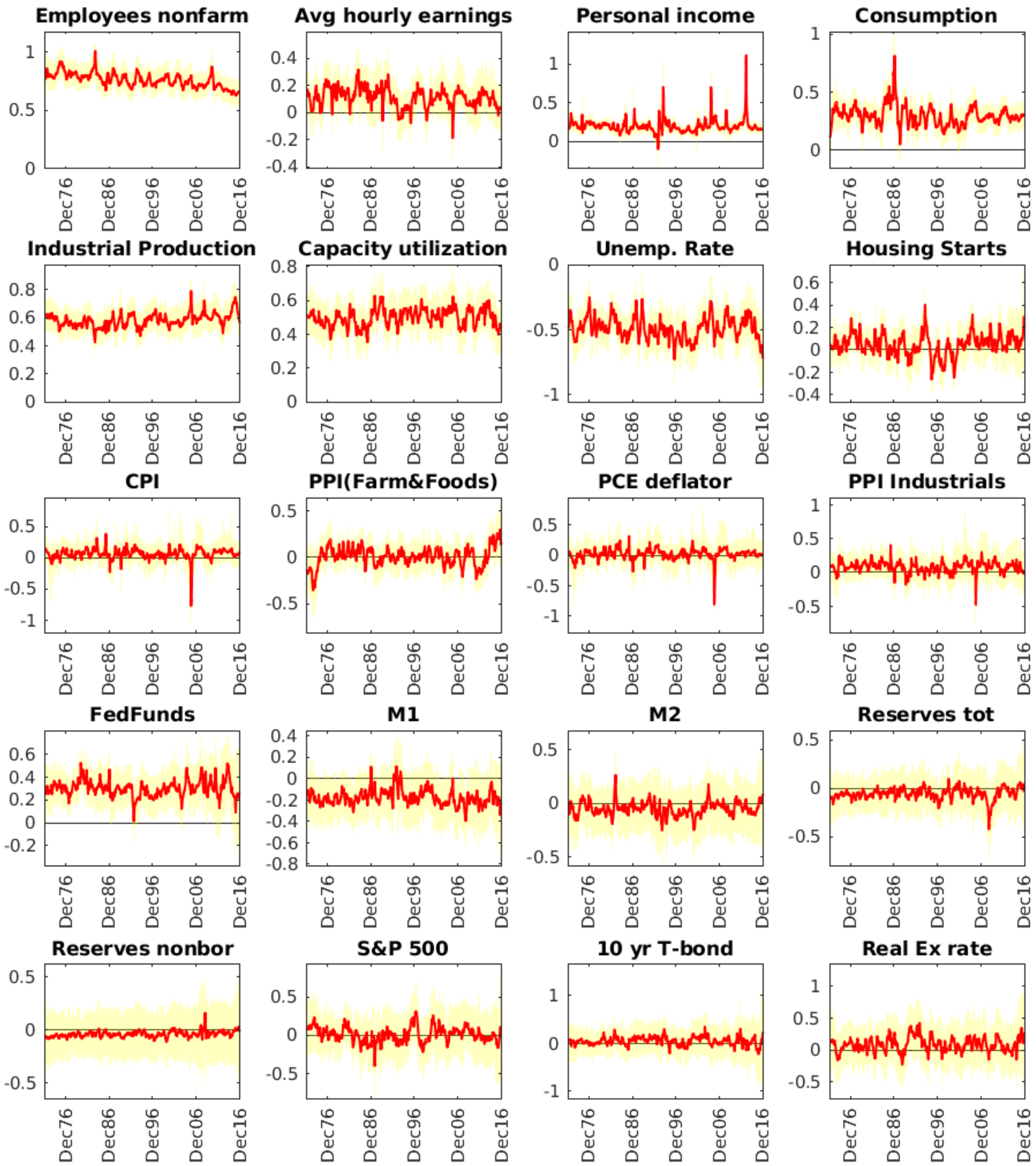

Figure 15: Response to a permanent Demand shock at horizon $h = 12$. Posterior bands.

