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A METHOD TO ESTIMATE POWER PARAMETER IN EXPONENTIAL POWER DISTRIBUTION VIA POLYNOMIAL REGRESSION

by Daniele Coin*

Abstract

The Exponential Power Distribution (EPD), also known as Generalized Error Distribution (GED), is a flexible symmetrical unimodal family belonging to the exponential family. The EPD becomes the density function of a range of symmetric distributions with different values of its power parameter β. A closed-form estimator for β does not exist, so the power parameter is usually estimated numerically. Unfortunately the optimization algorithms do not always converge, especially when the true value of β is close to its parametric space frontier. In this paper we present an alternative method for estimating β, based on the Normal Standardized Q-Q Plot and exploiting the relationship between β and the kurtosis. It is a direct method that does not require computational efforts or the use of optimization algorithms.

JEL Classification: C14, C15, C63.
Keywords: Exponential Power Distribution, kurtosis, normal standardized Q-Q plot.

Contents

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1 Introduction and Motivation

The density function of the Exponential Power Distribution (EPD) with mean $\mu \in (-\infty, \infty)$, scale parameter $\sigma \in (0, \infty)$ and power parameter $\beta \in (-1, 1]$ is

$$f_{EPD}(x; \mu, \sigma, \beta) = \frac{e^{-\frac{1}{2}\left|\frac{x-\mu}{\sigma}\right|^{\frac{2}{1+\beta}}}}{2^{\frac{\beta+3}{2}}\sigma\Gamma\left(\frac{\beta+3}{2}\right)}. \tag{1}$$

This family is also known as Generalized Error Distribution (GED) and it is a flexible symmetrical, with respect to the mean, unimodal member of the exponential family (Box and Tiao (1973), Harvey (1990)).

The value of β makes (1) become the density function of a range of symmetric distributions such as the uniform ($\beta \to -1$), the double exponential ($\beta = 1$) and the normal ones ($\beta = 0$) (i.e. to obtain the standard normal distribution we set $\beta = 0$, $\mu = 0$ and $\sigma = 1$ in (1)); tails are more platykurtic for $\beta < 0$ and more leptokurtic for $\beta > 0$ than the normal distribution. In statistical modeling the EPD has thus been used when the concentration of values around the mean or the tail are of particular interest.

Thanks to its flexibility properties, the EPD family has many applications such as models for atmospheric noise, for sub band encoding of audio and video signals (see Sharifi and Leon-Garcia (1995)) or for the error distribution in time series analysis (see Nelson (1991), Chen *et al.* (2008)).

The odd central moments are zero while the even moments are given by

$$E(X-\mu)^r = \left[\frac{\sigma^2\Gamma\left(\frac{\beta+1}{2}\right)}{\Gamma\left(\frac{3\beta+3}{2}\right)}\right]^r \frac{\Gamma\left(\frac{(r+1)(\beta+1)}{2}\right)}{\Gamma\left(\frac{\beta+1}{2}\right)}. \tag{2}$$

The EPD parametrization, reported in (1), was originally proposed by Box and Tiao (1973). Others are available in the literature. In particular let $v > 1$ the new power parameter; the following relationship links v with β, $v = \frac{2}{1+\beta}$.

Substituting v to β in (1), we obtain the widely diffused parametrization adopted in Nelson (1991).

We decided to adopt the parametrization reported in (1) because the power parameter has a finite domain, property useful in the next sections of the study.

The aim of this paper is to introduce a method to solve an open problem, regarding the EPD: the estimation of β. In fact in the literature maximum-likelihood (ML) and method of moments (MM) estimators have been studied but these estimators do not have closed-form solutions, hence parameter estimates need to be obtained by numerical methods. While the ML estimator is asymptotically more efficient than the MM estimator, the likelihood function does not always have a well-defined maximum. Thus, optimization algorithms do not always converge, especially when the true value of β is close to its boundary space and/or the number of observations is small (see Agro' (1995)). The MM estimator, on the other hand, does not necessarily exist for the whole parameter space of β and can only be approximated for certain ranges of β (see Varanasi and Aazhang (1989)); furthermore the probability of a real solution depends on the true value of β; for this reason we do not consider this approach in our work (see Dominguez-Molina *et al.* (2009)).

