Peer monitoring via loss mutualization

by Francesco Palazzo
Temi di discussione

(Working papers)

Peer monitoring via loss mutualization

by Francesco Palazzo

Number 1088 - October 2016
The purpose of the Temi di discussione series is to promote the circulation of working papers prepared within the Bank of Italy or presented in Bank seminars by outside economists with the aim of stimulating comments and suggestions.

The views expressed in the articles are those of the authors and do not involve the responsibility of the Bank.


Editorial Assistants: Roberto Marano, Nicoletta Olivanti.

ISSN 1594-7939 (print)
ISSN 2281-3950 (online)

Printed by the Printing and Publishing Division of the Bank of Italy
PEER MONITORING VIA LOSS MUTUALIZATION

by Francesco Palazzo*

Abstract

Several regulatory provisions increased the burden of losses from a bank's bankruptcy taken on by other banks. For example, bank resolution schemes and central counterparties introduced mechanisms to share losses among surviving banks, with an emphasis on ensuring a sufficient loss absorption capacity. Without understating this concern, the article suggests that a loss mutualization scheme may also foster market discipline among banks (peers). The optimal design imposes higher contributions on banks with closer interlinkages with the defaulter. The results highlight how the allocation of losses beyond the defaulter's initial contribution plays a considerable role in reinforcing peer monitoring.

JEL Classification: G23, G28.
Keywords: peer monitoring, default fund, resolution fund, clearinghouse, collateral.

Contents

1. Introduction ...................................................................................................................... 5
2. Related literature .............................................................................................................. 9
3. Baseline model ............................................................................................................... 10
4. Equilibrium analysis ...................................................................................................... 13
   4.1 Autarchy ................................................................................................................. 13
   4.2 Interbank equilibrium ............................................................................................. 14
      4.2.1 Interbank market ............................................................................................ 14
      4.2.2 Effort choice .................................................................................................. 15
      4.2.3 Contract with investors .................................................................................. 17
   4.3 Interbank collateral ................................................................................................. 19
5. Loss mutualization scheme ............................................................................................ 22
   5.1 Idiosyncratic credit statuses .................................................................................. 23
   5.2 Perfectly correlated credit statuses ....................................................................... 27
   5.3 Implementation challenges .................................................................................... 30
   5.4 Information acquisition ......................................................................................... 31
6. Conclusion ..................................................................................................................... 34
7. Appendix ....................................................................................................................... 35
   7.1 Baseline model ...................................................................................................... 35
   7.2 Loss mutualization scheme .................................................................................. 36
   7.3 Microfoundation matching process ....................................................................... 45
   7.4 Discrete N banks case .......................................................................................... 47
References .......................................................................................................................... 52

* Bank of Italy, Financial Stability Directorate.
1 Introduction

During the recent financial crisis, extensive bailout plans were implemented in order to prevent an even more damaging collapse of financial markets and bank credit intermediation. The US Treasury disbursed $313 billions to the financial industry through the TARP program to inject capital and guarantee assistance to troubled financial institutions. Similarly, during the 2008–2014 period, governments in the Euro Area incurred a net cost of €178 billions in recapitalizations, asset relief programs, guarantees and other liquidity measures. Although some public programs ended up being profitable, the initial intervention spurred a huge public debate because of its potential burden on taxpayers.

To reduce the possible impact of future bailout costs on taxpayers, new legislation has been passed to increase the share of losses that the financial sector should bear in case of bankruptcy of a financial institution. For example, both the Bank Recovery and Resolution Directive (BRRD) in Europe and the Dodd Frank Act in the US envisage a bank resolution fund financed with contributions from the banking sector.

Similarly, central counterparties (CCPs) require their members to contribute to a default fund which is part of their default waterfall. The design of a loss mutualization scheme among financial institutions is a complex

---

*Every exemplary punishment to individuals has in it some tincture of injustice, which gets compensated by the promotion of the public good. Publius Cornelius Tacitus, *Annales* XIV, 44, my translation.
†I thank Nicola Branzoli, Jason R. Donaldson, Taneli Mäkinen, Giorgia Piacentino, Matteo Piazza, Anatoli Segura, Enrico Sette, Javier Suarez, Min Zhang, as well as seminar participants at the Bank of Italy, the ESSFM 2015 Gerzensee, the Systemic Risk Center at LSE and the XI Annual Seminar on Risk, Financial Stability and Banking for useful comments and discussions. The views expressed in this article are those of the author and do not involve the responsibility of the Bank of Italy. All errors remain mine.
1CBO report on the Troubled Asset Relief Program March 2015. As of January 31, 2015 a sum of $294 billions was reimbursed, while $18 billions has been written off.
2Eurostat Supplementary Tables for Financial Crisis, April 2015.
3Bank resolution funds were established within the recent bank resolution frameworks in Europe and in the US, and they should cover all the expenses required to carry out the core activities of a bank under resolution. In the US the Orderly Liquidation Fund can only be used for liquidity support. It is funded by financial institutions with consolidated assets greater than $50 billion who have to reimburse ex post the original disbursement by the Treasury. In Europe the BRRD prescribes the Single Resolution Fund to provide both liquidity and capital support. Although ex post contributions are possible, the Single Resolution Fund has to be pre-funded for at least one per cent of the amount of covered deposits by all credit institutions. All financial institutions shall contribute *pro quota* with some additional corrections based on size and risk profile (see Annex to Delegated Regulation supplementing Regulation No (EU) 806/2014 released on 17/12/15.)
4CCPs perform several functions: multi-lateral netting, post-trade transparency, setting initial and variation margins, and upon the default of a member they can distribute the resulting losses among its surviving members. CCPs members are mostly banks. Despite their relevance for financial stability, netting and trade transparency may be independently achieved through trade compression services and central trade repositories, respectively. Therefore, the loss mutualization mechanism is a distinctive element of CCPs, and it goes under the name of ‘default waterfall’. This is a set of rules determining the hierarchy of funds used to cover losses from a member default, and the extent to which each surviving member is responsible for these losses. Current default waterfall envisage to first use the initial margins of the defaulting member, then it makes a direct contribution from its capital, and lastly it uses the resources available in a pre-funded default fund; if pre-funded financial resources are not sufficient, CCP members contribute with additional funds (rights of assessment); see Elliott (2013) for a detailed explanation.
task because it has to strike a balance among a number of potentially contrasting dimensions: (i) how much insurance should the participants to the scheme offer to outsiders; (ii) the mix between costly pre-committed funds and ex post contributions; (iii) the metrics used to compute individual bank contributions.

The prevalent approach of policymakers has considered the problem from an ex post perspective. Legislative provisions envisage criteria to determine the appropriate size of the common pool of funds and the individual bank contributions, where the latter mostly depend on bank size and riskiness. Initial disbursements create a common pool of funds to withdraw upon the default of a bank; if not sufficient, additional ex post contributions can be levied from surviving institutions, usually in proportion to what was originally paid by each bank. According to this design, losses are implicitly distributed among surviving institutions based on their ex ante size and risk. Certainly, imposing a larger ex ante contribution to a riskier participant is a sensible criterion because it increases its ‘skin in the game’ and it decreases the potential loss upon default. However, it seems that no clear economic principle has informed the discussion on how to distribute losses beyond the initial contribution of the defaulter.

In this article I consider the problem of designing a loss mutualization scheme from a market discipline perspective. Specifically, I consider how a loss mutualization scheme should distribute losses among surviving banks in order to enhance the efficacy of peer monitoring. Similarly to Stiglitz (1990) and Varian (1990), I use the term peer monitoring as the possibility of alleviating an agency problem through peers’ interactions, in this case interbank transactions. I consider the ex post concerns on loss absorption, but I extend the analysis to take into account how the design of the loss mutualization scheme may reinforce peer monitoring and, in turn, provide banks ex ante with sound incentives over the risk of their investment strategies. Hence, the ex ante and ex post perspectives are not opposite, but rather complementary. I analyze this topic with a theoretical model which incorporates a few well known features of the banking industry: first, financial institutions may engage in risk shifting practices when only incomplete contracts are possible; second, banks have superior skills to assess the credit risk of a similar institution; third, banks find essential to trade in the interbank market, for instance because they have to hedge their exposure with clients, or to access short-term funding. Importantly, peer discipline is possible only in interbank markets where the identity of counter parties is known. This is the case for over-the-counter markets, while in platforms with anonymous trading peer monitoring is not feasible.

The main policy implication of this article is that losses should be allocated based on information about the previous interlinkages among banks, as measured, for example, by bilateral interbank trade activity, and impose a larger share of losses to those peers who have carried
out a more intense interbank trading activity with the defaulter. In this way, peer discipline is more effective in preventing a risky conduct, thus reducing the need for alternative disciplining devices such as capital or collateral requirements that increase banks’ skin in the game. In my model, the optimal loss allocation scheme reinforces two different disciplining mechanisms in the interbank market. The first one is the threat of market exclusion: a risky bank may not trade—suffering a loss due to ‘autarchy’—if all other banks prefer not to be exposed to a high risk of incurring losses due to its default. In turn, the possibility to be excluded from the interbank market provides an ex ante incentive to avoid risky investment activities. The threat of market exclusion is an effective peer discipline mechanism when there is a small number of banks and/or credit risk statuses among banks are strongly correlated (conditional on not misbehaving), for example because they depend on common macroeconomic factors. The second disciplining mechanism is based on a positive assortative matching result, and I refer to it with the expression endogenous peer selection. Safe banks find it mutually beneficial to trade with each other’s in order to minimize the chances of incurring losses due to the default of a counter party; as a consequence, risky banks are more likely to match and trade with each other’s. Although risky banks are not excluded from the interbank market, trading with another risky bank is costly because of the higher risk of incurring losses. As a result, it is ex ante more convenient for banks to avoid a risky business conduct, maximizing the chances of being safe and match with another safe counter party. This second disciplining mechanism is more relevant when the interbank market has a large number of participants and/or there is a low correlation among banks’ credit risk statuses; under these circumstances, it is likely that some banks in the market are risky, and each bank assigns a low probability to the event of matching with a bank of different credit risk.

A well known alternative to peer discipline is to raise banks’ skin in the game. To make the problem relevant, I assume banks can finance part of the project with their own funds at a higher opportunity cost than outside investors. In the context of my model, self-financing can be equivalently interpreted as raising more capital or posting more collateral in advance. A first direct effect is easily predictable: more skin in the game decreases shirking incentives, and it acts as a substitute for peer discipline; more interestingly, it also reinforces peer discipline. Indeed, when banks hold a higher stake in the financing of their investments, they are also more cautious in their interbank trades because they would lose more in case of default of a bank counter party. As a result, the threat of market exclusion is more effective as safe banks are less inclined to accept an interbank trade with a risky counter party. The analysis also points out that an initial commitment of funds could be necessary for the implementation of a loss mutualization scheme, as it refrains banks from free riding on the insurance received by
other banks. Without a minimum mandatory requirement banks have a strategic incentive to offer investors a contract with less collateral, and subsequently shirk; although investors get a compensation for the higher risk, the shirking bank does not incur its full cost because the scheme provides partial insurance to investors. It is interesting to notice that a common feature of many loss mutualization mechanisms is to require ex ante mandatory contributions to a common pool of funds. Ex ante contributions have been justified on the grounds of reducing larger ex post disbursements which may cause a liquidity squeeze, whereas in my model they prevent free riding on the loss mutualization scheme.

The model prescription to impose a higher share of losses on the defaulter’s counter parties may resemble what would happen in a bilateral interbank market, but there are important differences. In the absence of a loss sharing scheme, peer discipline is stronger in a bilateral market the higher is the expected loss that banks suffer from the default of a bank counter party. On the one hand, nowadays interbank transactions are often collateralized or cleared through a central counterparty; this reduction in interbank exposures limits direct contagion,\(^5\) but it may have drastically reduced banks’ peer monitoring incentives. Indeed, a safe bank is less cautious in accepting a trade with a risky counter party, worsening the ex ante agency problem. This perverse incentive is maximal in a CCP because the novation\(^6\) of contracts hinders the incentives to screen counter parties; see also Antinolfi et al. (2014) on this effect. On the other hand, although it is an empirical question beyond the scope of this article, it is reasonable to argue that the magnitude of ex post contributions that would incentivize banks’ peer monitoring is likely to be lower than the bilateral exposures resulting from a fully unsecured interbank market. Large bilateral exposures could trigger direct contagion without improving peer monitoring, as losses would be also imposed on banks’ uninformed creditors, with no ex ante benefit for disciplining incentives. Conditioning ex post contributions on the previous interlinkages between the defaulter and other banks has the objective to reinforce the incentives to screen other bank counter parties, without relying on large bilateral exposures. In other words, a well designed loss allocation scheme may help to reinforce peer discipline also in a fully collateralized interbank market, hence trying to reconcile market discipline with a financial stability perspective. Although there are significant issues for the practical implementation of the suggested policy implications, this article is a first step in the direction of including a market discipline perspective in the design of a loss mutualization scheme.

In the next section I discuss the related literature. Section 3 presents the model setup.

---
\(^5\)The default of a bank poses a serious threat to its counter parties when they hold large mutual exposures. For some empirical evidence on contagion risk see Liedorp et al. (2010) and Craig et al. (2014).
\(^6\)The legal term ‘novation’ indicates the action to replace a party in an agreement with a new party: a CCP interposes itself between two clearing members, becoming buyer to every seller and vice versa.
Section 4 characterizes the equilibria of the baseline model. Section 5 studies the optimal loss mutualization scheme. Section 6 concludes. All proofs are presented in the Appendix.

2 Related literature

A main assumption of my model is the possibility for banks to access or possess superior information on other bank’s credit risk, as already suggested by several authors. It is often justified on the grounds that banks have a comprehensive understanding of the industry, real time information on market conditions, and possibly superior information on the risk exposure of other banks. A non-exhaustive list of articles includes Rochet and Tirole (1996), DeYoung et al. (1998), Peek et al. (1999), Berger et al. (2000) and Furfine (2002). The empirical evidence on the existence of banks’ peer monitoring is limited, but it tends to support this hypothesis; see Furfine (2001), Ashcraft and Bleakley (2006), King (2008), Dinger and von Hagen (2009), Affinito (2012) and Bräuning et al. (2014).

My results are mainly related to the literature on peer monitoring. Rochet and Tirole (1996) argue that the existence of a decentralized interbank market can be only motivated by peer monitoring. In turn, to be effective, the incentive to monitor other banks requires to be responsible for losses on interbank loans, and not to be protected by a lending of last resource policy. Similarly, in my model peer monitoring requires surviving banks to be exposed to the risk of paying higher loss contributions when they previously traded with a defaulter. Rochet and Tirole (1996) assume the existence of a monitoring action that directly reduces the benefit from shirking of the monitoree. I model more explicitly the peer monitoring mechanisms, i.e. the threat of market exclusion and endogenous peer selection. In particular, I show how the effectiveness of peer monitoring is endogenously determined by the loss sharing scheme.

Focusing on micro finance, Ghatak (2000) shows that joint liability leads to an endogenous selection among borrowers of similar credit risk. He points out that assortative matching can be exploited by lenders to screen borrowers and avoid distortions due to credit rationing or over-investment. In my model an analogous assortative matching result applies, and I characterize under which conditions it effectively provides incentives to undertake the first best action, i.e. when the number of banks is large and their credit risk shocks are not perfectly correlated.\footnote{Ghatak (2000) considers a setup with a continuum of agents with a predetermined share of risky and safe borrowers.} Varian (1990) presents another possible mechanism leading to assortative matching: the lender randomly selects and screens one group member, and if he turns out to be risky, all group members are denied credit. Both Ghatak (2000) and Varian (1990) consider an adverse selection
framework in which borrowers’ credit risk is predetermined and peer monitoring can be used as a screening device. In my setup, lenders cannot screen banks because they sign an incomplete investment contract before the effort choice determines the bank’s credit type. Hence, in my model peer discipline provides ex ante incentives to exert effort in order to minimize the chances of becoming risky. Stiglitz (1990) presents the idea that punishments may help to reinforce peer monitoring. In his model it is welfare enhancing to write a joint liability contract that specifies penalties on a borrower whenever a contractual partner defaults. My optimal loss sharing scheme prescribes an analogous penalty structure on the trading partner. However, in my model the decision to exert effort is not observable to any peer; in particular, matching takes place after the default probabilities realize, and no collusion on possible deviations is possible.

My article may also contribute to a recent literature on CCPs. The possibility to pool and diversify idiosyncratic default risk through a CCP provides a missing insurance market, and improves the allocation of risk among traders. For instance, this point is raised in Carapella and Mills (2012), Biais et al. (2012), Koeppl and Monnet (2013). Carapella and Mills (2012), Biais et al. (2012) and Antinolfi et al. (2014) all point out that a CCP reduces traders’ incentive to acquire information on counter party risk. In Carapella and Mills (2012) this common incentive to remain ignorant reduces the negative effects of adverse selection. However, Antinolfi et al. (2014) point out that this lack of screening incentives results in higher collateral requirements, as an instrument to mitigate limited commitment. Biais et al. (2012) stresses that in the presence of aggregate shocks the optimal clearing contract only offers partial insurance to maintain buyers’ incentives to screen only good counter parties. Differently from my model, this literature does not consider the possibility that each clearing member may have superior information on the credit risk of other members.

