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A QUANTITATIVE ANALYSIS OF RISK PREMIA
IN THE CORPORATE BOND MARKET

by Sara Cecchetti*

Abstract

We propose an econometric model to decompose corporate bond spreads into compensation required by investors for unpredictable future changes in the credit environment and for expected default losses. We use the model to understand whether the significant reduction in corporate bond spreads observed since the launch of the CSPP (Corporate Sector Purchase Programme) is attributable more to the fact that expansionary monetary policy measures tend to increase the risk appetite of investors and compress risk premia, or to the ability of unconventional measures to reduce expected default losses by improving investors’ expectations about the economic and financial conditions of issuers.

JEL Classification: B26, C02, F30, G12, G15.
Keywords: bond excess return, credit default swap, distress risk premium, expected default frequency, jump-at-default risk premium.

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1 Introduction¹

Corporate bond spreads are carefully monitored by central banks as they influence the transmission of the monetary policy decisions to the real economy, thus determining their effectiveness. From a monetary policy perspective, the aim of this paper is to evaluate the effectiveness of the Corporate Sector Purchase Programme (CSPP) launched by the ECB in March 2016 in the corporate bond market. In particular, the main objective of our analysis is to understand if the significant reduction in corporate bond spreads observed since the launch of the CSPP can be attributed more to the ability of unconventional measures to reduce expected losses by improving investors’ expectations about the economic and financial conditions of issuers or to the fact that expansionary monetary policy measures tend to increase the risk appetite of investors and consequently compress risk premia. It is well known that these two different aspects are possible and desirable effects of unconventional measures and both are reflected in corporate bond spreads as the compensation for the expected losses and the bond risk premium required by investors, respectively.²

Instead of using corporate bond spreads, we rely on the information embedded in CDS prices. In fact, default swap spreads approximate the spreads of referenced bonds.³ Moreover, empirical literature⁴ suggests that CDS spreads are better measures of default risk than bond spreads for a number of reasons. Roughly speaking, mainly because: the corporate CDS market is more liquid than the corresponding bond market; it is difficult to construct bond spreads in practice (for example in terms of maturity matching); it is well known that the CDS market plays a leading role in the price discovery process.⁵

We decompose the price of a credit default swap into an expected losses component and a risk premium component, which we then further decompose into two contributions.

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²For a discussion of the decomposition of corporate bond spreads in the expected losses and the risk premium components see, for example, Section 2.2 in Cecchetti and Taboga (2017).

³In Berndt and Obreja (2010) the payments of a CDS can be reproduced by a replicating portfolio consisting of a long position in a defaultable bond and a short position in a risk-free bond.

⁴See Blanco et al. (2005) and Forte and Pena (2009).

⁵Meaning that price variations of CDSs anticipate variations in bond spreads. This evidence is consistent with the standard hypothesis whereby CDS prices adjust more rapidly to the release of new information and that adjustment, in turn, generates an informative signal to which bond spreads react with a time lag.
In particular, building on the work of Díaz et al. (2013), we propose econometric models to decompose risk premia into two different sources of risk: 1) the compensation for changes in the credit environment associated with business and macro conditions (and thus for unexpected changes in the creditworthiness of the bond issuer), which investors receive for bearing risks associated with unpredictable variations in the underlying state variables; 2) the remuneration required for the risk associated with the restructuring of the entity (and thus for the risk that the bond’s price will drop in the event of default). Disentangling these two compensations allows us to quantify the different contributions of the sources of the excess return in a corporate bond.

We monitor the evolution over time of the estimated expected losses and two risk premia components and we focus on the most recent period to assess on which of them the CSPP has been more effective.

To estimate the expected losses component we only need CDS prices: in fact, this component is just approximated by the CDS price under the objective probability measure. To estimate the two abovementioned risk premium components we need both CDS prices and expected default frequency (EDF) data: in particular, we employ the only information embedded in CDS contracts by focusing on the default risk premium embedded in such spreads to quantify the compensation for those future changes in the creditworthiness of the bond issuer that might vary from expectations, the so-called *distress risk premium*; to quantify the remuneration for the surprise jump in the bond price at the event of default - the *jump-at-default-risk premium* - we also need EDF data, used as a proxy of the actual probability of default of an issuer.

From a methodological point of view, this article adopts the intensity approach of Lando (1998) and Duffie and Singleton (1999) to study the default risk premium. Our estimation strategy follows a two-step procedure, as initially developed by Driessen (2005) and employed by Díaz et al. (2013).

First, we obtain the maximum likelihood (ML) estimates of the risk-neutral mean of default arrival rates ($\lambda^Q$). To do this we use a dataset of source Capital IQ composed of the daily spreads of 1-, 3- and 5-year CDS contracts for the firms in the iTraxx Europe index, from January 2007 to February 2017. Under the assumption that the default event is diversifiable, we can obtain the estimates of the distress risk premium for each

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6The same decomposition is employed by Pan and Singleton (2006) for the credit spreads of Japanese banks.

7As in Pan and Singleton (2008) for sovereign CDS spreads.
firm. The same procedure was previously implemented by Pan and Singleton (2008) and Longstaff et al. (2011) in the case of sovereign CDS spreads, and by Díaz et al. (2013) for corporate CDS spreads. In this first step we also extrapolate the informations about the expected losses embedded in CDS prices by looking at the CDS price in terms of the risk neutral default intensity but under the objective probability measure $P$ (thus not reflecting investors’ risk preferences).

Second, we estimate the ML actual default intensity ($\lambda^P$) using the information at our disposal on the actual probabilities of default as those provided by Moody’s KMV expected default frequencies (EDFs). The jump-at-default risk premium for each firm is computed in terms of the ratio $\lambda^Q/\lambda^P$, as shown in Yu (2002), Driessen (2005) and Pan and Singleton (2006).

In this way we assess fully both the expected losses and the corporate default risk premium embedded in European CDS of each firm in the iTraxx index.

The two components of risk premium and the expected losses component are investigated to determine the main channel by which the CSPP has determined a significant reduction in corporate bond spreads. For this reason we conduct an event study analysis around the main dates of the announcements of the ECB related to the CSPP. We can confirm that the date on which both components of risk premium and the expected losses component decreased the most is 10 March 2016, when the ECB announced its decision to launch the programme. Looking at the variations between the day before and the day after the announcement, we find a more important effect of the CSPP on the risk premia component and, in particular, on the distress risk premium; however, the reduction of the expected losses is also not negligible. Moreover, using our firm-by-firm analysis, we can investigate the different behaviour of the risk premium and expected losses components in the financial and non-financial sectors, and we are able to assess the spillovers of the CSPP to financial firms (as the iTraxx Europe index also includes 25 CDSs of financial firms): one interesting result of our paper is that there are, in fact, spillovers to the financial sector, in particular in terms of a reduction of expected losses. Considering the entire time period between the first announcement of CSPP and

\footnote{For the dynamics of $\lambda^Q$, we also studied a different model (starting from Li and Zinna (2014) and Cheridito et al. (2007)) from the one used in those papers, but we found that this new model did not perform as well as the first: for this reason we continued our analysis using the consolidated model.}

\footnote{In the CSPP the object of the purchases are the investment grade bonds with residual life between 1 and 30 years, issued by non-financial firms or non-banking financial firms. The purchases can be made both in the primary and the secondary markets.}
February 2017, we can say that both expected losses and risk premia components have continued to diminish, in both the financial and non-financial sectors.

We contribute to the literature by developing an analysis of the sources of both expected losses and sources of excess returns of corporate bonds (also known in the literature as excess bond premium\(^{10}\)) during a time span that encompasses both periods of crisis (financial and sovereign debt) in the last ten years and the recent unconventional monetary policy measures implemented by the ECB. Our firm-by-firm analysis potentially allows different investigations, such as of differences between sectors of the firms or between different credit worthiness scores. From a monetary policy perspective, we are the first, to our knowledge, to use the consolidated framework for the CDS prices to assess the channels by which the recently launched CSPP has been effective in reducing corporate bond spreads. A significant contribution would derive from a comparison of the results obtained calibrating the model to the iTraxx index with those of a theoretical iTraxx index obtained from the aggregation of individual CDSs, in order to better disentangle systemic and idiosyncratic risks and contribute to the systemic risk literature; this will be subject of future research.

The paper is organized as follows. Section 2 presents the theoretical framework of the model used to decompose the sources of default risk premium of a general defaultable security and to identify the risk premia and expected losses components in a CDS price. Section 3 describes the data and the econometric framework for the estimation strategy. Section 4 contains the estimation results for the two components of the risk premium. Section 5 describes the event study analysis. Section 6 concludes. Proofs and technicalities are collected in the Appendix.

2 Two components of corporate bonds excess return: the model

Consider a defaultable security with price \(P(t, X_t)\) depending on a set of state variables \(X_t\) that follow a diffusion process under a probability measure \(\mathbb{P}\)

\[
dX_t = \mu_X(X_t, t)dt + \sigma_X(X_t, t)dW^\mathbb{P}_t,
\]

where \(\mu_X\) and \(\sigma_X\) are the drift and the instantaneous volatility, respectively, and \(W^\mathbb{P}_t\) is a standard Brownian motion under the actual measure \(\mathbb{P}\). According to Girsanov’s

\(^{10}\)A completely different approach that also analysis the excess bond premium is in Gilchrist and Zakrajsek (2012).
theorem, let define a price of risk $\Lambda_t$ and a Brownian motion under the risk-neutral measure $Q$

$$W_t^Q = W_t^P + \int_0^t \Lambda_s ds$$

so that the risk-neutral process for the state variables becomes

$$dX_t = \mu_X^Q(X_t, t)dt + \sigma_X(X_t, t)dW_t^Q,$$

with the drift under the risk neutral measure being $\mu_X^Q = \mu_X^P(X_t, t) - \sigma_X(X_t, t)\Lambda_t$.

