Macroeconomics determinants of the correlation between stocks and bonds

by Marcello Pericoli
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MACROECONOMIC DETERMINANTS OF THE CORRELATION BETWEEN STOCKS AND BONDS

by Marcello Pericoli*

Abstract

We analyze the correlation between the stock and bond markets in Germany and the US. We use a standard no-arbitrage affine model to decompose the correlation between these two assets into its main drivers. The correlation between bond yields and stock returns is a key determinant of asset allocation. Our results show that the correlation is primarily influenced by the uncertainty about inflation and real interest rates as well as by co-movement between inflation, real interest rates and dividend growth. Shocks to inflation, real interest rates and dividend growth can explain the correlation’s temporary deviation from its long-term dynamics.

JEL Classification: C32, E43, G12.
Keywords: bond market, stock market, macroeconomic shocks, money illusion.

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1 Introduction

The correlation between stock and bond prices is a key variable in choosing portfolio allocation, but it is also a driver of long-term government bond prices and of their term premia. Thus, detecting its main determinants is key for both investors and policymakers. In advanced economies, the correlation, which was positive until the end of the 1990s, changed sign at the turn of the century; since the late 1990s stock and bond prices have been moving in opposite directions. If we focus on the last ten years, we see that unconventional monetary policies introduced by the US and euro-area central banks have partially influenced this correlation by pushing long-term bond prices to positive levels and have weakened the negative link between the two assets. On the one hand, a negative correlation implies that bonds can hedge stock portfolio when the economy is in a bad state and this increases the room for portfolio immunization. On the other hand, an increased hedging demand for bonds can explain the extremely low level of bond term premia we have observed since around 2005.

In this paper we analyze the correlation between stock and bond markets in Germany and the US from 1990 to 2017. We borrow from the modern macro-finance literature a standard no-arbitrage affine model to decompose the correlation between these two assets into its main drivers. The model suggests that the correlation between bond and stock returns can be decomposed into the uncertainty about inflation and real interest rates, the covariance between inflation, real interest rates and dividend yields – a proxy of consumption growth. We test empirically this decomposition by regressing a time-varying measure of the stock/bond correlation on variables, which approximates the measures of uncertainty mentioned above.

This paper rests on a long stream of economic and financial literature on bond and stock determinants and on their interlinkages. The literature agrees that long-term bond prices are determined by the inflation rate, real interest rates, the term premium, and the corresponding shocks to inflation and real interest rates. Stocks share with bonds two of their factors (namely inflation and real interest rates) and two of their shocks (namely to inflation and real interest rates) but they are also affected by distributed earnings (i.e. dividends) and by shocks to them. In general, for a given level of inflation, a negative shock to the real interest rate will push bond and stock prices higher, but the former will also be influenced by dividend shocks, which may increase or offset the initial shock. Conversely, inflation impacts bonds negatively, since their payoff is in nominal terms, and stocks positively or neutrally.

*The views expressed are those of the author and do not necessarily represent those of the Bank of Italy. I would like to thank Nicola Borri, Francesco Corsello, Filippo Natoli and seminar participants at the Banca d'Italia workshop 'Financial factors in the context of economic recovery'. Of course, all errors are my own. Email address: marcello.pericoli@bancaditalia.it*
since dividends and stock payoffs may change with respect to variations in inflation.

One of the main debates in financial economics is the contribution of the co-movement between inflation and (dividend) consumption growth to the correlation between bonds and stocks. The literature separates the period up to the late 1990s from the subsequent period; in the first period, the cycle in advanced economies was characterized by supply shocks, which induced a countercyclical response of inflation to growth. Conversely, since 2000, the cycle has been characterized by demand shocks, both monetary and fiscal, and thus inflation has behaved in a procyclical manner, showing a positive correlation with output growth. Recently, some authors have also proposed a role for the degree of aggressiveness of monetary policy in influencing the correlation between bonds and stocks; according to Burkhardt and Hasseltoft (2012), Campbell et al. (2015) and Song (2017), the inflation targeting of the Federal Reserve since the chairmanship of Greenspan has partially influenced the joint dynamics of bonds and stocks.

Similarly, the co-movement between inflation and real interest rates is debated because, theoretically, real assets should be priced by real and not nominal discount factors. However, empirically, we have observed some correlation between these two factors and this may be partly explained by agents’ money illusion. A link between inflation and real interest rates may also be due to unexpected inflation when the central bank follows a Taylor rule as a monetary policy function.

Results show that the stock/bond correlation is caused primarily by uncertainty about expected inflation, in the short- and long-term, and real interest rates, by the covariance between inflation, dividends and real interest rates, and by the equity risk premium. The results are relevant to investors since forecasting the stock/bond correlation using macroeconomic factors helps to improve their asset allocation decisions; from the investor’s point of view, the new regime, which we have observed since just before the turn of the century, is extremely beneficial as diversification opportunities are available, making portfolio immunization more effective compared with the 1980s and the 1990s.

The paper is structured as follows. Section 2 presents some stylized facts about the correlation between bond and stock markets. Section 3 presents a brief survey of the literature on empirical and theoretical models that jointly price bonds and stocks. Section 4 introduces a very stylized affine model for bonds and stock prices and Section 5 presents the estimate. Section 6 concludes.
2 Facts

Four stylized facts can be presented to describe the dynamics of the stock/bond correlation (simply ‘correlation’ hereafter). First, the correlation changes sign from the end of the 1990s; since then investors have come to regard government bonds as hedges, assets that perform well when other assets lose value, and more generally when bad macroeconomic news arrives. Moreover, during both of the two most recent global recessions, in 2001 and 2007-09 (the global financial and the euro-area debt crises), government bonds performed well. In addition, since the turn of the century and particularly during these downturns, government bond returns have been negatively correlated with stock returns. In previous decades, however, government bonds performed very differently; they were either uncorrelated or positively correlated with stock returns. Figure 1 presents the time-varying correlations for the US, the UK, Germany, France and Italy since data availability. The correlation recorded positive figures until the late 1990s and dropped into negative territory just before 2000. Only in Italy has the correlation remained negative for few years (2002-09) thanks to the widening of the premium requested by investors to hold Italian government bonds.

Figure 1: bond/stock correlation – The Figure reports the correlation between returns on stock indices and bond indices for the US, Germany, France and Italy. We use monthly end-of-period data. Correlation is computed by means of an exponential weighted moving average with decay factor equal to 0.96.
Recently, the correlation has been affected by the unconventional monetary policies in the US and in the euro area. From 2009 to 2013, the Large Scale Asset Purchase Programme (LSAP) of the Federal Reserve caused a sharp increase in the correlation in the US; the purchase of long-term government bonds provoked an increase in bond prices which has been matched by a stock market recovery. Similarly in 2015 the start of the ECB’s Extended Asset Purchase Programme (EAPP) led to an increase in euro-area government bond prices, along with a recovery of the euro-area stock markets.

Second, inflation has played a role in driving the correlation; this result contrasts with that presented by Duffee (2017). We show this by evaluating the relative contribution of inflation to the correlation, computing the correlation and the covariance between returns on stocks and index-linked government bonds, securities whose payoff is computed net of realized inflation (see Figure 2). We also report the correlation coefficients for the UK markets, even though they are not considered in the empirical section, because UK index-linked bonds have been available since 1985 and their correlation with the stock market returns resembles very closely that of the US; data for index-linked bonds are in general available not before 2003 for the euro area and Germany and only from 1998 for the US. In the UK and the US the standard and real correlation coefficients and the covariances moved together, even with a spread between the two, until the global financial crisis in 2008-09 and moved in opposite directions thereafter; the stock/real-bond correlation and covariance was negative from 2000 to 2008 and positive thereafter, while the standard correlation and covariance has remained negative since 2008. The divergence between the two coefficients starts with the inception of the unconventional monetary policies in the two countries. In Germany, the stock/real-bond correlation has been always larger than the standard stock/nominal-bond correlation: close to zero just before the global financial crisis and positive since 2009; the spread can be partly explained by the fact that we jointly use French and German index-linked government bonds to compute the returns on German index-linked bonds. In general, in all three countries the two correlation coefficients (covariances) co-move even with a spread until the period 2008-17, when fears of low inflation or even deflation cause a decoupling. In general, this evidence documents that inflation plays a role in determining the dynamics of the standard correlation and covariance.

