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SEARCH COSTS AND THE SEVERITY OF ADVERSE SELECTION

by Francesco Palazzo*

Abstract

In view of some recent empirical evidence, I suggest a relationship between the magnitude of search costs and the severity of adverse selection in the context of a dynamic model with asymmetric information. In markets with small search costs sellers with low quality products misrepresent their quality and demand a high price. If instead search costs are not negligible and buyers receive sufficiently precise signals, sellers' price offers are truthful and all product qualities are traded over time. In markets with small search costs, a budget balanced mechanism can avoid to exacerbate adverse selection: sellers should pay a per period market participation tax and receive a rebate after trading.

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Contents

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1 Introduction*

Information asymmetry is a pervasive feature of real world markets: financial securities, real estate, electronics and secondhand vehicles are just a few examples. In these markets one side—usually buyers—may lack the information or experience to ascertain the true quality of a specific good. Since Akerlof's (1970) seminal paper, it is a well known result that high quality goods may not be traded if buyers believe there is a high chance of buying a 'lemon.' Clearly, real world markets present a much more complex environment than the static adverse selection model, and a growing literature has been reconsidering the effects of asymmetric information in a dynamic setting.

The theoretical literature offers contrasting views. On the one hand, Janssen and Roy (2002), Blouin (2003), Camargo and Lester (2014), Moreno and Wooders (2014) and Fuchs and Skrzypacz (2015) claim that sellers *signal* a higher product quality by delaying trade. In these models, low quality sellers trade early while high quality sellers demand higher prices and trade at a later date. I will refer to this economic mechanism as *intertemporal separation* (henceforth, ITS). On the other hand, Taylor (1999) claims that spending a longer time on the market is a negative 'stigma' for sellers, and leads to trading at a lower price. Differently from the first strand of literature, buyers accept goods based on a private informative signal about the product quality:[1] in equilibrium, high quality sellers trade more rapidly, and older cohorts of sellers include a higher share of 'lemons' whose price offers were rejected in previous trading rounds. Several papers consider a market setup in which buyers can infer how long a seller has been on the market, while others assume buyers lack this piece of information. Although in many traditional retail markets the latter case is more plausible, the former setup is gaining increasing attention because electronic platforms make it easy to provide buyers with information on sellers' previous time on the market.

The empirical literature on this topic is very limited and it documents contrasting findings. On the one hand, Ghose (2009) considers used electronics products sold on Amazon, and she finds that goods of higher quality take longer to trade than lower quality ones, but they manage to trade at a higher price as time goes by. On the other hand, Tucker et al. (2013) argues that a

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[1]Only a few other papers consider buyers equipped with informative signals: the early work by Taylor (1999) and the most recent contributions by Lauermann and Wolinsky (2015) and Kaya and Kim (2014); see the related literature.

longer time on the market leads to a negative stigma.[2]

In this paper, I suggest that the relationship between price dynamics and adverse selection may depend on the *magnitude* of search costs in a decentralized market. I consider a dynamic model in which every seller—endowed with a good of either high or low quality—finds a new buyer only by incurring a search cost; once a seller finds a buyer, the former proposes a price to the latter, who receives a private and imperfectly informative signal on the seller's product quality, and decides whether to accept or reject the price offer. My results point out that markets with *small* search costs suffer from a more severe adverse selection problem: low quality sellers prefer to pool on a high price as it is cheaper to wait until a buyer accepts. If buyers observe how long the good has been on sale, a long stay on the market penalizes sellers: prices decrease over time because high quality goods trade faster than lower quality ones. However, when time on market is not observable the adverse selection problem is even more severe because the average market quality worsens. On the contrary, marketplaces with non-trivial search costs may favour high quality sellers' market participation because the search cost acts as a separating device.[3] Although it takes longer to trade a high quality good than a low quality one, it ends up trading at a higher price. Low quality sellers demand a low price because imitation would lead to incur too high expected search costs before trade.

Linking the adverse selection problem to the magnitude of search costs may explain *why* the empirical evidence on intertemporal separation in Ghose (2009) concerns markets for small value goods such as electronics, while Tucker et al. (2013) find a negative stigma effect for expensive items such as residential homes.[4] The proposed relationship between adverse selection and search costs may also be relevant to interpret the growing literature on online trading platforms—where search costs have been almost eliminated—that documents a more severe adverse selection problem. Jin and Kato (2007) show that the lemons problem may induce different product qualities to segment across online and offline markets. Specifically, products offered online are more likely to be of *low* quality, unless certified by a professional third party, while higher quality products are usually sold through the retail channel.[5] Similarly, Wolf and

[2]Tucker et al. (2013) exploit a quasi experimental setup due to a sudden policy change: in 2006 the real estate listing service in Massachusetts decided to prevent home sellers from resetting their properties' days on market through relisting. Homes exposed to the policy change experienced a considerable reduction in sale price relative to comparable Rhode Island homes, with the effect being greater for older listings; in contrast, newer listings experienced a slight increase in prices after the policy change.

[3]Obviously, search costs should not be *too* high; otherwise no seller would participate in the market.

[4]Kaya and Kim (2014) recently provided a theoretical model that might also explain the conflicting empirical evidence. It suggests a relationship between the observed price pattern and the adverse selection problem based on the initial share of high quality sellers: if it is high, delaying trade penalizes sellers as prices decrease over time; if it is low, waiting signals better product quality and older cohorts of sellers trade at higher prices. Their mechanism would imply that, *ceteris paribus*, used electronics goods markets have relatively more high quality products than residential homes. Unfortunately, it is hard to test this implication because of the difficulties in observing the effective quality of goods offered in a real world market.

[5]Wolf and Muhanna (2005) provide similar evidence with data from eBay Motors: they find that for vehicles with higher quality uncertainty—older and high mileage vehicles—the price that eBay buyers are willing to pay for the vehicle decreases by

Muhanna (2005) and Overby and Jap (2009) find that newer cars and those with low mileage are less likely to be sold on eBay.[6]

The main economic mechanism underlying the negative relationship between search costs and adverse selection is fairly intuitive. Small search costs create greater incentives for sellers of low quality goods to adopt the same pricing strategy as high quality sellers, as it is cheap to search until a buyer wrongly perceives the product as being good. In equilibrium, buyers realize that all sellers pool on the same price, and they accept only if they receive a positive signal on the good quality *and* the price offer is sufficiently low. When sellers' time on market is observable, the longer a seller has been on the market the lower the price offers will be: high quality goods sell more rapidly—as they are more likely to receive a positive signal—and buyers realize that older sellers are more likely to offer a low quality good.[7] If a seller had been on the market for too long, buyers would only accept prices below the reservation value of high quality sellers who, in turn, prefer to exit the market. The model prediction of a decreasing price path is consistent with the empirical patterns reported in Tucker et al. (2013). Moreover, even if they do not discuss how their results relate to the adverse selection problem, Hendel et al. (2009) document that some real estate sellers in Madison, WI decided to switch to a realtor after some time spent on a for-sale-by-owner website, an online platform with a publicly observable posting day. In markets with non-trivial search costs, there is less scope for imitation because low quality sellers may prefer to truthfully reveal their type and trade immediately rather than continuing to search. In separating (or semi-separating) equilibria, a low quality good trades at a low price that buyers always accept, whereas a high quality one trades at a higher price only if a buyer receives a positive signal on its quality. Separation results from the difference in the expected search costs of pursuing a high price strategy for the two types of sellers.

In view of this unfavourable evidence on the functioning of online markets, I study how a market designer could overcome the lemons problem in a market with small search costs. In particular, I analyze to what extent a system of transfers may maximize sellers' market participation and trade. I focus on mechanisms that satisfy a series of properties: budget balance, informational efficiency of prices and interim individual rationality. The efficient market design intervention achieves separation through a constant market participation tax, and it relaxes a seller's individual rationality constraint through a final rebate after trading. Low quality sell-

more than what is observed on offline markets. Analogously, Dewan and Hsu (2004) find that identical stamps trade at a 10–15 percent discount on eBay compared to a speciality stamps auction with lower quality uncertainty.

[6] As documented by Dellarocas (2005), Dimoka et al. (2012) and Hui et al. (2015) eBay has been successful in solving asymmetric information problems with adequate mechanism design and feedback. My model is more closely related to less sophisticated online platforms such as Craigslist.com, Gumtree.com or Pianomart.com, where anonymous sellers place simple ads and buyers can individually contact the seller.

[7] Taylor (1999) is the first paper to exploit this social learning mechanism in the context of a two-period adverse selection model (see section 2).

ers do not find it profitable to post a high price because, on average, they are less likely to find a buyer who receives a positive signal; if they pursued a high price strategy, they would pay, on average, a cumulative amount of market participation taxes that would make imitation unprofitable. In terms of incentive compatibility constraints, the market participation tax is analogous to a per period search cost. Nevertheless, the former is not a waste of economic resources, and it can be partially recouped through a rebate, thus relaxing sellers' market participation constraints. Time on market observability does *not* affect the efficient mechanism: taxes and rebates are inversely proportional to the precision of buyers' signal, but they do not depend on sellers' time on market, although in principle they could. This market design intervention achieves full market participation in a large set of economies, and it is only unsuccessful when buyers' signals are close to being uninformative.

In separating equilibria, sellers' strategies signal their type, and market outcomes do not depend on the availability of public information on sellers' time on market. On the contrary, this information becomes crucial in a pooling equilibrium, i.e. when search costs are small. In particular, if time on market is not observable, sellers from all cohorts offer the same price because buyers cannot penalize 'old' sellers. There is no decreasing price path and no seller drops out once he has initially participated in the market. Nevertheless, the adverse selection problem is more severe *without* information on sellers' time on market, to the extent that neither the dynamic dimension nor the existence of private informative signals improves high quality sellers' market participation relative to a static model with uninformed buyers. The economic mechanism underlying this result depends on the interaction between sellers' pooling strategies and the precision of buyers' signals in determining the endogenous average quality of the goods offered on the market. Indeed, when search costs are small, low quality sellers demand a high price until a buyer receives a positive signal; because signals are informative, high quality goods trade more rapidly than low quality ones, and this difference is increasing in relation to the signal precision. Therefore, more precise signals lead low quality sellers to stay longer on the market, worsening the average market quality. When buyers cannot distinguish how long a seller has been on the market, the prior belief of receiving a high quality good is equal to the average quality of the products offered on the market. Greater signal precision is self-defeating as it worsens the average quality of the pool of products offered on the market; this negative effect on the prior belief offsets the positive effect of a more precise signal. On the contrary, if time on market is observable, buyers can distinguish different cohorts of sellers and a higher signal precision unambiguously alleviates the lemons problem. In this respect, Lewis (2011) points out that greater information disclosure on eBay motors increases sellers' chances of trading as well as final prices. My results point out that the benefits from information disclosure may be higher when information on sellers' time on market is publicly available.

A last word of caution on the set of equilibria considered. I analyze a dynamic signalling model, and it is an extremely challenging task to provide a complete equilibrium characterization. Similarly to many other papers, I focus on a particular set of perfect Bayesian equilibria. Specifically, I restrict attention to equilibria in which buyers play pure strategies on the equilibrium path; nonetheless, buyers' deviations and sellers' equilibrium play can be in mixed strategies. Within this set of equilibria, if search costs are sufficiently small the only admissible equilibrium strategy for sellers is to pool on the same price offer (Lemma 4.1). Separation *could* be possible if buyers played mixed strategies, but the pooling equilibria presented in Proposition 4.2 and 4.3 would still exist; in addition, when search costs are non-negligible pooling equilibria would fail to exist even if buyers could mix. I cannot claim to have a robust theoretical prediction based on equilibrium uniqueness; nevertheless, I focus on a set of equilibria that deserves special attention as it matches several empirical findings observed in real world markets.

In the next section I discuss the related literature. Section 3 presents the model. Section 4 characterizes the equilibria when search costs are close to zero. Section 5 derives the efficient market intervention. Section 6 concludes. All proofs are in Appendices A and B.

2 Related literature

This paper is mainly related to the theoretical literature on dynamic adverse selection in decentralized markets.[8] Janssen and Roy (2002), Blouin (2003), Camargo and Lester (2014), Moreno and Wooders (2014) and Fuchs and Skrzypacz (2015) present models in which postponing trade may signal good quality and open up the opportunity to sell at a higher price.[9] A key assumption underlying the intertemporal separating mechanism is the ability of buyers to infer how long a seller has been waiting on the market. They can either observe hard evidence of time on market or there is common knowledge of the initial date of the game. Equilibria are inefficient from a welfare perspective due to delays, but all sellers eventually trade. The literature mainly considers non-stationary equilibria, as the market starts at an initial date and

[8] The latter term defines a class of models that depart from the classic Walrasian price formation paradigm to explicitly model the bilateral interaction between buyers and sellers. A non-exhaustive list of previous papers on decentralized markets with complete product information includes Diamond (1971), Rubinstein and Wolinsky (1985), Gale (1986a,b), Duffie et al. (2005, 2007), Vayanos and Weill (2008), and Lagos and Rocheteau (2009). Wolinsky (1990) considers a decentralized market with asymmetric information on the common quality of all units. Serrano and Yosha (1993), Blouin and Serrano (2001), and Duffie et al. (2009, 2014) provide other contributions to this literature.

[9] Blouin (2003), Camargo and Lester (2014), and Moreno and Wooders (2014) characterize non-stationary equilibria in infinite horizon games. The main differences among these papers have to do with the division of trade surplus and are partly driven by alternative bargaining protocols. The former two papers adopt the exogenous price bargaining of Wolinsky (1990), while the latter assume buyers make take-it-or-leave-it offers.

strategies depend on time. In this paper time on market coincides with the number of buyers previously encountered, but I prefer to use the expression 'time on market' for lexical convenience.[10] Compared to models featuring ITS, I adopt an alternative assumption on the cost of finding a new trade opportunity, thereby capturing a different economic interpretation on the nature of delay costs. In models with ITS, postponing trade imposes a cost via time discounting, reducing the present value of a positive expected payoff. Trading late is costly, but market participation always provides a non-negative payoff. In contrast, in my model postponing trade imposes a per period cost in the form of an additive utility loss and, for simplicity, there is no discounting. Analogous results hold with discounting if the latter has a sufficiently small order of magnitude relative to search costs; see Example 7.1. Since finding a new trade opportunity imposes an additive and symmetric (w.r.t to sellers' types) cost, cumulative search costs may be larger than total gains from trade. In contrast, intertemporal separation crucially relies on discounting an *instantaneous* payoff as it guarantees two essential properties: first, a strict single crossing condition with respect to time, and, second, perpetual market participation as waiting costs cannot lead to a negative expected payoff. In section 3 I discuss the various implications in full. Several papers adopt a model setup with search costs and no discounting: Lauermann and Wolinsky (2015) in the dynamic adverse selection literature, Atakan (2006) in the matching literature, and many sequential search models (for example Stigler (1961), Rothschild (1973), Reingaum (1979) and Stahl (1989)).

My model is closely related to the literature on sequential trading between a long-lived seller and a sequence of short-lived buyers. Taylor (1999) considers a two-period model in which a single informed seller posts prices under different price observability assumptions.[11] His paper was the first to point out the negative information externality that affects older cohorts of sellers when buyers observe private informative signals. Kaya and Kim (2014) extend Taylor's model to an infinite horizon setup in which buyers observe sellers' time on market, receive private informative signals and make offers to sellers. In their setup, prices and beliefs converge to a steady state and the transition depends on the initial probability of trading with a high quality seller: if it is high, beliefs move downward as in Taylor (1999) and a high price is offered less often; if it is low, sellers separate over time. In my model, a similar downward price movement takes place when time on market is observable and search costs are small. Moreover, I explicitly compare the interactions among the magnitude of search costs, the pre-

[10] Kim (2014) shows that when market frictions are small (small discount rate), observing only time on market is welfare improving relative to public information on previous matches. This result stems from the fact that staying on the market strengthens reputation; in this respect, information on previous matches conveys a more precise signal than time on market.

[11] Hörner and Vieille (2009) and Fuchs et al. (2014) also study the effect of price history observability in models in which buyers have no informative signals.

cision of buyers' signals, and time on market observability. Lastly, Lauermann and Wolinsky (2015) consider a stationary sequential search model with informative private signals for buyers and additive search costs. They show the existence of a search friction that reduces price informativeness compared to a common auction environment. They consider buyers who receive signals sampled from a continuous distribution—possibly of unbounded precision—while I use a simple symmetric binary signal of bounded precision. My choice is motivated by tractability concerns, especially for non-stationary equilibria. Moreover, I focus on allocative efficiency and market exclusion, while they analyze informational efficiency.

My paper is also related to a new strand of literature on optimal market intervention for lemons markets. In particular, Fuchs and Skrzypacz (2015) consider government interventions through taxes and subsidies.[12] Their Pareto improving budget balanced policy suggests a short tax exempt trading window followed by a short lived period of positive taxes; sellers trade immediately and afterwards the tax goes back to zero. In equilibrium, no taxes are actually paid. They exploit the intertemporal separation mechanism, and the goal of taxes and subsidies is to reduce the amount of inefficient delay necessary to separate different sellers' types. My efficient intervention also points out the need to subsidize initial trade, but it prescribes a constant market participation tax thereafter. In equilibrium, revenues from the per period market participation tax are used to finance the rebate after trade.

3 Model

This section presents the model setup and discusses how the main assumptions relate to the research questions.

3.1 Model setup

Consider a decentralized market where trade is only possible in bilateral transactions between one buyer and one seller.[13] Each seller is endowed with a single indivisible good of high (H) or low (L) quality. A seller knows the quality of his product but nobody else can observe it. Let θ_λ and v_λ be buyers and sellers' valuations, respectively, for a product of quality $\lambda \in \{H, L\}$, and assume $\theta_H > v_H > \theta_L > v_L$. I use the terms H-sellers and L-sellers to refer to sellers with high and low quality goods.

[12] Their paper differs from Philippon and Skreta (2012) and Tirole (2012) because the latter consider a government intervention in the presence of a static competitive private market.

[13] As a convention, throughout the paper I refer to the seller as "he" and to the buyer as "she".

Time $t \in \{..,-1,0,1,..\}$ is discrete and in each period a set of sellers μ_t of unit mass is born. Only a fraction $q^0 \in (0,1)$, independent of t, of newly born sellers owns a high quality good. Sellers are long-lived and they can participate in the market until they trade or exit. Buyers live for a single period and they always outnumber sellers.

I denote the set of sellers participating in the market at time t as S_t, while $S_t^\kappa \subset S_t$ is the set of sellers who have been participating in the market for $\kappa \in \mathbb{N}_0$ previous periods; similarly, $S_{\lambda,t}^\kappa \subset S_t^\kappa$ is the subset of sellers of type λ in S_t^κ.[14] Sellers pay a search cost c to participate in the market and match with a buyer. Buyers match uniformly at random with sellers and have no search cost. For simplicity, they have no opportunity to buy a good and re-sell it on the market. All players are risk-neutral and have quasi-linear utilities with respect to monetary transfers. Sellers do not discount future payoffs.

Buyers and sellers trade according to a simple mechanism. Each seller $i \in S_{t-1} \cup \mu_t$ who has *not* traded at time $t-1$ takes an action $a_{S,i} \in A_S = \{\{D\},\mathbb{R}_+\}$, where D denotes the decision to irreversibly drop out of the market and $p \in \mathbb{R}_+$ is the posted price at which he commits to sell the good in period t. If $a_{S,i} = D$, seller i is not matched with a buyer and does not pay the cost c; however, he has no future possibility of participating in the market. If $a_{S,i} = p$, seller i pays c and gets matched with a buyer. A particular history for seller $i \in S^\kappa$ is indicated with $h_i^\kappa = (h_i^{\kappa-1}, a_{S,i}^\kappa \times a_{B,i}^\kappa)$ (with $h_i^{-1} = \varnothing$) and H^κ is the set of all possible histories.

Let $Z_i^\kappa \subset H_i^\kappa$ denote the set of terminal histories for seller i after κ previous periods (with $Z_i = \bigcup_{\kappa\in\mathbb{N}_0} Z_i^\kappa$). If $h_i^\kappa \in Z_i^\kappa$ seller i exits the market after κ previous periods in the market and he cannot choose any further action, i.e. $A_{S,i}^j = \varnothing$, $j \geq \kappa+1$. Let $Z_i^\kappa(D) \subset Z_i^\kappa$ include all terminal histories in which seller i drops out of the market after κ periods; similarly, $Z_i^\kappa(p) \subset Z_i^\kappa$ denotes the set of histories in which seller i trades at price p after κ previous periods in the market. The final payoff to seller $i \in S_{\lambda,t}^\kappa$ in $z \in Z_i^\kappa$ is

$$\tilde{u}_\lambda(z) = \begin{cases} -\kappa c & \text{if } z \in Z_i^\kappa(D) \\ p - v_\lambda - (\kappa+1)c & \text{if } z \in Z_i^\kappa(p) \end{cases}$$

Once matched with seller i, a buyer receives a private signal $\xi \in \{H,L\}$ on his product quality, but she cannot observe his previous price history.[15] Buyers' signals have precision $\gamma \in (\frac{1}{2},1)$, i.e. $\mathbb{P}_H(\xi = H) = \mathbb{P}_L(\xi = L) = \gamma$. For a given vector $(\theta_H, v_H, \theta_L, v_L)$, I parametrize a specific economy $\mathscr{E}(\gamma, q^0)$ by signal precision γ and a newly born measure q^0 of H-sellers.

I consider two different setups for publicly available information. If time on market is

[14] To simplify exposition, I slightly abuse notation using S, S^κ, S_λ^κ to denote both the set and the measure of sellers.

[15] This assumption significantly simplifies the set of possible equilibria. See Taylor (1999), Hörner and Vieille (2009) and Fuchs et al. (2014) for models that consider equilibria with price history observability.

observable (TMO), a buyer observes how long a seller has been participating in the market; i.e. it is common knowledge whether $i \in S_t^\kappa$ for some $\kappa \in \mathbb{N}_0$. In contrast, if time on market is *not* observable (TMN) no buyer can observe this information.

