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SELF-FULFILLING DEFLATIONS

by Roberto Piazza*

Abstract

What types of monetary and fiscal policy rules produce self-fulfilling deflationary paths that are monotonic and empirically relevant? This paper presents simple theoretical conditions that guarantee the existence of these paths in a general equilibrium model with sticky prices. These sufficient conditions are weak enough to be satisfied by most monetary and fiscal policy rules. A quantification of the model which combines a real shock à la Hayashi and Prescott (2002) with a simultaneous sunspot that deanchors inflation expectations matches the main empirical features of the Japanese deflationary process during the "lost decade". The results also highlight the key role of the assumption about the anchoring of inflation expectations for the size of fiscal multipliers and, in general, for any policy analysis.

JEL Classification: E31, E40, E43.
Keywords: deflation, liquidity trap, deanchoring, inflation target, sunspot.

Contents

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1 Introduction[1]

Since 2008, aggressive monetary policy has brought the short-term interest rate to the lower bound across the major advanced economies. Time and time again "deflation scares" and the occurrence of liquidity traps have been a source of concern for policy-makers around the world. While liquidity traps are an interesting empirical phenomenon and an important topic of policy discussions, the theoretical debate on their nature is still very much open. This paper contributes to this debate by presenting new theoretical and quantitative results on the properties of self-fulfilling liquidity traps.

Self-fulfilling liquidity traps may arise because, under rational expectations, monetary and fiscal feedback rules typically give rise to multiple equilibria. Indeed, the study of multiplicity in monetary models spans a small literature on its own (for an early overview see Michenera and Ravikumar [1998]). In particular, when the monetary authority pursues an inflation target additional equilibria exist where inflation expectations are deanchored from the target. A sunspot shock that deanchors inflation expectations may therefore be the reason why an economy is driven into a deflationary liquidity trap. The critical role of inflation expectations in shaping liquidity traps and driving policymakers to extreme measures emerges clearly in the words of Bank of Japan's Governor Kuroda: «Japan is different from countries like the United States, which has inflation expectations anchored at 2 percent. [...] There was a risk that despite having made steady progress, we could face a delay in *eradicating the public's deflation mindset* [...]. It's important for the BOJ [...] to *get its price target firmly embedded in people's mindset.*»[2]

What types of policy rules can give rise to self-fulfilling deflationary paths that are interesting from an empirical standpoint? The literature has not provided a conclusive answer to this question. The main reason is that the dynamic equation defining the evolution of deanchored inflation expectations is, as Fernandez-Villaverde (2014) puts it, «hard to characterize» in general terms. Thus, while a case-by-case approach focusing on specific functional forms for the policy rules has led to important seminal results (Benhabib, Schmitt-Grohé and Uribe [2001]), a more comprehensive answer is still lacking. The paper addresses this issue.

First, the paper proves that any monetary and fiscal policy rules that jointly satisfy certain easy-to-check sufficient conditions give rise to "reasonably looking" self-fulfilling deflationary paths. In my definition, "reasonably looking" is equivalent to monotonicity. Focusing on monotonic deflationary paths is attractive because, for instance, they match the empirical experience of Japan. By selecting paths that feature simple dynamics I am thus deliberately leaving aside complicated or chaotic expectational dynamics (Benhabib, Schmitt-Grohé and Uribe [2002b]). Such equilibria feature «a volatile sequence of interest rates and inflation rates followed by sudden arrival at the low nominal interest rate steady state» and may be more a curious mathematical object rather than an empirically relevant representation of how people's expectations actually evolve (Bullard [2010]). Second, the paper shows that the aforementioned sufficient conditions are satisfied by a wide set of potential policy rules. This set includes any static or forward looking rule that prescribes some minimal form of stimulus

[1]I would like to thank James Bullard, John Cochrane, Stephanie Schmitt-Grohé, David Andolfatto and one anonymous referee for their useful comments. The views expressed are those of the author and do not necessarily reflect those of the Bank of Italy. Contact information: Bank of Italy, via Nazionale 91, 00184, Roma, e-mail: roberto.piazza@bancaditalia.it

[2]Reuters Fri Oct 31, 2014 6:27am EDT.

when inflation, actual and/or expected, drifts below the target. These sufficient conditions are for instance satisfied even by "neo-Fisherian" monetary rules that call for an *increase* in the policy rate in some deflationary states.

These findings indicate that the existence of reasonably looking deflationary equilibria is a pervasive phenomenon in sticky-price models with rule-based policies. However, while all stimulative rules are subject to self-fulfilling and monotonic deflationary paths, it is nonetheless true that rules that are *more* stimulative have some advantages. In particular, the paper proves that a very stimulative rule requires a sizable initial deanchoring of inflation expectations for the self-fulfilling deflationary path to form (*Type II* equilibrium). Instead, when the rule is not very stimulative, the deflationary path forms even for an initial arbitrarily small deanchoring of expectations (*Type I* equilibrium). Simulations indicate, for instance, that a Taylor rule with coefficient 1.25 for the inflation gap and 0.5 for the output gap is not stimulative enough to prevent the occurrence of a *Type I* equilibrium.

How well can a basic sticky price model match the empirical counterpart of a deflationary process driven by a deanchoring of expectations? I choose to focus, as my empirical counterpart, on the Japanese deflation during the "lost decade" 1992-2002, for a number of reasons. First, the Japanese deflation is arguably the most studied and, up to recently, unique instance of a liquidity trap in the post-war era. Second, the Japanese deflationary process was monotonic. Third, there is a widespread perception, as also repeatedly stated by Japanese policymakers, that the deanchoring of inflation expectations is an indication of the self-fulfilling nature of the Japanese liquidity trap. This idea finds further support in the suggestive evidence that the deanchoring process occurred at a very early stage of the lost decade.

As an application of the theoretical results, I show that an appropriately calibrated general equilibrium sticky price model, albeit very simple in its elements, does a good job in matching the salient features of the Japanese deflationary process. This result is based on three crucial ingredients. The first is a sunspot that deanchors inflation expectations. The second is a fall in the natural real rate induced by a reduction in potential growth. For this part of the calibration, I rely on the estimates by Hayashi and Prescott (2002) of the evolution of market real rates and of TFP growth. The third ingredient is the calibration of the monetary policy rule, which is assumed to have a Taylor form. Once calibrated to match some initial conditions on the nominal interest rate in Japan, the inflation parameter of the rule equals 1.38, which is below the typical value of 1.5 reported by Taylor (1993). The rule is then "not very stimulative", a point made also by Bernanke and Gertler [2000]).

A final set of results concerns the effects of fiscal expansions. As for the case of stimulative monetary policies, the paper proves that anticipated expansionary fiscal policies cannot by themselves prevent the insurgence of monotonic self-fulfilling deflationary paths. Of particular interest is the effect of a fiscal expansion at the zero lower bound. The paper shows that, if inflation expectations are deanchored, short-term fiscal multipliers are small and equal to a standard value of 0.5 (longer-term multipliers are even smaller), in line with Mertens and Ravn (2014). This is not in contrast, however, with the findings that fiscal multipliers at the zero lower bound can be greater than 1 (Eggertsson [2010a], Christiano, Eichenbaum, Rebelo [2011]). The two results are in fact obtained under two different equilibrium selections (Christiano, Eichenbaum [2012]). Large long-run fiscal multipliers are obtained if inflation expectations are assumed to be anchored. If, instead, inflation expectations are deanchored, then fiscal multipliers have about the same size as outside the zero lower bound. Equilibrium selection choices, which often are only implicitly made, are therefore far from harmless for policy analyses in standard sticky price models (Cochrane [2011], Cochrane [2013]).

This paper is part of a wider effort to understand the properties and the empirical relevance of self-fulfilling deflation mechanisms. A prominent example is Aruoba, Cuba-Borda and Schorfheide (2016). The authors find that, after 1995, Japan likely experienced a sunspot switch to the deflationary steady state. The presence of unanticipated shocks to fundamentals plays, together with sunspot shocks, a prominent role in their analysis. Taken in isolation, their sunspot shock simply makes the inflation rate jump between the target and the deflationary steady state. This is different from, and complementary to, the quantitative application presented here. In this paper, while abstracting from unanticipated shocks to fundamentals, I focus on the self-fulfilling deflationary path in its entirety, and not just on its two endpoints (i.e. the target and liquidity traps steady states).

The rest of the paper proceeds as follows. The next section provides a brief review of the theoretical literature on liquidity traps and describes some stylized facts about inflation expectations in Japan. Section 3 presents the main theoretical results using a simple version of the aggregate equilibrium equations of the model. Section 4 provides a detailed microfoundation for the aggregate equilibrium equations. Section 5 explores the quantitative properties of the calibrated model and Section 6 concludes.

2 A primer on modeling liquidity traps

This section discusses some stylized facts about the Japanese liquidity trap and provides a summary of the main current theoretical approaches to modeling liquidity traps.

The most prominent example of a "liquidity trap" is provided by Japan. Starting with 1992, and in correspondence with a bust in house prices and at the onset of a severe banking crisis, CPI inflation began to fall (Figure 1). In response to these developments, the Bank of Japan progressively cut the nominal interest rate which, in a time span of about 10 years, reached the zero lower bound. While the overall peak-to-trough reduction in inflation was quite sizeable (about 4 percentage points), it took a relatively long time to fully materialize (10 years, from 1992 to 2002). Once the CPI inflation is netted of the effects of changes in the consumption tax, it becomes also apparent that the deflationary path was roughly monotonic. After 2002 a mild deflation remained in place, bar short periods of price increases mostly due to commodity price spikes. Because of these stylized facts, the deflationary process in Japan gained the adjective of "creeping". On the fiscal side, the government reacted to the deflation through various measures of fiscal stimulus, that eventually caused the gross public debt to soar to over 200 percent of GDP. While it is sometimes argued that these measures have prevented an even worse outcome for Japan, it remains true that, from a purely observational perspective, they did not lift the country out of the deflation. Interestingly, over this rather long period of time, the Japanese authorities have reiterated their commitment to put in place a (arguably, Ricardian) long-run fiscal strategy to repay the public debt and avoid both an outright default and debt monetization.[3]

Moving from the empirical to the theoretical description, the literature has mainly proposed two ways of explaining liquidity traps. The first sees liquidity traps as a consequence of *fundamental* shocks hitting the economy. The second, which is the one studied in this paper,

[3]For instance, recently Prime Minister Shinzo Abe has declared that the Bank of Japan's bond buying program is not tantamount to debt monetization (Thomson Reuters, May 24 2013, 02:45).

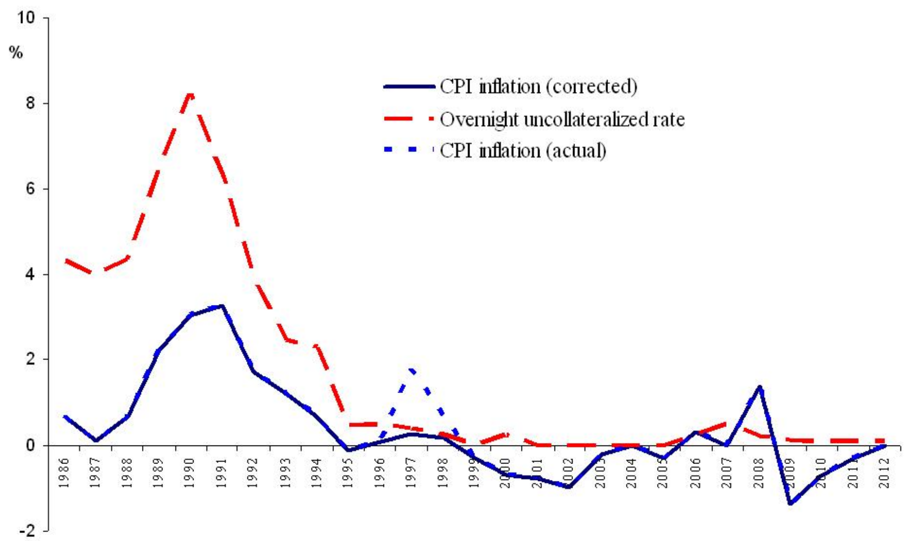

Figure 1: Inflation and the nominal short-term rate in Japan. In April 1997 the consumption tax in Japan was raised by 2 percent (from 3 to 5 percent). To obtain a measure of inflation net (corrected) of the change in the consumption tax, the graph assumes that it took 4 quarters for the tax hike to be completely incorporated into prices. Therefore, three fourths of the overall 2 percent VAT-induced inflation are attributed to the year 1997, and one fourth to 1998.

focuses on liquidity traps that are generated by the presence of multiple *self-fulfilling* equilibria, whose existence is induced by specific government policy rules. Both explanations arise from the same underlying model economy, i.e. from a standard dynamic general equilibrium model. This section provides a brief overview of these two alternative views on liquidity traps. As we shall see, depending on which of the two views is embraced, one may obtain quite different answers to the questions: how do liquidity traps come about? Are there any negative consequences for welfare when the economy ends up in a liquidity trap? What can and should policymakers do?

2.1 Shocks to fundamentals

Liquidity traps can be generated by preference shocks which push into negative territory the "natural real rate", here defined as the inverse of the subjective discount factor minus one (see Eggertsson and Woodford [2003, 2004] and Krugman [1998]). In the presence of sticky nominal prices and a zero low bound, movements in the nominal interest rate may not be enough to accommodate negative natural real rates. As the equilibrium real rate exceeds the natural rate, current aggregate demand falls. The ensuing economic recession puts downward pressure on nominal prices, generating deflation and exacerbating further the recession, with associated negative consequences on welfare. Even though the nominal rate has already reached the zero lower bound, policymakers still have tools to avoid the worst of the recession. One option is to promise ("forward guidance") to keep *future* nominal rates at lower than usual levels, even when the natural rate has exogenously returned to normal levels. This raises both *future*

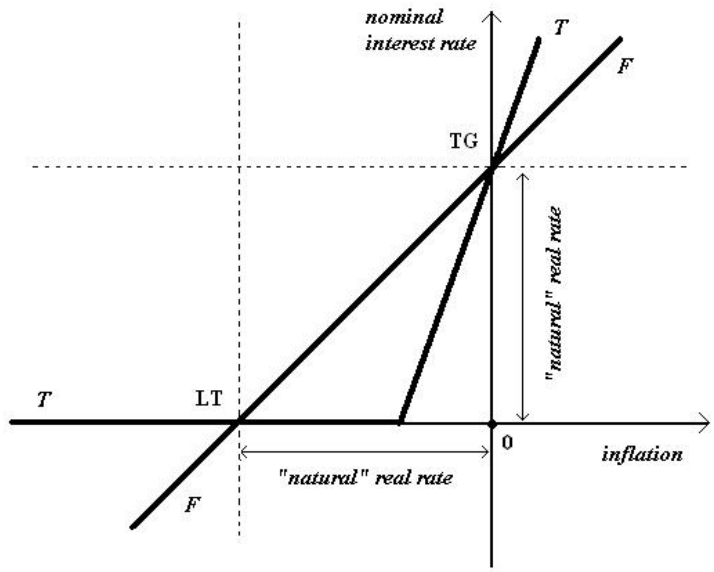

Figure 2: Taylor principle, zero lower bound and the liquidity trap steady state.

inflation and, via the expectation channel of monetary policy, *current* inflation, allowing the current real interest rate to fall. Alternatively, any policy that is able to produce a rise in current inflation has the potential of being beneficial. Examples are fiscal expansions or fine-tuned sequences of consumption tax hikes (Christiano, Eichenbaum, Rebelo [2011]), Correira, Farhi, Nicolini, Teles [2013]).

