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by Stefano Neri and Alessandro Notarpietro

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THE MACROECONOMIC EFFECTS OF LOW
AND FALLING INFLATION AT THE ZERO LOWER BOUND

by Stefano Neri* and Alessandro Notarpietro*

Abstract

This paper assesses the macroeconomic consequences of a prolonged period of low and falling inflation when monetary policy is constrained by the zero lower bound (ZLB) on short-term nominal interest rates, the private sector is indebted in nominal terms (debt deflation mechanism) and nominal wages are downward rigid. Cost-push shocks that in normal circumstances would reduce inflation and stimulate output have contractionary effects on economic activity, once the ZLB interacts with the debt deflation mechanism. The contractionary effects are larger and more persistent when nominal wages cannot be reduced and when the private sector is highly indebted.

JEL Classification: E21, E31, E37, E52.
Keywords: DSGE models, zero lower bound, debt-deflation mechanism, downward nominal wage rigidities.

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* Bank of Italy, Directorate General for Economics, Statistics and Research.
“An unexpected period of low inflation and low nominal income results in a higher actual and expected future real debt burden. Unless compensated for by expectations of higher future income, firms may reduce investment and households consumption. Banks may in turn respond to this situation with stricter credit standards, which reinforces disinflationary pressure and hence worsens debt burdens. This is fertile ground for a pernicious negative spiral, which then also affects expectations.”

Mario Draghi, President of the European Central Bank, “Monetary policy in a prolonged period of low inflation”, speech at the ECB Forum on Central Banking, Sintra, 26 May 2014.

1 Introduction

This paper analyses the effects of a negative shock to inflation when the short-term nominal interest rates are at their zero lower bound (ZLB) in an economy characterized by the presence of a non-negligible amount of private-sector debt and nominal wage rigidities. All these three elements characterized the euro-area economy in the period 2014-2015.

First, in September 2014 the European Central Bank (ECB) reduced the rate on the main refinancing operations to 0.05% – the effective ZLB – to counteract the risks related to a protracted period of too low inflation. Annual inflation turned negative in December 2014 (-0.2% per cent) in the euro area (Figure 1). With short-term nominal interest rates at the ZLB, real interest rates increased substantially in 2014 (Figure 2). Second, the overall amount of private-sector debt as a fraction of euro-area GDP has remained at historically high levels since 2009 (Figure 3). Third, evidence of downward nominal wage rigidity (DNWR) in several European countries is documented in the report of the Wage Dynamics Network of the Eurosystem (European Central Bank, 2009). Schmitt-Grohé and Uribe (2013) also report that nominal wages remained largely unchanged in some peripheral euro-area countries since 2008, despite an unprecedented contraction in aggregate

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2In 2014 low inflation rates characterized most developed economies; only 14 OECD countries out of 34 had annual headline inflation rates above 1%.
demand.

Our analysis is based on a New Keynesian dynamic general equilibrium model with credit frictions in the form of collateral constraints, close to Gerali et al. (2010) and Iacoviello (2005). A fraction of households and entrepreneurs act as net borrowers, facing borrowing constraints that link the maximum amount of funds they can obtain to the value of their available collateral. Borrowing takes the form of one-period nominal debt contracts, so that variations in the inflation rate modify the real value of debt. The economy features nominal and real rigidities. In order to study the effects of low and falling inflation, we simulate a negative cost-push shock when the short-term interest rate is stuck at the ZLB.\(^3\)

This paper expands and complements the analysis in Neri and Notarpietro (2014). Specifically, the importance of downward nominal wage rigidities in the labor market is explicitly taken into account and its effects are thoroughly studied. In addition, we analyze the role of movements in asset prices in determining endogenous variations in the borrowing limit and, as a result, in the transmission mechanism of shocks. The latter issue is studied by comparing the baseline model specification to an alternative one in which the borrowing limit is exogenous and invariant to changes in any macroeconomic variable.

The main results can be summarised as follows.

First, a cost-push shock that in normal circumstances (i.e. in the absence of the ZLB) would lower inflation and stimulate output is shown to have contractionary effects on economic activity, once the interplay of the ZLB and the debt deflation mechanism is considered. On the one hand, with the short-term nominal interest rate at the ZLB, a fall in inflation immediately translates into a higher real interest rate, which, through the intertemporal substitution effect, reduces aggregate demand. On the other hand, lower inflation also interferes with private sector debt deleveraging. Since debt contracts are signed in nominal terms, a fall in inflation increases the real debt burden for borrowers in terms of both principal repayment and interest payments, and induces the private sector to reduce debt, leading to a fall in asset prices. In turn, private sector net worth collapses, leading to a decrease in output and a further fall in inflation – which may quickly turn into deflation – and to a contraction in lending, thus generating a downward spiral. This mechanism is the well-known debt deflation channel (Fisher, 1932, 1933).

Second, the negative effects of the shock are larger and more persistent with

\(^3\)All simulations are run under the assumption of perfect foresight. Therefore, households and firms fully anticipate the dynamics of all variables after the initial period, in which the shock occurs.
downward nominal wage rigidities. Indeed, low inflation hampers the adjustment in the labor market, because of the downward rigidity of nominal wages, as suggested by Tobin (1972). As the shock becomes contractionary at the ZLB, the large cost implied by reducing nominal wages precludes a sufficient initial adjustment in the labor market, leading to persistent losses in output.\footnote{Focusing on real, as opposed to nominal wage rigidities, Nucci and Riggi (2015) show that, once endogenous labor force participation is allowed for in a New Keynesian model with search and matching frictions, the degree of real wage rigidities exerts little influence on inflation dynamics, different from the implications of a standard New Keynesian model. The authors also find that the higher the degree of real wage rigidity the larger is the fall in output following both demand and supply shocks, regardless of whether labor participation is endogenous or not. In this paper we focus instead on the role of nominal wage rigidities, so that the overall adjustment in real wages depends on the relative stickiness of prices and wages and does not reflect any imperfection or friction in the labor market. As such, in response to shocks to inflation the adjustment in the labor market mainly reflects nominal rigidities and their interplay with other model features, such as private debt and the ZLB.}

Finally, the amount of private debt is crucial in determining the size of the negative effects and the importance of the debt deflation mechanism in propagating the shock to inflation.

