The collateral channel of unconventional monetary policy

by Giuseppe Ferrero, Michele Loberto and Marcello Miccoli
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THE COLLATERAL CHANNEL OF UNCONVENTIONAL MONETARY POLICY

by Giuseppe Ferrero*, Michele Loberto* and Marcello Miccoli*

Abstract

We build a general equilibrium model - along the lines of Williamson (2012) - where financial assets can be used as collateral in secured interbank markets to obtain reserves (central bank money). In this framework, frictions in the exchange process give rise to a liquidity premium for assets. An open market operation that provides reserves in exchange for assets decreases the availability of collateral by increasing its liquidity premium (and decreasing its return). The magnitude of the effect depends on assets' pledgeability properties (haircuts). We explore the positive implications of the model shown in the data. Focusing on the period 2009-2014, we analyse the relationship between yields of euro-area government bonds and the relative amount of bonds and central bank reserves held by the euro-area banking sector. We find evidence consistent with our model: yields decrease when reserves increase relative to bonds, with the effect being stronger at lower levels of haircuts. The results are confirmed after several robustness checks.

JEL Classification: E43, E58, G12.
Keywords: unconventional monetary policy, secured interbank market, asset prices.

Contents

1. Introduction ......................................................................................................................... 5
   1.1 Related literature .......................................................................................................... 7
2. Model .................................................................................................................................. 8
   2.1 Environment ............................................................................................................... 9
   2.2 Problem of the financial intermediaries ..................................................................... 12
   2.3 Equilibrium characterization ...................................................................................... 14
   2.4 Empirical implications of the model .......................................................................... 18
3. Empirical analysis ............................................................................................................. 18
   3.1 Data ............................................................................................................................ 19
   3.2 Baseline estimation results ......................................................................................... 20
   3.3 Robustness checks ..................................................................................................... 23
4. Conclusion ........................................................................................................................ 29
References .............................................................................................................................. 30
Appendix A: Problem of the financial intermediaries ........................................................... 31
Appendix B: Equilibrium characterization............................................................................. 32

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1 Introduction

Since the beginning of the financial crisis, central banks in advanced economies have been very active in their conduct of monetary policy, complementing their conventional tool, the short term interest rate, with unconventional ones. While this is not the place to provide a taxonomy of these tools, most of them have a common element that is investigated in this work: they alter the size and the composition of the balance sheet of both the central bank and the private sector. Outright purchases, asset swaps and refinancing operations fall in this category. The Term Auction Facility (TAF), the Term Securities Lending Facility (TSLF), the Term Asset-Backed Facility (TALF) and the Large-Scale Asset Purchases (LSAP) are just some examples of measures undertaken by the Federal Reserve since December 2007; Securities Market Programme (SMP), Longer-Term Refinancing Operations (LTRO) and the Asset Purchase Programmes (APP) have been used repeatedly by the ECB to enhance the monetary transmission mechanism and to provide further monetary stimulus in the proximity of the lower bound of short-term interest rates during the global financial and the sovereign debt crises.

Through these monetary policy measures central banks reduce (or increase) the amount of certain assets in the market and expand (or reduce) the amount of other assets usually characterised by a relatively higher degree of liquidity. The central bank is the only economic player that can conduct this kind of intervention on a large scale since, in principle, it can expand its balance sheet indefinitely owing to its monopolistic power in the provision of the most liquid asset in the economy, the monetary base (e.g. currency in circulation and central bank’s reserves). There is now a vast litterature studying the direct effects of such measures on the relative prices of assets that are exchanged. The main channels of transmission are related to the signalling effect - the committment that the central banks will keep interest rates low even after the economy recovers, least be subject to losses on the assets it has bought - and to the portfolio-rebalance effect - the ability through asset purchases to alter duration risk in the economy and thereby alter the yield curve. In this case, the degree of imperfect substitutability among private sector’s balance sheet items, which arises in the presence of economic frictions due to preferred-habitat investors, is the crucial element underpinning the economic effect of asset purchases.

In this paper we focus on an additional, indirect, effect of unconventional monetary policy measures that hinges on the secured interbank money market (lending of reserves among financial institutions backed by financial assets) and exploits an imperfect substitutability between assets due to frictions in the exchange process and their intrinsic pledgeability properties. We propose a general equilibrium model of how relative avail-

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1 This is a revised and expanded version of a previously circulated paper titled “Open Market Operations, Interbank Market and Over-collateralization”. We thank participants to the Banca d’Italia and EIEF lunch seminar, to the 11th Workshop on Macroeconomic Dynamics, to the Workshop on frictions and monetary policy held at Banca d’Italia, to the Macro Banking and finance Workshop, and discussants Leo Ferraris, Fabrizio Mattesini and Bruno Parigi for comments on the older version. We thank participants to the European Central Bank and Banca d’Italia internal seminar series for comments on the revised version.

2 Another set of unconventional monetary policy tools is aimed at influencing expectations, like forward guidance. We will not be dealing with these unconventional tools here.


4 Since the beginning of the global financial crisis, collateralized lending has taken the prominence of money market transactions with respect to unsecured lending. For the euro area, most interbank lending is secured. See European Central Bank (2015).
ability of money and collateral can influence the price of collateral through secured lending markets. We do so by expanding Williamson (2012) with a collateralized interbank market and with different types of assets (in addition to money), each differing in their pledgeability property (haircut levels). In our framework, frictions in the exchange process require the use of money, while assets have the property of facilitating trades, either directly in some type of exchanges or indirectly as collateral in the interbank market to obtain money. Assets are the preferred mean of exchange, since they dominate money in rate of return. This implies that when collateral is scarce, it commands a liquidity premium. However, since the haircut decreases the value of the loan that a collateral can secure, assets with larger haircuts have a lower liquidity premium. The central bank, by decreasing (increasing) the relative availability of collateral with respect to money through open market operations, it thus increases (decreases) the liquidity premium of the assets used as collateral, the strength of the effect being different depending on haircut levels. The main takeaway of the theoretical model is a set of positive implications on how the return of assets used as collateral varies with respect to the total amount of money and collateral available in the economy and its haircut level.

In the second part of our work, we explore empirically the predictions of the model. We build a panel data set of yields of euro area government bonds at different maturities spanning the period 2009-2014, together with haircut levels applied on repo transactions with euro area government bonds as collateral on one of the central clearing platform for repo in the euro area, Cassa di Compensazione e Garanzia. In our empirical strategy we regress the basis, i.e. the difference between the yield of the sovereign with a risk free rate and the credit default swap premia against the empirical counterpart of the main state variable in the model, the relative availability of money and collateral in the economy and haircut levels. The result of the estimates are consistent with the implications of the model. The basis decreases when money becomes more abundant relative to collateral, this effect being stronger at lower haircut levels. The economic impact of the estimated effect is not negligible: in our baseline estimates 25 billions more of reserves relative to bonds translate into a decrease of 2 basis points of the basis for a sovereign with a haircut level of 10%. The effect increases to around 4 basis points at a haircut level of 1%. Finally, in order to deal with potential endogeneity and stationarity issues, we perform a number of robustness checks, which show that results are unaffected.

The structure of the work is the following. Section 1.1 discusses the closest relevant papers, pointing out our contribution with respect to the literature, while section 2 provides a description of the model; equilibrium implication for prices are analysed in section 2.3. In section 3 we perform the empirical analysis; section 4 concludes. In the appendix a formal derivation of the model’s equilibrium and proofs omitted from the main text are provided.

5It should be noted that the model applies not only to unconventional measures, but any open market operations which changes the relative size of money and collateral in the economy will have an effect on asset prices.

6The model’s role is to guide the empirical analysis in the second part of the paper, as such some simplifying assumptions are taken. A thorough micro-foundation of interbank markets and assets’ coexistence, as well as normative analysis, are out of the scope of this work.

7A repurchase agreement, or repo, is a contract whereby one party agrees to sell, and another to buy, a security at the spot price and a forward agreement to buy back the same security at a specified date and price. It is essentially an interbank loan backed by securities.

8Since the model abstracts from short-term interest rates decisions by the central bank or credit risk of the assets, this is the closest empirical counterpart to the return of the asset in the theoretical model.
1.1 Related Literature

The theoretical framework of our model is based on the research line started by Williamson (2012). This seminal paper analyzes monetary policy in a general equilibrium model where money is micro-founded à la Lagos and Wright (2005) and financial intermediaries emerge naturally to insure agents against idiosyncratic liquidity shock (as in Diamond and Dybvig (1983)). Central banks modify the composition of public liquidity (currency and government bonds) throughout open market operations and a liquidity trap equilibrium, where the nominal interest rate is zero and banks keep excesses reserves, can emerge for any long-run money growth rate, whenever currency is plentiful relative to government bonds. In addition to Williamson (2012) we build a specific role for transfer between banks, by having banks that face idiosyncratic liquidity needs and a secured interbank markets allows the redistribution of liquidity among banks. Moreover, as in Williamson (2016) we allow the economy to have more than one type of asset with different pledgeability properties. In our theoretical framework assets are valued not only for their return in the different states of the world, but also because they provide additional liquidity services that facilitates exchange. Other papers with similar features are Andolfatto and Williamson (2015), Williamson (2016), Rochetau, Wright and Xiao (2015).

Our empirical analysis is linked to works that study the effect of unconventional monetary policies. This literature is now becoming vast, however very few works consider the effects through the collateralized interbank market. Two exceptions are Corradin and Maddaloni (2015) and D’Amico, Fan and Kitsul (2014) which focus on special repo markets respectively in the euro area and the US. The former analyzes the impact of outright purchases of the European Central Bank on repo rates. In particular they find that purchases increased the degree of specialness, a measure linked to special repo rates which gauges scarcity of the asset used as collateral. However they do not provide evidence that scarcity in collateral has also an effect on their prices. D’Amico et al. (2014) analyzes instead the effects of the asset purchases conducted during the LSAP program of the Federal Reserve on the special collateral repo market using CUSIP level data. The authors find that anticipated central bank purchases, reducing the aggregate supply of a given security, create a significant and quite persistent reduction of the repo rate on that specific security. As a consequence, this scarcity premium is incorporated also in the asset price. Similar to this work we provide evidence that unconventional monetary policy affects prices of collateral, however we show that this effect is not limited to assets used in special repos, but involves more general classes of collateral.

The main theoretical explanation for the effects of central banks’ asset purchases on long-term interest rates has been the presence of preferred-habitat investors à la Modigliani and Sutch (1966). The seminal contribution on this topic is Vayanos and Vila (2009), in such setup, changes in the supply availability of assets influence their return. Based

\footnotesize
\begin{itemize}
  \item For a more general introduction to this type of models see Williamson and Wright (2010)
  \item In a special repo contract two counterparties agree not only on quantities (the amount borrowed/loaned), prices (the interest rate charged to the borrower) and maturity, but also on the typology of security used as collateral, precluding the possibility to deliver asset that are substitutes.
  \item A fundamental feature to be put in evidence is that through this channel open market operations work not when they are announced, but when the purchases of the assets take actually place.
  \item In Vayanos and Vila (2009) investors have preferences for particular assets and they do not engage in trading across different maturities. Instead, risk-averse arbitrageurs intermediate across maturities and make the term structure arbitrage-free, ensuring that bonds with nearby maturities trade at similar prices. However, as arbitrageurs are risk averse and carry trade is a risky activity, they do not completely eliminate
\end{itemize}
on the preferred habitat framework, Greenwood and Vayanos (2014) consider how the supply and maturity structure of government debt affect bond yields in the US. In their empirical analysis they show that a decrease in securities’ supply increases the return of the security, this effect being larger on long-term bonds than short-term bonds. In our empirical analysis we also find that an increase in the scarcity of the security (through open market operations) increases their return, with the effects being smaller on assets with higher levels of haircuts. Since normal practice in central clearing counterparties is to set haircuts based on the historical volatility of securities, those with longer maturities, being more subject to duration risk, are associated with higher haircuts. Thus our framework, which takes into account the specific role of assets as collateral, generates opposite effects to those of Greenwood and Vayanos (2014) and provides evidence of a new channel of the effect of scarcity not present in their work.

Christensen and Krogstrup (2016) argue that central banks asset purchases have an effect on interest rates not only through changes in the supply of assets, but also because the corresponding injection of central bank reserves in the financial system has a portfolio re-balancing effect per se. They test this hypothesis using as an experiment the unconventional monetary policies conducted by the Swiss National Bank (SNB) in August 2011, that consisted in a large increase in central bank reserves achieved without acquiring any long-lived securities or close substitutes thereof. After controlling for changing market expectations about future monetary policy (signalling channel), they find that the drop in long-term bond yields predominantly reflected a drop in the term premium, suggestive of reserve-induced portfolio balance effects. Indeed, our paper support this evidence, both theoretically and empirically. In our model what matters is the relative availability of central bank liquidity (reserves) with respect to eligible collateral that affects asset prices through repo markets: an increase of reserves, keeping unchanged the stock of securities, causes a reduction of bonds yields.

2 Model

In this paragraph we provide a bird’s eye view of the model. As in Williamson (2012), the model features buyers and sellers that can settle their exchanges by using either fiat money or credit claims issued by banks and backed by assets; however, only a fraction of sellers is assumed to accept credit claims as a medium of exchange and, when deciding the composition of their portfolio, buyers do not know which seller they will meet. Banks and interbank market play a specific and well defined role inside the payment system: as in Diamond and Dybvig (1983) banks can diversify, issue credit claims backed by assets they hold and thus provide insurance to depositors on the type of meeting they will face. Banks themselves face random shocks on the fraction of depositors that need to withdraw their deposits; the price differentials arising from demand shock to particular clienteles of investors. An important implication of this framework is that when risk aversion is high, demand effects are more "local", in the sense that shocks to particular maturities are transmitted only to the securities with nearby maturities. An empirical support to this mechanism was provided by D’Amico and King (2013), who analysed the effects of securities purchases made under LSAP program using CUSIP level data.

Note that both medium of exchange could be defined as “outside money”, as the former is issued directly by the central bank, while the latter is a claim issued by the banking system, but the asset that backs the claim is not in zero net supply within the private sector. See Lagos (2010) for a definition of inside and outside money.

13
interbank market provides an insurance against the risk of being unable to satisfy the demand of their depositors. The same frictions that operate between buyers and sellers, also characterize exchanges between banks; as a consequence, loans in the interbank market are collateralized. Assets, when used as collateral, have different degrees of pledgeability, that is, haircut values. Finally, through open market operations - the central bank and the banking system trade money for bonds either through outright purchases - the central bank is able to determine the relative amount of fiat money and assets in the economy.

In the followings section we provide a formal, but concise, description of the theoretical framework, leaving all model’s derivations to the appendix, and sum up its empirical implications. While we will resort to some simplifying assumptions in order to keep the analysis straightforward, the main positive implications can be retrieved also in a more micro-founded model.\[14\] The stark modelling choices are an analytical simplification to highlight two mechanisms. The first is that once we consider the role of secured interbank market, the amount of collateral available in the economy is an important factor to take into account; central banks open market operation, by changing the relative amount of collateral and reserves in the economy, are able to influence the price of the assets used as collateral. The second is that the pledgeability properties of an asset, the haircut, are an important parameter to consider once assets have a role as collateral.

