Low frequency drivers of the real interest rate: a band spectrum regression approach

by Fabio Busetti and Michele Caivano
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LOW FREQUENCY DRIVERS OF THE REAL INTEREST RATE:  
A BAND SPECTRUM REGRESSION APPROACH

by Fabio Busetti* and Michele Caivano*

Abstract

This paper presents an empirical analysis of the underlying drivers of the real interest rate in advanced economies over the last 35 years. We adopt a band spectrum regression approach, which allows to study the link between the real interest rate and its determinants only over low frequencies, leaving aside business cycle fluctuations and high frequency noise. Spectral regressions are pooled across countries, allowing for country fixed effects. Our findings indicate that important factors affecting the long-term movements of real interest rates are the evolution of total factor productivity (with a specific role for human capital accumulation) and demographic trends. Monetary policy developments and changes in income inequality, instead, appear to play a limited part. According to our estimates, over recent years the natural rate of interest fell below zero in the euro area. Finally, the paper provides an empirical contribution to the debate on secular stagnation, suggesting that supply-side mechanisms were one of the most significant factors behind the fall in income growth in the advanced economies over the last two decades.

JEL Classification: C22, E43.
Keywords: natural rate, secular stagnation, spectral analysis.

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* Bank of Italy, Directorate General for Economics, Statistics and Research.
1 Introduction

Real interest rates tend to comove among advanced countries. Theoretical arguments suggest that their long-term behavior is driven by supply side determinants, such as technology and demographic trends, that in turn follow broadly common dynamics at the international level. These factors directly enter the definition of the ‘natural’ or ‘equilibrium’ real rate of interest, i.e. the one that would keep the economy operating at full employment with stable inflation. In an influential paper, Laubach and Williams (2003) estimate the natural real rate of interest by relating it to the (unobserved) trend growth of output and a Phillips curve.

In addition to supply side drivers, other elements affecting the aggregate propensity to save can influence the long-term dynamics of real interest rates. Examples include: monetary policy developments, investors’ risk attitude, wealth and income inequality, changes in demographic composition. All these factors have been mentioned in the literature as possible explanations of the gradual but sizeable fall in real interest rates since the eighties; see e.g. Rachel and Smith (2015) for a comprehensive review. Indeed, according to the so-called ‘secular stagnation’ hypothesis, revived by Summers (2014), the fall in real interest rates and potential growth observed in the last two decades is mainly a result of demand shortfalls, driven by higher savings. Gordon (2015) has also stressed the contribution of a slowdown in technological progress.

In this paper we provide a novel empirical analysis to identify the relative importance of the various potential drivers of the long-term behavior of the real interest rate. We adopt the methodology of band spectrum regression, which allows to isolate the contributions of the various factors in explaining only the low frequency movements of the dependent variable leaving aside the business cycle fluctuations and the high frequency noise; see, inter alia, Hannan (1963) and Engle (1974). As restricting to low frequencies implies a loss of degrees of freedom, we pool band spectrum regressions across countries, allowing for country fixed effects.

Our findings indicate that at low frequencies most of the variation of real interest rates is explained by the evolution of demography and total factor productivity; within the latter a major driver is human capital accumulation. Monetary policy developments and changes in income inequality play on the other hand a limited role.

Several papers have used band spectrum regressions for empirical macroeconomic analyses. Engle (1974), Corbae, Ouliaris, and Phillips (1994) and Tan and Ashley (1999) investigate the permanent income hypothesis. Sum-
mbers (1986) studies the low frequency relationship between nominal interest rates and expected inflation. Erol and Balkan (1996) look at the responses of financial markets to money supply announcements. The link between money and inflation at various frequencies is examined in Assenmacher-Wesche and Gerlach (2008). Benati (2001) studies the relationship, at business cycle frequencies, between labor force participation and cyclical conditions for various sex and age groups. To our knowledge, no application has been proposed in the context of the secular stagnation hypothesis. The idea of pooling band spectrum regressions across countries seems novel as well.

