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by Flavia Corneli
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SOVEREIGN DEBT MATURITY STRUCTURE AND ITS COSTS
by Flavia Corneli*

Abstract

I propose a theoretical model of a debt contract between a sovereign and its international lenders that determines the optimal debt maturity structure and related costs. It is shaped by two financial frictions: limited liability (the country cannot guarantee that it will not dilute its obligations or default on them) and market incompleteness (only non-contingent assets can be issued). I find that, in equilibrium, debt dilution constrains the amount of long-term debt issuance. I then focus on two aspects that are currently widely debated in both academic and policy fora: the possibility of sovereign debt restructuring with private creditors and international official lending in the event of exclusion from the international capital markets. The possibility of restructuring after default stimulates long-term debt issuance. However, in equilibrium, the proposed crisis management tools are unable to loosen the constraint on long-term debt issuance. Consistently with the empirical literature, I find that even when these policy options for crisis resolution are available, the country tends to issue mainly short-term debt.

JEL Classification: E43, F33, G15, H63.
Keywords: sovereign debt, optimal maturity, strategic default, crisis management, lender of last resort.

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* Bank of Italy, Directorate General for Economics, Statistics and Research.
1 Introduction

Why do countries issue debt with different maturities? The literature analyzing emerging market economies’ decisions has provided both empirical and theoretical motivations to rationalize this behavior. Long-term debt reduces the need for debt roll-over, which could be problematic at times of liquidity shortages; therefore it can reduce the probability that a sudden stop in capital inflows could lead to a sovereign default (e.g. Rodrik and Velasco, 2003). Short-term debt is, however, considered to have a disciplinary role for the country (e.g. Jeanne, 2009, Arellano and Ramanarayanan, 2012) and this, in turn, reduces the risk premium paid on short-term debt with respect to long-term borrowing, especially during crises (Broner et al., 2013).

Several elements lay behind those explanations and need to be incorporated in a theoretical investigation of the optimal sovereign debt maturity structure and determine a trade-off between short- and long-term debt and the cost of debt at different maturities: (i) the possibility that a liquidity crisis may arise and lead to debt roll-over problems; (ii) a strictly positive probability of sovereign default; and (iii) the ability of capital markets to provide incentives or discipline to the sovereign through the premia set on the cost of debt.

The goal of the present work is to propose a framework to study the debt composition and the interest rate an emerging country pays, starting from the observations above. Then I make use of the proposed framework to analyze how the presence of mechanisms introduced to help counties dealing with debt burden in the event of liquidity and solvency crises can also condition ex-ante choices of all involved agents; I study whether these mechanisms help mitigate the trade-off between short- and long-term debt. In particular, I examine how the decisions on debt maturity structure and debt costs can be modified by the possibility of a partial repayment of international lenders after default (private debt restructuring, following Yue, 2010), and the intervention of an international financial institution (IFI) as lender of last resort (LOLR) in case a liquidity crisis excludes the country from international financial markets. I consider three different types of IFI intervention alternatively: the first option entails full repayment of private international lenders; the second, partial default on private debt through debt maturity extension; and the third full default on obligations held by private investors. The proposed mechanism reproduces in a very stylized way some key characteristics of the IMF lending framework approved in 2016. And in particular, the introduction of the possibility of maturity extension of private obligation instead on full default, in some specific circumstances. I show how any policy options available

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1The views expressed are those of the author and do not necessarily reflect those of the Bank of Italy.
at the moment of debt issuance (i.e. private debt restructuring and IFI intervention) and their interplay can change ex-ante country’s incentives and private investors’ decisions.

I propose a model that incorporates the elements mentioned, which I will then enrich to include the policy options of private debt restructuring and IFI intervention. In particular, in this static game there are a sovereign and its international lenders, all surviving for three-periods. The sovereign chooses between the issuance of short- and long-term debt and international lenders, with their decision on how much of those instruments to buy, determine the cost of the two types of debt. Moreover, two financial frictions shape the interaction between the country and its international lenders: limited liability (the country cannot commit not to dilute, Bolton and Jeanne, 2009, nor to default on its obligations, Aguiar and Amador, 2015) and market incompleteness (only non-contingent assets can be issued, Arellano, 2008). In the interim period, short-term debt issuance is not limited to roll-over and is only limited in equilibrium by the willingness of international lenders to buy those instruments.

Already in this basic model, a clear trade-off arises between short- and long-term obligations, which is manifested at the interim stage and from the point of view of international lenders it reads: more long-term bonds increase the possibility of debt dilution but reduce the need for debt roll-over. Here is where the disciplinary role of short-term debt arises. This trade-off in equilibrium translates into a limit on the amount of long-term bonds the country can issue, even in the absence of debt seniority (as instead hypothesized by Bolton and Jeanne, 2009) and with risk neutral agents (Broner et al, 2013, instead have risk-averse international investors).

I then introduce the crisis management tools mentioned above. First of all, the possibility of private debt restructuring provides that, when the country decides to default on its debt obligations it can partially repay its international lenders. From the point of view of the country, private debt restructuring is welfare improving because, consistently with empirical evidence (Cruces and Trebesch, 2013; Trebesch and Zabel, 2017; Asonuma and Trebesch 2016), a lower haircut on debt obligations is associated with better economic outcomes, or milder punishment.

Second, I introduce a third actor, an IFI that intervenes as a LOLR and lends whenever the country is excluded from international capital markets. As for the rest of the model, also in this case the action of the IFI is very stylized. However, in order to make it interesting for policy debate, I take same elements of the actual lending policy of the IMF. In particular, I consider that the IFI lends resources at concessional rates, below the market ones, and enjoys a preferred creditor status in case of country default. Importantly, IFI intervention does not eliminate the two financial frictions mentioned above. I consider three types of IFI intervention alternatively

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2I define debt dilution following Hatchondo et al. (2016): "Debt dilution refers to the reduction in the value of existing debt triggered by the issuance of new debt. Issuing new debt reduces the value of existing debt because it increases the probability of default".

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(i.e. the type is known at time zero, before lending takes place): (i) full repayment, i.e. the IFI provides enough resources to overcome the liquidity crisis and repay the debt matured at that time; (ii) restructuring, i.e. the country defaults on all private obligations and the IFI provides only the resources needed to face the crisis; and (iii) reprofiling (similar to the one introduced in 2016 in the IMF lending framework), i.e. the IFI provides only the resources needed to overcome the liquidity crisis, the country fails to repay short-term obligations in the interim period (maturity extension) and repays all obligations issued at time zero after the completion of the project, if it decides to do so.

Any proposed crisis management tool have an impact on country and international lenders’ decisions, taken ex-ante. When these tools are available, in equilibrium the overall cost of debt decreases, however this is not uniform as any intervention distort ex-ante price setting, compared to the basic framework. This is due to different forces at work. In particular, the possibility of partial repayment after default, when private debt restructuring is an option, makes long-term debt relatively cheaper and boosts debt dilution. The impact of IFI on the cost of debt depends on the type of intervention. With full repayment, short-term debt becomes cheaper due to the IFI’s repayment of those obligations and then its concessional rate; with reprofiling, instead, short-term debt cost raises due to the burden of maturity extension. Finally, the impact of IFI with restructuring is less distorting but more costly for the country than other IFI interventions due to higher but uniform haircut on debt burden. In equilibrium, however, none of these tools are able to lessen the trade-off between short- and long-term debt and in particular the impact of debt dilution. I find that the constraint on the amount of long-term debt still crucially shapes the debt maturity composition: the country issues mainly short-term debt, as also found in the data (Broner et al., 2013). Moreover, this result is in line with the empirical finding of Saravia (2013) that IMF lending can incentivize short-term debt issuance.

By construction, the IFI’s intervention is ex-ante always welfare improving, regardless of its type. Not surprisingly, full repayment is ex-ante preferred by the country from a welfare point of view but requires a larger IFI involvement in terms of resources. Reprofiling and restructuring are equivalent in terms of the resources the IFI provides to the country. Moreover, in equilibrium, the model cannot help discriminate between reprofiling and restructuring from a welfare point of view, for the country: while with restructuring long-term debt is more costly, in case of reprofiling the country needs to employ more resources to repay international lenders; in equilibrium those effects are ex-ante equivalent. This very simple setup is not suited to assess which type of IFI intervention should be finally chosen and in the spirit of the IMF lending framework the three interventions should be considered as complements rather than substitutes. However, since before the 2016 reform reprofiling was not included in the lending framework and those cases (debt deemed sustainable but not with high probability) would necessitate a debt restructuring, the proposed welfare analysis could be read as a comparison between pre- and
post-2016. The result of this very simple analysis warns against considering a priori reprofiling welfare improving. I need to introduce additional assumptions to this very simplified set-up, in order to be able to differentiate the welfare consequences of the two types of interventions. Finally here, with no asymmetries in agents’ information, moral hazard cannot arise; therefore uncertainty around the IFI intervention as a mean to discipline the country plays no role.

The paper is organized as follows. The next section review the received literature. Then I present the basic model. Section 4 introduces the possibility of private debt restructuring and in the following section an IFI is proposed, which can intervene in three different modalities. Section 6 combines private debt restructuring and IFI interventions. Section 7 discusses the role played by the main hypotheses of the model, the simplifying assumptions and their limitations. Finally, I conclude.

2 Literature Review

The contribution of the present work is to propose a very simple and tractable model of sovereign debt maturity that can be flexibly adapted to incorporate policy options for crisis management discussed in the actual debate. This basic framework is partially taken from Jeanne (2009). He also analyzes a liquidity crisis arising from debt-rollover problems. However, in his formulation the debt structure entails a unique bond that can be redeemed in the interim stage and no debt dilution, which shapes the results of the present analysis, with and without crisis management tools.

As mentioned above, debt dilution is a crucial assumption of the present model, it affect the agents’ decisions. Hatchondo et al. (2016) quantitatively show the importance of debt dilution and its impact on debt maturity: eliminating this possibility would lengthen the average maturity of sovereign debt, a result that my model is able to replicate. A close contribution is Bolton and Jeanne (2009) since they introduce debt dilution, debt restructuring and its impact on the ex-ante debt structure. They consider, however, the endogenous decision between renegotiable and non-renegotiable debt with the goal of studying debtors’ coordination and debt enforcement when international investors are segmented in the two categories. They moreover extend to the possibility of issuing short-term renegotiable debt that under some condition can lead to a first best result. In the present analysis, I show that, even with perfect information, short-term debt entails a cost since it exacerbate the impact of a liquidity crisis that can lead to country default. Fernandez and Martin (2014) as well offer a theoretical analysis of sovereign debt maturity with debt dilution. Differently from the present study, they assume that, even in the absence of an IFI, debt of different maturities have different seniority, which can be a very relevant topic as
proved by Chatterjee and Eyigungor (2015). Fernandez and Martin (2014) also introduce an IFI, which is not a LOLR and is closer to a voluntary Sovereign Debt Restructuring Mechanism (SDRM), since it doesn’t lend but enables the country to negotiate a reprofiling or restructuring with its lenders. I abstract instead from the analysis and possible impact of an SDRM.

