SECULAR STAGNATION, R&D, PUBLIC INVESTMENT AND MONETARY POLICY: A GLOBAL-MODEL PERSPECTIVE

ONLINE-ONLY APPENDIX

by Pietro Cova*, Patrizio Pagano§, Alessandro Notarpietro* and Massimiliano Pisani*

June 11, 2019

Contents

* Bank of Italy, Directorate General for Economics, Statistics and Research.
§ World Bank.

A Appendix: The model

The model is an Open Economy New Keynesian Model. The world economy consists of five regions: Home (EA), US (*), CH (China, **), JP (Japan, ***) and RW (rest of the world). The size of the world economy is normalized to 1. EA, US, CH, JP and RW have sizes equal to n, n^*, n^{**}, n^{***}, and $(1 - n - n^* - n^{**} - n^{***})$, respectively, with $n, n^*, n^{**}, n^{***} > 0$ and $n+n^*+n^{**}+n^{***} < 1$. For each region, the size refers to the overall households' population and to the number of firms operating in each sector (intermediate tradable, intermediate nontradable, final nontradable consumption, final nontradable investment). Each region has a central bank that sets the nominal interest rate according to a standard Taylor rule, and reacts to domestic consumer prices and GDP growth.[1]

Households consume a final good, which is a composite of intermediate nontradable goods and intermediate tradable goods. Intermediate tradables are domestically produced or imported. All households supply differentiated labor services to domestic firms and act as wage setters in monopolistically competitive labor markets, as they charge a wage mark-up over their marginal rate of substitution between consumption and leisure.

Households trade two bonds. One is traded domestically, and is denominated in the domestic currency. The other is internationally traded, and is denominated in US dollars. The related first–order conditions imply that an uncovered interest parity condition holds, linking the differential between domestic and US monetary policy rates to the expected depreciation of the exchange rate of the domestic currency vis-à-vis the US dollar.

On the production side there are firms that, under perfect competition, produce two final manufacturing goods (consumption and investment goods) and firms that, under monopolistic competition, produce intermediate (internationally) tradable and nontradable goods.

The final manufacturing goods are sold domestically and are produced combining all available intermediate goods using a constant-elasticity-of-substitution (CES) production function. The two resulting bundles can have different composition. Intermediate tradable and nontradable goods are produced combining capital and labor, supplied by the domestic households. Capital and labor are assumed to be mobile across the two intermediate sectors.

Given the assumption of differentiated intermediate goods, firms have market power, are price-setters and restrict output to create excess profits. Intermediate tradable goods can be sold domestically and abroad. It is assumed that markets for tradable goods are segmented, so that firms can set a different price in each of the three regions.

In line with other dynamic general equilibrium models of the EA (see, among the others, Warne et al. (2008) and Gomes et al. (2010), we include adjustment costs on real and nominal variables, ensuring that consumption, production, and prices react in a gradual way to a given shock. On the real side, habits and quadratic costs prolong the adjustment of consumption and

[1] We make the assumption of a cashless economy.

investment, respectively. On the nominal side, quadratic costs make wages and prices sticky.[2]

In what follows, we report the main equations for the Home country. Similar equations hold in the other regions (if not so, we report the differences).

A.1 Firms

We initially show the final good sectors (private consumption and investment good). Thereafter, the intermediate good sectors (intermediate nontradable goods, and intermediate tradable goods). We report only equations of the Home (EA) economy. Similar equations hold for the other regions. We explicitly state when this is not the case.[3]

A.1.1 Final private consumption good

There is a continuum of symmetric Home firms producing final nontradable consumption goods under perfect competition. Each firm producing the consumption good is indexed by $x \in (0, n]$, where the parameter $0 < n < 1$ measures the size of the Home region. Firms in the other regions are similarly indexed. The CES production technology used by the generic firm x is

$$A_t(x) \equiv \left(a_T^{\frac{1}{\phi_A}} \left(\begin{array}{c} a_H^{\frac{1}{\rho_A}} Q_{HA,t}(x)^{\frac{\rho_A-1}{\rho_A}} + \\ (1-a_H)^{\frac{1}{\rho_A}} \times \\ \left(\left(\sum_{i=1}^{4} a_{IMPA,i}^{\frac{1}{\rho_{IMPA}}} Q_{IMPA,i,t}(x)^{\frac{\rho_{IMPA}-1}{\rho_{IMPA}}} \right)^{\frac{\rho_{IMPA}}{\rho_{IMPA}-1}} \right)^{\frac{\rho_A-1}{\rho_A}} \end{array} \right)^{\frac{\rho_A}{\rho_A-1}\frac{\phi_A-1}{\phi_A}} \begin{array}{c} \\ \\ \\ + (1-a_T)^{\frac{1}{\phi_A}} Q_{NA,t}(x)^{\frac{\phi_A-1}{\phi_A}} \end{array} \right)^{\frac{\phi_A}{\phi_A-1}} \quad \text{(A.1)}$$

where Q_{HA}, $Q_{IMPA,i}$, and Q_{NA} are bundles of respectively intermediate tradables produced in Home, intermediate tradables produced in one among the other four regions and imported by Home, and intermediate nontradables produced in the Home country.

The parameter $\rho_{IMPA} > 0$ is the elasticity of substitution among imported consumption goods, $\rho_A > 0$ is the elasticity of substitution among tradable goods and $\phi_A > 0$ is the elasticity of substitution between tradable and nontradable goods. The parameter a_H ($a_H > 0$) is the weight of the Home tradable, the parameter $a_{IMPA,i}$ ($0 < a_{IMPAi,} < 1$, $\sum_{i=1}^{4} a_{IMPA,i} = 1$) the weight of the generic imported tradable from country i, and the parameter a_T ($0 < a_T < 1$) the weight of the tradable goods.

[2] See Rotemberg (1982).

[3] For a detailed description ot the main features of the model see also Pesenti (2008), which provides a description of the GEM (the International Monetary Fund Global Economy Model).

A.1.2 Final investment good

The production of the investment good is similar to that of the consumption bundle. There are symmetric Home firms under perfect competition indexed by $y \in (0, n]$. Output of the generic Home firm y is

$$E_t(y) \equiv \left(\begin{array}{c} v_T^{\frac{1}{\phi_E}} \left(\begin{array}{c} v_H^{\frac{1}{\rho_E}} Q_{HE,t}(x)^{\frac{\rho_E - 1}{\rho_E}} + \\ (1 - v_H)^{\frac{1}{\rho_E}} \times \\ \left(\left(\sum_{i=1}^{4} v_{IMPE,i}^{\frac{1}{\rho_{IMPE}}} Q_{IMPE,i,t}(x)^{\frac{\rho_{IMPE}-1}{\rho_{IMPE}}} \right)^{\frac{\rho_{IMPE}}{\rho_{IMPE}-1}} \right)^{\frac{\rho_E - 1}{\rho_E}} \end{array} \right)^{\frac{\rho_E}{\rho_E - 1} \frac{\phi_E - 1}{\phi_E}} \\ + (1 - v_T)^{\frac{1}{\phi_E}} Q_{NE,t}(x)^{\frac{\phi_E - 1}{\phi_E}} \end{array} \right)^{\frac{\phi_E}{\phi_E - 1}} \quad \text{(A.2)}$$

where Q_{HE}, $Q_{IMPE,i}$, and Q_{NE} are bundles of respectively intermediate tradables produced in Home, intermediate tradables produced in one among the other four regions and imported by Home, and intermediate nontradables produced in the Home country. The parameter $\rho_{IMPE} > 0$ is the elasticity of substitution among imported investment goods, $\rho_E > 0$ is the elasticity of substitution among tradable goods and $\phi_E > 0$ is the elasticity of substitution between tradable and nontradable goods.

