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CALIBRATING THE ITALIAN SMILE WITH TIME-VARYING VOLATILITY AND HEAVY-TAILED MODELS

by Michele Leonardo Bianchi,* Frank J. Fabozzi‡ and Svetlozar T. Rachev†

Abstract

In this paper we consider several time-varying volatility and/or heavy-tailed models to explain the dynamics of return time series and to fit the volatility smile for exchange-traded options where the underlying is the main 'Borsa Italiana' stock index. Given observed prices for the time period we investigate, we calibrate both continuous-time and discrete-time models. First, we estimate the models from a time-series perspective (i.e. under the historical probability measure) by investigating more than ten years of daily index price log-returns. Then, we explore the risk-neutral measure by fitting the values of the implied volatility for numerous strikes and maturities during the highly volatile period from April 1, 2007 (prior to the subprime mortgage crisis in the U.S.) to March 30, 2012. We assess the extent to which time-varying volatility and heavy-tailed distributions are needed to explain the behavior of the most important stock index of the Italian market.

JEL Classification: C02, C46, C58, C61, C63.

Keywords: volatility smile, option pricing, non-Gaussian Ornstein-Uhlenbeck processes, Lévy processes, tempered stable processes and distributions, stochastic volatility models, time-changed Lévy processes, GARCH model, filtered historical simulation, particle filter.

Contents

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* Bank of Italy, Specialized Intermediaries Supervision Department, Financial Innovation and Risk Analysis Division, Rome, Italy.

† College of Business and Department of Applied Mathematics and Statistics, Stony Brook University, NY, and FinAnalytica INC.

‡ EDHEC Business School, Nice, France.

1 Introduction[1]

One phenomenon that is inconsistent with the Black-Scholes model is the so-called "volatility smile" observed by option traders soon after the stock market crash in October, 19 1987 referred to as "Black Monday". Using implied volatility as a measure of volatility, one observes that at-the-money put options had lower volatility compared to both deep-in-the money and deep-out-of-the money put options. The implication is that for each strike and for each maturity, there may exist a different volatility assumption rather than a constant volatility assumption as assumed in the Black-Scholes framework.

There is a general consensus that asset log-returns exhibit volatilities that change through time, despite the classical Black-Scholes model assumption that it is constant. It is reasonable to consider a volatility whose evolution is not deterministic but depends on random events (i.e. a stochastic volatility or time-dependent volatility) rather than employ a simplistic model that is inconsistent with a stylized fact. Observed volatility moves in clusters (if it is high, it remains high, and if it is low, it remains low) and for this reason it is important to find a way to take such observed patterns into account when modeling asset prices. Stochastic volatility models have been proposed to allow for a time-varying volatility in a continuous-time framework. Alternatively, in the financial time-series literature, GARCH models are a popular choice to model changing variances.

Additionally, the underlying assumption made in most financial models is that the uncertainty in financial markets can be explained by a normal distribution. However there is an extensive body of empirical evidence that indicates that the normal distribution is not flexible enough to explain the dynamics of complex financial products. The criticism of the normal model is by no means recent (see Mandelbrot, 1963). Academic researchers, as well as practitioners, have increasingly applied more complex non-normal distribution models in finance, particularly since the turn of the century. The introduction of jumps and heavy tails into the dynamics of stock returns was followed by the introduction of jumps in volatility dynamics (see Rachev and Mittnik, 2000; Schoutens, 2003; Cont and Tankov, 2004; and Rachev et al., 2011).

In this paper, we empirically investigate some well-known option pricing models: the Heston (1993) continuous-time model, enhancements of the Heston model allowing for jumps proposed by Nicolato and Venardos (2003) and Yu et al. (2011), and the Heston and Nandi (2000) discrete-time model. These models, together with continuous-time and time-changed Lévy models (see Schoutens, 2003; Cont and Tankov, 2004; and Rachev et al., 2011) are compared to discrete-time GARCH models with normal and tempered stable innovations and with the non-parametric model proposed by Barone Adesi et al. (2008). In practice, we consider several continuous-time models: the classical tempered stable and the normal inverse Gaussian Lévy model, the Heston model, and two modifications of the Heston model that allow for jumps in stock log-returns, the Ornstein-Uhlenbeck stochas-

[1] The authors are grateful to Giuseppe Ferrero, Nicoletta Olivanti, Pietro Tommasino, an two anonymous referees at the Bank of Italy for their comments and suggestions. Michele Leonardo Bianchi acknowledges that the views expressed in the article are those of the author and do not involve the responsibility of the Bank of Italy.

tic volatility model, and the time-changed classical tempered stable model. We compare these models with different discrete-time models (i.e. the Heston and Nandi GARCH model, the filtered historical simulation Glosten-Jagannathan-Runkle model, the normal NGARCH model, and an alternative NGARCH model with asymmetric and heavy-tailed innovations).

Most empirical testing of European option pricing models has focused on the S&P 500 index given the ability to implement the necessary hedging strategy and the very active futures market for the index. In this paper we study instead the benchmark stock market index in Italy, the Financial Times Stock Exchange Milano Indice di Borsa (FTSE MIB), using a large data set of implied volatilities related to options written on this index. The FTSE MIB index is the reference index for numerous structured bonds, covered warrants, and certificates traded in Italy. Given this key role in the Italian financial market, a correct calibration of the smile is needed for pricing and hedging these products where this stock market index serves as the reference index. Notwithstanding there have been some recent papers that have dealt with an analysis of the Italian option market (see Ciccone et al., 2011; Muzzioli, 2011; and Centanni and Ongaro, 2011), to our knowledge we believe our paper is the first extensive empirical study that analyzes the statistical properties of daily log-returns and volatility surfaces of the major Italian index.

The principal purpose of this paper is to assess the extent to which the incorporation of stochastic volatility and heavy tails are needed to explain the behavior of the FTSE MIB index and to properly calibrate the related implied volatility surface. To do this, we consider two methodologies. First, we compare different time-varying volatility models in terms of fitting performance and computational tractability by extracting the historical volatility directly from daily log-returns. Second, we compare all models proposed in terms of calibration performance and computing time by fitting on a daily basis the implied volatility surface.

More specifically, after analyzing the distributional properties of FTSE MIB daily log-returns from a historical time-series perspective, we find that the models based on the normality assumption do not provide a reliable explanation of the historical distribution of returns. The empirical evidence indicates that, for the index that we analyze and for the time period that we investigate, the tempered stable GARCH model has better explanatory power in fitting daily log-returns compared to standard models based on the normal distribution assumption and other continuous-time models of the Heston type.

As far as the smile calibration is concerned, our findings indicate that there are not remarkable differences in terms of fitting errors between the continuous-time models analyzed. Although the discrete-time models show less flexibility in fitting the observed implied volatility surfaces compared to the continuous-time models, they exhibit a more stable calibration error over time. In most of the trading days analyzed, the overall error is around 6% for continuous-time models and slightly more for the discrete-time ones. However, all continuous-time models have spikes in the behavior of the pricing error corresponding to the three recent market turmoils (Lehman Brothers bankruptcy in September 2008, the worsening of the Greek sovereign debt crisis, and the Italian sovereign debt crisis in 2011 when the 10-year yield spread between Italian government bonds relative to German government bonds exceeded 550 basis points). Indeed, the dates for which the error is the greatest are located in the heart of the crisis. Note that, as it has already been shown in similar studies (e.g. Guillaume, 2012), our study confirms that models based on Lévy processes performed poorly during the crisis period. The GARCH models allowing for heavy tails are only partially affected by those events.

The remainder of this paper is organized as follows. Section 2 reviews the various continuous-time and discrete-time models considered in the empirical study. In Section 3 we describe the data analyzed in the empirical study and identify some computational issues. The historically based estimation and the calibration of the volatility surface together with the empirical results are discussed in Sections 3.1 and 3.2, respectively. Section 4 summarizes the principal conclusions of the paper and Appendix A contains the details of the main theoretical results.

2 Modeling stock price returns

2.1 The continuous-time framework

In this section we describe five models under a continuous-time framework. Given a probability space $(\Omega, \mathcal{F}, \mathrm{P})$ and a filtration $\{\mathcal{F}_t\}$ satisfying the usual conditions, the dynamics of stock price returns is defined as the exponential of a continuous process $(X_t)_{t\geq 0}$[2] starting from 0

$$S_t = S_0 \exp(X_t) \tag{2.1}$$

and where S_0 is the stock price at time 0. In the following we indicate the market measure by P (with parameters Θ) and the risk-neutral measure by Q (with parameters Θ^*). Since we are interested in calibrating both the market measure and the risk-neutral one, we will describe the relation between the two measures and the dynamics of stock price log-returns under P which is used to conduct an analysis from a time-series perspective, and under Q which is used to calibrate implied volatilities. However, as we describe in Section 3, we do not perform a jointly calibration of the model by fitting at the same time both the time series of index log-returns and the cross-section of implied volatilities. Instead, we calibrate the two measures separately.

2.1.1 The Lévy model

First we consider a simple Lévy based model with constant volatility in order to empirically prove whether we need a model that allows for stochastic volatility. As described by Kim and Lee (2007), the model can be written as

$$dX_t = (\mu_t - \psi_J(-i))dt + dJ_t \tag{2.2}$$

where μ_t represents the deterministic drift at time t, J_t is a classical tempered stable (CTS) process (see Rachev et al., 2011), and ψ_J is the characteristic exponent of J_1.[3] Under a possible risk-neutral measure, we obtain the following equality

$$dX_t = (r_t - d_t - \psi_{J^*}(-i))dt + dJ_t^* \tag{2.3}$$

[2] For simplicity in the following we refer to $(X_t)_{t\geq 0}$ as X_t.
[3] The characteristic exponent is defined in Appendix A.1.

where r_t and d_t represent the deterministic risk-free rate and the dividend yield at time t and J_t^* is a CTS process under Q. Under this setting, the equality

$$\mu_t - \psi_J(-i) = r_t - d_t - \psi_{J^*}(-i) \tag{2.4}$$

has to be fulfilled for each time t. This result directly follows from Theorem A.2 in the Appendix. The process J_t is said to be a CTS process with parameters (α, C, λ_+, λ_-, m) if its characteristic exponent is given by

$$\begin{aligned}\psi_{CTS}(u) &= \log E[\exp(iuX_t)] = iut(m - C\Gamma(1-\alpha)(\lambda_+^{\alpha-1} - \lambda_-^{\alpha-1})) \\ &\quad + Ct\Gamma(-\alpha)((\lambda_+ - iu)^\alpha - \lambda_+^\alpha + (\lambda_- + iu)^\alpha - \lambda_-^\alpha)\end{aligned} \tag{2.5}$$

where α, C, λ_+, and λ_- are positive constants, $0 < \alpha < 2$, and $m \in \mathbb{R}$ is the mean. We refer to this model as the *CTS model*.

In the empirical study we also consider a Lévy model based on the normal inverse Gaussian (NIG) process; that is, in equations (2.2) and (2.3) J_t (and J_t^*) is assumed to be a NIG process with parameters (α, β, δ, m),[4] given by

$$\begin{aligned}\psi_{NIG}(u) &= iut\left(m - \delta\beta(\alpha^2 - \beta^2)^{-\frac{1}{2}}\right) \\ &\quad + \delta t\left(\sqrt{\alpha^2 - \beta^2} - \sqrt{\alpha^2 - (\beta + iu)^2}\right)\end{aligned} \tag{2.6}$$

We refer to this model as the *NIG model*.

Additionally, we add to the dynamic described in equation (2.2) a Brownian component, thereby obtaining a model with one more parameter (α, C, λ_+, λ_-, σ, m) where σ represents the constant volatility of a standard Brownian motion. That is, under the risk-neutral measure the dynamic of the model is

$$dX_t = (r_t - d_t - \frac{1}{2}\sigma^2 - \psi_{J^*}(-i))dt + \sigma W_t + dJ_t^*. \tag{2.7}$$

We refer to this model as the *Bls-CTS model*. As demonstrated in Appendix A.1, the pure jumps CTS models enhanced by adding a diffusion component gives a more flexible change of measure.

2.1.2 The Heston model

A widely used approach among practitioners to price exotic and structured products has been proposed by Heston (1993) and analyzed in depth by Guillaume and Schoutens (2012). Heston extended the Black-Scholes model by making the volatility parameter σ stochastic. More specifically, the stock price process X_t follows the well-known Black-Scholes stochastic differential equation

$$\begin{aligned}dX_t &= (r_t - d_t)dt + \sigma_t dW_t \\ d\sigma_t^2 &= \kappa(\eta - \sigma_t^2)dt + \vartheta\sigma_t d\tilde{W}_t,\end{aligned} \tag{2.8}$$

where r_t and d_t represent the deterministic risk-free rate and the dividend yield at time t, respectively, W_t and $\tilde{W}_t$ are correlated Brownian motions with correlation

[4] In both the CTS and NIG cases we select a truncation function h, as defined in the Appendix A.1, such that if $m = 0$, than the process has zero mean.