2.2 Self-fulfilling deflations

Since at least Sargent and Wallace (1975), it has been known that monetary policy rules can give rise to self-fulfilling multiple equilibria. In particular, liquidity traps having a self-fulfilling nature are generated when monetary policy follows a static Taylor rule satisfying the “Taylor principle” (Benhabib, Schmitt-Grohé, Uribe [2001], [2002a], [2002b]). The Taylor principle prescribes that the nominal interest rate reacts more than one-to-one to deviations of current inflation from the target inflation rate (Taylor [1993]). To better see how this rule operates consider Figure 2. The FF line is the (linearized) Fisher equation, i.e. the locus of points such that the difference between the nominal interest rate i and the inflation rate π equals a given natural real rate r^n. The line TT is the (linearized) Taylor rule, indicating how the nominal rate deviates from its target level i^{TG} as inflation deviates from its target $\pi^{TG} = i^{TG} - r^n$. Since the nominal rate reacts, at least around the target inflation, more than one-to-one to deviations of π from π^{TG}, the line TT has locally a larger slope than FF. Given that by assumption the nominal rate is a continuous function of inflation and bounded below by zero, then TT and FF must cross again at $\pi^{LT} = -r^n$ and $i^{LT} = 0$, i.e. the liquidity trap equilibrium.

Self-fulfilling liquidity traps equilibria arise with or without sticky prices, but with sticky prices liquidity traps equilibria are in general inefficient.[4] However, “stimulative” monetary or

[4]Intuitively, the qualitative properties of the lines FF and TT in Figure 2 do not depend on assumptions

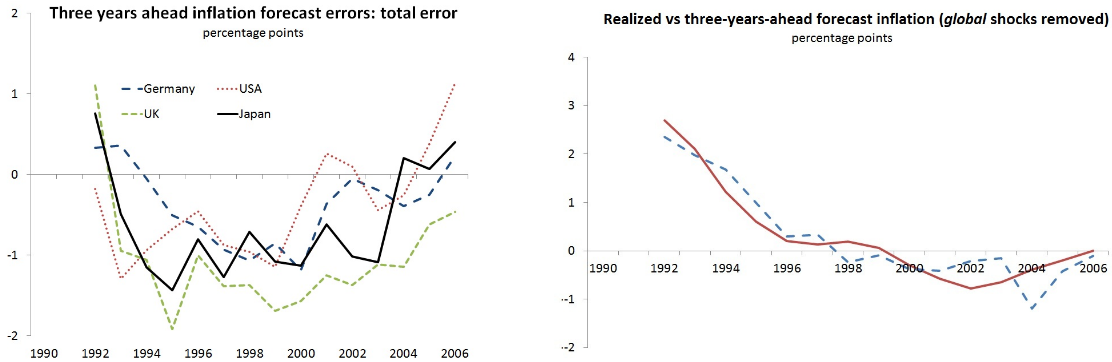

Figure 3: Left hand side panel: average realized inflation minus ex-ante average forecasted inflation for the three years ending in each t. Right hand side panel: three-year average realized inflation (solid line) and ex-ante average forecasted inflation (dashed line) once the global error is removed. Sources: Thomson Reuters Datastream, Statistics Bureau of Japan, Consensus Forecast (October survey).

fiscal measures of type discussed in Section 2.1 are here doomed to fail the task of pushing the economy out of the liquidity trap (Mertens and Ravn [2014]). In fact, self-fulfilling liquidity traps exist exactly *because* government policies respond strongly, e.g. by adopting the Taylor principle for the monetary rule, to a fall in the inflation rate. The literature has emphasized that, given an initial sunspot level of inflation, the dynamic convergence to the liquidity trap steady state π^{LT} is quite complicated. It can display spiral patterns or even chaotic behavior, as shown in Benhabib, Schmitt-Grohé, Uribe M. (2001, 2002b). Moreover, because of the presence of self-fulfilling sunspot, equilibrium selection issues typically arise in quantitative applications (Mertens and Ravn [2014]). Taken together, these problems have led some authors to question the practical relevance of self-fulfilling liquidity traps. In particular, a large body of literature has investigated the learnability of multiple rational expectations equilibria and found that various learning mechanisms could eliminate the multiplicity (Bullard and Mitra [2007], Eusepi [2007], Christiano and Eichenbaum [2012]). On the other hand Cochrane (2009) questions the relevance of equilibrium selection mechanisms based on learning techniques.

This paper contributes to the literature by presenting new results on self-fulfilling liquidity traps. In particular, it provides sufficient conditions for both monetary and fiscal rules such that, after an arbitrarily small deviation from the target, inflation monotonically converges to π^{LT}. Quantitative applications suggest that this type of equilibria can arise in empirically relevant situations, as in the case of Japan.

2.3 Expected or unexpected deflation?

A model that successfully replicates the main stylized facts about Japan must generate a long-lasting liquidity trap and deflation, coupled with an absence of above-trend output growth. Eggertsson and Woodford (2003) show that such features can be obtained if the economy is hit by a long series of negative *unexpected* shocks to the fundamental natural interest rate.

regarding price stickiness.

One implication of the unexpected nature of the shock is that, for the duration of the liquidity trap, actual inflation falls short of agents' ex-ante expectations.[5] After the shocks dissipate, expected and ex-post inflation coincide again and equal π^{TG}. Long-term inflation expectations remain then anchored to the inflation target. On the contrary, self-fulfilling liquidity traps are generated as non-fundamental (sunspot) shifts of inflation expectations in a perfect-foresight equilibrium. Ex-ante expectations, therefore, always coincide with ex-post inflation. Moreover, in the absence of new sunspot shocks, expected and actual inflation converge over time to π^{LT}. Long-run inflation expectations are then permanently deanchored from the target.

We turn to the data to assess whether the deflation in Japan was expected or unexpected. The left hand side panel of Figure 3 plots the difference between ex-post and ex-ante average inflation over a three-year forecasting period. Between 1992 and 2006 actual inflation persistently fell short of expected inflation. Should we conclude that Japan was hit by a sequence of unexpected shocks to fundamentals and thus to inflation?

While this is certainly possible, it can hardly represent a complete explanation of why Japan fell into a liquidity trap. Figure 3 makes in fact clear that unexpected inflation shocks in Japan were similar, in terms of magnitude and timing, to those experienced by other advanced countries, which nonetheless did *not* fall into a liquidity trap. There, inflation eventually returned to a positive target level, while in Japan it did not. In other words, the presence of a *global* component in Japan's inflation forecasting errors may account for temporarily low inflation and interest rates. It cannot, however, account for the *permanent* reduction in actual inflation and in long term inflation expectations. These are the defining and *idiosyncratic* facts specific only to Japan's experience. They can be explained if one considers the possibility that, along with global inflation shocks, Japan also experienced an underlying deanchoring of inflation expectations, which led to the fall into a self-fulfilling liquidity trap. To strengthen this point, the right hand side panel of Figure 3 depicts the result of removing the global forecasting error (proxied by the average forecasting errors across the US, the UK and Germany) from expected inflation in Japan. It is clear that this idiosyncratic-only measure of expected inflation tracks remarkably well actual inflation and the progressive fall into the permanent liquidity trap.[6] Under this light, the slow deanchoring of inflation expectations started at a very early stage of Japan's lost decade. Motivated also by the discussion in this section, we now move to a formal analysis of self-fulfilling liquidity traps in a standard sticky-prices general equilibrium model.

3 Self-fulfilling deflations in a three equation model

The full microfounded model on which the paper is based is, in its general parametrization form, quite cumbersome to present. To develop intuition for the main theoretical results I

[5] In the Eggertsson and Woodford (2003) framework, a fully anticipated sequence of positive shocks to the rate of time preference can generate a liquidity trap with, in addition, no deviation between actual and expected inflation. However, in this case, the model also predicts that during the duration of the liquidity trap the growth rate of output should be particularly large (Schmitt-Grohé and Uribe [2016]). For a discussion of the empirical challenges in modeling liquidity traps as driven by fundamental shocks see also Andolfatto and Williamson (2015).

[6] This conclusion is robust to the choice of a one-year forecast period or to the use of qualitative measures of Japanese households' inflation expectations, quantified through an appropriately modified Carlson-Parkin (1975) procedure (see Ueda [2010]). These results are available upon request to the author.

therefore start by introducing directly the three main equilibrium equations, obtained for a particularly convenient calibration of the model's parameters. In Section 4 I give a formal presentation of how these three equations are derived and generalized in the full microfounded model. The starting point of our discussion is the case where monetary policy is the only policy in the hands of the government. Section 3.3 extends the analysis to include fiscal policy.

3.1 Properties of the monetary policy rule

This section presents the main conceptual building blocks for our study of self-fulfilling liquidity traps. First of all, I introduce the three equations that are commonly used to describe the aggregate behavior of the economy when monetary policy follows an interest rate feedback-rule. I intuitively explain why this system of equations can give rise, depending on the particular interest rate rule assumed, to multiple equilibria that are uniquely pinned down by an initial *sunspot* state of the economy. I then move to provide a more concrete characterization of the set and dynamic properties of sunspot equilibria. Doing this requires postulating some minimal properties for the monetary policy rule.

The three equations that define the aggregate behavior of the economy are the following:

$$\frac{\hat{Y}_{t+1}}{\hat{Y}_t} = \beta(1 + r_{t+1}) \tag{1}$$

$$\pi_t = -\chi + \kappa \hat{Y}_t + \beta \pi_{t+1} \tag{2}$$

$$r_{t+1} = r(\pi_t, \pi_{t+1}) \tag{3}$$

where $\hat{Y}_t = \frac{Y_t}{Y_t^{TG}}$ is the deviation of output Y_t from the Pareto optimal (target) output Y_t^{TG}. Moreover, π_{t+1} and r_{t+1} are respectively the inflation rate and the real interest rate between time t and time $t+1$. The constants χ, κ and β are all strictly positive, with $\beta < 1$. The Wicksellian natural real interest rate r^n is defined as

$$r^n = \beta^{-1} - 1 > 0$$

Equation (1) can be interpreted as resulting from the consumption Euler equation of the household's maximization problem. Equation (2), instead, defines a forward-looking Phillips curve, relating current inflation to current output and to future inflation. Finally, equation (3) is the monetary policy rule, that defines the real rate as a function of current and future inflation. As I show in Section 4, equation (2) is the true forward-looking Phillips curve derived in the microfounded model.[7] This is important since, by avoiding linearizing the aggregate equations, I am thus able to legitimately use (1)-(3) to perform a global equilibrium analysis.

The interest rate rule in equation (3) was presented in the form of a *real* interest rate rule, rather than the more commonly used *nominal* interest rate rule $i_{t+1} = i(\pi_t, \pi_{t+1})$. This choice is made for the purpose of reducing the notation and simplifying the exposition, but is not restrictive in any way. The reason is that the Fisher condition always provides us with a one to one relation between a nominal and a real interest rate rule. In other words, for any given vector (π_t, π_{t+1}) we can always uniquely recover one rule from the other by using the

[7] Up to a harmless change of variables.

equivalence[8]

$$1 + i(\pi_t, \pi_{t+1}) = [1 + r(\pi_t, \pi_{t+1})](1 + \pi_{t+1}) \tag{4}$$

If we substitute the rule (3) into (1) we obtain a non-linear system of two first order difference equations in the variables $\hat{Y}_t$ and π_t. Hence, given two arbitrary conditions (both for $\hat{Y}_t$, or both for π_t, or one condition for each of the two variables), the dynamic evolution of the system is uniquely determined. Define $S_0 = (\pi_0, \pi_1)$ as the initial sunspot *state* of the economy. Given S_0, the system (1)-(2) gives a unique evolution of the state $S_t(\pi_t, \pi_{t+1})$ at all $t \geq 2$.[9] I postulate that any interesting candidate equilibrium for the economy must satisfy two sets of conditions:

$$\begin{aligned} S_t &\to S \in \mathbb{R}^2 \\ \hat{Y}(S_t) &> 0 \end{aligned} \tag{5}$$

The first of the two conditions restricts our attentions to dynamics where the variables converge to some finite steady state value. This is not necessarily an obvious requirement for a candidate equilibrium (for instance, this requirement rules out not only cyclical behaviors in inflation, but also explosive inflationary paths), but it is routinely done in most of the neo-Keynesian literature. The second condition naturally requires that output must always be strictly positive.