Third, the change in the sign of the stock/bond correlation has been a global phenomenon. In fact, Figure 1 shows that the correlation coefficients in the main advanced economies have been moving very closely, the only exception being Italy. In general, this result is in line with the large correlation between global government bond and equity markets.

Fourth, economic variables and their interlinkages play a pivotal role in driving the cor-
Figure 2: stock/bond and stock/real-bond correlation and covariance – The Figure reports the correlation coefficients (top panels) and the covariances (bottom panels) between returns on stock indices and index-linked government bond indices, and returns on stock indices and standard government bond indices for Germany, the UK and the US. The index-linked bonds are German Bunds for Germany, Treasury Inflation Protected Securities (TIPS) for the US, and index-linked Gilts for the UK. We use monthly end-of-period data. Correlation and covariance are computed by means of an exponential weighted moving average with decay factor equal to 0.96.

relation between stocks and government bonds. We can see that the change in the sign of the correlation coincides with a change in the sign of the correlation between real interest rates, inflation and growth. Table 1 reports the average correlation coefficients between the three main drivers of the stock/bond correlation in the periods 1990-99 and 2000-17. The correlation between dividends (approximated by the rate of growth of GDP) and real interest rates is negative in the two periods in Germany and the US. Conversely, the correlation coefficients dividend/inflation and real-interest-rate/inflation change considerably. The dividend/inflation relation moves from negative (countercyclical) to positive (procyclical) values in Germany and the US, confirming the changing dynamics of inflation over the two periods. Analogously, the real-interest-rate/inflation relation becomes negative in the second period,
from positive figures; since 2000 an increase in inflation causes a decrease in the real interest rate as the nominal interest rate does not increase like inflation.

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<tr>
<td>US</td>
<td>0.26</td>
<td>-0.20</td>
<td>-0.10</td>
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<td>Germany</td>
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<td>0.15</td>
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The Table reports the correlation coefficients between the 12-month change in the 5-year real interest rate, $r$, the monthly one-year ahead inflation surveyed by Consensus Forecasts, $\pi$, the monthly one-year ahead rate of growth of real GDP surveyed by Consensus Forecasts, $d$, over the samples shown in columns.

3 Literature

Jointly pricing stocks and bonds is a non-trivial task. Several papers have proposed different solutions. To address correlations from an econometric perspective the early literature used the Constant Correlation GARCH of Bollerslev et al. (1988), the BEKK GARCH of Engle and Kroner (1995), and the Dynamic Conditional Correlation (DCC) GARCH of Engle and Sheppard (2001). Focusing on stock and bond returns, Guidolin and Timmermann (2007) introduce multiple regimes to allow for a state-dependent co-movement structure between these asset classes. While financial econometrics has developed powerful and effective tools to analyze, describe, and predict the correlation of individual and aggregate financial series, this line of research has added little to the economic understanding of the factors driving the correlation. Thus, some authors have proposed using empirical proxies for the structural determinants of risk premia suggested by these models. Recently Asgharian et al. (2016) used a DCC model to introduce macro-finance factors and found that their forecasts predicted stock/bond correlation and correlation tends to be small and negative when the economy is weak, supporting the flight-to-quality phenomenon.

Three related branches of research explore time-varying correlations between bonds and stocks from a general equilibrium asset price perspective. One follows Mamaysky (2002b), Mamaysky (2002a), Li (2003), and d’Addona and Kind (2006) by searching for plausible conditioning information. What macroeconomic and financial variables observed at $t$ predict
the correlation at \( t+1 \)? Baele et al. (2010), David and Veronesi (2013) and David and Veronesi (2016) illustrate another branch that attempts to explain empirically the conditional second moments in Figure 1 with conditional second moments of plausible fundamentals such as output, inflation and liquidity. The third branch, proposed by Bekaert and Grenadier (1999), Campbell et al. (2017) and Duffee (2017), interprets the determinants of the stock/bond correlation introducing standard asset-pricing frameworks. Affine pricing models have clear economic interpretations, but closed-form solutions for stock price can only be derived in some special cases. In fact, a pitfall of these models is that factors for pricing jointly stocks and bonds are usually made of unobserved latent factors, which still lack some economic interpretation.

A key mechanism in much of this research is time-variation in the conditional correlation between expected inflation and expected aggregate cash flows to equity (Burkhardt and Hasseltoft, 2012; Connolly et al., 2005; Song, 2017). Macroeconomic dynamics swing from periods of countercyclical expected inflation – stagflation – to periods of procyclical expected inflation. During stagflation, good (bad) news about future cash flows tends to be accompanied by news of lower (higher) expected inflation. Thus when stock prices rise (fall), bond prices rise (fall). Such a pattern is consistent with what happened in the 1970s through the late 1990s in Figure 1. Procyclical shocks to expected inflation generate the opposite correlation, such as in the Great Recession. Some authors have also proposed a role for the degree of aggressiveness of monetary policy in influencing the correlation between bonds and stocks; according to Burkhardt and Hasseltoft (2012), Campbell et al. (2015), Song (2017), the inflation targeting of the Federal Reserve under Chairman Greenspan partially influenced the joint dynamics of bonds and stocks.

In the financial industry and in the central bank literature the topic has been examined by asset managers (PIMCO, 2013, 2018), the financial press and also by central banks (Rankin and Shah Idil, 2014).

4 Model

We borrow the model from the mainstream macro-finance literature (Li, 2003; Mamaysky, 2002b,a; d’Addona and Kind, 2006). In order to stress the role of changes in the factors and of their covariances we depart somewhat from the standard model by assuming a single factor for the short term rate – as opposed to the standard three-factor model – and a constant market price of risk. This feature allows us to consider the covariances between the factors.
as drivers of the correlation; however, extensions to models with several factors are clearly possible.

The short-term real interest rate, \( r_{t+1} \), inflation, \( \pi_{t+1} \), and the logarithm of the dividend yield, \( \delta = \ln(1 + D/P) \), follow affine mean reverting processes

\[
\begin{align*}
    r_{t+1} &= \bar{r} + k_r(r_t - \bar{r}) + \sigma_r \varepsilon_{r,t+1}, \\
    \pi_{t+1} &= \bar{\pi} + k_\pi(\pi_t - \bar{\pi}) + \sigma_\pi \varepsilon_{\pi,t+1} = \hat{\pi} + \sigma_\pi \varepsilon_{\pi,t+1}, \\
    \delta_{t+1} &= \bar{\delta} + k_\delta(\delta_t - \bar{\delta}) + \sigma_\delta \varepsilon_{\delta,t+1},
\end{align*}
\]

where \( \bar{r}, \bar{\pi}, \bar{\delta} \) are the long-run equilibrium levels, \( \varepsilon_{r,t+1}, \varepsilon_{\pi,t+1}, \varepsilon_{\delta,t+1} \) the corresponding shocks, \( k_r, k_\pi, k_\delta \) the speeds of adjustment, \( \hat{\pi} \) expected inflation; we assume that \( \Sigma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \), \( \varepsilon_{t+1} = [\varepsilon_r, \varepsilon_\pi, \varepsilon_\delta]_{t+1} \sim N(0, \rho) \), with \( \rho = E_t(\varepsilon_t \varepsilon_t') = \begin{bmatrix} 1 & \rho_{r\pi} & \rho_{r\delta} \\ \rho_{r\pi} & 1 & \rho_{\pi\delta} \\ \rho_{r\delta} & \rho_{\pi\delta} & 1 \end{bmatrix} \). It can be shown that every affine dividend yield process corresponds uniquely to a non-linear dividend growth process, and vice versa.