Arellano and Ramanarayanan (2012) and Aguiar et al. (2016) quantitatively study the optimal maturity structure, focusing in particular on the role of short-term maturities in time of crisis. Optimal debt maturity structure has also been studied by looking at the interaction with optimal taxation, as in Debortoli et al. (2016), or more generally policy reforms the country can implement to affect economic outcomes, as in Muller et al. (2016). My analysis abstracts from considerations on the internal decisions of the country and concentrates on the perception of international lenders that decide weather to invest looking solely at the final probability of being repaid.

In line with the contributions just mentioned, I abstract from the currency composition of debt (as in Engel and Park, 2017). Also, I focus only on external debt, a particularly relevant issue for the financial stability of emerging markets. While domestic debt is quantitatively important (e.g. Bolton, 2016), its dynamics are very different, as well as the cost associated with default, as shown in Gennaioli et al. (2014), Forni and Pisani (2018) and Mallucci (2015).

The role of official lending and how it should be structured has been widely debated. In particular, the trade-off between unlimited resources versus moral hazard is still unresolved (e.g. Corsetti et al., 2006). I touch upon this trade-off, by assuming that debt dilution is not allowed in the case of official lending, but focus on different lending policies and their impact on ex-ante choices. This is instead in common with Corsetti et al. (2018). However, while they differentiate IFI’s lending in terms of spread and maturity, I look at lending policies with different private debt treatment. Dellas and Niepelt (2016) focus on sovereign borrowing characteristics of a country that can borrow both from official and private agents. This differs from the present analysis since I assume that the country can borrow from the IFI only when it is excluded from private markets after a negative liquidity shock. International lenders and the country always weakly prefer IFI intervention. Here, the type of IFI intervention can change agents’ incentives.

Richer analysis on risk premia associated with the cost of debts with different maturities can be delivered by including risk-averse lenders, as in Broner et al. (2013) and Asonuma and Joo (2018), who also find a link between creditors’ behavior and the length and outcome (in terms of haircut) of the renegotiation process. I leave this for future work. The finite time set-up clearly marks the cost of punishment for defaulting and how this can affect ex-ante debt contracts. Moving to an infinite horizon could allow, as in Di Casola and Sichlimiris (2017)

3For a presentation and discussion of SDRM and Redemption Funds see Cioffi et al., 2018, and Committeri and Tommasino (2018).
and Phan (2017), to highlight the reputational costs of sovereign default. Also, it could help to enrich the debt sustainability definition from the one based only on debt stock to a new one incorporating also a flow metric with special focus on changing financing needs, which has become central in the policy debate, as shown in Gabriele et al. (2017). Finally, I consider a small open economy in isolation. I therefore abstract from the consequences of a crisis in a systemic country, like in Damiani (2015), and from contagion effects of sovereign defaults or clusters, which are empirically a relevant phenomenon, as documented by Reinhart and Rogoff (2009).

3 Basic Model

I propose a tractable model that delivers an analytical solution. I consider a sovereign country (a small open economy) over three periods \( t = 0, 1, 2 \), and with access to foreign borrowing. At stage zero, the country has to finance a project with a fixed amount of resources \( K \) by issuing debt, and it decides on the amounts of short-term (one period, \( b_{s,0} \)) or long-term (two periods, \( b_l \)) bonds. International lenders set the cost of short-term debt, \( q_{s,0} \), and long-term debt, \( q_l \). At stage one, the country is hit by a liquidity shock, which can be positive (bringing extra-resources \( Y_1 \)) or negative (requiring the country to inject the resources needed to offset the shock \(-Y_1\)). After observing the shock, the country decides whether to default on all its debt obligations or to roll-over short-term debt \( b_{s,1} \) and inject additional resources, if needed. International lenders set the cost of the newly issued short-term debt \( q_{s,1} \). If the country does not default at stage one, it brings the production technology to completion. At stage two, the productivity shock is realized and, after observing this second shock, the sovereign can again choose whether to default on all its issued debt (selective default is not allowed) or to repay it and keep the remaining resources.

\[
\begin{array}{c|c|c}
\text{t = 0} & \text{t = 1} & \text{t = 2} \\
\hline
\text{Lending} & \text{Lending} & \text{Repayment} \\
\text{Game} & \text{Game} & \text{or default} \\
\text{qs,0, ql} & \text{qs,1} & \\
\text{Investment} & \text{Investment} & \\
\text{Game} & \text{Game} & \\
\text{bs,0, bl} & \text{bs,1 or default} & \\
\end{array}
\]

**Figure 1:** Timeline

Figure 1 illustrates the timing of the game and the choices of the country and its lenders while figure 2 shows all the possible branches of the game-tree. Our timing structure is close to the three-period formulation in Diamond and Dybvig (1983): at time zero the long-term

\footnote{Long-term debt and its cost have no time index since this type of debt can be issued only at time zero and repaid in the last period, \( t = 2 \).}
investment decisions are taken, in the interim period a shock can hit the economy and in the last period the production technology yields the final output.

I solve the model by backward induction and the equilibrium concept I employ is the Sub-game Perfect Nash Equilibrium (SPNE) in pure strategies. In what follows, I present in detail how each relevant node is modeled and the main ingredients of the game.

![Game-tree](image)

**Figure 2**: Game-tree

3.1 Model Set-up

3.1.1 Lending and Investment Games

I follow Bolton and Jeanne (2007) and Corneli and Tarantino (2016) in the definition of the lending game. There is a continuum of atomistic and risk-neutral international lenders (indexed by $i \in I$). The total mass of lenders is large, ensuring that perfect competition prevails and lenders do not extract any rent; also, lenders have unlimited access to funds at the riskless interest rate (that, for simplicity, is assumed to be zero).

The sovereign borrows from a subset of mass 1 of lenders $b_{s,0}$ and $b_t$ at stage zero with the goal of raising $K$, and $b_{s,1}$ the period after. Lenders move first by each simultaneously submitting a bid. The sovereign then decides which bid(s) to accept.

The lenders’ payoff is equal to the value of the repayments $b_{s,0}$, $b_{s,1}$ and $b_t$ discounted by the probability that the sovereign repays in full the acquired portion of debt. For each debt maturity and period $b_j$, at the bidding stage of the game each lender $i$ makes an offer on the rate of return, $r_j(i)$, that insures break even in expectation. That is, lender $i$ solves a problem of the following sort:
\[ b_j = b_j (1 + r_j(i)) \text{Prob\{The Country is Solvent\}} \]
\[ 1/(1 + r_j(i)) = \text{Prob\{The Country is Solvent\}} \]
\[ q_j(i) = \text{Prob\{The Country is Solvent\}}. \]

Thus, throughout the model I will denote each debt cost \( q_j(i) = 1/(1 + r_j(i)) \) the inverse of the rate of return \( r_j(i) \) that lender \( i \) asks in exchange for a loan \( b_j \). Accordingly, the Nash equilibrium of the lending game is defined by a set of bids that maximizes the lender \( i \)'s payoff taking all the other bids as given. At equilibrium the sovereign squeezes all the surplus from the lending relationship and (randomly) selects among a set of identical bids, so I can focus on a representative sovereign-lender pair.

The investment game takes place at stage zero: the country decides on the debt composition, \( b_{s,0} \) and \( b_l \) in order to obtain \( K \) from the lenders. Specifically, the sovereign budget constraint is given by:

\[ K = q_{s,0} b_{s,0} + q_l b_l. \] (1)

The country is risk neutral. When choosing the debt composition, it maximizes its expected welfare at stage two, \( E_0(W) \), taking into account all possible branches of the game - tree. In particular, the welfare is given by the liquid resources accumulated at stage one (see below) plus the expected output of the project, \( Y_2 \), after repaying its creditors or the residual, after subtracting the deadweight losses the country incurs in case it decides to default.

### 3.1.2 Decision to Default

The economy is exposed to two shocks at two different stages and can decide to default after observing each of them. It can decide to default at stage one, after the realization of the liquidity shock. It can moreover default at stage two after the realization of the productivity shock.

The country is exposed to the risk that a liquidity crisis occurs at stage one, before the production technology’s receipts materialize. I follow the specification of Chang and Velasco (2000): the liquidity shock hits the country in the interim period, stage 1. With probability \( \beta \) the liquidity shock is positive, bringing additional liquid resources \( Y_1 \) to the country; while, with probability \( 1 - \beta \), the production technology needs a further injection of resources of the amount \( Y_1 \) in order for the production technology to be completed at stage two.\(^5\)

\(^5\)The two quantities the country has to repay or receives in the interim period do not need to be the same in
The very stylized formalization of the liquidity shock aims at capturing the idea that in a liquidity crisis it becomes more expensive for the country to obtain liquidity. In this model, the only way to obtain liquidity is through debt issuance therefore at stage one the cost of debt issuance increases and the country needs to issue more short-term debt to cover its financing needs, if hit by a negative liquidity shock. In an alternative formulation, instead of being fixed, the dimension of the liquidity shock could be linked to short-term debt (e.g. impossibility to roll-over debt with a certain probability). In that world, however, the liquidity shock would have a direct impact on the debt maturity structure, while in the present formulation I take the liquidity shock as an exogenous process, whose intensity is independent of the sovereign choices at stage zero. This specification captures therefore a process that is outside of the model equations. It could be thought as a reduced form for the definition of sudden capital surges or sudden stops that could hit a small open economy exposed to the global financial conditions, for example a change in the international investors’ confidence toward the domestic financial system, or a sudden and unanticipated movement in the exchange rate (e.g., Caballero and Panageas, 2007).

At stage one, after observing the liquidity shock, the sovereign can default on all the issued debt. In this case it loses all its resources, since the project is not yet realized. Alternatively, the country can try to borrow again from international lenders in order to repay the short-term debt, and to face the liquidity shortage \( -Y_1 \) in case of negative liquidity shock:

\[
qs,1b_{s,1} \geq b_{s,0} \pm Y_1.
\]

The equation above indicates also that the country can always decide to dilute its long-term debt, that is the amount of short-term debt issued in period one can be larger than the financing needs ( \( b_{s,0} \) possibly plus the liquidity shortage). Any liquid resource accumulated at stage one \( \max(0, qs,1b_{s,1} - (b_{s,0} \pm Y_1)) \) contributes to the country’s welfare (defined below in equation 5) the period after. Notice that those possible liquid resources cannot be seized by international lenders in case of default (those resources can be thought as sovereign reserves, see e.g. Aizenman and Marion, 2004), which gives the country an incentive to dilute its debt at stage one, even if it is risk neutral.

