Does trend inflation make a difference?

by Michele Loberto and Chiara Perricone
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DOES TREND INFLATION MAKE A DIFFERENCE?

by Michele Loberto* and Chiara Perricone§

Abstract

Although the average inflation rate of developed countries in the postwar period has been greater than zero, much of the extensive literature on monetary policy has employed models that assume zero steady-state inflation. In comparing four estimated medium-scale NK DSGE models with real and nominal frictions, we seek to shed light on the quantitative implications of omitting trend inflation, that is, positive steady-state inflation. We compare certain population characteristics and the IRFs for the four models by applying two loss functions based on a point distance criterion and on a distribution distance criterion, respectively. Finally, we compare the RMSE forecasts. We repeat the analysis for three sub-periods: the Great Inflation, the Great Moderation and the union of the two periods. We do not find clear evidence for always preferring a model that uses trend inflation.

JEL Classification: C1, C5, E4, E5.
Keywords: new Keynesian DSGE, trend inflation, loss function, entropy.

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* Bank of Italy, Directorate General for Economics, Statistics and Research, michele.loberto@bancaditalia.it.
§ LUISS Guido Carli, cperricone@luiss.it.
1 Introduction

Looking at the macroeconomic data of developed countries, it is clear that the average inflation rate in the postwar period was greater than zero and that it varied by country. However, much of the extensive literature on monetary policy rules has employed models that assume zero steady-state inflation. Ascari and Ropele (2007) suggest that monetary policy literature has centred on this particular assumption, even though it is both empirically unrealistic and theoretically special, for two reasons: it is analytically convenient and price stability is the optimal prescription in a cashless economy\(^1\). By relaxing the zero steady-state inflation assumption, we gain new insights. First, Ascari and Ropele (2007, 2009) show that even low trend inflation can affect optimal monetary policy and the dynamics of inflation, output and interest rates under a standard New Keynesian model. Moreover, trend inflation shrinks the determinacy region of a basic New Keynesian model when monetary policy is conducted by a contemporaneous interest rate rule\(^2\). Second, as shown by Cogley and Sbordone (2008), in small-scale models the inclusion of time-varying trend inflation seems to eliminate the need to include partial indexation schemes to produce a backward-looking dynamic.

Given the empirical practice and these theoretical caveats, the goal of this analysis is to shed light on the quantitative implications of omitting trend inflation in an estimated medium-scale DSGE model, whereas most of the literature on trend inflation involves calibrated models. We compare a NK DSGE model log-linearized around zero steady-state inflation and partially indexed to past inflation with an equivalent model using trend inflation\(^3\). Then, since trend inflation should, in theory, help to account for the backward-looking dynamic of inflation, we also compare these two models without indexing them to past inflation. The chosen NK DSGE model is based on two workhorse medium-scale DSGEs: Smets and Wouters (2007) and Schmitt-Grohe and Uribe (2004). These NK DSGE models add both real and nominal frictions to the standard textbook model. The real frictions are: monopolistic competition in goods and labour markets, habit formation in consumption preferences, capital utilization and investment adjustment cost. The nominal frictions are based on the Calvo mechanism for nominal price and wage. We have chosen this model since it fits well with the observed data, replicating the main US macro features.

We analyse three different periods: the entire span of time between 1966 and 2004, the period before the Great Moderation (1966–1982) and the years of the Great Moderation (1983–2004). These periods have different average levels of inflation and therefore we are able to test the quantitative implications of trend inflation for different levels of inflation in the steady state.

We compare the cross-correlations and the IRFs for the four models by applying the evaluation method proposed by Schorfheide (2000). Specifically, we compare the models by using two types of loss functions. The first one is based on a point distance criterion, as in Schorfheide (2000). The second, proposed as a novelty in this study, is a distribution distance criterion based on the idea of entropy suggested by Ullah (1996). The bench-

\(^1\)See Woodford (2003).

\(^2\)Other papers study the effects of changes in trend inflation, such as Hornstein and Wolman (2005) and Kiley (2007), concluding that the Taylor Principle breaks down when the trend inflation rate rises and that a more aggressive policy in response to inflation is needed to insure determinacy.

\(^3\)Ascari and Ropele (2007) show that with full indexation under the Calvo pricing scheme, log-linearization around zero trend inflation or positive trend inflation are identical. In this case the distortions due to positive trend inflation disappear when all the non-re-optimizing firms re-adjust their prices to past inflation and/or to the trend inflation.
mark against we compare the different models is a weighted average, computed from the
classical model. Moreover, since one of the advantages of the DSGE model is its use in forecasting, we
compare the in-sample forecast RMSEs of the DSGE models.
We do not find clear evidence for preferring a model that uses trend inflation. In all our
various comparisons, the presence of trend inflation does not produce results that differ
significantly from those of the classical model. These results are consistent with those
reported by Ascari, Branzoli and Castelnovo (2011). They studied the determinacy of the
inflation in a calibrated medium-scale New Keynesian framework and concluded
that trend inflation does not seem to offset the determinacy region when real frictions are
included.
When we studied the two sub-periods, we found that the models are almost equivalent
during the Great Moderation. However, the pre-1982 trend inflation is relevant since it
results in better forecasting and a good fit between the IRFs.
We have contributed to the trend inflation literature studying the effects of different levels
of inflation in an estimated NK model. Few articles have investigated trend inflation in a
calibrated model while focusing on the determinacy issue. The first paper that examined
the effects of trend inflation on the dynamics of the standard New Keynesian model was
cycle characteristics of the model vary with trend inflation. Ascari and Ropele (2007)
analysed how optimal short-run monetary policy changes with trend inflation, whereas,
in Ascari and Ropele (2009), moderate levels of trend inflation offset the determinacy
region, substantially altering the monetary policy rule. Kiley (2007) investigated how
trend inflation influences the determinacy region and the unconditional variance of inflation
in a model in which prices are staggered à la Taylor and monetary policy is described
by à Taylor rule. Coibion and Gorodnichenko (2011) showed that determinacy in New
Keynesian models under positive trend inflation depends not only on the central bank’s
response to inflation and output gap, as is the case under zero trend inflation, but also on
many other components of endogenous monetary policy.
The paper is organized as follows. In Section 2 we introduce the general DSGE model and
the nested models we will compare and we present the data, the Bayesian estimates for
the parameters, the relative short-run dynamics and forecasts. In Section 3 we explain the
procedure for comparing the models and the results for correlation and IRFs. In Section
4 we state our conclusions.

2 Model and introductory comparisons

2.1 General model

We base our analysis on a medium-scale DSGE model, similar to the well-known
model estimated by Smets and Wouters (2007). Households maximize a non-separable
utility function with two arguments (final goods and labour effort) over an infinite life
horizon. The presence of time-varying external habit formation means that the past also
affects current consumption. Labour decisions are made by a union, which supplies labour
monopolistically to a continuum of labour markets, sets nominal wages à la Calvo and dis-
tributes the markup applied over the marginal cost of labour to households. Households
rent capital services to firms and decide how much capital to accumulate given capital ad-
justment costs. Capital utilization is variable and chosen by the households in accordance with a cost schedule.

There is a sector of intermediate goods where there is a continuum of firms that produce differentiated goods in a monopolistic market à la Dixit and Stiglitz, decide on labour and capital inputs, and set prices, again in accordance with the Calvo model. The consumption good is a composite of intermediate goods. The final good producers buy the intermediate goods on the market, package them into units of the composite good, and resell them to consumers in a perfectly competitive market.

We assume that the central bank systematically reacts to inflation ($\tilde{\pi}_t$) and to output ($\tilde{y}_t$) growth in accordance with the rule:

$$\tilde{R}_t = \tilde{R}_{t-1}^{\rhoR} R_t^{\rhoR} \exp \epsilon_t^{R}$$

$$\tilde{R}_t = \left( \frac{\tilde{\pi}_t}{\pi_*} \right)^{\psi_\pi} \left( \frac{\tilde{y}_t}{y_{t-1}} \right)^{\psi_y}$$

where $\epsilon_t^{R}$ is a monetary policy shock that captures transitory deviations from the interest rate feedback rule that are unanticipated by the public\(^4\).

Labour decisions are made by a union, which supplies labour monopolistically to a continuum of labour markets of measure 1, indexed by $l \in [0, 1]$, and sets wages in accordance with the Calvo model. Their optimization problem yields the following wage equation:

$$\frac{\theta^w - 1}{\theta^w} \frac{\tilde{w}_t^o}{\tilde{w}_t} \tilde{f}_t^{1,w} = \tilde{f}_t^{2,w}$$

where $\tilde{f}_t^{1,w}$ and $\tilde{f}_t^{2,w}$ are defined as

$$\tilde{f}_t^{1,w} = \left( \frac{\tilde{w}_t^h}{\tilde{w}_t^o} \right)^{\theta^w} \tilde{L}_t^d + (\omega^w/\beta) \gamma^{-1} \left( \frac{\tilde{\pi}_t+1}{\tilde{\pi}_t^o} \right)^{\theta^w-1} \left( \frac{\tilde{w}_t+1}{\tilde{w}_t^o} \right)^{\theta^w} \tilde{\xi}_{t+1|t} \tilde{f}_t^{1,w}$$

and

$$\tilde{f}_t^{2,w} = \tilde{w}_t^h \left( \frac{\tilde{w}_t^h}{\tilde{w}_t^o} \right)^{\theta^w} \tilde{L}_t^d + (\omega^w/\beta) \gamma^{-1} \left( \frac{\tilde{\pi}_t+1}{\tilde{\pi}_t^o} \right)^{\theta^w} \left( \frac{\tilde{w}_t+1}{\tilde{w}_t^o} \right)^{\theta^w} \tilde{\xi}_{t+1|t} \tilde{f}_t^{2,w}$$

and $\theta^w$ is the intratemporal elasticity of substitution in the labour market, $\tilde{w}_t^o$ is the optimal wage, $\tilde{L}_t^d$ is a measure of aggregate labour demand by firms at time $t$, $\beta$ is the subjective discount factor, $\omega^w$ is the probability of not re-optimizing wages, $\epsilon_w$ is the indexation of wages to past consumer price inflation, $\gamma$ represents the labour–augmenting deterministic growth rate, $\sigma_c$ is the inverse of the elasticity of intertemporal substitution for constant labour, $\tilde{\xi}_{t+1|t}$ is the stochastic discount factor, $\tilde{w}_t$ is an index of nominal wages prevailing in the economy, and $\tilde{w}_t^h$ is the nominal wage received by households.

Firms in the intermediate sector produce a continuum of goods indexed by $i \in [0, 1]$ in a monopolistic competitive environment; each intermediate good is produced by a single firm. Prices are assumed to be sticky à la Calvo, indeed only a fraction $1 - \omega_p$ of firms can optimally set the price $P_{i,t}$ at time $t$, which is chosen to maximize the expected present discounted value of profits. The price equation obtained from their problem is:

$$\frac{\theta^p - 1}{\theta^p} \tilde{f}_t^{1,p} = \tilde{f}_t^{2,p}$$

\(^4\)The model used here is identical to the one estimated by Smets and Wouters, except for three departures. First, we assume that the final producers package their goods in accordance with the Dixit and Stiglitz aggregator instead of the Kimball aggregator. Second, in our model, the monetary authority adjusts the nominal interest rate in response to inflation and output growth, while Smets and Wouters use the output gap. Third, we log-linearize the model around a positive level of steady-state inflation. A detailed explanation is found in Appendix A.
where we define

\[
\tilde{f}_{t}^{1,p} = \tilde{y}_{t}(\tilde{p}_{t})^{-\theta_{p}} + \omega_{p}\beta\gamma^{1-\sigma_{c}}E_{t}
\left[
\left(\frac{\tilde{p}_{t}^{\gamma}}{\tilde{p}_{t+1}^{\gamma}}\right)^{1-\theta_{p}}\left(\frac{\tilde{p}_{t+1}^{\gamma}}{\tilde{p}_{t+1}^{\gamma}}\right)^{-\theta_{p}}\tilde{\xi}_{t+1}\right]\tilde{f}_{t+1}^{1,p}
\]

and

\[
\tilde{f}_{t}^{2,p} = \frac{\tilde{y}_{t}}{\tilde{p}_{t}}(\tilde{p}_{t})^{-\theta_{p}-1} + \omega_{p}\beta\gamma^{1-\sigma_{c}}E_{t}
\left[
\left(\frac{\tilde{p}_{t}^{\gamma}}{\tilde{p}_{t+1}^{\gamma}}\right)^{1-\theta_{p}}\left(\frac{\tilde{p}_{t+1}^{\gamma}}{\tilde{p}_{t+1}^{\gamma}}\right)^{-\theta_{p}-1}\tilde{\xi}_{t+1}\right]\tilde{f}_{t+1}^{2,p}
\]

where \(\tilde{p}_{t}\) is the optimal price, \(\omega_{p}\) is the probability of not re-optimizing prices, \(\theta_{p}\) is the price indexation parameter, \(\theta_{p}\) is the parameter of the Dixit–Stiglitz aggregator over the \(j\)-ths firms, and \(\tilde{\xi}_{t}\) is the price mark-up.