To solve this kind of difficulties we propose a method based on the Normal Standardized Q-Q Plot. The existence of our estimator does not depend on the true value of β. Furthermore, it does not need relevant computational efforts. We compare by simulation the properties of the Maximum Likelihood Estimation with those of our method up to 1,000 observations; we find that our proposal behaves better for small sample sizes.

This paper is organized as follows. In Section 2 we describe the Normal Standardized Q-Q Plot. Section 3 presents our proposal. In Section 4 an extensive Monte Carlo study comparing our proposal performances with the likelihood method is summarized. Concluding remarks are provided in Section 5.

2 Normal Standardized Q-Q Plot

Let $\boldsymbol{\alpha}_n = (\alpha_1, ..., \alpha_n)$ denote the vector of n expected values of standard normal order statistics, and let $N_{(1)}, ..., N_{(i)}, ..., N_{(n)}$ be an ordered random sample of size n from a standard normal distribution so that

$$\alpha_i = E\left(N_{(i)}\right) \qquad i = 1, ..., n. \tag{3}$$

Since α_i in (3) is unknown, we use the approximation proposed by Royston (1982). Please note that in any situation the values assumed by the elements of $\boldsymbol{\alpha}_n$ are function of n only.

Given a set of ordered observations $\mathbf{x}_{(.)} = (x_{(1)}, ..., x_{(n)})$ the Normal Standardized Q-Q Plot is constructed by plotting

$$z_{(i)} = \frac{x_{(i)} - \widehat{\mu}}{\widehat{\sigma}}$$

against α_i, where $\widehat{\mu}$ and $\widehat{\sigma}$ are the sample mean and the sample standard deviation, respectively.

If the estimates of location and scale parameters are selected such that $z_{(i)} = \frac{x_{(i)} - \widehat{\mu}}{\widehat{\sigma}}$ is location and scale invariant, then any linear transformation of the original data will not alter any point of the Normal Standardized Q-Q Plot. Furthermore the intercept and the slope of the best fit line of $\boldsymbol{z}$ in function of $\boldsymbol{\alpha}$ have to be 0 and 1 respectively.

A very interesting feature of the Normal Standardized Q-Q Plot is given by the fact that samples drawn from non normal symmetrical distributions tend to assume typical S-shaped curves. Consider for example Figure 1: in panel 1 we display different EPD density shapes for some β values, while in panel 2 we represent the same EPD on the Normal Standardized Q-Q Plot.

Analyzing this figure we see that the families of symmetrical distributions with tails heavier than the normal distribution are represented by symmetric, with respect to the origin of the axes, inverted S-shaped curves, while families of symmetrical alternatives with tails lighter or shorter than the normal distribution are represented by symmetrical, with respect to the origin of the axes, S-shaped curves.

The Normal Standardized Q-Q Plot properties summarized above are well known in the literature, for precise discussion of these familiar pattern see for instance Wilk and Gnanadesikan (1968), section 6.5 of Chambers *et al.* (1983) or pag. 382-383 of Bickel and Doksum (1977), furthermore for methods derived from these properties, see for example Kuczmarski and Rosenbaum (1999) or Coin (2008). In appendix A we provide a more formal proof of these empirical results. Here we would like to investigate the possibility of deriving an estimator for β exploiting the relationship, firstly between S-shaped curves and kurtosis and secondly, between the traditional measure of the kurtosis β_2 and the power parameter β given by the following relationship

$$\beta_2 = \frac{E\left(|X-\mu|^4\right)}{E\left\{\left[(X-\mu)^2\right]\right\}^2} = \frac{\Gamma\left(\frac{5\beta+5}{2}\right)\Gamma\left(\frac{\beta+1}{2}\right)}{\left[\Gamma\left(\frac{3\beta+3}{2}\right)\right]^2}, \tag{4}$$

which is easily derived from (2). This is the subject of the next section.