The productivity shock hits the country at stage two. With probability \( \alpha \) the output of the production technology is high, \( Y_{2,H} \), while with probability \( 1 - \alpha \) the project outcome is low, \( Y_{2,L} \), with \( Y_{2,H} > Y_{2,L} \). Output level and probabilities are independent of country’s and the international lenders’ choices. The cost of defaulting at stage two is that the country loses half absolute value. They are set equal but with opposite sign for simplicity and in order to reduce the number of parameters.

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of its realized output. As mentioned above, even after defaulting the country keeps its liquid resources accumulated at stage one.

I detail here some assumptions on the parameters of the model, in order to make the analysis interesting. The country defaults on the project whenever the revenue generated by the production technology is low, that is, the country always defaults at stage two with probability $1 - \alpha$. The sufficient condition for this default to occur is that $Y_{2,L} < K$. Also, at stage one and without international organization’s intervention, the country defaults if it is hit by a negative liquidity shock, or, in terms of parameters, I assume that $\frac{Y_{2,H}}{2} < Y_1 + \frac{K}{\alpha \beta}$ but $\frac{Y_{2,H}}{2} > Y_1 + K$. The two conditions are derived from the incentive compatibility constraint defined below, which establishes the condition for the country not to default after a positive productivity shock. The first condition states that in case of negative liquidity shock the country cannot credibly approach the financial market since at market prices (the ones of the basic model) the country will always default after observing the productivity shock. The second condition tells instead that, if all debt is financed at the risk-free interest rate (which is set to zero), even if hit by a negative liquidity shock, the country will not default after a high productivity shock. This allows me to make the IFI’s intervention with full repayment of foreign lenders feasible. This moreover makes country solvency dependent on the cost of its debt as in Corsetti et al. (2018). Yet, this condition does not guarantee that the country would never default in case of IFI intervention (see below). In fact, even with IFI intervention, the limited liability friction would still make the country able to choose to default if defaulting would entail higher welfare.

### 3.2 Model Solution

As mentioned above, the only interesting branch of the game-tree in this simple setup with no intervention by an IFI is the upper one in figure 2, when the country is hit by a positive shock in stage one and in stage two the expected project is high. Formally, at stage two the sovereign does not default after a positive liquidity shock, if

$$Y_{2,H} - b_l - b_{s,1} + Y_1 + (q_{s,1} b_{s,1} - b_{s,0}) \geq \frac{Y_{2,H}}{2} + Y_1 + (q_{s,1} b_{s,1} - b_{s,0}).$$

The term in the round bracket represents the resources obtained from dilution. Those resources, together with the liquidity inflow $Y_1$, remain untouched in case of default, therefore they appear on both sides of the inequality. This expression reduces to:

---

6See Alfaro and Kanczuk (2005) for a discussion on output losses in the event of default.
\[ \frac{Y_{2,H}}{2} \geq b_l + b_{s,1} \]

From the condition above I obtain the incentive compatibility constraint on \( b_{s,1} \) that needs to be satisfied at the time of debt issuance (stage one) in order for foreign lenders to be repaid at stage two in case of high productivity outcome:

**Incentive compatibility constraint**

\[ b_{s,1} \leq \frac{Y_{2,H}}{2} - b_l. \] \( (2) \)

The constraint above represents the maximum amount of debt the country can issue at stage one in order to get resources from foreign lenders.

Going backward, at stage one, the country needs to have enough resources to roll-over its debt. Given that it also has extra-liquidity \( Y_1 \), the feasibility constraint, for the game to move on to the final stage reads:

**Feasibility constraint**

\[ b_{s,1} \geq \frac{b_{s,0} - Y_1}{q_{s,1}}. \] \( (3) \)

The two constraints together limits \( b_{s,1} \): they define a range for the short-term debt issued at stage one.

\[ \frac{b_{s,0} - Y_1}{q_{s,1}} \leq b_{s,1} \leq \frac{Y_{2,H}}{2} - b_l. \] \( (4) \)

By combining the incentive compatibility constraint and the feasibility constraint, I derive the condition for the maximum amount of long-term bonds the country can borrow:

\[ b_l^{max} \leq \frac{q_{s,0} q_{s,1}}{q_{s,0} q_{s,1} - q_l} \left( \frac{Y_{2,H}}{2} + \frac{Y_1}{q_{s,1}} - \frac{K}{q_{s,0} q_{s,1}} \right). \] \( (5) \)

Consistently with Farhi and Tirole (2016), stating that "short-term bonds limit the possibilities of dilution, increase the incentives to pay down debt [...]", in this very simple model I
find that the country’s choice on short- and long-term debt is limited by the incentive compatibility constraint. Therefore, the possibility for the country to dilute long-term debt generates a long-term borrowing limit.

Moving backward to stage zero, the country chooses between short- and long-term debt in order to maximize its expected welfare, while satisfying the two constraints indicated above and the budget constraint in equation 1:

\[
\max_{b_{s,0}, b_{s,1}, b_l} E(W) = \beta [Y_1 + q_{s,1} b_{s,1} - b_{s,0} + \alpha (Y_{2,H} - b_l - b_{s,1}) + (1 - \alpha) \frac{Y_{2,L}}{2}].
\]  

(6)

As mentioned above, in the case of a negative liquidity shock the country defaults and loses all resources, therefore this case, which happens with probability \(1 - \beta\), does not appear in the expected welfare, equation 6. Moreover, in the case of a negative productivity shock the country defaults and keeps half of the realized project \(Y_{2,L}\).

After substituting \(b_{s,0}\) from the budget constraint, I maximize 6 with respect to \(b_{s,1}\) and \(b_l\). I obtain the following expressions:

\[
\max_{b_{s,1}} E(W) \rightarrow q_{s,1} - \alpha = 0. 
\]  

(7)

\[
\max_{b_l} E(W) \rightarrow \frac{q_l}{q_{s,0}} - \alpha = 0. 
\]  

(8)

If I instead maximize with respect to \(b_{s,0}\) I again obtain an expression that does not depend on the amounts chosen. This is due to the linearity of the welfare equation and risk-neutrality assumptions. The choice on how much to invest in short- and long-term bonds depends only on the cost of debt set by international lenders. I now look at the optimal choice of those agents. As mentioned in the lending game, they set the cost of each type of debt equal to the probability of being repaid. Therefore, those costs are:

\[
q_{s,0} = \text{Prob}\{\text{Country solvent at } t = 1\} = \beta \\
q_{s,1} = \text{Prob}\{\text{Country solvent at } t = 2\} = \alpha \\
q_l = \text{Prob}\{\text{Country solvent at } t = 1 \text{ and at } t = 2\} = \alpha \beta.
\]

In this very simple formulation, in which the two events (liquidity shock and productivity
shock) are independent, international lenders are risk neutral and have no constraint or rents, the cost of long-term debt is simply the product of the probabilities of the two positive events. Moreover, the interest rate equalization condition is verified: the cost of long-term debt is equal to the product of the return of the two short-term debts. This result could be also interpreted as a validation of the expectations hypothesis on the sovereign bond yield curve. The cost of debt just found makes also the country indifferent between short-term and long-term debt. Finally, when the interest rate equalization condition is verified, the limit on long-term debt always holds, as long as \( \frac{Y_2}{2} + \frac{Y_1}{q_{s,2}} - \frac{K}{q_{s,1}} \geq 0 \).

I will present in the next sections some cases in which the interest rate equalization is no longer verified. This has a consequence on the feasible and optimal debt composition. In what follows, I introduce the policy options available for the country: private debt restructuring and IFI intervention. In order to clearly show how each policy option acts on the agents’ choices I introduce them one by one and then combine in Section 6 and provide the main results. In Section \{ass12\} I then discuss the obtained results, their limitations and possible extensions.

4 Model with Private Debt Restructuring

In this section I introduce a simple formulation for debt restructuring, taking the main hypotheses from the findings of the empirical literature (e.g. Cruces and Trebesch, 2013). International lenders can now recover part of the invested resources, and in the negotiation process they try and recover as much as they can, provided the incentive for the country to choose partial repayment instead of full default is satisfied. While very stylized, this hypothesis is moreover consistent with Schumacher et al. (2018) documenting that legal developments (increased contract enforceability through the increase in sanctions) make full default more costly for the country, while reaching an agreement could be beneficial for the debtor as well as its creditors. However, partial repayment can occur only once the project is completed, while I assume for simplicity that in the case of default in the interim period the country has no liquid resources to partially repay its creditors.

Consistently with Asonuma et al. (2017), I stipulate that haircuts are uniform across bonds with the same residual maturity. However, I abstract from the possibility of multi-round bargaining negotiations, as in Asonuma and Joo (2018). At stage 2 the country can decide either to fully repay its international lenders, to default on the entire amount of debt (those are the two possibilities seen so far) or to restructure its debt (i.e. partial default). I make the hypothesis that, in the case of partial default, international lenders can set the haircut imposed on their debt repayment and they do so by considering the country’s ability and willingness to pay. In particular, they will ask to be paid the fraction of their original claims that makes the debtor country indifferent between restructuring or defaulting on the entire amount. This, in order to provide an incentive for the country to participate in the restructuring (instead of defaulting on the entire amount) and at the same time recover as much as feasible of the resources lent. Again following the empirical literature, I assume that, in case of partial default, the country’s output
loss is lower than in the case of full default (and set for simplicity to a third of the realized output).\(^7\) The rest of the model is equal to what shown before.