The *target* steady state $S^{TG} = (\pi^{TG}, \pi^{TG})$, with the target inflation given by $\pi^{TG} = 0$ is of particular importance in the analysis. By setting $\kappa = \chi$, an assumption that I maintain throughout, the target steady state becomes also the Pareto optimal level of output (i.e. $\hat{Y}_t = 1$). For S^{TG} to be a steady state of the economy, we must require that

$$r(0, 0) = r^n > 0 \tag{6}$$

The issue of the existence of multiple equilibria boils down to finding whether, in addition to $S_0 = S^{TG}$, we can find other initial states $S_0 \neq S^{TG}$ that induce a dynamic evolution of the

[8] It is worth emphasizing that casting monetary policy in terms of real interest rate rules is not equivalent to assuming that the monetary authority has the power to affect as it pleases the *equilibrium* real interest rate. To see this, consider the case of flexible prices where we know that, *in equilibrium*, the real interest rate must always equal the natural real rate, i.e. $r(\pi_t^*, \pi_{t+1}^*) = r^n$ for all t. Consider a nominal rule such that $1 + i(\pi_t, \pi_{t+1}) = (1 + r^n)(1 + \zeta\pi_{t+1})$ for some $\zeta > 1$. It is easy to verify that this nominal rule is equivalently expressed via a real rule $1 + r(\pi_t, \pi_{t+1}) = (1 + r^n)(1 + \zeta\pi_{t+1})/(1 + \pi_{t+1})$. The real rule prescribes a real rate strictly smaller (strictly greater) than the natural rate r^n whenever the future inflation rate π_{t+1} is strictly smaller (strictly greater) than the zero target. This is a perfectly legitimate choice for a rule, yet it does not imply that the monetary authority has the power to steer the real interest rate away from the natural rate. It only implies that any state where $\pi_{t+1} \neq 0$, and thus where $r(\pi_t, \pi_{t+1}) \neq r^n$, must be *off the equilibrium path*. Indeed, under such rule for any $t > 1$ there is a unique equilibrium for the economy, and this equilibrium coincides with the steady state $\pi_t^* = 0$, where the real rule is consistent with $r(\pi_t^*, \pi_{t+1}^*) = r^n$. It is straightforward to see that, once a zero lower bound on the nominal rate is added, the (modified) rule above gives again rise to a second steady state, and thus to multiple equilibria, exactly as explained in Figure 2.

[9] As a joint-product of the sequence $\{S_t\}_{t=0}^{\infty}$, we obtain also a unique sequence $\{\hat{Y}_t\}_{t=0}^{\infty}$. As mentioned above, the definition of an appropriate state of the system is arbitrary, and subject only to the requirement that it is a vector of two variables. For instance, instead of using as a state the vector (π_t, π_{t+1}), we could as well use the vector $(\pi_t, \hat{Y}_t)$. For any relevant purpose the particular choice we make is inconsequential. Still, I prefer to use the state $S_t = (\pi_t, \pi_{t+1})$ because notationally this choice lends itself better to collapsing, as I do below, the entire system (1)-(3) into one second order difference equation in just the variable π_t.

S_t that satisfies (1)-(3) and (5). To answer this question, we need to provide some minimal structure to the real interest rate rule $r(S_t)$ beyond condition (6). A first basic requirement is that the nominal interest rate must satisfies a zero lower bound condition. Using (4) this becomes,

$$\log[1 + r(\pi_t, \pi_{t+1})] \geq -\log(1 + \pi_{t+1}) \qquad \forall (\pi_t, \pi_{t+1}) \in \mathbb{R}^2 \tag{7}$$

Another assumption is that the monetary policy rule is a continuous function of the state of the economy and it is monotonically increasing in π_t, i.e. it calls for a reduction in the nominal (and thus also in the real) interest rate whenever current inflation goes down. Similarly to Figure 2, define π^{LT} implicitly as $\log(1 + r^n) = -\log(1 + \pi^{LT})$. Consider then the following assumption,

Assumption 1. *The real rate policy $r(\pi_t, \pi_{t+1})$ is continuous and decreasing in π_t and continuous in π_{t+1}. Moreover, $r(\pi, \pi) < r^n$ for every $\pi \in (\pi^{LT}, \pi^{TG})$.*

Imagine that the x-axis in Figure 2 represents the one-dimensional subspace of states $\bar{S}_t = (\pi_t, \pi_t)$ with the property that $\pi_t = \pi_{t+1}$. Then Assumption 1 requires that, at such states, the nominal interest rate $i(\bar{S})$ is below the 45-degree line (the FF curve), so that $r(\bar{S}) < r^n$. Along the lines of Figure 2, it is easy to show that Assumption 1 and the zero lower bound condition (1) imply $r(S^{LT}) = r^n$, where $S^{LT} \equiv (\pi^{LT}, \pi^{LT})$. The target and the liquidity trap states S^{TG} and S^{LT} are then the only stady states of the system.

The dynamic properties are obtained by studying the following second order difference equation, obtained by collapsing in one equation the system (1)-(3),

$$\frac{\chi + \pi_{t+1} - \beta\pi_{t+2}}{\chi + \pi_t - \beta\pi_{t+1}} = \frac{1 + r(\pi_t, \pi_{t+1})}{1 + r^n} \tag{8}$$

Let us restrict our attention to initial conditions S_0 with the property that $S_0 \in (S^{LT}, S^{TG})$. To characterize the possible equilibrium dynamic paths of the state S_t, it is useful to use the concept of *maximum reaction* of the monetary policy rule,

Definition 1. *The* maximum reaction $\bar{\phi}$ *of the monetary policy rule $r(S_t)$ is given by*

$$\bar{\phi} = \sup\left\{\phi \geq 1 | r(\pi, \phi\pi) < r^n, \quad \text{for some } \pi \in (\pi^{LT}, \pi^{TG})\right\} \tag{9}$$

The maximum reaction of monetary policy gives the highest possible gross rate $\bar{\phi} = \pi_{t+1}/\pi_t$ at which inflation could fall while still allowing the real interest rate to be smaller than the natural rate. Note that Assumption 1 guarantees that a $\bar{\phi} \geq 1$ actually exists. Monetary policy rules that are characterized by a higher value of $\bar{\phi}$ are referred to, in this paper, as policies that are more "stimulative".

It is instructive to show how $\bar{\phi}$ is characterized in the case of a monetary policy rule that has attracted much interest, i.e. a Taylor rule, defined by positive inflation and output gap parameters, respectively ϕ_π and ϕ_Y. Suppose, for the moment, that the static Taylor rule is unconstrained by the zero lower bound. Specifically, define the nominal interest rate policy $i_{t+1} = i(\pi_t, \pi_{t+1})$ in the following log-linear form

$$\log[1 + i(\pi_t, \pi_{t+1})] = \log(1 + r^n) + \phi_\pi[\log(1 + \pi_t) - \log(1 + \pi^{TG})] + \phi_Y \log \hat{Y}_t \tag{10}$$

Since $\pi^{TG} = 0$, the Fisher condition (4) gives the corresponding real interest rule as,

$$\log[1 + r(\pi_t, \pi_{t+1})] = \log(1 + r^n) + \phi_\pi \log(1 + \pi_t) + \phi_Y \log(\hat{Y}_t - 1) - \log(1 + \pi_{t+1}) \tag{11}$$

Linearizing around the target steady state, and using (2) to substitute for $\hat{Y}_t$, we obtain the approximate expression[10]

$$\log \frac{1+r(\pi,\phi\pi)}{1+r^n} \approx -\pi\left[\left(1+\frac{\beta\phi_Y}{\kappa}\right)\phi - \phi_\pi + \frac{\phi_Y}{\kappa}\right]$$

Hence, irrespective of which negative value for $\pi \in (\pi^{LT}, 0)$ we pick, the supremum $\bar{\phi}$ in (9) is found by setting to zero the difference $r^n(\pi, \bar{\phi}\pi) - r^n$, giving

$$\bar{\phi} = \frac{\phi_\pi + \frac{\phi_Y}{\kappa}}{1+\beta\frac{\phi_Y}{\kappa}} \tag{12}$$

If the nominal interest rate follows a Taylor rule then the maximum reaction $\bar{\phi}$ can be analytically expressed as a function of the Taylor parameters, up to a first order approximation. This result is unaffected by the inclusion of a zero lower bound.[11]

Finally, recall that Assumption 1 guarantees that $\bar{\phi} > 1$. For the special case (12) the condition $\bar{\phi} > 1$ takes a well-known significance: it is equivalent to the necessary and sufficient condition for a Taylor rule that satisfies the Taylor principle (see Woodford [2003], p. 254). This is a way to show that Taylor rules are, together with the parametric restrictions usually imposed on them, just a special case of the more general monetary rules considered here.

3.2 The equilibrium manifold

We are now ready to characterize the dynamic path of the economy starting from initial sunspots $S_0 \in (S^{LT}, S^{TG})$,

Proposition 1. *If Assumption 1 holds, then*

i) There exists a continuous function $\mathcal{M}(\pi)$ satisfying $\pi^{LT} < \mathcal{M}(\pi) < \pi$ and such that, if the initial sunspot state $S_0^* = (\pi_0^*, \pi_1^*)$ has $\pi_1^* = \mathcal{M}(\pi_0^*)$, the equilibrium inflation π_t^* monotonically decreases to π^{LT}.

ii) In addition, if $1 < \bar{\phi} \leq \beta^{-1}$, $\mathcal{M}(\cdot)$ satisfies

$$\mathcal{M}(\pi_0^*) \to \pi^{TG} \qquad \text{for } \pi_0^* \to \pi^{TG} \tag{13}$$

iii) Also, if $1 < \bar{\phi} \leq \beta^{-1}$ then along the equilibrium path $\hat{Y}_t^* < 1$.

Proof. See Appendix A. For the special case considered in this section the result is proved by setting $\gamma = \alpha = 1$, $\psi = 0$, and fiscal variables ω equal to 1. □

[10] Around the target steady state $\pi_t = \pi_{t+1} = 0$ and $\hat{Y}_t = 1$, so that the linearized equation takes the usual form $\log \frac{1+r(\pi_t,\pi_{t+1})}{1+r^n} = \phi_\pi \pi_t + \phi_Y(\hat{Y}_t - 1) - \pi_{t+1}$.

[11] If the rule is constrained by the zero lower bound, then the nominal rate cannot be decreased indefinitely. Hence, for a level of inflation π_t low enough, the constrained rule becomes *less* "stimulative" than the unconstrained rule. Still, the value of $\bar{\phi}$ is not affected when the zero lower bound constraint is added, since $\bar{\phi}$ is calculated using a *sup* operator, i.e. it is calculated over the states where the rule is *most* "stimulative".

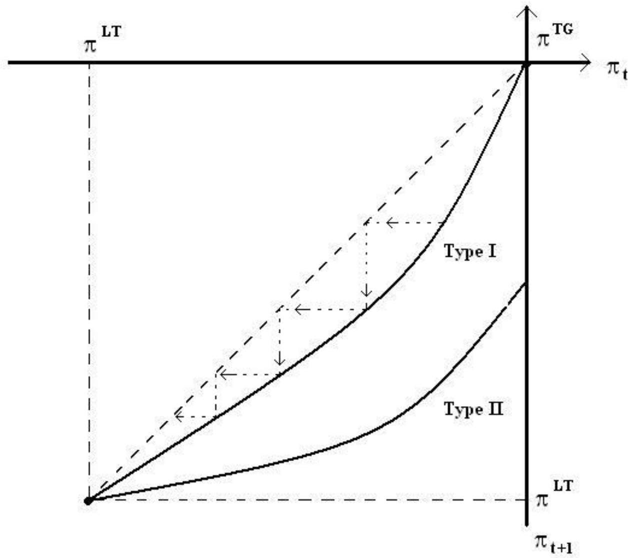

Figure 4: The two types of equilibrium manifold. The dashed line is the 45 degree line.

Figure 4 provides a graphical representation of the results in Proposition 1. The figure depicts the two steady states S^{LT} and S^{TG} alongside with two possible versions of the equilibrium manifold $\pi_{t+1} = \mathcal{M}(\pi_t)$.

A *sufficient* condition to obtain a *Type I* saddle connection is that $1 < \bar{\phi} < 1/\beta$, i.e. the monetary policy rule is "not too stimulative". In this case, any arbitrarily small sunspot deviation π_1^* of inflation from the target is associated with a monotonic deflationary path leading to the (deanchored) deflationary steady state. In addition, output is always below the target level. Notice that $1 < \bar{\phi} < 1/\beta$ is only a sufficient condition, meaning that *Type I* saddle connections *may* arise even for values $\bar{\phi} > 1/\beta$ (in this case, though, it is not guaranteed that $\hat{Y}_t^* < 1$ at all times). If the monetary rule is instead sufficiently stimulative, and thus $\bar{\phi}$ is large enough, then the equilibrium manifold is of *Type II.* In this case a discrete downward shift in inflation expectations π_1^* is required for a monotonic deflationary path to form.

Taylor rules: a special case. Taylor rules are a special case of monetary policy rules that allow to connect the general results of Proposition 1 to the seminal work by Benhabib, Schmitt-Grohé, Uribe (2001). For instance, to replicate the Taylor rule specification in Benhabib, Schmitt-Grohé, Uribe (2001), we need the additional restriction $\phi_Y = 0$, thus obtaining $\bar{\phi} = \phi_\pi$. In this case, Proposition 1 tells us that if ϕ_π is large enough, then the equilibrium manifold is of *Type II.* In this case, the manifold corresponds to the decreasing arm of the oscillatory path identified by Benhabib, Schmitt-Grohé, Uribe (2001).[12] Instead, if ϕ_π is greater than 1 but small enough, then Proposition 1 guarantees that the equilibrium is of *Type I.* The possible existence, in sticky price models, of a *Type I* saddle had not been previously

[12]If the stable manifold is of *Type II*, then the manifold clearly extends also to initial inflation levels $\pi > \pi^{TG}$. The characterization of this additional section of the manifold requires assumptions on the properties of $r(S)$ even at states that don't satisfy $S < S^{TG}$. Such analysis is beyond the scope of this paper, which focuses only on characterizing monotonic deflationary paths for initial sunspot expectations at or below the target.

identified in the literature, not even in the special setting of Benhabib et al. (2001). The existence of such paths was previously proved only when, in addition to the aforementioned Taylor assumptions, prices were assumed to be perfectly flexible (Benhabib et al. [2001], Cochrane [2011]).

While the sufficient condition $\bar{\phi} < \beta^{-1}$ appears to be quite tight, the quantitative analysis in Section 5 shows that *Type I* connections arise for empirically relevant calibrations. For instance, a "not too stimulative" Taylor rule with $\phi_\pi = 1.25$ and $\phi_Y = 0.5$ yields a maximum reaction $\bar{\phi} = 1.065$, just slightly above a standard threshold $\beta^{-1} = 1.031$. Numerical simulations show that $\bar{\phi}$ is so close to its sufficient condition threshold that the equilibrium manifold is indeed of *Type I.*

Example of a "neo-Fisherian" policy. Taylor rules certainly do no exhaust the set of monetary rules that one may want to consider. For instance, a central bank may be interested in knowing whether a proposed new monetary rule can rule out once and for all the possibility of self-fulfilling deflationary paths. The general applicability of Proposition 1 and of its sufficient conditions help tackling this issue in an easy way. Consider for example the following nominal rule,

$$i(\pi_t, \pi_{t+1}) = r^n + \phi_\pi \pi_t + \phi_\Delta(\pi_t - \pi_{t+1})$$

for $\phi_\pi > 1$ and $\phi_\Delta > 0$. The proposed rule is like a standard Taylor rule but with the addition of a term $\phi_\Delta(\pi_t - \pi_{t+1})$ which responds to changes in inflation. The proposed rule is "neo-Fisherian", in the sense that there are states where current and future inflations are strictly below the zero target, and yet the rule calls for *increasing* the nominal rate above r^n. This happens anytime the expected fall $\pi_t - \pi_{t+1}$ in inflation is large enough. Schmitt-Grohé and Uribe (2016) have recently advocated in favor of a (discontinuous) rule that *raises* the nominal interest rate once the economy is in a liquidity trap.