Let $\mathbf{x}_{(.)} = (x_{(1)}, ..., x_{(n)})$ denote an n dimensional vector of ordered random observations. If $\mathbf{x}_{(.)}$ is drawn from a normal distribution with unknown parameters μ and σ we can write

$$x_{(i)} = \mu + \sigma\alpha_i + \epsilon_i, \tag{5}$$

where μ and σ become the intercept and slope of the best fit line on a Normal Q-Q Plot and $\boldsymbol{\epsilon}$ is the vector of errors (see Balakrishnan and Cohen (1991)). For sufficiently large n, the $x_{(i)}$ may be considered independent and consequently $\boldsymbol{\epsilon}$ can be assumed to be homoscedastic, see Gupta (1952) and Shapiro and Francia (1972). Thus, the two parameters in (5) may be consistently estimated by the simple least squares method (LS).

We mentioned that if the estimates of location and scale parameters are selected such that $z_{(i)} = \frac{x_{(i)}-\widehat{\mu}}{\widehat{\sigma}}$ is location and scale invariant, then any linear transformation of the original data will not alter any point of the plot.

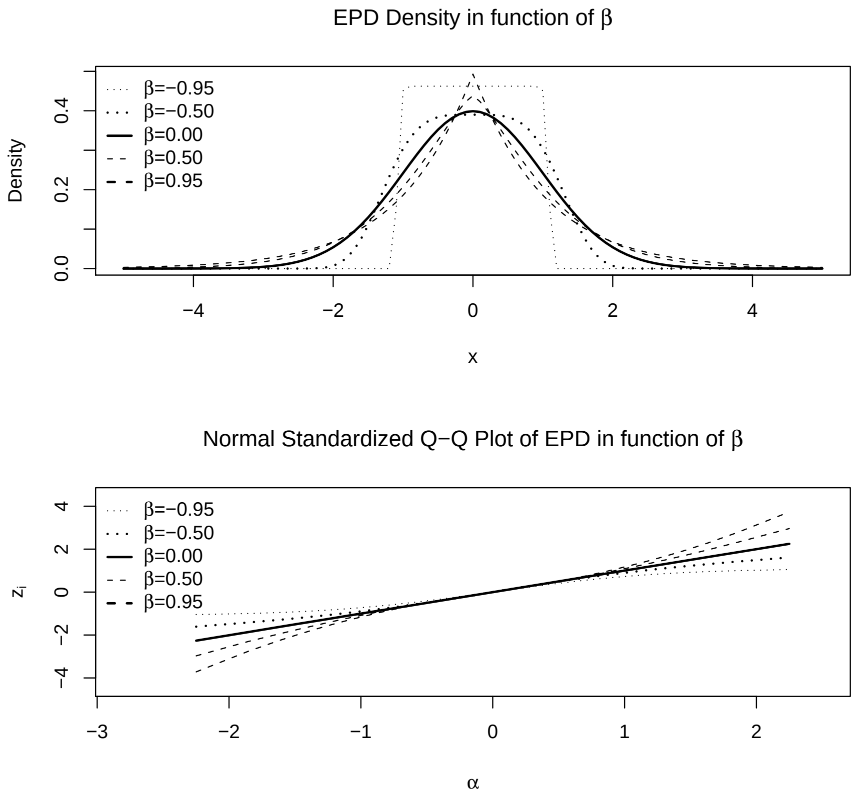

Figure 1: Densities and Normal Standardized Q-Q Plots of different EPD

Thanks to the properties of the Normal Standardized Q-Q Plot, mentioned above, if we use the standardized form for $x_{(i)}$ in (5), we should have $\widehat{\mu}_{LS} = 0$ and $\widehat{\sigma}_{LS} = 1$, where μ_{LS} and σ_{LS} indicate the values of the parameters of (5) estimated with least squares method.

In order to capture the S shapes of the Normal Standardized Q-Q Plot, Coin (2008) considered the following model

$$z_{(i)} - \alpha_i = \beta_3 \alpha_i^3 + \frac{\epsilon_i}{\sigma} \tag{6}$$

and proposed to estimate β_3 with least squares, obtaining $\widehat{\beta}_3$. Since $\widehat{\beta}_3$ values sensibly differ from zero when the Normal Standardized Q-Q Plot assumes S-shaped or inverted S-shaped curves, the author proposed to use a transformation of $\widehat{\beta}_3$ as statistic to test composite null hypothesis of normality.