The real pricing kernel, \( M_t \), is a positive stochastic process such that all assets \( i \) are priced by \( 1 = E_t[(1 + R_{i,t+1})M_{t+1}] \), where \( R_{i,t+1} \) is the percentage real return of asset \( i \) and \( E_t \) the conditional expectation operator. The existence of a pricing kernel is ensured in any arbitrage-free economy; Harrison and Kreps (1979) derive the condition for the uniqueness of \( M_t \). The logarithm of the real pricing kernel is defined by

\[
m_{t+1} = \ln(M_{t+1}) = -\mu_m - r_t + \lambda_r \sigma_r \varepsilon_{r,t+1} + \lambda_\pi \sigma_\pi \varepsilon_{\pi,t+1} + \lambda_\delta \sigma_\delta \varepsilon_{\delta,t+1} = -\mu_m - r_t + \lambda' \Sigma \varepsilon_{t+1},
\]

where \( \lambda = [\lambda_r, \lambda_\pi, \lambda_\delta]' \). In what follows we are agnostic about \( \lambda \): it can be time varying and dependent on the model factors, i.e. \( \lambda_t = \lambda_0 + \lambda_1 \varepsilon_{r,t}, \pi_t, \delta_t \)' as in Lemke and Werner (2009), or be constant, i.e. \( \lambda_1 = 0 \). The distribution of \( \varepsilon \) allows us to consider the covariance between shocks to real interest rates, inflation and dividend yields. Our model generalizes d’Addona and Kind (2006) who assume that \( \lambda_t = 0, \rho_{\pi\delta} = \rho_{r\delta} = 0 \). Hasseltoft (2012) and Campbell et al. (2017) introduce an additional state variable, \( \psi_{t+1} = \bar{\psi} + k_\psi(\psi_t - \bar{\psi}) + \sigma_\psi \varepsilon_{\psi,t+1} \), in

\[
\begin{bmatrix} k_r & 0 & 0 \\ 0 & k_\pi & 0 \\ 0 & 0 & k_\delta \end{bmatrix}, \quad \text{and the market price of risk } \lambda_t = \lambda_0 + \lambda_1 X_t'.
\]

\footnote{This corresponds to the general affine specification \( X_{t+1} = X + K(X_t - X) + \Sigma \varepsilon_{t+1}, \text{ where } X_{t+1} = [r_{t+1}, \pi_{t+1}, \delta_{t+1}]', K = \begin{bmatrix} k_r & 0 & 0 \\ 0 & k_\pi & 0 \\ 0 & 0 & k_\delta \end{bmatrix}, \text{ and the market price of risk } \lambda_t = \lambda_0 + \lambda_1 X_t'.}
equations (1.3) that multiplies the shocks, namely \( r_{t+1} = \bar{r} + k_r(r_t - \bar{r}) + \psi_t \varepsilon_t r_{t+1} + \varepsilon_{t+1}, \)
\( \pi_{t+1} = \bar{\pi} + k_\pi(\pi_t - \bar{\pi}) + \psi_\pi \varepsilon_t \pi_{t+1} \) and \( \delta_{t+1} = \bar{\delta} + k_\delta(\delta_t - \bar{\delta}) + \psi_\delta \varepsilon_{\delta,t+1}; \) this is equivalent to assuming stochastic volatility processes for real interest rates, inflation and dividend yields; the variable \( \psi_t \) enters bond prices quadratically. Without loss of generality, we may assume that the covariance matrix, \( \Sigma, \) or the correlation matrix, \( \rho, \) are affine function of some variables \( X, \) i.e. \( \Sigma_t = \sqrt{\Sigma_0} + \Sigma_1 X_t \) as in Bekaert and Grenadier (1999).

Following an usual assumption in this literature we assume that consumption – or the rate of growth of GDP – and dividends share the same data generating process. In formal notation this translates into \( \delta_{t+1} = \phi c_{t+1}, \) where \( \phi > 1 \) captures the idea that equity is leveraged. In the empirical application this implies that we can alternatively use information about the rate of growth of dividends or consumption and GDP.

We assume the existence of no arbitrage. Since the real interest rate is the return on one-period real bonds, the pricing kernel must satisfy the following non-arbitrage condition

\[
    r_t \equiv -\ln(E_t(M_{t+1})) = \mu_m + r_t - \frac{1}{2} \lambda' \Sigma \rho \Sigma' \lambda
\]

\[
    \mu_m \equiv \frac{1}{2} \lambda' \Sigma \rho \Sigma' \lambda
\]

### 4.1 Bond and stock prices

By simple passages, shown in Appendix A, nominal returns of a \( n \)-period bond are given by:

\[
    B_{t+1}^{n-1} = \begin{bmatrix} r_t & \hat{\pi}_t & -\frac{1}{2} \lambda^* \Sigma' \rho \Sigma \lambda^* \end{bmatrix} + \begin{bmatrix} A_{n-1}^r \sigma_r \varepsilon_{r,t+1} + A_{n-1}^\pi \sigma_\pi \varepsilon_{\pi,t+1} \end{bmatrix},
\]

where \( \lambda^* = \lambda + [A_{n-1}^r, (A_{n-1}^\pi - 1), 0]' \), \( A_{n-1}^r = -\frac{1}{1-k_r}, A_{n-1}^\pi = -\frac{k_\pi(1-k_{\pi}^{-1})}{1-k_\pi} \).

Nominal returns on stock prices are given by

\[
    S_{t+1} = \begin{bmatrix} r_t & \hat{\pi}_t & -\frac{1}{2} \lambda' \Sigma' \rho \Sigma \lambda \end{bmatrix} + \begin{bmatrix} a^r \sigma_r \varepsilon_{r,t+1} + \sigma_\pi \varepsilon_{\pi,t+1} + (a^d + 1) \sigma_\delta \varepsilon_{\delta,t+1} \end{bmatrix},
\]

where \( \lambda = \lambda + [a^r, 0, (a^d + 1)]', a^r = -\frac{1}{1-k_r}, a^d = -\frac{k_\delta}{1-k_\delta} \).
The upper part of (4) indicates that the expected bond return is the sum of the real interest rate, expected inflation and term premium. The lower part of (4) indicates how bond returns respond to interest rates and unexpected inflation shocks. Under normal conditions $|k_r| < 1$ and $|k_π| < 1$, we have $A^r_{n-1} < 0$ and $A^π_{n-1} < 0$, which means positive shocks to unexpected inflation and real interest rates cause bond returns to fall. Both $A^r_{n-1}$ and $A^π_{n-1}$ are increasing functions of maturity $n$ in absolute value, indicating that bonds with longer maturity are more vulnerable to these shocks.

The upper part of (5) shows the expected stock returns. Expected stock returns share two components with expected bond returns: the real interest rate and expected inflation. The lower part of (5) shows that unexpected stock returns are subject not only to shocks to inflation and real interest rates but also to shocks to dividend yields. Under normal conditions, we have $a^δ > 0$ and $a^r < 0$; which means that positive dividend shocks raise stock returns, and positive interest rate shocks reduce stock returns. Unexpected shocks of the price level also raise stock returns.

### 4.2 Conditional covariance and correlation

From equations (4-5) we compute the conditional covariance between returns on bond and stock prices $B_{t+1}^{n-1}$ and $S_{t+1}$. This is given by

$$
cov_t(B_{t+1}^{n-1}, S_{t+1}) = A^r_{n-1} a^r \cdot \sigma^2_r + A^π_{n-1} \cdot \sigma^2_π + A^π_{n-1} (a^δ + 1) \cdot \rho_δ \sigma_δ \sigma_π + [A^r_{n-1} a^r + A^r_{n-1}] \cdot \rho_r \sigma_r \sigma_π + A^r_{n-1} (a^δ + 1) \cdot \rho_δ \sigma_δ \sigma_r
$$

(6)

The first row shows that, since $A^r_{n-1} a^r \cdot \sigma^2_r > 0$, higher uncertainty about the real interest rate tends to increase the co-movement of stock and bond returns. This is intuitive because the real interest rate determines how an investor discounts stock and bond cash flows. Therefore, interest rate shocks are likely to move stock and bond prices in the same direction. The
second row, $A_{n-1}^\pi \cdot \sigma_\pi^2$, summarizes the effect of unexpected inflation on the co-movement through the nominal channel; it unambiguously reduces the stock/bond co-movement since investors prefer stocks to bonds as the payoffs of the former are hedged against inflation shocks while bond returns are negatively impacted by inflation growth.