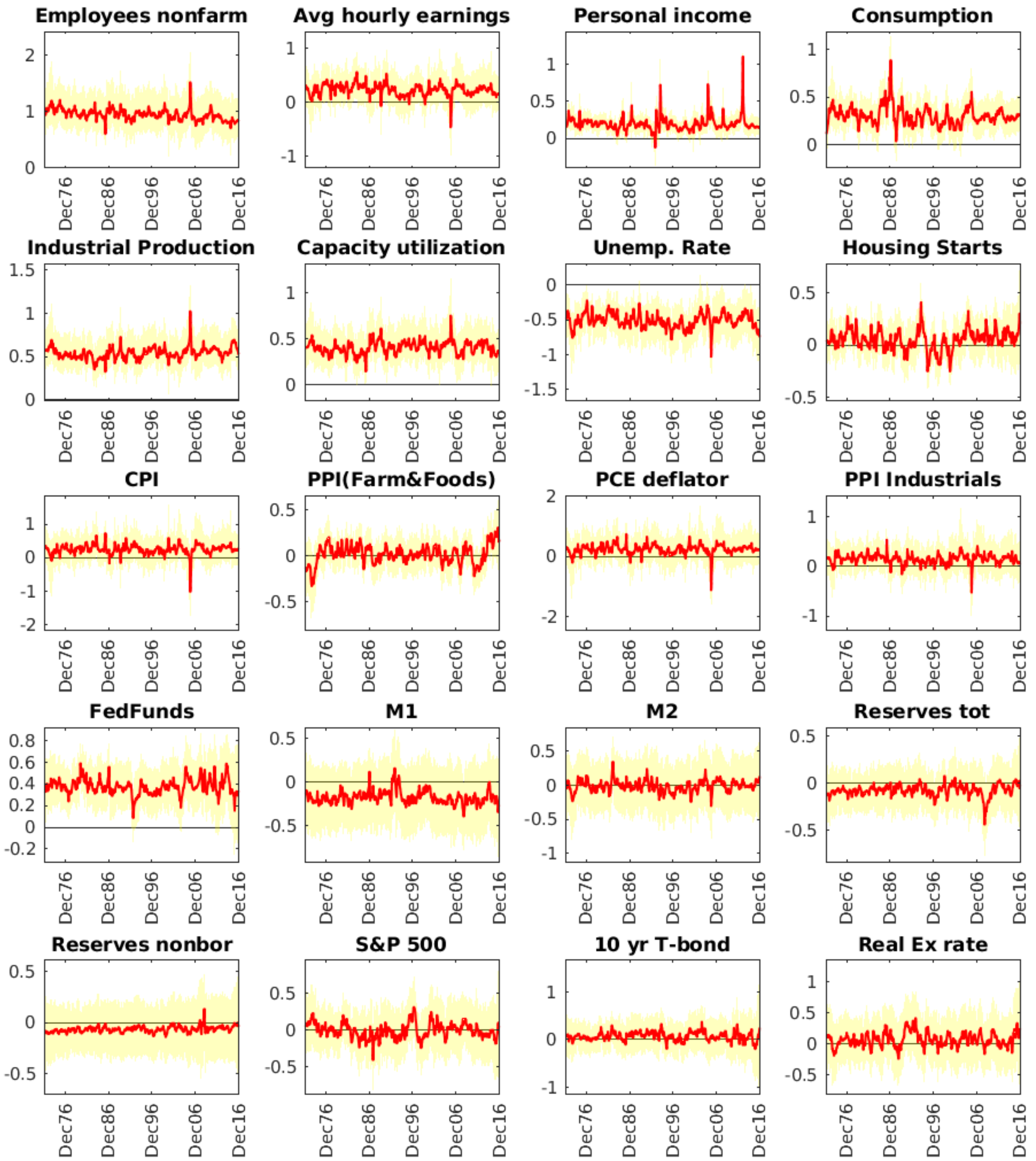

Figure 16: Response to a permanent Demand shock at horizon $h = 48$. Posterior bands.