The rest of the paper is organized as follows. Section 2 presents the main issues and some descriptive statistics. Section 3 contains the regression results. In section 4 our estimates of the natural rate of interest are given and compared with others taken from the literature. Section 5 concludes.

2 Underlying drivers of real interest rates

Real interest rates have been on a decreasing path over the past thirty years in most world economies. Figure 1 shows the average of the 3-month interbank rate in advanced economies (Canada, France, Germany, Italy, Japan, Spain, US and UK), deflated by the CPI inflation; the shaded band represents the range of values across countries. Besides some short term fluctuations, a downward trend is visible starting from the late eighties.

The debate on the sources of the decline in real interest rates was revived in the aftermath of the global financial crisis. Theoretical arguments suggest that the longer term movements of real interest rates are driven by supply side determinants, such as technology and demographic trends. These factors directly enter the definition of the ‘natural’ or ‘equilibrium’ real rate of interest, i.e. the one that would keep the economy operating at full employment with stable inflation.\footnote{In an influential paper, Laubach and Williams (2003) estimate the natural real rate of interest by relating it to the (unobserved) trend growth of output and a Phillips curve.} In a standard Solow growth model the natural rate would be equal to the real return to capital, which in steady state is given by

\[
r = \alpha \frac{(n + g + \delta)}{s},
\]

where \( r \) denotes the marginal product of capital, \( \alpha \) the capital share of the economy, \( n \) the rate of growth of the labor force, \( g \) the rate of growth of labor augmenting technical progress, \( \delta \) the depreciation rate and \( s \) the saving
rate. Declines in \( r \) can be interpreted in terms of a lower propensity to invest (driven by decline in the numerator of the right-hand side of equation 1, and in particular in the \( n + g \) term) or in terms of a higher propensity to save (as embedded in the saving rate \( s \)).

Numerous studies, that can be grouped under the ‘secular stagnation’ label, suggest that the global economy may have entered a long-lasting phase of slow growth, which has permanently decreased the propensity to invest. In the formulation of Summers (2014) the reduction in potential growth is mainly related to an increasing propensity to save, and decreasing propensity to invest, with associated demand shortfalls. A rise of income inequality might also be part of the story, through a shift of resources from low-income to high-income households who consume relatively less. Other authors, such as Gordon (2015), interpret the reduction in potential growth observed over the past 30 years mainly as a consequence of supply-side forces, including a slowdown in technological progress.\(^2\)

A different view, that essentially provides a ‘disequilibrium’ explanation, have instead emphasized the role of excessively expansionary monetary policies in determining the low level of interest rates. According to these pa-

\(^2\)In the 1930s, the then growing literature on long economic cycles suggested that demography may have a specific role in generating cycles of periodicity between 15 and 25 years (so-called Kuznets cycles, after Kuznets (1930)).
pers\textsuperscript{3} monetary policies, that have too aggressively reduced interest rates in downturns and too slowly increased them in upturns, might have induced a downward bias in interest rates, pushing them away from their 'natural' level. Eventually such a bias fuelled a build-up in debt, with resource misallocations and an impaired financial sector which in turn negatively affected potential output and the real rate.

The two interpretations we have briefly reviewed need not be seen as mutually exclusive, but they can both contribute in explaining the long-term fluctuations of real interest rates. The aim of this work is to assess the relative importance of various factors associated to either hypothesis.