The rows from the third to the fourth represent the interaction between the three factors; we name them the cash flow channel, $A_{n-1}^\pi (a^\delta + 1) \cdot \rho_\delta \sigma_\delta \sigma_\pi$, the discount factor channel, $[A_{n-1}^r a^r + A_{n-1}^r] \cdot \rho_{r\pi} \sigma_r \sigma_\pi$, and the portfolio rebalancing channel, $A_{n-1}^r (a^\delta + 1) \cdot \rho_\delta \sigma_\delta \sigma_r$. The effects of the cash flow and the discount factor channels are ambiguous and depend on parameter values. They are also among the most debated topics in finance and macroeconomics. If the economy is neutral to unexpected inflation shocks (i.e. $\rho_\delta \sigma_\delta \sigma_\pi = \rho_{r\pi} \sigma_r \sigma_\pi = 0$) then we expect the stock/bond correlation to decrease with higher uncertainty about unexpected inflation. Otherwise, the effect of unexpected inflation shocks cannot be determined. In general, the cash-flow channel has been debated as the main driver of the correlation; it relates to the macro-finance literature on supply and demand shocks and the corresponding countercyclical and pro-cyclical behavior of inflation. The discount factor channel relates to the existence of investors illused by money who use the nominal discount factor to discount real quantities, while rational (not-illuded) agents should use the real discount factor; in the latter case the covariance between the real interest rate and inflation may not be negligible.

Finally, the effect of the portfolio rebalancing channel, $A_{n-1}^r (a^\delta + 1) \cdot \rho_\delta \sigma_\delta \sigma_r$, depends on the covariance between the dividend yield and the real interest rate; in general we expect that this term approximates the relative performance of stocks with respect to bonds in real terms and that it is negative (positive) during expansions (recessions).

The stylized model (4-5) shows that expected inflation, real interest rates and their shocks are common drivers of stock and bond returns in the same direction, while dividend shocks are unique to stock returns. This outcome is consistent with the empirical work of Fama and French (1993) who show that the pricing factors for bonds are only a subset of those for stocks. In summary, this model points to the uncertainty of three macroeconomic factors and their interlinkages as the explanatory factors of the stock/bond correlation. Greater uncertainty about expected inflation and the real interest rate increases this correlation. The effect of unexpected inflation is ambiguous and depends on whether the dividend yield and the real interest rate are affected by unexpected inflation shocks. In addition, uncertainty about the stock unique component reduces the stock/bond correlation by changing the volatility of stock returns.

As a last point we should observe that the covariance between bond and stock returns
is different from the correlation, the latter also being influenced by the volatility of the two returns. As shown by Pericoli and Sbracia (2003), changes in correlation may be driven not only by an increase in covariance but also by a variation in the variances of stock and bond returns. However, Figure 2 shows that differences between correlation and covariance are not particularly significant in our sample since the volatility of bond returns is usually low while that of stocks increased only temporarily in 1987, 2000 and 2008-09, keeping the dynamics of correlation and covariance very similar. For this reason, in the empirical Section (5) we test equation (6) by regressing a measure of time-varying correlation on a set of variables that approximate uncertainties and covariances of macroeconomic determinants; we substitute correlation with covariance in the robustness checks.

From equation (4-5) we compute the variances of bond and stock returns as

$$Var_t(B_{t+1}^{n-1}) = (A_{n-1}^r \cdot \sigma_r)^2 + (A_{n-1}^\pi \cdot \sigma_\pi)^2 + 2A_{n-1}^r A_{n-1}^\pi \sigma_r \sigma_\pi \rho_r\pi$$
$$Var_t(S_{t+1}) = (a^r \cdot \sigma_r)^2 + \sigma_\pi^2 + (a^\delta + 1)^2 \sigma_\delta^2 + 2a^r \sigma_r \sigma_\pi \rho_r\pi + 2a^\delta (a^\delta + 1) \sigma_r \sigma_\pi \rho_\pi + 2(a^\delta + 1) \sigma_\delta \sigma_\pi \rho_\delta \pi.$$ (7)

Following Mamaysky (2002a) and d’Addona and Kind (2006), we assume that dividend shocks can be decomposed into the sum of three components:

$$\varepsilon_{t+1} = \rho_{\delta r} \varepsilon_{r,t+1} + \rho_{\delta \pi} \varepsilon_{\pi,t+1} + \sigma_u \varepsilon_{u,t+1},$$

with $$E_t(\varepsilon_{u,t+1}\varepsilon_{\pi,t+1}) = E_t(\varepsilon_{u,t+1}\varepsilon_{r,t+1}) = 0.$$ $$\varepsilon_{u,t+1}$$ is the stock unique component. The correlation between stock and bond returns is equal to

$$Corr_t(B_{t+1}^{n-1}S_{t+1}) = \frac{Cov_t(B_{t+1}^{n-1}S_{t+1})}{\sqrt{Var_t(B_{t+1}^{n-1})Var_t(S_{t+1})}}.$$ 

Greater uncertainty about the stock unique component, $$\sigma_u$$, say an increase in uncertainty about earnings, increases $$Var_t(S_{t+1})$$ without affecting $$Var_t(B_{t+1}^{n-1})$$ and $$Cov_t(B_{t+1}^{n-1}S_{t+1})$$ and mechanically reduces the stock/bond correlation.

4.3 Simulation

We simulate the change in the stock/bond correlation depending on the variation of the correlation coefficients between the three factors. In what follows we mainly focus on the center-top and center-bottom charts of Figure 3, where the remaining correlation coefficient is
set to zero. We see that the stock/bond correlation decreases as the relationship between the dividend yield and the real interest rate goes from negative to positive; a better performance of stocks with respect to bonds produces a portfolio rebalancing towards the former and pushes the correlation towards negative values. Analogously, the stock/bond correlation decreases as inflation tends to move in synchronicity with the dividend yield, i.e. as the inflation passes from being countercyclical to procyclical (center-top panel); this result is confirmed by the three bottom panels where the stock/bond correlation is always positive for countercyclical inflation ($\rho_{\delta\pi} = -1$, left-bottom panel), almost always positive for inflation independent from dividend yield ($\rho_{\delta\pi} = 0$, center-bottom panel) and negative for procyclical inflation ($\rho_{\delta\pi} = 1$, right-bottom panel) for larger values of the correlation, $\rho_{\delta r}$, between the dividend-yield and the real interest rate. Finally, a positive correlation between the real interest rate and inflation ($\rho_{rr} = 1$, right-top panel) almost always gives a positive stock/bond correlation; when this correlation becomes negative ($\rho_{rr} = -1$, left-top panel), the link between the real interest rate and inflation becomes inverted.

Figure 3: Stock/bond correlation sensitivity to parameters. The Figure reports the simulation of the correlation coefficient between returns on stocks and bonds for changing values of the correlation between inflation, dividend yield and real interest rate. We set long-term values and standard deviations equal to their sample values and $\lambda = [7, 7, 7]'$. The charts imply a term premium on the 10-year bond equal to 1.44 and an equity premium equal to 4.17.
4.4 Equity premium

The model (4.5) also gives the conditional risk premium, i.e. the difference between the expected stock return and the short-term interest rate. For 1 period we have \( rp_{t+1} = E_t(S_{t+1}) - r_t - \pi_t \), while for \( n \) periods \( rp_{t+n} = E_t(S_{t+n}) - r_t - \pi_t \). By solving the equation forward (see Appendix) we have

\[
(rp_{t+n}) = (1 - k_{t,n}^{n-1})\pi + (1 - k_{t,n}^{n-1})\pi - \frac{1}{2} [a^r, 0, a^\delta + 1] \Sigma \rho \Sigma' [a^r, 0, a^\delta + 1] + (1 - k_{t,n}^{n})r_t - (1 - k_{t,n}^{n})\pi_t \tag{8}
\]

The equity premium is driven by the current and past values of the factors. Equation (8) shows that the equity premium is increasing in the forecast horizon. Moreover, increases in short term real rates and inflation decrease the equity premium and widen the gap from its long term average.

5 Results

The simple theoretical model of Section 4 is built upon the extreme assumptions of affine state variables and homoskedastic shocks. We derive its implications using comparative statics. As a result, this model offers no direct guide as to how its implications can be tested. In this section, we use a formulation to examine empirically the link between the stock/bond correlation and the uncertainty about macroeconomic factors suggested by the model.