The paper relates to several strands of the macroeconomic literature that have studied the implications of the ZLB for macroeconomic stabilization policies and the role of the debt-deflation channel. Eggertsson (2010, 2012) shows that negative supply shocks can be expansionary in standard new Keynesian models when monetary policy is constrained by the ZLB, to the extent that they increase inflation expectations and hence lower expected real interest rates. Wieland (2014) empirically tests the prediction that temporary negative supply shocks are expansionary in a standard New Keynesian model at the ZLB, insofar as they stimulate consumption by raising inflation expectations, and finds that oil supply shocks and the Great Japanese Earthquake are contractionary despite satisfying these conditions. Iacoviello (2005) develops and estimates a small-scale monetary business cycle model with nominal loans and collateral constraints tied to housing values. Such analysis abstracts from the presence of the ZLB constraint on monetary policy. The role of nominal debt in modifying the response of output to inflation surprises is analysed in Eggertsson and Krugman (2012), which show, using a model with borrowing-constrained agents of a similar type to the one developed in Iacoviello (2005), that a rise in expected inflation can help the economy deal with a deleveraging shock, by lowering the real rate at the ZLB.

Our model features borrowing constraints on households and firms and, crucially, distinguishes between different mechanisms of wage setting. In the latter
respect, our contribution also relates to the strand of the macroeconomic literature assessing the role of wage rigidities and alternative wage setting mechanisms for the propagation of shocks. Abbritti and Fahr (2013) set up a new-Keynesian model with downward wage rigidities, which are introduced by means of an asymmetric adjustment cost function for wages. They show that such feature, besides capturing in a stylized way an empirical feature observed in several euro-area economies, can explain the observed evidence on wage adjustment along the business cycle and help understand, for instance, the differences in the effects on inflation of negative and positive monetary policy shocks. More recently, Schmitt-Grohé and Uribe (2013) analyze the role of downward rigid nominal wages in a model of the euro area, motivated by the empirical evidence that nominal wages have remained largely unchanged in peripheral euro-area countries since the outbreak of the global financial crisis in 2008. They abstract from the presence of nominal private debt and do not consider the effects of a negative shock to inflation at the ZLB.

Some of the consequences of low and falling inflation rates are not included in our analysis. If inflation remains low for too long, inflation expectations may de-anchor from the central bank’s target. Indeed, the risk of a de-anchoring in the euro area increased significantly in 2014. Five year five-year ahead inflation expectations based on inflation swap contracts reached 1.6 per cent in mid-December, down from 2.2 a year earlier.\footnote{Cecchetti et al. (2015) find that the sensitivity of five year five-year ahead swap-based inflation expectations to one-year one-year ahead expectations increased to an historical high in December 2014, after having remained close to zero in the previous years, consistently with well-anchored long-term expectations.} The debt-deflation channel and the risk of de-anchoring of inflation expectations may interact and result in a potentially dangerous spiral, as the quotation at the beginning of this section shows. Tackling this issue would require a departure from a rational expectations environment and is beyond the scope of our analysis.\footnote{Busetti et al. (2015) use a simple New Keynesian model to study the consequences of a prolonged period of low inflation. The authors find that low and falling inflation affects expectations in two ways: first, it reduces the perceived inflation target; second, it increases the share of agents which attach no role to the inflation target (model selection effect) in the expectation formation process.}

We also do not address the issue of self-reinforcing deflationary spirals (Eggertsson, 2012).

Finally, the analysis in this paper does not consider unconventional monetary policy measures. Burlon et al. (2015) analyse the recently-launched asset purchase programme of the European Central Bank in a richer setup. Fiscal policy is also ignored here, although it may be used, in principle, as a stabilization tool. We do not consider such option since in the euro area fiscal rules constrain the room for...
manoeuvre for national fiscal authorities.

The reminder of the paper is organized as follows. Section 2 briefly presents the model. Section 3 discusses the results of the simulations. Section 4 presents some sensitivity analysis. Section 5 concludes and suggests some possibilities for future research.

2 The model

Our analysis is based on a simple model with credit frictions, very close to the one in Iacoviello (2005). A close reference to our setup is the model in Gerali et al. (2010), which however provides a much richer description of the economy, in that it includes a banking sector. We deem the latter not essential for the purpose of understanding the main macroeconomic effects of shocks to inflation at the ZLB and thus we abstract from it.

Three types of agents populate the economy: patient households, impatient households and entrepreneurs. Impatient households and entrepreneurs act as net borrowers, facing borrowing constraints that link the maximum amount of funds they can obtain from patient households (the savers) to the value of their collateral (housing and the capital stock, respectively). Borrowers obtain their funds from the savers. Debt is issued in the form of one-period nominal contracts. Unexpected changes in inflation thus cause ex-post fluctuations in the real value of debt and in the real interest rate paid by borrowers. The economy features nominal and real rigidities. Nominal rigidities are introduced in the form of price and wage stickiness and indexation to inflation. Real rigidities include habit formation in consumption and adjustment costs on investment. The short-term rate, which represents the cost of funds to the borrowers, is set by the central bank according to a Taylor-type rule, subject to the ZLB constraint.

In this Section we describe the ingredients of the model that are important for our analysis, referring the reader to the Appendix for a more detailed description.

2.1 Households

Households consume and work. There are two types of households that differ in terms of their degree of impatience: the discount factor of patient households is higher than that of impatient households. This heterogeneity in preferences gives rise to positive financial flows in equilibrium, as patient households save and the impatient borrow. Housing, which is in fixed supply, is traded between
the two households. Households are monopolistic suppliers of homogeneous labor services that are sold to perfectly competitive labor packers selling the labor to entrepreneurs. Nominal wages are set by the unions, to which households belong.

The patient household maximizes her lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^P_t \left[ (1 - a^P) \log(c^P_t(i) - a^P c^P_{t-1}) + \log h^P_t(i) - \frac{t^P_t(i)^{1+\phi}}{1 + \phi} \right]$$

subject to the (real) budget constraint:

$$c^P_t(i) + q^h_t \Delta h^P_t(i) + b_t(i) \leq w^P_t l^P_t(i) + (1 + R_{t-1}) b_{t-1}(i)/\pi_t + t^P_t(i)$$

where $c^P_t$ is consumption, $q^h_t$ denotes the price of housing in terms of consumption goods, $h^P_t$ is housing, $b_t(i)$ the total amount of lending to impatient households and entrepreneurs (more below), $R_t$ their remuneration, $w^P_t$ the real wage, $\pi_t$ the inflation rate and $t^P_t$ transfers that include a labor union membership net fee and dividends from monopolistic competitive firms. The parameters $a^p$ and $\phi$ measure, respectively, the degree of (external) habit formation in consumption and the inverse of the Frisch elasticity.