2.1 Environment

Time is infinite and discrete. Each period is divided into two subperiods. In the first subperiod (day) agents trade in a decentralized market (DM), while in the second subperiod (night) they trade in a centralized market (CM). There are two non storable goods, one for each subperiod, called DM and CM good. There exists a continuum of buyers with unit mass. Each buyer has preferences given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t [u(q_t) - w_t]$$ (1)

where $q$ is the consumption of the DM good, produced by sellers through a linear technology, while $w$ is the difference between labor supply and consumption of the CM good, produced only by buyers during the night with a linear technology. We assume that $u(\cdot)$ is logarithmic.\[15\] There is also a continuum of sellers with unit mass. Each seller has preferences given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t [-q_t + c_t]$$ (2)

where $q$ is the disutility to produce $q$ units of the DM good, and $c$ is the consumption of the CM good.

In the DM, buyers and the seller meet randomly and trade pairwise. The terms of the trade are determined through a Nash Bargaining Process in which all the bargaining power is given to the buyer. During the CM, instead, buyers and sellers trade in a walrasian

\[14\]The interested reader can refer to Williamson (2012), Williamson (2016) and Rochetau et al. (2015) for a more general and extensive treatment of the same theoretical subjects.

\[15\]Most of our results hold under a more general utility functions, provided that it is a continuous and differentiable function, $u'(\cdot) > 0$, $u''(\cdot) < 0$, it satisfies the Inada conditions ($u'(0) = \infty$ and $u'(\infty) = 0$), that $\exists \hat{x} > 0$ such that $u(\hat{x}) = \hat{x}$ and that $-\frac{u''(\hat{x})}{u'(\hat{x})} \geq 1$. 

9
market. As in Lagos and Wright (2005), limited commitment and the absence of a record-keeping technology make unsecured credit unfeasible and every trade in the DM must be \textit{quid pro quo}: sellers want to exchange the DM good only for claims that can be exchanged for goods in the future. We assume that in the DM buyers and sellers can trade in two alternative (and mutually exclusive) forms. In one type of exchange, that we called \textit{cash-meetings}, the need for a tangible object that serves as a medium-of-exchange is satisfied only by fiat money issued by the government. In the other type, defined \textit{credit-meetings}, the need for a medium-of-exchange can be satisfied by secured credit claims (IOUs) that buyers can provide to sellers, and a costless technology is available to sellers to verify that claims are backed by holdings of some assets.

In the CM buyers, sellers and the government meet in a centralized walrasian market. All production and consumption decisions are made during the CM, but buyers discover their type of meeting only at the beginning of the following DM. This give rise to risk-sharing role for financial intermediaries as in Diamond and Dybvig (1983).

2.1.1 Nominal and real assets

There are three assets in the economy, all in exogenous positive net supply and traded only in the CM: fiat money, $M$, nominal government bonds, $B$ and a real asset $A$. Fiat money is a tangible object, without intrinsic value, issued by the central bank. Government bonds are nominal liabilities issued by the government: each unit of bond is issued at time $t$ in the CM with nominal price $\tilde{\psi}_t$ and pays one unit of fiat money in the CM at time $t + 1$. The real asset is a one-period-lived Lucas tree. In any period $t$ during the CM, buyers are endowed in equal proportion claims on the asset $A > 0$. The asset pays off at the beginning of the CM in period $t + 1$ one unit of the CM good.

The role of the real asset in the model is to introduce an additional type of security in the model which can be used as collateral but, as it will be described below, has different pledgeability properties than the government bond. Alternatively one could have posited a longer-term government or a foreign supplied asset. As long as the pledgeability properties of this assets are different, the results would be unchanged.

2.1.2 Financial Intermediaries and the Interbank Market

As in Williamson (2012) the uncertainty on the type of meeting the buyers will face creates a role for a financial intermediary sector that allows risk-sharing across its depositors. In the economy, there is a continuum of short-lived banks. Banks are formed in the CM at time $t$, offer deposit contract to buyers and invest the deposit received in money, bonds and real asset. In the DM, at $t + 1$, banks allow their depositors to withdraw a predetermined amount of money or credit claims on their deposit, that can be used as medium of exchange, depending on the type of meeting they face. In credit-meetings sellers accept claims on deposits (because they are collateralized by bank’s assets) and at the beginning of the CM in $t + 1$ they go to the banks and cash those claims, while any remaining money and asset are then redistributed to banks’ depositors and the banks are dissolved. We assume that

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16 The argument goes as follows. If you know in advance you will be in a cash-meeting, then you will bring only money, because other assets are useless. Otherwise, in a credit-meeting you will bring only other assets, as fiat money is always weakly dominated in rate of return.

17 As discussed also by Rochetau et al. (2015), there is no so much difference if the additional asset is nominal or real. We make this assumption just to keep the analysis as simple as possible.
the banking sector is perfectly competitive: banks offer deposit contracts that maximize the expected utility of the buyer and earn zero profits.

In the spirit of Bhattacharya and Gale (1987), banks are subject to idiosyncratic liquidity shocks: in the DM they discover the effective fraction of buyers that will be in cash-meetings and credit-meetings. More specifically, with probability $1/2$ a bank will have a fraction $\rho + \varepsilon (\rho - \varepsilon$, respectively) of buyers in a cash-meeting and a fraction $1 - \rho - \varepsilon (1 - \rho + \varepsilon)$ of buyers in a credit-meeting. We call a bank of type 1 (type 2) if it has relatively more (less) buyers in a cash-meeting. Given our modeling assumptions, in the aggregate a fraction $\rho$ of meetings is a cash-meeting while a fraction $1 - \rho$ is a credit-meeting.

Banks discover their type at the beginning of the DM, when their investment choices in nominal and real assets have been already made. However they can trade money among themselves during the DM in a walrasian and secured interbank market. In order to trade money banks need to post collateral, in the form of either the government bond or the real asset. We assume that the government bond and the real asset have different degrees of pledgeability - the extent to which an asset can be used to secure loans. The bond has the highest degree of pledgeability, as the real amount of loans that can be secured is assumed to be equal to the real value of the bond; the real asset has a relatively lower degree of pledgeability, implying that a haircut is applied when these assets are used as collateral. The amount of money in real terms, $l$, that a bank can borrow in this market is constrained by the present value of the assets they have in the balance sheet, taking into account the haircut, $h \in (0, 1)$,

$$l_{t+1} \leq \frac{b_{t+1} + (1-h)a_{t+1}}{R}$$

where $R$ is the gross nominal rate on interbank lending, $b_{t+1}$ and $a_{t+1}$ are the amount of government bonds in real terms and the amount of real assets bought by the bank in the CM in period $t$. Interbank loans are settled at the beginning of the CM.

2.1.3 Consolidated government and open market operations

The central bank and government are a consolidated entity. At time $t$, in the CM the fiscal-monetary authority injects an amount of money $M_t$, issues an amount $B_t$ of one-period government bonds and levies lump-sum taxes $T_t$, denominated in terms of the CM.

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18 As in all papers in the tradition of Bhattacharya and Gale (1987), the liquidity shock is not micro-founded. This can be done in the present setup by having an island model a la Gertler and Kiyotaki (2010) in which banks cannot perfectly diversify over all islands (due, say, to geographical proximity needs). In this model the fraction of sellers that accepts cash in any given island is random, and banks are thus subject to this liquidity shocks. The interbank market across islands allows banks to smooth the liquidity shocks, up to the uninsurable shock $\rho$. For sake of simplicity we did not pursue this modelling strategy here, also because the assumption of log utility implies full consumption risk sharing through the interbank market, so that equilibrium consumption allocation and prices do not depend on $\varepsilon$.

19 We are implicitly assuming that banks have full commitment versus their depositors, while they have limited commitment versus the sellers and the other banks. A more micro-founded model would allow in the banks’ problem an incentive constraint that precludes them from stealing from their depositor. But, as can be seen in Williamson (2015) and Williamson (2016) this constraint would be automatically satisfied whenever banks should meet regulatory capital requirements. If there are no exogenous limits for banks to accumulate capital, as in the above cited papers, the main results of our model are still valid.

20 See for instance Venkateswaran and Wright (2014).
Banks observe their liquidity shocks and enter the interbank market.

Buyers withdraw after observing their type.

Buyers consume good 1, produced by sellers.

Banks settle interbank debt and sellers claims.

Sellers consumes good 2.

Buyers produce good 2.

New banks offer a deposit contract to buyers.

Buyers make deposits and banks make their portfolio choice.

Figure 1: Timeline of the model

Good, on buyers in the CM. Letting $\phi_t$ denote the price of money in terms of the CM good, the consolidated government budget constraint is

$$\phi_t(M_t + \tilde{\psi}_t B_t + T_t) = \phi_t(M_{t-1} + B_{t-1}).$$

(3)

We assume that the consolidated entity commits to a policy such that the total stock of nominal government liabilities, $M_t + B_t$, grows at a constant gross rate $\mu$. Moreover, as in Williamson (2012) the monetary authority keeps the ratio of currency to the total nominal government debt, $\delta$, constant:

$$M_t = \delta(M_t + B_t).$$

(4)

Here, $B_t$ denotes the bonds held by the private sector. We consider $B_t \geq 0$ for all $t$ (the government is a net debtor), that it is equivalent to restrict $\delta$ in the interval $(0, 1]$. In this paper, as in Williamson (2012), we interpret a change in $\delta$ as a permanent open market operations conducted by the central bank, whereby it alters the relative amount of money and bonds in the economy\footnote{Since in our model what matters is the total amount of pledgeable collateral, it makes no difference if the central bank purchases government bonds or real assets. Indeed, the latter would be equivalent to a change of $A$ (see Rochetau et al. (2015)). Equivalently the problem could be rewritten defining $\delta$ as $M_t = \delta(M_t + B_t + A_t)$, all implications of the model remaining unchanged.}.

We assume that the government starts in period zero with no outstanding liabilities ($\phi_0(M_0 + \tilde{\psi}_0 B_0 + T_0) = 0$) and that fiscal policy is purely passive: the path of lump-sum taxes changes to support chosen paths for the nominal liabilities of the consolidated government.

Figure 1 shows the timeline of the model, summarizing its description.

2.2 Problem of the financial intermediaries

Since the financial intermediation sector is competitive, banks’ problem is equivalent to maximize the utility of the buyers. Banks are formed in the CM at period $t$ and get dissolved in the CM in period $t + 1$. Thus their problem in the CM is the portfolio choice of money, government bond and the real asset given the deposits received by the buyer.
Formally:

$$\max_{m_{t+1}, b_{t+1}, a_{t+1}} -w_t + \beta \left[ \frac{1}{2} F^1(m_{t+1}, b_{t+1}, a_{t+1}) + \frac{1}{2} F^2(m_{t+1}, b_{t+1}, a_{t+1}) \right]$$

s.t.  

$$d_t + \tau_t = w_t + W_t$$

$$d_t = \frac{\phi_t}{\phi_{t+1}} m_{t+1} + \frac{\phi_t}{\phi_{t+1}} \psi_t b_{t+1} + p_t a_{t+1}$$

where $m_{t+1}, b_{t+1}$ are real amount of money and bonds and $a_{t+1}$ is the amount of the real asset bought by the bank in the CM in period $t$. $F^1$ and $F^2$ are the continuation values of the utility of the buyer after the current CM if the bank is of type 1 or 2, and $\tau_t$ represents lump-sum real taxes, $d_t$ are the real deposits the buyer makes to the bank and $W_t$ is the wealth the buyer has in the centralized market at time $t$.

Banks enter the DM period with $\{m_{t+1}, b_{t+1}, a_{t+1}\}$, receive withdrawal demands of money or credit claims by their depositors and have the possibility to access the interbank market in order to satisfy their withdrawal requests. In the subsequent CM banks devolve any remaining asset to their depositors and are dissolved. Since under perfect competition banks maximize the utility of buyers, banks will optimally choose the amount of money and credit claims to give to their depositors (indirectly also choosing loans on the interbank market) in order to maximize buyers’ utility, taking into account the bargaining process between the buyer and seller. Formally thus the continuation value for type 1 bank is defined by:

$$F^1(m_{t+1}, b_{t+1}, a_{t+1}) = \max_{m_{t+1}, b_{t+1}, \psi_{t+1}, m_{t+1}} (\rho + \varepsilon)u(q^m_{t+1}) + (1 - \rho - \varepsilon)u(q^c_{t+1}) + e^1_{t+1}$$

s.t  

$$R_{t+1} \leq b_{t+1}^1 + (1 - h)a_{t+1}^1$$

$$q^m_{t+1} = \frac{m_{t+1}^1 + l_{t+1}}{\rho + \varepsilon}, \quad q^c_{t+1} = \frac{(b_{t+1} - b_{t+1}^1) + (m_{t+1} - m_{t+1}^1) + (1 - h)(a_{t+1} - a_{t+1}^1)}{1 - \rho - \varepsilon}$$

$$e^1_{t+1} = h a_{t+1} + b_{t+1}^1 + (1 - h)a_{t+1}^1 - R_{t+1}$$

$$0 \leq a_{t+1}^1 \leq a_{t+1}, \quad 0 \leq b_{t+1}^1 \leq b_{t+1}, \quad l_{t+1} \geq 0, \quad m_{t+1} \leq m_{t+1}$$

where $m_{t+1}^1 + l_{t+1}$ defines the real amount of currency given to its buyers in cash-meetings (the amount $l_{t+1}$ coming from operating on the interbank market); $b_{t+1}^1$ and $a_{t+1}^1$ are respectively the amount of bonds (in real terms) and real assets that are not given to buyers in credit-meetings and can be pledged on the interbank market. $e^1_{t+1}$ defines resources in excess of the withdrawals and of the settlements of the interbank market (if any), given back in equal proportion to their depositors in the following CM.

---

22Note that ex-ante the buyer does not know whether she will face a type 1 or type 2 bank. This however does not give rise to an aggregate shock since the presence of the interbank market and the assumption of log utility generate full consumption risk sharing, so that buyers facing either type of bank will consume the same in equilibrium.

23The wealth of buyers in the CM is represented by resources of banks born at $t-1$ not traded away in the DM, that banks give back to their depositors.