3 Baseline model

There are four periods $t = 0, 1, 2, 3$ and a continuum of measure one of banks. Each bank $B_i, i \in [0, 1]$, faces a population of competitive, deep pockets, and risk neutral investors. All banks have an identical project which requires an initial outlay of $I > 0$ dollars at $t = 0$ and pays either a return $R > 0$ (deterministic) or zero at $t = 3$. For simplicity, I first consider a model with no bankruptcy costs for investors; I introduce such costs in the analysis of the loss mutualization mechanism. In the Appendix I consider a model with a discrete number $N$ of banks: the underlying economic intuitions are analogous, but the analytical problem becomes less tractable.

At $t = 0$ each bank $B_i$ simultaneously offers a contract $(p_i, k_i)$ to outside investors. All
contracts are publicly observed. Each $B_i$ receives $I$ from investors, and promises a transfer $p_i$, to be paid at $t = 3$, and a quantity $k_i \geq 0$ of collateral posted at $t = 0$. Banks produce the collateral asset at $t = 0$ at a per unit cost $\mu > 1$, and at $t = 3$ one unit of collateral is worth one dollar (normalized). The discount rate of all market participants is normalized to one. If $B_i$ defaults, collateral $k_i$ is transferred to investors, while if it honors its obligations $B_i$ gets back the asset. Without a moral hazard problem no collateral is necessary because investors are risk neutral and they do not incur bankruptcy costs. It is implicit in the restriction of the contractual space $(p_i, k_i)$ that banks cannot offer contracts contingent on any future event. After observing the contract offers, investors decide whether to accept or reject each offer. I consider period $t = 0$ to be divided in two sub-periods: in the first one banks simultaneously offer contracts, and in the second one investors accept or reject.

I denote with $d_i$ the probability that $B_i$’s investment returns zero at $t = 3$; from $t = 0$ standpoint it is a random variable which realizes at $t = 2$. At $t = 1$, exerting effort ($e_i = 1$) imposes an immediate cost $c > 0$, and at $t = 2$ it leads to be safe with (marginal) probability $1 - \alpha$ and risky with probability $\alpha$; if a bank shirks ($e_i = 0$) it is risky with probability one. The effort cost can be interpreted as the cost of implementing good risk management techniques, or to select good proprietary investments. A safe bank has $d_i = 0$ and it defaults at $t = 3$ only for direct contagion by another bank (see later). If a bank is risky, at $t = 3$ it defaults with probability $d > 0$, and it survives with probability $1 - d$, unless there is contagion. The random variable $d$ comes from a differentiable cumulative distribution $G(\cdot)$ with expected value $m$. To simplify the exposition of the main economic mechanisms, I assume that at $t = 2$ the realized default probability $d$ is identical for all risky banks. As a result, at $t = 2$ there are two types of banks—safe and risky—with the riskiness level $d$ being a random variable before $t = 2$. The investment is profitable only if $B_i$ exerts effort. For simplicity, I consider two polar cases for the joint distribution of credit risk statuses (i.e. safe or risky) across banks conditional on exerting effort. If credit risks are independent across banks a share $\alpha > 0$ of banks is risky and a share $1 - \alpha$ is safe. If credit statuses are perfectly correlated, it is as if there are two aggregate states, say H and L; conditional on all banks exerting effort, with probability $1 - \alpha$ state H realizes and all banks are safe, while with complementary probability state L occurs and all banks are risky. In the Appendix I also consider the case with intermediate correlation.

Once banks observe at $t = 2$ the default probabilities of all other banks, they simultaneously choose to match with another bank. Default probabilities are not observable to investors or a judicial court but only to banks. The matching among banks is endogenous. Let $B_j, j \neq i$, denote the bank matched with $B_i$. A matched pair of banks have to agree on whether entering into an interbank trade or not. If both agree they avoid a cost $L > 0$, otherwise each bank
immediately incurs this cost. For example, \( L \) can be interpreted as the extra effort required to manage an unhedged position, or the cost to run the project without trading on the interbank market. Mutual agreement avoids the cost \( L \) to each bank, but it increases the risk of contagion: if at \( t = 3 \) bank \( B_j \) defaults, \( B_i \) may default as well with probability \( \gamma \in [0, 1] \). The parameter \( \gamma \) measures the contagion risk due to the insolvency of the trading partner. For example, it is likely to be high in the case of a large unsecured interbank loan, and low if secured. Similarly, an interest rate swap contract may cause a large loss if the defaulter has to pay its counter party, but it may not cause any contagion risk if the defaulter has a position ‘in the money’. The interbank transaction at \( t = 2 \) and the contagion probability \( \gamma \) are reduced forms to capture an interbank financial trade aimed at reducing operating costs, but leading to a potentially disrupting financial exposure towards another bank. I exclude from the analysis why banks exchange financial claims more or less sensitive to counter party risk, and the optimal loss mutualization scheme is designed taking this variable as given. Interbank trade can be ex-post verified—for example during a bankruptcy procedure—after a bank defaults. If both banks agree to trade and their credit risk profile is \((d_i, d_j)\), \( B_i \) defaults at \( t = 3 \) with probability \( d_i + (1 - d_i)d_j\gamma \). The timing of the game is illustrated in Figure 1. I assume the expected cost of trading with a counter party that defaults with a probability of one to be larger than refusing to trade, i.e. \( \gamma(R - I) > L \). To simplify the exposition of the main mechanisms, I restrict attention to \( L \geq \frac{\gamma R}{4} \) which is a sufficient condition to guarantee that two risky banks always trade. If two risky banks would find optimal not to trade for some values \( d \), the analysis would be more complicated without adding any relevant economic insight.

\[ B_i \text{ OFFERS } (p_i, k_i). \]
\[ B_i \text{ EXERTS UNOBSERVABLE EFFORT } e_i. \]
\[ B_i \text{ DEFAULT PROBABILITIES REALIZE. } \]
\[ \text{INVESTORS ACCEPT OR REJECT} \]
\[ \text{DEFAULT PROBABILITIES REALIZE. } \]
\[ \text{BANKS ENDOGENOUSLY MATCH AND AGREE TO TRADE OR NOT} \]
\[ \text{CONTRACTS ENFORCED UNLESS DEFAULT} \]

Figure 1: Timing of the game.

I only consider the symmetric subgame perfect equilibria of the game. A strategy profile is subgame perfect if its restriction is a Nash equilibrium in every proper subgame. The proper subgames coincide with each period \( t \) and sub-period (for \( t = 0 \)): at \( t = 2 \) all banks know the investors’ contracts and banks’ default probabilities, and they decide whether to trade or
not; at $t = 1$ banks decide the effort decision, knowing all investors’ contracts; in the second
sub-period of $t = 0$, investors accept a contract only after observing all contracts; in the first
sub-period of $t = 0$, each $B_i$ simultaneously offers $(p_i, k_i)$.

In conclusion, the model can be thought as a stylized representation of the bank lending
process: banks raise deposits, invest in a portfolio of loans they should carefully monitor, and
they trade in the interbank market to hedge from interest rate risk, or to receive short term
funding for liquidity purposes. Alternatively, investors could be thought as CCP clients, and
banks as CCP members that intermediate their trade while trading in a bilateral interdealer
market.

4 Equilibrium analysis

In a first best allocation each bank $B_i$ invests at $t = 0$ and it exerts effort at $t = 1$. Otherwise,
the project has a negative net present value (NPV) and it would not be undertaken. Collateral
is not used when effort is contractible because no incentive compatibility constraint arises, and
each unit of collateral costs $\mu > 1$ upfront. Hence, it is optimal to set $k_i = 0$.

To highlight the effects of peer monitoring, I first analyze the equilibrium contract in an
economy with a single bank (hence with no interbank market), then I present the equilibrium
with multiple banks.

4.1 Autarchy

I denote the final payoff at $t = 3$ for the single bank $B_i$ with $\pi_i := R - p_i + k_i$. In a first best, all
investments projects with positive $NPV = (1 - \alpha m)R - I - c$ are financed. If effort is not ob-
servable and verifiable, a bank exerts effort at $t = 1$ only if the contract is incentive compatible,
i.e. $(1 - \alpha m)\pi_i - c \geq (1 - m)\pi_i$ or

$$\pi_i \geq \frac{c}{(1 - \alpha m)}$$

(1)

An incentive compatible contract solves:

$$\max_{p_i \geq 0, k_i \geq 0} (1 - \alpha m)(R - p_i + k_i) - \mu k_i - c$$

s.t. $R - p_i + k_i \geq \frac{c}{(1 - \alpha m)}$ (IC)  (2)

$(1 - \alpha m)p_i + \alpha mk_i \geq I$ (IR)

The next Proposition characterizes the equilibrium contract:
Proposition 4.1 The unique subgame perfect equilibrium of the game is:

1. If $NPV \geq \frac{1-m}{m(1-\alpha)}c$ the investment is undertaken without collateral ($p = \frac{I}{1-\alpha m}, k = 0$).

2. If $\frac{1-m}{m(1-\alpha)}c > NPV \geq \frac{\mu-1}{\mu} \frac{1-m}{m(1-\alpha)}c$ the investment is undertaken but costly collateral is posted to investors ($p = \frac{I-\alpha mk}{1-\alpha m}, k = \frac{1-m}{m(1-\alpha)}c - NPV$).

3. If $NPV < \frac{\mu-1}{\mu} \frac{1-m}{m(1-\alpha)}c$ no investment is undertaken.

Compared to the first best contract, the equilibrium outcome is suboptimal in case 2 because costly collateral must be used, and in case 3 because it is not possible to finance the investment. The equilibria in the next sections display similar qualitative features, but thresholds change as peer discipline may alleviate the moral hazard problem.

4.2 Interbank equilibrium

I now extend the autarchic model by considering the effect of a stylized interbank market at $t = 2$. To solve for the equilibrium of this game I proceed by backward induction.

4.2.1 Interbank market

Consider a matched pair of banks $B_i$ and $B_j$ with corresponding default probabilities $d_i$ and $d_j$. By assumption, each bank knows the default probability of the matched counter party. Each bank decides whether to trade with the matched counter party or not. If $B_j$ is willing to trade, it is weakly optimal for $B_i$ to accept if and only if:

$$(1 - d_i)\pi_i - (1 - d_i)d_j\pi_i\gamma \geq (1 - d_i)d_i - L \quad \Rightarrow \quad d_j \leq \frac{L}{(1 - d_i)\gamma \pi_i} \quad (3)$$

In words, $B_i$ wants to hedge only if the default probability of the other bank is sufficiently low. The threshold value negatively depends on the expected loss due to contagion—$(1 - d_i)\gamma \pi_i$—and positively on $L$. In particular, $B_i$ is less willing to be financially exposed to $B_j$ when the loss $L$ from refusing to trade is small, contagion probability $\gamma$ is large, its own credit risk $d_i$ is low, and its future payoff $\pi_i$ is high.

The expected payoff in equation (3) is strictly decreasing in both $d_i$ and $d_j$ for every $\pi_i > 0$; hence, for each pair of banks the sum of banks’ expected payoff is strictly decreasing in the credit risk of each bank. This strict payoff monotonicity with respect to default probability $d_i$
and \(d_j\) implies that the only stable matching displays positive assortative matching.\(^8\) Therefore, safe banks match with safe banks and risky with risky. The underlying intuition is simple: safe banks have a mutual benefit to trade with each other and avoid contagion risk arising from a trade with a risky bank; as a result, risky banks can only trade among themselves unless there is no other risky bank available.\(^9\) In the Appendix I provide a non-cooperative game for the stable matching outcome that is also useful for the results on the incentives to acquire information in Section 5.4.

Notice that banks of identical credit risk always decide to trade. This is obvious when both banks are safe, while for risky banks it requires \(d(1-d) \leq \frac{L}{\gamma R} \), for every \(d \in [0,1]\). Since by assumption \(L > \frac{\gamma R}{4}\) and, in every equilibrium, \(\pi_i < R\) (i.e. self financing the project), this condition is always satisfied. Therefore, two banks may not trade only if a matched pair includes a safe and a risky bank. In this case, a safe bank \(i\) accepts to hedge with bank \(j\) only if the default probability \(d_j\) is sufficiently small. If banks self-financed the investment \(I\), a safe bank would reject to trade with a risky bank if \(d > \frac{L}{\gamma R} = d^*\), with \(d^* < 1\) as \(L < \gamma R\).

### 4.2.2 Effort choice

If all banks exert effort, the ex ante belief to end up matching with a bank is zero, irrespective of the correlation among banks’ credit risk statuses.\(^10\) Indeed, consider the two polar assumptions of (i) statistical independence and (ii) perfect correlation. In case (i), at \(t = 2\) a share \((1-\alpha)\) of banks is safe and a share \(\alpha > 0\) is risky, hence there is a positive measure of each type of bank, and the probability to have a match between a safe and a risky bank is zero almost surely. In case (ii), if all banks exercise effort they all have identical credit risk at \(t = 2\), either all safe or risky. Therefore, the equilibrium payoff when a bank exerts effort, conditional on all others doing the same, is:

\[
\mathbb{E}_{e=1}[u_i|\pi] = \left[ (1-\alpha) + \alpha \int_0^{1-x} (1-x)[(1-x)+x(1-\gamma)]g(x) \, dx \right] \pi_i - c
\]

\[
= \left( (1-m\alpha)\pi_i - \alpha \int_0^x (1-x)g(x) \, dx \gamma \right) \pi_i - c
\]

Although in equilibrium a bank exerting effort expects to match and trade with a bank of\(^8\)A matching is stable if there is no pair of banks who prefer to form a new union. Positive assortative matching results from a standard corollary of Gale and Shapley (1962) algorithm for matching with non-transferable utilities. The results in Becker (1973) guarantee that positive assortative matching continues to hold with transferable utility because expected payoffs in my model display strict supermodularity.

\(^9\)In the baseline model with a continuum of banks, a match between a safe and a risky bank does not occur in equilibrium. However, if shocks are perfectly correlated, out-of-equilibrium a shirking bank has probability \(1-\alpha\) to only face safe banks when the aggregate state is \(H\).

\(^{10}\)This is no longer true when \(N\) is finite; see section 7.4 in the Appendix.
identical credit risk, this may no longer be the case for a shirking bank. Let \( q_{rs} \) be the probability that, after shirking, a risky bank matches with a safe bank, assuming all other banks exert effort. The probability \( q_{rs} \) is zero in the case of statistical independence because there is a share \( \alpha > 0 \) of risky banks at \( t = 2 \), and a shirking bank expects to match with one of them almost surely; if shocks are perfectly correlated it is \( q_{rs} = 1 - \alpha \) because all the other banks have an identical safe credit status at \( t = 2 \), and they are going to be all safe with probability \( 1 - \alpha \). This effect of correlation on \( q_{rs} \) is going to play an important role in the incentive compatibility constraint as it determines which disciplining mechanism applies, either the threat of market exclusion or the endogenous peer selection.  

If bank \( B_i \) decides to shirk (\( e_i = 0 \)), it is risky at \( t = 2 \) and its expected payoff at \( t = 1 \) is:

\[
E[e_i=0|u_i] = q_{rs} \left[ (1-m)\pi_i - \left[ 1 - G\left( \frac{L}{\gamma\pi_j} \right) \right] L \right] + (1-q_{rs}) \left[ \int_0^1 (1-x)(1-\gamma x)g(x)dx \right] \pi_i 
\]

\[
= (1-m)\pi_i - q_{rs} \left[ 1 - G\left( \frac{L}{\gamma\pi_j} \right) \right] L - (1-q_{rs}) \left[ \int_0^1 x(1-x)g(x)dx \right] \gamma \pi_i 
\]

(5)

In words, if \( B_i \) matches with a safe bank, he is not exposed to contagion risk, but he may not hedge if \( d > \frac{L}{\gamma\pi_j} \). If both banks are risky they trade for sure, but there is contagion risk if one bank defaults and the other survives.

The contract at \( t = 0 \) is incentive compatible if \( E[e_i=1|u_i, \pi] \geq E[e_i=0|u_i, \pi] \). Rearranging equations (4) and (5) conveniently, it is equivalent to:

\[
\pi_i \geq \frac{c - q_{rs} \left[ 1 - G\left( \frac{L}{\gamma\pi_j} \right) \right] L}{m(1-\alpha) + \gamma(1-\alpha - q_{rs}) \int_0^1 x(1-x)g(x)dx} := \xi(\pi_j) 
\]

(6)

This incentive compatibility constraint highlights that banks’ interactions decrease the convenience of shirking via two different peer monitoring mechanisms.

1. **Threat of market exclusion.** The quantity \( q_{rs} \left[ 1 - G\left( \frac{L}{\gamma\pi_j} \right) \right] L \) captures the expected loss from shirking if no other bank accepts to trade on the interbank market. It depends on:

   (i) the probability \( q_{rs} \) to match with a safe bank after shirking; (ii) the probability that a safe bank rejects to trade whenever \( d > \frac{L}{\gamma\pi_j} \); and (iii) the additional loss \( L \) incurred when the investment project is run without using the interbank market.  

   11 With a continuum of banks the threat of market exclusion and the endogenous peer selection are two mutually exclusive disciplining mechanisms. However, in section 7.4 I show that both may simultaneously exist in a model with a finite number \( N \) of banks.