The parameter v_H ($v_H > 0$) is the weight of the Home tradable, the parameter $v_{IMPE,i}$ ($0 < v_{IMPE,i} < 1$, $\sum_{i=1}^{4} v_{IMPE,i} = 1$) the weight of the generic imported tradable from country i, and the parameter v_{TE} ($0 < v_{TE} < 1$) the weight of the tradable goods.

A.1.3 Final public consumption good

The public consumption good $C^g_{N,t}$ is fully biased towards the intermediate nontradable good

$$C^g_{N,t}(x) \equiv \left[\left(\frac{1}{n} \right)^{\theta_N} \int_0^n C^g_{N,t}(i, x)^{\frac{\theta_N - 1}{\theta_N}} di \right]^{\frac{\theta_N}{\theta_N - 1}}, \quad \text{(A.3)}$$

where $\theta_N > 1$ is the elasticity of substitution among brands in the nontradable sector.

A.1.4 Demand for intermediate goods

Final consumption goods are composed by CES bundles of differentiated intermediate goods, each produced by a single firm under conditions of monopolistic competition,

$$Q_{HA,t}(x) \equiv \left[\left(\frac{1}{n}\right)^{\theta_T} \int_0^n Q_t(h,x)^{\frac{\theta_T-1}{\theta_T}} dh \right]^{\frac{\theta_T}{\theta_T-1}}, \tag{A.4}$$

$$Q_{GA,t}(x) \equiv \left[\left(\frac{1}{n^*}\right)^{\theta_T} \int_n^{n+n^*} Q_t(g,x)^{\frac{\theta_T-1}{\theta_T}} dg \right]^{\frac{\theta_T}{\theta_T-1}}, \tag{A.5}$$

$$Q_{FA,t}(x) \equiv \left[\left(\frac{1}{n^{**}}\right)^{\theta_T} \int_{n+n^*}^{n+n^*+n^{**}} Q_t(f,x)^{\frac{\theta_T-1}{\theta_T}} df \right]^{\frac{\theta_T}{\theta_T-1}}, \tag{A.6}$$

$$Q_{MA,t}(x) \equiv \left[\left(\frac{1}{n^{***}}\right)^{\theta_T} \int_{n+n^*+n^{**}}^{n+n^*+n^{**}+n^{***}} Q_t(m,x)^{\frac{\theta_T-1}{\theta_T}} dm \right]^{\frac{\theta_T}{\theta_T-1}}, \tag{A.7}$$

$$Q_{RA,t}(x) \equiv \left[\left(\frac{1}{1-n-n^*-n^{**}-n^{***}}\right)^{\theta_T} \int_{n+n^*+n^{**}+n^{***}}^{1} Q_t(r,x)^{\frac{\theta_T-1}{\theta_T}} dr \right]^{\frac{\theta_T}{\theta_T-1}}, \tag{A.8}$$

$$Q_{NA,t}(x) \equiv \left[\left(\frac{1}{n}\right)^{\theta_N} \int_0^n Q_t(i,x)^{\frac{\theta_N-1}{\theta_N}} di \right]^{\frac{\theta_N}{\theta_N-1}}, \tag{A.9}$$

where firms in the Home intermediate tradable and nontradable sectors are respectively indexed by $h \in (0,n]$ and $n \in (0,n]$, firms in US by $g \in (n, n+n^*]$, firms in CH by $f \in (n+n^*, n+n^*+n^{**}]$, firms in JP by $m \in (n+n^*+n^{**}, n+n^*+n^{**}+n^{***}]$ and firms in the RW by $r \in (n+n^*+n^{**}+n^{***}, 1]$. Parameters θ_T, $\theta_N > 1$ are respectively the elasticity of substitution among brands in the tradable and nontradable sector. The prices of the intermediate nontradable goods are denoted $p(i)$. Each firm x producing the final consumption good takes these prices as given when minimizing production costs. The resulting demand for intermediate nontradable input i is

$$Q_{A,t}(i,x) = \left(\frac{1}{n}\right) \left(\frac{P_t(i)}{P_{N,t}}\right)^{-\theta_N} Q_{NA,t}(x), \tag{A.10}$$

where $P_{N,t}$ is the cost-minimizing price of one basket of local nontradable intermediates,

$$P_{N,t} = \left[\int_0^n P_t(i)^{1-\theta_N} di \right]^{\frac{1}{1-\theta_N}}. \tag{A.11}$$

Firms y producing the final investment goods have similar demand curves. Aggregating over x and y, it can be shown that total demand for intermediate nontradable good i by firms in the final consumption and investment sectors is

$$\int_0^n Q_{A,t}(i,x)\,dx + \int_0^n Q_{E,t}(i,y)\,dy + \int_0^n C_{N,t}^{g}(i,x)\,dx =$$
$$\left(\frac{P_t(i)}{P_{N,t}}\right)^{-\theta_N}\left(Q_{NA,t} + Q_{NE,t} + C_{N,t}^{g}\right), \qquad \text{(A.12)}$$

where C_N^g is public sector consumption.

Home demands for (intermediate) domestic and imported tradable goods and the cost-minimizing prices of the corresponding baskets can be derived in a similar way.

A.2 Supply of intermediate goods

We report the production function and the implied first-order conditions. Finally, we show the labor bundle.

Production function The supply of each Home intermediate nontradable good i is denoted by $N^S(i)$:

$$N_t^S(i) = \left(\alpha_{L_N}^{\frac{1}{\xi_N}}\left(TREND_t(i)\,L_{N,t}(i)\right)^{\frac{\xi_N-1}{\xi_N}} + \alpha_{K_N}^{\frac{1}{\xi_N}} K_{N,t}(i)^{\frac{\xi_N-1}{\xi_N}} + \left(1-\alpha_{L_N}-\alpha_{K_N}\right)^{\frac{\xi_N-1}{\xi_N}}\left(\frac{K_{t-1}^G}{gr_t}\right)^{\frac{\xi_N-1}{\xi_N}}\right)^{\frac{\xi_N}{\xi_N-1}}. \qquad \text{(A.13)}$$

The term K_{t-1}^G denotes public capital.

The growth rate of the labor-augmenting technology process, $gr_t = TREND_t/TREND_{t-1}$ is the source of endogenous growth. The term $TREND_t$ represents a unit-root labor-augmenting technology process, that is common to all sectors producing intermediate goods. Because of this process, all real variables have to be divided by the process to be stationary. In the following we describe the model equations by assuming that all real variables are made stationary after dividing them by the productivity level.

Firm i uses labor $L_{N,t}(i)$ and capital $K_{N,t}(i)$ supplied by domestic household and takes the stock of public capital K_{t-1}^G as given. The parameter $\xi_N > 0$ measures the corresponding elasticities of substitution. The parameters $0 < \alpha_{L_N} < 1$ and $0 < \alpha_{K_N} < 1$ are the weights of labor and capital, respectively. Firms producing intermediate goods take the prices of labor and capital and the stock of public capital K_{t-1}^G as given when minimizing their costs. [4]

[4]In the main text we consider the limiting case of $\xi_N = 1$, so that the generic CES production function A.13 becomes a Cobb-Douglas:

$$N_t^S(i) = \left(\left(TREND_t(h)\,L_{N,t}(i)\right)^{\alpha_{L_N}} K_{N,t}(i)^{\alpha_{K_N}}\left(\frac{K_{t-1}^G}{gr_t}\right)^{\left(1-\alpha_{L_N}-\alpha_{K_N}\right)}\right)^{\frac{\xi_N}{\xi_N-1}}. \qquad \text{(A.14)}$$