Solving again by backward induction, at stage 2 if the realized output is high, the constraint looks like equation 2, but it now incorporates also the option of partial default:

\[ Y_{2,H} - b_l - b_{s,1} \geq \frac{2}{3} Y_{2,H} - (b_l + b_{s,1})H_{2,H}. \]  

\(1 - H_{2,H}\) defines the haircut and \(H_{2,H}\) specifies how much each individual lender can recover out of one unit of investment. From the expression above it is clear that long and short-term bonds have the same seniority in this simple formulation (a different assumption will be added below for the IFI credits). The equality is due to the ability of foreign lenders to extract the maximum feasible resources from the restructuring and give an expression for \(H_{2,H}\):

\[ H_{2,H} = \frac{Y_{2,H}}{6(b_l + b_{s,1})}. \]  

The incentive compatibility constraint does not change given the equality (and therefore country’s indifference) between full and partial default. The main difference arises in the case of low output realization. With \(Y_{2,L}\), the country is not able to repay in full its creditors, but they can now extract some resources through debt restructuring. Analogously to what seen before, the haircut is found by equalizing the country’s outcome under full and partial default:

\[ \frac{Y_{2,L}}{2} = \frac{2}{3} Y_{2,L} - (b_l + b_{s,1})H_{2,L}. \]  

In this case the parameter defining the haircut is:

\[ H_{2,L} = \frac{Y_{2,L}}{6(b_l + b_{s,1})}. \]

Moving backward to stage one, the possibility of debt restructuring does not change the game. In the case of a negative liquidity shock the country has no incentive to partially repay its debt, since in any case it cannot move to stage 2 and loses all its resources. Therefore, the feasibility constraint does not change; it is still the one reported in equation 3, and in the case of a negative liquidity shock the country still defaults. The limit on long-term borrowing therefore does not change, it is still the one reported in equation 5.

At stage 0, the maximization problem is, after substituting \(b_{s,0}\) from the budget constraint:

\(^7\)In case of partial default the deadweight loss is lower that in the case of full default, therefore the overall welfare improvement for the system given by the possibility of partial default is imposed by construction and justified by the empirical evidence mentioned in the introduction.
\[
max_{b_s,1,b_l} E(W) = \beta[Y_1 + q_{s,1}b_{s,1} - \frac{K - q_l b_l}{q_{s,0}} + \alpha(Y_{2, H} - b_l - b_{s,1}) + \frac{(1 - \alpha)Y_{2, L}}{2}].
\] (13)

The maximization problem does not change with respect to the previous case:

\[
max_{b_s,1} E(W) \rightarrow q_{s,1} - \alpha = 0.
\] (14)

\[
max_{b_l} E(W) \rightarrow \frac{q_l}{q_{s,0}} - \alpha = 0.
\] (15)

Given the assumptions of the lending game, the expressions below represent the cost of debt set by international lenders for long-term bonds and short-term bonds issued at stage zero and one. Since even with a low output realization they can recover some resources, the optimal contracts offered by foreign lenders will differ from the ones of the basic model:

\[
q_{s,0} = \text{Prob}\{\text{Country solvent at } t = 1\} = \beta,
\]

\[
q_{s,1} = [\text{Prob}\{\text{Country solvent at } t = 2\} + \text{Prob}\{\text{Country restr. at } t = 2\}H_{2,L}] = \alpha + (1 - \alpha)H_{2,L},
\]

\[
q_l = [\text{Prob}\{\text{Country solvent at } t = 1 \text{ and } t = 2\} + \text{Prob}\{\text{Country solvent at } t = 1 \text{ and restr. at } t = 2\}H_{2,L}] = \beta(\alpha + (1 - \alpha)H_{2,L}).
\]

Compared to the costs of debt of the basic model, the possibility of restructuring decreases the cost of long-term obligations and the cost of short-term debt issued at stage one, which makes the country better off since it increases its expected net output. The interest rate equalization condition still holds and, as seen above, the constraints do not change, therefore the country’s incentives are still the ones of the basic model. The cost of issuing short-term debt at stage one is, however, lower than in the basic model and, more importantly, it is lower than the probability of repaying. The first order condition (i.e. the net gain from increasing issuance of short-term debt at stage one) is always positive and the country will therefore dilute all long-term obligations up to the incentive compatibility constraint: \(b_{s,1} = \frac{Y_{2,H}}{2} - b_l\). Finally, the first order condition of long-term debt (i.e. the net gain from increasing the issuance of long-term debt) is always positive; therefore at time zero the country prefers to issue only long-term debt (since the FOC for short-term obligations is equal to zero). It can also do so since, when the interest rate equalization condition holds, the constraint on long-term obligations is never binding. The fraction of lent resources \(H_{2,L}\) that international lenders are able to obtain in the low productivity case becomes then: \(H_{2,L} = \frac{Y_{2,L}}{3\gamma_{2,H}}\).

5 Model with IFI’s Intervention

So far I have assumed that, at stage 1 in the case of a negative liquidity shock, the country defaults because it is unable to roll-over its short-term debt through the financial markets. I
now introduce the possibility of requesting assistance to an IFI. The setup presented is very stylized but, in order to make the analysis useful for policy considerations, I provide some technical details that have some analogy with IMF financial assistance as defined by the last revision of the IMF lending framework. I present three types of intervention, which differ in terms of what happens to the short- and long-term debt issued at time zero: full repayment, restructuring or reprofiling. This is the only difference between the three interventions presented below, while the rest of the assumptions remain the same as presented so far. Crucially, the three interventions are alternative and at time zero all agents know whether and how the IFI would intervene in case of country request (for an alternative formulation with private information and uncertainty over IFI intervention see Corsetti et al., 2006). Also, the only cost for the country when applying for the IFI assistance is that it cannot dilute long-term debt since the IFI knows and lend the exact amount of resources needed to overcome the liquidity crisis.

The following assumptions are made:

- First of all, the IFI is a LOLR as it provides enough resources to face the liquidity shock, therefore the country can move on with the project.
- Second, the IFI program conditionality is modeled by assuming that policy advice helps the country to improve its project outcome and therefore the high productivity shock becomes more likely, with probability $\alpha'$, and $\alpha' > \alpha$.
- At stage one, after receiving IFI’s assistance, the country cannot issue short-term debt and dilute its obligations.
- IFI’s lending $b_{IFI}$ is concessional, the charged interest rate is below the ones charged by international lenders (at which the country would not be able to repay, as assumed above). For simplicity, the interest rate on IFI lending is set equal to the risk-free interest rate, i.e. at zero (a positive small charge could be assumed and would not change the results, but make the analysis more complicated).
- As mentioned above, the IFI enjoys a preferred creditor status, therefore at stage two, after observing the productivity shock, the country can decide to repay only the IFI in full, in this case it will lose a third of its output, as assumed above for partial default (restructuring). The full specification would entail repayment of the IFI up to the value of output that makes the country indifferent between full default and reprofiling. I make the simplifying assumption that in the case of a low output realization the country doesn’t have enough resources to repay all the IFI’s loan and also part of its obligations toward international lenders. In terms of parameters, suppose that $\frac{1}{6}Y_{2,L} \leq Y_1$ therefore the country at most repays the IMF but not international lenders (therefore the low output realization has no impact on the cost of debt).
- Finally, the limited liability friction still shapes the country choice even when the IFI intervenes since the country can decide to default also on IFI loan. As will be presented below, this friction determines two incentive compatibility constraints.

In the three sections that follow I will first focus on each type of IFI intervention (full repayment, restructuring, reprofiling) by analyzing debt limits, and the consequent choices of the international lenders on the cost of debt at time zero. I will then consider the three possible interventions together in order to compare country’s welfare consequences and the implications for how much short- and long-term debt is issued at time zero. As mentioned before, I abstract from considerations on the debt sustainability profile, which in the case of the IMF would make

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8See for details https://www.imf.org/en/News/Articles/2015/09/14/01/49/pr1631.
the IFI finally decide on the type of intervention. The three interventions therefore cannot be considered as substitute but complement in the spirit of the IMF lending framework. However, since before the 2016 reform, reprofiling was not included in the lending framework and those cases (debt deemed sustainable but not with high probability) would necessitate a debt restructuring, a joint analysis and welfare comparison of reprofiling and restructuring could be read as a comparison between pre- and post-2016. Notice that, from the point of view of the IFI, the two interventions entail the same disbursement.

5.1 Full Repayment

Suppose for simplicity that private restructuring of foreign lenders’ debt is not possible (as seen in section 4), then in case of positive liquidity shock, with probability $\beta$, the constraints are the ones of the basic model, reported in equations 2 and 3, and the limit on long-term debt is given by 5. In the case of a negative liquidity shock, having probability $1 - \beta$, the country can approach the IFI for financing assistance. The IMF exceptional access framework prescribes that, when it assesses that the sovereign debt is sustainable with high probability, it lends to the country with balance of payments problems without conditioning its assistance on any debt restructuring. Under the previous IMF policy, this was also the case when the systemic exemption clause was triggered. So, in terms of the parameters of the basic model, this means that, at stage one, after observing a negative liquidity shock, the country agrees with the IFI on an intervention such that all obligations maturing at time one are met through the resources lent by the IFI:

$$b_{IFI} = b_{s,0} + Y_1.$$  

Equation 16

At stage two, after observing the productivity shock, the country can take three different decisions: repay all creditors (IFI and long-term bond holders), default on all debt obligations and, finally, repay in full only the IFI (given its preferred creditor status) and default on foreign lenders’ obligations. If, with probability $\alpha'$, output is high, the country will repay in full if:

$$Y_{2,H} - b_l - b_{IFI} \geq \frac{Y_{2,H}}{2}. $$  

Equation 17

and also

$$Y_{2,H} - b_l - b_{IFI} \geq \frac{2}{3} Y_{2,H} - b_{IFI}. $$  

Equation 18

Equation 17 is the condition ensuring that the country does not default on all its obligations. Equation 18 ensures the country repays not only the IFI but also its international lenders. When lending to the country, the IFI looks at both constraints since debt has to be sustainable with high probability, therefore the constraints on the amount of long-term debt issued at time zero is now given by (after substituting $b_{IFI}$ from 16):
Incentive compatibility constraints

\[ b_t \leq \frac{Y_{2,H}}{2} - (b_{s,0} + Y_1), \]  
(19)

which becomes (by substituting \( b_{s,0} \)):

\[ b_t^{\text{max}} \leq \frac{q_{s,0}}{q_s,0 - q_l} (\frac{Y_{2,H}}{2} - Y_1 - \frac{K}{q_{s,0}}). \]  
(20)

and

\[ b_t^{\text{max}} \leq \frac{Y_{2,H}}{3}. \]  
(21)

The two conditions above define two long-term borrowing limits and are linked to the parameters of the model. In particular, the first limit is linked to the hypotheses made above on the size of output in the case of a high productivity shock and on the country’s liquidity and financing needs: \( \frac{Y_{2,H}}{2} < Y_1 + \frac{K}{\alpha \beta} \) but \( \frac{Y_{2,H}}{2} > Y_1 + K \). Those technical assumptions on the parameters ensure that the solution on debt limit is non-trivial.