### 2.2 Nested models

The model we outlined above is a standard NK DSGE model with trend inflation and partial indexation to past inflation, similar to that of Schmitt-Grohe and Uribe (2004). Here, trend inflation means that the model is log-linearized around a steady-state value of inflation equal to the mean value of the inflation series over the time horizon studied. We also do not consider a scenario of full indexation because, in this case, a model with trend inflation is equivalent to one without it, as shown by Ascari and Ropele (2009). Hereafter, we will label this model TI.\(^{5}\)

In order to investigate the empirical relevance of trend inflation we estimate four nested DSGE models. The models differ with respect to two characteristics: trend inflation and partial indexation of both prices and wages to past inflation.

When we remove trend inflation, we set the steady-state rate of inflation at 0. Where indexation is used, we estimate the two parameters; where we remove it, we set \(\theta_{p} = \theta_{w} = 0\), thereby obtaining a purely forward-looking NKPC.

Starting from model TI, we obtain the other scenarios by combining the presence or absence of both/either trend inflation and indexation, as summarized in Table 1.

<table>
<thead>
<tr>
<th>Model</th>
<th>Trend inflation</th>
<th>Indexation</th>
<th>Literature</th>
</tr>
</thead>
<tbody>
<tr>
<td>TI</td>
<td>Yes</td>
<td>Yes</td>
<td>Ascari and Ropele (2009), Schmitt-Grohe and Uribe (2004)</td>
</tr>
<tr>
<td>nTI</td>
<td>No</td>
<td>Yes</td>
<td>Ascari and Ropele (2007)</td>
</tr>
<tr>
<td>TnI</td>
<td>Yes</td>
<td>No</td>
<td>Smets and Wouters (2007), Gali and Gertler (1999)</td>
</tr>
<tr>
<td>nTnI</td>
<td>No</td>
<td>No</td>
<td>Clarida, Gali and Gertler (1999), Taylor (1993)</td>
</tr>
</tbody>
</table>

Table 1: Models characterization and examples of related literature.

We will compare TI (trend inflation with indexation) with nTI (no trend inflation with indexation) and TnI (trend inflation without indexation) with nTnI (no trend inflation without indexation) for different periods having different levels of inflation. We expect to find differences between models for the periods with higher inflation and similarity during the Great Moderation. Moreover, models without partial indexation are expected

\(^{5}\)The capital T means that the model features trend inflation, while I indicates there is partial indexation.
to perform worse than those with indexation, but in the absence of indexation a model
with trend inflation is preferred because the inclusion of trend inflation should help to
produce a backward-looking dynamic for inflation, as shown by Cogley and Sbordone
(2008).

2.3 Data and Bayesian Estimation

The seven variables used in our analysis are the quarterly data of the log of real GDP
per capita \( (y_t) \), the log of real consumption per capita \( (c_t) \), the log of real investment per
capita \( (i_t) \), the log of hours per capita \( (l_t) \), the log of the GDP deflator \( (\pi_t) \), the log of real
wages \( (w_t) \), and the federal funds rate \( (R_t) \). All the data are obtained from the FRED2
database maintained by the Federal Reserve Bank of St. Louis and first differences are
taken of all variables, with the exception of hours, federal funds rate and inflation, as
in Smets and Wouters. We consider three time horizons that are identified by different
means and variances for the inflation rate. The first period covers the years 1966 Q1 – 1982
Q4 and it is marked by high inflation (called the Great Inflation). The second interval,
representing the Great Moderation, runs from 1983 Q1 to 2004 Q4 and has a low level
and variance of inflation. Finally, we estimate the full sample, i.e., from 1966 Q1 to 2004
Q4. Table 2 reports the means and variances of the inflation series for the three periods.

<table>
<thead>
<tr>
<th>Period</th>
<th>Mean</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966 - 1982</td>
<td>6.08</td>
<td>0.3183</td>
</tr>
<tr>
<td>1983 - 2004</td>
<td>2.43</td>
<td>0.0620</td>
</tr>
<tr>
<td>1966 - 2004</td>
<td>4.02</td>
<td>0.3783</td>
</tr>
</tbody>
</table>

Table 2: Mean and variance for the quarterly GDP deflator over the three periods.

The Bayesian estimate of the DSGE models is based on the theoretical prescription of
An and Schorfheide (2007).\footnote{We set up a MATLAB routine performing a Random
Walk Metropolis–Hastings, with the algorithm samples using a variance and covariance
matrix obtained as the inverse of the Hessian previously computed using the Sims optimization
algorithm. In order to arrive at a solution to the dynamic system based on
time series behaviour and structural shocks, QZ decomposition is performed using Klein’s
algorithm.} As in Smets and Wouters (2007), we have set the depreciation
rate of capital, \( \delta = 0.025 \), the intratemporal elasticity of substitution in the labour market,
\( \theta^w = 3 \), and the steady–state exogenous spending-output ratio, \( \gamma_y = 0.18 \).
The priors for the parameters are kept equal across the models and the periods. In
Appendix B, we report the data, priors and posteriors for all 34 parameters.

Posterior Estimates

Explicitly modelling trend inflation seems to have only limited effect on the estimated
parameters. Looking at the results of the Metropolis–Hastings algorithm reported in
Tables 16-18, we observe major differences across different estimation periods and between
models that use and do not use indexation. When the only discriminating factor between
models is the presence of trend inflation (i.e., comparing TI with nTI and TnI with nTnI),
the posterior means for the structural parameters differ significantly only when we estimate
the model over the entire 1966–2004 period, as in Table 16. In this case, the estimated
response of the interest rate to inflation, \( \psi^* \), and the indexation of wages to past inflation,
\( \eta^w \), are greater in the case of TI than of nTI. Other significant differences arise concerning
the persistence of the wage shock and the elasticity of labour supply to real wages. In
comparing TnI and nTnI we also find various differences in the estimated parameters.
Looking at Table 18, instead, we can see that the estimated parameters are similar during the Great Moderation, as we expected. What is unexpected, however, is that there are limited differences during the 1966–1982 as well, as shown in Table 17.

<table>
<thead>
<tr>
<th></th>
<th>TI</th>
<th>nTI</th>
<th>TnI</th>
<th>nTnI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966-2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\iota^p$</td>
<td>0.32</td>
<td>0.31</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\iota^w$</td>
<td>0.60</td>
<td>0.46</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>0.87</td>
<td>0.87</td>
<td>0.87</td>
<td>0.89</td>
</tr>
<tr>
<td>$\omega_w$</td>
<td>0.81</td>
<td>0.84</td>
<td>0.89</td>
<td>0.82</td>
</tr>
<tr>
<td>$\psi_\pi$</td>
<td>1.37</td>
<td>1.26</td>
<td>1.13</td>
<td>1.23</td>
</tr>
<tr>
<td>$\psi_y$</td>
<td>0.20</td>
<td>0.20</td>
<td>0.19</td>
<td>0.21</td>
</tr>
<tr>
<td>$\rho_\iota$</td>
<td>0.27</td>
<td>0.28</td>
<td>0.29</td>
<td>0.25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>TI</th>
<th>nTI</th>
<th>TnI</th>
<th>nTnI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966-1982</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\iota^p$</td>
<td>0.43</td>
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<td>-</td>
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<tr>
<td>$\iota^w$</td>
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<td>0.60</td>
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<td>-</td>
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<tr>
<td>$\omega_p$</td>
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<td>0.80</td>
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<td>0.81</td>
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<tr>
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<td>0.77</td>
<td>0.78</td>
<td>0.78</td>
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<tr>
<td>$\psi_\pi$</td>
<td>1.39</td>
<td>1.45</td>
<td>1.38</td>
<td>1.41</td>
</tr>
<tr>
<td>$\psi_y$</td>
<td>0.14</td>
<td>0.13</td>
<td>0.13</td>
<td>0.14</td>
</tr>
<tr>
<td>$\rho_\iota$</td>
<td>0.23</td>
<td>0.24</td>
<td>0.23</td>
<td>0.23</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>TI</th>
<th>nTI</th>
<th>TnI</th>
<th>nTnI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1983-2004</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\iota^p$</td>
<td>0.18</td>
<td>0.17</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\iota^w$</td>
<td>0.55</td>
<td>0.54</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\omega_p$</td>
<td>0.87</td>
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<tr>
<td>$\omega_w$</td>
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<tr>
<td>$\psi_\pi$</td>
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<td>1.83</td>
<td>1.86</td>
<td>1.86</td>
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<tr>
<td>$\psi_y$</td>
<td>0.18</td>
<td>0.18</td>
<td>0.18</td>
<td>0.19</td>
</tr>
<tr>
<td>$\rho_\iota$</td>
<td>0.45</td>
<td>0.46</td>
<td>0.44</td>
<td>0.43</td>
</tr>
</tbody>
</table>

Table 3: Posterior means for the four models in the three periods.

In Table 3 we focus our attention on the parameters that are more relevant to monetary policy. When considering the full period with indexation, we observe that $\psi_\pi$, i.e., the reaction of the central bank to inflation is greater when we assume there is trend inflation. However, in the absence of indexation, the result is the opposite. The wage indexation parameters also differ significantly. Looking at the two sub-periods, instead, we see only small differences for $\psi_\pi$ during the Great Inflation period. We discuss our results in more detail in Appendix B.

**Short-run dynamics**

As we have seen in the previous section, the inclusion of trend inflation does not seem to dramatically affect the estimates of the structural parameters of the model. Next, we look to see if differences arise in the short-run dynamics of the model following a negative monetary policy shock (in Appendix C). Generally, we notice that whenever there is indexation to past inflation, taking trend inflation into account does not dramatically affect the short-run dynamics of the economy. Differences only emerge when we compare models with indexation to those without. Nevertheless, if we study the Great Moderation period, independent of the considered models, all the impulse response functions collapse. This general similarity across IRFs is at odds with what was proposed by Ascari and Ropele (2007). Thus, in order to check this result, we consider the trivariate textbook...
model as in Woodford (2003). We still conclude that trend inflation does not affect the short-run dynamics, as reported in Appendix E. We presume that this difference occurs for two reasons. Firstly, Ascarì and Ropele obtained the IRFs by keeping the calibrated parameters constant across different levels of trend inflation, but, as can be seen in Table 19, under Woodford’s trivariate model, the estimated parameters differ whether or not we take trend inflation into account. Secondly, the two authors observed the primary differences in IRFs when they compared the zero trend model with 8% or 10% trend inflation. Nevertheless, we never observed these levels of mean inflation in the sub-periods studied.

Forecasting

One of the features that make DSGE models attractive to central banks is their ability to produce reliable forecasts. The literature on DSGE forecasting evaluation has focused on point forecasts, predominantly evaluated by measuring the root mean square error (RMSE). Let us consider an in-sample forecast. The prediction horizon for the Great Inflation period covers the period from 1978 Q4 to 1980 Q4, whereas for both the Great Moderation period and the full sample, the forecast horizon is from 2000 Q4 to 2002 Q4. Tables 4 and 5 set out the forecast RMSEs, computed as:

\[ \text{RMSE} = \frac{1}{H} \sqrt{\sum_{h=1}^{H} (y_{t+h} - \hat{y}_{t+h|t})^2} \]

where the forecast horizon is \( H = 8 \). As a general observation, in most cases the four models generate similar RMSEs and the forecasts obtained for the period with low volatility, as in Table 4, show smaller errors than those for the period with high volatility, as in Table 5.

