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Limited credit records and market outcomes

by Margherita Bottero and Giancarlo Spagnolo

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# LIMITED CREDIT RECORDS AND MARKET OUTCOMES

by Margherita Bottero\* and Giancarlo Spagnolo^

## Abstract

Credit registers collect, store and share data on borrowers' past and current credit relations. Interestingly, the data are typically erased from the public records after a certain number of years, in accordance with privacy protection laws designed to enable people to make a fresh start. However, in order to give creditworthy but unlucky borrowers the chance for a new beginning, these provisions ultimately remove all the public information, including data that may still be relevant for purposes of screening. This paper assesses this trade-off, examining the impact of limited records on borrowers' behaviour and market outcomes in a stylized credit market for unsecured loans. In this setup, limited records endogenously produce beneficial reputation effects in the form of greater effort in equilibrium. That is, alleviate the limitations rather than worsen the distortions due to asymmetric information. Further, we show that when moral hazard is great, one-period records can increase welfare and lower the default rate by comparison with records that show either all of the past history or none.

**JEL Classification:** G24, G18, D82.

**Keywords:** privacy, data retention, credit registers, limited records.

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# 1 Introduction<sup>1</sup>

In most modern credit markets, credit registers collect and share data on borrowers' past behavior with other market participants, in an effort to reduce the well-known informational asymmetries that characterize lending relationships (Jappelli, Pagano 2002). These asymmetries take the form of adverse selection (Akerlof 1970) or moral hazard. Reliable information on a borrower's past behavior serves both as a screening device for lenders and as a "reputational" collateral for borrowers (Vogel, Burkett 1986), which they can use to signal to potential lenders their private information and intended choices.

Interestingly, from a legal point of view credit data fall under the umbrella of personal data and are thus subject to the privacy protection provisions that regulate the extent to which personal information can be handled, shared and stored. In particular, an important by-product of privacy protection is the principle of data retention, which prescribes that personal data, as collected by any data user, should be retained for a limited period of time only, and by any means for no longer than what is necessary for the intended purpose.<sup>2</sup> Although such a principle, which protects at the core many important individual rights, looks intuitively appealing, it is not immediately clear whether it is desirable from a strictly economic standpoint. In fact, data retention directly mandates the removal from the market of those very data that have a key role in disciplining the market asymmetries described above. Indeed, the issue has stirred an intense and so far unresolved debate about the implications of limited records for privacy, efficiency and reputation (Bottero, Spagnolo 2011; Sartor 2006).<sup>3</sup>

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<sup>2</sup>Data retention was first mentioned by the Council of Europe in 1973, where it was declared that "[T]hose responsible of these [personal data] files [...] should refrain from storing information which is not necessary for the given purposes". This principle was later on incorporated into the 1995 EU Directive (95/46/EC) on Data Protection, which states that "[M]ember States shall provide that personal data must be [...] adequate, relevant and not excessive in relation to the purposes for which they are collected and/or further processed".

<sup>3</sup>According to the anti-privacy position, privacy gives the possibility to manipulate one's reputation and to impede the autonomous function of the impersonal market mechanism that allocates credit at better rates to borrowers with better reputations (Posner 1983). From Posner's blog (May 8<sup>th</sup> 2005): "All that privacy means in the informational context [...] is that people want to control what is known about them by other people. They want to conceal facts about themselves that are embarrassing or discreditable [...] Such concealment is a species of fraud". Yet, other scholars argue that privacy may be needed when agents are boundedly rational, or even in cases of market failures with rational agents (Taylor 2004). With the former concern in mind, Jeffrey Rosen (2000) writes that "[P]rivacy protects us from being misdefined and judged out of context in a world of short attention spans, a world in which information can be easily confused with knowledge". For a reference to the debate in the context of the credit market, see the acts of the Congressional debate on the adoption of the Fair Credit Reporting Act, U.S. Senate 1969 and U.S. House 1970.

This paper develops a formal analysis of the effects of data retention provisions on borrowers' behavior and market outcomes. To this end, we study a stylized model of an unsecured credit market for firms (particularly suited to represent family-run and small businesses), where a credit register decides on the amount of information to be disclosed to the lenders. We compare three informational arrangements, labelled "full withholding", "full disclosure" and "limited records". Under full withholding, privacy is enforced and the register discloses no information about the borrowers. Under full disclosure, instead, all the relevant information is made available to the lenders. Finally, with limited records, the credit register implements a policy inspired by the data retention principle, by collecting, storing and sharing only the outcomes of the  $N$  most recent projects undertaken by a borrower.

We show that it is possible to characterize under which sufficient conditions each of the three arrangements leads to a lower (ex ante) default rate than the other two. Interestingly, when adverse selection is severe, full disclosure is always the most preferable arrangement, while full withholding delivers the lowest default rate when both adverse selection and moral hazard are mild. Finally, when the moral hazard in the market is a severe problem, and adverse selection is not a major concern, limited records outperform the two other arrangements, supporting the case for privacy protection legislations. In a situation of high moral hazard, in fact, the two policies of full disclosure and withholding hamper the performance of low-ability borrowers while not eliciting a sufficiently higher repayment rate from the high-ability group. Limited records, instead, provide borrowers of all types with the possibility to alleviate, at least in part, the moral hazard problem by equipping them with a credible device, the records, to build or restore a reputation for being a trustworthy borrower. By moderating the moral hazard problem, thus, the limited records arrangement leads to the lowest default rate.

Further, it can be proved that limited records may also lead to higher aggregate welfare, albeit under fairly restrictive conditions. More precisely, when moral hazard is severe and borrowers are not too different in their abilities, limited records are more efficient because the resulting interest rates succeed in increasing borrowers' aggregate welfare, while keeping lenders' welfare constant. Yet, under the model's assumptions, if these conditions are not met, it is preferable to disclose all information about the borrowers.

The main result of our analysis is that data-protection laws, by deleting key information on borrowers and thus preserving the uncertainty regarding their type, create scope for borrowers to build themselves a reputation for being of high ability. A good reputation has an economic value for any borrower, as it directly translates in lower requested interest rates. Further, to build a reputation, a borrower needs to repay rather than default on his loan, as high ability borrowers are by assumption less likely to default. In turn, as in order to repay borrowers choose higher effort than in the case without the possibility of reputation building, the whole market benefits in terms of higher repayment rates and,



possibly, welfare.<sup>4</sup>

The present work is connected to the literature on reputation and data retention. For what concerns the credit markets, the effects of information sharing on borrowers' behavior were first addressed by Diamond (1989), who documents that such policy leads to positive and persistent reputation effects on the equilibrium interest rate and the quality of the pool of borrowers. However, Diamond does not address the effects of data retention policies. The model we put forward, instead, builds on Vercammen's (1995) and (2002) papers. In his 1995 paper, Vercammen puts forward the first theoretical model that studies the effects on credit market outcomes of information censoring from part of a centralized credit bureau. More precisely, he demonstrates that in a market characterized by both adverse selection and moral hazard, transmitting limited, rather than unlimited, records on borrowers' past behavior gives rise to a sequential equilibrium (in finite time) in which borrowers are concerned about their reputation, and accordingly exert higher levels of effort. Further, a numerical analysis describes how welfare varies by setting different lengths for the records.

Our model moves from Vercammen's with the intent to generalize and extend his results in the following dimensions. First, we apply his framework to an infinitely repeated market setup to study and compare within the same framework the effects on equilibrium outcomes of the three different policies of full disclosure, full withholding and limited records. This exercise highlights how in the three equilibria the informational asymmetries in the market persist to different degrees, which motivates us to investigate which policy minimizes the ex ante default rate and total welfare for given initial levels of moral hazard and adverse selection in the market. In particular, when interpreting the conditions under which limited records lead to a higher welfare than the other policies, we refer to Vercammen's 2002 work where it is shown that it may be welfare improving to allowing for adverse selection in a market already characterized by moral hazard, which is precisely what happens when opting for limited records.

The effects of data retention provisions on market outcomes are also studied by Elul and Gottardi (2008). These authors provide a game theoretic analysis of the effects of forgetting some of the past information on borrowers' past behavior in a credit market where such information is transmitted to lenders by a credit register. Besides a few minor differences, our models focus on the same question and they both assume that the information shared by the register is a binary variable that reports if the previous loans have been repaid in full or defaulted upon.<sup>5</sup> Elul and Gottardi, however, model data retention by assuming that the credit register discloses the whole history of borrowers' past behavior,

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<sup>4</sup>Importantly, the removal of past data also circumvents the problem of disappearing reputation effects (Cripps et al. 2004) by continuously replenishing a borrower's incentives to sustain his reputation (for a game-theoretic discussion of this, see Ekmekci 2010; Liu, Skrzypacz 2011).

<sup>5</sup>Note however that in our model both types of borrowers can fail, while in Elul and Gottardi's model only low-ability borrowers can fail. The authors do however consider the possibility that both types fail in a numerical example.

but with a positive probability it registers a realized default as a repayments instead. The equilibrium they study, then, revolves around the lenders' posterior belief on the type of the borrowers, which they form and update by looking at these modified records. In our model, instead, the information is always correctly registered and fully censored after  $N$  periods. Although the authors argue at length that the two formulations lead to similar results, we believe that ours allows for a more straightforward discussion of the optimal length of the register's memory. In particular, it permits us to identify the trade-offs that have to be solved by the optimal memory design. Our model is also more suitable for accommodating relevant extensions such as whether, and how, borrowers strategically respond to limited records. The two models, however, address complementary aspects of the issue at hand. More precisely, Elul and Gottardi look at the conditions under which a policy of limited records, as opposed to full disclosure, improves the access to credit as well as welfare, while we seek to characterize the conditions under which different informational arrangements, among which limited records, improve on default rates and welfare.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 describes the equilibrium under full disclosure and full withholding, and section 4 characterizes the equilibrium with 1-period limited records. Sections 5 and 6 present some comparative statics for the three informational arrangements, in terms of the ex ante default rate and aggregate welfare respectively. Section 7 explores the case of  $N$ -period limited records with  $N$  larger than one (but finite), and goes through the effects of different choices of memory on borrowers' behavior. Section 8 concludes.

## 2 The model

The present model builds on the work of Vercammen (1995, 2002) and provides a stylized representation of an unsecured credit market for firms, where borrowers may switch between lenders and where the relevant information on a borrower's past behavior is transmitted via a credit register. The market features adverse selection, because borrowers' ability is private information, and moral hazard, because the effort choice is a hidden action taken after the contract has been negotiated. Time is infinite and discrete, and at the beginning of each period a competitive credit market opens, where borrowers and lenders interact as described below.

### 2.1 The borrowers

There is a large number of risk-neutral borrowers, that each period face the exogenous probability  $\zeta$  to leave the market.<sup>6</sup> In that case, a new borrower, identical in everything

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<sup>6</sup>The fact that the probability with which a borrower leaves the market is exogenous means that his "staying in" the market does not have any signalling value.

to the previous one, enters the market. Borrowers discount the future at rate  $\beta < 1$ , which is an effective discount rate that also accounts for the exit probability.

Borrowers are credit-worthy and identical in everything but in a privately known, fixed ability parameter  $\theta$  that can take two values, high,  $H$ , or low,  $L$ , and affects the disutility of effort. More precisely, in supplying effort  $e \in E = [0, \infty)$ , a borrower pays  $c(e, \theta)$ , but high-ability borrowers face a lower marginal cost,  $c'_e(e, H) < c'_e(e, L)$ .<sup>7</sup> Otherwise, the cost function is continuous, twice differentiable and strictly convex,  $c'_e > 0$ ,  $c''_{ee} > 0$ ; further  $c(0, \theta) = 0$  and for both types the function satisfies the Inada conditions, which are needed to ensure the existence of an interior solution in the effort choice problem.<sup>8</sup> High-ability borrowers are present in a fraction  $p \in (0, 1)$  of the population and the remainder is of low ability.

At the beginning of each period, a borrower seeks an indivisible loan and combines it with his own effort in a project that generates non-storable revenues  $Y(e, \omega)$  that are used for consumption. Revenues are also a function of a non-observable positive random shock  $\omega$ , i.i.d. across borrowers and periods, drawn from a distribution  $F$  defined on  $\Omega = [0, 1]$ . The revenue function is continuous, twice differentiable and strictly concave, meaning that revenues increase in effort,  $Y'_e > 0$ , and in the realization of the shock,  $Y'_\omega > 0$ .<sup>9</sup> Further, marginal productivity decreases in effort and increases in the realization of the shock,  $Y''_{ee} < 0$  and  $Y''_{e\omega} > 0$ . Finally, it is assumed that zero effort and the lowest shock realization both yield in zero revenues,  $Y(e, 0) = Y(0, \omega) = 0$ .

A borrower's expected utility is given by expected revenues net of the repayment of the loan and the promised interests,  $1 + r$ , minus the cost of effort. If the realized revenues fall short of the contracted repayment, the lender appropriates whatever has been generated and the borrower receives zero utility. Following Vercammen (1995), we define the "default threshold" for a borrower who exerted effort  $e$  and agreed to repay interest rate  $r$  as the realization of the shock  $\bar{\omega}(e, r)$  which permits him to just honor his debt,  $Y(e, \bar{\omega}(e, r)) = 1 + r$ . Note that higher effort and lower interest rates reduce the probability of default.<sup>10</sup> A type  $\theta$  borrower's life-time utility, then, can be written as,

$$U(e_t, r_t, \theta) = \sum_{t=0}^{\infty} \beta^t \left[ \int_{\bar{\omega}(e_t, r_t)}^1 [Y(e_t, \omega_t) - (1 + \tilde{r}_t)] dF(\omega) - c(e_t, \theta) \right] \quad (1)$$

<sup>7</sup>Assuming that a borrower's type only affects his cost of effort is intended to replicate Vercammen's set-up and continue his analysis, as discussed in the Introduction. However, this assumption also has the nice implication that heterogeneity only affects the current component of the payoff function, making easier to characterize the first order conditions and the equilibrium effort.

<sup>8</sup>Namely,  $\lim_{e \rightarrow 0} c'(e, \theta) = 0$  and  $\lim_{e \rightarrow \infty} c'(e, \theta) = \infty$ .

<sup>9</sup>Strict concavity simplifies the analysis, and reduces to assuming  $Y_{\omega\omega} < 0$  so that the determinant of the Hessian is  $\det H = Y_{ee}Y_{\omega\omega} - Y_{e\omega}^2 < 0$ .

<sup>10</sup>Precisely,  $\bar{\omega}_e(e, r) = -\frac{Y_e}{Y_{e\omega}} < 0$  and  $\bar{\omega}_r(e, r) = \frac{1}{Y_{\omega}} > 0$  for  $e \in [e^*(r), 1]$ , where  $e^*(r)$  is such that  $Y(e^*(r), 1) \equiv 1 + r$ .

where  $\theta = L, H$  and  $\tilde{r}_t$  is the anticipated interest rate. Borrowers are assumed to have perfect foresight, which allows to equal the anticipated with the realized interest rate in equilibrium. For all  $\theta$ , the function  $U$  is continuous, twice differentiable and strictly concave, because the cost function  $c$  is strictly convex. The reservation utility is zero for borrowers of both types.

Under the limited records arrangement, borrowers' expected utility also depends on past performance via the credit report  $c_N$ . More precisely, such report shows a binary variable specifying whether the loans obtained in the  $N$  most recent periods were repaid,  $S$ , or defaulted upon,  $D$ . As the information in  $c_N$  impacts the lenders' choice of interest rate  $r$ , borrowers face a recursive dynamic programming problem, and maximize the following value function,

$$V(c_N, \theta) = \max_e U(e, r(c_N), \theta) \tag{2}$$

$$+ \beta [\Pr(c'_N = c_{NS} \mid e, c_N) V(c_{NS}, \theta) + \Pr(c'_N = c_{ND} \mid e, c_N) V(c_{ND}, \theta)]$$

where the utility function  $U$  is defined in (1), and  $c_{NS}$  ( $c_{ND}$ ) is the credit report obtained from  $c_N$  by deleting the outcome realized  $N$  periods ago, which is the last displayed by the record, and adding a repayment (default) in front of the *most recent* position, and finally  $\Pr(c'_N = c_{NS} \mid e, c_N) = 1 - F(\bar{\omega}(e, r(c_N)))$ , while  $\Pr(c'_N = c_{ND} \mid e, c_N) = F(\bar{\omega}(e, r(c_N)))$ .<sup>11</sup> The shock  $\omega$  does not appear as an argument of the value function because  $V(c_N, \theta)$  gives the value of being in state  $c_N$  in a certain period, *before* the shock is realized. Tomorrow's credit report,  $c'_N$ , is a state under the borrower's control via the borrower's effort choice, which affects the probability of honoring the debt (as described by  $\Pr(c'_N = \cdot \mid e, c_N)$ ). The current-period report,  $c_N$ , is a proper state and the borrower's type  $\theta$  acts as a state chosen by nature and fixed over time.

## 2.2 The credit industry

There is a competitive credit industry consisting of an infinite number of lenders that each period interact with a different borrower. Financial contracts come only in the form of one-period debt contracts for loans of size one with limited liability, fully characterized by the rate of interest  $r$ . There is exclusivity in contracts, namely borrowers can interact with only one lender in each period. Loans have an opportunity cost  $\rho > 0$ , which derives from an outside investment opportunity with fixed return. When the market opens, lenders hold the common prior  $p_0$  on the fraction of high-ability borrowers, which they subsequently update using the information made available by the credit register.

Because of perfect competition, lenders choose  $r$  by setting their expected profit equal

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<sup>11</sup>Note that, thanks to the recursive nature of the problem, it is possible to substitute the time subscripts with the apostrophe notation when denoting a variable in the next period.

to the opportunity cost, so that  $r$  is a function that sends the common belief  $p \in P = [0, 1]$  into  $[0, \bar{Y}]$  where  $\bar{Y} = Y(1, 1)$ . When credit records are available, lenders also take into account  $c_N$ , so that  $r(c_N) : (0, 1) \times C^N \rightarrow [0, \bar{Y}]$  where  $C^N$  is the set of all possible credit records. A costless monitoring technology guarantees that borrowers cannot embezzle the realized revenues.

### 2.3 The credit register

We assume a long-lived credit register, which knows for each borrower whether he has repaid or defaulted upon the loans he obtained up to the current period.<sup>12</sup> The register can adopt three informational policies, full disclosure, full withholding and limited records. By opting for the policy of full disclosure, the register discloses the borrowers' type once and for all. The policy of full withholding, instead, consists of the register shutting down and not disclosing any information. When the register chooses to disclose limited records, instead, it produces for each borrower a credit report  $c_N$  displaying the outcomes of the last  $N$  projects. Since the outcome of a project can be either a success,  $S$ , when the loan is repaid, or a default,  $D$ , we let  $C^N = \{S, D\}^N$  be the set of all possible credit reports of length  $N$ .

These three scenarios let informational asymmetries persist in the market to different degrees. With full disclosure adverse selection is eliminated completely, while with full withholding both moral hazard and adverse selection remain in the market. Finally, limited records allow to soothe, at least in part, adverse selection.