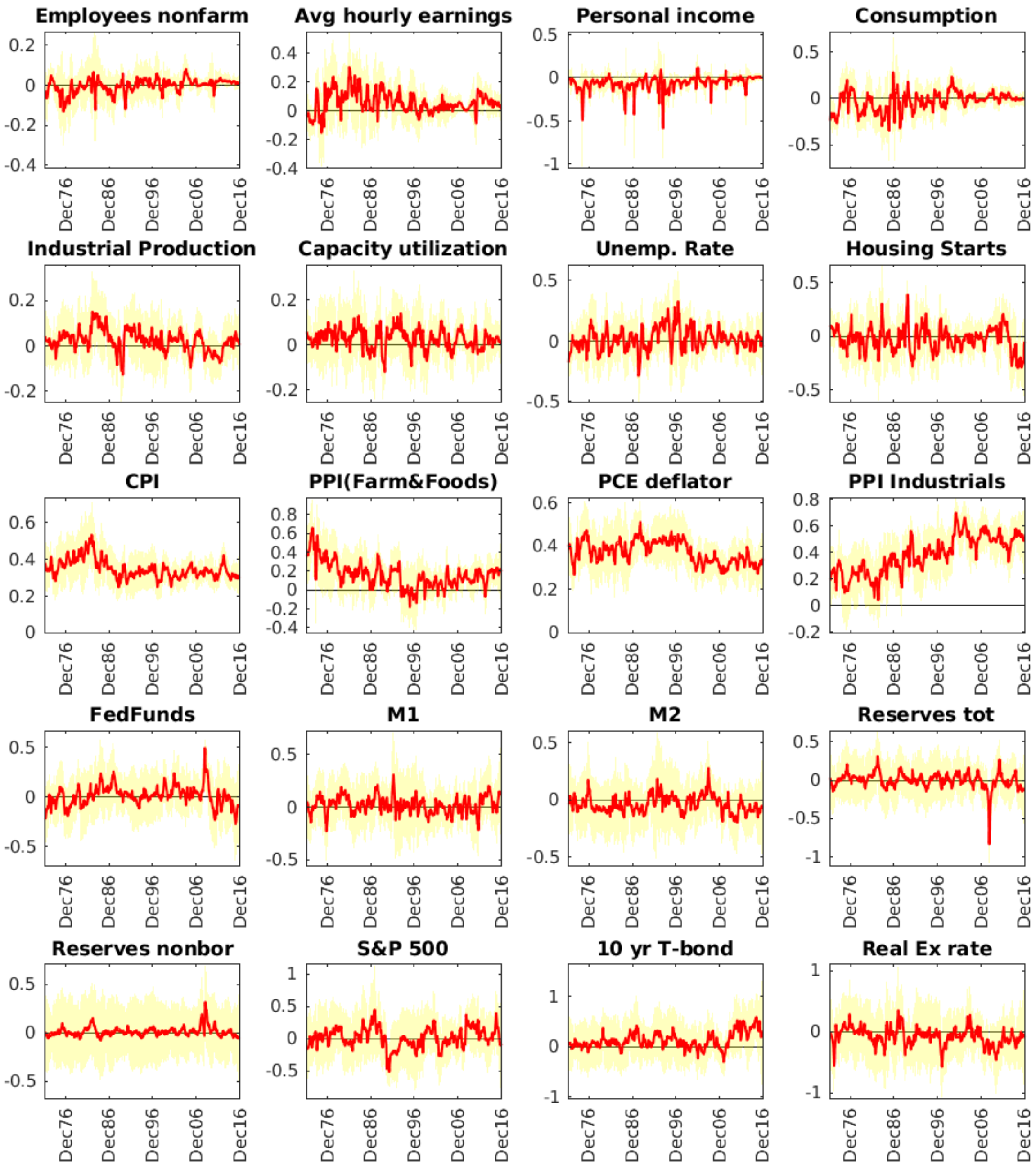

Figure 17: Response to a permanent Supply shock at horizon $h = 1$. Posterior bands.