Table 1 shows descriptive statistics for a set of indicators related to demographics, productivity growth, credit developments and income inequality; these will be used in the empirical analysis. The figures are averages over eight major advanced economies (Canada, France, Germany, Italy, Japan, Spain, USA and UK).\textsuperscript{4}

Regarding demographics developments, the observed pattern of the working age population points to a reduction of the real interest rate. On average over advanced economies, working age population growth has more than halved over the past 15 years compared to the 1980-89 decade. In parallel, the old age dependency ratio (defined as the share of population aged more than 64 over total population) shows a clear upward trend for all the economies considered, with increases between 1980 and 2015 that range from around 5 percentage points (for the UK and the US) to almost 30 percentage points (for Japan). How this trend can influence the movements of the real interest rate (\textit{a priori} in either direction) is a matter of empirical investigation.\textsuperscript{5}

Table 1 then shows a progressive slowdown of total factor productivity (measured by the so-called Solow residual in a standard growth accounting

\textsuperscript{3}Among else, Borio (2014) and Juselius, Borio, Disyatat, and Drehmann (2016).
\textsuperscript{4}The data have been retrieved from the FRED database of the Federal Reserve Bank of St. Louis (with the exception of credit-to-GDP data); the original source of the data are: (i) OECD, "Main Economic Indicators - complete database" for working age population (data of working age population for Germany have been corrected on the basis of Destatis data, to take into account the German re-unification), (ii) the BIS for credit-to-GDP ratios; (iii) the World Bank for the old age dependency ratio and the GINI indices of income distribution, (iv) the University of Groningen and University of California, Davis (Penn World Table 9.0) for TFP and human capital per person data.
\textsuperscript{5}On the one hand, standard life-cycle consumption theory suggests that the shift in composition towards older cohorts should decrease the propensity to save; on the other, the increase in life expectancy, which partly drives the trend in the old age-dependency ratio, should motivate higher savings to fund consumption for a longer retirement period.
framework) and of the OECD index of human capital per person. Consis-

tently with this evidence, some authors have suggested that the gains from
education might be approaching an upper bound in advanced economies,
due to the fact that there is a limit to the increase in education that a given
population may attain; cf. among else Gordon (2015).

Finally, for the advanced economies the table shows a rising trend in
income inequality (relatively stronger in the USA, UK and Germany) and
in the credit-to-GDP ratio. As the latter is partly related to an ongoing
process of financial innovation and liberalization, for the empirical analysis
carried out in the next section we will use a measure of gap in the credit-
to-GDP developments.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Working age population (% change)</td>
<td>0.88</td>
<td>0.63</td>
<td>0.36</td>
</tr>
<tr>
<td>Old age dependency ratio (%)</td>
<td>18.9</td>
<td>21.6</td>
<td>26.1</td>
</tr>
<tr>
<td>Total Factor Productivity (% change)</td>
<td>1.3</td>
<td>0.9</td>
<td>0.5</td>
</tr>
<tr>
<td>Human capital per person (% change)</td>
<td>0.74</td>
<td>0.62</td>
<td>0.46</td>
</tr>
<tr>
<td>Credit to GDP ratio</td>
<td>103.8</td>
<td>124.5</td>
<td>152.9</td>
</tr>
<tr>
<td>Income distribution (Gini index)</td>
<td>29.3</td>
<td>31.4</td>
<td>32.0</td>
</tr>
</tbody>
</table>

Table 1: Drivers of the real interest rate drivers: average values in advanced
economies

3  The empirical analysis

We start from a pooled linear regression in the time domain

\[ y_i = X_i \beta + \varepsilon_i \]  \hspace{1cm} (2)

where \( y_i \) is the \( T \times 1 \) vector of the real interest rate for country \( i \), \( i = 1, ..., n \)
and \( X_i \) is a \( T \times k \) matrix of country-specific explanatory variables; \( \varepsilon_i \) is
a \( T \times 1 \) country specific disturbance term. These variables include TFP
growth, the annual change in the age-dependency ratio, growth in working-
age population, the Gini index of income inequality and a measure of credit
to GDP gap; see Table 1. In the first subsection below it is shown how to

\[ ^6 \text{This is computed as the change over the previous decade of the ratio between credit to the non-financial private sector and GDP. Using a measure of gap helps to remove trend components related to financial liberalization and innovation.} \]
estimate model (2) across frequency bands. Subsections 3.2 and 3.3 contain the estimation results.