The excess return (or risk premium) of the defaultable security can be written as the difference of expectations of the relative price variation under $P$ and $Q$ measures, and computed by considering $P(t, X_t)$ as a function of the state variables $X_t$ and using standard techniques of stochastic calculus (basically apply Ito’s lemma for stochastic processes with drift, diffusion and jumps, under $Q$ and $P$ measures):

$$e_t = E^P\left[ \frac{dP(X_t, t)}{P(X_t, t)} \right] - E^Q\left[ \frac{dP(X_t, t)}{P(X_t, t)} \right] = \frac{1}{P_t} \frac{\partial dP_t}{\partial X_t} \sigma_X(X_t, t)\Lambda_t + \frac{R_t - P_t}{P_t} \lambda^P_t \Gamma_t$$

(1)

where $\Lambda_t$ represents the market price of risk (risk premium per unity of volatility), $\sigma_X(X_t, t)$ is the volatility of risk factors, $R_t$ is the recovery value, $\lambda^P_t$ is the default intensity under the objective or risk-adjusted measure (or the arrival rate of a credit event), $\Gamma_t = 1 - \lambda^Q_t / \lambda^P_t$ is the price of risk at the event of default ($\lambda^Q_t$ being the default intensity under the risk-neutral measure). The first term of equation (1) (price of risk multiplied by volatility of risk factors) accounts for changes in the risk environment, and is called distress risk premium; the second term represents the expected payoff associated with a (downward) jump in the price of the bond if the reference entity does restructure, and is called jump-at-default-premium. A sketch of the proof of equation (1) can be found in Appendix A.1.

We can see that the excess return in equation (1) is zero if there is no compensation for both distress and jump-at-default risks ($\Lambda_t = \Gamma_t = 0$).

2.1 Price of a Credit Default Swap

A Credit Default Swap (CDS) is a financial swap agreement between two parties (protection seller and protection buyer) to receive insurance against the default of a certain bond (the reference entity). In the event of default the protection seller receives the defaulted bond, and restores its amount to the protection buyer. To get such insurance,
the protection buyer pays a spread to the protection seller, usually quarterly up to the maturity of the contract, if there is no default.

Longstaff et al. (2005) and Pan and Singleton (2008) provide the following formula for the price of a CDS contract with maturity $M$ in an intensity based setting:

$$CDS_t^Q(M) = \frac{(1 - R_t^Q) \int_t^{t+M} E_t^Q[\lambda_u^Q e^{-\int_u^t (r_s + \lambda_s^Q) ds}] du}{\int_t^{t+M} E_t^Q[e^{-\int_u^t (r_s + \lambda_s^Q) ds}] du}$$

(2)

where $r_t$ is the risk-free interest rate, $\lambda_t^{Q,11}$ and $(1 - R_t^Q)$ are default intensity and the loss given default under the risk-neutral $Q$ measure at time $t$. As in Pan and Singleton (2008), we assume independence between $r_t$ and $\lambda_t$, and a constant recovery rate $R$. Additionally we consider that the annual spread is usually paid quarterly. It follows that we can rewrite equation (2) as

$$CDS_t^Q(M) = 4 \text{*} \frac{(1 - R_t^Q) \int_t^{t+M} D(t, u) E_t^Q[\lambda_u^Q e^{-\int_u^t \lambda_s^Q ds}] du}{\sum_{i=1}^{4M} D(t, t + 0.25i) E_t^Q[e^{-\int_t^{t+0.25i} \lambda_s^Q ds}] du}$$

(3)

where $E_t^Q$ denotes expectations based on $\lambda_t^Q$ following a risk-neutral stochastic process and $D(t, u)$ is the price of a default-free zero-coupon bond (issued at date $t$ and maturing at date $u$).

When we ask for the market price of a CDS, we refer to the price under the risk neutral measure $Q$. With simple algebra, we can write this observed price in terms of the price under the objective measure $P$ as

$$CDS^Q = CDS^P + (CDS^Q - CDS^P),$$

(4)

where $CDS^P$ represents the price component related to the objective probability of default, while $(CDS^Q - CDS^P)$ represents a risk premium component. The price of a credit default swap can thus be decomposed into an expected losses component, approximated by the CDS price under the objective probability measure, and a risk premium component. In the following sections, we are going to decompose the risk premium component into the different contributions of the distress risk premium and the jump-at-default risk premium components.

11 Discounting by $r_t + \lambda_t^Q$ captures the survival-dependent nature of the payments.

12 These strong technical assumptions are standard in the literature, and necessary to construct the discrete approximation for the solution of the CDS price introduced in Lando (2004), which we’ll see in Section 2.2.
2.2 Distress risk premium

In formula (1) we have seen that the excess return, or risk premium component of the price of a defaultable security, can be decomposed into a distress risk premium component and a jump-at-default risk component. In the first step we assume no compensation for the default event ($\Gamma_t = 0$), and thus a null jump-at-default risk. From a theoretical point of view we are assuming the conditionally diversifiable hypothesis of Jarrow et al. (2005): this assumption states that jump-at-default risk is purely idiosyncratic when risk-neutral and actual default probabilities are equal, conditional to the existence of an infinite number of bonds in the economy and independence between default processes. Considering the high number of bonds in the iTraxx and the very low probability of a simultaneous default given the investment grade ratings, these two conditions seem to be reasonably satisfied.

To estimate the distress risk premium, we rely on the information embedded in CDS prices. In fact default swap spreads approximate the spreads of referenced bonds and empirical literature suggests to use CDS spreads to measure default risk.\(^{13}\)

As in Díaz et al. (2013) we impose a Ornstein-Uhlenbeck process for the logarithm of the default intensity $\lambda^Q_t$ under the risk-neutral measure $Q$:

$$d \ln \lambda^Q_t = K^Q(\theta^Q - \ln \lambda^Q_t)dt + \sigma^Q dW^Q_t$$

(5)

where parameters $K^Q$, $\theta^Q$ and $\sigma^Q$ capture the mean-reversion rate, the long-run mean and the volatility of the process, respectively. By adopting this framework, the intensity is ensured to be positive. In order to deal with the same stochastic process under the objective (or historical) measure $P$, we assume a market price of risk $\Lambda_t$ underlying a change of measure from $P$ to $Q$ to be an affine function of $\ln \lambda_t$:

$$\Lambda_t = \gamma_0 + \gamma_1 \ln \lambda_t.$$  

(6)

In fact, the dynamics of the logarithm of the risk-neutral mean arrival rate of default $\lambda^Q_t$ under the objective measure $P$ results in\(^{14}\)

$$d \ln \lambda^P_t = K^P(\theta^P - \ln \lambda^Q_t)dt + \sigma^P dW^P_t,$$

(7)

\(^{13}\)See the Introduction for references and a brief explanation.

\(^{14}\)Technical proof can be found in Appendix A.3.
where the mean-reversion rate and the long-run mean of the process under the objective probability measure, in terms of the market price of risk parameters, are

\[ K^P = K^Q - \gamma_1 \sigma_\lambda \]

and

\[ K^P \theta^P = K^Q \theta^Q + \gamma_0 \sigma_\lambda. \]

Note that we are discussing the properties of \( \lambda^Q_t \), as a stochastic process, under two different measures, \( Q \) and \( P \). At this juncture, \( \lambda^P_t \), the arrival rate of default under the historical measure, is playing no role in our analysis. As highlighted by Jarrow et al. (2005) and Yu (2002), this information cannot be extracted from bond or CDS spread data alone. We will deal with \( \lambda^P_t \) and we will briefly comment on the relationship between \( \lambda^P_t \) and \( \lambda^Q_t \) in the subsequent section, when we will estimate the jump-at-default risk premium.

In our analysis we have also considered the CIR (square root) process for \( \lambda_t \), as used in Li and Zinna (2014):

\[ d\lambda^Q_t = K^Q(\theta^Q - \lambda^Q_t)dt + \sigma_\lambda \sqrt{\lambda^Q_t} dW^Q_t \tag{8} \]

Using a CIR process for \( \lambda_t \), has the advantage that both the expectations in the numerator and denominator of equation (3) can be computed in closed form (see Appendix A.5). Differently from Li and Zinna (2014), who adopt an essentially affine market price of risk as in Duffee (2002),\(^1\) we have assumed an extended market price of risk \( \Lambda_t \) from \( P \) to \( Q \) as in Cheridito et al. (2007):

\[ \Lambda_t = \frac{\gamma_0}{\sqrt{\lambda_t}} + \gamma_1 \sqrt{\lambda_t}. \]

The advantage of this choice of market price of risk is that again the dynamics for the risk neutral intensity \( \lambda^Q_t \) under the actual measure \( P \) is a CIR process \(^2\)

\[ d\lambda^Q_t = K^P(\theta^P - \lambda^P_t)dt + \sigma_\lambda \sqrt{\lambda^Q_t} dW^P_t \tag{9} \]

with

\[ K^P = K^Q - \sigma_\lambda \gamma_1 \]

\(^1\)According to the authors, the motivation for this choice is to avoid to impose the Feller condition required by the CIR dynamics to assure a \( \lambda^Q_t \) strictly positive: \( 2K^Q \theta^Q > \sigma^2 \). Such choice of market price of risk determines only one different parameter in the processes under the actual and the risk neutral measure.