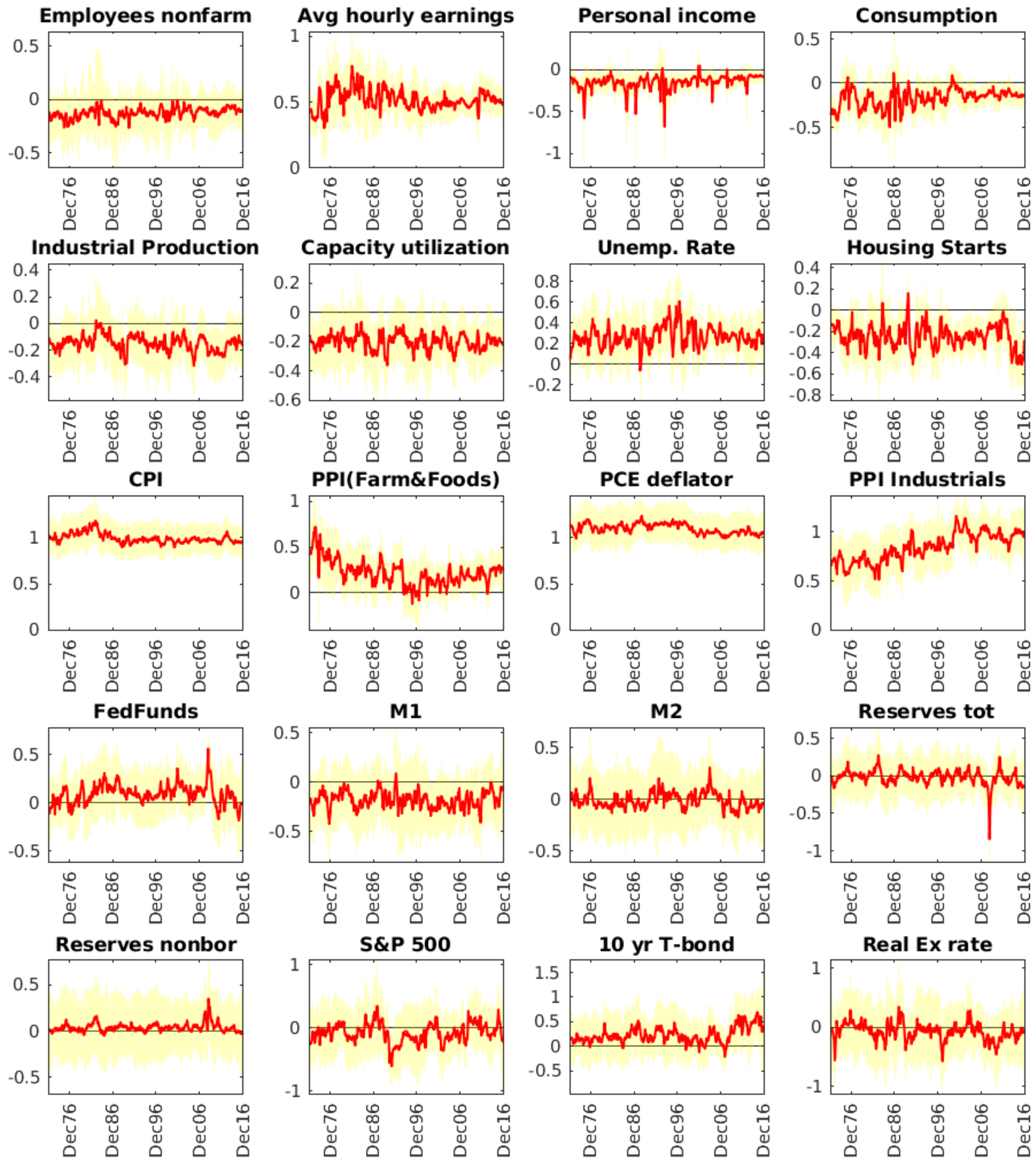

Figure 18: Response to a permanent Supply shock at horizon $h = 12$. Posterior bands.

