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STRATEGY AND TACTICS IN PUBLIC DEBT MANAGEMENT

by Davide Dottori[a] and Michele Manna[b]

Abstract

We examine the public debt management problem with respect to the maturity mix of new issues in a mean-variance framework. After identifying the main determinants of the long-run target (strategy), we focus on which interest rate conditions allow for a temporary deviation (tactics). The study is partly motivated by the apparent 'window of opportunity' to issue more heavily at longer maturities given the recent historically low yields. We show that the room for long tactical positions on the long-term bond is actually narrower than predicted by rules of thumb based on Sharpe-like ratios. Once the model is augmented to embed real world features such as no price-taking and transaction costs, the scope for tactical position shrinks further. We discuss the model results and its implications in terms of the principal-agent dilemma (government vs. debt manager); the paper also explores the financial stability implications arising from public debt issuance choices. All in all, our findings provide a rationale for the degree of caution often shown by many public debt managers in fulfilling their mandate.

JEL Classification: G1, H6, D4.
Keywords: public debt management, government bond market, issuance maturity, agency problem, financial stability.

Contents

1. Introduction 5
2. A selection of literature on debt management issues 6
   2.1 The general problem of public debt 6
   2.2 Issuance choices and interest rates 7
   2.3 Some implications of debt management for financial stability 8
3. The model 9
   3.1 Basic algebra of debt composition and average debt maturity 10
   3.2 The model with one unknwon (strategy only) 10
   3.3 The model with two unknowns (strategy and tactics) 14
   3.4 A discussion of the two models 17
4. Calibration and evaluation 18
   4.1 Calibration of the parameters 18
   4.2 Evaluation of tactical positions and the comparison with rules of thumb 20
   4.3 Issuance portfolio reallocation after parameter-perturbation 21
   4.4 Removing the price-taking assumption 22
5. Concluding remarks 23
   References 25
   Additional charts 27
   Appendix 30

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1. Introduction[1]

At the most basic level, public debt managers must choose which securities to issue with a view to collecting enough funds to honour the debt that will fall due and to address new government financing needs. There is ample consensus that this process should be geared towards a cost of serving the outstanding stock of debt that is as low as possible over the medium term while taking on only tolerable risks (IMF-WB, 2014). In this paper we examine a stylised version of the manager's choice, where the alternatives are a shorter bill vs. a longer bond and the stock of debt is assumed to be constant over time. A key element of the proposed approach is that the manager solves a cost minimization function so as to set a long-run target for the relative weight of the bond (the 'strategy' referred to in the paper's title) while retaining the flexibility to adopt in the current period a different mix between the two securities (the 'tactics') to take advantage of today's interest rate levels.

A good example of why a debt manager could find it advantageous to take a tactical position is offered by the market conditions prevailing in spring 2013. At the end of April of that year, the zero yield on the 10-year US bond stood at 1.83%, 2.0 standard deviations below its long-run average which we work out to be 4.6% using a 1995-2013 sample. By comparison, at 0.16% the yield of the 1-year US Treasury bill was 'only' 1.3 standard deviations below its average. Arguably, these statistics are not set in stone, depending on estimation techniques and data samples, but few analysts would disagree that at the time the US rates stood well below long-run means. Hence, a temporary reduction in the supply of bills to make room for more bonds could have been a smart move to lock in a low cost of debt for a prolonged period. Roughly similar results are observed in data for Germany, another top-rated country (Chart A.1).[2]

The allocation between two (or more) financial instruments, each with its own level and variability of returns, is widely studied in finance. Nevertheless, to the best of our knowledge few papers discuss the optimal portfolio in public debt management. In fact, this subject looks interesting for several reasons. First, the placement of government securities is a key driver in financial markets, not least due to the sheer size of public debt in many countries. Second, the optimal portfolio problem exhibits some peculiar traits where public debt is concerned. Indeed, the uncertainty faced by the agent is not much about the interest rate of the security currently being issued – the secondary market provides a good benchmark – but rather about the interest rate which will prevail when the current security will fall due and a new one will need to be issued. We suggest a specific interpretation of the transaction costs borne by the Treasury if it were to enact swift and large changes in the supply mix of its securities. Finally, across government securities, short-term bills provide their holders with distinct quasi-monetary services; as a consequence changes in the maturity mix pursued by the public debt manager are deemed to be non-neutral for the economic system as a whole (**Angeletos et al., 2013**, **Farhi and Tirole, 2011**).

This paper sets out to fill what we regard as a gap between two strands in the literature on public debt management, along the lines of the recent contribution by **Debortoli, Nunes and Yared (2014)**. At one end of the spectrum, there are established papers of a scholarly nature (e.g. **Barro**

1 The views expressed in this article are those of the authors alone and do not necessarily reflect those of the Bank of Italy. The authors wish to thanks participants in seminars held at the US and Italian Treasuries, at the ECB and Bank of Italy as well as two anonymous referees for a number of helpful suggestions. Thanks go also to Alice Mary Agnes Chambers that read carefully the text.

2 Throughout the paper a few key charts and tables are displayed in the main text, while additional tables and charts are presented in the annex; the latter are coded with a final 'A' (e.g. Chart A.1).

1999) which lay down key concepts in this field but tend to be insufficiently detailed to meet the real needs of the debt manager.[3] At the other end, there is a small but increasing number of works which identify least-cost refinancing strategies through hard number crunching and simulations (**Bolder 2008**, **Larson and Lessard 2011**, **Pick and Anthony 2006**).[4]

Essentially we model public debt management as an insurance problem, where bond issuance implies a cost (the premium of the insurance due to the prevailing positive slope of the yield curve) but it hedges against the higher market risks associated with the bill's issuance. We argue that the choice between the shorter bill and the longer bond should not just be seen in its "static" dimension – two financial instruments each with its own return profile – but also in its "dynamic" one, since the issuance of government securities gives rise to a repeated game between the Treasury and the investors. Here, we consider well-known features of financial markets, namely segmentation and clienteles (**Stigum 1990**, **Collins and Mack 1994**). Both imply that a change in the maturity mix of securities being offered can imply additional costs for the issuer, who needs to compensate investors who otherwise would be reluctant to substitute one horizon bucket with another.

Anticipating some of our main results, when the model is calibrated taking into account some of these real world frictions the optimal tactical position (in terms of over-weight of the bond) based on US data in April 2013 is roughly half of what we obtain in a "frictionless" version of the model. This may shed some light on why debt managers operating in the real world exerted a degree of caution in exploiting the recent patterns in interest rates.

Furthermore, we argue in favour of a broad perspective that acknowledges the link between public debt management and financial stability. To bring forward just one element of this link, the issuance of bills by the Treasury tends to crowd out short-term paper from private firms and this makes their balance-sheet better able to weather the storm in financial crises (**Holmström and Tirole 1998, Greenwood, Hanson and Stein 2010b**).

We wish to stress that this paper is not meant to be an appraisal of US debt management even though we use mainly US data to exploit previous results from a rich literature in finance. Moreover, the choice of a top-rated country such as the US fits the requirements of a model that actually deals with market risk management, while refinancing risk (i.e. the odds that investors would no longer be willing to buy/roll-over the supplied securities or they would do so only at prohibitive yields) is beyond the scope of the current research.

The rest of the paper is organised as follows. Section 2 recaps a number of key issues in public debt management drawing from previous literature. This lays the theoretical and empirical groundwork for Section 3 on the model framework and Section 4 on the calibration and empirical evaluation of the model. Section 5 concludes.

2. A selection of literature on debt management issues

2.1 The general problem of public debt

In a seminal paper **Barro (1999)** sets out the general problem of debt management noting that uncertainty tends to motivate the government to issue securities whose payoffs are contingent on the relevant risks. For instance, the government would like to issue bonds that pay off badly when the level of public outlay is high (all else being equal). In practice, moral hazard problems rule out

[3] A discussion of the literature on the theory of public debt management is in **Faraglia, Marcet and Scott** (2010).

[4] A survey of these works is in **Manna, Bernardini, Bufano and Dottori** (2013).

recourse to state-contingent bonds. Even when such problems were effectively dealt with, **Lucas and Stokey (1983)** remind us that "the idea of trying to write bond contracts or set monetary standards in a way that is optimal under *all possible* realizations of shocks would not (even if one knew what that meant) be of any practical interest". Luckily enough, it is widely acknowledged that non-contingent debt of different maturities can offer a 'synthetic' equivalent to contingent debt (**Angeletos, 2002, Buera and Nicolini, 2004, Shin, 2007**).

Hence debt management can be regarded as an instance of portfolio allocation of securities with heterogeneous maturities, with the obvious caveat that here the objective is to minimize costs rather than maximize profits. Two basic concepts are worth noting. First, in choosing a maturity mix the debt manager is effectively taking out insurance since he trades the risks of the roll-over of shorter bills with the higher cost of the issuance of longer bonds, due to the usually positive slope of the yield curve (**Barro, 1979**). This insurance argument is backed by data proving a well-established positive correlation between the average term to maturity of the outstanding debt and the magnitude of the debt itself in proportion to GDP: **Greenwood, Hanson and Stein (2012a)** find a correlation of 0.71 between the two series using US data from 1952-2009; similar results are obtained by **Krishnamurthy and Vissin-Jorgensen (2012).** However, and this brings us to the second concept, changes in the maturity structure of the government securities being offered tend to add extra costs to the Treasury (**Vayanos and Vila, 2009**), due to a well-entrenched maturity segmentation in the financial market – owing in turn to so-called inelastic investors (**Stigum, 1990 and Collins and Mack, 1994,** just to mention a few)[5] – where these extra costs cannot immediately be inferred from the current rates prevailing in the secondary market (**Greenwood, Hanson and Stein, 2010b**).

2.2 Issuance choices and interest rates

Based on the above, a model of public debt management can be arranged in a mean-variance asset-allocation framework, not unlike the CAPM approach. If that is the benchmark set up, we take issue with two of its simplifying assumptions given the criticisms levelled at it in the literature.

The first of these is the assumption that the investor is a price-taker. When applied to the Treasury as issuer, this requires the yield-at-issuance of the government securities to be invariant, all else being equal, to changes in the amounts being offered. However, the odds are that such invariance does not hold true. Evidence on the relationship between the supply of government bonds and their yields is provided by **Greenwood, Hanson and Stein (2012a)** who regress 4-week changes in the z-spread of US T-Bills on changes on the bills' stock to GDP and find that a one-percentage point change in the latter ratio leads to an increase of the spread by 6.3 basis points using a 1983-2009 sample (if the sample ends in 2007, to mark the break-up of the financial crisis, the coefficient rises to 8.1 basis points). Earlier research by **Fleming (2002)** had shown that larger issues of T-Bills are associated with higher yields even in a set-up which allows for indirect liquidity benefits of the bigger issue size. As to the longer bonds, **Greenwood and Vayanos (2010)** examine two events to shed light on how long-term interest rates experienced large and long-lasting shifts because of changes in demand by the relevant clientele in one case or in supply by the government in another.

The second assumption we take issue with is the absence of transactions costs. In fact, such costs can arise in public debt management, when the Treasury changes the mix of securities being

[5] Examples of these investors include corporations funding specific liabilities that mature on a given date (**Ogden, 1987**), money market mutual funds (**Cook and Duffield, 1993**), and entities such as foreign central banks and individual direct purchasers of Treasury bills.

offered, due to segmentation in the government bond market between different clienteles, an argument rooted in a line of research going back to **Culbertson (1957)** and **Modigliani and Sutch (1966**). If investors have a preferred maturity horizon, the transfer in demand necessary to match a change in the supply mix needs to be remunerated by a higher yield (**Guibaud, Nosbuch and Vayanos, 2013** is a recent contribution on the clientele argument).

An additional important fact when dealing with public debt is the so-called liquidity premium featured by Treasuries. The idea is that investors are willing to underwrite government securities at a yield-to-maturity well below the one predicted by standard asset-pricing models (thus at a higher price), given all other conditions and foremost the securities issuer's merit of credit. Indeed, compared to corporates, government securities offer their holders some distinct 'liquidity services'. Think, for example, about the fact that a government security is widely accepted as collateral in a number of transactions. Using 1926-2008 US data, **Krishnamurthy and Vissing-Jorgensen (2012)** find that the interest rate gap between short-term corporates and T-Bills amounts to 73 basis points and argue that more than half of this gap is attributable to the liquidity of the latter. **Grinblatt (1995)** notes that government securities have a liquidity advantage over privately-issued instruments because Treasuries are the desired mechanism for hedging interest rate risk. Hence owners of such securities receive a convenience yield in addition to dividends and price appreciation.[6] **Longstaff (2004)** relates the magnitude of the liquidity premium to the amount of Treasury debt available to investors. Finally, **Duffee (1996)** found peculiarities of T-Bills with respect to both short term privately-issued bills and longer-term government bonds: the bills feature an idiosyncratic variation and a breakdown of the principal components reveals distinctive elements with respect to other securities.[7]

2.3 Some implications of debt management for financial stability

Based on the above, public debt management can be thought of as a debt portfolio problem with peculiarities related to the market structure and agents involved. However, it does have a further peculiar feature insofar as it is strongly linked with financial stability, as is well established both in the literature and in practice.

Short-term securities issued by Treasuries are commonly seen as the most suitable financial instrument to meet market demand for safe money-like securities. As such they give their holders a distinctive advantage, which securities issued by a private firm cannot do; this competitive edge is felt more strongly by highly-rated companies (**Graham, Leary and Roberts, 2014**). As a result, by issuing short-term debt, the government crowds out private firms that as a result will tend to issue less short-term and more long-term debt (**Greenwood, Hanson and Stein, 2010a**). However, were the Treasury to retrench and issue smaller volumes of bills, it is expected that the private sector would eventually step into the vacuum (**Holmström and Tirole, 1998**). The process of replacement from government-to-private short-term issues is non-neutral in financial stability terms. The private sector is, indeed, a less efficient provider of money-like securities because in this scenario it would over-issue short-term instruments by not internalizing the effects of fire sales in bad states of the world. **Angeletos et al. (2013)** argue that even before the onset of any crisis, transaction costs in the economy would rise since private short-term debt does not match the Treasury bills' moneyness. Moreover, and more ominously, the new environment would be more financially fragile, given the

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6 This argument is reckoned to apply more strongly to the three-month bill, which is more heavily traded.

7 According to **Duffee (1996)** , idiosyncrasy concerns only bills with very short maturities (one or two months).

scarcer supply of outside liquidity and the more limited pledgeability of corporate income (**Farhi and Tirole, 2011),** as well its increased vulnerability to fire sales by private agents **(Tirole 2011 and Stein 2012)**. On the contrary, through a sustained and significant offer of Treasury bills, the public sector can crowd out the private one, pushing it to towards longer liabilities.

Issuing 'too little' short-term government bills is not only dangerous in itself, for the reasons just outlined, but it also implies 'too much' issuance of long-term bonds, which is not good either, from a financial stability perspective. If the Treasury increases the supply of bonds when their yields are low (and accordingly their prices high), the risks increase that at the next turn of the business cycle when yields will go up again and prices fall, intermediaries will record losses in their balance sheets, to the detriment of their lending power to the economy (**Bernanke and Gertler, 1989, Kiyotaki and Moore, 1997, Martin and Ventura, 2011)**.

In fact, a vicious circle can arise when the fall in bonds' prices may force banks to deleverage, triggering further reductions in prices. This channel is likely to be exacerbated for intermediaries in the shadow banking system which tend to be less sheltered by support actions by the authorities (**Adrian and Shin, 2010**). However, it also works for banks, not least because backstop interventions are not entirely cost- or risk-free, nor can they be taken for granted. **Brunnermeier (2009)** argues that as asset prices drop, financial institutions' capital erodes while lending standards and margins tighten. Altogether, this causes fire sales, pushing down prices and tightening funding even further.[8]

Probably, most public debt managers would object to formally including financial stability within their remit. Nevertheless, this view is likely to change if the general government's broader perspective is taken, since government is concerned with public costs arising from the management of bouts of financial instability.[9] This divergence is nothing but a first of several instances where the principal-agent dilemma may arise in public debt management, where the State acts as the principal while the debt manager is the agenda (see **Tirole, 1994**, **and Dewatripont et al., 1999** for a formal treatment of agency problems applied to government**)**.[10]

3. The model

In this section we first briefly sketch the basic algebra linking average debt to maturity to issuance policy. This is warranted as average debt maturity is often used as a synthetic indicator for the composition of the debt stock, but what is less clear is that even small changes in average debt maturity may imply substantial changes in the issuance policy. Next, we present the baseline version of our model with one unknown: basically, in this case the debt manager can choose only the strategy, meant as its fundamental target for the issuance mix. Then, we allow for a temporary deviation by extending the model through a second unknown (tactics). Finally, we discuss the two models.