FOCs: demand of inputs Denoting W_t the nominal wage index and R_t^K the nominal rental price of capital, cost minimization implies that

$$L_{N,t}(i) = \alpha_{L_N} \left(\frac{W_t}{MC_{N^S,t}(i)} \right)^{-\xi_N} N_t^S(i), \tag{A.15}$$

and

$$K_{N,t}(i) = \alpha_{K_N} \left(\frac{R_t^K}{MC_{N^S,t}(i)} \right)^{-\xi_N} N_t^S(i), \tag{A.16}$$

where $MC_{N^S,t}(i)$ is the nominal marginal cost:

$$MC_{N^S,t}(i) = \left(\alpha_{L_N} W_t^{1-\xi_N} + \alpha_{K_N} \left(R_t^K\right)^{1-\xi_N} \right)^{\frac{1}{1-\xi_N}}. \tag{A.17}$$

The production of each Home tradable good, $T^S(h)$, is similarly characterized. Specifically, firms operate a CES production function:

$$T_t^S(h) = \left(\alpha_{L_T}^{\frac{1}{\xi_T}} \left(TREND_t(i) L_{T,t}(i)\right)^{\frac{\xi_T-1}{\xi_T}} + \alpha_{K_T}^{\frac{1}{\xi_T}} K_{T,t}(i)^{\frac{\xi_T-1}{\xi_T}} + \left(1-\alpha_{L_T}-\alpha_{K_T}\right)^{\frac{\xi_T-1}{\xi_T}} \left(\frac{K_{t-1}^G}{gr_t} \right) \right)^{\frac{\xi_T}{\xi_T-1}}. \tag{A.18}$$

The labor-augmenting technology shock specific to the generic US firm f, $TREND^{US}\left(f^{US}\right)$, is defined as

$$\begin{aligned} TREND_t(h) \quad = \quad & A \left(\left(R\&D_t(h)\right)^{\eta} \left(R\&D_t\right)^{1-\eta} \right)^{\gamma} \times \\ & \left(R\&D_t^*\right)^{\gamma^*} \times \left(R\&D_t^{**}\right)^{\gamma^{**}} \times \left(R\&D_t^{***}\right)^{1-\gamma-\gamma^*-\gamma^{**}-\gamma^{***}}, \forall h, \end{aligned} \tag{A.19}$$

where $A > 0$ is a scaling parameter, $R\&D(h)$ is the Home firm h's demand for the stock of R&D (accumulated by domestic households), while the aggregate stock of R&D in Home is

$$R\&D \quad = \quad \int_0^n R\&D_t(h)\, dh, \tag{A.20}$$

where $0 < n < 1$ is the number of firms in the Home tradable sector. Similar equations hold for US, JP and CH.

The $TREND(h)$ is positively affected by the stock of R&D optimally chosen by the generic firms in Home, US, JP, and CH intermediate tradable sectors. When choosing the optimal $R\&D(h)$, the generic firm h takes into account its direct contribution to $TREND(h)$ (measured by the parameter η, $0 < \eta < 1$). The parameters γ measure the elasticity of $TREND(f)$ with respect to country-specific R&D ($0 < \gamma, \gamma^*, \gamma^{**}, \gamma^{***} < 1$, $\gamma + \gamma^* + \gamma^{**} + \gamma^{***} < 1$). The generic firm h optimally demands capital, labor, and R&D (all of them are supplied by domestic households), taking as given prices, the stock of public capital (accumulated by domestic government), the R&D accumulated by other (domestic and foreign) individual firms, and the aggregate R&D in each domestic and foreign sector.[5]

A similar trend holds for every firm in US (and JP, CH, and RW):

$$\begin{aligned} TREND_t^*(h^*) &= A\left(\left(R\&D_t^*(h^*)\right)^{\eta^*}\left(R\&D_t^*\right)^{1-\eta^*}\right)^{\gamma^*} \times \\ & \left(R\&D_t\right)^{\gamma} \times \left(R\&D_t^{**}\right)^{\gamma^{**}} \times \left(R\&D_t^{***}\right)^{1-\gamma-\gamma^*-\gamma^{**}-\gamma^{***}}, \end{aligned} \tag{A.21}$$

As we consider a symmetric equilibrium, in which all firms belonging to the same sector make the same choices, $TREND_t(h)$ will end up being the same for every Home firm. The same is true for every firm in US, JP, CH, and RW (even if RW firms are assumed to not invest in R&D). Thus, $TREND_t(h)$ will be equal to all trends in other regions. This implies that there is a (common) global trend of labor-augmenting technology shock $TREND_t^{world}$, i.e. in the symmetric equilibrium the trend is common across all firms producing intermediate tradable and nontradable goods in all regions of the global economy,
$TREND_t = TREND_t^* = TREND_t^{**} = TREND_t^{***} = TREND_t^{****} = TREND_t^{world}$.

Firms in the nontradable sector demand physical capital and labor supplied by domestic households, and take public capital and the (global common) labor-augmenting technology as given. They do not invest in R&D, in line with the empirical evidence.

FOCs: supply of intermediate tradables Firms operating in the Home intermediate tradable sector solve a similar problem. We assume that there is market segmentation because nominal prices are invoiced and sticky in the currency of the destination market (local currency pricing).

The (generic) Home firm producing the brand h chooses the optimal prices $P_t(h)$ in the Home market, $P_t^*(h)$ in the US, $P_t^{**}(h)$ in CH, $P_t^{***}(h)$ in JP and $P_t^{****}(h)$ in the RW to maximize the expected flow of profits (in terms of domestic consumption units),

$$E_t \sum_{\tau=t}^{\infty} \beta^{\tau} \lambda_{t,\tau} \left[\begin{array}{c} \frac{P_\tau(h)}{P_\tau} Q_\tau(h) + \frac{S_\tau P_\tau^*(h)}{P_\tau} Q_\tau^*(h) \\ + \frac{S_\tau P_\tau^{**}(h)}{S_\tau^{**} P_\tau} Q_\tau^{**}(h) + \frac{S_\tau P_\tau^{***}(h)}{S_\tau^{***} P_\tau} Q_\tau^{***}(h) + \frac{S_\tau P_\tau^{****}(h)}{S_\tau^{****} P_\tau} Q_\tau^{****}(h) \\ - \frac{MC_{H,\tau}(h)}{P_\tau} \left(Q_\tau(h) + Q_\tau^*(h) + Q_\tau^{**}(h) + Q_\tau^{***}(h) + Q_\tau^{****}(h)\right) \end{array} \right], \tag{A.22}$$

where the term E_t denotes the expectation operator conditional on the information set at time t, $\lambda_{t,\tau}$ is the appropriate stochastic discount factor, and $MC_{T,t}(h)$ is the nominal marginal cost, S, S^{**}, S^{***}, S^{****} are the nominal exchange rates of the Home, CH, JP, and RW currency vis-à-vis the US dollar, respectively.

The maximization is subject to the demand of the destination market and (destination-

[5] Firms do not demand public capital and there is no price or tariff paid for its use.

specific) quadratic price adjustment costs.

The demand curves for the generic Home intermediate good h depend on local demand for all Home goods and on relative prices. Thus, the demand curves in Home country, US, CH, JP, and RW are

$$Q_{\tau}(h) = \left(\frac{P_{H,\tau}(h)}{P_{H,\tau}}\right)^{-\theta_H} Q_{H,\tau}, \tag{A.23}$$

$$Q_{\tau}^{*}(h) = \left(\frac{P_{H,\tau}^{*}(h)}{P_{H,\tau}^{*}}\right)^{-\theta_H} Q_{H,\tau}^{*}, \tag{A.24}$$

$$Q_{\tau}^{**}(h) = \left(\frac{P_{H,\tau}^{**}(h)}{P_{H,\tau}^{**}}\right)^{-\theta_H} Q_{H,\tau}^{**}, \tag{A.25}$$

$$Q_{\tau}^{***}(h) = \left(\frac{P_{H,\tau}^{***}(h)}{P_{H,\tau}^{***}}\right)^{-\theta_H} Q_{H,\tau}^{***}, \tag{A.26}$$

$$Q_{\tau}^{****}(h) = \left(\frac{P_{H,\tau}^{****}(h)}{P_{H,\tau}^{****}}\right)^{-\theta_H} Q_{H,\tau}^{****}, \tag{A.27}$$

respectively.