The impatient household maximizes her lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^I_t \left[ (1 - a^I) \log(c^I_t(i) - a^I c^I_{t-1}) + \log h^I_t(i) - \frac{t^I_t(i)^{1+\phi}}{1 + \phi} \right]$$

with $\beta^I < \beta^P$, subject the budget constraint:

$$c^I_t(i) + q^h_t \Delta h^I_t(i) + (1 + R_{t-1}) b^I_{t-1}(i)/\pi_t \leq w^I_t l^I_t(i) + b^I_t(i) + t^I_t(i)$$

where $b^I_t(i)$ is the amount of borrowing from the patient households and $t^I_t(i)$ is a transfer from the labor unions, and the borrowing constraint:

$$(1 + R_t) b^I_t(i) \leq m^I E_t \left[ q^h_{t+1} h^I_t(i) \pi_{t+1} \right]$$

where $m^I$ is the loan-to-value (LTV) ratio and the term in brackets represents the value of the housing stock than can be pledged as collateral for the loan (see Iacoviello 2005). The value of the collateral must be such that it can cover a fraction $m^I$ of the amount borrowed and the interest rate payment. As in Kiyotaki and Moore (1997) and Iacoviello (2005), households do not default in equilibrium.
2.2 Entrepreneurs

Entrepreneur \((i)\)'s utility depends only on consumption:

\[
E_0 \sum_{t=0}^{\infty} \beta_E^t \log(c_t^E(i) - a^E c_{t-1}^E)
\]  \(6\)

where \(a^E\) is the degree of habit formation and \(\beta_E\) is the discount factor (with \(\beta_E < \beta_P\)). Entrepreneur \((i)\) maximizes her lifetime utility under the budget constraint:

\[
c_t^E(i) + w_t P_t^E P(i) + w_t I_t^E I(i) + \left(\frac{1 + R_t}{\pi_t} b_{t-1}^E(i) + q_t^k k_t^E(i) + \phi(u_t(i)) k_{t-1}^E(i)\right) \leq \ldots \tag{7}
\]

\[
\ldots \leq \frac{y_t^E(i)}{x_t} + b_t^E(i) + q_t^k (1 - \delta) k_{t-1}^E(i)
\]

where \(q_t^k\) is the price of capital in terms of consumption, \(\phi(u_t(i)) k_{t-1}^E(i)\) is the real cost of setting a level \(u_t\) of utilization rate, \(\delta\) is the depreciation rate of capital \(k_t^E\) and \(\frac{P^W}{P_t} = 1/x\) is the relative price of the wholesale good \(y^E\) produced using the technology:

\[
y_t^E(i) = [k_{t-1}^E(i) u_t(i)]^\alpha I_t^E(i)^{1-\alpha} \tag{8}
\]

where aggregate labor is a combination of inputs from patient and impatient households according to \(l_t^E = (l_t^E P (l_t^E I))^{\mu} (l_t^E I)^{1-\mu}\), with parameter \(\mu\) calibrated to measure the labor income share of unconstrained households as in Iacoviello and Neri (2010).\footnote{The functional form for the adjustment cost for capacity utilization is \(\phi(u_t) = \xi_1 (u_t - 1) + 0.5 \cdot \xi_2 (u_t - 1)^2\).}

Entrepreneurs are also subject to the borrowing constraint:

\[
(1 + R_t) b_t^E(i) \leq m^E E_t \left[ (q_t^k \pi_{t+1} (1 - \delta) k_t^E(i)) \right] \tag{9}
\]

where \(m^E\) is the loan-to-value ratio and the term in brackets represents the value of collateral, given by the market value of installed physical capital. In the saving/debt market the amount of borrowing by the impatient households and the entrepreneurs is equal to the amount of savings by the patient households. Also entrepreneurs do not default.
2.3 Final goods producers

The retail goods market is assumed to be monopolistically competitive as in Bernanke, Gertler, and Gilchrist (1999). Retailers’ prices are sticky and are indexed to a combination of past and steady-state inflation, with relative weights parameterized by $\iota_p$; specifically, retailers face a quadratic price adjustment cost parameterized by $\kappa_p$. Retailers solve the following problem

$$\max_{\{P_t(j)\}} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^P \left[ P_t(j) y_t(j) - P_t^W y_t(j) - \frac{\kappa_p}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - \pi_t^{i_p} \pi_{t-1}^{1-i_p} \right)^2 P_t y_t \right]$$

subject to a demand coming from consumers maximization of a consumption aggregator

$$y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon_t^Y} y_t$$

where the elasticity of substitution between individual goods, $\varepsilon_t^Y$ is time-varying, reflecting the presence of cost-push shocks. In a symmetric equilibrium, the first-order conditions give rise to a non-linear Phillips curve:

$$1 - \varepsilon_t^Y + \frac{\varepsilon_t^Y}{x_t} - \kappa_P (\pi_t - \pi_{t-1}^{i_p} \pi_1^{1-i_p}) \pi_t + \beta P E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \kappa_P (\pi_{t+1} - \pi_t^{i_p} \pi_{t+1}^{1-i_p}) \pi_{t+1} y_{t+1} y_t \right] = 0$$

(12)

2.4 The labor market

We consider two alternative modelling of nominal wage rigidities in the labor market.\(^8\)

In the first case we adopt the standard framework for nominal wage setting in which changes in nominal workers compensation are subject to quadratic adjustment costs. Therefore, changes to wages are possible, but costly; moreover, these costs are the same, for given change, regardless of the direction. In the second case, we assume, following Abbritti and Fahr (2013), that the costs for adjusting nominal wages are convex and asymmetric, implying larger costs for cutting than for increasing them by the same size.\(^9\) This specification of the cost function en-

\(^8\)A complete description of the labor market setup is contained in the Appendix.

\(^9\)Benigno and Ricci (2011) consider the extreme case in which nominal wages can never be cut and show that such feature has implications for the slope of the long-run Phillips curve. Schmitt-Grohé and Uribe (2013) adopt the same assumption in a model for the euro area.
compasses the quadratic one. Abbritti and Fahr (2013) find that the asymmetry, which is more consistent with observed nominal wage adjustments, explains the asymmetry in the effects of positive and negative monetary policy shocks on wages and inflation.