8 **Gorton (2010)** points out that a decrease in the value of bond prices implies that less liquidity can be borrowed and hence a lower leverage. The effect is stronger if the decrease in prices is accompanied by a fall in the credit rating of the issuer which implies an increase in the applied haircuts (**Gorton and Metrick, 2009**).

9 The crisis that began in 2007 proves beyond any doubt that the management of a financial crisis may at some point require recourse to taxpayers' money. Admittedly there is no novelty in this statement as shown by earlier surveys of crisis management (**Goodhart and Shoenmaker, 1995**).

10 Sections 3.4 and 4.2 discuss other examples of the principal-agent dilemma.

3.1 *Basic algebra of debt composition and average debt maturity*

Let public debt be composed of two securities, one maturing over 1 period (year) and the other with maturity at issue of T periods (years) with weights out of the overall stock as a proportion of (1– β) and β.

$$[1] \qquad V \equiv S + L = (1-\beta)\,V + \beta\,V \qquad\qquad \beta \in [0,1]$$

where V is the total stock of debt, S is the stock of the short-term security and L that of the long-term one. In the steady state, in each period one long-term bond (issued T periods ago) falls due, together with the outstanding short-term bill. If both securities are renewed in kind, total reimbursements and gross issues amount to:

$$[2] \qquad \text{reimbursements} = \text{gross issues} = 1/T \times L + S = V \times [\beta + (1-\beta)\,T] / T$$

The weighted average term to maturity (WATM) of this debt is[11]

$$[3] \qquad \text{WATM} = \frac{1}{V}\left[\left(1-\frac{1}{2}\right)S+\left(\frac{1+T}{2}-\frac{1}{2}\right)L\right] = \frac{1}{2}\left[(1-\beta)+\beta\,T\right]$$

Setting T = 10 years would appear to be an obvious choice for the maturity of the representative long-term bond. In fact, for such T eq. [3] proves that even if β = 1, the implausible assumption of no bills, WATM is 5 years, below current measures of public debt in USA and Germany (5.4 and 6.3 years respectively at the end of 2012; Chart A.2). If, instead, one wishes β to be (close to) 0.8, T must be 13.5 years to obtain WATM = 5.4 (and T needs to be 16 to obtain WATM = 6.3 years). As a first outcome of this easy algebra, the representative bond should be understood as a rather long one. Next, to increase the WATM by just 0.5 years from 5.5 to 6, 1–β needs to go down from 0.20 to 0.12: namely, the stock of the bills needs to shrink by roughly half. Or, if 1–β is to stay put, the maturity of representative bond has to climb to 30 years to achieve the desired WATM increase in one year. Thus, WATM tends to be very persistent and a dramatic change in the structure of the stock of the debt itself is warranted to achieve an even apparently moderate rise in the WATM itself.

Additional standard algebra in public debt is set out in Appendix A.

3.2 *The model with one unknown (strategy only)*

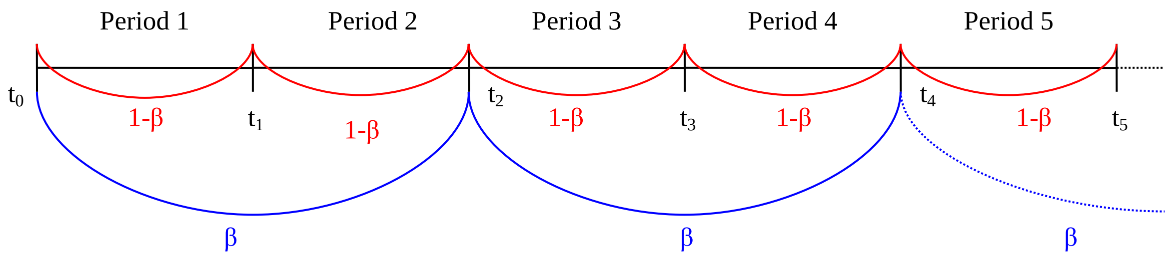
The baseline version of our model is organised as follows. At time t_0 a rationale risk-averse debt manager – whose mandate is to finance a 1-unit debt – sets the volume β of the issue of a security maturing after two periods while the residual 1–β is financed through a one-period security (in this fictitious world, the two securities stand respectively for the long-term bond and the short-term bill; Chart 1). The same weights will be applied in future roll-overs, up to period N.

[11] We deduct 1/2 from each element because we want to work out the WATM as the average over the entire period and not just at the time of issuance.

Chart 1

Issuance and maturity of bonds and bills in a long run model

β and (1–β) denote the weight of the long- and short-term security respectively



The weight β is set so as to minimize the expected costs in serving the debt over the entire planning horizon, be they of a direct nature – the issuer pays an interest on the issued securities – or an indirect one – since the debt manager is risk averse, uncertainty about future interest rate translates into a monetary cost through a risk shadow price λ.

Besides λ, the parameters driving the decision on β are the current short- and long-term interest rates, denoted *i* and *r* respectively, the corresponding expected values E(i) and E(r) – which for the scope of this paper should be understood as the equilibrium level of the short- and long-term interest rate expected to prevail on average through the business cycle – plus the uncertainty surrounding these expectations, σ_i^2 and σ_r^2, and finally the associated covariance $\sigma_{i,r}$. Arguably, this is a simplified set-up. First, the interest rate levels set at the auctions are assumed to coincide with those observed in the secondary market: in fact, this may not be necessarily the case due to the balance between supply and demand at each maturity bucket and because the Treasury is not a price-taker.[12] Secondly, after t_0 we posit an immediate transition from the current interest rates i and r to their longer run norms E(i) and E(r). A richer set-up would allow for a more gradual transition, where the expectation as of time 0 of the interest rate prevailing in, say, time t differs from the one prevailing in time t+1 and so on and so forth. Thirdly, like any agent in financial markets, the public debt manager ought to be concerned not only with the central values of the interest rate expectations and their variance / covariance, but also with higher moments too. We shall discuss the implications of these assumptions in Section 3.4 and later in the concluding section.

In algebraic terms, the setting of β boils down to the following cost minimization problem:

[4] $$\min\ f_\beta = \sum_{j=0}^{N-1} f_\beta[j]$$

$$\text{s.t. } \beta \in [0,1],\ \lambda > 0$$

where[13]

[12] As an additional and more technical element, the amounts offered – the weights β and 1-β in our set-up – are usually announced a few days earlier than the execution of the auctions so that even if the Treasury were a price-taker, the supply decision taken is based on market rates observed before the auctions take place.

[13] One could consider a version of the model where β can be higher than 1 while still maintaining non-negative the supply of the bill. This would boil down to letting the Treasury issue securities in excess of the government's financing needs. A similar variant could be promising if one were interested in the problem of public debt

[4a] $f_\beta[j] = (\omega[j])' z[j] + \lambda (\omega[j]' \Sigma[j] \omega[j])$

[4b] $\omega[j]'=(\beta; 1-\beta)\ \forall j$; $z[j]' = (r; i)$ and $\Omega[j] = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ for $j = 0$; $z[j]' = (r; E(i))$ and $\Omega[j] = \begin{pmatrix} 0 & 0 \\ 0 & \sigma_i^2 \end{pmatrix}$ for $j = 1$; $z[j]' = (E(r); E(i))$ and $\Omega[j] = \begin{pmatrix} \sigma_r^2 & \sigma_{i,r} \\ \sigma_{i,r} & \sigma_i^2 \end{pmatrix}$ for $j \geq 2$

Thus, $\omega[j]$ and $z[j]$ are respectively the vector of loads and the vector of yields at time j while $\Omega[j]$ is the variance covariance matrix of the yields.

Proposition 1. Solution of the long run model with one unknown. The optimal weight of the bond under [4] when $N \to \infty$ is

[5] $$\hat{\beta}_L = \frac{\sigma_i^2 - \sigma_{i,r}}{\Sigma} - \frac{E(s)}{2\lambda\Sigma} \qquad \text{with } \Sigma \equiv \sigma_i^2 - 2\sigma_{i,r} + \sigma_r^2 > 0;\ s \equiv r - i$$

Proof. See Appendix B1.

Infinite-horizon solution. Result [5] is the difference between two terms that both have an economic interpretation. In the first term, the difference $\sigma_i^2 - \sigma_{i,r}$ is the net hedge from the riskiness of the bill: this is a reminder that the optimal β is not related to the absolute uncertainty over future levels of the short-term interest rate but rather to the relative magnitude of this statistic net of the covariance with the uncertainty of the bond's rate. This adheres to a tenet in portfolio theory – i.e. correlation matters – which is quite important in the context of public debt management as the yields of short- and long-term government maturities tend to be highly correlated (see Section 4.1). Incidentally, this highlights a link between public debt and monetary policy: consider two twin countries which mirror one another except that in one case the central bank operates under an exchange-rate target, which brings about a more active stance in the management of its key interest rates and as a result more volatility in the domestic money market, while in the other case an inflation target is pursued and the central bank needs to be less active. Then, all other things being equal, the debt manager in the former country is advised to hedge the enhanced volatility through a larger β. Of course, there is no free lunch, since this course of action implies additional costs owing to the positive slope in the yield curve.

That brings us to the second term of [5], the slope E(s) of the yield curve expected to prevail in the long run. This is the premium in the insurance purchased by the public debt manager. As intuition would suggest, the higher the premium, the less insurance would be purchased and the smaller is the optimal β. Note, however, that the inverse link between the optimal β and E(s) is discounted by the λ parameter: a more risk averse debt manager would look down to the cost of the premium and would tend to buy more of the insurance i.e. to issue more of the bond for given E(s), compared to a less cautious manager.

More nuances in the model laid down in [4]-[4a]-[4b] can be found when exploring the links between the optimal β and the variance of the short-term interest rate *i*, the covariance between *i*

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management at high frequency – say, over intervals of single months and quarters – but becomes less relevant over longer horizons since it would imply a systematic over issuance.

and r and the shadow price of risk aversion. Indeed, the signs of the partial derivatives of $\hat{\beta}_L$ vis-à-vis σ_i^2 and, in turn, $\sigma_{i,r}$ and λ are not unambiguous (Table 1, proofs are in Appendix B2). We try to offer here an intuition about the first and second of these results (the third is trivial). The more uncertain E(i) is, hence the larger the exposure to the bill's market risk, the larger the bond's weight ought to be, provided the correlation coefficient ρ between E(i) and E(r) is not too high. Otherwise, the hedging offered by the bond is limited and more likely to be dwarfed by its higher cost. Regarding the impact of the covariance $\sigma_{i,r}$ on $\hat{\beta}_L$, based on what has been pointed out so far one would expect that the higher the former, the lower should be the latter, again because of the hedging argument. However, this result holds true only so long as the parameter λ is below a threshold which depends, in turn, on the expected yield curve slope E(s) and the difference $\sigma_i^2 - \sigma_r^2$. Otherwise, when λ is "high", i.e. the public debt manager is utterly risk averse, he would disregard the correlation argument and purchase the insurance provided by the bond anyway, no matter what its cost and effective hedging.

Table 1

Effect of perturbing parameters on $\hat{\beta}_L$

	E(i)	E(r)	σ_i^2	σ_r^2	σ_{ir}	λ
Sign (1)	+	-	+	–	–	+
Condition (2)			$\rho < \frac{E(s)}{2\lambda\sigma_i\sigma_r} + \frac{\sigma_r}{\sigma_i}$		$\lambda < \frac{E(s)}{\sigma_i^2 - \sigma_r^2}$	E(s)>0

(1) Sign of the partial derivative of $\hat{\beta}^L$ vs. the specified parameter. (2) The condition under which the sign holds. The indicated sign holds always if the corresponding "condition" cell is blank.

Shortest-horizon solution. The version of this model with N =2 yields additional insights. This is a set-up when the debt manager is concerned with a time horizon so short that it involves no roll-over of the bond and uncertainty remains associated only with the issuance of the bill at t_1. In this set-up, the solution of the cost minimization problem is:

$$[6] \qquad \hat{\beta}_2 = 1 - \frac{s - \frac{E(i) - i}{2}}{\lambda\sigma_i^2}$$

The bulk of the conclusions reached for result [5] apply to [6] as well. For instance, the optimal β increases with both the risk aversion parameter λ and with the uncertainty of the short-term interest rate. However, through [6] we also understand that it increases when the short-term interest rate is expected to rise, E(i) – i > 0. That is, not only the structure of the yield curve, as summarized by its slope, matters, but also its expected movements.

However, the latter results singles out that the slope s must be neither too small nor too large to obtain inner solutions of $\hat{\beta}_2$ (the number in the subscript is informative about the number of periods). If s is less than half of the expected change in the short-term interest rate – this could be the case when the yield curve is flat or negatively inverted and the more ordinary positive inclination is not expected to prevail soon – then the purchase of insurance through the issuance of the bond would come quite cheaply and $\hat{\beta}_2 = 1$. Conversely, if the yield curve is steep, notably s is higher than half of the expected change in the short-term interest rate *plus* the monetary value $\lambda\sigma_i^2$ of the uncertainty over this change, then the purchase of the insurance turns out to be too costly and the opposite stance is taken, $\hat{\beta}_2 = 0$. In turn, what "too costly" means depends precisely on λ: the more debt managers are risk averse, the more they are willing to pick intermediate values of $\hat{\beta}_2$. Conversely, when they are not very risk averse (and their planning horizon is short), they will swing back and forth between either of the two corner solutions with bills only or bonds only.

[7] $$0 < \hat{\beta}_2 < 1 \quad \Leftrightarrow \quad \frac{E(i)-i}{2} < s < \lambda\sigma_i^2 + \frac{E(i)-i}{2}$$

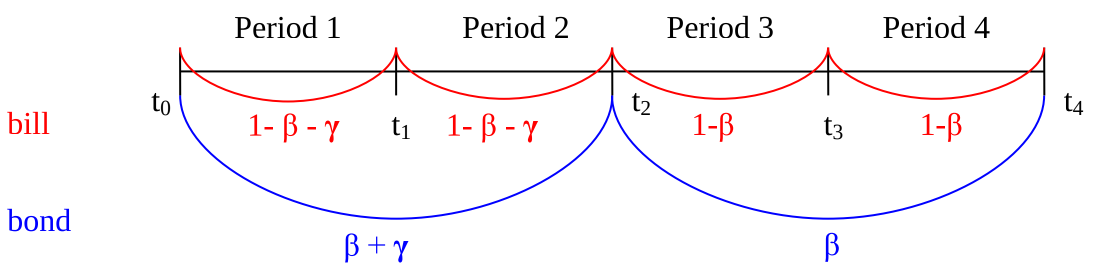
[7a] $$\hat{\beta}_2 = 1 \quad \Leftrightarrow \quad s \leq \frac{E(i)-i}{2} \quad \text{while} \quad \hat{\beta}_2 = 0 \quad \Leftrightarrow \quad s \geq \lambda\sigma_i^2 + \frac{E(i)-i}{2}$$

3.3 *The model with two unknowns (strategy and tactics)*

We now put forward a version of the model where the debt manager can differentiate between strategy and tactics. The intuition is that he may see in the current level of interest rates an opportunity to cut the cost of serving the debt, through a cunning call of the issuance mix, while still longing to switch to the target weights later on. In the algebra of this set-up, at t_0 the manager issues $\beta+\gamma$ of the bond while at t_2 (and thereafter) he issues β; the new parameter γ, which can take either sign, accounts for the tactical position. It may be convenient to discuss this new set-up over a planning horizon that stops at time 4 (or anyway is finite), as sketched in Chart 2.

Chart 2

Supply of bills and bonds over a four-period horizon



The cost minimization function is now

[8] min $$f_{\beta,\gamma} = \sum_{j=0}^{3} f_{\beta,\gamma}[j]$$

s.t. $\beta \in [0,1]$; $\gamma \in [-\beta, 1-\beta]$, $\lambda > 0$

where

[8a] $$f_{\beta,\gamma}[j] = (\omega[j])' \, z[j] + \lambda \, (\omega[j])' \, \Omega[j] \, \omega[j]$$

replaces [4a] while the statements laid down in [4b] apply here as well.

Proposition 2. Solution of the model with two unknowns. The optimal weights of [8] when N = 4 are

[9] $$\hat{\beta} = \frac{\sigma_i^2 - \sigma_{r,i}}{\Sigma} - \frac{E(s)}{2\lambda\Sigma}$$

[10] $$\hat{\gamma} = 1 - \left[\frac{\sigma_i^2 - \sigma_{r,i}}{\Sigma} - \frac{E(s)}{2\lambda\Sigma} + \frac{s - \frac{E(i) - i}{2}}{\sigma_i^2 \lambda} \right] = 1 - \hat{\beta} - \frac{s - \frac{E(i) - i}{2}}{\sigma_i^2 \lambda}$$

Proof. See Appendix B2.