ρ, $\sigma_0 > 0$, the mean reversion rate κ, the long-run variance η and the volatility of the variance ϑ are positive parameters. Additionally, one has $2\kappa\eta > \vartheta^2$ in order to ensure that the origin is inaccessible, that is, the volatility process does not reach zero. The dynamics of σ_t is the well-known Cox-Ingersoll-Ross (CIR) process, that is a positive mean-reverting process driven by a Brownian motion. As described by Christoffersen et al. (2010), a possible risk-neutral dynamic is given by

$$\begin{aligned} dX_t &= (r_t - d_t)dt + \sigma_t dW_t^* \\ d\sigma_t^2 &= (\kappa - \lambda)\left(\frac{\kappa\eta}{\kappa - \lambda} - \sigma_t^2\right)dt + \vartheta\sigma_t d\tilde{W}_t^*, \end{aligned} \tag{2.9}$$

where $\lambda\sigma^2$ is the volatility risk premium and W_t^* and $\tilde{W}_t^*$ are correlated Brownian motions with correlation ρ under the risk-neutral measure. We refer to this model as the Heston model. The characteristic exponent of this model with parameters (κ, η, ϑ, ρ, σ_0, m) can be computed in closed form by (see Albrecher et al., 2007)

$$\begin{aligned} \psi_{Heston}(u) &= \log E[\exp(iuX_t)] \\ &= iutm + \eta\kappa\vartheta^{-2}\left((\kappa - \rho\vartheta iu - g_1)t - 2\log\left(\frac{1 - g_2 e^{-g_1 t}}{1 - g_2}\right)\right) \\ &+ \frac{\sigma_0^2\vartheta^{-2}(\kappa - \rho\vartheta iu - g_1)(1 - e^{-g_1 t})}{1 - g_2 e^{-g_1 t}} \\ g_1 &= \sqrt{(\rho\vartheta ui - \kappa)^2 + \vartheta^2(iu + u^2)} \\ g_2 &= \frac{\kappa - \rho\vartheta iu - g_1}{\kappa - \rho\vartheta iu + g_1} \end{aligned} \tag{2.10}$$

2.1.3 A jump-diffusion stochastic volatility model

The previously described stochastic volatility Heston model can be enhanced by adding a jump component J_t^x to the return dynamics (jump diffusion stochastic volatility - JD-SV). In this paper we explore the jump-diffusion stochastic volatility model recently defined by Yu et al. (2011); that is, the stochastic differential equation defining X_t has the following form

$$\begin{aligned} dX_t &= \mu_t dt + \sigma_t dW_t + dJ_t^x \\ \mu_t &= r_t - d_t - \frac{1}{2}\sigma_t^2 - \psi_{J^x}^{\mathrm{Q}}(-i) + \lambda\sigma_t^2 \\ d\sigma_t^2 &= \kappa(\eta - \sigma_t^2)dt + \vartheta\sigma_t(\rho dW_t + \sqrt{1 - \rho^2}d\tilde{W}_t), \end{aligned} \tag{2.11}$$

where μ_t is a deterministic function of the time t, respectively, W_t and $\tilde{W}_t$ are independent Brownian motions, $\sigma_0 > 0$, and κ, η and ϑ are positive parameters, where, one has $2\kappa\eta > \vartheta^2$ in order to ensure that the origin is inaccessible for the CIR component of the volatility. The parameter ρ measures the correlation between volatility and returns. Additionally, $\psi_{J^x}^{\mathrm{Q}}$ is the characteristic exponent of the return jump component and $\lambda\sigma^2$ represents the volatility premium. It can be proven that under the risk-neutral measure the stock price model is described by the following stochastic differential equation

$$\begin{aligned} dX_t &= \left(r_t - d_t - \frac{1}{2}\sigma_t^2 - \psi^{Q}_{J^x}(-i)\right) dt + \sigma_t dW_t^* + dJ_t^{x*} \\ d\sigma_t^2 &= (\kappa - \lambda)\left(\frac{\kappa\eta}{\kappa-\lambda} - \sigma_t^2\right) dt + \vartheta\sigma_t(\rho dW_t^* + \sqrt{1-\rho^2}d\tilde{W}_t^*), \end{aligned} \tag{2.12}$$

where W_t^* and $\tilde{W}_t^*$ are independent under the risk-neutral measure Q and J_t^{x*} is the jump process under Q.

Yu et al. (2011) considered that J_t^x is a variance gamma (VG)[5] process with parameters (C, G, M), that is

$$\begin{aligned} \psi_{VG}(u) = iu(m - C(\lambda_- - \lambda_+)\lambda_+^{-1}\lambda_-^{-1}) \\ - C\log(\lambda_+\lambda_- + (\lambda_+ - \lambda_-)iu + u^2) + C\log(\lambda_+\lambda_-). \end{aligned} \tag{2.13}$$

By applying Theorem A.2 of Appendix A.1 we would find that the parameter C has to be the same under both measures P and Q while the parameters G and M can freely change. As observed by Cont and Tankov (2004), the presence of a diffusion component provides considerable freedom in changing both the Lévy measure and the drift, while preserving the equivalent condition between the two measures. In practice this means that a different choice for μ_t is also possible. The model can be easily enhanced by assuming that J_t^x is a CTS process with parameters α, C, λ_+, and λ_-. Under this assumption, it can be proven that α and C have to be the same under both measures P and Q, and, as in the above considered VG case, the parameters λ_+ and λ_- can freely change.

Yu et al. (2011) shows that the characteristic exponent of this model under the risk-neutral measure has the following form

$$\begin{aligned} \psi_{JDSV}(u) &= iut(r_t - d_t - \psi^{Q}_{J^x}(-i)) + \psi^{Q}_{J^x}(u)) - b(t)\sigma_0 - c(t) \\ b(t) &= \frac{(iu+u^2)(1-e^{-\delta t})}{\delta + \kappa_M + (\delta - \kappa_M)e^{-\delta t}} \\ c(t) &= \frac{\kappa\eta}{\vartheta}\left(2\log\frac{2\delta - (\delta-\kappa_M)(1-e^{-\delta t})}{2\delta} + (\delta - \kappa_M)t\right) \\ \kappa_M &= \kappa - \lambda - iu\vartheta\rho \\ \delta &= \sqrt{\kappa_M^2 + (iu+u^2)\vartheta^2} \end{aligned} \tag{2.14}$$

We refer to these models as the *Heston-VG model* when the jump component is VG distributed, and the *Heston-CTS model* when the jump component is CTS distributed.

2.1.4 Ornstein-Uhlenbeck stochastic volatility model

The CIR model can be enhanced by adding jumps by considering the so-called jump diffusion CIR (JCIR) model as described by Brigo and Mercurio (2006)

[5] A VG process can be viewed as the limiting case of a CTS with α going to zero.

and by Lando (2004). Alternatively, one can consider pure jumps mean reverting processes of the Ornstein-Uhlenbeck family. Over the past decade, non-Gaussian Ornstein-Uhlenbeck (OU) processes introduced by Barndorff-Nielsen and Shephard (2001) have been widely studied by practitioners and academia from both empirical and theoretical points of view and used in applications in finance, economics, engineering, and other applied sciences. This family of processes can capture important distributional properties observed in real data and offer a more flexible structure with respect to Gaussian-based models. This flexibility, the possibility to explain certain *stylized facts* of financial time series, and a suitable degree of computational tractability have increased the number of applications in finance, in particular, to stochastic volatility (see Nicolato and Venardos, 2003, among others) together with a vast amount of theoretical research papers.

As defined by Barndorff-Nielsen and Shephard (2001), an OU process v_t is a solution of a stochastic differential equation of the form[6]

$$dv_t = -\lambda v_t dt + dZ_{\lambda t}. \tag{2.15}$$

If Z_t is an increasing Lévy process with finite variation starting from 0 and if $v_0 > 0$, it can be proven that the process v_t is strictly positive and bounded from below by $v_0 \exp(-\theta t)$. If v_t is an OU process with marginal law D,[7] then it is referred to as a D-OU process. Under certain assumptions[8] and given a marginal law for D, one can compute the characteristic function of the process Z_t (the so-called *background driving Lévy process* (BDLP)). The process v_t (σ_t^2 in the following) can be used to model volatility.

In this part of the paper, we assume that volatility can be described by a Gamma-OU process with parameters (λ, C, a).[9] This choice is also motivated by the fact that this leads to a closed-form solution for the characteristic function of X_t. Under the market measure P the model has the following dynamics

$$\begin{aligned} dX_t &= \left(\mu_t - \frac{1}{2}\sigma_t^2\right) dt + \sigma_t dW_t + \rho dZ_{\lambda t} \\ \mu_t &= r_t - d_t - \lambda \psi_Z^*(\rho) \\ d\sigma_t^2 &= -\lambda \sigma_t^2 dt + dZ_{\lambda t} \end{aligned} \tag{2.16}$$

where μ_t is a deterministic function of time t, W_t is a Brownian motion, Z_t is the BDLP corresponding to the Gamma-OU process, $\sigma_0 > 0$, λ is a positive parameter, and $\rho \leq 0$. Under the risk-neutral measure Q it becomes

$$\begin{aligned} dX_t &= \left(r_t - d_t - \lambda \psi_Z^*(\rho) - \frac{1}{2}\sigma_t^2\right) dt + \sigma_t dW_t^* + \rho dZ_{\lambda t}^* \\ d\sigma_t^2 &= -\lambda \sigma_t^2 dt + dZ_{\lambda t}^* \end{aligned} \tag{2.17}$$

[6] The unusual timing λt is deliberately chosen so that the marginal distribution of v_t is independent of the choice of θ (see Barndorff-Nielsen and Shephard, 2001).

[7] This means that if one starts the process with an initial value sampled from the D distribution, at each future time t, v_t is distributed as D.

[8] One has to assume the law D *self-similar*. For the definition of self-similarity, see Sato (1999).

[9] See Schoutens (2003) and Bianchi (2012) for more details on this process.

where r_t and d_t represent the deterministic risk-free rate and the dividend yield at time t, respectively, W_t^* is a Brownian motion, Z_t is the BDLP corresponding to the Gamma-OU process, σ_0 and λ are positive parameters, and $\rho \in \mathbb{R}$. Also in this case Theorem A.2 of Appendix A.1 allows one to find the relation between parameters under the two measures P and Q. We point out that the presence of the diffusion component allows us to freely change the drift while preserving the equivalent condition between measures. In practice this means that a different choice for μ_t is also possible. In the IG-OU and Gamma-OU cases, the relations between market and risk-neutral parameters are given by Corollary 3.3 in Nicolato and Venardos (2003). In particular, in the Gamma-OU case the parameters C and a can freely change. The characteristic exponent of this model with parameters (ρ, λ, C, a, σ_0, m) has been evaluated by Nicolato and Venardos (2003) and its closed form is given by

$$
\begin{aligned}
\psi_{JDSV}(u) &= iut\left(m - \frac{\lambda C \rho}{a-\rho}\right) - b(t)g(u)\sigma_0^2 \\
&\quad + \frac{C}{a - f_2(u)}\left(a \log \frac{a - f_1(u)}{a - iu\rho} + \lambda t f_2(u)\right) \\
b(t) &= \frac{1 - e^{-\lambda t}}{\lambda} \\
g(u) &= \frac{1}{2}(u^2 + iu) \\
f_1(u) &= iu\rho - \lambda g(u) b(t) \\
f_2(u) &= iu\rho - g(u)
\end{aligned}
\tag{2.18}
$$

In the following, we refer to this model as the *SV-GammaOU model*.

2.1.5 The time-changed classical tempered stable model

A large part of modern finance has been concerned with modeling the evolution of return processes over time. By subordination, it is possible to capture empirically observed anomalies that contradict the classical log-normality assumption for asset prices. In periods of high volatility, time runs faster than in periods of low volatility. The subordinator models operational time and provides the so-called fat-tail effects, often observed in financial markets. The subordination approach has been widely studied in the literature (see Hurst et al., 1997; Hurst et al., 1999; Geman et al., 2001; and Geman et al., 2002). In periods of high volatility, time runs faster than in periods of low volatility.