\(^2\)Technical proof can be found in Appendix A.4.
and

\[ K^P \theta^P = K^Q \theta^Q + \sigma \lambda_0 \]

so that we let vary both the speed of adjustment and the long run parameters of the process for the default intensity under the two measures \( Q \) and \( P \).

The model based on the CIR dynamics however, empirically provides a worst performance when calibrated to the data, with respect to the model based on the Ornstein-Uhlenbeck process. For this reason, after looking at the results, we have decided to rely on the Ornstein-Uhlenbeck process, and in the following we will refer to this model.

To measure the size of the distress risk premium, we follow Longstaff et al. (2011): as the dynamics of the objective (under \( P \) measure) and risk-neutral (under \( Q \)) processes for \( \lambda^Q_t \) coincide when there is no risk premium (\( \Lambda_t = 0 \)) since, from the above discussion, they would have the same parameters, the size of the risk premium can be inferred by simply taking the difference

\[ DRP_t = CDS^Q_t(M) - CDS^P_t(M) \quad (10) \]

where \( CDS^Q_t(M) \) is the price of the CDS implied by the risk neutral process \( \lambda^Q_t \) (taking expectations in equation (3) using the risk-neutral probability distribution \( Q \) implied by equation (5)), \( CDS^P_t(M) \) is the price of the CDS implied by the objective process (taking expectations in equation (3) but using the probability distribution \( P \) implied by the objective process in equation (7)). Let us remark that the CDS price under the objective measure \( CDS^P_t(M) \) provides an estimate of the expected losses.

For the CDS price computation we use the general approximation formula described in Lando (2004):

\[ CDS^Q_t(M) = 4 \sum_{i=1}^{M} \frac{D(0,i)(S(0,i-1) - S(0,i))}{\sum_{i=1}^{M} D(0,i)S(0,i)} \quad (11) \]

where

\[ S(0,t) = \mathbb{E}_Q^Q[e^{-\int_0^t \lambda^Q_s ds}] \]

and we compute this expectation numerically using the Crank–Nicholson implicit finite-difference method to solve the associated Feynman-Kac partial differential equation:\footnote{See Appendix A.6 for details of this method.}

\[
\begin{cases} 
S_t + S_x (K^Q \theta^Q - K^Q \ln \lambda_t) + \frac{1}{2} S_{xx} \sigma^2 - \lambda_t S = 0 \\
S(t) = 1
\end{cases}
\]
2.3 Jump-at-default risk premium

Separate from the risk related to unexpected risk-neutral default arrivals because of changes in the credit environment (which we called *distress risk premium*), is the risk of a jump in the price of the underlying bond in the event that a firm restructures. We refer to this risk as the *jump-at-default risk*, for which investors also demand a *default risk premium*.

This compensation, as reflected in excess returns, is captured in the second term of equation (1), as

\[ JADRP_t = \frac{R_t - P_t}{P_t} \lambda^P_t \Gamma_t \]

where \( \frac{R_t - P_t}{P_t} \) is the percentage loss of value due to default, \( \lambda^P_t \) is the actual default intensity and \( \Gamma_t \) is the market price of jump-at-default risk.

It turns out theoretically\(^ {18}\) that the risk premium associated with the jump-at-default risk is the ratio between the risk-neutral and historical arrival rate of credit events \( \lambda^Q_t / \lambda^P_t \), and the price of this risk can be defined as\(^ {19}\)

\[ \Gamma_t^P = 1 - \frac{\lambda^Q_t}{\lambda^P_t} \]

The ratio \( \lambda^Q_t / \lambda^P_t \) is usually referred to as the jump-at-default premium and has been previously studied in Driessen (2005) and Berndt et al. (2005).

Typically, the ratio \( \lambda^Q_t / \lambda^P_t \) is larger than one since, assuming that investors are averse to jump-at-default risk, to obtain the correct market prices using risk-neutral valuation, \( \lambda^Q_t \) must be set larger than \( \lambda^P_t \). Effectively, the investment environment must, risk-neutrally, be much riskier (default must be more likely) than what has been experienced historically.

If the jump-at-default premium is one (risk-neutral and actual default intensities coincide), then \( JADRP \) is zero and does not affect excess returns: in other words, there is no concern about jumps at the time of credit events and no compensation for the event of default. On the other hand, if the jump-at-default premium is greater than one, then \( \Gamma_t \) is negative. Since the term \( \frac{R_t - P_t}{P_t} \) is negative if prices jump down when there is a credit event, the overall contribution of \( JADRP \) to the excess return is positive.

\(^ {18}\)See Yu (2002) for a heuristic discussion.

\(^ {19}\)See Appendix A.7 for a technical explanation.
While $\lambda_Q^P$ (and the distress risk premium component) are obtained using only CDS data, according to general literature, to estimate $\lambda_P^P$ we rely on the market’s expected probabilities of default over the next year, or Expected Default Frequencies (EDFs), calculated by Moody’s KMV using a Merton-style balance-sheet model of credit events, as proxies for actual default probabilities.

More specifically, we assume that the actual default intensity process $\lambda_P^P$ comes from the definition of the EDF over $M$ years

$$ EDF(M) = 1 - E_t^P[e^{-\int_t^{t+M} \lambda_P^P ds}] \quad (12) $$

and, as for the default intensity under the risk neutral measure, we assume an Ornstein-Uhlenbeck process for the logarithm of the objective default intensity:

$$ d \ln \lambda_P^P = \alpha_P^P (\beta_P^P - \ln \lambda_P^P) dt + \sigma_{\lambda_P^P} dW_P^P \quad (13) $$

where parameters $\alpha_P^P$, $\beta_P^P$ and $\sigma_{\lambda_P^P}$ represent the mean-reversion rate, the long-run mean and the volatility of the process, respectively.

### 3 Data and econometric framework

We use daily time series of closing prices of CDSs with maturities 1 year, 3 years and 5 years, for the firms in the iTraxx Europe Series 26, downloaded from Capital IQ, and for these firms the related EDFs at 1 year, 3 years and 5 years, with source Moody’s KMV. The sample period goes from 1 January 2007 to 6 February 2017.

From the 125 firms in the iTraxx index, in order to have the entire time series dataset for both CDS prices and EDFs, we end up with 102 firms. Among these, we have 25 financial firms and 77 non-financial firms. In terms of credit rating, all of them are investment grade, but 63 have a Moody’s rating between Baa3 and Baa1 (and we refer to them as being in the B investment grade class), and 39 are classified by Moody’s between A3 and Aaa rating class (and we refer to them as being in the A investment grade class). Given the high variation of CDS prices not only across firms but also in time, instead of reporting static descriptive statistics, we show in Figure 1 the time series of percentiles of 5-year CDS prices, as well as the average prices in the two sectors,

*Empirical studies corroborate the accuracy of EDFs for predicting default (see Kealhofer (2003), Bharath-Shumway (2008), Korablev-Dwyer (2007)) and using EDFs to determine the default premium has been also referred to in Berndt et al. (2005) for US corporate CDS spreads, Vassalou and Xing (2004) for stock prices and Pan and Singleton (2006) for CDS spreads of Japanese banks.*
financial and non financial, and in the two $A$ and $B$ investment grade rating classes. The same information about 5-year EDFs is reported in Figure 2.

Figure 1 – Time series statistics of 5-year observed CDS prices.
The figure shows in the three panels the observed 5-year CDS prices displayed as percentiles, averages of financial and non financial firms, and averages of $A$ and $B$ investment grade class. The sample period covers from 1 January 2007 to 6 February 2017. The red vertical bar coincides with the date of the first announcement of the ECB related to the CSPP, on 10 March 2016.
Expected Default Frequencies

(per cent)

Figure 2 – Time series statistics of Moody’s 5-year expected default frequencies.
The figure shows in the three panels the Moody’s KMV 5-year expected default frequencies
displayed as percentiles, averages of financial and non financial firms, and averages of A and B
investment grade class. The sample period covers from 1 January 2007 to 6 February 2017. The
red vertical bar coincides with the date of the first announcement of the ECB related to the
CSPP, on 10 March 2016.

As in Driessen (2005) and Diaz et al. (2013) the estimation strategy follows a two-step procedure:

1. We obtain the maximum likelihood (ML) estimates of the risk-neutral mean of
default arrival rates $\lambda^Q_t$ from the CDS spreads. Assuming that the default event is
diversifiable, we obtain a estimate of the distress risk premium.

2. We estimate the ML actual default intensity $\lambda^P_t$ using the information on the actual
probabilities of default embedded in the Moody’s KMV EDFs. We thus compute
the jump-at-default risk premium in terms of the ratio $\frac{\lambda^Q_t}{\lambda^P_t}$, as shown by Driessen

Our maximum likelihood estimation strategy is based on Chen and Scott (1993), and has also been employed in a similar context by Berndt et al. (2005), Pan and Singleton (2008) and Longstaff et al. (2011).

---

21 Which are based on the Merton model for pricing corporate debt (see Merton (1974)).
22 See also Hamilton and Wu (2012) for a description of the method.
3.1 Maximum likelihood estimates of risk-neutral default intensity

Let us first consider the estimates of $\lambda^Q_t$ from the CDS spreads. For each time $t$, we have one unobservable factor, $\lambda_t$, and three corresponding CDS prices, with different maturities. Consequently, we must include additional random variables in order to perform a change of variables from the unobservable state variables to the CDS prices. To this aim, we assume CDSs with maturity 3 years ($CDS^{3Y}$) as being perfectly priced, and we add standard normal measurement errors $u^{1Y}$ and $u^{5Y}$ for CDSs with maturities 1 and 5 years ($CDS^{1Y}$ and $CDS^{5Y}$), respectively.

$$CDS_t^{sY} = f^{CDS^{sY}} + u_t^{sY} \quad s = 1, 3, 5$$ (14)

where

$$u_t^{3Y} = 0$$

$$\phi(u_t^{1Y}, u_t^{5Y}) = \frac{1}{\sqrt{(2\pi)^2|\Omega|}} e^{-\frac{1}{2}u_t^\top \Omega^{-1} u_t}$$

and $\Omega$ is the diagonal covariance matrix for the measurement errors, with the variances of the measurement errors $\sigma_{1Y}^2$ and $\sigma_{5Y}^2$ on the diagonal.