3.1 Pooled band spectrum regressions

A linear regression model can be equivalently set up in the frequency domain, by applying a finite Fourier transform to the dependent and independent variables. This creates a set of observations that are not indexed by time, but by frequency; see e.g. Hannan (1963) and Engle (1974). Spectrum regression then simply amounts to regressing the transformed dependent variables on the transformed covariates. We follow the suggestion of Duncan and Jones (1966) and Harvey (1978) to carry out the finite Fourier transform in real terms: the $T$-dimensional time series data are mapped into the frequency domain by pre-multiplying them by an orthogonal $T \times T$ matrix $Z$ with typical element

$$z_{tj} = \begin{cases} 
T^{-\frac{1}{2}} & \text{for } j = 1 \\
2T^{-\frac{1}{2}} \cos \left( \frac{\pi(j-1)}{T} \right) & \text{for } j > 1, \ j \text{ even} \\
2T^{-\frac{1}{2}} \sin \left( \frac{\pi(j-1)(t-1)}{T} \right) & \text{for } j > 1, \ j \text{ odd} \\
T^{-\frac{1}{2}} (-1)^{t+1} & \text{for } j = T, \ T \text{ even}
\end{cases}$$

(3)

Using this matrix, the transformed data can be handled by any regression package. Band spectrum regression can then be easily done by selecting the relevant frequencies through a selection matrix that contains the corresponding rows of $Z$.

The estimation of model (2) in a low frequency band (identified by fluctuations with periodicity $P$ greater than some threshold, here denoted as $P^{\text{min}}$) implies a loss of degrees of freedom, that could lead to very noisy estimates. This is the reason why we have imposed equal regression coefficients.

---

7 Although carried out in a straightforward way, the idea of pooling band spectrum regressions across countries seems novel.

8 Let $y$ be the $Tx1$ vector of time series data and let $\tilde{y} = Zy$. Assume for simplicity that $T$ is even. Then, for $0 < h < T/2$, the pairs of transformed observations $[\tilde{y}_{2h-1}], [\tilde{y}_{2h}]$ provide the contribution of frequency $\lambda_h = \frac{2\pi h}{T}$ (and period $P_h = \frac{T}{2\lambda_h}$); for $\lambda_1 = 0$ and $\lambda_{T/2} = \pi$ there is a single transformed observation, $[\tilde{y}_1]$ and $[\tilde{y}_T]$ respectively. See Harvey (1978) and Tan and Ashley (1999) for further details.
across countries; some heterogeneity is allowed by including country fixed effects.

It must be noted that the theoretical properties of band spectrum regression require that the data are stationary. While for nominal interest rates non-stationarity may arise as a consequence of a possible unit root in inflation, the real interest rate can usually be approximated by a stationary process, in line with theoretical economic arguments. The statistical evidence obtained from applying multivariate unit root tests (as in Abuaf and Jorion (1990) and Harvey and Bates (2003)) and multivariate stationarity tests (as in Nyblom and Harvey (2000)) indicates the real interest rate is level-stationary over the period 1980-2014; further details are in the appendix.9

3.2 Estimation results

The empirical analysis is carried out on annual data for Canada, France, Germany, Italy, Japan, Spain, US, UK, over the period 1980-2014. The real short term interest rate is computed using the nominal 3-month interbank rate, deflated by a measure of inflation expectations. The latter is obtained from the fitted data of an AR(1) model for annual CPI inflation, as in Hamilton, Harris, Hatzius, and West (2016).

Model (2) is estimated for the period 1980-2014, on demeaned data. The estimation has been conducted over two frequency bands, corresponding to periodicity larger than \( P_{\text{min}} = 7 \) years and \( P_{\text{min}} = 15 \) years: the total number of observations are, respectively, 88 and 40. The results are shown in table 2 for both a pooled OLS regression and a panel data regression with country fixed effects.