The preferred model for the full period employs trend inflation, but without indexation (TnI), whereas, for the Great Moderation, we tend to prefer a model that uses indexation, with or without trend inflation, since the forecast RMSEs in these two cases are very close. These results are attributable to the different features of the two periods. When we consider the full sample, we avoid indexation since we are in a period with heterogeneous values for inflation and so we do not want to be anchored too much to the past. On the other hand, when considering a more homogeneous period, such as that of the Great Moderation, there is a forecast gain in introducing dependence on the past through indexation. We are almost indifferent as to whether or not trend inflation is present, probably because the mean value of inflation over this period is very low and the gain made by adding trend inflation is not so clear. Finally, with respect to the Great Inflation period, we prefer the model that uses trend inflation, which is potentially able to capture the consistently high level of inflation during those years.

3 Comparison of models via loss function analysis

In this section we further compare the models by applying the quantitative evaluation procedure proposed by Schorfheide (2000). We will compare certain correlations and IRFs, penalizing deviations of each model from population characteristics in different ways. More specifically, we will study two loss functions: (i) the point distance, taken from Schorfheide (2000), which compares two points (in this case, the modes); (ii) the distribution distance, which we introduce as a novelty by applying the idea of entropy proposed by Ullah (1996) in order to summarize the divergence between two distributions.
Forecast Period: 2000 Q4 - 2002 Q4

<table>
<thead>
<tr>
<th>Estimation Period:</th>
<th>Y</th>
<th>nTI</th>
<th>TnI</th>
<th>nTnI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966 - 2004</td>
<td>Y</td>
<td>0.2238</td>
<td>0.2405</td>
<td>0.2032</td>
</tr>
<tr>
<td></td>
<td>π</td>
<td>0.0700</td>
<td>0.0806</td>
<td>0.0590</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>0.1490</td>
<td>0.1563</td>
<td>0.1348</td>
</tr>
</tbody>
</table>

Table 4: In-sample forecast RMSEs for output, inflation and interest rate using the posterior mean of the parameters for the sub-periods: 1966–2004 and 1983–2004.

Forecast Period: 1978 Q4 - 1980 Q4

<table>
<thead>
<tr>
<th>Estimation Period:</th>
<th>Y</th>
<th>nTI</th>
<th>TnI</th>
<th>nTnI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966 - 1982</td>
<td>Y</td>
<td>0.3671</td>
<td>0.3778</td>
<td>0.3548</td>
</tr>
<tr>
<td></td>
<td>π</td>
<td>0.1582</td>
<td>0.2095</td>
<td>0.2061</td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>0.2576</td>
<td>0.2739</td>
<td>0.3016</td>
</tr>
</tbody>
</table>

Table 5: In-sample forecast RMSEs for output, inflation and interest rate using the posterior mean of the parameters for the period 1966–1982.

3.1 Methodology

This comparison procedure consists of three steps. In the first step we compute the posterior distributions \( p(\theta_i|Y_T, \mathcal{M}_i) \) for model parameters \( \theta_i \) and the posterior model probabilities:

\[
\pi_{i,T} = \frac{\pi_{i,0} p(Y_T|\mathcal{M}_i)}{\sum_{i=0}^{7} \pi_{i,0} p(Y_T|\mathcal{M}_i)}
\]

where \( p(Y_T|\mathcal{M}_i) \) is the marginal data density:

\[
p(Y_T|\mathcal{M}_i) = \int p(Y_T|\theta_{(i)}, \mathcal{M}_i)p(\theta_{(i)}|\mathcal{M}_i)d\theta_{(i)}
\]

under model \( \mathcal{M}_i \), in which the labels are:

<table>
<thead>
<tr>
<th>i</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>DSGE TI</td>
</tr>
<tr>
<td>1</td>
<td>DSGE nTI</td>
</tr>
<tr>
<td>2</td>
<td>DSGE TnI</td>
</tr>
<tr>
<td>3</td>
<td>DSGE nTnI</td>
</tr>
<tr>
<td>4</td>
<td>VAR(1)</td>
</tr>
<tr>
<td>5</td>
<td>VAR(2)</td>
</tr>
<tr>
<td>6</td>
<td>VAR(3)</td>
</tr>
<tr>
<td>7</td>
<td>VAR(4)</td>
</tr>
</tbody>
</table>

This approach takes into account the potential misspecifications of the candidate model, since neither population moments nor IRFs are directly observable in the data. Therefore, a probabilistic representation of the data that serves as a benchmark for the comparison of DSGE models has to be constructed. To implement the procedure, we also include a structural VAR in our analysis because it is more densely parametrized than the DSGE models and therefore can avoid dynamic misspecifications\(^7\).

\(^7\)For a detailed discussion of VAR estimation and identification see Appendix D.
In order to compute the marginal data density for the DSGE models, we need to choose a numerical approximation approach. Taking into account the results of Schorfheide (2000) and An and Schorfheide (2007), we decide to approximate the marginal data density using Geweke (1999) modified harmonic mean estimator, as shown in Appendix D. Whereas for the Bayesian VAR, we can recover the marginal data density in closed-form solution since we adopt a natural conjugate prior.

In the second step we compute the population characteristics \( \phi \), which are a function \( f(\theta_{(i)}) \) of the model parameters \( \theta_{(i)} \). Based on the posterior distribution of \( \theta_{(i)} \), one can obtain a posterior for \( \phi \) conditional on model \( M_i \), denoted by \( p(\phi|Y_T, M_i) \). Since we are considering six different models, the overall posterior of \( \phi \) is given by the mixture:

\[
p(\phi|Y_T) = \sum_{i=0}^{7} p(\phi|Y_T, M_i)\pi_{i,T}
\]

where the posterior probabilities \( \pi_{i,T} \) during the previous step, determine the weights of the densities \( p(\phi|Y_T, M_i) \).

In the third step, we introduce a loss function in order to assess the ability of the DSGE models to replicate patterns of co-movements among key macroeconomic variables and impulse responses to structural shocks. The loss function penalizes deviations of model moments from the population characteristics that were computed in the two preceding steps. Given a specific definition of loss function, we need to provide a measure of how well model \( M_i \) reproduces the population characteristics \( \phi \), i.e., we want to compare the four DSGE models based on a posterior risk of deviating from the population characteristics. We will discuss two types of loss functions based on different ideas of divergence from the population characteristics.

**Loss function 1: point distance**

The first loss function we present, \( L^1(\phi, \tilde{\phi}) \), penalizes deviations of DSGE model predictions \( \tilde{\phi} \) from population characteristics \( \phi \). The prediction from DSGE model \( M_i \) is obtained as follows: we suppose a decision maker bases decisions exclusively on DSGE model \( M_i \) and the optimal predictor is thus:

\[
\hat{\phi}_i = \arg \min_{\phi \in \mathbb{R}^{m}} \int L^1(\phi, \tilde{\phi})p(\phi|Y_T, M_i)d\phi
\]

The loss function we use is taken from Schorfheide (2000) and it is defined as:

\[
L^1(\phi, \tilde{\phi}) = \mathbb{I}\left\{ p(\phi|Y^T) > p(\tilde{\phi}|Y^T) \right\}
\]

Indeed, it penalizes point predictions that lie in regions of low posterior density, i.e., \( L^1 \) identifies a distribution for the model akin to the distribution of the population characteristic if the two relative modes are close, on the assumption that the distributions are unimodal.

**Loss function 2: distribution distance**

While the first loss function reduces to a comparison of two individual points, we propose a different concept of loss function that allows us to use all the information held in the posterior distributions, i.e., \( p(\phi|Y_T) \) and \( p(\phi|Y_T, M_i) \).

When we study the distributions of the characteristics generated by the models, we observe
that they can be asymmetric. Therefore a simple comparison of summary statistics, like the mode in the previous example of $L^1(\hat{\phi}, \hat{\phi})$, could lead to a biased conclusion. Starting with this observation, we decide to consider the entire distribution, i.e., $p(\phi|Y^T, M_i)$. For this purpose, our loss function is inspired by the generalized entropy proposed by Ullah (1996). A divergence measure can be derived in terms of the ratio:

$$\lambda \equiv \frac{p(\phi|Y^T, M_i)}{p(\phi|Y^T)}$$

such that the difference in the distributions is large when $p(\phi|Y^T, M_i)$ is far from $p(\phi|Y^T)$ and is equal to 1 if and only if $p(\phi|Y^T, M_i) = p(\phi|Y^T)$. Therefore an alternative measure of divergence can be developed in terms of the information, or entropy, content in $\lambda$. Let us consider a convex function $g(\lambda)$ such that $g(1) = 0$. The information content in $p(\phi|Y^T, M_i)$ with respect to $p(\phi|Y^T)$, or the divergence of $p(\phi|Y^T, M_i)$ with respect to $p(\phi|Y^T)$, is then:

$$H_g (p(\phi|Y^T, M_i), p(\phi|Y^T)) = g \left( \frac{p(\phi|Y^T, M_i)}{p(\phi|Y^T)} \right)$$

This divergence measure can be considered an extension of the entropy function. Specifically, we consider the family of functions:

$$g_\alpha(\lambda) = \begin{cases} \frac{1}{\alpha - 1} \left[ 1 - \lambda^{\alpha - 1} \right] & \text{if } \alpha > 0 \text{ and } \alpha \neq 1 \\ -\log \lambda & \text{if } \alpha = 1 \end{cases}$$

where $g_\alpha(\lambda)$ has two characteristics: $g_0(1) = 0$ and $g_\alpha(\lambda)$ is monotonic. Therefore the loss function is:

$$L_\alpha^2(\phi, \phi, M_i) = \begin{cases} \frac{1}{\alpha - 1} \left[ 1 - \left( \frac{p(\phi|Y^T, M_i)}{p(\phi|Y^T)} \right)^{\alpha - 1} \right] & \text{if } \alpha > 0 \text{ and } \alpha \neq 1 \\ -\log \left( \frac{p(\phi|Y^T, M_i)}{p(\phi|Y^T)} \right) & \text{if } \alpha = 1 \end{cases}$$

A drawback to this entropy-based approach is the role of the support of distribution. Since one of the interpretations of entropy is the information that the distribution $p(\phi|Y^T, M_i)$ carries about the benchmark distribution $p(\phi|Y^T)$, the support of the former must lie in the support of the latter. Indeed, the part of the compared distribution that lies outside the support of the benchmark distribution embodies no information about the benchmark distribution\(^8\). Since, in our analysis, the distribution of the population characteristics always shows a larger variance than in the models compared, the support of the former is always greater than the support of the latter and we do not suffer this drawback.