   12 The effect of a larger \( L \) is ambiguous: it increases the loss \( L \), but, at the same time, it decreases the chances that a safe bank refuses to trade, since he is also going to suffer a high loss without a hedge. This ambiguity is the result of the simplifying assumption that the loss \( L \) does not depend on banks’ credit risk. It is straightforward to introduce two distinct losses, \( L_s \) for safe banks and \( L_r \) for risky ones, that reflect different costs. For example, it might be the case that safe dealers find less costly
exclusion is ex ante less credible when contagion risk $\gamma$ is low, as the expression $G\left(\frac{L_{rs}}{\pi_j}\right)$ points out. Importantly, this channel is more effective when the other bank has a larger stake $\pi_j$ at $t = 3$.

2. **Endogenous peer selection.** If credit risk statutes are not too correlated, exerting effort leads a bank to match with another bank of identical credit quality. Being risky leads to trade with another risky bank, thus to be exposed to its counterparty risk. Therefore, shirking leads not only to increase the individual probability to default, but also to have a higher exposure to contagion risk. This peer discipline mechanism is captured by the term $\gamma(1 - \alpha - q_{rs}) \int_{0}^{1} x(1 - x) g(x) dx$.

The relative importance of these two disciplining mechanism depends, among other parameters, on the probability $q_{rs}$ to match after a deviation with a safe bank. In the baseline model with a continuum of banks, the threat of market exclusion arises only if there is perfect correlation among credit risk statuses; otherwise, for intermediate correlation there is always a positive measure of risky banks at $t = 2$ and the only peer discipline incentives would be provided by the endogenous peer selection mechanism. However, this extreme conclusion is driven by the simplifying assumption of a continuum of banks. With a finite number $N$ of banks the probability for a risky bank to trade only with safe banks exist also for intermediate correlation levels, and the threat of market exclusion would still be relevant; see section 7.4 in the Appendix.

### 4.2.3 Contract with investors

I turn to characterize the financial contracting problem between a bank and investors, taking into account the equilibrium strategies on the interbank market, and the incentive compatibility constraint in equation (6).

In a first best, banks can write contracts contingent on the effort choice and no collateral has to be used for incentive purposes. As previously explained, a bank exerting effort finds almost surely a counterparty of equal credit risk with whom he can trade on the interbank market. The survival probability after exerting effort is:

$$s := 1 - \alpha + \alpha \left[ \int_{0}^{1} (1 - x)(1 - \gamma x)g(x) \, dx \right] = 1 - \alpha \left[ m + \gamma \int_{0}^{1} x(1 - x)g(x) \, dx \right]$$

(7)

...to find an alternative hedge, so $L_s < L_r$. In this slightly more general model, the incentive compatibility constraint would have an expected loss from being excluded from the market equal to $\left[ 1 - G\left(\frac{L_{rs}}{\pi_j}\right) \right] L_s$.  

17
i.e. the sum of the probabilities to be safe and match with another safe bank \((1 - \alpha)\), or to be risky \((\alpha)\), trade almost surely with a risky bank and survive. The latter event occurs only if a bank does not default and it does not suffer contagion in case of default of the other counterparty. At \(t = 0\) the first best contract solves:

\[
\max_p \quad s(R - p) - c \\
\text{s.t.} \quad sp \geq I
\]

The optimal contract has \(p = I/s\) and the project is financed if it has positive NPV, i.e. \(sR - I - c \geq 0\). If effort is not observable the contract \((p_i, k_i)\) has to satisfy also the incentive compatibility constraint in equation (6).

\[
\max_{p_i \geq 0, k_i \geq 0} \quad s(R - p_i + k_i) - \mu k_i - c \\
\text{s.t.} \quad R - p_i + k_i \geq \xi(\pi_j) \quad \text{(IC)} \\
sp_i + (1 - s)k_i \geq I \quad \text{(IR)}
\]

Proposition 4.2 characterizes the symmetric subgame perfect equilibrium of the game.

**Proposition 4.2** Let \(\pi_{ib}\) be the unique solution \(\pi = \xi(\pi)\) of equation (6). The only symmetric subgame perfect equilibrium is:

1. If \(NPV \geq s\xi(R - \frac{l}{s}) - c\) each bank invests and no distortion arises \((p = \frac{l}{s}, k = 0)\).

2. If \(s\xi(R - \frac{l}{s}) - c > NPV \geq \frac{\mu - 1}{\mu}[s\pi_{ib} - c]\) each bank invests only if it posts a minimum amount of collateral to investors \((p = \frac{l - (1 - s)k}{s}, k = s\pi_{ib} - c - NPV)\).

3. If \(NPV < s\pi_{ib} - c\) no investment is undertaken.

The equilibrium shares similar features with the autarchic case: if the net present value of the project is sufficiently high, the first best contract is incentive compatible; otherwise, collateral has to be posted to restore incentives by increasing the skin in the game. However, collateral imposes an additional cost and it is no longer profitable to finance all positive NPV projects.

Compared to the autarchic equilibrium, the equilibrium interactions on the interbank market—due to banks’ superior information on credit risk—create additional incentives to exert effort. Trading with another bank reduces operating costs, but it also creates contagion risk. Banks have an incentive to avoid risky counter parties. In turn, this ex post convenience raises the ex
ante cost of becoming risky—relaxing the incentive compatibility constraint—via two channels: (i) being risky may lead not to find any bank willing to trade; (ii) risky banks tend to match with each other, hence they are also exposed to a higher contagion probability. The relative importance between these two economic forces depends on $q_{rs}$ which, in turn, depends on the correlation among banks’ credit status after exerting effort. The stronger the incentives induced by the interbank market, the lower the need to restore incentives through the use of costly collateral. From the perspective of a single bank, peer monitoring is a substitute of collateral. However, collateral has an additional positive spillover effect on other banks. Posting collateral raises the final payoff at $t = 3$, and it increases the loss in case of contagion. Therefore, higher collateral posted by bank $i$ increases its incentives to reject an interbank trade with a risky bank. In turn, a stronger motive to look for a safe counter party relaxes the incentive compatibility constraint of other banks: it is now less convenient for other banks to shirk because it increases the chances to incur the additional cost $L$. This additional positive spillover effect of capital on peer monitoring incentives can be observed from the threshold in item 2. of Proposition 4.2: posting collateral $k$ increases the final payoff $\pi$, and it decreases the quantity $\xi(\pi)$ whenever $q_{rs} > 0$, leading to $\pi_{ib} = \xi(\pi_{ib}) < \xi(R - \frac{L}{3})$, i.e. incentives can be restored with a lower $k$. In words, an increase in the overall quantity of collateral posted by all banks not only improves their individual incentives to exert effort by increasing their ‘skin in the game’ in the project, but it also enhances the threat of exclusion mechanism, leading to post less collateral relative to an autarchic situation.

### 4.3 Interbank collateral

I turn to discuss how the possibility to post collateral to other banks change the moral hazard problem. The baseline model has a sharp representation of contagion costs: after the default of a counter party, either contagion leads to lose entirely the final return $R$ (with prob. $\gamma$), or there is no loss at all. This assumption provides a tractable way to gain several insights, but it is not well suited to discuss collateral posting among banks and its effect on incentives. In this extension I consider a slightly more general representation of losses to highlight the role of collateral posting among banks. Upon the default of a counter party, a bank is exposed to a stochastic loss $l$ distributed according to a cumulative density function $F(\cdot)$. Losses from the default of a counter party can be mitigated by an amount $k^D$ of collateral posted by the defaulting bank at $t = 2$. Posting collateral to the other bank at $t = 2$ has a cost $\mu^D > 1$ per unit. In case of default, the trading partner of a defaulting bank can use the posted collateral to cover as much as possible its losses $l$. 

19
Let \( \pi_i = R - p_i + k_i \) be the return to bank \( i \) at \( t = 3 \) coming from the project (\( R \)), after repaying (\( p_i \)) and collecting previously posted collateral (\( k_i \)) to outside investors. Let \((d_i, d_j, k_i^D, k_j^D)\) the default probabilities of two banks and the amount of collateral posted to each other. The expected payoff for bank \( i \) if both accept trade is:

\[
\mathbb{E}[u_i|d_i, d_j, k_i, k_j] = (1 - d_i) \left\{ (1 - d_j) (\pi_i + k_i^D) + d_j \int_0^{\infty} \max \left\{ 0, \pi_i + k_i^D - l + \min\{l, k_j^D\} \right\} dF(l) \right\} - \mu^D k_i^D
\]

\[
= (1 - d_i) \left\{ \pi_i + k_i^D - d_j \left[ (1 - F(\pi_i + k_i^D + k_j^D)) (\pi_i + k_i^D) + \int_{k_j^D}^{\pi_i+k_i^D+k_j^D} l - k_j^D dF(l) \right] \right\} - \mu^D k_i^D
\]

If bank \( i \) decides not to trade its expected payoff is simply \((1 - d_i) \pi_i - L\). Therefore, bank \( i \) accepts to trade only if:

\[
d_j \leq \frac{L - k_i^D (\mu^D - 1 + d)}{(1 - d_i) \left[ (1 - F(\pi_i + k_i^D + k_j^D)) (\pi_i + k_i^D) + \int_{k_j^D}^{\pi_i+k_i^D+k_j^D} l - k_j^D dF(l) \right]} := \xi(d_i, d_j, k_i^D, k_j^D, \pi_i) \tag{11}
\]

It is easy to show that the RHS of equation (11) is decreasing in \( k_j^D \) and increasing in \( k_i^D \). In other words, bank \( i \) is more willing to trade with a riskier bank \( j \) if the latter posts more collateral \( k_j^D \) to him, and less willing if he has to post more collateral \( k_i^D \).

Equation (11) provides an acceptance rule that helps to determine how much collateral is posted in equilibrium by two banks. Since collateral is costly, I consider they agree to trade at the minimum amount of collateral \((k_i, k_j)\) that satisfy equation (11) for both banks. It is immediate to realize that two safe banks do not post any collateral and they trade for sure. If instead a safe bank \( i \) matches with a risky bank \( j \) who defaults with probability \( d \), bank \( i \) does not post any collateral to \( j \). Indeed, \( k_i \) does not provide any advantage to \( j \) as \( i \)'s project succeeds for sure, but—from equation (11)—bank \( i \) would be less willing to accept to trade with \( j \). The minimum amount of collateral that bank \( j \) has to post to bank \( i \) is:

- \( k_j = 0 \) if \( d \leq d^N(\pi_i) \) where \( d^N(\pi_i) \) solves:

\[
d^N(\pi_i) = \frac{L}{[1 - F(\pi_i)] \pi_i + \int_0^{\pi_i} l dF(l)} \tag{12}
\]
• $k_j (d, \pi_i) > 0$ if $d \in (d^N(\pi_i), \bar{d}(\pi_i)]$ where $k_j (d, \pi_i)$ solves

$$
d = \frac{L}{[1 - F(\pi_i + k_j)] \pi_i + \int_{k_j}^L l - k_j \, dF(l)}$$

(13)

and $\bar{d}(\pi_i)$ is the minimum value between 1 and the value $d$ that satisfies:

$$
d = \frac{L}{[1 - F(\pi_i + \frac{L}{\mu^{\mu-1+d}})] \pi_i + \int_{\mu^{\mu-1+d}}^L l - \frac{L}{\mu^{\mu-1+d}} \, dF(l)}$$

(14)

• If $d > d(\pi_i)$ bank $i$ would pretend an amount of collateral $k_j$ that makes trading unprofitable for bank $j$. As a result there is no trade.

Lastly, if two risky banks $i$ and $j$ have $\pi_i = \pi_j$, as it is the case in a symmetric equilibrium, then no collateral is posted. To see this point, it is sufficient to show that the negative effect on the willingness to trade due to an increase in $k_i$ outweighs the positive effect due to an increase in $k_j$. Indeed, from equation (11) it is immediate to get:

$$\frac{\partial \xi (d, d, k_i, k_j, \pi_i)}{\partial k_i} + \frac{\partial \xi (d, d, k_i, k_j, \pi_i)}{\partial k_j} < 0$$

(15)

Next, I use the equilibrium collateral posting just derived in the effort decision. Exerting effort provides expected payoff:

$$E_{e_i=1}[u_i|\pi] = (1 - \alpha)\pi_i + \alpha \left[ \pi_i \int_0^1 (1 - x)[1 - x(1 - F(\pi_i))]g(x) \, dx - \int_{\mu^{\mu-1+d}}^L \int_{\mu^{\mu-1+d}}^L 1 - \frac{L}{\mu^{\mu-1+d}} \, dF(l) \right]$$

(16)

Shirking leads to:

$$E_{e_i=0}[u_i|\pi] = q_{rs} \left[ (1 - m)\pi_i - \int_{\pi_i}^{\bar{d}(\pi_i)} k'(\pi_j, x)(\mu - 1 + x)g(x) \, dx - \int_{\mu^{\mu-1+d}}^L \int_{\mu^{\mu-1+d}}^L 1 - G(\bar{d}(\pi_j)) \right] L$$

(17)
As a result, exerting effort is incentive compatible if and only if:

\[
c - q_{rs} \left[ \int k(\pi_j, x)(\mu - 1 + x)g(x)dx + \left[ 1 - G(\bar{d}(\pi_j)) \right] \right] - (1 - \alpha - q_{rs}) \int_{0}^{1} f(l)dl \int_{0}^{1} x(1 - x)g(x)dx
\]

\[
\pi_j \geq \frac{\left[ \int k(\pi_j, x)(\mu - 1 + x)g(x)dx + \left[ 1 - G(\bar{d}(\pi_j)) \right] \right] - (1 - \alpha - q_{rs}) \int_{0}^{1} f(l)dl \int_{0}^{1} x(1 - x)g(x)dx}{(1 - \alpha)m + (1 - \alpha - q_{rs})[1 - F(\pi_j)] \int_{0}^{1} x(1 - x)g(x)dx}
\]

(18)

Notice that for every \( d > d^N(\pi_j) \) a safe bank would not trade with a risky one without the possibility to post collateral. Moreover, for \( d \in (d^N(\pi_j), \bar{d}(\pi_j)] \) it holds \( k(\pi_j, d) < \frac{L}{\mu^\alpha + \bar{d}} \); hence, the incentive compatibility constraint in equation (18) would be less binding if banks could not post any collateral, especially when the probability \( q_{rs} \) is large, i.e. when banks’ shocks are more correlated. Intuitively, interbank collateral is used to "bribe" a safe bank by decreasing its loss given default, in turn reducing the peer discipline incentives operating through the threat of market exclusion.

As it clearly emerges from this extension, there is a difference between collateral posted before the effort choice to uninformed investors, and collateral posted to informed banks after the credit risk realizes. The former alleviates the agency problem, while the latter reduces peer discipline as it makes a safe dealer more willing to trade with a risky counter party.\(^{13}\) In general, peer discipline is effective when an informed bank is more severely affected by the default of another financial counter party. Interbank collateral reduces this sensitivity, harming peer discipline, and making the ex ante agency problem more severe.

5 Loss mutualization scheme

In this section I explore how a loss mutualization mechanism can be designed to alleviate the risk shifting problem. The scheme requires surviving banks to reimburse, at least partially, investors who suffered losses from the default of a bank. I consider a system of transfers that differentiate payments among banks based on their previous interbank trading: surviving banks who traded with a defaulting bank should pay a loss contribution \( \tau_D \), while other banks pay a contribution \( \tau_S \). In the previous section all agents were risk neutral and investors did not incur any bankruptcy cost other than a loss equal to \( p - k \). Although the baseline model provides a neat intuition of the main economic mechanisms at work, it may not be realistic because bankruptcy is likely to impose losses on investors beyond the actual monetary costs. For example, risk aversion or difficulties to comply with the payment cycle create utility losses.

\(^{13}\) In my model banks cannot commit to refrain from posting collateral to other dealers. Commitment would make the agency problem less severe.
beyond the actual amount $p - k$ unpaid at $t = 3$. In this section, I include additional bankruptcy costs on investors proportional to the loss incurred. In particular, upon the default of a bank, its investors may suffer a monetary loss $l \geq 0$, incurring a utility loss of $b(l)$, with $b(\cdot)$ being a strictly convex increasing function satisfying $b(0) = 0$ and $b'(0) = 1$.

I consider separately the two polar case of idiosyncratic and perfectly correlated credit risk statuses. As a remainder, if credit statuses are idiosyncratic across banks peer discipline operates through the endogenous peer selection mechanism, while with perfect correlation the threat of market exclusion is the only relevant mechanism. Nonetheless, the resulting optimal loss mutualization scheme has similar features: peer discipline incentives require to impose the maximum feasible level of contributions to banks who traded with a defaulter ($\tau_D = \pi_i = R - p + k$). This last result is reminiscent of the maximal punishment principle first established in Baron and Besanko (1984); nonetheless, differently from a standard auditing model, the ‘auditor’ is selected at a later stage of the game as the result of an endogenous matching process. Loss contributions $\tau_S$ negatively impact banks’ incentives to exert effort, and they are used only to pool idiosyncratic risk and avoid costly losses on investors.