To build the maximum likelihood estimator for the parameters of our model, we develop a likelihood function for the observed CDS prices as functions of the unobserved default intensities. It is important to remark that while for CDS prices we consider the dynamics under the risk neutral measure $Q$, when we deal with the time series of default intensities, we must use the dynamics under the objective measure $P$. The vector of parameters that we are going to estimate is thus $\Theta = (K^Q, \theta^Q, K^P, \theta^P, \sigma_\lambda, \sigma_{1Y}, \sigma_{5Y})$.

We first obtain a possible path of $\lambda^Q_t$ by numerically inverting equation (11).

---

23The same hypothesis is in Longstaff et al. (2011) and Diaz et al. (2013), but is not related to different liquidity consistions of CDS contracts with different maturities.

24We can consider this dynamics as the continuous time version of the corresponding AR(1) process, or

$$\lim_{\Delta t \to 0} \ln \lambda_t - \ln \lambda_{t-1} = \theta(1 - e^{-K}) + (e^{-K} - 1) \ln \lambda_{t-1} + \epsilon_t,$$

with $\epsilon_t \sim N(0, \sigma_\epsilon^2)$ and $\sigma_\epsilon^2 = \frac{1}{2K}(1 - e^{-2K})$.

25A non-central $\chi^2$ distribution, defined in terms of the modified Bessel function of first kind, is instead involved in case of a CIR dynamics for $\lambda_t$ (see Chen and Scott (1993)).
measurement errors in our model, the log-likelihood function for the entire framework results in the following form:

$$\log L(\lambda_1, \ldots, \lambda_T) = \sum_{t=1}^{T} \left( \log |B_t| + \log(2\pi) + \frac{1}{2} \log \sigma_{1y}^2 + \frac{1}{2} \log \sigma_{5y}^2 + \frac{(u_1^t)^2}{2\sigma^2} + \frac{(u_5^t)^2}{2\sigma^2} \right)$$

where the elements of the matrix $B$ are such that the Jacobian of the transformation from $\lambda_t^Q$ to $CDS_t$ is $|B_t|^{-1}$, and $\sigma_{1y}$ and $\sigma_{5y}$ are the standard deviations of the measurement errors for the CDSs with maturity 1- and 5-years, respectively.

### 3.2 Maximum likelihood estimates of actual default intensity

For the estimates of the objective default intensity $\lambda_P^t$ our dataset consists of 1-, 3- and 5-year EDFs from Moody’s KMV. We employ the same methodology as in Section 3.1 of which in the following we briefly recall the main steps. We assume that 3-year EDFs are observed without error. To estimate the parameters of $\lambda_P^t$ we maximize the likelihood of the joint density of process (13) and the mispricing errors, related to 1- and 5-year EDFs. To this aim we first obtain a possible path of $\lambda_P^t$ by inverting equation (12). Mispricing errors of 1- and 5-year EDFs are normally distributed with zero mean and standard deviations $\sigma_{1y}$ and $\sigma_{5y}$, respectively. To compute the expectation in equation (12) for the intensity process (13) we employ as before the finite difference method of Crank-Nicholson.

### 4 Estimates of the two risk premia components

In this section we describe the estimation results related to the two sources of excess return.

#### 4.1 Estimation results of the distress risk premium

Table 1 provides summary statistics of maximum likelihood estimates of the parameters of the risk-neutral process $\lambda_t^Q$. 

19
Table 1 – Parameters of risk neutral default intensity process under risk neutral and objective probability measures.

<table>
<thead>
<tr>
<th></th>
<th>( K^Q )</th>
<th>( \theta^Q )</th>
<th>( \sigma_\lambda )</th>
<th>( K^P )</th>
<th>( \theta^P )</th>
<th>( \sigma_{1y} )</th>
<th>( \sigma_{5y} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.3288</td>
<td>-4.5333</td>
<td>1.1908</td>
<td>0.4314</td>
<td>-6.6636</td>
<td>0.0016</td>
<td>0.0013</td>
</tr>
<tr>
<td>Std</td>
<td>0.0339</td>
<td>0.4705</td>
<td>0.0677</td>
<td>0.0047</td>
<td>0.0717</td>
<td>0.0016</td>
<td>0.0007</td>
</tr>
<tr>
<td>Median</td>
<td>0.3294</td>
<td>-4.5036</td>
<td>1.2000</td>
<td>0.4304</td>
<td>-6.6799</td>
<td>0.0012</td>
<td>0.0012</td>
</tr>
</tbody>
</table>

Looking at these results we can see that on average, mean-reversion rates are higher under actual than under risk-neutral measure (\( K^P > K^Q \)). Conversely, long-term mean parameters are higher under \( Q \) than \( P \) measure (\( \theta^Q > \theta^P \)). These results are consistent with empirical evidence in the literature (see, for instance, Pan and Singleton (2008) and Diaz et al. (2013)) and imply that \( \lambda^Q \) will tend to be larger under \( Q \) than under \( P \); in other words the arrival of credit events is more intense in the risk-neutral (higher long-run means) than in the actual environment. Moreover, for a given level of \( \lambda^Q \), there is more persistence under \( Q \) than under \( P \) (bad times last longer in the risk-neutral world as the speed of mean reversion is lower under \( Q \)).

Remembering the choice of the market price of risk \( \Lambda_t \) and the related relationship between the parameters under \( Q \) and \( P \) measures, we can compute the average market price of risk parameters:

\[
\gamma_0 = \frac{K^P \theta^P - K^Q \theta^Q}{\sigma_\lambda} = -1.1749
\]

and

\[
\gamma_1 = \frac{K^Q - K^P}{\sigma_\lambda} = -0.0862.
\]

These parameters determine a negative \( \Lambda_t \), for each \( t \): on average we have

\[
\bar{\Lambda} = \gamma_0 + \gamma_1 \ln \lambda^Q = -0.6699.
\]

We can say that the negative sign of coefficients \( \gamma_0 \) and \( \gamma_1 \) confirm that the credit environment is worst under risk-neutral than under actual measure. According to Pan and Singleton (2008), it is this pessimism about the credit environment that allows risk-neutral pricing to recover market prices in the presence of investors who are averse to default risk.
From the estimates of the parameters of the process $\lambda_t^Q$ under both $Q$ and $P$ measures, we can compute the distress risk premium as in equation (10).

In Figure 3 are displayed the average 5 year CDS prices estimated under the risk-neutral measure $Q$ and the objective measure $P$, and the observed prices. The green line, representing the CDS price under the objective measure, provides the time series of the estimated expected losses.

**5-year estimated CDS prices**

*(basis points)*

![Graph showing 5-year estimated CDS prices](image)

**Figure 3 – Average 5-year CDS prices observed and estimated under the two probability measures.**
The figure shows the average 5-year CDS prices, estimated under the $Q$ and $P$ measures, and observed. The sample period covers from 1 January 2007 to 6 February 2017. The red vertical bar coincides with the date of the first announcement of the ECB related to the CSPP, on 10 March 2016.

Figure 4 shows the estimated average and the percentiles of the distress risk premia for the firms in the iTraxx index in basis points.
Distress risk premium
(basis points)

Figure 4 – Distress risk premium: average and percentiles.
The figure shows the average and percentiles of the distress risk premium embedded in 5-year
CDS prices. The sample period covers from 1 January 2007 to 6 February 2017. The red vertical
bar coincides with the date of the first announcement of the ECB related to the CSPP, on 10
March 2016.

According to the intuition and the empirical evidence in the literature, we can see
that the distress risk premium increases markably during the periods of the financial
crisis and the sovereign debt crisis. Moreover there is a considerable dispersion between
the firms in the index, as we can see when looking at the percentiles.

Figure 5 displays the different behaviour of the distress risk premia for the financial
and non financial firms and according to the rating class.
Figure 5 – Distress risk premium for financial and non financial firms and for rating class.

The figure shows the average distress risk premium, embedded in 5-year CDS prices, in the financial and non financial subsets and in two rating classes subsets. The sample period covers from 1 January 2007 to 6 February 2017. The red vertical bar coincides with the date of the first announcement of the ECB related to the CSPP, on 10 March 2016.

We can say that while both financial and non financial firms were deeply affected by the financial crisis, the sovereign debt crisis hit most the financial firms. Finally, as we would expect, the distress risk premium of the firms in the lower rating classes is higher than in the higher rating classes.

4.2 Estimation results of the jump-at-default risk premium

Table 2 displays the average parameters of the actual default intensity process $\lambda_t$:  

23
Table 2 – Parameters of objective default intensity process.
Summary statistics of maximum likelihood estimates of actual $\lambda_t^P$ process. $K^P$ denotes the mean-reversion rate of $\lambda_t^P$ under the actual measure. $\theta^P$ is the long-run mean of $\lambda_t^P$ under the actual measure. $\sigma_\lambda$ is the instantaneous volatility of the process. $\sigma_{1y}$ and $\sigma_{5y}$ represent the volatility of the errors in the EDFs for 1- and 5-year maturities.