5.1 Data

We use end-of-month returns on government-bond and stock market indices available from Thomson Reuters Datastream and Bloomberg from January 1990 to December 2017, for a total of 336 observations. Data for the US are available from 1961, but we limit our analysis to the period 1990-2017, the sample common to continental Europe. Inflation rates, industrial production indices and 3-month interest rates are from Thomson Reuters Datastream. The correlation is computed as the exponential moving average of the first differences of the logarithm of the stock and bond indices, with a decay equal to 0.96.

The most challenging part of the selection of variables is the choice of the uncertainty and covariance measures. In fact, financial variables generally have a larger variability than macroeconomic variables and apparently the two sets may seem unrelated. See the Appendix.
for a detailed list of potential variables that proxy uncertainty.

For uncertainty about inflation we use a proxy for short-term uncertainty and a proxy for long-term uncertainty and mean reversion. For the first we use the cross-sectional standard deviation of one-year-forward expected inflation surveyed by Consensus Forecast (Figure 4, panel A); for the second, we use the second principal component score of the term structure of expected inflation in one, two, three, four, and five-years’ time of Consensus Forecasts, which, like the bond-yield curve, approximates the slope of the term structure and the speed of the mean reversion to the long-term average, Figure 5.

For uncertainty about interest rates we use the difference between the 3-month and the 10-year interest rates disagreement between participants in the monthly Consensus Forecast survey, Figure 6 panel E, whereas [Duffee (2017)] uses the difference between the option-implied volatility on 3-month and 10-year interest rates.

For covariance between inflation, real interest rates and dividend yield we use the covariance obtained from an exponential weighted moving average with decay equal to 0.96 of the first differences of one-year-forward expected inflation, 10-year real interest rates and the rate of growth of GDP surveyed by Consensus Forecast. The covariance between stock and bond returns is also explained by the equity risk premium, computed as the inverse of the one-year-forward price-earnings ratio less the one-year-forward 10-year real interest rate (one-year-forward nominal 10-year interest rate less one-year-forward inflation), like [Blanchard and Gagnon (2016)].

Additional indicators of global uncertainty are given by the stock market volatility index (e.g. VIX for the US and VStoxx for Germany) and the Economic Policy Uncertainty index by [Baker et al. (2016)]. Some authors also use the level of inflation and of the real interest rate as a proxy of their uncertainty, under the assumption that their uncertainty is positively linked to their levels.

5.2 Regressions

We run White’s adjusted heteroskedastic consistent Least-squares Regression of the Fisher transformation of the correlation coefficient between stock and bond returns on a set of variables that proxy the uncertainties and their co-movements presented in equation (6) for the US and for Germany. Our regression uses the Fisher transformation of the correlation coefficient, $corr$, that maps the domain $[-1, +1]$ into the domain $[-\infty, +\infty]$, defined by

$$y_F = \frac{1}{2} \ln \left(\frac{1 + corr}{1 - corr}\right).$$
Thus, our base regression is given by

\[ y_F = \alpha + X'\beta + \Omega u , \]  

where \( u \sim N(0, I) \), \( X \) is a vector of regressors and \( (\alpha, \beta, \Omega) \) parameters to be estimated. Results are shown in columns (1-8) of Table (2).

### 5.3 Empirical results

We regress our measure of realized correlation on a set of regressors. First, we start with the proxies for short- and long-term inflation uncertainty, which, according to the theoretical model, should be negative. Second, we add, one by one, measures of interest-rate uncertainty, covariance between inflation, dividend and real interest rate. Finally, we add a measure of the earning/price ratio, \( ep \), and insert a dummy variable for the inception of the unconventional monetary policies in the euro area and in the US, indicated by \( QE \).

Results for Germany are shown in the upper part of Table (2): the volatility of inflation has the expected negative impact on the correlation: short-term, \( \sigma(\pi)ST \), and long-term, \( \sigma(\pi)LT \), inflation uncertainties have negative and significant loadings even when other variables are added. Moreover, the uncertainty about the interest rate has the expected positive sign; its loading decreases in size when the earning/price, \( ep \), and the unconventional monetary policy dummy, \( QE \), are considered. This implies that the correlation moves more than one-to-one with the gap between uncertainty about short-term and long-term interest rates. If we interpret lower uncertainty about long-term interest rates as a proxy of higher GDP growth, we can interpret the relative contribution of lower long-term interest rate uncertainty as a clear indication of higher growth and, for a given inflation level, an increase in both stock and bond prices. Considering only uncertainties about inflation and interest rates, the explained variance, measured by the \( R^2 \), stands at 27%. When we add the three covariances, \( \sigma(r, \pi), \sigma(r, d), \sigma(d, r) \), between inflation, dividends and real interest rates we observe an increase in explained variances; all covariances are very significant and negatively influence the correlation. Finally, the forward earning/price ratio helps to explain a large fraction of the correlation, with the \( R^2 \) rising to 70%. The equity premium, \( ep \), has a very negative and significant impact on the correlation in the two markets; in general, equity risk premiums reflect the degree of risk aversion of investors between equities and bonds: an increase in the equity premium usually heralds a decrease in equity markets, typically matched by an increase in bond yields. The unconventional monetary policy dummy, \( QE \), is also significant.
and contributes to the increase in the correlation between stock and bond returns.

Results for the US are shown in the lower part of Table [2]: differently from Germany, the short-term inflation uncertainty has a positive impact while the long-term inflation uncertainty has the expected and significant negative sign. The interest-rate uncertainty, $\sigma(r)$, measured by the difference between short- and long-term disagreements, positively affects the correlation even if the size of the coefficient is considerably lower than that for Germany and changes sign when the forward earning/price ratio is added. In general, uncertainties about inflation and interest rate explain a much smaller fraction of the variation in correlation – only 13%. When we add the three covariances, $\sigma(r, \pi), \sigma(r, d), \sigma(d, r)$, we see that the main contribution comes from the covariance between inflation and real interest rates with a positive coefficient, while those on the covariances inflation/growth and real-interest-rate/growth are negative but much less significant; in total, the three covariances explain up to 44% of the total variance (not shown). Similarly to Germany, the forward earning/price ratio is very significant and contributes to the total variance – the $R^2$ goes to 63%; the impact of the unconventional monetary policy, $QE$, is similar to the German case.

Comparing results for Germany and the US we note that i) uncertainty about short-term inflation, $\sigma(\pi)ST$, has the expected negative sign in Germany while it has a positive loading in the US that changes significance with the introduction of additional regressors; this difference between the two areas may be due to the greater persistence of short-term inflation in Germany with respect to the US; ii) however, the uncertainty about long-term inflation, $\sigma(\pi)LT$, is negative and similar in the two economic areas; iii) uncertainty about interest rates, $\sigma(r)$, which proxies the uncertainty channel, has a stable loading in Germany while it is more variable in the US since it becomes negative when the forward earning/price ratio is added; we ascribe this result for the US to the role of risk aversion, measured by the earning/price ratio, which may partly compensate for the uncertainty about long-term interest rates; iv) the role played by the covariance between inflation and real interest rate, $\sigma(\pi, r)$, is very different in the two countries: negative in Germany and positive in the US. If we interpret this variable as a proxy of money illusion we may argue that it seems significant in the US; we leave this finding to further research.

5.4 Robustness checks

We run a battery of robustness checks. First, we compute several alternative measures of the correlation coefficients and the covariances between stock and bond returns: we use 20-day moving averages and the DCC GARCH and BEEK GARCH models. Second, we use different
variables for uncertainties about inflation, interest rates and covariances among factors – a list of these variables can be found in Appendix B. Last, we use current historical variables, i.e. industrial production, leading indicators, consumer and business confidence, in place of their expectations. In general, results are not very dissimilar to those presented in Table (2).

Regressions are run using correlations and not covariances. Thus, we run the same regressions adding the volatility of stocks and bonds; they are not particularly informative.