### 2.4 Time-line of the events and equilibrium

Time is discrete and infinite and in each period the events unfold as follows. At the beginning of the period, the credit register discloses the information on borrowers according to the chosen policy. Then lenders offer contracts to the borrowers, specifying the rate of interest asked in exchange for the loan, and simultaneously borrowers choose which contract to accept and which effort level to exert. Next, the outcome of the project is realized and the loan is either repaid or defaulted. The credit register observes the projects' outcomes and updates the records. Then, a new period begins. Figure 1 displays the

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<sup>12</sup>Note that the model's assumptions are such that a borrower borrows each period.

relevant time-line.

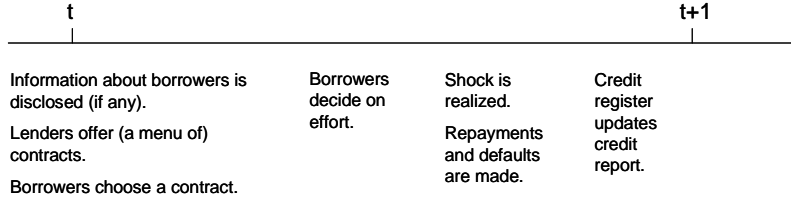


Fig. 1. The time-line of the events.

Under the policies of full disclosure and full withholding, the market equilibrium can be described by the infinite repetition of the (Bayesian) Nash equilibrium, with complete and incomplete information respectively.<sup>13</sup> More precisely this equilibrium, if it exists, will consist of an interest rate such that lenders make zero profit in expectation and an effort choice such that, given the equilibrium interest rate, borrowers maximize their revenues.

When the register opts for a limited records policy, it is appropriate to make use of the following rational-expectations, recursive, competitive equilibrium notion, which we call  $N$ -period equilibrium.

**Definition 1** *A  $N$ -period equilibrium consists of a policy function  $e^* : C^N \times \{H, L\} \rightarrow [0, 1]$ , an interest rate function  $r^* : C^N \rightarrow [0, \bar{Y}]$ , a belief function  $p^* : C^N \rightarrow [0, 1]$  and two distribution functions  $(G_H, G_L)$  over the state space  $C^N$  such that*

- *borrowers behave optimally: given the interest rate function  $r^*$  and the belief function  $p^*$ ,  $e^*$  maximizes the borrower's utility*

- *the credit industry is competitive: for each state  $c_N$ , given that the proportion of high type is  $p_{c_N}^*$  and given that type- $\theta$  borrowers exert  $e_{\theta c_N}^*$ , the interest rate  $r_{c_N}^*$  ensures zero profits*

- *consistency: if the current distributions of high and low types over the state space  $C^N$  is given by  $(G_H, G_L)$  and borrowers use the policy function  $e^*$ , the next period distribution of high and low types is also given by  $(G_H, G_L)$ . The belief function  $p^*$  is consistent with this steady state distribution.*

In all three informational arrangements, we will focus only on the steady-state and dispense of the dynamics outside the stationary path. For the time being, we also disregard the periods of initial history, when  $t < N$ .

The notation used in the paper is summarized in the Appendix.

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<sup>13</sup>Note how in both information arrangements, since borrowers cannot affect future payoffs with current actions, the equilibrium of the repeated game boils down to the repetition of the static equilibrium.

### 3 Full disclosure and full withholding

The full disclosure case corresponds to a situation in which there is no incomplete information in the market. Since the problem is stationary, in every period the borrowers choose effort in order to maximize their period- $t$  utility  $U$ , which, accordingly, is written as a function of  $r$  because  $c_N$  is not available. The maximization of (1) leads to the following first order condition,

$$U'_e(e, r, \theta) = \int_{\bar{\omega}(e, r)}^1 Y'_e(e, \omega) dF(\omega) - c'_e(e, \theta) = 0 \quad (3)$$

which states that borrowers supply effort until marginal return equals marginal cost.<sup>14</sup> The solution to (3) is unique and interior, as the second order conditions are satisfied, and it implicitly defines each type's conditional effort function  $e_\theta = e(r, \theta)$ .<sup>15</sup>

Lenders expect the following return from dealing with a type  $\theta$  borrower,

$$\Pi(e, r, \theta) = \int_0^{\bar{\omega}(e_\theta^*, r)} Y(e_\theta, \omega) dF(\omega) + (1 - F(\bar{\omega}(e_\theta^*, r)))(1 + r) \quad (4)$$

which consists of the probability-weighted sum of what will be appropriated in case of a default (first addend) and the interest rate in the case of a repayment (second addend). Since borrowers' types are known, the lenders are able to charge to the high- and low-ability types different interest rates, called  $r_H$  and  $r_L$  respectively. Then, under full disclosure there exists a unique stationary equilibrium in which high-ability borrowers exert more effort and face a lower interest rate than low-ability borrowers.<sup>16</sup>

**Proposition 1** *The pairs  $(e_H^*, r_H^*)$  and  $(e_L^*, r_L^*)$  are the unique stationary equilibrium of the market under full disclosure. Further,  $e_H^* > e_L^*$  and  $r_H^* < r_L^*$ .*

**Proof.** This and all the following proofs are relegated to the Appendix. ■

Despite the fact that the full disclosure policy eliminates the adverse selection problem from the market, moral hazard persists and the resulting equilibrium effort is a second best with respect to the case of perfect monitoring.<sup>17</sup> By preventing lenders to make the

<sup>14</sup>Note that the discount factor disappears due to the absence of any intertemporal link.

<sup>15</sup>Corner solutions are not maxima thanks to the Inada conditions assumed on the cost function.

<sup>16</sup>Note that the uniqueness of the equilibrium follows also from the assumption that lenders cannot write long-term contracts or condition current contracts on past performance in a relational fashion. Were this not the case, more complex dynamics could arise.

<sup>17</sup>The intuition behind such a distortion goes as follows. With moral hazard, the interest rate cannot be conditioned on the effort choice, meaning that the expected value of marginal effort is lower than in the perfect information case. Additional effort, in fact, benefits the lender, but such benefits are not

interest rate contingent on the level of effort, moral hazard implies that when a default occurs the marginal value of effort to a borrower is zero. While this type of distortion affects both types' choice of effort, it is particularly severe for the low-ability borrowers. Due to their higher marginal cost of effort, in fact, low-ability borrowers are charged a higher interest rate. In turn, this increases the likelihood of defaults and further reduces the marginal value of effort for borrowers in this group.<sup>18</sup>

Under the alternative policy of full withholding, lenders know nothing about the borrowers and the equilibrium interest rate will be a function of the prior only.<sup>19</sup> The borrowers' optimization problem does not change with respect to the previous case and it results in a conditional effort choice  $e_{\theta_0} = e(r_0^*, \theta)$  which depends on the anticipated interest rate  $r_0^*$  and the borrower's own type. The expected return on lending is now given by,

$$p_0 \Pi(e_{H0}^*, r_0^*, H) + (1 - p_0) \Pi(e_{L0}^*, r_0^*, H) = 1 + \rho \quad (5)$$

where the prior  $p_0$  is used as a weight. Then, by solving together the equations (3) and (5), it is possible to find the borrowers' optimal effort choices,  $e_{H0}^*$  and  $e_{L0}^*$ , and the corresponding equilibrium interest rate,  $r_0^*$ . The next proposition demonstrates that under full withholding, there exists a unique stationary equilibrium in which borrowers of both types are charged the same interest rate but high-ability borrowers exert more effort than low-ability borrowers.

**Proposition 2** *The pairs  $(e_{H0}^*, r_0^*)$  and  $(e_{L0}^*, r_0^*)$  are the unique stationary equilibrium of the market under full withholding. Further,  $e_{H0}^* > e_{L0}^*$ .*

The proof is straightforward so it is omitted. Due to the assumption of fixed investment, in equilibrium lenders offer exactly one interest rate. If they were trying to separate borrowers by offering a menu of two contracts, one with a higher interest rate for the low-ability type and viceversa for high-ability borrowers, both types of borrowers would choose the lower interest rate, which would result in negative expected returns for the lenders. As we will discuss more extensively in section 6, under this informational arrangement high-ability types are cross-subsidizing low-ability types. This follows because they are charged an interest rate higher than the one in the complete information case while low-ability types are charged a comparatively lower interest rate.

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internalized by the borrower because the interest rate cannot be made dependent on the effort choice. Besides reducing the expected value of marginal effort, moral hazard also results in higher interest rates, because these are lower for higher levels of effort (for a more detailed discussion, see Brander, Spencer 1989).

<sup>18</sup>As the lower bound of the integral depends on the default threshold, the marginal value of effort decreases in the promised interest rate  $r$ , since  $\bar{\omega}_r' > 0$ .

<sup>19</sup>Borrowers can do nothing to credibly signal their type. In particular, they cannot do so by accepting higher interest rates, because lenders interact with the same borrower for one period only and multi-period contracts are not available.



Before moving on to study the equilibrium with limited records, it is worth pointing out that it is always the case that, comparing the equilibrium effort choices for the same borrower under the two arrangements studied so far, high-ability borrowers exert higher effort under full disclosure than under full withholding, and viceversa for low-ability borrowers.

**Remark 1** *It is always true that  $e_L^* < e_{L0}^*$ ,  $e_{H0}^* < e_H^*$  and  $e_L^* < e_H^*$ .*

The remark above follows from the assumption on marginal costs and the structure of the interest rates.

## 4 One-period limited records

Here we analyze the case of  $N = 1$ , in which, at the beginning of each period, the register discloses to the credit industry credit reports  $c_1$ . These display only the outcome, either a success  $S$  or a default  $D$ , realized by a borrower in the preceding period. This information is used by the industry to update the prior  $p_0$  on the borrower's type. Here we characterize the players' choices on the stationary path, along which their equilibrium strategies depend on the state  $c_1$  and are otherwise history- and time-independent.

A type- $\theta$  borrower's problem can be formulated by the following time-invariant Bellman equation  $V(c_1, \theta)$ ,

$$V(c_1, \theta) = \max_e U(e, r(c_1), \theta) + \beta [\Pr(c'_1 = S | e, c_1)V(S, \theta) + \Pr(c'_1 = D | e, c_1)V(D, \theta)] \quad (6)$$

where  $c'_1 = S, D$  is the credit report obtained from  $c_1$  by replacing the last reported outcome with the current-period project's outcome, weighted by the appropriate probabilities,  $\Pr(c'_1 = S | e, c_1) = 1 - F(\bar{\omega}(e, r_{c_1}))$ . Note that now the interest rate  $r$  depends on the public record,  $r(c_1)$ . The solution to the problem in (6) will be a stationary policy function  $e_{\theta c_1} : C^1 \times \{H, L\} \rightarrow [0, 1]$ .

To prove that the one-period equilibrium defined in section 2 exists and is unique, we demonstrate that the borrowers' value function has a unique fixed point, which is the (unique) stationary policy function  $e_{\theta c_1}^*$  that solves the dynamic program described above. The fact that borrowers' equilibrium strategies are stationary allows us to prove that they induce a unique distribution of borrowers across credit scores. Accordingly, in equilibrium, lenders have a unique menu of posterior beliefs, one for each  $c_1$ , and thus offer a unique and time-invariant menu of interest rates.

By applying Banach fixed point theorem, the next proposition shows that the value function in (6) is a contraction mapping with a unique fixed point which can be found by iteration.

**Proposition 3** *There exists a unique stationary one-period equilibrium, with effort choices  $e_{HS}^*, e_{HD}^*, e_{LS}^*, e_{LD}^*$ , interest rates  $r_S^*, r_D^*$  and beliefs  $p_{c_1}^*$ .*

When  $N = 1$ , borrowers can be in one of two states and in the unique stationary equilibrium they exert one of two levels of effort depending on their current-period credit report,  $e_{\theta c_1} = e(c_1, \theta)$  where  $c_1 = S, D$ . Optimal effort can be found by solving the borrowers' dynamic optimization in (6), the first-order condition of which is given by,

$$Y'_e(e, \omega) - \beta E F' \bar{\omega}'_e(e_{\theta c_1}, r_{c_1}) (V(S, \theta) - V(D, \theta)) = c'_e(e, \theta) \quad (7)$$

According to equation (7), the unique optimal effort policy  $e_{\theta c_1}^*$  is determined by balancing its marginal benefit (LHS) and its marginal cost (RHS). The former is given by a direct increase in revenues today,  $Y'_e(e, \omega)$ , and by a marginal improvement in today's likelihood of repaying the loan,  $F' \bar{\omega}'_e(e_{\theta c_1}, r_{c_1})$ , which in turn leads to an increase in the future expected stream of utility. Without making additional assumptions on the functional forms, it is possible to derive (see Appendix) the following expression for a type- $\theta$  borrower's optimal effort,

$$U'_e(e_{\theta S}^*, r_S^*) = \frac{F' \bar{\omega}'_{e_{\theta S}^*}}{F' \bar{\omega}'_{e_{\theta D}^*}} U'_e(e_{\theta D}^*, r_D^*) \quad (8)$$

which implicitly defines the effort policy  $e_{\theta c_1}^*$  for a type  $\theta$  borrower. Inspecting equation (8) it can be seen that in the one-period equilibrium borrowers exert more effort when a repayment, rather than a default, is going to be erased in the next period. Mathematically, this is demonstrated in the proof of corollary 1 (see Appendix), which shows that the ratio  $\frac{F' \bar{\omega}'_{e_{\theta S}^*}}{F' \bar{\omega}'_{e_{\theta D}^*}}$  is larger than 1. The interpretation for this result relies on the fact that the interest rate charged in equilibrium to a borrower with a repayment in his credit record is lower than if his report shows a default; accordingly, such borrower is better able to internalize the return on his effort, for which reason he chooses to exert a higher level of it. Besides this, according to equation (8), high-ability borrowers exert more effort than low-ability borrowers for any credit report. The next corollary formalizes these two considerations,

**Corollary 1** *In the one-period equilibrium,  $e_{\theta S}^* > e_{\theta D}^*$  for  $\theta = H, L$  and  $e_{Hc}^* > e_{Lc}^*$  for  $c = \{S, D\}$ .*

The fact that high-ability borrowers exert more effort than those of low-ability is also true for the other two informational arrangements discussed in section 3, and it follows directly from the assumption on the cost function. That the optimal effort differs between the two states is, however, clearly a novelty that belongs to the limited records

arrangement. Effort for both types is higher when they are in the repayment state. This result is prompted by the fact that in the repayment state borrowers are faced with a more attractive interest rate, which increases the marginal value of effort by lowering the probability of default.

Lenders, as borrowers, base their equilibrium behavior on the state  $c_1$ . In particular, for each state the credit industry anticipates the borrowers' policy function  $e_{\theta c_1}^*$  described above, and uses it to update the belief  $p_{c_1}^* = p(H | p_0, e_{\theta c_1}^*, r_{c_1}^*)$  that a borrower is of high-ability. The policy  $N = 1$  allows for two credit reports to be observed by the lenders, thus in equilibrium there will be two interest rates,  $r_{c_1}^* = \{r_S^*, r_D^*\}$ , which can be found by solving for each  $c_1 = S, D$  the following zero-profit condition together with the first order condition in (7),

$$p_{c_1}^* \Pi(e_{Hc_1}^*, r_{c_1}^*) + (1 - p_{c_1}^*) \Pi(e_{Lc_1}^*, r_{c_1}^*) = 1 + \rho \quad (9)$$

where  $\Pi(e_{\theta c_1}^*, r_{c_1}^*, \theta)$  is defined in equation (4).

More generally, in any period, the borrowers' strategies and the interest rates induce a probability mass function  $g_\theta$  which gives the probability  $g_\theta(c_N)$  that a borrower of type  $\theta$  has credit report  $c_N$ . Let this function have a cumulative distribution  $G$ . Although individual borrowers will not have the same credit report over time, in the stationary equilibrium the distribution of reports at the population level will remain stable. Let this stationary distribution be  $G^*$ , and let  $g^*$  be the corresponding stationary density. When  $N = 1$ , it is the case that  $g_\theta(D) = 1 - g_\theta(S)$ , so that the steady state probability  $g_\theta^*(c_N)$  can be rewritten (see the Appendix for the complete derivation) as,

$$g_\theta^*(S) = \frac{1 - F(\bar{\omega}(e_{\theta D}^*, r_D^*))}{1 - (F(\bar{\omega}(e_{\theta D}^*, r_D^*)) - F(\bar{\omega}(e_{\theta S}^*, r_S^*)))}$$

which is an explicit expression for the steady state densities. It follows that, according to Bayes' rules, the credit industry's posterior upon seeing report  $c_1 = S, D$  is given by,

$$p_S^* = \frac{p_0 g_H^*(S)}{p_0 g_H^*(S) + (1 - p_0) g_L^*(S)} \quad (10)$$

$$p_D^* = \frac{p_0 g_H^*(D)}{p_0 g_H^*(D) + (1 - p_0) g_L^*(D)}$$

where  $p_0$  is the industry's prior on the borrower being of high ability. The numerator in equation (10) represents the stationary-state probability that the observed outcome  $c_1$  can be ascribed to a high-ability borrower. The denominator can be explained in a similar vein, and acts as the normalization factor.

Importantly it can be demonstrated that, on the stationary path, the distribution

of high-ability borrowers' credit reports first-order stochastically dominates that of low-ability borrowers. This feature guarantees that in equilibrium borrowers of the former type are more likely to have better reports than borrowers of the latter type, that is,

**Proposition 4**  $G_H^*$  first-order stochastically dominates  $G_L^*$ .

The result follows from the assumption on the types' marginal costs, which in equilibrium ensures that high-ability borrowers exert more effort and hence achieve better reports more frequently than low-ability ones.

Proposition 4 is important because it implies that having a better record has a tangible value for the borrower. Lenders, in fact, will correctly believe that a good record is a signal of high ability and will offer better contractual terms to applicants with better records. Anticipating this, borrowers of both types will strive to obtain a good report, in order to benefit from the lower cost of capital associated with it.

In other words, borrowers use their records to build themselves a *reputation* for being of high ability, which will ensure them access to capital at a lower interest rate. It is possible to find the counterpart of the concern for reputation in the term  $V(S, \theta) - V(D, \theta)$  which appears in the borrowers' first order condition (equation (7)). This term encapsulates the borrower's concern for the effect that his current actions will have on his future payoffs. In particular,  $V(S, \theta) - V(D, \theta)$  represents a part of the marginal benefits of today's effort, which, besides leading to a higher utility today, also helps to instill in the lenders the belief to be dealing with a high-ability borrower. Thus, under limited records, and differently from the other two informational arrangements (equation 3), all borrowers share a concern for reputation, which arises endogenously and that can be capitalized in terms of higher future payoff.

## 5 Comparative statics: default rates

This section presents a comparative statics exercise concerning the expected default rate induced by the regimes of full disclosure, full withholding and one-period records. By default rate it is meant the ex ante probability of default under the different equilibrium interest rates and effort choices induced by the arrangement under study.<sup>20</sup> To give more economic meaning to the analysis, we restrict our attention to the case  $p \leq \frac{1}{2}$ , which allows high- and low-ability borrowers to be present at most in equal shares, otherwise it restricts low-ability types to be more frequent.<sup>21</sup> The next propositions pin down the sufficient condition(s) for each informational arrangement to minimize the expected default rate.

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<sup>20</sup>Note that we do not compare the arrangements in terms of the amount that is defaulted upon, but only on the basis of the default events that are induced ex-ante.

<sup>21</sup>Further, when high-ability borrowers outnumber those of low-ability it is difficult to find conditions under which full disclosure is outperformed by the other two arrangements.