As preliminary evidence, the results of an OLS regression in the time domain is shown in the first column of the table; this regression is equivalent to the band spectrum regression when the selection matrix is equal to the identity matrix, i.e. when all frequencies included. From the OLS estimation in the time domain, the most important determinants of the real interest rate are the total factor productivity, the working age population and the credit-to-GDP ratio.

---

9The evidence of non-stationarity in the US real interest rate found by Hamilton, Harris, Hatzius, and West (2016) refers to a very long time series of data (from 1861 to 2014), subject to regime changes in the institutional and monetary policy framework.
Table 2: Baseline regression results in the time domain and over low frequencies

The estimates over low frequency bands offer a number of insights when compared with the regression in the time domain. For periodicity higher than 7 years, the coefficients on both total factor productivity and working age population increase substantially, while that on the credit-to-GDP ratio slightly decreases and is no longer significant at the standard 5% confidence level (although in the pooled regression it remains significant at the 10% level). The effect of inequality increases somewhat but remains insignificant. Focusing on even longer fluctuations, the estimates for periodicity longer than 15 years confirm the importance of total factor productivity and working age population as long run determinants of the real interest rate, while the role of the credit-to-GDP ratio appears limited. The effect of the age dependency ratio is now larger and statistically significant.\textsuperscript{10} Note also that the results of the pooled and the fixed effects spectral regressions are similar to each other.

\textsuperscript{10}The positive sign of the coefficient could be taken as an indication that in the long run the most relevant effect of the shift towards older cohorts of the population composition is to reduce the aggregate propensity to save, in line with standard life-cycle theories of consumption.
Generally speaking, table 2 shows that low frequency movements of real interest rates are largely driven by the determinants of the natural interest rate, i.e. by productivity and demographic developments. Disequilibrium factors, as proxied by the credit-to-GDP gap, may have long-lasting effects, but are basically unimportant in the long run. Inequality does not seem to play a role, although the pattern displayed by the coefficient estimate over progressively lower frequency bands suggests that it could be a relevant factor in the very long run.

It is also interesting to look at the role of these different factors over business cycle frequencies. Table 3 shows the estimation results for a frequency band corresponding to periodicities between 2 and 7 years.

<table>
<thead>
<tr>
<th></th>
<th>$P_{\min} = 2$, $P_{\max} = 7$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Pooled</td>
</tr>
<tr>
<td>Total factor productivity</td>
<td>0.206</td>
</tr>
<tr>
<td></td>
<td>(2.443)</td>
</tr>
<tr>
<td>Old age dependency ratio</td>
<td>-1.539</td>
</tr>
<tr>
<td></td>
<td>(-4.043)</td>
</tr>
<tr>
<td>Working age population</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>(0.022)</td>
</tr>
<tr>
<td>Credit-to-GDP ratio</td>
<td>0.048</td>
</tr>
<tr>
<td></td>
<td>(2.112)</td>
</tr>
<tr>
<td>Gini index</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>(0.764)</td>
</tr>
<tr>
<td>R-square</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Note: t-statistics in parenthesis

Table 3: Panel estimation results: business cycle frequencies

While the role of total factor productivity remains sizeable, there are substantial differences for the other explanatory variables: the effect of working age population growth is now tiny and insignificant, while that of the change in the age dependency ratio is negative and significant.\(^{11}\) The credit-to-GDP

\(^{11}\)The negative sign on the age dependency ratio regressor contrasts with the evidence
gap has a positive sign, but it is not significant in the fixed effects regression. Inequality is not significant also at the business cycle frequencies.

Finally, both credit-to-GDP and inequality movements turn out to be insignificant over the business cycle, which may depend on the limited variability of these variables in the short-to-medium run.