**Risk function**

Let us now define the posterior risk for the two types of loss function. Under the first definition of loss the DSGE models are judged according to the expected loss of $\hat{\phi}$ under the overall posterior distribution $p(\phi|Y_T)$, with the posterior risk function:

$$\mathcal{R}^1(\hat{\phi}|Y_T) = \int L^1(\phi, \hat{\phi}) p(\phi|Y_T) d\phi$$

\(^8\)This drawback is particularly severe when $\alpha = 1$. In this case, the comparison can be made only when the two distributions have the same support. Since the implied loss function involves a logarithmic function, if the support of $p(\phi|Y^T, M_i)$ is larger, then we have $L_1^1(\phi, \phi, M_i) = -\log(\infty)$, or if the support of $p(\phi|Y^T)$ is larger, then $L_1^1(\phi, \hat{\phi}, M_i) = -\log(0)$. 
where $\mathcal{R}^{1}(\hat{\phi}_{i}|Y_{T}) \in [0,1]$. The model $i^{th}$ is preferred to the model $j^{th}$ if:

$$\mathcal{R}^{1}(\hat{\phi}_{i}|Y_{T}) < \mathcal{R}^{1}(\hat{\phi}_{j}|Y_{T})$$

Instead, according to the second definition of loss function, we need to summarize the information relative to the distance between the kernel distributions of the characteristics for the population and the $i^{th}$ model. Since the loss function takes both positive and negative values, we propose two functions in order to summarize the posterior risk, i.e., the sum of the squared values or the sum of the absolute values:

$$\mathcal{R}_{a}^{S}(\phi_{M_{i}}|Y_{T}) = \int \left[L^{2}_{a}(\phi, \phi_{M_{i}})p(\phi|Y_{T})\right]^{2} d\phi$$

or

$$\mathcal{R}_{a}^{A}(\phi_{M_{i}}|Y_{T}) = \int |L^{2}_{a}(\phi, \phi_{M_{i}})p(\phi|Y_{T})| d\phi$$

The final measure of whether one DSGE model is a better fit than another is given by the ratio of the posterior risks associated with model $(M_{i})$ and model $(M_{j})$:

$$\text{Ratio}^{S}_{a} = \frac{\mathcal{R}_{a}^{S}(\phi_{M_{i}}|Y_{T})}{\mathcal{R}_{a}^{S}(\phi_{M_{j}}|Y_{T})}$$

$$\text{Ratio}^{A}_{a} = \frac{\mathcal{R}_{a}^{A}(\phi_{M_{i}}|Y_{T})}{\mathcal{R}_{a}^{A}(\phi_{M_{j}}|Y_{T})}$$

When the ratio is smaller than one, $M_{i}$ is preferred to $M_{j}$, whereas the converse is true when the ratio is higher than one. We take into account the following ratios:

$$\frac{M_{1}}{M_{0}} \text{ and } \frac{M_{3}}{M_{2}}$$

### 3.2 Results

The model with the highest value for the marginal data density is VAR(4) in all periods, as shown in Tables 6, 7, and 8. In comparing the DSGE models, we can conclude that, in the sub-periods, the marginal data densities, conditional on the presence or absence of indexation, are very close, especially during the Great Moderation. A second general observation is that models that employ indexation better fit the data than those without indexation.

| Model | $i$ | Prior Prob. $\pi_{i,0}$ | $\ln p(Y_{T}^{*}|Y_{T},M_{i})$ | Harmonic Mean | Post Prob. $\pi_{i,T}$ |
|-------|----|----------------|-----------------|---------------|----------------|
| TI    | 0  | 1/8            | -               | -741.5273     | $1e^{-10}$     |
| nTI   | 1  | 1/8            | -               | -745.9096     | $2e^{-10}$     |
| TnI   | 2  | 1/8            | -               | -756.0987     | $8e^{-15}$     |
| nTnI  | 3  | 1/8            | -               | -746.0043     | $2e^{-10}$     |
| VAR(1)| 4  | 1/8            | -773.5461       | -             | $2e^{-22}$     |
| VAR(2)| 5  | 1/8            | -744.7351       | -             | $7e^{-10}$     |
| VAR(3)| 6  | 1/8            | -736.2191       | -             | $3e^{-06}$     |
| VAR(4)| 7  | 1/8            | -723.6860       | -             | ~ 1           |

Table 6: Results for the first step, full sample (1966–2004).

Tables 9 (TI versus nTI) and 10 (TnI versus nTnI) show the results of the comparisons, considering the correlations among output, inflation and interest rate in terms of $R^{1}(\hat{\phi}_{i}|Y_{T})$
Table 7: Results for the first step, Great Inflation (1966–1982).

<table>
<thead>
<tr>
<th>Model</th>
<th>i</th>
<th>Prior Prob. ( \pi_{i,0} )</th>
<th>( \ln p(Y_t^\prime Y_{\cdot t}, M_i) )</th>
<th>Harmonic Mean</th>
<th>Post Prob. ( \pi_{i,T} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TI</td>
<td>0</td>
<td>1/8</td>
<td>-</td>
<td>-349.7733</td>
<td>7 ( \times 10^{-24} )</td>
</tr>
<tr>
<td>nTI</td>
<td>1</td>
<td>1/8</td>
<td>-</td>
<td>-347.5604</td>
<td>6 ( \times 10^{-23} )</td>
</tr>
<tr>
<td>TnI</td>
<td>2</td>
<td>1/8</td>
<td>-</td>
<td>-357.0488</td>
<td>5 ( \times 10^{-27} )</td>
</tr>
<tr>
<td>nTnI</td>
<td>3</td>
<td>1/8</td>
<td>-</td>
<td>-354.9165</td>
<td>4 ( \times 10^{-26} )</td>
</tr>
<tr>
<td>VAR(1)</td>
<td>4</td>
<td>1/8</td>
<td>-346.0916</td>
<td>-</td>
<td>2 ( \times 10^{-22} )</td>
</tr>
<tr>
<td>VAR(2)</td>
<td>5</td>
<td>1/8</td>
<td>-321.9535</td>
<td>-</td>
<td>8 ( \times 10^{-12} )</td>
</tr>
<tr>
<td>VAR(3)</td>
<td>6</td>
<td>1/8</td>
<td>-309.6899</td>
<td>-</td>
<td>1 ( \times 10^{-06} )</td>
</tr>
<tr>
<td>VAR(4)</td>
<td>7</td>
<td>1/8</td>
<td>-296.4994</td>
<td>-</td>
<td>~ 1</td>
</tr>
</tbody>
</table>

Table 8: Results for the first step, Great Moderation (1983–2004).

<table>
<thead>
<tr>
<th>Model</th>
<th>i</th>
<th>Prior Prob. ( \pi_{i,0} )</th>
<th>( \ln p(Y_t^\prime Y_{\cdot t}, M_i) )</th>
<th>Harmonic Mean</th>
<th>Post Prob. ( \pi_{i,T} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>TI</td>
<td>0</td>
<td>1/8</td>
<td>-</td>
<td>-212.9269</td>
<td>2 ( \times 10^{-05} )</td>
</tr>
<tr>
<td>nTI</td>
<td>1</td>
<td>1/8</td>
<td>-</td>
<td>-213.6370</td>
<td>1 ( \times 10^{-05} )</td>
</tr>
<tr>
<td>TnI</td>
<td>2</td>
<td>1/8</td>
<td>-</td>
<td>-217.3644</td>
<td>2 ( \times 10^{-07} )</td>
</tr>
<tr>
<td>nTnI</td>
<td>3</td>
<td>1/8</td>
<td>-</td>
<td>-217.6114</td>
<td>2 ( \times 10^{-07} )</td>
</tr>
<tr>
<td>VAR(1)</td>
<td>4</td>
<td>1/8</td>
<td>-237.1459</td>
<td>-</td>
<td>6 ( \times 10^{-16} )</td>
</tr>
<tr>
<td>VAR(2)</td>
<td>5</td>
<td>1/8</td>
<td>-209.9751</td>
<td>-</td>
<td>4 ( \times 10^{-04} )</td>
</tr>
<tr>
<td>VAR(3)</td>
<td>6</td>
<td>1/8</td>
<td>-207.3185</td>
<td>-</td>
<td>0.0061</td>
</tr>
<tr>
<td>VAR(4)</td>
<td>7</td>
<td>1/8</td>
<td>-202.2180</td>
<td>-</td>
<td>0.9935</td>
</tr>
</tbody>
</table>

Table 9: Percentage of cases in which TI is preferred to nTI, considering the correlations among \( Y_t \) and \( Y_{t-h} \) for \( h = 0, 1, \ldots, 12 \).

We do not find clear evidence that one particular model should always be preferred over the others, whether across different periods or within each period. If we consider the comparison of the models that use indexation, we notice that, in the two sub-periods, a model with trend in inflation seems to be preferred, although there is some ambiguity with respect to the Great Inflation. Contrast this with the full period, for which a model
Table 10: Percentage of cases in which $TnI$ is preferred to $nTnI$, considering the correlations among $t$ and $t-h$ ($h = 0, \ldots, 12$). $R^1$ indicates the first type of risk function and $Ratio^2$ the ratio of the second type of posterior risk.

<table>
<thead>
<tr>
<th>$\mathcal{M}_2 \succ \mathcal{M}_3$</th>
<th>$R^1$</th>
<th>$Ratio^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Y_{t-h}$</td>
<td>$\pi_{t-h}$</td>
</tr>
<tr>
<td>1966-2004</td>
<td>$Y_t$</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>$\pi_t$</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>$R_t$</td>
<td>0.08</td>
</tr>
<tr>
<td>1966-1982</td>
<td>$Y_t$</td>
<td>0.15</td>
</tr>
<tr>
<td></td>
<td>$\pi_t$</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>$R_t$</td>
<td>0.54</td>
</tr>
<tr>
<td>1983-2004</td>
<td>$Y_t$</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>$\pi_t$</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>$R_t$</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Moving on to the second comparison ($TnI$ versus $nTnI$), we notice that, in the absence of rigidity caused by indexation, a model with trend inflation is preferred for the Great Inflation period. The other cases are more ambiguous.

We also want to stress the difference between the two classes of loss functions and the associated posterior risk. For example, let us consider the correlation between $Y_t$ and $\pi_t$ for the period 1966–1983, reported in 1.

![Box plots for the distribution of $\Gamma_{Y_t, \pi_t}$ obtained for the four DSGE models. The solid lines are the associated modes. The shaded green area corresponds to the interval $[0.25; 0.75]$ for the population characteristic and the solid green line is the relative mode.](image-url)
Under the first definition of loss function, the associated posterior risk suggests that \( nTI \) is preferred to \( TI \), whereas under the second definition of loss function, the ratio of the posterior risks demonstrates that \( TI \) is to be preferred to \( nTI \). The result for the first type of loss function is driven by the nearness of the population characteristics mode and the mode for the model without indexation, even if the distribution of the correlation implied by the \( TI \) model is closer to the distribution of population characteristics than those generated by the \( nTI \) models, as shown by the box plots. It is also important to consider this information on distribution because there are no restrictions that guarantee the normality of the distributions, and all the implied properties, for the correlations.

The same exercise is repeated for cumulative impulse response functions over ten periods for a negative monetary policy shock. In Tables 11 and 12 we summarize the results for the comparison between \( TI \) and \( nTI \), under the two types of posterior risk, respectively. Whereas in Tables 13 and 14, we compare \( TnI \) and \( nTnI \). In both cases, for the loss function based on entropy, we consider the sum of the absolute values with \( \alpha = 2 \), i.e. \( Ratio^2_\alpha \).

For the first comparison we observe that, with respect to the Great Moderation, the two models are almost identical, as we have already seen with the IRFs plots. The main differences appear with respect to the Great Inflation period where the model using trend inflation very well fits the response of some of the variables. For the second comparison, under the first type of loss function we prefer a model that uses trend inflation for the Great Inflation period. As for the other comparison, the two models are identical for the Great Moderation. On the other hand, the results for the full period are puzzling since they strictly depend on the type of loss function adopted.

<table>
<thead>
<tr>
<th>Period</th>
<th>Models</th>
<th>Output</th>
<th>Inflation</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966 - 2004</td>
<td>( M_0 )</td>
<td>0.60</td>
<td>0.16</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td>( M_1 )</td>
<td>0.53</td>
<td>0.26</td>
<td>0.39</td>
</tr>
<tr>
<td>1966 - 1982</td>
<td>( M_0 )</td>
<td>0.01</td>
<td>0.80</td>
<td>0.04</td>
</tr>
<tr>
<td></td>
<td>( M_1 )</td>
<td>0.30</td>
<td>0.68</td>
<td>0.15</td>
</tr>
<tr>
<td>1983 - 2004</td>
<td>( M_0 )</td>
<td>1</td>
<td>0.20</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>( M_1 )</td>
<td>0.99</td>
<td>0.16</td>
<td>0.99</td>
</tr>
</tbody>
</table>

Table 11: Results under \( R^1 \) for the IRFs to a monetary shock for the three periods assuming indexation. The preferred model is the one that shows the lower value.