We start off with investigating whether full withholding ever leads to a lower default rate than full disclosure. Normalizing the borrowers' population to one, this would be the case when,

$$pF(\bar{\omega}(e_H^*, r_H^*)) + (1 - p)F(\bar{\omega}(e_L^*, r_L^*)) \geq pF(\bar{\omega}(e_{H0}^*, r_0^*)) + (1 - p)F(\bar{\omega}(e_{L0}^*, r_0^*))$$

The LHS represents the proportion of loans that are defaulted upon under the full disclosure arrangement. More precisely, it sums up to the (expected) defaults from part of both high-,  $pF(\bar{\omega}(e_H^*, r_H^*))$ , and low-ability borrowers,  $(1 - p)F(\bar{\omega}(e_L^*, r_L^*))$ . The RHS instead displays the default rate under full withholding, and it has a similar interpretation.

Assuming a uniform distribution for the shock,  $\omega \sim U[0, 1]$ , the inequality above can be rearranged to read,

$$p[\bar{\omega}(e_L^*, r_L^*) - \bar{\omega}(e_{L0}^*, r_0^*)] \geq 1 - p[\bar{\omega}(e_{H0}^*, r_0^*) - \bar{\omega}(e_H^*, r_H^*)] \quad (11)$$

Equation (11) states that the rate of default is lower under full withholding if the improvement in the "default threshold"  $\bar{\omega}(e_L^*, r_L^*) - \bar{\omega}(e_{L0}^*, r_0^*)$  that is brought about for the low-ability group by switching from the full disclosure to the full withholding regime (LHS) is larger than the corresponding decrease for the high-ability group (RHS), weighted by the proportion of high and low-ability borrowers in the market.

Given its importance in proving the following propositions, before proceeding we discuss the default threshold  $\bar{\omega}(e, r)$  in further details. According to the definition given in section 2, the default threshold is the shock which, for any  $e$  and  $r$ , delivers just enough revenues to repay the loan plus the interests, and it is implicitly defined by  $Y(e, \bar{\omega}(e, r)) = 1 + r$ . It follows that  $\bar{\omega}(e, r)$  corresponds to the level curves of  $Y(e, \cdot)$  evaluated at the different  $r$ 's of interest, that is,  $\bar{\Omega}(r) = \{(e, \omega) \in [0, 1] \times [0, 1] : Y(e, \omega) = 1 + r\}$ . Due to the concavity of  $Y(e, \omega)$ , its (upper) level sets are convex sets.<sup>22</sup> These properties can be seen in figure A1 in the Appendix, which draws a few level curves for an arbitrary concave function.

Each level curve (isoquant) is drawn holding fixed a value of  $r$  and it plots all the combinations of effort and the threshold  $\bar{\omega}$  such that the realized revenues are just enough to repay the interest rate  $r$  to which the curve corresponds. Note that for any pair  $(e, r)$  there is only one threshold for which this is true. From the concavity of the revenue function it also follows that the distance between the level sets of fixed increments in the value of  $Y$  (or  $1 + r$ ) increases with  $(e, \omega)$ . As mentioned, these considerations will be important in proving the next proposition, that outlines a sufficient condition for full withholding to induce a lower default rate than full disclosure.

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<sup>22</sup>For the level set to be convex, the following has to hold:  $\bar{\omega}_{ee}''\bar{\omega}_{rr}'' - (\bar{\omega}_{er}'')^2 > 0$ , or  $\bar{\omega}_{ee}''\bar{\omega}_{rr}'' > (\bar{\omega}_{er}'')^2$ . As the first term is positive, we can assume, respecting the condition above that  $\bar{\omega}_{er}''(e, r) < 0$ , namely that the marginal effect that effort has on lowering the default threshold ( $\bar{\omega}_e'(e, r) < 0$  as shown in footnote 8) decreases with the interest rate.

**Proposition 5** *Full withholding induces a lower default rate than full disclosure if  $r_0^* < \frac{1}{2}(r_H^* + r_L^*)$  holds.*

The proof behind proposition 5 is fairly straightforward. We know that full withholding leads to a lower default rate than full disclosure as long as the decrease in defaults from the low-ability group due to the shift from  $r_L^*$  to  $r_0^*$  is higher than the change (increase) in defaults for the high-ability group, accruing due to the shift from  $r_H^*$  to  $r_0^*$ . Then, the condition on  $r_0^*$ , together with the concavity of the revenue function, and thus the convexity of its contour sets, ensures that low-ability borrowers will increase their effort supply by more than what the high-ability borrowers will reduce their own, thus delivering the result.

The finding in proposition 5 paves the way to the question whether or not allowing a one-period memory, when condition  $r_0^* < \frac{1}{2}(r_H^* + r_L^*)$  holds, would reduce the default rates even further. Differently from the case of full withholding, the choice of a one-period memory leads to two equilibrium interest rates,  $r_D^*$  and  $r_S^*$ , which depend on the borrower's credit report. Proposition 6 shows that the policy  $N = 1$  can indeed lead to a lower default rate than both full disclosure and full withholding, but only if an additional condition holds.

**Proposition 6** *If  $r_0^* < \frac{1}{2}(r_H^* + r_L^*)$  holds, limited records induce a lower default rate than both full disclosure and full withholding if  $\bar{\omega}(e_{\theta D}^*, r_D^*) - \bar{\omega}(e_{\theta S}^*, r_S^*)$  is small.*

In words, when the policy of full withholding leads to a lower default rate than full disclosure, the policy of limited records can outperform both alternatives, provided that the additional condition on  $\bar{\omega}(e_{\theta c_1}^*, r_{c_1}^*)$  holds. This second condition concerns the fact that the new equilibrium interest rates  $r_D^*$  and  $r_S^*$ , are such that  $r_S^* < r_0^*$  but  $r_D^* > r_0^*$ . In proposition 5, we have shown that the condition  $r_0^* < \frac{1}{2}(r_H^* + r_L^*)$  is enough to make sure that the shift from  $(r_H^*, r_L^*)$  to  $r_0^*$  leads to an overall lower default rate. The same condition would remain sufficient for proposition 6 to hold if the  $r_S^*$  was the only equilibrium interest rate, since we know that  $r_S^* < r_0^*$ . However, an additional condition is needed to make sure that the shift from  $r_0^*$  to  $r_D^*$  does not undermine the result. To this end, it is enough to require that the difference in default thresholds between the default state  $\bar{\omega}(e_{\theta D}^*, r_D^*)$  and the repayment state  $\bar{\omega}(e_{\theta S}^*, r_S^*)$  is small.

The next natural question, then, is whether limited records can outperform full disclosure when  $r_0$  is *not* closer to  $r_H^*$  than to  $r_L^*$ , in which case we know from proposition 5 that full withholding may not be preferable to full disclosure.

To this end, we can directly compare the default rate under full disclosure with that under limited records. The latter is lower when,

$$p \left[ \bar{\omega}(e_L^*, r_L^*) - \frac{\bar{\omega}(e_{LS}^*, r_S^*)}{1 - (\bar{\omega}(e_{LD}^*, r_D^*) - \bar{\omega}(e_{LS}^*, r_S^*))} \right] \geq 1 - p \left[ \frac{\bar{\omega}(e_{HS}^*, r_S^*)}{1 - (\bar{\omega}(e_{HD}^*, r_D^*) - \bar{\omega}(e_{HS}^*, r_S^*))} - \bar{\omega}(e_H^*, r_H^*) \right] \quad (12)$$

where equation (12) has been derived by assuming a uniform distribution of the shock, and simplified using the equations in (10). Inequality (12) has the same interpretation as inequality (11) and it requires that the absolute difference in the thresholds  $\bar{\omega}(e_{\theta_{c_1}}^*, r_{c_1}^*)$  between complete information and limited records for the low-ability group is larger than that for the high-ability group, accounting for the proportion of high- and low-ability types in the economy. In other words, the fall in the defaults of low-ability borrowers, accruing from a decrease in the interest rates from  $r_L^*$  to  $(r_S^*, r_D^*)$ , should more than compensate the increase in the high-ability defaults, that is brought about by the increase from  $r_H^*$  to  $(r_S^*, r_D^*)$ .

It turns out that this result can be obtained under the same condition that was identified in proposition 6 for limited records to be preferable to full withholding, that is,

**Proposition 7** *If  $r_0^* > \frac{1}{2}(r_H^* + r_L^*)$ , limited records induce a lower default rate than full disclosure if  $\bar{\omega}(e_{\theta_D}^*, r_D^*) - \bar{\omega}(e_{\theta_S}^*, r_S^*)$  is small.*

Again, the proof behind this result is simple. Since by the concavity of the revenue function the decrease in low-ability borrowers' default rate is larger than the corresponding increase in that of high-ability borrowers, to obtain the result it is only needed to make sure that the possibility of ending in a default state does not upset the original result.

As mentioned in the beginning of the section, the results in propositions 5-7 are only sufficient conditions, which moreover impinge on the assumptions of  $p \leq \frac{1}{2}$  and uniform distribution of the shock, so they have to be taken with a grain of salt. However, they do provide some insights regarding how a credit register should set the length of its records in order to minimize the default rate, as displayed in Figure 2, which plots the informational arrangements that minimize the ex ante default rate as a function of the market's possible equilibrium interest rates and default thresholds. In the figure, A is the point that satisfies the condition " $\bar{\omega}(e_{\theta_D}^*, r_D^*) - \bar{\omega}(e_{\theta_S}^*, r_S^*)$  is small". When this is not the case, the policy choice to minimize the ex ante default rate is between the full disclosure and the full withholding arrangements. The latter is preferable if the resulting equilibrium interest rate  $r_0^*$  is below the threshold  $\frac{1}{2}(r_H^* + r_L^*)$ . Alternatively, when the conditions on the thresholds  $\bar{\omega}$  is satisfied (i.e.  $r_S^*$  and  $r_D^*$  and the effort choices  $e_{\theta_S}^*$  and  $e_{\theta_D}^*$  are such that  $\bar{\omega}(e_{\theta_D}^*, r_D^*) - \bar{\omega}(e_{\theta_S}^*, r_S^*)$  is small) limited records is always the informational regime that minimizes the ex ante default rate.

Concerning the interpretation, it is possible to argue that among the three arrangements studied here, full disclosure is always preferable in a market where adverse selection is severe, or, at least, more severe than moral hazard. This is the case when  $r_0^* > \frac{1}{2}(r_H^* + r_L^*)$  and when  $\bar{\omega}(e_{\theta_D}^*, r_D^*) - \bar{\omega}(e_{\theta_S}^*, r_S^*)$  is large. Both statements, in fact, are true when the difference in cost structure between the two types is large and when moral hazard is not a particularly serious concern.<sup>23</sup>

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<sup>23</sup>This last statement can be seen from the fact that since the default thresholds in the two states are far apart monitoring works quite well.

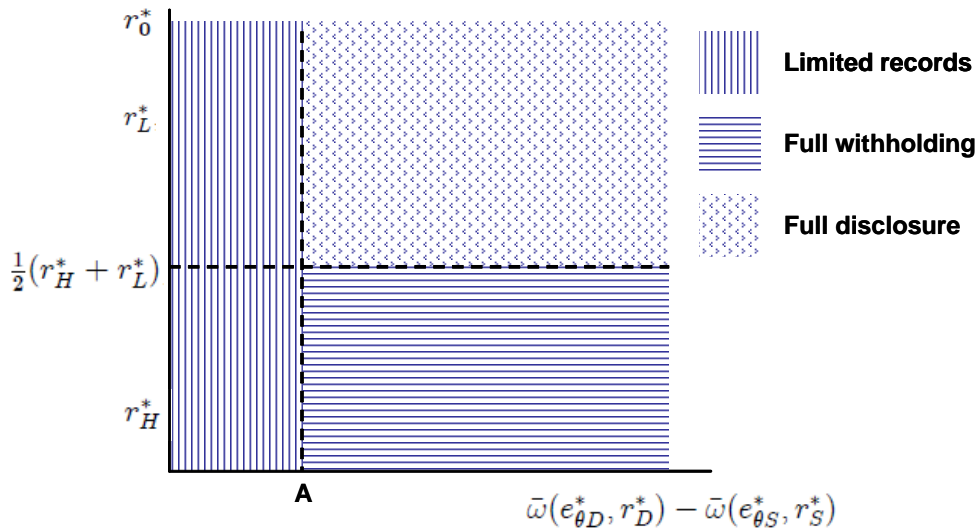


Fig. 2. Comparative statics: default rates

Instead, condition  $r_0^* < \frac{1}{2}(r_H^* + r_L^*)$  is likely to hold in a market where adverse selection is not particularly grave, that is, when the two types face similar costs of effort. When this is the case, the full disclosure arrangement may excessively distort the effort choice of the low-ability borrowers, without succeeding in eliciting from the high-ability borrowers enough effort to compensate this distortion.<sup>24</sup> Full withholding, on the other hand, may successfully lead to an equilibrium interest rate that alleviates the moral hazard problem for the low-ability group without distorting too much the effort supplied by the high-ability borrowers.

However, as moral hazard becomes more severe, limited records becomes the arrangement that delivers the lowest default rate. A situation of high moral hazard, and not too high adverse selection, is captured in this model by the statement that  $\bar{\omega}(e_{\theta D}^*, r_D^*) - \bar{\omega}(e_{\theta S}^*, r_S^*)$  is small. The fact that the two thresholds are close, in fact, means that monitoring is noisy, and even when charged two different interest rates, the correspondingly different effort choices do not lead to different outcomes in a clear-cut way. When this is the case, limited records leads to the lowest default rate, by extracting the highest effort from borrowers of both types, thus counteracting the moral hazard problem in the most efficient way.<sup>25</sup>

<sup>24</sup>Recall that higher interest rates worsen the moral hazard problem more for low-ability than for high-ability borrowers (see discussion in section 3).

<sup>25</sup>The condition on  $\bar{\omega}(e_{\theta c_1}^*, r_{c_1})$  has other implications that are worth mentioning. First, it holds when both types face similar moral hazard problems, and thus implicitly it makes sure that high ability types do not have to cross-subsidize low ability types too much. Secondly, it is possible to argue that when it holds, signalling (learning) is noisy, so it is more effective to enforce an arrangement that alleviates the



Again, the interpretation just given is subject to all the assumptions of the model and has to be subjected to further scrutiny. In particular, it has to be stressed that in this model adverse selection and moral hazard are to a certain extent intertwined (as can be seen in equation (1)), which calls for further care in extrapolating the results to the real world.

## 6 Comparative statics: welfare

As agents are risk-neutral, welfare in this model is computed as the sum of the utilities of all market participants. In a market characterized by adverse selection, full disclosure looks appealing in terms of welfare, as it directly reduces the informational asymmetry. However, such a policy does not deal with the problem of moral hazard, which results in an inefficiently low choice of effort for both types and distorts in particular the low-ability borrowers' effort supply, as discussed in section 3.

As first shown by Vercammen (2002), the distortionary effects of moral hazard under full disclosure could be mitigated by charging a higher interest rate to the high-ability borrowers and simultaneously lowering the interest rate charged to the low-ability group. This manipulation, in fact, can be shown to increase borrowers' *aggregate* welfare while leaving lenders' aggregate returns constant. As discussed in section 3, a cross-subsidization of this kind occurs naturally under full withholding where, thanks to adverse selection and the fixed-investment assumption, the equilibrium interest rate  $r_0^*$  falls in  $r_H^* < r_0^* < r_L^*$ . Thus in line of principle, full withholding *could* lead to a higher aggregate welfare, which may be interpreted as supporting strict privacy protection.

It turns out that under the assumptions of our model it is possible to write a sufficient, nonnecessary, condition under which aggregate welfare is higher with full withholding than with full disclosure. The proof is relegated to the Appendix, while here we only summarize its fundamental steps. We begin with considering the full disclosure interest rates and show that if  $r_H^*$  is increased marginally, while keeping constant the aggregate return for the lenders,  $r_L^*$  necessarily decreases. That is,  $\left. \frac{dr_L^*}{dr_H^*} \right|_{Lenders} < 0$ . Then we demonstrate that such a marginal increase in  $r_H^*$  produces an increase in the borrowers' aggregate welfare, or  $\left. \frac{U^A}{dr_H^*} \right|_{Lenders} > 0$ , provided that a condition, which we call *CS* for "cross-subsidization", holds. The *CS* condition requires that  $\frac{1-F(\bar{\omega}(e_H^*, r_H^*))}{1-F(\bar{\omega}(e_L^*, r_L^*))}$  to be greater than one, but not too large, and it can be shown to hold if the two full disclosure interest rates are close to each other. When this holds, the interest rate  $r_0^*$  that arises under the full withholding equilibrium can be interpreted as the result of cross-subsidization with respect to the full

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moral hazard problem. Finally, due to the high noise, the probability of an unfair transition (the case in which a high ability borrowers defaults due to a high negative shock) to the low state is high, thus a choice of  $N = 1$  may be also preferable as it prevents borrowers from being trapped in states with high interest rates.

disclosure equilibrium rates  $r_H^*$  and  $r_L^*$  and, according to the reasoning above, it leads to a higher aggregate welfare. It follows that,

**Proposition 8** *Full withholding is ex-ante welfare-improving with respect to full disclosure if the CS condition holds.*

By making sure that the full disclosure default thresholds are not too far away from each other, the CS condition allows cross-subsidization to work in a welfare-enhancing way. In particular, it implies that the decrease in effort from part of the high-ability borrowers can be compensated by the increase in effort from the low-ability borrowers, so that lenders' expected returns remain constant. Moreover, it implies that the respective decrease and increase in high- and low-ability borrowers' utility result in an higher aggregate welfare for the group of borrowers. Further, in order to interpret correctly Proposition 8, it is useful to recall that in this model moral hazard and adverse selection are intertwined, because by assumption the moral hazard is less severe for the high-ability borrowers, who face a lower marginal cost of effort. Thus, by requiring the interest rates  $r_H^*$  and  $r_L^*$  to be not too far from each another, the CS condition also implicitly constrains the two types not to be too unlike. Differently stated, the CS condition is satisfied in a credit market where adverse selection is low.

Informally, when the two types are not too unlike, the benefits from screening are not large and under CS they are outweighed by the costs of moral hazard. For this reason, the full withholding interest rate may provide the low-ability borrowers with the right incentives while not distorting "too much" the high type borrowers' effort choices, leading to a higher aggregate welfare.<sup>26</sup> This result is essentially an instance of the theory of second-best, here driven by the assumption of fixed investment-size that, by implying the uniqueness of the equilibrium interest rate under the full withholding arrangement, prevents adverse selection from causing net welfare losses.

In proving proposition 8, it has been shown that when the CS condition holds the borrowers' aggregate utility increases in  $r_H^*$ , and we have argued that it is possible to induce such an increase by withholding all the relevant information from the market, in order to force the credit industry to offer a unique interest rate  $r_0^* \in (r_H^*, r_L^*)$ . However, this is an extreme scenario. The next corollary shows that for the welfare result to hold, any policy that induces an increase in the interest rate faced by the high-ability types and a decrease in that faced by the low-ability types leads to higher welfare than the full disclosure case, provided that the original CS condition is satisfied. In particular, a policy mandating  $N = 1$  leads to an increase in aggregate welfare. Thus, we have,

**Corollary 2** *Under CS, limited records with  $N = 1$  are ex-ante welfare-improving with respect to full disclosure.*

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<sup>26</sup>Note that anyway in the full withholding equilibrium, however, high types will still be exerting a higher level of effort, and will be having a higher probability of success than low types.