Is the rule effective in preventing the economy from falling into monotonic deflationary paths? Answering this question in a traditional way would require trying to characterize the set of solutions to the nonlinear system of difference equations (1)-(3) under this new, previously unexplored, rule. This task may be hard (Fernandez-Villaverde [2014]), but Proposition 1 provides us with a simple shortcut. It is easy to verify that, under the usual ZLB constraint, the proposed rule satisfies Assumption 1, and the only steady states are, as usual, π^{TG} and π^{LT}. By Proposition 1, these observations are enough to conclude that the proposed rule is still subject to self-fulfilling and monotonic deflationary paths for any initial sunspot $\pi^* \in (\pi^{LT}, \pi^{TG})$. But we can also say something more once, by the usual approximation techniques, we compute the *maximum reaction* parameter of the rule as

$$\bar{\phi} = \frac{\phi_\pi + \phi_\Delta}{1 + \phi_\Delta} > 1$$

For any possible value of $\phi_\pi > 1$, there always exists a ϕ_Δ large enough for which $\bar{\phi} < \beta^{-1}$. Proposition 1 therefore implies that if the "neo-Fisherian" response ϕ_Δ is large enough, then the saddle connection becomes of *Type I.* The policy fails in preventing the occurrence of self-fulfilling deflationary paths.

The mechanics of self-fulfilling deflations. It is worth concluding this section with some intuition on why a monetary policy rule that satisfies the properties laid out in Assumption 1 allows the emergence of deflationary equilibrium paths. Suppose that from time $t+1$ onwards inflation is *anchored* at the target level, which implies, in particular, that $\hat{Y}_{t+1} = 1$. Can a deflation at time t temporarily push the economy to a sub-optimal level (i.e. $\hat{Y}_t < 1$)? The answer is no, as long as monetary policy is sufficiently "stimulative of current demand". In fact, if $\hat{Y}_t < 1$ and simultaneously $r_{t+1} < r^n$, then the left hand side of (1) would be strictly bigger than one, while the right hand side would be smaller than one. Hence, as long as $\hat{Y}_{t+1} = 1$ and monetary policy stands ready to react in a sufficiently stimulative way, no equilibrium can form with $\hat{Y}_t < 1$. The target steady state is dynamically unstable and thus, locally, it is the unique equilibrium (Woodford [2003]).

However, as King (2000) points out, in the neo-Keynesian framework, which incorporates the rational expectations framework, «macroeconomic analysis can[not] be conducted by simple curve-shifting», since it is «necessary to solve simultaneously for current and expected future variables». This means that, in our example, the expected value of output $\hat{Y}_{t+1}$ has to be solved simultaneously with the rest of the equilibrium variables. Consider again the possibility that $\hat{Y}_t < 1$. If $r_{t+1} < r^n$ then monetary policy is stimulating the *relative* demand of goods at time t. However, differently from before, assume now that this intertemporal stimulus, instead of being associated with an increase in current demand $\hat{Y}_t$, is associated with a self-fulfilling pessimistic expectation of reduction in future demand, i.e. $\hat{Y}_{t+1} < \hat{Y}_t < 1$. An equilibrium of this type, where now $\hat{Y}_{t+1}$ is away from the target, can indeed exist. In conclusion, once we select an equilibrium with expectations *deanchored* from the target, then a sequence of low real (and nominal) interest rates is endogenously consistent with a series of pessimistic expectations about the evolution of future demand and thus, in turn, with an expected fall in inflation.

3.3 Fiscal policy

When the nominal interest rate has already reached the zero lower bound and inflation expectations are *anchored*, fiscal policy can play a useful role in fighting a deflation triggered by shocks to fundamentals (Christiano, Eichenbaum, Rebelo [2011]). However, this conclusion does not hold when inflation expectations are *deanchored*. This section generalizes Proposition 1 and shows in fact that monotonic and self-fulfilling deflationary paths are not eliminated when "stimulative" fiscal policies are brought into the picture. The introduction of fiscal policy can even make things worse. For instance, if fiscal policy is required to be time-consistent then the deflationary equilibrium is always of *Type I*, irrespective of the parameter $\bar{\phi} < \infty$ for the monetary policy rule.

Fiscal policies, in the form of either distortionary taxation on consumption and production inputs, or in the form of wasteful government spending, have the effect of introducing "wedges" which distort the dynamics of the system (1)-(3) (Chari, Kehoe, McGrattan [2007], Correia, Farhi, Nicolini, Teles [2013]). Such distortions show up, in a microfounded model, in the form of time-varying and endogenous variables $\tilde{\beta}_{t+1} = \beta\omega^d_{t+1}$ and $\tilde{\kappa}_t = \kappa\omega^s_t$ which replace, respectively, the constants β and κ in (1)-(3). I call the variables ω^d_{t+1} and ω^s_t, respectively, the *dynamic* and *static wedges* at time t induced by fiscal policy in the following way (for a detalied derivation see Section 4),

$$\begin{aligned} \omega^d_{t+1} = \omega^G_{t+1}\omega^C_{t+1} &\equiv \frac{1+\sigma^G_{t+1}}{1+\sigma^G_t}\frac{1+\tau^C_t}{1+\tau^C_{t+1}} \\ \omega^s_t &\equiv \frac{1+\tau^C_t}{(1+\sigma^G_t)(1-\tau^N_t)} \end{aligned} \tag{14}$$

where τ^C_t is the tax rate on consumption, τ^N_t is the tax rate on labor (the only input in production), and σ^G_t indicates the amount of government consumption expressed as a share of private consumption. Notice that we can decompose the dynamic wedge into the product of a government spending wedge $\omega^G_{t+1} = (1+\sigma^G_{t+1})/(1+\sigma^G_t)$ and a consumption tax wedge $\omega^C_{t+1} = (1+\tau^C_t)/(1+\tau^C_{t+1})$. The dynamic and static wedges are related by,

$$\omega^d_{t+1} = \omega^N_{t+1}\frac{\omega^s_t}{\omega^s_{t+1}}$$

where $\omega^N_{t+1} = (1-\tau^N_t)/(1-\tau^N_{t+1})$. If wedges $\omega(S)$ are a function of the state of the economy, then fiscal policy is expressed in the form of rules. The Euler equation and forward-looking Phillips curve become,

$$\frac{\hat{Y}_{t+1}}{\hat{Y}_t} = \beta\omega^d(S_t)[1+r(S_t)] \tag{15}$$

$$\pi_t = -\chi + \kappa\omega^s(S_t)\hat{Y}_t + \beta\omega^d(S_t)\pi_{t+1} \tag{16}$$

In order to solve (15)-(16) we need to impose some structure on the wedge policies. Let us focus first on the dynamic wedge. Notice that monetary policy and fiscal policy interact in equation (15) simply through the product $\omega^d_t(1+r_t)$. Then, from the viewpoint of the demand equation (15) government interventions through the monetary policy $r(S)$ or the fiscal policy tools $\omega^d(S)$ are equivalent. Given this complementarity between monetary and fiscal policy along the dynamic dimension, it is then natural to require that Assumptions 1 made for the monetary policy rule are inherited by the rule $\omega^d(S) = \omega^G(S)\omega^C(S)$.

Assumption 2. *Policies $\omega^G(S_t)$ and $\omega^C(S_t)$ are continuous and increasing in π_t and continuous in π_{t+1}. Moreover, $\omega^G(\pi,\pi) \leq 1$ and $\omega^C(\pi,\pi) \leq 1$ for every $\pi \leq \pi^{LT}$, with equality if $\pi \in \{\pi^{TG}, \pi^{LT}\}$.*

At any time t a strictly positive fiscal stimulus is provided if $\omega^C(S_t) < 1$ or $\omega^G(S_t) < 1$. In turn, this requires that consumption taxes increase, and the share of government spending falls, between time t and $t+1$. This is consistent with standard results in the neo-Keynesian literature: to stimulate aggregate demand at time t the government should engineer a sequence of consumption tax hikes or front-load its public spending (Correia, Farhi, Nicolini, Teles [2013]). Similarly to the case of the interest rate policy $r(\cdot)$, the requirement that wedges are increasing in current inflation means that the stimulus is smaller the closer current inflation $\pi_t < \pi^{TG}$ is to the target. To complete the characterization of the dynamic wedge ω^d we add the following,

Assumption 3. *At any state S_t with $\pi^{LT} \leq \pi_{t+1} \leq \pi_t \leq 0$, $v_1(S_t) < 0$ and $v_2(S) > 0$, where the function $v(S_t)$ is defined implicitly by*

$$\chi v(\pi_t, \pi_{t+1}) = \tilde{\beta}(\pi_t, \pi_{t+1})\pi_{t+1} - \pi_t \tag{17}$$

I will discuss momentarily the intuitive rationale for imposing Assumption 3. For now notice that, under the definition $\tilde{\beta}(S_t) \equiv \beta\omega^d(S_t)$ provided above, Assumption 2 already ensures that $v_1(S) < 0$ for $S < S^{TG}$. Moreover, while Assumption 2 imposes no condition on the behavior of $\omega^d(\pi_t, \pi_{t+1})$ with respect to π_{t+1}, Assumption 3 instead requires $v_2(\pi_t, \pi_{t+1}) > 0$, i.e. the (negative) product $\omega^d(\pi_t, \pi_{t+1})\pi_{t+1}$ must be increasing in π_{t+1}.

To characterize the static wedge $\omega^s(S)$ we are now left with the task of postulating some properties about the labor tax policy. Consider the following,

Assumption 4. *The policy $\omega^N(S_t)$ is continuous and increasing in π_t and continuous in π_{t+1}. Moreover, $\omega^N(\pi, \pi) \leq 1$ for every $\pi \leq \pi^{LT}$, with equality if $\pi \in \{\pi^{TG}, \pi^{LT}\}$.*

Remember that we want to restrict our attention to policies whose intended goal is to stimulate the economy when inflation falls below target. At the very least, this stimulus must be provided at deflationary states $S(\pi, \pi)$ with $\pi < \pi^{TG}$. In these cases, Assumption 4 requires that $\omega^N(\pi, \pi) \leq 1$, i.e. the labor tax rate at time t is higher than that at time $t+1$. Similarly to the case of government spending policies, economic activity is stimulated at time t if the labor tax path is front-loaded at time t. It may appear counterintuitive to say that raising labor taxation at time t is a way to stimulate the economy. Yet this is a standard result, also known as "paradox of toil", which has been highlighted in the context of neo-Keynesian models with anchored long-run inflation expectations (Eggertsson [2010b]). By creating an artificial reduction in today's supply curve - either by taxing factors of production or by wasting output (government spending) - inflation is increased, the real interest rate is reduced, and today's aggregate demand is stimulated.

Assumptions 3-4 work in the direction of preserving standard features of neo-Keynesian analyses. To see this assume, for instance, that the dynamic wedge is constant and equal to 1 and that the labor tax policy is represented by a decreasing function $\tau_t^N = \tau^N(\pi_t)$, for $\pi_t \leq \pi^{TG}$. Notice that this labor tax policy gives rise to a wedge $\omega^N(\pi_t, \pi_{t+1})$ that satisfies Assumption 4. Re-arranging the forward-looking Phillips curve, (16) we have,

$$\hat{Y}_t = [1 - \tau^N(\pi_t)][1 - v(\pi_t, \pi_{t+1})]$$

Assumptions 3-4 then guarantee that an increase in current inflation π_t is associated with an improvement in the current output gap $\hat{Y}_t$. This relation between current output and current inflation is a defining element of neo-Keynesian models. Notice also that, in our example, the assumption $v_2(\pi_t, \pi_{t+1}) > 0$ implies that a credible future disinflation increases current output, which is another standard property of neo-Keynesian models (Ball [1994]).[13]

Define,

$$\bar{\beta} \equiv \max_{\{\pi^{LT} \leq \pi_{t+1} \leq \pi_t \leq 0 | v(\pi_t, \pi_{t+1}) = 0\}} \tilde{\beta}(\pi_t, \pi_{t+1}) \tag{18}$$

[13]For a constant dynamic wedge, from equation (16) we can write $\hat{Y}_t = \omega^N(\pi_t, \pi_{t+1})[1 - v(\pi_t, \pi_{t+1})]/[1 - v(\pi_{t+1}, \pi_{t+2})]\hat{Y}_{t+1}$. Iterating forward one period we have,

$$\hat{Y}_t = \omega^N(\pi_t, \pi_{t+1})\omega^N(\pi_{t+1}, \pi_{t+2})\frac{1 - v(\pi_t, \pi_{t+1})}{1 - v(\pi_{t+1}, \pi_{t+2})}\hat{Y}_{t+2}$$

$\hat{Y}_{t+2}$ is a function of inflation rates $\pi_{t+2}, \pi_{t+3}, \ldots$. By Assumptions 3-4, $\hat{Y}_t$ is then increasing in π_t. To guarantee, in addition, that a credible disinflation has expansionary effects, further assumptions would have to be placed on ω^N such, as in the example provided in the text, $\omega^N(\pi_t, \pi_{t+1})\omega^N(\pi_{t+1}, \pi_{t+2})$ weakly decreasing in π_{t+1}.

$$1 + \tilde{r}_t \equiv \omega_t^N (1 + r_t) \tag{19}$$

$$1 + \tilde{f}_t \equiv \omega_t^d (1 + r_t) \tag{20}$$

In the presence of fiscal policy rules we need to make two modifications to the way we calculated, in Proposition 1, the threshold condition $\bar{\phi} < \beta^{-1}$. These modifications take into account that additional "stimulus" is now provided by fiscal policy, through both the dynamic and the static (the labor tax policy, in particular) wedges. First, the threshold value for the maximum reaction parameter is not β^{-1} anymore, since the quantity $\tilde{\beta}(S) = \beta\omega^d(S)$ is now state dependent. It is possible to show that to account for this it is sufficient to set the new threshold to $\bar{\beta}^{-1}$, where $\bar{\beta}$ is the maximum reached by $\tilde{\beta}(\pi_t, \pi_{t+1})$ over all the decreasing sequences $\pi_{t+1} \leq \pi_t$ such that $v(\pi_t, \pi_{t+1}) = 0$.[14] The second modification is that the relevant maximum reaction parameter $\bar{\phi}^{\tilde{r}}$ is now calculated with respect to the function $\tilde{r}(\cdot)$ in (19).