It is easy to observe that the $\hat{\beta}$ of [9] is the same as the $\hat{\beta}_L$ of [5]. Namely, the optimal weight of the bond in the long run does not change whether the 'long run' is worked out under a finite or infinite horizon;[14] moreover, $\hat{\beta}$ is not dependent on $\hat{\gamma}$, i.e.: $\hat{\beta}$ and $\hat{\gamma}$ can be solved in a closed form solution of a simple system of two equations in two unknowns.[15] The latter remark may need to be reappraised once some friction in switching from $\hat{\beta} + \hat{\gamma}$ to $\hat{\beta}$ at t_2 is added to the picture (see Section 3.4).

By design, the novelty of this second set-up is in result [10]. We can easily note that $\hat{\gamma}$ cannot be higher than $1 - \hat{\beta}$: quite understandably, the higher the target weight of the bond in the long run, the less room it leaves for further increases in the current period. The incentive to move towards this ceiling is an inverse function of the yield curve slope *s* as observed today and a direct function of the expected movement of the curve at its short end, *E(i)–i*. There is no doubt that current levels of interest rates have a bearing on the tactical position, namely on the extent to which the volume of the bond to be issued at t_0 deviates from the target volume in the long run.

We define the tactical stance as neutral when $\hat{\gamma}$ = 0 and we use symbol r^e to denote the level of the long-term interest rate that verifies this condition . It can be shown (see Appendix B3) that:

14 In order to highlight this finding we opted to discuss here the model with two unknowns under finite horizons, in juxtaposition to the infinite horizon of the model with one unknown. In Appendix B5 we prove that this equivalence holds true also with values of N other than 4.

15 Additional insights across the different set-ups may be gained by solving the model with one parameter, that is only β and no γ, when the number of periods N is 4. Here we obtain:

$$\hat{\beta} = \frac{\sigma_i^2 - \sigma_{r,i}}{\Sigma} - \frac{E(s)}{2\lambda\Sigma} + \frac{\sigma_i^2}{4\Sigma} - \frac{s - \frac{E(i) - i}{2}}{2\lambda\Sigma} = \hat{\beta}^L + \frac{\sigma_i^2}{4\Sigma} - \frac{s - \frac{E(i) - i}{2}}{2\lambda\Sigma}$$

The first two terms of the sum yield $\hat{\beta}^L$, that is the optimal beta when N tends to infinite and there is one unknown in the model or, which is the same as just pointed out in the text, the optimal beta when N = 4 and the model is about two unknowns. Conversely, the final two terms tell us that, if there is just one unknown, its solution keeps track of what should be done now and not only in the longer term. In other words, if the debt manager is confronted with a finite horizon and is bound to offer a constant supply mix (there is no room for tactics, in our parlance), then his choice will be a sort of average between short- and long-term considerations.

[11] $\hat{\gamma} = 0 \quad \Leftrightarrow \quad r^e = \frac{1}{2} i + \frac{1}{2}\theta E(r) + \frac{1}{2}(1-\theta)E(i) + \theta\left(\sigma_r^2 - \sigma_{r,i}\right)\lambda$

[11a] $\hat{\gamma} > 0 \quad \Leftrightarrow \quad r < r^e \quad$ while $\quad \hat{\gamma} < 0 \quad \Leftrightarrow \quad r > r^e$

where $\theta \equiv \sigma_i^2 / \Sigma$ and $\Sigma \equiv \sigma_r^2 - 2\sigma_{r,i} + \sigma_i^2$.

Thus, [11] defines the locus where $\hat{\gamma} = 0$ as the current long-term rate *r* being a linear combination of the current short-term rate *i*, the expected values E(r) and E(i) – according to weights inversely related to the relative uncertainty surrounding the two expectations – plus a final term which embodies again the variance-covariance parameters as well as the shadow price of risk λ. It is worth examining in some detail the partial derivatives of r^e against the other parameters in [11], namely by how much r^e should change when *i*, E(r), … vary to keep $\hat{\gamma} = 0$ (Table 2; proofs are in Appendix B4).

Table 2

Effect of perturbing the locus of values r such that $\hat{\gamma} = 0$

	i	E(i)	E(r)	λ
Sign (1)	+	–	+	+
Condition (2)	always	$\rho > 0.5\sigma_r / \sigma_i$	always	$\sigma_i^2 - \sigma_{i,r} > 0$

	σ_i^2	σ_r^2	σ_{ir}
Sign (1)	–	+	+
Condition (2)	$0.5\frac{\sigma_r}{\sigma_i} < \rho < \frac{E(r) - E(i)}{2\lambda\sigma_i\sigma_r} + \frac{\sigma_r}{\sigma_i}$	$\rho < \frac{\sigma_i}{\sigma_r} - \frac{E(r) - E(i)}{2\lambda\sigma_i\sigma_r}$	$\lambda < \frac{E(r) - E(i)}{\sigma_i^2 - \sigma_r^2}$

(1) Sign of the partial derivative of r^e vs. the specified parameter. (2) Condition under which the sign holds.

To sketch the implications of [11] consider, for instance, a scenario where the long-term growth potential of the economy is revised downwards, lowering the real rate component in the interest rates and E(i) in turn.[16] Starting from a neutral position $\hat{\gamma} = 0$, *r* rises above r^e provided the correlation ρ is larger than half of the ratio σ_r / σ_i, as is the case in our data set of US 1995-2013 monthly observations. Then, the said decrease in E(i) would make it convenient to take a short tactical position $\hat{\gamma} < 0$ by under-weighting the bond and relying more on the bill. As a second example, let's assume that the central bank engages in a policy of outright operations in the secondary market aimed at checking the volatility σ_r^2 of the bond, to enhance the transmission of the monetary policy impulse. Then, *r* falls under r^e (provided the correlation ρ is not too high) and the debt manager is advised to go long on the bond, $\hat{\gamma} > 0$.

[16] Of course, this may also have an impact on E(r) which, however, could be at least partially offset by term premia. For the sake of the example we are putting forward it is enough if E(i) decreases more than E(r).

3.4 A discussion of the two models

Yields at issuance. In model [8] any positive tactical choice has no effect on r. This means that the market is willing to buy any further supply of bonds at the same borrowing cost for the issuer. The Treasury is then a price-taker so that yields at auction (r^A) do not change with the amount supplied through γ, $r^A = r$ always. If this is not the case,

[12] $$r^A - r = f(\gamma) \quad \text{where} \quad \frac{\partial\left(r^A - r\right)}{\partial \gamma} > 0 \quad \text{if } \gamma > 0 \;\text{ and }\; r^A = r \;\text{ otherwise}$$

then [10] becomes

[10a] $$\hat{\gamma}^A = 1 - \left[\frac{\sigma_i^2 - \sigma_{r,i}}{\Sigma} - \frac{E(s)}{2\lambda\Sigma} + \frac{r^A - \dfrac{i + E(i)}{2}}{\sigma_i^2 \lambda} \right]$$

so that $0 < \hat{\gamma}^A < \hat{\gamma}$. Note that $\hat{\gamma}^A$ is lower than $\hat{\gamma}$ since $r^A > r$ when $\hat{\gamma}^A$ and $\hat{\gamma}$ are higher than zero (this effect disappears when a neutral tactical position is taken).

Varying horizon. One item worth examining in detail is the role of the planning horizon, since in the real world the debt manager is not committed to a predefined horizon (see infra). In Section 3.3, we set N = 4, being the lowest number of periods (hence, easier algebra) that could nonetheless introduce uncertainty over the roll-over of the bond, besides that of the bill. What happens, however, if N gets larger? If, say, N = 6, the solution of the cost minimization problem is (Appendix B5)

[13] $$\hat{\gamma}_6 = \left(1 - \hat{\beta}_6\right) - \sqrt{\frac{s - \dfrac{\left[E(i_1) - i\right]}{2}}{\lambda\,\sigma_i^2}} \qquad \hat{\beta}_6 + \hat{\gamma}_6 \in [0,1]$$

where the subscript "6" earmarks the number of periods under the current horizon. Result [13] resembles [10] except for the square root operator. Hence, conditional on $\hat{\beta}_6$ the optimal tactical position $\hat{\gamma}_6$ is higher than the $\hat{\gamma}$ which was the result of the game plan with N = 4. Put differently, the longer the planning horizon, the more the debt manager relies on the bond. Furthermore, so long as we maintain the assumption that rate expectations are constant over the time horizon, one has

[13a] $$\hat{\beta}_6 = \hat{\beta}$$

That is, while the tactics are affected by the planning horizon, the strategy is not. However, things change if we relax the assumption on interest rate expectations. If, say, the long-term rate converges immediately to its long-run norm, $E(r_2) = E(r_4) = E(r)$, while the short-term rate follows a more gradual path, $E(i_2) \leq E(i_3) \leq E(i_4) \leq E(i_5) = E(i)$, then $\hat{\beta}_6 \leq \hat{\beta}$. As the planning horizon stretches further, different solutions will be found. This again raises the agency problem: who sets the planning horizon in public debt management? The principal who could be expected to care for the long run (well, long enough until the public debt is reimbursed in full if ever) or the agent who could be keener to focus on the end of his term, hoping for a renewal of the contract?

Time consistency. Another moot point is whether the debt manager should stick to his plans and what are the implications if he does not. Indeed, in our set-up there is nothing that binds the debt manager: at time t_2 he is free to set a weight for the bond other than the β value he had planned in t_0. What the model points out is rather that he will change his strategic allocation only if his rate that "structural" parameters have changed, otherwise he will probably just rephrase the tactical allocation in light of the new current yields. Metaphorically speaking, the strategy represents the direction in which the debt manager wants to move, while the tactics is the actual pathway he or she chooses to walk on in order to get to his destination.

However, even ruling out pre-commitment on future issuance decisions by the Treasury, the literature cited in Section 2 suggests that even a temporary deviation (i.e. a non-neutral tactical position) may bring about additional costs so that in practice the room for manoeuvre is limited. The intuition is the following. Investors are known to be willing to underwrite government securities, bills first and foremost, at interest rates well below the levels predicted by pricing models. In turn, this behaviour appears to reflect the sheer size of the outstanding stock of such securities, which makes them highly liquid and an eligible guarantee in many transactions. If the debt manager enacts a $\beta+\gamma$ with $\gamma \neq 0$ at time t_0, investors, being aware that the manager is not bound to actually implement β in the future, may sense that such a move could have a lasting impact. Say, if $\gamma > 0$, then they can't rule out that the stock of bills is going to be slashed for good, to the detriment of their liquidity. Hence, they could become less willing to grant the Treasury a large premium when underwriting the bills. The bottom line is that a positive γ could lead to an increase in E(i), where at this stage of qualitative investigation it does not matter whether such a reaction is non-linear or whether it embeds some threshold effect.

To explore how this would work in our set-up, let's take the partial derivative of the optimal β and γ as regards E(i) in [9] and [10]:

[14] $$\frac{\partial\hat{\beta}}{\partial E(i)} = \frac{1}{2\lambda\Sigma} > 0$$

[14a] $$\frac{\partial\hat{\gamma}}{\partial E(i)} = \frac{\sigma_r^2 - \sigma_{i,r}}{2\lambda\sigma_i^2\Sigma}$$

The target weight β will certainly increase with E(i). Conversely, we can't be sure about the sign of the change in γ except in general [14a] is non-zero. Evidence based on the whole 1995-2013 set of US zero yields suggests that the difference $\sigma_r^2 - \sigma_{i,r}$ is negative. Hence, further to the increase in E(i) the weight of the bond in the current issue would increase less than in the longer run, as the rise in the first term of the sum $\hat{\beta} + \hat{\gamma}$ would be offset, at least in part, by the decrease in the second term.[17]

4 Calibration and evaluation

4.1 Calibration of the parameters

To calibrate the parameters of the model, one obvious option is to search for sample counterparts whenever these are readily available. For example, the historical average of short- and long-term

[17] This finding depends critically on the length of the sample over which we work out the covariance: if this is measured over shorter periods, say as the rolling average over five-year periods rather than over the entire 18 years of the sample, the sign of [14a] turns positive.

yields over a sufficiently long period of time are good candidates for E(i) and E(r).[18] To this end, we use a dataset of end-month observations January 1995 – December 2013 of the 1-year US T-Bill and 10-year US Bond (Chart A.1).[19] From this dataset, we also work out empirical counterparts to the parameters σ_i, σ_r and $\sigma_{i,r}$.[20] In real-world data, the correlation between the short- and long-term yields is surprisingly high, being close to 0.9 on US data (higher than 0.8 in a parallel dataset for Germany) but it tends to fall in shorter samples. Since in practice the debt manager may think in terms of a relatively short horizon, we also measure variance-covariance statistics referred to a 5-year rolling window, which are then averaged out over the entire sample. In the final calibration of the model, the latter estimate is used (the results based on the former are available from the authors upon request).

Table 3

Sample statistics, based on US zero yields at 1 and 10 years (1995 – 2013)

	E(i)	E(r)	σ^2_i	σ^2_r	σ_{ir}	ρ
Whole sample	3.12	4.60	5.38	2.00	2.90	0.89
5-year rolling averages			2.28	0.44	0.73	0.73

More tricky is the estimation of the risk aversion parameter λ which has no straightforward sample counterpart. In this respect, we follow a 'revealed preference argument', by solving [7] with respect to λ and imposing $\beta_L = 0.817$ where this is the figure which in our set-up returns the actual WATM of the US debt at end 2012:

[15] $$\hat{\lambda} = \frac{E(s)}{2\left(\sigma_i^2 - \sigma_{r,i} - 0.817\Sigma\right)},$$

In practice, we are assuming that the maturity structure of the outstanding stock tells the level of risk aversion.[21] If one plugs the parameters shown in Table 3 in [15], then $\hat{\lambda} = 1.415$.

18 Of course, this is a simplification which neglects issues of structural breaks, non-stationarity, and so on.

19 The WATM of US debt is close to 5.5 years, maturity which in our set-up requires a representative bond in the area of 12 years (see result [3]). In fact, we used a time series of zero yields at the more liquid 10-year maturity; given the limited and relatively stable slope of the yield curve in the 10 to 12 year region, this choice is unlikely to affect the empirical results much.

20 The use of a time series compiled with end-month observations is meant to get rid of the high frequency noise often found in series of daily data. As a further data issue, one might like to rely on measures of *real* interest rates in lieu of the *nominal* figures we have used. However, using real rates would imply inferring one way or the other a monthly series of expectations of inflation over different maturities, a process that can hardly be unaffected by modelling decisions. In any case, throughout the period being examined the inflation rate did not show any extreme volatility in the US and Germany so the odds are that the use of either nominal or real interest rates would alter the main findings.

21 This procedure can be challenged on the grounds that we are measuring λ from one data point, while risk aversion can be deemed to change over time. However, it must be borne in mind that the objective here is not running inference on the 'true' value of λ but rather and more simply a value of λ which is consistent within our algebraic set-up with the other parameters. See Appendix B6 for further details on calibration steps.

4.2 Evaluation of tactical positions and the comparison with rules of thumb

To evaluate the opportunity of tactical position we draw in the {i,r} space the locus of points that brings the tactical position down to zero. In this way it can be readily checked whether a given pair of *i* and *r* falls in the region where γ is positive or negative. The interest rate data as of April 2013 (the red dot in Chart 3a) stand below the $\gamma = 0$ line, i.e. the model suggests taking a long (positive) tactical position on the bond. Through Eq. [10] we find that $\gamma = 12.6\%$. A positive tactical position is the same response that one could expect by simply applying a rule of thumb, such as a comparison of the Sharpe ratios of the bill and the bond (the green line in Chart 3a). According to this simple rule, it is good to issue a long term bond “more than usual” whenever $[r - E(r)]/\sigma_r < [i - E(i)]/\sigma_i$, that is

$$[16] \qquad \mathrm{r} < \frac{\sigma_{\mathrm{r}}}{\sigma_{\mathrm{i}}}[\mathrm{i} - \mathrm{E(i)}] + \mathrm{E(r)}$$

In this case both the model and the rule of thumb point to the same decision (i.e. an increase in bond issuance). However, this is not necessarily the case. If we consider data as of November 2013 the rule of thumb would insist with that choice, while the model points at the opposite conclusion (i.e. a reduction in bond issuance). Hence, rules of thumb, even when relatively sophisticated as in the Sharpe ratio version, may offer misleading signals by failing to take into account distinctive elements that the model considers. Anyhow, even when the signal is right, the rule-of-thumb does just that – it suggests to issue more or less of the bond – but, unlike the model, it does not provide the quantitative dimension of the optimal tactical position.