24In order to see where this term come from is useful to consider each transaction the bank does at the beginning of the CM. The bank has $a_{t+1}$ units of the asset which give payoff at the beginning of the CM 1 and $b_{t+1}$ units of government bonds that pays off one unit of money each. Therefore real resources for the bank at the beginning of the CM are $a_{t+1}+b_{t+1}$. Then the bank pays to sellers who were in credit-meetings with their depositors $(b_{t+1} - b_{t+1}^1) + (1 - h)(a_{t+1} - a_{t+1}^1)$ and to other banks with which it operated on the interbank market $R_{t+1}$. Summing up these terms one obtains resources that might be redistributed to depositors at the beginning of the CM.
\(q_{1t}^m\) and \(q_{1t}^c\) are the quantities of the DM good consumed respectively by buyers in a cash-meeting and in a credit-meeting (\(m\) is mnemonic for money and \(c\) for credit). These variables are obtained in the following way. The bank offers to each buyer in a cash-meeting to withdraw currency in the nominal amount of \(m_{t+1}^{l+1}\). The buyer makes a take-it-or-leave-it offer to the seller, who is going to use these nominal resources to buy the consumption good in the following CM. The seller will accept the offer as long as the marginal cost of producing the consumption good in the DM is not greater than the marginal benefit of consuming the CM good given the offer received. Linear utility of the seller and the price level of the CM good then implies that the quantity of the consumption good the buyer can consume in the DM is given by \(q_{1t}^m = \frac{m_{t+1}^{l+1}}{\beta}\). Similarly, the bank offers to its depositors in a credit-meeting credit claims up to the amount of available resources on its balance sheet (therefore not considering bonds and real assets pledged on the interbank market), so that, given the take-it-or-leave-it offer to the seller, consumption of buyers in a credit-meeting is given by \(q_{1t}^c = \frac{(b_{t+1} - b_{t+1}^h) + (m_{t+1} - m_{t+1}^h) + (1-h)(a_{t+1} - a_{t+1}^h)}{1-h\epsilon}\). In a similar way we can write down the continuation value of type 2 bank, we refer the reader to equation (15) in the appendix for the precise formulation.

We will confine our attention of the problem defined in (5), (6) and (15) to stationary equilibria solutions where real quantities are constant over time. In this equilibrium, the supply of the real asset is constant and the inflation rate is a constant defined by \(\phi = \frac{\mu}{\phi_{t+1}}\). Hereafter we will refer to the real price of government bonds, defined as \(\psi = \mu\psi\). The definition of the equilibrium is the following:

**Definition 1 (Equilibrium Definition)** Given a monetary policy rule \((\mu, \delta)\), a quantity of real assets \(A > 0\) and a level of haircut \(h \in [0,1]\), a stationary equilibrium consists of real quantities of currency \(m\) and government bonds \(b\), bank transfers \(m^i, b^i, a^i\) for each bank type \(i = 1, 2\) and real interbank loans \(l\) and \(n\) such that, for given an initial tax \(T_0\), a gross interest rate on interbank market \(R\), bond price \(\psi\) and asset price \(p\), \(\{m, b, a, m^i, b^i, a^i, l, n\}\) i) solve problems (5), (6) and (15) when \(\frac{\phi_{t+1}}{\phi_{t+1}} = \mu\), ii) prices are such that all markets clear \(l = n, b = m(1/\delta - 1), A = a\), iii) \(T_1\) adjusts so that the government budget constraint (7) holds at \(t = 1, 2, \ldots\).

### 2.3 Equilibrium characterization

A thorough derivation of the solution of the model can be found in the appendix, here we will just sketch the features of the equilibrium and its positive implication for prices of the government bond and the real asset. Given \(\mu, \delta\) and the amount of real asset \(A\), the model features a unique equilibrium. However the equilibrium quantities and prices will differ depending on the value of \(\mu, \delta\) and \(A\). Intuitively, the inflation rate determines the consumption possibilities of the buyers in cash-meetings, while the real amount of interest bearing asset in the economy, given their role as collateral and thus as facilitator of exchanges, determine the consumption possibilities of buyers in credit-meetings, which in turn through their marginal utilities determine asset prices.

In the appendix we show that necessary condition for the equilibrium to exist is that \(\mu \geq \beta\), which implies that the nominal interest rate on the government bond is weakly

---

25 The role of the model is purely to provide a positive analysis. Normative questions are not taken into considerations in this work.

26 Uniqueness obtains since the problem is strictly concave.
positive. If $\mu = \beta$ then, independently of the values of $\delta$ and $A$, the model has a unique equilibrium in which $\psi = p = \beta$. This is the Friedman rule, in which the inflation rate is equal to the rate of time preference. In this equilibrium consumption is at its first best and there is no role for the banking system. In what follows, we thus restrict the parametrization to the case when $\mu > \beta$. Moreover we will assume that the amount of real asset $A$ is not too large, the motivation for such assumption will be clearer after the description of the equilibrium. Since our main object is to derive implications on asset prices of open market operations, in what follows we will focus on prices for different values of $\delta$. The following proposition provides a first general characterization of asset prices in equilibrium.

**Proposition 1 (Equilibrium prices)** For any $\delta$, in equilibrium $\beta \leq \psi \leq \mu$ and $p = h\beta + (1 - h)\psi$. Moreover, whenever the volume of interbank lending is positive $R = \frac{p}{\psi}$.

**Proof.** In the appendix

In the appendix we provide a formal proof of this proposition, here we just provide some intuition for the result. In equilibrium money, the bond and the real asset must be held by agents. Therefore $\psi$ cannot be greater than $\mu$ otherwise there would be no demand for the asset since at that point it would be better to carry only money, and $\psi$ cannot be less than $\beta$ otherwise there would be an infinite demand for bonds. Note that $\psi = \beta$ would be the price of the government bonds in a standard frictionless general equilibrium model, which we denote as price at fundamentals. When $\psi > \beta$ bonds have a liquidity premium, that is a premium commanded by the government bond given its role as collateral that facilitates consumption in the decentralized market. The government bond and the real asset will either both feature a liquidity premium, or both will be valued at fundamentals. However, the haircut decreases the value of the real asset in exchanges, so that when there is a liquidity premium ($\psi > \beta$), only a fraction $(1 - h)$ of the real asset is valued in the exchange process, and hence it must have the same real return of the bond, while a fraction $h$ will be valued for the dividend it pays off during the follow centralized market. Equivalently, when the government and the real asset are valued also for their liquidity properties in the exchange process, then the real asset will dominate in rate of return the government bonds given its inferior pledgeability properties.

The result that $R = \frac{\mu}{\psi}$ comes from a no-arbitrage condition: the bank must be indifferent between having a bond to pledge on the interbank market to obtain money at a nominal price $R$ and carrying one more unit of money from the CM foregoing the nominal return on the bond $1/\psi$ (since the bond price and the asset price are related through the no-arbitrage condition, an equivalent reasoning can be done in terms of the real asset).

We now illustrate the different equilibrium values of prices of the bond and the real asset depending on the value of $\delta$. Figure 2 provides a synoptic view. A thorough derivation can be found in the appendix.

**Plentiful interest bearing assets equilibrium.** When the quantity of bonds and the real assets is plentiful enough ($\delta$ low for given $A$), there are enough assets in the economy to back first-best level of consumption of buyers in credit-meetings. Banks will access the interbank market, however the quantity of assets and bond in the economy is large enough

---

27 A precise quantification of the statement is provided in the appendix.

28 In the appendix, a graph showing the equilibrium values of consumption and real quantities exchanged on the interbank market can be found.
the borrowing constraint on the interbank market is slack. In this case the prices of assets
do not incorporate any premium and $\psi = p = \beta$, that is, the yield of the government
bond and the real asset is positive and equal to $1/\beta$. It is worthwhile to note that the
two assets have the same return, although they have different pledgeability: as we will
see later, haircuts matter only if interest bearing assets are scarce. In this equilibrium a
marginal change of $\delta$ does not influence prices. Even if an open market operation lowers
the amount of bonds in the economy, there are still enough interest bearing assets such
that consumption is at the first best and the borrowing constraint in the interbank market
is slack.

**Scarcity interest bearing assets equilibrium.** As $\delta$ increases interest bearing assets
become scarce and banks cannot give to their depositors in credit-meetings enough claims
to consume the first best quantity of goods. The scarcity of interest bearing assets now
implies that the collateral constraint on the interbank market is binding. In this situation
banks will trade-off consumption of their depositors in credit-meetings and cash-meetings,
and the interbank interest rate $R$ has the role to equate marginal utilities of the buyers
in the different meetings. Since buyers in credit-meetings are not able to consume first-
best quantities, the prices of the assets now include a liquidity premium: the price of
the government bond is greater than $\beta$, similarly for the price of the real asset, $p = h\beta + (1-h)\psi > \beta$. However, $p < \psi$ because only a fraction of the real asset is valued
for consumption allocation. In this equilibrium open market operations, by changing the
relative size of money and bonds available in the economy, affect consumption allocation
of buyers and thus asset prices. A marginal increase in $\delta$, by increasing scarcity, implies
an increase of all asset prices, although the effect is stronger on government bonds because

$29$ For these parameter values, one can show that the yield of the government bond can be expressed in
closed form as

$$
\frac{1}{\psi} = \frac{\rho}{1-\rho} \left[ \left( \frac{1}{\delta} - 1 \right) + \frac{(1-h)A}{\rho u^{-1}(\mu/\beta)} \right]
$$


Figure 2: Equilibrium yields with respect to $\delta$ when $\mu > \beta$ and $A < A$. Threshold values
are defined in the appendix.
they are superior as collateral. Thus, an increase in $\delta$ implies a decrease of both yields and, given the different pledgeability values, the decrease of the yield of the government is higher than that of the real asset.\cite{footnote:13}

**Liquidity trap equilibrium.** In this case the quantity of interest bearing assets is so scarce that liquidity premium they command drives the nominal yields on the government bond to zero. The real price of the government bond reaches thus its upper threshold, $\psi = \mu$, as the price of the real asset, $p = h\beta + (1 - h)\mu$. Money and the government bond are perfect substitutes. Banks exchange collateral one-to-one for money on the interbank market and thus they are able to equalize consumption across buyers in all types of meetings (though consumptions levels are lower than first-best). In this case changes in the monetary policy choice $\delta$ have no real effect on consumption allocation and prices, only the volume of interbank loans is affected, since as $\delta$ increases money is so abundant that there is no need anymore for bank to access the interbank market.

Given the mechanics of the model, it is now clear the role of the assumption that $A$ should not be too large. If $A$ was large enough, only the plentiful interest bearing asset equilibrium will be a feasible equilibrium for the economy. However, as the supply of the real asset decreases, then both $A$ and $\delta$ will determine the type of stationary equilibrium for the economy. Figure\cite{figure:3} provides a synoptic view of the equilibrium as function of $\delta$ and $A$.

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\textsuperscript{30}The assumption of log utility is particularly useful in this case since it allows for full consumption sharing of the buyers across banks’ types, that is, consumption quantities and prices do not depend on the variable $\varepsilon$. However, according to numerical simulations, the results about the impact of changes in $\delta$ on $R$ and asset prices are robust to a constant relative risk aversion utility specification that satisfies the assumptions stated in footnote\textsuperscript{15} provided risk aversion is not excessively large.
2.4 Empirical implications of the model

The basic implications that we want to take out of the model are that, everything else equal, the haircut defines the value of the collateral in the interbank market, and so agents want to be compensated with higher yields in response to higher haircuts since these imply a lower pledgeability value of the bond. Given this property, open market operations have different effect on securities yields depending on haircut levels in secured interbank markets. In particular, our model gives rise to three empirical implications. Everything else equal:

• securities with higher haircut have higher yields;
• securities’ yields weakly decrease as the amount of relative liquidity ($\delta$) available in the economy increases;
• this decrease is less pronounced for securities that have higher haircuts.

3 Empirical analysis

We are going to test whether the model’s implications can be found in the data using a panel dataset consisting of sovereign bonds’ yields of some euro area country at different maturities and their corresponding haircut levels applied in repo transactions on commonly used trading platforms. There are some limitations of using these data for the empirical analysis. The most important is that the European Central Bank operates in a multi-country multi-assets environment. Thus, firstly other assets are used as collateral in open market operations with respect to those present in our dataset, implying that liquidity injections operations, for instance, might not decrease the amount of government bonds held by banks. Secondly liquidity conditions might vary among countries: aggregate euro area liquidity conditions might not be a good proxy for local liquidity, and thus for the need of banks to access the interbank markets. Both effects could bias downward the magnitude of the estimated effects of relative liquidity on the yields of the assets.

The main object of our analysis is the basis, i.e. the difference between the spread of the sovereign yield with a proxy for the risk free rate on the same horizon of the sovereign, and the premium on the credit default swap (CDS) contract on the same type of sovereign. Given that our model abstracts from risk free rates, expectations of future monetary policy short-term rate decisions\(^{31}\) and default risk, the basis represents the closest empirical counterpart to the yields in our model\(^{32}\). The basis represents the return one investor would obtain by borrowing at the risk free rate and buying a sovereign and its CDS:

$$b_{c,i,t} \equiv (y_{c,i,t} - IRS_{i,t}) - CDS_{c,i,t}, \quad (7)$$

where $y_{c,i,t}$ is the yield on sovereign of country $c$ with maturity $i$ at time $t$, $IRS_{i,t}$ is the risk free rate (maturity but not country specific), as measured by the rate on the zero-coupon Interest Rate Swap contract with maturity $i$, and $CDS_{c,i,t}$ is the premium for the CDS on

\(^{31}\)Note that by controlling for a risk-free interest rate of the same maturity of the sovereign, we are implicitly netting out all expectations about future short-term rates decisions of the central bank, which includes the signalling effect of unconventional monetary policy measures.

\(^{32}\)Results on regressions analysis where the dependent variables is the sovereign yield itself, controlling for CDS and a risk free rate, are similar.
the country \(c\) sovereign with maturity \(i\). In a frictionless market and absent any liquidity premium on the bonds, the basis should be zero. Our empirical analysis will ascertain whether deviations of the basis from zero are correlated with variations in the amount of liquidity, bonds and in haircut levels in the economy, consistently with results of the theoretical model presented in section 2. We therefore estimate an equation of the form:

\[
b_{c,i,t} = \beta_0 + \beta_1 \delta_t + \beta_2 h_{c,i,t} + \beta_3 \delta_t \ast h_{c,i,t} + \mu' X_{c,i,t} + \varepsilon_{c,i,t}
\]  

(8)

where \(b_{c,i,t}\) is the basis on sovereign of country \(c\) with maturity \(i\) at time \(t\), \(\delta_t\) is, as specified in the model, our measure of relative liquidity, \(h_{c,i,t}\) is the haircut applied on sovereign of country \(c\) with maturity \(i\) at time \(t\) and \(X_{i,t}\) is a set of controls, which includes country, maturity and quarter-year dummies in our baseline specification, but will include other variables in the robustness section.

In order to construct the empirical counterpart of the variable \(\delta\), the measure of relative liquidity, we use as the empirical counterpart of \(M\) reserves issued by the European Central Bank and held by euro area banking sector in the deposit facility and current account at the Eurosystem. Since reserves are issued as counterpart of open market operations and they constitute the object exchanged on the collateralized interbank market, they provide the closest empirical representation to \(M\) in the model. We chose not to include currency (physical banknotes) in circulation since it is not exchanged on the interbank market; moreover it is a well known fact that its time evolution is very stable, therefore it would constitute only a level shift. As the empirical counterpart of \(B\) we are going to use the amount of sovereign bonds on the balance sheet of Monetary and Financial Institutions (MFI) in euro area.