<table>
<thead>
<tr>
<th>Period</th>
<th>Output</th>
<th>Inflation</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966 - 2004</td>
<td>2.51</td>
<td>19.22</td>
<td>1.05</td>
</tr>
<tr>
<td>1966 - 1982</td>
<td>0.63</td>
<td>3.84</td>
<td>0.55</td>
</tr>
<tr>
<td>1983 - 2004</td>
<td>2.23</td>
<td>0.82</td>
<td>1.29</td>
</tr>
</tbody>
</table>

Table 12: Results under \( Ratio^2_\alpha \) for the IRFs to a monetary shock for the three periods and in comparing \( TI \) vs \( nTI \). Where the value is less than one, \( nTI \) is preferred to \( TI \).
### Table 13: Results under $R^1$ for the IRFs to a monetary shock for the three periods without indexation. The preferred model is the one that shows the lower value.

<table>
<thead>
<tr>
<th>Period</th>
<th>Models</th>
<th>Output</th>
<th>Inflation</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966 - 2004</td>
<td>$M_2$</td>
<td>0.49</td>
<td>0.32</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>$M_3$</td>
<td>0.48</td>
<td>0.28</td>
<td>0.15</td>
</tr>
<tr>
<td>1966 - 1982</td>
<td>$M_2$</td>
<td>0.27</td>
<td>0.35</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>$M_3$</td>
<td>0.414</td>
<td>0.45</td>
<td>0.11</td>
</tr>
<tr>
<td>1983 - 2004</td>
<td>$M_2$</td>
<td>0.33</td>
<td>0.17</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td>$M_3$</td>
<td>0.33</td>
<td>0.17</td>
<td>0.67</td>
</tr>
</tbody>
</table>

### Table 14: Results under Ratio$^3$ for the IRFs to a monetary shock for the three periods and in comparing $T_{nI}$ vs $nT_{nI}$. Where the value is less than one, $nT_{nI}$ is preferred to $T_{nI}$.

<table>
<thead>
<tr>
<th>Period</th>
<th>Output</th>
<th>Inflation</th>
<th>Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1966 - 2004</td>
<td>1.25</td>
<td>5.86</td>
<td>1.27</td>
</tr>
<tr>
<td>1966 - 1982</td>
<td>0.0004</td>
<td>0.29</td>
<td>1.71</td>
</tr>
<tr>
<td>1983 - 2004</td>
<td>1.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

### 4 Conclusion

The assumption of positive levels of steady-state inflation in a New Keynesian model shrinks the determinacy region and affects the short-run dynamics with respect to the standard textbook model approximated around zero steady-state inflation. Nevertheless, the empirical relevance of trend inflation in estimating a DSGE is not clear since most of the analyses are performed on calibrated models. In this analysis we estimate, using Bayesian techniques, four declinations of a medium-scale New Keynesian DSGE model with real and nominal frictions and we study the quantitative implications of trend inflation over different time horizons, which are identified by different average levels of inflation. More specifically, we compare models both with and without trend inflation, conditional on the presence (or absence) of partial indexation to past inflation. The posterior estimates for the structural parameters and the short-run dynamics do not appear to be affected by the inclusion of trend inflation when we analyse the three time horizons.

Beyond these introductory observations, we compare the models based on two loss functions: the first based on a point distance criterion and the second on the idea of entropy. In comparing whether the four DSGE models fit the data by using marginal data density, we observe that in the sub-periods the marginal data densities, conditional on the presence or absence of indexation, are very close, especially during the Great Moderation. Moreover, models that employ indexation better fit the data than those without indexation, even when trend inflation is used.

The results for the cross-correlations and IRFs based on loss functions do not present clear evidence for preferring one model over another, whether across different periods or within each period.

Taking into account the correlations, we observe that, assuming there is indexation, the model with trend inflation appears preferable for the two homogeneous sub-periods, whereas for the full period a model without trend inflation should be chosen. Therefore, models with trend inflation are less precise in the presence of an underlying change of regime that is not explicitly modelled. It is important to note that, in absence of rigidity caused by indexation, a model with trend inflation is preferred for the Great Inflation period.

With respect to the IRF analysis, we find that the two models are almost identical for
the Great Moderation period. Contrast this with the Great Inflation period, for which a model with trend inflation very well fits the response of some of the variables.

A final comment should be made on our comparison procedure: the two classes of loss functions and the associated posterior risk verify different characteristics and thus the two results are not always aligned, making it difficult to identify a model that should be always preferred.

In conclusion, we do not find strong evidence that a model with trend inflation should always be preferred with respect to estimated medium-scale DSGE. During periods of high inflation or when a backward-looking component is not incorporated in the model that is indexed to past inflation, using a model that employs trend inflation can improve the analysis. Nevertheless, where there is uncertainty concerning the change of an inflation regime, such as the recent drop, we suggest adopting a traditional approach that does not use trend inflation.
Appendix A: the general model with trend inflation and partial indexation to past inflation for price and wage

In this section we present the log-linearized equations that characterize the model. As compared with Smets and Wouters (2007), the main difference lies in the fact that we consider a steady-state level of gross inflation, $\pi_*$, greater than one. Given that, we should in particular consider that price and wage dispersion also affect the dynamic up to the first-order approximation.

The log-linearized aggregate resource constraint of this closed economy is given by:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \epsilon_t^g$$

where $y_t$ is real GDP, absorbed by real private consumption $c_t$, real private investments $i_t$, capital utilization rate $z_t$, and exogenous government spending $\epsilon_t^g$. The parameter $c_y$ is the steady-state consumption-output ratio and $i_y$ is the steady-state investment-output ratio, where:

$$c_y = 1 - g_y - i_y$$

and $g_y$ is the steady-state exogenous spending-output ratio. The steady-state investment-output ratio is determined by:

$$i_y = (\gamma - 1 + \delta)k_y$$

where $k_y$ is the steady-state capital-output ratio, $\gamma$ is the steady-state labour-augmenting growth rate, and $\delta$ is the depreciation rate of capital; the parameter $z_y$ is equal to $r_k^* k_y$, where $k_y = \frac{k^*}{y^*}$.

The dynamics of consumption follow from the consumption Euler equation given by:

$$c_t = c_1 c_{t-1} + (1 - c_1)E_t c_{t+1} + c_2 (l_t - E_t l_{t+1}) - c_3 (r_t - E_t \pi_{t+1} + \epsilon^b)$$

where $l_t$ is hours worked, $r_t$ is the nominal interest rate and the coefficients are:

\[
\begin{align*}
    c_1 &= \frac{\lambda}{\gamma} \left(1 + \frac{\lambda}{\gamma}\right) \\
    c_2 &= \left[ (\sigma_c - 1) \frac{w^h L_*}{\sigma_c} \right] \frac{1}{\sigma_c \left(1 + \frac{\lambda}{\gamma}\right)} \\
    c_3 &= \left(1 - \frac{\lambda}{\gamma}\right) \frac{1}{\sigma_c \left(1 + \frac{\lambda}{\gamma}\right)}
\end{align*}
\]

where $\lambda$ measures external habit formation, $\sigma_c$ is the inverse of the elasticity of intertemporal substitution for constant labour, while $\frac{w^h L_*}{\sigma_c}$ is the steady-state hourly real wage bill to consumption ratio.\(^9\)

The log-linearized investment Euler equation is given by:

$$i_t = i_1 i_{t-1} + (1 - i_1)E_t i_{t+1} + i_2 q_t + \epsilon^i_t$$

where $q_t$ is the real value of the existing capital stock, while $\epsilon^i_t$ is an exogenous investment-specific technology variable. The parameters are given by:

\[
\begin{align*}
    i_1 &= \frac{1}{1 + \beta^2 \gamma^{1-\sigma_c}} \\
    i_2 &= \frac{1}{1 + \beta^2 \gamma^{1-\sigma_c}}
\end{align*}
\]

\(^9\) If $\sigma_c = 1$ (log-utility) and $\lambda = 1$ (no external habit), then the above equation reduces to the familiar purely forward-looking consumption Euler equation.
where $\beta$ is the discount factor used by households and $\phi$ is the steady-state elasticity of the capital adjustment cost function.

The dynamic equation for the value of the capital stock is:

$$q_t = q_1 E_t q_{t+1} + (1 - q_1) E_t r^k_{t+1} - (r_t - E_t \tau_{t+1} + \epsilon^i_t)$$

where $r^k_t$ is the rental price of capital. The parameter $q_1$ is given by:

$$q_1 = \beta \gamma^{-\sigma_c}(1 - \delta)$$

Turning to the supply side of the economy, the log-linearized aggregate production function can be expressed as:

$$s^p_t + y_t = \alpha k^s_t + (1 - \alpha) l^d_t + \epsilon^s_t$$

where $k^s_t$ is capital services used in production, $l^d_t$ represents labour demand and $\epsilon^s_t$ an exogenous total factor productivity variable. Parameter $\alpha$ reflects the share of capital in production, while $s^p_t$ is the relative price dispersion evolution due to the Dixit-Stiglitz aggregator:

$$s^p_t = \theta_p\left(\pi^{1-\iota_p} - 1\right) - \frac{\omega_p \pi^{(\theta_p - 1)(1-\iota_p)}}{1 - \omega_p \pi^{(\theta_p - 1)(1-\iota_p)}} (\pi_t - \iota_p \pi_{t-1}) + \omega_p \theta_p (1-\iota_p) s^p_{t-1}$$

where $s_t$ has a lower bound equal to one and $\pi_*$ is the inflation at the steady-state. From the Calvo pricing mechanism, $1 - \omega_p$ is the probability that a firm can re-optimize its price at time $t$, whereas $\theta_p > 1$ is the parameter of the Dixit-Stiglitz aggregator over the $j$-ths firms:

$$\tilde{P}_t = \left[ \int \tilde{P}^{1-\theta_p}_{j,t} dj \right]^{1-\theta_p}$$

Moreover $\iota_p \in [0, 1]$ is the price index such that non-optimizing firms could re-adjust their prices to past inflation:

$$\tilde{Y}_{t,j} = \left[ \frac{\tilde{P}^{1-\theta_p}_{j,t} \tilde{\Omega}_{t,j}^{-1}}{\tilde{P}^{-1}_{j,t} \tilde{\Omega}_{t,j}^{-1}} \right]^{-\theta_p} \tilde{Y}_{t+j}$$

$$\tilde{\Omega}_{t,j}^{-1} = \Pi_{j=0}^{\infty} \tilde{\alpha}_{t+j}^{1-\theta_p}$$

The capital services variable is used to reflect the fact that newly installed capital becomes effective only with a one period lag. This means that:

$$k^s_t = k_{t-1} + z_t$$

where $k_t$ is the installed capital. The degree of capital utilization is determined from the cost minimization by households that provide capital services, and it is therefore a positive function of the rental rate of capital. Specifically,

$$z_t = z_1 r^k_t$$

where

$$z_1 = \frac{1 - \psi}{\psi}$$

and $\psi$ is a positive function of the elasticity of the capital adjustment cost function and normalized to be between zero and one.

The log-linearized equation that describes the development of installed capital is:

$$k_t = k_1 k_{t-1} + (1 - k_1) i_t + k_2 \epsilon^i_t$$
The two parameters are given by

\[ k_1 = \frac{1 - \delta}{\gamma} \]
\[ k_2 = \left( 1 - \frac{1 - \delta}{\gamma} \right) (1 + \beta \gamma^{1-\sigma_c}) \gamma^{2 \phi} \]

In the monopolistically competitive goods market, the price markup \( \mu^p_t \) is equal to negative

\[ \mu^p_t = \alpha (k_1^s - l^d_t) + \epsilon^s_t - w_t \]

where the real wage is represented by \( w_t \).

Cost minimization by firms also implies that the rental rate of capital is related to the capital–labour ratio and to the real wage, according to:

\[ r^k_t = -(k_t - t^d_t) + w_t \]

In the monopolistically competitive labour market the wage markup is equal to the difference between the real wage and the marginal rate of substitution between labour and consumption:

\[ \mu^w_t = w_t \left[ \sigma_t l_t + \frac{c_t - \frac{\lambda}{\gamma} c_{t-1}}{1 + \frac{\lambda}{\gamma}} \right] \]

where \( \sigma_t \) is the elasticity of labour supply with respect to the real wage.