Therefore it seems reasonable to consider $\widehat{\beta}_3$ as a statistic measuring the propensity to the S or the inverted S shapes of the Normal Standardized Q-Q Plot. Above we have suggested a reasonable relationship between S-shaped curves and kurtosis; furthermore (4) states a relationship between the kurtosis and β. It emerges that both β and $\widehat{\beta}_3$ are linked to the kurtosis, hence we deduce that a function connecting $\widehat{\beta}_3$ and β should exist.

3 Polynomial Estimator of the Power Parameter

Our proposal consists in deriving a plug-in estimator based on an appropriate function of $\widehat{\beta}_3$ and n in β. Formally, let $\boldsymbol{g}_n = (g_{(1)}, ..., g_{(n)})$ be an n-size ordered sample drawn from G, an EPD with unknown β, μ and σ, in symbol

$$G \sim EPD\left(\mu, \sigma, \beta\right).$$

Denote with $\boldsymbol{g}_n^*$ the corresponding standardized ordered sample given by

$$\boldsymbol{g}_n^* = \frac{\boldsymbol{g}_n - \widehat{\mu}}{\widehat{\sigma}}, \tag{7}$$

where, as usual, $\widehat{\mu}$ and $\widehat{\sigma}$ denote the sample mean and the sample standard deviation.

Replacing $g_{(i)}^*$ to $z_{(i)}$ in (6) we get

$$g_{(i)}^* - \alpha_i = \beta_3 \alpha_i^3 + \frac{\epsilon_i}{\sigma}, \tag{8}$$

obviously it is possible to estimate β_3 by ordinary least squares. If we assume the existence of a function $f(.)$ such as

$$\beta = f\left(n, \beta_3\right), \tag{9}$$

we can use as plug-in estimator for β the following

$$\hat{\beta} = f\left(n, \hat{\beta}_3\right), \tag{10}$$

where $\hat{\beta}_3$ is the estimation with least squares of β_3 in (8).

In order to define $f\left(n, \hat{\beta}_3\right)$ we need $\boldsymbol{\alpha}\left(\boldsymbol{\beta}\right)_n = \left(\alpha\left(\beta\right)_1, ..., \alpha\left(\beta\right)_n\right)$, the vector of n expected values of a standard ordered EPD with power parameter β. It is clear that $\boldsymbol{\alpha}\left(\boldsymbol{\beta}\right)_n$ is a function of n and β only. If we replace $\alpha\left(\beta\right)_i$ to $g^*_{(i)}$ in (8) we get

$$\alpha\left(\beta\right)_i - \alpha_i = \beta_3 \alpha_i^3, \tag{11}$$

a deterministic relationship by which we will obtain $f(.)$.

Since $\boldsymbol{\alpha}\left(\boldsymbol{\beta}\right)_n$ is unknown, we estimated it by simulation. We considered the following sample sizes

n = (50, 60, 70, 80, 90, 100, 150, 200, 250, 300, 350, 400, 450, 500, 600, 700, 800, 900, 1000)

and the following values for β

β =(-0.99, -0.95, -0.90, -0.85, -0.80, -0.75, -0.70, -0.65, -0.60, -0.55, -0.50, -0.45, -0.40, -0.35, -0.30, -0.25, -0.20, -0.15, -0.10, -0.05, 0.00, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30, 0.35, 0.40, 0.45, 0.50, 0.55, 0.60, 0.65, 0.70, 0.75, 0.80, 0.85, 0.90, 0.95, 1.00).

For each combination of n and β we generated 200,000 standard ordered samples from an EPD with $\mu = 0$ and $\sigma = 1$ by the function `rnormp` available in the R package by Mineo (2007). Given a specific n-size sample, let $s\left(\beta\right)_{(i)j}$ denote the i^{th} ordered observation in the j^{th} simulated samples: then we will estimate $\alpha\left(\beta\right)_i$ with

$$\widehat{\alpha}\left(\beta\right)_i = \frac{\sum_{j=1}^{200{,}000} s\left(\beta\right)_{(i)j}}{200,000}. \tag{12}$$

In Table 1 we report as an example some of the estimated $\widehat{\alpha}\left(\beta\right)_i$ for $n = 50$.

After having replaced $\alpha\left(\beta\right)_i$ with $\widehat{\alpha}\left(\beta\right)_i$ in (11) we estimate β_3 with ordinary least squares for any combination of β and n. In any case we systematically obtain a coefficient of determination $R^2 > 0.9985$.