The interbank stage at $t = 2$ is analogous to the one in the previous section. For a generic tuple $(\tau_S, \tau_D)$ a bank $B_i$ is willing to trade with a bank $B_j$ if and only if:

\[(1 - d_i)(1 - d_j)(\pi_i - \tau_S) + (1 - d_i)d_j(1 - \gamma)(\pi_i - \tau_D) \geq (1 - d_i)(\pi_i - \tau_S) - L\] (19)

Simplifying and rearranging:

\[d_j \leq \frac{L}{(1 - d_i)(\gamma \pi_i + (1 - \gamma)(\pi_i - \tau_D - \tau_S))}\] (20)

The expression in equation (20) is analogous to the one in (3), but it additionally features the effect of $\tau_D$ and $\tau_S$: the former decreases the denominator as it increases the likelihood to reject to trade with a risky bank, while the latter has the opposite effect. Moreover, the assortative matching result continues to hold as safe banks find mutually convenient to trade among themselves.

### 5.1 Idiosyncratic credit statuses

If credit risk statuses are idiosyncratic, at $t = 2$ there is always a strictly positive measure $\alpha > 0$ of risky banks, although in equilibrium all banks exerted effort at $t = 1$. In turn, only the endogenous peer selection operates as a disciplining device. Peer discipline operates because a shirking bank will trade at $t = 2$ only with a risky counter party, increasing the chances to
default as well (with probability $\gamma$) or to pay a loss contribution $\tau_D$. In this setup the threat of exclusion is not a relevant disciplining device, and it is convenient to restrict attention to a distribution $G(\cdot)$ which is degenerate around a single value $x > 0$. Indeed, this additional assumption allows to derive explicit analytical solutions without affecting the underlying economic mechanism in place, as the particular value of $x$ does not affect the resulting assortative matching outcome.

In this setup with a unit measure of agents, the individual probabilities to default ($y_L$), to survive and trade with a defaulting bank ($y_D$), and to survive and trade with another non-defaulting bank ($y_S$), are equal to the share of banks in the population. Taking into account the endogenous assortative matching outcome at $t = 2$:

\[
y_L = \alpha x [1 + \gamma (1 - x)] \quad y_D = \alpha x (1 - x) (1 - \gamma) \quad y_S = 1 - \alpha x (2 - x)
\]  

(21)

I consider the possibility to impose transfers $\tau_S$ and $\tau_D$ on surviving banks up to the amount of resources available to them at $t = 3$, i.e. $\tau_S, \tau_D \leq R - p + k$.

I first solve for the optimal loss mutualization mechanism. This scheme is implementable when a central planner can directly choose $p$ and $k$ for all banks or, alternatively, when it can condition transfers $\tau_S$ and $\tau_D$ on the tuple $(p, k)$ offered by each bank to its investors. In either case, the optimal tuple $(p, k, l, \tau_S, \tau_D)$ solves the following concave program:

\[
\begin{align*}
\max_{p, k, l, \tau_S, \tau_D} & \quad (1 - y_L)(R - p + k) - \mu k - c - y_S \tau_S - y_D \tau_D \\
\text{s.t.} & \quad R - p + k \geq \frac{c + (1 - \alpha) x [(2 - x) \tau_S - (1 - x)(1 - \gamma) \tau_D]}{(1 - \alpha) x [1 + \gamma (1 - x)]} \quad \text{(IC)} \\
& \quad p - y_L b(l) \geq I \quad \text{(IR)} \\
& \quad y_S \tau_S + y_D \tau_D \geq y_L (p - k - l) \quad \text{(BB)} \\
& \quad p \geq 0 \quad k \geq 0 \quad l \in [0, p] \quad \tau_S, \tau_D \in [0, R - p + k]
\end{align*}
\]  

(22)

If banks were not subject to a moral hazard problem—i.e. without the IC constraint—it is immediate to realize every positive NPV project (i.e. $(1 - y_L)R - I - c \geq 0$) could be financed with $p = I$, $k = 0$, $l = 0$, $\tau_S = \tau_D = \frac{y_L I}{1 - y_L}$. The loss mutualization scheme would be implemented only because it provides investors with full insurance against the idiosyncratic risk that a bank defaults.\footnote{If investors were risk neutral, i.e. $b(l) = l$, and effort contractible, the loss mutualization scheme does not improve the allocation because no insurance is desirable and no incentives should be indirectly provided.} In particular, it is not relevant to set $\tau_S$ and $\tau_D$ differently because effort is directly contractible. This conclusion may no longer be valid if effort is not contractible, as
stated in the following proposition.

**Proposition 5.1** In the presence of moral hazard, the optimal loss mutualization scheme is:

1. If $\text{NPV} \geq \frac{1-x(2-x)}{(1-\alpha)x(2-x)}c$ then:
   \[ p^* = I \quad k^* = 0 \quad l^* = 0 \]
   and $\tau^*_S, \tau^*_D$ satisfy IC weakly and BB strictly.

2. If $\frac{1-x(2-x)}{(1-\alpha)x(2-x)}c > \text{NPV} \geq \frac{\mu - 1 - x(2-x)}{\mu (1-\alpha)x(2-x)}c$ then:
   \[ p^* = I \quad k^* = \frac{1-x(2-x)}{(1-\alpha)x(2-x)}c - \text{NPV} \quad l^* = 0 \quad \tau^*_D = R - p^* + k^* \quad \tau^*_S = \tau^*_D - \frac{c}{(1-\alpha)x(2-x)} \]

3. If $\text{NPV} < \frac{\mu - 1 - x(2-x)}{\mu (1-\alpha)x(2-x)}c$ no investment takes place.

When the net present value of the investment is sufficiently high the first best outcome is implementable (item 1). There are infinitely many combinations $\tau^*_S, \tau^*_D$ that satisfy the IC constraint and cover all remaining losses. In particular, it is always an admissible solution to first cover losses with contributions $\tau_D$ from banks who traded with defaulters, and to raise additional contributions $\tau_S$ from other banks in order to cover all remaining losses. The most interesting case arises when providing incentives is costly, i.e. when the incentive compatibility constraint is binding (item 2). Investors should not bear any loss ($l^* = 0$) and they require a risk free repayment ($p^* = I$): losses should be first covered with the collateral $k^*$ posted by defaulters, and all remaining losses $p^* - k^*$ should be paid by other surviving banks: those who previously traded with a defaulter should be fully expropriated of their final returns ($\tau^*_D = R - p^* + k^*$), while banks who traded with other surviving peers should pay less ($\tau^*_S < \tau^*_D$). The loss contribution $\tau^*_D$ relaxes banks’ incentive compatibility constraint, but the transfer $\tau_S$ worsens banks’ agency problem by reducing the final payoff when a bank is safe; hence, $\tau_S$ is raised only to avoid imposing losses on investors, who would otherwise require a higher repayment to compensate for the default risk. Interestingly, it is never optimal to impose losses on investors when their marginal utility $b'(l)$ is higher than one, i.e. for every $l > 0$. Full insurance on investors is feasible because the total returns from successful projects $(1 - y_L)R$ are higher than the total amount of debt to reimburse (i.e. $p^* = I$) since $\text{NPV} = (1 - y_L)R - I - c \geq 0$. The risk insurance motive to protect investors from losses has been already pointed out in Biais et al. (2012) and Antinolfi et al. (2014). However, in my model investors do not play any direct role in disciplining banks, hence exposure to banks’ default risk does not provide any useful monitoring incentive. On the contrary, a large loss contribution $\tau_D$ provides peer discipline.
incentives by exploiting the superior information held by banks on their peers. Lastly, when the net present value is too low (item 3), no investment is undertaken because the incentive compatible contract would make the implementation of the project unprofitable.

The optimal loss mutualization scheme prescribes to impose the highest possible penalty $\tau^*_D = R - p^* + k^*$ only to banks that traded with defaulters. The loss contribution $\tau^*_D$ discourages shirking because risky banks trade with other risky banks, and, unless they default, they are more likely to pay this higher penalty. If a counter party defaults, the trading partner not only receives a zero payoff when there is contagion (with probability $(1 - x)x\gamma$), but also when it survives, since the penalty $\tau^*_D$ reduces its final payoff to zero (full expropriation). It is no longer crucial for peer discipline to have a higher contagion probability $\gamma$, because $\tau^*_D$ achieves the same incentive effects even without contagion risk.

The scheme in Proposition 5.1 relies on the assumption that the central planner can control, directly or indirectly, the contractual terms between banks and their investors. In particular, it can make banks’ participation to the loss mutualization scheme conditional on posting an amount of collateral $k^* = \frac{1-x(2-x)}{(1-\alpha)(2-x)}c - NPV$. Pre-funded resources play a fundamental role for three reasons: they increase banks’ skin in the game by raising the final payoff in non-default states; they reduce losses to investors upon default; and they increase the maximum feasible amount of ex post loss contributions $\tau_S$ and $\tau_D$.

If a bank could privately set $p$ and $k$ without affecting the reimbursement $p^* - k^*$ offered by the scheme to investors, and entirely funded by surviving banks, then it is no longer possible to implement the optimal loss mutualization scheme. Indeed, each bank would have a profitable deviation if it offers a strictly lower amount of collateral to its investors, and it shirks at $t = 1$. Investors anticipate the higher risk of the contract proposed by a bank that will subsequently shirk, and they demand a higher return. Nonetheless, the partial insurance $p^* - k^*$ offered by the loss mutualization scheme reduces the extent to which investors demand compensation for the higher risk, and it makes this deviation profitable. In other words, the loss mutualization scheme is a public good, but banks may free ride on it when the scheme cannot be made contingent on the private contracts offered to investors.

The following proposition provides a formal result of this intuition.

**Proposition 5.2** Let $\frac{1-x(2-x)}{(1-\alpha)x(2-x)}c > NPV \geq \frac{\mu - 1}{\mu} \frac{1-x(2-x)}{(1-\alpha)x(2-x)}c$. If $k$ can be observed only by investors, the optimal scheme in Proposition 5.1 is not a subgame perfect Nash equilibrium.

Proposition 5.2 points out that it is crucial to have an effective minimum capital/collateral regulation as a preliminary step to put in place a welfare enhancing loss mutualization scheme.
In the absence of minimum requirements, individual strategic incentives to play opportunistically would prevent a Pareto efficient outcome, similarly to what happens in a classic Prisoners’ dilemma game. Compulsory contributions to pre-funded pool of resources are often seen as beneficial because they provide a readily available pool of resources in case of a bank default. The result in Proposition 5.2 stresses a different role of a minimum collateral/capital requirement insofar it also creates the necessary conditions to support the existence of a loss mutualization scheme.

It is worth noting that a loss mutualization mechanism further strengthens the complementarity between collateral posted to investors and peer discipline. As before, by increasing the final payoff, posting more collateral at $t = 0$ enhances peer discipline as it makes safe banks less willing to trade with risky banks at $t = 2$. Moreover, posting more collateral reduces the loss suffered by investors in case of default and, as a consequence, it may reduce the ex post transfer from surviving banks not connected with any defaulter (i.e. $\tau_3$), which harms peer discipline because it reduces surviving banks’ final payoff. Lastly, higher collateral requirements increase the amount of ex post transfers available (since $\tau_5, \tau_D \leq R - p + k$); as a result, banks have more incentives to exert effort to increase the chances to match with another safe bank and avoid the higher ex post loss contribution $\tau_D$.

5.2 Perfectly correlated credit statuses

When credit risk shocks are perfectly correlated, it is convenient to think about two aggregate states, H and L, that realize with probability $1 - \alpha$ and $\alpha$ respectively. If a bank exercises effort it is safe in state H and risky in state L. To simplify exposition, I assume each bank defaults with probability $x > 0$ if state L realizes at $t = 3$. If a bank shirks, it is risky also in state H, and its default probability is drawn from a continuous distribution $G(\cdot)$ on $[0, 1]$ with expected value $m$. In the case of perfectly correlated statuses, it is possible to condition ex post loss contributions on the aggregate state. Let $(\tau^H_D, \tau^L_D)$ and $(\tau^H_S, \tau^L_S)$ be the loss contributions imposed on banks in state H or L, depending on whether their trading counter party defaulted or not respectively.

Suppose all other banks exert effort at $t = 1$. For a given $\pi = (\pi_i, \pi_j)$, bank $B_i$’s expected
payoff is:

\[
\begin{align*}
\mathbb{E}_{t-1}[u_t | \pi] &= (1 - \alpha)(R - p + k - \tau_S^H) + \alpha(1 - x)((1 - x)(R - p + k - \tau_S^H) + x(1 - \gamma)(R - p + k - \tau_D^H)) - c \\
\mathbb{E}_{t=0}[u_t | \pi] &= (1 - \alpha) \left[ (1 - m)(R - p + k - \tau_S^H) - \left[ 1 - G \left( \frac{L}{\varepsilon \gamma (1-\gamma) \tau_D^H - \tau_S^H} \right) \right] L \right] \\
&+ \alpha(1 - x)((1 - x)(R - p + k - \tau_S^H) + x(1 - \gamma)(R - p + k - \tau_D^H)) - c
\end{align*}
\] (25)

As previously pointed out, a bank always trades after exerting effort because at \( t = 2 \) all banks have identical credit risk, either all safe or all risky. If instead a bank shirks, then with probability \( 1 - \alpha \) state \( H \) realizes, all other banks are safe, and another bank accepts to trade with a shirking bank only if the realized credit risk is below \( \frac{L}{\varepsilon \gamma (1-\gamma) \tau_D^H - \tau_S^H} \); see equation (20). From the above expressions it is immediate to get the incentive compatibility constraint. The optimal loss mutualization scheme solves:

\[
\begin{align*}
\max_{p,k,l,\{\tau_S^H, \tau_D^H\} \neq H,L} & \quad [1 - \alpha x][1 + \gamma (1 - x)][(R - p + k) - \mu k - c - (1 - \alpha) \tau_S^H] \\
&- \alpha[1 - x(2 - x)] \tau_S^H - \alpha x (1 - x) (1 - \gamma) \tau_D^H \\
\text{s.t.} & \quad R - p + k \geq \frac{c + (1 - \alpha) m \tau_S^H - \left[ 1 - G \left( \frac{L}{\varepsilon \gamma (1-\gamma) \tau_D^H - \tau_S^H} \right) \right] L}{(1 - \alpha) m} \quad \text{(IC)} \\
& \quad p - \alpha x[1 + \gamma (1 - x)] b(l) \geq l \quad \text{(IR)} \\
& \quad [1 - x(2 - x)] \tau_S^H + x(1 - x)(1 - \gamma) \tau_D^H \geq x[1 + \gamma (1 - x)](p - k - l) \quad \text{(BB)} \\
& \quad p \geq 0 \quad k \geq 0 \quad l \in [0, p] \quad \tau_S^H, \tau_D^H \in [0, R - p + k] \quad \theta = H, L
\end{align*}
\] (26)

The program maximizes the expected profit of a bank, net of the loss mutualization contributions. Notice that in equilibrium every bank exerts effort and no bank defaults in state \( H \), so \( \tau_D^H \) does not appear in the objective function. However, it does appear in the incentive compatibility constraint. The investors’ individual rationality (IR) constraint takes into account the potential loss \( l \) in case of default when state \( L \) realizes. The budget balance (BB) constraint for the loss mutualization mechanism is a simple accounting identity. The following proposition summarizes the main results.

**Proposition 5.3** The optimal loss mutualization mechanism satisfies:

1. If \( 1 - x[1 + \gamma (1 - x)] R - l \geq 0 \) surviving banks cover all losses from defaulting banks in state \( L \).
• If the condition
  \[ R - I \geq \frac{c - \left[ 1 - G \left( \frac{l}{R - I} \right) \right] L}{(1 - \alpha)m} \]  
  (27)
is satisfied then \( p^* = I, k^* = 0, l^* = 0 \) with loss mutualization transfers satisfying:

  \[ \tau^H_S = 0 \quad \tau^H_D = R - I \quad [1 - x(2 - x)]\tau^H_S + x(1-x)(1-\gamma)\tau^H_D = x[1 + \gamma(1-x)]I \quad \tau^H_S, \tau^H_D \leq R - I \]  
  (28)

  In particular, it is always admissible to set \( \tau^H_S = \tau^H_D = \frac{x[1 + \gamma(1-x)]}{1 - x[1 + \gamma(1-x)]}I \).

• If equation (27) is not satisfied then \( p^* = I, l^* = 0 \) and \( k^* \) solves:

  \[ R - I + k^* = \frac{c - \left[ 1 - G \left( \frac{l}{R - I + k^*} \right) \right] L}{(1 - \alpha)m} \]  
  (29)

  The project is undertaken only if the net present value is sufficiently high.\(^{15}\)

2. If \( 1 - x[1 + \gamma(1-x)] \) \( R - I < 0 \) surviving banks cannot cover all losses in state \( L \).

Let \( l_1, l_2, l_3 \) be the unique solutions to:

  \[ x[1 + \gamma(1-x)](l_1 - \alpha b(l_1)) = I - [1 - x][1 + \gamma(1-x)]R \]

  \[ b'(l_2) = \frac{1}{\mu} \left[ 1 + \frac{\mu - 1}{\alpha} \right] \]  
  \[ x[1 + \gamma(1-x)](R - l_3) = \frac{c - \left[ 1 - G \left( \frac{l}{1 + \gamma(1-x)} \right) \right] L}{(1 - \alpha)m} \]  
  (30)

An incentive compatible loss mutualization scheme is feasible only if \( l_3 \geq 0 \). In this case, it is:

  \[ \tau^H_S = 0 \quad \tau^H_D = \tau^D_S = \tau^D_D = R - p^* + k^* \]  
  (31)

where \( p^*, k^*, l^* \) satisfy \( l^* = \min\{l_1, l_2, l_3\} \), \( p^* = I + \alpha x[1 + \gamma(1-x)]b(l^*) \) and

  \[ k^* = x[1 + \gamma(1-x)](\alpha b(l^*) - l^*) + I - [1 - x][1 + \gamma(1-x)]R \]  
  (32)

The project is undertaken if and only if \( (1 - \alpha)(R - p^* + k^*) - \mu k^* - c \geq 0 \). Moreover, the solution coincides with the one available when effort is contractible only if \( l_3 \geq \min\{l_1, l_2\} \).