The corollary directly follows from proposition 8 and the proof is omitted. If we let the shock be uniformly distributed, it is possible to combine the results in the previous section and the corollary above, to have,

**Corollary 3** *Under CS, limited records with  $N = 1$  is the arrangement that guarantees both the highest welfare ex ante and induces the lowest default rate.*

When *CS* holds in the full disclosure equilibrium, in fact, imposing a policy with limited records improves on the default rate of the low-ability group, while not distorting too much the effort supply, and hence the default rate, of the high-ability borrowers. This follows because  $r_{c_1}^* \in (r_H^*, r_L^*)$ , and  $r_H^*, r_L^*$  are already close to one another by *CS*. Then from the previous section, we know that, by the concavity of the revenue function, the positive effect on the performance of low-ability borrowers overcompensates the negative effect on the performance of the high-ability group.

Corollary 3 means that it is possible to outline sufficient conditions for one-period limited records to be the policy that performs best in both dimensions.<sup>27</sup> Although this result appears to speak in favor of limited records, we argued above that the *CS* condition may not be representative of all credit markets, and that  $N = 1$  is in itself a very simple case. The question now becomes what are the effects of limited records, with  $N > 1$ , on defaults and welfare. In general, limited records elicit higher effort from part of the low-ability borrowers than full disclosure. In fact, if a lender knew the borrower's type, he would not offer him a lower interest rate because of his better records, since the borrower's future incentives would be unrelated to his credit history. Similarly, a borrower would not anymore be able to influence his future payoff by choosing a higher effort, and he would reduce it due to moral hazard. However, limiting memory also entails erasing past information and accordingly it prevents any screening, impeding borrowers from being charged the rate of interest that reflects the true value of their investment. This suggests that limited records deliver a higher welfare if the high-ability types have enough room to signal themselves out, and if simultaneously they provide low-ability types with the correct incentives to deal with the moral hazard problem. The findings in this section support these speculations, and suggest that censoring credit records is also viable alternative for credit registers to reduce the overall default rate.

## 7 Extension: the equilibrium with multi-period records

Having characterized the equilibrium with one-period records, this section studies the general case for finite choices of  $N$ . As argued for the case  $N = 1$ , to prove the existence and uniqueness of the stationary equilibrium defined in section 2.4, it is enough to

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<sup>27</sup>Precisely, under *CS*, limited records score better than full disclosure in both dimensions, by corollary 3 and proposition 6 and 7. With respect to full withholding, instead, limited records certainly score better in terms of the default rate, proposition 6 and 7, while the effects on welfare are ambiguous.

demonstrate that the value function defined in (2) has a unique fixed point. The next proposition makes sure that this is indeed the case, and hence that the borrowers' problem has a unique stationary solution.

**Proposition 9** *There exists a unique stationary  $N$ -period equilibrium.*

To replicate the results obtained for the case  $N = 1$ , the main technical difficulty is posed by the fact that, for an arbitrary  $N$ , borrowers can find themselves in any of  $2^N$  states, corresponding to the equally many possible credit reports that might have been filed. As there is no natural way to order the different combinations of credit reports, the following analysis studies effort choices, lenders' beliefs and interest rates for arbitrarily long pairs of credit reports that are alike in each entry but the less recent one.<sup>28</sup> Reports that only differ in the last outcome, in fact, are associated with *the same* continuation payoff, since the last outcome will be erased by the passage of time. Thanks to this, we can isolate the effect that today's choice has on the agent's future payoffs, which, before the choice is made, are equivalent for different histories. All of the following results accordingly apply only to an environment where borrowers have records of length  $N$ , with  $N > 1$  and finite, which only differ in the oldest displayed outcome.

To simplify notation, we define the following. Given a credit report  $c_N$ , let  $c_{DN}$  be the same report with the most recent outcome erased and a default appended before the *last* outcome displayed in the report. Similarly, let  $c_{SN}$  be computed in the same way but with a repayment appended. This notation allows to specify the possible reports which the borrowers could have had the period before the current one. Precisely, assuming we are in period  $t$ ,

$$\begin{aligned} c_N &= \{ y_{t-N}, y_{t-N+1}, \dots, y_{t-1}, y_t \} \\ c_{DN} &= \{ D, y_{t-N}, y_{t-N+1}, \dots, y_{t-1} \} \\ c_{SN} &= \{ S, y_{t-N}, y_{t-N+1}, \dots, y_{t-1} \} \end{aligned}$$

In such a simplified world, borrowers solve a dynamic program similar to that in (6). Here however, a borrower that finds himself in any state  $c_N$  can move either to  $c_{NS}$  or to  $c_{ND}$ , and the first order condition now becomes,

$$Y'_e(e, \omega) - \beta F'_e \bar{\omega}'_e(e, r_{c_N}) (V(c_{NS}, \theta) - V(c_{ND}, \theta)) = c'_e(e, \theta) \quad (13)$$

According to equation (13), as in the case  $N = 1$ , the difference in future payoff streams associated with repaying or defaulting,  $V(c_{NS}, \theta) - V(c_{ND}, \theta)$ , changes with the state the borrower finds himself in. By extending the analysis in the previous section, it is possible

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<sup>28</sup>It is not clear, for instance, if report  $\{S, S, D\}$  is better than report  $\{S, D, S\}$  or  $\{D, S, S\}$ , in the sense of being more likely to belong to high- rather than a low-ability borrower.

to demonstrate (see proof of Corollary 4) that the optimal policy function solves the following,

$$U'_{e_{c_{SN}}^*}(e_{c_{SN}}^*, r_{c_{SN}}, \theta) = \frac{F'\bar{\omega}'_{e_{c_{SN}}^*}}{F'\bar{\omega}'_{e_{c_{DN}}^*}} U'_{e_{c_{DN}}^*}(e_{c_{DN}}^*, r_{c_{DN}}, \theta) \quad (14)$$

for any two credit reports  $c_{SN}$ ,  $c_{DN}$  that differ only in the outcome realized  $N$  periods ago. Equation (14) confirms the results for the  $N = 1$  case that borrowers exert more effort when a repayment, rather than a default, is going to be erased in the next period. That is, for any finite  $N > 1$ ,

**Corollary 4** *In the stationary  $N$ -period equilibrium,  $e_{\theta c_{SN}}^* > e_{\theta c_{DN}}^*$  for  $\theta = H, L$  and  $e_{Hc_N}^* > e_{Lc_N}^*$  for  $c_N \in C^N$ .*

Lenders' beliefs that a borrower is of high ability can be found by proceeding in the same way as in the one-period case, leading to the expression for the equilibrium posterior belief  $p_{c_N}^*$ ,

$$p_{c_N}^* = \frac{p_0 g_H^*(c_N)}{p_0 g_H^*(c_N) + (1 - p_0) g_L^*(c_N)} \quad (15)$$

which can be explained in the familiar way. Anticipating that borrowers will follow the policy  $e_{\theta c_N}^*$  described above, lenders will use these beliefs to derive the equilibrium (zero profit) interest rate menu  $\langle r_{c_N}^* \rangle$ ,

$$p_{c_N}^* \Pi(e_{Hc_N}^*, r_{c_N}^*) + (1 - p_{c_N}^*) \Pi_L(e_{Lc_N}^*, r_{c_N}^*) = 1 + \rho \quad (16)$$

where  $\Pi(e_{\theta c_N}^*, r_{c_N}^*)$  is defined in equation (4). A policy of  $N > 1$ , then, leads to an exponential increase in the number of interest rates, which, anticipating the results in the next section, are also closer to one another. This consideration will be important when gauging the effects of different choices of  $N$  on the welfare in the market.

The general case of  $N$  periods also preserves the feature that high-ability borrowers are more likely to have better reports than low-ability borrowers, that is,

**Proposition 10**  $G_H^*$  *first-order stochastically dominates*  $G_L^*$ .

The same considerations made in section 4 apply: the stochastic dominance demonstrated in proposition 10 is the key to understanding why borrowers care about having good reports. Given that lenders in equilibrium offer more favorable contractual terms to borrowers with better reports, these will exert more effort in order to build themselves a good reputation and enjoy the monetary benefits associated with it.

Concluding this section, it is worth pointing out yet again that the main novelty brought about by the limited records arrangement is that it allows borrowers to link their current effort choices to future gains, via building, maintaining and possibly restoring a

reputation for being of the high type. For limited records to act as such a "reputational" device both adverse selection and moral hazard are necessary. Thanks to the former distortion, each borrower is in the position to influence his future contractual terms by taking certain actions in the current period, while thanks to the second, the borrower can actually affect the distribution of the returns.

## 7.1 Interest rates and effort choices with limited records

This section investigates the effects of the choice of different lengths  $N$  of the credit reports on lenders' beliefs, equilibrium interest rates and borrowers' behavior. We begin with the lenders' beliefs. As can be seen from equation (15), for any possible credit report  $c_N$ , lenders form a corresponding belief  $p_{c_N}$  that the report belongs to a high-ability borrower. Denote with  $\bar{c}_N$  the ("clean") report of length  $N$  with only repayments, and similarly  $\underline{c}_N$  the ("dirty") report with only defaults. Then, we have that the difference in posterior beliefs assigned to the clean and the dirty record is larger the larger is  $N$ . Further, for any two credit reports that differ only in the last outcome, the difference in posterior beliefs is smaller the larger is  $N$ .

**Lemma 1**  $p_{\bar{c}_N}^* - p_{\underline{c}_N}^*$  increases in  $N$  and  $p_{c_{NS}}^* - p_{c_{ND}}^*$  decreases in  $N$ .

The result above has a direct repercussion on the structure of the interest rates. Any of the  $2^N$  equilibrium beliefs, in fact, directly gives rise to an interest rate,  $r_{c_N}^*$ , defined according to equation (16). The menu of equilibrium interest rates  $\langle r_{c_N}^* \rangle$  falls in the interval  $(r_H^*, r_L^*)$  bounded by the two full-disclosure interest rates. In fact, as  $N \rightarrow \infty$  and  $p_{\bar{c}_N} \rightarrow p$ , the interest rates corresponding to the clean and dirty records approach the boundaries.

The next Remark 2 follows from the previous lemma and equation (16). It shows that the choice of a larger  $N$  induces a larger menu of more fine-grained interest rates, of which those corresponding to the clean and the dirty record are relatively closer to  $(r_H^*, r_L^*)$ . Conversely, a smaller  $N$  results in a smaller and coarser menu of interest rates, the extremes of which are relatively further away from  $(r_H^*, r_L^*)$ . That is,

**Remark 2**  $|r_H^* - r_{\bar{c}_N}^*|$ ,  $|r_L^* - r_{\underline{c}_N}^*|$  and  $|r_{c_{NS}}^* - r_{c_{ND}}^*|$  decrease in  $N$ .

This last remark and the previous lemma have implications for the difference in future payoff streams associated with a repayment and a default,  $V_\theta(c_{NS}) - V_\theta(c_{ND})$ , which appears in the borrowers' first order condition, in equation (13). Lemma 1, in fact, states that the difference in the posterior beliefs for any two credit reports that differ only in the last outcome increases as  $N$  decreases. Remark 3 extends this consideration for the interest rate schedule, that becomes coarser as  $N$  decreases. Then, we have that following an exogenous decrease in  $N$ , the difference between the future payoff stream associated with a repayment, and default, increases, while an increase in  $N$  acts symmetrically.

**Remark 3**  $V(c_{NS}, \theta) - V(c_{ND}, \theta)$  decreases in  $N$ .

Note that the remark above disregards the "size" of the future payoff streams, which depends on the borrower's credit score, and, for the clean (dirty) record becomes higher (lower) as  $N$  grows. Rather, it states that the difference between filing a repayment rather than a default is larger the smaller  $N$  is.

We are now ready to state the main result of this section. The next proposition makes use of the remarks above to characterize how different choices of  $N$  affect the supply of effort from part of the borrowers. It turns out that borrowers' effort choices are most responsive to limited records when  $N = 1$ , that is,

**Proposition 11** For any  $N$ -period equilibrium,  $\frac{e_S^*}{e_D^*} > \frac{e_{SN}^*}{e_{DN}^*}$ .

To understand the proposition, consider a type- $\theta$  borrower's first order condition (13), which governs his policy function  $e_{\theta c_N}^*$ . Optimal effort results from solving the trade-off between current and future benefit of marginal effort (LHS) and its cost (RHS). By looking at the LHS of the first order condition, it is possible to see that the benefit of marginal effort is increasing in the difference between the future payoff streams associated with a repayment and a default respectively. By setting  $N = 1$ , the register contributes to giving maximum value to today's marginal effort. In turn, this implies that the borrowers care a lot for not having a default in their records, and accordingly they exert their highest effort when a repayment is about to be erased. Note that the fact that effort is higher when a repayment rather than a default is about to be erased is true of any choice of  $N$ , because in the former state the interest rate is lower. A one-period memory policy further contributes to this effect, by maximizing the return of effort in terms of future payoff streams.

Note that proposition 11 does not make any reference to the default rate, or the welfare, that different choices of  $N$  achieve, but it just states that reporting only the last-period performance is the policy that is most likely to elicit the highest effort from the borrowers. A one-period limited-record policy, in fact, maximizes the difference in the future payoff stream that follows from realizing a repayment rather than a default, and thus it allows borrowers to internalize most of the value of their effort.

## 8 Conclusions

With this paper, we put forward a model that tries to assess the impact of data removal legislation on credit markets, where it applies to the data on borrowers' past behavior stored by credit registers. We studied a stylized credit market wherein long-lived borrowers repeatedly seek funding from short-lived lenders and where a credit register regulates the disclosure of relevant information. By comparing different privacy protection arrangements, we demonstrate that a policy of limited records may lead to both a lower (ex ante)

default rate and a higher (ex ante) aggregate welfare than either full disclosure or full withholding.

The model presented here is in an extremely stylized form and several extensions come to mind. While we have tried to justify the most stringent assumptions, more realistic modelling choices may improve our understanding of the reputational dynamics brought about by data retention policies. In particular, the next topics on the research agenda should include a thorough study of the design of credit registers' optimal memory and the relaxation of the assumption that borrowers are of a certain type which is fixed over time.



## Tables and figures

The following table summarizes the notation used in the paper,

Table 2. Notation used in the paper

Notation	Definition	
$c(e, \theta)$	cost of effort	$c : E \times \{H, L\} \rightarrow \mathbb{R}_+$
$Y(e, \omega)$	revenues	$Y : E \times \Omega \rightarrow \mathbb{R}_+$
$r(c_N)$	interest rate	$\begin{cases} r : p_0 \rightarrow [0, \bar{Y}] \text{ without limited records} \\ r : C^N \rightarrow [0, \bar{Y}] \text{ with limited records} \end{cases}$
$U(e, c_N, \theta)$	utility	$U : R \times C^N \times \{H, L\} \rightarrow \mathbb{R}_+$
$\bar{\omega}(e, c_N)$	default threshold	$\bar{\omega} : E \times R \rightarrow [0, 1]$
$\Pi(e, c_N, \theta)$	profits	$\Pi : E \times R \times \{H, L\} \rightarrow \mathbb{R}_+$
$e_\theta^*, e_{0\theta}^*, e_{\theta c_N}^*$	equilibrium effort, in full disclosure, full withholding and lim. records	
$r_\theta^*, r_0^*, r_{c_N}^*$	equilibrium int. rate, in full disclosure, full withholding and lim. records	
$p, p_0, p_{c_N}^*$	equilibrium beliefs, in full disclosure, full withholding and lim. records	
$c_N$	credit report of length $N$	

Note that  $r$  is a function of  $c_N$  only when limited records are made available by the credit register, otherwise it is a fundamental of the model. Notationally, first and second derivatives are indicated with both the apostrophes and the subscripts, indicating the variables with respect to which the derivative is taken. Subscript notation alone is used to identify the equilibrium variables under different informational arrangements. This should not create confusion because the latter will always be defined in the text and are marked with an asterisk. In the recursive equilibrium, the apostrophe notation indicates future values.

Figure A1 shows an illustrated example of the default thresholds. It displays the isoquants for an arbitrary concave revenue function  $Y$ , drawn in correspondence of constant increases in  $r$  (precisely, 0.1, 0.2, ..., 1). Note that the distance between the level sets of fixed increments in the value of  $Y$  (or  $1 + r$ ) increases with  $(e, \omega)$ .

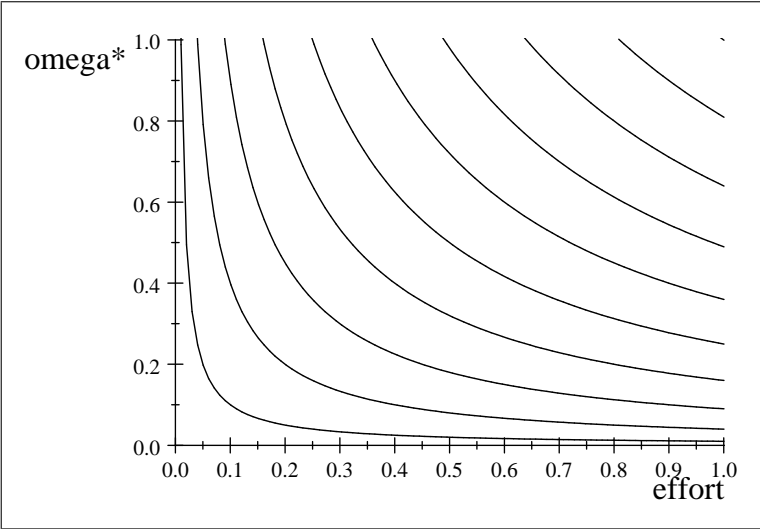


Figure A1. Level curves for  $Y(e, \bar{\omega}) = 1 + r$

## Appendix

Here follows the proofs of the propositions, lemmas and corollaries in the text.

**Proposition 1** *The pairs  $(e_H^*, r_H^*)$  and  $(e_L^*, r_L^*)$  are the unique stationary equilibrium of the market under full disclosure. Further,  $e_H^* > e_L^*$  and  $r_H^* < r_L^*$ .*

**Proof.** Equation (3) describes the borrowers' conditional effort functions  $e_H(r)$  and  $e_L(r)$ . As under full disclosure borrowers' types are known, the equilibrium interest rates  $r_H^*, r_L^*$  are found by solving the following zero-profit conditions, obtained by setting the (expected) profit function  $\Pi(e_\theta, r_\theta)$  (equation (4)) equal to the opportunity cost,

$$\begin{cases} \Pi(e_H, r_H^*) = 1 + \rho \\ \Pi(e_L, r_L^*) = 1 + \rho \end{cases}$$

In equilibrium, borrowers correctly anticipate  $r_H^*, r_L^*$  and exert  $e_H^*$  and  $e_L^*$ . Lenders cannot deviate from offering  $r_\theta^*$ , because charging a lower interest rate would mean incurring in losses, while no borrower would accept a higher interest rate due to perfect competition. At the interest rate  $r_\theta^*$  it is optimal for all borrowers to accept the contract. In particular, this happens because both types' reservation utility is zero, meaning that a type  $\theta$  borrower's participation constraint is,

$$\int_{\bar{\omega}(e_\theta, r)}^1 [Y(e_\theta^*, \omega) - (1 + r)] dF(\omega) - c(e_\theta^*, \theta) \geq 0$$

which by construction is always satisfied, in expectation, by  $r = r_\theta^*$ . The second part of the proposition follows directly from the assumption on marginal costs,  $c'_e(e, H) < c'_e(e, L)$ .