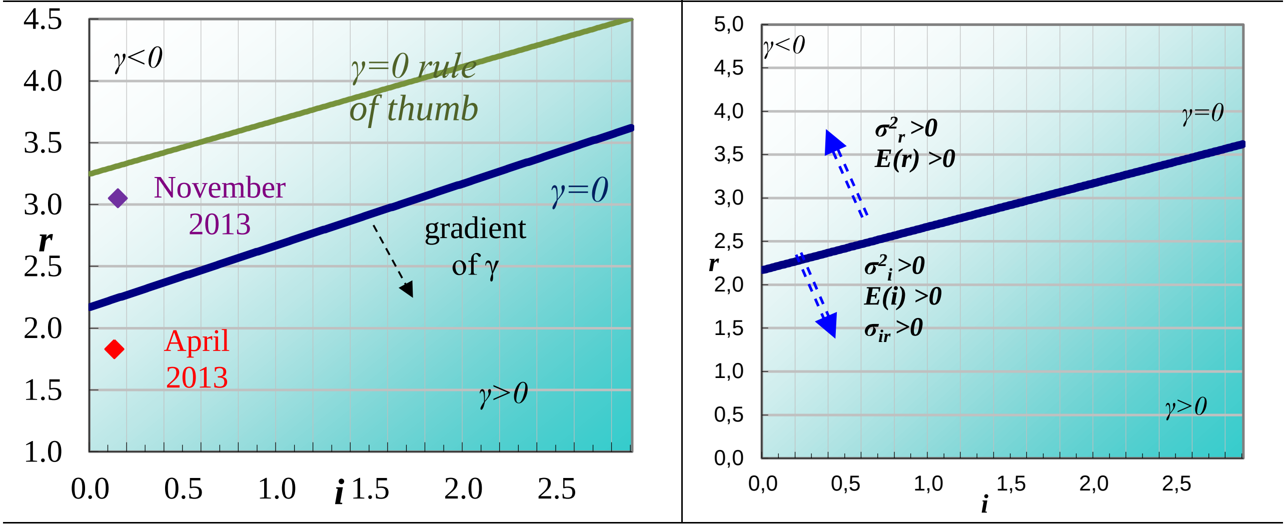
The assessment of strategic and tactical positions can be interestingly linked to a principal-agent problem in public debt management. We have shown above that different results can be obtained depending on the length of the planning horizon, and the fact that the principal’s planning horizon may well differ from the agent’s: while the State-principal ought to think in terms of a very long horizon (in principle it should stretch farther in the future until debt is paid back in full), debt manager-agents could consider a shorter horizon for their own agenda. Moreover, we have shown that a measure related to risk aversion, such as the shadow price of risk λ, affects the results both for the strategic and tactical position; yet, λ has no direct sample counterpart – and a State rarely bothers to utter its risk aversion precisely (or even vaguely). Again, debt manager-agents are not committed to a pre-defined level of risk tolerance, thus being ex-ante free to use their own one, and ex-post less accountable for their choices. Debt management is indeed monitored by means of simple proxies (such as the WATM of debt) or rules of thumb that, as shown, may give misleading signals.

Chart 3

The γ=*0* locus, current yields and the tactical positions (1)

(percentage)

a) The model vs the rule of thumb | b) Impact of change in parameters



(1) Parameters are calibrated as shown in the second row of Table 3 (rolling window version).

When we consider the impact of a perturbation in the parameters from the baseline setting (Chart 3b), it emerges that a lower covariance moves the blue line toward the line of the rule of thumb. In practice, a debt manager adopting such a rule would de facto be underestimating the correlation between the two securities. As a result, they are driven to think that there is a higher-than-real benefit from issuing the bond because of its lower volatility, neglecting however that this volatility is partly linked to the one of the bill.

4.3 Issuance portfolio reallocation after parameter-perturbation

In Charts A.3 we show how the issuance mix reacts to changes in the parameters. In each panel, moving from left to right, the area represents the portfolio allocation at three stages. The first one coincides with the initial steady state obtained under the calibration used in Chart 3a. The second stage represents the interim allocation after the parameter-perturbation, where a tactical position is pursued in addition to the possible revision of the strategic position. And finally, in the right-hand part of each panel only the new strategy is shown (representing the new long run equilibrium). The following findings can be highlighted.[22]

An increase in E(r) brings about a new strategic allocation in the long run where the weight of the bond is reduced as it becomes more costly; nevertheless, from a tactical point of view, it is convenient to delay the transition to the new strategy as the current bond yield r is still relatively low. Therefore, in the short run the issuance breakdown is unchanged (panel a).

Conversely, when it comes to E(i) to be revised, the model suggests to adapt the mix of issues with no delay (panel b), reflecting the fact that from [8] $\hat{\gamma}$ is a function also of shifts of the curve at its

[22] Without loss of generality, we consider parameter changes in the direction that implies a smaller or equal bond's portfolio-share in the new steady state.

short end. In non-algebraic terms, the idea is that by expanding already at t_0 the pool of bills, there is more to harvest in terms of decreasing interest expenditure when in t_1 the bills are to be renewed.

However, if both E(i) and E(r) move upward by the same magnitude, than the debt manager is advised to adopt $\hat{\gamma}>0$ but otherwise to keep $\hat{\beta}$ unchanged (panel c). This is because when the two interest rates increase by the same amount, there is no reason to revise the steady state strategy, as it depends on the "structural slope" which is by definition unaffected by a parallel shift (see [7]). However, it is convenient take a temporary long tactical position on the bond to exploit its current rates, which are lower than those expected to prevail in the future.

As to the parameters on the uncertainty of long-run values, both an increase in σ_r^2 and a reduction in σ_i^2 (respectively panels d and e) bring about a reduction in the bond share, ultimately because these changes make the bond less effective in insuring against the fluctuations of the bill yield. Note that the strategic position changes more widely with an increase in σ_r^2 than a decrease in σ_i^2.[23]

Finally, a decrease in λ decreases the issuance of the bond in the new strategic position; however, in the short run, it is not convenient to move straight to the new steady state but rather to keep on issuing the bond tactically (panel f). This is again due to the lock-in effect of current known yields. Therefore, the transition to the new steady state is smoothed through an interim stage where the issuance of the bond is somewhere in between the old and the new strategy.

4.4 Removing the price-taking assumption

In this section we remove the assumption that the volumes offered by the Treasury do not affect the interest rates set at the auctions. Notably, we assume that: (i) the yield r of the bond rises if the Treasury offers a larger than normal amount of this security, which in our framework is the supply suitable to keep the steady state (strategy); (ii) the yield i of the bill rises / decreases if the Treasury offers more / less than the strategy amount;[24] (iii) the future magnitude of the liquidity premium decreases, and all else being equal the expected yield E(i) of the bill rises, if the Treasury shrinks the outstanding stock of the bill.

Taken together, these additional assumptions make the process iterative. Let's denote $\gamma_0 = f(i_0, r_0, E_0(i))$ the solution in γ when the Treasury is a price-taker. Now, however, we expect interest rates to change: $i_0 \rightarrow i_1$, $r_0 \rightarrow r_1$ and $E_0(i) \rightarrow E_1(i)$. In turn, a new solution $\gamma_1 = f(i_1, r_1, E_1(i))$ is found. This brings about a new change in the interest rates and so on and so forth. If the process converges and, say, $\gamma_0 > 0$, one obtains the series of solutions: $0 < \gamma_0 < \gamma_2 < .. \gamma_n ..< \gamma_3 < \gamma_1$. Quite crucially, the process converged for 'ordinary' sets of the starting parameters, say when $\lambda > 1$, but not for all sets.

In (i), using estimates on Italy's BTPs (**Manna, Bernardini, Bufano and Dottori, 2013**), we set *r* to increase by 35 basis points for every 100% in the supply of the bond compared to the steady state solution which in our framework amounts to β/2 per year. As to (ii), we follow **Greenwood,**

[23] In [9] both the hedging and the cost components are scaled down by Σ and the portfolio total risk depends positively on both σ^2_r and σ^2_i When σ^2_r increases, Σ gets larger too, scaling down the magnitude of the issuance strategy by dwarfing the comparison between hedging advantages vs. slope costs. The opposite occurs following a decrease in σ^2_i but in this case there is another contrasting effect which prevails: i.e. the weakening of the hedging advantage.

[24] In (i) and (ii) the short-term interest rate decreases if the Treasury reduces the supply of the bill while the long-term interest rate stays unchanged when the bond's volume declines. This asymmetry ultimately reflects the larger advantage the Treasury holds compared to the private sector in issuing the bill. One alternative approach would have been to posit that both interest rates decline but the short-term one declines more. Hence, the described asymmetry should be understood as a relative measure of the different elasticity.

Hanson and Stein (2010a), who estimate the short-term interest rates to change by 8.09 times the change in the bill's stock to GDP ratio. If the overall stock of debt is comparable to the size of GDP and the bill's weight out of total stock of debt is $(1–\beta)$, the change of the interest rate is in the order of 8.09 times the change in the stock of bills, where the latter amounts to $\gamma/(1–\beta)$. Finally, as regards (iii), according to **Krishnamurthy and Vissing-Jorgesen (2012)** the liquidity premium of the US Treasury Bills is 73 basis points. As concluded at the end of Section 2.2, our thinking is that were the stock of bills to shrink significantly, the liquidity premium could shrink and E(i) rise accordingly.

To sum up, the iterations in the numerical exercise are carried out according to the following 'laws of motion':

[17.1] $\Delta r_k = 0.35 \times \gamma_{k-1} / (\beta_0/2)$ if $\gamma_0 > 0$

0 otherwise

[17.2] $\Delta i_k = 0.0809 \times \gamma_{k-1} / (1–\beta_0)$

[17.3] $\Delta E(i)_k = – \Delta i_k + 0.73 \times \gamma_{k-1} / (1–\beta_0)$ if $\gamma_0 > 0$

$– \Delta i$ otherwise

where $\beta_0 = 0.817$ in the calibration using the US data and $\gamma_k = f\ (\Delta r_k,\ \Delta i_k.\ \Delta E(i)_k \mid E(r))$. The numerical solution for k = 65 is $\gamma_{65} = 8.0\%$ (where γ_{65} is our fit after the latest iteration we tried out, the 65th) which compares to $\gamma_0 = 12.6\%$.[25] Namely, the optimal tactical position in a set-up where interest rates of government securities are elastic to volumes of supply is between a half and two thirds of the result obtained if interest rates were inelastic.

While there is no uncertainty as to the sign of the interest rate changes laid down in [17.1], [17.2] and [17.3], the exact magnitude of such changes is debateable. As outlined above, we rely on the extant literature to select the numerical values of 0.35 in [17.1], 0.0809 in [17.2] and 0.73 in [17.3]; these are estimates from models and as such they are surrounded by unavoidable uncertainty. Thus, we carried out a number of checks to shed light on how sensitive our end-result is in terms of γ_{65} to some of the underlying assumptions: e.g. we cut to half the parameter of 0.73 in [17.3] and we increased the ratio in the maturities of the bond to the bill from 2:1 to 3.1 and then to 4:1. As a result, we obtained new outcomes for γ_{65} ranging from 6.6% to 8.4%. While this range is not exceedingly narrow, as a matter of fact it does not change the bottom line: when frictions are added in the numerical exercise, the increase in the weight of the bond measured by the parameter γ_{65} falls substantially, perhaps to half.

5. Concluding remarks

This paper aims to answer the following question: under which conditions should public debt managers opt for a maturity mix in the issuance of the government securities other than a long-run target? Empirically, the quest for an answer to this question owes much to the constellation of market interest rates prevailing in 2012-2013. At that time, the (zero yield) interest rate on the US 1-year bill was well below a long-run average but in relative terms the corresponding rate on the 10-year bond was even lower. Hence, there was apparently a "window of opportunity" to lock in the

[25] In most of the numerical exercises we carried out γ_k stabilises already when k=10.

cost of debt at very low levels for a prolonged period of time, by increasing the supply of bonds at the expense of bills.

We structured the algebra to analyse this problem in terms of a cost minimization function where the debt manager simultaneously seeks to achieve the optimal weight on the bond in the current period and in the long run, while debt is assumed to stay constant. The model is examined first in a frictionless world, where the Treasury is a price-taker and there are no implications for the planned switch from one maturity mix to another as time goes by. Then, we discuss how the removal of these assumptions could affect our main results. As a main numerical result based on the US interest rate data as at April 2013, we find that the long tactical position – by which we mean a temporary increase in the weight of the bond – is close to 13% in a world with frictions, falling to 6-8% when price elasticity and potential developments in the T-Bills liquidity premium come into the picture. It is noteworthy that it is the penalty associated with the flattening of such premiums that keeps in check the supply mix. This does not go as far as implying a full commitment – in fact, no debt manager ties his or her hands entirely – but it slows down the extent to which the current "tactical" choice, in the sense used in the introduction, would diverge from the "strategic" one. Results like these could help explain the degree of caution exercised by many debt managers.

While carrying out this research, we could not help but see how often a principal (government) – agent (debt manager) dilemma emerged. In an ex-ante perspective, it appears in the definition and disclosure of risk tolerance, which in our set-up amounts to calibrating the shadow price of risk; or in the missing statement of an explicit planning horizon, about which we observed that taking a short-horizon can make the maturity mix especially volatile. In an ex-post perspective, the principal-agent dilemma arises when monitoring the performance of the agent, which is not straightforward. Few statistics are available to double check whether the actual debt management is meeting the widely accepted definition of "low cost over the medium term with a tolerable degree of risk" in serving the debt.

This lack of structural information is compounded by the fact that rules of thumb can give misleading signals. For instance, again using US data as of November 2013, we show that while a model-based solution suggested to retrench towards shorter maturities, a sort of Sharpe ratio rule would have given the opposite signal.

The model we put forward allows us to identify at least some links between public debt management and monetary policy. For instance, all other things being equal, were the central bank to pursue an exchange rate target then the debt manager is advised to seek a structure of the debt with longer maturity, compared to a scenario where the central bank can control the short-term rates more steadily.

We also urge actors not to overlook the financial stability implications of debt management. A shrinking of the bills' stock on the scale suggested by the aforementioned results when no frictions are added could make the whole financial system more fragile. The odds are that private agents would fill in the lighter supply of securities at shorter maturities, making their balance sheets more fragile. Moreover, investors would take a substantial amount of market risk by underwriting larger sums of bonds, with long maturity and duration, when yields are very low and thus prices very high.

The model we propose offers opportunities for interesting extensions and further research. For instance, in presenting this paper to several audiences we have been encouraged to explore the impact of shocks on the size of the debt. A crisis scenario of that type could make the analysis of higher moments in the distribution of interest rates more important (hence, fat tails) but also of time-varying correlations.