In this way we got for any n and β their relative β_3. In table 2 we present a selection of the estimated β_3 for some β and n.

Afterwards we plot the data partially reported in Table 2 in figure 2, which clearly shows a regular functional relationship between β and $\widehat{\alpha}\left(\beta\right)_i$.

We used a Taylor approximation in $\hat{\beta}_3$ and n to define $f(.)$ in (10).

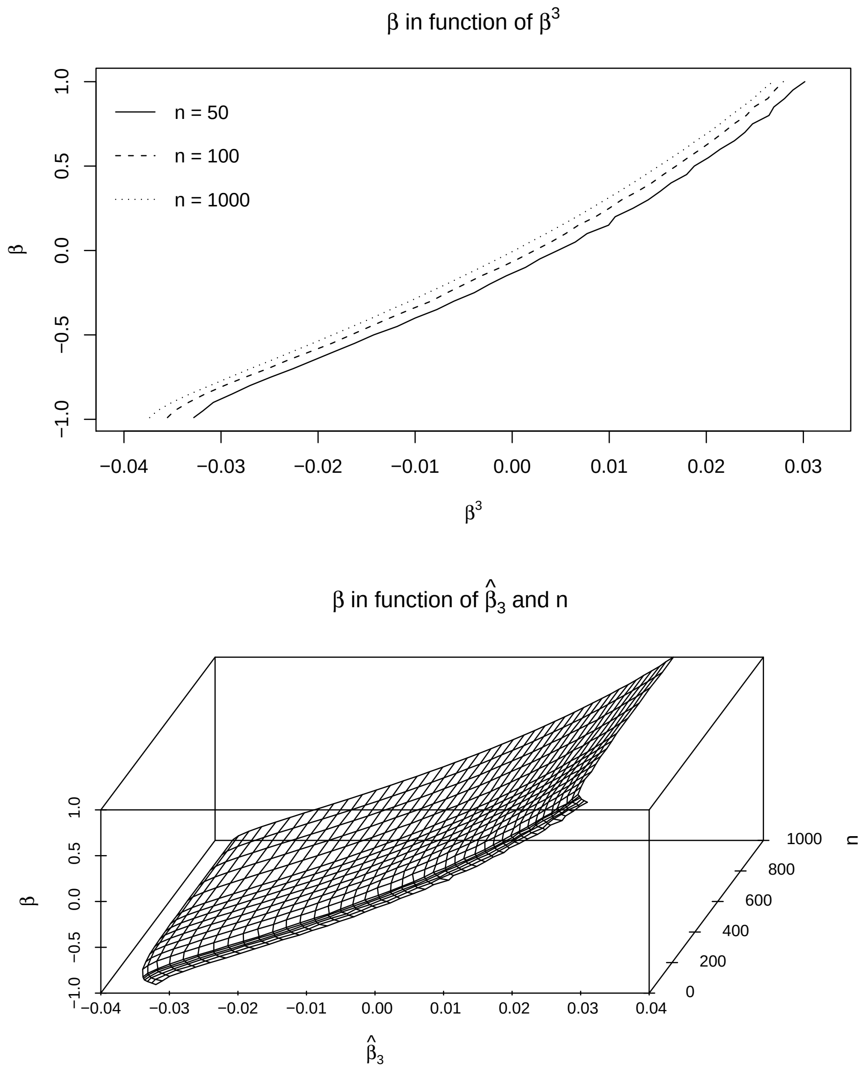

Figure 2: β in function of $\hat{\beta}_3$ and n.