When credit risk shocks are perfectly correlated, the loss mutualization mechanism affects the effort choice only through the loss contributions in state \( H \); although in equilibrium \( \tau^H_S \) and \( \tau^H_D \) are never paid, they have an important role to deter a deviation. The optimal loss allocation

\(^{15}\)See the proof in the Appendix for a precise characterization.
resembles, in an extreme form, the one in section 5.1: banks that traded with a defaulter should contribute as much as possible \((\tau^H_D = R - p^* + k^*)\), while other surviving banks should not bear any loss \((\tau^H_S = 0)\).\(^{16}\) In state L loss transfers do not affect effort incentives, hence they are used only to cover investors’ losses, and both \(\tau^L_S\) and \(\tau^L_D\) are set to the maximum feasible level \((R - p^* + k^*)\), unless all banks have collectively a final return sufficient to cover all losses from defaulting banks. In the latter case—which requires \([1 - x(1 + \gamma(1 - x))]R - I \geq 0\)—investor do not incur any loss as \(l^* = l_1 = 0\). If instead all surviving banks do not have enough funds to cover all losses in state L—i.e. \([1 - x][1 + \gamma(1 - x)])R - I < 0\)—investors have to incur a loss \(l^* > 0\). This loss is optimally set taking into account the relative cost between increasing the loss \(b(l)\) to investors, hence increasing the repayment \(p\) at \(t = 3\), and raising additional collateral \(k\) in order to reduce investors’ losses in state L. If \(l_1 \leq \min\{l_2, l_3\}\) it is optimal not to raise any collateral, neither to insure investors nor to provide effort incentives to banks. If instead \(l_1 > \min\{l_2, l_3\}\) and \(l_3 \geq 0\) then an incentive compatible scheme is possible with \(k^* > 0\): if \(l_2 < l_3\) collateral is used only to provide partial insurance to investors, while if \(0 < l_3 < l_2\) collateral is posted to raise banks’ skin in the game and enhance the incentive to exert effort at \(t = 1\). This scheme is effectively implemented if only if the expected profit is positive, and the condition \((1 - \alpha)(R - p^* + k^*) - \mu k^* \geq 0\) is equivalent to this requirement. Lastly, the moral hazard problem prevents to finance the investment—despite the benefits of loss mutualization in terms of investors’ insurance in state L—when \(l_3 < 0\) because final returns for banks are not enough to provide the necessary incentives to exert effort.\(^{17}\)

5.3 Implementation challenges

The model provides a straightforward policy implication for the design of a loss sharing scheme: losses should be first paid by the banks that previously traded with the defaulter, and its amount should be as high as possible, i.e. the model suggests to expropriate all profits. A first implication of this result is that another financial institution should not be unlimitedly liable for losses to outside investors. A loss mutualization scheme has to foster peer discipline by sufficiently penalize a bank who traded with a defaulter, but it should not trigger a new default, also because a regulator may not find a credible commitment to implement such a policy.

There are other issues related to the practical implementation of my suggested policy recommendation. A first important issue concerns the determination of the total amount that each

\(^{16}\)Intuitively, in equilibrium no default takes place in state H, hence no investor suffers a loss, and there is no reason to raise funds through \(\tau^H_D\) because it would only decrease effort incentives.

\(^{17}\)For example, if \([1 - x][1 + \gamma(1 - x)])R - I = 0\) and \([1 - \alpha x[1 + \gamma(1 - x)])R - I - c = 0\) (i.e. \(NPV = 0\)), then \(c = (1 - \alpha)x[1 + \gamma(1 - x)]R\) and it is immediate to verify that the IC constraint is always violated for \(l_3 \geq 0\).
bank has to disburse. In my static model the definition of total profits is easily applicable; in a
dynamic setting it is unrealistic to punish a bank for an amount equal to its total market value,
because it would create obvious debt overhang problems in the future business conduct. As a
result, default contributions should be such as banks incorporate the additional cost of trading
with a risky counter party, but it should not exacerbate too much future agency problems. Strik-
ing a balance between these two opposite forces is a challenging task, but this difficult trade-off
is not a reason to ignore any market discipline consideration in the distribution of losses beyond
the defaulter’s initial contributions.

A second issue concerns the distribution of losses among the set of financial institutions
with previous trading relationships with the defaulter. For tractability, I restrict attention to
bilateral interactions in the interbank market, but in reality each financial institution trades
with a large number of other financial institutions, and in a variety of financial instruments and
maturities. It is an open question how these losses should be distributed among different trading
counter parties in order to maximize peer discipline.

A third issue relates to the timing of the disbursement of loss sharing contributions. If de-
fault contributions are large, it is extremely unlikely that a financial institution may pay this
sum promptly to immediately cover losses. This requirement would trigger additional liquidity
issues for surviving banks, especially during an episode of market turmoil. To cope with this
issue, the design of the new bank resolution fund in Europe and in the US adopt two different
strategies: in Europe the financial industry has to contribute in advance and additional contri-
butions may be required at the time of a bank default; in the US the Treasury should extend
credit in advance to the resolution authority, and banks are expected to reimburse later. A CCP
has a more complex system which goes under the name of ‘default waterfall’ (see Elliott (2013)
for a detailed explanation). Current default waterfall envisage to first use pre-funded resources
and later to call members to contribute with additional funds (rights of assessment). Moreover,
CCPs are developing sophisticated rules to rapidly run successful auctions to allocate the posi-
tions of the defaulter to other clearing members. As the CCP default waterfall highlights, it is
a complex task to balance the requirements of closing out positions rapidly without impairing
the liquidity position of other members. The most convenient way to tackle this issue is an
important future research question.

5.4 Information acquisition

I extend the model to include a decision to acquire costly information on the default probabili-
ties of other banks. After exerting effort, but before default probabilities realize, each bank can
pay a cost \( c_d > 0 \) to observe the default probabilities of other banks.

I first consider the situation in which the project is entirely self-financed and it is optimal to have all banks collect this information. If all banks observe the default probabilities of every other bank, the equilibrium outcome displays assortative matching. Therefore, if all banks collect information on other banks the equilibrium expected payoff for a bank is:

\[
E_{e_i=1}[u_i|R,\text{info}] = \left[ 1 - \alpha(m + \gamma \int_{0}^{1} x(1-x)g(x) \, dx) \right] R - I - c - c_d \tag{33}
\]

If instead no bank collects information, it is plausible to assume banks match uniformly at random because there is no possible action to signal their credit risk. Let \( n_s \) be the share of safe banks. I denote with \( \mathbb{E}[n_s|d_i] \) the belief to match with a safe bank if bank \( i \) has default probability \( d_i \).

It is optimal to have banks acquire information if

\[
E_{e_i=1}[u_i|R,\text{info}] \geq E_{e_i=1}[u_i|R,\text{no info}].
\]

Simplifying and rearranging:

\[
c_d \leq \left( (1 - \alpha)(1 - \mathbb{E}[n_s|d_i = 0])m - \alpha \mathbb{E}[n_s|d_i > 0] \int_{0}^{1} x(1-x)g(x) \, dx \right) \gamma R \tag{35}
\]

Clearly, it is never optimal to collect information on other banks when credit statuses are perfectly correlated, because in equilibrium every bank has the same credit risk. In this case, the RHS of equation (35) is zero since \( \mathbb{E}[n_s|d_i = 0] = 1 \) and \( \mathbb{E}[n_s|d_i > 0] = 0 \). If instead shocks are idiosyncratic, \( \mathbb{E}[n_s|d_i] = 1 - \alpha \) for every \( d_i \), and equation (35) becomes \( c_d \leq \alpha(1 - \alpha)\gamma R \int_{0}^{1} x^2 g(x) \, dx \).

In the reminder, I show that the optimal loss mutualization design increases the incentives to collect information on other banks. Indeed, if the project is not self financed, the incentives to acquire information may not coincide with the first best, and an additional incentive compatibility constraint on information acquisition arise. In this respect, because of the endogenous matching among banks, I have to carefully model what the possible matching outcomes are for a single bank which does not to acquire the costly technology, given that all other banks are

\[\text{The expectation is taken with respect to the data generating process of the correlated credit risk statuses across banks.}\]
informed. Let \( j \in \{ss, sr, rs, rr\} \) denote the possible matching pairs for a bank, where the first letter indicates its credit risk (\( s \) for safe, \( r \) for risky), and the second letter the one of its trading partner. I denote with \( \tilde{p}_j \) the out-of-equilibrium probability of a matching \( j \in \{ss, sr, rs, rr\} \) when a bank does not acquire information on other banks, conditional on all other banks acquiring information. I construct an explicit micro foundation of the matching process in section 7.3 of the Appendix. For the sake of this section, I first derive the incentive compatibility constraint keeping \( \tilde{p}_j \) unspecified.

Suppose all banks have a final payoff \( \pi \) and consider a scheme \((\tau_S, \tau_D)\). If a bank does not acquire information on other bank's credit risk, it can either (i) always accept to trade, or (ii) process. Under the plausible assumption that \( \alpha \) on this case as it is always satisfied for \( \gamma > 0 \) sufficiently small.

In case (ii) the expected payoff is

\[
\tilde{p}_{ss}(\pi - \tau_S) + \tilde{p}_{rr} \int_0^1 (1-x)(\pi - \tau_S + x(1-\gamma)(\pi_i - \tau_D))g(x) \, dx + \\
+ \tilde{p}_{rs} \left[ (1-m)(\pi - \tau_S) - \left[ 1 - G\left( \frac{L}{\gamma \tau + (1-\gamma)\tau_D - \tau_S} \right) \right] L \right] + \tilde{p}_{sr} [(1-m)(\pi - \tau_S) + m(1-\gamma)(\pi - \tau_D)] - c
\]

(36)

In case (ii) the expected payoff is

\[
\tilde{p}_{ss}(\pi - \tau_S - L) + \tilde{p}_{rr} \int_0^1 (1-x)[(\pi - \tau_S + x(1-\gamma)(\pi_i - \tau_D))g(x) \, dx + \\
+ \tilde{p}_{rs} \left[ (1-m)(\pi - \tau_S) - \left[ 1 - G\left( \frac{L}{\gamma \tau + (1-\gamma)\tau_D - \tau_S} \right) \right] L \right] + \tilde{p}_{sr} [\pi - L - \tau_S] - c
\]

(37)

The first deviation is more profitable if and only if \( \frac{\tilde{p}_{ss} + \tilde{p}_{rr}}{\tilde{p}_{rr}} m = \frac{\tilde{p}_{sr}}{1-\alpha} m. \)

I focus on this case as it is always satisfied for \( \alpha \) sufficiently small.

Therefore, acquiring information is incentive compatible if and only if:

\[
c_d \leq \left[ \tilde{p}_{sr} m + (\tilde{p}_{rr} - \alpha) \int_0^1 x(1-x)g(x) \, dx \right] \left( \gamma \pi + (1-\gamma)\tau_D - \tau_S \right) + \tilde{p}_{rs} \left[ 1 - G\left( \frac{L}{\gamma \pi + (1-\gamma)\tau_D - \tau_S} \right) \right] L
\]

(38)

In section 7.3 of the Appendix I describe a plausible micro foundation of the matching process. Under the plausible assumption that \( \alpha < \frac{1}{2} \), I show that \( \tilde{p}_{sr} = (1-\alpha)(1-\mathbb{E}[n_s|d_i = 0]) \), \( \tilde{p}_{rr} = \alpha \) and \( \tilde{p}_{rs} = 0 \). In other words, if a bank does not acquire the information technology, it has strictly positive probability to match with a risky counter party when safe, while this is not the case when it pays the cost and observes the credit risk of other banks. The RHS of equation (38) becomes:

\[
c_d \leq (1-\alpha)(1-\mathbb{E}[n_s|d_i = 0]) m(\gamma \pi_i + (1-\gamma)\tau_D - \tau_S)
\]

(39)

\footnote{Notice that \( \tilde{p}_{ss} + \tilde{p}_{rr} = 1-\alpha \) and \( \tilde{p}_{rs} + \tilde{p}_{ss} = \alpha \) always holds.}
This incentive compatibility constraint is relaxed when $\tau_D$ is as large as possible and $\tau_S$ as little as possible. Therefore, allocating a disproportionate amount of losses to banks who traded with defaulters—as suggested by the results in section 5—also maximize the incentives to collect information on other banks.

6 Conclusion

In this article I study peer monitoring incentives and its relationship with loss sharing schemes. An optimal design should take into account banks’ superior information, and reduce the use of costly commitment devices such as initial capital or collateral posted. Interbank markets introduce two main disciplining mechanisms. First, becoming risky may lead to be excluded by other (safe) banks, imposing an additional cost when running the project in autarchy. Second, there is endogenous peer selection, and a hedge with a safe bank reduces contagion risk. I analyze when each of these two mechanisms dominates. The threat of market exclusion is more effective when banks’ credit shocks are highly correlated, for instance due to a macroeconomic shock, while avoiding to trade with a risky bank is relevant when shocks are predominantly idiosyncratic.

Both disciplining mechanisms can be made more effective with a well designed loss sharing scheme. Importantly, to maximize peer monitoring incentives, a higher share of losses should be covered by those members with closer interlinkages with defaulters. Ignoring previous trade patterns may harm peer monitoring incentives, and require higher initial margins to restore good incentives. This mechanism has practical implications for the design of a clearinghouse default waterfall or in the newly established bank resolution funds.

In future research, the model could be extended in several directions. In particular, a more realistic modelling of the interactions in the interbank market should lead to include several financial instruments and an endogenous determination of contagion risk.
7 Appendix

7.1 Baseline model

Proof Proposition 4.1. In the optimum the IR constraint binds. Substituting $p_i$ into the objective function and the IC constraint:

$$\max_{k_i \geq 0} (1 - \alpha m) \left( R - \frac{I - k_i}{1 - \alpha m} \right) - \mu k_i - c$$

s.t. $R - \frac{I - k_i}{1 - \alpha m} \geq \frac{\mu}{(1 - \alpha) m}$ (IC)

(40)

The first best contract is incentive compatible only if $k_i = 0$, i.e.:

$$(1 - \alpha m) R - I - c = \text{NPV} \geq \frac{1 - m}{(1 - \alpha) m} c$$

(41)

If $\text{NPV} < \frac{1 - m}{(1 - \alpha) m} c$ the first best contract is not implementable and collateral $k_i$ must be used to satisfy equation (1). An incentive compatible contract requires to post at least an amount of collateral:

$$k_i = \frac{1 - \alpha m}{(1 - \alpha) m} c - [(1 - \alpha m) R - I] = \frac{1 - m}{(1 - \alpha) m} c - \text{NPV}$$

(42)

Substituting equation (42) in the objective function, it is profitable to invest only if:

$$(1 - \alpha m) R - I - c = \text{NPV} \geq \frac{\mu - 1}{\mu} \frac{1 - m}{(1 - \alpha) m} c$$

(43)

Proof of Proposition 4.2. In a symmetric equilibrium it holds $\pi_i = \pi_j$ for every bank $i, j$, with $i \neq j$. In equilibrium all banks offer the minimum to investors, i.e. the IR constraint binds. Thanks to this constraint, it is easy to substitute an expression for $p_i$ into the profit function and the IC constraint. The problem becomes:

$$\max_{k_i \geq 0} s R - I - (\mu - 1) k_i - c$$

s.t. $s R - I + k_i \geq s \xi (\pi_i)$ (IC)

(44)

The optimal contract minimizes the amount of collateral $k_i$ necessary to satisfy the IC constraint. The first best solution has $k_i = 0$ and $p = \frac{I}{s}$ (so $\pi = R - \frac{I}{s}$) and it is implementable if and only if:

$$s R - I \geq \xi \left( R - \frac{I}{s} \right)$$

(45)

or $\text{NPV} \geq \xi \left( R - \frac{I}{s} \right) - c$. If this inequality is violated, collateral has to be used to satisfy the
IC constraint. As $k_i$ decreases utility at a rate $\mu - 1$, the optimal choice is to set it as low as possible, i.e. to satisfy the IC constraint with equality:

$$k_i = s\xi(\pi) - (sR - I)$$  \hspace{1cm} (46)$$

where $\pi$ is the unique value that satisfies $\pi = \xi(\pi)$ ($\xi(\pi)$ is weakly decreasing in $\pi$). Substituting it into the profit function and rearranging, the investment is profitable if and only if:

$$sR - I - c = NPV \geq \frac{\mu - 1}{\mu} [s\xi(\pi) - c]$$  \hspace{1cm} (47)$$