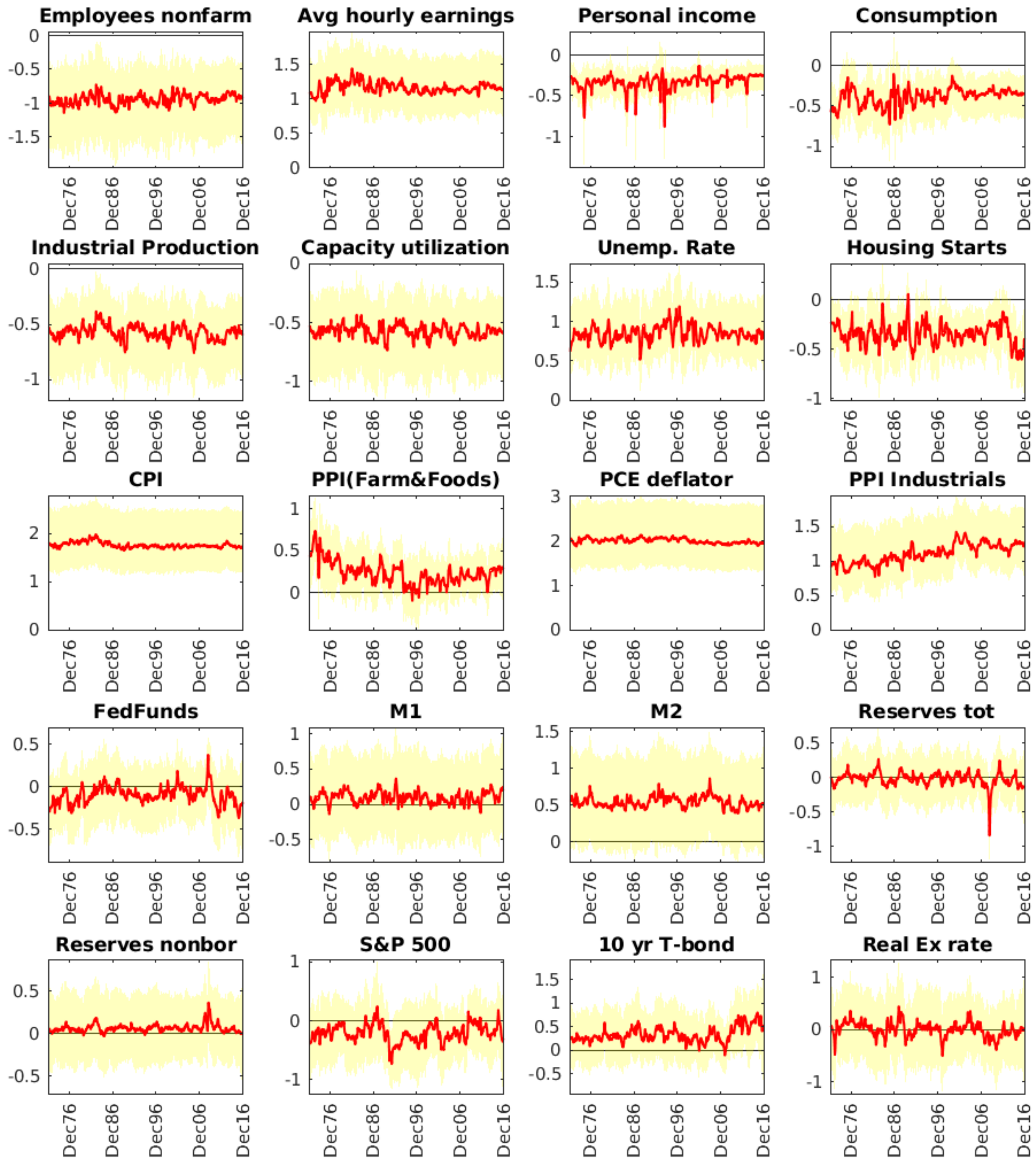

Figure 19: Response to a permanent Supply shock at horizon $h = 48$. Posterior bands.