3.1 Data

We build a panel dataset for sovereigns of some euro area countries, namely Austria, Belgium, France, Germany, Ireland, Italy, Netherlands, Spain. We use yields on zero-coupon sovereigns at the 2-, 3-, 4-, 7-, 10-, 15-, 30-year maturity as provided on the Bloomberg platform. The choice of countries, maturities and time sample is dictated by the availability of data on haircuts. Haircut levels come from Cassa di Compensazione e Garanzia (CCG), which acts as central clearing counterparty for operations conducted on the MTS, EuroMTS and BrokerTec repo trading platform.\(^{33}\) For each country, CCG differentiates sovereigns into different classes according to an interval of maturity: for instance a class includes bonds with residual maturity between 4 years minus one day and 7 years. Each class is then associated to a haircut level. We match the haircut level in that class with the yield of the bond of the highest maturity within that class. Unfortunately CCG did not act as central clearing counterparty for repo conducted with all euro area sovereigns as collateral until recently, therefore the available series of haircuts span a different time sample depending on the country. The sample starts in January 2009 for Italy, in March 2010 for France and Germany, in June 2014 for Austria, Belgium, Netherlands and Spain, and in September 2014 for Ireland.

The sample ends in December 2014, before the decision of the ECB to start the quantitative easing program involving the purchase of euro area sovereigns (Public Sector Purchase Program) announced on the 22nd of January 2015. Our choice is motivated by the

\(^{33}\)We thank Stefano Corradin and Cassa Compensazione e Garanzia for kindly sharing these data with us.
fact that the channels of transmission of the quantitative easing programs highlighted in
the empirical literature (see for instance, Krishnamurthy and Vissing-Jorgensen (2011)),
while implying, as in our model, that an increase in reserves is associated with a decrease
in yields, are different from the ones derived in this work. Since our econometric pro-
dure does not allow to separately estimates the contribution of the different channels of
transmissions, we chose to end the sample before the announcement of the program.

Given the frequency limitation on the availability of data on the amount of sovereign
bonds in the balance sheet of banks, our panel dataset will have monthly frequency. How-
ever, in order to avoid unusual variation in the last day of the month, for variables available
at daily frequency we compute averages over the week spanning the end of the month. In
particular, weekly averages are computed from Wednesday to the Tuesday of the following
week. This is so in order to average reserves held by banks between the weekly Main
Refinancing Operations auctions of the ECB (which are allotted on Wednesdays). Thus,
for every maturity and country, an observation in our dataset is the yield of the sovereign,
reserves, price of CDS and IRS rate averaged over the week that spans the end of the
month; amount of bonds on balance sheet of the MFIs and haircut levels are as of the last
working day of the month.

Table 1 provides summary statistics for variables in our dataset (values are in percent-
age points). For each country \( N \) represents the number of observations for each sovereign
maturity. The dataset has 1505 observations. Average haircuts increase with maturity
of the bond for all countries, ranging from a minimum of 1% on the 2-year maturity for
Austrian, German and Dutch bonds during the last months of 2014 to 30% on the Italian
30-year maturity bond during 2012 and 2013. Time variation in haircuts is low: for
Italy (the country with the longest time span in our sample) we have only 12 changes
in the haircuts in our sample. The basis is negative on average in our sample for most
sovereigns and maturities, the exception being Italian, Spanish and French sovereigns at
longer maturities. Figure 4 shows the empirical counterpart of the measure of relative
liquidity in the model, \( \delta \). The average value in the sample is 0.2 and it ranges from 0.1
to 0.37 in March 2012, after the ECB implemented its two 3-year Long-Term Refinancing
Operations (LTOs).

3.2 Baseline estimation results

The upper panel of table 2 provides estimates of our regression equation, while the lower
panel of Table 2 provides marginal effects estimate at different percentile of the distribution
of Haircut and \( \delta \). Column (1) provides estimates of (8) with no control variables included
in the equation; Column (2) and (5) provide baseline estimates after controlling for country,
maturity and quarter-year fixed effects, respectively, obtained with OLS and Panel Fixed
Effects regression methods, the latter with the cross-section defined as the couple country-
maturity. The coefficient on the constant is not reported but included in every regression.

All estimated coefficients and marginal effects are significant, with magnitudes constant
across estimation methods. Consistently with the implication of the model, an increase
in the amount of relative liquidity, \( \delta \), is linked to lower levels of the basis, the effect being

\[34\] Data for the CDS premia and the IRS rates come from the Thomson Reuters (CDS data on 15-year
maturity sovereigns was not available. We used the CDS on the 20-year maturity sovereigns instead).
Data on the amount of reserves held at the deposit facility and the current account for euro area banks is
provided at daily frequency on the ECB website. The series for euro area sovereigns held by the MFIs is
available from the ECB’s Statistical Data Warehouse with monthly frequency (end of month).
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<td>2.679</td>
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<td>-0.716</td>
<td>-0.740</td>
<td>-0.618</td>
<td>-0.682</td>
<td>-0.589</td>
<td>-0.334</td>
</tr>
<tr>
<td><strong>Ireland (N=4)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yield</td>
<td>0.123</td>
<td>0.269</td>
<td>0.388</td>
<td>0.992</td>
<td>1.594</td>
<td>1.866</td>
<td>1.855</td>
</tr>
<tr>
<td>Haircut</td>
<td>0.070</td>
<td>0.075</td>
<td>0.075</td>
<td>0.090</td>
<td>0.100</td>
<td>0.100</td>
<td>0.270</td>
</tr>
<tr>
<td>CDS</td>
<td>17.960</td>
<td>26.100</td>
<td>33.990</td>
<td>59.400</td>
<td>75.300</td>
<td>84.670</td>
<td>87.350</td>
</tr>
<tr>
<td>IRS</td>
<td>0.198</td>
<td>0.248</td>
<td>0.321</td>
<td>0.632</td>
<td>0.987</td>
<td>1.398</td>
<td>1.749</td>
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<tr>
<td>Basis</td>
<td>-0.254</td>
<td>-0.240</td>
<td>-0.273</td>
<td>-0.235</td>
<td>-0.146</td>
<td>-0.379</td>
<td>-0.767</td>
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<td><strong>Italy (N=67)</strong></td>
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<tr>
<td>Yield</td>
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<td>2.716</td>
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<td>3.906</td>
<td>4.590</td>
<td>5.145</td>
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<tr>
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<td>0.091</td>
<td>0.112</td>
<td>0.137</td>
<td>0.187</td>
<td>0.196</td>
<td>0.266</td>
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<td>165.000</td>
<td>177.100</td>
<td>195.800</td>
<td>199.700</td>
<td>194.400</td>
<td>191.600</td>
</tr>
<tr>
<td>IRS</td>
<td>1.051</td>
<td>1.241</td>
<td>1.448</td>
<td>2.015</td>
<td>2.419</td>
<td>2.831</td>
<td>2.842</td>
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<td>Basis</td>
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<td>-0.175</td>
<td>-0.132</td>
<td>-0.068</td>
<td>0.174</td>
<td>0.369</td>
<td>0.828</td>
</tr>
<tr>
<td><strong>Netherlands (N=7)</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Yield</td>
<td>0.017</td>
<td>0.070</td>
<td>0.158</td>
<td>0.618</td>
<td>1.110</td>
<td>1.504</td>
<td>1.928</td>
</tr>
<tr>
<td>Haircut</td>
<td>0.010</td>
<td>0.010</td>
<td>0.015</td>
<td>0.020</td>
<td>0.030</td>
<td>0.045</td>
<td>0.095</td>
</tr>
<tr>
<td>CDS</td>
<td>5.398</td>
<td>8.018</td>
<td>11.700</td>
<td>23.290</td>
<td>31.600</td>
<td>37.870</td>
<td>39.620</td>
</tr>
<tr>
<td>IRS</td>
<td>0.243</td>
<td>0.299</td>
<td>0.384</td>
<td>0.743</td>
<td>1.127</td>
<td>1.556</td>
<td>1.886</td>
</tr>
<tr>
<td>Basis</td>
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<td>-0.310</td>
<td>-0.343</td>
<td>-0.358</td>
<td>-0.332</td>
<td>-0.431</td>
<td>-0.354</td>
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<tr>
<td><strong>Spain (N=7)</strong></td>
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<tr>
<td>Yield</td>
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<td>2.263</td>
<td>3.037</td>
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<tr>
<td>Haircut</td>
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<td>0.050</td>
<td>0.055</td>
<td>0.090</td>
<td>0.111</td>
<td>0.174</td>
<td>0.266</td>
</tr>
<tr>
<td>CDS</td>
<td>32.400</td>
<td>42.620</td>
<td>51.920</td>
<td>79.890</td>
<td>100.400</td>
<td>111.600</td>
<td>114.500</td>
</tr>
<tr>
<td>IRS</td>
<td>0.243</td>
<td>0.299</td>
<td>0.384</td>
<td>0.743</td>
<td>1.127</td>
<td>1.556</td>
<td>1.886</td>
</tr>
<tr>
<td>Basis</td>
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<td>-0.087</td>
<td>-0.091</td>
<td>-0.031</td>
<td>0.133</td>
<td>0.366</td>
<td>0.903</td>
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</tbody>
</table>
stronger at lower haircut levels. The marginal effects of an increase in haircut levels is positive with the OLS estimate: as the theory predicts, an increase in haircut decreases the liquidity value of the asset, so that its return has to increase in order for agents in the economy to hold it. The marginal effect of haircut is not significant with the panel fixed effect estimates. This is probably due to the low time variation of our haircut variables: once the within maturity mean is subtracted, there is not enough variability to precisely estimate the effect.

The economic impact of estimated marginal effects is fairly large. To relate the change in relative liquidity to a more direct variable, reserves injected by the ECB, an increase in $\delta$ by 0.01 is, using December 2014 values, tantamount to an increase in reserves of around 25 billions of euro (assuming bonds held by banks do not change), that is an increase of around 7% in reserves. Thus, at a haircut level of 10% (approximately the 50th percentile in the distribution of haircuts in our sample) the increase in relative liquidity implies a reduction in the basis of around 2 basis points. In order to gauge this magnitude, consider that a haircut level of 10% was applied, for instance, on the 10 year maturity Spanish sovereign in December 2014: an increase of 2 basis points would thus have implied an increase of around 10% of the basis on the 10 year Spanish sovereign at that date. The impact of a 1% change in the level of haircut (around 60 percent of the changes in haircut are within this magnitude) is around 1 basis point (using the OLS estimates), that is around 4% of the average value of the basis in the dataset.

Our estimates support the theoretical prediction that changes in the relative amount of money and assets in the economy due to open market operations have an impact on assets prices given their use as collateral in interbank trading. In our framework the role of securities’ haircuts is crucial, as they represent the extent to which assets can be used by banks as collateral for funding, and in the empirical analysis it allows to distinguish the channels at works in our model from those obtained in the literature based on the preferred-habitat framework, as in Greenwood and Vayanos (2014). In their work,
scarcity has a stronger effect at longer maturities since it changes the amount of duration risk and long-term bonds are more sensitive to this risk than short-term bonds. Since it is normal practice in central clearing counterparties to set haircuts based on the historical volatility of securities, longer maturities, being more prone to duration risk, are associated with higher haircuts. If our estimation procedure were to erroneously pick-up scarcity effect more linked to preferred-habitat frictions, the effect of scarcity on yields should increase with haircuts. However our results are exactly the opposite and thus suggest that the channel at work in our empirical analysis are different from those highlighted in the preferred-habitat literature.

3.3 Robustness checks

We list here a number of robustness checks that we perform, analyzing subsequently each in turn:

1. Endogeneity of reserve injections and banks’ holdings
2. Stationarity issues
3. Endogeneity of haircuts
4. Haircut and relative liquidity vs. convenience yield

3.3.1 Endogeneity of reserve injections and banks’ holdings

The baseline estimates can present endogeneity issues. In particular reserve levels and amount of sovereigns held by banks might be endogenous to price developments of the sovereign. Our time sample includes the euro area sovereign crisis: at the end of December 2012, the Italian zero-coupon yield on the 10-year maturity reached 7.5%, from around 5% in June of the same year. The turbulent times of the euro area sovereign crisis might have led to different incentives to hold bonds and reserves by banks and for the conduct of monetary policy with respect to the mechanics of our model.

With respect to policy intervention, the European Central Bank was not idle during this period. In May 2010 it introduced the Securities Market Programme. This implied outright purchases of government securities in order to sustain sovereign bond markets liquidity, which was hampering the transmission of the monetary policy stance. Purchases lasted intermittently until August 2012 (they involved Italian bonds only from August 2011), when, in order to quell fears of break up of the euro area which were priced increasingly in sovereigns, the ECB introduced the Outright Monetary Transactions, by which it could buy unlimited amount of government bonds of a country, if some conditions were satisfied. Even though the OMTs were never applied in practice, just their availability as a monetary policy instrument was already very successful in bringing down yields in non-core countries. Moreover, in December 2011 and March 2012, the ECB also conducted two Long Term Refinancing Operations (LTROs) of the duration of 3 years with total allotted amount of around 1 trillion of euros. While purchases under the SMP program were sterilized (and thus liquidity did not actually increase) and the OMT was never activated, the large increase in reserves through the 3-year LTROs might have created a negative correlation between the yields and relative liquidity not because of scarcity, as in our model, but because it helped in calming tensions in the markets.
<table>
<thead>
<tr>
<th></th>
<th>Pooled OLS</th>
<th>Panel FE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1) Full Sample</td>
<td>(2) Full Sample</td>
</tr>
<tr>
<td></td>
<td>(0.337)</td>
<td>(1.035)</td>
</tr>
<tr>
<td>Haircut</td>
<td>-0.557</td>
<td>-1.532*</td>
</tr>
<tr>
<td></td>
<td>(1.599)</td>
<td>(0.789)</td>
</tr>
<tr>
<td>δ*Haircut</td>
<td>22.74***</td>
<td>16.80***</td>
</tr>
<tr>
<td></td>
<td>(2.307)</td>
<td>(2.132)</td>
</tr>
<tr>
<td>Country FE</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Maturity FE</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Quarter-Year FE</td>
<td>Y</td>
<td>Y</td>
</tr>
<tr>
<td>Observations</td>
<td>1505</td>
<td>1505</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.307</td>
<td>0.759</td>
</tr>
</tbody>
</table>

**Marginal effects**

|                | (1) Full Sample  | (2) Full Sample    | (3) Core        | (4) Non-core | (5) Full Sample | (6) Core | (7) Non-core |
|                | (0.327)          | (1.030)            | (0.658)         | (0.634)      | (0.958)         | (0.654)  | (0.998)      |
|                | (0.304)          | (1.017)            | (0.540)         | (0.666)      | (0.955)         | (0.547)  | (0.990)      |
| δ (haircut = 10%)| -1.949***       | -2.379**           | -1.242**        | -3.918**     | -2.297**        | -1.212** | -3.772*      |
|                | (0.305)          | (1.003)            | (0.362)         | (0.800)      | (0.949)         | (0.360)  | (0.974)      |
| Haircut (at 50p of δ) | 3.165**       | 1.218**           | 0.986           | -0.855       | 0.217           | 0.733    | -1.024**     |
|                | (1.305)          | (0.496)            | (0.531)         | (0.568)      | (0.412)         | (0.520)  | (0.179)      |

Robust and clustered by country standard errors in parentheses. Significance values based on small sample statistics; *** p<0.01, ** p<0.05, * p<0.1. Columns (1) through (4) estimated with OLS; (5) through (7) with Panel Fixed Effects method where the cross-section is defined as the couple country-maturity. Core countries are Austria, Belgium, France, Germany, Netherlands; Peripheral countries are Ireland, Italy, Spain.
In addition to monetary policy actions, also banks behaviour during the crisis might have induced negative correlations between yields and the measure of relative liquidity for reasons that are not related to scarcity. Figure 5 plots, for instance, the ratio of Italian sovereigns held by Italian banks to total amount of Italian sovereigns outstanding. This ratio increased suddenly in December 2011, with the onset of the first 3-year LTRO. The availability of cheap financing from the ECB might have induced banks to use the liquidity provided by the LTROs in order to buy sovereigns.