Additionally a new result is obtained when the maximum reaction $\bar{\phi}^{\tilde{f}}$ is calculated with respect to $\tilde{f}(\cdot)$ and the labor tax policy is *time consistent*. The definition of time consistency can be understood quite intuitively. Recall that Y^{TG} indicates the target (the "Pareto optimal", in a microfounded model) level of output. Consider an exogenously given function $\Xi(\pi_t) \geq 0$ that represents the time t loss, in percentage of Y^{TG}, of economic efficiency due to the fact that inflation π_t deviates from the zero target $\pi^{TG} = 0$. Conditional on this loss, the ("constrained Pareto optimal") target level output is reduced to $Y_t = Y^{TG}/[1 + \Xi(\pi_t)]$, or $\hat{Y}_t = [1 + \Xi(\pi_t)]^{-1}$. The *time consistent* labor tax policy is thus defined from (16) as the unique function $\tau^N(S_t)$ such that $\hat{Y}(\pi_t, \pi_{t+1}) = [1 + \Xi(\pi_t)]^{-1}$ for all states $S_t = (\pi_t, \pi_{t+1})$, taking as given the other fiscal policies. We then have the following generalization of Proposition 1,

Proposition 2. *Under Assumptions 2-4 result* i) *of Proposition 1 holds. Result* ii) *holds under the new condition* $\bar{\phi}^{\tilde{r}} < \bar{\beta}^{-1}$. *In addition, result* iii) *holds if the labor tax policy keeps the static wedge constant across all states. Moreover, if* $\bar{\phi}^{\tilde{f}} < \infty$ *and the labor tax policy is* time-consistent*, then the deflationary path is always of* Type I*, with* $\hat{Y}_t^* < 1$ *if* $\sigma_t^G = 0$.

Proof. See appendix A. For the treatment of the cases with constant static wedge and time consistent labor tax see appendix B. □

Assumptions 2-4 encompass the set of typical expansionary fiscal policies aimed at fighting deflations in standard neo-Keynesian analyses. Proposition 2 states that, under such policies and appropriate modifications of the threshold parameters, the main results of Proposition 1 still hold. In particular, result *iii)* is extended to all cases when the static wedge is constant (the situation presented in Proposition 1 is just a special case among them). Finally, a new result shows that if labor taxation is time-consistent, then the equilibrium is of *Type I* under the very mild condition that $\bar{\phi}^{\tilde{f}} < \infty$.[15] This indicates that the introduction of fiscal policy can have important effects in relaxing the condition for which, given a certain monetary policy rule, a *Type I* equilibrium emerges.

[14] Notice that $v(0,0) = 0$ and $\tilde{\beta}(0,0) = \beta$, so that $\bar{\beta} \geq \beta$.

[15] For instance, if $\omega^G(0, \pi_{t+1}) \geq 1$, $\omega^C(0, \pi_{t+1}) \geq 1$ and $r(0, \pi_{t+1}) \geq r^n$ for any $\pi_{t+1} < 0$ (i.e. no stimulus is provided when current inflation is at the target $\pi^{TG} = 0$) then $\bar{\phi}^{\tilde{f}} < \infty$.

4 The microfounded model

The equilibrium equations analyzed so far are a special case of those derived from a standard microfounded neo-Keynesian model where labor is the only productive factor and sticky prices are modeled à la Rotemberg. Some of the key parameters are the coefficient of relative risk aversion $\gamma \geq 1$, the inverse of the Frisch elasticity of labor supply $\psi \geq 0$ and the labor share $\alpha \in (0,1)$. The results of Proposition 2 are proved in appendix A and B under this general calibration, provided $\tilde{\beta}$, $\tilde{r}$ and $\tilde{f}$ in (18)-(20) are replaced, respectively, with the generalized versions (21)-(23):

$$\tilde{\beta}_t = (\beta \omega_t^C)^{\frac{1}{\gamma}} \omega_t^G (1+r_t)^{\frac{1-\gamma}{\gamma}} \tag{21}$$

$$1+\tilde{r}_t = \left(\omega_t^N\right)^{\frac{\gamma\alpha}{1+\psi+\alpha(\gamma-1)}} \left(\omega_t^G \omega_t^\Xi\right)^{\frac{\gamma(1+\psi-\alpha)}{1+\psi+\alpha(\gamma-1)}} \left(\omega_t^C\right)^{\frac{1+\psi-\alpha}{1+\psi+\alpha(\gamma-1)}} (1+r_t) \tag{22}$$

$$1+\tilde{f}_t = \left(\omega_t^G\right)^{\frac{\gamma(1+\psi)}{1+\psi+\alpha(\gamma-1)}} \omega_{t+1}^C (1+r_t) \tag{23}$$

where the adjustment cost wedge is defined as $\omega_t^\Xi = \frac{1+\Xi(\pi_t)}{1+\Xi(\pi_{t-1})}$. The equilibrium equations and functional forms discussed in the previous sections are obtained for the special calibration $\gamma = \alpha = 1$ and $\psi = 0$. The rest of this section presents in detail the microfounded model and provides the elements necessary to derive the equations above.

4.1 The problem of the agents

There are three classes of economic agents: households, a government and firms.

Households. The household maximizes its lifetime utility

$$\sum_{t=0}^{\infty} \beta^t u(C_t, N_t) \tag{24}$$

$$u(C_t, N_t) = \frac{C_t^{1-\gamma} - 1}{1-\gamma} - \frac{N_t^{1+\psi}}{1+\psi}$$

$$C_t = \left[\int_{[0,1]} c_t^{\frac{\theta-1}{\theta}}(j)dj\right]^{\frac{\theta}{\theta-1}}$$

subject to the budget and cash constraints

$$(1+\tau_t^C)P_tC_t + B_{t+1} + M_{t+1} = M_t + (1+i_t)B_t + \frac{\theta}{\theta-1}(1-\tau_t^N)W_tN_t + \Pi_t - P_tT_t$$

$$\frac{M_{t+1}}{P_{t+1}} \geq m_{t+1}$$

with $m_t \geq 0$ an exogenous bounded sequence of real balances, and total spending P_tC_t on each of the consumption good variety j is defined by $P_tC_t = \int_{j\in[0,1]} p_t(j)c_t(j)dj$. Distortionary tax wedges on consumption and labor are, respectively, $1+\tau_t^C$ and $\frac{\theta}{\theta-1}(1-\tau_t^N)$. Lump sum taxation is T_t, Π_t are profits received by the household from firms, and B_t is nominal government debt purchased by the household. The coefficients of relative risk aversion satisfies $\gamma \geq 1$.

In equilibrium all firms charge the same price P_t, therefore the household's optimal demand for variety j produced by a (deviant) firm charging the price $p_t(j)$ is given by

$$\frac{c_t(j)}{C_t} = \left[\frac{P_t}{p_t(j)}\right]^{\theta} \tag{25}$$

Since in equilibrium we will have $p_t(i) = P_t$ and then $c_t(i) = C_t$, the optimal intertemporal allocation of consumption requires that the following Euler equation holds

$$\left(\frac{C_{t+1}}{C_t}\right)^{\gamma} = \beta \frac{1+\tau_t^C}{1+\tau_{t+1}^C}(1+r_{t+1}) \tag{26}$$

where $1 + r_{t+1} = \frac{P_t}{P_{t+1}(1+i_{t+1})}$ is the real interest rate between time t and time $t+1$.

The household's optimal choice of time allocation between leisure and labor gives the labor supply schedule

$$C_t^{\gamma} N_t^{\psi} = \frac{\theta}{\theta - 1} \frac{(1-\tau_t^N)W_t}{(1+\tau_t^C)P_t} \tag{27}$$

Optimal monetary holdings are given by

$$\frac{M_{t+1}}{P_{t+1}} = m_t \qquad i_{t+1} > 0 \tag{28}$$

while the combination of M_t and B_t is undetermined if $i_{t+1} = 0$. As underlined below, the money quantity M_t will play no role in our analysis so that, following much of the current Neo-Keynesian literature, we may even consider a cashless economy where $M_t = 0$. Still, to stress the assumed Ricardian nature of the economy, it is instructive to explicitly maintain, at least for the moment, the presence in the equilibrium equations of the money quantity. In particular, to guarantee the optimality of the household's consumption plan, I impose the following transversality condition to the problem,

$$\lim_{t\to\infty} \beta^t u_{C,t} \frac{B_t + M_t}{P_t} = 0 \tag{29}$$

Equations (26), (27) and (28) provide, together with the individual demand (25) and the transversality condition (29), the necessary and sufficient conditions for the optimality of the household's plan.

Government. I consider a Ricardian environment where government policies are divided into two groups, *instrumental* and *non-instrumental*. Instrumental policies comprise the nominal interest rate i_{t+1}, the government spending on final goods, and the distortionary taxes (on consumption and labor) levied. Instrumental policies are those that the government uses to directly influence the equilibrium of the economy. In particular, government's aggregate purchases of final goods are given by

$$G_t = \left[\int_{[0,1]} g_t^{\frac{\theta-1}{\theta}}(j)dj\right]^{\frac{\theta}{\theta-1}}$$

while purchases of the individual variety j are

$$\frac{g_t(j)}{G_t} = \left[\frac{P_t}{p_t(j)}\right]^\theta \tag{30}$$

Conversely, non-instrumental policies are either set arbitrarily or are just an endogenous by-product of instrumental policies. Specifically, the set of non-instrumental policies comprises the supply M_t^s and B_t^s of money and of one period bonds, and the levying of lump sum taxation T_t. I assume that, given the evolution of prices P_t, the stock of money issued is set to

$$M_t^s = \bar{m}P_t \tag{31}$$

for given constant real balances $m_t = \bar{m} \geq 0$. Similarly, I assume that the nominal government debt B_t^s issued equals a constant $\bar{B} \geq 0$. Lump sum taxes T_t are raised so that the following budget constraint of the government is satisfied period by period

$$P_tG_t + i_tB_t^s + \left[\frac{\theta}{\theta-1}(1-\tau_t^N) - 1\right]W_tN_t = B_{t+1}^s + \tau_t^C C_t + P_tT_t + M_{t+1}^s - M_t^s$$

I will return on the distinction between instrumental and non-instrumental policies in Section 4.2. For now, it is enough to point out that this distinction qualifies the model as Ricardian. The model is Ricardian in the sense that the particular mix between debt issuance and lump-sum taxation is irrelevant for the equilibrium of the economy. This justifies fixing, as I did, an arbitrary evolution of nominal debt, while letting lump sum taxes T_t automatically adjust to cover the higher (lower) interest rate expenditure stemming from an arbitrary higher (lower) level of public debt $\bar{B}$.

A somewhat subtler issue is whether, under the assumed evolution of the government's nominal liabilities B_t^S and M_t^S, the transversality condition (29) represents or not a redundant constraint for the equilibrium of the economy. To settle this point, which is beyond the scope of this paper, I take $\bar{B} = 0$ which, together with (31), guarantees that the transversality (29) is always automatically satisfied. The restrictions on the evolution of nominal public liabilities B_t and M_t are assumed, here, only to guarantee that the government does not commit to follow, *at the infinity*, policies that violate (29) and thus "blow up the economy". For more details see Cochrane (2011), who clarifies why, if the economy is assumed to behave in a Ricardian way (rather than, for instance, behaving in a non-Ricardian way as in Cochrane [2001]), then the use of the transversality condition as an equilibrium selection device (as in Benhabib, Schmitt-Grohé, Uribe [2002a]) is problematic.

Firms. Each firm in the production sector is indexed by the variety j it produces. Firms sell their output to households and to the government in a monopolistically competitive final goods market. Firms also sell part of their output as intermediate goods to other firms.

The production technology of each monopolistically competitive firm transforms n_t units of labor input into $y_t = A_t n_t^\alpha$ of output, with $\alpha \in (0, 1]$. Firms are subject to Rotemberg (1982) price adjustment costs.[16] In particular, a firm j has to buy $\Xi_t(j)Y_t$ units of intermediate goods

[16]The model may also be formulated with price adjustment costs in the utility function (as in Benhabib, Schmitt-Grohé and Uribe M. [2001]), rather than in the production set. The particular formulation of the Rotemberg adjustment costs is not crucial for the theoretical results obtained in the paper. Quantitatively, instead, the formulation with adjustment costs in the utility function allows for an equilibrium evolution of the labor input which is empirically more appealing, as emphasized in Section 5.

if it wants to change its selling price from p_{t-1} to p_t, where $Y_t = C_t + G_t$ is the GDP and $\Xi_t(j) = \frac{\xi}{2}\left[\frac{p_t(j)}{p_{t-1}(j)} - 1\right]^2$ is a quadratic cost, for a strictly positive constant ξ.