At stage zero the country chooses its debt structure by maximizing the following expression:

\[
\max_{b_{s,0}, b_{s,1}, b_t} E(W) = \beta [Y_1 + q_{s,1}b_{s,1} - b_{s,0} + \alpha (Y_{2,H} - b_l - b_{s,1}) + (1 - \alpha) \frac{Y_{2,L}}{2}] + \\
+ (1 - \beta) [\alpha^\prime (Y_{2,H} - b_l - b_{IFI}) + (1 - \alpha^\prime) \frac{Y_{2,L}}{2}],
\]

or

\[
\max_{b_{s,0}, b_{s,1}, b_t} E(W) = \beta [Y_1 + q_{s,1}b_{s,1} - b_{s,0} + \alpha (Y_{2,H} - b_l - b_{s,1}) + (1 - \alpha) \frac{Y_{2,L}}{2}] + \\
+ (1 - \beta) [\alpha^\prime (Y_{2,H} - b_l - Y_1 - b_{s,0}) + (1 - \alpha^\prime) \frac{Y_{2,L}}{2}],
\]

The first part (in case \( \beta \)) coincides with equation 6 of the basic model. The second part shows that in case of full IFI intervention, there is no profit from debt dilution at stage one, and the probability of high output increases, by assumption. By substituting short-term debt issued at stage zero and maximizing output with respect to long-term bond and short-term bond issued at stage one, I obtain the following expressions:

\[ \max_{b_{s,1}} E(W) \rightarrow q_{s,1} - \alpha = 0. \]  
(22)

This corresponds to the one obtained in the basic model, since this type of debt is issued only in the case of a positive liquidity crisis and, therefore, of no IFI intervention.
\[ \max_{b_t} E(W) \rightarrow \frac{q_t}{q_{s,0}} (\beta + (1 - \beta)\alpha') - \alpha\beta - (1 - \beta)\alpha' = 0. \] (23)

By substituting \( b_t \) from the budget constraint and maximizing with respect to \( b_{s,0} \) I instead obtain:

\[ \max_{b_{s,0}} E(W) \rightarrow \frac{q_{s,0}}{q_t} (\alpha\beta + (1 - \beta)\alpha') - \beta - (1 - \beta)\alpha' = 0. \] (24)

Suppose all debt constraints are verified, then, as above, international lenders look at the probability and the amount of repayment in order to set the interest rate required for each type of debt at time zero and at time one in case of positive liquidity shock. In this case, of full IFI intervention, short-term debt will always be repaid therefore:

\[
\begin{align*}
q_{s,0} &= \text{Prob}\{ \text{Country solvent at } t = 1 \} = 1, \\
q_{s,1} &= \text{Prob}\{ \text{Country solvent at } t = 2 \} = \alpha, \\
q_t &= \left[ \text{Prob}\{ \text{Country solvent at } t = 1 \text{ and at } t = 2 \} + \text{Prob}\{ \text{IFI intervention at } t = 1 \text{ and country solvent at } t = 2 \} \right] = \beta\alpha + (1 - \beta)\alpha'.
\end{align*}
\]

Given the assumptions made above, that the IFI intervention increases the probability of high project outcome, and the IFI lends at concessional rates, the interest rates on debt with different maturity are not equalized: in the case of a positive liquidity shock, the product of the two short-term interest rates is higher (or the discount is lower) than the interest rate on long-term debt, while it is lower in the case of a negative liquidity shock and IFI intervention. Overall, in expectation issuing short-term debt is cheaper than issuing long-term one. Moreover, by substituting the cost of debt set by lenders in the first order conditions of the country I obtain that at stage zero the country prefers to issue only short-term debt (the derivative is positive while the one for long-term debt is negative, independently of the amount of issued debt). The IFI intervention with concessional lending rates makes the country opportunistically choose to issue the maximum amount of short-term debt whose renewal, in case \( 1 - \beta \), will cost less than the probability of repaying.

5.2 Restructuring

As assumed above, in case the IFI requires the country to default on all its private obligations, in order to intervene (i.e. at time one and regardless of the productivity outcome at time two), there is no partial repayment to international lenders. In the case of a positive liquidity shock, i.e. with probability \( \beta \), the constraints are the ones of the basic model, reported in equations 2 and 3, and the limit on long-term debt is given by 5. In the case of a negative liquidity shock with probability \( 1 - \beta \), the country can approach the IFI but in this situation, even if the probability of high output increases, this is not enough to judge the debt to be sustainable. Given the parameters of the model, this could be given by a different condition on high output: \( \frac{Y_2\mu}{2} < Y_1 + K \). The IFI therefore intervenes by providing only the resources that enable the country to face the liquidity shock and move to stage two:
\[ b_{IFI} = Y_1. \] (25)

At stage two, after observing the productivity shock, the country can decide whether to repay the IFI or to default on all its obligations (it already agreed to default on international lenders’ obligations). The country will repay the IFI if:

\[ \frac{2}{3} Y_{2,H} - b_{IFI} \geq \frac{Y_{2,H}}{2}. \] (26)

From the equation above (as mentioned before) I impose that partial default entails lower output loss for the country, compared with full default, following the empirical literature. In this situation there is no condition on long-term debt, the only condition is on the parameters of the model:

\[ \frac{Y_{2,H}}{6} \geq Y_1. \] (27)

In this case therefore the only constraint is the one of the case \( \beta \), with no IFI intervention, equation 5.

The country maximizes its expected welfare:

\[
\max_{b_{s,1}, b_l} E(W) = \beta \left[ Y_1 + q_{s,1} b_{s,1} - \frac{K - q_l b_l}{q_{s,0}} + \alpha (Y_{2,H} - b_l - b_{s,1}) + (1 - \alpha) \frac{Y_{2,L}}{2} \right] + \\
(1 - \beta) \left[ \alpha' \left( \frac{2}{3} Y_{2,H} - Y_1 \right) + (1 - \alpha') \frac{Y_{2,L}}{2} \right].
\]

The maximization problem corresponds to the one of the basic model. The cost of debt is also equal to the one of the basic model (9) since in case \( 1 - \beta \) the country would anyway default on private obligations. The interest rates on debt of different maturities are equalized. What changes is however the expected welfare of the country since it might bring the project to completion and earn some output even in the case of a negative liquidity shock (as I expose below).

### 5.3 Reprofiling

If debt is judged to be sustainable, but not with high probability, the IFI can ask the country to extend the maturity instead of repaying \( b_{s,0} \) at stage one, before lending the amount needed to face the liquidity crisis and move to stage two \( (Y_1) \). Reprofiling entails the postponement of payments of short-term debt from stage one to stage two. Also, reprofiling is considered a credit event since the country, at stage one, fails to honor its short-term obligations, therefore, independently of the realization of the productivity shock, there will be a deadweight loss in terms of output, once output is realized. At stage two, if the productivity shock is high ( with probability \( \alpha' \)), the country will repay if:
\[ \frac{2}{3}Y_{2,H} - b_{s,0} - b_l - b_{IFI} \geq \frac{Y_{2,H}}{2}, \]  

(28)

Incentive compatibility constraint

\[ b_l^{\text{max}} \leq \frac{q_{s,0}}{q_{s,0} - q_l} \left( \frac{Y_{2,H}}{6} - Y_1 - \frac{K}{q_{s,0}} \right). \]  

(29)

In this case again, the incentive compatibility constraint limits the amount of the country’s long-term borrowing.

At stage zero the country chooses its debt structure by maximizing the following expression:

\[
\max_{b_{s,0}, b_{s,1}, b_l} E(W) = \beta [Y_1 + q_{s,1} b_{s,1} - b_{s,0} + \alpha (Y_{2,H} - b_l - b_{s,1}) + (1 - \alpha) \frac{Y_{2,L}}{2}] + \\
+ (1 - \beta) [\alpha' (\frac{2Y_{2,H}}{3} - b_l - b_{s,0} - b_{IFI}) + (1 - \alpha') \frac{Y_{2,L}}{2}].
\]

or

\[
\max_{b_{s,0}, b_{s,1}, b_l} E(W) = \beta [Y_1 + q_{s,1} b_{s,1} - b_{s,0} + \alpha (Y_{2,H} - b_l - b_{s,1}) + (1 - \alpha) \frac{Y_{2,L}}{2}] + \\
+ (1 - \beta) [\alpha' (\frac{2Y_{2,H}}{3} - b_l - Y_1 - b_{s,0}) + (1 - \alpha') \frac{Y_{2,L}}{2}].
\]

The first order condition on \( b_{s,1} \) is again:

\[
\max_{b_{s,1}} E(W) \rightarrow q_{s,1} - \alpha = 0. \]  

(30)

This corresponds to the one obtained in the basic model, since this type of debt is issued only in case of positive liquidity shock and therefore no IFI intervention.

\[
\max_{b_l} E(W) \rightarrow \frac{q_l}{q_{s,0}} (\beta + (1 - \beta)\alpha') - \alpha \beta - (1 - \beta)\alpha' = 0. \]  

(31)

The first order conditions are identical to the case of full repayment. What changes is the amount of output the country can recover in the case of a negative liquidity shock. The cost of debt set by international lenders is, however, different from the full repayment case:

\[
q_{s,0} = \text{Prob}\{\text{Country solvent at } t = 1 \text{ or it is not but is solvent at } t = 2\} = \beta + (1 - \beta)\alpha' \]
\[
q_{s,1} = \text{Prob}\{\text{Country solvent at } t = 2\} = \alpha \]
\[
q_l = \left[ \text{Prob}\{\text{Country solvent at } t = 1 \text{ and at } t = 2\} + \right. \\
\left. \text{Prob}\{\text{IFI intervention at } t = 1 \text{ and country solvent at } t = 2\} \right] = \beta \alpha + (1 - \beta)\alpha'.
\]
The important difference with respect to the previous cases is what happens to the cost of debt in the case of a negative liquidity shock. Reprofiling makes the short-term debt issued at time zero more expensive with respect to the full repayment case, since international lenders know that with probability $1 - \beta$ the repayment will be subject to the realization of the productivity shock. In the case of a negative liquidity shock, short-term debt becomes therefore equivalent to long-term debt. Also, the interest rates on debt of different maturities are not equalized; in this case the product of the two short-term interest rates is lower (or the price is higher) than the interest rate on long-term debt. By looking at the maximization problem, and substituting the debt costs, the country is now indifferent between issuing short-term or long-term debt and the borrowing limit can be binding.

The distortion in debt costs of this type of intervention comes from the fact that implicitly again the country is paying less for short-term obligations, since \textit{de facto} short-term debt issued at time zero is extended with no additional (proportional) cost for the country and yet the country is not fully defaulting after observing the negative liquidity shock. International lenders bear the cost of credit extension. It has to be noticed that, in this very simple setup, one crucial assumption is made: the interest rate on alternative investments for international lenders is zero (or symmetrically, the discount rate is one), that is, waiting one period before obtaining the credit does not entail any additional cost.