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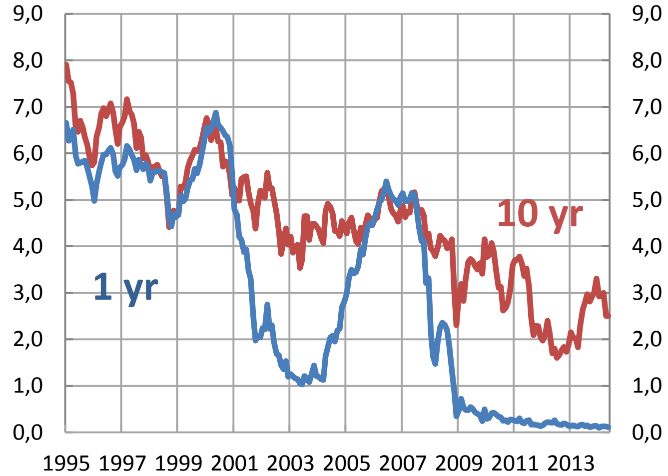
Chart A.1

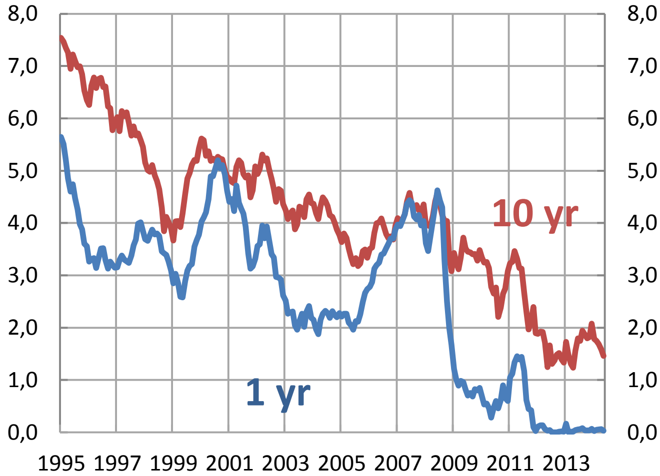
1- and 10-year USA and German zero yields on sovereign benchmarks
(end-of-month data)

A) yields to maturity (per cent)

USA

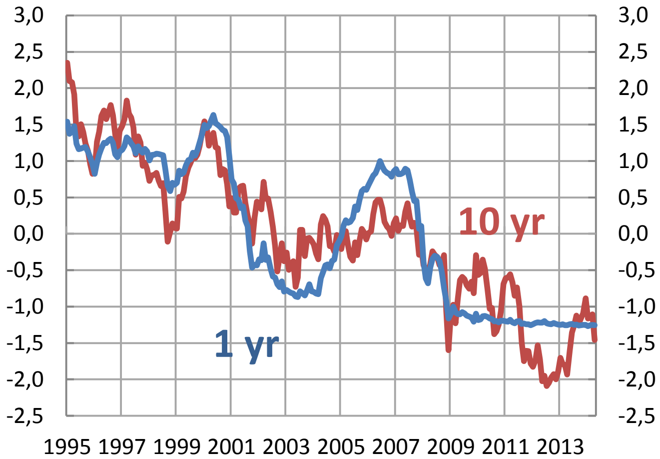


GERMANY

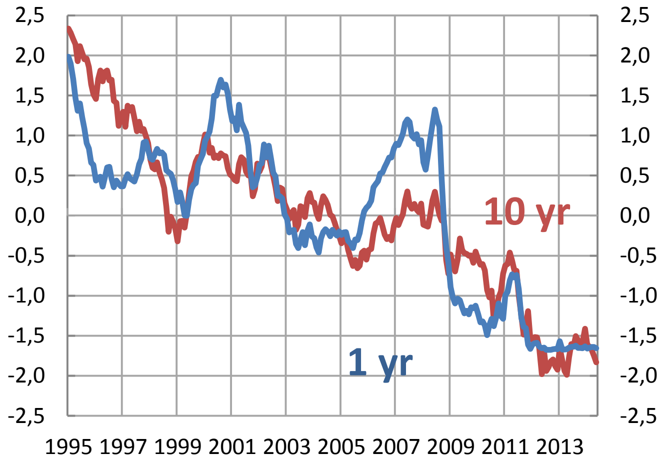


B) deviations from mean (number of standard deviations)

USA

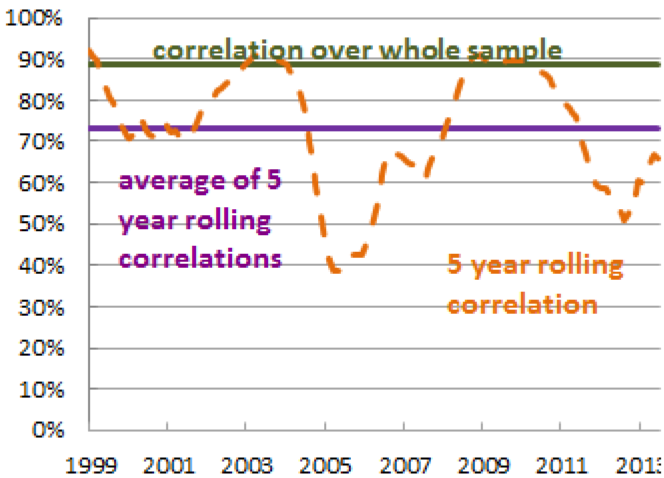


GERMANY

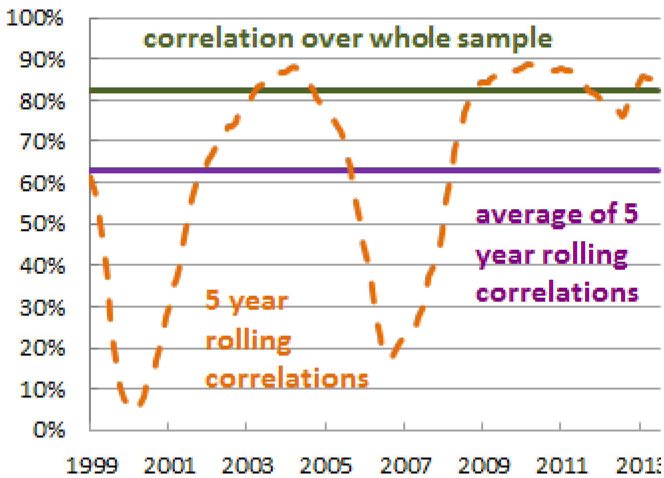


C) correlations

USA



GERMANY

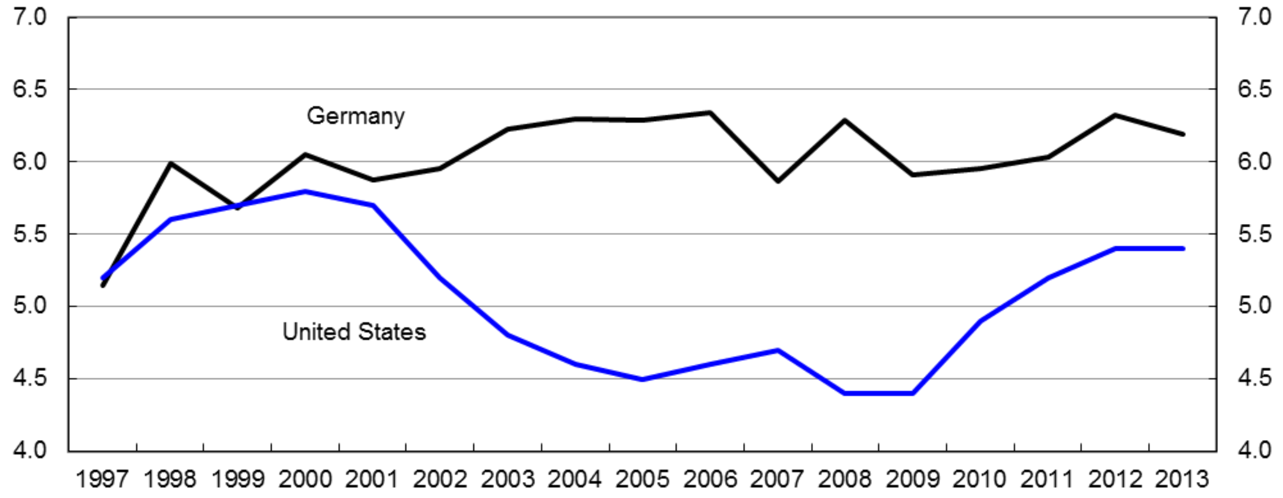


Source: Authors' calculations based on Bloomberg data.

Chart A.2

Average term to maturity for domestic debt

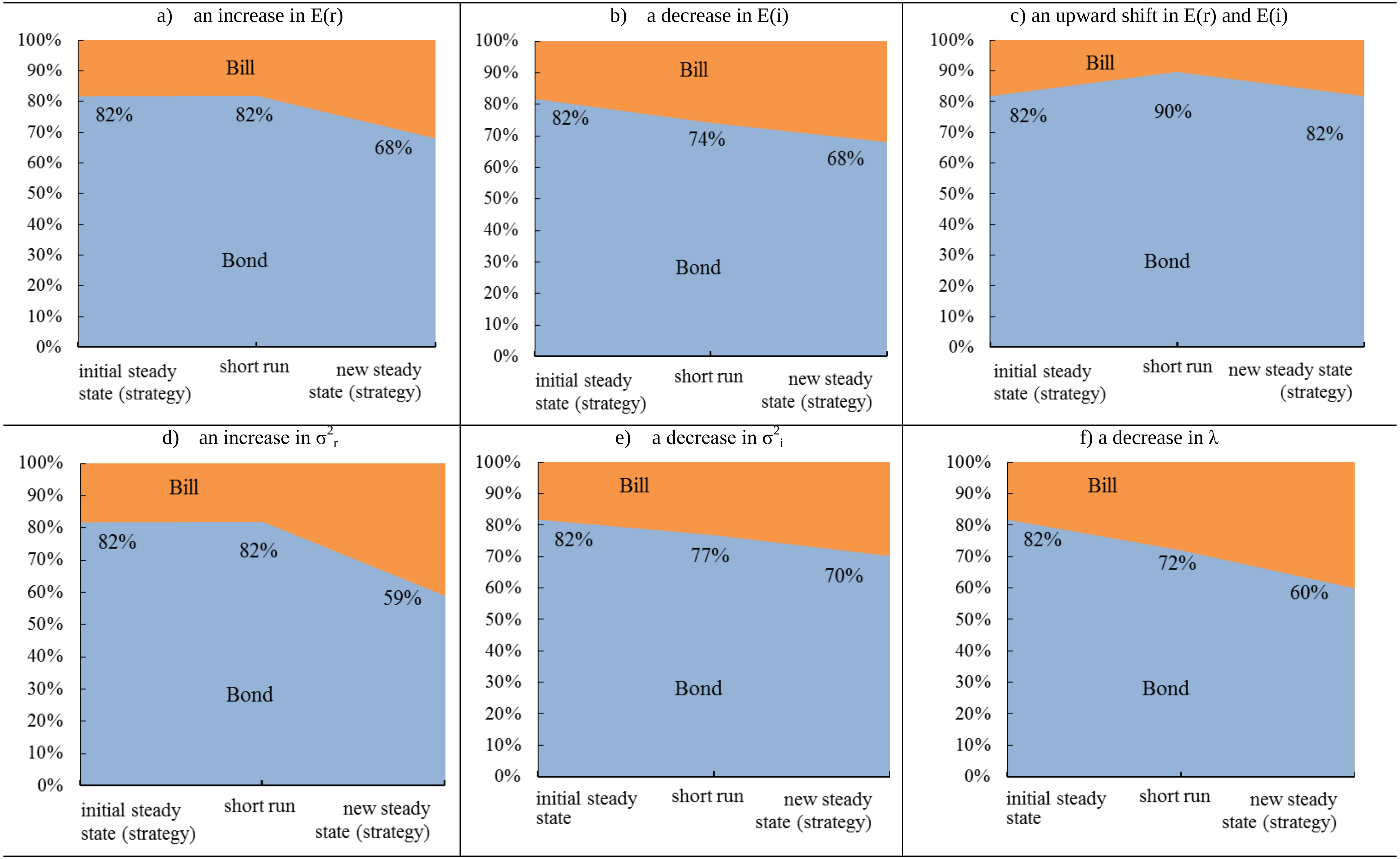
(number of years; measured on outstanding stock at year end)



Sources: OECD, US Treasury and author’s calculation on Thomson Reuters data.

Chart A.3

Debt portfolio reallocation following … (1)



(1) Changes of 50 basis points are assumed for panels from a) to e), 0.2 for panel f). The initial (baseline) scenario is obtained with parameter calibration as in Table 4

APPENDIX

A. Basic algebra in public debt

Let G_t be government spending on goods and services in period t, T_t the tax revenues, i_t the weighted average cost of serving the debt, I_t and R_t the gross issues and the reimbursements of government securities while B_{t-1} is the stock of debt outstanding at the end of t–1 (here, we are following with some minor adaptations **Neck and Egbert, 2008**, but the reader could easily find many good references on the ensuing algebra). Hence, the budget constraint

[A.1] $G_t + i_t \times B_{t-1} = T_t + (I_t - R_t)$

Taking ratios to GDP so that e.g. $g_t \equiv G_t / GDP_t$, and introducing the additional symbols $d_t \equiv g_t - t_t$ to refer to the primary budget deficit ratio and $\hat{y}_t$ to the growth rate of GDP, [1] becomes

[A.2] $$d_t + \frac{i_t}{1+\hat{y}_t} b_{t-1} = b_t$$

Once a No Ponzi Game condition is imposed, whereby government cannot rely on ever increasing issuance of new debt in proportion to GDP to repay old debt, one obtains the government intertemporal budget constraint:

[A.3] $$\sum_{t=1}^{\infty}\left(d_t \prod_{s=1}^{t} \frac{1+\hat{y}_s}{i_s} \right) + b_0 = 0$$

where b_0 is the current debt ratio.

PROOFS

B1. Solution of the model with one unknown (β)

By expanding $f_\beta[j]$ we have

$$f_\beta[0] = \beta r + (1-\beta) i$$

$$f_\beta[1] = \beta r + (1-\beta) E(i) + \lambda (1-\beta)^2 \sigma_i^2$$

$$f_\beta[j] = \beta E(r) + (1-\beta) E(i) + \lambda [\beta^2 \sigma_r^2 + 2 \beta (1-\beta) \sigma_{i,r} + (1-\beta)^2 \sigma_i^2], \text{ for } j=2,\ldots,N-1$$

Summing up over j and taking the partial derivatives with respect to β we find the condition for interior solutions:

$$\frac{\partial f_\beta}{\partial \beta} = 2r - i - E(i) - 2\lambda(1-\beta)\sigma_i^2 + (N-2)\left\{E(r) - E(i) - 2\lambda\left[(1-\beta)\sigma_i^2 - (1-2\beta)\sigma_{i,r} - \beta\sigma_r^2\right]\right\} = 0$$

$$= 2r - i - E(i) - (N-2)[E(r) - E(s)] - 2\lambda\sigma_i^2 + 2\lambda\sigma_i^2\beta + 2\lambda\beta(\sigma_i^2 - 2\sigma_{i,r} + \sigma_r^2) - 2\lambda(\sigma_i^2 - 2\sigma_{i,r}) = 0$$

Setting $\Sigma \equiv \sigma_i^2 - 2\sigma_{ir} + \sigma_r^2$ and solving for β we have:

$$\hat{\beta} = \frac{\sigma_i^2}{\sigma_i^2 + (N-2)\Sigma} + \frac{(N-2)(\sigma_i^2 - \sigma_{ir})}{\sigma_i^2 + (N-2)\Sigma} - \frac{(N-2)[\frac{E(r)-E(i)}{2\lambda}]}{\sigma_i^2 + (N-2)\Sigma} - \frac{[2r - i - E(i)]/2\lambda}{\sigma_i^2 + (N-2)\Sigma}$$

Taking the limit for $N\to\infty$, we get Eq. [5]. Introducing the correlation coefficient $\rho = (\sigma_i^2 \times \sigma_r^2) / \sigma_{i,r}$, we also have $\Sigma = \sigma_i^2 + \sigma_r^2 - 2\rho\sigma_i \sigma_r$. Since $\Sigma > 0$, it must be $\rho < (\sigma_i^2 + \sigma_r^2) / (2 \sigma_i \sigma_r)$. Using the notation $x \equiv \sigma_i / \sigma_r$, we have $\rho < (x + 1/x) / 2$ which always holds true if $(x + 1/x) / 2 > 1$ as $\rho \le 1$. Now $(x + 1/x) / 2 > 1 \Leftrightarrow x^2 - 2x + 1 > 0 \Leftrightarrow (x - 1)^2 > 0$, except for $x = 1$, i.e. when $\sigma_i = \sigma_r$. In this particular situation, however, the problem becomes trivial as the two instruments would feature the

same volatility and the issuer would prefer the security with the lowest yield, having no advantage from diversification.
The second order condition for a maximum requires that the second derivative is positive. It always holds true as:

$$\frac{\partial^2}{\partial\beta^2} f \equiv f_\beta^{"} = 2\lambda[\sigma_i^2 + (\sigma_i^2 - 2\sigma_{i,r} + \sigma_r^2)] = 2\lambda(\sigma_i^2 + \Sigma) > 0$$

where the sign is unambiguous as λ, σ_i^2 and Σ are all strictly positive.
The partial derivatives of $\hat{\beta}^L$ are:

$$\frac{\partial}{\partial\sigma_r^2}\hat{\beta}^L = \frac{-(\sigma_i^2 - \sigma_{i,r}) + [E(r) - E(i)]/(2\lambda)}{\Sigma^2} = -\frac{1}{\Sigma}\hat{\beta}^L < 0$$

$$\frac{\partial}{\partial\sigma_i^2}\hat{\beta}^L = \frac{\Sigma - (\sigma_i^2 - \sigma_{i,r})}{\Sigma^2} + \frac{1}{\Sigma^2}\frac{E(r) - E(i)}{2\lambda} = \frac{\sigma_r^2 - \sigma_{i,r}}{\Sigma^2} + \frac{1}{\Sigma^2}\frac{E(r) - E(i)}{2\lambda}$$

Hence

$$\frac{\partial}{\partial\sigma_i^2}\hat{\beta}^L > 0 \Leftrightarrow \sigma_r^2 - \sigma_{i,r} + \frac{E(r) - E(i)}{2\lambda} > 0 \Leftrightarrow \rho < \frac{E(r) - E(i)}{2\lambda\sigma_i\sigma_r} + \frac{\sigma_r}{\sigma_i}$$

$$\sigma_{i,r} = \rho\,\sigma_i\,\sigma_r$$

$$\frac{\partial}{\partial\sigma_{i.r}}\hat{\beta}^L = \frac{-\Sigma + 2(\sigma_i^2 - \sigma_{i,r})}{\Sigma^2} - \frac{2}{\Sigma^2}\frac{E(r) - E(i)}{2\lambda} = \frac{\sigma_i^2 - \sigma_r^2}{\Sigma^2} - \frac{1}{\Sigma^2}\frac{E(r) - E(i)}{\lambda}$$

Hence

$$\frac{\partial}{\partial\sigma_{i,r}}\beta^L > 0 \Leftrightarrow \sigma_i^2 - \sigma_r^2 - \frac{E(r) - E(i)}{\lambda} > 0 \Leftrightarrow \lambda > \frac{E(r) - E(i)}{\sigma_i^2 - \sigma_r^2}$$

Where we used the fact that $\Sigma > 0$;

$$\frac{\partial}{\partial\lambda}\beta^L = \frac{2[E(r) - E(i)]}{2\lambda^2\Sigma} =$$

$$\frac{\partial}{\partial\lambda}\beta^L > 0 \Leftrightarrow E(r) > E(i)$$

The effects of E(r) and E(i) are straightforward.