Given the price set by a firm, the quantity produced is demand-determined. As noted, total demand for a firm's output has two components. The first is the quantity $c_t(j) + g_t(j)$ demanded in the monopolistically competitive final goods market, and defined by (25) and (30). The second is the demand in the form of intermediate good by other firms, which use the goods purchased to pay for their adjustment costs. Since price adjustment costs are only a small share of GDP, I simplify the exposition and assume that firms disregard this part of their output demand when deciding the optimal price $p_t(j)$ at which to sell their output. This is equivalent to assuming that firms take as given the selling price P_t in the intermediate goods market, where equilibrium quantities are demand-determined, and demand $\Xi_t Y_t$ is assumed to be spread equally across firms. Hence, the problem of each firm is to choose sequences of prices $p_t(j)$ in order to maximize the firm's discounted flow of profits, which is equivalent to solving the following problem

$$\max_{\{p_t(j)\}} \sum_{t=0}^{\infty} \mathcal{D}_t \Pi_t(j) \tag{32}$$

where nominal period profits are given by

$$\Pi_t(j) = P_t \Xi_t Y_t + p_t(j) y_t(j) - W_t n_t(j) - \frac{\xi}{2}\left(\frac{p_t(j)}{p_{t-1}(j)} - 1\right)^2 P_t Y_t \tag{33}$$

the discount factor is

$$\mathcal{D}_t = \prod_{s=1}^{t} \frac{1}{1+i_t}$$

and firm's labor demand satisfies $n_t(j) = \left[\frac{y_t(j)+\Xi_t Y_t}{A_t}\right]^{\frac{1}{\alpha}}$. In a symmetric equilibrium, the optimal price $p_t^*(j)$ is the same for all firms, hence $p_t^*(j) = P_t$ and $y_t^*(j) = Y_t$. The first order condition for the optimal price then gives

$$P_t = \left[\frac{\theta}{\theta-1}\right] \cdot \left[\frac{1}{1-v_t}\right] \cdot \left[\left(\frac{Y_t}{A_t}\right)^{\frac{1}{\alpha}-1} \frac{(1+\Xi_t)^{\frac{1}{\alpha}-1}}{\alpha A_t}\right] W_t \tag{34}$$

where

$$v_t = \frac{\xi}{\theta-1}\left[-\pi_t + \frac{Y_{t+1}}{(1+r_{t+1})Y_t}\pi_{t+1}\right] \tag{35}$$

With a harmless abuse of terminology, I refer to $\pi_t = \frac{P_t}{P_{t-1}}\left(\frac{P_t}{P_{t-1}} - 1\right)$ as the inflation rate at time t.[17] Equation (34) indicates that the mark-up charged by firms is the product of three components: the elasticity of substitution between final goods (first term in brackets), deviations of current and future inflation from the zero inflation target (second term in brackets),

[17] The true price inflation between time $t-1$ and time t is $\tilde{\pi}_t = \frac{P_t}{P_{t-1}} - 1 > -1$. We can then write the relation between π_t and $\tilde{\pi}_t$ as $\pi_t = \tilde{\pi}_t(1+\tilde{\pi}_t)$. As long as $\tilde{\pi}_t > -50\%$, which for any practical purpose we assume to be the case in our analysis, there is a monotonically increasing relation between π_t and $\tilde{\pi}_t$. Moreover, the sign of π_t is always identical to the sign of $\tilde{\pi}_t$ and, up to a first order approximation, there is no difference between π_t and $\tilde{\pi}_t$ in a neighborhood of $\tilde{\pi}_t = 0$. Overall, in the context of this paper, there is no relevant loss of information in referring to π_t as the inflation rate.

and the nominal marginal cost of production, which equals the inverse of labor productivity (third term in brackets) at the level $(1+\Xi_t)Y_t$ of gross production, times the nominal wage. Finally, equilibrium demand of labor and profits are, respectively,

$$N_t^d = \left[\frac{Y_t}{A_t}(1+\Xi_t)\right]^{\frac{1}{\alpha}} \tag{36}$$

$$\Pi_t = P_t Y_t - W_t N_t^d \tag{37}$$

4.2 Equilibrium

Definition 2. *An equilibrium is a sequence of exogenous productivities $\{A_t\}_{t=0}^{\infty}$, prices $\{P_t, W_t, i_{t+1}\}_{t=0}^{\infty}$, household quantities $\{C_t, L_t, M_t, B_t,\}_{t=0}^{\infty}$, taxes and government spending $\{\tau_t^N, \tau_t^C, T_t, G_t\}_{t=0}^{\infty}$, government assets $\{M_t^s, B_t^s\}_{t=0}^{\infty}$, and firms' labor demand and profits $\{N_t^d, \Pi_t\}_{t=0}^{\infty}$ such that i) given prices, taxes, and profits, the household quantities solve the household problem; ii) the government is solvent at every period; iii) given taxes and wages and final demand $Y_t = C_t + G_t$, the evolutions of aggregate prices P_t and of labor demand N_t^d solve the firms' problem, giving Π_t as the maximal profits; iv) markets clear, i.e. $M_t^s = M_t$, $B_t^s = B_t$, $N_t = N_t^d$, $C_t + G_t = Y_t$.*

There are two crucial equations that characterize any equilibrium path for the economy. The first is the Euler equation (26). The second is the forward-looking Phillips curve, which incorporates the optimal pricing condition for firms and is derived as follows. Define $\sigma_t^G = \frac{G_t}{C_t}$ the ratio of government spending over private spending. It is then just a matter of simple algebra to derive the forward-looking Phillips curve as[18]

$$\pi_t = -\chi + \tilde{\kappa}_t \hat{Y}_t^{\frac{1+\psi}{\alpha}+\gamma-1} + \tilde{\beta}_t \pi_{t+1} \tag{38}$$

where $\hat{Y}_t \equiv \frac{Y_t}{Y_t^{TG}}$ and the unconstrained Pareto optimal level of output (and consumption) can be easily computed as $Y_t^{TG} = \alpha^{\frac{\alpha}{1+\psi+\alpha(\gamma-1)}} A_t^{\frac{1+\psi}{1+\psi+\alpha(\gamma-1)}}$. The function $\tilde{\beta}_t$ is given by (21) and

$$\begin{aligned} \chi &\equiv \frac{\theta-1}{\xi} > 0 \\ \tilde{\kappa}_t &\equiv \chi(1+\Xi_t)^{\frac{1+\psi}{\alpha}-1} \frac{1+\tau_t^C}{(1+\sigma_t^G)^{\gamma}(1-\tau_t^N)} \end{aligned} \tag{39}$$

Finally, substituting for the definition of $\hat{Y}_t$ and $\tilde{\beta}_t$, we can re-write the (26) as

$$\frac{\hat{Y}_{t+1}}{\hat{Y}_t} = \tilde{\beta}_t(1+r_{t+1}) \tag{40}$$

where, for simplicity, (40) was derived assuming a constant A_t.

Parameter	Description	Calibration
γ	Relative risk aversion	1
$\frac{1}{\psi}$	Frisch elasticity	1
θ	Demand elasticity	10
ξ	Adjustment cost coefficient	50
α	Labor share in production	0.7

Table 1: Calibration parameters. The model is calibrated in annual terms.

5 Calibration

The calibrations in this section explore the quantitative properties of the self-fulfilling path to the liquidity trap steady state.[19] The model is calibrated in annual terms, with the parameters in Table 1 common across all exercises, while the values of subjective discount factor β and of productivity growth $g_t = A_t/A_{t-1} - 1$ vary across calibrations. All calibrations assume that monetary policy is conducted according to a Taylor rule (11).[20] As shown in the theoretical part of the paper, the qualitative characteristics of the deflationary path do not depend on this specific assumption. Yet the choice of focusig on Taylor rules is justified here since, in order to coherently compare the model to actual data, it is proper to assume rules that have, arguably, the ability to replicate actual past policy behaviors (Taylor [1993]).

The section presents four calibrations. The first explores the empirical relevance of *Type I* versus *Type II* saddle connections under the Taylor framework. The second looks at the full dynamics of the main macro variables under a *Type I* saddle connections. The third calibration is the most comprehensive and assesses the ability of the model to explain the Japanese "lost decade" 1992-2002. The fourth calculates fiscal multipliers at the zero lower bound along a (deanchored) deflationary path.

Type I and Type II manifolds. Under the additional calibration $\beta = 0.97$ and $g_t = 0$, Figure 5 depicts the equilibrium manifold for two different parametrizations of the Taylor parameters. By presenting the manifold in the $(\tilde{\pi}_t, \hat{Y}_t - 1)$ space, Figure 5 provides information on both inflation $\tilde{\pi}_t = P_t/P_{t-1} - 1$ and on the net output gap $\hat{Y}_t - 1$. By virtue of (11), the rule calibrated on the left-hand side of the figure yields a maximum reaction parameter of $\bar{\phi} = 1.5$ which is much more stimulative than the calibration on the right-hand side, which instead yields $\bar{\phi} = 1.065$. Consistently with our theoretical results, the calibration with $\bar{\phi} = 1.5$ gives rise to a *Type II* manifold while the "not too stimulative" calibration with $\bar{\phi} = 1.065$

[18] To obtain the forward-looking Phillips curve start by eliminating the real wage W_t/P_t in the labor supply (27) using the value of the real wage derived from the labor demand (36). Then notice that consumption is written as $C_t = Y_t/(1 + \sigma_t^G)$, and that by (26) we can write $\tilde{\beta}_t = \frac{Y_{t+1}}{Y_t(1+r_{t+1})}$.

[19] The algorithm used for the numerical calculation is based on finding the saddle path solution to equation (41) in appendix A. The algorithm exploits the fact that the value of π_{t+2} can be expressed explicitly as a function of π_{t+1} and π_t. Recall that the initial sunspot π_0 is chosen exogenously. Solving (41) given the initial condition π_0 is equivalent to finding the unique value for π_1 such that the uniquely identified sequence π_0, π_1, π_2, ... converges monotonically to π^{LT}. Given π_0, the algorithm then searches for the value π_1 over the range (π^{LT}, π_0) such that the corresponding sequence of π_t is a) monotonically decreasing; b) as time grows large it lays between a radius of 0.005% from π^{LT} (by comparison, the absolute value of π^{LT} typically equals 3%).

[20] In all exercises and accompanying discussions actual inflation $\tilde{\pi}_t$ now replaces the convenient variable π_t.

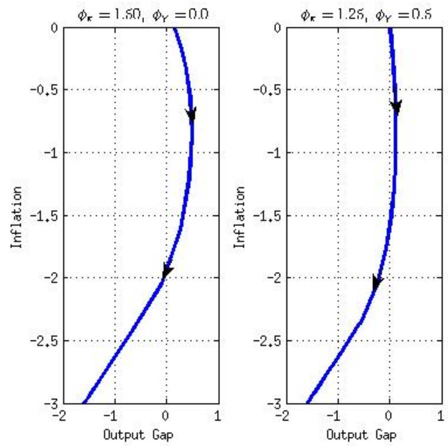

Figure 5: Equilibrium manifold in the $(\tilde{\pi}, \hat{Y} - 1)$ space. The manifold is of *Type II* for $\phi_\pi = 1.50$ and $\phi_Y = 0$, while it is of *Type I* for $\phi_\pi = 1.25$ and $\phi_Y = 0.5$. The subjective discount factor and productivity growth are, respectively, calibrated to $\beta = 0.97$ and $g_t = 0$.

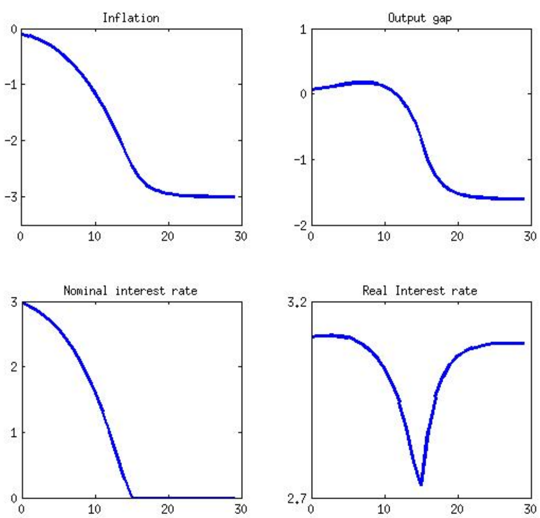

Figure 6: Dynamics of the *Type I* manifold ($\phi_\pi = 1.25$, $\phi_Y = 0.5$). The figure depicts the evolution of inflation $\tilde{\pi}_t$, the net output gap $\hat{Y}_t - 1$, the nominal rate i_t and the equilibrium real rate r_t starting from a sunspot $\tilde{\pi}_0 = -10^{-3}$. The subjective discount factor and productivity growth are, respectively, calibrated to $\beta = 0.97$ and $g_t = 0$. Values in the figure are expressed in percentage points.

generates a *Type I* saddle connection. This conclusion can be drawn by observing that only in the right hand side case the saddle connection reaches continuously the target steady state $(\tilde{\pi}^{TG}, \hat{Y}^{TG} - 1) = (0, 0)$. Notice also that the *Type I* saddle connection arises even though $\bar{\phi} = 1.065$ is slightly above the *sufficient* condition threshold $\beta^{-1} = 1.031$. Since in this case $\bar{\phi} > \beta^{-1}$, it is not guaranteed that $\hat{Y}_t < 1$ for all t, and in fact output gaps are marginally positive when inflation is not too far away from the target.

Dynamics along a Type I saddle. Figure 6 presents the time evolution of the main macro variables along the *Type I* saddle connection identified in Figure 5, assuming an initial sunspot inflation $\tilde{\pi}_0^* = -0.1\%$. Starting from such value, it takes approximately 20 years for the inflation rate $\tilde{\pi}_t$ to be close to its liquidity trap steady state value. In about 15 years the nominal interest rate hits the zero lower bound. Overall, the deflationary path is slow, qualitatively replicating the features of a "creeping" deflation. With an output gap of roughly zero for the first 10 years, the monetary authority keeps lowering the nominal rate as inflation slowly falls. Since the natural real rate is calibrated to a constant value the equilibrium real interest rate, while moving quantitatively little, follows a V-shaped path, with its initial and final values coinciding with an annual $r^n = \beta^{-1} - 1 = 3.1\%$.

Simulating the Japanese case: expectations deanchoring and a falling natural real rate. A casual observation of Figure 1, confirmed by a more thorough analysis, suggests that the equilibrium real interest rate in Japan has fallen in a roughly monotonic way since the early '90s. This is in sharp contrast with the simulation in Figure 6 where, as discussed above, the constancy of the natural real rate forces V-shape dynamics for the equilibrium real rate. A falling natural real rate seems to be a crucial element for any model that attempts to replicate the Japanese experience. Since the natural real rate depends on potential growth, I turn to Hayashi and Prescott (2002) for their analysis of growth in Japan. In particular, I take advantage of their approach to ask the following question: can a real shock to potential growth, jointly with a sunspot shock to inflation expectations, account for the evolution of both real and nominal variables during the lost decade 1992-2002?

Hayashi and Prescott (2002) report that TFP growth in Japan fell from 3.7% during the booming years of the late '80s, to just 0.3% during the '90s. To roughly replicate this observation I calibrate the TFP growth rate in the year $t = 1990$ to $g_t = 3.7\%$ and I further assume that subsequently TFP growth falls monotonically and reaches a long-run value of 0.3%, according to the law

$$g_t = 0.3\% + 3.4\% \cdot 2^{-\frac{t-1990}{2}}$$

The top-left panel in Figure 7 shows that this exogenous TFP process reaches the the desired value of just above 0.3% by the year 2000. The simulations below are carried out under perfect foresight, i.e. assuming that agents fully anticipate the progressive slowdown in the economy's potential growth.