When time on market is observable, a buyer's information set $\mathscr{I}_B(p,\kappa,\xi)$ includes the seller's offer p, his previous κ periods in the market and the buyer's signal ξ. If time on market is not observable, it only includes p and ξ (i.e. $\mathscr{I}_B(p,\xi)$). Given her information set, a matched buyer takes an action $a_{B,i} \in A_B = \{A,R\}$, where A denotes acceptance and R rejection of seller i price offer. If she accepts offer p, trade occurs and they leave the market; in case of rejection, no exchange takes place and seller i moves to period $t+1$.

I only consider equilibria in which players' equilibrium strategies do not depend on time t. Strategies may depend on seller's type λ, cohort κ and history $h_i^{\kappa-1}$. Notice that invariance with respect to t does not exclude non-stationary equilibria, as strategies may be different for each cohort κ and, within each cohort κ, for different histories $h_i^{\kappa-1}$. In equilibrium, the mass of sellers S_t, S_t^κ and $S_{\lambda,t}^\kappa$ is constant over time—i.e. $S_t = S$, $S_t^\kappa = S^\kappa$ and $S_{\lambda,t}^\kappa = S_\lambda^\kappa$ for every $\kappa \in \mathbb{N}_0$ and $t \in \mathbb{Z}$—and I omit the subscript t in the remainder of this paper. I denote a strategy profile with σ and a belief system with π. A strategy profile σ and a belief system π form an assessment (σ,π). I use σ_{-i} and π_{-i} to indicate the strategy profile and the belief system of any agent other than i.

Let $q^\kappa = \mathbb{P}(\theta_H|S^\kappa)$ be the prior probability under uniform random matching that a seller in S^κ offers a high quality good. On the equilibrium path, a buyer incorporates her private signal into the publicly available information according to Bayes' rule.

Definition 3.1 *A assessment (σ,π) is an equilibrium of the game if it is a weak Perfect Bayesian Equilibrium (PBE) with the following restrictions:*

1. *Symmetry: if sellers $i,j \in S_\lambda^\kappa$ have $h_i^{\kappa-1} = h_j^{\kappa-1}$, they play the same strategy.*
2. *In equilibrium buyers play pure behavioural strategies.*

I adopt a standard notion of Perfect Bayesian Equilibrium with two restrictions. First, strategies do not depend on time t.[16] This restriction does not prevent strategies from being non-stationary as they may depend on κ, i.e. how long a seller has been on the market. I introduce time t but distinguish among cohorts κ in order to encompass both the TMO and the TMN case in the same model setup. Second, conditional on their information set, buyers play pure strategies.[17] Nonetheless, sellers *can* play mixed strategies in equilibrium, and buyers can

[16] See Definition 7.1 in Appendix A for a formal definition.

[17] Buyers' strategies depend on the information set, hence strategies may depend on signal ξ despite the fact that the latter may not change the belief π_B.

deviate using a mixed strategy. The pure strategy restriction on buyers' equilibrium strategies simplifies the set of possible outcomes, and it delivers equilibrium predictions consistent with the available empirical evidence. Relaxing this restriction may lead to the existence of other equilibria. For example, it is straightforward to show that a separating equilibrium in mixed strategies exists for all $c > 0$ when $\gamma \geq \frac{\theta_H - \theta_L + c}{2\theta_H - v_H - \theta_L + c} > \frac{1}{2}$.[18] For simplicity, I do not specify out-of-equilibrium beliefs in the proposition statements. The main result for the admissible equilibrium strategies—Lemma 4.1—rules out other strategies without relying on any specific out-of-equilibrium belief. The admissible equilibrium strategies only require buyers to hold sufficiently pessimistic beliefs out of the equilibrium path.

My main departure from the previous literature—with the notable exception of Lauermann and Wolinsky (2015)—is the explicit introduction of a per period search cost. This assumption is common in the sequential search literature, while the dynamic adverse selection literature has mostly considered a specific preference specification in which the discount rate can be interpreted as a measure of search frictions. In particular, the standard utility specification is $\delta^{\kappa}(p - v_{\lambda})$, where κ is the time index. This utility function is justified on the grounds that sellers receive a flow payoff during the time spent on the market or, alternatively, sale and production occur contemporaneously. Thanks to this utility specification, in equilibrium sellers always find it convenient to participate in the market as long as they can trade at a price greater or equal to their reservation value v_{λ}, since $\delta^{\kappa}(p - v_{\lambda}) \geq 0$ for every κ. In several models (Moreno and Wooders (2010, 2014), Kaya and Kim (2014)) H-sellers only trade at $p = v_H$, eliminating de facto their temporal preferences. This property is crucial to support intertemporal separation of types; in fact, high quality sellers accommodate any period of delay deemed necessary to prevent low quality sellers from deviating. Final allocations are inefficient because H-sellers delay their trades, but all sellers eventually trade.

In my model I explicitly consider search costs as an additive utility loss and I ignore time discounting, i.e. $u(\lambda, \kappa, p) = p - v_{\lambda} - (\kappa + 1)c$. This per period cost could be interpreted as a search cost, a market participation fee or a maintenance cost for displaying the good on the market. Other than in the aforementioned paper by Lauermann and Wolinsky (2015), it has been routinely used in the sequential search literature.[19] This preference specification has two main properties: (i) sellers stay out of the market if they expect to trade after a long time because the cumulative amount of search costs would be larger than their total gains from

[18] After receiving a high signal, buyers should play a specific randomization strategy between accepting or rejecting θ_H. A lower search cost requires a higher rejection probability. Even with mixed strategies, separation is not possible in the limit case of $c = 0$, while the equilibria in section 4.1 continue to exist.

[19] The introduction of discounting would increase the number of equilibria by introducing additional possibilities to use time as a signalling device. However, in my analysis I focus on some equilibria that match a few empirical patterns presented in section 1, and whose main characteristics are not affected by the possibility to introduce time discounting in the model.

trade; and (ii) all sellers suffer the same utility loss if they postpone trade.[20] Introducing an additive search cost is not the only possibility to satisfy these two properties. For example, it is also the case for $\delta^\kappa p - v_\lambda$, a utility specification that can easily be interpreted as a seller who discounts future prices but incurs the production cost v_λ before entering the market. More formally, in models that support intertemporal separation, preferences satisfy a single crossing property with respect to time (κ) which makes high quality sellers relatively more patient. For a utility function $u(\lambda,\kappa,p)$, the strict single-crossing condition is satisfied if $\frac{u_p(\lambda,\kappa,p)}{|u_\kappa(\lambda,\kappa,p)|}$ is strictly increasing in λ and it has the same sign for all (λ,κ,p) (see Milgrom and Shannon (1994)). If $u(\lambda,\kappa,p)=\delta^\kappa(p-v_\lambda)$ the ratio of partial derivatives is $\frac{\delta^\kappa}{|(p-v_\lambda)\delta^\kappa \ln\delta|}$, always positive and strictly increasing in λ as $v_H > v_L$. In contrast, in my model $u(\lambda,\kappa,p)=p-v_\lambda-(\kappa+1)c$ this ratio is equal to $\frac{1}{c}$. Similarly, if $u(\lambda,\kappa,p)=\delta^\kappa p - v_\lambda$, then $\frac{\delta^\kappa}{|p\,\delta^\kappa \ln\delta|}$ is constant for all λ.

3.2 Static benchmark

To understand how the temporal dimension may alleviate the adverse selection problem, it is customary to exclude economies in which a static model predicts that all sellers trade. In this case no allocative efficiency problem arises. This efficient equilibrium outcome exists only if all sellers post the same price p and buyers always accept. If buyers do not have informative signals, all sellers trade only if the maximum price they are willing to pay—equal to their expected value for the good—is higher than the reservation value of H-sellers. Formally,

$$q^0\theta_H + (1-q^0)\theta_L - c \geq v_H \quad \rightarrow \quad q^0 \geq \frac{v_H - \theta_L + c}{\theta_H - \theta_L} := q_c^S \text{ [21]} \tag{1}$$

Buyers pay at least $v_H + c$ only if they hold a sufficiently high prior probability q^0 of matching with a high quality seller. In the remainder of this paper, unless specified, I assume $q^0 < q_c^S$.

If buyers have informative signals, all sellers trade only if buyers accept the pooling price p after a low signal realization. This equilibrium is possible only if $p \leq \mathbb{E}_{\pi_B}[\theta|\mathscr{I}_B(p,0,L)]$ and $p \geq v_H + c$. These two conditions imply $\mathbb{E}_{\pi_B}[\theta|\mathscr{I}_B(p,0,L)] \geq v_H + c$, i.e.:

$$q^0 \geq \frac{\gamma(v_H + c - \theta_L)}{(1-\gamma)(\theta_H - v_H - c) + \gamma(v_H + c - \theta_L)} := q_c^{IP}$$

The highest possible pooling price, $p = \mathbb{E}_{\pi_B}[\theta|\mathscr{I}_B(p,0,L)]$, is decreasing in γ because a more informative signal has a stronger negative impact on the posterior expectation. Therefore,

[20] Atakan (2006) highlights the role of asymmetric delay costs in a model of assortative matching.

[21] The subscript c indexes threshold values for q^0 to the search cost c. Later I use q_0^{IP} to denote the value of the threshold for $c = 0$.

q_c^{IP} increases in γ and $q_c^{IP} > q_c^S$ for every $\gamma > \frac{1}{2}$.

4 Equilibrium analysis

One of the most common results in the dynamic adverse literature is the possibility for all sellers to trade. Different types of sellers are more likely to trade at different points in time: low quality goods trade earlier and, on average, at lower prices, while high quality goods trade less rapidly and at higher prices. In this respect, waiting is a signalling device analogous to education in the classic Spence (1973) model. Buyers find this separating mechanism credible, and they are willing to pay higher prices to sellers who have been on the market longer. Importantly, intertemporal separation works when buyers do not have any informative signals. This result is no longer valid if delay costs enter utility in an additive way as in the present model setup.

Proposition 4.1 *If buyers have uninformative signals, no intertemporal separation is possible.*

Intuitively, time could credibly signal higher quality only if H-sellers incur a cumulative utility loss larger than all gains from trade for high quality goods. This delay makes market participation unprofitable, and they prefer to stay out of the market. Formally, the incentive compatible delay period leads to a utility loss which violates the individual rationality constraint of H-sellers.[22]

4.1 Negligible search costs

This section provides a complete characterization of equilibria—under the restrictions of Definition 7.1—when the search cost c is close to zero. I analyze each public information setup (TMO and TMN) separately although the underlying economic intuition is similar.

The main result if c is small is that L-sellers have strong incentives to pool on H-sellers' price offers. Lemma 4.1 states the admissible behavioural strategies on the equilibrium path.

Lemma 4.1 *There exists $c^* > 0$ such that for every $c \leq c^*$ every equilibrium path only admits the following behavioural strategies:*

- *TMO and $q^0 < q_c^S$:*

[22]The impossibility of this result depends on the additive specification of delay costs, and it is unchanged even if I consider the set of equilibria in which buyers may play fully mixed strategies (see the proof of Propositon 4.1).

– *H- and L-sellers in S^κ post a price p^κ that buyers accept only after a high signal, or all prices they offer are rejected with a probability of one.*[23]

– *H-sellers stay out of the market and L-sellers trade at price θ_L.*

- *TMN and $q^0 < q_c^{IP}$:*

  – *H and L-sellers in every cohort S^κ post the same price $\bar{p}$ which buyers accept only after a high signal.*

  – *H-sellers stay out of the market and L-sellers trade at price θ_L.*

Irrespective of time on market observability, if H-sellers participate in the market and post a price accepted with positive probability, equilibria are only possible in pooling strategies.[24] Both types of seller offer the same price and buyers accept only if they receive a high signal. When time on market is observable, prices may be different among different cohorts of sellers, but they post a single price when this information on time on market is not available.[25] Low search costs reduce the cost of finding a new buyer, and low quality sellers find it profitable to demand a high price, looking for a buyer who receives a wrong signal. Even if a low quality seller is unlikely to receive a high signal, this event has a strictly positive probability $1-\gamma > 0$. If they happen not to sell at a high price, they can always reveal their type and trade at θ_L. The absence of a credible signalling device precludes separation, and different types of sellers pool on the same action.

Lemma 4.1 helps to characterize the set of equilibria of this dynamic signalling game, especially in the non-stationary TMO case. The game cannot be solved recursively because current strategies depend on future continuation values and, vice versa, strategies endogenously determine the share of high quality sellers in each cohort. Figure 1 represents this equilibrium interaction between current strategies and future continuation values. To further illustrate this point, consider how buyers form expectations. First, they have a prior probability of being randomly matched with a high quality seller in S^κ. In equilibrium, this prior is determined endogenously and is equal to the share of H-sellers in cohort S^κ because of uniform random matching (i.e. $q^\kappa = \frac{S_H^\kappa}{S^\kappa}$). Once matched with seller i, a buyer observes the seller's age and posted price, and updates her beliefs according to signal ξ. The value q^κ plays a substantial

[23] The last case is not interesting, and it is a pathological result of signalling games. The PBE notion allows for these 'sudden stops' in trade when buyers hold pessimistic beliefs on sellers in cohort S^κ and accept only if $p \leq \theta_L$; in equilibrium, both types of seller prefer not to trade and move to period $\kappa+1$. These behavioural strategies are ruled out by the undefeated refinement. I do not use the refinement at this stage to stress that the pooling result does not rely on the refinement or other specific restrictions on out-of-equilibrium beliefs.

[24] The *if* clause is crucial. Propositions 4.2 and 4.3 show that H-sellers may *not* participate in the market.

[25] Lemma 4.1 has different thresholds for q^0; see the proof of Lemma 4.1 for further details.

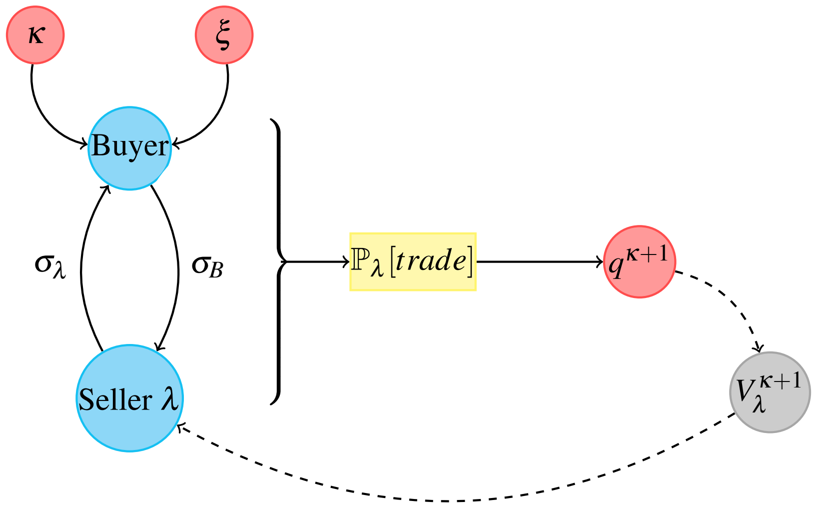

Figure 1: Equilibrium interactions with TMO.

role in forming expectations, and it contributes to determine the maximum price that buyers accept from a seller belonging to cohort S^κ. In turn, equilibrium prices determining whether H-sellers want to participate in the market, while equilibrium strategies determine the type-dependent trade probabilities and the evolution of q^κ across different cohorts S^κ.

Thanks to Lemma 4.1—whose long proof is presented in Appendix B—the set of admissible behavioural strategy is tractable and the equilibrium characterization straightforward. The next two subsections illustrate how final market outcomes depend on time on market observability. In Propositions 4.2 and 4.3, I am going to use the undefeated equilibrium refinement introduced by Mailath et al. (1993).[26] I adopt this refinement for two simple purposes: (i) to rule out the self-fulfilling PBE in which only L-sellers trade because buyers believe only L-sellers participate, whenever there exists another PBE in which H-sellers participate in the market; and (ii) to select the one with the highest possible price among the set of pooling PBE, i.e. $p^\kappa = \mathbb{E}_{\pi_B}[\theta|\mathscr{I}_B(p^\kappa,\cdot,H)]$. Therefore, the refinement is *not* used to rule out separating or semi-separating strategy profiles, differently from what happens in the Spence (1973) model.[27] Indeed, Lemma 4.1 does not rely on the undefeated refinement to show that the only admissible behavioural strategies have both types of sellers in the set S^κ pool strategies on the same price. *De facto* this refinement selects the equilibrium in which H-sellers' market participation constraint is satisfied for the lowest possible q^0.[28]

[26] See Definition 7.2 in the Appendix for a formal definition.

[27] See Mailath et al. (1993) for a discussion of this point.

[28] Roughly speaking, H-sellers in S^κ participate only if $\mathbb{E}_{\pi_B}[\theta|\mathscr{I}_B(p^\kappa,\cdot,H)] \geq p^\kappa \geq v_H$, and, in a pooling equilibrium, this expectation is strictly increasing in π_B, which, in turn, is weakly increasing in q^0.

4.1.1 Time on market observability

Time on market observability (TMO) refers to buyers' ability to observe how long each seller has been participating in the market. Although this information is specific to each individual seller, it plays a crucial role in shaping overall market dynamics. The model provides a tractable framework to analyze how the bilateral asymmetric information problem affects aggregate market dynamics and—reciprocally—how market dynamics influence the possible terms of trade in bilateral transactions.

Proposition 4.2 *Let $q^0 < q_c^S$. There exists $c^* > 0$ such that $\forall c \leq c^*$ there is a unique undefeated equilibrium.*

1. *If* $q^0 \geq q_c^O := \dfrac{(1-\gamma)(v_H + \frac{c}{\gamma} - \theta_L)}{\gamma(\theta_H + \frac{c}{\gamma} - v_H) + (1-\gamma)(v_H - \theta_L - \frac{c}{\gamma})}$

   - *For $\kappa \leq \kappa^*(q^0) < \infty$, $\kappa^*(q^0) = \max\{\kappa \in \mathbb{N}_0 : q^\kappa \geq q_c^O\}$, H- and L-sellers in S^κ post*

     $$p^\kappa = \mathbb{E}_{\pi_B}[\theta | \mathscr{I}_B(p^\kappa, \kappa, H)]$$

     and buyers accept if and only if $\xi = H$. Prices p^κ are strictly decreasing in κ.

   - *After $\kappa^*(q^0) + 1$ periods H-sellers exit the market while L-sellers post θ_L and trade.*

   - *For $\kappa < \kappa^*(q^0)$ the share of H-sellers across different cohorts is decreasing in κ:*

     $$q^{\kappa+1} = \frac{(1-\gamma)q^\kappa}{(1-\gamma) + (2\gamma - 1)(1 - q^\kappa)}$$

2. *If $q^0 < q_c^O$ only L-sellers participate in the market and trade at price θ_L.*

Proposition 4.2 proves the existence of a unique undefeated equilibrium for a sufficiently small c. All sellers from cohort S^κ post the same price p^κ, and buyers only accept if they receive a high signal $\xi = H$. H-sellers are more likely to trade and, on average, they exit the market more rapidly than L-sellers. The share of H-sellers decreases in κ: the longer a seller has been on the market, the lower buyers' prior belief in matching with a high quality seller. Once this belief falls below the minimum threshold q_c^O, no buyer would be willing to pay a price above H-sellers' reservation utility, and the latter prefer to drop out of the market.

Taylor (1999) was the first to point out a negative price externality for older cohorts of sellers. Kaya and Kim (2014) recently obtained a similar dynamic when the initial prior belief in meeting a high quality seller is sufficiently high. Proposition 4.2 suggests a declining price

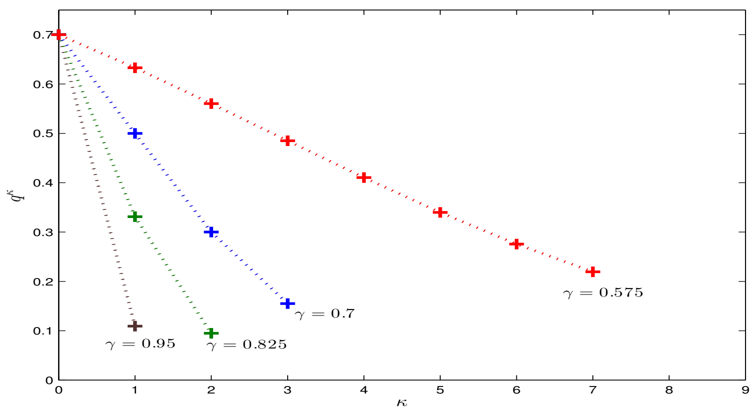

Figure 2: Number of periods on the market for H-sellers $\kappa^*(q^0)$ and signal precision γ.

path *and* a decision to exit the market after a finite number of periods. No market dropout occurs in Taylor (1999) or Kaya and Kim (2014).

Greater signal precision γ leads to a more rapid decrease in q^κ (see Figure 2). However, γ's effect on the measure of H-sellers that exit the market without trading is ambiguous: in this discrete time model an increase in γ may reduce $\kappa^*(q^0)$, but it also increases the share γq^κ of H-sellers who trade in every period $\kappa \leq \kappa^*(q^0)$.

4.1.2 Time on market not observable

When time on market is not observable, Lemma 4.1 guarantees that all sellers post the same price and buyers only accept if they receive a high signal. Buyers do not distinguish sellers' cohorts, so their prior belief in matching with a high quality seller does not depend on κ and is equal to the share of H-sellers in the overall market. In a stationary equilibrium, this share does not change over time because the mass of each type of seller is constant, i.e. $\bar{q}_t = \bar{q}$ and $S^\kappa_{\lambda,t} = S^\kappa_\lambda$ for every t and λ. This is possible if and only if the entry and exit flows are equal for each type. The entry and exit conditions impose a pair of equations that jointly determine $\overline{q}$ and the overall measure of sellers, say $\overline{S}$.

$$\begin{aligned} \text{H-sellers:} \quad & q^0 = \overline{S}\gamma\overline{q} \\ \text{L-sellers:} \quad & (1-q^0) = \overline{S}(1-\gamma)(1-\overline{q}) \end{aligned}$$

The following proposition describes the equilibrium.