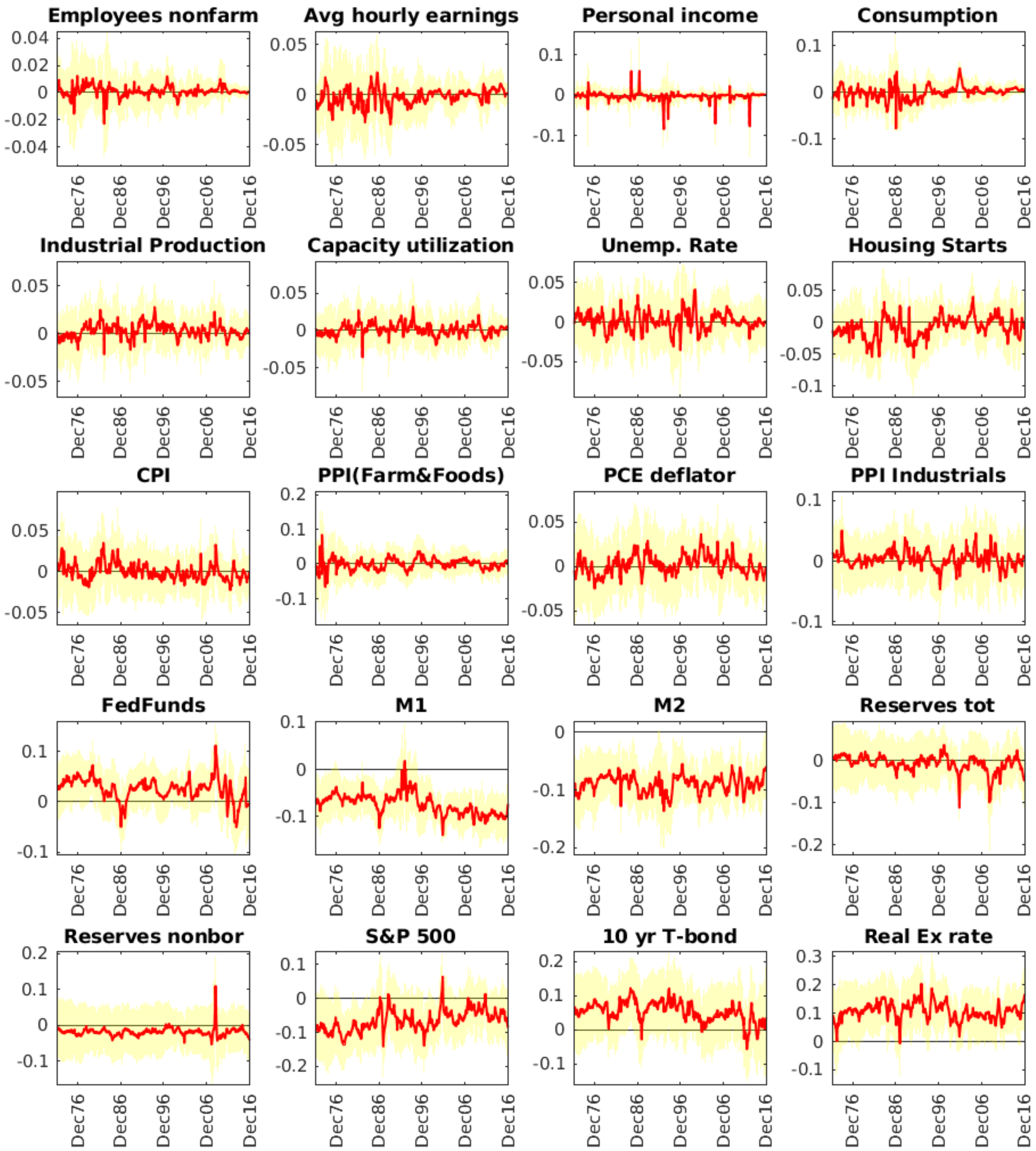

Figure 20: Response to a permanent Financial shock at horizon $h = 1$. Posterior bands.