In addition, Hayashi and Prescott (2002) also calculate that in 1990 the post-tax real return on capital was 5%. For the same year, the difference in Figure 1 between the uncollateralized overnight rate and the inflation rate gives a real return of 4.3%. I take the average of these two numbers, and I set to 4.65% the value of the natural real rate in 1990. Using the condition $1 + r^n_{1990} = (1 + g_{1990})/\beta$ I can then back out a calibration of $\beta = 0.991$ for the subjective discount factor. Given β and the evolution of g_t, the top-left panel of Figure 7 depicts the

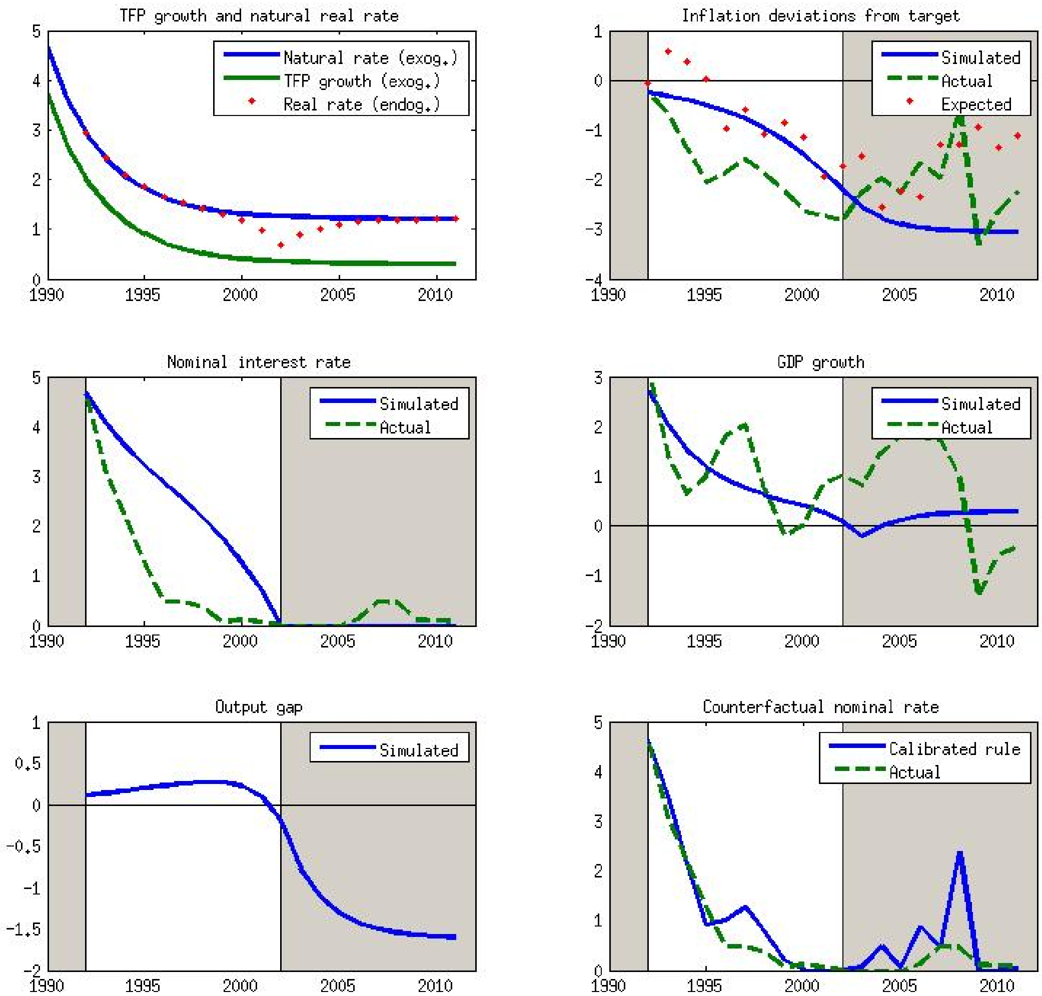

Figure 7: A calibrated model for the Japanese lost decade 1992-2002. The "potential" growth rate g_t falls from 3.7% in 1990 towards 0.3%, following a process with constant half-life equal to 2 years. The subjective discount factor is calibrated to $\beta = 0.991$, the initial (normalized) inflation sunspot is $\pi_0^* = -0.24\%$, and the Taylor rule has parameters $\phi_\pi = 1.38$, $\phi_Y = 0.5$. Values in the figure are expressed in percentage points.

exogenous evolution of the calibrated natural real rate. To check whether the entire path for r_t^n, and not just its initial value, is reasonable I compare the calibrated value of r_t^n against the data in the year $t = 2000$, i.e. ten years after the initial calibration date. It turns out that the calibrated value of $r_{2000}^n = 1.28\%$ falls exactly between the 2% value calculated by Hayashi and Prescott (2002) for the year 2000 and the 0.8% computed, as above, using the overnight uncollateralized rate. Similar conclusions are reached if the data are compared to the equilibrium real interest rate generated by the model (dotted line in the top left-hand side panel of Figure 7).

Having calibrated the productivity shock, we need to calibrate the sunspot shock to inflation expectations. To do this, we first have to define a target inflation rate in the early '90s in Japan. This is a problematic task since the Bank of Japan was not operating under an explicit target at that time. I deal with this issue by assuming that the inflation target was 2%, a value consistent with both the inflation targets adopted at the time by various advanced countries' central banks, and with the current Bank of Japan's target. With this target in mind, I set the sunspot inflation at the beginning of the lost decade to a small deviation $\tilde{\pi}_{1992} = -0.24\%$ from the target. I take 1992 as the first year of the deflationary path because 1992 was in fact the first when actual inflation (1.76%) fell below the presumed 2% target. We also need to define a monetary policy rule and calibrate its parameters. Once more, we face the issue that the Bank of Japan did not explicitly follow a monetary policy rule. I resort to the assumption, common in the literature, that the BoJ was following a standard Taylor rule.[21] I calibrate the reaction parameter to the output gap, which is an unobservable variable, to the standard value of $\phi_Y = 0.5$. Given this, the reaction parameter ϕ_π is calibrated using observables. Specifically I set ϕ_π so that, given the initial inflation deviation $\pi_{1992} = -0.24\%$, the endogenous nominal rate i_{1992} generated by the model matches exactly the observed value of 4.66% for the uncollateralized overnight rate in 1992. This calibration strategy for the monetary policy rule yields a parameter value of $\phi_\pi = 1.38$, which is "not too stimulative" relative to a standard calibration of 1.5 as in Taylor (1993).

Figure 7 compares to the data the deflationary path obtained from the calibrated model. The period of interest is the *lost decade* 1992-2002. As mentioned in the introduction, this decade is defined by the peak-to-trough monotonic fall in the inflation rate, and by a substantial reduction in GDP growth.[22] The model matches almost perfectly the timing of the trough reached by actual inflation and it matches exactly the year when the nominal rate reached the zero lower bound. Notice also that the simulated inflation is, for most of the lost decade, very close to measured expected inflation but is higher than actual inflation. This makes sense in light of my interpretation of Figure 3. Recall, in fact, that the model is simulated under a perfect foresight assumption, i.e. in the absence of *unexpected* shocks to inflation. A fair comparison between the model and the empirical values would then require us to filter

[21]The model can be easily modified to include a non-zero target inflation rate. This simply amounts to a normalization of the inflation variables. In particular, call $\bar{\pi}^{TG}$ the target (net) growth rate of prices. Re-normalize the actual inflation rate so that now $1+\tilde{\pi}_t = \frac{P_t}{P_{t-1}(1+\bar{\pi}^{TG})}$. With this normalization, price adjustment costs continue to be expressed as $\Xi(\tilde{\pi}_t) = \xi/2\tilde{\pi}_t^2$. Similarly, the convenient variable π_t that we have used in Sections 3-4 continues to be defined as $\pi_t = \tilde{\pi}_t(1+\tilde{\pi}_t)$. Finally, under the new definition of the actual inflation $\tilde{\pi}_t$, the real policy rule derived from the Taylor rule is still expressed by (11), with $\tilde{\pi}_t$ replacing, as usual, the convenient variable π_t.

[22]The decade 2003-2012 is also reported in the graphs. This decade is marked by important exogenous shocks (e.g. the rise in commodity prices prior to 2008, the global financial crisis, the Japan earthquake and tsunami of 2011) that are not incorporated in the model simulation.

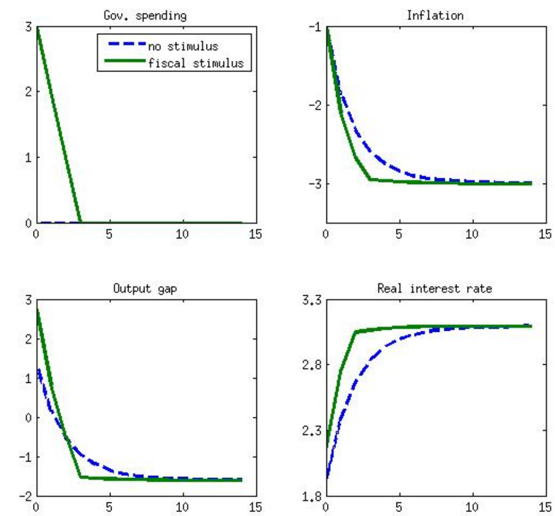

Figure 8: Dynamics with a fiscal expansion at the ZLB. The calibrated parameters are as in Table 1. The baseline scenario without stimulus is thus the same as the one in Figure 6, but holding fixed to zero the nominal interest rate. Values in the figure are expressed in percentage points.

out any unexpected inflation shock from the data or, equivalently, to compare the simulated inflation path to that of measured *expected* inflation rather than to actual inflation. When this latter comparison is done then, as Figure 3 shows, the model not only captures the overall and monotonic reduction in inflation, but it also matches the slow speed of such process. Another way to phrase this conclusion is to say that the model simulates the counterfactual path for actual inflation and the nominal interest rate in the absence of the negative and *global* sequence of inflation shocks that, among other countries, hit Japan during its lost decade (see the discussion in Section 2.3). Indeed, these global shocks fully account for the somewhat slower fall, relative to the data, of the simulated nominal interest rate. To see this, assume that the (unobservable) output gap was zero during the lost decade, as roughly suggested by the model's simulation (bottom-left panel of Figure 7). Under this assumption, we can plug the sequence of actual inflation into our calibrated Taylor rule, thus obtaining an implied sequence of nominal rates that now incorporate also the reaction of the monetary authority to the unexpected global inflation shocks. As the figure makes clear (bottom-right panel), once these unexpected global shocks are factored in, the rule-implied nominal rate practically coincides with the data.

Overall the double-shock approach – a shock to potential growth plus a simultaneous sunspot deanchoring of inflation expectations – simulated by the model accounts quite well for the evolution of key variables, both real and nominal, during the Japanese lost decade.

Government spending at the ZLB. Consider again the calibration where the natural real rate is constant, with $\beta = 0.97$ and $g_t = 0$, and set the initial arbitrary sunspot inflation deviation to $\tilde{\pi}_0 = -1\%$. Consider now the effect of carrying out an anticipated fiscal expansion

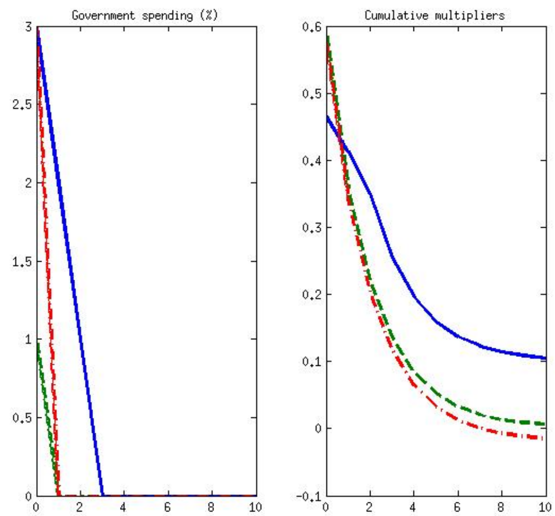

Figure 9: The left-hand side panel shows the path of the share of government spending σ^G starting from an inflation $\tilde{\pi}_0^* = -1\%$, while the right-hand side gives the corresponding cumulative spending multipliers at different horizons.

while the nominal interest rate is kept fixed at the ZLB. Specifically, assume that government spending σ^G initially increases to 3% of private spending and then, over the following four years, progressively returns to zero.

The solid lines in Figure 8 depict the evolution of the equilibrium variables under the announced fiscal stimulus plan, while the dashed line represents the corresponding baseline path in the absence of stimulus, i.e. for $\sigma_t^G = 0$. The simulation indicates that, while on impact the fiscal expansions allows for a sharp improvement in the output gap, inflation is nonetheless always lower than without fiscal stimulus. This can be interpreted in light of the result of the theoretical part of the paper. As seen, an expansion in government spending creates a dynamic wedge that operates in a way very similar to a lowering of the nominal interest rate. Hence, we can apply to a spending expansion the same intuition as the one developed in the last two paragraphs of Section 3.1: a more "stimulative" policy today is associated with an increase in current *relative* demand, which translates into a faster fall in future demand, and thus into a faster fall in inflation.

The association of an anticipated fiscal expansion with a fall in the inflation rate is not in contrast with other results in the literature (Eggertsson [2010a], Christiano, Eichenbaum, Rebelo [2011]). There is no contrast because the two types of results are obtained from essentially the same model but under two different assumptions for the equilibrium selection issue (Christiano, Eichenbaum [2012]). In particular, under the assumption that long-term inflation expectations are *anchored*, a fiscal expansion at the ZLB generates an increase in the inflation rate, a reduction in the real interest rate, and thus a stimulus to *private* demand: government spending crowds-in private spending and thus government spending multipliers are greater than one. The opposite holds, instead, when an equilibrium with *deanchored* inflation expectations is selected: fiscal expansions are anticipated to occur in conjunction with a faster fall in inflation, an increase in the real rate and thus with a crowding-out of private spending, leading to fiscal multipliers that are smaller than one.