5.4 Debt Limits, Debt Costs and Welfare

In this section I compare the debt cost, debt composition and country’s welfare implications of the three types of interventions (see the table in Section 7 for a quick comparison of debt costs and composition in all presented setups). The expected profits for international lenders are always zero. The IFI instead is expected to make at most no gains, given the two assumptions above: $\frac{1}{6} Y_{2,L} \leq Y_1$ (repayment in case of low output realization), and the concessional lending rate below the one of the international capital markets. In expectation full intervention is more costly since it entails higher disbursement.

In the simple framework chosen so far, from the point of view of the country the more expensive type IFI intervention in terms of debt costs is the one that entails full default on private obligations. At the other extreme is the full intervention, in which short-term debt issued at stage zero is costless. The reprofiling represents an intermediate case in terms of debt costs, since long-term debt is as expensive as in the full repayment case, but short-term debt at time zero is also costly.

I now consider the maturity structure finally chosen depending on the type of intervention. For all three cases the debt limit found in the basic model, the one of equation 5, is still binding, since the IFI does not intervene in case of positive liquidity shock. This limit is reported below.

$$b_{t}^{\text{max}} \leq \frac{q_{s,0}q_{s,1}}{q_{s,0}q_{s,1} - \gamma t} \left( \frac{Y_{2,H}}{2} + \frac{Y_1}{q_{s,1}} - \frac{K}{q_{s,0}q_{s,1}} \right).$$

In the case of full repayment and reprofiling there are additional long-term debt limits reported respectively in equations 20 - 21 and 29. Those limits need to be combined with the general assumptions that insure that only the IFI can and will be able to intervene in case of negative liquidity shock presented in section 3.1.2:
\[ Y_1 + K < \frac{Y_{2,H}}{2} < Y_1 + \frac{K}{\alpha} \]

It emerges then that, in the case of reprofiling, only for very special values of the parameters the model’s assumption and the incentive compatibility constraint deliver a positive long-term debt limit. Therefore, in most of the calibrations, in the case of debt reprofiling the country would be able to issue very little or no long-term debt. Also, by looking at the constraint in equation 32 in the case of full intervention, the price distortion generated by concessional lending determines a negative debt limit; therefore also in this case the country issues only short-term debt. This moreover corresponds to the optimal choice of the country as shown above. In the case of restructuring instead, as in the basic model, the constraint is not binding and the country is indifferent with respect to the debt maturity structure.

In order to facilitate the welfare comparison among the three types of intervention I therefore consider that also in the restructuring case the country issues only short-term debt. I substitute the cost of debt chosen by the international lenders and obtain, in the case of full intervention:

\[
E(W_{\text{full}}) = \beta[Y_1 - K + \alpha Y_{2,H} + (1 - \alpha) \frac{Y_{2,L}}{2}] +
\]
\[
(1 - \beta)[\alpha'(Y_{2,H} - Y_1 - K) + (1 - \alpha') \frac{Y_{2,L}}{2}].
\]

For default on all private obligations:

\[
E(W_{\text{restr}}) = \beta[Y_1 - K \frac{Y_{2,H}}{\beta} + \alpha Y_{2,H} + (1 - \alpha) \frac{Y_{2,L}}{2}] +
\]
\[
(1 - \beta)[\alpha'(\frac{2}{3} Y_{2,H} - Y_1) + (1 - \alpha') \frac{Y_{2,L}}{2}].
\]

For reprofiling:

\[
E(W_{\text{repr}}) = \beta[Y_1 - K \frac{Y_{2,H}}{\beta + (1 - \beta)\alpha'} + \alpha Y_{2,H} + (1 - \alpha) \frac{Y_{2,L}}{2}] +
\]
\[
(1 - \beta)[\alpha'(\frac{2}{3} Y_{2,H} - Y_1 - Y_1) + (1 - \alpha') \frac{Y_{2,L}}{2}].
\]

The difference between the three equations is in the cost of debt, as mentioned above. Also, with full intervention there is no deadweight loss in the case of a negative liquidity shock and a high productivity shock, while in the other two cases there is the loss associated with partial default. From a welfare point of view, in this very simple model, the country is indifferent between restructuring and reprofiling, because the higher indebtedness cost in the restructuring case is compensated by the lower repayment of private lenders in case of high productivity shock and IFI intervention. Not surprisingly, full intervention is instead, for the two elements just mentioned, always preferred to any form of restructuring. In this very simple setup, therefore, the IFI intervention with reprofiling is not welfare improving for the country and it entails the same amount of IFI resources involved. Finally, by assumption, any possible IFI intervention is preferred to no intervention since in the latter case the game ends at stage one with negative liquidity shock.
6 Model with both IFI Intervention and Private Debt Restructuring

As seen in the previous sections, the possibility of restructuring changes the cost of debt set by international lenders and the debt maturity profile chosen by the country. I consider now IFI intervention together with the possibility of private debt restructuring. I maintain the crucial assumption that the IFI enjoys a de facto preferred creditor status, and this limits the amount of resources available for foreign lenders in the restructuring. In particular, and for simplicity, I maintain the assumption that, in the case of a low productivity realization and after a negative liquidity shock, the resources available for restructuring, which makes the country indifferent between full default and restructuring, cover at most IFI credits ($\frac{1}{6}Y_{2,L} \leq Y_1$). I consider again separately the three different types of IFI intervention and then compare debt limits, debt costs and welfare.

6.1 Full Repayment

In the case of a positive liquidity shock, the game proceeds as described in section 4, while in the case of a negative liquidity shock the IFI intervenes as described in section 5.1. In this second case, the constraints do not change, since in the case of a high productivity shock the country will not default, while in the case of a low productivity the country will at most repay its obligations toward the IFI. The differences arise in the debt composition and in the cost of debt, since international lenders now consider that in the case of a positive liquidity shock and a low productivity they can still obtain some resources.

The maximization problem, after substituting short-term debt issued at time zero, becomes:

$$max_{b_{s,1}, b_l} E(W) = \beta[Y_1 + q_{s,1}b_{s,1} - \frac{K - q_l b_l}{q_{s,0}} + \alpha(Y_{2,H} - b_l - b_{s,1}) + \frac{(1 - \alpha)Y_{2,L}}{2}]$$

$$+(1 - \beta)[\alpha'(Y_{2,H} - b_l - Y_1 - \frac{K - q_l b_l}{q_{s,0}}) + (1 - \alpha')\frac{Y_{2,L}}{2}].$$

The first order conditions with respect to the short-term debt issued at time one after a positive liquidity shock is the one obtained in equation 14. The first order condition with respect to long-term bonds is equivalent to 23:

$$max_{b_l} E(W) \rightarrow \frac{q_l}{q_{s,0}}(\beta + (1 - \beta)\alpha') - (\alpha\beta + (1 - \beta)\alpha') = 0.$$

The cost of debt set by international lenders is:

28
\[ q_{s,0} = \text{Prob}\{\text{Country solvent at } t = 1\} = 1, \]
\[ q_{s,1} = \left[ \text{Prob}\{\text{Country solvent at } t = 2\} + \text{Prob}\{\text{Country restr. at } t = 2\} H_{2,L} \right] = \alpha + (1 - \alpha) H_{2,L}, \]
\[ q_l = \left[ \text{Prob}\{\text{Country solvent at } t = 1 \text{ and at } t = 2\} + \text{Prob}\{\text{Country solvent at } t = 1 \text{ and restr. at } t = 2\} H_{2,L} + \text{Prob}\{\text{IFI intervention at } t = 1 \text{ and country solvent at } t = 2\} \right] = \beta(\alpha + (1 - \alpha) H_{2,L}) + (1 - \beta) \alpha'. \]

Even with the possibility of debt restructuring, the interest rate equalization condition does not hold and in expectation short-term debt is cheaper than long-term debt. The first order condition and the cost of borrowing short-term at time one do not change, with respect to the case without IFI intervention, therefore the country still prefers to dilute all its long-term obligations after a positive liquidity shock and set \( b_{s,1} = \frac{Y_{2,H}}{2} - b_l \). Moreover, the amount of debt that international lenders can recover in the case of a low productivity shock and of a positive liquidity shock does not change with or without IFI intervention \( (H_{2,L} = \frac{Y_{2,L}}{3Y_{2,H}}) \) since it does not depend on the amount of long-term borrowing. The sign of the first order condition for long-term debt depends on the parameters of the model, while the one for short-term debt issued at time zero is always positive. Suppose that the one on long-term debt is also positive, then the country can issue long-term bonds as long as the two constraints on long-term borrowing are met, 5, 20. Finally, by looking at the constraint in 5, if the cost of short-term debt in the case of a positive liquidity shock is higher than the one of long-term debt \( (H_{2,L} < \frac{\alpha - \alpha'}{1 - \alpha}) \), then the long-term borrowing constraint 5 is verified only if long-term debt is zero.

Price distortions generated by the possibility of restructuring and by the IFI intervention force then the country to issue only short-term debt at time zero. As a result, the IFI has then to intervene with more resources in the case of a negative liquidity shock, since it has to cover all obligations issued by the country at time zero.

### 6.2 Restructuring

As presented in the subsection above, in case of a positive liquidity shock the game proceeds as seen in section 4. Now, however, after a negative liquidity shock, the IFI injects only the resources needed to move on with the investment project. The country therefore has to default on short-term obligations coming due but it can go on with the project. The new element, with respect to the previous analysis, is that the country, after observing a positive productivity shock, repays in full the IFI and uses part of the remaining resources to repay long-term bonds’ holders but not short-term ones since it has already defaulted on those obligations.\(^9\) At stage two the country repays if:

\[
\frac{2}{3} Y_{2,H} - b_{IFI} - H_{2,H} b_l \geq \frac{Y_{2,H}}{2}, \quad (32)
\]

\(^9\)If the country repays also its short-term obligations with delay, the case would become analogous to the reprofiling case of the next section.
Long-term bond holders set the repayment fraction of their debt such that the country is indifferent between full default and partial default with restructuring:

\[ H_{2,H} = \frac{(Y_{2,H} - Y_1)}{b_l}. \]  

(33)

Since the IFI is repaid in full, the higher the IFI intervention, the lower the share of resources long-term bond holders can recover. Notice that, if the repayment is possible in the reprofiling case, without restructuring (in equation 28) then the amount defined above, \( H_{2,H} \), could become larger than one, therefore the country could repay all its long-term obligations and \( H_{2,H} = 1 \). The maximization problem, after substituting short-term debt issued at time zero and the haircut, and with the assumption that \( H_{2,H} = 1 \) becomes:

\[
\begin{align*}
\max_{b_s,1,b_l} E(W) &= \beta[Y_1 + q_{s,1}b_{s,1} - \frac{K - q_l b_l}{q_{s,0}} + \alpha(Y_{2,H} - b_l - b_{s,1}) + \\
&\quad + \frac{(1 - \alpha)Y_{2,L}}{2} + (1 - \beta)[\alpha' \left(\frac{2}{3}Y_{2,H} - Y_1 - b_l\right) + (1 - \alpha') \frac{Y_{2,L}}{2}].
\end{align*}
\]