We also consider state-dependent coefficients during expansions and recessions. This is a natural extension of the baseline model, specified by pre-multiplying each regressor by a dummy, $I^+$, and its complement, $I^- = (1 - I^+)$, that takes the value 1 during economic

<table>
<thead>
<tr>
<th>Table 2: Main regression</th>
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</thead>
<tbody>
<tr>
<td>(1)</td>
</tr>
<tr>
<td>Germany</td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$\sigma(\pi)ST$</td>
</tr>
<tr>
<td>$\sigma(\pi)LT$</td>
</tr>
<tr>
<td>$\sigma(r)$</td>
</tr>
<tr>
<td>$\sigma(\pi,d)$</td>
</tr>
<tr>
<td>$\sigma(r,d)$</td>
</tr>
<tr>
<td>$ep$</td>
</tr>
<tr>
<td>$QE$</td>
</tr>
<tr>
<td>$R^2$</td>
</tr>
<tr>
<td>US</td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$\sigma(\pi)ST$</td>
</tr>
<tr>
<td>$\sigma(\pi)LT$</td>
</tr>
<tr>
<td>$\sigma(r)$</td>
</tr>
<tr>
<td>$\sigma(\pi,d)$</td>
</tr>
<tr>
<td>$\sigma(r,d)$</td>
</tr>
<tr>
<td>$ep$</td>
</tr>
<tr>
<td>$QE$</td>
</tr>
<tr>
<td>$R^2$</td>
</tr>
</tbody>
</table>

The Table reports estimates of the White’s adjusted heteroskedastic consistent Least-squares Regression of the Fisher transformation of the correlation coefficient between government-bond and stock returns and the regressors indicated in the left column. In parenthesis the t-statistics. $\sigma(\pi)ST$ is the analysts’ disagreement on one-year ahead inflation forecast surveyed by Consensus Forecasts; $\sigma(\pi)LT$ is the second principal component score of inflation expectations one, two, three, four, and five-years ahead; it measures the slope of the term structure of inflation expectations which approximates the speed of mean reversion of inflation towards its long-term average; $\sigma(r)$ is the difference between the 3-month and the 10-year interest rate analysts’ disagreement surveyed by Consensus Forecasts; $\sigma(\pi,d)$, $\sigma(\pi,d)$, $\sigma(\pi,d)$ are the covariances between one-year-forward expected inflation, GDP rate of growth and 10-year real interest rate – surveyed by Consensus Forecasts – computed with an exponential moving average with decay equal to 0.96; $ep$ is the inverse of the price earning ratio less the 5-year real interest rate; $QE$ is a dummy variable that takes unity value from March 2009 until June 2003 for the US and from September 2014 until December 2017 for Germany.
expansions. Results are shown in Table (3). The state-dependent regression shows no changes for Germany; conversely, for the US we find a large and positive short-term nominal channel, $\sigma(\pi) ST$, a negative cash-flow channel during recessions, $\sigma(\pi, d)$, while the portfolio channel loses significance both in expansion and recession. All in all, state-dependent results are not very different from those of the main regression.

Equation 6 states that $A^\pi, A^r, a^r, a^\delta$ are functions of expected inflation, expected real interest rates and expected dividends; we then add these variables in the regressions and interact them with their standard deviations.

As an additional test, we run the exercise for France and Italy. As shown in Figure (1), France looks very similar to Germany, while for Italy the correlation coefficient becomes positive again in 2010 owing to a deterioration in sovereign credit quality during the euro-area debt crisis. Thus, a worsening of the sovereign credit risk is intuitively similar to an increase in inflation since it does not allow investors to hedge their stock-portfolio exposure with government bonds.

6 Conclusions

In this paper we analyze the correlation between stock and bond markets in Germany and the US from 1990 to 2017. We borrow from the modern macro-finance literature a standard no-arbitrage affine model to decompose the correlation between these two assets into its main drivers. The model suggests that the correlation between bond yields and stock returns can be decomposed into the uncertainty about inflation and real interest rates, the covariance between inflation, real interest rates and dividend yields – a proxy of consumption growth. We test empirically this decomposition by regressing a time-varying measure of the stock/bond correlation on variables that approximates the measures of uncertainty depicted above.

Results show that the stock/bond correlation is determined primarily by uncertainty about expected inflation, in the short- and long-term, and real interest rates, by the covariance between inflation, dividends and real interest rates, and by equity risk premia. Results support the role of inflation across the business cycle in determining the sign of the stock/bond correlation. A key mechanism in much of this research is time-variation in the conditional correlation between expected inflation and expected aggregate cash flows to equity. Macroeconomic dynamics swing from periods of countercyclical expected inflation – stagflation – to periods of procyclical expected inflation. During stagflation, good (bad) news about future cash flows tends to be accompanied by news of lower (higher) expected inflation.
The Table reports estimates of the White’s adjusted heteroskedastic consistent Least-squares Regression of the Fisher transformation of the correlation coefficient between government-bond and stock returns and the regressors indicated in the left column. In parenthesis the t-statistics. With respect to equation 9, we run the regression

\[ y_F = \alpha + I^+ X' \beta + I^- X' \beta + \Omega u \]

where \( I^+ \) is equal to 1 during expansion, nil otherwise, \( I^- \) equal to 1 during recession, nil otherwise; the expansion and recession phases are computed by means of a Hodrick-Prescott filter and are shown by + and − in the Table. \( \sigma(\pi)ST \) is the analysts’ disagreement on one-year ahead inflation forecast surveyed by Consensus Forecasts; \( \sigma(\pi)LT \) is the second principal component score of inflation expectations 1-, 2-, 3-, 4-, and 5-years ahead; it measures the slope of the term structure of inflation expectations which approximates the speed of mean reversion of inflation towards its long-term average; \( \sigma(r) \) is the difference between the 3-month and the 10-year interest rate analysts’ disagreement surveyed by Consensus Forecasts; \( \sigma(\pi, d) \), \( \sigma(\pi, d) \), \( \sigma(\pi, d) \), \( \sigma(\pi, d) \) are the covariances between one-year-forward expected inflation, GDP rate of growth and 10-year real interest rate – surveyed by Consensus Forecasts – computed with an exponential moving average with decay equal to 0.96; \( ep \) is the inverse of the price earning ratio less the 5-year real interest rate; \( QE \) is a dummy variable that takes unity value from March 2009 until June 2003 for the US and from September 2014 until December 2017 for Germany.

<table>
<thead>
<tr>
<th>Cycle</th>
<th>Germany</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>α</td>
<td>0.07 (0.43)</td>
<td>0.68 (8.65)</td>
</tr>
<tr>
<td>( \sigma(\pi)ST )</td>
<td>+ -0.45 (-2.05)</td>
<td>1.67 (8.17)</td>
</tr>
<tr>
<td></td>
<td>- -1.34 (-4.98)</td>
<td>1.59 (10.20)</td>
</tr>
<tr>
<td>( \sigma(\pi)LT )</td>
<td>+ -0.11 (-4.42)</td>
<td>-0.16 (-5.06)</td>
</tr>
<tr>
<td></td>
<td>- -0.00 (-0.18)</td>
<td>-0.08 (-2.74)</td>
</tr>
<tr>
<td>( \sigma(r) )</td>
<td>+ 0.65 (5.06)</td>
<td>-0.13 (-0.85)</td>
</tr>
<tr>
<td></td>
<td>- 0.89 (6.18)</td>
<td>-0.20 (-1.21)</td>
</tr>
<tr>
<td>( \sigma(\pi, d) )</td>
<td>+ -0.55 (-2.56)</td>
<td>-0.02 (-0.67)</td>
</tr>
<tr>
<td></td>
<td>- 0.76 (2.90)</td>
<td>-0.31 (-9.83)</td>
</tr>
<tr>
<td>( \sigma(\pi, r) )</td>
<td>+ -0.46 (-5.78)</td>
<td>0.36 (2.95)</td>
</tr>
<tr>
<td></td>
<td>- -0.63 (-7.80)</td>
<td>0.42 (3.79)</td>
</tr>
<tr>
<td>( \sigma(r, d) )</td>
<td>+ -0.87 (-3.83)</td>
<td>0.43 (1.95)</td>
</tr>
<tr>
<td></td>
<td>- -0.14 (-0.59)</td>
<td>0.29 (1.95)</td>
</tr>
<tr>
<td>( ep )</td>
<td>+ -0.10 (-12.75)</td>
<td>-0.10 (-8.76)</td>
</tr>
<tr>
<td></td>
<td>- -0.07 (-12.45)</td>
<td>-0.08 (-5.84)</td>
</tr>
<tr>
<td>( QE )</td>
<td>+ 0.40 (7.32)</td>
<td>0.23 (3.02)</td>
</tr>
<tr>
<td></td>
<td>- 0.35 (6.45)</td>
<td>0.13 (2.04)</td>
</tr>
</tbody>
</table>

\( \sum^2 \) 0.78 0.64
Thus when stock prices rise (fall), bond prices rise (fall). Such a pattern is consistent with what happened from the 1970s to the late 1990s. Pro-cyclical shocks to expected inflation generate the opposite correlation, such as in the Great Recession.