The framework of the labor market is the following. Workers provide differentiated labor types, sold by unions to perfectly competitive labor packers who assemble them in a constant-elasticity-of-substitution (CES) aggregator with elasticity of substitution \( \varepsilon_l \) and sell the homogeneous labor to entrepreneurs. For each labor type \( m \), there are two unions, one for patient households and one for impatient households (indexed by \( s \)). Each union \((s, m)\) sets nominal wages \( W^s_t(m) \) for its members by maximizing their utility subject to a downward sloping demand and to quadratic adjustment costs (parameterized by \( \kappa_w \)). The adjustment cost is given by:

\[
AC^w_t = \frac{\kappa_w - 1}{2} \left( \frac{W^s_t(m)}{W^s_{t-1}(m)} - 1 \right)^2 \frac{W^s_t}{P_t} \tag{13}
\]

so that the cost is proportional to aggregate real wages. In the case of downward nominal rigidities, the adjustment cost function is:

\[
AC^w_t = \frac{\kappa_w - 1}{2} \left[ \left( \frac{W^s_t(m)}{W^s_{t-1}(m)} - 1 \right)^2 + \frac{1}{\psi^2} \left( e^{-\psi(\pi^w_t - 1)} + \psi \left( \pi^w_t - 1 \right) - 1 \right) \right] \frac{W^s_t}{P_t} \tag{14}
\]

where \( \psi \) measures the degree of asymmetry. Moreover, equation (14) nests the symmetric cost function case (13) as \( \psi \to 0 \). In order to better highlight the role of the different types of wage settings, we assume no indexation of wages to inflation (neither past nor steady-state inflation).

### 2.5 Monetary policy

In normal times, when monetary policy can adjust the policy rate to offset a shock to inflation, the central bank sets the short-term nominal interest rate \( R_t \) according to a Taylor-type rule:

\[
(1 + R_t) = (1 + \bar{R})^{(1-\phi_R)} (1 + R_{t-1})^{\phi_R} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi(1-\phi_R)} \left( \frac{y_t}{y_{t-1}} \right)^{\phi_y(1-\phi_R)} \tag{15}
\]

where \( \phi_\pi, \phi_y \) and \( \phi_R \) are the weights assigned to inflation, output growth and the lagged interest rate, respectively, \( \bar{R} \) is the steady-state policy rate and \( \bar{\pi} \) is the
inflation target.

When, instead, the central bank cannot adjust its policy rate, the policy rule is defined by the following equation:

\[
(1 + R_t) = \max \left[ (1 + \bar{R})^{(1-\phi_R)} (1 + R_{t-1})^{\phi_R} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\phi_{\pi} (1-\phi_R)} \left( \frac{y_t}{y_{t-1}} \right)^{\phi_y (1-\phi_R)} , (1 + \bar{R}) \right]
\]

which shows that the central bank cannot choose an interest rate \( R_t \) that is lower than its steady-state value \( \bar{R} \), even though this would be consistent with the observed developments in inflation and output according to equation (15). In this way, we constrain the behaviour of the central bank in a way that mimics the presence of the ZLB, even though, from a technical point of view, we do not set the lower bound to zero. From an economic perspective, the key feature is that the central bank cannot react to a negative shock to inflation by lowering the short-term nominal interest rate (as equation 15 would suggest), but instead remains passive. As such, when the policy rate does not react to the shock, any decrease in the inflation rate from its steady-state value implies a corresponding rise in the real interest rate from its steady state.

2.6 Calibration

Table 1 reports the calibration of the structural parameters, which is based on Gerali et al. (2010).

3 The macroeconomic effects of a negative shock to inflation

This section illustrates the response of the economy to a negative cost-push shock. The nature of the shock is such that, while it always reduces inflation, it may or may not be accompanied by a fall in economic activity. Specifically, the shock \( \varepsilon_t^y \) enters as a negative additive term in the linear Phillips curve obtained by log-linearizing equation (5). Hence, it moves inflation and output in opposite directions, under normal conditions. We focus on such shock since it is likely to have hit the euro-area economy in 2014, when a persistent fall in inflation has resulted in a sequence of negative surprises, even though economic activity has slowly recovered in the same period. To be sure, other sources of disturbances may have been
responsible for the observed macroeconomic developments in the euro area, such as a negative demand shock. The latter would have different effects in our setup, which would deserve a separate treatment and is beyond the scope of our analysis. The negative cost-push shock is assumed to hit the economy for four periods and vanish afterwards. All simulations are carried out under the assumption of perfect foresight. Therefore, households and firms fully anticipate the dynamics of all variables. The monetary policy rule (16) endogenously determines the duration of the ZLB. Thus, the economy exits the ZLB at different speeds in the various scenarios we consider. The simulations are meant to be only illustrative and should not be taken to provide an accurate quantification of the macroeconomic effects of a fall in the euro-area inflation rate. However, they show that the interaction between the debt-deflation channel, the ZLB, and wage setting is likely to result in a negative cost-push shock having contractionary effects on the economy.

3.1 A negative cost-push shock under normal conditions

Under normal conditions, a cost-push shock such as the one considered here drives inflation and output in opposite directions and temporarily modifies the trade-off faced by the central bank between maintaining the inflation rate close to its target and stabilizing cyclical fluctuations. A negative cost-push shock induces a fall in the inflation rate, which raises real disposable income, allowing consumers to buy more goods and thus resembling a positive supply shock (one that drives inflation below its steady-state level). In normal circumstances, the central bank can offset the shock by adjusting the policy rate. When the ZLB prevents this action, the real interest rate is bound to increase. Such effect, per se, induces households and firms to postpone their consumption and investment decisions, through an intertemporal substitution effect. Moreover, in the presence of private borrowing, the debt-deflation mechanism may result in a further negative reaction of economic activity, as shown in the simulations below.

In order to illustrate how the transmission of a negative inflation shock is affected by the state of the economy, the following cases are considered. First, it is assumed that the central bank can freely adjust the short-term nominal interest rate in response to the shock. Second, the impulse responses of the main macroeconomic variables are compared to those obtained when all debt contracts are indexed to the inflation rate, so that unexpected changes in the price level do not

\[^{10}\text{Using an autoregressive process with persistence equal to or above 0.8 would give similar results.}\]
affect the real value of debt and the real cost of servicing it.\textsuperscript{11} The comparison aims at highlighting the contribution of the debt-deflation channel to the transmission of the shock.

Figures 4 to 7 report the responses of the main macroeconomic variables to a negative inflation shock (green line). Inflation falls on impact by between 20 and 30 basis points (in annualized terms) and gradually returns to steady state. The central bank lowers the short-term rate and the real interest rate falls (with some lag, due to the presence of interest-rate inertia in the monetary policy rule). This translates into lower interest rate expenses for the borrowers (impatient households and entrepreneurs; Figure 5), who increase their debt, taking advantage of the lower interest payment. Borrowing households benefit from higher labor income and cheaper borrowing and therefore increase consumption. Entrepreneurs also take advantage of the lower cost and increase their borrowing, which allows them to invest more and increase their productive capacity. Labour income increases, reflecting higher labor demand (Figure 6). Consumption, investment and output rise above their steady-state levels for more than two years (Figure 7).