### 7.2 Loss mutualization scheme

**Proof Proposition 5.1.**

Before solving the FOCs of the Lagrangean, I derive the incentive compatibility constraint. Conditional on all other banks exerting effort, the expected payoff from exerting effort for a bank $i$ with final payoff $\pi = R - p + k$ is:

$$E_{e_i=1}[\pi] = (1 - \alpha)(\pi - \tau_S) + \alpha(1 - x)((1 - x)(\pi - \tau_S) + x(1 - \gamma)(\pi - \tau_D)) - c$$

$$= [1 - \alpha x(1 + x(1 - \gamma))]\pi - [1 - \alpha x(2 - x)]\tau_S - \alpha(1 - x)x(1 - \gamma)\tau_D - c$$  \hspace{1cm} (48)$$

If the bank shirks it gets an expected payoff equal to:

$$E_{e_i=0}[\pi] = (1 - x)[(1 - x)(\pi_i - \tau_S) + x(1 - \gamma)(\pi - \tau_D)]$$  \hspace{1cm} (49)$$

A bank exercises effort only if $E_{e_i=1}[\pi] \geq E_{e_i=0}[\pi]$. Simplifying the expression and substituting $\pi = R - p + k$, I get the incentive compatibility constraint:

$$R - p + k \geq \frac{c + (1 - \alpha)x[(2 - x)\tau_S - (1 - x)(1 - \gamma)\tau_D]}{(1 - \alpha)x[1 + \gamma(1 - x)]}$$  \hspace{1cm} (50)$$
The Lagrangean of this concave maximization problem is:
\[
\max_{p,k,l,\tau_5,\tau_D,\{\lambda_i\}} \mathcal{L} = \left[1 - \alpha x(1 + \gamma (1 - x))\right] (R - p + k) - \mu k - c - \left[1 - \alpha x(2 - x)\right] \tau_5 - \alpha x(1 - x)(1 - \gamma) \tau_D + \lambda_1 \left[ R - p + k - \frac{c(1 - \alpha x)(2 - x) - (1 - x)\tau_5 - (1 - x)(1 - \gamma) \tau_D}{(1 - \alpha x)(1 + \gamma (1 - x))} \right] + \lambda_2 \left[ p - \alpha x[1 + \gamma (1 - x)]\tau_5 - \alpha x[1 + \gamma (1 - x)]\tau_D - \alpha x[1 + \gamma (1 - x)](p - k - l) \right] + \lambda_4 \tau_5 + \lambda_5 \tau_D + \lambda_6 (R - p + K - \tau_5) + \lambda_7 (R - p + k - \tau_D) + \lambda_8 l + \lambda_9 (p - l) + \lambda_{10} k
\]

(51)

The first-order conditions of the Lagrangean with respect to the policy variables are sufficient for the global maximum.

\[
\frac{\partial \mathcal{L}}{\partial p} = -1 + \alpha [1 + \gamma (1 - x)] - \lambda_2 - \lambda_3 \alpha x[1 + \gamma (1 - x)] - \lambda_6 - \lambda_7 + \lambda_9 = 0 \quad \text{(i)}
\]
\[
\frac{\partial \mathcal{L}}{\partial k} = 1 - \alpha [1 + \gamma (1 - x)] + \lambda_7 - \mu + \lambda_3 \alpha x[1 + \gamma (1 - x)] + \lambda_6 + \lambda_7 + \lambda_{10} = 0 \quad \text{(ii)}
\]
\[
\frac{\partial \mathcal{L}}{\partial \tau_5} = -[1 - \alpha x(2 - x)] - \lambda_4 \frac{2 - x}{1 + \gamma (1 - x)} + \lambda_3 [1 - \alpha x(2 - x)] + \lambda_4 - \lambda_6 = 0 \quad \text{(iii)}
\]
\[
\frac{\partial \mathcal{L}}{\partial \tau_D} = -\alpha x(1 - x)(1 - \gamma) + \lambda_4 \frac{(1 - x)(1 - \gamma)}{1 + \gamma (1 - x)} + \lambda_3 \alpha x(1 - x)(1 - \gamma) + \lambda_5 - \lambda_7 = 0 \quad \text{(iv)}
\]
\[
\frac{\partial \mathcal{L}}{\partial l} = -\lambda_2 b'(l) \alpha x[1 + \gamma (1 - x)] + \lambda_3 \alpha x[1 + \gamma (1 - x)] + \lambda_8 - \lambda_9 = 0 \quad \text{(v)}
\]

From (iii) and (iv) I get:
\[
\lambda_4 = \alpha x[1 + \gamma (1 - x)][1 - \alpha x(2 - x)] \left[ \frac{\lambda_7 - \lambda_5}{\alpha x(1 - x)(1 - \gamma)} - \frac{\lambda_6 - \lambda_4}{1 - \alpha x(2 - x)} \right]
\]

(53)

Summing (i) and (ii) I get \( \lambda_2 = \mu - \lambda_{10} - \lambda_9 \), while from (v) it is \( \lambda_3 = \lambda_2 b'(l) - \frac{\lambda_8 - \lambda_9}{\alpha x[1 + \gamma (1 - x)]} \).

I use a guess and verify method to characterize the solutions of this optimization problem.

1. Consider the case in which \( \lambda_1 = 0 \), i.e. the IC constraint is not binding, and suppose \( \lambda_4 = \lambda_5 = \lambda_6 = \lambda_7 = 0 \). From (iii) or (iv) it follows that \( \lambda_3 = 1 \). Substituting this value in (i) it must hold \( \lambda_3 + \lambda_9 = \mu - \lambda_{10} = 1 \), hence \( \lambda_{10} > 0 \), i.e. \( k = 0 \). Moreover, substituting \( \lambda_9 = 1 - \lambda_2 \) and \( \lambda_3 = 1 \) in (v) it is
\[
1 = \lambda_2 b'(l) - \frac{\lambda_8 - 1 + \lambda_2}{\alpha x[1 + \gamma (1 - x)]}
\]

(54)
2. Suppose \( \lambda_l \) follows the values of the Lagrange multipliers in (ii) it follows: 

\[
\lambda_2 = \frac{\frac{1}{\alpha(1+x(1-x))} - 1}{\alpha(1+y(1-x))} > 1
\]  

(55)

as \( b'(l) > 1 \) for \( l > 0 \). However, \( \lambda_2 \geq 1 \) as \( \lambda_2 + \lambda_9 = 1 \) and \( \lambda_9 \geq 0 \). Therefore it must be \( l = 0 \), hence \( \lambda_2 = 1 \) and \( \lambda_8 = 0 \). Since \( \lambda_2 > 0 \) then investors’ individual rationality constraint binds and it follows that \( p = I \) as \( l = 0 \). Substituting \( p = I \) and \( k = 0 \) in the IC and BB constraint I get the admissible solutions for \( \tau_S \) and \( \tau_D \):

\[
R - I \geq \frac{c}{(1-\alpha)(2-x)} + \frac{(2-x)\tau_l - (1-x)(1-\gamma)\tau_0}{1+\gamma(1-x)} \\
[1 - \alpha x(2-x)]\tau_5 + \alpha x(1-x)(1-\gamma)\tau_D = \alpha x[1+x(1-\gamma)]I
\]  

(56)

It is easy to realize that this solution is admissible for the largest set of parameters whenever \( \tau_D = R - I \) (maximum value). Substituting this value in the BB constraint I get \( \tau_5 = \frac{-\alpha x}{1-\alpha x(2-x)}[(2-x)R - (1-x)(1-\gamma)R] \). Substituting these values in the IC constraint, it is satisfied whenever \( NPV = [1 - \alpha x(1+\gamma(1-x))]R - I - c \geq \frac{1-x(2-x)}{(1-\alpha)(2-x)}c \).

2. Suppose \( \lambda_1 > 0 \) (IC binding), \( \lambda_7 > 0 \) (i.e. \( \tau_D = R - p + k \)). From these restrictions, it follows \( \tau_5 = R - p + k - \frac{c}{(1-\alpha)x(2-x)} \), so \( \lambda_6 = 0 \) and \( \lambda_9 = 0 \). Moreover, consider \( \lambda_{10} = 0 \) (i.e. \( k > 0 \)). Summing (i) and (ii) it is \( \lambda_2 = \mu \). From (iii) it is \( \lambda_3 = \frac{1 + \lambda_1}{1-\alpha x(2-x)} \frac{2-x}{1+\gamma(1-x)} \), and substituting this result in (iv) it implies \( \lambda_7 = \frac{(1-x)(1-\gamma)}{1+\gamma(1-x)} \frac{1}{1-\alpha x(2-x)} \lambda_1 > 0 \). Substituting the values of the Lagrange multipliers in (ii) it follows:

\[
\lambda_1 = \frac{[1 - \alpha x(2-x)][1 + \gamma(1-x)]}{2-x}(\mu - 1)
\]  

(57)

Substituting back these values in the previous expressions, it is \( \lambda_3 = \mu \) and \( \lambda_7 = \frac{(1-x)(1-\gamma)}{2-x}(\mu - 1) \). Substituting \( \lambda_3 = \lambda_2 = \mu \) in (v) it is:

\[
\mu = \mu b'(l) - \frac{\lambda_8}{\alpha x[1+\gamma(1-x)]}
\]  

(58)

which is satisfied only if \( l = 0 \) and \( \lambda_8 = 0 \).

Since \( \lambda_2 > 0 \) (IR investors binding) and \( \lambda_3 > 0 \) (BB binding), then \( p = I + \alpha x[1+x(1-\gamma)] \), and substituting in BB it holds:

\[
b(l) - l = \frac{NPV + k}{\alpha x[1+\gamma(1-x)]} - \frac{1-x(2-x)}{\alpha x[1+\gamma(1-x)](1-\alpha x(2-x))C}
\]  

(59)

Since \( l = 0 \) then \( k = \frac{1-x(2-x)}{(1-\alpha)x(2-x)} c - \text{NPV} \), which is positive only if \( \text{NPV} < \frac{1-x(2-x)}{(1-\alpha)x(2-x)} c \).

It is immediate to verify that this value also implies \( \tau_5 > 0 \). Lastly, I derive when this incentive compatible solution is profitable. Substituting in the objective function all the previous optimal values, it is:

\[
E_{\alpha=1} = \frac{1-x(2-x)}{(1-\alpha)x(2-x)} c - \mu \left[ \frac{1-x(2-x)}{(1-\alpha)x(2-x)} c - \text{NPV} \right] \tag{60}
\]

which is positive only if \( \text{NPV} \geq \frac{\mu-1}{\mu} \frac{1-x(2-x)}{(1-\alpha)x(2-x)} c \). Therefore, this solution is admissible only if \( \frac{1-x(2-x)}{(1-\alpha)x(2-x)} c > \text{NPV} \geq \frac{\mu-1}{\mu} \frac{1-x(2-x)}{(1-\alpha)x(2-x)} c \). If the NPV is below this threshold no investment is undertaken.

\[
\quad
\]

Proof of Proposition 5.2.

From Proposition 5.1 \( p^* = I \) and \( k^* = A^* - \text{NPV} \) (where \( A^* = \frac{1-x(2-x)}{(1-\alpha)x(2-x)} c \)) are the equilibrium contract terms under the optimal loss mutualization scheme, and \( \tau_D = R - p^* + k^* \), \( \tau_5^* = \tau_D^* - \frac{c}{(1-\alpha)x(2-x)} \) the resulting loss mutualization transfers.

It is straightforward to realize that no deviation would provide a higher payoff if a bank continues to exert effort. Given \( \tau_D^* \) and \( \tau_5^* \), the IC constraint is satisfied only if the final payoff from the deviation, say \( \tilde{\pi} \), satisfies \( \tilde{\pi} \geq R - I + k^* \). However, this requires a higher collateral, say \( \tilde{k} > k^* \), so it would be clearly unprofitable. Therefore, let’s consider the deviation in which the bank shirks. As a result, it matches with probability one with another risky bank.

Let \( (\tilde{p}, \tilde{k}, \tilde{l}) \) be the most profitable contract a bank can offer to investors when it will subsequently shirk at \( t = 1 \) (i.e. the IC is violated) and its investors receive \( I - k^* \) from the loss mutualization mechanism upon its default. Since the IC constraint is violated, it must hold \( R - \tilde{p} + \tilde{k} < R - I + k^* = \tau_D^* \). Hence, a bank receives a positive payoff only when it survives together with its trading counter party. Since the bank is infinitesimal relative to the overall measure of banks in the economy, if it deviates it can ignore its effect on the overall budget balance of the loss mutualization mechanism. Therefore, conditional on shirking, the best contract a bank can offer to its investor solves the following Lagrangean program:

\[
\max_{p,k,l} \mathcal{L} = [1-x(2-x)](R-p+k-\tau_5^*) - \mu k + \lambda_1[p-x(1+\gamma(1-x))b(l)-l] + \lambda_2(I-k^*-p+k+l) + \lambda_3 k \tag{61}
\]
FOCs:
\[
\frac{\partial \tilde{c}}{\partial \tilde{p}} = -[1 - x(2 - x)] + \lambda_1 - \lambda_2 = 0 \quad (i)
\]
\[
\frac{\partial \tilde{c}}{\partial \tilde{k}} = 1 - x(2 - x) - \mu + \lambda_2 + \lambda_3 = 0 \quad (ii)
\]
\[
\frac{\partial \tilde{c}}{\partial \tilde{l}} = -\lambda_1 x[1 + \gamma(1 - x)]b'(\tilde{l}) + \lambda_2 = 0 \quad (iii)
\]

Suppose \( k = 0 \). By (i) and (iii) both \( \lambda_1 > 0 \) and \( \lambda_3 > 0 \) are positive. Hence, it holds \( I - k^* - p + l = 0 \) and \( p = I + x[1 + \gamma(1 - x)] \). Substituting \( p \) in the first expression it must hold that losses \( \tilde{l} \) satisfy \( \tilde{l} - x[1 + \gamma(1 - x)]b'(\tilde{l}) = k^* \), and \( \tilde{p} = I + \tilde{l} - k^* \). Substituting (iii) in (i) I get
\[
\lambda_1 = \frac{1 - x(2 - x)}{1 - x[1 + \gamma(1 - x)]b'(\tilde{l})}
\]

Substituting in (ii) it must hold:
\[
\lambda_3 = \mu - \frac{1 - x(2 - x)}{1 - x[1 + \gamma(1 - x)]b'(\tilde{l})} > 0
\]

So this solution is valid only if \( \tilde{l} < \tilde{l} \) where \( \tilde{l} \) is the value which makes \( \lambda_3 = 0 \). The expected payoff from this deviation is:
\[
[1 - x(2 - x)][R - \tilde{p} - \tau^*] = [1 - x(2 - x)]\left[\frac{c}{(1 - \alpha)\lambda(2 - x)} - \tilde{l}\right]
\]

while from Proposition 5.1 the payoff from exerting effort is \( \mu NPV - \frac{1 - x(2 - x)}{(1 - \alpha)\lambda(2 - x)}c \). The deviation is profitable if:
\[
NPV < A^* - I - \frac{1 - x(2 - x)}{\mu}
\]

Notice that for \( NPV = A^* \) it is \( k^* = 0 \) and \( \tilde{l} = 0 \), hence the inequality (66) is satisfied with equality. Then, it is sufficient to show that the derivative of the RHS with respect to \( NPV \) is lower than one whenever \( \tilde{l} < \tilde{l} \) and \( NPV < A^* \). From the implicit function theorem it is
\[
\frac{\partial \tilde{l}}{\partial NPV} = -\frac{1}{1 - x[1 + \gamma(1 - x)]b'(\tilde{l})},
\]

so the derivative of the RHS of equation (66) is lower than one whenever
\[
\frac{1 - x(2 - x)}{1 - x[1 + \gamma(1 - x)]b'(\tilde{l})} < \mu
\]

which is verified when \( \lambda_3 > 0 \), i.e. \( \tilde{l} < \tilde{l} \).