Proposition 4.3 *Let $q^0 < q^{IP}_c$. There exists $c^* > 0$ such that $\forall c \leq c^*$ there is a unique undefeated equilibrium.*

1. *If* $q^0 \geq q_c^N := \dfrac{v_H - \theta_L + \frac{c}{\gamma}}{\theta_H - \theta_L} > q_c^S$

   - *Both types of sellers post price*

   $$p^k = \overline{p} = \mathbb{E}_{\pi_B}[\theta | \mathscr{I}_B(\overline{p}, H)] = q^0\theta_H + (1-q^0)\theta_L$$

   for all $\kappa \in \mathbb{N}_0$ *and buyers accept if and only if* $\xi = H$.

   - *In every period*

   $$\bar{S} = \frac{\gamma - q^0(2\gamma - 1)}{\gamma(1-\gamma)} \qquad \bar{q} = \frac{q^0(1-\gamma)}{\gamma - q^0(2\gamma-1)} < q^0$$

2. *If* $q^0 < q_c^N$ *only L-sellers participate in the market and they trade at price* θ_L.

Similarly to Proposition 4.2, H-sellers do not participate in the market when their initial share is too small ($q^0 < q_c^N$). In comparison with the TMO case, they participate in the market for a smaller set of economies as the threshold q_c^N is strictly higher than q_c^O.[29] If the initial share of high quality sellers is sufficiently high, all sellers post a unique price $\overline{p}$ irrespective of their previous periods in the market. Sellers are not penalized if they trade late because buyers do not observe previous time on market and they hold a single prior probability $\bar{q}$. If high quality sellers participate in the market, they will continue to do so until they trade because sellers are forward looking and previous search costs are sunk.

The equilibrium share of H-sellers $\bar{q}$ is strictly lower than q^0. The underlying economic intuition is simple: on average, H-sellers trade before L-sellers ($\frac{1}{\gamma}$ versus $\frac{1}{1-\gamma}$ periods, respectively); the latter stay longer on the market and decrease H-sellers' market share to below q^0. The value of $\bar{q}$ is negatively related to signal precision γ because it decreases the average time on market for H-sellers and increases it for L-sellers. The negative impact of γ on $\bar{q}$ perfectly outweighs the positive effect that higher signal precision has on buyers' posterior beliefs after $\xi = H$. This feedback effect makes signal precision irrelevant for equilibrium prices; in fact, the pooling price $\overline{p} = q^0\theta_H + (1-q^0)\theta_L$ does not depend on γ. In particular, it is equal to buyers' expected value for a good offered by *newly* born sellers (S^0) *before* receiving a signal.

When $\gamma > \frac{1}{2}$ all sellers trade immediately only if $q^0 \geq q_c^{IP} > q_c^S$. Proposition 4.3 implies that H-sellers participate only if $q^0 \geq q_c^N > q_c^S$. Therefore, when c is small neither the temporal dimension nor buyers' informative signals mitigate the adverse selection problem. Actually,

[29] Precisely, this holds when $\frac{c}{\gamma} < \theta_H - v_H$. This is a necessary condition to obtain $q_c^N < 1$.

because $q_c^N > q_c^S$, all sellers trade for a strictly smaller set of economies compared to the classic static adverse selection model (even if $q_0^N = q_0^S$). Janssen and Roy (2004) point out that the infinite repetition of the static equilibrium is the only stationary equilibrium of a dynamic adverse selection model with uninformed buyers. Proposition 4.3 suggests that this result also applies when buyers have informative signals. Even though this conclusion seems extreme, the mechanism in place is interesting and it might be worth assessing its empirical validity.

4.1.3 TMO vs. TMN with negligible search costs

I now compare whether the availability of public information on time on market may be useful to increase H-sellers market participation when search costs are close to zero.

When time on market is not observable and $q^0 < q_0^N = q_0^S$, only L-sellers trade and final allocations are identical to the ones in Akerlof (1970) model. Obviously the equilibrium allocation does not maximize total welfare. When time on market is observable and $q^0 \in [q_0^O, q_0^S)$ all H-sellers initially participate in the market, but a strictly positive measure does not trade their goods because they drop out after a finite number of periods (see Proposition 4.2). Not all mutually beneficial exchanges take place, resulting in allocative inefficiency. Nevertheless, H-sellers participate and trade with positive probability at least for one period, but they always stay out of the market if time on market is not observable. To sum up, when c is small, a dynamic model with private informative signals achieves a welfare improvement compared with a static model with uninformed buyers only when time on market is observable and $q^0 \in [q_0^O, q_0^S)$.

Figure 4.1.3 illustrates the regions in which pooling equilibria exist. Each economy is parametrized by a (γ, q^0) coordinate. Depending on time on market observability, there are different pooling equilibria: immediate pooling (white; $q^0 \geq q_0^{IP}$), pooling with TMN (white and blue; $q^0 \geq q_0^S$), pooling with TMO[30] (white, blue and green; $q^0 \geq q_0^O$), and no pooling equilibrium irrespective of time on market observability (red; $q^0 < q_0^O$).

4.2 Non-negligible search costs

In this section I consider whether markets with non-negligible search costs are more or less likely to mitigate the adverse selection problem, by inducing high quality sellers to participate and trade. Naively speaking, lower search costs increase final payoffs and relax H-sellers' individual rationality constraint. However, this intuition fails to take into account how equilibria might change. As discussed in section 4.1, when search costs are small all sellers pool on the

[30] For $c < c^*$ the equilibria in Proposition 4.2 and 4.3 exist for every $q^0 \geq q_0^O$ and $q^0 \geq q_0^N$, respectively. However, they may not be unique when $q^0 \geq q_0^S$ or $q^0 \geq q_0^{IP}$, respectively.

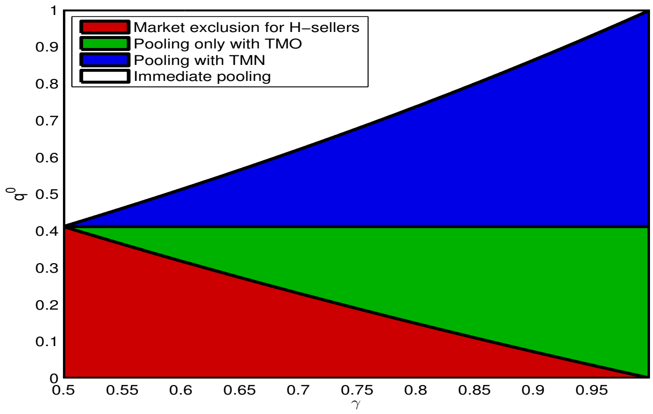

Figure 3: Equilibria in different economies $\mathscr{E}(\gamma, q^0)$ for $c \leq c^*$.

same price, and H-sellers only participate in the market if their initial share is sufficiently high. In this section I show how the combined effect of buyers' informative signals and search costs may lead to separation between the two sellers' types, and mitigate the extent to which the market breaks down. Before stating the main results, I first introduce the notion of a separating equilibrium.

Definition 4.1 *An equilibrium assessment is separating if H- and L-sellers post different prices after every history $h_i^\kappa \in H^\kappa$ and buyers accept them with positive probability.*

Proposition 4.4 characterizes the unique separating equilibrium of the game.

Proposition 4.4 *Irrespective of time on market observability, a separating equilibrium exists for every $q^0 \in (0,1)$ if and only if*

$$c \in \left[\frac{1-\gamma}{\gamma}(\theta_H - \theta_L), \gamma(\theta_H - v_H)\right]$$

In equilibrium, high quality sellers post θ_H and low quality sellers post θ_L. Buyers accept θ_H after a high signal, but they always accept θ_L.

A separating equilibrium exists when both search costs and signal precision are sufficiently *high*. Importantly, this equilibrium exists irrespective of the initial share q^0 of H-sellers entering the market in every period. However, if the signal precision γ is too low, the interval in Proposition 4.4 does not exist. For example, for γ close to $\frac{1}{2}$, the lower bound of the interval is higher than the upper bound. If separation is possible, low quality sellers do not post θ_H, waiting until a buyer receives a high signal, because they expect to pay too high a search cost compared to the immediate payoff for revealing their type and trading at a lower price θ_L. On average, H-sellers receive a high signal after $\frac{1}{\gamma}$ periods, while L-sellers do so after $\frac{1}{1-\gamma}$ periods; as a result, informative private signals create an asymmetric cost of delay between seller' types. In other words, differences in the probability of receiving a high signal restore a single crossing condition and allow separation. The temporal dimension is a necessary condition, but it contributes to the creation of a credible signalling device only with sufficiently informative signals *and* an adequate level of search costs. Final allocations do not maximize welfare because sellers pay strictly positive search costs, but all sellers eventually trade.

The beneficial signalling effect of search costs may extend to intermediate values of c, i.e. when search costs are too small to create a separating equilibrium but too large to support a pooling equilibrium. Unfortunately, providing a complete equilibrium characterization for all values of c is a complex endeavour, especially in the TMO case. As a result, I justify this claim through a specific semi-separating equilibrium.

Proposition 4.5 *Irrespective of time on market observability, there exists a region of parameters* $(\theta_H, v_H, \theta_L, v_L, q^0, \gamma)$ *where*

- *Only L-sellers trade for sufficiently small* c.
- *For*
$$c \in \left[\frac{\gamma(1-\gamma)}{\gamma^2+\gamma-1}(v_H-\theta_L), \frac{1-\gamma}{\gamma}(\theta_H-\theta_L)\right)$$
there exists at least one semi-separating equilibrium in which all sellers trade.

In this semi-separating equilibrium all H-sellers participate in the market and trade. However, γ and c should be high enough to exclude complete pooling on the same action. In equilibrium, H-sellers only post a high price and L-sellers mix between the high price and θ_L. Posted prices do not depend on κ and all sellers trade over time.[31]

To sum up, search costs can be beneficial by discouraging low quality sellers from pretending to have a high quality good. Although a small c makes participation cheaper, it may worsen adverse selection and leave high quality goods out of the market when their initial share is low. In the next section, I consider whether it is possible to enjoy the welfare benefits of low search costs without exacerbating the adverse selection problem.

[31] See the proof of Proposition 4.5 for a complete characterization of the equilibrium.

5 Market design

As previously explained, when the cost c is small H-sellers may have less incentive to participate in the market. Even when all sellers participate, equilibria are in pooling strategies and prices do not provide any information on product quality. If informational efficiency is considered relevant, this is another loss to take into account.

From a policy perspective, understanding how a benevolent market designer can intervene to promote full market participation for the largest possible set of economies is crucial. I adopt a stringent benchmark for the objectives of the market design intervention: the resulting equilibrium has to achieve both allocative (i.e. all sellers trade over time) and informational efficiency (i.e. prices reveal sellers' types). In my setup, an allocative efficient equilibrium maximizes utilitarian welfare when $c = 0$.

The market designer is subject to a series of reasonable limitations. First, the mechanism has to be budget balanced on the equilibrium path. This restriction seems natural as the market should not depend on any external amount of resources to induce participation and trade. Second, transfers cannot be conditional on any posted price. Differently from buyers, the market designer cannot observe currently posted prices. This restriction is consistent with the idea that bilateral transactions involve elements of private negotiation that are difficult to verify externally.[32] Therefore, transfers can only be conditional on market participation (τ), trade (r) and, possibly, time on market (κ). If time on market is observable transfers $(\tau^{\kappa}, r^{\kappa})$ can vary across different sellers' cohorts. If instead time on market is not observable, transfers are constant, i.e. $(\tau^{\kappa}, r^{\kappa}) = (\tau, r)$. A budget balanced mechanism that satisfies these properties is considered *feasible*. For every cohort of sellers, a feasible mechanism has to be ex interim individually rational as sellers know their type when they participate in the market. I consider a feasible mechanism to be *efficient* if it leads to an allocative and informationally efficient equilibrium.

Proposition 5.1 *Let $c = 0$. When time on market is observable, there is an efficient mechanism only in economies with*

$$q^0 \geq q^* := \max\left\{0, 1 - \left(\frac{\gamma}{1-\gamma}\right)^2 \frac{\theta_H - v_H}{\theta_H - \theta_L}\right\}$$

The efficient market mechanism implements a separating equilibrium with:

[32] For example, parties may exchange side payments in order to misreport posted prices. Setting up a market mechanism with price contingent transfers and which is robust to side payments goes beyond the scope of this paper. Moreover, if a designer could observe currently posted prices, he would be able to reconstruct the price history for each seller and would have more information than buyers.

- *a constant market participation tax* $\tau^* = \frac{1-\gamma}{\gamma}(\theta_H - \theta_L)$.
- *a fixed tax rebate* $r^* = \tau^* \left(1 + \frac{1-\gamma}{\gamma} q^*\right)$ *once the seller trades.*

For $q^0 < q^*$ *no feasible mechanism improves market outcomes and only L-sellers trade.*

In equilibrium, low quality sellers post θ_L and buyers accept this price for every signal realization, while high quality sellers post θ_H and trade once they match with a buyer who receives a high signal. Prices reveal sellers' types and the equilibrium is informationally efficient. The efficient market intervention is invariant with respect to κ because transfers do not depend on cohort S^κ, although they are not ex ante restricted to being equal across cohorts. This feature is related to the fact that prices reveal types, and information on the specific cohort becomes irrelevant for inferring product quality. As (τ^*, r^*) is κ-invariant, the same mechanism is efficient when time on market is not observable.

The green and yellow areas in Figure 4 illustrate the improvement due to (τ^*, r^*). Without a market intervention, an equilibrium is allocative efficient only if $q^0 \geq q_0^S$ (blue and white areas). When time on market is observable, H-sellers could also participate in the market in economies in the green area $q^0 \in [q_0^O, q_0^S)$, but they would not trade for certain. No high quality seller participates in economies in the yellow and red areas.

Proposition 5.1 points out that the mechanism (τ^*, r^*) may support an allocative and informationally efficient allocation for every $q^0 \in (0,1)$ only if

$$\gamma \geq \frac{\sqrt{\theta_H - \theta_L}}{\sqrt{\theta_H - \theta_L} + \sqrt{\theta_H - v_H}} := \gamma^*$$

Despite the improvement, it is still not possible to implement a first best allocation in every economy.[33] If $\gamma < \gamma^*$ an efficient allocation is only possible if $q^0 \geq q^* > 0$ (see the red area in Figure 4). Otherwise, it is not possible to mitigate adverse selection with a feasible market intervention. In these economies, the mechanism (τ^*, r^*) violates the individual rationality constraint of H-sellers because of the budget balance restriction. High and low quality sellers have different expected times on market ($\frac{1}{\gamma}$ and 1 periods respectively), and budget balance leads to an implicit transfer from high to low quality sellers, reducing the former's expected payoff. High quality sellers prefer to stay out of the market when this expected transfer outweighs their gains from trade.

[33] However, notice that γ^* is lower than $\frac{\theta_H - \theta_L + c}{2\theta_H - v_H - \theta_L + c}$, i.e. the minimum signal precision necessary to support a separating equilibrium with buyers playing a mixed strategy; see section 3.

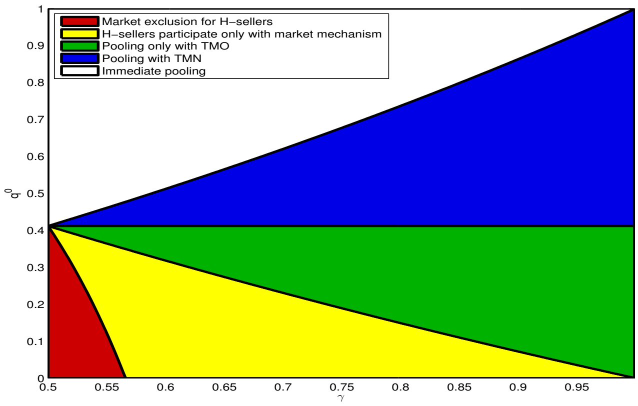

Figure 4: Efficient market intervention (τ^*, r^*).

6 Conclusion

I study a dynamic adverse selection model with sequential search: sellers incur a cost to find a new trade opportunity, and the market participation decision is non-trivial. The resulting analysis provides two main benefits. First, it highlights the role of search costs in the provision of a credible signalling device; second, it suggests market design policies to enhance participation in markets, such as online trading platforms. which are currently looking for ways to attract high quality products.

I present a framework with only two types of goods and binary signals. Despite its simplicity, the model allows me to uncover the main economic mechanisms at work. Future extensions may broaden the setup to multiple goods and more general signal distributions. I would expect that the main economic intuition would continue to hold. Another extension would be to introduce buyers who search for different product qualities in a directed search environment.[34]

7 Appendix A

7.1 Extended notation

A behavioural strategy for seller $i \in S^{\kappa}_{\lambda}$ is a function $\sigma^{\kappa}_{\lambda,i} : H^{\kappa-1} \to \Delta(A_S)$. Let $\Sigma^{\kappa}_{\lambda,i}$ be the set of all strategies $\sigma^{\kappa}_{\lambda,i}$ (let $\Sigma_{\lambda,i} = \bigcup_{\kappa=0}^{\infty} \Sigma^{\kappa}_{\lambda,i}$ and $\sigma_{\lambda,i} \in \Sigma_{\lambda,i}$). A behavioural strategy for a buyer matched with seller i is a function $\sigma_{B,i} : \mathscr{I}_B \to \Delta(A_B)$, where $\mathscr{I}_B$ denotes her information set. It is $\mathscr{I}_B(p,\kappa,\xi)$ if time-on

[34] Guerrieri and Shimer (2014) characterize a competitive search equilibrium where high quality sellers separate because they are more willing to accept a lower probability to trade. Jullien and Mariotti (2006) present a similar mechanism for auctions, and separation results from setting different type-dependent reservation prices.

market is observable and $\mathscr{I}_B(p,\xi)$ if it is not. When it is not relevant to specify whether TMO or TMN applies I denote an information set with $\mathscr{I}_B(p,\cdot,\xi)$.

Let $\pi_{\lambda,i}(\xi|h_i^{\kappa-1})$ be seller's $i \in S_\lambda^\kappa$ belief that his matched buyer receives signal ξ after history $h_i^{\kappa-1}$. I denote with $\mathbb{P}^{\sigma^*\pi^*}(z|h_i^{\kappa-1})$ the probability of reaching terminal history $z \in Z_i$ from $h_i^{\kappa-1} \in H^{\kappa-1}$ under the assessment (σ^*,π^*). I use $V_{\lambda,i}^\kappa(\sigma^*,\pi^*|h_i^{\kappa-1})$ for the continuation value of a seller $i \in S_\lambda^\kappa$ with previous history $h_i^{\kappa-1}$. It uses the utility function $u_\lambda^\kappa(z) := \tilde{u}_\lambda(z) + \kappa c$, which ignores the previous κc sunk costs.

Each seller maximizes his intertemporal expected utility after every history $h_i^{\kappa-1}$, $\kappa \in \mathbb{N}_0$.

$$V_{\lambda,i}^\kappa(\sigma_i^*,\sigma_{-i}^*,\pi_i^*,\pi_{-i}^*|h_i^{\kappa-1}) = \max_{\sigma_i \in \Sigma_{\lambda,i}} \sum_{z \in \{\cup Z_i^j\}_{j=\kappa}^\infty} \mathbb{P}^{\sigma_i\sigma_{-i}^*\pi_i^*\pi_{-i}^*}(z|h_i^{\kappa-1}) u_\lambda^\kappa(z)$$

In equilibrium, if a seller $i \in S_\lambda^\kappa$ posts price p, it is possible to write the value function as:[35]

$$V_{\lambda,i}^\kappa(\sigma^*,\pi^*|h_i^{\kappa-1}) = \sum_{\xi \in \{H,L\}} \pi_{\lambda,i}^*(\xi|h_i^{\kappa-1}) \left[\sigma_B^*(A|\mathscr{I}_B(p,\cdot,\xi))(p-v_\lambda) + \sigma_B^*(R|\mathscr{I}_B(p,\cdot,\xi)) V_\lambda^{\kappa+1}(\sigma^*,\pi^*|h_i^\kappa)\right] - c$$

A buyer can accept or reject an offer. Her expected payoff is simply:

$$\begin{cases} \mathbb{E}_{\pi_{\mathbb{B}}}[\theta] - p & if\ a_B = A \\ 0 & if\ a_B = R \end{cases}$$

where $\mathbb{E}_{\pi_B}[\theta] = \pi_B\theta_H + (1-\pi_B)\theta_L$ is the expectation under her posterior belief π_B.

Definition 7.1 is a formal statement of the equilibrium concept in Definition 3.1.