B2. Solution of the model with two unknowns (β and γ), when N = 4

The algebra is laid down using the transforms $\beta' = \beta + \gamma/2$ and $\gamma' = \gamma/2$ where β and γ are defined as in [9] and [10] respectively. This eases the presentation by introducing an element of symmetry.

B2.1 We expand [6] for j=0,…,3

[A.4a] $f_{\beta,\gamma}[0] = \begin{bmatrix}\beta' + \gamma' & 1 - \beta' - \gamma'\end{bmatrix}\begin{bmatrix} r \\ i \end{bmatrix}$

[A.4b] $f_{\beta,\gamma}[1] = \begin{bmatrix}\beta' + \gamma' & 1 - \beta' - \gamma'\end{bmatrix}\begin{bmatrix} r \\ E(i) \end{bmatrix} + \lambda\begin{bmatrix}\beta' + \gamma' & 1 - \beta' - \gamma'\end{bmatrix}\begin{bmatrix} 0 & 0 \\ 0 & \sigma_i^2 \end{bmatrix}\begin{bmatrix} \beta' + \gamma \\ 1 - \beta' - \gamma' \end{bmatrix}$

[A.4c] $f_{\beta,\gamma}[2] = f_{\beta,\gamma}[3] = \begin{bmatrix}\beta' - \gamma' & 1 - \beta' + \gamma'\end{bmatrix}\begin{bmatrix} E(r) \\ E(i) \end{bmatrix} + \lambda\begin{bmatrix}\beta' - \gamma' & 1 - \beta' + \gamma'\end{bmatrix}\begin{bmatrix} \sigma_r^2 & \sigma_{i,r} \\ \sigma_{i,r} & \sigma_i^2 \end{bmatrix}\begin{bmatrix} \beta' - \gamma' \\ 1 - \beta' + \gamma' \end{bmatrix}$

where $f_{\beta,\gamma}$ [2] = $f_{\beta,\gamma}$ [3] is a straightforward implication of the simplifying assumption that the expected short term rate is the same over time.
The constraint $\beta' \pm \gamma' \in [0,1]$ implies that

$$0 \le \beta' + \gamma' \le 1 \cup 0 \le \beta' - \gamma' \le 1 \quad \Rightarrow \quad -\beta' \le \gamma' \le 1 - \beta' \cup -\beta' \le -\gamma' \le 1 - \beta'$$

$$\Rightarrow \quad -\beta' \le \gamma' \le 1 - \beta' \cup -1 + \beta' \le \gamma \le \beta'$$

Which can be written as:

$$\begin{cases} -\beta \le \gamma \le \beta & \text{if} \quad \beta \in [0, 1/2] \\ -(1-\beta) \le \gamma \le (1-\beta) & \qquad \beta \in [1/2, 1] \end{cases}$$

B2.2 Taking derivatives

The functions [7a]-[7c] can be written as:

$$f_{\beta,\gamma}[0] = (\beta' + \gamma')\, r + (1 - \beta' - \gamma')\, i$$

$$f_{\beta,\gamma}[1] = (\beta' + \gamma')\, r + (1 - \beta' - \gamma')\, E(i) + \lambda\, (1 - \beta' - \gamma')^2\, \sigma_i^2$$

$$f_{\beta,\gamma}[2] = f_{\beta,\gamma}[3] = (\beta' - \gamma')\, E(r) + (1 - \beta' + \gamma')\, E(i) + \lambda\, [(\beta' - \gamma')^2 \sigma_r^2 + 2\, (\beta' - \gamma')\, (1 - \beta' + \gamma')\, \sigma_{i,r} + (1 - \beta' + \gamma')^2 \sigma_i^2\,]$$

Taking the partial derivatives we have:

[A.5a] $$\frac{\partial}{\partial \beta'} f \equiv f'_\beta = [r - i] + [r - E(i) - 2\,\lambda\, (1 - \beta - \gamma)\,\sigma_i^2\,] + 2\, \{E(r) - E(i) + 2\,\lambda\, (\beta' - \gamma')\,\sigma_r^2 + 2\,\lambda\, [(1 - \beta' + \gamma') - (\beta' - \gamma')]\,\sigma_{i,r} - 2\,\lambda\, (1 - \beta' + \gamma')\,\sigma_i^2\, \}$$

$$\Rightarrow \quad f'_\beta = r - i + r - E(i) - 2\,\lambda\, (1 - \beta' - \gamma')\,\sigma_i^2 + 2\, E(r) - 2\, E(i) + 4\,\lambda\, (\beta' - \gamma')\,\sigma_r^2 + 4\,\lambda\, (1 - \beta' + \gamma' - \beta' + \gamma')\,\sigma_{i,r} - 4\,\lambda\, (1 - \beta' + \gamma')\,\sigma_i^2$$

$$\Rightarrow \quad f'_\beta = 2\, r + 2\, E(r) - i - 3\, E(i) - 2\,\lambda\, (1 - \beta' - \gamma')\,\sigma_i^2 + 4\,\lambda\, (\beta' - \gamma')\,\sigma_r^2 + 4\,\lambda\, (1 - 2\,\beta' + 2\,\gamma')\,\sigma_{i,r} - 4\,\lambda\, (1 - \beta' + \gamma')\,\sigma_i^2$$

[A.5b] $$\frac{\partial}{\partial \gamma} f \equiv f'_\gamma = [r - i] + [r - E(i) - 2\,\lambda\, (1 - \beta' - \gamma')\,\sigma_i^2\,] + 2\, \{-E(r) + E(i) - 2\,\lambda\, (\beta' - \gamma')\,\sigma_r^2 + 2\,\lambda\, [-(1 - \beta' + \gamma') + (\beta' - \gamma')]\,\sigma_{i,r} + 2\,\lambda\, (1 - \beta' + \gamma')\,\sigma_i^2\, \}$$

$$\Rightarrow \quad f'_\gamma = r - i + r - E(i) - 2\,\lambda\, (1 - \beta' - \gamma)\,\sigma_i^2 - 2\, E(r) + 2\, E(i) - 4\,\lambda\, (\beta' - \gamma')\,\sigma_r^2 + 4\,\lambda\, (-1 + \beta' - \gamma' + \beta' - \gamma')\,\sigma_{i,r} + 4\,\lambda\, (1 - \beta' + \gamma')\,\sigma_i^2$$

$$\Rightarrow \quad f'_\gamma = 2\, r - 2\, E(r) - i + E(i) - 2\,\lambda\, (1 - \beta' - \gamma')\,\sigma_i^2 - 4\,\lambda\, (\beta' - \gamma')\,\sigma_r^2 + 4\,\lambda\, (-1 + 2\,\beta' - 2\,\gamma')\,\sigma_{i,r} + 4\,\lambda\, (1 - \beta' + \gamma')\,\sigma_i^2$$

[A.5c] $$\frac{\partial}{\partial \beta\, \partial \beta} f \equiv f''_{\beta,\beta} = 2\,\lambda\,\sigma_i^2 + 4\,\lambda\,\sigma_r^2 - 8\,\lambda\,\sigma_{i,r} + 4\,\lambda\,\sigma_i^2 = 4\,\lambda\,\sigma_r^2 - 8\,\lambda\,\sigma_{i,r} + 6\,\lambda\,\sigma_i^2$$

[A.5d] $$\frac{\partial}{\partial \gamma\, \partial \gamma} f \equiv f''_{\gamma,\gamma} = 2\,\lambda\,\sigma_i^2 + 4\,\lambda\,\sigma_r^2 - 8\,\lambda\,\sigma_{i,r} + 4\,\lambda\,\sigma_i^2 = 4\,\lambda\,\sigma_r^2 - 8\,\lambda\,\sigma_{i,r} + 6\,\lambda\,\sigma_i^2$$

[A.5e] $$\frac{\partial}{\partial \beta\, \partial \gamma} f \equiv f''_{\beta,\gamma} = 2\,\lambda\,\sigma_i^2 - 4\,\lambda\,\sigma_r^2 + 8\,\lambda\,\sigma_{i,r} - 4\,\lambda\,\sigma_i^2 = -4\,\lambda\,\sigma_r^2 + 8\,\lambda\,\sigma_{i,r} - 2\,\lambda\,\sigma_i^2$$

where [A.5e] can be obtained by deriving [A.5a] with respect to γ or [A.4b] with respect to β.

B2.3 Second order conditions for a minimum

The Hessian matrix is given by: $\begin{bmatrix} \mathrm{Ic} & \mathrm{Ie} \\ \mathrm{Ie} & \mathrm{Id} \end{bmatrix}$. Necessary conditions are that, evaluated in the critical points, (i) Ic>0 and (ii) the determinant Ic×Id–Ie×Ie >0.

First, we note that none of these terms depend upon β and γ, therefore if the conditions hold they hold for the critical points too. Condition (i) requires that

$$\sigma_{i,r} < \frac{3\sigma_i^2 + 2\sigma_r^2}{4}$$

It can be proved the inequality is always verified. Since $\sigma_{i,r} = \rho\sigma_i\sigma_r$, we have:

$$\sigma_{i,r} = \rho\sigma_i\sigma_r < \frac{3\sigma_i^2 + 2\sigma_r^2}{4}$$

$$\Rightarrow \qquad \rho < \frac{3\frac{\sigma_i}{\sigma_r} + 2\frac{\sigma_r}{\sigma_i}}{4}$$

$$\Rightarrow \qquad \rho < \frac{3x + 2\frac{1}{x}}{4} \qquad \text{where } \frac{\sigma_i}{\sigma_r} \equiv x$$

Since $\rho \le 1$, the inequality is certainly true if the right hand side is above one. This amount to state 3 x +2/x > 4 $\Rightarrow 3x^2 - 4x + 2 > 0$, which has no real roots and is always positive.

As regards condition (ii), we have [*Ic*]×[*Id*]–[*Ie*]×[*Ie*]>0

$$\Rightarrow \qquad (4\lambda\sigma_r^2 - 8\lambda\sigma_{i,r} + 6\lambda\sigma_i^2)(4\lambda\sigma_r^2 - 8\lambda\sigma_{i,r} + 6\lambda\sigma_i^2) - (-4\lambda\sigma_r^2 + 8\lambda\sigma_{i,r} - 2\lambda\sigma_i^2)^2$$

$$\Rightarrow \qquad (4\lambda\sigma_r^2 - 8\lambda\sigma_{i,r} + 6\lambda\sigma_i^2)^2 - (4\lambda\sigma_r^2 - 8\lambda\sigma_{i,r} + 2\lambda\sigma_i^2)^2$$

$$\Rightarrow \qquad (2\lambda)^2(2\sigma_r^2 - 4\sigma_{i,r} + 3\sigma_i^2)^2 - (2\lambda)^2(2\sigma_r^2 - 4\sigma_{i,r} + \sigma_i^2)^2$$

$$\Rightarrow \qquad (2\lambda)^2[(2\sigma_r^2 - 4\sigma_{i,r})^2 + (3\sigma_i^2)^2 + 2\times(2\sigma_r^2 - 4\sigma_{i,r})\times(3\sigma_i^2) - (2\sigma_r^2 - 4\sigma_{r,i})^2 - (\sigma_i^2)^2 - 2\times(2\sigma_r^2 - 4\sigma_{i,r})\times(\sigma_i^2)]$$

$$\Rightarrow \qquad 4\lambda^2[8(\sigma_i^2)^2 + 4\times(2\sigma_r^2 - 4\sigma_{i,r})\times\sigma_i^2]$$

$$\Rightarrow \qquad 16\lambda^2\sigma_i^2[2\sigma_i^2 + 2\sigma_r^2 - 4\sigma_{i,r}]$$

$$\Rightarrow \qquad 32\lambda^2\sigma_i^2(\sigma_i^2 + \sigma_r^2 - 2\sigma_{i,r})$$

from which: $\sigma_i^2 + \sigma_r^2 - 2\sigma_{i,r} \equiv \Sigma > 0$ as proved in Proof 1.

B2.4 The interior solution.

We solve the system [A.4a] and [A.4b]

[A.5a] $2r + 2E(r) - i - 3E(i) - 2\lambda(1-\beta'-\gamma')\sigma_i^2 + 4\lambda(\beta'-\gamma')\sigma_r^2 + 4\lambda(1-2\beta'+2\gamma')\sigma_{i,r} - 4\lambda(1-\beta'+\gamma')\sigma_i^2 = 0$

[A.5b] $2r - 2E(r) - i + E(i) - 2\lambda(1-\beta'-\gamma')\sigma_i^2 - 4\lambda(\beta'-\gamma')\sigma_r^2 + 4\lambda(-1+2\beta'-2\gamma')\sigma_{i,r} + 4\lambda(1-\beta'+\gamma')\sigma_i^2 = 0$

To have a light notation, we introduce the symbols

$$A \equiv 2r + 2E(r) - i - 3E(i)$$

$B \equiv 2\,r - 2\,E(r) - i + E(i)$

it follows:

$A + B = (2\,r + 2\,E(r) - i - 3\,E(i)) + (2\,r - 2\,E(r) - i + E(i)) = 2\,r + 2\,E(r) - i - 3\,E(i) + 2\,r - 2\,E(r) - i + E(i) = 4\,r - 2\,i - 2\,E(i)$

$A - B = (2\,r + 2\,E(r) - i - 3\,E(i)) - (2\,r - 2\,E(r) - i + E(i)) = 2\,r + 2\,E(r) - i - 3\,E(i) - 2\,r + 2\,E(r) + i - E(i) = 4\,E(r) - 4\,E(i)$

Thus, the system [Ia] e [Ib] becomes

[A.6a] $A - 2\,\lambda\sigma_i^2 + 2\,\lambda\,\beta'\sigma_i^2 + 2\,\lambda\,\gamma'\sigma_i^2 + 4\,\lambda\,\beta'\sigma_r^2 - 4\,\lambda\,\gamma'\sigma_r^2 + 4\,\lambda\sigma_{i,r} - 8\,\lambda\,\beta'\sigma_{i,r} + 8\,\lambda\,\gamma'\sigma_{i,r} - 4\,\lambda\sigma_i^2 + 4\,\lambda\,\beta'\sigma_i^2 - 4\,\lambda\,\gamma'\sigma_i^2 = 0$

$\Rightarrow$ $A - 6\,\lambda\sigma_i^2 + 6\,\lambda\,\beta'\sigma_i^2 - 2\,\lambda\,\gamma'\sigma_i^2 + 4\,\lambda\,\beta'\sigma_r^2 - 4\,\lambda\,\gamma'\sigma_r^2 + 4\,\lambda\sigma_{i,r} - 8\,\lambda\,\beta'\sigma_{i,r} + 8\,\lambda\,\gamma'\sigma_{i,r} = 0$

[A.6b] $B - 2\,\lambda\sigma_i^2 + 2\,\lambda\,\beta'\sigma_i^2 + 2\,\lambda\,\gamma'\sigma_i^2 - 4\,\lambda\,\beta'\sigma_r^2 + 4\,\lambda\,\gamma'\sigma_r^2 - 4\,\lambda\sigma_{i,r} + 8\,\lambda\,\beta'\sigma_{i,r} - 8\,\lambda\,\gamma'\sigma_{i,r} + 4\,\lambda\sigma_i^2 - 4\,\lambda\,\beta'\sigma_i^2 + 4\,\lambda\,\gamma'\sigma_i^2 = 0$