Figure 5: Italian sovereign held by Italian banks over total outstanding

One first line of defense against this argument is that our explanatory variable, the basis, already controls for some of the effects on yields of the sovereign crisis, namely the effects of default risk since it is computed by subtracting CDS premia from yields of sovereign. Moreover country and quarter-year fixed effects should control for other sorts of country-specific and time varyings effects, as, for instance the fear of the break up of the euro area (re-denomination risk). Therefore the buildup of tensions and the following return to calm should be already taken into account in our baseline regression.

However, as a further check that the negative correlation between relative liquidity and basis is not due to the positive effects of ECB actions on sovereign yields of the countries most affected by the crisis, we split the sample into core (Austria, Belgium, France, Germany, Netherlands) and non-core (Ireland, Italy, Spain) countries, the ones most affected by the crisis. Columns (3) and (4), (6) and (7), respectively with OLS and Panel fixed effects estimation method, we estimate the baseline equation in each subsample. Baseline results hold: an increase in relative liquidity as a negative effect on the basis, the larger so at lower levels of haircuts. The marginal effect of a change in haircuts is however not significant anymore.

As a second way to address the endogeneity issues we restrict our sample, taking away the whole period starting from June 2011 to June 2013. This periods includes the sovereign crisis, most of the purchases under the SMP program, the two 3-year LTROs, the introduction of the OMTs. The date of June 2013 was chosen since it is the month when the ratio of Italian sovereigns held by Italian banks stopped increasing with respect
to the total outstanding. Estimates are not provided here, however results are similar in significance, sign and magnitude.

3.3.2 Stationarity

A second source of concern for our result might be non-stationarity in the time series. In order to take this concern into account we are going to perform two robustness checks. First we add the lagged value of the basis as an additional regressor. The results are provide in Column (1) of table 3. The coefficient on the lagged basis is significant and positive, however results continue to hold, an increase in the amount of relative liquidity implies a decrease in the basis, the effect being stronger at lower haircut levels. In the panel fixed effect estimation of the baseline estimations, the marginal effect of a change in haircut is not significant. Second we perform the analysis on first differences of each variable, but for country, maturity and country year fixed effects. Our estimating equation thus becomes

\[
\Delta b_{c,i,t} = \gamma_1 \Delta \delta_t + \gamma_2 \Delta h_{c,i,t} + \gamma_3 \Delta \delta_t \times \Delta h_{c,i,t} + \nu'X_{c,i,t} + \varepsilon_{c,i,t} \tag{9}
\]

The results are reported in Column (2) and (3).\(^{36}\) Coefficients on the relative liquidity and the interactions term are negative and significant, while coefficient on the change in haircut is not. A positive change in relative liquidity is related to a negative change in the basis, confirming our baseline results.

3.3.3 Endogenous haircuts

Also changes in haircuts might be endogenous to yields. While in our model haircuts are exogenously set, higher riskiness implies higher and more volatile yields, and therefore central clearing counterparties optimally minimize the risks by setting higher haircuts. Even though this concern is lessened in our estimates since, by using as dependent variable the basis, we are already controlling for CDS prices, which are by themselves a measure of riskiness, in order to take the potential endogeneity into account we are going to estimate our regression equation with a 2SLS approach, using the lagged value for sovereign yields as instrument for haircut levels.\(^{37}\) Column (1) of Table 4 provides the estimates. Coefficients on the haircut variables is now highly significant, however the interaction coefficient is not significant anymore. Marginal effects are in magnitude not dissimilar from the baseline estimate, but for the one on the haircut, which is positive and large. When we add as an additional regressor to the instrumental variable estimation the lagged value of the basis, in order to take into account both endogeneity and stationarity issues (Column (2)), the magnitude of the marginal effect decreases but remains significant. All other coefficients maintain significance, sign and magnitude.

\(^{35}\)Results are similar if we exclude larger subperiods around those dates, as for instance, taking away the period starting from June 2010 to June 2013.

\(^{36}\)We do not provide marginal effects estimates since their interpretation is misleading given the estimation in first differences.

\(^{37}\)Using an instrumental variable approach when model includes an interaction term makes obtaining the estimates more cumbersome. Here we relied on the approach that if \(z\) is a good instrument for \(x_1\), then \(z \times x_2\) is a good instrument for \(x_1 \times x_2\). Therefore technically we have two instruments in our regression: the lagged value of sovereign yields and the lagged value of sovereign yields interacted with our relative liquidity measure.
Table 3: Robustness analysis: stationarity

<table>
<thead>
<tr>
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<th>(1) OLS</th>
<th>(2) OLS in changes</th>
<th>(3) FE in changes</th>
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<tbody>
<tr>
<td>δ</td>
<td>-2.918**</td>
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<td></td>
</tr>
<tr>
<td>Haircut</td>
<td>-0.530</td>
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<td></td>
</tr>
<tr>
<td>δ*Haircut</td>
<td>4.094***</td>
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<tr>
<td>Basis_{t-1}</td>
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<td></td>
</tr>
<tr>
<td>Δδ</td>
<td></td>
<td>-2.804**</td>
<td>-2.805**</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.935)</td>
<td>(0.930)</td>
</tr>
<tr>
<td>ΔHaircut</td>
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<td>0.00570</td>
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</tr>
<tr>
<td></td>
<td>(0.282)</td>
<td>(0.271)</td>
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<tr>
<td>Δδ*ΔHaircut</td>
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<td>-213.9***</td>
<td>-215.2***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(20.84)</td>
<td>(20.21)</td>
</tr>
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</tr>
<tr>
<td>Maturity FE</td>
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<td>Y</td>
<td></td>
</tr>
<tr>
<td>Quarter-Year FE</td>
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<td>Y</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>1505</td>
<td>1449</td>
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<tr>
<td>R-squared</td>
<td>0.885</td>
<td>0.221</td>
<td>0.216</td>
</tr>
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</table>

Marginal effects

| δ (haircut= 1%) | -2.877*** |
|                 | (0.992)   |
| δ (haircut= 4%) | -2.754**  |
|                 | (0.967)   |
| δ (haircut= 10%)| -2.508**  |
|                 | (0.919)   |
| Haircut (at 50p of δ) | 0.140 |
|                  | (0.151)   |

Robust and clustered by country standard errors in parentheses. Significance values based on small sample statistics: *** p<0.01, ** p<0.05, * p<0.1.

Column (3) estimated with Panel fixed effects methods, the cross-section being defined as the couple country-maturity.
Table 4: Robustness analysis: Instrumental variable

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<td></td>
<td>IV</td>
<td>IV</td>
<td>OLS</td>
</tr>
<tr>
<td>δ</td>
<td>-2.687**</td>
<td>-2.593**</td>
<td>-3.542**</td>
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<td></td>
<td>(0.923)</td>
<td>(0.856)</td>
<td>(1.054)</td>
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<tr>
<td>Haircut</td>
<td>15.21***</td>
<td>2.741**</td>
<td>-1.472</td>
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<tr>
<td></td>
<td>(3.212)</td>
<td>(1.016)</td>
<td>(0.790)</td>
</tr>
<tr>
<td>δ*Haircut</td>
<td>1.188</td>
<td>0.821</td>
<td>16.99***</td>
</tr>
<tr>
<td></td>
<td>(3.856)</td>
<td>(1.375)</td>
<td>(2.115)</td>
</tr>
<tr>
<td>Basis_{t-1}</td>
<td>0.674***</td>
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<tr>
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<td>(0.103)</td>
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<td>Y</td>
<td>Y</td>
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<tr>
<td>Quarter-Year FE</td>
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<td>Y</td>
<td>Y</td>
</tr>
<tr>
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<td>1505</td>
</tr>
<tr>
<td>R-squared</td>
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**Marginal effects**

<table>
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<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>IV</td>
<td>OLS</td>
</tr>
<tr>
<td>δ (haircut = 1%)</td>
<td>-2.676**</td>
<td>-2.585**</td>
<td>-3.372**</td>
</tr>
<tr>
<td></td>
<td>(0.930)</td>
<td>(0.860)</td>
<td>(1.048)</td>
</tr>
<tr>
<td>δ (haircut = 4%)</td>
<td>-2.640**</td>
<td>-2.560**</td>
<td>-2.863**</td>
</tr>
<tr>
<td></td>
<td>(0.958)</td>
<td>(0.873)</td>
<td>(1.035)</td>
</tr>
<tr>
<td>δ (haircut = 10%)</td>
<td>-2.569**</td>
<td>-2.511**</td>
<td>-1.844</td>
</tr>
<tr>
<td></td>
<td>(1.052)</td>
<td>(0.903)</td>
<td>(1.019)</td>
</tr>
<tr>
<td>Haircut (at 50p of δ)</td>
<td>15.41***</td>
<td>2.875**</td>
<td>1.309**</td>
</tr>
<tr>
<td></td>
<td>(2.765)</td>
<td>(0.869)</td>
<td>(0.504)</td>
</tr>
</tbody>
</table>

Robust and clustered by country standard errors in parentheses. Significance values based on small sample statistics; *** p<0.01, ** p<0.05, * p<0.1. Columns (1) and (2) estimated with 2SLS methods, the variable haircut being instrumented by the lagged value of the sovereign yield.
3.3.4 Haircut and relative liquidity vs. convenience yield

The difference in return between the yield of bonds which can be used as collateral and bonds that cannot is sometimes called "convenience yield" in the finance literature. Pericoli and Taboga (2015) show that yields on the 10-year Italian sovereign display a sizable convenience yield. They use as a proxy of the convenience yield the 3-month Euribor-Eurepo spread. Since the Euribor is the rate on unsecured interbank borrowing while Eurepo represents the cost of secured borrowing through repos, the spread indicates how much more expensive is uncollateralized borrowing with respect to collateralized borrowing. Read through the finance literature, our model provides an explanation of how the convenience yield arises and how it relates to interbank lending features (haircuts) and monetary policy decisions (relative liquidity). However, if differential changes in yields between pledgeable and not pledgeable bonds are entirely captured by the Euribor-Eurepo spread, then one could possibly use only this measure and how it relates to relative liquidity in order to explain changes in sovereigns yields due to scarcity. We thus check whether the haircut and relative liquidity remain significant once a proxy for the convenience yield is also inserted in the estimating equation. Column (3) in table 4 shows the estimation results once the 3-month Euribor-Eurepo spread is added as regressor. The estimated coefficient is negative and significant, as in Pericoli and Taboga (2015), however all other coefficients remain broadly unchanged with respect to previous estimates. The results thus confirm the analysis that changes in relative liquidity and haircuts impact the yields of sovereigns, even when controlling for a commonly used proxy of the convenience yield.

4 Conclusion

We built a general equilibrium model in which frictions in the exchange process give rise to an essential role of money. The banking sector pledges assets as collateral on interbank markets to obtain liquidity for their depositors. In this framework we show that i) central banks open market operations, by altering the relative amount of collateral and money in the economy, are able to influence the price of the assets used as collateral; ii) pledgeability properties (haircuts) of the collateral are an important parameter in determining the effects of open market operations on its price. We take the model to the data, analyzing how the yield of a selected sample of euro area sovereigns changes with the relative amount of money and collateral available in the economy. Predictions of the model are confirmed by the empirical analysis.

This paper points out to a channel of transmission of unconventional monetary policies little analysed so far in the literature, as to the best knowledge of the authors38 the impact of unconventional policies on prices of assets through their role as collateral on the interbank market. Differently than a preferred-habit model, the imperfect substitutability between assets is not driven by investors preferences but by assets’ role in the exchange process and by their intrinsinc pledgeability properties. While our empirical analysis is only able to highlight the impact of monetary policy through the latter type of frictions, both frictions are likely at work in the real economy, the relative strength of each being uncertain. In this respect, an empirical strategy which is able to jointly estimate the impact

38 As commented in the related literature paper, the closest empirical analysis to the one provided here is in D’Amico et al. (2014), which is based on a different analysis technique and data, and is however missing a theoretical support.
on asset prices of unconventional monetary policies through preferred-habitat channel and collateral channel should shed light on the issue. This is left for future work.

References


A Problem of the financial intermediaries

Given our equilibrium definition (Definition 1 in the main text), we can rewrite the problem of the financial intermediaries as:

$$\max_{m, b, a} -\mu m - \psi b - pa + \beta \left[ \frac{1}{2} F^1(m, b, a) + \frac{1}{2} F^2(m, b, a) \right]$$

(10)

Bank 1 solves the following maximization problem:

$$F^1(m, b, a) = \max_{m, b, a} (\rho + \varepsilon)u\left( \frac{m + l}{\rho + \varepsilon} \right) + (1 - \rho - \varepsilon)u\left( \frac{b - b^1 + (1 - h)(a - a^1) + m - m^1}{1 - \rho - \varepsilon} \right) +$$

$$+ (a - a^1) - (1 - h)(a - a^1) + (b^1 + a^1 - R_l)$$

s.t. $a^1 \geq 0 (\xi^1), b^1 \geq 0 (\mu^1), l \geq 0 (\nu^1)$

$a \geq a^1 (\theta^1), b \geq b^1 (\omega^1), m \geq m^1 (\lambda^1)$

$R_l \leq b^1 + (1 - h)a^1 (\zeta^1)$

where in parenthesis we put the associated Lagrange multiplier. The first order conditions

$$u'\left( \frac{m + l}{\rho + \varepsilon} \right) = u'\left( \frac{b - b^1 + (1 - h)(a - a^1) + m - m^1}{1 - \rho - \varepsilon} \right) + \lambda^1$$

(11)

$$u'\left( \frac{b - b^1 + (1 - h)(a - a^1) + m - m^1}{1 - \rho - \varepsilon} \right) = 1 + \mu^1 - \omega^1 + \zeta^1$$

(12)

$$u'\left( \frac{b - b^1 + (1 - h)(a - a^1) + m - m^1}{1 - \rho - \varepsilon} \right) (1 - h) = (1 - h)(1 + \zeta^1) + \xi^1 - \theta^1$$