Definition 7.1 *An equilibrium of the game with TMO is a stationary assessment (σ^*,π^*) such that for every $i \in S_\lambda^\kappa$, $\lambda \in \{H,L\}$, and $\kappa \in \mathbb{N}_0$:*

1. $\sigma_\lambda^{*\kappa}(a_{S,i}|h_i^{\kappa-1}) \in \arg\max_{\sigma_i \in \Sigma_{\lambda,i}} V_\lambda^\kappa(\sigma_i,\sigma_{-i}^*,\pi^*|h_i^{\kappa-1})\ \forall h_i^{\kappa-1} \in H^{\kappa-1}$.
2. $\sigma_B^*(a_{B,i}|\mathscr{I}_B(p^\kappa,\kappa,\xi))$ *is a pure-strategy best response.*
3. $\pi_B(p,\kappa,\xi)$ *is updated according to Bayes' rule whenever possible.*
4. $\pi_\lambda(\xi|h_i^{\kappa-1}) = \mathbb{P}_\lambda(\xi)$ *for every $h_i \in H^{\kappa-1}$, $\kappa \in \mathbb{N}_0$ and $\lambda \in \{H,L\}$.*

The definition slightly restricts the weak Perfect Bayesian Equilibrium concept. Condition 1 allows best response strategies to depend on λ, κ and $h_i^{\kappa-1}$. Condition 2 only considers pure strategy best responses for buyers. Condition 3 requires buyers to update beliefs according to Bayes' rule on the equilibrium path. Since buyers are short-lived, it is not necessary to impose any additional restriction on their off the equilibrium path beliefs in order to have a *reasonable assessment* (see Definition 3

[35] Lemma 7.1 adapts the results in Hendon et al. (1996) to ensure that the one-shot deviation property holds in this model setup.

of Fudenberg and Tirole (1991)). Finally, condition 4 restricts sellers to not changing their beliefs on the likelihood that future matched buyers receive signal $\xi \in \{H, L\}$. This restriction seems natural as buyers' signal realizations are independent from H-sellers' previous history. As a result, it is equivalent to a "no signalling what you don't know" condition on sellers' posterior beliefs. Adapting this definition to the TMN setup is straightforward and I omit it in the interests of space. The only difference relates to buyers' impossibility to condition on κ, so they form beliefs using a single prior probability q. I always keep the possibility that behavioural strategy profiles may differ among sellers' cohorts and histories.

Definition 7.2 states the concept of undefeated equilibrium originally presented in Mailath et al. (1993) in the context of my framework.

Definition 7.2 *An equilibrium assessment* (σ^*, π^*) *defeats* $(\tilde{\sigma}, \tilde{\pi})$ *if* $\exists \{p^\kappa\}_{\kappa=0}^{\infty}$ *such that:*

1. $\exists \kappa \in \mathbb{N}_0$ *with* $\tilde{\sigma}_\lambda^\kappa(p^\kappa | h_i^{\kappa-1}) = 0$ *for every* $\lambda \in \{H, L\}$ *and* $h_i^{\kappa-1} \in H^{\kappa-1}$ *while* $\Lambda(p^\kappa) := \{\lambda : \exists h_i^{\kappa-1} \in H^{\kappa-1} \text{ s.t } \sigma_\lambda^{*\kappa}(p^\kappa | h_i^{\kappa-1}) > 0\} \neq \varnothing$.
2. $\forall \lambda \in \Lambda(p^\kappa)$ *it holds* $V_\lambda^j(\sigma^*, \pi^* | h_i^{j-1}) \geq V_\lambda^j(\tilde{\sigma}, \tilde{\pi} | h_i^{j-1})\ \forall h_i^{j-1} \in H^{j-1}$ *and* $j \in \mathbb{N}_0$. *Moreover,* $\exists \lambda \in \Lambda(p^\kappa)$ *s.t.* $V_\lambda^\kappa(\sigma^*, \pi^* | h_i^{\kappa-1}) > V_\lambda^\kappa(\tilde{\sigma}, \tilde{\pi} | h_i^{\kappa-1})$ *for some* $h_i^{\kappa-1} \in H^{\kappa-1}$.
3. $\tilde{\pi}(p^\kappa, q^\kappa, \xi) \neq \frac{\mathbb{P}_H(\xi) q^\kappa \sigma_H^\kappa(p^\kappa)}{\mathbb{P}_H(\xi) q^\kappa \sigma_H^\kappa(p^\kappa) + \mathbb{P}_L(\xi)(1-q^\kappa)\sigma_L^\kappa(p^\kappa)}$ *with* $\sigma_\lambda^\kappa(p^\kappa)$ *satisfying:*
   - $\lambda \in \Lambda(p^\kappa)$ *and* $V_\lambda^\kappa(\sigma^*, \pi^* | h_i^{\kappa-1}) > V_\lambda^\kappa(\tilde{\sigma}, \tilde{\pi} | h_i^{\kappa-1}) \Rightarrow \sigma_\lambda^\kappa(p^\kappa) = 1$.
   - $\lambda \notin \Lambda(p^\kappa) \Rightarrow \sigma_\lambda^\kappa(p^\kappa) = 0$.

An equilibrium assessment (σ^*, π^*) is undefeated if there is no other equilibrium that defeats (σ^*, π^*) according to Definition 7.2.

For notational simplicity I omit to explicitly specify π^* when it is obvious from the context. For instance, I sometimes use $V_\lambda^\kappa(\sigma^*)$ to denote $V_\lambda^\kappa(\sigma^*, \pi^*)$.

7.2 Preliminary results

Example 7.1 *Intertemporal separating equilibrium.*

The example explains how ITS works. It is extremely simple and its goal is to make the main backbone mechanism as transparent as possible.

Suppose sellers discount future payoffs at rate δ and they pay a per period search cost c. In equilibrium low quality sellers trade immediately while high quality sellers wait until a future period $t > 0$ to trade at a higher price.[36] This equilibrium exists only if:

$$IR_H: \quad \delta^t(\theta_H - v_H) - \tfrac{1-\delta^{t+1}}{1-\delta}c \geq 0 \qquad \rightarrow \quad t \leq f(\delta, c)$$

$$IC_L: \quad \theta_L - v_L - c \geq \delta^t(\theta_H - v_L) - \tfrac{1-\delta^{t+1}}{1-\delta}c \qquad \rightarrow \quad t \geq h(\delta, c)$$

[36] To simplify derivation, I assume sellers have full bargaining power but notice that the argument also applies to other bargaining protocols.

The functions $f(\delta,c)$ and $h(\delta,c)$ are continuous in both arguments with limits $\lim_{c\searrow 0} f(\delta,c)=+\infty$, $\lim_{\delta\nearrow 1} f(\delta,c)=\frac{\theta_H-v_H-c}{c}$, $\lim_{c\searrow 0} h(\delta,c)=\frac{1}{\ln\delta}\ln\frac{\theta_L-v_L}{\theta_H-v_L}$ and $\lim_{\delta\nearrow 1} h(\delta,c)=\frac{\theta_H-v_H}{c}$.

The ITS equilibrium only exists if $f(\delta,c)\geq h(\delta,c)$. This is always the case if $c=0$ and $\delta<1$ while it is never so if $c>0$ and $\delta=1$ as $v_H>\theta_L$.

Lemma 7.1 *A strategy profile σ_i^* is a sequential best reply to (σ_{-i}^*,π^*) for seller $i\in S$ if and only if $\sigma_i^{*^\kappa}(a_{S,i}|h_i^{\kappa-1})$ is a local best reply to (σ_{-i}^*,π^*) for all $\kappa\in\mathbb{N}_0$ and $h_i^{\kappa-1}\in H^{\kappa-1}$.*

Proof Lemma 7.1.

Necessity. It follows directly from the definition of sequential best reply.

Sufficiency. Suppose on the contrary that σ_i^* is a local best reply for every $h_i^{\kappa-1}\in H^{\kappa-1}$ and $\kappa\in\mathbb{N}_0$, but there exists a strategy σ_i' that strictly improves on σ_i^* after history $h_i^{\kappa-1}$. Let this increment be equal to $\varepsilon>0$. Seller's i expected payoff at $h_i^{\kappa-1}$ is:

$$\begin{aligned} V_{\lambda,i}^{\kappa}(\sigma_i',\sigma_{-i}^*,\pi^*|h_i^{\kappa-1}) &= \sum_{z\in\{\cup Z_i^j\}_{j=\kappa}^{\infty}} \mathbb{P}^{\sigma_i'\sigma_{-i}^*\pi^*}(z|h_i^{\kappa-1})u_\lambda^\kappa(z)=\sum_{z\in Z_i^\kappa}\mathbb{P}^{\sigma_i'\sigma_{-i}^*\pi^*}(z|h_i^{\kappa-1})u_\lambda^\kappa(z)\\ &+ \sum_{h^\kappa\in H^\kappa/Z_i^\kappa}\mathbb{P}^{\sigma_i'\sigma_{-i}^*\pi^*}(h_i^\kappa|h_i^{\kappa-1})\sum_{z\in\{\cup Z_i^j\}_{j=\kappa+1}^{\infty}}\mathbb{P}^{\sigma_i'\sigma_{-i}^*\pi^*}(z|h_i^\kappa)u_\lambda^\kappa(z)\end{aligned}$$

Let $c(z|h_i^{\kappa-1})$ be the search costs from $h_i^{\kappa-1}\in H^{\kappa-1}$ to terminal history $z\in Z_i^j$, i.e. $c(z|h_i^{\kappa-1})=(j-\kappa+1)c$.

An upper bound on $\sum_{z\in\{\cup Z_i^j\}_{j=\kappa+1}^{\infty}}\mathbb{P}^{\sigma_i'\sigma_{-i}^*\pi^*}(z|h_i^\kappa)u_\lambda^\kappa(z)$ is:

$$\sum_{z\in\{\cup Z_i^j(p)\}_{j\geq\kappa+1}}\mathbb{P}^{\sigma_i'\sigma_{-i}^*\pi^*}(z|h_i^\kappa)[\theta_H-v_\lambda-c(z|h_i^\kappa)]+\sum_{z\in\{\cup Z_i^j(D)\}_{j\geq\kappa+1}}\mathbb{P}^{\sigma_i'\sigma_{-i}^*\pi^*}(z|h_i^\kappa)[-c(z|h_i^\kappa)] \tag{2}$$

Observe that σ_i^* is a local best reply at $h_i^{\kappa-1}$, and a seller can always get a zero continuation value if he drops out of the market. Therefore, for some $h_i^\kappa\in H^\kappa/Z_i^\kappa$ it must be $\mathbb{P}^{\sigma_i'\sigma_{-i}^*\pi^*}(h_i^\kappa|h_i^{\kappa-1})>0$ and σ_i' is a profitable deviation only if

$$\sum_{z\in\{\cup Z_i^j\}_{j\geq\kappa+1}}\mathbb{P}^{\sigma_i'\sigma_{-i}^*\pi^*}(z|h_i^\kappa)u_\lambda^\kappa(z)>\sum_{z\in\{\cup Z_i^j\}_{j\geq\kappa+1}}\mathbb{P}^{\sigma_i^*\sigma_{-i}^*}(z|h_i^\kappa\pi^*)u_\lambda^\kappa(z)\geq -c \tag{3}$$

As a result, $\sum_{z\in\{\cup Z_i^j(p)\}_{j\geq\kappa+1}}\mathbb{P}^{\sigma_i'\sigma_{-i}^*}(z|h_i^\kappa)>0$ otherwise sellers would have the same expected payoff at $h_i^{\kappa-1}$

because σ_i^* is a local best reply. Hence, equations (2) and (3) imply:

$$(\theta_H - v_\lambda) > \frac{\sum\limits_{z\in\{\cup Z_i^j\}_{j\geq\kappa+1}} \mathbb{P}^{\sigma_i'\sigma_{-i}^*}(z|h_i^\kappa)c(z|h_i^\kappa) - c}{\sum\limits_{z\in\{\cup Z_i^j(p)\}_{j\geq\kappa+1}} \mathbb{P}^{\sigma_i'\sigma_{-i}^*}(z|h_i^\kappa)} \geq \frac{\mathbb{P}^{\sigma_i'\sigma_{-i}^*}(z|h_i^\kappa)c(z|h_i^\kappa) - c}{\sum\limits_{z\in\{\cup Z_i^j(p)\}_{j\geq\kappa+1}} \mathbb{P}^{\sigma_i'\sigma_{-i}^*}(z|h_i^\kappa)}$$

for all $z \in Z_i^j$ and $j \geq \kappa + 1$.

Since $c(z \in Z_i^j|h_i^\kappa) \to +\infty$ as $j \to +\infty$ the inequality only holds if $\lim\limits_{j\to+\infty} \mathbb{P}^{\sigma_i'\sigma_{-i}^*}(z \in Z_i^j|h_i^\kappa) = 0$. Therefore, there is a finite $\hat{t}$ and history $h_i^{\kappa+\hat{t}}$ such that the strategy

$$\hat{\sigma}_i = \begin{cases} \sigma_i' & \forall j < \kappa + \hat{t} \text{ and } \forall h_i^j \in H^j \\ \sigma_i^* & \forall j \geq \kappa + \hat{t} \text{ and } \forall h_i^j \in H^j \end{cases}$$

improves by at least $\frac{\varepsilon}{2}$ on σ_i^* with a finite number of deviations; i.e

$$V_{\lambda,i}^\kappa(\hat{\sigma}_i, \sigma_{-i}^*|h_i^{\kappa-1}) - V_{\lambda,i}^\kappa(\sigma_i^*, \sigma_{-i}^*|h_i^{\kappa-1}) \geq \frac{\varepsilon}{2}$$

However, the main result in Hendon, Jacobsen and Sloth (1996) ensures that no finite sequence of deviations can improve on σ_i^*, contradiction. ■

Proof Proposition 4.1.

Suppose per contra there is an ITS equilibrium (σ^*, π^*). I denote with $\mathbb{P}_\lambda^{\sigma^*\pi^*}(z^j(p)|h_i^{\kappa-1})$ the probability that seller $i \in S_\lambda^\kappa$ reaches the terminal history $z \in Z_i^j(p)$ under this separating equilibrium (σ^*, π^*). For a L-seller $i \in S_L^\kappa$ a deviation strategy $\sigma_i'^j(p|h_i^{j-1}) = \sigma_H^{*j}(p|h_i^{j-1})$, $j \geq \kappa$, for all $h_i^{j-1} \in H^{j-1}$ implies

$$\mathbb{P}_H^{\sigma^*\pi^*}(z^j(p)|h_i^{\kappa-1}) = \mathbb{P}_L^{\sigma_i'\sigma_{-i}^*\pi^*}(z^j(p)|h_i^{\kappa-1}) \tag{4}$$

$\forall z^j(p) \in Z^j(p)$, $j \geq 0$, since signals are not informative ($\gamma = \frac{1}{2}$) and both sellers have identical chances to trade if they play the same strategy profile.

In equilibrium L-sellers do not find strictly profitable to play σ_i' only if

$$\theta_L - v_L - c \geq \sum_{j=\kappa}^{\infty} \int_0^\infty \mathbb{P}_L^{\sigma_i'\sigma_{-i}^*\pi^*}(z^j(p)|h_i^{\kappa-1})\,[p - v_L - (j-\kappa+1)c]\,\mathrm{d}p \tag{5}$$

All H-sellers eventually trade hence

$$\sum_{j=\kappa}^{\infty} \int_0^\infty \mathbb{P}_H^{\sigma^*\pi^*}(z^j(p)|h_i^{\kappa-1})\,\mathrm{d}p = \sum_{j=\kappa}^{\infty} \int_0^\infty \mathbb{P}_L^{\sigma_i'\sigma_{-i}^*\pi^*}(z^j(p)|h_i^{\kappa-1})\,\mathrm{d}p = 1 \tag{6}$$

where the first equality follows from equation (4). Therefore, L-sellers no deviation condition can be rewritten as

$$\theta_L - c \geq \sum_{j=\kappa}^{\infty} \int_0^{\infty} \mathbb{P}_H^{\sigma^*\pi^*}(z^j(p)|h_i^{\kappa-1})\,[p-(j-\kappa+1)c]\,\mathrm{d}p \tag{7}$$

H-sellers' market participation constraint is satisfied only if

$$\sum_{j=\kappa}^{\infty} \int_0^{\infty} \mathbb{P}_H^{\sigma^*\pi^*}(z^j(p)|h_i^{\kappa-1})\,[p-v_H-(j-\kappa+1)c]\,\mathrm{d}p \geq 0$$

However, by equations (6) and (7) an upper bound on H-sellers' equilibrium expected payoff is $\theta_L - v_H - c < 0$, a violation of H-sellers' individual rationality constraint. ■

Lemma 7.2 *In every equilibrium assessment (σ^*, π^*) it holds:*

$$V_\lambda^\kappa(\sigma^*,\pi^*|h_i^{\kappa-1}) = V_\lambda^\kappa(\sigma^*,\pi^*|\tilde{h}_i^{\kappa-1}) := V_\lambda^\kappa(\sigma^*,\pi^*)$$

for every $h_i^{\kappa-1}, \tilde{h}_i^{\kappa-1} \in H^{\kappa-1}$.

Proof Lemma 7.2.
Previous histories $h_i^{\kappa-1} \in H^{\kappa-1}$ are not observable to buyers so their best responses with respect to any price offer p from sellers in S^κ are identical for sellers with different histories. Moreover, final payoffs only depend on how many periods κ were previously spent on the market and—in case of trade—on the last price offer p. Therefore, in equilibrium all sellers in S_λ^κ must have the same expected payoff irrespective of $h^{\kappa-1}$ otherwise there would be a profitable and unobservable deviation for a subset of sellers. ■

7.3 Negligible search costs

Proof Lemma 4.1.
See Appendix B. ■

Lemma 7.3 *For a sufficiently small c, under TMO there is no undefeated equilibrium (σ^*, π^*) in which $S_H^\kappa \neq \emptyset$, $S_L^\kappa \neq \emptyset$ and H- and L-sellers in S^κ only post prices rejected with a probability of one.*

Proof. By Lemma 4.1 if all sellers participate in the market the only admissible equilibrium strategies are: (i) H- and L-sellers post the same price and buyers accept only if $\xi = H$; or (ii) under TMO, H- and L-sellers only post prices rejected with a probability of one.

Let $K^N(\kappa) := \{j \geq \kappa : \text{sellers in } S^j \text{ play strategy (ii)}\}$. Obviously sellers in S_λ^κ, $\lambda \in \{H, L\}$ will not play strategy profile (ii) if they drop out of the market in the subsequent period $\kappa+1$. Therefore, there is at least one future period in which they trade with positive probability, i.e. there is at least one $l \geq \kappa$ such that sellers in S^l play behavioural strategy (i). Without loss of generality, consider j such that $j \in K^N(\kappa)$ and $j+1 \notin K^N(\kappa)$. Since no seller in S^j trades then $q^j = q^{j+1}$. Consider an alternative equilibrium $(\tilde{\sigma}, \tilde{\pi})$ in which each seller $i \in S^j$ plays: $\tilde{\sigma}^l(p|h_i^{l-1}) = \sigma^{*^l}(p|h_i^{l-1})$ for every $l < j$ and $\forall h_i^{l-1} \in H^{l-1}$; and $\tilde{\sigma}^l(p|h_i^{l-1}) = \sigma^{*^{l+1}}(p|h_i^l)$ for every $l \geq j$ and $\forall h_i^{l-1} \in H^{l-1}$.[37]. Let $\tilde{\pi}(p,l,\xi) = \pi^*(p,l,\xi)$ for every $l < j$, and $\tilde{\pi}(p,l,\xi) = \pi^*(p,l+1,\xi)$ for every $l \geq j$. It is easy to observe that, if (σ^*, π^*) is an equilibrium assessment, $(\tilde{\sigma}, \tilde{\pi})$ is also an equilibrium as it satisfies analogous no deviation conditions. However, $V_\lambda^l(\tilde{\sigma}, \tilde{\pi}) > V_\lambda^l(\sigma^*, \pi^*)$ for every $l \leq j$ as they do not incur an extra cost c in period j. As a result, (σ^*, π^*) would be defeated by $(\tilde{\sigma}, \tilde{\pi})$. ■

Proof Proposition 4.2.

1. By Lemma 4.1 and 7.3 the only admissible equilibrium strategy for $c \leq c^*$ is

$$\sigma_H^{*^\kappa}(p^\kappa) = \sigma_L^{*^\kappa}(p^\kappa) = 1 \qquad \sigma_B^*(A|\mathscr{I}_B(p^\kappa,\kappa,H)) = 1 \qquad \sigma_B(A|\mathscr{I}_B(p^\kappa,\kappa,L)) = 0$$

Playing this strategy profile implies:

$$\left.\begin{array}{l} S_H^{\kappa+1} = (1-\gamma)S_H^\kappa \\ S_L^{\kappa+1} = \gamma S_L^\kappa \end{array}\right\} \tfrac{S^{\kappa+1}}{S^\kappa} = (1-\gamma)\tfrac{S_H^\kappa}{S^\kappa} + \gamma\tfrac{S_L^\kappa}{S^\kappa} = (1-\gamma) + (2\gamma-1)(1-q^\kappa)$$

Therefore,

$$q^{\kappa+1} = \frac{S_H^{\kappa+1}}{S^{\kappa+1}} = (1-\gamma)\frac{S_H^\kappa}{S^\kappa}\frac{S^\kappa}{S^{\kappa+1}} = \frac{(1-\gamma)q^\kappa}{(1-\gamma)+(2\gamma-1)(1-q^\kappa)} := g(q^\kappa) := g^\kappa(q^0)$$

As $q^{\kappa+1}$ does not depend on p^κ and buyers cannot observe previously posted prices, future continuation values $V_\lambda^j(\sigma^*, \pi^*)$, $j > \kappa$, do not depend on p^κ. As a result, $p^\kappa = \mathbb{E}_{\pi_B}[\theta|\mathscr{I}(p^\kappa,\kappa,H)]$ supports the unique undefeated equilibrium (σ^*, π^*) as both types of seller get the highest possible payoff in the class of admissible equilibria (see Lemma 4.1). The undefeated equilibrium is unique because prices $\{p^\kappa\}_{\kappa \in \mathbb{N}_0}$ are unique.