We conclude this section by describing another well-known technique to build stochastic volatility models. Indeed we define under the market measure the following log-return process

$$
\begin{aligned}
dX_t &= \mu_t dt + dJ_{T_t} \\
T_t &= \int_0^t \lambda_s ds \\
d\lambda_s &= -\theta \lambda_s ds + dZ_{\theta s}
\end{aligned}
\tag{2.19}
$$

where λ_s is an an Ornstein-Uhlenbeck process as defined in Section 2.1.4 and μ_t is a deterministic function of time. Since we do not perform an historical calibration on this model, we do not provide further details on the choice of μ_t. If the characteristic function of both processes J_t and T_t is known in closed form and if they are independent, then it is possible to compute the charactiristic function of the process X_t. A possible log-return price dynamics under the risk-neutral measure is given by the following stochastic differential equation

$$\begin{aligned} dX_t &= (r_t - d_t - \psi_{J_T^*}(-i))dt + dJ_{T_t}^* \\ T_t &= \int_0^t \lambda_s ds \\ d\lambda_s &= -\theta \lambda_s ds + dZ_{\theta s}^* \end{aligned} \tag{2.20}$$

where $J_{T_t}^*$ and $Z_{\theta s}^*$ are the processes defined under the risk-neutral measure. The characteristic exponent of the model which assumes that J_t is a CTS process with parameters (C, λ_+, λ_-, α, m) and T_t is an integrated Gamma-OU process with parameters (θ, a, b) starting from λ_0 can be computed in closed form. It is given by (see Cont and Tankov, 2004)

$$\psi_{CTS-Gamma-OU}(u) = \log E[\exp(iuX_t)] = \log \psi_T(-i\psi_{CTS}(u)) \tag{2.21}$$

where ψ_{CTS} is defined in equation (2.5) and the characteristic exponent of the integrated Gamma-OU process T is given by

$$\begin{aligned} \psi_{Int\ Gamma-OU}(u) = \exp\Bigg(&- \frac{iu\lambda_0}{\theta}(1 - \exp(-\theta t)) \\ &+ \frac{\theta a}{iu - \theta b}\left(b \log\left(\frac{b}{b - iu\theta^{-1}(1 - \exp(-\theta t)}\right) - iut \right) \Bigg). \end{aligned} \tag{2.22}$$

We refer to this model as the *CTS-GammaOU model.*

2.2 The discrete-time framework

In this section, we review three parametric GARCH models and a nonparametric GARCH model. First we consider the classical normal based Heston and Nandi GARCH model, then the filtering historical simulation GARCH model where the empirical innovation needed for pricing purposes is extracted by a likelihood based estimation on historical returns time series, and finally we describe under both a normal and a heavy tails framework the nonlinear GARCH dynamic for the conditional volatility proposed by Engle and Ng (1993).

As shown by Kim et al. (2010), the GARCH stock price model is defined over a filtered probability space (Ω, $\mathcal{F}$, $(\mathcal{F}_t)_{t\in\mathbb{N}}$, P) which is constructed by considering the σ-algebras generated by a sequence $(\varepsilon_t)_{t\in\mathbb{N}}$ of independent and identically distributed (i.i.d.) real random variables, such that ε_t is a random variable with zero mean and unit variance, and assume that $E[e^{x\varepsilon_t}] < \infty$ where $x \in (-a, b)$ for some $a, b > 0$. A similar condition on the moments is necessary for the construction of exponential Lévy models, see Cont and Tankov (2004). Thus, we assume that the stock price log-returns have the form

$$\log\left(\frac{S_t}{S_{t-1}}\right) = g(r_t, d_t, \lambda_t, \sigma_t) + \sigma_t \varepsilon_t, \quad 1 \leq t \leq T$$

and the conditional variance process is defined as

$$\sigma_t^2 = h(\sigma_{t-1}, \varepsilon_{t-1}; \Theta), \quad 1 \leq t \leq T, \quad \varepsilon_0 = 0.$$

The function g explains the behavior of the log-returns, while the function h provides the conditional variance dynamic depending on parameters Θ. This second function h defines the behavior of the conditional variance varying over time.

In the following we deal with three different dynamics. The process r_t represents the interest rate and the process d_t represents the dividend yield process. The process λ_t may be viewed as the market price of risk that we consider constant. A market price of risk varying over the time is difficult to estimate. To do this one has to jointly analyze both the return distributions implicit in the time series of returns and option prices. We do not perform this joint calibration in this paper. The product $\sigma_t \varepsilon_t$ represents the error term with zero mean and variance σ_t. In the normal case it is simple to prove that for each t it is normally distributed with zero mean and conditional variance σ_t.

2.2.1 The Heston-Nandi GARCH model

In the normal based model we empirically investigate the well-known models for option pricing with GARCH proposed by Heston and Nandi (2000), where the stock price log-returns under the market measure are as follows

$$\log\left(\frac{S_t}{S_{t-1}}\right) = r_t - d_t + \lambda\sigma_t + \frac{\sigma_t}{2} - \frac{\sigma_t^2}{2} + \sigma_t \varepsilon_t, \quad 1 \leq t \leq T \tag{2.23}$$

and the conditional variance is defined as

$$\sigma_t^2 = \alpha_0 + \alpha_1(\varepsilon_{t-1} - \gamma\sigma_t)^2 + \beta_1 \sigma_{t-1}^2, \quad t \in \mathbb{N}. \tag{2.24}$$

and we refer to it as the *HN-GARCH model*. We assume $\beta_1 + \alpha_1\gamma^2 < 1$ in order to guarantee the existence of a strong stationary solution with finite mean and variance. The set of constant parameters is $(\alpha_0,\ \alpha_1,\ \beta_1,\ \lambda,\ \gamma)$. The particular conditional volatility structure allows one to obtain a recursive formula to calculate the price of a European option.

A possible risk-neutral dynamic, as proposed by Duan (1995), is

$$\log\left(\frac{S_t}{S_{t-1}}\right) = r_t - d_t - \frac{\sigma_t^2}{2} + \sigma_t \xi_t, \quad 1 \leq t \leq T$$

where the conditional variance has the form

$$\sigma_t^2 = \alpha_0^* + \alpha_1^*(\xi_{t-1} - \omega^*\sigma_t)^2 + \beta_1^* \sigma_{t-1}^2, \quad t \in \mathbb{N}$$

where $\omega^* = \gamma^* + \lambda^* + 1/2$ and ξ_t is standard normally distributed.

2.2.2 A nonparametric GARCH model

A possible alternative to the classical parametric GARCH models where a distributional assumption is always assumed is the *filtering historical simulation* (FHS) approach. This algorithm has been applied to the study of option pricing in the GARCH framework by Barone Adesi et al. (2008). Permutations of the historical series are considered as the source of the randomness without the necessity of any distributional assumption. The idea comes from the observation that Monte Carlo simulations drawn from a particular distribution impose the risk structure that one is supposed to investigate. In particular, with the normal distribution hypothesis we cannot incorporate excess skewness and kurtosis, and cannot capture extreme events. Empirical studies show that residuals are not normally distributed; therefore, one possibility to overcome this drawback is not to impose any theoretical distribution. Historical simulations usually sample from past data assuming that returns are i.i.d.. Thus, one needs to remove any serial correlation and volatility clusters present in the historical series.

Under the market measure, the stock price has the following dynamics

$$\log\left(\frac{S_t}{S_{t-1}}\right) = \mu + \eta_t \tag{2.25}$$

where $\eta_t = \sigma_t \varepsilon_t$. Additionally, Barone Adesi et al. (2008) assume the asymmetric Glosten-Jagannathan-Runkle (GJR) model for the conditional variance, that is

$$\sigma_t^2 = \alpha_0 + \alpha_1 \eta_{t-1}^2 + \gamma I_{t-1} \eta_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \tag{2.26}$$

where $I_{t-1} = 1$ for negative residuals, otherwise it is zero. The set of constant parameters is $(\alpha_0, \alpha_1, \beta_1, \gamma)$. If $\gamma > 0$, then the model considers the *leverage effect*, that is, bad news raises the future volatility more than good news. This information is captured by the indicator function I_t, since volatility increases when a negative event occurs. In order to obtain a strong stationary solution, it is necessary to assume that $(\alpha_1 + \gamma/2) + \beta_1 < 1$. Furthermore, the model requires the condition $\alpha_1 + \gamma \geq 0$.

The empirical innovation density captures potential non-normalities in the true innovation density. In order to use the estimated residuals for historical simulation, one needs to scale them with respect to the volatility, that is $\varepsilon_t = \eta_t / \sigma_t$. It is clear that the first step is the estimation of the model's parameters by assuming that η_t is normally distributed with zero mean and variance σ_t^2, and then extracting the estimated residuals. The historical simulation is provided by a random choice within the set of estimated residuals after an opportune scaling by σ_t as explained above. For each step t, the value of the innovation ε_t is choosen and the conditional variance is updated until the entire path is generated.

The dynamic under the risk-neutral measure is

$$\begin{aligned} \log\left(\frac{S_t}{S_{t-1}}\right) &= \mu^* + \eta_t \\ \sigma_t^2 &= \alpha_0^* + \alpha_1^* \eta_{t-1}^2 + \gamma^* I_{t-1} \eta_{t-1}^2 + \beta_1^* \sigma_{t-1}^2. \end{aligned} \tag{2.27}$$

Note that ε_t is the same under both the market and risk-neutral measures. This means that the set of parameters of this model is $(\alpha_0^*,\ \alpha_1^*,\ \beta_1^*,\ \gamma^*)$. The empirical martingale simulation method proposed by Duan and Simonato (1998) ensures that parameter μ^* is chosen so that under the risk-neutral measure Q the equality $E[S_t/S_{t-1}|\mathcal{F}_{t-1}] = e^{r_t}$ holds. We note that the log-returns of the market prices of the underlying asset are used to estimate parameters of the risk-neutral model. More specifically, random choices of the estimated innovation process ε_t are generated to simulate the underlying stock price process under the risk-neutral measure. In the following, we refer to this model as the *FHS model*.

2.2.3 The classical tempered stable NGARCH model

By following the approach of Kim et al. (2010), we propose the following stock price dynamics under the market measure

$$\log\left(\frac{S_t}{S_{t-1}}\right) = r_t - d_t + \lambda_t\sigma_t - l_{\varepsilon_t}(\sigma_t) + \sigma_t\varepsilon_t, \quad t \in \mathbb{N}, \tag{2.28}$$

where ε_t is CTS distributed with zero mean and unit variance (stdCTS) and the function l is the log-Laplace transform of ε_t, that is, $l(u) = log(E[e^{u\varepsilon_t}])$.[10] The one-period ahead conditional variance σ_t^2 follows a NGARCH(1,1) process with a restriction $0 < \sigma_t < b$, i.e,

$$\sigma_t^2 = (\alpha_0 + \alpha_1\sigma_{t-1}^2(\varepsilon_{t-1} - \gamma)^2 + \beta_1\sigma_{t-1}^2) \wedge \rho, \quad t \in \mathbb{N},\ \varepsilon_0 = 0, \tag{2.29}$$

where α_0, α_1, β_1, and γ are non-negative, $\alpha_1 + \beta_1 < 1$, $\alpha_0 > 0$, and $0 < \rho < b^2$.

Now we briefly describe the behavior of the stock price process under a risk-neutral measure. Further details are provided by Rachev et al. (2011). The stock price dynamics under a possible risk-neutral measure can be written as

$$\log\left(\frac{S_t}{S_{t-1}}\right) = r_t - d_t - l_{\xi_t}(\sigma_t) + \sigma_t\xi_t, \quad 1 \le t \le T$$

and the variance process has the form

$$\sigma_t^2 = (\alpha_0^* + \alpha_1^*\sigma_{t-1}^2(\xi_{t-1} - k_t)^2 + \beta_1^*\sigma_{t-1}^2) \wedge \rho, \quad 1 \le t \le T, \quad \xi_0 = 0.$$

where for each $1 \le t \le T$, P_t is a measure under which ε_t is stdCTS distributed with parameters Θ and Q_t is a measure under which $\xi_t = \varepsilon_t + k_t$ is stdCTS distributed with parameters Θ_t^*, where k_t is defined as

$$k_t := \lambda_t + \gamma + \frac{1}{\sigma_t}(l_{\xi_t}(\sigma_t) - l_\varepsilon(\sigma_t)) \tag{2.30}$$

and $T \in \mathbb{N}$ is the time horizon (see Kim et al., 2010). We refer to this model as the *CTS-NGARCH model*. Furthermore, in order to assess whether the CTS innovation distributional assumption plays a role in the modelling of stock price returns, we also consider the NGARCH model with normal innovation and refer to it as the *NGARCH model*. Since the normal distribution has exponential moments of any order, the restriction $0 < \sigma_t < b$ is always satisfied because under the normal distributional assumption $b = \infty$.