With debt contracts indexed to the inflation rate (Figure 7, solid blue line), the effects of the shock are qualitatively similar. The response of consumption and output is somewhat more front-loaded. The difference is due to the redistribution effect induced by the change in inflation. With nominal debt contracts, the initial fall in inflation results in a transfer of wealth from the borrowers to the savers, who are more patient and do not immediately increase consumption and investment. Therefore, when debts contracts are indexed, the response of consumption and investment is more rapid. The initial increase in output is larger and the response of the demand components is less persistent. Interestingly, the overall debt-to-output ratio increases, as the reduction in the real interest rate induces a large expansion in borrowing, which more than offsets the increase in output.

3.2 A negative cost-push shock at the zero lower bound

This section considers the case in which the central bank cannot adjust the policy rate in response to the shock. Specifically, two scenarios are considered. In the first one it is assumed that the central bank is constrained by the ZLB. In the second one, the response of the economy with indexed debt contracts is investigated. At the ZLB, the real interest rate increases after a negative cost-push shock (Figure

\textsuperscript{11}See Iacoviello (2005) for the details on how to neutralize the debt-deflation channel.
4, red-dashed and grey-dotted lines), thus counteracting the expansionary effects on aggregate demand of the decrease in the price level. Borrowers are forced to reduce their debt to limit the consequences of the rise in the real interest rate on interest rate expenses (Figure 5, red and grey lines). Entrepreneurs, facing lower demand for consumption goods by households, reduce the accumulation of capital (Figure 6) and the demand for labor. As a result, households’ wage income declines (Figure 6). The response of real wages is now different across households. In particular, the real wage of impatient households increases less than in the absence of the ZLB, despite the fact that the fall in inflation is larger. The different response reflects the reduction in nominal wages (which is limited by nominal wage rigidities). As impatient households face larger debt repayment costs after the negative income shock, they would try to reduce nominal wages, to increase the demand for their labor, by relatively more than impatient agents. However, labor demand by entrepreneurs is sizeably reduced by the fall in aggregate demand.\textsuperscript{12}

The overall effect is a large decrease in wage income, which clearly hurts the borrowers relatively more. Aggregate consumption falls and the overall effect on output is contractionary, as opposed to what is observed when the central bank can lower the policy rate.

The debt-deflation channel operates as an amplification mechanism: the increase in the real interest rate forces borrowers to deleverage more, since the real value of existing debt also increases. The larger reduction in borrowing induces a fall in asset prices and in the value of collateral, which further accelerates the deleveraging process. The ensuing fall in consumption and investment is larger and more rapid compared to the case of no debt deflation (Figure 7, grey lines). The overall contractionary effect on output is correspondingly larger (Figure 7, grey and red line, respectively).

A clear message emerges from the exercise. A shock that drives inflation and output in opposite directions when the interest rate is free to react can instead induce a fall in both variables at the ZLB. In addition, the interplay of the ZLB with the debt-deflation channel amplifies the contractionary effects on economic activity.

Some observations are in order. First, the assumption of one-period debt contracts implies that all existing debt is rolled over in each period and the issuance of new bonds immediately adjusts to the shock. With longer debt maturities,

\textsuperscript{12}The fall in labor demand concerns patient and impatient households approximately in the same amount, due to the complementarity of the two types of labor in the production function.
the effects of a negative inflation shock on economic activity may be stronger and more persistent.\textsuperscript{13} Second, the model does not include the possibility of default by borrowers.\textsuperscript{14} Therefore, unexpected variations in the real cost of servicing the debt only affect consumption and investment decisions, but do not trigger any debt restructuring. The latter would likely imply that prolonged periods of falling inflation rates at the ZLB would increase the probability of default, so that the contractionary effects of the initial shock could be further magnified. Third, as Figure 5 shows, the adjustment in the labor market plays a crucial role. At the ZLB, lower labor demand by firms implies a large fall in hours worked for both types of households. Moreover, the increase in real wages induced by lower inflation is relatively smaller for the borrowers, who try to reduce their nominal wages to accommodate more labor demand and thus contain the loss in their wage income. However, nominal wage rigidities hamper the necessary reduction in real wages. In the next section we consider the alternative modelling of the wage setting process (equation 14), in which cuts in nominal wages are much more costly then increases and in practice prevent nominal wages from responding to the negative cost-push shock.

### 3.3 The role of downward nominal wage rigidities

Having discussed the impact of negative shocks to inflation when nominal wages are sticky, we now look deeper into the role of downward nominal wage rigidities in shaping the adjustment of the economy to the same shock to inflation. As already noted, the presence of limits in the adjustment of nominal wages in the face of a deflationary shock is particularly relevant at the ZLB, when monetary policy cannot further reduce the short-term nominal interest rate. The assumption of downward nominal wage rigidities captures the limit-case in which nominal wages cannot be further reduced. As observed in Schmitt-Grohé and Uribe (2013), nominal wages have remained largely unchanged in some peripheral euro-area countries (namely Cyprus, Greece, Ireland, Portugal and Spain) since 2008, despite a sharp contraction in aggregate demand. Crucially, the combination of weak aggregate demand and high labor costs was associated with a rise in unemployment. Our model is not suited for a formal analysis of unemployment, which is anyway beyond the scope

\textsuperscript{13}See Andrés, Arce and Thomas (2014) for an analysis of the role of multi-period debt contracts and slow debt-deleveraging at the ZLB.

\textsuperscript{14}Clerc et al. (2015) develop a model in which households, entrepreneurs and banks can all default on their debt obligations. The model is used to provide a positive and normative analysis of macroprudential policies.
of our analysis. In the following, we limit ourselves to analysing how downward nominal wage rigidities may influence labor market adjustment and hence affect the response of the main macroeconomic variables to a negative inflation shock at the ZLB. The assumption of asymmetric adjustment costs captures the extreme case in which nominal wages are not adjusted and real wages must necessarily increase. In this respect, it represents a limit-case of the assumption of nominal wage rigidities.

In this section we assume that the costs for adjusting nominal wages is given by eq. (14). We keep the parameter $\kappa_w$ at the same value used in the previous simulations and set the value of $\psi$ to 78,000, which is sufficient to neutralize the response of nominal wages to the shock.\(^{15}\) Hence, the overall cost of adjusting nominal wages in response to the shock is much larger than in the previous simulations and, as a matter of fact, nominal wages cannot fall.