Let $V_\lambda^\kappa(q) := V_\lambda^\kappa(\sigma^*, \pi^*)$ be the continuation value in (σ^*, π^*) for a seller $i \in S_\lambda^\kappa$ when $q^\kappa = q$. Let $\kappa^*(q^0)+1$ be the maximum number of periods on the market for a H-seller. If $\kappa^*(q^0) = 0$ he participates for just one period, while if $\kappa^*(q^0) = +\infty$ he participates until he trades. For every seller $i \in S^\kappa$, $\kappa \leq \kappa^*(q^0)$, the maximization problem can be rewritten as follows:

$$V_\lambda^\kappa(q^\kappa) = \mathbb{P}_\lambda(H)(p^\kappa - v_\lambda) + [1-\mathbb{P}_\lambda(H)]V_\lambda^{\kappa+1}(q^{\kappa+1}) - c$$

As q^κ is decreasing in κ then p^κ is also decreasing in κ as $\mathbb{E}_{\pi_B}[\theta|\mathscr{I}_B(p^\kappa,\kappa,H)]$ is monotonically increasing in q^κ. Hence, $V_\lambda^\kappa(q^\kappa)$ is decreasing in q^κ and $V_\lambda^{\kappa+1}(q^{\kappa+1}) < V_\lambda^\kappa(q^\kappa)$ as $q^{\kappa+1} < q^\kappa$. As a result,

[37] The s-th element in h_i^{l-1} is equal to the s-th element in h^l for $s < j$ and equal to the $s+1$-th for $s \geq j$.

H-sellers do not find it profitable to postpone trade to a future period.

Market participation requires $V_{\lambda}^{\kappa}(q^{\kappa}) \geq 0$. Let's consider the following bounds on $V_H^{\kappa}(q^{\kappa})$:

$$\begin{aligned}&\text{Upper bound: } U_H^{\kappa}(q^{\kappa}) = \gamma(p^{\kappa} - v_H) + (1-\gamma)U_H^{\kappa}(q^{\kappa}) - c \Rightarrow U_H^{\kappa}(q^{\kappa}) = p^{\kappa} - v_H - \tfrac{c}{\gamma}\\ &\text{Lower bound: } L_H^{\kappa}(q^{\kappa}) = \gamma(p^{\kappa} - v_H) - c\end{aligned}$$

$U_H^{\kappa}(q^{\kappa})$, $V_H^{\kappa}(q^{\kappa})$ and $L_H^{\kappa}(q^{\kappa})$ are monotonically increasing in q^{κ} and $U_H^{\kappa}(q^{\kappa}) \geq V_H^{\kappa}(q^{\kappa}) \geq L_H^{\kappa}(q^{\kappa})$. Notice that $U_H^{\kappa}(q^{\kappa}) = \frac{L_H^{\kappa}(q^{\kappa})}{\gamma}$ so H-sellers exit the market whenever $L_H^{\kappa}(q^{\kappa}) \leq 0$.

Let $q_c^O := \underset{q\in(0,1)}{\arg}\ L_H^{\kappa}(q) = 0$. Then

$$L_H^{\kappa}(q_c^O) = \gamma(\mathbb{E}_{\pi_B}[\theta|\mathscr{I}_B(p^{\kappa},\kappa,H)] - v_H) - c = \gamma\left(\frac{q_c^O\gamma\theta_H + (1-q_c^O)(1-\gamma)\theta_L}{q_c^O\gamma + (1-\gamma)(1-q_c^O)} - v_H\right) - c = 0$$

Solving for q_c^O

$$q_c^O = \frac{(1-\gamma)(v_H + \frac{c}{\gamma} - \theta_L)}{\gamma(\theta_H - v_H - \frac{c}{\gamma}) + (1-\gamma)(v_H + \frac{c}{\gamma} - \theta_L)}$$

H-sellers only participate for a finite number of periods

$$\kappa^*(q^0) = \max\left\{\kappa \in \mathbb{N}_0 : g^{\kappa}(q^0) \geq q_c^O\right\}$$

since $q^{\kappa+1} = g^{\kappa}(q^0)$ is strictly decreasing in κ.

2. H-sellers do not participate in economies $\mathscr{E}(\gamma, q^0)$ such that $q^0 < q_c^O$. L-sellers always trade because there are gains from trade ($\theta_L > v_L$), and the only possible sequential best response is to post θ_L which buyers accept with a probability of one.[38] ■

Proof Proposition 4.3.

1. By Lemma 4.1, for $c \leq c^*$ the equilibrium strategy of sellers is $\sigma_{\lambda}(p) = \sigma_{\lambda}^{\kappa}(p) = 1$ for every $\lambda \in \{H, L\}$ and $\kappa \in \mathbb{N}_0$. Price p is accepted only if $\xi = H$.

In equilibrium, strategies do not depend on time and, by Lemma 4.1, the price p is posted by all cohorts of sellers. Thus, this outcome is only possible if the economy is in a stationary state, i.e. if the measure of sellers S, say $\bar{S}$, and the fraction of H-sellers, say $\bar{q} = \frac{\bar{S}_H}{\bar{S}}$, are constant over time. In turn, in every period an equal measure of each type of seller must enter and exit the market:

$$\begin{cases} q^0 = \bar{S}\gamma\bar{q} \\ (1-q^0) = \bar{S}(1-\gamma)(1-\bar{q}) \end{cases} \Rightarrow \begin{cases} \bar{S} = \frac{\gamma - q^0(2\gamma-1)}{\gamma(1-\gamma)} \\ \bar{q} = \frac{q^0(1-\gamma)}{\gamma - q^0(2\gamma-1)} \end{cases}$$

Notice that $\bar{S}$ and $\bar{q}$ do not depend on c or p. A similar argument to the one presented in the proof of Proposition 4.2 ensures that the undefeated equilibrium is unique and sellers post a price $\bar{p}$ equal to

[38] The price θ_L must be always accepted with a probability of one; otherwise, L-sellers would deviate and post $\theta_L - \varepsilon$, for ε arbitrary small, which buyers would always accept. Indeed, under any belief, their minimum valuation for the good is θ_L.

buyers' posterior valuation when $\xi = H$.

$$\bar{p} = \mathbb{E}_{\pi_B}[\theta|\mathscr{I}_B(\bar{p},H)] = \frac{\gamma\bar{q}\theta_H + (1-\gamma)(1-\bar{q})\theta_L}{\gamma\bar{q} + (1-\gamma)(1-\bar{q})} = \frac{\theta_H + \frac{(1-\gamma)}{\gamma}\frac{(1-\bar{q})}{\bar{q}}\theta_L}{1 + \frac{(1-\gamma)}{\gamma}\frac{(1-\bar{q})}{\bar{q}}}$$

Observe that $\frac{(1-\gamma)}{\gamma}\frac{(1-\bar{q})}{\bar{q}} = \frac{1-q^0}{q^0}$, so $\bar{p} = \mathbb{E}_{\pi_B}[\theta|\mathscr{I}_B(\bar{p},H)] = q^0\theta_H + (1-q^0)\theta_L$.

Every cohort of sellers S^κ posts price $\bar{p}$ and previous search costs are sunk, hence continuation values are constant $\forall\kappa$, i.e. $V_\lambda^\kappa(\sigma^*,\pi^*) = V_\lambda(\sigma^*,\pi^*)$. H-sellers participate in the market until they trade because their forward looking decision problem is unchanged. The individual rationality constraint for H-sellers is:

$$V_H(\sigma^*,\pi^*) = \gamma(\bar{p} - v_H) + (1-\gamma)V_H(\sigma^*,\pi^*) - c \geq 0$$

i.e.

$$V_H(\sigma^*,\pi^*) = \bar{p} - v_H - \frac{c}{\gamma} = q^0\theta_H + (1-q^0)\theta_L - v_H - \frac{c}{\gamma} \geq 0.$$

This is satisfied only if $q^0 \geq q_c^N = \frac{v_H - \theta_L + \frac{c}{\gamma}}{\theta_H - \theta_L}$.

2. See point 2. in the proof of Proposition 4.2. ∎

7.4 Non-negligible search costs

Proof Proposition 4.4.

Consider sellers in S^κ, $\kappa \geq 0$. Definition 4.1 requires separating strategies for every seller $i \in S^\kappa$—after all possible histories $h_i^{\kappa-1} \in H^{\kappa-1}$—on and off the equilibrium path.

Step 1. Let $S^\kappa \neq \emptyset$. In equilibrium, separating strategies are:

$$\begin{aligned}
&\sigma_H^{*^\kappa}(\theta_H|h_i^{\kappa-1}) = 1 && \sigma_L^{*^\kappa}(\theta_L|h_i^{\kappa-1}) = 1\\
&\sigma_B^*(A|\mathscr{I}_B(\theta_H,\cdot,H)) = 1 && \sigma_B^*(A|\mathscr{I}_B(\theta_H,\cdot,L)) = 0\\
&\sigma_B^*(A|\mathscr{I}_B(\theta_L,\cdot,L)) = 1 && \sigma_B^*(A|\mathscr{I}_B(p,\cdot,\xi)) = 0 \, \textit{for } p \in (\theta_L,\theta_H)
\end{aligned}$$

for every $h_i^{\kappa-1} \in H^{\kappa-1}$.

Let $p_H^\kappa > p_L^\kappa$ be separating prices for H- and L-sellers in S^κ. As $S^\kappa \neq \emptyset$, on the equilibrium path sellers may stay on the market at least κ previous periods. In equilibrium, separation implies $\mathbb{E}_{\pi_B}[\theta|\mathscr{I}(p_H^\kappa,\cdot,\xi)] = \theta_H$ and $\mathbb{E}_{\pi_B}[\theta|\mathscr{I}(p_L^\kappa,\cdot,\xi)] = \theta_L$ for every $\xi \in \{H,L\}$.

If $\sigma_B(A|\mathscr{I}_B(p_H^\kappa,\cdot,\xi)) = \sigma_B(A|\mathscr{I}_B(p_L^\kappa,\cdot,\xi))$ for every $\xi \in \{H,L\}$, buyers accept p_H^κ and p_L^κ with equal probability. In turn L-sellers would deviate as $p_H^\kappa > p_L^\kappa$. Therefore, separation can occur only if buyers accept (p_H^κ, p_L^κ) with different probabilities. By point 2. in Definition 7.1 the only plausible equilibrium strategy is: buyers accept p_L^κ for every $\xi \in \{H,L\}$ and p_H^κ only if $\xi = H$. This is only possible if $p_H^\kappa = \theta_H$ otherwise buyers would always accept. Sequential rationality requires L-sellers to ask the highest price accepted by buyers, i.e. $p_L^\kappa = \theta_L$.

Suppose *per contra* that separating behavioural strategies are not in pure strategies. From the argument in Step 1. of the proof of Lemma 4.1 sellers in S_λ^κ can mix: (i) between two prices p_λ^κ and $p_{\lambda,2}^\kappa$ both accepted with positive probability; and (ii) between p_λ^κ and a price rejected with a probability of one.

If H-sellers play strategy (i), L-sellers would not post $p_L^\kappa = \theta_L$ as $p_{H,2}^\kappa \geq v_H$ is accepted with the same probability. Similarly, if L-sellers play strategy (i) and mix between θ_L and $p_{L,2}^\kappa$ they prefer not to post $p_{L,2}^\kappa$ but $\theta_H > p_{L,2}^\kappa$ as both prices are accepted only if $\xi = H$, thus contradicting the hypothesis that H-sellers play a separating strategy.

If sellers in S_λ^κ play strategy (ii) then at least one of these two indifference conditions hold:

$$\begin{aligned}\gamma(\theta_H - v_H) + (1-\gamma)V_H^{\kappa+1}(\sigma^*,\pi^*|h_i^{\kappa-1}) - c &= V_H^{\kappa+1}(\sigma^*,\pi^*|h_i^{\kappa-1}) - c \\ \theta_L - v_L - c &= V_L^{\kappa+1}(\sigma^*,\pi^*|h_i^{\kappa-1}) - c\end{aligned}$$

The first equation implies $V_H^{\kappa+1}(\sigma^*,\pi^*|h_i^{\kappa-1}) = \theta_H - v_H$ but in all possible equilibria it must be $V_H^{\kappa+1}(\sigma,\pi) \leq \theta_H - v_H - c$ otherwise buyers have to pay a price higher than θ_H, violating their individual rationality. The second equation cannot hold either because a L-seller would get a higher payoff by deviating to θ_H since

$$(1-\gamma)(\theta_H - v_L) + \gamma V_L^{\kappa+1}(\sigma^*,\pi^*|h_i^{\kappa-1}) - c = (1-\gamma)(\theta_H - v_L) + \gamma(\theta_L - v_L) - c > \theta_L - v_L - c$$

Therefore, in equilibrium all L-sellers trade immediately while a share $1-\gamma$ of H-sellers in S^κ moves to period $\kappa+1$, i.e. $S^{\kappa+1} \neq \emptyset$ if H-sellers continue to participate in the market. Definition 4.1 requires separating behavioural strategies for every cohort S^κ, $\kappa \in \mathbb{N}_0$. As the separating problem for sellers in $S^{\kappa+1}$ is identical to the one in S^κ the same behavioural strategy is played for every $\kappa \geq 0$. To support separation, off the equilibrium path beliefs should be sufficiently negatively, say $\pi_B(p,\cdot,\xi) = 0$, for every price above θ_L but below θ_H.

Step 2. A separating behavioural strategy profile exists if and only if

$$c \in \left[\frac{1-\gamma}{\gamma}(\theta_H - \theta_L), \gamma(\theta_H - v_H)\right]$$

By Step 1 and Lemma 7.2 it follows that $V_\lambda^\kappa(\sigma^*,\pi^*|h_i^{\kappa-1}) = V_\lambda(\sigma^*,\pi^*)$. Using Lemma 7.1 L-sellers in S_L^κ post θ_L and trade if and only if for every $h_i^{\kappa-1} \in H^{\kappa-1}$:

$$V_L(\sigma^*,\pi^*) = \theta_L - v_L - c \geq (1-\gamma)(\theta_H - v_L) + \gamma(\theta_L - v_L - c) - c$$

i.e. $c \geq \frac{1-\gamma}{\gamma}(\theta_H - \theta_L)$.

H-sellers' participate in the market until they trade if and only if:

$$V_H(\sigma^*, \pi^*) = \gamma(\theta_H - v_H) + (1-\gamma)V_H(\sigma^*, \pi^*) - c \geq 0$$

i.e. $V_H(\sigma^*, \pi^*) = \theta_H - v_H - \frac{c}{\gamma} \geq 0$ or $c \leq \gamma(\theta_H - v_H)$. ■

Proof Proposition 4.5.
I construct an equilibrium assessment (σ, π) such that:

1. For every $\kappa \in \mathbb{N}_0$ H-sellers always post the price $p_H^\kappa = p_H = \theta_L + \frac{\gamma}{1-\gamma}c$ while L-sellers post p_H with probability $\frac{1-\gamma}{\gamma}$ and θ_L with probability $\frac{2\gamma-1}{\gamma}$.

2. Buyers always accept θ_L while they accept p_H only if $\xi = H$.

Step 1. Equilibrium characterization
In a semi-separating equilibrium sellers evolve across cohorts S^κ according to

$$\left.\begin{aligned} S_H^{\kappa+1} &= (1-\gamma)S_H^\kappa \\ S_L^{\kappa+1} &= \sigma_L^\kappa(p_H)\gamma S_L^\kappa \end{aligned}\right\} \frac{S^{\kappa+1}}{S^\kappa} = (1-\gamma)\frac{S_H^\kappa}{S^\kappa} + \sigma_L^\kappa(p_H)\gamma\frac{S_L^\kappa}{S^\kappa} = (1-\gamma) + [\gamma(1+\sigma_L^\kappa(p_H)] - 1](1-q^\kappa)$$

Hence,

$$q^{\kappa+1} = \frac{S_H^{\kappa+1}}{S^{\kappa+1}} = (1-\gamma)\frac{S_H^\kappa}{S^\kappa}\frac{S^\kappa}{S^{\kappa+1}} = \frac{(1-\gamma)}{(1-\gamma) + [\gamma(1+\sigma_L^\kappa(p_H)) - 1](1-q^\kappa)}q^\kappa$$

If $q^{\kappa+1} = q^\kappa$ $\forall \kappa \in \mathbb{N}_0$ L-sellers' strategy satisfies $[\gamma(1+\sigma_L^\kappa(p_H)] - 1] = 0$, i.e. $\sigma_L^k(p_H) = \frac{1-\gamma}{\gamma}$. In order to play a mixed behavioural strategy L-sellers are indifferent as to whether they post p_H or θ_L. Since $q^\kappa = q^0$ for every $\kappa \in \mathbb{N}_0$, time on market observability is irrelevant because buyers' prior probability of matching with an H-seller is constant across cohorts. Buyers' optimal strategy does not change across cohorts S^κ and—in turn—sellers' continuation values do not depend on κ, i.e. $V_\lambda^\kappa(q^\kappa) = V_\lambda(q^0)$[39] $\forall \kappa \in \mathbb{N}_0$. The indifference condition for L-sellers is:

$$V_L(q^0) = (1-\gamma)(p_H - v_L) + \gamma V_L(q^0) - c = \theta_L - v_L - c$$

or

$$V_L(q^0) = (1-\gamma)(p_H - v_L) + \gamma[\theta_L - v_L - c] - c = \theta_L - v_L - c$$

The latter expression implies $p_H = \theta_L + \frac{\gamma}{1-\gamma}c$. By substituting this price into

$$V_H(q^0) = \gamma(p_H - v_H) + (1-\gamma)V_H(q^0) - c$$

[39] I adopt the notation already used in the proof of Proposition 4.2.

we get

$$V_H(q^0) = p_H - v_H - \frac{c}{\gamma} = \theta_L - v_H + \frac{\gamma^2+\gamma-1}{\gamma(1-\gamma)}c$$

H-sellers individual rationality constraint is satisfied only if $c \geq \frac{\gamma(1-\gamma)}{\gamma^2+\gamma-1}(v_H - \theta_L)$.

Buyers follow their equilibrium strategy if and only if

$$\mathbb{E}_{\pi_B}[\theta|\mathscr{I}_B(p_H,\kappa,H)] \geq p_H \geq \mathbb{E}_{\pi_B}[\theta|\mathscr{I}_B(p_H,\kappa,L)]$$

i.e.

$$\frac{q^0\gamma\theta_H + (1-q^0)\frac{(1-\gamma)^2}{\gamma}\theta_L}{q^0\gamma + (1-q^0)\frac{(1-\gamma)^2}{\gamma}} \geq \theta_L + \frac{\gamma}{1-\gamma}c \geq q^0\theta_H + (1-q^0)\theta_L$$

This set of inequalities can be rewritten as:

$$q_{SSU} := \frac{\frac{\gamma}{1-\gamma}c}{\theta_H - \theta_L} \geq q^0 \geq \frac{1-\gamma}{\gamma\left[\frac{\theta_H-\theta_L}{c}\right] - \frac{2\gamma-1}{1-\gamma}} := q_{SS}$$

Notice that $\gamma\left[\frac{\theta_H-\theta_L}{c}\right] - \frac{2\gamma-1}{1-\gamma} > 0$ since $\frac{\theta_H-\theta_L}{c} \geq \frac{\gamma}{1-\gamma} > \frac{2\gamma-1}{\gamma(1-\gamma)}$.

Step 2. There exist a set of economies $\mathscr{E}(\gamma,q^0)$ that supports the equilibrium in Step 1. but not the pooling equilibria in Propositions 4.2 and 4.3.

It is sufficient to find a set of parameters $(\theta_H,\theta_L,v_H,v_L,\gamma,c)$ such that $q^{SS} < q_0^O$, i.e.

$$\frac{1-\gamma}{\gamma\left[\frac{\theta_H-\theta_L}{c}\right] - \frac{2\gamma-1}{1-\gamma}} < \frac{(1-\gamma)(v_H-\theta_L)}{\gamma(\theta_H-v_H)+(1-\gamma)(v_H-\theta_L)}$$

and c is such that there is a semi-separating equilibrium, i.e. $c \geq \frac{\gamma(1-\gamma)}{\gamma^2+\gamma-1}(v_H - \theta_L)$.

As $v_H - \theta_L > 0$, the signal precision γ must be high enough to satisfy $\frac{\gamma^2+\gamma-1}{\gamma(1-\gamma)} > 0$, i.e. $\gamma > \frac{\sqrt{5}-1}{2}$. For a given set of parameters $(\theta_H,\theta_L,v_H,v_L,\gamma)$, the lower bound q^{SS} is increasing in c so its value is minimum for $c = \frac{\gamma(1-\gamma)}{\gamma^2+\gamma-1}(v_H-\theta_L)$. Then, $q^{SS} < q_0^O$ requires:

$$\frac{1-\gamma}{\gamma\left[\frac{\theta_H-\theta_L}{c}\right] - \frac{2\gamma-1}{1-\gamma}} = \frac{(1-\gamma)^2(v_H-\theta_L)}{(\gamma^2+\gamma-1)\theta_H + \gamma(1-\gamma)\theta_L - (2\gamma-1)v_H} < \frac{(1-\gamma)(v_H-\theta_L)}{\gamma(\theta_H-v_H)+(1-\gamma)(v_H-\theta_L)}$$

After a series of calculations, the above inequality is equivalent to:

$$2\frac{\theta_H-v_H}{v_H-\theta_L}\gamma^2 + \gamma - \frac{\theta_H-\theta_L}{v_H-\theta_L} > 0$$

Using the standard quadratic formula:

$$\gamma > \hat{\gamma} = \frac{-1+\sqrt{1-8\frac{(\theta_H - v_H)(\theta_H-\theta_L)}{(v_H-\theta_L)^2}}}{4\frac{\theta_H - v_H}{v_H-\theta_L}}$$

The discriminant $\Delta \equiv 1-8\frac{(\theta_H-v_H)(\theta_H-\theta_L)}{(v_H-\theta_L)^2}$ is strictly positive if $\frac{(\theta_H-v_H)(\theta_H-\theta_L)}{(v_H-\theta_L)^2} < \frac{1}{8}$ (for example: $\theta_H = 1$, $v_H = 0.98$ and $\theta_L = 0.8$). If the latter inequality holds, then $\Delta < 1$ and $\hat{\gamma} < 1$ because:

$$\frac{-1+\sqrt{\Delta}}{4\frac{\theta_H-v_H}{v_H-\theta_L}} < 1 \Leftrightarrow \sqrt{\Delta} - 4\frac{\theta_H - v_H}{v_H - \theta_L} < 1$$

Therefore, if $\frac{(\theta_H-v_H)(\theta_H-\theta_L)}{(v_H-\theta_L)^2} < \frac{1}{8}$ for every $\gamma > \max(\hat{\gamma}, \frac{\sqrt{5}-1}{2})$ there is at least an economy such that $q^{SS} < q^O_0$. ■

7.5 Market design

Proof Proposition 5.1.