[10] A stdCTS law has distribution with zero mean and unit variance (see Scherer et al., 2012). In the following we will refer to it as stdCTS with parameters (λ_-, λ_+, α).

3 The empirical study

Here we provide a description of the data used in the empirical analysis. We obtained from Bloomberg the implied volatilities[11] extracted from European call and put options written on the FTSE MIB index from April 1, 2007 to March 30, 2012 with a maturity between one month and two years and with moneyness between 80% and 120%. We obtained more than 70,000 observations (from April 1, 2007 to June 1, 2009 we have only options with moneyness between 90% and 110%). We also obtained the closing prices of the FTSE MIB index and the estimated 1-year dividend yield for that index. The time period in this study includes the high volatility period after the Lehman Brothers filing for Chapter 11 bankruptcy protection (September 15, 2008) and the recent sovereign debt crisis in November 2011 when the 10-year Italian BTP was more than 500 basis points over the German bund with the same maturity. Risk-free interest rates are extracted from the EURIBOR swap rate for short term maturities up to nine months and the EU swap curve for maturities from one year to two years. For each observation day and for each maturity, the discount factor is computed by linear interpolation of the risk-free term structure.

For each maturity, the implied dividend is extracted by considering the well-known put-call parity (see Chapter 13 in Hull, 2002) for stocks (or indexes) providing a continuous dividend yield equal to q

$$C + Ke^{-rT} = P + S_0 e^{-qT} \tag{3.1}$$

and the estimated 1-year dividend yield provided by Bloomberg is used as starting point for the algorithm that finds a value for q such that the equality given by (3.1) is fulfilled.[12]

There are two possible methodologies to estimate a continuous-time option pricing model: (1) one can fit the model to the daily implied volatilities calculated from market prices and check both the model capabilities and the stability of the parameters, or; (2) one can calibrate both daily returns and implied volatilities by using a filter. The Kalman filter and its extensions can be taken into consideration when the model is Gaussian, otherwise a more sophisticated framework has to be applied (see Christoffersen et al., 2010; Yu et al., 2011; and Li, 2011).

The first approach can be viewed as a short-term static estimation, for the purpose of pricing and hedging on a daily basis, and for this reason we refer to it as the *market maker approach*. Under this approach at a given point in time model prices have to be as close as possible to the market prices observed and even if the market is quoting unreasonable prices, market makers have to find the parameters that replicate those prices. The market maker needs to achieve static consistency in order to provide at each given point in time two-sided quotes.

The second approach is a long-term dynamic estimation where the stock price process is estimated by filtering the information coming from the stock market by looking at the underlying under the market measure and from the derivatives

[11] We consider the market implied volatility surface and not Bloomberg evaluations (see Cui and Zhang, 2011).

[12] We use the Matlab r2011a function *fzero*.

market by looking at the implied volatilities under the risk-neutral measure for the purpose of assessing the long-term behavior. Estimated model parameters can be used to make projections on price movements and then used to trade the difference between the model prediction and the market quotes. These investment strategies (known as *statistical arbitrage strategies*) are commonly used by hedge funds and banks' proprietary trading desks and, for this reason, we refer to this second estimation methodology as the *long-term convergence trader approach*. Under this second approach, one assumes that the model can beat the market, or, more precisely, one trusts the model prices and tries to find possible arbitrage opportunities by looking at the differences between model and market prices. Dynamic consistency is important for long-term convergence trading and pricing errors represent trading opportunities (see Wu, 2008).

A similar approach can be also considered under a discrete-time setting. By taking into consideration a suitable change of measure (or pricing kernel), Christoffersen et al. (2013) jointly analyze both the return distributions implicit in the time series of returns and option prices and calibrate a GARCH model by maximizing the sum of the return likelihood and a likelihood based on successive cross-sections of option prices. They fit their proposed model to weekly data.

As already pointed out, the computational cost of the calibration of stochastic volatility models under the dynamic approach is much greater than under the static approach. Even if the computational cost strictly depends on both the hardware and on the software employed, most of the theoretical research works on more complex models that apply a nonstandard statistical algorithm to calibrate both daily returns and implied volatilities, does not report the computational time needed for the calibration. We suspect that in some cases the computational time is so high that the algorithm cannot be implemented without having a huge computational power (i.e. a cluster) and it is difficult to use such models in practice, for example when a daily estimation is needed for the valuation of a derivatives portfolio. Mainly for this reason, in this paper we consider only a static approach to determine the differences between the 12 competitor models we test in this paper.

In Table 1 we briefly summarize the main distinctive features of the models analyzed in the empirical study. We start from continuous-time Lévy models (NIG, CTS and Bls-CTS), then we study stochastic volatility models without jumps (Heston) and with jumps (Heston-CTS, Heston-VG, SV-GammaOU, CTS-GammaOU), and finally we analyze GARCH models without heavy tails (HN-GARCH, NGARCH) and with heavy tails (FHS, CTS-NGARCH).

3.1 The historical volatility

The empirical analysis we conduct starts from the estimation of the historical volatility, that is the volatility extracted by considering only the time series of the FTSE MIB index log-returns. This empirical study is motivated by the need that under the discrete-time setting, the first value of the instantaneous volatility σ_0 will be the starting value of the option pricing algorithm. Additionally, for both the FHS and the CTS-GARCH models the innovation estimates based on the historical data will be used in the valuation of option prices. More specifically, in the FHS model the risk-neutral innovations are randomly drawn by taking into account the historical one and in the CTS-GARCH model the parameter of the CTS innovations are assumed to be the same under both the market and the risk-neutral measures.

We estimate the parameters of the discrete-time models using the classical maximum likelihood estimation (MLE) procedure. The log-likelihood function to be maximized is of the form

$$\log L(\Theta) = \sum_{t=1}^{T} \frac{1}{\sigma_t} \log f \left(y_t - \frac{g(r_t, d_t, \lambda_t, \sigma_t)}{\sigma_t} \right) \tag{3.2}$$

where $y_t = \log(S_t/S_{t-1})$ and f is the density function of the innovation. Such function can be easily written in the normal case, but in general it has a complex structure or cannot be written in analytic form. Note that because the conditional volatility σ_t strictly depends on parameters Θ, the likelihood function to be maximized is viewed as a function of this set of parameters.

In order to find the parameters of these discrete-time models that allow for volatility clustering, we have to maximize the likelihood function (3.2). In the optimization problem, the log-likelihood function could be evaluated using the following recursive method:

1. Set a starting point Θ_0 for the model's parameters and let us consider $t = 1$.
2. Choose a value for σ_0 and set $\varepsilon_0 = 0$.
3. Calculate $\varepsilon_t = \frac{g(r_t,d_t,\lambda_t,\sigma_t)^2}{\sigma_{t-1}}$.
4. Calculate $\sigma_t^2 = h(\sigma_{t-1}, \varepsilon_{t-1}; \Theta)$ and the sum in equation (3.2) until t.
5. If $t = T$, return $\log L(\Theta)$, otherwise go to the next step.
6. Set $t = t + 1$ and return to step 3.

The optimization procedure will move the starting point in a suitable direction until it reaches the optimal solution of the problem. Since in the CTS-GARCH model only the characteristic function is known, a discrete evaluation of the density function f together with an interpolation algorithm is used. That is, by means of the classical fast Fourier transform procedure, the characteristic function is inverted to calculate the density function (see Scherer et al., 2012). For the CTS-NGARCH model, the one-step MLE procedure which estimates both GARCH parameters and innovation parameters at the same time is a complex optimization problem (see also Rachev et al., 2011). For this reason, we first estimate the NGARCH parameters by assuming that the innovation is normally distributed. Then, we find the parameters of the stdCTS distribution that better fit the innovation extracted under the normal NGARCH model. This two-step procedure is more efficient, stable, and provides similar estimates to the one-step MLE procedure, in which one calibrates all the CTS-NGARCH parameters together through the maximization of (3.2). The results of these two estimation approaches are reported in Table 2.

After having dealt with the four different GARCH models, we want to compare these estimates with the historical volatility estimated by considering continuous-time stochastic volatility models. As observed by Christoffersen et al. (2010), the empirical challenge in stochastic volatility models is that the unobserved spot volatility σ_t is a latent factor. Thus, in order to extract the volatility, we need to apply to these models a filtering technique using observed index log-returns. Filtering methods are standard tools for exploring the behavior of the unobservable factors from observed data and they have been successfully applied to finance particularly in interest rate term structure and in stochastic volatility models (see, for example, Bhar, 2010). Inference with filtering methods has been widely studied, see for example the work of Lopes and Tsay (2010), and applied in engineering and finance.

In continuous-time stochastic volatility models, the unobserved volatility can be inferred by the observed stock returns. Since one deals with discrete observations, in our case with daily returns, the model has to be discretized. More specifically, the model can be written as

$$\begin{aligned} v_t &= f(v_{t-1}, \Theta, \xi_{t-1}) \\ y_t &= h(v_t, \Theta, \varepsilon_t) \end{aligned} \tag{3.3}$$

where $v_t = \sigma_t^2$, t is the day counter and v_t is the state variable modeled as a Markov process with initial distribution $p(v_0)$ and transition law $p(v_t|v_{t-1})$. The state variable follows the dynamics described by the transition function f. The variable y_t represents the set of given observations (in our case the observed index log-returns). It is assumed to be conditionally independent given the state v_t and with distribution $p(y_t|v_t)$. Then, ξ_{t-1} depends on the volatility dynamics and the noise ε_t is normally distributed noise with mean zero and unit variance, at least in the stochastic volatility models we are going to study. The function h is the so-called measurement function, that in our case is given by the stock price returns model and Θ is a set of static parameters.

If the measurement function h is linear and the state is Gaussian, one can use the Kalman filter for state and parameter estimation. In all cases we are interested in, we have a non-Gaussian state and for this reason we apply the the particle filter (PF) approach. The PF algorithm relies on the approximation of the true density of the state v_t by a set of particles that are updated iteratively through the dynamics described by the functions f and h in the system defined in equation (3.3). Given equation (3.3) and since ε_t is normally distributed, one has a simple way to evaluate the likelihood that the observation y_{t+1} has been generated by v_{t+1}. Hence, one is able to compute the weight given to each particle and to recursively evaluate the likelihood to be maximized in order to obtain a state and parameters estimation. The algorithm is briefly described in Appendix A.2 and the estimated parameters are reported in Table 2.

The two Heston models allowing for jumps in the returns (Heston-VG and Heston-CTS) can be also estimated by considering similar arguments. In these two cases, the algorithm that simulates the particles has to generate the CIR dynamics for the volatility and VG or CTS random numbers for the dynamics of stock returns. Drawing VG random numbers is of minor concern as they can be generated as the difference between gamma random numbers: a VG process can be viewed as the difference between two independent gamma processes (see Schoutens, 2003). More challenging is the simulation of CTS random numbers, but they can be efficiently drawn by the algorithm described in the Appendix A.3. The estimation of the two Heston models allowing for jumps in log-return dynamics is more challenging and ad-hoc algorithms may be more efficient and provide better convergence properties, as discussed by Yu et al. (2011). However, an exhaustive analysis of all computational issues with which one has to deal in estimating these models is beyond the scope of this paper.

In Figure 1 we show the dynamics of three GARCH models (HN-GARCH, FHS, and NGARCH) and four continuous-time stochastic volatility models (three Heston based and SV-GammaOU). When it is possible, we compare them with the FTSE MIB index implied volatility extracted from at-the-money options with the shortest maturity (one month). Even if each model provides distinctive features, the pattern of the historical-based spot volatility is not far from that of the implied volatility considered here (except for the HN-GARCH model). The volatility estimated under the SV-GammaOU model and all GARCH models shows jumps larger than the Heston-based model, mainly because by construction they concentrate more probability mass on tail events. Recall that for both the Heston-VG and Heston-CTS models the jumps are only in the return dynamics and not in the volatility.

In Table 2, we report the estimated parameters and the value of the log-likelihood for all competitor models. We consider 5,000 particles in all continuous-time models. Besides estimating the parameters, we apply the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) to identify the superior model. The AIC and the BIC are evaluated as

$$AIC = 2np - 2LL$$

and

$$BIC = np \log(no) - 2LL$$

where np is the number of parameters, no is the number of observations, and LL is the model's log-likelihood. According to both the AIC and the BIC, the CTS-NGARCH model is better because its AIC and BIC values are smaller compared with all other competitor models. Regarding the continuous-time models, the Heston-VG model slightly improves the Heston model, at least for the data considered in this study.