<table>
<thead>
<tr>
<th></th>
<th>$\alpha^P$</th>
<th>$\beta^P$</th>
<th>$\sigma_\lambda$</th>
<th>$\sigma_{1y}$</th>
<th>$\sigma_{5y}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.3035</td>
<td>-12.5442</td>
<td>1.1360</td>
<td>0.0024</td>
<td>0.0012</td>
</tr>
<tr>
<td>Std.</td>
<td>0.0948</td>
<td>12.6319</td>
<td>0.1721</td>
<td>0.0046</td>
<td>0.0016</td>
</tr>
<tr>
<td>Median</td>
<td>0.3079</td>
<td>-9.3997</td>
<td>1.1735</td>
<td>0.0011</td>
<td>0.0006</td>
</tr>
</tbody>
</table>

In Figure 6 are displayed the average 5-year EDFs reported by Moody’s KMV and estimated with our model.

5-year expected default frequencies
(per cent)

Figure 6 – Average 5-year EDFs of source Moody’s and estimated.
The figure shows the average 5-year EDFs of source Moody’s KMV and estimated. Data are in percentages. The sample period covers from 1 January 2007 to 6 February 2017. The red vertical bar coincides with the date of the first announcement of the ECB related to the CSPP, on 10 March 2016.

Once we have estimated the actual default intensities, we can compute the jump-at-default premium $\lambda_t^Q/\lambda_t^P$ as well as the market price of jump-at-default risk $\Gamma_t$. The jump-at-default premium ratio results in approximately 2.9 on (averaged) median for the firms under study. This estimate is consistent with those previously reported in the literature for the US market (see, among others, Driessen (2005)) and for the European
market (see for instance Diaz et al. (2013)). Employing the definition of jump-a-default-risk premium in equation (1), and having assumed a recovery rate of 40%, we have

\[ JADRP_t = (0.4 - 1) \ast \lambda_t^P \ast (1 - \frac{\lambda_t^Q}{\lambda_t^P}) \] (15)

Figures 7 and 8 show the time-series of jump-at-default risk premium in basis points, displayed as average and percentiles, and divided in the two classes, financial and non financial firms and by rating class, respectively. Our (averaged) median jump-at-default estimate is 11 basis points, that is in line with the existing literature (Diaz et al. (2013) estimate a (averaged) median jump-at-default of 13 basis points). This result indicates that the default event itself carries a risk premium and that this premium increases during the stress periods, which agrees the intuition about the jump-at-default risk premium.

![Jump-at-default risk premium](image)

**Figure 7 – Jump-at-default risk premium: average and percentiles.**
The figure shows the average and the percentiles of the jump-at-default risk premium. Data are in basis points. The sample period covers from 1 January 2007 to 6 February 2017. The red vertical bar coincides with the date of the first announcement of the ECB related to the CSPP, on 10 March 2016.
Figure 8 – Jump-at-default risk premium for financial and non financial firms and for rating class.

The figure shows the average jump-at-default risk premium in the financial and non financial subsets and in the two rating classes subsets. Data are in basis points. The sample period covers from 1 January 2007 to 6 February 2017.

It is evident that the average jump at default risk premium is slightly negative in calm periods. In fact, according to formula (15), jump at default risk premium is negative if $\lambda^P > \lambda^Q$, and this happens, on average, during calm periods, as displayed in Figure 9.
Default intensity under $\mathbb{P}$ and $\mathbb{Q}$ measures

\textit{per cent}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{default_intensity.png}
\caption{Average physical and risk-neutral default intensity.}
\end{figure}

The figure shows the average of the estimated default intensity under $\mathbb{P}$ and $\mathbb{Q}$ measures, in percentages. The sample period covers from 1 January 2007 to 6 February 2017. The red vertical bar coincides with the date of the first announcement of the ECB related to the CSPP, on 10 March 2016.

The jump-at-default-risk-premium is not statistically different from zero during the calm periods. Negative values might arise from the problem of realibility of $\lambda^\mathbb{P}$ estimates, a problem that persists in the existing literature: in fact, the estimation of the actual default intensity is a really difficult task, and we do not count on optimal estimates of it. However, we employ EDFs because they have a higher predictive power than credit ratings, depending on stock prices and being time-variant. Several empirical studies corroborate the accuracy of EDFs for predicting default.\textsuperscript{26} Additionally, using EDFs to determine the default premium has also been referred to in Berndt et al. (2005) for US corporate CDS spreads, in Vassalou and Xing (2004) for stock prices and in Pan and Singleton (2006) for the CDS spreads of Japanese banks.

The problem of negative jump-at-default risk premium disappears when looking at the median, as we can see in Figure \textsuperscript{7}

\textsuperscript{26}See, for instance, Kealhofer (2003), Bharath and Shumway (2008) and Korablyev and Dwyer (2007).
5 Analysis of the two risk premium components and event study exercise on the risk premia and the expected losses

Putting together the two estimated risk premia, the distress risk premium and the jump-at-default risk premium, we obtain the excess return of the corporate bond:

\[ e_t = DRP_t + JADRP_t \]

Figure 10 displays the time series of the estimated average excess return.

![Excess return](image)

**Figure 10 – Average excess return.**
The figure shows the average excess return, obtained as the sum of the distress risk premium and the jump-at-default risk premium. The sample period covers from 1 January 2007 to 6 February 2017. The red vertical bar coincides with the date of the first announcement of the ECB related to the CSPP, on 10 March 2016.

According to our estimates, the (averaged) median distress risk premium in the entire sample period is around 60 basis points and accounts for 88% of the excess return, while the remaining (averaged) median jump-at-default risk premium is around 11 basis points and accounts for the residual 12%. If we look at the most recent period, the contribute of the distress risk premium is even higher: in the data sample starting on January 2016 the (averaged) median distress risk premium is around 53 basis points and accounts for 93%, while the (averaged) median jump-at-default risk premium is around 3 basis points.
and accounts for 7%. An interesting comparison is between the behaviour of the two risk premium components in the financial and non-financial sectors. Looking at the same recent period, we can say the following:

- The (averaged) median distress risk premium is higher for the financial firms (around 69 basis points, versus 49 basis points of the non-financial counterparties);

- On the contrary, the (averaged) median jump-at-default risk premium is higher for the non-financial firms (being around 4 basis points, versus a negative value of the financial counterparties);

- There is a higher heterogeneity between financial financial firms with respect to the non-financial counterparties, as both the dispersion in the distress risk premium and in the jump-at-default risk premium are higher for the financial firms: in particular the average standard deviation of the distress risk premium is around 30 basis points for the financial firms, versus 22 for the non-financial; the average standard deviation of the jump-at-default risk premium is around 73 basis points, versus 19 for the non-financial firms.

We conduct an event study exercise around key ECB announcements related to the Corporate Sector Purchase Programme (CSPP) during the press releases to investigate the effects of these announcements, and thus of the launch of this programme, in the decrease of the two risk premium components. We consider the following dates:\[27\]

- 10 March 2016: ECB adds corporate sector purchase programme (CSPP) to the asset purchase programme (APP);\[28\]

- 21 April 2016: Press Release: ECB announces details of the corporate sector purchase programme (CSPP), mainly on the eligibility of bonds;\[29\]

- 2 June 2016: ECB announces remaining details of the corporate sector purchase programme (CSPP) on the issuer eligibility.\[30\]

\[27\] Of course we expect that the main impact on the components of CDS prices arises at the date of the first announcement of the CSPP (as in fact is confirmed by the results), but we also consider the dates of the other two press releases in which the ECB provides details of the programme.


We use a time window of ten days, five before and five after the announcements. Unfortunately, we cannot use a difference in difference methodology\textsuperscript{31} to adjust for changes in financial sector risk because also corporate spreads of financial institutions benefited from ECB decisions in those dates. In terms of rating, all the bonds of the firms in the iTraxx are eligible for the purchases in the CSPP, having an investment grade rating.

Figure 11 shows the median of the corporate bond excess return around announcement dates, while Figure 12 shows separately the two components of the excess return - the median of the distress risk premium and the median of the jump-at-default risk premium - around the same dates. Looking at these figures it is evident that most of the decrease in both risk premia was observed on 10 March 2016. In fact, also in the ECB Economic Bulletin (Issue 5, 2016) is pointed out that "the announcement of the CSPP on 10 March was followed by a significant contraction in the spread between yields on bonds issued by non-financial corporations (NFCs) and a risk-free rate". A further spread contraction can be observed on 21 April (when was confirmed the eligibility of insurance corporations), while in June, spreads developments where also related to the uncertainty generated by the UK referendum. Table 3 displays the average variations of the excess return and its components between the day after the ECB announcement on 10 March and the day before, while in Table 4 we can see the historical daily variations. Looking at the different magnitudes of the variations reported in the two tables, we can say that the reductions observed on 10 March where significant.

<table>
<thead>
<tr>
<th>$\Delta ExRet$</th>
<th>$\Delta DRP$</th>
<th>$\Delta JADRP$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-16.75</td>
<td>-10.87</td>
<td>-5.88</td>
</tr>
</tbody>
</table>

Table 3 – Average variations around ECB announcements
This table shows the average variations between the day after the ECB announcement on 10 March 2016 and the day before, of the excess return, the distress risk premium and the jump-at-default risk premium. Basis points.

<table>
<thead>
<tr>
<th>$h\Delta ExRet$</th>
<th>$h\Delta DRP$</th>
<th>$h\Delta JADRP$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.010</td>
<td>0.009</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 4 – Historical daily variations
This table shows the historical daily variations of the excess return, the distress risk premium and the jump-at-default risk premium. Basis points.

\textsuperscript{31}As often employed in the literature for event study exercises, see for instance Kelly at al. (2012).
Table 5 reports the percentage variations of the excess return and its components between the day after the ECB announcement on 10 March and the day before.