Under TMO the set of feasible transfers $(\tau^\kappa, r^\kappa)_{\kappa=0}^{+\infty}$ can vary across sellers belonging to different cohorts S^κ. I denote a transfer from a participating seller in S^κ to the designer with τ^κ and a transfer from the designer to a seller with r^κ, if the seller trades after κ previous periods in the market.[40] For a constellation $(\theta_H, v_H, \theta_L, v_L, \gamma)$, let $(\tau^{*^\kappa}, r^{*^\kappa})_{\kappa=0}^{+\infty}$ be the efficient mechanism existing for the lowest possible q^0. Denote this minimum value with q^*.

To implement an equilibrium with informationally efficient prices, the mechanism has to implement a separating equilibrium. By Step 1 and 2 of the proof of Proposition 4.4, in equilibrium, L-sellers trade immediately while H-sellers trade once they match with a buyer who receives a high signal. By the appropriate law of large numbers, a system of transfers is budget balanced (henceforth BB) on the equilibrium path if and only if

$$(1-q^0)(r^0-\tau^0) \leq q^0 \sum_{\kappa=0}^{+\infty} (1-\gamma)^\kappa (\tau^\kappa - \gamma r^\kappa)$$

Since utility is quasi-linear in transfers and agents are risk neutral, if the BB constraint holds with equality a feasible mechanism does not change the total trade surplus between buyers and sellers, i.e. $q^0(\theta_H - v_H) + (1-q^0)(\theta_L - v_L)$. Moreover, for $c = 0$ delaying trade does not reduce the total surplus. In a separating equilibrium assessment, say (σ^*, π^*), sellers of type λ trade at price θ_λ and the buyers' expected payoff is zero. Therefore, if BB holds with equality a mechanism implementing a separating

[40] For example $\tau^\kappa > 0$ is the amount that sellers in S^κ have to pay to the designer in order to participate in the market. On the contrary $r^\kappa > 0$ is the positive transfer that sellers receive if they trade after κ previous periods in the market. In this vein, I often call τ^κ a market participation tax and r^κ a tax rebate that a seller receives upon trade.

equilibrium (σ^*, π^*) satisfies:

$$q^0 V_H^0(\sigma^*, \pi^*) + (1-q^0)V_L^0(\sigma^* \pi^*) = q^0(\theta_H - v_H) + (1-q^0)(\theta_L - v_L)$$

where the LHS is sellers' equilibrium payoffs while the RHS is total trade surplus.

If BB is slack it is possible to have an efficient mechanism for every q^0, i.e. $q^* = 0$. If $q^* > 0$, BB holds with equality, otherwise the market designer may use his profit to relax sellers' individual rationality constraints. In the remainder of the proof I consider a binding BB.

Step 1. If $V_H^0(\sigma^*, \pi^*) = 0$ then $V_H^\kappa(\sigma^*, \pi^*) = 0$ for every $\kappa \in \mathbb{N}_0$.

If $V_H^0(\sigma^*, \pi^*) = 0$ then $(1-q^0)V_L^0(\sigma^*, \pi^*) = q^0(\theta_H - v_H) + (1-q^0)(\theta_L - v_L)$, i.e. L-sellers get all the trade surplus available in the economy. If $V_H^\kappa(\sigma^*, \pi^*) < 0$ for some κ then H-sellers in S_H^κ would exit the market, and the resulting allocation would not be efficient. If $V_H^\kappa(\sigma^*, \pi^*) > 0$ for some $\kappa \geq 1$, then H-sellers in cohort S^κ enjoy a strictly positive payoff only if their matched buyers get a negative expected payoff, contradicting the optimality of buyers' strategy.

Step 2. If $q^* > 0$ then $V_H^0(\sigma^*, \pi^*) = 0$.

As $(\tau^{*^\kappa}, r^{*^\kappa})_{\kappa=0}^{+\infty}$ supports a separating equilibrium H-sellers' equilibrium payoff is:[41]

$$V_H^0(q^0) = \theta_H - v_H - \sum_{\kappa=0}^{\infty}(1-\gamma)^\kappa(\tau^{*^\kappa} - \gamma r^{*^\kappa}) = \theta_H - v_H - \frac{1-q^0}{q^0}(r^{*^0} - \tau^{*^0})$$

where the last equality follows from the BB constraint. Observe that τ^{*^0} only affects the initial market participation decision for sellers in cohorts $\kappa = 0$, but it does not change the future incentive compatibility constraints. Rearranging the BB constraint gives:

$$\tau^{*^0} = (1-q^0)r^{*^0} - q^0[\gamma r^{*^0} + \sum_{\kappa=1}^{\infty}(1-\gamma)^\kappa(\tau^{*^\kappa} - \gamma r^{*^\kappa})]$$

For different values of q^0 we can leave r^{*^0} unchanged and $(\tau^{*^\kappa}, r^{*^\kappa})_{\kappa=1}^{\infty}$ and achieve BB through τ^0. Substituting τ^{*^0} in sellers' expected payoff gives:[42]

$$\begin{aligned}
V_H^0(q^0) = & \quad \theta_H - v_H - \tfrac{1-q^0}{q^0}(r^{*^0} - \tau^{*^0}) \\
= & \quad \theta_H - v_H - (1-q^0)[(1+\gamma)r^{*^0} + \sum_{\kappa=1}^{\infty}(1-\gamma)^\kappa(\tau^{*^\kappa} - \gamma r^{*^\kappa})] \\
V_L^0(q^0) = & \quad \theta_L - v_L + \tfrac{q^0}{1-q^0}[\theta_H - v_H - V_H^0(\sigma^*)] \\
= & \quad \theta_L - v_L + q^0[(1+\gamma)r^{*^0} + \sum_{\kappa=1}^{\infty}(1-\gamma)^\kappa(\tau^{*^\kappa} - \gamma r^{*^\kappa})]
\end{aligned}$$

[41] Similarly to the proof of Proposition 4.2 I stress the importance of q^0 and I expand the notation using $V_\lambda^\kappa(q^0)$ rather than $V_\lambda^\kappa(\sigma^*, \pi^*)$.

[42] Notice that I use the trade surplus equation to rewrite L-sellers expected payoff.

Suppose on the contrary that $V_H^0(q^*) > 0$ with $q^* > 0$. If $[(1+\gamma)r^{*^0} + \sum_{\kappa=1}^{\infty}(1-\gamma)^\kappa(\tau^{*^\kappa} - \gamma r^{*^\kappa})] > 0$ then it is easy to observe that there is a sufficiently small $\varepsilon > 0$ such that for every $q^0 \geq q^* - \varepsilon$ a market intervention that only adjusts τ^0 to preserve budget balance achieves $V_H^\kappa(q^0) \geq 0$ and $V_L^\kappa(q^0) \geq 0$ for every $\kappa \in \mathbb{N}_0$, contradicting the definition of q^*. If instead $[(1+\gamma)r^{*^0} + \sum_{\kappa=1}^{\infty}(1-\gamma)^\kappa(\tau^{*^\kappa} - \gamma r^{*^\kappa})] < 0$ then for every $q^0 < q^*$ we can adjust τ^0 such that $V_H^0(q^0) > V_H^0(q^*)$ and $V_L^0(q^0) > V_L^0(q^*) \geq 0$, contradicting the definition of q^* again.

Step 3. For every $(\theta_H, v_H, \theta_L, v_L)$ there is a γ^* such that for $\gamma \in (\frac{1}{2}, \gamma^*)$ it is $q^* > 0$.

Let $(\tau_\gamma^{*^\kappa}, r_\gamma^{*^\kappa})_{\kappa=0}^{\infty}$ be a mechanism that implements a separating equilibrium in an economy with signal precision γ. Sellers in S_L^0 post θ_L only if:

$$\theta_L - v_L + r_\gamma^{0^*} - \tau_\gamma^{0^*} \geq \theta_H - v_L - \sum_{\kappa=0}^{\infty}\gamma^\kappa[\tau_\gamma^{\kappa^*} - (1-\gamma)r_\gamma^{\kappa^*}] \tag{8}$$

From the BB constraint I can substitute $r_\gamma^{0^*} - \tau_\gamma^{0^*}$ with $\frac{q^0}{1-q^0}\sum_{\kappa=0}^{\infty}(1-\gamma)^\kappa(\tau_\gamma^{*^\kappa} - \gamma r_\gamma^{*^\kappa})]$ to get the no-deviation condition:

$$\sum_{\kappa=0}^{\infty}\gamma^\kappa[\tau_\gamma^{\kappa^*} - (1-\gamma)r_\gamma^{\kappa^*}] + \frac{q^0}{1-q^0}\sum_{\kappa=0}^{\infty}(1-\gamma)^\kappa(\tau_\gamma^{\kappa^*} - \gamma r_\gamma^{\kappa^*})] \geq \theta_H - \theta_L$$

Notice that for every $\varepsilon > 0$ it is possible to find γ_ε^* such that for every $\gamma \in (\frac{1}{2}, \gamma_\varepsilon^*)$:

$$|\Delta_\gamma| := \left|\sum_{\kappa=0}^{\infty}\gamma^\kappa[\tau_\gamma^{\kappa^*} - (1-\gamma)r_\gamma^{\kappa^*}] - \sum_{\kappa=0}^{\infty}(1-\gamma)^\kappa(\tau_\gamma^{*^\kappa} - \gamma r_\gamma^{*^\kappa})]\right| < \varepsilon$$

where $\Delta_\gamma \to 0$ for $\gamma \to \frac{1}{2}$.
Adding $\sum_{\kappa=0}^{\infty}(1-\gamma)^\kappa(\tau_\gamma^{*^\kappa} - \gamma r_\gamma^{*^\kappa})]$ to both sides of equation (8) and simplifying:

$$\sum_{\kappa=0}^{\infty}(1-\gamma)^\kappa(\tau_\gamma^{*^\kappa} - \gamma r_\gamma^{*^\kappa})] \geq (1-q^0)(\theta_H - \theta_L - \Delta_\gamma)$$

Now observe that the individual rationality constraint for H-sellers requires:

$$V_H^0(\sigma^*, \pi^*) = \theta_H - v_H - \sum_{\kappa=0}^{\infty}(1-\gamma)^\kappa(\tau_\gamma^{*^\kappa} - \gamma r_\gamma^{*^\kappa})] \geq 0$$

However for γ sufficiently close to $\frac{1}{2}$ (i.e. $\Delta_\gamma \to 0$) and q^0 small enough we have:

$$\theta_H - v_H - \sum_{\kappa=0}^{\infty}(1-\gamma)^\kappa(\tau_\gamma^{*^\kappa} - \gamma r_\gamma^{*^\kappa})] \leq \theta_H - v_H - (1-q^0)(\theta_H - \theta_L - \Delta_\gamma) < 0$$

as $v_H > \theta_L$. Therefore, no feasible system of transfers can implement an efficient allocation for values of q^0 and γ sufficiently close to zero and $\frac{1}{2}$, respectively.

Step 4. No separating equilibrium exists for

$$q^0 < \max\left\{0, 1-\left(\frac{\gamma}{1-\gamma}\right)^2 \frac{\theta_H - v_H}{\theta_H - \theta_L}\right\} = q^*$$

A first best-allocation is implemented with a market participation tax $\tau^\kappa = \tau^* = \frac{1-\gamma}{\gamma}(\theta_H - \theta_L)$ and, once a seller trades, a tax rebate $r^\kappa = r^* = \tau^*\left(1 + \frac{1-\gamma}{\gamma} q^*\right)$.

By Step 1 and 2 if $q^* > 0$ we must have $V_H^\kappa(q^*) = 0$ for every $\kappa \in \mathbb{N}_0$, hence

$$V_H^\kappa(q^*) = \gamma(\theta_H - v_H + r^{*^\kappa}) - \tau^{*^\kappa} = 0 \;\Rightarrow\; \tau^{*^\kappa} - \gamma r^{*^\kappa} = \gamma(\theta_H - v_H)$$

By substituting this expression into the BB constraint:

$$(1-q^0)(r^{*^0} - \tau^{*^0}) = q^0 \sum_{\kappa=0}^{+\infty} (1-\gamma)^\kappa (\tau^{*^\kappa} - \gamma r^{*^\kappa}) = q^0 \sum_{\kappa=0}^{+\infty} (1-\gamma)^\kappa \gamma(\theta_H - v_H) = q^0(\theta_H - v_H)$$

Hence $r^{*^0} - \tau^{*^0} = \frac{q^0}{1-q^0}(\theta_H - v_H)$. Combining with $\tau^{*^0} - \gamma r^{*^0} = \gamma(\theta_H - v_H)$ I get

$$r^{*^0} = \frac{\theta_H - v_H}{1-\gamma} \frac{\gamma + q^0(1-\gamma)}{1-q^0} \qquad\qquad \tau^{*^0} = \frac{\gamma}{1-\gamma} \frac{\theta_H - v_H}{1-q^0} \tag{9}$$

Using the one-shot deviation property, L-sellers no deviation condition has to satisfy:

$$\theta_L - v_L + r^{*^\kappa} - \tau^{*^\kappa} \geq (1-\gamma)(\theta_H - v_L + r^{*^\kappa}) + \gamma(\theta_L - v_L + r^{*^{\kappa+1}} - \tau^{*^{\kappa+1}}) - \tau^{*^\kappa}$$

$$r^{*^\kappa} + \tau^{*^{\kappa+1}} - r^{*^{\kappa+1}} \geq \frac{1-\gamma}{\gamma}(\theta_H - \theta_L) \tag{10}$$

As $\tau^{*^{\kappa+1}} = \gamma(\theta_H - v_H + r^{*^{\kappa+1}})$ for every $\kappa \in \mathbb{N}_0$, substituting this value in equation gives: (10):

$$r^{*^{\kappa+1}} \leq \frac{r^{*^\kappa}}{1-\gamma} + \frac{\gamma}{1-\gamma}(\theta_H - v_H) - \frac{\theta_H - \theta_L}{\gamma}$$

Observe that sellers in S_H^κ prefer not to postpone trade to period $\kappa + 1$ only if

$$\gamma(\theta_H - v_H + r^{*^\kappa}) - \tau^{*^\kappa} \geq V_H^{\kappa+1}(\sigma^*, q^*) - \tau^{*^\kappa} = -\tau^{*^\kappa} \Rightarrow\; r^{*^\kappa} \geq -(\theta_H - v_H)$$

where $V_H^{\kappa+1}(\sigma^*, q^*) = 0$ follows from Step 1. Therefore, for every $\kappa \geq 1$, r^{*^κ} satisfies

$$-(\theta_H - v_H) \leq r^{*^\kappa} \leq \frac{r^{*^{\kappa-1}}}{1-\gamma} + \frac{\gamma}{1-\gamma}(\theta_H - v_H) - \frac{\theta_H - \theta_L}{\gamma} \tag{11}$$

In order to satisfy the LHS of equation (11), I consider the RHS inequality binding. In this case, r^{*^κ} satisfies a first-order difference equation with the solution:

$$r^{*^\kappa} = \left(\frac{1}{1-\gamma}\right)^\kappa \left[r^{*^0} + (\theta_H - v_H) - \left(\frac{1-\gamma}{\gamma^2}\right)(\theta_H - \theta_L)\right] - \left[(\theta_H - v_H) - \left(\frac{1-\gamma}{\gamma^2}\right)(\theta_H - \theta_L)\right]$$

Substituting r^{*^0} from equation (9) and rearranging it as follows gives:

$$r^{*^\kappa} = \left(\frac{1}{1-\gamma}\right)^\kappa \left[\frac{\theta_H - v_H}{(1-\gamma)(1-q^0)} - \left(\frac{1-\gamma}{\gamma^2}\right)(\theta_H - \theta_L)\right] - \left[(\theta_H - v_H) - \left(\frac{1-\gamma}{\gamma^2}\right)(\theta_H - \theta_L)\right]$$

As $\frac{1}{1-\gamma} > 1$, the solution is not explosive towards $-\infty$—violating equation (11)—only if the first term in squared brackets is non-negative. Simplifying the expression:

$$q^0 \geq 1 - \left(\frac{\gamma}{1-\gamma}\right)^2 \frac{\theta_H - v_H}{\theta_H - \theta_L}$$

Let $q^* := \max\left\{0, 1 - \left(\frac{\gamma}{1-\gamma}\right)^2 \frac{\theta_H - v_H}{\theta_H - \theta_L}\right\}$. All the previous inequality constraints are binding when $q^0 = q^* > 0$. Then:

$$\tau^\kappa = \tau^* = \frac{1-\gamma}{\gamma}(\theta_H - \theta_L) \qquad r^\kappa = r^* = \tau^*\left(1 + \frac{1-\gamma}{\gamma} q^*\right)$$

■

8 Appendix B

Outline of the proof for Lemma 4.1.

I show that no other behavioural strategy is admissible except for the ones in Lemma 4.1. To prove this result, I first obtain a bound on the difference between continuation values, i.e. $V_L^\kappa - V_H^\kappa$. This preliminary result is obtained in three lemmata. First, I show that, for c sufficiently small, there is a common price path played by H- and L-sellers (Lemma 8.1 and 8.2); then, I derive an expression for V_λ^κ and a bound on their difference (Lemma 8.3).

Lemma 8.1 *For c sufficiently small, no equilibrium path has L-sellers in $S_L^\kappa \neq \emptyset$ trade with positive probability, and H-sellers in $S_H^\kappa \neq \emptyset$ only post prices rejected with a probability of one.*

Proof Lemma 8.1.

Consider an equilibrium (σ^*, π^*) and sellers $i \in S_H^\kappa$ and $l \in S_L^\kappa$ with histories $h_i^{\kappa-1}$ and $h_l^{\kappa-1}$. Let $\bar{\kappa} := \arg\max_{m \in \mathbb{N}_0} \left\{\exists \{p^j\}_{j=\kappa}^m : \sigma_H^{*^j}(p^j | h_i^{j-1}) > 0, h_i^{j-1} = \{h_i^{\kappa-1}, p^\kappa, ..., p^{j-1}\}\right\}$.[43] Let $\tilde{h}_i^{\bar{\kappa}-1}$ be the history resulting from $h_i^{\kappa-1}$ and the sequence of prices $\{p^j\}_{j=\kappa}^{\bar{\kappa}-1}$ up to period $\bar{\kappa}$. Denote with $\tilde{h}_i^{j-1}$, $j \leq \bar{\kappa}$, a sub-history of $\tilde{h}_i^{\bar{\kappa}-1}$. If $\bar{\kappa} < +\infty$ the seller drops out after $\bar{\kappa}+1$ periods.

[43] If $\bar{\kappa} = +\infty$ there is at least one history in which a high quality seller participates in the market forever.

Suppose, per contra, that sellers in S^{κ}, $\kappa \leq \bar{\kappa} < \infty$, play the behavioural strategy described in the Lemma statement. First, sequential rationality implies $\kappa < \bar{\kappa}$: posting a price rejected with a probability of one in period $\bar{\kappa}$ before dropping out in period $\bar{\kappa}+1$ is not optimal, since dropping out after $\bar{\kappa}$ previous periods saves one search cost c. As a consequence, price $p^{\bar{\kappa}}$ is accepted with positive probability. Similarly, if $\bar{\kappa} = +\infty$, there is at least a $\kappa' > \kappa$ such that H-sellers in $S^{\kappa'}$ post a price $p^{\kappa'}$ accepted with positive probability.