Similar indications can be drawn from the analysis of the share of variance of the various explanatory variables. Figure 2 reports the percentage of total $R^2$ attributable to each explanatory variable\(^{12}\) for the frequency bands considered here.

\(^{12}\)The figures are obtained by re-scaling (so they sum up to one) the so-called 'semipartial correlations', i.e. the correlation coefficients between the dependent variable and the residuals of the prediction of each independent variable by the other ones. The semipartial correlation also corresponds to the increase in $R^2$ when this variable is added to a regression that already included all the others. It can therefore be used to assess the specific contribution of each independent variable, net of the co-movements with the other regressors.

---

Figure 2: Specific contributions to total $R^2$
At low frequencies TFP growth is the variable that mostly contributes to explain the variability of the real interest rate, with a weight of 40% in the pooled regression for periodicities larger than 7 years, which increases up to 60% for periodicities larger than 15 years (the share is around 60% for both frequency bands in the fixed effects model). Demographic factors, as summarized by working age population and age dependency ratio, together provide a specific contribution of about 40% in total $R^2$ over long periodicities. In all cases inequality has a negligible predictive power, while the credit-to-GDP gap appears to provide some contribution at the middle frequencies, but not in the long run.

3.3 The role of human capital

The regression results presented so far did not consider human capital as a specific driver of real interest rates (although it was implicitly included in the measure of total factor productivity) given the considerable degree of uncertainty surrounding available measures of human capital, such as the one reported in table 1. Having some empirical evidence on the role of human capital is however of intrinsic interest. We have thus run a further regression including among the explanatory variables both human capital and TFP developments, where the latter has been orthogonalized by taking the residuals of a preliminary regression on human capital growth itself. The results are presented in table 4. The coefficient on human capital is positive and strongly significant for these low frequency bands, suggesting the importance of human capital in the longer term fluctuations of real interest rates. The coefficients on TFP and working age population are now smaller in size compared with the benchmark specification, but they remain significant. The overall conclusions, pointing to a minor role of monetary policy developments and of inequality, remain unchanged.
<table>
<thead>
<tr>
<th></th>
<th>Pooled</th>
<th>Fixed effects</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time domain</td>
<td>$P_{\text{min}} = 7$</td>
<td>$P_{\text{min}} = 15$</td>
<td>$P_{\text{min}} = 7$</td>
</tr>
<tr>
<td>Total factor productivity</td>
<td>0.363</td>
<td>0.640</td>
<td>1.225</td>
<td>0.746</td>
</tr>
<tr>
<td></td>
<td>(3.582)</td>
<td>(2.706)</td>
<td>(3.241)</td>
<td>(3.042)</td>
</tr>
<tr>
<td>Human capital</td>
<td>6.921</td>
<td>7.007</td>
<td>7.352</td>
<td>6.578</td>
</tr>
<tr>
<td>Age dependency ratio</td>
<td>0.430</td>
<td>0.593</td>
<td>1.111</td>
<td>0.590</td>
</tr>
<tr>
<td></td>
<td>(3.041)</td>
<td>(2.620)</td>
<td>(3.172)</td>
<td>(2.583)</td>
</tr>
<tr>
<td>Working age population</td>
<td>0.873</td>
<td>1.335</td>
<td>1.900</td>
<td>1.166</td>
</tr>
<tr>
<td></td>
<td>(2.722)</td>
<td>(2.302)</td>
<td>(2.160)</td>
<td>(1.918)</td>
</tr>
<tr>
<td>Credit-to-GDP ratio</td>
<td>-0.008</td>
<td>-0.006</td>
<td>-0.003</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>(-1.387)</td>
<td>(-0.713)</td>
<td>(-0.279)</td>
<td>(-0.841)</td>
</tr>
<tr>
<td>Gini index</td>
<td>-0.014</td>
<td>-0.046</td>
<td>-0.074</td>
<td>-0.032</td>
</tr>
<tr>
<td></td>
<td>(-0.535)</td>
<td>(-0.894)</td>
<td>(-0.956)</td>
<td>(-0.616)</td>
</tr>
<tr>
<td>R-square</td>
<td>0.30</td>
<td>0.33</td>
<td>0.56</td>
<td>0.365</td>
</tr>
</tbody>
</table>

Note: t-statistics in parenthesis, computed with heteroscedasticity-consistent standard errors.