Market clearing on the labour market implies:

\[ L_t = \int_0^1 \left( \frac{W_t(j)}{W_t} \right)^{\theta_w} L_i d_j \]
\[ = \tilde{s}^w_i L_t^d \Rightarrow l_t = s^w_i + t^d_i \]

where \( \tilde{s}^w_i \) is the relative wage dispersion, characterized by the log-linearized dynamics:

\[ s^w_t = -\theta^w (1 - \omega_w) \pi^w \sigma_t \left( w^p_t - w_t \right) + \omega_w \pi^w (1 - \omega_w) \theta^w + \]
\[ s^w_{t-1} + \theta^w (\pi_t - \omega_t \pi_{t-1}) - \theta^w (w_{t-1} - w_t) \]

Firms in the intermediate sector produce a continuum of goods indexed by \( i \in [0, 1] \) in a monopolistic competitive environment. Each intermediate good is produced by a single firm.

Prices are assumed to be sticky à la Calvo, indeed only a fraction \( 1 - \omega_p \) of firms can optimally set the price \( \tilde{P}^o_{i,t} \) at time \( t \), which is chosen to maximize the expected present discounted value of profits:

\[ \max_{\tilde{P}^o_{i,t}} \sum_{j=0}^{\infty} (\omega_p \beta)^j \tilde{E}_{t+j}[\frac{\tilde{P}^o_{i,j}}{\tilde{P}_{t+j}}] \tilde{\Omega}_{t,j}^{p} - \frac{\tilde{\mu}^p_{t}}{\tilde{\mu}^p_{t+j}} \tilde{Y}_{t,j} \]

subject to the aggregate demand for good \( i \):

\[ \tilde{Y}_{t,j} = \left[ \frac{\tilde{P}^o_{i,j}}{\tilde{P}_{t+j}} \tilde{\Omega}_{t,j}^{p} \right]^{-\theta_p} \tilde{Y}_{t,j} \]

where \( \tilde{\mu}_t \) is the price markup and \( \theta^p > 1 \) is the parameter of the Dixit–Stiglitz aggregator over the \( j - ths \) firms, i.e.:

\[ \tilde{P}_t = \left[ \int \tilde{P}_{j,t}^{1-\theta^p} d_j \right]^{\frac{1}{1-\theta^p}} \]
Moreover, \( \iota_p \in [0,1] \) is the price indexation parameter such that the non optimizing firms can re-adjust their prices to past inflation:

\[
\tilde{\Omega}_{t,t+j-1} = \Pi_{j=0}^{\iota_p} \pi_{t+j-1}^{\iota_p}
\]

The first-order condition, after rearrangement, is:

\[
0 = E_t \sum_{j=0}^{\infty} (\omega_p \beta \gamma^{1-\sigma_c})^j \xi_t \tilde{p}_{t+j}^{-\iota_p} \left( \prod_{k=1}^{j} \frac{\tilde{\pi}_{t+k-1}^{\iota_p}}{\pi_{t+k}} \right)^{-\iota_p} \tilde{y}_{t+j} \left[ \frac{\theta_p - 1}{\theta_p} \tilde{p}_t^{\iota_p} \left( \prod_{k=1}^{j} \frac{\tilde{\pi}_{t+k-1}^{\iota_p}}{\pi_{t+k}} \right) - \frac{\mu_s^p}{\iota_p \mu_t} \right]
\]

which can be rewritten as:

\[
\frac{\theta_p - 1}{\theta_p} f_t^{1,p} = f_t^{2,p}
\]

where we define:

\[
f_t^{1,p} = \tilde{y}_t (\tilde{p}_t^{\iota_p})^{-\iota_p} + \omega_p \beta \gamma^{1-\sigma_c} E_t \left[ \left( \frac{\tilde{\pi}_t^{\iota_p}}{\pi_{t+1}} \right)^{1-\iota_p} \left( \frac{\tilde{p}_t^{\iota_p}}{\tilde{p}_t^{\iota_p+1}} \right)^{-\iota_p} \tilde{\xi}_{t+1} \right] f_t^{1,p}
\]

and

\[
f_t^{2,p} = \frac{\tilde{y}_t}{\mu_t} (\tilde{p}_t^{\iota_p})^{-\iota_p-1} + \omega_p \beta \gamma^{1-\sigma_c} E_t \left[ \left( \frac{\tilde{\pi}_t^{\iota_p}}{\pi_{t+1}} \right)^{1-\iota_p} \left( \frac{\tilde{p}_t^{\iota_p}}{\tilde{p}_t^{\iota_p+1}} \right)^{-\iota_p-1} \tilde{\xi}_{t+1} \right] f_t^{2,p}
\]

Thus, the log-linearization of the generalized NKPC is given by:

\[
f_t^{1,p} = f_t^{2,p}
\]

where

\[
f_t^{1,p} = (1 - A_1^p) \left[ \theta_p \tilde{p}_t^{\iota_p} + y_t \right] + A_1^p \left[ \frac{\iota_p}{\theta_p + 1} \pi_t + \theta_p \tilde{p}_t^{\iota_p} - \theta_p \pi_{t+1} - \theta_p \pi_{t+1} + f_t^{1,p} + \tilde{\xi}_{t+1} \right]
\]

and

\[
f_t^{2,p} = (1 - A_2^p) \left[ (\theta_p - 1) \tilde{p}_t^{\iota_p} + y_t - \mu_t^p \right] + A_2^p \left[ \theta_p \tilde{p}_t^{\iota_p} - (\theta_p - 1) \tilde{p}_t^{\iota_p} + (1 - \theta_p) \tilde{p}_t^{\iota_p+1} + (1 - \theta_p) \tilde{p}_t^{\iota_p+1} + f_t^{2,p} + \tilde{\xi}_{t+1} \right]
\]

and the coefficients are:

\[
A_1^p = \omega_p \beta \gamma^{1-\sigma_c} \left[ \frac{1}{\pi_t} \right] (1-\iota_p)(\theta_p+1)
\]

\[
A_2^p = \omega_p \beta \gamma^{1-\sigma_c} \left[ \frac{1}{\pi_t} \right] (1-\iota_p)\theta_p
\]

whereas \( p_t^{\iota_p} \), the optimal price, evolves according to:

\[
p_t^{\iota_p} = \frac{\omega_p \pi_t^{\theta_p-1}(1-\iota_p)}{1 - \omega_p \pi_t^{\theta_p-1}(1-\iota_p)} (\tilde{\pi}_t - \iota_p \tilde{\pi}_{t-1})
\]

By setting \( \pi_* = 1 \), it is possible to recover the standard NKPC, named Hybrid NKPC in Ascari and Ropele (2007):

\[
\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3 \mu_t^p
\]
The first order condition

\[ \pi_1 = \frac{\pi}{1 + \beta \gamma^{1-\sigma}} \]

\[ \pi_2 = \frac{\beta \gamma^{1-\sigma}}{1 + \beta \gamma^{1-\sigma}} \]

\[ \pi_3 = (1 - \beta \gamma^{1-\sigma})(1 - \omega_p) \]

Labour decisions are made by a union, which supplies labour monopsonistically to a continuum of labour markets of measure 1, indexed by \( l \in [0, 1] \), and sets wages according to the Calvo model. The problem of the union is:

\[
\max_{\tilde{W}_t(l)} E_t \sum_{s=0}^{\infty} \left( \omega_w \beta^s \right) T^{t+s} \tilde{P}_t \left( \tilde{W}_t(l) - \tilde{W}_t^h \right) L_t^d \]

subject to the demand curve:

\[
L_{t+s}(l) = \left[ \frac{\tilde{W}_t(l)}{W_t} \right]^{-\theta^w} L_t^d
\]

and wage setting, through the optimal wage \( \tilde{W}_t^o(l) \):

\[
\tilde{W}_t(l) = \tilde{W}_t^o(l) \prod_{k=1}^{s} \gamma_{t+s-k}^{t+w}
\]

Here \( \tilde{W}_t(l) \) denotes the nominal wage charged by the union in labour market \( l \) at time \( t \), \( \tilde{W}_t^o \) is an index of nominal wages prevailing in the economy, \( \tilde{W}_t^p \) is the nominal wage received by the households, \( L_t^d \) is a measure of aggregate labour at time \( t \) demanded by firms, \( \tilde{P}_t \) is the nominal price index, \( \beta \) is the subjective discount factor, \( \omega_w \) is the probability of not re-optimizing wages, \( \gamma \) is the wage indexation to past consumer price inflation, \( \beta \) represents the labour-augmenting deterministic growth rate and \( \sigma \) is the inverse of the elasticity of intertemporal substitution for constant labour, whereas the stochastic discount factor, \( \tilde{\Xi}_{t+s} \), is defined as:

\[
\tilde{\Xi}_{t+s} = \tilde{\Xi}_{t+s} - \tilde{\Xi}_t = \gamma^{\sigma}(t) \tilde{\xi}_t
\]

The first order condition is:

\[
0 = E_t \sum_{s=0}^{\infty} \left( \omega_w \beta^s \right) T^{t+s} \tilde{P}_t \left( \tilde{W}_t(l) - \tilde{W}_t^h - \theta^w \left[ \tilde{W}_t(l) \right]^{-\theta^w} L_t^d \right) \]

where

\[
\tilde{X}_{s} = \prod_{k=1}^{s} \gamma_{t+s-k}^{t+w}
\]

Defining \( \tilde{W}_t = \gamma^{t} \tilde{w}_t \), after applying some algebra:

\[
0 = E_t \sum_{s=0}^{\infty} \left( \omega_w \beta^{1-\sigma} \right)^{s} T^{t+s} \left( \tilde{w}_t^o \tilde{w}_t^{1,w} \right) \]

Starting from the first order condition, we can write the wage equation as:

\[
\frac{\theta^w - 1}{\theta^w} \tilde{w}_t^o \tilde{f}_t^{1,w} = \tilde{f}_t^{2,w}
\]
where $\tilde{f}_{t}^{1,w}$ and $\tilde{f}_{t}^{2,w}$ are defined as:

$$\tilde{f}_{t}^{1,w} = \left(\frac{\tilde{w}_t}{\tilde{w}_t^0}\right)^{\theta^w} \tilde{L}_t + (\omega_w \beta \gamma^{1-\sigma}) \left(\frac{\tilde{\pi}_{t+1}}{\tilde{\pi}_t^w}\right)^{\theta^w-1} \left(\frac{\tilde{w}_t^0}{\tilde{w}_t^0}\right)^{\theta^w} \tilde{\xi}_{t+1/|f_{t+1}^{1,w}}$$

and

$$\tilde{f}_{t}^{2,w} = \tilde{w}_t^h \left(\frac{\tilde{w}_t}{\tilde{w}_t^0}\right)^{\theta^w} \tilde{L}_t^d + (\omega_w \beta \gamma^{1-\sigma}) \left(\frac{\tilde{\pi}_{t+1}}{\tilde{\pi}_t^w}\right)^{\theta^w} \left(\frac{\tilde{w}_t^0}{\tilde{w}_t^0}\right)^{\theta^w} \tilde{\xi}_{t+1/|f_{t+1}^{2,w}}$$

Therefore, the log-linearized wage equation with trend inflation is given by:

$$f_{t}^{1,w} + w_t^o = f_{t}^{2,w}$$

where

$$f_{t}^{1,w} = (1 - A_{1}^{w}) \left[\theta^w (w_t - w_t^o) + L_t^d\right] + A_{1}^{w} \left[(\omega^w - 1) (\pi_{t+1} - \pi_w \pi_t) + \theta^w (w_{t+1} - w_t^o) + \xi_{t+1/|f_{t+1}^{1,w}}\right]$$

and

$$f_{t}^{2,w} = (1 - A_{2}^{w}) \left[\theta^w (w_t - w_t^o) + L_t^d + w_t - \mu_t^w\right] + A_{2}^{w} \left[\theta^w (\pi_{t+1} - \pi_w \pi_t) + \theta^w (w_{t+1} - w_t^o) + \xi_{t+1/|f_{t+1}^{2,w}}\right]$$

and the coefficients are:

$$A_{1}^{w} = \omega_w \beta \gamma^{1-\sigma} \pi_*^{(1-\omega^w)(\theta^w-1)}$$

$$A_{2}^{w} = \omega_w \beta \gamma^{1-\sigma} \pi_*^{(1-\omega^w)\theta^w}$$

whereas the equation for the optimal wage $w_t^o$ is:

$$w_t^o = \frac{w_\pi}{\pi_* (1-\omega_w)} w_t - \frac{\omega_w}{1 - \omega_w \pi_*^{(1-\omega^w)(\theta^w-1)}} \frac{w^t_\pi}{w^t_\pi} \left[w_{t-1} + \omega \pi_{t-1} - \pi_t\right]$$