■

**Proposition 3** *There exists a unique stationary one-period equilibrium, with effort choices  $e_{HS}^*, e_{HD}^*, e_{LS}^*, e_{LD}^*$ , interest rates  $r_S^*, r_D^*$  and beliefs  $p_{c_1}^*$ .*

**Proposition 9** *There exists a unique stationary  $N$ -period equilibrium.*

**Proof.** To prove these two propositions, we use Banach's fixed point theorem to show that the value function (2) has a unique fixed point. The proof will be carried out for the general case of  $N$ -period limited records, and it applies similarly to the 1-period records.

The equilibrium in question can be thought as the fixed point of the value function (2), and accordingly the proof shows that equation (2) can be seen as a contraction mapping on a complete metric space which, by Banach's fixed point theorem, has a unique fixed point, hence proving the existence and uniqueness of the equilibrium.

(1) *Showing that the value function is a contraction mapping.*

First we show that equation (2) is a contraction mapping, by proving that Blackwell's (sufficient) conditions apply:

*Blackwell's sufficient conditions:* Let  $T$  be an operator on a metric space  $(S, d_\infty)$  where  $S$  is a space of functions with domain  $X$  and  $d_\infty$  is the sup metric. Then  $T$  is a contraction mapping with modulus  $\beta$  if it satisfies the following

- Monotonicity: For any pair of functions  $(V_1, V_2) \in S$ ,  $V_1(x) \geq V_2(x)$  for all  $x \Rightarrow TV_1(x) \geq TV_2(x)$

- Discounting: For any function  $V(x) \in S(X)$ , positive real numbers  $q > 0$  and  $\beta \in (0, 1)$  it is true that for any  $x$  on which  $V$  is defined  $T(V + c) \leq T(V) + \beta c$ .

We begin with describing the appropriate metric space for the case at hand and showing that it is complete. Secondly, we show that the value function satisfies the conditions of monotonicity and discounting.

(i) *Defining the metric space  $(V, d_\infty)$*

Abusing notation somewhat, the value function (2) can be thought as a functional equation,

$$V(c_N, \theta) = \max_e U(e, r_{c_N}, \theta) + \beta \sum_{c'_N} \Pr(c'_N | e, c_N) V_\theta(c'_N, \theta)$$

$$TV(c_N, \theta) = \max_e U(e, r_{c_N}, \theta) + \beta \sum_{c'_N} \Pr(c'_N | e, c_N) V_\theta(c'_N, \theta)$$

where  $T$  defines a mapping with domain equal to the space of functions  $V$ . We can restrict our attention to the space  $\mathcal{V}$  of functions  $V$  which are continuous and bounded. Given that  $U$  is by assumption a continuous and bounded function, and so is  $V$ ,  $TV$  is also bounded. Furthermore, by the theorem of the Maximum,  $TV$  is continuous as well. Then, the fact that the space of continuous and bounded real functions defined over some set  $X$ ,

$$\mathcal{B} \equiv \{V : X \rightarrow \mathbb{R} | V \text{ is continuous and } m < V(x) < M, \forall x \text{ for some } m, M \in \mathbb{R}\}$$

and equipped with the sup metric

$$d_\infty(V_1, V_2) = \sup_x | V_1(x) - V_2(x) |$$

is a complete metric space, allows us to conclude that also  $(\mathcal{V}, d_\infty)$  is a complete metric space. It follows that  $T$  is an operator that maps the metric space  $(\mathcal{V}, d_\infty)$  to itself.

(ii) *Establishing that the value function is a contraction mapping*

To establish that  $T$  is a contraction, we are left with proving that it satisfies the monotonicity and discounting conditions. Monotonicity can be proven as follows. Take any two  $(V_1, V_2) \in \mathcal{V}$  such that  $V_1(x) \geq V_2(x)$ . Then,

$$\begin{aligned} TV_1(c_N) &= \max_e U(e, c_N, \theta) + \beta \sum_{c'_N} \Pr(c'_N | e, c_N) V_1'(c'_N, \theta) \\ &\geq \max_e U(e, c_N, \theta) + \beta \sum_{c'_N} \Pr(c'_N | e, c_N) V_2'(c'_N, \theta) \\ &= TV_2(c_N, \omega_t) \end{aligned}$$

As for the Discounting condition, take any function  $V(x) \in S(X)$ , positive real numbers  $q > 0$  and  $\beta \in (0, 1)$

$$\begin{aligned} T(V + q) &= \max_e U(e, c_N, \theta) + \beta \sum_{c'_N} \Pr(c'_N | e, c_N) (V_1'(c'_N, \theta) + q) \\ &= \max_e U(e, c_N, \theta) + \beta \sum_{c'_N} \Pr(c'_N | e, c_N) V_1'(c'_N, \theta) + \beta E \sum_{c'_N} \Pr(c'_N | e, c_N) q \\ &= \max_e U(e, c_N, \theta) + \beta \sum_{c'_N} \Pr(c'_N | e, c_N) V_1'(c'_N, \theta) + \beta q \\ &= T(V) + \beta q \end{aligned}$$

where  $\sum_{c'_N} \Pr(c'_N | e, c_N) = 1$  by definition.

(2) Showing that the value function is a contraction mapping with a unique fixed point

Finally, as  $T$  is a contraction mapping on a complete metric space, we know, via Banach's Theorem, that it has a unique fixed point. ■

**Corollary 1** *In the one-period equilibrium,  $e_{\theta S}^* > e_{\theta D}^*$  for  $\theta = H, L$  and  $e_{Hc}^* > e_{Lc}^*$  for  $c = \{S, D\}$ .*

**Proof.** For the case  $N = 1$ , to compute explicitly the optimal effort schedule  $e_{\theta c_1}^*$  in each state  $c_1 = \{S, D\}$ , the following system has to be solved,

$$\begin{cases} V(S, \theta) = \max_e U(e_{\theta S}^*, r_S^*, \theta) + \beta E \sum_{c'_1=S,D} \Pr(c'_1 | e_{\theta S}^*, r_S^*) V(c'_1, \theta) \\ V(D, \theta) = \max_e U(e_{\theta D}^*, r_D^*, \theta) + \beta E \sum_{c'_1=S,D} \Pr(c'_1 | e_{\theta D}^*, r_D^*) V(c'_1, \theta) \\ 0 = \frac{dU(e_{\theta S}^*, r_S^*, \theta)}{de_{\theta S}^*} + \beta E F' \frac{d\bar{\omega}(e_{\theta S}^*, r_S^*)}{de_{\theta S}^*} (V(S, \theta) - V(D, \theta)) \\ 0 = \frac{dU(e_{\theta D}^*, r_D^*, \theta)}{de_{\theta D}^*} + \beta E F' \frac{d\bar{\omega}(e_{\theta D}^*, r_D^*)}{de_{\theta D}^*} (V(S, \theta) - V(D, \theta)) \end{cases}$$

Assuming that borrowers have perfect foresight of the steady state interest rates  $\langle r_S^*, r_D^* \rangle$ , the system above consists of four equations in four unknowns,  $\{V(S, \theta), V(D, \theta), e_S^*, e_D^*\}$ . Rearranging the last two first order conditions, it follows that

$$U'_e(e_{\theta S}^*, r_S^*) = \frac{F' \bar{\omega}'_{e_{\theta S}^*}}{F' \bar{\omega}'_{e_{\theta D}^*}} U'_e(e_{\theta D}^*, r_D^*)$$

The expression  $F' \frac{d\bar{\omega}}{de^*}$  stands for the change in the default likelihood that follows from an increase in effort. In the case  $N = 1$ , since  $\bar{\omega}'_e < 0$  and  $\bar{\omega}''_{er} < 0$  (see footnote 22) and as  $r_S^* < r_D^*$  in equilibrium (as lenders correctly expect a borrower with a repayment to be more likely to have filed a success instead of a default -see proposition 4), it is also true that  $\left| F' \frac{d\bar{\omega}(e_S^*, r_S^*)}{de^*} \right| > \left| F' \frac{d\bar{\omega}(e_D^*, r_D^*)}{de^*} \right|$ , and we can conclude that for both types,  $e_{\theta S}^* > e_{\theta D}^*$ .

We now only have to show that  $e_{Hc}^* < e_{Lc}^*$  for  $c = \{S, D\}$ . Take equation (8) and rewrite,

$$\frac{dU(e_c, r_c, \theta)}{de_c} = \frac{dY(e_c, \omega)}{de_c} - \frac{dc(e_c, \theta)}{de_c}$$

Since  $\frac{dc(e, L)}{de} > \frac{dc(e, H)}{de}$  by assumption, it follows that although the two types of borrowers are charged the same interest rate, it the case that,

$$\frac{dU(e_{Hc}^*, r_c)}{de_{Hc}^*} > \frac{dU(e_{Lc}^*, r_c)}{de_{Lc}^*}$$

meaning that the marginal value of effort is higher for high-ability borrowers rather than for low-ability ones. It follows that  $e_{Hc}^* > e_{Lc}^*$  for  $c = \{S, D\}$ . ■

**Corollary 4** *In the stationary  $N$ -period equilibrium,  $e_{\theta c_{SN}}^* > e_{\theta c_{DN}}^*$  for  $\theta = H, L$  and  $e_{Hc_N}^* > e_{Lc_N}^*$  for  $c_N \in C^N$ .*

**Proof.** By extending the analysis above, it is possible to set up a system of equations to prove the result of corollary 1 for a world where the records are of arbitrary length but only differ in the less recent outcome. To facilitate reading, we substitute momentarily the notation  $e_{c_N}^*$  with  $e^*(c_N)$ , and we use  $c_{NS}, c_{ND}$  to indicate the report  $c_N$  with farthest outcome erased and  $S, D$  added in front of the most recent position,

$$\left\{ \begin{array}{l} V(c_{SN}, \theta) = \max_e U(e^*(c_{SN}), r^*(c_{SN}), \theta) + \beta \sum_{c'_N = c_{NS}, c_{ND}} \Pr(c'_N | e^*(c_{SN}), r^*(c_{SN})) V(c'_N, \theta) \\ V(c_{DN}, \theta) = \max_e U(e^*(c_{DN}), r^*(c_{DN}), \theta) + \beta \sum_{c'_N = c_{NS}, c_{ND}} \Pr(c'_N | e^*(c_{DN}), r^*(c_{DN})) V(c'_N, \theta) \\ 0 = \frac{dU(e^*(c_{SN}), r^*(c_{SN}), \theta)}{de^*(c_{SN})} + \beta F' \frac{d\bar{\omega}(e^*(c_{SN}), r^*(c_{SN}))}{de^*(c_{SN})} (V(c_{NS}, \theta) - V(c_{ND}, \theta)) \\ 0 = \frac{dU(e^*(c_{DN}), r^*(c_{DN}), \theta)}{de^*(c_{DN})} + \beta F' \frac{d\bar{\omega}(e^*(c_{DN}), r^*(c_{DN}))}{de^*(c_{DN})} (V(c_{NS}, \theta) - V(c_{ND}, \theta)) \end{array} \right. \quad (17)$$

To prove the first part of the corollary 4 consider again the system in (17). Rearranging the four equations, it is possible to find out that the policy function solves the following,

$$\frac{dU(e^*(c_{SN}), r^*(c_{SN}), \theta)}{de^*(c_{SN})} = \frac{F' \frac{d\bar{\omega}}{de^*(c_{SN})}}{F' \frac{d\bar{\omega}}{de^*(c_{DN})}} \frac{dU(e^*(c_{DN}), r^*(c_{DN}), \theta)}{de^*(c_{DN})} \quad (18)$$

for any two  $c_{SN}, c_{DN}$ . The expression  $F' \frac{d\bar{\omega}}{de^*}$  stands for the change in the default likelihood that follows from an increase in effort. It is immediate to extend the considerations made for the case  $N = 1$ , i.e. that  $\left| F' \frac{d\bar{\omega}(e^*, r_{SN}^*)}{de^*} \right| > \left| F' \frac{d\bar{\omega}(e^*, r_{DN}^*)}{de^*} \right|$ , and we can conclude that for both types,  $e_{\theta S}^* > e_{\theta D}^*$ .

We now only have to show that  $e_{Hc}^* < e_{Lc}^*$  for arbitrarily long  $c$ . Take equation (13) and rewrite,

$$\frac{dU(e_c, r_c, \theta)}{de_c} = \frac{dY(e_c, \omega)}{de_c} - \frac{dc(e_c, \theta)}{de_c}$$

Since  $\frac{dc(e,L)}{de} > \frac{dc(e,H)}{de}$  by assumption, it follows that although the two types of borrowers are charged the same interest rate, it is the case that,

$$\frac{dU(e_{Hc}^*, r_c)}{de_{Hc}^*} > \frac{dU(e_{Lc}^*, r_c)}{de_{Lc}^*}$$

meaning that the marginal value of effort is higher for high-ability borrowers rather than for low-ability ones. It follows that  $e_{Hc}^* > e_{Lc}^*$ . ■

### Computations of the equilibrium stationary density functions $g_\theta(c_N)$

Here we compute the stationary density function of the distribution of credit reports at the population level (p. 13, section 4), for the cases  $N = 1$ . Let the densities are  $g_{Lt}$  and  $g_{Ht}$  be in period  $t$ , then in the next period it will be the case that,

$$\begin{aligned} g_{Ht+1}(S) &= \Pr(S | e_{HS}^*, H)g_{Ht}(S) + \Pr(S | e_{HD}^*, H)g_{Ht}(D) \\ g_{Lt+1}(S) &= \Pr(S | e_{LS}^*, L)g_{Lt}(S) + \Pr(S | e_{LD}^*, L)g_{Lt}(D) \end{aligned}$$

and similarly for  $g_{Ht+1}(D), g_{Lt+1}(D)$ . In steady state, the densities  $\langle g_\theta \rangle$  solve,

$$\begin{aligned} g_\theta(S) &= \Pr(S | e_{\theta S}^*, \theta)g_\theta(S) + \Pr(S | e_{\theta D}^*, \theta)g_\theta(D) \\ g_\theta(D) &= \Pr(D | e_{\theta S}^*, \theta)g_\theta(S) + \Pr(D | e_{\theta D}^*, \theta)g_\theta(D) \end{aligned}$$

Since  $g_\theta(D) = 1 - g_\theta(S)$ , the steady state distribution can be rewritten as

$$\begin{aligned} g_\theta(S) &= \frac{\Pr(S \mid e_{\theta D}^*, \theta)}{1 + \Pr(S \mid e_{\theta D}^*, \theta) - \Pr(S \mid e_{\theta S}^*, \theta)} \\ &= \frac{1 - F(\bar{\omega}(e_{\theta D}^*, r_D))}{1 - (F(\bar{\omega}(e_{\theta D}^*, r_D)) - F(\bar{\omega}(e_{\theta S}^*, r_S)))} \end{aligned}$$

**Proposition 4**  $G_H^*$  first-order stochastically dominates  $G_L^*$ .

**Proof.** We momentarily suppress the asterisk notation to indicate equilibrium variables, with the understanding that  $\langle e_{\theta c_1} \rangle$  and  $\langle r_{c_1} \rangle$  indicate equilibrium quantities. Take any two  $G_{Ht}, G_{Lt}$ . Next period distribution is given by,

$$\begin{aligned} G_{Ht+1} &= TG_{Ht} \\ G_{Lt+1} &= TG_{Lt} \end{aligned}$$

and

$$\begin{aligned} G_{\theta t+1}(S) &= G_{\theta t}(S)(1 - F(\bar{\omega}(e_{\theta S}, r_S))) + G_{\theta t}(D)(1 - F(\bar{\omega}(e_{\theta D}, r_D))) \\ G_{\theta t+1}(D) &= G_{\theta t}(S)(F(\bar{\omega}(e_{\theta S}, r_S))) + G_{\theta t}(D)(F(\bar{\omega}(e_{\theta D}, r_D))) \end{aligned}$$

We consider in order the cases of (a)  $G_{Lt}(D) = G_{Ht}(D)$ , (b)  $G_{Lt}(D) \geq G_{Ht}(D)$  and (c)  $G_{Lt}(D) \leq G_{Ht}(D)$  and show that in either one it is the case that  $G_{Lt+1}(D) - G_{Ht+1}(D) \geq 0$ , namely that  $G_H$  first-order stochastically dominates  $G_L$

Case (a) Let us assume that  $G_{Lt}(D) = G_{Ht}(D)$  so at time  $t$  high- and low-ability borrowers are distributed in the same way over the credit reports  $S$  and  $D$ . Then in the next period,

$$\begin{aligned} G_{Lt+1}(D) - G_{Ht+1}(D) &= G_{Lt}(S)(F(\bar{\omega}(e_{LS}, r_S))) + G_{Lt}(D)(F(\bar{\omega}(e_{LD}, r_D))) \\ &\quad - G_{Ht}(S)(F(\bar{\omega}(e_{HS}, r_S))) - G_{Ht}(D)(F(\bar{\omega}(e_{HD}, r_D))) \\ &= G_t(S) [F(\bar{\omega}(e_{LS}, r_S)) - F(\bar{\omega}(e_{HS}, r_S))] \\ &\quad + G_t(D) [F(\bar{\omega}(e_{LD}, r_D)) - F(\bar{\omega}(e_{HD}, r_D))] \\ &> 0 \end{aligned}$$

where the last step follows from the assumption on marginal costs that makes sure that,

$$F(\bar{\omega}(e_{Lc_1}, r_{c_1})) \geq F(\bar{\omega}(e_{Hc_1}, r_{c_1}))$$

in any equilibrium, and for any  $c_1 = \{S, D\}$ .