To confirm this intuitive conclusion, Figure 9 calculates government spending multipliers at various horizons for three different sizes of the initial spending expansion. The solid line corresponds to the large and prolonged fiscal expansion in Figure 8, the dotted-dashed line corresponds to a large and short-lived expansion, while the dashed line gives a small and short-lived stimulus. The comparison among fiscal multipliers for different sizes of the stimulus is interesting because, contrary to what is often done in the neo-Keynesian literature, my solution to the model is fully non-linear, leaving scope for non-linearity in the multipliers as well. For a given time horizon of t years, the *cumulative* multipliers are calculated as the ratio of two quantities. The numerator is set equal to the cumulative difference from time 0 up to time t between output under stimulus and output under the baseline of no stimulus (the numerator is thus the integral between the solid and the dashed output gap lines in Figure 8). Similarly, the denominator is equal to the cumulative difference between the absolute level of government spending under stimulus and spending under no stimulus (zero spending). Figure 9 confirms that, under the assumption of deanchored inflation expectations, spending multipliers are always well below 1. In the short-run there is no evidence of strong non-linear effects, and on impact multipliers are clustered around 0.5 in all three cases. Over the longer run, sharper (i.e. larger and short-lived) fiscal expansions are associated with significantly lower cumulative multipliers.

The concept of fiscal "multipliers" is arguably quite problematic, and maybe of little use, in our context with multiple equilibria. Still, the calculations in Figure 9 are useful to confirm that fiscal multipliers are rather small in self-fulfilling liquidity traps (Mertens and Ravn [2014], Christiano and Eichenbaum [2012]). The theoretical analysis of Sections 3.1-3.3 helps, in turn, to account for this result. Equilibrium selection choices between anchored versus dis-anchored equilibria are far from harmless for policy analysis in standard sticky price models (Cochrane [2011], Cochrane [2013]).

6 Conclusions

This paper provides a general analysis of deflationary paths in a standard sticky price model. It finds sufficient conditions on the form of the monetary and fiscal policy rules that guarantee the existence of a monotonic deflationary path, triggered by a sunspot that deanchors inflation expectations. This path can take two forms, depending on the configuration of the parameters. The *Type I* equilibrium, in particular, directly connects the target and the liquidity trap steady states and had not been previously studied in the literature. Monotonic self-fulfilling deflationary paths are not only pervasive in theoretical models, they could also capture real-world situations. For instance, a quantitative application of the results shows that a self-fulfilling deanchoring of expectations plays an important role in replicating the evolution of the main macro variables during the Japanese lost decade.

The findings of the paper open the door to further issues. One is how to model the triggers of sunspot inflation shocks. Sunspots may be linked, for instance, to the occurrence of other, real or nominal, shocks. Another issue is the exit from the self-fulfilling deflationary steady state. For example, in a world of multiple equilibria, can "shock and awe" policy announcements act as a coordination device that re-anchors inflation expectations? Answering these questions is grounds for future research.

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8 Appendix A

Proof. To reduce notation define $\hat{\psi} \equiv (1+\psi)/\alpha > 1$. Combining (38) and (40) we obtain

$$\left[\frac{1-v(\pi_{t+1},\pi_{t+2})}{1-v(\pi_t,\pi_{t+1})}\right]^{\frac{\gamma}{\hat{\psi}+\gamma-1}} = \frac{1+\tilde{r}(\pi_t,\pi_{t+1})}{1+r^n} \tag{41}$$

where $v(\cdot)$ is derived by substituting (21) into (17). We begin by proving that around the liquidity trap steady state the equilibrium is a monotonic saddle path. For brevity, let us make the auxiliary assumption that government fiscal and monetary policies are constant not just at S^{LT} but also in a small neighborhood of S^{LT}. Then, in such small neighborhood $\omega^G_{t+1} = \omega^C_{t+1} = \omega^N_{t+1} = 1$ and $i(S_t) = 0$, so that (41) becomes $1+\tilde{r}_{t+1} = (\omega^{\Xi}_{t+1})^{\gamma(\hat{\psi}-1)/(\hat{\psi}+\gamma-1)}(1+r_{t+1})$. Linearizing (41) around π^{LT} yields the second order difference equation,

$$\hat{\pi}_{t+2} - \chi_1\hat{\pi}_{t+1} + \chi_0\hat{\pi}_t = 0$$

where

$$\chi_0 = \frac{1}{\chi v_2(\pi^{LT},\pi^{LT})}\left[1-\chi(\hat{\psi}-1)\Xi'(\pi^{LT})\frac{1-v(\pi^{LT},\pi^{LT})}{1+\Xi(\pi^{LT})}\right] > 1$$

$$\chi_1 = 1+\chi_0 - r_2(\pi^{LT},\pi^{LT})(\hat{\psi}+\gamma-1)\frac{1-v(\pi^{LT},\pi^{LT})}{(1+r^n)\gamma v_2(\pi^{LT},\pi^{LT})} > 1+\chi_0$$

and $\hat{\pi}_t = \pi_t - \pi^{LT}$. To establish the above inequalities notice that, since by assumption $i(S_t) = 0$ in a neighborhood of S^{LT}, then the real gross interest rate is approximately $1 + r(\pi_t,\pi_{t+1}) \approx (1+\pi_{t+1})^{-1}$. This observation leads to four results: *a)* $r_1(S^{LT}) = 0$. Hence, $\tilde{\beta}_1(S^{LT}) = 0$ and $v_1(S^{LT}) = -1/\chi$; *b)* $r_2(S^{LT}) = -(1+r^n)^2 < 0$; *c)* $0 < v(S^{LT}) < 1$; *d)* $0 < \chi v_2(S^{LT}) < 1$, since as usual $-0.5 < \pi^{LT} < 0$, $\gamma \geq 1$ and

$$\chi v_2(\pi^{LT},\pi^{LT}) \approx \frac{1}{1+r^n}\left(1+\frac{\gamma-1}{\gamma}\frac{\pi^{LT}}{1+\pi^{LT}}\right)$$

Finally, recall that $\Xi'(\pi^{LT}) < 0$. The difference equation has two roots,

$$\lambda_1 = \frac{\chi_1+\sqrt{\chi_1^2-4\chi_0}}{2}$$

$$\lambda_2 = \frac{\chi_1-\sqrt{\chi_1^2-4\chi_0}}{2}$$

Since $\chi_1 > 1+\chi_0$ then $\chi_1^2 - 4\chi_0 > (\chi_0-1)^2 > 0$, so that both roots are positive and real. Moreover, since λ_1 is increasing in χ_1, the condition $\chi_1 > 1+\chi_0$ implies that $\lambda_1 > \chi_0 > 1$. Similarly, since λ_2 is strictly decreasing in χ_1, the condition $\chi_1 > 1+\chi_0$ implies $\lambda_2 < 1$. We have proven that around the liquidity trap equilibrium there is a saddle path (approximately) described by the stable manifold $\pi_{t+1} = \mathcal{M}(\pi_t) = \pi^{LT} + \lambda_2(\pi_t - \pi^{LT})$. Notice that, locally, $\mathcal{M}(\pi)$ is continuous, monotonically increasing and such that $\pi^{LT} < \mathcal{M}(\pi) < \pi$ for $\pi > \pi^{LT}$.

I now show how to extend the function $\mathcal{M}(\pi)$ from a neighborhood of π^{LT} to the entire interval (π^{LT},π^{TG}). Start by choosing a value π^*_{t+1} close to π^{LT}, so that we know that a monotonic equilibrium manifold $\mathcal{M}$ exists and is linearly approximated as shown above.

Specifically, for such π^*_{t+1} we know that $\pi^*_{t+2} = \mathcal{M}(\pi^*_{t+1}) < \pi^*_{t+1}$. Given π^*_{t+1}, we can show that there is one and only one value π^*_t that solves (41). In fact, for $\pi_t = \pi^*_{t+1}$, the left hand side is greater than one ($v_2 > 0$ by Assumption 3) and thus it is strictly bigger than the right hand side, which instead is strictly smaller than one (Assumptions 1, 2 and 4). Moreover, for π_t large enough (possibly positive), the left hand side is smaller than the right hand side since, by non-negativity of nominal rates, the right hand side is bounded away from zero by some strictly positive constant. Continuity then implies the existence of one value $\pi^*_t = \mathcal{M}^{-1}(\pi^*_{t+1})$ that satisfies (41). Moreover, since both sides are monotonic in π_t, one decreasing (Assumption 3) and the other increasing (Assumption 1), then the solution is unique. Proceeding in this way, we can construct a continuous and monotonic function $\mathcal{M}(\pi_t)$ on the interval $\pi_t \in [\pi^{LT}, \pi^{TG}]$. If $\mathcal{M}(\pi^{TG}) < \pi^{TG}$ then the equilibrium is of *Type II*, while if $\mathcal{M}(\pi^{TG}) = \pi^{TG}$ then the equilibrium is of *Type I.*

Next, I assume that $1 \leq \bar{\phi}^{\tilde{r}} \leq \bar{\beta}^{-1}$ and I show that in this case we can guarantee the existence of a *Type I* equilibrium. Given that $v(S^{LT}) > 0$, for π^*_{t+1} close enough to π^{LT} we have $v(\pi^*_{t+1}, \pi^*_{t+2}) > 0$ and hence the numerator of the left hand side of (41) is strictly smaller than 1. Moreover, since $v_2 > 0$ and $\pi^*_{t+1} > \pi^*_{t+2}$, then also $v(\pi^*_{t+1}, \pi^*_{t+1}) > 0$. Since by assumption $\bar{\beta} < 1$ then there exists a unique $\pi_t > \pi^*_{t+1}$ such that $v(\pi_t, \pi^*_{t+1}) = 0$. For such value of π_t the denominator of the left hand side of (41) equals 1, and the whole ratio is therefore strictly smaller than 1. On the contrary, for such value of π_t the right hand side of (41) is greater than one. Recall in fact that $\tilde{\beta}(\pi_t, \pi^*_{t+1}) < \bar{\beta} \leq 1/\bar{\phi}$, where $\bar{\beta}$ is defined by (18), and thus

$$\tilde{r}(\pi_t, \pi^*_{t+1}) = \tilde{r}(\tilde{\beta}(\pi_t, \pi^*_{t+1})\pi^*_{t+1}, \pi^*_{t+1}) > \tilde{r}(\bar{\beta}\pi^*_{t+1}, \pi^*_{t+1}) \geq \tilde{r}(\pi^*_{t+1}/\bar{\phi}^{\tilde{r}}, \pi^*_{t+1}) \geq r^n$$

We conclude that the equilibrium value π^*_t must satisfy $\pi^*_{t+1} < \pi^*_t < \pi_t$, with $v^*_t > 0$. Repeating this procedure, we obtain an infinite sequence $\pi^*_{t-2}, \pi^*_{t-1}, \pi^*_t ...$ of equilibrium values. Since at any point of this infinite iteration $v^*_t > 0$ then necessarily $\pi^*_t < 0$ and thus $\mathcal{M}^{-1}(\pi) < 0$ for all $\pi \in (\pi^{LT}, 0)$, showing that the equilibrium is of *Type I.*

□

9 Appendix B

Constant static wedge. In appendix A we proved that, under the condition $1 \leq \bar{\phi}^{\tilde{r}} \leq \bar{\beta}^{-1}$, we have $0 < v(\pi_t^*, \pi_{t+1}^*) < 1$ along the equilibrium manifold, with $\pi_{t+1}^* < \pi_t^* < 0$. If labor taxes are set so that the static wedge is constant then (38) gives

$$\hat{Y}_t^{*\hat{\psi}+\gamma-1} = [1 - v(\pi_t^*, \pi_{t+1}^*)][1 + \Xi(\pi_t^*)]^{-(\hat{\psi}-1)} < 1$$

Time-consistent labor tax. To obtain the equilibrium under a time-consistent labor tax calculate, at any time t, the welfare maximizing tax rates τ_t^{N*}, taking as given the sequences of future quantities and labor taxes, and the series of current and future prices, consumption taxes, and government spending shares. Since there is no "stock" variables (such as capital) in the economy, it is easy to show that the problem of the fiscal authority is static, meaning that τ_t^{N*} is chosen to maximize current utility. Moreover, the availability of just the labor tax instrument turns out to be enough to allow the government to reach, period by period, the *constrained* Pareto optimal allocation. The constraints, in particular, are represented by the time t level of output costs $\Xi(\pi_t)$ associated with the given level of time t inflation, and by the time t share of wasteful government spending σ_t^G (note $\sigma_t^{G*} = 0$ would be the solution if the government were also allowed to choose, period by period, the optimal share of wasteful government spending). Since in practice both σ_t^G and Ξ_t have the same effect on the time t constrained Pareto optimal allocation as a corresponding reduction in productivity A_t, then the time t constrained Pareto optimal consumption equals

$$C_t^* = [(1 + \sigma_t^G)(1 + \Xi_t)]^{-\frac{\hat{\psi}}{\hat{\psi}+\gamma-1}} Y^{TG}$$

where Y^{TG} is the usual *unconstrained* Pareto optimal consumption (and output) and $\hat{\psi} = (1+\psi)/\alpha$. Call $\hat{Y}_t^* = Y_t^*/Y^{TG} = (1+\sigma_t^G)C_t^*/Y^{TG}$ and substitute $\hat{Y}_{t+1}^*/\hat{Y}_t^*$ into (40). Along the equilibrium with time consistent tax policy we must then have,

$$1 = \left[\omega^{\Xi}(\pi_t^*, \pi_{t+1}^*)\right]^{\frac{\gamma\hat{\psi}}{\hat{\psi}+\gamma-1}} \frac{1 + \tilde{f}(\pi_t^*, \pi_{t+1}^*)}{1 + r^n} \tag{42}$$

with $1 + \tilde{f}(\pi_t^*, \pi_{t+1}^*)$ given by (22). Fix a value for $\pi_{t+1}^* \in (\pi^{LT}, \pi^{TG})$ and consider possible candidates π_t for the equilibrium π_t^*. For $\pi_t = \pi_{t+1}^* < 0$ the right hand side of (42) is strictly smaller than 1 (Assumptions 1, 2 and 4). Instead, for $\pi_t = \pi^{TG} = 0$ the right hand side of (42) is strictly larger than 1, since $\omega^{\Xi}(0, \pi_{t+1}^*) > 1$ and $\tilde{f}(0, \pi_{t+1}^*) > r^n$ by the assumption $\bar{\phi}^{\tilde{f}} < \infty$. Since the right hand side of (42) is strictly increasing in $\pi_t < 0$ there exists then one and only one value $\pi_t^* \in (\pi_{t+1}^*, 0)$ such that (42) holds. The finding that $\mathcal{M}^{-1}(\pi_{t+1}^*)$ is strictly smaller than 0 for all $\pi_{t+1}^* \in (\pi^{LT}, 0)$ implies that the equilibrium manifold under time consistent tax policies is of *Type I*. Notice that if $\sigma_t^G = 0$ then $\hat{Y}_t^* = Y_t^*/Y^{TG} = C_t^*/Y^{TG} < 1$.

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