β	$i=1$	$i=2$	…	$i=25$	$i=26$	…	$i=49$	$i=50$
-0.99	-0.983	-0.942	…	-0.022	0.018	…	0.943	0.984
-0.95	-1.049	-1.000	…	-0.025	0.019	…	0.998	1.048
…	…	…	…	…	…	…	…	…
-0.75	-1.308	-1.199	…	-0.023	0.025	…	1.196	1.303
…	…	…	…	…	…	…	…	…
-0.50	-1.611	-1.419	…	-0.029	0.023	…	1.420	1.610
…	…	…	…	…	…	…	…	…
-0.25	-1.918	-1.637	…	-0.026	0.024	…	1.638	1.923
…	…	…	…	…	…	…	…	…
0.00	-2.264	-1.865	…	-0.029	0.021	…	1.853	2.246
…	…	…	…	…	…	…	…	…
0.25	-2.604	-2.086	…	-0.025	0.024	…	2.086	2.607
…	…	…	…	…	…	…	…	…
0.50	-2.971	-2.303	…	-0.026	0.021	…	2.315	2.958
…	…	…	…	…	…	…	…	…
0.75	-3.374	-2.545	…	-0.023	0.023	…	2.558	3.367
…	…	…	…	…	…	…	…	…
0.95	-3.717	-2.761	…	-0.020	0.025	…	2.772	3.731
1.00	-3.806	-2.804	…	-0.025	0.019	…	2.814	3.833

Table 1: Simulated expected values of a standard ordered EPD $\widehat{\alpha}\left(\beta\right)_i$ in function of β, $n=50$.

In order to estimate efficiently β we adopt a bivariate polynomial model whose degrees were chosen with a forward stepwise approach. The resulting model is in the follows:

$$\beta = a_1\frac{1}{n} + a_2\frac{1}{n^2} + a_3\beta_3 + a_4\beta_3^2 + a_5\beta_3^3 + a_6\frac{\beta_3}{n} + \epsilon, \qquad (13)$$

where n is the sample size. In Table 3 we report the estimates of the parameters in (13) obtained with least squares. We pointed out that we obtain a coefficient of determination $R^2 > 0.9998$.

We are now able to define our power parameter estimation procedure. Let $\boldsymbol{g}_n$ be an ordered n-size sample drawn from an EPD with unknown parameters. The first step consists in standardize $\boldsymbol{g}_n$ obtaining $\boldsymbol{g}_n^*$. Secondly the coefficient β_3 in (8) is estimated with ordinary least squares method obtaining $\widehat{\beta}_3$.

Finally we substitute the estimation $\widehat{\beta}_3$ to β_3 in (13) in order to obtain a plug-in estimation for β, in symbol

$$\widehat{\beta} = f\left(\widehat{\beta}_3\right) = \hat{a}_1\frac{1}{n} + \hat{a}_2\frac{1}{n^2} + \hat{a}_3\widehat{\beta}_3 + \hat{a}_4\widehat{\beta}_3^2 + \hat{a}_5\widehat{\beta}_3^3 + \hat{a}_6\frac{\widehat{\beta}_3}{n}. \qquad (14)$$

4 Monte Carlo Study

In order to investigate some properties and performances of the $f\left(\widehat{\beta}_3\right)$ estimator we performed a Monte Carlo study.

	$\beta =$									
n	-0.99	-0.95	-0.5	-0.25	0	0.25	0.5	0.75	0.95	1
50	-3281	-3187	-1427	-391	469	1244	1876	2476	2892	3015
60	-3357	-3303	-1506	-485	372	1155	1830	2417	2871	2953
70	-3436	-3384	-1576	-524	312	1109	1797	2386	2819	2896
80	-3497	-3417	-1595	-625	279	1060	1735	2325	2742	2864
90	-3526	-3457	-1622	-648	232	1025	1727	2285	2744	2832
100	-3552	-3481	-1660	-669	231	999	1688	2285	2707	2794
150	-3640	-3565	-1729	-736	156	919	1625	2233	2669	2756
200	-3676	-3601	-1773	-769	97	899	1593	2193	2644	2728
250	-3702	-3635	-1802	-811	85	882	1569	2174	2626	2742
300	-3713	-3639	-1815	-804	71	860	1565	2168	2613	2723
350	-3720	-3649	-1824	-815	67	839	1553	2160	2602	2710
400	-3724	-3642	-1832	-837	57	845	1550	2160	2610	2713
450	-3731	-3650	-1837	-840	47	844	1539	2158	2593	2705
500	-3726	-3652	-1837	-844	34	832	1536	2149	2590	2692
600	-3734	-3658	-1839	-856	33	827	1532	2137	2585	2690
700	-3734	-3659	-1843	-863	31	815	1522	2145	2592	2690
800	-3735	-3659	-1853	-865	26	821	1510	2143	2578	2681
900	-3735	-3658	-1845	-865	32	817	1522	2137	2584	2691
1000	-3735	-3659	-1851	-869	24	812	1510	2143	2577	2686

Table 2: Estimated $\hat{\beta}_3$ for some β and n. Values $\times 100,000$.