Case (b) Let us now assume that  $G_{Lt}(D) \geq G_{Ht}(D)$  so that at time  $t$  the distribution of  $L$  borrowers is already dominated by that of  $H$  borrowers. Then,

$$\begin{aligned}
G_{Lt+1}(D) - G_{Ht+1}(D) &= G_{Lt}(S)(F(\bar{\omega}(e_{LS}, r_S)) + G_{Lt}(D)(F(\bar{\omega}(e_{LD}, r_D))) \\
&\quad - G_{Ht}(S)(F(\bar{\omega}(e_{HS}, r_S)) - G_{Ht}(D)(F(\bar{\omega}(e_{HD}, r_D))) \\
&= G_{Lt}(D)(F(\bar{\omega}(e_{LD}, r_D)) - G_{Ht}(D)(F(\bar{\omega}(e_{HD}, r_D))) \\
&\quad + (1 - G_{Lt}(D))(F(\bar{\omega}(e_{LS}, r_S)) - (1 - G_{Ht}(D))(F(\bar{\omega}(e_{HS}, r_S))) \\
&= G_{Lt}(D)(F(\bar{\omega}(e_{LD}, r_D)) - G_{Ht}(D)(F(\bar{\omega}(e_{HD}, r_D))) \\
&\quad + (F(\bar{\omega}(e_{LS}, r_S)) - F(\bar{\omega}(e_{HS}, r_S))) \\
&\quad + G_{Ht}(D)(F(\bar{\omega}(e_{HS}, r_S)) - G_{Lt}(D)(F(\bar{\omega}(e_{LS}, r_S))) \\
&\geq 0
\end{aligned}$$

where the last step follows from acknowledging that,

$$\begin{aligned}
&G_{Lt}(D)(F(\bar{\omega}(e_{LD}, r_D)) - G_{Ht}(D)(F(\bar{\omega}(e_{HD}, r_D))) + (F(\bar{\omega}(e_{LS}, r_S)) - F(\bar{\omega}(e_{HS}, r_S))) \\
&\geq G_{Lt}(D)(F(\bar{\omega}(e_{LS}, r_S)) - G_{Ht}(D)(F(\bar{\omega}(e_{HS}, r_S)))
\end{aligned}$$

Case (c) Finally, consider the case  $G_{Lt}(D) \leq G_{Ht}(D)$ . Using the same algebra as in case (b), we have once again that  $G_{Lt+1}(D) - G_{Ht+1}(D) \geq 0$ . ■

**Proposition 10**  $G_H^*$  first-order stochastically dominates  $G_L^*$ .

**Proof.** In proving stochastic dominance for the case  $N = 1$ , we demonstrated that regardless of whether  $G_{Lt}(D) \leq G_{Ht}(D)$  in period  $t$ , the equilibrium strategies are such that in the following period  $t + 1$  it is going to be the case that  $G_{Lt+1}(D) \geq G_{Ht+1}(D)$ . To prove the current proposition it is enough to apply the same computations to each  $c_N \in C^N$  -recalling however that we are considering records of arbitrary length which differ only in the less recent outcome-, with the difference that now, the expressions

$$\begin{aligned}
G_{\theta t+1}(S) &= G_{\theta t}(S)(1 - F(\bar{\omega}(e_{HS}, r_S)) + G_{\theta t}(D)(1 - F(\bar{\omega}(e_{HD}, r_D))) \\
G_{\theta t+1}(D) &= G_{\theta t}(S)(F(\bar{\omega}(e_{HS}, r_S)) + G_{\theta t}(D)(F(\bar{\omega}(e_{HD}, r_D)))
\end{aligned}$$

become (using a slightly different notation),

$$\begin{aligned}
G_{\theta t+1}(c_{NS}) &= G_{\theta t}(c_{SN})(1 - F(\bar{\omega}(e_{\theta}(c_{SN}), r(c_{SN}))) + G_{\theta t}(c_{DN})(1 - F(\bar{\omega}(e_{\theta}(c_{DN}), r(c_{DN})))) \\
G_{\theta t+1}(c_{ND}) &= G_{\theta t}(c_{SN})(F(\bar{\omega}(e_{\theta}(c_{SN}), r(c_{SN}))) + G_{\theta t}(c_{DN})(F(\bar{\omega}(e_{\theta}(c_{DN}), r(c_{DN}))))
\end{aligned}$$

Using the same algebra it is possible to show that regardless of whether  $G_{Lt}(c_N) \leq G_{Ht}(c_N)$  in period  $t$ , in the next period  $G_{Ht+1}$  will first-order stochastically dominate  $G_{Lt+1}$ . ■

**Proposition 5** *Full withholding induces a lower default rate than full disclosure if  $r_0^* < \frac{1}{2}(r_H^* + r_L^*)$  holds.*

**Proof.** Take any two separating full disclosure equilibrium interest rates,  $r_L^*$  and  $r_H^*$ . We know from Proposition 1 that  $r_H^* < r_L^*$  and  $e_H^* > e_L^*$ . Further, by (5) we have that the full withholding equilibrium interest rate lies in  $r_0^* \in (r_L^*, r_H^*)$ . Under the condition  $r_0^* < \frac{1}{2}(r_H^* + r_L^*)$ , we can write,

$$|r_H^* - r_0^*| < |r_L^* - r_0^*|$$

from which we also have that, for any  $e \in [0, 1]$ ,

$$|\bar{\omega}(e, r_0^*) - \bar{\omega}(e, r_H^*)| < |\bar{\omega}(e, r_L^*) - \bar{\omega}(e, r_0^*)|$$

by the concavity of the revenue function. Together, the inequalities above lead to,

$$|e_H^* - e_{0H}^*| < |e_L^* - e_{0L}^*|$$

as  $\bar{\omega}_{er}''(e, r) < 0$ .

Then, using the convexity of  $\bar{\omega}(e_\theta, r_\theta)$  and knowing that  $r_H^* < r_0^* < r_L^*$  implies  $e_H^* < e_{H0}^* < e_{L0}^* < e_L^*$  (where the intermediate step follows from the assumption on marginal cost), we also have that,

$$\bar{\omega}(e_{H0}^*, r_0^*) - \bar{\omega}(e_H^*, r_H^*) < \bar{\omega}(e_L^*, r_L^*) - \bar{\omega}(e_{L0}^*, r_0^*)$$

which proves the proposition whenever  $p \leq \frac{1}{2}$ . ■

**Proposition 6** *If  $r_0^* < \frac{1}{2}(r_H^* + r_L^*)$  holds, limited records induce a lower default rate than both full disclosure and full withholding if  $\bar{\omega}(e_{\theta D}^*, r_D^*) - \bar{\omega}(e_{\theta S}^*, r_S^*)$  is small.*

**Proof.** The proof unfolds in two parts. First, we show that under the conditions above limited records induce a lower default rate than full disclosure; secondly, we compare limited records with full withholding.

(1) *Limited records vs. full disclosure*

By the same computations as for (11), the limited records arrangement with  $N = 1$  leads to a lower default rate than full disclosure when,

$$p \left[ \bar{\omega}(e_L^*, r_L^*) - \frac{\bar{\omega}(e_{LS}^*, r_S^*)}{1 - (\bar{\omega}(e_{LD}^*, r_D^*) - \bar{\omega}(e_{LS}^*, r_S^*))} \right] \geq (1 - p) \left[ \frac{\bar{\omega}(e_{HS}^*, r_S^*)}{1 - (\bar{\omega}(e_{HD}^*, r_D^*) - \bar{\omega}(e_{HS}^*, r_S^*))} - \bar{\omega}(e_H^*, r_H^*) \right]$$

From the previous proposition, we already know that, under  $r_0^* < \frac{1}{2}(r_H^* + r_L^*)$ , the following inequality holds,

$$p [\bar{\omega}(e_L^*, r_L^*) - \bar{\omega}(e_{L0}^*, r_0^*)] \geq (1 - p) [\bar{\omega}(e_{H0}^*, r_0^*) - \bar{\omega}(e_H^*, r_H^*)]$$

so to demonstrate the proposition we only need to show that,

$$\begin{aligned} \bar{\omega}(e_{L0}^*, r_0^*) &> \frac{\bar{\omega}(e_{LS}^*, r_S^*)}{1 - (\bar{\omega}(e_{LD}^*, r_D^*) - \bar{\omega}(e_{LS}^*, r_S^*))} \\ \bar{\omega}(e_{H0}^*, r_0^*) &> \frac{\bar{\omega}(e_{HS}^*, r_S^*)}{1 - (\bar{\omega}(e_{HD}^*, r_D^*) - \bar{\omega}(e_{HS}^*, r_S^*))} \end{aligned} \quad (19)$$

is verified. By (10), we know that,

$$p_D < p_0 < p_S$$

due to the lenders' updating process and the stochastic dominance of  $G_H$  over  $G_L$ . Then it follows that,

$$r_S^* < r_0^* < r_D^*$$

which implies that  $\bar{\omega}(e, r_S^*) < \bar{\omega}(e, r_0^*)$  by the concavity of the revenue function. By comparing the first order equations for the cases of full disclosure (equation (3)) and for limited records  $N = 1$ , (equation (7)), it is possible to see that  $e_{\theta 0}^* < e_{\theta S}^*$ . Then, it follows that,

$$\begin{aligned} \bar{\omega}(e_{L0}^*, r_0^*) &> \bar{\omega}(e_{LS}^*, r_S^*) \\ \bar{\omega}(e_{H0}^*, r_0^*) &> \bar{\omega}(e_{HS}^*, r_S^*) \end{aligned}$$

which together with the assumption that  $\bar{\omega}(e_{\theta D}^*, r_D^*) - \bar{\omega}(e_{\theta R}^*, r_S^*)$  is small proves the inequalities (19) and hence the first part of the proposition.

(2) *Limited records vs. full withholding*

Limited records with  $N = 1$  leads to lower default rate full withholding when the following holds,

$$p \left[ \bar{\omega}(e_{L0}^*, r_0^*) - \frac{\bar{\omega}(e_{LS}^*, r_S^*)}{1 - (\bar{\omega}(e_{LD}^*, r_D^*) - \bar{\omega}(e_{LS}^*, r_S^*))} \right] \geq (1 - p) \left[ \frac{\bar{\omega}(e_{HS}^*, r_S^*)}{1 - (\bar{\omega}(e_{HD}^*, r_D^*) - \bar{\omega}(e_{HS}^*, r_S^*))} - \bar{\omega}(e_{H0}^*, r_0^*) \right]$$

Under  $r_0^* < \frac{1}{2}(r_H^* + r_L^*)$  and  $\bar{\omega}(e_{\theta D}^*, r_D^*) - \bar{\omega}(e_{\theta S}^*, r_S^*)$  small, this which follows directly from (19). ■

**Proposition 7** *If  $r_0^* > \frac{1}{2}(r_H^* + r_L^*)$ , limited records induce a lower default rate than full disclosure if  $\bar{\omega}(e_{\theta D}^*, r_D^*) - \bar{\omega}(e_{\theta S}^*, r_S^*)$  is small.*

**Proof.** Having assumed that  $\bar{\omega}(e_{\theta D}^*, r_D^*) - \bar{\omega}(e_{\theta S}^*, r_S^*)$  is small, to prove inequality (12) we only need to show that  $\bar{\omega}(e_L^*, r_L^*) - \bar{\omega}(e_{LS}^*, r_S^*)$  is greater than  $\bar{\omega}(e_{HS}^*, r_S^*) - \bar{\omega}(e_H^*, r_H^*)$ . When  $r_0^* > \frac{1}{2}(r_H^* + r_L^*)$ , we also have that,  $|r_0^* - r_S^*| > |r_0^* - r_L^*|$  which means that,

$$|\bar{\omega}(e, r_0) - \bar{\omega}(e, r_S)| > |\bar{\omega}(e, r_0) - \bar{\omega}(e, r_L)|$$

Further, the assumption on  $\bar{\omega}(e_{\theta D}^*, r_D^*) - \bar{\omega}(e_{\theta R}^*, r_S^*)$  implies that  $\bar{\omega}(e, r_D) - \bar{\omega}(e, r_S)$  are close, which means that,

$$|\bar{\omega}(e_L^*, r_L^*) - \bar{\omega}(e_{LR}^*, r_S^*)| > |\bar{\omega}(e_{HS}^*, r_S^*) - \bar{\omega}(e_H^*, r_H^*)|$$

Then if  $\bar{\omega}(e_{\theta D}^*, r_D) - \bar{\omega}(e_{\theta S}^*, r_S)$  is small enough, inequality (12) holds. ■

**Proposition 8** *Full withholding is ex-ante welfare-improving with respect to full disclosure if the CS condition holds.*

**Proof.** The proof unfolds in three steps. In order to simplify the notation, we suppress the asterisks that denote equilibrium variables (such as  $e_{\theta}^*, r_{\theta}^*$ ), with the convention that variables are understood in equilibrium unless otherwise stated.

*Step (i) In the full disclosure equilibrium, allowing for a marginal increase in  $r_H^*$  and for a marginal reduction in  $r_L^*$  keeps lenders' aggregate returns constant.*

We begin with considering the full disclosure equilibrium, to show that a marginal increase in  $r_H$  leads to a marginal decrease in  $r_L$ , holding the lenders' aggregate return constant. We suppress the time index, with the understanding that we are referring to the same period  $t$ . The lenders' *aggregate* break even (*BE*) condition reads,

$$p\Pi(\bar{\omega}(e_H, r_H), e_H, r_H) + (1 - p)\Pi(\bar{\omega}(e_L, r_L), e_L, r_L) = 1 + \rho$$

and its total derivative is given by,

$$\begin{aligned} dBE = & p \left( \frac{dBE}{d\Pi} \frac{d\Pi}{d\bar{\omega}} \frac{d\bar{\omega}}{de_H} \frac{de_H}{dr_H} + \frac{dBE}{d\bar{\omega}} \frac{d\bar{\omega}}{de_H} \frac{de_H}{dr_H} + \frac{dBE}{de_H} \frac{de_H}{dr_H} + \frac{dBE}{dr_H} \right) dr_H \\ & + (1 - p) \left( \frac{dBE}{d\Pi_L} \frac{d\Pi_L}{d\bar{\omega}} \frac{d\bar{\omega}}{de_L} \frac{de_L}{dr_L} + \frac{dBE}{d\bar{\omega}} \frac{d\bar{\omega}}{de_L} \frac{de_L}{dr_L} + \frac{dBE}{de_L} \frac{de_L}{dr_L} + \frac{dBE}{dr_L} \right) dr_L \end{aligned} \quad (20)$$

Setting  $dBE = 0$ , in order to hold the return for the lenders constant, the above can be rearranged into,

$$\left. \frac{dr_L}{dr_H} \right|_{Lenders} = - \frac{p}{1 - p} \frac{\left( \frac{dBE}{d\Pi} \frac{d\Pi}{d\bar{\omega}} \frac{d\bar{\omega}}{de_H} \frac{de_H}{dr_H} + \frac{dBE}{d\bar{\omega}} \frac{d\bar{\omega}}{de_H} \frac{de_H}{dr_H} + \frac{dBE}{de_H} \frac{de_H}{dr_H} + \frac{dBE}{dr_H} \right)}{\left( \frac{dBE}{d\Pi_L} \frac{d\Pi_L}{d\bar{\omega}} \frac{d\bar{\omega}}{de_L} \frac{de_L}{dr_L} + \frac{dBE}{d\bar{\omega}} \frac{d\bar{\omega}}{de_L} \frac{de_L}{dr_L} + \frac{dBE}{de_L} \frac{de_L}{dr_L} + \frac{dBE}{dr_L} \right)} \quad (21)$$

which describes how  $r_L$  changes following an increase in  $r_H$ . Next, we proceed with computing separately the terms within brackets. In doing so, we will suppress the type index because each term is referring to the same type. For the time being, we will also suppress the  $p$ , to reintroduce them later. Beginning with  $\frac{dBE}{d\Pi} \frac{d\Pi}{d\bar{\omega}} \frac{d\bar{\omega}}{de} \frac{de}{dr}$ , it turns out that this term disappears in both numerator and denominator, since,

$$\frac{d\Pi}{d\bar{\omega}} = Y(e, \bar{\omega}(e, r))f(\bar{\omega}(e, r)) - (1 + r)f(\bar{\omega}(e, r)) = 0$$

where we made use of the fact that  $dF(\omega) = f(\omega)d(\omega)$ , and because by assumption  $Y(e, \bar{\omega}(e, r)) = (1 + r)$ . We now consider  $\frac{dBE}{d\bar{\omega}} \frac{d\bar{\omega}}{de} \frac{de}{dr}$ . First note that the shock  $\bar{\omega}$  affects the  $BE$  condition only through  $\Pi$ . Then, the derivative can be rewritten as  $\frac{d\Pi}{d\bar{\omega}} \frac{d\bar{\omega}}{de} \frac{de}{dr}$  and by the same reasoning as before, this is equivalent to zero. We now turn our attention now to  $\frac{dBE}{de} \frac{de}{dr}$ . The effort  $e$  of type  $\theta$  affects the  $BE$  only via the expected profit function  $\Pi$ , so that,

$$\begin{aligned} \frac{dBE}{de} &= \frac{d}{de} \left( \int_0^{\bar{\omega}(e(r), r)} Y(e, \omega) dF(\omega) + \int_{\bar{\omega}(e(r), r)}^1 (1 + r) dF(\omega) \right) \\ &= Y(e, \bar{\omega})f(\bar{\omega}) \frac{d\bar{\omega}(e, r)}{de} + \int_0^{\bar{\omega}(e, r)} Y'_e(e, \omega) f(\omega) d\omega - (1 + r)f(\bar{\omega}) \frac{d\bar{\omega}(e, r)}{de} \\ &= \int_0^{\bar{\omega}(e, r)} Y'_e(e, \omega) f(\omega) d\omega \end{aligned}$$

by Leibniz rule. Since  $\frac{de}{dr} = e_r(r)$ , we can write,

$$\frac{dBE}{de} \frac{de}{dr} = \int_0^{\bar{\omega}(e, r)} Y'_e(e, \omega) dF(\omega) e_r(r)$$

Similarly we can compute  $\frac{dBE}{dr}$  as,

$$\frac{dBE}{dr} = \frac{d}{dr} \left( \int_0^{\bar{\omega}(e(r), r)} Y(e, \omega) dF(\omega) + \int_{\bar{\omega}(e(r), r)}^1 (1 + r) dF(\omega) \right)$$

The first derivative can be found by using the chain rule,

$$\begin{aligned} \frac{d}{dr} \int_0^{\bar{\omega}(e(r),r)} Y(e, \omega) f(\omega) d\omega &= Y(e, \bar{\omega}(e(r), r)) f(\bar{\omega}(e(r), r)) \bar{\omega}'_r(e(r), r)) \\ &= (1+r) \bar{\omega}'_r(e(r), r) f(\bar{\omega}(e(r), r)) \end{aligned}$$

The second derivative can instead be computed using the Leibniz' rule,

$$\begin{aligned} \frac{d}{dr} \int_{\bar{\omega}(e(r),r)}^1 (1+r) f(\omega) d\omega &= -f(\bar{\omega}(e(r), r)) \bar{\omega}'_r(e(r), r) (1+r) + \int_{\bar{\omega}(e(r),r)}^1 f(\omega) d\omega \\ &= -(1+r) \bar{\omega}'_r(e(r), r) f(\bar{\omega}(e(r), r)) + 1 - F(\bar{\omega}(e(r), r)) \end{aligned}$$

Thus,

$$\frac{dBE}{dr} = 1 - F(\bar{\omega}^*(e(r), r))$$

Merging the results above we get that for each type  $\theta$ ,

$$\begin{aligned} &\left( \frac{dBE}{d\Pi} \frac{d\Pi}{d\bar{\omega}} \frac{d\bar{\omega}}{de} \frac{de}{dr} + \frac{dBE}{d\bar{\omega}} \frac{d\bar{\omega}}{de} \frac{de}{dr} + \frac{dBE}{de} \frac{de}{dr} + \frac{dBE}{dr} \right) \\ &= \hat{Y} + (1 - F(\bar{\omega}^*(e(r), r))) \end{aligned}$$

where  $\hat{Y} = \int_0^{\bar{\omega}(e,r)} Y'_e(e, \omega) dF(\omega) e_r(r)$ . This expression describes lenders' marginal return with respect to  $r_\theta$  and is positive in equilibrium. Reintroducing the types' indexes, equation (21) now reads,

$$\left. \frac{dr_L}{dr_H} \right|_{Lenders} = - \frac{p_0 \hat{Y}_H + (1 - F(\bar{\omega}(e_H(r_H), r_H)))}{1 - p_0 \hat{Y}_L + (1 - F(\bar{\omega}(e_L(r_L), r_L)))} < 0 \quad (22)$$

demonstrating that in the full disclosure equilibrium a marginally higher value of  $r_H$  necessarily implies a marginal reduction in  $r_L$ .