We conducted an empirical study on all discrete-time models by considering 1,271 rolling windows containing 1,500 daily observations. That is, for each trading day in which the implied volatility data are available, all GARCH models are estimated by considering the time series of log-returns of the previous 1,500 trading days. As we will discuss later, these market estimates will be used in Section 3.2. Table 3 provides summary statistics of the results of the empirical study conducted over time and across the four GARCH models. As already observed in previous empirical studies (e.g., see Kim et al., 2010), the CTS-NGARCH model outperforms its competitors models. In particular, the values of the Kolmogorov-Smirnov (KS) statistic show the better fit performances of the non-normal model analyzed here. In the CTS-based GARCH model, since λ_+ is always greater than λ_-, the fitted CTS is always skewed to the left. Additionally, the difference between λ_+ and λ_- is large (1.6, on average). This difference between the positive and negative tails in the CTS-GARCH model seems to be the factor that decreases the estimation error in terms of KS distance and likelihood function value. We point out that the asymmetry of the innovation process is captured also by the non-parametric FHS model. To visualize this phenomenon, in Figure 2 we show the histogram based on 5 million random simulated innovations drawn from the NGARCH, FHS, and CTS-GARCH model based on the parameters estimated from the historical time-series of FTSE MIB log-returns between January 4, 2000 to March 30, 2012. Recall that FHS innovations are drawn by randomly selecting from the finite set of historical returns.

3.2 The calibration of the implied volatilities

We consider in the empirical study the market maker approach, in order to study the pricing performance on a daily basis and pricing error during market downturns. From a practical perspective, on each trading day we minimize the root mean square error (RMSE) given by

$$RMSE(\Theta^*) = \sqrt{\sum_{T_i}\sum_{K_j} \frac{(iVol^{market}_{T_iK_j} - iVol^{model}_{T_iK_j}(\Theta^*))^2}{\text{NumObs}}} \tag{3.4}$$

where T_i (with i =1,. . . ,7) and K_j (with j =1,. . . ,9) are the different maturities and strikes, respectively, $NumObs$ is the number of observations (63), and Θ^* is the parameter vector according to a given model.

In practice, we want to find a Θ^* such that the model implied volatility ($iVol^{model}$) is as close as possible to the market implied volatility ($iVol^{market}$). Since the minimization of equation (3.4) with respect to the parameters vector Θ^* has neither a closed-form solution nor a global minimum, a numerical optimization routine is needed to find a local minimum.[13] As already observed by Fang et al. (2010), the minimization of equation (3.4) is a well-known ill-posed problem, mainly because the solution is not necessarily unique and there is no guarantee that a solution exists. Consequently, it is not a simple numerical procedure. For this reason we consider a regularization term of the form

$$f(\Theta) = \rho\|(\Theta^* - \Theta_0)\|^2 \tag{3.5}$$

where ρ is a given constant parameter, Θ_0 is a given set of model parameters, and the optimization problem becomes

$$\hat{\Theta} = \min_{\Theta^*} \big(RMSE(\Theta^*)^2 + \rho\|(\Theta^* - \Theta_0)\|^2\big). \tag{3.6}$$

Furthermore, this approach leads to more parameter stability over time (Fang et al., 2010). Note that while in the first calibration day $t = 1$ we set $\rho = 0$, in the following days t we set Θ_0 equal to the parameters estimated on the previous day, that is $\Theta_0 = \Theta^*_{t-1}$.

[13] We use the Matlab r2011a function *fmincon* for the optimization routine and the function *blsimpv* to find the implied volatilities from the values of option prices. The procedure was run on an 8 cores AMD FX processor with 16GB of Ram with a Linux based 64-bit operating system.

The choice of the parameter ρ influences the model calibration; however, ρ cannot be fixed in advance but depends on the data at hand and the level of error present in the data (see Cont and Tankov, 2004). In the calibration exercise, we consider two different values for ρ. First, we solve the optimization problem (3.6) without regularization techniques, that is $\rho = 0$. Then, we solve it with $\rho = 1$. This last value for ρ shows a good balance between pricing performance and parameter stability. In the following we show the results only under the regularized approach, as the regularized approach shows satisfactory performance compared to the no-regularized one in terms of calibration error and parameters stability over time. A similar approach was considered in Bianchi (2012).

Contrary to the classical Black-Scholes case, in the continuous-time models we note that there is no explicit formula for European call and put option prices. However, one does not need to recur to Monte Carlo simulation since, thanks to the closed-form solution for the characteristic functions of models we consider, we can follow the widely-known analytical (up to an integration) pricing method for standard vanilla options proposed by Carr and Madan (1999) (see also Schoutens, 2003).

In the discrete-time case, we consider a Monte Carlo simulation routine to find the price of the options and, thus, the implied volatilities. The simulation algorithm is inserted into the optimization procedure to find a solution to the problem in equation (3.4).[14] As already noted, the value of the initial conditional variance is obtained by the historical time series estimation. As already observed in Section 3.1, in the CTS-GARCH model we assume that the parameters of the stdCTS innovations are the same under both the market and the risk-neutral measure; all other model parameters are estimated by solving the optimization problem (3.4). At each time step, we simulate 20,000 random paths for each model that are kept fixed in the optimization algorithm in order to reduce the variance in the option valuation and increase the computational speed. At each time step, three matrices of dimension $20{,}000 \times 517$ are allocated: the first contains standard normal random numbers for both the HN-GARCH and the NGARCH model, the second randomly selected innovations for the FHS model, and the third stdCTS random numbers. The computational cost needed to simulate the innovations is negligible compared to the time needed to find a solution for the optimization problem: 0.24 seconds in the normal case, 0.53 seconds in the FHS case, and 2.23 seconds in the CTS case. The random stdCTS innovations are simulated by considering the inverse transform algorithm described in Appendix A.3.

Even though for the HN-GARCH model one may use a recursive formula (see Heston and Nandi, 2000), we prefer to consider the same simulation algorithm for all four models in order to have a more precise model comparison that is not influenced by numerical issues. As observed by Barone Adesi et al. (2008), the computational time of the Monte Carlo pricing algorithm is roughly the same as for the competing GARCH pricing models proposed by Heston and Nandi (2000), where one considers a recursive pricing algorithm and whether Monte Carlo op-

[14] As in the continuous-time case, we use the Matlab r2011a function *fmincon* for the optimization routine and the function *blsimpv* to find the implied volatilities from the values of option prices.

tion prices are sufficiently accurate for pricing purposes will be reflected in the empirical pricing performance of our approach. As we will observe in the following of the section, given the wide range of moneyness and maturities considered, the calibration is quite satisfactory.

Based on the average relative percentage error (ARPE)[15] over the entire sample on successive cross-sections of implied volatilities, the CTS-GammaOU model is the best performing model (ARPE equal to 5.56%), and the HN-GARCH model is the worst one (ARPE equal to 8.99%). All other continuous-time models have an ARPE of about 6% and the other discrete-time models of about 8%.

As observed by Lehar et al. (2002), the calibration error varies across moneyness and maturity. In Tables 4 and 5 we show the value of the ARPE across moneyness and maturities for each model analyzed. The model parameters are calibrated on a daily basis by considering the whole volatility surface. For the continuous-time models, the best results are obtained for moneyness between 100% and 105%. For moneyness between 80% and 120%, the error is larger. For all other moneyness, the error increases as one moves away from the moneyness 100-105%. Regarding maturity, for most of the models, the 6-month implied volatility is the best calibrated in term of pricing error. As already observed, the best performing model is the CTS-GammaOU model across almost all moneyness and maturities, with errors less than 6% in most of the cases.

The diffusion component in the Bls-CTS model slightly enhances the pure jumps CTS model. Surprisingly, the pure jumps CTS model shows results comparable to the stochastic volatility models. The addition of jumps into the Heston model does not markedly affect the performance of the model in term of pricing error, at least for the data and the estimation method considered in this study.[16] The calibration of both the Heston-VG and the Heston-CTS models is not an easy task as the number of parameters increases dramatically. The NGARCH model based on the normality assumption of the innovation shows similar results compared to the heavy-tailed CTS-NGARCH model based on the stdCTS assumption. We observe that under our setting, the parameters of the stdCTS innovation are calibrated on the time series of the index log-returns and not on the cross-section of implied volatilities. In Figure 4, we report the implied volatility surface estimates on a randomly selected date for all models analyzed and they are compared to the market surface; as also shown in Table 4, for the 80% moneyness the calibration error is large in all discrete-time models.

The pricing error varies across time as shown in Figure 3. The three GARCH models seem to be more stable over time with pricing error greater than 15% in few cases compared with the other competitor models. There were only five times

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[15] ARPE is defined as

$$ARPE = \frac{1}{\text{NumObs}} \sum_{T_i} \sum_{K_j} \frac{|iVol_{T_iK_j}^{market} - iVol_{T_iK_j}^{model}|}{iVol_{T_iK_j}^{market}}.$$

[16] As discussed above, Yu et al. (2011) showed that the Heston-VG model can efficiently be applied to capture the joint dynamics of stock and short-term options closest to the money (i.e., with a strike-to-spot price ratio close to one). Here we consider a larger option dataset in terms of moneyness and maturities on a different underlying.

that the FHS model error exceeded 15% (17 times for both the NGARCH and the CTS-NGARCH model). There were 19 times that the CTS-GammaOU model error exceeded 15%. For all other models, there were more than 46 times that the ARPE was greater than 15% with a maximum for the HN-GARCH model (63 times). On September 29, 2008 all models had a large calibration error, as the volatility surface was particularly undulating. Additionally, as shown in Figure 3, all continuous-time models had three spikes in the behavior of the pricing error corresponding to the Lehman Brothers bankruptcy, the worsening of the Greek sovereign debt crisis, and the recent Italian sovereign debt crisis when the 10-year BTP-bund spread exceeded 550 basis points. The three GARCH models (FHS, NGARCH, and CTS-GARCH models) were only partially affected by those events. The FHS model seems to be the best performing among the four GARCH models analyzed.

In Table 6 we report the risk-neutral parameters estimates and the computing time (median, 2.5th and 97.5th percentile) across 1,271 trading days. The calibration on a daily basis is not able to capture the dynamics of the volatility surface over time and, for this reason, the values reported in Table 6 may differ from those reported in Table 2. As proposed in Yu et al. (2011), a joint calibration of index log-returns and implied volatilities has to be considered in order to explore the dynamics of the historical volatility together with those of the volatility surface.

The NIG model is the best performing in terms of computing time: to calibrate an observed volatility surface one needs, in median, 168.9 seconds. The computing time increases if one considers more complex models less parsimonious in terms of the number of parameters. The Heston-based models have a larger computational complexity, greater, in median, than 319 seconds in all cases analyzed. For all GARCH models, the time needed for the calibration is in median less than 500 seconds. The SV-GammaOU model has an acceptable calibration time, in median less than 249 seconds. The time necessary to calibrate these models generally increases if one does not consider regularization techniques,[17] additionally, allowing one to obtain more stable parameters over time. The calibration algorithm becomes faster if one fits directly the option prices without inverting the Black-Scholes formula to obtain the implied volatility. The procedure that involves direct valuation of option prices without finding the corresponding implied volatility is seven times faster than the procedure used in this study. However, our choice of objective function in the optimization problem (i.e., equation (3.4)) is an integral part of model specification (see also Christoffersen et al., 2010).

As far as the behavior of the risk-neutral estimated parameters over time is concerned, as expected, more parsimonious models in terms of number of parameters show more stable dynamics over time because, in general, they are simpler to calibrate, at least with the algorithm considered in this study. The choice of the regularization function in equation (3.5) implies that the parameters with greater values are generally more stable over time compared to those with smaller values. Although for each model an ad-hoc choice of the regularization function is needed to obtain a similar stability across all model parameters, our results are sufficiently accurate even with the regularized function in equation (3.5).

[17] See Bianchi (2012) and the references therein for a discussion on this topic.

4 Conclusions

The objective of this paper is twofold. First, we analyzed the dynamics of the FTSE MIB index log-returns volatility from a historical time-series perspective by considering both continuous-time and discrete-time stochastic volatility models. Second, we analyzed the pricing error in calibrating on a daily basis the observed implied volatility surface of a wide range of models that allow for stochastic volatility and/or heavy tails. The calibration exercise is conducted on the implied volatilities related to options with a maturity between one month and two years, with moneyness between 80% and 120% and traded in the period April 1, 2007 and March 30, 2012. We are aware that a proper risk analysis should take into account a dynamic joint calibration of both the time series of historical returns and the cross-section of implied volatilities. However, more efficient algorithms and huge computational power are needed to conduct this kind of empirical study in real-world applications. Mainly for these reasons, we analyzed the time series of historical returns and the cross-section of implied volatilities for options separately.