Parameter	Estimates
$\hat{a}_1$	-6.03758
$\hat{a}_2$	-48.41451
$\hat{a}_3$	29.25522
$\hat{a}_4$	219.36466
$\hat{a}_5$	3410.16169
$\hat{a}_6$	-51.11288

Table 3: Estimated parameters of (13).

We simulated 200,000 standard ordered samples with the function `rnormp` for any combination of n and β included in the set considered in Section 3.

Then, for any sample, we estimated β using $f\left(\widehat{\beta}_3\right)$; furthermore, for comparison purposes, we did the same using the function `gedFit` which maximizes the likelihood function of the EPD by an optimization algorithm. This function is included in the R package by Wuertz and Miklovic (2008).

The performances of this two methods were evaluated through their mean and their square error (MSE).

In the previous sections, we stated that the likelihood function does not always have a well-defined maximum and in this case the ML estimations are unreliable. This is clearly proved by the results reported in table 4, 5 and 6. In fact table 4 reports the percentage of convergence of the ML algorithm while tables 5 and 6 present the corresponding estimated mean and MSE. By the analysis of table 4 we see that the convergence ratio is very low especially when the true β is close to its parametric space frontier and for moderate sample sizes. Furthermore when the algorithm has not converged the estimations returned are unreliable. Analyzing tables 5 and 6 we see that MSE could be extremely large and the punctual estimation completely wrong. This kind of problem is overtaken by our procedure, in tables 7 and in table 8 we report the estimated mean and MSE of our procedure performed over the corresponding samples where the ML showed to be unreliable. It emerges an overall better performance. The MSE of our method never diverges since it is a closed form plug-in estimator, on the contrary the ML needs the convergence of an optimization algorithm; when we are not in such situation it can return point estimations that are outside from the space parameter bounds (see table 5).

We now focus on the situation where the ML algorithm converges. In table 9 and 11 we report the mean of $f\left(\widehat{\beta}_3\right)$ and maximum likelihood estimates, while in table 10 and 12 we report the mean square errors. In this case both the methods seem to be consistent and asymptotically correct even if the MLE seems to perform a little bit better in terms of MSE.

Finally, in order to evaluate the properties of our plug-in estimator we report the Estimated Expected Values (see table 13) and the MSE (see table 14) computed on the whole simulated samples.

Analyzing table 13 it emerges that the $f\left(\widehat{\beta}_3\right)$ is biased but asymptotically correct, its rate convergence depends on the true value of β, it is faster for β closer to -1. The results reported in table 14 prove that $f\left(\widehat{\beta}_3\right)$ is consistent, since the MSE tends to 0. Even in this case the rate of convergence depends on the true value of β and it is again faster for β closer to -1. This phenomenon is not only caused by the bias but mainly because the $f\left(\widehat{\beta}_3\right)$ is less accurate when $\beta \to 1$. In order to show this statement we derive bias and variance, the two component of the MSE and thanks to the following relationship we can get the variance (The bias could be easily obtained from table 13):

$$MSE\left(f\left(\widehat{\beta}_3\right)\right) = Var\left(f\left(\widehat{\beta}_3\right)\right) + Bias\left(f\left(\widehat{\beta}_3\right)\right)^2. \tag{15}$$

The results, reported in table 15, again show that the value of this indicator is connected with the true value of β.

By the comparison with the performances of $f\left(\widehat{\beta}_3\right)$ and MLE it emerges that if the ML algorithm converges it is preferable to adopt its estimation: on the contrary in the not rare situation of no convergence our estimator could supply more reliable estimations.

The computational time of both the procedures are negligible.

5 Concluding Remarks

In this paper an original method to estimate the power parameter for Exponential Power Distribution is presented. This approach shows some appealing features such as definiteness, negligible computational time, asymptotic correctness and consistency.

We proved it is very useful because it is always able to provide reliable estimation in any situation, even when the existing methodologies are affected by different problems.

Moreover we think that our procedure can be easily implemented in any statistical package.