$\Rightarrow$ $B - 2\,\lambda\sigma_i^2 - 2\,\lambda\,\beta'\sigma_i^2 + 6\,\lambda\,\gamma'\sigma_i^2 - 4\,\lambda\,\beta'\sigma_r^2 + 4\,\lambda\,\gamma'\sigma_r^2 - 4\,\lambda\sigma_{i,r} + 8\,\lambda\,\beta'\sigma_{i,r} - 8\,\lambda\,\gamma'\sigma_{i,r} = 0$

Let us introduce another two symbols to group terms that do not depend on β e γ

$C \equiv -6\,\lambda\sigma_i^2 + 4\,\lambda\sigma_{r,i}$

$D \equiv 2\,\lambda\sigma_i^2 - 4\,\lambda\sigma_{r,i}$

it follows

$C + D = (-6\,\lambda\sigma_i^2 + 4\,\lambda\sigma_{i,r}) + (2\,\lambda\sigma_i^2 - 4\,\lambda\sigma_{i,r}) = -4\,\lambda\sigma_i^2$

$C - D = (-6\,\lambda\sigma_i^2 + 4\,\lambda\sigma_{i,r}) - (2\,\lambda\sigma_i^2 - 4\,\lambda\sigma_{i,r}) = -8\,\lambda\sigma_i^2 + 8\,\lambda\sigma_{i,r}$

$\Rightarrow$ $A + C + 2\,(2\sigma_r^2 - 4\sigma_{i,r} + 3\sigma_i^2)\,\lambda\,\beta' + 2\,(-2\sigma_r^2 + 4\sigma_{i,r} - \sigma_i^2)\,\lambda\,\gamma' = 0$

$B + D + 2\,(-2\sigma_r^2 + 4\sigma_{i,r} - \sigma_i^2)\,\lambda\,\beta' + 2\,(2\sigma_r^2 - 4\sigma_{i,r} + 3\sigma_i^2)\,\lambda\,\gamma' = 0$

Summing up the former and the latter equations, we obtain

$A + B + C + D + 4\,\lambda\,\beta'\sigma_i^2 + 4\,\lambda\,\gamma'\sigma_i^2 = 0$

$\Rightarrow$ $4\,r - 2\,i - 2\,E(i) - 4\,\lambda\sigma_i^2 + 4\,\lambda\,\beta'\sigma_i^2 + 4\,\lambda\,\gamma'\sigma_i^2 = 0$

[A.6c] $2\,r - i - E(i) - 2\,\lambda\sigma_i^2 + 2\,\lambda\,\beta'\sigma_i^2 + 2\,\lambda\,\gamma'\sigma_i^2 = 0$

While subtracting the latter from the former, we get

$A - B + C - D + (8\sigma_r^2 - 16\sigma_{i,r} + 8\,\sigma_i^2)\,\lambda\,\beta' + (-8\sigma_r^2 + 16\sigma_{i,r} - 8\,\sigma_i^2)\,\lambda\,\gamma' = 0$

$\Rightarrow$ $4\,E(r) - 4\,E(i) - 8\,\lambda\sigma_i^2 + 8\,\lambda\sigma_{r,i} + (8\sigma_r^2 - 16\sigma_{i,r} + 8\,\sigma_i^2)\,\lambda\,\beta' + (8\sigma_r^2 - 16\sigma_{i,r} - 8\,\sigma_i^2)\,\lambda\,\gamma' = 0$

[A.6d] $E(r) - E(i) - 2\,\lambda\sigma_i^2 + 2\,\lambda\sigma_{i,r} + 2\,(\sigma_r^2 - 2\sigma_{i,r} + \sigma_i^2)\,\lambda\,\beta' - 2\,(\sigma_r^2 - 2\sigma_{i,r} + \sigma_i^2)\,\lambda\,\gamma' = 0$

Note that in [IIc] the slope of the function β′ with respect to γ′ is –1, whereas in [A.6d] it is +1. The two functions hence cross each other and the system has a solution.

From [A.6c] we have:

$-2\,\lambda\,\gamma'\sigma_i^2 = -2\,\lambda\sigma_i^2 + 2\,\lambda\,\beta'\sigma_i^2 + (2\,r - i - E(i))$

$\Rightarrow$ $\gamma' = 1 - \beta' - (2\,r - i - E(i))\,(2\,\lambda\,\sigma_i^2)^{-1}$

[A.7a] $\hat{\gamma}(\beta') = 1-\beta' - \dfrac{r - \dfrac{i+E(i)}{2}}{\lambda\sigma_i^2}$

which, if we denote by *s* the slope of the yield curve, can be rewritten as

[A.7a'] $\hat{\gamma}(\beta') = 1-\beta' - \dfrac{s - \dfrac{E(i)-i}{2}}{\lambda\sigma_i^2}$

By solving [A.6d] with respect to γ we get

$$E(r) - E(i) - 2\,\lambda\,\sigma_i^2 + 2\,\lambda\,\sigma_{i,r} + 2\,(\sigma_r^2 - 2\,\sigma_{i,r} + \sigma_i^2)\,\lambda\,\beta' - 2\,(\sigma_r^2 - 2\,\sigma_{i,r} + \sigma_i^2)\,\lambda\,\gamma' = 0$$

$$\Rightarrow \quad 2\,(\sigma_r^2 - 2\,\sigma_{i,r} + \sigma_i^2)\,\lambda\,\gamma = -2\,\lambda\,\sigma_i^2 + 2\,\lambda\,\sigma_{i,r} + 2\,(\sigma_r^2 - 2\,\sigma_{i,r} + \sigma_i^2)\,\lambda\,\beta' + (E(r) - E(i))$$

[A.7b] $\hat{\gamma}(\beta') = \beta' - \dfrac{\sigma_i^2 - \sigma_{i,r}}{\left(\sigma_r^2 - 2\sigma_{i,r} + \sigma_i^2\right)} + \dfrac{E(r)-E(i)}{2\left(\sigma_r^2 - 2\sigma_{i,r} + \sigma_i^2\right)\lambda}$

Then, by equating [A.7a] and [A.7b] we find the solution for β

$$1-\beta' - \frac{2r-i-E(i)}{2\lambda\sigma_i^2} = \beta' - \frac{\sigma_i^2 - \sigma_{i,r}}{\left(\sigma_r^2 - 2\sigma_{i,r} + \sigma_i^2\right)} + \frac{E(r)-E(i)}{2\left(\sigma_r^2 - 2\sigma_{i,r} + \sigma_i^2\right)\lambda}$$

$$\Rightarrow \quad -2\beta' = -1 - \frac{\sigma_i^2 - \sigma_{i,r}}{\left(\sigma_r^2 - 2\sigma_{i,r} + \sigma_i^2\right)} + \frac{E(r)-E(i)}{2\left(\sigma_r^2 - 2\sigma_{i,r} + \sigma_i^2\right)\lambda} + \frac{2r-i-E(i)}{2\lambda\sigma_i^2}$$

$$\Rightarrow \quad \beta' = \frac{1}{2} + \frac{\sigma_i^2 - \sigma_{i,r}}{2\left(\sigma_r^2 - 2\sigma_{i,r} + \sigma_i^2\right)} - \frac{E(r)-E(i)}{4\left(\sigma_r^2 - 2\sigma_{i,r} + \sigma_i^2\right)\lambda} - \frac{2r-i-E(i)}{4\lambda\sigma_i^2}$$

$$\Rightarrow \quad \beta' = \frac{1}{2} + \frac{\sigma_i^2 - \sigma_{i,r}}{2\Sigma} - \frac{E(s)}{4\Sigma\,\lambda} - \frac{2r-i-E(i)}{4\lambda\sigma_i^2}$$

By plugging the latest result indifferently into [A.7a] and [A.7b] we get

$$\hat{\gamma} = 1 - \frac{1}{2} - \frac{1}{2}\left\{\frac{\sigma_i^2 - \sigma_{i,r}}{\Sigma} - \frac{E(s)}{2\Sigma\lambda} - \frac{s - \dfrac{[E(i)-i]}{2}}{\sigma_i^2\,\lambda}\right\} - \frac{s - \dfrac{E(i)-i}{2}}{\lambda\sigma_i^2}$$

$$\Rightarrow \quad \hat{\gamma} = \frac{1}{2} - \frac{1}{2}\frac{\sigma_i^2 - \sigma_{i,r}}{\Sigma} + \frac{1}{4}\frac{E(s)}{2\Sigma\lambda} + \frac{1}{2}\frac{s - \dfrac{[E(i)-i]}{2}}{\sigma_i^2\,\lambda} - \frac{s - \dfrac{E(i)-i}{2}}{\lambda\sigma_i^2}$$

$$\Rightarrow \quad \hat{\gamma} = \frac{1}{2} - \frac{1}{2}\frac{\sigma_i^2 - \sigma_{i,r}}{\Sigma} + \frac{1}{4}\frac{E(s)}{2\Sigma\lambda} - \frac{1}{2}\frac{s - \dfrac{E(i)-i}{2}}{\lambda\sigma_i^2}$$

Going back from the pairs {β′, γ′} to {β, γ}, the results

$$\hat{\beta}' = \frac{1}{2} + \frac{\sigma_i^2 - \sigma_{i,r}}{2\Sigma} - \frac{E(s)}{4\Sigma\,\lambda} - \frac{2r-i-E(i)}{4\lambda\sigma_i^2}$$

$$\hat{\gamma} = \frac{1}{2} - \frac{1}{2}\frac{\sigma_i^2 - \sigma_{i,r}}{\Sigma} + \frac{1}{4}\frac{E(s)}{2\Sigma\lambda} - \frac{1}{2}\frac{s - \dfrac{E(i)-i}{2}}{\lambda\sigma_i^2}$$

are written out as

$$\hat{\beta} = \frac{\sigma_i^2 - \sigma_{i,r}}{\Sigma} - \frac{E(s)}{2\lambda\Sigma}$$

$$\hat{\gamma} = 1 - \left[\frac{\sigma_i^2 - \sigma_{i,r}}{\Sigma} - \frac{E(s)}{2\lambda\Sigma} + \frac{s - \frac{E(i) - i}{2}}{\lambda\sigma_i^2} \right]$$

which are results [9] and [10].

B3. Condition for a tactical long position on the bond

The condition such that the tactical deviation in [11] is positive is the following

$$1 - \left\{ \frac{\sigma_i^2 - \sigma_{i,r}}{\Sigma} - \frac{E(s)}{2\lambda\Sigma} + \frac{s - \frac{[E(i) - i]}{2}}{\sigma_i^2 \lambda} \right\} > 0$$

$$\Rightarrow \qquad \frac{r - \frac{[E(i) + i]}{2}}{\sigma_i^2} < \lambda \left[1 - \frac{\sigma_i^2 - \sigma_{i,r}}{\Sigma} + \frac{E(r) - E(i)}{2\lambda\Sigma} \right]$$

$$\Rightarrow \qquad \frac{r - \frac{[E(i) + i]}{2}}{\sigma_i^2} < \lambda \left[\frac{2\lambda\Sigma - 2\lambda(\sigma_i^2 - \sigma_{i,r}) + E(r) - E(i)}{2\lambda\Sigma} \right]$$

$$\Rightarrow \qquad \frac{r - \frac{[E(i) + i]}{2}}{\sigma_i^2} < \frac{2\left(\sigma_r^2 - \sigma_{i,r}\right)\lambda + E(r) - E(i)}{2\Sigma}$$

$$\Rightarrow \qquad \frac{r - \frac{[E(i) + i]}{2}}{\sigma_i^2} < \frac{\left(\sigma_r^2 - \sigma_{i,r}\right)}{\Sigma}\lambda + \frac{E(r) - E(i)}{2\Sigma}$$

$$\Rightarrow \qquad r < \frac{\sigma_i^2}{\Sigma}\left(\sigma_r^2 - \sigma_{r,i}\right)\lambda + \frac{\sigma_i^2}{2\Sigma}\left(E(r) - E(i)\right) + \frac{[E(i) + i]}{2}$$

$$\Rightarrow \qquad r < \frac{1}{2} i + \frac{\sigma_i^2}{2\Sigma} E(r) + \frac{\sigma_r^2 - 2\sigma_{i,r}}{2\Sigma} E(i) + \frac{\sigma_i^2}{\Sigma}\left(\sigma_r^2 - \sigma_{i,r}\right)\lambda$$

Using the notation $\Sigma \equiv \sigma_r^2 - 2\sigma_{i,r} + \sigma_i^2$ and $\theta \equiv \sigma_i^2 / \Sigma$ we obtain

$$r < \frac{1}{2} i + \frac{1}{2}\theta E(r) + \frac{1}{2}(1 - \theta)E(i) + \theta\left(\sigma_r^2 - \sigma_{r,i}\right)\lambda$$

B4. Partial derivatives in result [11]

- $$\frac{\partial r^e}{\partial E(r)} = \frac{1}{2}\theta > 0$$
- $$\frac{\partial r^e}{\partial E(i)} = \frac{1}{2}(1 - \theta) \rightarrow ?$$

The effect is ambiguous since (1–θ) can be either larger or lower than zero. It holds that:

$$\frac{\partial r^e}{\partial E(i)} < 0 \text{ if } 1-\theta < 0 \Leftrightarrow \frac{\sigma_r^2 - 2\sigma_{r,i}}{\Sigma} < 0 .$$

From $\Sigma > 0$, it follows:

$$\sigma_r^2 - 2\rho\sigma_r\sigma_r < 0 \Leftrightarrow \rho > \frac{1}{2}\frac{\sigma_r}{\sigma_i}$$

$$\Rightarrow \qquad \frac{\partial r^e}{\partial E(i)} < 0 \text{ if } \rho > \frac{1}{2}\frac{\sigma_r}{\sigma_i}$$

- $$\frac{\partial\theta}{\partial\sigma_i^2} = \frac{\Sigma - \sigma_i^2}{\Sigma^2} = \frac{\sigma_r^2 - 2\sigma_{ir}}{\Sigma^2} = \frac{1-\theta}{\Sigma}$$

now, computing the partial derivative of r with respect to σ_i^2 we get

$$\frac{\partial r}{\partial\sigma_i^2} = \frac{1}{2}E(r)\frac{1-\theta}{\Sigma} - \frac{1}{2}E(i)\frac{1-\theta}{\Sigma} + \frac{1-\theta}{\Sigma}(\sigma_r^2 - \sigma_{i,r})\lambda =$$

$$\frac{\partial r}{\partial\sigma_i^2} > 0 \Leftrightarrow (1-\theta)\left[E(r) - E(i) + 2(\sigma_r^2 - \sigma_{i,r})\lambda\right] > 0$$

The sign depends on the correlation. There are three cases:

- $\rho < \frac{1}{2}\frac{\sigma_r}{\sigma_i}$: then $1-\theta > 0$ and $\sigma_r^2 - \sigma_{i,r} > 0$ so the overall effect is unambiguously positive (at least as long as E(r) > E(i))
- $\frac{1}{2}\frac{\sigma_r}{\sigma_i} < \rho < \frac{E(r)-E(i)}{2\lambda\sigma_i\sigma_r} + \frac{\sigma_r}{\sigma_i}$, then the term above in square brackets has a positive sign and the overall effect is thus negative;
- $\rho > \frac{E(r)-E(i)}{2\lambda\sigma_i\sigma_r} + \frac{\sigma_r}{\sigma_i}$, then the term in square brackets has a negative sign and the overall effect is thus positive. Of course, the implicit requirement is $E(r) - E(i) < 2\lambda\sigma_i\sigma_r$, otherwise the condition can never hold as ρ is at most zero.