The results are important for investors since forecasting this stock/bond correlation using macroeconomic factors helps to improve investors’ asset allocation decisions; from the investor’s point of view, the new regime, observed just before the turn of the century, is extremely beneficial since diversification opportunities are available, making portfolio immunization more effective with respect to the 1980s and the 1990s. Although central banks do not have specific price targets for bonds or stocks, they are increasingly using the information contained in the prices of these assets to gauge market participants’ growth and inflation expectations. Hence, stock/bond return correlation estimates may offer policymakers useful complementary information to determine whether markets are changing their views on inflation or economic activity prospects.

References


### Appendix – pricing

#### A.1 Bonds

Following the literature on affine term structure models we denote the nominal bond price with maturity $n$ with

$$ P_{B,t}^n = \exp \left( A_0^n + A_r^n r_t + A^\pi_r^n \pi_t \right). $$

(10)
The logarithm of the pricing kernel for nominal asset is $m_{t+1} - \pi_{t+1}$ and the bond price satisfies

$$P^n_{B,t} = E_t \left[ \exp (m_{t+1} - \pi_{t+1}) P^{n-1}_{B,t+1} \right]$$

$$= E_t \left[ \exp \left( -\mu_m - r_t + \lambda' \sum \xi_{t+1} - \pi_{t+1} + A^0_{0} - 1 + A^0_{t} r_{t+1} + A^{n-1}_{\pi} \pi_{t+1} \right) \right]$$

$$= E_t \left[ \exp \left( -\mu_m - r_t + \lambda' \sum \xi_{t+1} + A^0_{0} - 1 + A^0_{r} r_{t+1} + (A^{n-1}_{\pi} - 1) \pi_{t+1} \right) \right]$$

$$= E_t \left[ \exp \left( -\mu_m - r_t + \lambda' \sum \xi_{t+1} + A^0_{0} - 1 \\
+ A^0_{r} (\tau + k_r(r_t - \tau) + \sigma_r \xi_{t+1}) \\
+ (A^{n-1}_{\pi} - 1)(\pi + k_\pi(\pi_t - \pi) + \sigma_\pi \xi_{t+1}) \right) \right]$$

$$= E_t \left[ \exp \left( -\mu_m + \lambda' \sum \xi_{t+1} + A^0_{0} - 1 \\
+ A^0_{r} (1 - k_r) \tau + (A^{n-1}_{r} - 1) k_r - 1) r_{t} + A^{n-1}_{\pi} \sigma_r \xi_{t+1} \\
+ (A^{n-1}_{\pi} - 1)(1 - k_\pi) \pi + (A^{n-1}_{\pi} - 1) k_\pi \pi_t + (A^{n-1}_{\pi} - 1) \sigma_\pi \xi_{t+1} \right) \right]$$

$$= \exp \left( A^0_{0} + A^0_{r} r_{t} + A^{n}_{\pi} \pi_t \right),$$

since $r_{t+1} = \tau + k_r(r_t - \tau) + \sigma_r \xi_{t+1}$ and $\pi_{t+1} = \pi + k_\pi(\pi_t - \pi) + \sigma_\pi \xi_{t+1}$. Define $\lambda^* = \lambda + \left[ A^{n-1}_{r}, (A^{n-1}_{\pi} - 1), 0 \right]$ and we have

$$= E_t \left[ \exp \left( -\mu_m + A^0_{0} - 1 \\
+ A^0_{r} (1 - k_r) \tau + (A^{n-1}_{r} - 1) k_r - 1) r_{t} \\
+ (A^{n-1}_{\pi} - 1)(1 - k_\pi) \pi + (A^{n-1}_{\pi} - 1) k_\pi \pi_t \right) \right]$$

$$= E_t \left[ \exp \left( -\mu_m + A^0_{0} - 1 \\
+ A^0_{r} (1 - k_r) \tau + (A^{n-1}_{r} - 1) k_r - 1) r_{t} \\
+ (A^{n-1}_{\pi} - 1)(1 - k_\pi) \pi + (A^{n-1}_{\pi} - 1) k_\pi \pi_t \right) \right]$$

Since $E_t(a + bx) = a + bE_t(x) + b^2 Var_t(x)$ if $x$ is lognormal. Equating 10 and 11 we have that

$$A^0_{0} = -\mu_m + A^0_{0} - 1 + \lambda' \sum \rho \Sigma \lambda^* + A^{n-1}_{r} (1 - k_r) \tau + (A^{n-1}_{\pi} - 1)(1 - k_\pi) \pi$$

$$A^0_{r} = (A^{n-1}_{r} - 1) = -\frac{1 - k^{n}_{r}}{1 - k^{r}_{r}}$$

$$A^{n}_{\pi} = (A^{n-1}_{\pi} - 1) k_\pi = -\frac{1 - k^{n}_{\pi}}{1 - k^{\pi}_{\pi}}$$
The return is

\[ B_{t+1}^{n-1} = \ln P_{B,t+1}^{n-1} - \ln P_{B,t}^{n} \]

\[ = (A_{0}^{n-1} - A_{0}^{n}) + (A_{r}^{n-1}r_{t+1} - A_{r}^{n}r_{t}) + (A_{\pi}^{n-1}\pi_{t+1} - A_{\pi}^{n}\pi_{t}) \]

\[ = r_{t} + \bar{\pi}_{t} + \mu_{m} - \lambda^{*}\Sigma\rho\Sigma'\lambda^{*} - \frac{1 - k_{r}^{n-1}}{1 - k_{r}}\sigma_{r}\varepsilon_{r,t+1} - \frac{1 - k_{\pi}^{n-1}}{1 - k_{\pi}}k_{\pi}\sigma_{\pi}\varepsilon_{\pi,t+1} . \]

The unexpected bond return is

\[ (E_{t+1} - E_{t})B_{t+1}^{n-1} = -\frac{1 - k_{r}^{n-1}}{1 - k_{r}}\sigma_{r}\varepsilon_{r,t+1} - \frac{1 - k_{\pi}^{n-1}}{1 - k_{\pi}}k_{\pi}\sigma_{\pi}\varepsilon_{\pi,t+1} . \]

### A.2 Stocks

We denote the real price of a stock that pays dividends in \( n \) periods

\[ P_{S,t}^{n} = \exp (a_{0}^{n} + a_{r}^{n}r_{t} + a_{\delta}^{n}\delta_{t}) . \quad (12) \]

If we assume a transversality condition for \( 12 \) we can extend equation for \( n \to \infty \) and obtain

\[ \lim_{n \to \infty} P_{S,t}^{n} = \lim_{n \to \infty} \left[ \exp (a_{0}^{n} + a_{r}^{n}r_{t} + a_{\delta}^{n}\delta_{t}) \right] = P_{S,t} . \]