I turn to consider the solution when \( \tilde{k} > 0 \), hence \( \lambda_3 = 0 \). Summing (i) and (ii) it is \( \lambda_1 = \mu \). Substituting \( \lambda_2 \) from (iii) in (i) and using \( \lambda_1 = \mu \), the optimal amount of losses to investors solves
\[
\frac{1 - x(2 - x)}{1 - x[1 + \gamma(1 - x)]b'(\tilde{l})} = \mu
\]
i.e. \( \tilde{l} = \bar{l} \). Since \( \lambda_1 > 0 \) and \( \lambda_2 > 0 \) then it is \( \tilde{k} = k^* + x[1 + \gamma(1 - x)]b'(\bar{l}) - \bar{l} \) and \( \bar{p} = \bar{l} + x[1 + \gamma(1 - x)]b'(\bar{l}) = \bar{l} + \tilde{k} - k^* + \lambda l \). Substituting this optimal values in the objective function we get the value of deviating:

\[
[1 - x(2 - x)][R - \bar{p} + \tilde{k}] - \mu \tilde{k} = [1 - x(2 - x)][\frac{c}{(1 - \alpha) \bar{x}(2 - x)} - \bar{l}] - \mu [A^* - NPV + x[1 + \gamma(1 - x)]b'(\bar{l}) - \bar{l}]
\]

From Proposition 5.1 the payoff from exercising effort is \( \mu NPV - \frac{1 - x(2 - x)}{(1 - \alpha) \bar{x}(2 - x)}c \). Therefore, the deviation is profitable if and only if:

\[
\mu[\tilde{l} - x[1 + \gamma(1 - x)]b'(\bar{l})] - [1 - x(2 - x)]\bar{l} \geq 0
\]

The LHS of this inequality as a function of \( l \) is equal to zero at \( l = 0 \), and it has a global maximum at \( l \) such that:

\[
\frac{1 - x(2 - x)}{1 - x[1 + \gamma(1 - x)]b'(l)} = \mu
\]

i.e. at \( l = \tilde{l} = \bar{l} \) the equilibrium values of losses. Therefore, the inequality is always satisfied strictly, and the deviation is profitable. \( \blacksquare \)

**Proof of Proposition 5.3.**

The Lagrangean of the concave maximization problem is:

\[
\max L = [1 - \alpha x[1 + \gamma(1 - x)]](R - p + k) - \mu k - (1 - \alpha) \tau^H - \alpha[1 - x(2 - x)] \tau^L - \alpha x(1 - x)(1 - \gamma) \tau^B - c
\]

\[
+ \lambda_1 \left[R - p + k - \frac{c + (1 - \alpha) \tau^L}{(1 - \alpha) \bar{x}(2 - x)} \left[1 - G\left(\frac{p - \tau^B}{1 - \gamma(1 - x)}\right)\right] \right] + \lambda_2 [p - \alpha x[1 + \gamma(1 - x)]b(l) - l]
\]

\[
+ \lambda_3 \left[1 - x(2 - x) \tau^L + x(1 - x)(1 - \gamma) \tau^B - x[1 + \gamma(1 - x)](p - k - l) \right] + \sum_{\theta = H,L} \lambda_4^\theta \tau_5^\theta + \sum_{\theta = H,L} \lambda_5^\theta \tau_6^\theta
\]

\[
+ \sum_{\theta = H,L} \lambda_6^\theta (R - p + k - \tau_5^\theta) + \sum_{\theta = H,L} \lambda_7^\theta (R - p + k - \tau_6^\theta) + \lambda_8 l + \lambda_9 k
\]  

The FOCs are:

\[
41
\]
\[
\frac{\partial \mathcal{L}}{\partial p} = -1 + \alpha \beta_1\gamma(1-x) - \lambda_1 \left[1 + g \left( \frac{L}{\gamma(R-p+k)+(1-\gamma)\tau_0^s - \tau_0^t} \right) \right] + \lambda_2 \left[1 + g \left( \frac{L}{\gamma(R-p+k) + (1-\gamma)\tau_0^s - \tau_0^t} \right) \right] + \lambda_3 \beta_3(1-x) \]  

\]  

From (iii) it is \( \lambda_3 H > 0 \), so \( \tau_3^H = 0 \) and \( \lambda_6 = 0 \). Equation (iv) implies \( \lambda_7^H > 0 \) iff \( \lambda_1 > 0 \). Using (v) and (vi), it is possible to write (vii) as:

\[
\lambda_3 = \alpha - \frac{\lambda_3 L - \lambda_6 L}{x(1-x)(1-\gamma)} = \alpha - \frac{\lambda_3 L - \lambda_6 L}{1-x(2-x)} = \frac{\alpha b'(l)}{x(1+\gamma(1-x))} \]

I separately characterize two cases depending on the possibility for surviving banks at \( t = 3 \) to cover all losses incurred by investors of defaulting banks.

1. **Surviving banks can cover all losses from defaulting banks.**

I first characterize the solutions in which the incentive compatibility constraint does not bind, i.e. \( \lambda_1 = 0 \) and from equation (iv) it follows \( \lambda_7^H = \lambda_5^H = 0 \). Consider the case in which loss mutualization transfers in state L are not at their maximum level, i.e. \( \lambda_L^5 = \lambda_L^6 = \lambda_L^7 = 0 \). From (v) or (vi) it follows \( \lambda_3 = \alpha \). Substituting the values of these Lagrangean multipliers in (i) and (ii) it follows that \( \lambda_2 = 1 \) and \( \lambda_9 = \mu - 1 > 0 \) so \( k^* = 0 \). As a result, equation (74) becomes \( \alpha = \alpha b'(l) - \frac{\lambda_8}{x[1+\gamma(1-x)]} \), which is satisfied only if \( l^* = 0 \) and \( \lambda_8 = 0 \) since \( b'(0) = 1 \) and \( b'(l^*) > 1 \) for \( l^* > 0 \). Since \( l^* = 0 \) the binding investors’ constraint implies \( p^* = l \). Substituting \( p^* = l, k^* = 0 \) and \( l^* = 0 \) in the BB
constraint, transfers have to satisfy:

\[ [1 - x(2 - x)] \tau_S^L + x(1 - x)(1 - \gamma) \tau_D^L \geq x[1 + \gamma(1 - x)]I \]  

(75)

Since \( \tau_S^L, \tau_D^L \leq R - p^* + k^* = R - I \), this solution is admissible only if \( [1 - x[1 + \gamma(1 - x)]]R - I \geq 0 \). Lastly, the incentive compatibility constraint is satisfied only if

\[ R - I \geq \frac{c - [1 - G\left(\frac{l}{R - I + \gamma}\right)] L}{(1 - \alpha)m} \]  

(76)

If this inequality does not hold then \( \lambda_1 > 0 \) and \( k > 0 \). Summing (i) and (ii) it is \( \lambda_2 = \mu \). Moreover, observe that if there are enough resources to cover losses when \( k = 0 \), then \textit{a fortiori} it is possible to cover all losses when \( k > 0 \). Therefore, it must hold \( \tau_S^L, \tau_D^L < R - p + k \), hence \( \lambda_3^L = \lambda_7^L = 0 \). From (v) or (vi) it follows that \( \lambda_3 = \alpha \). Together with \( \lambda_2 = \mu \), equation (vii) implies \( \lambda_8 > 0 \), i.e. \( l^* = 0 \), and investors’ binding IR constraint implies \( p^* = I \). Substituting these results in the binding IC constraint the optimal \( k^* \) satisfies:

\[ R - I + k^* = \frac{c - [1 - G\left(\frac{l}{R - I + \gamma}\right)] L}{(1 - \alpha)m} \]  

(77)

From the BB and IC constraint it follows that:

\[ [1 - x(2 - x)] \tau_S^L + x(1 - x)(1 - \gamma) \tau_D^L = x[1 + \gamma(1 - x)](I - k^*) = x[1 + \gamma(1 - x)] \left( R - \frac{c - [1 - G\left(\frac{l}{R - I + \gamma}\right)] L}{(1 - \alpha)m} \right) \]  

(78)

Substituting the expressions in equations (77) and (78) into the objective function, it is straightforward to get a positive expected payoff only if:

\[ NPV = [1 - \alpha x[1 + \gamma(1 - x)]][R - I - c \geq \frac{\mu - 1}{\mu} \left[ \frac{c - [1 - G\left(\frac{l}{R - I + \gamma}\right)] L}{(1 - \alpha)m} - \alpha x[1 + \gamma(1 - x)]R - c \right] \]  

(79)

2. \textit{Surviving banks cannot cover all losses from defaulting banks.}

First, consider the solution in which the IC is not binding, i.e. \( \lambda_1 = 0 \). Since there are not enough funds to cover all losses at \( t = 3 \) then \( l^* > 0 \), hence \( \lambda_8 = 0 \). Suppose transfers in state L \( \tau_S^L \) and \( \tau_D^L \) are both set at the maximum level. From (v) and (vi) it is possible to get expressions for \( \lambda_6^L \) and \( \lambda_7^L \); substituting their values in (ii) it is \( \lambda_3 = \mu - 1 + \alpha - \lambda_9 \). Summing (i) and (ii) it is \( \lambda_2 = \mu - \lambda_9 \), hence \( \lambda_3 = \lambda_3 + 1 - \alpha > 0 \) and \( p = I + \alpha x[1 + \gamma(1 - x)]b(l^*) \). Substituting in (vii) it is \( \lambda_3 = \alpha(\lambda_3 + 1 - \alpha)b(l^*) \), that is \( \lambda_3 = \frac{\alpha(1 - \alpha)b(l^*)}{1 - \alpha b(l^*)} \).
Notice that \( \lambda_6^L = [1 - x(2 - x)](\lambda_3 - \alpha) = \alpha[1 - x(2 - x)](b'(l^*) - 1) > 0 \), and similarly
\[ \lambda_7^L = \alpha x(1 - x)(1 - \gamma)(b'(l^*) - 1) > 0. \]

If \( k^* = 0 \), then the loss \( l^* \) satisfies the BB equation, and it is immediate to verify it is equal to \( l_1 \). In order to be an admissible solution, it must be:
\[ \lambda_9 = \mu - 1 + \alpha - \lambda_3 = \frac{\mu - 1 + \alpha - \mu \alpha b'(l_1)}{1 - \alpha b'(l_1)} \geq 0 \quad (80) \]
i.e. \( b'(l_1) \leq \frac{1}{\mu} \left[ 1 + \frac{\mu - 1}{\alpha} \right] = b'(l_2) \), or \( l_1 \leq l_2 \). Notice that \( R - p^* = x[1 + \gamma(1 - x)](R - l_1) \), so it satisfies the IC constraint only if:
\[ x[1 + \gamma(1 - x)](R - l_1) \geq \frac{c - \left[ 1 - G \left( \frac{x[1 + \gamma(1 - x)](R - l_1)}{1 - \alpha} \right) \right] L}{(1 - \alpha)m} \quad (81) \]

Since the LHS (RHS) is decreasing (increasing) in \( l \) then it is equivalent to \( l_1 \leq l_3 \).

If \( l_2 < l_1 \) then \( k^* > 0 \) and \( \lambda_9 = 0 \). Then, \( \lambda_2 = \mu \) and \( \lambda_3 = \mu - 1 + \alpha \). From (vii) it follows that \( l^* = l_2 \). From the BB constraint it is immediate to get:
\[ k^* = I - [1 - x[1 + \gamma(1 - x)]R + x[1 + \gamma(1 - x)](\alpha b(l_2) - l_2) \quad (82) \]

Notice that \( R - p^* = k^* = x[1 + \gamma(1 - x)](R - l_2) \). To be admissible solutions it must be satisfy the IC constraint, that is \( l_2 \leq l_3 \).

From the previous discussion, the IC constraint must be binding when \( l_3 < \min\{l_1, l_2\} \). In turn, from (i) and (ii) it is \( \lambda_2 = \mu \). As the IC constraint is binding it is:
\[ R - p^* + k^* = \frac{c - \left[ 1 - G \left( \frac{x[1 + \gamma(1 - x)](R - p^* + k^*) - x[1 + \gamma(1 - x)](p^* - k^* - l^*)}{1 - \alpha} \right) \right] L}{(1 - \alpha)m} \quad (83) \]

From the BB constraint it is \[ [1 - x[1 + \gamma(1 - x)]](R - p^* + k^*) - x[1 + \gamma(1 - x)](p^* - k^* - l^*) = 0 \], i.e.:
\[ p^* - k^* = [1 - x[1 + \gamma(1 - x)]R + x[1 + \gamma(1 - x)]l^* \quad (84) \]

Substituting this expression in the IC constraint, then:
\[ x[1 + \gamma(1 - x)](R - l^*) = \frac{c - \left[ 1 - G \left( \frac{x[1 + \gamma(1 - x)](R - l^*)}{1 - \alpha} \right) \right] L}{(1 - \alpha)m} \quad (85) \]
i.e. \( l^* = l_3 \). To check it is an admissible solution, substitute \( \lambda_6^L \) and \( \lambda_7^L \) from (v) and (vi).
in (i) to get:

$$\lambda_3 = \mu - 1 + \alpha - \lambda_1 \left[ 1 + g \left( \frac{L}{x[1 + \gamma(1-x)](R-l_3)} \right) \frac{L\gamma}{[x[1 + \gamma(1-x)](R-l_3)]^2} \right]$$  (86)

Substituting in (vii) I get:

$$\mu - 1 + \alpha - \lambda_1 \left[ 1 + g \left( \frac{L}{x[1 + \gamma(1-x)](R-l_3)} \right) \frac{L\gamma}{[x[1 + \gamma(1-x)](R-l_3)]^2} \right] = \alpha \mu b'(l_3)$$  (87)

which implies \( l_3 < l_2 \). Lastly, it is immediate to get \( k^* = 1 - [1 - x[1 + \gamma(1-x)]]R + x[1 + \gamma(1-x)](\alpha b(l_3) - l_3) \), which is positive only if \( l_3 < l_1 \). Since the optimal loss mutualization scheme has \( \tau_S^* = \tau_D^* = R - p^* + k^* \), the maximum value of the objective function is equal to \( (1 - \alpha)(R - p^* + k^*) - \mu k^* - c \), hence the investment is undertaken only if its value is positive.

7.3 Microfoundation matching process

I present an explicit game of the matching process to provide a formal model for the out of equilibrium matching probabilities \( \tilde{p}_j \) in section 5.4.

I consider the following protocol. Once default probabilities realize, banks have the possibility to match with other banks. If a bank trades with another bank he avoid the loss \( L \) but it is exposed to the contagion risk when a counter party defaults. There are possibly infinitely many rounds of matching. In each round, all banks who have not previously traded are randomly matched in pairs. Let \( N \) (even) be the number of banks. Denote with \( N_k(s) \) the number of safe banks remaining in round \( k \). Once matched in a pair, a bank can accept or reject to trade with the matched counter party. Trade takes place if and only if both banks accept. If at least one bank rejects, both are randomly matched in round \( k+1 \).

From the discussion in section 4, trading with a safe bank provides the highest payoff, say \( s_i \), while trading with a bank who defaults with probability \( d \) provides a strictly lower payoff, say \( r_i(d) < s_i \) for every \( d \in (0,1) \); lastly, refusing to trade forever leads to a final expected payoff \( l_i < s_i \). For a safe bank it is \( l_i \leq r_i(d) \) if and only if \( d \leq \frac{L}{\mu \pi_{11} + (1-\gamma)\pi_{12} - \tau_S} := d_i \); for a risky bank \( r_i(d) \geq l_i \) for every \( d \in (0,1) \).

**Lemma 7.1** If matched with a safe bank, it is a weakly dominant strategy to accept trade.
Proof. Consider a bank $i$ that matches in round $k$ with a safe bank and denote with $V_i^{k+1}$ its continuation value if he does not trade. Clearly, $V_i^{k+1} \leq s_i$. Rejecting to trade yields $V_i^{k+1}$. For any probability $p_i$ that the safe counter party accepts to trade, proposing to trade yields $p_is_i + (1-p_i)V_i^{k+1} \geq V_i^{k+1}$. ■

Lemma 7.2 Suppose banks play the weakly dominant strategy in Lemma 7.1. In any round $k$, if at least two safe banks have not traded, it is a best response strategy for a safe bank to accept trade only with a safe counter party.

Proof. Consider a safe bank in round $k$ that matches with a risky counter party. By Lemma 7.1 a safe bank always has the future possibility to trade with a bank (safe or risky). Hence, it must be $V_i^{k+1} \geq \max\{r_i(d), l_i\}$ with the inequality strict if there is a strictly positive probability of a future match with another safe bank. If other safe banks follow the proposed strategy and there are at least two safe banks in round $k$, then a safe bank has a strictly positive probability to match with another safe bank. Accepting to trade immediately with the risky bank leads to a payoff $r_i(d) \leq V_i^{k+1}$. Hence, rejecting is a best response strategy. ■

By Lemmata 7.1 and 7.2, a safe bank accepts to trade only with another safe bank until he is the only safe bank left. In equilibrium, the possibility not to match with another safe bank may occur only if the number of safe banks at the beginning of the matching game is odd. In this case, one safe bank ends up either trading with a risky bank (if $r_i(d) \geq l_i$) or rejecting forever. The next lemma describes the equilibrium best response for risky banks.

Lemma 7.3 Suppose banks play the weakly dominant strategy in Lemma 7.1. The best response strategy of a risky bank $i$ matching with another risky bank in round $k$ is:

1. If $N_k(s)$ is even all equilibrium strategies are payoff equivalent and lead to get $r_i(d)$.

2. If $N_k(s)$ is odd he rejects to trade if $d \leq \tilde{d}$ and accept otherwise.

Proof. In case 1. by Lemma 7.2 all safe banks are going to match with another safe bank. For a risky bank $i$ it is $r_i(d) > l_i$ for every $d \in (0,1)$. In equilibrium he expects to trade with probability one with another risky bank. Any strategy that assigns positive probability to a rejection delays trade without consequences on final payoffs.