(13)

$$u'\left( \frac{m + l}{\rho + \varepsilon} \right) + \nu^1 = R(1 + \zeta^1)$$

(14)

together with the complementary slackness conditions: $\xi^1 a^1 = 0, \mu^1 b^1 = 0, \nu^1 l = 0, \theta^1 (a - a^1) = 0, \omega^1 (b - b^1) = 0, \lambda^1 (m - m^1) = 0, \zeta^1 (b^1 + (1 - h)a^1 - R_l) = 0$ are necessary and sufficient to solve the problem.
Bank 2 solves the following maximization problem:

\[
F^2(m, b, a) = \max_{m^2, b^2, \omega^2, \mu} \left( \rho - \varepsilon \right) u \left( \frac{m^2}{\rho - \varepsilon} \right) + (1 - \rho + \varepsilon) u \left( \frac{b - b^2 + (1 - h)(a - a^2) + m - m^2 - n + Rn}{1 - \rho + \varepsilon} \right) +
\]

\[
+ (a - a^2) - (1 - h)(a - a^2) + (b^2 + a^2)
\]

s.t. \( a^2 \geq 0 (\xi^2), b^2 \geq 0 (\mu^2), n \geq 0 (\nu^2) \)

\( a \geq a^2 (\theta^2), b \geq b^2 (\omega^2), m \geq m^2 + n (\lambda^2) \)

where in parenthesis we put the associated Lagrange multiplier. The first order conditions

\[
\begin{align*}
\psi &= 1 \left[ \frac{u'}{\beta} \left( \frac{\beta - b^2 + (1 - h)(a - a^2) + m - m^2 - n + Rn}{1 - \rho + \varepsilon} \right) + \omega^{1} \right] +
\end{align*}
\]

\[
\begin{align*}
\frac{\mu}{\beta} &= \frac{1}{2} \left[ u' \left( \frac{\beta - b^2 + (1 - h)(a - a^2) + m - m^2 - n + Rn}{1 - \rho + \varepsilon} \right) + \lambda^{1} \right] +
\end{align*}
\]

\[
\begin{align*}
\frac{\psi}{\beta} &= \frac{1}{2} \left[ u' \left( \frac{\beta - b^2 + (1 - h)(a - a^2) + m - m^2 - n + Rn}{1 - \rho + \varepsilon} \right) + \omega^{2} \right] +
\end{align*}
\]

\[
\begin{align*}
\frac{p}{\beta} &= \frac{1}{2} \left[ u' \left( \frac{\beta - b^2 + (1 - h)(a - a^2) + m - m^2 - n + Rn}{1 - \rho + \varepsilon} \right) (1 - h) - (1 - h) + \theta^{1} \right] +
\end{align*}
\]

\[
\begin{align*}
\frac{p}{\beta} &= \frac{1}{2} \left[ u' \left( \frac{\beta - b^2 + (1 - h)(a - a^2) + m - m^2 - n + Rn}{1 - \rho + \varepsilon} \right) (1 - h) - (1 - h) + \theta^{2} \right]
\end{align*}
\]

B Equilibrium characterization

The following propositions give a first characterization of the equilibria.

Proposition 2 (Necessary condition for equilibrium) Necessary condition for an equilibrium to exist is \( \mu \geq \beta \).

Proof. Consider \( \frac{\psi}{\beta} \), after having substituted \( \frac{\mu}{\beta} \) and \( \frac{\psi}{\beta} \): \( \mu = \beta \left[ \frac{1}{2} u'(q_m^1) + \frac{1}{2} u'(q_m^2) \right] \). The terms in parentheses on the right hand side have to be at least one, since \( q_m^1, q_m^2 \leq q^* \) and \( u'(q^*) = 1 \). Therefore \( \frac{\psi}{\beta} \geq 1 \) is necessary for the existence of an equilibrium. ■

The requirement of proposition 2 is that the nominal interest rate on bonds is weakly positive.
Lemma 1 (Indeterminacy of \( a^i \) and \( b^i \)) In any equilibrium for \( i = 1, 2 \) \( b^i = 0 \) implies \( a^i = 0 \), \( b^i = b \) implies \( a^i = a \) and \( b > b^i > 0 \) implies \( a > a^i > 0 \). In the last case, banks are indifferent between using bonds or real asset.

Proof. Consider the problem of bank 1. Combining (12) and (13) we obtain \( \mu^1 + \frac{\theta^1}{(1 - \pi^1)} = \omega^1 + \xi^1 (1 - \pi^1) \). Note that \( \mu^1 \) and \( \omega^1 \) (\( \xi^1 \) and \( \theta^1 \)) cannot be both strictly greater than zero, because otherwise we would have both \( b^1 = 0 \) (\( a^1 = 0 \)) and \( b^1 = b > 0 \) (\( a^1 = a > 0 \)). Suppose that \( \mu^1 = 0 \) and \( \omega^1 > 0 \) (\( b^1 = b \)). Then it must be that \( \theta^1 > 0 \) (\( a^1 = a \)), therefore \( \xi^1 = 0 \). Suppose instead that \( \mu^1 > 0 \) and \( \omega^1 = 0 \) (\( b^1 = 0 \)). Then it must be that \( \theta^1 = 0 \) (\( a^1 = 0 \)) and, therefore, \( \xi^1 > 0 \) (note that the converse is also true). Consider finally the case when \( \mu^1 = 0 \) and \( \omega^1 = 0 \), that is \( b > b^1 > 0 \), then it must be the case that also \( \xi^1 = 0 \) and \( \theta^1 = 0 \), implying that \( a > a^1 > 0 \). In this case both (12) and (13) are equal to \( u'(q_1^i) = 1 + \zeta^1 \), giving rise to the indeterminacy of \( a^1 \) and \( b^1 \). Using the same argument with (17) and (18), the same indeterminacy result for \( a^2 \) and \( b^2 \) it is easily obtained.

The intuition for the previous lemma is straightforward. Type 1 bank will always use either both real assets and bonds or neither of them in the interbank market. Bonds and real assets give rise to the same trade-off between the marginal cost of reducing deposits available for buyers' consumption and marginal benefit of posting one unit more of either of them on the interbank market as collateral. This is so since one unit of pledged asset increases the collateral pool by \((1 - h)\) and reduces deposits by \((1 - h)\), and one unit of pledged bond increases the collateral pool by one and reduces deposits by one, or said differently, they have the same opportunity-cost (relatively speaking). A corollary of the lemma is that it cannot be the case that the bank pledges all its bonds (real assets) in the interbank market but holds some positive amount of real assets (bonds) as excess reserves. Note that the previous lemma is true also for bank 2, even though the reason is slightly different: real assets and bonds have the same marginal effect on utility when used in the DM or when kept as excess reserves.

The previous lemma tells us that we cannot pin down exactly \( b^i \) and \( a^i \) (unless both of them are zero or \( b \) and \( a \) respectively), therefore we introduce in our analysis a new variable, \( \pi \), that represents the total value of interest bearing assets in the portfolio. Specifically we let:

\[
\pi \\equiv b + (1 - h)a, \quad \pi^i \\equiv b^i + (1 - h)a^i, \quad i = 1, 2.
\]

The following proposition, already reported in the main text, provides a characterization of the prices in equilibrium.

Proposition 3 (Equilibrium prices) In every equilibrium \( \beta \leq \psi \leq \mu \) and \( p = h\beta + (1 - h)\psi \). Moreover, whenever the volume of interbank lending is positive \( R = \frac{\psi}{\beta} \).

Proof. We start by proving the second part of the proposition. If an interbank market exists we have \( \nu^1 = \nu^2 = 0 \). Combining (19) with (16) obtains: \( u'(q^m_1) = Ru'(q^m_1) \). Putting together instead (14) with (12) we get \( u'(q^m_2) = R [u'(q^m_2) - \mu^1 + \omega^1] \).

Since we are assuming the existence of the interbank market, then by lemma 1 we know that both asset and bonds will be used, therefore \( \mu^1 = 0 \). We want to show that \( \omega^1 = 0 \). Suppose \( \omega^1 > 0 \) and \( \lambda^1 > 0 \). Then from lemma 1 we would have \( \theta^1 > 0 \) and \( q^i_1 = 0 \), implying \( u'(q^i_1) = \infty \) and so this cannot be a solution, because \( \frac{\mu^1}{R} > \frac{\mu^1}{\mu^1} > 0 \). Suppose instead that \( \omega^1 > 0 \) and \( \lambda^1 = 0 \), from (11) we have \( u'(q^m_1) = u'(q^m_1) \). Using it in the equation found above we have \( u'(q^m_2) = R [u'(q^m_2) + \omega^1] \), implying \( R < 1 \). However when \( R < 1 \) from (19) we have that \( (1 - R)u'(q^m_2) + \lambda^2 = \nu^2 \) which implies \( \nu^2 > 0 \) and therefore that there is no interbank market, which is a contradiction.

Therefore \( \mu^1 = 0 \) and \( \omega^1 = 0 \), and we can rewrite the previous equation as \( u'(q^m_1) = R [u'(q^m_1)] \).

By using a similar argument and lemma 1 one can show that \( \omega^2 = 0 \). Using these results and given \( u'(q^m_2) = u'(q^m_2)/R \) in (21) we obtain

\[
R^\beta_\psi = \frac{1}{2} [u'(q^m_1)] + \frac{1}{2} [u'(q^m_2)] \quad \text{(23)}
\]
Substituting (11) and (16) in (20) we can rewrite it as:

$$\frac{\mu}{\beta} = \frac{1}{2} [u'(q_1^m)] + \frac{1}{2} [u'(q_2^m)]$$

Therefore from (23) and (24) necessarily $R = \frac{\beta}{\mu}$.

We can now prove the first part of the lemma. From equation (21) we can see that, since $u'(q_1^l) \geq 1$, $u'(q_2^p) \geq 1$, $\omega^1 \geq 0$ and $\omega^2 \geq 0$, the RHS has to be at least equal to 1. Therefore $\frac{\omega^1}{\beta} \geq 1$ or equivalently $\psi \geq \beta$. Using the same argument on equation (22) we can see that $\beta \geq \beta$.

We want to show that $\psi \leq \mu$. Suppose there is an interbank market, then by the argument in the first part of the proof we know that $\omega^1 = \omega^2 = 0$. Comparing equation (20) and (21) we can see that since $\lambda^1 \geq 0$ and $\lambda^2 \geq 0$ then $\psi \leq \mu$.

Suppose now the interbank market does not exist, and consider the case $\omega^1 > 0$. Then from lemma 1 it must be $\mu^1 = 0$ and from (12) it has to be $\zeta^1 > 0$. Therefore $b^1 = b$ and since the borrowing constraint is binding $l > 0$, but this is a contradiction since we assumed $l > 0$.

From (17), since by lemma 1 $\mu^2$ and $\omega^2$ cannot be both strictly greater than zero, then the solution will always imply $\mu^2 > 0$ and $\omega^2 = 0$. Therefore $\omega^1 = \omega^2 = 0$ and using the same argument as before when we assumed that there is no interbank market, we have $\psi \leq \mu$.

We now turn to the upper bound on the asset price $p$. By lemma 1 $\omega^1 = \omega^2 = 0$ implies $\theta^1 = \theta^2 = 0$. By combining (21) and (22) we obtain $p = h\beta + (1 - h)\psi$, and since (for $\mu \geq \beta$) $\beta \leq \psi \leq \mu$, we have that $\beta \leq p \leq h\beta + (1 - h)\mu$.

Before to move to the derivation of the different equilibria, we introduce an useful lemma that will help in simplifying the first order conditions.

**Lemma 2** In any equilibrium, $\omega^1 = \omega^2 = \theta^1 = \theta^2 = 0$.

**Proof.** Suppose $\omega^2 > 0$, which implies $\mu^2 = 0$ from lemma 1 and then consider (17). We have $u'(q_2^p) = 1 - \omega^2$, which implies $u'(q_2^p) < 1$, but this is a contradiction. Therefore $\omega^2 = 0$ and by lemma 1 also $\theta^2 = 0$.

Now let $\psi = \beta$ and considering (21) we get $2 = u'(q_1^l) + \omega^1 + u'(q_2^p) \geq 2 + \omega^1$, where the first inequality comes from $u'(q_2^p) \geq 1$ for $i = 1, 2$, which implies $\omega^1 \leq 0$. But $\omega^1 \geq 0$ by definition and we have a contradiction. Then it must be $\omega^1 = 0$.

Instead now consider $\beta < \psi < \mu$ and suppose $\omega^1 > 0$ (that is $b^1 = b$, which also implies $\mu^1 = 0$ and by lemma 1 $\theta^1 > 0$, or $a^2 = a$), then from (12): $u'(q_1^l) = 1 - \omega^1 + \zeta^1$. Since $u'(q_1^l) \geq 1$, then $\omega^1 \leq \zeta^1$ and $\lambda^1 > 0$, that is, the interbank market constrain is binding, $Rl = b + (1 - h)a$, hence $l > 0$ and $\nu^1 = 0$. By combining (12) and (14) we obtain $u' \left( \frac{m - b}{1 - p - \epsilon} \right) = R u' \left( \frac{m - b}{1 - p - \epsilon} \right) + \lambda^1$. Notice that $m^1 < m$, otherwise by the Inada condition the RHS of the previous expression tends to infinity, therefore $\lambda^1 = 0$. Using equation (11) in the previous equation we have $(1 - R) u' \left( \frac{m - b}{1 - p - \epsilon} \right) = R \omega^1$, which is a contradiction since $R > 1$ by proposition 3 and the LHS is negative, while the RHS is assumed to be positive. Therefore $\omega^1 = 0$.

Finally let $\psi = \mu$ and suppose $\omega^1 > 0$ (from lemma 1 also $\theta^1 > 0$, or $a^2 = a$), from (21) and (24) we have $2\frac{\beta}{\mu} = u'(q_1^l) + \omega^1 + u'(q_2^p)$ and $2\frac{\beta}{\mu} = u'(q_1^l) + \lambda^1 + u'(q_2^p) + \lambda^2$, which implies $\omega^1 \geq \lambda^1 \geq \lambda^2 > 0$. Moreover, $\omega^1 > 0$ implies $\mu^1 = 0$ and therefore from (12) we have $\zeta^1 > 0$. Since we are assuming $b^1 = b$ and $a^1 = a$, then $q_1^l = \frac{m - b}{1 - p - \epsilon}$ and therefore $m^1 < m$, otherwise by the Inada conditions $u'(q_1^l) \to \infty$. Therefore $\lambda^1 = 0$ and $\lambda^2 = \omega^1 > 0$. As $\zeta^1 > 0$, $b^1 = b$ and $a^1 = a$ we also have $l = n > 0$ and $\nu^2 = 0$. But since by proposition 3 when $\psi = \mu$ we have $R = 1$, equation (19) is violated. Therefore $\omega^1 = 0$ for $\beta \leq \psi \leq \mu$, and by lemma 1 also $\theta^1 = 0$.

**B.1 Plentiful interest bearing assets equilibrium: $\psi = p = \beta$**

In this equilibrium interest bearing assets are not scarce and buyers in credit-meetings will be able to consume the first best quantity $q^*$ independently of the type of bank they are facing. Moreover,
since buyers in credit-meetings are already consuming the first best quantity, banks will carry excess reserves to the next centralized market $\pi^1 \geq 0$ and $\pi^2 \geq 0$, and the constraint on the interbank market is slack $R_l \leq \pi^1$.