<table>
<thead>
<tr>
<th>(\frac{\Delta ExRet}{ExRet_{t-1}})</th>
<th>(\frac{\Delta DRP}{DRP_{t-1}})</th>
<th>(\frac{\Delta JADR}{JADR_{t-1}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>-21.41</td>
<td>-15.84</td>
<td>-34.47</td>
</tr>
</tbody>
</table>

Table 5 – Average relative variations around ECB announcements

This table shows the average relative variations between the day after the ECB announcement on 10 March 2016 and the day before, of the excess return, the distress risk premium and the jump-at-default risk premium. Percentages.

The decreases in the distress risk premium and the jump-at-default risk premium were on average of 15.8 and 34.5 per cent, respectively, between the day following this announcement and the day before. We can thus say that jump-at-default component was more affected, in relative terms, by the announce of the CSPP.

Excess return around ECB announcements

(basis points)

Figure 11 – Event study: median of the excess return.

The figure shows the median of the corporate bond excess return around announcement dates. The black line shows the average risk premium response over all announcements, and the coloured lines show the responses to each individual announcement.
Risk premia around ECB announcements

\textit{(basis points)}

Figure 12 – Event study: median of the distress and jump-at-default risk premia.
The figure shows the median of the distress risk premium and the jump-at-default risk premium around announcement dates. In each figure, the black line shows the average risk premium response over all announcements, and the coloured lines show the responses to each individual announcement.

With the same event study exercise we can also analyze the different responses of the financial and non financial sectors to the CSPP announcements.

Figure 14 shows the median of the corporate bond excess return of the financial and non financial sectors, around announcement dates, while Figure 15 shows separately the two components of the excess return - the median of the distress risk premium and the median of the jump-at-default risk premium in the two sectors - around the same dates.

Table 6 displays the average variations of the excess return and its components, for the financial and non financial sectors, between the day after the ECB announcement on 10 March and the day before; in Table 7 we can see the related percentage variations.

<table>
<thead>
<tr>
<th>$\Delta ExRet_f$</th>
<th>$\Delta DRP_f$</th>
<th>$\Delta JADRP_f$</th>
<th>$\Delta ExRet_n$</th>
<th>$\Delta DRP_n$</th>
<th>$\Delta JADRP_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-11.75</td>
<td>-13.14</td>
<td>-1.87</td>
<td>-16.68</td>
<td>-7.12</td>
<td>-4.71</td>
</tr>
</tbody>
</table>

Table 6 – Average variations around ECB announcements in the financial and non financial sectors
This table shows the average absolute variations between the day after the ECB announcement on 10 March 2016 and the day before, of the excess return, the distress risk premium and the jump-at-default risk premium in the financial and non financial sectors. Basis points.
Table 7 – Average relative variations around ECB announcements in the financial and non financial sectors
This table shows the average relative variations between the day after the ECB announcement on 10 March 2016 and the day before, of the excess return, the distress risk premium and the jump-at-default risk premium in the financial and non financial sectors. Percentages.

What is evident is that the spillovers of the launch of the CSPP to the financial sector have been rather important: in relative terms the drop of the excess return of the financial sector has even been slightly higher than in the non financial sector (−26% vs −25%), as well as the decrease of the distress risk premium component; the percentage variation of the jump-at-default risk premium component has instead been more pronounced in the non financial sector.

To verify that the changes in the estimated risk premia components after the announcements were substantial, Table 8 displays the historical daily variations of the excess return and its components for the financial and non financial sectors, and we can confirm that they are significantly smaller.

Table 8 – Historical daily variations in the financial and non financial sectors
This table shows the average historical daily variations of the excess return, the distress risk premium and the jump-at-default risk premium in the financial and non financial sectors. Basis points.

In addition to the risk premia, we know that a CDS price embeds the compensation for the expected losses. If we also add the estimates of the expected losses, we obtain the decomposition of the CDS price into its three components. Figure 13 displays at selected dates the average estimates of the expected losses, the distress risk premium and the jump-at-default risk premium.
Decomposition of the average CDS price
(basis points)

Figure 13 – Components of average CDS price.
The figure shows the three components of the average CDS price (the average expected losses, the average distress risk premium and the average jump-at-default risk premium) at selected dates: before the financial crisis (January 2007), at the peaks of the financial crisis (January 2009) and of the sovereign debt crisis (January 2012); before the announcement of the programme (March 2016); when the purchases begin (June 2016); at the most recent date (February 2017).

We can also estimate the effect of the CSPP on the expected losses component of the CDS prices, by looking at the variation of the CDS price under the $\mathbb{P}$ measure: on average, between the day before the CSPP announcement and the day after, the expected losses decreased by 3.7 basis points; the reduction for the financial firms was more pronounced than for the non financial counterparties (5.2 versus 3.2 basis points). These variations are quite important, given that the historical daily variations of the expected losses components are 0.003 basis points on average for the entire sample and 0.006 and 0.002 basis points for the financial and non financial firms, respectively.
Excess return around ECB announcements in the financial and non-financial sectors (basis points)

Figure 14 – Event study: median of the excess return in the financial and non-financial sectors.
The figures show the median of the corporate bond excess return around announcement dates in the financial and non-financial sectors. In each figure the black line shows the average risk premium response over all announcements, and the coloured lines show the responses to each individual announcement.

Risk premia around ECB announcements in the financial and non-financial sectors (basis points)

Figure 15 – Event study: median of the distress and jump-at-default risk premia in the financial and non-financial sectors.
The figures show the median of the distress risk premium and the jump-at-default risk premium around announcement dates. In each figure, the black line shows the average risk premium response over all announcements, and the coloured lines show the responses to each individual announcement.

6 Conclusion

Among the unconventional monetary policy measures recently adopted by the ECB, the Corporate Sector Purchase Programme (CSPP) has especially exerted a downward
pressure on the corporate bond spreads. We contribute to the recent literature on
the assessment of the effectiveness of unconventional monetary policy measures, with a
focus on the corporate bond market. Corporate bonds are carefully monitored by central
banks because they constitute an important channel of transmission of monetary policy
decisions to the real economy.

The purpose of this paper is to investigate if the reduction observed in corporate bond
spreads after the launch of the programme can be attributed more to a decrease of the
expected losses or to a reduction of the risk premia, and in particular the compensation
required by investors for changes in the the credit environment associated with business
and macro conditions and the compensation for the risk associated with a jump in price
of the bond in case of default.

To this aim, following Diaz et al. (2013), we use the information embedded in the
Credit Default Swaps of the firms in the iTraxx Europe index, and the corresponding
Expected Default Frequencies, to decompose the excess return of the corporate bond
of each firm into two sources of risk premia: the distress risk premium and the jump-
at-default risk premium. Our estimates are in line with those previously reported in
the literature. In relative terms, we find that the distress risk premium accounts for
approximately 88% of the total excess return on (averaged) median, historically. On
the other end, the jump-at-default risk premium accounts for the remaining 12% on
(averaged) median. If we restrict our time horizon to the most recent period (since
January 2016), the relative contribute of the distress risk premium has even increased,
reaching the 93% of the excess return. The estimated risk premia show significant and
interesting variations not only in time, with marked increases in both risk premium
components during the two main crisis periods of the last ten years (the financial crisis
and the sovereign debt crisis), but also across type of firms (although the bonds are all
investment grade rated) and sectors. In particular, in the recent period the dispersion
of both risk premium components is higher for financial firms; moreover, this sector
exhibits a higher distress risk premium, while the jump-at-default risk premium is more
pronounced for non-financial firms.

Our analysis also allows us to estimate the expected losses component of the CDS
price, provided by the CDS price obtained under the objective measure (that does not
reflect the investors’ risk preferences).

To assess on which source of risk premium the CSPP has been more effective, we
perform an event study exercise around the dates of the announcements of the ECB related to the launch of this programme and we can see, as we would expect, that the date correspondent to the main decrease of the CDS price and its components, is the date of the first announcement of the programme, on 10 March 2016. Looking at the variations between the day before and the day after the first announcement of the programme, we find that on average the decrease of the risk premia was about 16.7 basis points (−10.9 and −5.8 basis points for the distress risk premium and the jump-at-default risk premium, respectively), while the decrease of the expected losses was about 3.7 basis points, suggesting a more important effect of the CSPP on the risk premia component, and in particular on the distress risk premium component; however, also the reduction of the expected losses component is not negligible, proving that the CSPP has also contributed to diminish the default probability of the issuers, by improving the financial conditions and the expectations on the economic outlook. From the same exercise we also identify important spillovers of the launch of the CSPP to the financial sector, in particular in terms of a decrease of the distress risk premium and of the expected losses.

Subject of future research will be an analysis of the systemic and idiosyncratic risk in corporate bond market, based on our firm-by-firm analysis and a comparison of the results obtained calibrating the model to the overall iTraxx index with those of a theoretical iTraxx index obtained from the aggregation of individual CDSs.
Appendix

A.1 Decomposition of the excess return into two components

In this appendix we provide mathematical evidence to explain the different sources of the excess return of a defaultable security. One first approach consists of looking at the equation for the price dynamics under actual and risk neutral measures.\(^{32}\) The price of a default-risky bond can be considered to depend on a set of state variables \(X_t\) the follow a diffusion process

\[
\frac{dX_t}{\mu^P(X_t, t)dt + \sigma_X(X_t, t)dW_t^P} \tag{A.1.16}
\]

where \(\mu^P\) and \(\sigma_X\) are the drift and instantaneous volatility, respectively, under the actual measure \(P\). According to Girsanov’s theorem the same equation can be represented under the risk-neutral measure \(Q\):

\[
\frac{dX_t}{\mu^Q(X_t, t)dt + \sigma_X(X_t, t)dW_t^Q} \tag{A.1.17}
\]

where

\[
W_t^Q = W_t^P + \int_0^t \Lambda_t ds,
\]

and

\[
\mu^Q_X = \mu^Q_X(X_t, t) - \sigma_X(X_t, t)\Lambda_t
\]

are the Brownian motion and the drift under the risk-neutral measure \(Q\), with \(\Lambda_t\) being the price of risk.