All in all, the conditions can be summarized as follows:

$$\frac{\partial r^e}{\partial\sigma_i^2} < 0 \text{ if } \frac{1}{2}\frac{\sigma_r}{\sigma_i} < \rho < \frac{E(r)-E(i)}{2\lambda\sigma_i\sigma_r} + \frac{\sigma_r}{\sigma_i}); \ \frac{\partial r^e}{\partial\sigma_i^2} < 0 \text{ otherwise.}$$

To discuss the partial derivatives of r^e on σ_r^2, it is useful to assess beforehand the effect of σ_r^2 on θ

$$\frac{\partial\theta}{\partial\sigma_r^2} = -\frac{\sigma_i^2}{\Sigma^2} = -\frac{\theta}{\Sigma}$$

now, computing the partial derivative of r with respect to σ_r^2 we get

$$\frac{\partial r^e}{\partial\sigma_r^2} = \frac{1}{2}E(i)\frac{\theta}{\Sigma} - \frac{1}{2}E(r)\frac{\theta}{\Sigma} - \left(\sigma_r^2 - \sigma_{i,r}\right)\frac{\theta}{\Sigma}\lambda + \theta\lambda =$$

$$-\frac{1}{2}\frac{\theta}{\Sigma}\left(E(r) - E(i)\right) + \frac{\lambda}{\Sigma}\left[-\theta\left(\sigma_r^2 - \sigma_{i,r}\right) + \theta\left(\sigma_r^2 - 2\sigma_{i,r} + \sigma_i^2\right)\right] =$$

$$-\frac{1}{2}\frac{\theta}{\Sigma}\left(E(r) - E(i)\right) + \frac{\lambda}{\Sigma}\theta\left(\sigma_i^2 - \sigma_{i,r}\right) > 0$$

$$E(r) - E(i) < 2\lambda\left(\sigma_i^2 - \sigma_{i,r}\right)$$

$$E(r)-E(i)<2\lambda\left(\sigma_i^2-\rho\sigma_i\sigma_r\right)$$

the inequality can be rewritten by highlighting the correlation:

$$\frac{\sigma_r}{\sigma_i}\rho<1-\frac{E(r)-E(i)}{2\lambda\sigma_i^2}$$

therefore:

$$\frac{\partial r^e}{\partial\sigma_r^2}>0 \text{ if } \rho<\frac{\sigma_i}{\sigma_r}-\frac{E(r)-E(i)}{2\lambda\sigma_i\sigma_r}$$

- By the same token, to discuss the partial derivatives of r^e on $\sigma_{i,r}$, it is useful to assess beforehand the effect of $\sigma_{i,r}$ on θ

$$\frac{\partial\theta}{\partial\sigma_{i,r}}=\frac{2\sigma_i^2}{\Sigma^2}=\frac{2\theta}{\Sigma}$$

now, computing the partial derivative of r with respect to σ_{ir} we get

$$\frac{\partial r}{\partial\sigma_{i,r}}=\frac{\theta}{\Sigma}\left(E(r)-E(i)\right)+2\frac{\theta}{\Sigma}\left(\sigma_r^2-\sigma_{i,r}\right)\lambda-\theta\lambda=$$

$$=\frac{\theta}{\Sigma}\left(E(r)-E(i)\right)+\frac{\lambda}{\Sigma}\left[\left(2\sigma_r^2-2\sigma_{i,r}\right)\theta-\theta\Sigma\right]=$$

$$=\frac{\theta}{\Sigma}\left(E(r)-E(i)\right)+\frac{\lambda}{\Sigma}\left[\left(2\sigma_r^2-2\sigma_{i,r}\right)\frac{\sigma_i^2}{\Sigma}-\sigma_i^2\right]=$$

$$=\frac{\theta}{\Sigma}\left(E(r)-E(i)\right)+\frac{\lambda}{\Sigma^2}\left[\left(2\sigma_r^2-2\sigma_{i,r}\right)\sigma_i^2-\sigma_i^2\left(\sigma_r^2-2\sigma_{i,r}+\sigma_i^2\right)\right]=$$

$$=\frac{\theta}{\Sigma}\left(E(r)-E(i)\right)+\frac{\lambda}{\Sigma^2}\left(\sigma_r^2-\sigma_i^2\right)\sigma_i^2=$$

$$=\frac{\theta}{\Sigma}\left(E(r)-E(i)\right)+\frac{\lambda\theta}{\Sigma}\left(\sigma_r^2-\sigma_i^2\right)=$$

$$=\frac{\theta}{\Sigma}\left(E(r)-E(i)\right)+\frac{\lambda\theta}{\Sigma}\left(\sigma_r^2-\sigma_i^2\right)>0 \text{ if } E(r)-E(i)>\lambda\left(\sigma_i^2-\sigma_r^2\right)$$

therefore:

$$\frac{\partial r^e}{\partial\sigma_{i,r}}>0 \text{ if } \lambda<\frac{E(r)-E(i)}{\sigma_i^2-\sigma_r^2}$$

B5 ***(Approximated) solution of the model with two unknowns (beta and gamma), when N = 6***

We expand [8] for j=0,…,5 and, at the same time, we relax the assumption on the expected interest rates.

[A.8a] $f_{\beta,\gamma}[0]=\begin{bmatrix}\beta+\gamma & 1-\beta-\gamma\end{bmatrix}\begin{bmatrix}r\\ i\end{bmatrix}$

[A.8b] $f_{\beta,\gamma}[1]=\begin{bmatrix}\beta+\gamma & 1-\beta-\gamma\end{bmatrix}\begin{bmatrix}r\\ E_o(i_1)\end{bmatrix}+\lambda\begin{bmatrix}\beta+\gamma & 1-\beta-\gamma\end{bmatrix}\begin{bmatrix}0 & 0\\ 0 & \sigma_i^2\end{bmatrix}\begin{bmatrix}\beta+\gamma\\ 1-\beta-\gamma\end{bmatrix}$

[A.8c] $f_{\beta,\gamma}[2]=\begin{bmatrix}\beta & 1-\beta\end{bmatrix}\begin{bmatrix}E_o(r_2)\\ E_0(i_2)\end{bmatrix}+\lambda\begin{bmatrix}\beta & 1-\beta\end{bmatrix}\begin{bmatrix}\sigma_r^2 & \sigma_{i,r}\\ \sigma_{i,r} & \sigma_i^2\end{bmatrix}\begin{bmatrix}\beta\\ 1-\beta\end{bmatrix}$

[A.8d] $f_{\beta,\gamma}[3] = \begin{bmatrix}\beta & 1-\beta\end{bmatrix}\begin{bmatrix}E_o(r_3)\\ E_0(i_3)\end{bmatrix} + \lambda\begin{bmatrix}\beta & 1-\beta\end{bmatrix}\begin{bmatrix}\sigma_r^2 & \sigma_{i,r}\\ \sigma_{i,r} & \sigma_i^2\end{bmatrix}\begin{bmatrix}\beta\\ 1-\beta\end{bmatrix}$

[A.8e] $f_{\beta,\gamma}[4] = \begin{bmatrix}\beta & 1-\beta\end{bmatrix}\begin{bmatrix}E_o(r_4)\\ E_0(i_4)\end{bmatrix} + \lambda\begin{bmatrix}\beta & 1-\beta\end{bmatrix}\begin{bmatrix}\sigma_r^2 & \sigma_{i,r}\\ \sigma_{i,r} & \sigma_i^2\end{bmatrix}\begin{bmatrix}\beta\\ 1-\beta\end{bmatrix}$

[A.8f] $f_{\beta,\gamma}[5] = \begin{bmatrix}\beta & 1-\beta\end{bmatrix}\begin{bmatrix}E_o(r_5)\\ E_0(i_5)\end{bmatrix} + \lambda\begin{bmatrix}\beta & 1-\beta\end{bmatrix}\begin{bmatrix}\sigma_r^2 & \sigma_{i,r}\\ \sigma_{i,r} & \sigma_i^2\end{bmatrix}\begin{bmatrix}\beta\\ 1-\beta\end{bmatrix}$

Hence (hereinafter we drop the subscript ‘0’ to refer to the time when the expectation is taken, to keep the notation a bit lighter)

$$f_{\beta,\gamma}[0] = (\beta + \gamma)\, r + (1 - \beta - \gamma)\, i$$

$$f_{\beta,\gamma}[1] = (\beta + \gamma)\, r + (1 - \beta - \gamma)\, E(i_1) + \lambda\, (1 - \beta - \gamma)^2\, \sigma_i^2$$

$$f_{\beta,\gamma}[2] = \beta\, E(r_2) + (1 - \beta)\, E(i_2) + \lambda\, [\beta^2 \sigma_r^2 + 2\, \beta\, (1 - \beta)\, \sigma_{i,r} + (1 - \beta)^2 \sigma_i^2]$$

$$f_{\beta,\gamma}[3] = \beta\, E(r_2) + (1 - \beta)\, E(i_3) + \lambda\, [\beta^2 \sigma_r^2 + 2\, \beta\, (1 - \beta)\, \sigma_{i,r} + (1 - \beta)^2 \sigma_i^2]$$

$$f_{\beta,\gamma}[4] = \beta\, E(r_4) + (1 - \beta)\, E(i_4) + \lambda\, [\beta^2 \sigma_r^2 + 2\, \beta\, (1 - \beta)\, \sigma_{i,r} + (1 - \beta)^2 \sigma_i^2]$$

$$f_{\beta,\gamma}[5] = \beta\, E(r_4) + (1 - \beta)\, E(i_5) + \lambda\, [\beta^2 \sigma_r^2 + 2\, \beta\, (1 - \beta)\, \sigma_{i,r} + (1 - \beta)^2 \sigma_i^2]$$

Taking the partial derivatives we have:

$$\frac{\partial}{\partial\beta} f \equiv f'_\beta = [r - i] + [r - E(i_1) - 2\, \lambda\, (1 - \beta - \gamma)^2 \sigma_i^2] + E(r_2) - E(i_2) + 2\, \lambda\, [\beta\, \sigma_r^2 + (1 - \beta - \beta)\, \sigma_{i,r} - (1 - \beta)\, \sigma_i^2] + E(r_2) - E(i_3) + 2\, \lambda\, [\beta\, \sigma_r^2 + (1 - \beta - \beta)\, \sigma_{i,r} - (1 - \beta)\, \sigma_i^2] + E(r_4) - E(i_4) + 2\, \lambda\, [\beta\, \sigma_r^2 + (1 - \beta - \beta)\, \sigma_{i,r} - (1 - \beta)\, \sigma_i^2] + E(r_4) - E(i_5) + 2\, \lambda\, [\beta\, \sigma_r^2 + (1 - \beta - \beta)\, \sigma_{i,r} - (1 - \beta)\, \sigma_i^2] =$$

$$2\, [r + E(r_2) + E(r_4)] - [i + E(i_1) + E(i_2) + E(i_3) + E(i_4) + E(i_5)] + 8\, \lambda\, [\beta\, \sigma_r^2 + (1 - 2\beta)\, \sigma_{i,r} - (1 - \beta)\, \sigma_i^2] - 2\, \lambda\, (1 - \beta - \gamma)^2 \sigma_i^2$$

$$\frac{\partial}{\partial\gamma} f \equiv f'_\gamma = r - i + r - E(i_1) - 2\, \lambda\, (1 - \beta - \gamma)^2\, \sigma_i^2 = 2\, r - [i + E(i_1)] - 2\, \lambda\, (1 - \beta - \gamma)^2\, \sigma_i^2$$

Subtracting f'_γ from f'_β, we have

$$2\, [r + E(r_2) + E(r_4)] - [i + E(i_1) + E(i_2) + E(i_3) + E(i_4) + E(i_5)] + 8\, \lambda\, [\beta\, \sigma_r^2 + (1 - 2\beta)\, \sigma_{i,r} - (1 - \beta)\, \sigma_i^2] - 2\, \lambda\, (1 - \beta - \gamma)^2 \sigma_i^2 - 2\, r + [i + E(i_1)] + 2\, \lambda\, (1 - \beta - \gamma)^2\, \sigma_i^2 = 0$$

$$2\, [E(r_2) + E(r_4)] - [E(i_2) + E(i_3) + E(i_4) + E(i_5)] + 8\, \lambda\, [\beta\, \sigma_r^2 + (1 - 2\beta)\, \sigma_{i,r} - (1 - \beta)\, \sigma_i^2] = 0$$

[A.9a] $+ 8\, \lambda\, [\beta\, \sigma_r^2 + (1 - 2\beta)\, \sigma_{i,r} - (1 - \beta)\, \sigma_i^2] = - 2\, [E(r_2) + E(r_4)] + [E(i_2) + E(i_3) + E(i_4) + E(i_5)]$

Note that [A.9a] yields a solution for the optimal β which, once more, is not a function of current interest rates.

Using in [A.9a] the notation $f(r,i) \equiv 2\, [E(r_2) + E(r_4)] - [E(i_2) + E(i_3) + E(i_4) + E(i_5)]$, we have

$$[\beta\, \sigma_r^2 + (1 - 2\beta)\, \sigma_{i,r} - (1 - \beta)\, \sigma_i^2] = -f(r,i) / 8\, \lambda$$

$$\Rightarrow \quad \beta\, \sigma_r^2 - 2\beta\, \sigma_{i,r} + \sigma_{i,r} + \beta\, \sigma_i^2 - \sigma_i^2 = -f(r,i) / 8\, \lambda$$

$$\Rightarrow \quad \beta\, (\sigma_r^2 - 2\, \sigma_{i,r} + \sigma_i^2) + (\sigma_{i,r} - \sigma_i^2) = -f(r,i) / 8\, \lambda$$

[A.9b] $$\hat{\beta}_6 = \frac{-\frac{f(r,i)}{8\lambda} - \left(\sigma_{i,r} - \sigma_i^2\right)}{\sigma_i^2 - 2\sigma_{i,r} + \sigma_r^2} = \frac{\sigma_i^2 - \sigma_{i,r}}{\Sigma} - \frac{f(r,i)}{8\lambda\Sigma}$$

If $E(r_2) = E(r_4) = E(r)$ and $E(i_2) = E(i_3) = E(i_4) = E(i_5) = E(i)$, then

[A.9c] $$\hat{\beta}_6 = \frac{\sigma_i^2 - \sigma_{i,r}}{\Sigma} - \frac{f(r,i)}{8\lambda\Sigma} = \frac{\sigma_i^2 - \sigma_{i,r}}{\Sigma} - \frac{2[2E(r)] - 4E(i)}{8\lambda\Sigma} = \frac{\sigma_i^2 - \sigma_{i,r}}{\Sigma} - \frac{E(r) - E(i)}{2\lambda\Sigma} = \hat{\beta}$$

which shows that under constant interest rate expectations, the optimal β is invariant to the length of the planning horizon. If, however, this assumption is removed, whether $\hat{\beta}_6$ is bigger or smaller than $\hat{\beta}$ depends on how fast the convergence of short- and long-term interest rates are to their long-run averages. For the sake of the example, if, say, the long-term rate converges immediately, so that $E(r_2) = E(r_4) = E(r)$, while the short-term rate follows a more gradual path, so that $E(i_2) \leq E(i_3) \leq E(i_4) \leq E(i_5) = E(i)$, then $\hat{\beta}_6 \leq \hat{\beta}$.

Turning to f'_γ, we have

$$2r - [i + E(i_1)] - 2\lambda(1 - \hat{\beta}_6 - \gamma)^2 \sigma_i^2 = 0$$

$$\Rightarrow \quad 2\lambda(1 - \hat{\beta}_6 - \gamma)^2 \sigma_i^2 = 2r - [i + E(i_1)]$$

$$\Rightarrow \quad (1 - \hat{\beta}_6 - \gamma)^2 = \{2r - [i + E(i_1)]\} / (2\lambda\sigma_i^2)$$

$$\Rightarrow \quad 1 + \hat{\beta}_6^2 + \gamma^2 - 2\hat{\beta}_6 - 2\gamma + 2\hat{\beta}_6\gamma = E \quad \text{where} \quad E \equiv \frac{2r - [i + E(i_1)]}{2\lambda\sigma_i^2} = \frac{s - \frac{[E(i_1) - i]}{2}}{\lambda\sigma_i^2}$$

[A.9d] $\gamma^2 - 2(1 - \hat{\beta}_6)\gamma - (E - 1 - \hat{\beta}_6^2 + 2\hat{\beta}_6^2) = 0$

an equation of second order in γ whose solutions are

[A.9e] $\hat{\gamma}_6 = \left(1 - \hat{\beta}_6\right) \pm \sqrt{E}$

Where, to meet the constraint $\hat{\beta}_6 + \hat{\gamma}_6 \in [0,1]$, the solution with the + sign in front of the square root can be ruled out.

B6. Calibration steps

The analysis on the calibrated example proceeds as follows:

1. from a sample data, a calibration for $\{\sigma_i^2, \sigma_r^2, \sigma_{i,r}, E(r), E(i)\}$ is obtained.
2. the calibration for λ as a function of $\{\sigma_i^2, \sigma_r^2, \sigma_{i,r}, E(r), E(i), \hat{\beta}^*\}$ is obtained from the equation $\hat{\beta} = \hat{\beta}^*$, where $\hat{\beta}$ is given by [7] and $\hat{\beta}^*$ denotes the target level of the optimal β that matches, in the model metrics, the empirically observed maturity-composition of debt, i.e., the average life to maturity of the outstanding debt.
3. the effect of parameters perturbation is assessed by checking the conditions in Table 2.

The pair of current short- and long-term yields is identified in the {i,r} space and checked whether it is located in the region of positive/null/negative tactical position. The exact amount of the tactical position is given by [10], while the total portion of long-term bonds to issue (tactical plus strategic) is given by [9].

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