Regarding the historically based estimation, among discrete-time and continuous-time models the CTS-NGARCH is the best performing in terms of both the Akaike and Bayesian information criterion and it can be effectively used to explain the dynamics of historical volatility. The estimation algorithm related to the CTS-NGARCH model is simpler to implement and faster to run in comparison to the competitor continuous-time stochastic volatility models analyzed in this study. Stochastic volatility and heavy-tailed models can be useful to explain observed patterns of the Italian index we investigated.

As far as the implied volatility risk-neutral calibration is concerned, our findings indicate that there are not remarkable differences in terms of pricing errors between pure jumps Lévy models and continuous-time stochastic volatility models of the Heston type. However, in theory, the presence of a diffusion component offers more flexibility to the change of measure needed to jointly analyze the time series of historical returns and the cross-section of implied volatilities for options. Although the discrete-time models show less flexibility in fitting the observed smirk compared to the competitor continuous-time models, they exhibit a more stable calibration error over time. In calibration exercises of this type an algorithm that considers a regularization technique is needed to obtain stable parameters over time, particularly then the number of parameters to be estimated is large. As expected, more parsimonious models in terms of number of parameters show more stable parameters over time and they are the best performing in terms of computing time. In general, they are simpler to calibrate, at least with the algorithm considered in this study.

Since we show that the skewness and fat-tail properties of the Italian daily log-returns are important to explain the index historical dynamics and to explain the shape of the implied volatility surface, these empirical findings should be taken into consideration for pricing and hedging of financial instruments related to the major Italian market index and for a proper risk assessment of these products. Not properly accounting for these stylized facts can result in models that may incorrectly assess the tail risk related to this market.

A Appendix

A.1 Change of measure for Lévy process

Before explaining how to find a proper change of measure between the market measure P and the risk-neutral measure Q, we review, some useful definitions.

Theorem A.1 (Lévy-Khintchine formula). *A probability law μ of a real-valued random variable X on $\mathbb{R}$ is infinitely divisible with characteristic exponent ψ,*

$$\int_{\mathbb{R}} e^{i\theta x}\mu(dx) = e^{\psi(\theta)} \quad \textit{for } \theta \in \mathbb{R}$$

if and only if there exists a triple (a_h, σ, ν) where $a_h \in \mathbb{R}$, $\sigma \geq 0$, ν is a measure on $\mathbb{R}\backslash\{0\}$ satisfying

$$\int_{\mathbb{R}\backslash\{0\}} (1 \wedge x^2)\nu(dx) < \infty$$

and h is a given truncation function such that

$$\psi(\theta) = ia\theta - \frac{1}{2}\sigma^2\theta^2 + \int_{\mathbb{R}\backslash\{0\}} (e^{i\theta x} - 1 - i\theta h(x))\nu(dx) \tag{A.1}$$

for every $\theta \in \mathbb{R}$.

We say that our infinitely divisible distribution μ has *Lévy triplet* (a_h, σ, ν). The measure ν is called the *Lévy measure* of μ, σ represents the Gaussian component, and a is a constant depending from the truncation function h. If the Lévy measure is of the form $\nu(dx) = u(x)dx$, we call $u(x)$ the *Lévy density.* If μ is an infinitely divisible distribution, there exists a Lévy process $(X_t)_{t\geq 0}$ such that the distribution of X_1 is μ. Conversely, if $(X_t)_{t\geq 0}$ is a Lévy process, there is always a Lévy triplet (a_h, σ, ν), such that $E[e^{iuX_t}] = e^{t\psi(u)}$.

Now, we want to find conditions under which the Lévy process X_t under the measure P is still a Lévy process under a new measure Q. In order to find an equivalent measure, we will consider the general result of density transformation between Lévy processes proven in Sato (1999). Even if we restrict our attention to structure-preserving measures, the class of probabilities equivalent to a given one is surprisingly large. Nonetheless, as stated in the following theorem (statement 3.), we cannot freely change the drift a_h if a diffusion component is not present, that is if $\sigma = 0$.

Theorem A.2. *Let (X_t, P) and (X_t^*, Q) be Lévy processes on $\mathbb{R}$ with generating triplets (a_h, σ, ν) and (a_h^*, σ^*, ν^*), respectively. Then,* P *and* Q *are equivalent for each t if and only if*

1. *$\sigma = \sigma^*$;*

2. *The following integral is finite*

$$\int_{\mathbb{R}} (e^{\varphi(x)/2} - 1)^2\nu(dx) < \infty,$$

with the function $\varphi(x)$ defined by $\frac{d\tilde{\nu}}{d\nu} = e^{\varphi(x)}$;

3. *The constant b is such that*

$$a_h^* - a_h - \int_{\mathbb{R}} h(x)(\nu^* - \nu)(dx) = b\sigma^2$$

if $\sigma > 0$ and zero if $\sigma = 0$.

Proof. See Theorem 33.1 in Sato (1999) and Cont and Tankov (2004). □

A.2 Particle filter

In continuous-time stochastic volatility models, the unobserved volatility can be inferred by the observed stock returns. Since in most of the cases one deals with daily returns the model has to be discretized. In all the cases we are interested in, the model can be written as

$$\begin{aligned} v_t &= f(v_{t-1}, \Theta, \xi_{t-1}) \\ y_t &= h(v_t, \Theta) + \sqrt{v_t}\varepsilon_t \end{aligned}$$

where $v_t = \sigma_t^2$, t is the day counter, v_t is the square of the stochastic volatility modeled as a Markov process with initial distribution $p(v_0)$, and transition law $p(v_t|v_{t-1})$. Both p and the transition function f depends on the dynamics described by model chosen. The variable y_t represents the set of given observations (in our case the observed index log-returns). It is assumed to be conditionally independent given the state v_t and with distribution $p(y_t|v_t)$. Then, ξ_{t-1} depends on the volatility dynamics and the noise ε_t is normally distributed noise with mean zero and unit variance, at least in the stochastic volatility models we are interested in studying. The function h is the so-called measurement function, that in our case is given by the stock price returns model and Θ is a set of static parameters.

Particle filter is a sequential Monte Carlo method for recursively approximating the posterior density $p(v_t|y_{1:t})$ by assuming a known measurement density $h(y_t|v_t)$ and the ability to simulate from the Markov transition density $f(v_{t+1}|v_t)$. The algorithm estimates the posterior density by considering a set of random samples with associated weights $\{v_t^i, w_t^i\}_{i=1}^N$ where N is the number of samples at each given point in time t. The algorithm, also known as the bootstrap filter, includes three main steps: (a) sampling, (b) weights computation, and (c) resampling. In our empirical test we proceed as follows:

1. we sample v_t^i from the distribution $p(v_t|v_{t-1})$ with i ranges from 1 to N, where N is equal to 5,000;

2. we compute the weight as follows

$$w_t^i = -\frac{1}{\sqrt{2\pi v_t^i}} \exp\left(-\frac{(y_t^{observed} - h(v_t^i, \Theta))^2}{2v_t^i}\right),$$

evaluate the likelihood estimate

$$\widehat{L}_t = \frac{1}{N}\sum_{i=1}^N w_t^i$$

and then normalize the weights in order to have $\sum_{i=1}^N w_t^i = 1$.

3. we resample by taking into consideration the smooth resampling algorithm (see Douc and Cappé, 2005; and Malik and Pitt, 2011) and we obtain a new set of samples $\tilde{v}_t^i$ approximately distributed according to $p(v_t|y_{1:t})$ and evaluate the state
$$\widehat{v}_t = \frac{1}{N}\sum_{i=1}^{N} v_t^i;$$

To estimate the parameters Θ, we build the joint log-likelihood over the entire observation period, that is

$$LL(\Theta, y_{1:T}) = \sum_{t=1}^{T} \log\left(\widehat{L}_t\right),$$

and, finally, we insert this function into an optimization procedure. We use the Matlab r2011a function *fmincon* as the optimization routine. The classical Heston and the SV-GammaOU models have a better convergence property compared to other competitor stochastic volatility continuous-time models. This is mainly due to the number of parameters to be estimated.

A.3 Simulate classical tempered stable random numbers

The simulation of a tempered stable random draw is not a simple task. A series representation algorithm has been proposed in Rosinski (2007) and empirically studied in Bianchi et al. (2010) and Imai and Kawai (2011). Since there exists an efficient algorithm to draw random samples from stable distributions (see Chambers et al., 1976), the problem of generating random numbers from a tempered stable law X can be solved by using a stable law Y possessing a probability density g similar to the probability density f of X. One can generate a value for Y and accept (reject) this value, if a given condition is satisfied (not satisfied). This acceptance-rejection simulation method has been widely studied in the literature (see Kawai and Masuda, 2011a, 2011b, 2012, and references therein). By applying this algorithm, one can sample tempered stable random numbers in an exact (or approximate) way if the tail index α is less (or greater) than 1. The computational cost strictly depends on the parameters. In the case when $\alpha < 1$, a double rejection sampling algorithm that does not depend upon the model parameters has been proposed by Devroye (2009).

Furthermore, one can use a similar acceptance-rejection algorithm proposed by Rosinski (2001) and based on a comparison between the Lévy measures of stable and tempered stable laws. In general, the probability of an acceptance event depends on the parameters of the tempered stable distribution to simulate.

Alternatively, one can consider the inverse transform algorithm: that is, given the cumulative distribution function F, one can use the following three-step procedure:

1. generate a sequence $U_1, \ldots, U_n$ of i.i.d. uniform variables;

2. find a root X_i of the equation $F(X_i) - U_i = 0$ for each $i = 1, \ldots, n$;

3. return the sequence $X_1, \ldots, X_n$.

In the tempered stable case, it easy to see that the function F is not available in closed form. To find values of F, first we have to invert the characteristic function,[18] to find both the density (f) and the cumulative distribution (F), and then find the value X_i satisfying the equality $F(X_i) - U_i = 0$. Even if this method may seem computational demanding, an efficient procedure can be written in order to increase the speed and make the time necessary for the simulation of large matrices of minor concern. This approach can be efficiently applied for all values of the parameters and can compete with the acceptance-rejection method when a huge matrix of tempered stable random numbers have to be drawn (see also the recent work of Ballotta and Kyriakou, 2011). To prove this statement, we test both the acceptance-rejection and the inverse transform algorithm by drawing stdCTS random numbers. We consider three different parameter sets (α, λ_+, λ_-): (0.75, 0.5, 0.5), (1.5, 1, 0.5), and (1.75, 0.1, 0.05).[19] The computational time needed for generating a matrix with dimension 20,000 $\times$ 1000 is 15 (56, and 11) seconds with the acceptance-rejection algorithm, slightly more than 5 seconds with the inverse algorithm in all the cases considered.

Since the computational cost of the inverse transform algorithm is not influenced by the parameter values of the stdCTS distribution and is of minor concern compared with the cost of the optimization algorithm, we consider it in the calibration of implied volatility under the CTS-GARCH model.

[18] Here the term inversion means Fourier inversion. More details on this method can be found in Scherer et al. (2012) and Bianchi et al. (2013).

[19] The computational time can be much greater if one selects other parameters.

Figure 1: Maximum likelihood estimated historical annualized volatilities from FTSE MIB index log-returns data between January 4, 2000 to March 30, 2012: (1) HN-GARCH, (2) FHS, (3) NGARCH, (5) Heston, (6) Heston-VG, (7) Heston-CTS, (8) SV-GammaOU. The estimated volatilities are compared with the (4) in-the-money 30 days implied volatility, available since April 1, 2007.