The country-specific adjustment costs paid by the generic firm h are

$$AC_{H,\tau}^{p}(i) \equiv \frac{\kappa_H^p}{2}\left(\frac{P_{H,\tau}(h)/P_{H,\tau-1}(h)}{\pi_{H,\tau-1}^{ind_H}\pi_{target}^{1-ind_H}} - 1\right)^2 \frac{P_{H,\tau}}{P_{\tau}} Q_{H,\tau}, \tag{A.28}$$

$$AC_{H,\tau}^{p*}(h) \equiv \frac{\kappa_H^{p*}}{2}\left(\frac{P_{H,\tau}^{*}(h)/P_{H,\tau-1}^{*}(h)}{\left(\pi_{H,\tau-1}^{*}\right)^{ind_H^*}\pi_{target^*}^{1-ind_H^*}} - 1\right)^2 \frac{S_{\tau}P_{H,\tau}^{*}}{P_{\tau}^{*}} Q_{H,\tau}^{*}, \tag{A.29}$$

$$AC_{H,\tau}^{p**}(i) \equiv \frac{\kappa_H^{p**}}{2}\left(\frac{P_{H,\tau}^{**}(h)/P_{H,\tau-1}^{**}(h)}{\left(\pi_{H,\tau-1}^{**}\right)^{ind_H^{**}}\pi_{target^{**}}^{1-ind_H^{**}}} - 1\right)^2 \frac{S_{\tau}P_{H,\tau}^{**}}{S_{\tau}^{**}P_{\tau}^{**}} Q_{H,\tau}^{**}, \tag{A.30}$$

$$AC_{H,\tau}^{p***}(i) \equiv \frac{\kappa_H^{p***}}{2}\left(\frac{P_{H,\tau}^{***}(h)/P_{H,\tau-1}^{***}(h)}{\left(\pi_{H,\tau-1}^{***}\right)^{ind_H^{***}}\pi_{target^{***}}^{1-ind_H^{***}}} - 1\right)^2 \frac{S_{\tau}P_{H,\tau}^{**}}{S_{\tau}^{***}P_{\tau}^{***}} Q_{H,\tau}^{***}, \tag{A.31}$$

$$AC_{H,\tau}^{p****}(i) \equiv \frac{\kappa_H^{p****}}{2}\left(\frac{P_{H,\tau}^{****}(h)/P_{H,\tau-1}^{****}(h)}{\left(\pi_{H,\tau-1}^{****}\right)^{ind_H^{****}}\pi_{target^{****}}^{1-ind_H^{****}}}-1\right)^2 \frac{S_\tau P_{H,\tau}^{****}}{S_\tau^{****}P_\tau^{****}}Q_{H,\tau}^{****}, \tag{A.32}$$

in the domestic, US, CH, JP, and RW markets, respectively. The parameters κ_H^p,$\kappa_H^{p\ *}$,$\kappa_H^{p\ **}$, $\kappa_H^{p\ ***}$, $\kappa_H^{p\ ****} > 0$ measure the degree of nominal rigidity in the Home country, US, CH, JP, and RW, respectively, whereas ind_H, ind_H^*, ind_H^{**} ind_H^{***}, ind_H^{****} are the corresponding indexation parameters. Moreover, π_{target}, π_{target^*}, $\pi_{target^{**}}$ $\pi_{target^{***}}$, $\pi_{target^{****}}$denote the long-run (consumer-price) inflation targets set by the central bank in EA, US, CH, JP and RW, respectively.

First order condition with respect to the domestic price of the Home tradable good. The first order condition with respect to the Home price set in the domestic market, $P_t(h)$, is:

$$\begin{aligned}
0 = {} & \frac{\lambda_t}{P_t}\left(\frac{P_{H,t}(h)}{P_{H,t}}\right)^{-\theta_H} Q_{H,t} \\
& -\frac{\lambda_t}{P_t}\theta_H \frac{(P_{H,t}(h))^{-\theta_H-1}}{P_{H,t}^{-\theta_H}}\left(P_{H,t}(h) - MC_{H,t}(h)\right)Q_{H,t} \\
& -\lambda_t \kappa_H^p \frac{1/P_{H,t-1}(h)}{\pi_{H,t-1}^{ind_H}\pi_{target}^{1-ind_H}}\left(\frac{P_{H,t}(h)/P_{H,t-1}(h)}{\pi_{H,t-1}^{ind_H}\pi_{target}^{1-ind_H}}-1\right)\frac{P_{H,t}}{P_t}Q_{H,t} \\
& +\beta\kappa_H^p E_t\left[\lambda_{t+1}\frac{P_{H,t+1}(h)/P_{H,t}(h)^2}{\pi_{H,t}^{ind_H}\pi_{target}^{1-ind_H}}\left(\frac{P_{H,t+1}(h)/P_{H,t}(h)}{\pi_{H,t}^{ind_H}\pi_{target}^{1-ind_H}}-1\right)\frac{P_{H,t+1}}{P_{t+1}}Q_{H,t+1}\right],
\end{aligned} \tag{A.33}$$

where $\theta_H > 1$ is the elasticity of substitution among intermediate tradable brands, while the terms multiplied by κ_H^p are related to the presence of price adjustment costs.

In symmetric equilibrium (i.e., $P_{H,t}(h) = P_{H,t}$ for every h) and dividing all terms by $\lambda_t Q_{H,t}$, the FOC becomes

$$\begin{aligned}
0 = {} & \frac{1}{P_t}\left(\frac{P_{H,t}}{P_{H,t}}\right)^{-\theta_H} \\
& -\frac{1}{P_t}\theta_H \frac{(P_{H,t})^{-\theta_H-1}}{P_{H,t}^{-\theta_H}}\left(P_{H,t}(h) - MC_{H,t}\right) \\
& -\kappa_H^p \frac{1/P_{H,t-1}}{\pi_{H,t-1}^{ind_H}\pi_{target}^{1-ind_H}}\left(\frac{P_{H,t}/P_{H,t-1}}{\pi_{H,t-1}^{ind_H}\pi_{target}^{1-ind_H}}-1\right)\frac{P_{H,t}}{P_t} \\
& +\beta\kappa_H^p E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\frac{P_{H,t+1}/P_{H,t}^2}{\pi_{H,t}^{ind_H}\pi_{target}^{1-ind_H}}\left(\frac{P_{H,t+1}/P_{H,t}}{\pi_{H,t}^{ind_H}\pi_{target}^{1-ind_H}}-1\right)\frac{P_{H,t+1}}{P_{t+1}}\frac{Q_{H,t+1}}{Q_{H,t}}\right];
\end{aligned} \tag{A.34}$$

using the definition $\pi_{H,t} \equiv P_{H,t}/P_{H,t-1}$ and rearranging terms we get

$$
\begin{aligned}
\frac{P_{H,t}}{P_t} &= \theta_H \left(P_{H,t} - MC_{H,t}\right) \frac{1}{P_t} \\
&+ \frac{P_{H,t}}{P_t} \kappa_H^p \frac{\pi_{H,t}}{\pi_{H,t-1}^{ind_H} \pi_{target}^{1-ind_H}} \left(\frac{\pi_{H,t}}{\pi_{H,t-1}^{ind_H} \pi_{target}^{1-ind_H}} - 1 \right) \\
&- \beta \kappa_H^p E_t \left[\frac{P_{H,t}}{P_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{H,t+1}^2}{\pi_{H,t}^{ind_H} \pi_{target}^{1-ind_H}} \left(\frac{\pi_{H,t+1}}{\pi_{H,t}^{ind_H} \pi_{target}^{1-ind_H}} - 1 \right) \frac{Q_{H,t+1}}{Q_{H,t}} \right].
\end{aligned} \tag{A.35}
$$