Figure 8 reports the results. We repeat the same simulation as in the previous sections, but we only focus on the case in which the ZLB holds and the debt-deflation channel is active. The role of asymmetries in the wage adjustment cost only arises when the ZLB is in place. In fact, when the central bank is not constrained by the ZLB, the negative cost-push shock has expansionary effects and, by reducing real marginal costs, it allows real wages to increase. As such, no relevant differences arise compared to the case of a symmetric adjustment cost function.\(^{16}\) To the opposite, when the ZLB holds, the increase in the real interest rate and, if present, the debt-deflation channel make the same shock contractionary, as already observed. Therefore, nominal wages should fall to allow the labor market to adjust to the lower aggregate demand and hence to the lower demand for labor. However, the large cost incurred by unions when reducing nominal wages precludes the required adjustment. Facing an increase in the real cost of borrowing, at the ZLB impatient households reduce their debt, pushing it below the steady state for the whole simulation horizon, thus countering the increase in interest rate expenses. Entrepreneurs also persistently reduce their borrowing. At the ZLB, deleveraging starts right after the shock, regardless of the type of nominal rigidities. On the contrary, when the real cost of borrowing falls, entrepreneurs increase their debt in the short run. As a result, in the ZLB scenario, capital accumulation falls persistently below the steady state. Entrepreneurs also reduce labor demand, which

\(^{15}\)Using values such as 26,000 as in Fahr and Smets (2010), or 24,100 as in Abbritti and Fahr (2013) we obtain qualitatively similar results. We use a larger value to highlight the different effects of the shock under alternative wage adjustment cost functions.

\(^{16}\)We do not report the results of the simulations, which are available upon request.
causes households to receive a lower labor income, despite the increase in the real wage. In equilibrium, households work and consume less and reduce their borrowing; entrepreneurs also borrow and invest less. A clear difference stands out compared with the case of symmetric nominal wage adjustment costs discussed in the previous sections. In all the cases the effects of the negative shocks to inflation are more persistent. Importantly, the cumulated loss in output is much larger under DNWR after 20 quarters, while differences are much smaller up to 10 quarters after the shock. The larger and more protracted contractionary effects reflect the delayed labor market adjustment due to higher nominal wage rigidities.

4 Sensitivity analysis

In this Section we discuss the results of some sensitivity analysis. First, we explore the asset price channel, namely the contribution of fluctuations in asset prices in the propagation of the shock at the zero lower bound. Second, we consider the role of the level of private sector indebtedness.

4.1 The asset price channel

The formulation of the collateral constraints (5) and (9) links the maximum amount of funds that borrowers can obtain to the value of their collateral (housing and capital stock). Hence, fluctuations in asset prices ($q^h$ and $q^k$) contribute to modify the collateral value, over and above variations in the inflation rate. In this section we study the role of the asset price channel by studying the responses to the shock under the assumption that the borrowing limit is constant, independent of the asset value.\footnote{Iacoviello (2005) performs a similar analysis to highlight the role of asset prices.} Specifically, we replace equations (5) and (9) with:

\begin{equation}
(1 + R_t) b^l_t(i) \leq \bar{b}^l
\end{equation}

and

\begin{equation}
(1 + R_t) b^E_t(i) \leq \bar{b}^E
\end{equation}

where $\bar{b}^l$ and $\bar{b}^E$ are calibrated at the steady-state values of $b^l_t(i)$ and $b^E_t(i)$, respectively.

We study the effects of a negative cost-push shock at the ZLB, since this case
has been shown to be the most interesting one in the previous section. Figure 9 reports the responses of the main macroeconomic variables under four different assumptions: (i) the baseline case with the debt deflation mechanism and the asset price channel (red dashed line), (ii) no debt deflation and asset price channel (solid blue line), (iii) no debt deflation and no asset price channel (green line), (iv) debt deflation but no asset price channel (grey line). In the latter case, variations in the inflation rate do not determine any ex-post change in the real value of debt, as already illustrated in Section 3.

Without the asset price channel, the shock has virtually no effect on debt, for both types of borrowers. Hence, deleveraging cannot take place as both impatient households and entrepreneurs are stuck at the borrowing limit (which is assumed to bind at all times). The increase in the real interest rate, due to the ZLB, induces households and entrepreneurs to reduce consumption and investment, respectively, in order to repay their existing debt. In particular, the initial fall in investment is larger than the one observed when the asset price channel is activated. The overall fall in output is very similar with and without the asset price channel (compare the red dashed and grey lines). All in all, while the behaviour of debt (not reported) changes significantly when asset prices can adjust, the macroeconomic effects are largely unchanged when the asset price channel is shut off. A clear result stands out. At the ZLB, the presence of the debt deflation mechanism is sufficient for a negative cost-push shock to have contractionary effects. Variations in asset prices are not necessary for the contractionary effects to materialize.

4.2 The role of private sector indebtedness

Two parameters are particularly relevant in determining the role of private sector indebtedness: the LTV ratios faced by households and entrepreneurs and the relative share of indebted households. In this section we carry out a sensitivity analysis by varying each of these two parameters. In order to assess the role of private debt, we first reduce the LTV ratio faced by private borrowers (both households and firms), thus also reducing the overall amount of private debt in equilibrium. In an alternative scenario, we increase the labor share of indebted households, which, to the opposite, increases steady-state private debt. We focus only on the ZLB case, which was shown to be the most interesting one for our analysis. Moreover, we only report the results for the standard calibration, i.e.

\[\text{We focus on the case of symmetric adjustment costs for nominal wages. Results for the case of DNWR are available upon request.}\]
the case of a symmetric adjustment cost function for nominal wages.\footnote{The simulations for the case of DNWR are qualitatively similar and are available upon request.}

In the first simulation we reduce the LTV ratio faced by households and firms by half, to 0.35 and 0.175, respectively. As a result, the overall amount of private debt as a share of (annualized) GDP is also reduced by approximately the same magnitude, falling from 128\% (under the benchmark calibration) to 57\%. The main results of our previous analysis remain valid. Clearly, with a lower LTV ratio, the deleveraging effect induced by the shock has a smaller impact on the economy, compared to the benchmark simulations (Figure 10). The marginal contribution of the debt deflation channel is correspondingly smaller, and so is the fall in output.

In a similar way, a higher share of indebted agents in the economy contributes to strengthen the amplification effects. When the labor share of impatient households moves from 0.2 to 0.35, the steady-state private debt to (annualized) GDP ratio increases to 170\%, from 128 in the baseline case. The corresponding effects on the main macroeconomic variables are quantitatively very similar to those obtained in the benchmark simulations (Figure 11).

5 Conclusions

This paper has illustrated the macroeconomic consequences of a prolonged period of low and falling inflation when the nominal short-term rate is stuck at the ZLB, the private sector is indebted in nominal terms and nominal wages are downward rigid.