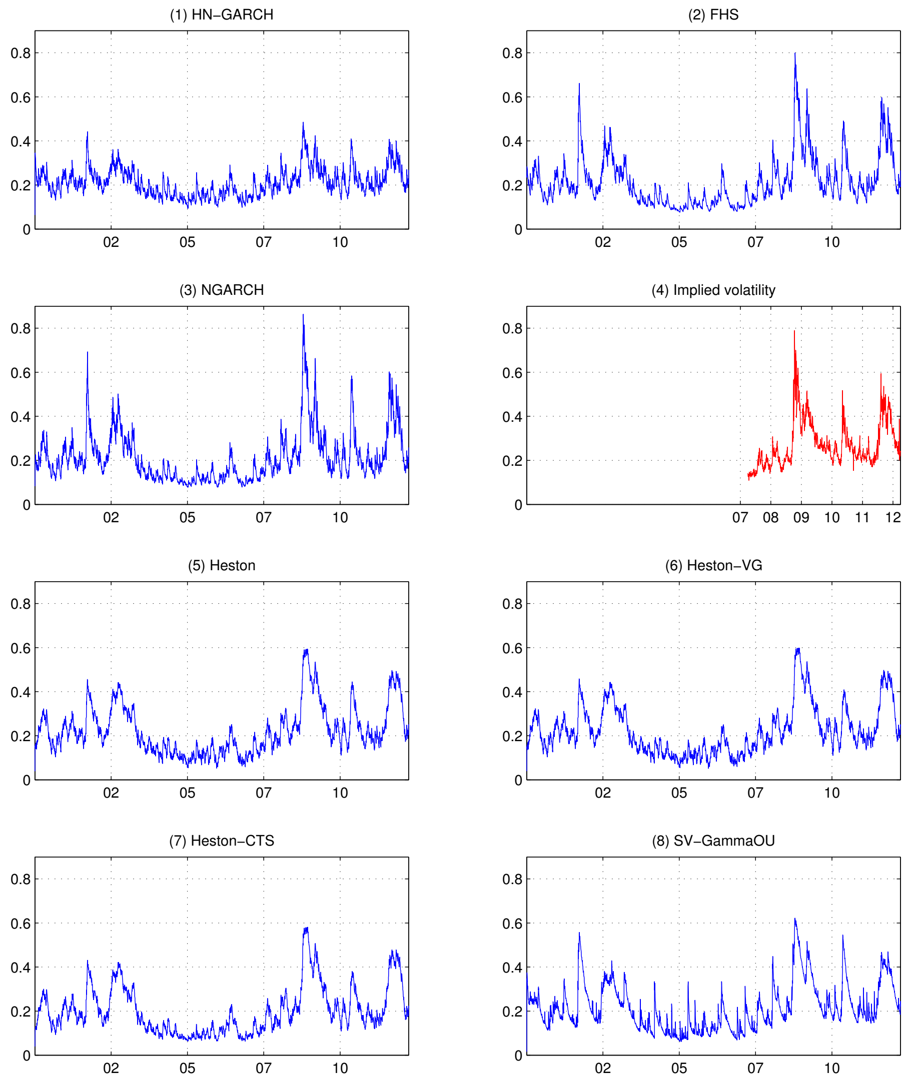


Figure 2: Histogram of the simulated innovations for the (1) NGARCH model, (2) FHS model, (3) CTS-GARCH model. Estimates are based on the historical time-series of FTSE MIB log-returns between January 4, 2000 to March 30, 2012. On the left (right) side, a detail of the left (right) tail is shown.

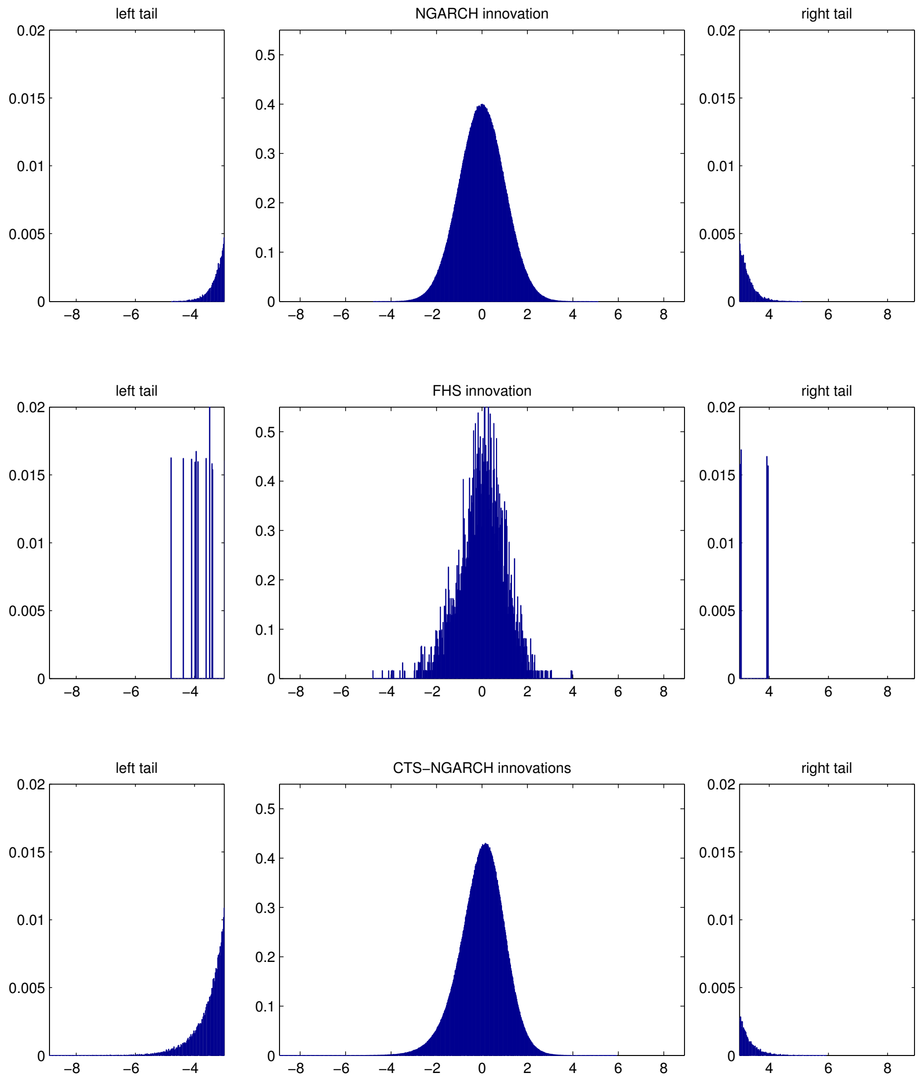


Figure 3: Implied volatility calibration error for all models analyzed. The calibration was conducted on a daily basis for each trading between April 1, 2007 and March 30, 2012.

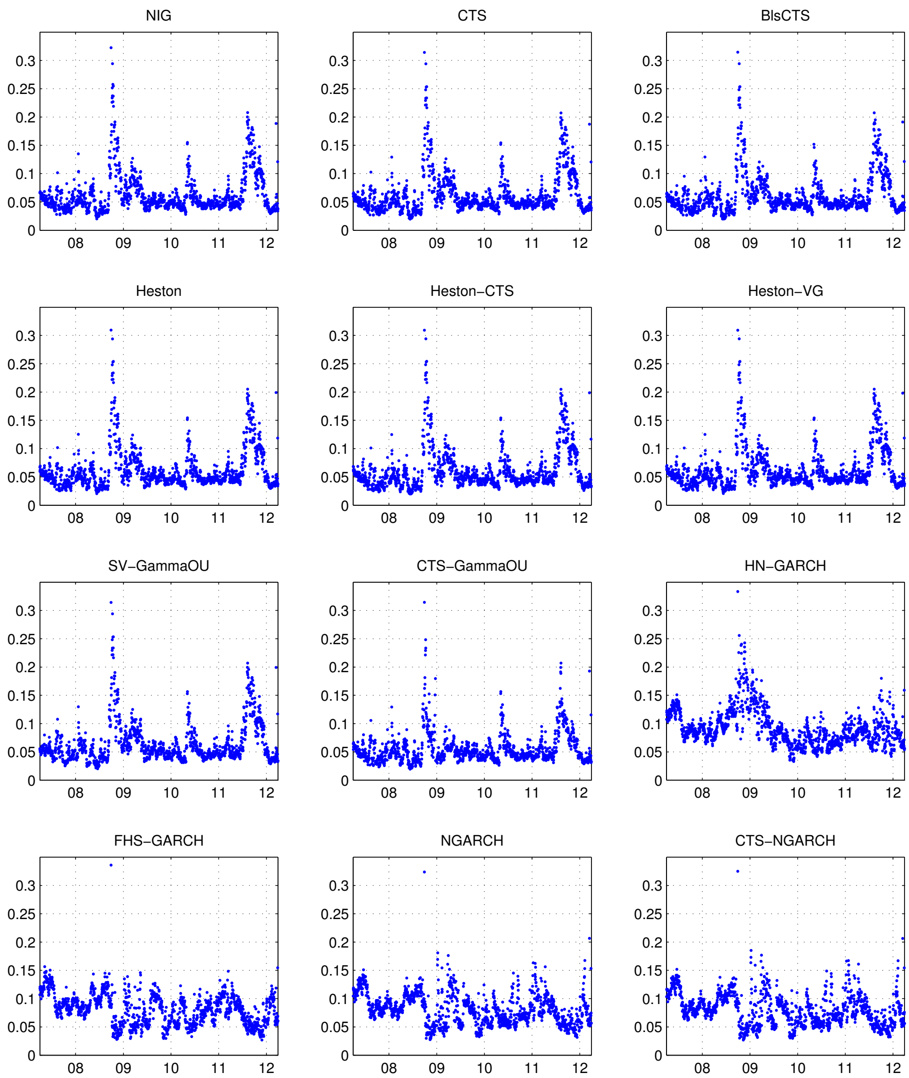
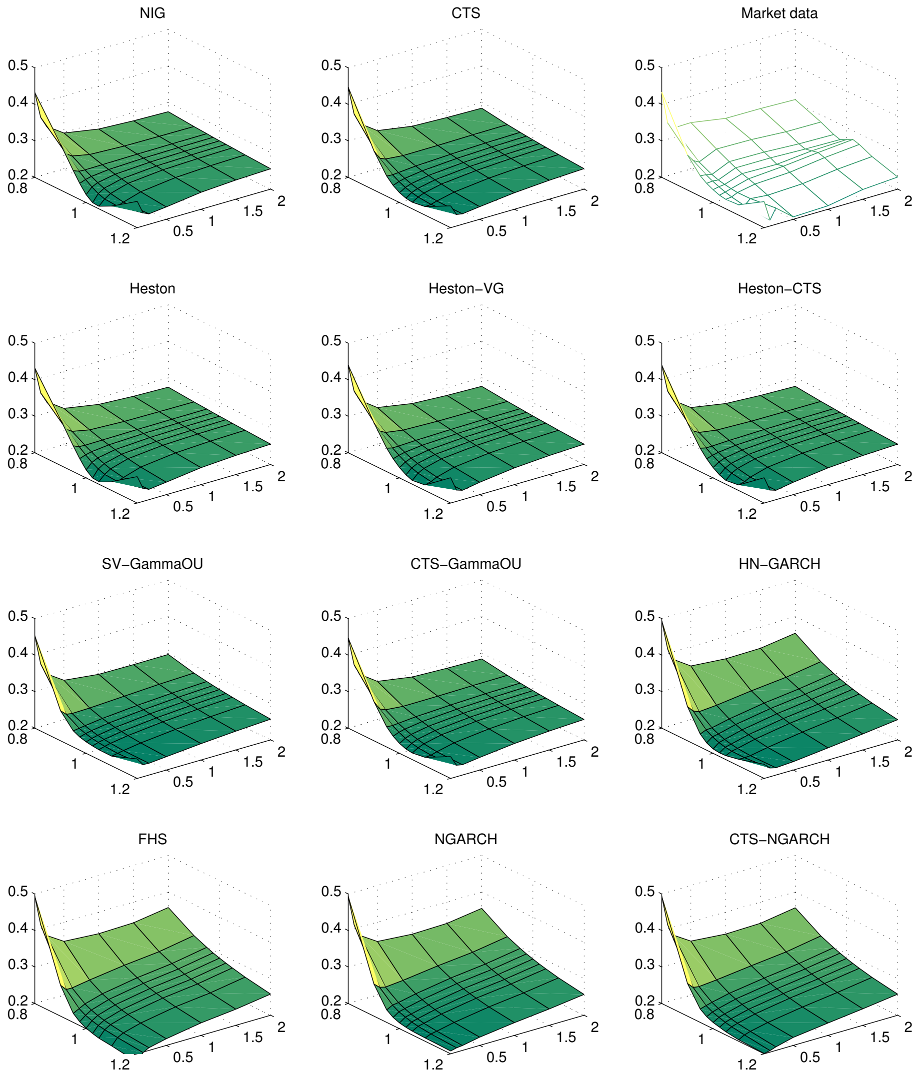


Figure 4: Implied volatility surface market data and estimates for all models analyzed on September 30, 2010. The surface estimated by the Bls-CTS model is not shown because it is similar to that of the CTS model. The moneyness ranges from 80% to 120% and the maturity ranges from one month to two years.



Models characteristics				
	Continuous	Discrete	Stochastic volatility	Fat-tail
Normal inverse Gaussian	✓			✓
Classical tempered stable	✓			✓
Black&Scholes – classical tempered stable	✓			✓
Heston	✓		✓	
Heston with classical tempered stable jumps	✓		✓	✓
Heston with variance gamma jumps	✓		✓	✓
Black&Scholes with Gamma-OU stochastic volatility	✓		✓	✓
Time changed classical tempered stable with integrated Gamma-OU time	✓		✓	✓
Heston and Nandi GARCH		✓	✓	
Filtering historical simulation Glosten-Jagannathan-Runkle		✓	✓	✓
Normal nonlinear GARCH		✓	✓	
Classical tempered stable nonlinear GARCH		✓	✓	✓

Table 1: Models considered in the empirical study.