The equation, in real terms (i.e., in units of domestic consumption) becomes

$$
p_{H,t} = \theta_H \left(p_{H,t} - mc_{H,t}\right) + A_{H,t}, \tag{A.36}
$$

where $p_{H,t} \equiv P_{H,t}/P_t$ (similar expressions hold for other relative prices), $A_{H,t}$ is defined as

$$
\begin{aligned}
A_{H,t} &\equiv p_{H,t} \kappa_H^p \frac{\pi_{H,t}}{\pi_{H,t-1}^{ind_H} \pi_{target}^{1-ind_H}} \left(\frac{\pi_{H,t}}{\pi_{H,t-1}^{ind_H} \pi_{target}^{1-ind_H}} - 1 \right) \\
&- p_{H,t} \beta \kappa_H^p E_t \left[\frac{1}{\pi_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{H,t+1}^2}{\pi_{H,t}^{ind_H} \pi_{target}^{1-ind_H}} \left(\frac{\pi_{H,t+1}}{\pi_{H,t}^{ind_H} \pi_{target}^{1-ind_H}} - 1 \right) \frac{Q_{H,t+1}}{Q_{H,t}} \right].
\end{aligned} \tag{A.37}
$$

First order condition with respect to the price of the Home tradable good in the US market. The FOC with respect to price of the Home good in the US $P_t^*(h)$ is

$$
\begin{aligned}
0 &= \frac{\lambda_t}{P_t} \left(\frac{P_{H,t}^*(h)}{P_{H,t}^*} \right)^{-\theta_H^*} Q_{H,t}^* S_t^* \\
&- \frac{\lambda_t}{P_t} \theta_H^* \frac{\left(P_{H,t}^*(h)\right)^{-\theta_H^*-1}}{{P_{H,t}^*}^{-\theta_H^*}} \left(P_{H,t}^*(h) - MC_{H,t}^*(h) \right) Q_{H,t}^* S_t^* \\
&- \lambda_t {\kappa_H^p}^* \frac{1/P_{H,t-1}^*(h)}{{\pi_{H,t-1}^*}^{ind_H^*} {\pi_{target}^*}^{1-ind_H^*}} \left(\frac{P_{H,t}^*(h)/P_{H,t-1}^*(h)}{{\pi_{H,t-1}^*}^{ind_H^*} {\pi_{target}^*}^{1-ind_H^*}} - 1 \right) \frac{P_{H,t}^*}{P_t} Q_{H,t}^* S_t^* \\
&+ \beta {\kappa_H^p}^* E_t \left[\lambda_{t+1} \frac{P_{H,t+1}^*(h)/P_{H,t}^*(h)^2}{{\pi_{H,t}^*}^{ind_H^*} {\pi_{target}^*}^{1-ind_H^*}} \left(\frac{P_{H,t+1}^*(h)/P_{H,t}^*(h)}{{\pi_{H,t}^*}^{ind_H^*} {\pi_{target}^*}^{1-ind_H^*}} - 1 \right) \frac{P_{H,t+1}^*}{P_{t+1}^*} Q_{H,t+1}^* S_{t+1}^* \right].
\end{aligned} \tag{A.38}
$$

In symmetric equilibrium, the previous equation becomes

$$
p_{H,t}^* = \theta_H^* \left(p_{H,t}^* - mc_{H,t}^* \right) + A_{H,t}^*, \tag{A.39}
$$

where $A^*_{H,t}$ is

$$A^*_{H,t} \equiv p^*_{H,t}{\kappa^*_H}^p \frac{\pi^*_{H,t}}{{\pi^*_{H,t-1}}^{ind^*_H}{\pi^*_{target}}^{1-ind^*_H}} \left(\frac{\pi^*_{H,t}}{{\pi^*_{H,t-1}}^{ind^*_H}{\pi^*_{target}}^{1-ind^*_H}} - 1 \right)$$
$$-\beta\kappa^p_H E_t \left[p^*_{H,t} \frac{1}{\pi_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \frac{\pi^2_{H,t+1}}{{\pi^*_{H,t}}^{ind^*_H}{\pi^*_{target}}^{1-ind^*_H}} \left(\frac{\pi^*_{H,t+1}}{{\pi^*_{H,t}}^{ind^*_H}{\pi^*_{target}}^{1-ind^*_H}} - 1 \right) \frac{Q^*_{H,t+1}}{Q^*_{H,t}} \frac{S^*_{t+1}}{S^*_t} \right]. \tag{A.40}$$

First order conditions with respect to the price of the Home tradable good in CH, JP and RW. Similar equations hold for the prices $P^{**}_t(h)$, $P^{***}_t(h)$ and $P^{****}_t(h)$set in the CH, JP and RW market, respectively.

First order condition with respect to the price of the Home nontradable good. Each firm i sets the price $P_{N,t}(i)$ in the Home intermediate nontradable sector to maximize the present discounted value of profits

$$E_t \sum_{\tau=t}^{\infty} \beta^t \lambda_{t,\tau} \left[Q_{N,\tau}(i) \left(\frac{P_{N,\tau}(i)}{P\tau} - \frac{MC_{N,\tau}(i)}{P\tau} \right) \right], \tag{A.41}$$

where the term E_t denotes the expectation operator conditional on the information set at time t, $\lambda_{t,\tau}$ is the appropriate discount rate, and $MC_{N,t}(h)$ is the nominal marginal cost.

The maximization is subject to the demand constraint

$$Q_{N,\tau}(i) = \left(\frac{1}{n}\right) \left(\frac{P_{N,\tau}(i)}{P_{N,\tau}}\right)^{-\theta_N} Q_{N,\tau}, \tag{A.42}$$

and the quadratic adjustment cost,

$$AC^p_{N,\tau}(i) \equiv \frac{\kappa^p_N}{2} \left(\frac{P_{N,\tau}(i)/P_{N,\tau-1}(i)}{\pi^{ind_N}_{N,\tau-1}\pi^{1-ind_N}_{target}} - 1 \right)^2 \frac{P_{N,\tau}}{P_\tau} Q_{N,\tau}. \tag{A.43}$$

The adjustment cost is paid in unit of sector-specific product $Q_{N,t}$, where $\kappa^p_N \geq 0$ is a parameter that measures the degree of price stickiness, $\pi_{N,t-1}$ is the previous-period gross inflation rate of nontradable goods $(\pi_{N,t} \equiv P_{N,t}/P_{N,t-1})$, π is the long-run (consumer-price) inflation target set by the central bank, and $0 \leq ind_N \leq 1$ is a parameter that measures indexation to previous-period inflation.