Table 4: Regression results including human capital accumulation

4 Estimates of the natural rate of interest

Fitted values for the low frequency component of the real interest rate are reported in figures 3 for the case of $P_{\text{min}} = 15$. The red dotted lines are the actual values of the real interest rate. The green dashed (red solid) lines are referred to the pooled (fixed effect) fitted values.\(^{13}\)

The estimated low-frequency component of the real interest rate tend to show a downward pattern over the sample, which is clearer for the the euro area and Japan where demographic factors seem to play a comparatively stronger role. Taking these estimates as proxies for the natural rate of interest, the figure shows a marked decline of the natural rate since the

\(^{13}\)The euro area is approximated by taking a (GDP-weighted) average of Germany, France, Italy and Spain.
mid-eighties, especially in the euro area, where the fall is near 5 percentage points. Over recent years the natural rate remained positive in US and UK, but fell below zero in the average of the euro area and in Japan.

Figure 4 shows a comparison between the low-frequency components of our real interest rates at periodicities larger than 7 and 15 years and the corresponding estimates of the natural rate of interest in Holston, Laubach, and Williams (2016) for the United States, the Euro area, the United Kingdom and Canada. The series are not fully comparable as we use slightly different definitions of short-term interest rate (e.g. for the US we consider the 3-month Libor rate, while Holston, Laubach, and Williams (2016) use the federal funds rate) as well as a different price index for computing inflation expectations: while Holston, Laubach, and Williams (2016) use core inflation, we use the total consumer price index; the two measures may differ in period of high volatility of commodity prices. Two main differences
can be highlighted between our estimates and those of Holston, Laubach, and Williams (2016): first, our estimates are considerably smoother, as they exclude by construction high frequency movements, while those by Holston, Laubach, and Williams (2016) seem to capture also fluctuations of the real rate occurring at business cycle frequencies; second, our estimates tend to anticipate some of the turning points of those of Holston, Laubach, and Williams (2016); this is especially evident in the case of the United Kingdom. Notwithstanding these differences and the above caveats, our estimates are generally comparable with those of Holston, Laubach, and Williams (2016): for all countries the natural rate of interest is on a decreasing pattern, that becomes especially pronounced in the past ten years; differently from Holston, Laubach, and Williams (2016), however, our estimates suggest that the natural rate of interest may have recovered over the most recent years. The comparison at the boundaries of the sample has to be taken with caution however: both our estimates and those by Holston, Laubach, and Williams (2016) are indeed based on two-sided filters, which typically display some instability at the very beginning and at the very end of the sample. Another strand of literature estimates the natural rate of interest by means of dynamic stochastic general equilibrium models (DSGE); this approach, that identifies the natural rate of interest as the real interest rate that would prevail in a flexible price equilibrium, generally delivers estimates characterized by a lower degree of smoothness compared with ours. Gerali and Neri (2017) use a DSGE model for estimating the natural rate in the United States and the euro area, finding a generalized decline of the natural rate starting in the 1980s; over recent years they estimate a negative value for the natural rate both in the euro area and, differently from our results, also in the United States.
According to our estimates, over recent years the natural rate of interest remained positive in US and UK but fell below zero in the euro area and Japan. This is similar to what has been found by Holston, Laubach, and Williams (2016) using an unobserved components model approach.