Type 1 banks will go on the interbank market to obtain more cash for its depositors. But since the constraint on the interbank market is not binding, the marginal cost and benefit of having one more unit of money for both banks are equal, therefore also consumptions of depositors in cash-meetings across the two banks will be equalized. Therefore $q_1^m = q_2^m = \frac{m}{\rho}$ and the quantity exchanged on the interbank market is $l = \frac{\epsilon}{\rho}$, where the value of $m$ is fixed by the first order condition with respect to $m$:

$$\frac{\mu}{\beta} = u'(\frac{m}{\rho})$$  \hspace{1cm} (25)

Note that buyers in cash-meetings are not consuming the first best quantity, since $\mu > \beta$ implies that $u'(\frac{m}{\rho}) > 1 = u'(q^*)$.

For this equilibrium to exists $\delta$ must be low enough such that in the market there is plentiful of bonds, or that there is a high amount of real assets, so that buyers in credit-meetings can consume the first best level of consumption. This implies that for bank of type 1 it must be $\pi - R_l \geq (1 - \rho - \epsilon)q^*$ and for bank of type 2: $\pi + R_l \geq (1 - \rho + \epsilon)q^*$. Using these inequalities a plentiful interest bearing equilibrium exists in the set $A \geq \tilde{\delta}$ for any $\delta \in (0,1]$, where $\tilde{\delta} \equiv \frac{\epsilon}{\rho}$, and when $A < \tilde{\delta}$ for $\delta < \tilde{\delta}$ where $\delta \equiv \frac{1}{1 - \rho + \epsilon - \Gamma(h)}$ and $\Gamma \equiv u''(\mu/\beta)$.

When $A < \tilde{\delta}$ a change of $\delta$ in $[0, \tilde{\delta}]$ does not influence real quantities. Even if an open market operation lowers the amount of bonds in the economy, there are still enough interest bearing assets such that consumption is first best and the borrowing constraint is slack. Moreover, from (25) the real amount of money is independent of $\delta$. Therefore, an injection of fiat money would result in a proportional increase in the price level, without affecting the consumption of buyers in cash-meetings. This implies that conventional open market operations, a change in $\delta$, do not have any effect on the interbank market price and quantities exchanged.

**Formal derivation:** When $\psi = \beta$, by lemma 2, $\omega^1 = \omega^2 = \theta^1 = \theta^2 = 0$ and from (21) and (22) we have $p = \beta$ and $u'(q_1^* = u'(q_2^* = 1$, that implies $q_1^* = q_2^* = q^*$. From (12), (17), (13) and (18), this also implies $\mu^1 = \mu^2 = \xi^1 = \xi^2 = \zeta^1 = 0$, or $\beta^1 > 0$, $\beta^2 > 0$, $a^1 > 0$, $a^2 > 0$ and $R_l < b^1 + (1 - h)\alpha^1$.

From proposition 1, $R = \frac{b}{\rho} = \frac{\beta}{\rho} > 1$, as we are considering only equilibria where the Friedman rule does not hold. Using (10) and (19), and the fact that in this equilibrium $u'(q_1^* = 1$ we find that necessarily $\lambda^2 > 0$ and $u'(q_2^*) = \frac{R + u^2}{\rho}$, while substituting (12) and (14) we get $u'(q_2^*) = \frac{R - \nu^2}{\rho - \nu^2}$. Substituting for $R$ we have that $u'(q_2^*) - u'(q_2^*) = \nu^1 + \nu^2$. Now suppose that banks are not using the interbank market, so that $l = n = 0$ and $\nu^1, \nu^2 > 0$. Since $\lambda^2 > 0$ and then $m = m$, $\nu^2 < \nu^2$ it is not possible as $\frac{m}{\rho - \epsilon} > \frac{m}{\rho - \epsilon}$. Therefore banks must be effectively using the interbank market, $l = n > 0$, and $\lambda^2 > 0$. This implies $q_1^m = q_2^m$. Since $\lambda^2 > 0$ and by (10) $q_2^m < q^*$, consequently in (11) $\lambda^1 > 0$. Therefore, $m = m$ and $m^2 = m$ and $m = n = m - l$.

Then, we have $q_1^m = \frac{m}{\rho + \epsilon}$, $q_2^m = \frac{m}{\rho + \epsilon}$ and given $q_1^m = q_2^m$, $l$ must be necessarily equal to $\frac{1}{\rho}$.

Therefore, $q_1^m = q_2^m = \frac{m}{\rho}$ and from (20) the equilibrium value of $m$ satisfies $u'(\frac{m}{\rho}) = \frac{\epsilon}{\rho}$.

The existence of this equilibrium requires that there are sufficient resources to consume the first best quantity for buyers in credit-meetings in each bank, that is $\frac{\pi - R_l}{\Gamma - \rho} \geq q^*$ and $\frac{\pi + R_l}{\Gamma + \rho} \geq q^*$. Using the monetary policy rule $b = m \left(\frac{1}{\rho} - 1\right)$, the market clearing condition for the Lucas tree.

\[39\] The reader might notice that there is one inequality for each type of bank that have to be satisfied, while the definition of $\tilde{\delta}$ involves only type 1 bank. With log utility both inequalities will bind at the same value of $\delta$ so it is irrelevant which one we choose. Under a more general utility this will not be the case, however there exist conditions on utility such that we can order the inequalities. In particular if $-u'(\frac{m}{\rho}) > 0$ it can be proven that the inequality for bank of type 1 will be the relevant one.
and the equilibrium values of $R$ and $l$ the previous inequalities can be rewritten as

$$
\frac{(1-h)A + \left(\frac{1}{\delta} - 1\right)m - \frac{\rho}{\rho + \epsilon} m}{1 - \rho - \epsilon} \geq q^* \quad \text{and} \quad \frac{(1-h)A + \left(\frac{1}{\delta} - 1\right)m + \frac{\rho}{\rho + \epsilon} m}{1 - \rho + \epsilon} \geq q^*
$$

(26)

Since we consider only equilibria in which the government is a net debtor of the private sector, $B_t \geq 0 \ \forall t$, then $\delta \in (0,1]$. Therefore we now define the thresholds values on $A$ and $\delta$ that characterize the set in which the plentiful interest bearing asset equilibrium exists. Firstly, we will show that under log-utility the two inequalities in (26) are equivalent, meaning that we can keep track of just one of them. Rearranging the two inequalities and isolating $A$ we get

$$
A \geq \frac{(1-\rho - \epsilon) q^* + \frac{\rho}{\rho + \epsilon} m - \left(\frac{1}{\delta} - 1\right)m}{(1-h)} \quad \text{and} \quad A \geq \frac{(1-\rho + \epsilon) q^* - \frac{\rho}{\rho + \epsilon} m - \left(\frac{1}{\delta} - 1\right)m}{(1-h)}
$$

Under log-utility $q^* = 1$ and from (25) $\frac{m}{\rho} = \frac{\beta}{\mu}$, then in the previous expressions both conditions for $A$ are equivalent and can be rewritten as $A \geq \frac{1-\rho - (\frac{1}{\delta} - 1)m}{1 + \frac{1}{h}}$. Now, setting $\delta = 1$ we can define $\bar{A} \equiv \frac{1-\rho}{1-\delta}$ as the amount of real asset such that for $A \geq \bar{A}$ then the only equilibrium entails $\psi = p = \beta$.

Then, let’s suppose that $A < \bar{A}$. By rearranging the same inequalities in (26) for $\delta$, in this case an equilibrium with $\psi = p = \beta$ exists if $\delta < \tilde{\delta}$, where $\tilde{\delta}$ is defined as

$$
\tilde{\delta} \equiv \frac{\rho \Gamma}{1 - \rho + \rho \Gamma - (1-h)A}
$$

(27)

where $\Gamma = (u')^{-1}(\mu/\beta) = \frac{\beta}{\mu}$.

### B.2 Liquidity trap equilibrium: $\psi = \mu$ and $p = h\beta + (1-h)\mu$

When $\psi = \mu$ the return of government bonds is the same as that of money, that is money and government bonds are perfect substitutes. This happens when interest bearing assets are so scarce with respect to money (or equivalently that money is so abundant) that banks use money also to back claims issued to their buyers in credit-meetings. Then the marginal value of giving one more unit of money to each type of buyers must be equal, that is:

$$
u'\left(q^*_m\right) = u'\left(q^*_c\right), \text{ for } i = 1, 2.
$$

(28)

Since banks have the same amount of resources (they are homogenous in the CM when they are created) then also consumption of buyers in each type of meeting will be equalized across banks.

In this equilibrium, since $R = 1$, banks exchange collateral one-to-one for money on the interbank market. This implies that bank of type 1 is indifferent, for instance, to pledge any amount of its $\pi$ units of collateral on the interbank market, get $l = \pi$ units of money and then give this money to both buyers in credit-meetings and cash-meetings. Therefore there is an indeterminacy of the quantities exchanged on the interbank market. Note that the indeterminacy comes from the assumption that, once interest bearing assets and money are perfect substitutes, then the interbank market is frictionless. As such, it would not be robust to adding for instance, arbitrarily small costs of operating on the interbank markets. We break the indeterminacy by assuming that if banks access the interbank market, they do it for the smallest quantity of money needed to satisfy (28).

Therefore while $R = 1$ always, there can be positive or no quantities exchanged at all on the interbank market depending on the relative abundance of money and interest bearing assets. This is intuitive: suppose that there was no real asset in the economy ($A = 0$). If the relative amount of money with respect to bonds is greater than $\rho + \epsilon$, so that there is enough currency to provide consumption to buyers of type 1 bank (the bank that has the relatively larger fraction of buyers in cash-meetings meetings), then type 1 bank has no need to access the interbank market. Therefore for $A$ sufficiently small and $\delta$ sufficiently large the liquidity trap equilibrium will entail
no quantities exchanged on the interbank market \((l = n = 0)\). However for a \(\delta\) smaller, money is not so abundant anymore, and positive quantities will be traded on the interbank market.

This implies that, for \(A < A\) (defined below) we have two types of equilibria in the liquidity trap: when \(\hat{\delta} < \delta < \hat{\delta}\) then \(l = (\rho + \varepsilon)(1 - h)A + (\rho + \varepsilon - \delta)\frac{\nu}{\beta}\) and \(\delta \geq \hat{\delta}\) then \(l = 0\), where \(\hat{\delta}\) is the necessary value of \(\delta\) in order to have a liquidity trap equilibrium and \(m\) is fixed by:

\[
\frac{\mu}{\beta} = u'(\frac{m}{\beta} + (1 - h)A)
\]  

(29)

In this equilibrium monetary policy choice \(\delta\) has no real effect even if real money holdings are not constant anymore. By (29) an increase in \(\delta\) increases proportionally real money holding \(m\), that is, the relative price of goods with respect to money does not change in a liquidity trap. Still consumption of buyers does not change, since it determined by (29). However monetary policy does influence activity on the interbank market, since \(l\), the amount exchanged, is decreasing in \(\delta\).

**Formal derivation:** Since \(\psi = \mu\), money and government bonds are equivalent. From lemma 2 we know that \(\omega^1 = \omega^2 = \theta^1 = \theta^2 = 0\) and using (20) and (21), together with \(\lambda^1, \lambda^2 \geq 0\) by definition, we have that \(\lambda_1 = \lambda_2 = 0\). Therefore, from (11) we have \(q_1^m = q_1^*\) and from (16) we have \(q_2^m = q_2^*\).

Suppose first that there is no interbank market. Then since both type 1 and type 2 banks enter the DM with the same amount of real resources it must be that \((1 - \rho - \varepsilon)q_1^m + (\rho + \varepsilon)q_2^m = (1 - \rho + \varepsilon)q_2^m + (\rho - \varepsilon)q_1^m\), and since consumption levels are equal in each bank across buyers in credit-meetings and cash-meetings, we have that \(q_1^m = q_2^m = q_1^m = q_2^m\). From (20) we then have, since \(\mu > \beta\), \(q_1^{m,c} < q^*\), which implies, from (17) and (18), that \(\mu_1^2 > 0\). Both \(\mu^2\) and \(\xi^2\) are both greater than zero (or \(b^2 = a^2 = 0\)). Setting from the expressions for \(q_2^m\) and \(q_2^m\), since \(q_2^m = q_2^m\) we have \(m = (\rho - \varepsilon)(\pi + m) = (\rho - \varepsilon)(\frac{\nu}{\beta} + (1 - h)A)\) and therefore \(q_2^m = \frac{\nu}{\beta} + (1 - h)A = q_1^m = q_2^m = q_2^m\).

The existence of this equilibrium requires \(m_1^1 \leq m, m_2^2 \leq m\) and \(m > 0\). From (20) using equilibrium consumption we see that \(m\) is fixed by:

\[
\frac{\mu}{\beta} = u'(\frac{m}{\beta} + (1 - h)A)
\]  

(30)

and because \(\delta \in [0, 1]\) for \(A\) excessively high we can have no positive solution for \(m\). Define at this moment the solution for \(m\) as \(m(\delta, \mu, A)\) and assume it is positive. Given \(m_1^1 > m_2^2\) (because \(q_1^m = q_2^m\)) we need to check only \(m_1^1 \leq m, (\rho + \varepsilon)\left(\frac{\nu}{\beta} + (1 - h)A\right) \leq m\) :

\[
\frac{1}{\delta} \leq \frac{1}{\rho + \varepsilon} - \frac{(1 - h)A}{m(\delta, \mu, A)}
\]  

(31)

Equations (30) and (31) have to hold simultaneously for this equilibrium to exist. The RHS of (31) is decreasing in \(A\), because from (30) also \(m\) is decreasing in \(A\). Therefore also \(\delta\) should increase in order to satisfy (30), but \(\delta\) has upper bound 1. Therefore, we have to look for a threshold \(A\) such that the equilibrium exists only for \(\delta = 1\). Setting \(\delta = 1\) and solving (31) as equality for \(A\) we get \(A = \frac{1 - \rho - \varepsilon}{(\rho + \varepsilon)(1 - h)}\). Substituting this expression in (30) we obtain \(\frac{\mu}{\beta} = u'(\frac{m}{\beta} + (1 - h)A)\), that implies \(m = (\rho + \varepsilon)u'^{-1}(\mu/\beta) > 0\). Putting back \(m\) in \(A\) we finally end up with \(A = \frac{(1 - \rho - \varepsilon)\Gamma}{(1 - h)A}\), where \(\Gamma \equiv u'^{-1}(\mu/\beta)\). Given \(A\), it can be seen from (31) that when \(A\) decreases, \(m\) increases and a lower \(\delta\) is sufficient to satisfy (31), therefore defining our threshold on \(\delta\) as \(\frac{1}{\delta} = \frac{1}{\rho + \varepsilon} - \frac{(1 - h)A}{m(\delta, \mu, A)}\). This can be rewritten as \(\delta = \frac{1}{1 - \rho - \varepsilon - \frac{(1 - h)A}{m(\delta, \mu, A)}}\).