The FOC with respect to $P_{N,t}(i)$ is

$$
\begin{aligned}
0 = {} & \frac{\lambda_t}{P_t}(1-\theta_N)\frac{P_{N,t}(i)^{-\theta_N}}{P_{N,t}{}^{-\theta_N}}Q_{N,t} - \frac{\lambda_t}{P_t}\theta_N \frac{P_{N,t}(i)^{-\theta_N - 1}}{P_{N,t}{}^{-\theta_N}} MC_{N,t}(i)\, Q_{N,t} \\
& -\lambda_t \kappa_N^p \frac{1/P_{N,t-1}(i)}{\pi_{N,t-1}^{ind_N}\pi_{target}^{1-ind_N}} \left(\frac{P_{N,t}(i)/P_{N,t-1}(i)}{\pi_{N,t-1}^{ind_N}\pi_{target}^{1-ind_N}} - 1 \right) \frac{P_{N,t}}{P_t} Q_{N,t} \\
& +\beta \kappa_N^p E_t \left[\lambda_{t+1} \frac{P_{N,t+1}(i)/P_{N,t}^2(i)}{\pi_{N,t}^{ind_N}\pi_{target}^{1-ind_N}} \left(\frac{P_{N,t+1}(i)/P_{N,t}(i)}{\pi_{N,t}^{ind_N}\pi_{target}^{1-ind_N}} - 1 \right) \frac{P_{N,t+1}}{P_{t+1}} Q_{N,t+1} \right]; \qquad \text{(A.44)}
\end{aligned}
$$

which becomes

$$
p_{N,t} = \frac{\theta_N}{\theta_N - 1} mc_{N,t} - \frac{A_{N,t}}{\theta_N - 1}, \qquad \text{(A.45)}
$$

where $A_{N,t}$ contains terms related to the presence of price adjustment costs

$$
\begin{aligned}
A_{N,t} \equiv {} & p_{N,t}\kappa_N^p \frac{\pi_{N,t}}{\pi_{N,t-1}^{ind_N}\pi_{target}^{1-ind_N}} \left(\frac{\pi_{N,t}}{\pi_{N,t-1}^{ind_N}\pi_{target}^{1-ind_N}} - 1 \right) \\
& -\beta \kappa_N^p E_t \left[p_{N,t} \frac{1}{\pi_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \frac{\pi_{N,t+1}^2}{\pi_{N,t}^{ind_N}\pi_{target}^{1-ind_N}} \left(\frac{\pi_{N,t+1}}{\pi_{N,t}^{ind_N}\pi_{target}^{1-ind_N}} - 1 \right) \frac{Q_{N,t+1}}{Q_{N,t}} \right]. \qquad \text{(A.46)}
\end{aligned}
$$

Price of EA imports from the US The US producer of the tradable brand f solves the following profit maximization problem (similar to the one solved by the EA exporter):

$$
E_t \sum_{\tau=t}^{\infty} \beta^{\tau} \lambda_{t,\tau} \left[\begin{array}{c}
\frac{P_\tau(f)}{P_\tau S_\tau^*} Q_\tau(f) + \frac{P_\tau^*(f)}{P_\tau^*} Q_\tau^*(f) + \frac{P_\tau^{**}(f)}{P_\tau^{**} S_\tau^{**}} Q_\tau^{**}(f) + \\
\frac{P_\tau^{***}(f)}{P_\tau^{***} S_\tau^{***}} Q_\tau^{***}(f) + \\
\frac{P_\tau^{****}(f)}{P_\tau^{****} S_\tau^{****}} Q_\tau^{****}(f) + \\
-\frac{MC_{F,\tau}^*(f)}{P_\tau} \left(Q_\tau(f) + Q_\tau^*(f) + Q_\tau^{**}(f) + Q_\tau^{***}(f) + Q_\tau^{****}(f) \right)
\end{array} \right]. \qquad \text{(A.47)}
$$

The demand curve in Home country for US goods is

$$
Q_\tau(f) = \left(\frac{P_{F,\tau}(f)}{P_{F,\tau}} \right)^{-\theta_F} Q_{F,\tau}, \qquad \text{(A.48)}
$$

while the (country-specific) adjustment costs paid by the generic firm f in the Home country, invoiced in the currency of the Home country, are

$$
AC_{F,\tau}^p(f) \equiv \frac{\kappa_F^p}{2} \left(\frac{P_{F,\tau}(f)/P_{F,\tau-1}(f)}{\pi_{F,\tau-1}^{ind_F}\left(\pi^{target}\right)^{1-ind_F}} - 1 \right)^2 \frac{P_{F,\tau}}{P_\tau} Q_{F,\tau}. \qquad \text{(A.49)}
$$

The FOC with respect to $P_{F,t}(f)$ is

$$\begin{aligned}0 = & \frac{\lambda_t}{P_t}\left(\frac{P_{F,t}(f)}{P_{F,t}}\right)^{-\theta_F} Q_{F,t} \\ & -\frac{\lambda_t}{P_t}\theta_F \frac{(P_{F,t}(f))^{-\theta_F-1}}{P_{F,t}^{-\theta_F}}\left(P_{F,t}(f) - MC_{F,t}^*(h) S_t\right) Q_{F,t} \\ & -\lambda_t \kappa_F^p \frac{1/P_{F,t-1}(f)}{\pi_{F,t-1}^{ind_F}\pi_{target}^{1-ind_F}}\left(\frac{P_{F,t}(f)/P_{F,t-1}(f)}{\pi_{F,t-1}^{ind_F}\pi_{target}^{1-ind_F}} - 1\right)\frac{P_{F,t}}{P_t} Q_{F,t} S_t \\ & +\beta\kappa_F^p E_t\left[\lambda_{t+1}\frac{P_{F,t+1}(f)/P_{F,t}(f)^2}{\pi_{F,t}^{ind_F}\pi_{target}^{1-ind_F}}\left(\frac{P_{F,t+1}(f)/P_{F,t}(f)}{\pi_{F,t}^{ind_F}\pi_{target}^{1-ind_F}} - 1\right)\frac{P_{F,t+1}}{P_{t+1}} Q_{F,t+1} S_{t+1}\right]. \qquad \text{(A.50)}\end{aligned}$$

In the symmetric equilibrium, the previous equation becomes

$$p_{F,t} = \theta_F\left(P_{F,t} - mc_{F,t} S_t\right) + A_{F,t}, \qquad \text{(A.51)}$$

where $A_{F,t}$ contains terms related to the presence of price adjustment costs

$$\begin{aligned}A_{F,t} \equiv & \; p_{F,t}\kappa_F^p \frac{\pi_{F,t}}{\pi_{F,t-1}^{ind_F}\pi_{target}^{1-ind_F}}\left(\frac{\pi_{F,t}}{\pi_{F,t-1}^{ind_F}\pi_{target}^{1-ind_F}} - 1\right) \\ & -\beta\kappa_F^p E_t\left[p_{F,t}\frac{1}{\pi_{t+1}}\frac{\lambda_{t+1}}{\lambda_t}\frac{\pi_{F,t+1}^2}{\pi_{F,t}^{ind_F}\pi_{target}^{1-ind_F}}\left(\frac{\pi_{F,t+1}}{\pi_{F,t}^{ind_F}\pi_{target}^{1-ind_F}} - 1\right)\frac{Q_{F,t+1}}{Q_{F,t}}\frac{S_{t+1}}{S_t}\right]. \qquad \text{(A.52)}\end{aligned}$$

Labor bundle In the case of the generic firm i operating in the intermediate nontradable sector, the labor input $L_N(i)$ is a CES combination of differentiated labor inputs supplied by domestic agents and defined over a continuum of mass equal to the country size ($j \in [0, n]$):

$$L_{N,t}(i) \equiv \left(\frac{1}{n}\right)^{\frac{1}{\sigma_L}}\left[\int_0^n L_t(i,j)^{\frac{\sigma_L-1}{\sigma_L}} dj\right]^{\frac{\sigma_L}{\sigma_L-1}}, \qquad \text{(A.53)}$$

where $L(i,j)$ is the demand of the labor input of type j by the producer of good i and $\sigma_L > 1$ is the elasticity of substitution among labor inputs. Cost minimization implies that

$$L_t(i,j) = \left(\frac{1}{n}\right)\left(\frac{W_t(j)}{W_t}\right)^{-\sigma_L} L_{N,t}(j), \qquad \text{(A.54)}$$

where $W(j)$ is the nominal wage of labor input j and the wage index W is

$$W_t = \left[\left(\frac{1}{n}\right)\int_0^n W_t(j)^{1-\sigma_L} dj\right]^{\frac{1}{1-\sigma_L}}. \qquad \text{(A.55)}$$

Similar equations hold for firms producing intermediate tradable goods. Each household is the monopolistic supplier of a labor input j and sets the nominal wage facing a downward-sloping demand obtained by aggregating demand across domestic firms.