Setting $\pi_* = 1$, it is possible to recover the standard wage equation:

$$w_t = w_1 w_{t-1} + (1 - w_1)(E_{t} w_{t+1} + E_{t} \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w$$

where

$$w_1 = \frac{1}{1 + \beta \gamma^{1-\sigma}}$$

$$w_2 = \frac{1 + \beta \gamma^{1-\sigma}}{1 + \beta \gamma^{1-\sigma}}$$

$$w_3 = \frac{w^t_\pi}{1 + \beta \gamma^{1-\sigma}}$$

$$w_4 = \frac{(1 - \beta \gamma^{1-\sigma} \omega_w)(1 - \omega_w)}{(1 + \beta \gamma^{1-\sigma} \mu w^t)}$$

The sticky price and wage part of the model is closed by adding the monetary policy reaction function:

$$R_t = \rho_R R_{t-1} + (1 - \rho_R) \left[\psi \pi_t + \psi y (y_t - y_{t-1})\right] + \epsilon_t$$

There are seven exogenous processes in the Smets and Wouters (2007) model. These are generally modelled as AR(1) processes with the exception of the exogenous spending process (where the process is the result of exogenous spending shock $\eta_t^\theta$ and the total factor productivity shock $\eta_t^\mu$) and the exogenous price and wage markup processes, which are treated as ARMA(1,1) processes. Therefore we have:
The shocks $\eta^j_t \sim N(0, 1)$ for $j = \{a, b, g, i, p, r, w\}$.

**Appendix B: data and Bayesian estimates**

**Data**

Figure 2 sets out the 7 series analyzed. The red line indicates the cut-off between the two Great Inflation and Great Moderation sub-periods.

Figure 2: Time series (right to left, top to bottom): output, consumption, investment, real wage, inflation, interest rate, hours. The red line corresponds to 1983 Q1.
Prior and posterior

Table 15 sets out the priors, whereas the following tables show the posterior mode and 95 probability intervals, taking into account the entire sample (Tab. 16), the Great Inflation period (Tab. 17) and the Great Moderation period (Tab. 18) for the four models.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Mean</th>
<th>Standard Deviation</th>
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<tr>
<td>$\sigma^b$</td>
<td>Inverse Gamma</td>
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<td>2.0</td>
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<tr>
<td>$\sigma^a$</td>
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<td>2.0</td>
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<td>2.0</td>
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<td>2.0</td>
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<td>0.2</td>
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<td>0.2</td>
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<td>0.2</td>
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<td>0.2</td>
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<td>Beta</td>
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<td>0.2</td>
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<td>0.1</td>
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Table 15: Priors.

Note that

$$\bar{\gamma} = 100(\gamma - 1) \quad \bar{\beta} = 100(\beta^{-1} - 1)$$

and $\bar{l}$, the steady-state hours worked, is normalized to be equal to zero.

In analysing the posterior, beyond the comparisons presented in the main text, we notice relevant differences between sub-periods, but with equal impact across models. However, since they are not the main object of this analysis, we briefly summarize them as follows:

- The standard deviations strongly decrease between the Great Inflation and the Great Moderation periods, except for the volatility of the wage shock, which increases
mildly\textsuperscript{10}.

- The reaction of the central bank to inflation is stronger during the Great Moderation, whereas its response to output growth increases only slightly. This result is consistent with Boivin and Giannoni (2006), who find evidence of a more stabilizing monetary policy during the Great Moderation, which is almost entirely explained by an increasing responsiveness to inflation. Instead, Smets and Wouters (2007) observe that the responses to inflation are only marginally higher and the reaction to the output gap is lower.

- A significant increase in persistence is observed in the coefficient $\rho^R$, which relates the past nominal interest rate to the actual one in the monetary policy function.

- The degree of price stickiness increases during the Great Moderation period, whereas the degree of wage stickiness is slightly reduced\textsuperscript{11}. This result is consistent with works such as those by Blanchard and Galí (2007) or Blanchard and Riggi (2009), which found an overall reduction in the degree of real wage rigidity, although it is at odds with the estimates of Smets and Wouters (2007), who concluded that $\omega_w$ increases during the Great Moderation.

- The degree of indexation to past inflation for both prices and wages decreases during the Great Moderation, reporting similar values in both models. Cogley and Sbordone (2008) demonstrated that by taking into account time-varying shifts in trend inflation, there is no need to include indexation in a VAR. In our medium-scale DSGE, and similarly in Aruoba and Schorfheide (2011), we do not observe such a situation. This may be due to the fact that we maintain a time–invariant level of trend inflation or, more likely, due to the presence of more frictions\textsuperscript{12}.

\textsuperscript{10}This result is supported by Heathcote, Storesletten and Violante (2010), who show a rising instability in US male earnings for recent decades.

\textsuperscript{11}It is tempting to compare our estimates with the microeconomic evidence on the average duration of prices, such as Bils and Klenow (2004) or Nakamura and Steinsson (2006), however this comparison is difficult because we only have partial indexation.

\textsuperscript{12}In a preliminary version of this work, we analysed the basic Woodford (2003) trivariate model, without wage rigidity, and we were able to recover the result at zero indexation. For the Great Moderation period, we estimated $\nu^p$ as equal to 0.2 and 0.0025 for the nTI and the TI models, respectively. See Appendix E.
<table>
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<td>0.12</td>
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<td>0.87</td>
<td>0.91</td>
<td>0.84</td>
</tr>
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<td>0.86</td>
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<td>0.81</td>
<td>0.86</td>
<td>0.76</td>
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<td>0.66</td>
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<td>0.28</td>
<td>0.13</td>
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<td>0.22</td>
<td>0.13</td>
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<td>1.47</td>
<td>2.14</td>
<td>1.03</td>
</tr>
<tr>
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<td>0.70</td>
<td>0.86</td>
<td>0.42</td>
</tr>
<tr>
<td>$\lambda$</td>
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<td>-0.47</td>
<td>1.86</td>
<td>-2.55</td>
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</table>

Table 16: Posteriors for the models with $\pi_* = 1$ (no trend inflation - nT), with $\pi_* = 1 + (4.02/400)$ (trend inflation - T) with or without Indexation (I or nI), period 1966 - 2004.
| Parameter | $\gamma$ | $\beta$ | $\sigma_c$ | $\phi$ | $\lambda$ | $\theta_p$ | $\epsilon_p$ | $\omega_p$ | $\omega_w$ | $\psi_r$ | $\psi_j$ | $\psi_w$ | $\sigma_w$ | $\sigma_b$ | $\sigma_a$ | $\sigma_g$ | $\sigma_p$ | $\sigma_v$ | $\sigma_{\varepsilon}$ | $\sigma_r$ | $\rho$ | $\psi_x$ | $\psi_j$ | $\alpha$ | $\sigma_l$ | $\psi$ | $\tilde{l}$ |
|-----------|---------|---------|-----------|-------|--------|--------|----------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| **5\textsuperscript{th}** percentile | 0.24 | 0.13 | 1.22 | 3.43 | 0.65 | 1.78 | 0.18 | 0.78 | 0.32 | 0.71 | 1.06 | 0.06 | 0.16 | 1.02 | 0.23 | -4.86 |
| **Mean** | 0.30 | 0.22 | 1.39 | 4.19 | 0.73 | 2.17 | 0.43 | 0.82 | 0.61 | 0.78 | 1.39 | 0.14 | 0.20 | 1.39 | 0.42 | -2.94 |
| **95\textsuperscript{th}** percentile | 0.36 | 0.34 | 1.55 | 4.97 | 0.80 | 2.71 | 0.74 | 0.86 | 0.87 | 0.85 | 1.70 | 0.21 | 0.24 | 2.12 | 0.66 | -3.62 |
| **5\textsuperscript{th}** percentile | 0.23 | 0.13 | 1.19 | 3.43 | 0.65 | 1.79 | 0.18 | 0.76 | 0.33 | 0.71 | 1.05 | 0.05 | 0.15 | 1.01 | 0.22 | -4.97 |
| **Mean** | 0.30 | 0.23 | 1.39 | 4.20 | 0.73 | 2.23 | 0.40 | 0.80 | 0.60 | 0.78 | 1.45 | 0.13 | 0.20 | 1.36 | 0.43 | -2.68 |
| **95\textsuperscript{th}** percentile | 0.40 | 0.37 | 1.57 | 5.00 | 0.80 | 3.00 | 0.65 | 0.84 | 0.85 | 0.85 | 1.80 | 0.21 | 0.25 | 1.97 | 0.69 | -1.01 |
| **5\textsuperscript{th}** percentile | 0.23 | 0.13 | 1.27 | 3.46 | 0.67 | 1.77 | 0.30 | 0.76 | 0.46 | 0.71 | 1.75 | 0.06 | 0.15 | 1.01 | 0.20 | -4.60 |
| **Mean** | 0.31 | 0.22 | 1.43 | 4.21 | 0.74 | 2.25 | 0.41 | 0.81 | 0.65 | 0.78 | 1.81 | 0.14 | 0.20 | 1.32 | 0.41 | -2.11 |
| **95\textsuperscript{th}** percentile | 0.41 | 0.35 | 1.59 | 4.98 | 0.80 | 2.91 | 0.65 | 0.86 | 0.84 | 0.85 | 2.00 | 0.21 | 0.24 | 1.98 | 0.68 | -0.80 |