We can now move to the case in which an interbank market exists. We require \(l = n > 0\) and \(\nu^1 = \nu^2 = 0\). Given \(q_1^m < q^*\), from (12) we need \(\mu^1 = 0\) (\(q_1^m = 0\)) and \(\zeta > 0\). Substituting (12) or (13) in (14), given (11) and \(\lambda_1 = 0\) we have that \(R\) must be equal to one. The same result is obtained substituting (16) in (19). In order to avoid equilibrium indeterminacy, we assume that when a bank is indifferent between reducing his amount of borrowing or keeping excesses reserves she prefers to reduce her borrowing. This allow us, using \(q_1^m = q_1^*, \) or \(\frac{m + \varepsilon}{\rho + \varepsilon} = \frac{\nu - l}{\rho - \varepsilon}\) to derive the amount of borrowing \(l = (\rho + \varepsilon)(1 - h)A + (\frac{\rho + \varepsilon - \delta}{\beta})m\). Therefore \(q_1^m = \frac{m + l}{\rho + \varepsilon} = \frac{m}{\beta} + (1 - h)A = q_2^m\). Obviously \(q_1^m = q_2^m\) and given \(\mu > \beta\), from (21) we have also that \(q_1^m = q_2^m < q^*\).
This equilibrium requires \( m^2 + l \leq m \). Since \( q_{1}^m = q_{2}^m \), we have that \( m^2 = (\rho - \varepsilon)(\pi + m) \) and, using the expression for \( l \), \( m^2 + l \leq m \) implies \( \frac{1}{3} \leq \frac{1}{\rho} - \frac{(1-h)A}{m} \).

Assuming \( A < A \), we know that exists a \( \tilde{\delta} \) in the interval \((\rho + \varepsilon, 1)\) such that \( \frac{1}{\delta} = \frac{1}{\rho + \varepsilon} - \frac{(1-h)A}{m(\delta, \mu, A)} \) and therefore, keeping \( A \) and \( \mu \) constant, \( \frac{1}{\delta} < \frac{1}{\rho} - \frac{(1-h)A}{m(\delta, \mu, A)} \). This implies that exists a \( \tilde{\delta} < \delta \) such that \( \frac{1}{\delta} = \frac{1}{\rho} - \frac{(1-h)A}{m(\delta, \mu, A)} \), where \( m(\delta, \mu, A) \) is the solution to \( \frac{\delta}{\rho} = u'(\frac{m}{\delta} + (1-h)A) \) and it is lower than \( m(\delta, \mu, A) \). This condition can be rewritten as \( \tilde{\delta} = \frac{\rho}{\rho - (1-h)A} \).

For \( \delta > \tilde{\delta} \) the LHS decreases and \( m(\delta, \mu, A) \) increases, therefore the condition is satisfied with a strict inequality. An equilibrium with a liquidity trap and an interbank market exists for \( \delta \geq \tilde{\delta} \). In order to verify that the upper thresholds for the region of this equilibrium is \( \delta \), we take the expression for \( l \) and we take the limit for \( l \) that goes to zero, getting \( \frac{1}{\delta} = \frac{1}{\rho + \varepsilon} - \frac{(1-h)A}{m(\delta, \mu, A)} = \frac{1}{\delta} \).

Before to conclude, it is important to remark that \( A < A \) is a sufficient condition for the existence of this equilibrium, but not necessary. In fact, this equilibrium can exists also for an \( A > A \) but sufficiently low such that \( \delta < 1 \).

**B.3 Scarcity interest bearing assets equilibrium:** \( \frac{\rho}{\mu} < \psi < \mu \) and \( \beta < p = h\beta + (1-h)\psi < h\beta + (1-h)\mu \)

When interest bearing assets are scarce, banks cannot give to their depositors in credit-meetings enough claims to consume the first best quantity. This implies that interest bearing assets are valued not only for their payoff, but also because at the margin they can facilitate consumption of buyers in credit-meetings. This implies that the prices of bonds and real assets now includes a liquidity premium.

The scarcity of interest bearing assets now implies that the collateral constraint of the interbank market is binding. In this situation both banks will trade-off consumption of their depositors in credit-meetings and cash-meetings according to:

\[
\psi(q_{1}^m) = Ru'(q_{1}) \quad \psi(q_{2}^m) = Ru'(q_{2})
\]  

(32)

where, for instance, for type 1 banks, the marginal benefit of borrowing money on the interbank market is given by the marginal utility of the buyers in cash-meetings that will use it, and its marginal cost is given by the interest rate on the interbank \( R \) and the marginal effects of posting more collateral that decreases the consumption of the buyers in credit-meetings. As in general equilibrium model, the price, the interbank interest rate \( R \) has the role equate relative marginal utilities of buyers.

Using the assumption on log utility, banks provide complete insurance to depositors, so that buyers in cash-meetings consume \( q^m = \frac{m}{\rho} \) and buyers in credit-meetings consume a quantity \( q \) such that \( \frac{m}{\rho} < q < q^{*} \), that is they will consume less than the first best quantity but more than buyers in cash-meetings.

The amount of interbank lending is unchanged from the plentiful government bonds equilibrium, \( l = \frac{1}{2}m \), but now the interest rate on the interbank market will equate marginal utilities of both type of buyers, being in equilibrium:

\[ R = \frac{\rho}{1 - \rho} \left[ \left( \frac{1}{\delta} - 1 \right) + \frac{(1-h)A}{m} \right] \]  

(33)

Note that \( R \) is increasing in the aggregate fraction \( \rho \) of buyers in cash-meetings (the larger buyers who need money, the larger its price on the interbank market), and it is increasing in \((1-h)A\): the higher the possibility to use the real assets as collateral in the interbank market, the larger the demand of money of type 1 bank, pushing up the interest rate. More importantly for our study, \( R \) is decreasing in \( \delta \): open market operations by changing the relative size of money and bonds available in the economy affect equilibrium consumption of buyers in credit-meetings (since it affects the real amount of government bonds in the economy), and through the liquidity premium and the real asset, the interbank interest rate.

38
The assumption of log utility is clearly critical in determining complete insurance to depositors. With a more general utility function consumption level would be different across credit-meetings and cash-meetings and also across banks’ types. However numerical simulations show that the result we care most in explaining, that is how $R$ and asset prices change with changes in $\delta$ and haircuts, is robust to a constant or increasing relative risk aversion utility specification that satisfies the assumptions stated in the main text of the paper. Log utility here buys also an explicit solution for $R$, which allows a direct understanding of its determinants.

The existence of this equilibrium requires interest bearing assets to be sufficiently scarce, so that $1 < R < \frac{\rho}{\delta}$. Therefore this equilibrium exists in the set $A < A$ and $\tilde{\delta} < \delta < \delta$, where $\tilde{\delta} \equiv \frac{\delta R - \rho}{\rho^2 - (\delta + \lambda A)}$ and $\delta \equiv \frac{\rho R - \lambda A}{\rho^2 - (\delta + \lambda A)}$.

**Formal derivation:** For this equilibrium we are going to consider log utility and we guess $q^m_1 = q^m_2$ and $q^l_1 = q^l_2$. At the end we will see that under this specific functional form for the utility function this equilibrium is unique.

Consider (21): given our guess and lemma 2 it must be that $\frac{q^m_2}{q^l_2} = u'(q^l_2)$ and, as $\psi > \beta$, $q^l_2 < q^m_2$. From (17) and (18) we then have $\mu^2 > 0$ and $\xi^2 > 0$ (equivalently $b^2 = 0$ and $a^2 = 0$).

Now consider (12). Since buyers in credit-meetings are consuming less than the optimal quantity, at least one between $\mu^1$ and $\xi^1$ must be greater than zero. Suppose $\xi^1 \geq 0$ and $\mu^1 > 0$, or $b^1 = 0$. We know from Lemma 1 that in this case also $\xi^1 > 0$ and $\mu^1 = 0$. This implies that $l = 0$ and $\nu^1 > 0$ and therefore $q^m_1 = \frac{m^1}{\rho + \psi}$ and $q^m_2 = \frac{m^2}{\rho + \psi}$. From (20) and (21), since $\psi < \mu$ and given our guess, at least one between $\Lambda_1$ and $\Lambda_2$ must be greater than zero. Suppose $\Lambda_1 = 0$ and $\Lambda_2 > 0$. This implies $m^1 < m$ and, since $n = l = 0$, $m^2 = m$. But then, for any $\varepsilon > 0$, it will never be possible that $q^m_1 = q^m_2$ since $q^m_1 = \frac{m^1}{\rho + \psi} < \frac{m}{\rho - \varepsilon} < \frac{m^2}{\rho - \varepsilon} = q^m_2$. Suppose instead that $\lambda^1 > 0$ and $\lambda^2 = 0$. From (11) we have $u'(q^m_1) = u'(q^m_2) + \lambda^1$, and from (16), $u'(q^m_2) = u'(q^l_2)$, that would violate our guess of perfect risk sharing. Therefore both $\lambda^1 > 0$ and $\lambda^2 > 0$. But then $\lambda_1 > 0$ and $\lambda_2 > 0$ imply that $m^1 = m$ and, since $n = 0$, $m^2 = m$. Therefore $q_1^m = \frac{m}{\rho + \psi} < \frac{m}{\rho - \varepsilon} = q^m_2$, which violates again our guess.

Therefore, it must be the case that $\zeta > 0$ and $\mu^1 = \theta^1 = 0$, then $b^1 = 0$ and $m^2 > 0$. The binding constraint for the interbank market implies that $l > 0$ ($\nu^1 = 0$) and $n > 0$ ($\nu^2$). From (12) and (14) substituting out for $\zeta^1$ we have that

$$u'\left(\frac{m^1 + l}{\rho + \varepsilon}\right) = Ru'\left(\frac{\pi - \pi^1 + m - m^1}{1 - \rho - \varepsilon}\right)$$

(34)

and from (16) and (19) substituting out of $\lambda_2$ we have

$$u'\left(\frac{m^2}{\rho - \varepsilon}\right) = Ru'\left(\frac{\pi - \pi^2 + m - m^2 - n + Ru}{1 - \rho - \varepsilon}\right)$$

(35)

Since $R > 1$ and $u'(q^l_2) > u'(q^m_2) > 0$, then by the previous equations $u'(q^m_1) > u'(q^m_2)$ and $u'(q^m_2) > u'(q^l_2)$, that from (11) and (16) implies $\lambda_1 > 0$ ($m^1 = m$) and $\lambda_2 > 0$ ($m^2 = n + m$). Therefore, using our guess $q^l_1 = q^m_2$ and using $l = n$ we can solve for the optimal amount of money exchanged in the interbank market and retrieve $l = \frac{\pi}{\rho} m$. From (20), using (11) and (16) the optimal amount of money is the solution to $\frac{\pi}{\rho} = u'(\frac{m}{\rho})$.

The final object to find is the interbank interest rate $R$. This must be such that $q^m_1 = q^m_2$. Given that $RL$ = $\pi^1$, $\pi^2 = 0$, $m^1 = m$ and $m^2 = l = m$, using our guess $\frac{\pi - RL}{1 - \rho - \varepsilon} = \frac{\pi + RL}{1 - \rho - \varepsilon}$. Using monetary policy $\delta = m (\frac{1}{2} - 1)$ and $l = \frac{\pi}{\rho} m$ we finally obtain $R = \frac{\pi}{1 - \rho} \left(\frac{1}{2} - 1 + \frac{(1 - \beta)A}{\rho}\right)$. Under the assumption of log utility, one can easily check that with the $l$ and $R$ found, both conditions (34) and (35) are satisfied.

The existence of this equilibrium requires $R > 1$, which from the expression for $R$ implies $\frac{1}{\delta} > \frac{1}{\rho} - \frac{(1 - \beta)A}{\rho} = \frac{1}{\rho} - \frac{(1 - \beta)A}{\rho} = \frac{1}{\delta}$. We also require $R < \frac{\mu}{\rho}$, than from the expression for $R$ is equivalent to $\frac{1}{\delta} < \frac{1}{\rho} + \frac{(1 - \beta)A}{\rho} = \frac{1}{\delta}$. Therefore $\tilde{\delta} \leq \delta \leq \delta$. 39
It is also possible to derive the threshold value \( \tilde{A} \) for which the economy reach the equilibrium interest rate \( R = 1 \) only at the limit, i.e. the lower \( A \) for which there is no possibility to have a liquidity trap equilibrium. Considering the condition \( \frac{1}{\delta} = \frac{1}{\rho} - \frac{(1-h)A}{\rho l} \) and assuming \( \delta = 1 \) it is possible to derive \( \tilde{A} = \frac{(1-\rho)\Gamma}{(1-h)\rho} \). It can be easily checked that \( \tilde{A} > \frac{A}{4} \) and that \( \tilde{A} < \bar{A} \).

Finally, we should prove that under the assumption of log-utility there exists no equilibrium in which consumption is different depending on the type of bank. Consider the case in which \( q_1 = q' \), \( q_2 = q'' \) and \( q' < q'' \leq q' \) (we would get the same result assuming \( q' > q'' \) ). The case in which the interbank market is not active is not relevant. Since by proposition 3 if the interbank market is active \( R > 1 \), for type 2 banks is always convenient to make interbank loans because reducing by \( n \) the amount of money used in cash-only meetings they get \( Rn \) resources in the credit meetings. Otherwise, they can trade off resources between the two type of meetings only one to one. Therefore, we consider only the case in which the interbank market is active.

Since \( R > 1 \) from (19) \( \lambda^2 > 0 \). Moreover, \( \nu^1 = \nu^2 = 0 \) and by lemma 1 \( \xi^1 = 0 \). From (11), (13) and (14) also \( \lambda^1 > 0 \). This implies that \( m^1 = m \) and \( m^2 = m - l \). Combining (13) with (14) and (16) and (19) we get

\[
R = \frac{q'(\rho + \varepsilon)}{m + l} \quad R = \frac{q''(\rho - \varepsilon)}{m - l}
\]

Since \( q' < q'' \), then \( \frac{m+l}{\rho + \varepsilon} < \frac{m-l}{\rho - \varepsilon} \) and therefore \( l < \frac{\varepsilon}{\rho} m \). Moreover \( q' < q'' \) is equivalent to \( \frac{\pi - Rl}{1 - \rho - \varepsilon} < \frac{\pi + Rl}{1 - \rho + \varepsilon} \), that implies \( R > \frac{\rho \pi}{1 - \rho m} \), if \( l < \frac{\varepsilon}{\rho} m \). Substituting \( q'' \) in the second equation in (36) we get \( R = \frac{(\rho - \varepsilon)\pi}{(1 - \rho + \varepsilon)m - 1} \), and, given the previous result about \( R \), the following inequality must be true: \( \frac{(\rho - \varepsilon)\pi}{(1 - \rho + \varepsilon)m - 1} > \frac{\rho \pi}{1 - \rho m} \). However, solving the inequality we get \( l > \frac{\varepsilon}{\rho} m \), but this is a contradiction. Therefore, \( q' = q'' \).
Plentiful interest bearing asset equilibrium
Scarce interest bearing asset equilibrium
Liquidity trap

Figure 6: Equilibria on interbank market with respect to $\delta$ when $\mu > \beta$ and $A < A_0$.

Figure 7: Consumption of agents with respect to $\delta$ when $\mu > \beta$ and $A < A_0$. Solid line: consumption of buyers in credit-meetings. Dotted line: consumption of buyers in cash-meetings. $q^* : u'(q^*) = 1$, $\tilde{q} : u'(\tilde{q}) = \mu/\beta$
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