The first order conditions for short-term debt is analogous the the case without IFI intervention, and for long-term becomes:

\[
\begin{align*}
\max_{b_{s,1}} E(W) &\to q_{s,1} - \alpha = 0. \quad (34) \\
\max_{b_l} E(W) &\to \beta\left(\frac{q_l}{q_{s,0}} - \alpha\right) - (1 - \beta)\alpha' = 0. \quad (35)
\end{align*}
\]

The cost of borrowing set by international lenders, which consider that short-term obligations are not repaid in case of negative liquidity shock and long-term obligations are repaid in full, becomes:

\[
\begin{align*}
q_{s,0} &= \text{Prob}\{\text{Country solvent at } t = 1\} = \beta, \\
q_{s,1} &= \left[\text{Prob}\{\text{Country solvent at } t = 2\} + \text{Prob}\{\text{Country restr. at } t = 2\} H_{2,L}\right] = \alpha + (1 - \alpha)H_{2,L}, \\
q_l &= \left[\text{Prob}\{\text{Country solvent at } t = 1 \text{ and at } t = 2\} + \text{Prob}\{\text{Country solvent at } t = 1 \text{ and restr. at } t = 2\} H_{2,L} + \text{Prob}\{\text{IFI intervention at } t = 1 \text{ and country solvent at } t = 2\}\right] = \\
&\quad \beta(\alpha + (1 - \alpha)H_{2,L}) + (1 - \beta)\alpha'.
\end{align*}
\]

As seen above, even in this case the first order condition and cost of borrowing for short-term debt at time one do not change, with respect to the case without IFI intervention, therefore the country still prefers to dilute all its long-term obligations after a positive liquidity shock and set \( b_{s,1} = \frac{Y_{2,H}}{2} - b_l \). The interest rates on debt of different maturities are not equalized, and in
this case long-term borrowing is always less expensive than short-term one, given the default on short-term obligations when a negative liquidity shock hits. The long-term debt constraint is again binding; therefore, even in this situation, the country will be able to issue only short-term obligations.

As a more general case, consider that \( H_{2,H} < 1 \) therefore in the case of a negative liquidity shock and positive productivity shock, foreign lenders are not able to recover all the borrowed resources. The maximization problem becomes:

\[
\max_{b_s,1, b_l} E(W) = \beta[Y_1 + q_{s,1}b_{s,1} - \frac{K - q_l b_l}{q_{s,0}} + \alpha(Y_{2,H} - b_l - b_{s,1}) + (1 - \alpha)Y_{2,L}] + (1 - \beta)[\alpha'(Y_{2,H}) + (1 - \alpha')Y_{2,L}].
\]

And the first order condition on long-term debt becomes analogous to what seen in the section on model solution, in equation 8:

\[
\max_{b_l} E(W) \rightarrow \frac{q_l}{q_{s,0}} - \alpha = 0.
\]

In setting the cost of debt, foreign lenders anticipate that after a negative liquidity shock and positive productivity shock they would be only partially repaid; therefore, they set a cost of debt lower than the one of full repayment above while the others do not change:

\[
q_{s,0} = \text{Prob}\{\text{Country solvent at } t = 1\} = \beta,
q_{s,1} = \text{Prob}\{\text{Country solvent at } t = 2\} + \text{Prob}\{\text{Country restr. at } t = 2\}H_{2,L} = \alpha + (1 - \alpha)H_{2,L},
q_l = \text{Prob}\{\text{Country solvent at } t = 1 \text{ and at } t = 2\} + \text{Prob}\{\text{Country solvent at } t = 1 \text{ and restr. at } t = 2\}H_{2,L} + \text{Prob}\{\text{IFI interv. at } t = 1 \text{ and country restr. at } t = 2\}H_{2,H} = \beta(\alpha + (1 - \alpha)H_{2,L}) + (1 - \beta)\alpha'H_{2,H}.
\]

Even in this case the interest rate equalization condition does not hold. Moreover, the long-term debt constraint is again verified only if long-term debt is zero, so only short-term debt is issued and the solution is analogous to the situation above with \( H_{2,H} = 1 \).

### 6.3 Reprofiling

In the case of a positive liquidity shock, the game proceeds as seen above when debt restructuring is possible. In the case of a negative liquidity shock, at stage one short-term obligations are not repaid. At stage two, if the country is hit by a high productivity shock it repays all its obligations, also the ones delayed from stage one, therefore the constraints set in section 5.3 still hold. The maximization problem becomes:
\[
\max_{b_s, b_l} E(W) = \beta [Y_1 + q_{s,1} b_{s,1} - \frac{K - q_l b_l}{q_{s,0}} + \alpha (Y_{2,H} - b_l - b_{s,1}) + (1 - \alpha) Y_{2,L}] + (1 - \beta) [\alpha' (\frac{2Y_{2,H}}{3} - b_l - b_{s,0} - b_{IFI}) + (1 - \alpha') \frac{Y_{2,L}}{2}].
\]

The first order conditions with respect to the short-term debt issued at time one after positive liquidity shock is the one obtained in equation 14. The first order condition with respect to long-term bonds instead becomes:

\[
\max_{b_l} E(W) \rightarrow \frac{q_l}{q_{s,0}} (\beta + (1 - \beta) \alpha') - \alpha \beta - (1 - \beta) \alpha' = 0. \tag{37}
\]

The cost of borrowing set by international lenders is again different from the previous case, since lenders anticipate partial repayment in the case of a positive liquidity shock and a negative productivity shock and repayment (with delay in case of short-term obligations) in the case of a negative liquidity shock and a positive productivity shock:

\[
q_{s,0} = \text{Prob}\{\text{Country solvent at } t = 1 \text{ or it is not but is solvent at } t = 2\} = \beta + (1 - \beta) \alpha',
\]

\[
q_{s,1} = [\text{Prob}\{\text{Country solvent at stage at } t = 2\} + \text{Prob}\{\text{Country restr. at } t = 2\} H_{2,L}] = \alpha + (1 - \alpha) H_{2,L},
\]

\[
q_l = [\text{Prob}\{\text{Country solvent at } t = 1 \text{ and at } t = 2\} + \text{Prob}\{\text{Country solvent at } t = 1 \text{ and restr. at } t = 2\} H_{2,L} + \text{Prob}\{\text{IFI intervention at } t = 1 \text{ and country solvent at } t = 2\}] = \beta (\alpha + (1 - \alpha) H_{2,L}) + (1 - \beta) \alpha'.
\]

As seen above, the country dilutes all obligations at stage one after a positive liquidity shock. Moreover, the first order condition for long-term borrowing is positive, however the interest rates on debt of different maturities are not equalized; the cost of long-term borrowing is lower than the cost of short-term obligations, therefore the constraints on borrowing force the country to issue only short-term obligations.

### 6.4 Debt Limits, Debt Costs and Welfare

When foreign lenders can recover part of their funds in the case where the country defaults, the cost set for different types of debt instruments changes significantly. In particular, short-term obligations have very different costs, with full repayment being the least expensive for the country and restructuring the most expensive, reprofiling lying in between. More importantly, the interest rate equalization condition on the debt structure never holds, and the type of games causes long-term obligations not to be viable from the point of view of the incentive compatibility constraint: due to the possibility of debt dilution, the country can therefore issue only short-term obligations that it has to roll-over at time one.

In comparing welfare under the three different types of intervention, I therefore consider that at time zero the country can issue only short-term debt. Also at time one, after observing a
positive liquidity shock, the country optimally decides to issue the maximum amount of debt that makes it still able to repay in full its obligations in case of high productivity shock, \( b_{s,1} \) = \( \frac{Y_{2,H}}{2} \). In case of full repayment of short-term obligations by the IFI, the country welfare is given by:

\[
E(W_{\text{full}}) = \beta [Y_1 - K + (\alpha + (1 - \alpha)H_{2,L})\frac{Y_{2,H}}{2} + \alpha \frac{Y_{2,H}}{2} + (1 - \alpha)\frac{Y_{2,L}}{2}] + (1 - \beta)[\alpha' (Y_{2,H} - Y_1 - K) + (1 - \alpha')\frac{Y_{2,L}}{2}].
\]

In case of restructuring on long-term obligations and full default on short-term obligations, the country welfare becomes (this does not depend on the value of \( H_{2,H} \) since the country issues only short-term obligations):

\[
E(W_{\text{restr}}) = \beta [Y_1 - \frac{K}{\beta} + (\alpha + (1 - \alpha)H_{2,L})\frac{Y_{2,H}}{2} + \alpha \frac{Y_{2,H}}{2} + (1 - \alpha)\frac{Y_{2,L}}{2}] + (1 - \beta)[\alpha' \frac{2}{3}Y_{2,H} - Y_1) + (1 - \alpha')\frac{Y_{2,L}}{2}].
\]

For reprofiling:

\[
E(W_{\text{repr}}) = \beta [Y_1 - \frac{K}{\beta + (1 - \beta)\alpha'} + (\alpha + (1 - \alpha)H_{2,L})\frac{Y_{2,H}}{2} + \alpha \frac{Y_{2,H}}{2} + (1 - \alpha)\frac{Y_{2,L}}{2}] + (1 - \beta)[\alpha' \frac{2}{3}Y_{2,H} - Y_1 - \frac{K}{\beta + (1 - \beta)\alpha'} + (1 - \alpha')\frac{Y_{2,L}}{2}].
\]

What makes the three equations different is mainly the cost of debt. Also, as already seen above, with full intervention there is no deadweight loss in the case of a high productivity shock, while there is a loss associated with partial default in the second and last interventions. From a welfare point of view, while full intervention always brings the highest level of welfare, the other two are again equivalent and for the same reason: in this very simple set-up the higher indebtedness cost in the restructuring case is compensated by the lower repayment in case of high productivity shock and IFI intervention.\(^{10}\) Also, as mentioned before, in the restructuring and reprofiling cases, the IFI involvement is equivalent; it provides the resources needed to move on with the project \( Y_1 \) at time one, moreover, the probability of being repaid and the amounts are the same, given the preferred creditor status. Finally, even if in the reprofiling case it would pay only a fraction of its private obligations, the country would still be indifferent between reprofiling and restructuring, from a welfare point of view.