In case 2. by Lemma 7.2 one safe bank does not match with another safe bank. If $d \leq \tilde{d}$ the last safe bank prefers to trade with a risky bank. In this case, all risky banks delay trade in order to have a positive probability to match with the remaining safe bank and get a higher payoff. If $d > \tilde{d}$ the last safe bank rejects to match with risky banks. For a risky bank it is $r_i(d) > l_i$ for
every \( d \in (0, 1) \), and he finds strictly convenient to trade as soon as possible. Indeed, the last risky bank does not succeed in trading with the remaining safe bank and gets \( l_i < r_i(d) \). ■

It is straightforward to realize that Lemmata 7.1, 7.2 and 7.3 imply the matching outcome used in section 4. I now exploit the equilibrium behavior of this matching game to derive consistent out of equilibrium probabilities \( \tilde{p}_{ij} \), to be used in section 5.4. If an uninformed bank turns to be risky, all safe banks reject to trade with him unless there is no other safe bank left. However, as \( N \to +\infty \), the probability to be the very one risky bank who trades with the safe bank (case 2. in Lemma 7.3) approaches zero, hence \( \tilde{p}_{rs} = 0 \). As a result, a risky bank is going to trade with a risky bank almost surely, hence \( \tilde{p}_{rr} = \alpha \). If an uninformed bank turns out to be safe, its offers are always accepted by other banks who know he is safe. Then he has to decide whether to accept or reject trade in every round \( k \). To derive its optimal strategy, it is necessary to consider how its beliefs of matching with a safe bank evolve from the initial round of matching. Denote the measure of safe and risky banks in round \( k \) with \( S_k \) and \( R_k \), respectively. For \( N \to \infty \), and taking into account the uniform random matching, it is easy to express the evolution of banks. First, notice that safe banks trade once they match with another safe bank. Hence,

\[
S_{k+1} = S_k \left( 1 - \frac{S_k}{S_k + R_k} \right) \tag{88}
\]

Second, observe that the fastest rate at which risky banks trade corresponds to the equilibrium strategy in which they immediately accept to trade with other risky banks. Therefore, a lower bound on the measure of risky banks in round \( k \), say \( R_L^k \), is

\[
R_{k+1} \geq R^L_{k+1} = R_k \left( 1 - \frac{R_k}{S_k + R_k} \right) \tag{89}
\]

Hence, the probability to match with a safe bank after the first round is at most \( \frac{S_{k+1}}{S_{k+1} + R^L_{k+1}} = \frac{1}{2} \).

In the first round \( k = 0 \) the safe bank holds a belief to match with another safe bank equal to its expectation of the share of safe banks, conditional on its default probability being \( d_i = 0 \). This quantity is \( \mathbb{E}[n_s|d_i = 0] \), as defined in section 5.4. Since banks’ credit risk may be positively correlated, then \( \mathbb{E}[n_s|d_i = 0] \geq 1 - \alpha \). Since \( \alpha < \frac{1}{2} \), then \( \mathbb{E}[n_s|d_i = 0] \geq \frac{1}{2} \), and it is strictly convenient to accept in the first round rather than accepting later; hence \( \tilde{p}_{sr} = 1 - \mathbb{E}[n_s|d_i = 0] \).

The previous discussion leads to the incentive compatibility constraint in equation (39).

### 7.4 Discrete \( N \) banks case

In this section I consider a model with a finite number \( N \) of banks. Conditional on \( N \) banks exerting effort, the probability to have \( l \leq N \) safe banks comes from a correlated binomial
distribution with probability distribution function $P_N(l)$. Since banks are identical and the probability to become risky for a bank exerting effort is $\alpha$, it must hold $\alpha = \sum_{l=0}^{n} P_n(l) \frac{n-l}{n}$ for every $n \leq N$. I assume the random variable $d$ to be independent from the realization $l$ of safe banks.

Let $l \leq N$ be the number of safe banks at $t = 2$. If $l$ is even, by positive assortative matching each matched pair includes banks of identical credit risk; if $l$ is odd, each safe/risky bank has the same probability to be included in the single match which includes banks of different credit risk. In other words, all pairs include banks of the same credit risk when $l$ is even, and there is only one ‘mismatched’ pair if $l$ is odd.

Let $j \in \{ss, sr, rs, rr\}$ denote the possible matching pairs for a bank, where the first letter indicates its credit risk ($s$ for safe, $r$ for risky), and the second letter the one of its trading partner. If all $N$ banks exert effort, the probability $p_j$ at $t = 1$ is:

\[
p_{ss} = \sum_{l=0}^{N} \frac{P_N(l)}{N} \left[ I_{\{l \text{ even}\}} + \left( 1 - \frac{1}{T} \right) I_{\{l \text{ odd}\}} \right] = 1 - \alpha - \frac{1}{N} \sum_{l=0}^{N} P_N(l) I_{\{l \text{ odd}\}} \\

p_{rr} = \sum_{l=0}^{N} \frac{P_N(l)}{N-1} \left[ \frac{N-1-l}{N-l} I_{\{l \text{ odd}\}} \right] = \alpha - \frac{1}{N} \sum_{l=0}^{N} P_N(l) I_{\{l \text{ odd}\}} \\

p_{rs} = p_{sr} = \frac{1}{N} \sum_{l=0}^{N} P_N(l) I_{\{l \text{ odd}\}}
\]

The probabilities of a mismatch $p_{sr}$ and $p_{rs}$ rapidly decrease in $N$ (an upper bound is $\frac{1}{N}$). In the baseline model with a continuum of banks it is $p_{sr} = p_{rs} = 0$.

The probability that a shirking bank (hence risky) matches with a safe bank is equal to:

\[
q_{rs} = \sum_{l=0}^{N-1} P_{N-1}(l) \frac{1}{N-1} I_{\{l \text{ odd}\}}
\]

This quantity may not tend to zero for $N$ large, and its value depends on: (i) the number of banks $N$; (ii) whether the credit risk of banks exerting effort depends on aggregate or idiosyncratic factors. At the end of the section I provide an explicit characterization of this relationship (Figure 2).

For each bank, the expected payoff from exerting effort at $t = 1$, conditional on the belief
that all other $N - 1$ banks exert effort and have final payoff $\pi_j$, is:

$$
\mathbb{E}_{e_i=1}[u_i|\pi] = \pi_i \mathbb{P}(d) + \mathbb{E} \left[ \int_0^1 (1-x)(1-\gamma x)g(x)\,dx \right] \pi_i \\
+ \mathbb{P}(r) \left\{ G \left( \frac{L}{\pi_i} \right) \mathbb{E} \left[ 1 - d_i|d_i \leq \frac{L}{\pi_i} \right] \pi_i + \mathbb{E} \left[ 1 - G \left( \frac{L}{\pi_i} \right) \right] \left( \mathbb{E} \left[ 1 - d_i|d_i \geq \frac{L}{\pi_i} \right] \pi_i - L \right) \right\} \\
+ \mathbb{P}(r) \left\{ G \left( \frac{L}{\pi_i} \right) \left[ \mathbb{E} \left[ 1 - d_j|d_j \leq \frac{L}{\pi_i} \right] \pi_i + \mathbb{E} \left[ d_j|d_j \leq \frac{L}{\pi_i} \right] (1-\gamma)\pi_i \right] + \mathbb{E} \left[ 1 - G \left( \frac{L}{\pi_i} \right) \right] (\pi_i - L) \right\} - c
$$

(92)

The expected payoff at $t = 1$ includes all the possible equilibrium outcomes of the game. With probability $p_{ss}$ bank $B_i$ is safe and matches with another safe bank: they trade, avoiding the extra cost $L$, and enjoy for sure their final profit $\pi_i$ at $t = 3$. If instead $B_i$ is safe, and matches with a risky bank—an outcome whose probability is $p_{sr}$—he accepts to trade only if $d \leq \frac{L}{\pi_i}$. If this is the case, he expects at $t = 1$ to receive its final profit $\pi_i$ only if the counter party does not default (with expected prob. $\mathbb{E} \left[ d_j|d_j \leq \frac{L}{\pi_i} \right]$), or the default does not lead to contagion (with prob. $\mathbb{E} \left[ d_j|d_j \leq \frac{L}{\pi_i} \right] (1-\gamma)$). If $B_i$ refuses to trade, he incurs a loss $L$ but he is not exposed to contagion risk, and enjoys $\pi_i$ for sure. If $B_i$ turns out to be risky, he can match with another risky bank (with prob. $p_{rr}$). In the relevant parameter space, two risky banks always find it optimal to trade and avoid the cost $L$; then, if $B_i$ survives at $t = 3$, he enjoys $\pi_i$ either when its counter party does not default (with prob. $1 - d$), or when it is not affected by contagion (with prob. $d(1-\gamma)$). Finally, if $B_i$ turns out to be risky, and matches with a safe bank (with prob. $p_{rs}$), he enjoys $\pi_i$ if he survives at $t = 3$, but he incurs an extra cost $L$ at $t = 2$ if the matched bank refuses to trade (with prob. $1 - G \left( \frac{L}{\pi_i} \right)$).

Simplifying the expression for $\mathbb{E}_{e_i=1}[u_i|\pi]$, it is:

$$
\mathbb{E}_{e_i=1}[u_i|\pi] = (1 - m\alpha)\pi_i - \left[ p_{sr} \int_0^1 x g(x)\,dx + p_{rr} \int_0^1 x(1-x)g(x)\,dx \right] \gamma\pi_i \\
- \left[ p_{sr} \left( 1 - G \left( \frac{L}{\pi_i} \right) \right) + p_{rs} \left( 1 - G \left( \frac{L}{\pi_i} \right) \right) \right] L - c
$$

(93)

If bank $B_i$ decides to shirk ($e_i = 0$) a bank is always risky at $t = 2$, and the problem is equivalent to the baseline model. The expected payoff at $t = 1$ is:

$$
\mathbb{E}_{e_i=0}[u_i|\pi] = q_{rs} \left[ (1-m)\pi_i - \left[ 1 - G \left( \frac{L}{\pi_i} \right) \right] L \right] + (1-q_{rs}) \left[ \int_0^1 (1-x)(1-\gamma x)g(x)\,dx \right] \pi_i \\
= (1-m)\pi_i - q_{rs} \left[ 1 - G \left( \frac{L}{\pi_i} \right) \right] L - (1-q_{rs}) \left[ \int_0^1 x(1-x)g(x)\,dx \right] \gamma\pi_i
$$

(94)

The contract at $t = 0$ is incentive compatible if $\mathbb{E}_{e_i=1}[u_i|\pi] \geq \mathbb{E}_{e_i=0}[u_i|\pi]$. Rearranging equations conveniently, it is equivalent to:
\[ \pi_i \geq \frac{c - (q_{rs} - p_{rs}) \left[ 1 - G \left( \frac{L}{y_i} \right) \right] L + p_{sr} \left[ 1 - G \left( \frac{L}{y_i} \right) \right] L}{m(1 - \alpha) + \gamma \left[ (1 - q_{rs} - p_{rr}) \int_0^1 x(1-x)g(x)\,dx - p_{sr} \int_0^1 xg(x)\,dx \right]} \] (95)

The incentive compatibility constraint for the discrete \( N \) case in equation (95) is less analytically tractable because of the presence \( p_{sr} \) and \( p_{rs} \), i.e. the probabilities of a match between a risky and a safe banks on the equilibrium path. Both \( p_{sr} \) and \( p_{rs} \) make the incentive compatibility constraint more stringent, as the relative convenience between shirking and exerting effort is reduced because of the possibility, on the equilibrium path, of matching with a risky bank also when a bank is safe. Nonetheless, the threat of market exclusion and endogenous peer selection are still present, and they represent the main economic forces. The relative importance between the two peer discipline mechanisms is determined by the probability to match with a safe counter party after shirking \( q_{sr} \): higher values of \( q_{sr} \) increase the effectiveness of the threat of market exclusion, and decrease the relevance of endogenous peer selection.

In the remainder I provide a plausible analytical characterization for \( q_{sr} \) and I display its value in Figure 2. The probability \( q_{sr} \) is higher when the number of banks is small and/or the correlation among banks’ credit risks is high, for example because it depends on an aggregate macroeconomic variable. On the contrary, endogenous peer selection is the relevant peer discipline channel when the number of banks is high and/or banks’ credit risk depends more on idiosyncratic factors.

Consider that bank \( B_i \) default probability, after exerting effort, is described by:

\[ d_i = \beta_i d^A + (1 - \beta_i) d^I_i \] (96)

- The random variables \( d^A \) and \( d^I_i \) are both equal to zero with probability \( 1 - \alpha \), while with probability \( \alpha \) their realized values come from the cumulative density function \( G(\cdot) \). The random variable \( d^A \) is a default probability common to all banks, while \( d^I_i \)’s are i.i.d. across banks; \( d^A \) and \( d^I_i \) are independent for every bank \( i \).

- The Bernoulli random variable \( \beta_i \) has realizations \( \{0, 1\} \), i.i.d across banks, with probability of success \( (\beta_i = 1) \) equal to \( b \in [0, 1] \). If \( b = 1 \) banks’ credit risk shocks are perfectly correlated, if \( b = 0 \) they are purely idiosyncratic. Intermediate values of \( b \) reflect intermediate correlation among banks’ credit risk statuses.

Notice that the marginal probability to be risky, i.e. the probability to have \( d_i > 0 \), is equal to \( \alpha \) for every \( b \in [0, 1] \). Denote with \( B(h,N,z) \) the probability mass function for \( h \) successes
Figure 2: Probability to match with a safe bank after shirking

Simulation with $\alpha = 0.2$ for different values of $N$ and $b$. A higher value of $q_{sr}$ increases the importance of the threat of market exclusion relative to the endogenous peer selection mechanism.

out of $N$ independent trials, with success probability equal to $z$. Notice that the probability to have $l$ risky banks out of $N$ is equivalent to the probability of $N - l$ safe banks out of $N$. Hence, the probability that $l$ out of $N$ banks are risky is:

$$P_N(N-l) = P(d^A > 0) \sum_{h=0}^{l} B(h,N,b)B(l-h,N-h,\alpha) + P(d^A = 0) \sum_{h=0}^{N-l} B(h,N,b)B(l,N-h,\alpha)$$

Equation (97) has the following logic: if the aggregate factor leads banks to be risky ($d^A > 0$) then $l$ banks are risky (or $N - l$ safe) if the credit risk for $h \leq l$ banks out of $N$ depends on the aggregate state (i.e. $\beta_i = 1$), and, out of the remaining $N - h$ banks, $l - h$ are risky because of the idiosyncratic component ($d^I_i > 0$); if the aggregate factor is not risky ($d^A = 0$) then all $h$ banks with $\beta_i = 1$ are safe, and out of the remaining $N - h$ there must be $l$ banks risky due to idiosyncratic factors. Figure 2 is constructed for a range of parameters $N$ and $b$ using equation (91) and (97). The market exclusion threat is more effective with a limited number $N$ of banks and/or a high dependence of banks’ credit risk statuses from the aggregate factor ($b$).
References


RECENTLY PUBLISHED “TEMI” (*)


N. 1065 – *How excessive is banks’ maturity transformation?*, by Anatoli Segura Velez and Javier Suarez (April 2016).

N. 1066 – *Common faith or parting ways? A time-varying factor analysis*, by Davide Delle Monache, Ivan Petrella and Fabrizio Venditti (June 2016).


N. 1068 – *The labor market channel of macroeconomic uncertainty*, by Elisa Guglielminetti (June 2016).

N. 1069 – *Individual trust: does quality of public services matter?*, by Silvia Canussi and Anna Laura Mancini (June 2016).

N. 1070 – *Some reflections on the social welfare bases of the measurement of global income inequality*, by Andrea Brandolini and Francesca Carta (July 2016).


N. 1072 – *Bank quality, judicial efficiency and borrower runs: loan repayment delays in Italy*, by Fabio Schiantarelli, Massimiliano Stacchini and Philip Strahan (July 2016).

N. 1073 – *Search costs and the severity of adverse selection*, by Francesco Palazzo (July 2016).


N. 1075 – *Quantifying the productivity effects of global sourcing*, by Sara Formai and Filippo Vergara Caffarelli (July 2016).


N. 1079 – *Parents, schools and human capital differences across countries*, by Marta De Philippis and Federico Rossi (September 2016).

N. 1080 – *Self-fulfilling deflations*, by Roberto Piazza, (September 2016).

N. 1081 – *Dealing with student heterogeneity: curriculum implementation strategies and student achievement*, by Rosario Maria Ballatore and Paolo Sestito, (September 2016).


N. 1083 – *BTP futures and cash relationships: a high frequency data analysis*, by Onofrio Panzarino, Francesco Potente and Alfonso Puorro, (September 2016).

N. 1084 – *Women at work: the impact of welfare and fiscal policies in a dynamic labor supply model*, by Maria Rosaria Marino, Marzia Romanelli and Martino Tasso, (September 2016).

(*) Requests for copies should be sent to:
2014


G. Micucci and P. Rossi, Il ruolo delle tecnologie di prestito nella ristrutturazione dei debiti delle imprese in crisi, in A. Zazzaro (a cura di), Le banche e il credito alle imprese durante la crisi, Bologna, Il Mulino, TD No. 763 (June 2010).


L. Gambacorta and P. E. Mistrulli, Bank heterogeneity and interest rate setting: what lessons have we learned since Lehman Brothers?, Journal of Money, Credit and Banking, v. 46, 4, pp. 753-778, TD No. 829 (October 2011).


M. Porqueddu and F. Venditti, Do food commodity prices have asymmetric effects on euro area inflation, Studies in Nonlinear Dynamics and Econometrics, v. 18, 4, pp. 419-443, TD No. 878 (September 2012).


M. Bugamelli, S. Fabiani and E. Sette, *The age of the dragon: the effect of imports from China on firm-level prices*, Journal of Money, Credit and Banking, v. 47, 6, pp. 1091-1118, TD No. 737 (January 2010).


D. Fantino, A. Mori and D. Scalise, *Collaboration between firms and universities in Italy: the role of a firm's proximity to top-rated departments*, Rivista Italiana degli economisti, v. 1, 2, pp. 219-251, TD No. 884 (October 2012).


2016


**FORTHCOMING**