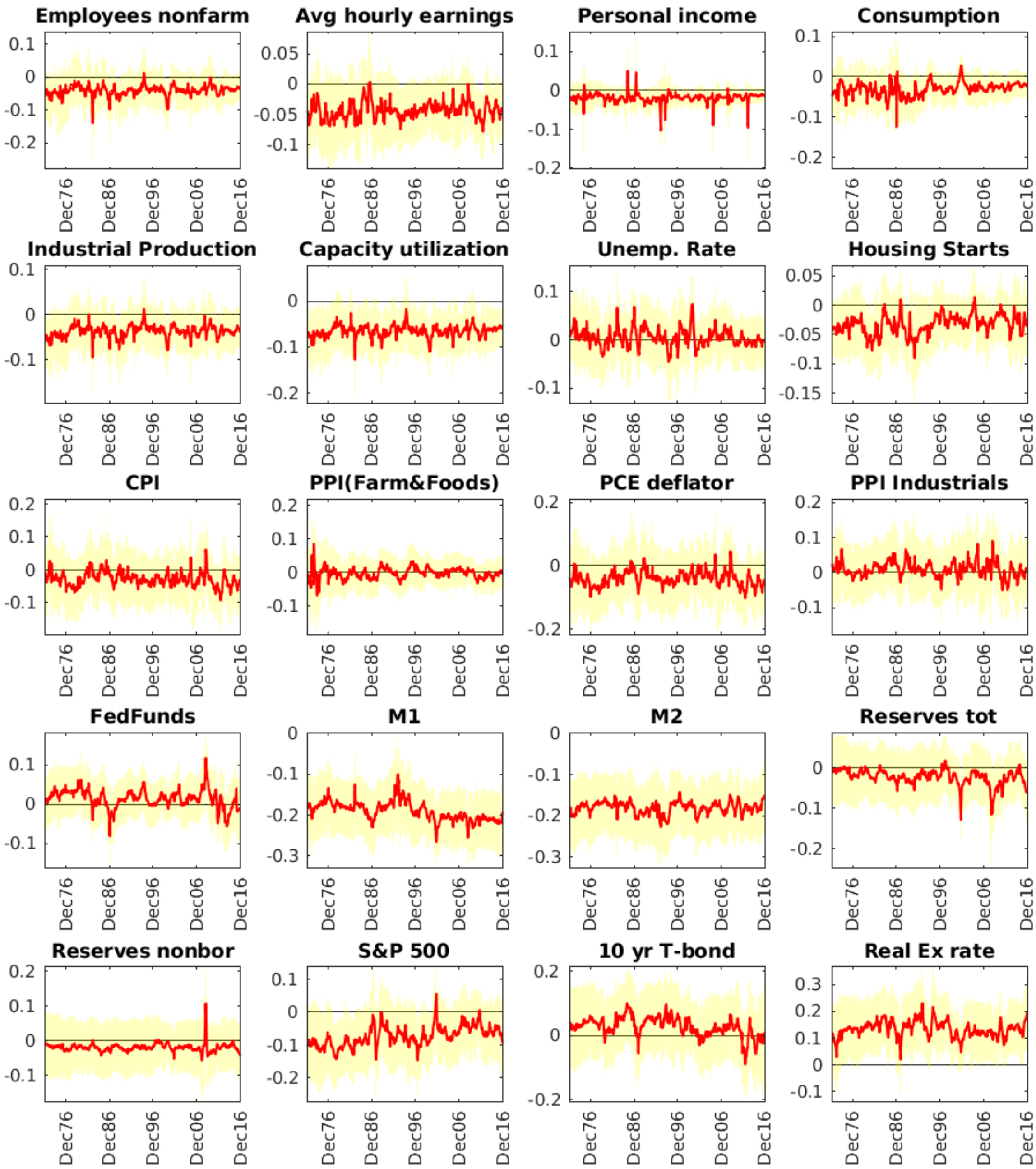

Figure 21: Response to a permanent Financial shock at horizon $h = 12$. Posterior bands.