We use a real pricing kernel since stocks pay real dividends. The real stock price satisfies

\[ P_{S,t} = E_{t} \left[ M_{t+1} (P_{S,t+1} + D_{t+1}) \right] \]

\[ = E_{t} \left[ M_{t+1}P_{S,t+1} \left( 1 + \frac{D_{t+1}}{P_{S,t+1}} \right) \right] \]

\[ = \lim_{n \to \infty} E_{t} \left[ \exp \left( m_{t+1} + a_{0}^{n-1} + a_{r}^{n-1}r_{t+1} + a_{\delta}^{n-1}\delta_{t+1} + \delta_{t+1} \right) \right] \]

\[ = \lim_{n \to \infty} \exp \left( \frac{1}{2}\lambda' \Sigma \rho \Sigma \lambda^{*} - \mu_{m} + a_{0}^{n-1} \right) \]

\[ + a_{r}^{n-1}(1 - k_{r})\bar{F} + (a_{\delta}^{n-1} + 1)(1 - k_{\delta})\bar{D} \]

\[ + (a_{r}^{n-1}k_{r} - 1)r_{t} + (a_{\delta}^{n-1} + 1)k_{\delta}\delta_{t} \]

\[ = \exp (a_{0} + a_{r}r_{t} + a_{\delta}\delta_{t}) . \]
where $\bar{\lambda} = \lambda + [a_n^{n-1}, 0, (a_\delta^{n-1} + 1)]'$. Equating [12] and [13] we have

$$a_0^n = \frac{1}{2} \bar{\lambda} \Sigma \rho \Sigma' \bar{\lambda} - \mu_m + a_0^{n-1} + a_r^{n-1}(1 - k_r)\bar{\pi} + (a_\delta^{n-1} + 1)(1 - k_\delta)\bar{\pi},$$

$$a_r^n = (a_r^{n-1}k_r - 1) \Rightarrow a_r = \lim_{n \to \infty} a_r^n = -\frac{1}{1 - k_r},$$

$$a_\delta^n = (a_\delta^{n-1} + 1)k_\delta \Rightarrow a_\delta = \lim_{n \to \infty} a_\delta^n = \frac{k_\delta}{1 - k_\delta}.$$  

The real return on the stock is $S_{t+1} = \ln \left( \frac{P_{S,t+1} + D_{t+1}}{P_{S,t}} \right) = \ln \left( \frac{P_{S,t+1}(1 + D_{t+1})}{P_{S,t}} \right)$ and, given our definition of $\delta_{t+1} = \ln \left( 1 + \frac{D_{t+1}}{P_{S,t}} \right)$, we obtain the nominal stock return by adding inflation, namely

$$S_{t+1} = \ln(P_{S,t+1} \exp(\delta_{t+1})) - \ln P_{S,t} + \pi_{t+1} = (a_0^{n-1} - a_0^n) + a_r(r_{t+1} - r_t) + a_\delta(\delta_{t+1} - \delta_t) + \delta_{t+1} + \pi_{t+1} = r_t + \pi_t + \mu_m - \frac{1}{2} \bar{\lambda} \Sigma \rho \Sigma' \bar{\lambda} - \frac{1}{1 - k_r} \sigma_r \pi_{r,t+1} + \sigma_\pi \pi_{x,t+1} + \frac{1}{1 - k_\delta} \sigma_\delta \pi_{\delta,t+1}.$$  

The expected nominal stock return is

$$E_t(S_{t+n}) = E_t(r_{t+n}) + E_t(\pi_{t+n}) + \frac{1}{2} [a_r, 0, a_\delta + 1]' \Sigma \rho \Sigma' [a_r, 0, a_\delta + 1] = (1 - k_r^{n-1})\bar{\pi} + (1 - k_\pi^{n-1})\bar{\pi} + k_r^{n}r_t + k_\pi^{n}\pi_t + \frac{1}{2} [a_r, 0, a_\delta + 1]' \Sigma \rho \Sigma' [a_r, 0, a_\delta + 1] = (1 - k_r^{n-1})\bar{\pi} + (1 - k_\pi^{n-1})\bar{\pi} + k_r^{n}r_t + k_\pi^{n}\pi_t + \frac{1}{2} [a_r, 0, a_\delta + 1]' \Sigma \rho \Sigma' [a_r, 0, a_\delta + 1].$$

The realized stock return is

$$S_{t+1} - E_t(S_{t+1}) = -\frac{1}{1 - k_r} \sigma_r \pi_{r,t+1} + \sigma_\pi \pi_{x,t+1} + \frac{1}{1 - k_\delta} \sigma_\delta \pi_{\delta,t+1}.$$  

This representation is equivalent to that presented in [Campbell et al. (2017)] for the pricing of equities. We see that expected stock returns increase when there are negative shocks to the real interest rate and positive shock to inflation and dividend yields.

**B Appendix – data for the robustness checks**

For uncertainty about inflation we consider the following variables: i) cross-sectional standard deviation of inflation rate expected in one, two, three, four, and five-years’ time by
participants in the monthly Consensus Forecast survey (a proxy for long-term disagreement and uncertainty; Figure 4, panel B); ii) the second principal component score of the term structure of expected inflation in one, two, three, four, and five-years’ time of Consensus Forecast, which, analogously to the bond-yield curve, approximates the slope of the term structure and the speed of the mean reversion to the long-term average, Figure 5; iii) yearly variation of inflation computed as the yearly change with respect to a time-varying average: \( SD_t(\pi) = \sqrt{\sum_{j=0}^{11} (\pi_{t-j} - \frac{1}{12}\sum_{j=0}^{11}\pi_{t-j})^2} \), Figure 4, panel C; iv) volatility obtained from a GARCH(1,1) model, Figure 4, panel D; v) volatility obtained from an exponential weighted moving average with decay equal to 0.96; vi) residuals from a 1-lag Bayesian VAR with inflation, industrial-production and 3-month interest rate (unexpected inflation shock), Figure 4, panel E.

For uncertainty about interest rates we consider the following variables: i) cross-sectional standard deviation of the 3-month and 10-year interest rates expected in one-year’s time by participants in the monthly Consensus Forecast survey (a proxy for short-term disagreement and uncertainty), Figure 6, panels A-B; ii) volatility obtained from a GARCH(1,1) model, Figure 6, panel C; iii) volatility obtained from an exponential weighted moving average with decay equal to 0.96; iv) residuals from a 1-lag Bayesian VAR with inflation, industrial-production and 3-month interest rate (unexpected interest rate shock), Figure 6, panel D; v) the difference between the 3-month and the 10-year interest rates disagreement among participants in the monthly Consensus Forecast survey, Figure 6 panel E, whereas Duffee (2017) uses the difference between the option-implied volatility on 3-month and 10-year interest rates.

For covariance between inflation, real interest rates and dividend yield we consider the following variables: i) the covariance obtained from an exponential weighted moving average with decay equal to 0.96 of the first (and twelfth) differences of the variables: inflation, real interest rates and industrial production growth – see Figure 7 we also compute the correlation using the one-year-forward expectations surveyed by Consensus Forecast; ii) the covariance obtained from a multivariate GARCH and with a DCC GARCH model with first differences of inflation, real interest rates and industrial production growth; iii) the covariance between the real interest rate and dividend yield is approximated by the forward equity premium, computed as the inverse of the IBES 12-month forward price/earning less the real interest rate.

Additional indicators of global uncertainty are given by the stock market volatility index (e.g. VIX for the US and VStoxx for Germany), the Economic Policy Uncertainty index by Baker et al. (2016). Some authors also use the level of inflation and of the real interest rate
as a proxy for their uncertainty, under the assumption that their uncertainty is positively linked to their levels.

Figure 4: Uncertainty of inflation – The Figure reports measures of uncertainty for inflation. Panel A reports the yearly standard deviation; panel B the cross-sectional standard deviation of forecasts by Consensus Forecasts; panel C the volatility from a GARCH (1,1) model; panel D reports the residual for inflation estimated from a Bayesian VAR model.
Figure 5: PCA of expected inflation – The Figures report the coefficient of a Principal Component Analysis of expected inflation 1-, 2-, 3-, 4-, and 5-years ahead, surveyed by Consensus Forecast every six months. In parentheses, the percentage of variance explained by each factor.
Figure 6: Uncertainty of interest rates – The Figure reports measures of uncertainty for inflation. Panel A reports the Consensus Forecasts disagreement on one-year ahead 3-month interest rates; panel B reports the Consensus Forecasts disagreement on one-year ahead 10-year interest rates; panel C the volatility from a GARCH (1,1) model; panel D reports the residual for inflation estimated from a Bayesian VAR model; panel E the difference between the two series in panels A and B.
Figure 7: Correlation of factors – The Figure reports the correlation coefficients between inflation, $\pi$, industrial production growth, $\delta$, and real interest rate, $r$, for differences between period $t$ and period $t - 1$, m-o-m, and between period $t$ and period $t - 12$, y-o-y.
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