In terms of the debate on secular stagnation, our results indicate the predominance of supply side mechanisms behind the fall of income growth in advanced economies over the last two decades.
References


A Stationarity properties and additional estimates

The statistical evidence obtained from applying multivariate unit root tests (as in Abuaf and Jorion (1990) and Harvey and Bates (2003)) and multivariate stationarity tests (as in Nyblom and Harvey (2000)) indicate the real interest rate is (level) stationary over the period 1980-2014. The evidence is however more mixed if univariate tests are used. Country results are presented in the table below. For all countries the null hypothesis of stationarity of the real interest rate is never rejected at 1% significance, although in some cases rejections occur at 5% significance. On the other hand, the null hypothesis of non-stationarity in levels could be rejected at least at 10% significance only for Canada and Japan according to a standard Augmented Dickey-Fuller test; allowing for a linear trend yields more rejections of the unit root hypothesis.

<table>
<thead>
<tr>
<th>Country</th>
<th>KPSS (test statistics)</th>
<th>ADF (p-value)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Level</td>
<td>Detrended</td>
</tr>
<tr>
<td>US</td>
<td>0.67</td>
<td>0.04</td>
</tr>
<tr>
<td>Canada</td>
<td>0.72</td>
<td>0.09</td>
</tr>
<tr>
<td>UK</td>
<td>0.44</td>
<td>0.12</td>
</tr>
<tr>
<td>Japan</td>
<td>0.56</td>
<td>0.12</td>
</tr>
<tr>
<td>Germany</td>
<td>0.68</td>
<td>0.06</td>
</tr>
<tr>
<td>France</td>
<td>0.59</td>
<td>0.11</td>
</tr>
<tr>
<td>Italy</td>
<td>0.45</td>
<td>0.13</td>
</tr>
<tr>
<td>Spain</td>
<td>0.49</td>
<td>0.11</td>
</tr>
</tbody>
</table>

Note: bold numbers correspond to the null of stationarity (non-stationarity) being not rejected (rejected) at 1% (10%) significance for the KPSS (ADF) test.

Table 5: Unit root tests

To investigate the possibility of trend-stationarity in real interest rates, we have also run band spectrum regressions after removing a deterministic linear trend from the data. The results, provided in table 6, confirm from a qualitatively and, to a large extent, quantitative point of view those presented in section 3.2. The estimates at the business cycle frequencies (not shown here but available upon request) are also similar, which is not surprising as in both cases the long run movements in the data have been removed.
<table>
<thead>
<tr>
<th></th>
<th>Pooled</th>
<th>Fixed effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time domain</td>
<td>$P_{\text{min}} = 7$</td>
</tr>
<tr>
<td>Total factor productivity</td>
<td>0.48</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>(3.97)</td>
<td>(3.29)</td>
</tr>
<tr>
<td>Age dependency ratio</td>
<td>0.12</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>(0.71)</td>
<td>(1.42)</td>
</tr>
<tr>
<td>Working age population</td>
<td>1.59</td>
<td>2.24</td>
</tr>
<tr>
<td></td>
<td>(4.27)</td>
<td>(3.24)</td>
</tr>
<tr>
<td>Credit-to-GDP ratio</td>
<td>-0.020</td>
<td>-0.018</td>
</tr>
<tr>
<td></td>
<td>(-3.05)</td>
<td>(-1.73)</td>
</tr>
<tr>
<td>Gini index</td>
<td>0.00</td>
<td>-0.05</td>
</tr>
<tr>
<td></td>
<td>(-0.01)</td>
<td>(-0.80)</td>
</tr>
<tr>
<td>R-square</td>
<td>0.16</td>
<td>0.14</td>
</tr>
</tbody>
</table>

Note: t-statistics in parenthesis

Table 6: Panel estimation results - linearly detrended series
RECENTLY PUBLISHED “TEMI” (*)


N. 1126 – The cyclicality of the income elasticity of trade, by Alessandro Borin, Virginia Di Nino, Michele Mancini and Massimo Sbracia (July 2017).


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(*) Requests for copies should be sent to:


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