The dynamics of the price \(P(t, X_t)\) can be represented by applying Ito’s lemma\(^{33}\) under \(Q\) measure:

\[
dP(X_t, t) = \frac{\partial P(X_t, t)}{\partial t} dt + \frac{\partial P(X_t, t)}{\partial X_t} r_t dt + \frac{\partial P(X_t, t)}{\partial X_t} \sigma_X(X_t, t) dW_t^Q \tag{A.1.18}
\]

and under \(P\) measure:

\[
dP(X_t, t) = \frac{\partial P(X_t, t)}{\partial t} dt + \frac{\partial P(X_t, t)}{\partial X_t} (r_t + \sigma_X(X_t, t)\Lambda_t) dt + \frac{\partial P(X_t, t)}{\partial X_t} \sigma_X(X_t, t)dW_t^P
\]

\[
+ \frac{1}{2} \frac{\partial^2 P(X_t, t)}{\partial X_t^2} \sigma_X^2(X_t, t) dt + (R_t - P(X_t, t))(\Lambda_t^Q - \psi(t)\Lambda_t^P) dt \tag{A.1.19}
\]

\[
\text{Technical details can be also found in Cont-Tankov (2004).}
\]

\[
\text{Terms come from Ito’s lemma for a process with drift, diffusion and Poisson jumps (due to default) and Girsanov’s theorem; see A.2 for a brief overview and Cont-Tankov (2004) for details.}
\]

\[
32\text{Technical details can be also found in Cont-Tankov (2004).}
\]

\[
33\text{Terms come from Ito’s lemma for a process with drift, diffusion and Poisson jumps (due to default) and Girsanov’s theorem; see A.2 for a brief overview and Cont-Tankov (2004) for details.}
\]

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with \( \psi(t) = \frac{\lambda}{X} - 1 \). The expected excess return of the risky bond can be defined as the difference of expectations under \( Q \) and \( P \) measures:

\[
\mathbb{E}_P\left[ \frac{dp(X_t,t)}{P(X_t,t)} \right] - \mathbb{E}_Q\left[ \frac{dp(X_t,t)}{P(X_t,t)} \right] = \frac{1}{P_t} \partial P_t \partial X_t \sigma_X(X_t,t) \Lambda_t + \frac{P(X_t,t) - R_t}{P(X_t,t)} \lambda_t^\psi \quad (A.1.22)
\]

### A.2 Ito’s lemma for a process with drift, diffusion and Poisson jumps

In this appendix we just want to give the main intuition/ingredients underlying the Ito’s lemma when we have a process that includes jumps.\(^{34}\) Consider a Poisson process with jump intensity \( \lambda_1 \).\(^{35}\) The probability of one jump in the interval \([t, t + \Delta t]\) is \( \lambda_1 \Delta t \). The survival probability \( \bar{p}(t) \) (i.e. the probability that no jump has occurred in the interval \([0, t]\)) is

\[
\bar{p}(t) = e^{-\int_0^t \lambda_1(s) \, ds}.
\]

Let \( X(t) \) be a discontinuous stochastic process and denote \( X(t^-) \) the value of \( X(t) \) as we approach \( t \) from the left. Denote by \( d_jX(t) \) the non-infinitesimal change in \( X(t) \) as a result of a jump. Then

\[
d_jX(t) = \lim_{\Delta t \to 0} (X(t + \Delta t) - X(t^-)).
\]

Let \( z \) be the magnitude of the jump and let \( \eta(X(t^-), z) \) be the distribution of \( z \). The expected magnitude of the jump is

\[
\mathbb{E}[d_j(X(t))] = \lambda_1(X(t^-)) dt \int_z z \eta(X(t^-), z) \, dz.
\]

Define \( dJ_X(t) \), a compensated process and martingale, as

\[
dJ_X(t) = d_jX(t) - \mathbb{E}[d_jX(t)].
\]

Then

\[
d_jX(t) = \mathbb{E}[d_jX(t)] + dJ_X(t) = \lambda_1(X(t^-)) \left( \int_z z \eta(X(t^-), z) \, dz \right) + dJ_X(t).
\]

Consider now a function \( f(X(t),t) \) of the jump process \( dX(t) \). If \( X(t) \) jumps by \( \Delta x \) then \( f(X(t)) \) jumps by \( \Delta f \). \( \Delta f \) is drawn from distribution \( \eta_f() \). The jump part of \( f \) is

\[
f(X(t)) - f(X(t^-)) = \lambda_1(t) dt \int_{\Delta f} \Delta f \eta_f(\cdot) \, d\Delta f + dJ_f(t).
\]

\(^{34}\) Technical details can be found in Cont-Tankov (2004).

\(^{35}\) \( \lambda_1 \) could be a constant, a deterministic function of time, or a stochastic process.
Let $X_t$ be a diffusion process with jumps, defined as the sum of a drift term, a Brownian stochastic integral and a compound Poisson process:

$$X_t = X_0 + \int_0^t \mu_s ds + \int_0^t \sigma_s dW_s + \sum_{i=1}^{N_t} \Delta X_i.$$ 

Then, the process $f(t, X_t)$ can be represented in differential notation as:

$$df(t, X_t) = \frac{\partial f}{\partial t}(t, X_t) dt + \mu_t \frac{\partial f}{\partial x}(t, X_t) dt + \frac{\sigma_t^2}{2} \frac{\partial^2 f}{\partial x^2}(t, X_t) dt$$

$$+ \frac{\partial f}{\partial x}(t, X_t) \sigma_t dW_t + [f(X_{t-} + \Delta X_t) - f(X_{t-})].$$

The term linked to the Girsanov transform is related to the derivative term (in addition to the usual derivative in case of a diffusion process) that comes from the change of the intensity of jumps: to change from a Poisson variable $N$ of parameter $\lambda_1$ to a variable of parameter $\lambda_2$, the derivative is

$$dP_2/dP_1(N) = (\lambda_2/\lambda_1)^N \exp(-\lambda_2 - \lambda_1).$$

### A.3 Ornstein Uhlenbeck dynamics for the risk neutral default intensity under the objective measure

Consider the Ornstein Uhlenbeck dynamics for the logarithm of the default intensity $\lambda_t$ under the risk neutral measure $Q$:

$$d \ln \lambda^Q_t = K^Q(\theta^Q - \ln \lambda^Q_t) dt + \sigma_\lambda dW^Q_t$$

(A.3.25)

Assume a market price of risk of the form

$$\Lambda_t = \gamma_0 + \gamma_1 \ln \lambda_t$$

We can write the relationship between the Wiener process under the risk neutral measure $W^Q_t$ and the Wiener process under the objective measure $W^P_t$ as

$$W^Q_t = W^P_t + \int_0^t \gamma_0 + \gamma_1 \ln \lambda_s ds$$

and, in differential terms,

$$dW^Q_t = dW^P_t + (\gamma_0 + \gamma_1 \ln \lambda_t) dt.$$
We can thus express (A.3.25) in terms of $W^Q_t$ as

\[ d\lambda_t = (K^Q Q + \sigma \gamma_0 - (K^Q - \sigma \gamma_1) \ln \lambda_t) dt + \sigma \lambda dW^P_t \]  

(A.3.26)

where

\[
K^P \theta^P = K^Q \theta^Q + \sigma \gamma_0; \quad K^P = K^Q - \sigma \gamma_1.
\]

### A.4 CIR dynamics for the risk neutral default intensity under the objective measure

Consider the CIR dynamics for the default intensity $\lambda_t$ under the risk neutral measure $Q$:

\[ d\lambda_t = K^Q (\theta^Q - \lambda_t) dt + \sigma \sqrt{\lambda_t} dW^Q_t \]  

(A.4.27)

$K^Q$ is the speed of adjustment, and must be strictly positive; to assure a $\lambda^Q_t$ strictly positive we must also impose the Feller condition: $2K^Q \theta^Q > \sigma^2 \lambda$.