A.3 Households

In the Home country there is a countinuum of households of mass $j \in [0, n]$. Each household j maximizes its lifetime expected utility subject to the budget constraint. The lifetime utility, in consumption C and labor L, is

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\log \left(C_t(j) - b_c \frac{C_{t-1}}{gr_t} \right) - \frac{1}{1+\zeta} L_t\left(j\right)^{1+\zeta} \right), \tag{A.56}$$

where $0 < \beta < 1$ is the discount factor, $b_c \in (0, 1)$ is the external habit parameter, $\zeta > 0$ is the reciprocal of the Frisch elasticity of labor supply, and L_t is the demand of the household-specific labor type by domestic firms,

$$L_t\left(j\right) = \left(\frac{W_t(j)}{W_t} \right)^{-\sigma_L} L_t, \tag{A.57}$$

where the parameter $\sigma_L > 1$ measures the elasticity of substitution among different labor brands supplied by different households. The budget constraint is

$$\begin{aligned} B_{H,t}(j) + S_t B_{F,t}(j) \leq R_{t-1} \frac{B_{H,t-1}(j)}{gr_t} \\ + R^*_{t-1} Z_{USRP,t-1} \left[1 - \Gamma_{B_F,t-1}\right] S_t \frac{B_{F,t-1}(j)}{gr_t} + R^k_t \frac{K_{t-1}(j)}{gr_t} + W_t(j) L_t(j) \\ - \frac{\kappa_W}{2} \left(\frac{W_t(j)/W_{t-1}(j)}{\pi_{W,t}^{ind_W} \pi_t^{1-ind_W}} - 1 \right)^2 W_t L_t + \Pi^{prof}_t(j) - P_{C,t} C_t(j) - P_{I,t} I_t(j) - P_{C,t} I_{R\&D,t}(j), \end{aligned} \tag{A.58}$$

where the parameter $\kappa_W > 0$ measures the degree of nominal wage rigidity and L_t is the total amount of labor in the Home economy. $B_{H,t}$ is the end-of-period t position in a nominal bond denominated in the Home currency, $B_{F,t}$ is the end-of period position in a nominal bond denominated in US dollars. The two bonds respectively pay the domestic R_t and US R^*_t (gross nominal) policy rates at the beginning of period $t+1$. The interest rates are known at time t (consistent with the riskless bond assumption). The variable S_t is the bilateral nominal exchange rate of the domestic currency vis-à-vis the US dollar, defined as number of Home currency units per unit of US dollar.

The function $\Gamma_{B_F,t}$ captures the costs of undertaking positions in the international asset market and pins down a well-defined steady-state. It has the following functional form:

$$\Phi \left(\frac{S_t B_{F,t}}{P_t} - b \right) \equiv \exp \left(\phi_b \left(\frac{S_t B_{F,t}}{P_t} - b \right) \right) \qquad \phi_B \geq 0.$$

The parameter ϕ_B controls the speed of convergence to the non-stochastic steady state.[6]

The sources of the household income are physical capital $K_t(j)$, which is rented to domestic intermediate firms at the net rate R_t^k, labor $L_t(j)$, which is supplied to domestic firms and earns the nominal wage $W_t(j)$, and $\Pi_t^{prof}(j)$, which represents profits from ownership of domestic firms (the profits are rebated in a lump-sum way to households).

The variable $I_t(j)$ is investment in physical capital. The latter is accumulated according to the following law:

$$K_t(j) \leq (1-\delta)\frac{K_{t-1}(j)}{gr_t} + \left[1 - \frac{\psi}{2}\left(\frac{I_t(j)}{I_{t-1}(j)} - gr_t\right)^2 I_t(j)\right], \tag{A.59}$$

where $0 < \delta < 1$ is the depreciation rate and investment is subject to a quadratic adjustment cost.

R&D is accumulated by the generic household according to

$$R\&D_t(j) = (1-\delta_{R\&D})\frac{R\&D_{t-1}(j)}{gr_t} + \left(1 - \frac{\psi_{R\&D}}{2}\left(\frac{I_{R\&D,t}(j)}{I_{R\&D,t-1}(j)} - gr_t\right)\right)^2 I_{R\&D,t}(i), \tag{A.60}$$

The parameter $0 < \delta_{R\&D} < 1$ is the depreciation rate; $\psi_{R\&D} > 0$ is a parameter measuring R&D investment adjustment costs and $I_{R\&D,t}$ is the investment in $R\&D$ (whose composition is assumed to be the same as that of private consumption).

A.3.1 First-order conditions

Household maximizes the intertemporal utility with respect to consumption $C_t(j)$, $B_{H,t}(j)$, $B_{F,t}(j)$, $W_t(j)$, subject to the budget constraint, the capital accumulation law, the R&D accumulation law, and the adjustment costs.

The corresponding FOCs in the generic period t are:

- with respect to domestic bond $C_t(j)$

$$\lambda_t(j) = \left(C_t(j) - b_c\frac{C_{t-1}}{gr_t}\right)^{-1}, \tag{A.61}$$

- with respect to domestic euro-denominated bond $B_{H,t}(j)$

$$\lambda_t(j) = \beta E_t \frac{\lambda_{t+1}(j)}{gr_{t+1}} R_t \pi_{t+1}^{-1}, \tag{A.62}$$

[6] The function $\Phi(.)$ depends on real holdings of the foreign assets in the entire Home economy. Hence, domestic households take it as given when deciding on the optimal holding of the foreign bond. We require that $\Phi(0) = 1$ and that $\Phi(.) = 1$ only if $S_t B_{F,t}/P_t = b$, where b is the steady state real holdings of the foreign assets in the entire home economy. The function $\Phi(.)$ is assumed to be differentiable and decreasing at least in the neighborhood of the steady state. The payment of this cost is rebated in a lump-sum fashion to foreign agents.

- with respect to US-dollar bonds $B_{F,t}(j)$

$$\lambda_t(j) = \beta E_t \frac{\lambda_{t+1}(j)}{gr_{t+1}} R_t^*(1-\Gamma_{B,t}) \frac{\Delta S_{t+1}}{\pi_{t+1}}, \tag{A.63}$$

- with respect to the end-of-period capital $K_t(j)$

$$Q_t(j) = \beta E_t \left[\lambda_{t+1} \frac{r_{t+1}^K}{gr_{t+1}} + Q_{t+1}(j) \frac{(1-\delta)}{gr_{t+1}} \right], \tag{A.64}$$

where $Q(j)$ is the Tobin's Q (i.e., the multiplier of the capital accumulation law),

- with respect to investment $I_t(j)$

$$\begin{aligned} \lambda_t(j) p_{I,t} = Q_t(j) &\left[1 - \frac{\psi}{2} \left(\frac{I_t(j)}{I_{t-1}(j)} - gr_t \right)^2 - \psi \left(\frac{I_t(j)}{I_{t-1}(j)} - gr_t \right) \frac{I_t(j)}{I_{t-1}(j)} \right] \\ &+ \beta E_t Q_{t+1}(j) \psi \left[\left(\frac{I_{t+1}(j)}{I_t(j)} - gr_{t+1} \right) \frac{I_{t+1}^2(j)}{I_t^2(j)} \right], \end{aligned} \tag{A.65}$$