*Step (ii) Deriving the conditions under which a marginal increase in  $r_H^*$  has a positive marginal impact on the borrowers' aggregate welfare.*

Aggregate borrowers' welfare  $\mathcal{W}_b$  in a group of  $(1-p_0)N$  low-ability borrowers and  $p_0N$  high-ability, risk-neutral borrowers can be written as,

$$\mathcal{W}_b = p_0 NU(\bar{\omega}(e_H, r_H), e_H, r_H) + (1-p_0) NU(\bar{\omega}(e_L, r_L), e_L, r_L)$$

where,

$$U(e, r, \theta) = \max_{e \in [0,1]} \int_{\bar{\omega}(e_\theta, r)}^1 Y(e_\theta, \omega) - (1 + r_\theta) dF(\omega) - c(e, \theta)$$

Writing down the total derivative and rearranging, we get,

$$\begin{aligned} \frac{U^A}{dr_H} &= p_0 N \left( \frac{dU}{d\bar{\omega}} \frac{d\bar{\omega}}{de_H} \frac{de_H}{dr_H} + \frac{dU}{de_H} \frac{de_H}{dr_H} + \frac{dU}{dr_H} \right) \\ &\quad + (1 - p_0) N \left( \frac{dU}{d\bar{\omega}} \frac{d\bar{\omega}}{de_L} \frac{de_L}{dr_L} + \frac{dU}{de_L} \frac{de_L}{dr_L} + \frac{dU}{dr_L} \right) \frac{dr_L}{dr_H} \end{aligned}$$

We can now make use of equation (22) and (3) to rearrange and simplify, yielding,

$$\begin{aligned} \frac{U^A}{dr_H} &= \frac{p_0 N}{\hat{Y}_L + (1 - F(\bar{\omega}(e_L, r_L)))} \tag{23} \\ &\quad \left( \hat{Y}_H [(1 - F(\bar{\omega}(e_L, r_L))) + c'_e(e_L) e_r(r_L)] - \hat{Y}_L [(1 - F(\bar{\omega}(e_H, r_H))) + c'_e(e_H) e_r(r_H)] \right. \\ &\quad \left. + (1 - F(\bar{\omega}(e_L, r_L))) \frac{dU}{de_H} \frac{de_H}{dr_H} - (1 - F(\bar{\omega}(e_H, r_H))) \frac{dU}{de_L} \frac{de_L}{dr_L} \right) \end{aligned}$$

where as before where  $\hat{Y}_\theta$  has been defined above.

When equation (23) is positive, raising the interest rate for the high-ability group (and thus lowering it for the low-ability ones) results in higher aggregate utility for borrowers, while leaving lenders' expected returns constant.

We have already argued in step (i) that the denominator is positive. It remains to establish the conditions under which the term within brackets is non-negative. This is the case when,

$$\begin{aligned} &\hat{Y}_H [(1 - F(\bar{\omega}(e_L, r_L))) + c'_e(e_L) e_r(r_L)] + (1 - F(\bar{\omega}(e_L, r_L))) \frac{dU}{de_H} \frac{de_H}{dr_H} \\ &\geq \hat{Y}_L [(1 - F(\bar{\omega}(e_H, r_H))) + c'_e(e_H) e_r(r_H)] + (1 - F(\bar{\omega}(e_H, r_H))) \frac{dU}{de_L} \frac{de_L}{dr_L} \end{aligned}$$

or, rearranging, when,

$$\begin{aligned} &(1 - F(\omega^*(e(r_L), r_L))) \left( \hat{Y}_H + \frac{dU}{de_H} \frac{de_H}{dr_H} \right) + \hat{Y}_H c'_e(e_L) e_r(r_L) \\ &\geq (1 - F(\omega^*(e(r_H), r_H))) \left( \hat{Y}_L + \frac{dU}{de_L} \frac{de_L}{dr_L} \right) + \hat{Y}_L c'_e(e_H) e_r(r_H) \end{aligned}$$

Consider first  $\hat{Y}_H c'_e(e_L) e_r(r_L)$  and  $\hat{Y}_L c'_e(e_H) e_r(r_H)$ , and rearrange and expand the first term to  $\int_{\bar{\omega}(e_H, r_H)}^1 Y_e(e(r_H), \omega_t) dF(\omega) e_r(r_H) c'_e(e_L) e_r(r_L)$  and proceed in the same way with the second. Since in equilibrium  $r_H < r_L$ ,  $e_t(r_H) > e_t(r_L)$ , and  $c'_e(e, L) > c'_e(e, H)$  for all  $e \in [0, 1]$  it follows that,

$$\hat{Y}_H c'_e(e_L) e_r(r_L) > \hat{Y}_L c'_e(e_H) e_r(r_H)$$

Since we are after a sufficient, not necessary, condition, we can focus on when the following holds,

$$\hat{Y}_H + \frac{dU}{de_H} \frac{de_H}{dr_H} \geq \mathcal{F}(\bar{\omega}_H, \bar{\omega}_L) \left( \hat{Y}_L + \frac{dU}{de_L} \frac{de_L}{dr_L} \right) \quad (24)$$

where  $\mathcal{F}(\bar{\omega}_H, \bar{\omega}_L) = \frac{1-F(\bar{\omega}(e_H, r_H))}{1-F(\bar{\omega}(e_L, r_L))} > 1$ . Since all the terms in the RHS are larger than their counterparts in the LHS, and since  $\frac{1-F(\bar{\omega}(e_H, r_H))}{1-F(\bar{\omega}(e_L, r_L))}$  satisfies the model's hypotheses, a sufficient condition for  $\frac{U^A}{dr_H} > 0$  is given by constraining  $\frac{1-F(\bar{\omega}(e_H, r_H))}{1-F(\bar{\omega}(e_L, r_L))}$  to be greater than one, but not too large.

*Step (iii) Full withholding is ex-ante welfare-improving with respect to full disclosure if the CS condition holds.*

When there is incomplete information in the market, lenders in equilibrium offer a single contract  $r_0^*$ , which falls in  $r_H^* < r_0^* < r_L^*$ . When the full information interest rates  $\langle r_\theta^* \rangle$  satisfy the CS condition, any  $r_0^*$  such that  $(r_0^* - r_H^*)$  and  $(r_L^* - r_0^*)$  is compatible with equation (22), incomplete information increases ex ante aggregate welfare. ■

**Corollary 3** *Under CS, limited records with  $N = 1$  is the arrangement that guarantees both the highest welfare ex ante and induces the lowest default rate.*

**Proof.** The first part of this corollary follows from corollary 2. The second follows from proposition 7, since the condition CS below,

$$\frac{1 - F(\bar{\omega}(e_H^*, r_H^*))}{1 - F(\bar{\omega}(e_L^*, r_L^*))} > 1 \text{ but not too large}$$

may be taken to imply that, under a uniform distribution of the shock,

$$\bar{\omega}(e_L^*, r_L^*) - \bar{\omega}(e_H^*, r_H^*) > 1 \text{ but close}$$

which also implies,

$$\bar{\omega}(e_{\theta D}^*, r_D) - \bar{\omega}(e_{\theta S}^*, r_S) \text{ small}$$

Then, under CS, the result obtains. ■



**Lemma 1**  $p_{\bar{c}_N}^* - p_{c_N}^*$  increases in  $N$  and  $p_{c_{NS}}^* - p_{c_{ND}}^*$  decreases in  $N$ .

**Proof.** The proof is articulated in two parts.

(1) *Demonstrating that  $p_{\bar{c}_N}^* - p_{c_N}^*$  increases in  $N$*

Denote with  $\bar{c}_N$  the "clean" report of length  $N$  with only repayments. Consider the statement,

$$p_{\bar{c}_N} - p_{\bar{c}_{N-1}} > 0 \text{ for any finite } N$$

This is true as long as,

$$\begin{aligned} \frac{g_H(\bar{c}_N)}{g_H(\bar{c}_{N-1})} &> \frac{p_0(g_H(\bar{c}_N) - g_L(\bar{c}_N)) + g_L(\bar{c}_N)}{p_0(g_H(\bar{c}_{N-1}) - g_L(\bar{c}_{N-1})) + g_L(\bar{c}_{N-1})} \\ &= \frac{p_0 g_H(\bar{c}_N) + (1 - p_0) g_L(\bar{c}_N)}{p_0 g_H(\bar{c}_{N-1}) + (1 - p_0) g_L(\bar{c}_{N-1})} \end{aligned}$$

To prove that the inequality above holds for any (finite)  $N$ , consider first that, thanks to the assumption on costs and the stochastic dominance result, we have,

$$g_H(\bar{c}_N) = 1 - \sum_{c_N \neq \bar{c}_N} g_H(c_N) > 1 - \sum_{c_N \neq \bar{c}_N} g_L(c_N) = g_L(\bar{c}_N)$$

Further, for any  $c_N$ ,

$$g(c_{N-1}) = g(c_{SN}) + g(c_{DN})$$

because when the register discloses one period less (moving from  $N$  to  $N - 1$ ), effectively it collapses into the same density,  $g(c_{N-1})$ , the densities of the two credit reports of length  $N$  equal in everything but in the farthest outcome,  $g(c_{SN})$  and  $g(c_{DN})$ . So we have that,

$$g_{\theta}(\bar{c}_N) \leq g_{\theta}(\bar{c}_{N-1})$$

Together, these results demonstrate the initial inequality holds, and that  $p_{\bar{c}_N} - p_{\bar{c}_{N-1}} > 0$  for any finite  $N$  (because  $p_0 \in (0, 1)$ ).

Building on these results, we have that,

$$p_{\bar{c}_1} < p_{\bar{c}_2} < p_{\bar{c}_3} < \dots$$

implying that the longer the history of repayment, the higher the belief in the high ability of the borrower. A similar reasoning can be applied to the report with only defaults,  $\underline{c}_N$ , to conclude that,

$$p_{\underline{c}_1} > p_{\underline{c}_2} > p_{\underline{c}_3} > \dots$$

Thus, putting these two results together,

$$p_{\bar{c}_1} - p_{\underline{c}_1} < p_{\bar{c}_2} - p_{\underline{c}_2} < \dots < p_{\bar{c}_{N-1}} - p_{\underline{c}_{N-1}} < p_{\bar{c}_N} - p_{\underline{c}_N}$$

which demonstrates the first part of the lemma.

(2) *Demonstrating that  $p_{c_{NS}}^* - p_{c_{ND}}^*$  decreases in  $N$ .*

As for the second part of the proof, we have to show that  $p_{c_N^S} - p_{c_N^D}$  decreases in  $N$ . This is equivalent to showing that,

$$\frac{p_0 g_H(c_{NS})}{p_0 g_H(c_{NS}) + (1 - p_0) g_L(c_{NS})} - \frac{p_0 g_H(c_{ND})}{p_0 g_H(c_{ND}) + (1 - p_0) g_L(c_{ND})} \rightarrow 0 \text{ as } N \rightarrow \infty$$

or, that,

$$g_\theta(c_{NS}) - g_\theta(c_{ND}) \rightarrow 0 \text{ as } N \rightarrow \infty$$

Note that,

$$g(c_{N-1}) = g(c_{SN}) + g(c_{DN})$$

for any  $c_N$ . That is, when the register chooses to disclose one period less (moving from  $N$  to  $N - 1$ ) effectively it collapses into the same density,  $g(c_{N-1})$  the densities of the two credit reports of length  $N$  equal in everything but in the farthest outcome,  $g(c_{SN})$  and  $g(c_{DN})$ . Then, we have that,

$$\begin{aligned} g_\theta(S) - g_\theta(D) &= (g_\theta(SS) + g_\theta(DS)) - (g_\theta(DD) + g_\theta(SD)) \\ &= (g_\theta(SS) - g_\theta(SD)) + (g_\theta(DS) - g_\theta(DD)) \end{aligned}$$

Thus,

$$\begin{aligned} g_\theta(S) - g_\theta(D) &\geq g_\theta(SS) - g_\theta(SD) \\ g_\theta(S) - g_\theta(D) &\geq g_\theta(DS) - g_\theta(DD) \end{aligned}$$

Applying recursively this decomposition, it is easy to prove that  $g_\theta(c_{NS}) - g_\theta(c_{ND}) \rightarrow 0$  as  $N \rightarrow \infty$ . ■

**Proposition 11** *For any  $N$ -period equilibrium,  $\frac{e_S^*}{e_D^*} > \frac{e_{c_{SN}}^*}{e_{c_{DN}}^*}$ .*

**Proof.** To prove the proposition consider the results in corollaries 1 and 4. For  $N = 1$ ,

$$\frac{dU(e_S^*, r_S^*, \theta)}{de_S^*} = \frac{F' \frac{d\bar{\omega}(e_S^*, r_S^*)}{de_S^*}}{F' \frac{d\bar{\omega}(e_D^*, r_D^*)}{de_D^*}} \frac{dU(e_D^*, r_D^*, \theta)}{de_D^*}$$

while for any arbitrary choice of  $N$ ,

$$\frac{dU(e_{c_{SN}}^*, r_{c_{SN}}^*, \theta)}{de_{c_{SN}}^*} = \frac{F' \frac{d\bar{\omega}(e_{c_{SN}}^*, r_{c_{SN}}^*)}{de_{c_{SN}}^*}}{F' \frac{d\bar{\omega}(e_{c_{DN}}^*, r_{c_{DN}}^*)}{de_{c_{DN}}^*}} \frac{dU(e_{c_{DN}}^*, r_{c_{DN}}^*, \theta)}{de_{c_{DN}}^*}$$

Since

$$\frac{F' \frac{d\bar{\omega}(e_S^*, r_S^*)}{de_S^*}}{F' \frac{d\bar{\omega}(e_D^*, r_D^*)}{de_D^*}} > \frac{F' \frac{d\bar{\omega}(e_{c_{SN}}^*, r_{c_{SN}}^*)}{de_{c_{SN}}^*}}{F' \frac{d\bar{\omega}(e_{c_{DN}}^*, r_{c_{DN}}^*)}{de_{c_{DN}}^*}}$$

because  $|r_S - r_D| > |r_{c_{SN}} - r_{c_{DN}}|$  and  $\bar{\omega}_e < 0$  and  $\bar{\omega}_{er} < 0$ . it follows also that

$$\frac{\frac{dU(e_S^*, r_S^*, \theta)}{de_S^*}}{\frac{dU(e_D^*, r_D^*, \theta)}{de_D^*}} > \frac{\frac{dU(e_{c_{SN}}^*, r_{c_{SN}}^*, \theta)}{de_{c_{SN}}^*}}{\frac{dU(e_{c_{DN}}^*, r_{c_{DN}}^*, \theta)}{de_{c_{DN}}^*}}$$

meaning that the ratio of the marginal value of effort between a state where a success is about to be erased (numerator) and one where a default is about to be erased (denominator) decreases with  $N$ . Then it follows that,

$$\frac{e_S^*}{e_D^*} > \frac{e_{c_{SN}}^*}{e_{c_{DN}}^*}$$

demonstrating that borrowers are most responsive to limited records when  $N = 1$ . ■

## References

- Akerlof, G. A., 1970. The market for "lemons": quality uncertainty and the market mechanism. *Quarterly Journal of Economics*, 84, 488-500.
- Bottero, M. and G. Spagnolo, 2011. Reputation, privacy and limited records: a survey. Manuscript.
- Brander, J. and B. Spencer, 1989. Moral hazard and limited liability: implications for the theory of the firm. *International Economics Review*, 30, 833-849.
- Cripps, M. W., Mailath, G. J. and L. Samuelson, 2004. Imperfect monitoring and impermanent reputation. *Econometrica*, 72, 407-432.
- Diamond, D. W., 1989. Reputation acquisition in debt markets. *Journal of Political Economy*, 97, 828-862.
- Elul, R. and P. Gottardi, 2008. Bankruptcy: is it enough to forgive or must we also forget? Federal Reserve Bank of Philadelphia, Working Paper, No. 11-14.
- Ekmekci, M., 2010. Sustainable reputations with rating systems. *Journal of Economic Theory*, 146, 479-503.
- Jappelli, T. and M. Pagano, 2002. Information Sharing, Lending and Defaults: Cross-Country Evidence. *Journal of Banking and Finance*, 26, 2017-2045.
- Liu, Q. and A. Skrzypacz, 2011. Limited records and reputation. Stanford University Graduate School of Business Research Paper, No. 2030.
- Posner, R. A., 1983. Privacy and related interests. In: *The economics of Justice*. Harvard University Press.
- Rosen, J., 2000. *The unwanted gaze: The destruction of privacy in America*. Random House: New York.
- Sartor, G., 2006. Privacy, reputation, and trust: some implications for data protection. Trust Management, Lecture Notes in Computer Science, 2006, 354-366.
- Taylor, C. , 2004. Consumer privacy and the market for customer information. *RAND*, 35, 631-650.
- United States House of Representatives. Committee on Banking and Currency. Subcommittee on Consumer Affairs (1970). Fair Credit Reporting: Hearings before the Subcommittee on Consumer Affairs of the Committee on Banking and Currency of the House of

Representatives. Ninety-First Congress, Second Session on H.R. 16340, March 17, 19, 20, 23 and 24; April 8 and 8, 1970.

United States Senate. Committee on Banking and Currency. Subcommittee on Financial Institutions (1969). Fair Credit Reporting: Hearings before the Subcommittee on Financial Institutions of the Committee on Banking and Currency of the United States Senate. Ninety-First Congress, First Session on S. 823, May 19, 20, 21, 22 and 23, 1969.

Vercammen, J., 1995. Credit register policy and sustainable reputation effects in credit markets. *Economica*, 62, 461-478.

\_\_\_\_\_ 2002. Welfare improving adverse selection in the credit markets. *International Economic Review*, 43, 1017-1033.

Vogel, R. C. , Burkett, P. , 1986. Deposit mobilization in developing countries: the importance of reciprocity in lending. *Journal of Developing Areas*, 20, 425-438.

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- A. BRANDOLINI, *On applying synthetic indices of multidimensional well-being: health and income inequalities in France, Germany, Italy, and the United Kingdom*, in R. Gotoh and P. Dumouchel (eds.), Against Injustice. The New Economics of Amartya Sen, Cambridge, Cambridge University Press, **TD No. 668 (April 2008)**.
- G. FERRERO and A. NOBILI, *Futures contract rates as monetary policy forecasts*, International Journal of Central Banking, v. 5, 2, pp. 109-145, **TD No. 681 (June 2008)**.
- P. CASADIO, M. LO CONTE and A. NERI, *Balancing work and family in Italy: the new mothers' employment decisions around childbearing*, in T. Addabbo and G. Solinas (eds.), Non-Standard Employment and Quality of Work, Physica-Verlag. A Springer Company, **TD No. 684 (August 2008)**.
- L. ARCIERO, C. BIANCOTTI, L. D'AURIZIO and C. IMPENNA, *Exploring agent-based methods for the analysis of payment systems: A crisis model for StarLogo TNG*, Journal of Artificial Societies and Social Simulation, v. 12, 1, **TD No. 686 (August 2008)**.
- A. CALZA and A. ZAGHINI, *Nonlinearities in the dynamics of the euro area demand for M1*, Macroeconomic Dynamics, v. 13, 1, pp. 1-19, **TD No. 690 (September 2008)**.
- L. FRANCESCO and A. SECCHI, *Technological change and the households' demand for currency*, Journal of Monetary Economics, v. 56, 2, pp. 222-230, **TD No. 697 (December 2008)**.