7 Discussion of the Main Results and Key Assumptions

The goal of the analysis is to understand what are the forces that shape the debt maturity structure and debt costs in the lending game between a sovereign and its international lenders

\(^{10}\)The repayment of short-term obligations in case of full default would not change the equivalence result.
and possibly an IFI intervention. The table below compares the main results in terms of those elements, namely the decision to issue short- versus long-term debt and the cost associated with each issuance in equilibrium.

<table>
<thead>
<tr>
<th>Model</th>
<th>Debt cost</th>
<th>Debt composition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Basic model</td>
<td>( q_{i,0} = \beta )</td>
<td>Either short- or long-term debt</td>
</tr>
<tr>
<td></td>
<td>( q_{i,1} = \alpha )</td>
<td>Long-term debt constraint not binding</td>
</tr>
<tr>
<td></td>
<td>( q_{i} = \alpha \beta )</td>
<td></td>
</tr>
<tr>
<td>Private restructuring</td>
<td>( q_{i,0} = \beta )</td>
<td>Long-term debt only</td>
</tr>
<tr>
<td></td>
<td>( q_{i,1} = \alpha + (1 - \alpha) H_{12} )</td>
<td>Long-term debt constraint not binding</td>
</tr>
<tr>
<td></td>
<td>( q_{i} = \beta [\alpha + (1 - \alpha) H_{12}] )</td>
<td></td>
</tr>
<tr>
<td>IFI</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full intervention</td>
<td>( q_{i,0} = 1 )</td>
<td>Short-term debt only</td>
</tr>
<tr>
<td></td>
<td>( q_{i,1} = \alpha )</td>
<td>Long-term debt constraint can be binding</td>
</tr>
<tr>
<td></td>
<td>( q_{i} = \alpha \beta + (1 - \beta) \alpha' )</td>
<td></td>
</tr>
<tr>
<td>Restructuring</td>
<td>( q_{i,0} = \beta )</td>
<td>Either short- or long-term debt</td>
</tr>
<tr>
<td></td>
<td>( q_{i,1} = \alpha )</td>
<td>Long-term debt constraint not binding</td>
</tr>
<tr>
<td></td>
<td>( q_{i} = \alpha \beta )</td>
<td></td>
</tr>
<tr>
<td>Reprofiling</td>
<td>( q_{i,0} = \beta + (1 - \beta) \alpha' )</td>
<td>Either short- or long-term debt</td>
</tr>
<tr>
<td></td>
<td>( q_{i,1} = \alpha + (1 - \alpha) H_{12} )</td>
<td>Long-term debt constraint not binding</td>
</tr>
<tr>
<td></td>
<td>( q_{i} = \beta [\alpha + (1 - \alpha) H_{12}] + (1 - \beta) \alpha' )</td>
<td></td>
</tr>
<tr>
<td>Private restructuring</td>
<td></td>
<td></td>
</tr>
<tr>
<td>and IFI</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Full intervention</td>
<td>( q_{i,0} = 1 )</td>
<td>Short-term debt only</td>
</tr>
<tr>
<td></td>
<td>( q_{i,1} = \alpha + (1 - \alpha) H_{12} )</td>
<td>Long-term debt constraint is binding</td>
</tr>
<tr>
<td></td>
<td>( q_{i} = \beta [\alpha + (1 - \alpha) H_{12}] + (1 - \beta) \alpha' )</td>
<td></td>
</tr>
<tr>
<td>Restructuring</td>
<td>( q_{i,0} = \beta )</td>
<td>Short-term debt only</td>
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<td></td>
<td>( q_{i,1} = \alpha + (1 - \alpha) H_{12} )</td>
<td>Long-term debt constraint is binding</td>
</tr>
<tr>
<td></td>
<td>( q_{i} = \beta [\alpha + (1 - \alpha) H_{12}] + (1 - \beta) \alpha' )</td>
<td></td>
</tr>
</tbody>
</table>

From the table above agents’ choices in the different situations clearly emerge. Any policy option available for debt crisis management weakly improves the term structure profile, however this is always at the expense of ex-ante price setting distortion. In particular, the possibility of partial repayment after default, when private debt restructuring is an option, makes long-term debt and short-term debt issued at the interim period relatively cheaper, boosting debt dilution. When IFI intervenes with full repayment, investing short-term becomes cheaper for the country, due to IFI’s concessional lending. Reprofiling instead makes short-term debt relatively more costly since international lenders anticipate that they would not be compensated in case of maturity extension. Finally, IFI intervention with restructuring is less distorting but more costly for the country than other IFI interventions due to higher haircut on debt burden. The final decision on debt composition is in any case influenced by debt dilution and the consequent
constraint on long-term debt. IFI intervention in particular, does not alleviate the constraint and the country tends to issue short-term debt.

The assumptions of agents’ risk-neutrality and of a linear welfare function simplify the analytics of the model. In the basic framework the result appears highly predictable, and could seem trivial; however, it is precisely done in order to then easily identify the effect of the financial frictions and the available policy options on the model’s results. Risk-averse lenders would affect the yield curve by increasing the cost of long-term debt due to a risk-premium on those issuances. The interest rate equalization condition would no longer hold and the long-term debt limit could become binding in all analyzed cases. The directions of policy interventions should however not change. Private debt restructuring would still make long-term debt relatively less expensive than in the basic case; moreover, any IFI intervention would distort debt costs and maturity structure in the same directions.

A risk-averse country has an even stronger incentive to dilute long-term obligations in order to obtain liquid resources at time one, given that those resources are certain at \( t = 1 \). Also, a risk-averse country or, in general, a non-linear welfare function could generate first order conditions dependent on the amount of each type of issued debt. This could potentially change the final mix of short versus long-term debt. However, the forces at play are still the ones presented in the introduction: the disciplinary role of short-term debt but an increasing probability of rollover problems.

As in Corneli and Tarantino (2016) the choice on the amount of debt could be made endogenous, and it could be allowed to impact on the probability that a liquidity crisis hits the country. This enrichment, however, would come at the expenses of an analytic solution for debt composition and costs, and in particular for the main result of the basic model; i.e. that the long-term debt limit derives directly from the financial frictions of the game.

The assumptions made on the country’s deadweight loss in case of different types of default (full, partial or with reprofiling) are crucial for the result that IFI intervention with reprofiling or restructuring are welfare equivalent. Fernandez and Martin (2014) assume that reprofiling, being considered a lighter form of restructuring, is less costly for the country in terms of deadweight loss and this leads to their conclusion that reprofiling is preferable to restructuring from a welfare point of view. Even in the present analysis, assuming that reprofiling entails a lower deadweight loss would make this type of IFI intervention ex-ante welfare enhancing for the country, compared to restructuring. Mine is a simplifying assumption, but it is also due to the lack of a clear message from the empirical literature: while, as already mentioned, Cruces and Trebesch (2013) find that lower haircuts are associated with lower output losses, Reinhart and Trebesch (2016) differentiate between debt relief operations, which are followed by improved output and debt cost conditions, and softer forms of debt relief, like maturity extensions, for which they do not find such gains.

Another assumption is made, in this very simple setup: the interest rate on alternative risk-free investments for international lenders is zero (or symmetrically, the discount rate is one). In the basic model and in the model with private debt restructuring this assumption is innocuous, since a positive risk-free rate would simply shift up all debt costs. By contrast, when comparing the welfare implications of IFI intervention this assumption is no longer neutral and can distort the conclusions, since it implies that, from the point of view of international lenders, waiting one extra period before being repaid does not entail any additional cost. In the case of IFI
intervention with reprofiling, a positive risk-free interest rate would make short-term debt issued at time zero more expensive and this would reduce the ex-ante country welfare and make the long-term debt limit more binding.

As mentioned in the introduction, limiting the model to a three-period analysis does not allow to analyze the costs of reputation in the case where the country defaults. In particular, as found by Cruces and Trebesch (2013) a higher haircut on sovereign debt delays market re-access. In this sense, reprofiling, which entails a lower haircut on debt, would allow a swifter market access, compared to a deeper debt restructuring. The three-period setup also prevents any discussions on the impact of accumulated debt on future outcomes, after the conclusion of the crisis. In this sense a full debt restructuring might allow a fresh start for the country, while reprofiling entails a higher legacy debt after the crisis has passed.

Finally, the analysis proposed does not help to discriminate what type of IFI intervention is more suitable while it takes it as given and known ex-ante if and how the IFI could intervene. The results of the game would sensibly change if the possibility of an IFI intervention or the type of intervention were established after agents’ decisions at $t = 0$. The full information setup also makes it impossible to discuss how to attenuate moral hazard and stigma concerns linked to the presence of a LOLR. Moreover, this hypothesis together with the three-period setup prevent any considerations on the best actions the government should undertake to improve on its reputation, which would be reflected in international lenders’ pricing. I leave all this for future research.

8 Final Remarks

I have presented a model of optimal debt structure of a risk neutral country that is subject to liquidity and productivity shocks. Two frictions, i.e. limited liability (the impossibility to commit to refrain from debt dilution and default) and no contingent assets, restrict and shape the optimal choice of the sovereign. The simple formulation of the basic model is used as a tool to disentangle the forces at work: on the one side, short-term debt plays a disciplinary role against debt dilution, on the other side, it increases the exposure to a liquidity crisis. Compared to previous studies, already in the basic model a long-term debt constraint arises and can shape the debt maturity structure.

I show with this simple set-up that any debt crisis management tool introduced to deal with a liquidity or solvency crisis has not only ex-post consequences on country’s welfare but also modifies optimal decisions taken ex-ante by the country and its international lenders. In particular, compared to the basic model, the possibility of partial repayment after default pushes the country to increase its long-term obligations and therefore the scope for debt dilution. The intervention of an IFI in case of a liquidity crisis can instead lead to a larger short-term debt issuance, depending on the type of IFI lending framework.

This result holds true not only when the IFI’s loans are employed to cover country’s short-term obligations coming due at the time of the liquidity crisis, but also in the case where the IFI imposes as a condition for intervening that the country defaults on those obligations. It emerges, in equilibrium, that none of the proposed crisis management tools are able to remove the trade-off between short- and long-term debt and in particular the impact of debt dilution on the debt...
maturity structure. The country issues mainly short-term debt even with IFI intervention whose mere presence is therefore unable to decrease the impact of a liquidity crisis.

Reinhart et al. (2017) have referred to the most recent years, since 2011, as a period of “missing sovereign defaults”. Based on historical data, they find that recent capital flow and commodity price movements would have justified a larger number of sovereign defaults than what observed. One possible explanation is the current low and stable global interest rate. The present situation should, however, not lead to complacency. Sovereign defaults and how best to intervene are still open issues for academics and policy makers.

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