By assumption, L-sellers trade with positive probability. Sequential rationality implies that they post θ_L. In turn, seller $i \in S_L^{\kappa}$ finds it profitable to trade at θ_L rather than postponing trade only if:

$$\theta_L - v_L - c \geq V_L^{\kappa+1}(\sigma^*, \pi^* | (h_l^{\kappa-1}, p^{\kappa} \times R)) - c \tag{12}$$

A possible deviation for a L-seller $l \in S_L^{\kappa+1}$ is to imitate the H-sellers' strategy until $\bar{\kappa}$, posting $\{p^j\}_{j=\kappa+1}^{\bar{\kappa}}$, and θ_L in period $\bar{\kappa}+1$. I denote this imitating strategy with σ_l'. In equilibrium σ_l' cannot provide a strictly higher expected payoff than σ_L^*, i.e. for every $h_l^{\kappa} \in H^{\kappa}$:

$$\begin{aligned}
V_L^{\kappa+1}(\sigma^*, \pi^* | h_l^{\kappa}) \geq\ & \sum_{j=\kappa+1}^{\bar{\kappa}} \mathbb{P}_L^{\sigma_l' \sigma_{-l}^* \pi^*} \left(z(p^j) | \left(h_l^{\kappa}, \{p^s \times R\}_{s=\kappa+1}^{j} \right) \right) \left[p^j - v_L - (j - \kappa + 1)c \right] \\
+\ & \left[1 - \sum_{j=\kappa+1}^{\bar{\kappa}} \mathbb{P}_L^{\sigma_l' \sigma_{-l}^* \pi^*} \left(z(p^j) | \left(h_l^{\kappa}, \{p^s \times R\}_{s=\kappa+1}^{j} \right) \right) \right] [\theta_L - v_L - (\bar{\kappa} - \kappa + 1)c] - c \\
\geq\ & \sum_{j=\kappa+1}^{\bar{\kappa}} \mathbb{P}_L^{\sigma_l' \sigma_{-l}^* \pi^*} \left(z(p^j) | \left(h_l^{\kappa}, \{p^s \times R\}_{s=\kappa+1}^{j} \right) \right) [v_H - v_L - (j - \kappa + 1)c] \\
+\ & \left[1 - \sum_{j=\kappa+1}^{\bar{\kappa}} \mathbb{P}_L^{\sigma_l' \sigma_{-l}^* \pi^*} \left(z(p^j) | \left(h_l^{\kappa}, \{p^s \times R\}_{s=\kappa+1}^{j} \right) \right) \right] [\theta_L - v_L - (\bar{\kappa} - \kappa + 1)c] - c
\end{aligned} \tag{13}$$

Equations (12) and (13) imply

$$(\bar{\kappa} - \kappa + 1)\, c \geq \sum_{j=\kappa+1}^{\bar{\kappa}} \mathbb{P}_L^{\sigma_l' \sigma_{-l}^* \pi^*} \left(z(p^j) | \left(h_i^{\kappa}, \{p^s \times R\}_{s=\kappa+1}^{j} \right) \right) [v_H - \theta_L + (\bar{\kappa} - j)c] \tag{14}$$

As $\kappa < \bar{\kappa}$, starting from history $\tilde{h}_i^{\kappa} \in H^{\kappa}$ it holds that $\sum_{j=\kappa+1}^{\bar{\kappa}} \mathbb{P}_H^{\sigma^* \pi^*} \left(z(p^j) | \left(\tilde{h}_i^{\kappa}, \{p^s \times R\}_{s=\kappa+1}^{j} \right) \right) > 0$. Hence, for every L-seller $i \in S_L^{\kappa}$ it must also hold that:

$$\sum_{j=\kappa+1}^{\bar{\kappa}-1} \mathbb{P}_L^{\sigma_l' \sigma_{-l}^* \pi^*} \left(z(p^j) | \left(h_l^{\kappa}, \{p^s \times R\}_{s=\kappa+1}^{j} \right) \right) [v_H - \theta_L] > 0$$

because previous price history $h_l^{\kappa-1}$ is not observable to buyers and, once matched with a L-seller, they receive every signal $\xi \in \{H, L\}$ with positive probability. Therefore, inequality (14) cannot hold for c sufficiently small. ■

Lemma 8.2 *For c sufficiently small, if there exists an equilibrium path in which H-sellers in $S_H^\kappa \neq \emptyset$ post a set of prices $P \subset \mathbb{R}_+$, all accepted with positive probability, then L-sellers in $S_L^\kappa \neq \emptyset$ post at least one price in P, unless they move to cohort $S^{\kappa+1}$ with a probability of one.*

Proof Lemma 8.2.
Consider sellers $i \in S_H^\kappa$ and $l \in S_L^\kappa$ with histories $h_i^{\kappa-1}$ and $h_l^{\kappa-1}$. Suppose on the contrary that L-sellers in S_L^κ do not post any price in P. Then, they can only trade at price θ_L. A seller $l \in S_L^\kappa$ prefers to post θ_L rather than the deviation strategy $\sigma_l'^\kappa(p|h_l^{\kappa-1}) = 1$ for some $p \in P$ only if

$$\begin{aligned}
\theta_L - v_L - c \geq \;& \mathbb{P}_L^{\sigma_l' \sigma_{-l}^* \pi^*}(z(p)|h_l^{\kappa-1})\,[p - v_L] + \left(1 - \mathbb{P}_L^{\sigma_l' \sigma_{-l}^* \pi^*}(z(p)|h_l^{\kappa-1})\right) V_L^{\kappa+1}(\sigma^*, \pi^*|h_l^\kappa) - c \\
\geq \;& \mathbb{P}_L^{\sigma_l' \sigma_{-l}^* \pi^*}(z(p)|h_l^{\kappa-1})\,[v_H - v_L] + \left(1 - \mathbb{P}_L^{\sigma_l' \sigma_{-l}^* \pi^*}(z(p)|h_l^{\kappa-1})\right) [\theta_L - v_L - c] - c
\end{aligned}$$

The second inequality holds because L-sellers can always trade at θ_L in period $\kappa + 1$, and $p \geq v_H$ because H-sellers only trade at prices greater or equal to v_H. The inequality can be rewritten as

$$c\left(1 - \mathbb{P}_L^{\sigma_l' \sigma_{-l}^* \pi^*}(z(p)|h_l^{\kappa-1})\right) \geq \mathbb{P}_L^{\sigma_l' \sigma_{-l}^* \pi^*}(z(p)|h_l^{\kappa-1})(v_H - \theta_L) \tag{15}$$

If p is accepted with positive probability, $\mathbb{P}_L^{\sigma_l' \sigma_{-l}^* \pi^*}(z(p)|h_l^{\kappa-1}) > 0$ as L-sellers may receive every signal $\xi \in \{H, L\}$, and buyers do not observe previous histories $h_i^{\kappa-1}$ and $h_l^{\kappa-1}$. By hypothesis, $\mathbb{P}_H^{\sigma^* \pi^*}(z(p)|h_i^{\kappa-1}) > 0$ and inequality (15) cannot hold for c sufficiently small. ■

Lemma 8.3 *For c sufficiently small, an equilibrium assessment (σ^*, π^*) satisfies*

$$V_L^\kappa(\sigma^*, \pi^*|h_l^{\kappa-1}) - V_H^\kappa(\sigma^*, \pi^*|h_i^{\kappa-1}) \leq v_H - v_L \qquad \forall \kappa \in \mathbb{N}_0 \text{ and } \forall h_i^{\kappa-1} \in H^{\kappa-1}, \forall h_l^{\kappa-1} \in H^{\kappa-1}$$

The condition holds with equality only if: (i) H- and L-sellers post a common price and trade with a probability of one; or (ii) H- and L-sellers are both indifferent between postponing trade and posting a price p accepted only after a high signal.

Proof Lemma 8.3.
Consider an equilibrium (σ^*, π^*) and sellers $i \in S_H^\kappa$ and $l \in S_L^\kappa$. If seller $i \in S_H^\kappa$ prefers to stay out of the market it trivially holds that:

$$V_H^\kappa(\sigma^*, \pi^*|h_i^{\kappa-1}) = 0 \qquad\qquad V_L^\kappa(\sigma^*, \pi^*|h_l^{\kappa-1}) = \theta_L - v_L - c$$

By Lemma 7.2 it holds for every seller in S_H^κ and S_L^κ, irrespective of previous histories.

Let:

$$\begin{aligned}
K_H^N(\kappa) :=\;& \left\{ j \geq \kappa : \forall h_i^{j-1} \in H^{j-1} \text{ and } \forall p \text{ with } \sigma_H^{*j}(p|h_i^{j-1}) > 0 \text{ it holds } \sigma_B^*(A|\mathscr{I}(p,\cdot,\xi)) = 0 \right\} \\
K_L^N(\kappa) :=\;& \left\{ j \geq \kappa : \forall h_l^{j-1} \in H^{j-1} \text{ and } \forall p \text{ with } \sigma_L^{*j}(p|h_l^{j-1}) > 0 \text{ it holds } \sigma_B^*(A|\mathscr{I}(p,\cdot,\xi)) = 0 \right\}
\end{aligned}$$

include all $j \geq \kappa$ such that all sellers in S_λ^j only post prices rejected with a probability of one. For simplicity, I denote with n the action to post a price rejected with a probability of one.

By definition, if $j \in K_\lambda^N(\kappa)$ for every $z \in Z^j(p)$ it is $\mathbb{P}_\lambda^{\sigma^*\pi^*}(z|h^{\kappa-1}) = 0$. By Lemma 8.1, it is $K_H^N(\kappa) \subseteq K_L^N(\kappa)$. By Lemma 8.2, if $j \notin K_H^N(\kappa)$ then either L-sellers do not trade with a probability of one, i.e. $j \in K_L^N(\kappa)$, or there exist $h_l^{j-1} \in H^{j-1}$, $h_i^{j-1} \in H^{j-1}$, and p^j such that $\sigma_L^{*j}(p^j|h_l^{j-1}) > 0$, $\sigma_H^{*j}(p^j|h_i^{j-1}) > 0$ and buyers accept p^j with positive probability. Consider such a sequence $\{p^j\}_{j\geq\kappa}$, $j \notin K_H^N(\kappa)$.

For $\{p^j\}_{j\geq\kappa}$, $j \notin K_H^N(\kappa)$, let $\bar{\kappa} = \arg\max_{j\in\mathbb{N}_0} \sigma_H^{*j}(p^j|h_i^{j-1}) > 0$, be the latest period of market participation for H-sellers along this price path. I denote with $\tilde{h}_i^{j-1}$ ($\tilde{h}_l^{j-1}$), $j \geq \kappa$, the history for seller $i \in S_H^\kappa$ ($l \in S_L^\kappa$) in which buyers reject price p^j if $j \notin K_H^N(\kappa)$ ($j \notin K_L^N(\kappa)$), or sellers play n if $j \in K_H^N(\kappa)$ ($K_L^N(\kappa)$). For $\kappa > \bar{\kappa}$, L-sellers' optimal best response is to post price θ_L and trade, so $\tilde{h}_L^{\bar{\kappa}+1} = (\tilde{h}_L^{\bar{\kappa}}, \theta_L \times A)$.

By Lemma 7.2, expected payoffs $V_\lambda^\kappa(\sigma^*,\pi^*)$ are independent from previous histories. Therefore, in equilibrium, sellers in S_λ^j are indifferent among all actions played with positive probability by any seller in S_λ^j. As a result, the price path along histories $\tilde{h}_i^{\bar{\kappa}}$ and $\tilde{h}_l^{\bar{\kappa}+1}$ provides an expected payoff equal to $V_H^\kappa(\sigma^*,\pi^*)$ and $V_L^\kappa(\sigma^*,\pi^*)$, respectively. Hence, it is possible to express sellers' expected payoff as:

$$
\begin{aligned}
V_H^\kappa(\sigma^*,\pi^*|h_i^{\kappa-1}) &= \sum_{j=\kappa}^{\bar{\kappa}} \mathbb{P}_H^{\sigma^*\pi^*}(z^j(p^j)|h_i^{\kappa-1}) u_H^\kappa(z^j(p^j)) = \sum_{j\notin K_H^N(\kappa)}^{\bar{\kappa}} \mathbb{P}_H^{\sigma^*\pi^*}(z^j(p^j)|h_i^{\kappa-1}) u_H^\kappa(z^j(p^j)) \\
V_L^\kappa(\sigma^*,\pi^*|h_l^{\kappa-1}) &= \sum_{j\notin K_L^N(\kappa)}^{\bar{\kappa}} \mathbb{P}_L^{\sigma^*\pi^*}(z^j(p^j)|h_l^{\kappa-1}) u_L^\kappa(z^j(p^j)) + \mathbb{P}_L^{\sigma^*\pi^*}(z^{\bar{\kappa}+1}(\theta_L)|h_l^{\kappa-1}) u_L^\kappa(z^{\bar{\kappa}+1}(\theta_L))
\end{aligned}
$$

To prove the statement I briefly state two preliminary observations:

a. $\mathbb{P}_L^{\sigma^*\pi^*}(z^{\bar{\kappa}+1}(\theta_L)|h_l^{\kappa-1}) \leq \left[1 - \sum_{j\notin K_L^N(\kappa)}^{\bar{\kappa}} \mathbb{P}_L^{\sigma^*\pi^*}(z^j(p^j)|h_l^{\kappa-1})\right]$

   Conditional on a history $h^{\kappa-1}$ the probability that a L-seller trades at price θ_L is *at least* equal to the complementary probability of trading in period j, $\kappa \leq j \leq \bar{\kappa}$, at price p^j along the equilibrium price path $\{p^j\}_{j=\kappa}^{\bar{\kappa}}$.

b. It holds that:

$$
\sum_{j\notin K_L^N(\kappa)}^{\bar{\kappa}} \mathbb{P}_L^{\sigma^*\pi^*}(z^j(p^j)|h_l^{\kappa-1}) u_H^\kappa(z^j(p^j)) - \sum_{j\notin K_H^N(\kappa)}^{\bar{\kappa}} \mathbb{P}_H^{\sigma^*\pi^*}(z^j(p^j)|h_i^{\kappa-1}) u_H^\kappa(z^j(p^j)) \leq 0 \qquad (16)
$$

   In equilibrium, buyers accept a posted price p^j either with a probability of one (for every ξ) or only if they receive a high signal $\xi = H$. Therefore, it is not possible to have:

$$
\sum_{j\notin K_L^N(\kappa)}^{\bar{\kappa}} \mathbb{P}_L^{\sigma^*\pi^*}(z^j(p^j)|h_l^{\kappa-1}) u_H^\kappa(z^j(p^j)) > \sum_{j\notin K_H^N(\kappa)}^{\bar{\kappa}} \mathbb{P}_H^{\sigma^*\pi^*}(z^j(p^j)|h_i^{\kappa-1}) u_H^\kappa(z^j(p^j)) = V_H^\kappa(\sigma^*,\pi^*|h_i^{\kappa-1})
$$

   If this were the case, a H-seller could profitably deviate by decreasing the probability of trading at earlier times in favour of later periods.[44]

Thanks to a. and b. it is possible to conclude that:

$$
\begin{aligned}
V_L^\kappa(\sigma^*,\pi^*|h_l^{\kappa-1}) \; & - V_H^\kappa(\sigma^*,\pi^*|h_i^{\kappa-1}) = \sum_{j\notin K_L^N(\kappa)}^{\bar\kappa} \mathbb{P}_L^{\sigma^*\pi^*}(z^j(p^j)|h_l^{\kappa-1})u_L^\kappa(z^j(p^j)) \\
& + \mathbb{P}_L^{\sigma^*\pi^*}(z^{\bar\kappa+1}(\theta_L)|h_l^{\kappa-1})u_L^\kappa(z^{\bar\kappa+1}(\theta_L)) - \sum_{j\notin K_H^N(\kappa)}^{\bar\kappa} \mathbb{P}_H^{\sigma^*\pi^*}(z^j(p^j)|h_i^{\kappa-1})u_H^\kappa(z^j(p^j)) \\
& \le \sum_{j\notin K_L^N(\kappa)}^{\bar\kappa} \mathbb{P}_L^{\sigma^*\pi^*}(z^j(p^j)|h_l^{\kappa-1})(v_H-v_L) + \left[1-\sum_{j\notin K_L^N(\kappa)}^{\bar\kappa} \mathbb{P}_L^{\sigma^*\pi^*}(z^j(p^j)|h_l^{\kappa-1})\right](\theta_L - v_L) \\
& + \sum_{j\notin K_L^N(\kappa)}^{\bar\kappa} \mathbb{P}_L^{\sigma^*\pi^*}(z^j(p^j)|h_l^{\kappa-1})u_H^\kappa(z^j(p^j)) - \sum_{j\notin K_H^N(\kappa)}^{\bar\kappa} \mathbb{P}_H^{\sigma^*\pi^*}(z^j(p^j)|h_i^{\kappa-1})u_H^\kappa(z^j(p^j)) \\
& \le v_H - v_L
\end{aligned}
$$

The first inequality holds because: (i) $\theta_L - v_L \ge u_L^\kappa(z^{\bar\kappa+1}(\theta_L))$; (ii) $\mathbb{P}_L^{\sigma^*\pi^*}(z^{\bar\kappa+1}(\theta_L)|h_l^{\kappa-1}) \le \left[1-\sum_{j\notin K_L^N(\kappa)}^{\bar\kappa} \mathbb{P}_L^{\sigma^*\pi^*}(z^j(p^j)|\right.$ (iii) for every $z^j(p)\in Z^j$ it holds that $u_L^\kappa(z^j(p)) = u_H^\kappa(z^j(p)) + v_H - v_L$. The second inequality follows from $\theta_L < v_H$ and equation (16).

Lastly, I show under which equilibrium strategies it is possible to have $V_L^\kappa(\sigma^*,\pi^*|h_i^{\kappa-1}) - V_H^\kappa(\sigma^*,\pi^*|h_i^{\kappa-1}) = v_H - v_L$. For simplicity, I write V_λ^κ rather than $V_\lambda^\kappa(\sigma^*,\pi^*|h_i^{\kappa-1})$. Case (i) in the Lemma statement is immediate because $V_\lambda^\kappa = p - v_\lambda - c$. To prove case (ii) consider that, if a price p^κ is not accepted with a probability of one, then in equilibrium it is accepted only if $\xi = H$. The indifference condition requires that:

$$V_L^\kappa - V_H^\kappa = (1-\gamma)(p^\kappa - v_L) + \gamma V_L^{\kappa+1} - \gamma(p^\kappa - v_H) - (1-\gamma)V_H^{\kappa+1} = v_H - v_L$$

Hence $p^\kappa = \frac{1}{2\gamma-1}\left[\gamma\left(V_L^{\kappa+1}+v_L\right) - (1-\gamma)\left(V_H^\kappa + v_H\right)\right]$.
Suppose, per contra, a type λ is not indifferent and strictly prefers to post p^κ. Then,

$$\mathbb{P}_\lambda(H)(p^\kappa - v_\lambda) + (1-\mathbb{P}_\lambda(H))V_\lambda^{\kappa+1} - c > V_\lambda^{\kappa+1} - c$$

i.e. $p^\kappa - v_\lambda > V_\lambda^{\kappa+1}$. Substituting p^κ into this expression and solving gives:

$$\frac{\mathbb{P}_\lambda(H)}{2\gamma-1}\left[V_L^{\kappa+1} - V_H^{\kappa+1} - (v_H - v_L)\right] > 0$$

which cannot hold as $V_L^{\kappa+1} - V_H^{\kappa+1} \le v_H - v_L$. ■

Proof Lemma 4.1.
To prove the statement I show that there exists a $c^* > 0$ such that $\forall c \le c^*$ no other behavioural strategy

[44] For example, he could play an out-of-equilibrium strategy that includes prices rejected with a probability of one.

is admissible as an equilibrium of the game. By Proposition 4.4, no separating equilibrium exists for $c < \frac{1-\gamma}{\gamma}(\theta_H - \theta_L)$, otherwise L-sellers would deviate. By Lemma 7.2 I simplify the notation and just write $V_\lambda^\kappa(\sigma^*, \pi^*)$ throughout the proof.

Step 1. If $q^\kappa < 1$ there is no equilibrium path in which H- and L-sellers in S^κ both use a mixed behavioural strategy.

Buyers play best responses in pure strategies and, for any posted price, they can either (i) accept with a probability of one; (ii) accept only if $\xi = j$, $j \in \{H, L\}$; or (iii) reject with a probability of one. Their best responses cannot depend on sellers' histories because they are not observable. Sequential rationality implies that—in equilibrium—no seller mixes between two different prices accepted with identical, positive, probability as he would strictly prefer the highest price. If seller $i \in S_\lambda^\kappa$ mixes among prices p_1, p_2, p_3, and buyers play $\sigma_B^\kappa(A|\mathscr{I}(p_1, \cdot, \xi)) = 1$ for every ξ, $\sigma_B(A|\mathscr{I}(p_2, \cdot, j)) = 1$ only for signal $\xi = j$ and reject otherwise, and $\sigma_B^\kappa(A|\mathscr{I}(p_3, \cdot, \xi) = 0$ for every ξ, then the following indifference conditions must hold:

$$\begin{aligned} p_1 - v_\lambda - c = &\quad \mathbb{P}_\lambda(j)\,(p_2 - v_\lambda) + (1 - \mathbb{P}_\lambda(j))\,V_\lambda^{\kappa+1}(\sigma^*, \pi^*) - c \\ p_1 - v_\lambda - c = &\quad V_\lambda^{\kappa+1}(\sigma^*, \pi^*) - c \end{aligned}$$

However, this system of equations implies $p_1 = p_2$, but an identical price cannot be accepted with different probabilities. Therefore, in equilibrium, H- and L-sellers can only mix between: (i) two prices accepted with positive probability; (ii) one price accepted with positive probability and one (or more) rejected with a probability of one.

(i) Assume H-sellers mix between two prices (p_1, p_2). Buyers always accept p_2, but they accept p_1 only after signal $\xi = j, j \in \{H, L\}$. Mixing requires to be indifferent:

$$\mathbb{P}_H(j)(p_1 - v_H) + (1 - \mathbb{P}_H(j))\,V_H^{\kappa+1}(\sigma^*, \pi^*) - c = p_2 - v_H - c \tag{17}$$

Moreover, H-sellers should prefer to trade rather than to move to period $\kappa + 1$, hence:

$$p_2 - v_H \geq V_H^{\kappa+1}(\sigma^*, \pi^*) \qquad p_1 - v_H \geq V_H^{\kappa+1}(\sigma^*, \pi^*) \tag{18}$$

In turn, inequalities (17) and (18) imply $p_1 \geq p_2 \geq v_H$. The inequality $p_1 \geq p_2$ holds strictly because buyers cannot accept the same price with different probabilities. Buyers only accept after signal j and reject otherwise only if $j = H$. Indeed, when H- and L-sellers mix on the same prices and $q^\kappa < 1$ buyers' beliefs satisfy $\pi_B(p_1, \cdot, H) > \pi_B(p_1, \cdot, L)$; as a result, $\mathbb{E}_{\pi_B}[\theta|\mathscr{I}(p_1, \cdot, H)] > \mathbb{E}_{\pi_B}[\theta|\mathscr{I}(p_1, \cdot, L)]$. If $j = L$ they would always accept p_1.

By hypothesis L-sellers also mix. Let's consider each possible mixed strategy.

a. L-sellers mix between two prices that are both accepted with positive probability. It is easy to realize that L-sellers pool on H-sellers' prices (p_1, p_2). Otherwise, in equilibrium, they can only trade at price θ_L; however, it would be a profitable deviation to trade with a probability of one posting $p_2 \geq v_H > \theta_L$.