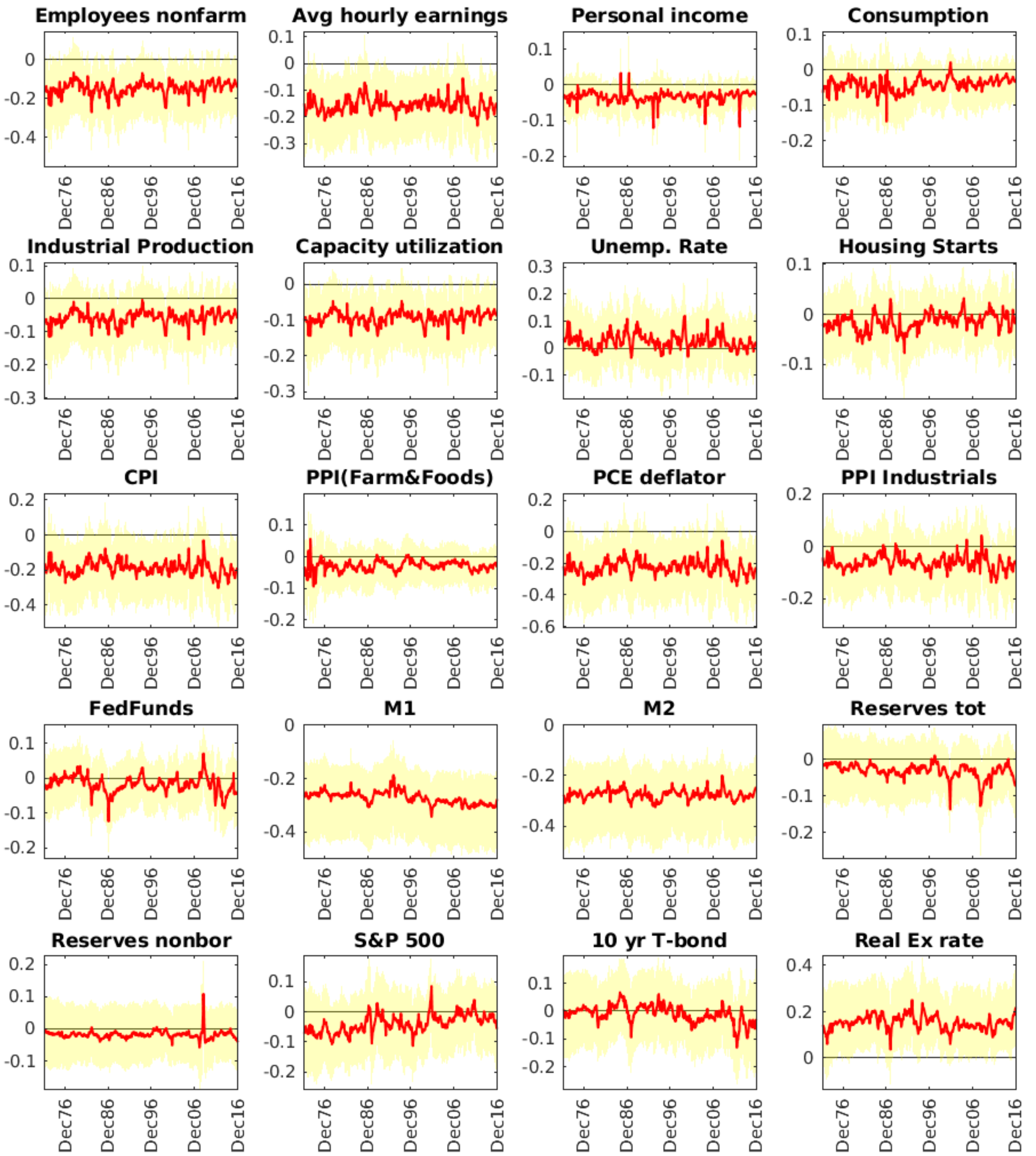

Figure 22: Response to a permanent Financial shock at horizon $h = 48$. Posterior bands.

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