The interaction of the ZLB and the debt-deflation mechanism may well revert the implications of a “good” shock, i.e., a shock that in normal circumstances would lower the inflation rate and expand economic activity. The negative effects of shocks to inflation are larger when nominal wages cannot be reduced. The effects are also larger when the private sector debt is large relatively to output.

The results are obtained in a very stylized setup in which financial intermediation plays no role. The inclusion of a banking sector may help shed some light on the dynamics of credit supply in a period of low and falling inflation and on the related macroeconomic effects. We leave the investigation of these issues to future research.
References


<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
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Figure 1: Harmonized Index of Consumer Prices (HICP) inflation in the euro area and components

Note: monthly data; 12-month-percentage changes and percentage points
Figure 2: Nominal and real short-term money market rates

Note: monthly data and percentage points. The real rate is computed as the nominal Euribor rate minus one-quarter ahead inflation expectations from Consensus Economics (linearly interpolated at monthly frequency.)
Figure 3: Euro-area total private debt as a percentage of GDP

*Note:* quarterly data.
Figure 4: Responses of inflation, real and nominal interest rates

Note: absolute deviations from steady state.
Figure 5: Responses of interest rate expenses and borrowing

Note: percentage deviations from steady state.
Figure 6: Responses of real wages, labor and labor income

Note: percentage deviations from steady state.
Figure 7: Responses of debt-to-output ratio, consumption, investment and output

**Note:** percentage deviations from steady state.
Figure 8: The role of downward nominal wage rigidities with ZLB and debt deflation channel

Note: percentage deviations from steady state.
Figure 9: Sensitivity. Responses of main variables with and without the asset price channel

Note: percentage deviations from steady state.
Figure 10: Sensitivity. Responses of main variables with lower LTV ratios

Note: percentage deviations from steady state.
Figure 11: Sensitivity. Responses of main variables with larger share of borrowers

Note: percentage deviations from steady state.
Appendix

In this Appendix we report a detailed description of the model.

Patient households

The representative patient household $i$ maximizes the expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ (1 - a^P) \log(c^P_t(i) - a^P c^P_{t-1}^i) + j \log h^P_t(i) - \frac{b^P_t(i)^{1+\phi}}{1+\phi} \right]$$

subject to the budget constraint

$$c_t^P(i) + q h^P_t(i) + b(i) \leq w_t^P l^P_t(i) + (1 + R_{t-1}) b_{t-1}(i)/\pi_t + t^P_t(i)$$

$j$ is the weight of housing services in the utility function.

Impatient households

The representative impatient household $i$ maximizes the expected utility

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ (1 - a^I) \log(c^I_t(i) - a^I c^I_{t-1}^i) + j \log h^I_t(i) - \frac{b^I_t(i)^{1+\phi}}{1+\phi} \right]$$

subject to the budget constraint:

$$c_t^I(i) + q^h_t \Delta h^I_t(i) + (1 + R_{t-1}) b_{t-1}(i)/\pi_t \leq w_t^I l^I_t(i) + b^I_t(i) + t^I_t(i)$$

and to a borrowing constraint

$$(1 + R_t) b^I_t(i) \leq m^I_t E_t \left[ q^h_{t+1} l^I_t(i) \pi_{t+1} \right]$$

Entrepreneurs

Each entrepreneur $i$ maximizes the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t_E \log(c^E_t(i) - a^E c^E_{t-1})$$
subject to the budget constraint (with multiplier $\lambda^E_t$)

$$c^E_t(i) + u^E_t P^E_t(i) + w^E_t E^I_t(i) + \frac{1 + R_{t-1}}{\pi_t} E^{E}_{t-1}(i) + q^k_t k^E_t(i) + \phi(u_t(i)) k^E_{t-1}(i) \leq \frac{y^E_t(i)}{x_t} + b^E_t(i) + q^k_t(1-\delta) k^E_{t-1}(i)$$

with $\phi(u_t) = \xi_1(u_t - 1) + \frac{\xi_2}{2}(u_t - 1)^2$. His available technology has

$$y^E_t(i) = a_t E^E_{t-1}(i) u_t(i)^{\alpha} (1 - \alpha)^{k^E_t(i)}$$

where aggregate labor is a combination of inputs from patient and impatient households according to $l^E_t = (l^{E,P}_t)^\mu (l^{E,I}_t)^{1-\mu}$, with parameter $\mu$ calibrated to measure the labor income share of unconstrained households as in Iacoziello and Neri (2009). The entrepreneur faces a borrowing constraint of the kind

$$(1 + R_t)b^E_t(i) \leq m^E_t E_t[(q^k_{t+1}(1-\delta) k^E_t(i))]$$

### Capital goods producers

Capital goods producing firms solve the following problem

$$\max_{\{\bar{x}_t, i_t\}} \sum_{t=0}^{\infty} \Lambda^E_{0,t} \left( q^k_t \Delta \bar{x}_t - i_t \right)$$

subject to

$$\bar{x}_t = \bar{x}_{t-1} + \left[ 1 - \frac{\kappa_t}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t$$

where $\Delta \bar{x}_t = k_t - (1-\delta) k_{t-1}$ is the flow output.

From FOCs, the amount of new capital that CGP firms can produce is given by:

$$k_t = (1-\delta) k_{t-1} + \left[ 1 - \frac{\kappa_t}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t$$

and the real price of capital $q^k_t$ is determined by

$$1 = q^k_t \left[ 1 - \frac{\kappa_t}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right)^2 - \kappa_t \left( \frac{i_t}{i_{t-1}} - 1 \right) \left( \frac{i_t}{i_{t-1}} \right)^{\alpha} \right] + \beta E_t \left[ \frac{\lambda^E_t}{\lambda^E_t} q^k_{t+1} \kappa_t \left( \frac{i_{t+1}}{i_t} - 1 \right) \left( \frac{i_t}{i_{t-1}} \right)^{\alpha} \right]$$
**Labor market**

**Labor packers**

A perfectly competitive labor packer $s \in \{P, I\}$ demands differentiated labor services from unions $(s, m)$ ($m \in [0, 1]$) in order to assemble them into a CES composite labor input to be supplied to entrepreneurs. He solves

$$
\max_{l_t^s(m)} \quad l_t^s = \left[ \int_0^1 l_t^s(m) \frac{\epsilon'}{\epsilon} \, dm \right]^{\frac{\epsilon}{\epsilon'-1}}
$$

s.t.  
$$
\int_0^1 W_t^s(m)l_t^s(m) \, dm \leq E_t
$$

for a given level of the overall wage bill $E_t$. The solution gives the demand for each kind of differentiated labor service $l_t^s(m)$:

$$
l_t^s(m) = \left( \frac{W_t^s(m)}{W_t^s} \right)^{-\epsilon} l_t^s
$$

where

$$
W_t^s = \left[ \int_0^1 W_t^s(m)^{1-\epsilon} \, dm \right]^{\frac{1}{1-\epsilon}}
$$