Historical market estimates										
HN										
β_1	α_1	α_0	λ	γ				LL	AIC	BIC
0.8923	1.37e-5	1.18e-6	-0.5000	50.0000				9179.13	-18348.26	-18318.04
FHS										
β_1	α_1	α_0	λ	γ				LL	AIC	BIC
0.9225	2.78e-3	1.39e-6	0.1319	-3.69e-5				9314.24	-18618.48	-18588.27
NGARCH										
β_1	α_1	α_0	λ	γ				LL	AIC	BIC
0.8582	0.0721233562	2.17e-6	1.01e-6	0.9298				9315.86	-18621.71	-18591.50
CTS-innovation										
λ_-	λ_+	α						LL	AIC	BIC
1.7140	3.1407	0.7502						9353.15	-18690.31	-18641.96
CTS-NGARCH										
β_1	α_1	α_0	λ	γ	λ_-	λ_+	α	LL	AIC	BIC
0.8590	0.0718	1.76e-6	1.00e-3	0.9551	1.6948	3.1162	0.7500	9354.17	-18692.33	-18643.99
Heston										
κ	η	ϑ	ρ					LL	AIC	BIC
0.3048	0.0015	0.0303	-0.6894					9276.87	-18545.74	-18521.56
Heston-CTS										
κ	η	ϑ	ρ	C	λ_+	λ_-	α	LL	AIC	BIC
0.2979	0.0016	0.0305	-0.6517	0.0106	0.7513	1.8577	1.5253	9235.93	-18463.86	-18439.69
Heston-VG										
κ	η	ϑ	ρ	C	λ_+	λ_-		LL	AIC	BIC
0.3068	0.0016	0.0306	-0.6844	0.5491	11.9661	18.1258		9284.58	-18561.16	-18536.99
SV-GammaOU										
λ	a	b	ρ					LL	AIC	BIC
4.0117	0.8021	3340.5647	-0.9215					9244.17	-18486.63	-18462.46

Table 2: Market parameters estimated using the MLE approach considering the time series of the FTSE MIB log-returns from January 1, 2000 to March 30, 2012. In the CTS-innovation case, we fit the stdCTS distribution to the innovations extracted by the NGARCH model. In the CTS-NGARCH case, we jointly estimate the GARCH dynamics and the stdCTS innovation parameters. For each model the value of the log-likelihood (LL), of the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) are reported.

GARCH parameter estimates								
						KS	p-value	LL
HN-GARCH								
	β_1	α_1	α_0	λ	γ			
median	0.8887	1.06e-5	7.63e-7	-0.4964	49.9997	0.0598	4.18e-5	4703.45
2.5th percentile	0.8695	6.36e-6	6.98e-8	-0.5393	49.9630	0.0504	2.91e-7	4232.31
97.5th percentile	0.9188	1.93e-5	3.10e-6	-0.4686	50.0000	0.0722	9.33e-4	4884.34
FHS								
	β_1	α_1	α_0	λ	γ			
median	0.9061	0.0000	1.70e-6	0.1543	1.92e-4			4798.54
2.5th percentile	0.8928	0.0000	1.16e-6	0.1148	-2.66e-4			4280.12
97.5th percentile	0.9270	0.0063	2.35e-6	0.1875	2.92e-4			4944.45
NGARCH								
	β_1	α_1	α_0	λ	γ			
median	0.8425	0.0676	2.19e-6	0.0038	0.9942	0.0459	0.0034	4799.79
2.5th percentile	0.7614	0.0298	1.55e-6	1.01e-6	0.8741	0.0304	0.0002	4287.69
97.5th percentile	0.8605	0.0837	3.30e-6	2.10e-2	2.3535	0.0561	0.1226	4937.31
CTS-NGARCH								
	λ_-	λ_+	α					
median	1.5882	3.3189	0.6375			0.0164	0.8082	4831.04
2.5th percentile	0.6880	1.8673	0.2501			0.0131	0.3876	4306.01
97.5th percentile	2.4629	4.3740	1.4000			0.0232	0.9553	4975.31

Table 3: FTSE MIB market parameters estimated using the MLE approach for each trading day from April 1, 2007 to April 1, 2012. For each trading day, a window of fixed size is considered (1,500 trading days) for a total of 1,271 rolling windows estimations for each GARCH model. For each model, the median, minimum and maximum values of the parameters, Kolmogorv-Smirnov statistic (with the corresponding p-value), and the log-likelihood (LL) are reported. For the non-parametric FHS model, the log-likelihood of the GJR-GARCH model with normal innovation is reported.

	Moneyness									
	80%	90%	95%	97.5%	100%	102.5%	105%	110%	120%	all
NIG	0.0749	0.0602	0.0575	0.0584	0.0530	**0.0520**	0.0525	0.0725	0.1129	0.0631
CTS	0.0738	0.0603	0.0574	0.0585	0.0527	**0.0517**	0.0523	0.0702	0.1107	0.0625
Bls-CTS	0.0753	0.0594	0.0555	0.0552	0.0498	**0.0496**	0.0522	0.0708	0.1106	0.0612
Heston	0.0714	0.0596	0.0564	0.0577	0.0529	**0.0524**	0.0525	0.0691	0.1115	0.0620
Heston-CTS	0.0708	0.0580	0.0549	0.0570	0.0519	**0.0505**	0.0511	0.0693	0.1112	0.0610
Heston-VG	0.0718	0.0595	0.0561	0.0574	**0.0530**	**0.0530**	0.0531	0.0695	0.1113	0.0622
SV-GammaOU	0.0695	0.0624	0.0643	0.0661	0.0553	0.0484	**0.0476**	0.0699	0.1136	0.0637
CTS-GammaOU	0.0739	0.0488	0.0446	0.0477	0.0441	**0.0451**	0.0483	0.0694	0.1097	**0.0556**
HN	0.1413	0.1299	0.0988	0.0897	0.0744	0.0699	**0.0682**	0.0727	0.0830	0.0899
FHS	0.1445	0.1160	0.0829	0.0726	**0.0608**	0.0655	0.0689	0.0768	0.0815	0.0826
NGARCH	0.1338	0.1141	0.0810	0.0698	**0.0561**	0.0597	0.0644	0.0759	0.0956	0.0801
CTS-NGARCH	0.1356	0.1139	0.0806	0.0694	**0.0558**	0.0601	0.0646	0.0752	0.0922	0.0798

Table 4: Average relative percentage error (ARPE). For each model, the ARPE as a function of the moneyness is reported. The daily calibration is performed from April 1, 2007 to March 30, 2012.

Maturity								
	1M	2M	3M	6M	1Y	1.5Y	2Y	all
NIG	0.0990	0.0537	0.0436	**0.0433**	0.0563	0.0688	0.0767	0.0631
CTS	0.0973	0.0534	0.0433	**0.0427**	0.0559	0.0685	0.0761	0.0625
Bls-CTS	0.0963	0.0512	**0.0426**	0.0438	0.0576	0.0704	0.0668	0.0612
Heston	0.0934	0.0539	0.0441	**0.0430**	0.0554	0.0682	0.0763	0.0620
Heston-CTS	0.0898	0.0532	0.0442	**0.0428**	0.0547	0.0672	0.0751	0.0610
Heston-VG	0.0939	0.0542	0.0440	**0.0430**	0.0555	0.0682	0.0764	0.0622
SV-GammaOU	0.0911	0.0634	0.0527	**0.0431**	0.0541	0.0673	0.0744	0.0637
CTS-GammaOU	0.0810	0.0451	**0.0419**	0.0493	0.0533	0.0568	0.0621	**0.0556**
HN	0.1200	0.0903	0.0792	0.0680	**0.0638**	0.0881	0.1198	0.0899
FHS	0.1041	0.0711	**0.0638**	0.0653	0.0664	0.0920	0.1157	0.0826
NGARCH	0.1063	0.0768	0.0681	0.0650	**0.0638**	0.0783	0.1023	0.0801
CTS-NGARCH	0.1036	0.0707	0.0623	**0.0615**	0.0646	0.0816	0.1056	0.0798

Table 5: Average relative percentage error (ARPE). For each model, the ARPE as a function of the maturity is reported. The daily calibration is performed from April 1, 2007 to March 30, 2012.

Risk-neutral estimates										
NIG	α	β	δ							time
median	18.55	-9.86	0.72							168.9
2.5th percentile	13.36	-13.66	0.25							105.9
97.5th percentile	24.70	-6.54	3.61							281.1
CTS	C	λ_-	λ_+	α						time
median	0.86	7.12	26.63	0.72						221.0
2.5th percentile	0.01	2.78	21.56	0.38						149.9
97.5th percentile	2.48	8.37	28.95	1.95						395.5
BlsCTS	C	λ_-	λ_+	α	σ					time
median	0.38	5.62	28.10	1.00	0.02					330.8
2.5th percentile	0.01	1.41	22.23	0.46	0.00					181.0
97.5th percentile	2.90	9.14	31.56	1.95	0.32					849.4
Heston	κ	η	ϑ	ρ	σ_0					time
median	5.11	0.07	0.48	-0.55	0.08					319.0
2.5th percentile	1.70	0.01	0.13	-1.00	0.01					161.6
97.5th percentile	6.70	0.22	0.80	-0.39	0.63					607.9
Heston-CTS	κ	η	ϑ	ρ	σ_0	C	λ_-	λ_+	α	time
median	0.53	0.13	0.25	-0.68	0.01	0.76	34.86	99.01	0.74	458.8
2.5th percentile	0.01	4.41e-3	0.05	-1.00	0.01	0.10	34.51	99.00	0.25	252.0
97.5th percentile	0.95	0.67	0.48	-0.44	0.20	2.10	34.93	99.10	1.37	1020.3
Heston-VG	κ	η	ϑ	ρ	σ_0	C	λ_-	λ_+		time
median	1.67	0.07	0.24	-0.58	0.03	88.54	1100.25	1470.68		349.1
2.5th percentile	0.12	2.50e-3	0.05	-1.00	0.01	88.54	1100.24	1470.68		213.3
97.5th percentile	2.84	0.28	0.52	-0.41	0.34	88.54	1100.25	1470.69		625.4
SV-GammaOU	λ	a	b	ρ	σ_0					time
median	0.22	6.40	9.39	-0.75	0.20					249.0
2.5th percentile	0.10	0.69	1.49	-1.22	0.12					151.1
97.5th percentile	0.36	10.00	14.92	-0.17	0.43					411.6
CTS-GammaOU	C	λ_-	λ_+	α	λ	a	b			time
median	0.09	2.09	198.07	1.45	0.93	0.18	2.09			370.6
2.5th percentile	0.03	0.01	197.12	0.91	0.10	0.10	0.10			177.9
97.5th percentile	1.05	3.80	198.52	1.71	1.98	0.80	3.99			841.7
HN	β_1	α_1	α_0	$\lambda+\gamma$						time
median	0.98	1.00e-6	1.01e-8	26.68						387.4
2.5th percentile	0.50	1.00e-6	1.00e-8	-1.00						162.9
97.5th percentile	0.99	5.25e-5	1.00e-4	58.80						857.7
FHS	β_1	α_1	α_0	γ	μ					time
median	0.95	2.96e-2	1.43e-6	1.48e-6	4.18e-4					354.7
2.5th percentile	0.87	1.00e-10	1.00e-10	1.00e-10	-4.93e-2					178.7
97.5th percentile	0.95	0.05	1.00e-5	0.09	0.05					760.0
NGARCH	β_1	α_1	α_0	$\lambda+\gamma$						time
median	0.97	1.03e-6	1.02e-6	25.26						371.7
2.5th percentile	0.75	1.00e-6	1.00e-10	1.91						200.6
97.5th percentile	0.99	1.43e-2	5.03e-5	99.97						780.8
CTS-NGARCH	β_1	α_1	α_0	$\lambda+\gamma$						time
median	0.97	1.03e-6	9.47e-7	25.28						484.3
2.5th percentile	0.75	1.00e-6	1.00e-10	1.89						252.4
97.5th percentile	0.99	1.09e-2	5.16e-5	99.96						1093.7

Table 6: Risk-neutral parameters estimated on the FTSE MIB implied volatility and computing time to solve the optimization problem. Median, 2.5th, and 97.5th percentile values for each model evaluated over 1,271 implied volatility surface observations.

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