- with respect to the end-of-period R&D stock $R\&D_t(j)$

$$Q_{R\&D,t}(j) = \beta E_t \left[\lambda_{t+1} \frac{r_{t+1}^{R\&D}}{gr_{t+1}} + Q_{R\&D,t+1}(j) \frac{(1-\delta_{R\&D})}{gr_{t+1}} \right], \tag{A.66}$$

where $Q_{R\&D}(j)$ is the R&D-specific Tobin's Q (i.e., the multiplier of the R&D accumulation law),

- with respect to R&D investment $I_{R\&D,t}(j)$

$$\begin{aligned} \lambda_t(j) = Q_{R\&D,t}(j) &\left[1 - \frac{\psi_{R\&D}}{2} \left(\frac{I_{R\&D,t}(j)}{I_{R\&D,t-1}(j)} - gr_t \right)^2 - \psi_{R\&D} \left(\frac{I_{R\&D,t}(j)}{I_{R\&D,t-1}(j)} - gr_t \right) \frac{I_{R\&D,t}(j)}{I_{R\&D,t-1}(j)} \right] \\ &+ \beta E_t Q_{R\&D,t+1}(j) \psi_{R\&D} \left[\left(\frac{I_{R\&D,t+1}(j)}{I_{R\&D,t}(j)} - gr_{t+1} \right) \frac{I_{R\&D,t+1}^2(j)}{I_{R\&D,t}^2(j)} \right], \end{aligned} \tag{A.67}$$

- with respect to nominal wage $W_t(j)$

$$\begin{aligned} \sigma_L \frac{W_t(j)^{-\sigma_L(1+\zeta)-1}}{W_t^{-\sigma_L(1+\zeta)}} L_t^{\zeta} + (1-\sigma_L) \frac{W_t(j)^{-\sigma_L}}{W_t^{-\sigma_L}} &= \lambda_t \kappa_W \left(\frac{W_t(j)/W_{t-1}(j)}{\pi_{W,t-1}^{ind_w} \pi^{1-ind_w}} - 1 \right) \frac{W_t/W_{t-1}(j)}{\pi_{W,t-1}^{ind_I} \pi^{1-ind_w}} \\ &- \beta \lambda_{t+1} \kappa_W \left(\frac{W_{t+1}(j)/W_t(j)}{\pi_{W,t}^{ind_w} \pi^{1-ind_w}} - 1 \right) \frac{W_{t+1} W_{t+1}(j)/W_t(j)^2 L_{t+1}}{\pi_{W,t}^{ind_w} \pi^{1-ind_w} L_t}. \end{aligned} \tag{A.68}$$

A.4 Monetary policy

The Home central bank sets the policy rate according to the following Taylor rule:

$$\left(\frac{R_t}{\bar{R}}\right)^4 = \left(\frac{R_{t-1}}{\bar{R}}\right)^{4\rho_R} \left(\frac{\Pi_{t,t-3}}{\bar{\Pi}^4}\right)^{(1-\rho_R)\rho_\pi} \left(\frac{GDP_t}{GDP_{t-1}}\right)^{(1-\rho_R)\rho_{GDP}}. \tag{A.69}$$

where R_t is the gross monetary policy rate. The parameter ρ_R ($0 < \rho_R < 1$) captures inertia in interest rate setting, while the parameter $\bar{R}$ represents the steady-state gross nominal policy rate. The parameters ρ_π and ρ_{GDP} are respectively the weights of Home consumer price index (CPI) inflation rate (π_t) (taken as a deviation from its long-run constant target π_{target}) and GDP.

A.5 Fiscal policy

The Home fiscal authority exogenously decides the amount of investment in infrastructure and, thus, the accumulation of public capital, $K_{G,t}$, according to

$$K_{G,t} = (1 - \delta_G)\frac{K_{G,t-1}}{gr_t} + I_{G,t}, \tag{A.70}$$

where $0 < \delta_G < 1$ is the depreciation rate, and $I_{G,t}$ is public investment.[7]

The government budget constraint is

$$B_{G,t} - \frac{B_{G,t-1}}{gr_t}R_{t-1} \leq P_{N,t}C_{G,t} + P_t I_{G,t} - TAX_t, \tag{A.71}$$

where $B_{G,t} > 0$ is public debt, which is financed by a one-period nominal bond issued in the domestic bond market, paying the (gross) monetary policy interest rate R_t. The variable $C_{G,t}$ represents government purchases of goods and services, while $TAX_t > 0$ (< 0) are lump-sum taxes (transfers) to households. Consistent with the empirical evidence, $C_{G,t}$ is fully biased towards the nontradable intermediate good. Therefore, it is multiplied by the corresponding price index $P_{N,t}$.[8] Investment in public capital $I_{G,t}$ is assumed to have the same composition as private consumption, in line with the existing literature. Thus, it is pre-multiplied by the consumption price deflator P_t.

The government follows a fiscal rule defined on lump-sum taxes to bring the public debt as a % of domestic GDP, $b_G > 0$, in line with its long-run (steady-state) target $\bar{b}_G$ and to stabilize its rate of change.[9]

[7] We do not explicitly consider the possibility that public investment takes time to accumulate into physical capital. For the public capital projects with delay between the authorization of a government spending plan and the completion of an investment project, see Kydland and Prescott (1982) and Leeper et al. (2010).

[8] See Corsetti and Muller (2006).

[9] The definition of nominal GDP is

$$GDP_t = P_t C_t + P_t^I I_t + P_t I_{R\&D,t} + P_t I_{G,t} + P_{N,t}C_{G,t} + P_t^{EXP}EXP_t - P_t^{IMP}IMP_t, \tag{A.72}$$

where P_t, is the price of private consumption, public investment, and investment in R&D, given that we assume that public investment and R&D investment bundles have the same composition as private consumption. P_t^I, $P_{N,t}$, P_t^{EXP}, P_t^{IMP} are prices of private investment in physical capital, public consumption, exports, and imports, respectively.

The rule is

$$\frac{TAX_t}{TAX_{t-1}} = \left(\frac{b_{G,t}}{\bar{b}_G}\right)^{\phi_1} \left(\frac{b_{G,t}}{b_{G,t-1}}\right)^{\phi_2}, \tag{A.73}$$

where parameters ϕ_1, ϕ_2 are greater than zero, calling for a increase (reduction) in lump-sum taxes whenever the current-period public debt (as a ratio to GDP) is above (below) the target and the previous-period public debt, respectively. We choose lump-sum taxes to stabilize public finance as they are non-distortionary and, thus, allow for a "clean" evaluation of the macroeconomic effects of public investment.

References

Corsetti, Giancarlo and Gernot J. Muller (2006) Twin deficits: squaring theory, evidence and common sense. *Economic Policy* 21 (48), 597–638.

Gomes, Sandra, Pascal Jacquinot and Massimiliano Pisani (2010) The EAGLE. A model for policy analysis of macroeconomic interdependence in the euro area. ECB working paper 1195.

Kydland, Finn E. and Edward C. Prescott (1982) Time to Build and Aggregate Fluctuations. *Econometrica* 50 (6), 1345–1370.

Leeper, Eric M., Todd B. Walker and Shu-Chun S. Yang (2010) Government investment and fiscal stimulus. *Journal of Monetary Economics* 57 (8), 1000–1012.

Pesenti, Paolo (2008) The Global Economy Model: Theoretical Framework. *IMF Staff Papers* 55 (2), 243–284.

Rotemberg, Julio J. (1982) Monopolistic Price Adjustment and Aggregate Output. *Review of Economic Studies* 49 (4), 517–531.

Warne, Anders, Gunter Coenen and Kai Christoffel (2008) The new area-wide model of the euro area: a micro-founded open-economy model for forecasting and policy analysis. ECB Working paper 944.