- G. ASCARI and T. ROPELE, *Trend inflation, taylor principle, and indeterminacy*, Journal of Money, Credit and Banking, v. 41, 8, pp. 1557-1584, **TD No. 708 (May 2007)**.
- S. COLAROSSO and A. ZAGHINI, *Gradualism, transparency and the improved operational framework: a look at overnight volatility transmission*, International Finance, v. 12, 2, pp. 151-170, **TD No. 710 (May 2009)**.
- M. BUGAMELLI, F. SCHIVARDI and R. ZIZZA, *The euro and firm restructuring*, in A. Alesina e F. Giavazzi (eds): Europe and the Euro, Chicago, University of Chicago Press, **TD No. 716 (June 2009)**.
- B. HALL, F. LOTTI and J. MAIRESSE, *Innovation and productivity in SMEs: empirical evidence for Italy*, Small Business Economics, v. 33, 1, pp. 13-33, **TD No. 718 (June 2009)**.

2010

- A. PRATI and M. SBRACIA, *Uncertainty and currency crises: evidence from survey data*, Journal of Monetary Economics, v. 57, 6, pp. 668-681, **TD No. 446 (July 2002)**.
- L. MONTEFORTE and S. SIVIERO, *The Economic Consequences of Euro Area Modelling Shortcuts*, Applied Economics, v. 42, 19-21, pp. 2399-2415, **TD No. 458 (December 2002)**.
- S. MAGRI, *Debt maturity choice of nonpublic Italian firms*, Journal of Money, Credit, and Banking, v.42, 2-3, pp. 443-463, **TD No. 574 (January 2006)**.
- G. DE BLASIO and G. NUZZO, *Historical traditions of civicness and local economic development*, Journal of Regional Science, v. 50, 4, pp. 833-857, **TD No. 591 (May 2006)**.
- E. IOSSA and G. PALUMBO, *Over-optimism and lender liability in the consumer credit market*, Oxford Economic Papers, v. 62, 2, pp. 374-394, **TD No. 598 (September 2006)**.
- S. NERI and A. NOBILI, *The transmission of US monetary policy to the euro area*, International Finance, v. 13, 1, pp. 55-78, **TD No. 606 (December 2006)**.
- F. ALTISSIMO, R. CRISTADORO, M. FORNI, M. LIPPI and G. VERONESE, *New Eurocoin: Tracking Economic Growth in Real Time*, Review of Economics and Statistics, v. 92, 4, pp. 1024-1034, **TD No. 631 (June 2007)**.
- U. ALBERTAZZI and L. GAMBACORTA, *Bank profitability and taxation*, Journal of Banking and Finance, v. 34, 11, pp. 2801-2810, **TD No. 649 (November 2007)**.
- M. IACOVIELLO and S. NERI, *Housing market spillovers: evidence from an estimated DSGE model*, American Economic Journal: Macroeconomics, v. 2, 2, pp. 125-164, **TD No. 659 (January 2008)**.
- F. BALASSONE, F. MAURA and S. ZOTTERI, *Cyclical asymmetry in fiscal variables in the EU*, Empirica, **TD No. 671**, v. 37, 4, pp. 381-402 (**June 2008**).
- F. D'AMURI, O. GIANMARCO I.P. and P. GIOVANNI, *The labor market impact of immigration on the western german labor market in the 1990s*, European Economic Review, v. 54, 4, pp. 550-570, **TD No. 687 (August 2008)**.
- A. ACCETTURO, *Agglomeration and growth: the effects of commuting costs*, Papers in Regional Science, v. 89, 1, pp. 173-190, **TD No. 688 (September 2008)**.
- S. NOBILI and G. PALAZZO, *Explaining and forecasting bond risk premiums*, Financial Analysts Journal, v. 66, 4, pp. 67-82, **TD No. 689 (September 2008)**.
- A. B. ATKINSON and A. BRANDOLINI, *On analysing the world distribution of income*, World Bank Economic Review, v. 24, 1, pp. 1-37, **TD No. 701 (January 2009)**.
- R. CAPPARIELLO and R. ZIZZA, *Dropping the Books and Working Off the Books*, Labour, v. 24, 2, pp. 139-162, **TD No. 702 (January 2009)**.
- C. NICOLETTI and C. RONDINELLI, *The (mis)specification of discrete duration models with unobserved heterogeneity: a Monte Carlo study*, Journal of Econometrics, v. 159, 1, pp. 1-13, **TD No. 705 (March 2009)**.
- L. FORNI, A. GERALI and M. PISANI, *Macroeconomic effects of greater competition in the service sector: the case of Italy*, Macroeconomic Dynamics, v. 14, 5, pp. 677-708, **TD No. 706 (March 2009)**.
- V. DI GIACINTO, G. MICUCCI and P. MONTANARO, *Dynamic macroeconomic effects of public capital: evidence from regional Italian data*, Giornale degli economisti e annali di economia, v. 69, 1, pp. 29-66, **TD No. 733 (November 2009)**.
- F. COLUMBA, L. GAMBACORTA and P. E. MISTRULLI, *Mutual Guarantee institutions and small business finance*, Journal of Financial Stability, v. 6, 1, pp. 45-54, **TD No. 735 (November 2009)**.
- A. GERALI, S. NERI, L. SESSA and F. M. SIGNORETTI, *Credit and banking in a DSGE model of the Euro Area*, Journal of Money, Credit and Banking, v. 42, 6, pp. 107-141, **TD No. 740 (January 2010)**.
- M. AFFINITO and E. TAGLIAFERRI, *Why do (or did?) banks securitize their loans? Evidence from Italy*, Journal



- of Financial Stability, v. 6, 4, pp. 189-202, **TD No. 741 (January 2010)**.
- S. FEDERICO, *Outsourcing versus integration at home or abroad and firm heterogeneity*, Empirica, v. 37, 1, pp. 47-63, **TD No. 742 (February 2010)**.
- V. DI GIACINTO, *On vector autoregressive modeling in space and time*, Journal of Geographical Systems, v. 12, 2, pp. 125-154, **TD No. 746 (February 2010)**.
- L. FORNI, A. GERALI and M. PISANI, *The macroeconomics of fiscal consolidations in euro area countries*, Journal of Economic Dynamics and Control, v. 34, 9, pp. 1791-1812, **TD No. 747 (March 2010)**.
- S. MOCETTI and C. PORELLO, *How does immigration affect native internal mobility? new evidence from Italy*, Regional Science and Urban Economics, v. 40, 6, pp. 427-439, **TD No. 748 (March 2010)**.
- A. DI CESARE and G. GUAZZAROTTI, *An analysis of the determinants of credit default swap spread changes before and during the subprime financial turmoil*, Journal of Current Issues in Finance, Business and Economics, v. 3, 4, pp., **TD No. 749 (March 2010)**.
- P. CIPOLLONE, P. MONTANARO and P. SESTITO, *Value-added measures in Italian high schools: problems and findings*, Giornale degli economisti e annali di economia, v. 69, 2, pp. 81-114, **TD No. 754 (March 2010)**.
- A. BRANDOLINI, S. MAGRI and T. M. SMEEDING, *Asset-based measurement of poverty*, Journal of Policy Analysis and Management, v. 29, 2, pp. 267-284, **TD No. 755 (March 2010)**.
- G. CAPPELLETTI, *A Note on rationalizability and restrictions on beliefs*, The B.E. Journal of Theoretical Economics, v. 10, 1, pp. 1-11, **TD No. 757 (April 2010)**.
- S. DI ADDARIO and D. VURI, *Entrepreneurship and market size. the case of young college graduates in Italy*, Labour Economics, v. 17, 5, pp. 848-858, **TD No. 775 (September 2010)**.
- A. CALZA and A. ZAGHINI, *Sectoral money demand and the great disinflation in the US*, Journal of Money, Credit, and Banking, v. 42, 8, pp. 1663-1678, **TD No. 785 (January 2011)**.

2011

- S. DI ADDARIO, *Job search in thick markets*, Journal of Urban Economics, v. 69, 3, pp. 303-318, **TD No. 605 (December 2006)**.
- F. SCHIVARDI and E. VIVIANO, *Entry barriers in retail trade*, Economic Journal, v. 121, 551, pp. 145-170, **TD No. 616 (February 2007)**.
- G. FERRERO, A. NOBILI and P. PASSIGLIA, *Assessing excess liquidity in the Euro Area: the role of sectoral distribution of money*, Applied Economics, v. 43, 23, pp. 3213-3230, **TD No. 627 (April 2007)**.
- P. E. MISTRULLI, *Assessing financial contagion in the interbank market: maximum entropy versus observed interbank lending patterns*, Journal of Banking & Finance, v. 35, 5, pp. 1114-1127, **TD No. 641 (September 2007)**.
- E. CIAPANNA, *Directed matching with endogenous markov probability: clients or competitors?*, The RAND Journal of Economics, v. 42, 1, pp. 92-120, **TD No. 665 (April 2008)**.
- M. BUGAMELLI and F. PATERNÒ, *Output growth volatility and remittances*, Economica, v. 78, 311, pp. 480-500, **TD No. 673 (June 2008)**.
- V. DI GIACINTO e M. PAGNINI, *Local and global agglomeration patterns: two econometrics-based indicators*, Regional Science and Urban Economics, v. 41, 3, pp. 266-280, **TD No. 674 (June 2008)**.
- G. BARONE and F. CINGANO, *Service regulation and growth: evidence from OECD countries*, Economic Journal, v. 121, 555, pp. 931-957, **TD No. 675 (June 2008)**.
- R. GIORDANO and P. TOMMASINO, *What determines debt intolerance? The role of political and monetary institutions*, European Journal of Political Economy, v. 27, 3, pp. 471-484, **TD No. 700 (January 2009)**.
- P. ANGELINI, A. NOBILI e C. PICILLO, *The interbank market after August 2007: What has changed, and why?*, Journal of Money, Credit and Banking, v. 43, 5, pp. 923-958, **TD No. 731 (October 2009)**.
- L. FORNI, A. GERALI and M. PISANI, *The Macroeconomics of Fiscal Consolidation in a Monetary Union: the Case of Italy*, in Luigi Paganetto (ed.), Recovery after the crisis. Perspectives and policies, VDM Verlag Dr. Muller, **TD No. 747 (March 2010)**.
- A. DI CESARE and G. GUAZZAROTTI, *An analysis of the determinants of credit default swap changes before and during the subprime financial turmoil*, in Barbara L. Campos and Janet P. Wilkins (eds.), The Financial Crisis: Issues in Business, Finance and Global Economics, New York, Nova Science Publishers, Inc., **TD No. 749 (March 2010)**.
- A. LEVY and A. ZAGHINI, *The pricing of government guaranteed bank bonds*, Banks and Bank Systems, v. 6, 3, pp. 16-24, **TD No. 753 (March 2010)**.

- G. GRANDE and I. VISCO, *A public guarantee of a minimum return to defined contribution pension scheme members*, *The Journal of Risk*, v. 13, 3, pp. 3-43, **TD No. 762 (June 2010)**.
- P. DEL GIOVANE, G. ERAMO and A. NOBILI, *Disentangling demand and supply in credit developments: a survey-based analysis for Italy*, *Journal of Banking and Finance*, v. 35, 10, pp. 2719-2732, **TD No. 764 (June 2010)**.
- G. BARONE and S. MOCETTI, *With a little help from abroad: the effect of low-skilled immigration on the female labour supply*, *Labour Economics*, v. 18, 5, pp. 664-675, **TD No. 766 (July 2010)**.
- A. FELETTIGH and S. FEDERICO, *Measuring the price elasticity of import demand in the destination markets of italian exports*, *Economia e Politica Industriale*, v. 38, 1, pp. 127-162, **TD No. 776 (October 2010)**.
- S. MAGRI and R. PICO, *The rise of risk-based pricing of mortgage interest rates in Italy*, *Journal of Banking and Finance*, v. 35, 5, pp. 1277-1290, **TD No. 778 (October 2010)**.
- M. TABOGA, *Under/over-valuation of the stock market and cyclically adjusted earnings*, *International Finance*, v. 14, 1, pp. 135-164, **TD No. 780 (December 2010)**.
- S. NERI, *Housing, consumption and monetary policy: how different are the U.S. and the Euro area?*, *Journal of Banking and Finance*, v.35, 11, pp. 3019-3041, **TD No. 807 (April 2011)**.
- V. CUCINIELLO, *The welfare effect of foreign monetary conservatism with non-atomistic wage setters*, *Journal of Money, Credit and Banking*, v. 43, 8, pp. 1719-1734, **TD No. 810 (June 2011)**.
- A. CALZA and A. ZAGHINI, *welfare costs of inflation and the circulation of US currency abroad*, *The B.E. Journal of Macroeconomics*, v. 11, 1, Art. 12, **TD No. 812 (June 2011)**.
- I. FAIELLA, *La spesa energetica delle famiglie italiane*, *Energia*, v. 32, 4, pp. 40-46, **TD No. 822 (September 2011)**.
- R. DE BONIS and A. SILVESTRINI, *The effects of financial and real wealth on consumption: new evidence from OECD countries*, *Applied Financial Economics*, v. 21, 5, pp. 409-425, **TD No. 837 (November 2011)**.

2012

- F. CINGANO and A. ROSOLIA, *People I know: job search and social networks*, *Journal of Labor Economics*, v. 30, 2, pp. 291-332, **TD No. 600 (September 2006)**.
- G. GOBBI and R. ZIZZA, *Does the underground economy hold back financial deepening? Evidence from the italian credit market*, *Economia Marche, Review of Regional Studies*, v. 31, 1, pp. 1-29, **TD No. 646 (November 2006)**.
- S. MOCETTI, *Educational choices and the selection process before and after compulsory school*, *Education Economics*, v. 20, 2, pp. 189-209, **TD No. 691 (September 2008)**.
- F. LIPPI and A. NOBILI, *Oil and the macroeconomy: a quantitative structural analysis*, *Journal of European Economic Association*, v. 10, 5, pp. 1059-1083, **TD No. 704 (March 2009)**.
- S. FEDERICO, *Headquarter intensity and the choice between outsourcing versus integration at home or abroad*, *Industrial and Corporate Change*, v. 21, 6, pp. 1337-1358, **TD No. 742 (February 2010)**.
- S. GOMES, P. JACQUINOT and M. PISANI, *The EAGLE. A model for policy analysis of macroeconomic interdependence in the euro area*, *Economic Modelling*, v. 29, 5, pp. 1686-1714, **TD No. 770 (July 2010)**.
- A. ACCETTURO and G. DE BLASIO, *Policies for local development: an evaluation of Italy's "Patti Territoriali"*, *Regional Science and Urban Economics*, v. 42, 1-2, pp. 15-26, **TD No. 789 (January 2006)**.
- F. BUSETTI and S. DI SANZO, *Bootstrap LR tests of stationarity, common trends and cointegration*, *Journal of Statistical Computation and Simulation*, v. 82, 9, pp. 1343-1355, **TD No. 799 (March 2006)**.
- S. NERI and T. ROPELE, *Imperfect information, real-time data and monetary policy in the Euro area*, *The Economic Journal*, v. 122, 561, pp. 651-674, **TD No. 802 (March 2011)**.
- G. CAPPELLETTI, G. GUAZZAROTTI and P. TOMMASINO, *What determines annuity demand at retirement?*, *The Geneva Papers on Risk and Insurance – Issues and Practice*, pp. 1-26, **TD No. 808 (April 2011)**.
- A. ANZUINI and F. FORNARI, *Macroeconomic determinants of carry trade activity*, *Review of International Economics*, v. 20, 3, pp. 468-488, **TD No. 817 (September 2011)**.
- M. AFFINITO, *Do interbank customer relationships exist? And how did they function in the crisis? Learning from Italy*, *Journal of Banking and Finance*, v. 36, 12, pp. 3163-3184, **TD No. 826 (October 2011)**.
- R. CRISTADORO and D. MARCONI, *Household savings in China*, *Journal of Chinese Economic and Business Studies*, v. 10, 3, pp. 275-299, **TD No. 838 (November 2011)**.

- V. DI GIACINTO, G. MICUCCI and P. MONTANARO, *Network effects of public transport infrastructure: evidence on Italian regions*, Papers in Regional Science, v. 91, 3, pp. 515-541, **TD No. 869 (July 2012)**.
- A. FILIPPIN and M. PACCAGNELLA, *Family background, self-confidence and economic outcomes*, Economics of Education Review, v. 31, 5, pp. 824-834, **TD No. 875 (July 2012)**.

2013

- F. BUSETTI and J. MARCUCCI, *Comparing forecast accuracy: a Monte Carlo investigation*, International Journal of Forecasting, v. 29, 1, pp. 13-27, **TD No. 723 (September 2009)**.

*FORTHCOMING*

- M. BUGAMELLI and A. ROSOLIA, *Produttività e concorrenza estera*, Rivista di politica economica, **TD No. 578 (February 2006)**.
- P. SESTITO and E. VIVIANO, *Reservation wages: explaining some puzzling regional patterns*, Labour, **TD No. 696 (December 2008)**.
- P. PINOTTI, M. BIANCHI and P. BUONANNO, *Do immigrants cause crime?*, Journal of the European Economic Association, **TD No. 698 (December 2008)**.
- F. CINGANO and P. PINOTTI, *Politicians at work. The private returns and social costs of political connections*, Journal of the European Economic Association, **TD No. 709 (May 2009)**.
- Y. ALTUNBAS, L. GAMBACORTA and D. MARQUÉS-IBÁÑEZ, *Bank risk and monetary policy*, Journal of Financial Stability, **TD No. 712 (May 2009)**.
- G. BARONE and S. MOCETTI, *Tax morale and public spending inefficiency*, International Tax and Public Finance, **TD No. 732 (November 2009)**.
- I. BUONO and G. LALANNE, *The effect of the Uruguay Round on the intensive and extensive margins of trade*, Journal of International Economics, **TD No. 743 (February 2010)**.
- G. BARONE, R. FELICI and M. PAGNINI, *Switching costs in local credit markets*, International Journal of Industrial Organization, **TD No. 760 (June 2010)**.
- E. COCOZZA and P. PISELLI, *Testing for east-west contagion in the European banking sector during the financial crisis*, in R. Matoušek; D. Stavárek (eds.), Financial Integration in the European Union, Taylor & Francis, **TD No. 790 (February 2011)**.
- E. GAIOTTI, *Credit availability and investment: lessons from the "Great Recession"*, European Economic Review, **TD No. 793 (February 2011)**.
- A. DE SOCIO, *Squeezing liquidity in a "lemons market" or asking liquidity "on tap"*, Journal of Banking and Finance, **TD No. 819 (September 2011)**.
- O. BLANCHARD and M. RIGGI, *Why are the 2000s so different from the 1970s? A structural interpretation of changes in the macroeconomic effects of oil prices*, Journal of the European Economic Association, **TD No. 835 (November 2011)**.
- S. FEDERICO, *Industry dynamics and competition from low-wage countries: evidence on Italy*, Oxford Bulletin of Economics and Statistics, **TD No. 879 (September 2012)**.
- F. D'AMURI and G. PERI, *Immigration, jobs and employment protection: evidence from Europe before and during the Great Recession*, Journal of the European Economic Association, **TD No. 886 (October 2012)**.