**Unions**

Workers $(i)$ sell their slightly differentiated labor types through unions, which for each labor type $m$ exist in two kinds: one for patient and one for impatient households (indexed by $s \in \{P, I\}$). Each union $(s, m)$ sets nominal wages $\{W_t^s(m)\}_{t=0}^\infty$ by maximizing

$$
E_0 \sum_{t=0}^\infty \beta_t \left\{ U^c_{i(t,m)} \left[ \frac{W_t^s(m)}{P_t} l_t^s(i, m) - \frac{\kappa_w}{2} \left( \frac{W_t^s(m)}{W_{t-1}^s(m)} - \pi_{t-1} \pi_w \right)^2 \frac{W_t^s}{P_t} \right] - \frac{l_t^s(i, m)^{1+\phi}}{1+\phi} \right\}
$$

subject to demand

$$
l_t^s(i, m) = l_t^s(m) = \left( \frac{W_t^s(m)}{W_t^s} \right)^{-\epsilon} l_t^s
$$

In a symmetric equilibrium, the labor choice for a household of type $s$ will be given by an ensuing (non-linear) wage-Phillips curve

$$
\kappa_w \left( \pi_t^w - \pi_{t-1} \pi_{1-w} \right) \pi_t^w = \beta_s E_t \left[ \frac{\lambda_t^s}{\lambda_{t+1}^s} \kappa_w \left( \pi_{t+1}^w - \pi_t^w \pi_{1-w} \right) \frac{\pi_{t+1}^w \pi_{1-w}}{\pi_{t+1}^w} \right] + \left( 1 - \epsilon' \right) l_t^s + \frac{\epsilon' l_t^s \phi} {\pi_t^w \lambda_t^s w_t}
$$
where $\omega_t^s$ is the real wage and nominal type $s$ wage inflation is equal to

$$\pi_t^w = \frac{w_t^s}{w_{t-1}^s}$$

**Final goods producers**

The retail goods market is assumed to be monopolistically competitive as in Bernanke, Gertler, and Gilchrist (1999). Retailers prices are sticky and are indexed to a combination of past and steady-state inflation, with relative weights parameterized by $\iota_p$; specifically, retailers face a quadratic price adjustment cost parameterized by $\kappa_p$. Retailers solve the following problem

$$\max_{(P_t(j))} E_0 \sum_{t=0}^{\infty} \Lambda_{0,t} \left[ P_t(j)y_t(j) - P_t^W y_t(j) - \frac{\kappa_p}{2} \left( \frac{P_t(j)}{P_{t-1}(j)} - \pi_{t-1} \pi_{1-\iota_p} \right)^2 P_t y_t \right]$$

subject to a demand coming from consumers maximization of a consumption aggregator

$$y_t(j) = \left( \frac{P_t(j)}{P_t} \right)^{-\varepsilon_t^y} y_t$$

where the elasticity of substitution between individual goods, $\varepsilon_t^y$ is time-varying, reflecting the presence of cost-push shocks. In a symmetric equilibrium, FOCs give rise to a non-linear Phillips curve

$$1 - \varepsilon_t^y + \frac{\varepsilon_t^y}{x_t} - \kappa_p (\pi_t - \pi_{t-1} \pi_{1-\iota_p}) \pi_t + \beta P E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \kappa_p (\pi_{t+1} - \pi_{t} \pi_{1-\iota_p}) \pi_{t+1} \frac{y_{t+1}}{y_t} \right] = 0$$

(21)

Profits are given by:

$$j_t^R = y_t \left[ 1 - \frac{1}{x_t} - \frac{\kappa_p}{2} (\pi_t - \pi_{t-1} \pi_{1-\iota_p})^2 \right]$$

**Monetary policy**

In normal times, when monetary policy can adjust the policy rate to offset a shock to inflation, the central bank sets the short-term nominal interest rate $R_t$ according to a Taylor-type rule:

$$(1 + R_t) = (1 + \bar{R})^{(1-\phi_R)} (1 + R_{t-1})^{\phi_R} \left( \frac{\pi_t}{\bar{\pi}} \right)^{\phi_t} \left( \frac{y_t}{y_{t-1}} \right)^{\phi_y}$$

(22)
where $\phi_x$ and $\phi_y$ are the weights assigned to inflation and output growth, respectively, $\bar{R}$ is the steady-state policy rate and $\bar{\pi}$ is the inflation target.

When, instead, the central bank cannot adjust its policy rate, the policy rule is defined by the following equation:

$$\left( 1 + R_t \right) = \max \left[ \left( 1 + \bar{R} \right) \left( 1 + R_{t-1} \right)^{\phi_R} \left( \frac{\pi_t}{\bar{\pi}} \right) \frac{\phi_y(1-\phi_R)}{(1-\phi_R)} \left( \frac{yz^t}{y_{t-1}} \right) \frac{\phi_y(1-\phi_R)}{(1-\phi_R)} \right], \left( 1 + \bar{R} \right)$$

which shows that the central bank cannot choose an interest rate $R_t$ that is lower than its steady-state value $\bar{R}$, even though this would be consistent with the observed developments in inflation and output according to equation (22). In this way, we constrain the behaviour of the central bank in a way that mimics the presence of the ZLB, even though, from a technical point of view, we do not set the lower bound to zero. From an economic perspective, the key feature is that the central bank cannot react to a negative shock to inflation by lowering the short-term nominal interest rate (as equation 22 would suggest), but instead remains passive. As such, when the policy rate does not react to the shock, any decrease in the inflation rate from its steady-state value implies a corresponding rise in the real interest rate from its steady state.

**Aggregation and market clearing**

In the final good market, the equilibrium condition is given by the following resource constraint

$$y_t = c_t + q^k_t \left[ k_t - (1 - \delta) k_{t-1} \right] + k_{t-1} \phi (u_t) + Adj_t$$

where $c_t = c_t^p + c_t^d + c_t^f$ and $k_t = \gamma^s k_t^s(i)$ ($\gamma^s$, $s \in P, I, E$, is the measure of each subset of agents). Equilibrium in the housing market is given by

$$\bar{h} = \gamma^p h_t^p(i) + \gamma^l h_t^l(i)$$
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