Table 17: Posterior distributions for the models with $\pi_* = 1$ (no trend inflation - nT), with $\pi_* = 1 + (6.08/400)$ (trend inflation - T) with or without Indexation (I or nI), period 1966 - 1982.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>TI 5&lt;sup&gt;th&lt;/sup&gt; Mean 95&lt;sup&gt;th&lt;/sup&gt;</th>
<th>nTI 5&lt;sup&gt;th&lt;/sup&gt; Mean 95&lt;sup&gt;th&lt;/sup&gt;</th>
<th>TnI 5&lt;sup&gt;th&lt;/sup&gt; Mean 95&lt;sup&gt;th&lt;/sup&gt;</th>
<th>nTnI 5&lt;sup&gt;th&lt;/sup&gt; Mean 95&lt;sup&gt;th&lt;/sup&gt;</th>
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<tbody>
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<td>σ&lt;sub&gt;i&lt;/sub&gt;</td>
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<td>0.38 0.49 0.62</td>
<td>0.38 0.49 0.62</td>
<td>0.38 0.50 0.63</td>
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<td>0.13 0.19 0.24</td>
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<td>σ&lt;sub&gt;α&lt;/sub&gt;</td>
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<td>σ&lt;sub&gt;ρ&lt;/sub&gt;</td>
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<td>0.11 0.13 0.15</td>
<td>0.11 0.13 0.15</td>
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<td>ρ&lt;sub&gt;i&lt;/sub&gt;</td>
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<td>0.42 0.56 0.70</td>
<td>0.42 0.56 0.70</td>
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<td>0.07 0.26 0.56</td>
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<td>0.88 0.93 0.96</td>
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<td>0.51 0.67 0.81</td>
<td>0.47 0.62 0.76</td>
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<tr>
<td>γ</td>
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<td>0.31 0.35 0.40</td>
<td>0.30 0.35 0.40</td>
<td>0.31 0.36 0.40</td>
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<tr>
<td>β</td>
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<td>0.15 0.25 0.39</td>
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<tr>
<td>σ&lt;sub&gt;ε&lt;/sub&gt;</td>
<td>1.19 1.33 1.48</td>
<td>1.18 1.33 1.48</td>
<td>1.19 1.33 1.48</td>
<td>1.19 1.33 1.47</td>
</tr>
<tr>
<td>φ</td>
<td>4.12 4.83 5.56</td>
<td>4.10 4.83 5.57</td>
<td>4.12 4.83 5.56</td>
<td>4.15 4.86 5.59</td>
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<td>λ</td>
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<td>0.67 0.76 0.82</td>
<td>0.68 0.76 0.82</td>
<td>0.68 0.76 0.82</td>
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<td>1.86 2.07 2.31</td>
<td>1.85 2.05 2.27</td>
</tr>
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<td>ι</td>
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<td>0.05 0.17 0.32</td>
<td>0.05 0.17 0.32</td>
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<tr>
<td>ω&lt;sub&gt;τ&lt;/sub&gt;</td>
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<td>0.84 0.86 0.89</td>
<td>0.84 0.87 0.89</td>
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<tr>
<td>ω&lt;sub&gt;τπ&lt;/sub&gt;</td>
<td>0.23 0.55 0.85</td>
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<td>0.22 0.54 0.84</td>
</tr>
<tr>
<td>ω&lt;sub&gt;τπ&lt;/sub&gt;</td>
<td>0.66 0.74 0.81</td>
<td>0.66 0.74 0.81</td>
<td>0.68 0.75 0.83</td>
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</tr>
<tr>
<td>ρ</td>
<td>0.76 0.80 0.84</td>
<td>0.75 0.80 0.84</td>
<td>0.76 0.80 0.83</td>
<td>0.76 0.80 0.84</td>
</tr>
<tr>
<td>ψ&lt;sub&gt;τ&lt;/sub&gt;</td>
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<td>1.63 1.86 2.11</td>
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<td>ψ&lt;sub&gt;τπ&lt;/sub&gt;</td>
<td>0.11 0.19 0.26</td>
<td>0.11 0.18 0.26</td>
<td>0.11 0.18 0.26</td>
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<td>α</td>
<td>0.17 0.20 0.24</td>
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<td>0.17 0.21 0.24</td>
<td>0.17 0.21 0.25</td>
</tr>
<tr>
<td>σ&lt;sub&gt;λ&lt;/sub&gt;</td>
<td>1.11 1.95 2.91</td>
<td>1.14 1.94 2.88</td>
<td>1.10 1.81 2.71</td>
<td>1.23 1.97 2.89</td>
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<tr>
<td>ψ</td>
<td>0.68 0.81 0.92</td>
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<td>0.68 0.81 0.92</td>
<td>0.67 0.81 0.91</td>
</tr>
<tr>
<td>λ</td>
<td>-4.33 -2.38 -0.36</td>
<td>-4.28 -2.33 -0.33</td>
<td>-4.32 -2.34 -0.32</td>
<td>-4.30 -2.39 -0.43</td>
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</table>

Table 18: Posteriors for the models with \( \pi_s = 1 \) (no trend inflation - nT), with \( \pi_s = 1 + (2.43/400) \) (trend inflation - T) with or without Indexation (I or nI), period 1983 - 2004.
Relationship between indexation and stickiness at different levels of steady-state inflation

By considering the joint distributions of \((\rho^p, \omega_p)\) and \((\nu^w, \omega_w)\), we better understand the different relationships between price or wage rigidity and long-run inflation.\(^{13}\) During the Great Inflation period, we observe a positive correlation between \(\rho^p\) and \(\omega_p\), whereas for the Great Moderation, the correlation drops to zero, as in Figure (3).

Figure 3: Contour plot for the joint distribution \((\rho^p, \omega_p)\) broken down by periods and models. The periods are shown by line, full sample (1\(^{st}\) line), Great Inflation (2\(^{nd}\) line) and Great Moderation (3\(^{rd}\) line), and the models are divided by column, nTI (left column) and TI (right column).

\(^{13}\)Under the assumption that the posteriors of the estimated parameters are normally distributed, the joint distribution is a bivariate normal. Thus, the contour gives us information on the correlation between the distributions of the two parameters. The relationship between coefficients is due to the use of the inverse Hessian in the Random Walk Metropolis–Hastings.
It is interesting to note that the information on the link between indexation and stickiness is consistent between the two models when we consider sub-periods with homogeneous behaviour for inflation (i.e., Great Moderation or Great Inflation), whereas when we study the full sample, a positive correlation is evident only for the model with trend inflation. This result reinforces the idea suggested by Gali and Gertler (1999): with high, volatile inflation the average price duration decreases due to the higher cost of not adjusting. Therefore, in order to hold down this cost, when stickiness increases, price indexation to past inflation must increase more than proportionally. Since the presence of a positive level of inflation is not enough to account for price stickiness, this positive relationship is more obvious for the full period, suggesting the need to study the periods separately as they have different levels of stickiness.

If we consider the joint distribution for the labour market parameters ($\omega_w$, $\omega_{lw}$), we observe a different relationship between the level of inflation and correlation between the parameters. In periods with low and stable inflation there is a negative correlation between $\omega_w$ and $\omega_{lw}$, whereas during the Great Inflation period the correlation is close to zero.

Appendix C: Impulse response functions to negative monetary policy shock

The following figures show the IRFs to a negative monetary policy shock for the three periods: full sample (Fig. 4), Great Inflation (Fig. 5) and Great Moderation (Fig. 6). The blue line represents the nTI model, the green line is the TI model, the red line is the TnI model and the magenta is the nTnI model.

Figure 4: IRFs to a monetary policy shock for the full sample, 1966 - 2004.
Figure 5: IRFs to a monetary policy shock for the Great Inflation period, 1966 - 1982.

Figure 6: IRFs to a monetary policy shock for the Great Moderation period, 1983 - 2004.
Appendix D

Bayesian Structural VAR

We obtain the estimates and the IRFs for the structural VAR by following the two-step procedure of Koop (1992). In the first step, following Koop and Korobilis (2009), we estimate the reduced-form VAR using the Bayesian technique with a natural conjugate prior, whereas in the second step we recover the structural form. Given the uncertainty about the right lag length, we consider all the possible lags from one to four.

Following Koop and Korobilis (2009) in the first step, we consider the multivariate version of the Wold decomposition theorem, which states that any covariance stationary \( m \times 1 \) vector time series, \( y_t \), can be rewritten as a possibly infinitely ordered vector moving average:

\[
y_t = \sum_{j=1}^{\infty} A_j y_{t-j} + \varepsilon_t \]

where \( y_t \) for \( t = 1, \ldots, T \), is a \( (7 \times 1) \) vector containing observations of the seven time series, \( \varepsilon_t \) is a \( (7 \times 1) \) vector of errors and \( A_j \) is a \( (7 \times 7) \) matrix of coefficients. We assume \( \varepsilon_t \) to be i.i.d. \( N(0, \Sigma) \) and, since there is uncertainty with respect to the appropriate lag length \( p \) of the VAR, we use a mixture of four vector autoregressive models.

In order to estimate this VAR we use the Bayesian technique with the natural conjugate prior. We rewrite our VAR(p) as:

\[
y_{mt} = z_{mt}^T \beta_m + \varepsilon_{mt} \]

where \( m = 1, \ldots, 7 \) variables. Stacking all equations into vectors/matrices, i.e. \( y_t = (y_{1t}, \ldots, y_{7t})' \), and defining:

\[
y = \begin{pmatrix} y_1 \\ \vdots \\ y_T \end{pmatrix} \]

we can rewrite:

\[
y = Z \beta + \varepsilon \]

where \( \varepsilon \) is a \( N(0, I \otimes \Sigma) \). As prior for this model we use the independent Normal-Wishart:

\[
\beta \sim N(\beta, \Sigma \otimes V) \\
\Sigma^{-1} \sim W(S^{-1}, \psi) 
\]

With this technique we obtain the estimates for \( \hat{A}_j \), where \( j = 1, \ldots, p \), and \( \hat{\Sigma} \).

Since we are interested in the structural IRFs, in the second step we rewrite our VAR as:

\[
y_t = \sum_{j=0}^{p} C_j e_{t-j} \]

where \( e_t \) is a structural error. The two representations are related by noting that \( C_j = A_j C_0 \), where \( \Sigma = C_0 C_0' \). However, because \( \Sigma \) is a symmetric matrix, an estimation of \( \Sigma \) is not enough to obtain \( C_0 \). We identify the response to a negative monetary policy shock via sign restrictions. Following Uhlig (2005), for the first four observations we impose a positive response for the interest rate and a negative reaction for inflation, output and investment. In papers such as Mountford and Uhlig (2009), the authors are agnostic about the response of output. Nevertheless, we have decided to impose a negative restriction,
considering the results obtained for the DSGE models. Figure 7 displays the IRFs to a monetary shock for the seven observed variables for a SVAR(4) on the full sample (1966–2004). The black line is the posterior mean, whereas the blue area denotes the 90% confidence interval.

\[ \frac{1}{p(Y)} = \int \frac{f(\theta)}{C(\theta|Y)p(\theta)} p(\theta|Y) d\theta \]

where \( f(\theta) \) has the property that \( \int f(\theta) d\theta = 1 \). Conditional on the choice of \( f(\theta) \), an estimator is:

\[ \hat{p}(Y) = \left[ \frac{1}{n_{\text{sim}} - n_{\text{burn}}} \sum_{s=n_{\text{burn}}+1}^{n_{\text{sim}}} \frac{f(\theta^{(s)})}{C(\theta^{(s)}|Y)p(\theta^{(s)})} \right]^{-1} \]

where \( \theta^{(s)} \) is drawn from the posterior \( p(\theta|Y) \). To make the numerical approximation efficient, \( f(\theta) \) should be chosen so that the addends are of equal magnitude. Geweke
(1999) proposed using the density of a truncated multivariate normal distribution:

\[
f(\theta) = \tau^{-1}(2\pi)^{-d/2}|\tilde{V}_\theta|^{-1/2} \exp \left\{ -\frac{1}{2} (\theta - \tilde{\theta})' \tilde{V}_\theta^{-1} (\theta - \tilde{\theta}) \right\} \\
\times \mathbb{I} \left\{ ((\theta - \tilde{\theta})' \tilde{V}_\theta^{-1} (\theta - \tilde{\theta}) \leq F_{\chi^2_d}^{-1}(\tau) \right\}
\]

Here \( \tilde{\theta} \) and \( \tilde{V} \) are the posterior mean and covariance matrix computed from the output of the posterior simulator, \( d \) is the dimension of the parameter vector, \( F_{\chi^2_d} \) is the cumulative density function of a \( \chi^2 \) random variable with \( d \) degrees of freedom, and \( \tau \in (0, 1) \).
Appendix E

We consider the textbook model in Woodford (2003) with sticky prices à la Calvo and capital in a Cobb-Douglas production function. Time is discrete and continues forever. In the economy, there is a continuum of three types of infinitely-lived agents: households, intermediate good producers and retailers. The three observed variables are output, interest rate and inflation. The estimates for the parameters of interest are found in Table 19, based on a TI model and a TnI model.

<table>
<thead>
<tr>
<th></th>
<th>nTI</th>
<th>TI</th>
</tr>
</thead>
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<tr>
<td></td>
<td>5th percentile</td>
<td>mean</td>
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<tr>
<td>( \pi^p )</td>
<td>0.037197</td>
<td>0.22497</td>
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<td>( \omega_p )</td>
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<td>( \psi_\pi )</td>
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<td>( \psi_y )</td>
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<tr>
<td>( \rho_r )</td>
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<td>0.49808</td>
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</table>

Table 19: Results for the Woodford textbook model assuming trend inflation and indexation (TI) or no trend inflation and indexation (nTI).

Figure 8 sets out the IRFs to a negative monetary policy shock for the three observables. We found that short-run dynamics are not affected by trend inflation if there is partial indexation.
Figure 8: IRFs to a negative monetary policy shock for the three periods: full sample 8(a), Great Inflation 8(b) and Great Moderation 8(c).
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