As H- and L-sellers mix on (p_1, p_2), the following indifference conditions hold:

$$\begin{aligned} V_H^\kappa(\sigma^*, \pi^*) = &\quad \gamma(p_1 - v_H) + (1-\gamma) V_H^{\kappa+1}(\sigma^*, \pi^*) - c = p_2 - v_H - c \\ V_L^\kappa(\sigma^*, \pi^*) = &\quad (1-\gamma)(p_1 - v_L) + \gamma V_L^{\kappa+1}(\sigma^*, \pi^*) - c = p_2 - v_L - c \end{aligned}$$

As $p_1 > p_2$ each equation implies $V_\lambda^{\kappa+1}(\sigma^*, \pi^*) < p_2 - v_\lambda$ for $\lambda \in \{H, L\}$. Using the first equation, I get $p_1 = \frac{1}{\gamma}[p_2 - (1-\gamma)(v_H + V_H^\kappa(\sigma, \pi)]$. By substituting this expression into the second equation and simplifying, I get:

$$p_2 = v_H + V_H^{\kappa+1}(\sigma, \pi) - \frac{\gamma^2}{2\gamma - 1}\left[(v_H - v_L) - \left(V_L^{\kappa+1}(\sigma, \pi) - V_H^{\kappa+1}(\sigma, \pi)\right)\right]$$

By Lemma 8.3 $V_L^{\kappa+1}(\sigma, \pi) - V_H^{\kappa+1}(\sigma, \pi) \leq v_H - v_L$. Hence $V_H^{\kappa+1}(\sigma, \pi) \geq p_2 - v_H$ contradicting the previous implication $V_H^{\kappa+1}(\sigma, \pi) < p_2 - v_H$.

b. L-sellers mix between a price accepted with positive probability and a price rejected with a probability of one. As in point a. it is straightforward to realize that L-sellers play either p_1 or p_2. Two cases are possible:

   1. L-sellers mix between p_2 and no trade. The indifference condition requires that:

   $$p_2 - c = V_L^{\kappa+1} - c$$

   Moreover, posting p_1 is not a profitable deviation, hence

   $$(1-\gamma)(p_1 - v_L) + \gamma V_L^{\kappa+1}(\sigma^*, \pi^*) - c \leq V_L^{\kappa+1}(\sigma^*, \pi^*) - c$$

   i.e. $p_1 - v_L \leq V_L^{\kappa+1}(\sigma^*, \pi^*)$, contradicting $p_1 > p_2$.

   2. L-sellers mix between p_1 and no trade. In turn it must hold that:

   $$(1-\gamma)(p_1 - v_L) + \gamma V_L^{\kappa+1}(\sigma^*, \pi^*) - c = V_L^{\kappa+1}(\sigma^*, \pi^*) - c$$

   i.e. $p_1 - v_L = V_L^{\kappa+1}(\sigma^*, \pi^*)$. In turn, using equation (18) and $p_1 > v_H$:

   $$V_L^{\kappa+1}(\sigma^*, \pi^*) - V_H^{\kappa+1}(\sigma^*, \pi^*) > v_H - v_L$$

   contradicting Lemma 8.3.

(ii) H-sellers mix between a price accepted with positive probability and no trade. Therefore one of the two following indifference conditions holds:

$$\begin{aligned} \gamma(p_1 - v_H) + (1-\gamma)V_H^{\kappa+1}(\sigma^*,\pi^*) - c &= V_H^{\kappa+1}(\sigma^*,\pi^*) - c \\ p_2 - v_H - c &= V_H^{\kappa+1}(\sigma^*,\pi^*) - c \end{aligned}$$

By rearranging this, I get $p_1 - v_H = V_H^{\kappa+1}(\sigma^*,\pi^*)$ and $p_2 - v_H = V_H^{\kappa+1}(\sigma^*,\pi^*)$, respectively. L-sellers' mixed strategy may either (a) offer two prices accepted with positive probability; or (b) offer a price accepted with positive probability or postpone trade.

a. Assume L-sellers mix between $(p_{1,L}, p_{2,L})$. Without loss of generality assume $p_{1,L}$ is accepted only after a signal $j \in \{H, L\}$ and $p_{2,L}$ is always accepted. Clearly, it is not optimal to post both prices different from that of H-sellers. Indeed, L-sellers would prefer to deviate and pool on H-sellers' price because it is accepted with the same probability of one between $p_{1,L}$ and $p_{2,L}$, but it provides a higher payoff than $\min\{p_1, p_2\} \geq v_H$. Therefore, L-sellers strategy can only mix between θ_L and the price posted by H-sellers. If H-sellers post p_2, it is never a best response to mix between θ_L and $p_2 \geq v_H > \theta_L$ as the latter is accepted with a probability of one. If H-sellers post p_1, L-sellers' indifference condition is

$$\mathbb{P}_L(j)(p_1 - v_L) + (1-\mathbb{P}_L(j))V_L^{\kappa+1}(\sigma^*,\pi^*) - c = \theta_L - v_L - c \tag{19}$$

Since $p_1 \geq v_H$, equation (19) implies $V_L^{\kappa+1}(\sigma^*,\pi^*) \leq \theta_L - v_L - \frac{\mathbb{P}_L(j)}{1-\mathbb{P}_L(j)}(v_H - \theta_L)$. L-sellers' continuation value always satisfies $V_L^{\kappa+1}(\sigma^*,\pi^*) \geq \theta_L - v_L - c$, because they can always trade at price θ_L. Therefore, for c sufficiently small both inequalities cannot contemporaneously hold.

b. Assume L-sellers mix between a price accepted with positive probability and no trade. As in point a., L-sellers pool on H-sellers' posted price. Hence, one of these two equations holds:

$$\begin{aligned} V_L^{\kappa}(\sigma^*,\pi^*) = (1-\gamma)(p_1 - v_L) + \gamma V_L^{\kappa+1}(\sigma^*,\pi^*) - c &= V_L^{\kappa+1}(\sigma^*,\pi^*) - c \\ V_L^{\kappa}(\sigma^*,\pi^*) = p_2 - v_L - c &= V_L^{\kappa+1}(\sigma^*,\pi^*) - c \end{aligned} \tag{20}$$

i.e. $p_1 - v_L = V_L^{\kappa+1}(\sigma^*,\pi^*)$ or $p_2 - v_L = V_L^{\kappa+1}(\sigma^*,\pi^*)$, respectively. H-sellers indifference conditions imply $p_1 - v_H = V_H^{\kappa+1}(\sigma^*,\pi^*)$ or $p_2 - v_H = V_H^{\kappa+1}(\sigma^*,\pi^*)$. Hence, $V_L^{\kappa+1}(\sigma^*,\pi^*) - V_H^{\kappa+1}(\sigma^*,\pi^*) = v_H - v_L$, and $V_L^{\kappa}(\sigma^*,\pi^*) - V_H^{\kappa}(\sigma^*,\pi^*) = v_H - v_L$. By Lemma 8.3, the only admissible strategies for sellers in S^j, $j > \kappa$, which satisfy this condition on continuation values are: (i) H- and L-sellers mix between a commonly posted price p^j and no trade; or (ii) H- and L-sellers in S^j post the same price p^j which is accepted with a probability of one. If sellers play strategy (i), $V_\lambda^j(\sigma^*,\pi^*)$ and p^j have to increase by c from j to $j+1$; see equation (20). Under TMN this strategy profile is not an equilibrium because sellers from different cohorts must have the same continuation value. Let's restrict attention to the TMO case. Observe that strategy (i) cannot be played for every $j \geq \kappa$, as it would eventually require to go above the upper bound $V_{\lambda}^{j}(\sigma^*, \pi^*) = \theta_H - v_{\lambda} - c$. Therefore, sellers eventually have to play strategy (ii); let $j^* \geq \kappa$ denote this future period. In turn, this requires to have $q^{j^*} > q_c^{IP}$. Clearly, the price posted under strategy (i) for $j < j^*$ has to be accepted only if $\xi = H$.

By assumption, $q^0 < q_c^S < q_c^{IP}$. In order to increase H-sellers' share from q^0 to $q^{j^*} \geq q_c^{IP}$, sellers should play a behavioural strategy of type (i) for $j^* - 1$ initial periods in order to increase q^j from q^0 to at least q_c^{IP}; then all sellers in S^{j^*} should trade with a probability of one at p^{j^*}. Let α^j and β^j denote the probability that H- and L-sellers, respectively, play p^j for $j < j^*$; the complementary probability denotes the probability of posting a price rejected with a probability of one. In equilibrium, for $j < j^*$ buyers accept p^j after $\xi = H$, i.e.

$$\frac{q^j \alpha^j \gamma \theta_H + (1-q^j)\beta^j(1-\gamma)\theta_L}{q^j \alpha^j \gamma + (1-q^j)\beta^j(1-\gamma)} \geq p^j \;\rightarrow\; \frac{\beta^j}{\alpha^j} \leq \frac{q^j}{1-q^j}\frac{\gamma}{1-\gamma}\frac{\theta_H - p^j}{p^j - \theta_L} \tag{21}$$

The share q^j increases only if

$$\frac{q^{j+1}}{1-q^{j+1}} = \frac{q^j[(1-\alpha^j)+\alpha^j(1-\gamma)]}{(1-q^j)[(1-\beta^j)+\beta^j\gamma]} > \frac{q^j}{1-q^j} \quad \rightarrow \quad \frac{\beta^j}{\alpha^j} > \frac{\gamma}{1-\gamma} \tag{22}$$

Equations (22) and (21) together imply:

$$q^j\theta_H + (1-q^j)\theta_L > p^j \geq v_H$$

This is possible only if $q^0 > \frac{v_H - \theta_L}{\theta_H - \theta_L}$. However, by assumption $q^0 < q_c^S = \frac{v_H - \theta_L + c}{\theta_H - \theta_L}$ and for c small enough the two inequalities cannot both hold.

Step 2. There is no equilibrium path in which L-sellers in $S_L^{\kappa} \neq \emptyset$ only post prices rejected with a probability of one, and H-sellers in $S_H^{\kappa} \neq \emptyset$ trade with positive probability.[45]

H-sellers in S_H^{κ} post a price p accepted with positive probability, so $p \leq \mathbb{E}_{\pi_B}[\theta | \mathscr{I}(p, \cdot, \xi)] = \theta_H$ for all $\xi \in \{H, L\}$, because, by hypothesis, L-sellers do not pool on this price. If $p < \theta_H$ all buyers accept with a probability of one. H-sellers' possible behavioural strategies are:

a. H-sellers post $p \leq \theta_H$ with a probability of one and buyers always accept. Therefore, it must hold $p - v_L - c \leq V_L^{\kappa+1} - c$ and $p - v_H - c \geq V_H^{\kappa+1} - c$. Both inequalities imply $V_L^{\kappa+1} - V_H^{\kappa+1} \geq v_H - v_L$. By Lemma 8.3 the inequality cannot hold strictly; moreover, the argument in Step 1(ii) b. excludes the equality case under TMO (when $q^0 < q_c^N$) and under TMN (always).

b. H-sellers post θ_H with a probability of one and buyers accept only for $\xi = j$, $j \in \{H, L\}$. L-sellers

[45] Notice that it is a more restrictive statement than Lemma 8.2.

prefer to postpone trade only if

$$V_L^{\kappa+1}(\sigma^*,\pi^*) \geq \mathbb{P}_L(j)(\theta_H - v_L) + (1-\mathbb{P}_L(j))\,V_L^{\kappa+1}(\sigma^*,\pi^*)$$

i.e. $V_L^{\kappa+1}(\sigma^*,\pi^*) \geq \theta_H - v_L$. However, in every equilibrium an upper bound on the expected payoff is $V_L^{\kappa+1} \leq \theta_H - v_L - c$. A similar argument applies if L-sellers mix between two prices both accepted with positive probability.

c. H-sellers mix between a price p accepted with positive probability and no trade. If p is always accepted then H-sellers' indifference requires

$$p - v_H = V_H^{\kappa+1}(\sigma^*,\pi^*)$$

L-sellers do not trade at p if $V_L^{\kappa+1}(\sigma^*,\pi^*) \geq p - v_L$. Then,

$$V_L^{\kappa+1}(\sigma^*,\pi^*) - V_H^{\kappa+1}(\sigma^*,\pi^*) \geq v_H - v_L$$

If the inequality is strict then it is inconsistent with Lemma 8.3. If it holds with equality, see Step 1(ii) b.

If H-sellers mix between θ_H and no trade then

$$\mathbb{P}_H(j)(\theta_H - v_H) + (1-\mathbb{P}_H(j))\,V_H^{\kappa+1}(\sigma^*,\pi^*) = V_H^{\kappa+1}(\sigma^*,\pi^*)$$

i.e. $V_H^{\kappa+1}(\sigma^*,\pi^*) = \theta_H - v_H$. But in every equilibrium an upper bound on expected payoffs is $V_H^{\kappa+1}(\sigma^*,\pi^*) \leq \theta_H - v_H - c$.

Step 3. There is no equilibrium path in which sellers in $S_H^\kappa \neq \emptyset$ and $S_L^\kappa \neq \emptyset$ play semi-separating behavioural strategies, and all posted prices are accepted with positive probability.

Suppose, on the contrary, that such a behavioural strategy is played in equilibrium. Let's consider the two different cases:

(i) L-sellers mix and H-sellers play a pure strategy. The only relevant case to consider is when L-sellers mix between θ_L (accepted with a probability of one) and p (accepted only if $\xi = H$), and H-sellers only post p. L-sellers mix between the two prices only if:

$$V_L^\kappa(\sigma^*,\pi^*) = (1-\gamma)(p - v_L) + \gamma V_L^{\kappa+1}(\sigma^*,\pi^*) - c = \theta_L - v_L - c$$

L-sellers can always trade at θ_L so $V_L^{\kappa+1}(\sigma^*,\pi^*) \geq \theta_L - v_L - c$. Hence,

$$\theta_L - v_L - c \geq (1-\gamma)(p - v_L) + \gamma(\theta_L - v_L - c)$$

i.e. $\theta_L \geq p - \frac{c}{1-\gamma}$. For c small enough $p < v_H$ as $v_H > \theta_L$. In equilibrium H-sellers would not post this price because lower than their reservation value.

(ii) H-sellers mix and L-sellers play a pure strategy. Sellers in S^κ play this behavioural strategy only if H-sellers mix between θ_H (played with probability α and accepted only if $\xi = H$) and a price p^κ (always accepted). L-sellers post p^κ with a probability of one.

I provide a separate proof for each assumption on time on market observability.

a. Under TMO, buyers accept p^κ irrespective of signal ξ only if $\mathbb{E}_{\pi_B}[\theta|(p^\kappa,\kappa,L)] \geq p^\kappa$. In turn, a necessary condition is $q^\kappa > q_c^{IP}$ (for $\alpha = 0$, it is higher for $\alpha > 0$).[46] As $q^0 < q_0^{IP} < q_c^{IP}$ it is sufficient to prove that $q^\kappa < q^0$ for every $\kappa \geq 1$. Consider $j \in \mathbb{N}_0$ such that $q^j < q_0^{IP}$. Therefore, in period j the proposed behavioural strategy cannot be played. By steps 1, 2 , 3 (i) and Proposition 4.4: (i) either both H- and L-sellers do not trade with a probability of one; or (ii) $\sigma_H^j(p) = \sigma_L^j(p) = 1$, $\sigma_B(A|\mathscr{I}_B(p,\kappa,H)) = 1$ and $\sigma_B(A|\mathscr{I}_B(p,\kappa,L)) = 0$. In case (i) $q^{j+1} = q^j$. In case (ii) buyers only accept if $\xi = H$, so H- and L-sellers trade with probability γ and $1-\gamma$, respectively. By the law of large numbers, a higher share of H-sellers trades and exits the market, so $q^{j+1} < q^j$. Let $j = 0$ concludes the argument.

b. Under TMN, buyers do not distinguish sellers beloning to different cohorts S^κ. In equilibrium, two cohorts of sellers $S_\lambda^{\kappa'}$ and $S_\lambda^{\kappa''}$, $\kappa' \neq \kappa''$ cannot play strategy profiles leading to different expected payoffs. If this were the case, there would be a profitable deviation for one cohort of sellers as buyers cannot observed previous prices. For every $\kappa \in \mathbb{N}_0$ such that $S_\lambda^\kappa \neq \emptyset$, $V_\lambda^\kappa(\sigma^*,\pi^*) = V_\lambda(\sigma^*,\pi^*)$ must hold, and H-sellers' indifference condition requires:

$$V_H(\sigma^*,\pi^*) = \gamma(\theta_H - v_H) + (1-\gamma)V_H(\sigma^*,\pi^*) - c = p^\kappa - v_H - c$$

hence $p^\kappa = \bar{p} = \theta_H - \frac{1-\gamma}{\gamma}c$.

Let $\bar{q} = \sum\limits_{\kappa\in\mathbb{N}_0} \frac{S^\kappa}{S} q^\kappa$. Buyers accept $\bar{p}$ only if

$$\frac{\bar{q}(1-\alpha)(1-\gamma)\theta_H + (1-\bar{q})\gamma\theta_L}{\bar{q}(1-\alpha)(1-\gamma) + (1-\bar{q})\gamma} \geq \theta_H - \frac{1-\gamma}{\gamma}c$$

hence

$$\frac{\bar{q}}{1-\bar{q}} \geq \frac{\gamma}{(1-\gamma)(1-\alpha)} \frac{\theta_H - \theta_L - c}{c} \tag{23}$$

By Step 1, 2, 3(i), the only admissible behavioural strategy for sellers in cohorts S^j, $j \neq \kappa$, are: (i) both H- and L-sellers do not trade with a probability of one; (ii) H- and L-sellers post the same price p_H, say, and buyers accept only if $\xi = H$; (iii) H- and L-sellers post the

[46] See section 4 for a characterization of q_c^{IP}. The subscript c refers to the associated search cost c.

same price p_L, say, and buyers accept with a probability of one; (iv) H-sellers mix between two prices and L-sellers post one price with a probability of one. Since all cohorts receive the same expected payoff, it must be $p_H = \theta_H$ and $p_L = \bar{p} = \theta_H - \frac{1-\gamma}{\gamma}c$. Importantly, the behavioural strategies (i), (ii) and (iii) imply that H-sellers in S_H^j trade at least with the same probability as L-sellers in S_L^j. As a result, $\bar{q}$ cannot be higher than q^0 if sellers only play strategies (i), (ii) and (iii). Therefore, if it is not possible to satisfy equation (23) in an equilibrium in which, for every $\kappa \in \mathbb{N}_0$, H-sellers in S_H^κ mix between θ_H and $\bar{p}$, and L-sellers in S_L^κ play $\bar{p}$ with a probability of one, then it is never possible. In the proposed equilibrium, stationarity requires exit and entry flows for each type of sellers to be equal. I denote with $\bar{S}$ the equilibrium mass of sellers:

$$\begin{cases} q^0 = \bar{S}\bar{q}[\alpha\gamma + (1-\alpha)] \\ (1-q^0) = \bar{S}(1-\bar{q}) \end{cases} \Rightarrow \begin{cases} \bar{S} = \frac{1-q^0}{(1-\bar{q})} \\ \frac{\bar{q}}{1-\bar{q}} = \frac{1}{\alpha\gamma-(1-\alpha)}\frac{q^0}{1-q^0} \end{cases}$$

Therefore, this equilibrium only exists if equation (23) is satisfied, i.e.

$$\frac{1}{\alpha\gamma + (1-\alpha)}\frac{q^0}{1-q^0} \geq \frac{\gamma}{(1-\gamma)(1-\alpha)}\frac{\theta_H - \theta_L - c}{c}$$

This expression is more likely to hold for α close to zero. Therefore, a necessary condition is:

$$\frac{q^0}{1-q^0} \geq \frac{\gamma^2}{1-\gamma}\frac{\theta_H - \theta_L - c}{c} \tag{24}$$

For $q^0 = q_c^{IP} = \frac{\gamma}{1-\gamma}\frac{v_H + c - \theta_L}{\theta_H - v_H - c}$[47] equation (24) becomes

$$\frac{v_H + c - \theta_L}{\theta_H - v_H - c} \geq \gamma\frac{\theta_H - \theta_L - c}{c}$$

However, for c sufficiently small this inequality cannot hold.

Step 4. There is no equilibrium path in which H- and L-sellers in $S_H^\kappa \neq \emptyset$ and $S_L^\kappa \neq \emptyset$ post a price accepted with a probability of one.

I provide a separate proof for each assumption on time on market observability.

a. Under TMO the argument is analogous to the one in Step 3 part (ii) point a.

b. Under TMN, an analogous argument to the one in Step 3 part (ii) point b establishes that all cohorts get the same continuation value. By Step 1, 2, 3 the only admissible behavioural strategy profiles for sellers in cohorts S^j, $j \neq \kappa$, are: (i) both H- and L-sellers do not trade with a probability of one; or (ii) H- and L-sellers post the same price p_H and buyers accept only if $\xi = H$; or (iii) H- and L-sellers post the same price p_L and buyers accept with a probability of one.

[47] See section 4 for a defnition.

By definition of q_c^{IP}, the behavioural strategy (iii) is possible only if $\bar{q} = \sum_{\kappa \in \mathbb{N}_0} \frac{S^\kappa}{S} q^\kappa \geq q_c^{IP}$, i.e. there must be at least one cohort $S^{\tilde{\kappa}}$, $\tilde{\kappa} \neq \kappa$, such that L-sellers trade with a higher probability than H-sellers. However, all admissible behavioural strategies (i), (ii) and (iii) imply that H-sellers' probability to trade is at least equal to that of L-sellers.

Step 5. If $S_H^\kappa \neq \emptyset$ then all sellers in S^κ post the same price. If time on market is not observable all sellers post the same price.

By steps 1, 2, 3, 4 and Lemma 8.2 the only admissible behavioural strategy profiles are: (i) H- and L-sellers only post prices rejected with a probability of one; (ii) H- and L-sellers in S^κ post the same price p^κ and buyers accept only if $\xi = H$. Under TMN, buyers do not observe the sellers' cohort, and sellers cannot post different prices accepted with the same probability; hence, $p^\kappa = \bar{p}$, for all $\kappa \in \mathbb{N}_0$. Postponing trade does not change the future trade price but it increases search costs. As a result, they find it strictly convenient to post $\bar{p}$ until they trade. ■

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