We assume a market price of risk as in Cheridito et al. (2007)

\[ \Lambda_t = \frac{\gamma_0}{\sqrt{\lambda_t}} + \gamma_1 \sqrt{\lambda_t} \]

We can write the relationship between the Wiener process under the risk neutral measure $W^Q_t$ and the Wiener process under the objective measure $W^P_t$ as

\[ W^Q_t = W^P_t + \int_0^t \left( \frac{\gamma_0}{\sqrt{\lambda_s}} + \gamma_1 \sqrt{\lambda_s} \right) ds \]

and, in differential terms,

\[ dW^Q_t = dW^P_t + \frac{\gamma_0}{\sqrt{\lambda_t}} dt + \gamma_1 \sqrt{\lambda_t} dt. \]

We can thus express (A.4.27) in terms of $W^Q_t$ as

\[ d\lambda_t = K^Q (\theta^Q + \sigma \gamma_0 - (K^Q - \sigma \gamma_1) \ln \lambda_t) dt + \sigma \sqrt{\lambda_t} dW^P_t \]  

(A.4.28)

where

\[
K^P \theta^P = K^Q \theta^Q + \sigma \gamma_0; \quad K^P = K^Q - \sigma \gamma_1.
\]

In order to satisfy the Novikov condition we must impose $\gamma_0 \leq K^P \theta^P - \frac{1}{2}$.
A.5 Explicit solution for the CDS price in the CIR model

Under the risk neutral measure $\mathbb{Q}$ consider the equation for the CDS price

$$CDS_t^Q(M) = \frac{(1 - R^Q) \int_t^{t+M} D(t,u) \mathbb{E}^Q_t[\lambda^Q_u e^{-\int_u^t \lambda^Q_s ds}] du}{\int_t^{t+M} D(t,u) \mathbb{E}^Q_t[e^{-\int_u^t \lambda^Q_s ds}] du}$$  \hfill (A.5.29)

and the CIR dynamics for $\lambda^Q_t$ as in (9)

We can explicitly solve the two expectations in (A.5.29)\textsuperscript{36} as

$$\mathbb{E}^Q_t[\lambda^Q_u e^{-\int_u^t \lambda^Q_s ds}] = (K^Q \theta^Q A(u - t) + \frac{\partial A(u - t)}{\partial u} \lambda^Q_t) \cdot C(u - t) e^{-A(u-t)\lambda^Q_t}$$

where

$$A(u - t) = \frac{2(e^{\gamma(u-t)} - 1)}{(\gamma + K^Q)(e^{\gamma(u-t)} - 1) + 2\gamma}$$

$$\frac{\partial A(u - t)}{\partial u} := B(u - t) = \frac{4\gamma^2 e^{\gamma(u-t)}}{((\gamma + K^Q)(e^{\gamma(u-t)} - 1) + 2\gamma)^2}$$

$$C(u - t) = \left[ \frac{2\gamma e^{\frac{1}{2}(\gamma+K^Q)(u-t)}}{(\gamma + K^Q)(e^{\gamma(u-t)} - 1) + 2\gamma} \right]^{\frac{2K^Q\theta^Q}{\sigma^2}}$$

and

$$\gamma = \sqrt{K^Q \theta^Q + 2\sigma^2_\lambda}.$$ 

A.6 PDE associated to the expectation and Crank Nicholson scheme

Consider a stochastic process

$$dy_t = p(y, t) dt + \sigma(y, t) dW_t,$$

with $W_t$ standard Brownian motion, and the partial differential equation

$$S_t + S_y p(y, t) + \frac{1}{2} S_{yy} \sigma(y, t)^2 = c(y, t) S$$  \hfill (A.6.30)

with boundary condition $g(y, T)$ at $T$, where $p(y, t)$ and $c(y, t)$ are two functions of $y$ and $t$. According to the Feynman-Kac theorem, the solution of equation A.6.30 is given by

$$S(y, T) = \mathbb{E}^Q[e^{-\int_y^T c(y_s, u) du} g(y_T, T)].$$  \hfill (A.6.31)

\textsuperscript{36}Technical details can be found in Duffie and Singleton (2003) and Schonbucher (2003).
In our paper we want to compute $\mathbb{E}_Q[e^{-\int_t^\tau \lambda_d ds}]$ so we set

$$y_t := x_t = \ln \lambda_t$$

$$p(y, t) := K \theta - K x_t$$

$$\sigma(y, t) := \sigma$$

$$c(y_u, u) := \lambda_s = e^{x_s}$$

$$g(y_T, T) := 1.$$  

Our expectation is then solution of the following PDE:

$$S_t + S_x(K \theta - K x_t) + \frac{1}{2} S_{xx} \sigma^2 = e^{x_t} S$$  \hspace{1cm} (A.6.32)

with boundary condition $S(x, u) = 1$.

At this point we can employ the Crank-Nicholson scheme to solve it. This is a finite difference method that consists of replacing terms in equation A.6.30 by approximation in discrete time and space. Consider: $N$ time steps with $t_i - t_{i-1} = \Delta t$ constant; $L$ space steps with $x_j - x_{j-1} = \Delta x$ constant. We can approximate the derivatives in time and space as

$$\frac{\partial S}{\partial t} \approx \frac{S_{j,i+1} - S_{j,i}}{\Delta t}$$

$$\frac{\partial S}{\partial x} \approx \frac{1}{2} \frac{S_{j+1,i} - S_{j-1,i}}{2 \Delta x} + \frac{1}{2} \frac{S_{j+1,i+1} - S_{j-1,i+1}}{2 \Delta x}$$

$$\frac{\partial^2 S}{\partial x^2} \approx \frac{1}{2} \frac{S_{j+1,i} - 2S_{j,i} + S_{j-1,i}}{\Delta x^2} + \frac{1}{2} \frac{S_{j+1,i+1} - 2S_{j,i+1} + S_{j-1,i+1}}{\Delta x^2}$$

$$S \approx \frac{S_{j,i} + S_{j,i+1}}{2}.$$  

Using these approximations we can rewrite equation A.6.32 as

$$\frac{S_{j,i+1} - S_{j,i}}{\Delta t} + (K \theta + K X_j) \frac{S_{j+1,i} - S_{j-1,i} + S_{j+1,i+1} - S_{j-1,i+1}}{4 \Delta x}$$

$$+ \frac{1}{2} \frac{S_{j+1,i} - 2S_{j,i} + S_{j-1,i}}{2 \Delta x^2} + \frac{1}{2} \frac{S_{j+1,i+1} - 2S_{j,i+1} + S_{j-1,i+1}}{\sigma^2}$$

$$= e^{x_t} \frac{S_{j,i} + S_{j,i+1}}{2}.$$  \hspace{1cm} (A.6.33)

After some simple algebra calculations equation A.6.33 becomes

$$A_i S_{j+1,i+1} + B_i S_{j,i+1} + C_i S_{j-1,i+1} = -A_i S_{j+1,i} D_i S_{j,i} - C_i S_{j-1,i},$$

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that can be written in matrix form as

\[ M_2 S_{i+1} = M_1 S_i \]

where \( S_i = [S_{1,i}, S_{2,i}, \ldots, S_{L-1,i}]^T \),

\[
M_1 = \begin{bmatrix}
D_1 & -C_1 \\
-A_2 & D_2 & -C_2 \\
& -A_3 & D_3 & -C_3 \\
& & & \ddots & \ddots \\
& & & -A_{M-2} & -D_{M-2} & -C_{M-2} \\
& & & & -A_{M-1} & -B_{M-1}
\end{bmatrix}
\]

and

\[
M_2 = \begin{bmatrix}
B_1 & C_1 \\
A_2 & B_2 & C_2 \\
& A_3 & B_3 & C_3 \\
& & & \ddots & \ddots \\
& & & & A_{M-2} & B_{M-2} & C_{M-2} \\
& & & & & A_{M-1} & B_{M-1}
\end{bmatrix}
\]

A.7 Price of jump-at-default risk

Consider that the jumps in price occur from time to time: the first jump occurs at time \( \tau_1 \), the second occurs \( \tau_2 \) time units after the first, the third occurs \( \tau_3 \) units after the second, etc. The variables \( \tau_1, \tau_2, \tau_3, \ldots \) are independent exponential random variables, all with the same mean \( \frac{1}{\lambda} \). Because the expected time between jumps is \( \frac{1}{\lambda} \), the jumps are arriving at an average rate of \( \lambda \) per unit time. The process \( N_t \) counts the number of jumps that occur at or before time \( t \), and we say that has intensity \( \lambda \).

Suppose that under the probability measure \( \mathbb{P} \) the Poisson process \( N_t \) has intensity \( \lambda^P_t \), and thus distribution:

\[
P(N_t = k) = \frac{(\lambda^P_t)^k}{k!} e^{-\lambda^P_t}
\]  
(A.7.34)

We have that the compensated process \( M_t := N_t - \lambda^P_t \) is a martingale; if the intensity is time varying, the process \( N_t - \int_0^t \lambda^P_s \) is a \( \mathbb{P} \)-martingale. Just as we can use Girsanov’s Theorem to change the measure so that a Brownian motion with drift becomes a Brownian motion without drift, and in terms of this change of measure we define the market price of risk, we can change the measure for a Poisson process and define the related jump price of risk. For a Poisson process, the change of measure affects the intensity.
We want to change to a measure $\mathbb{Q}$ such that $N_t \ (0 \leq t \leq T)$ is a Poisson process with intensity $\lambda^Q_t$, that means that

$$P(N_t = k) = \frac{(\lambda^Q_t)^k}{k!} e^{-\lambda^Q_t}$$

(A.7.35)

and the process $N_t - \int^t_0 \lambda^Q_s \, ds$ is a $\mathbb{Q}$-martingale. Let us fix the time $T > 0$. The Radon-Nykodim derivative that allows the change of measure is the function

$$Z(T) = e^{(\lambda^P - \lambda^Q)T} \left( \frac{\lambda^Q}{\lambda^P} \right)^{N_T}$$

(A.7.36)

and

$$\mathbb{Q}(A) = \int_A Z(T) \, d\mathbb{P}$$

In fact it can be proved\textsuperscript{37} that the process $Z(t)$ satisfies

$$dZ(t) = \frac{\lambda^Q - \lambda^P}{\lambda^P} Z(t-) \, dM(t),$$

is a martingale under $\mathbb{P}$ and $\mathbb{E}Z(t) = 1$ for all $t$, that is necessary to be able to change the measure.

The jump price of risk can thus be defined as

$$\Gamma^Q_t = \frac{\lambda^Q_t}{\lambda^P_t} - 1 \quad \text{or} \quad \Gamma^P_t = 1 - \frac{\lambda^Q_t}{\lambda^P_t}.$$  

(A.7.37)

\textsuperscript{37}See Shreve (2008) for technical details.
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