It remains to find the asymptotic distribution of our estimator. This task awaits further research.

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A In-depth examination of the S-Shapes

Samples drawn from unimodal and symmetric respect to the mean distributions with lighter or shorter (heavier) tails than a normal density tend to assume symmetrical, with respect to the origin of the axes, S-shaped (inverted S-shaped) curves when represented on the Normal Standardized Q-Q Plot. This feature, based on empirical observations, is widely accepted in the literature and it can be more formally proved. In the following we will only explore the case of distribution with lighter or shorter tails (the other can be obtained by similar arguments). Consider $f_X(x)$ to be a continuous, symmetrical and unimodal density and a normal density $f_N(x)$, without loss of generality we set mean and median equal to 0 and variance equal to 1 both for X and N. If $f_X(x)$ has lighter or shorter tails than $f_N(x)$ then we have

$$f_X(x) < f_N(x) \;\; for\; |x| > a$$

where a is a positive constant. Furthermore we get

$$\begin{aligned} F_X(x) < F_N(x) \;\; for\; x < -b\;;\; x \in (0, b) \\ F_X(x) > F_N(x) \;\; for\; x > b;\; x \in (-b, 0) \end{aligned} \tag{16}$$

where $0 < b < a$ and $F_X(x)$ and $F_N(x)$ are the cumulative distribution function of X and N respectively. If $f_X(x)$ and $f_N(x)$ have only two intersection in $-a$ and a then $b = 0$. Let $\boldsymbol{\alpha}(X)_n$ be the n-size vector ordered expected values of X. Since the (16) means that X is stochastically smaller than N for $x < -b$, thanks to the properties of stochastic orderings (see for example theorem 4.4.1 in David and Nagaraja (1981)) then exists an integer values $k \le \frac{n}{2}$ ($k = \frac{n}{2}$ if $f_X(x)$ and $f_N(x)$ have only two intersection) such that:

$$\begin{aligned} \alpha(X)_i < \alpha_i \;\; for\; i \le k \\ \alpha(X)_i > \alpha_i \;\; for\; i \ge n - k + 1. \end{aligned} \tag{17}$$

Furthermore thanks to the properties of symmetry of X and N we have

$$\alpha_i - x_{(i)} = \alpha_{n-i+1} - x_{(n-i+1)} \;\; \forall i \tag{18}$$

If $f_X(x)$ has a monotonic growth for $x < 0$ the conditions reported in (17) and in (18) determine that the vector of $\alpha(X)_i$ plotted over a Normal Standardized Q-Q Plot assume symmetrical, with respect the origin of the axes, S-shaped curves for $i \le k$ and $i \ge n-k+1$ (for any i if $f_X(x)$ and $f_N(x)$ have only two intersection). Finally we would like to point out that there is a direct relationship between light or heavy tails and kurtosis, this is clearly proved in Finucan (1964) and Dyson (1943).

B Tables of the Monte Carlo Study

	$n = 50$	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 1000$
-0.99	6.66	7.82	8.26	9.60	12.22	15.88	36.75
-0.95	9.56	16.50	37.16	57.74	73.64	83.08	98.30
-0.9	16.76	37.42	72.84	89.62	95.66	98.22	99.75
-0.85	25.28	57.86	88.26	97.26	98.80	99.38	99.65
-0.8	36.20	70.78	95.68	98.68	99.28	99.30	99.35
-0.75	47.12	82.68	97.82	99.08	99.70	99.26	99.60
-0.7	56.04	88.06	98.62	99.52	99.46	99.42	99.40
-0.65	63.18	92.86	99.36	99.48	99.60	99.58	99.80
-0.6	70.16	94.86	99.50	99.70	99.54	99.70	99.65
-0.55	76.20	96.20	99.52	99.66	99.68	99.66	99.60
-0.5	79.08	97.82	99.64	99.30	99.56	99.74	99.75
-0.45	82.36	98.26	99.60	99.42	99.50	99.50	99.45
-0.4	86.88	98.74	99.58	99.42	99.04	99.14	99.30
-0.35	88.70	98.88	99.54	99.58	99.28	99.16	99.10
-0.3	90.20	99.22	99.50	99.42	99.02	98.82	98.40
-0.25	91.76	98.86	99.44	99.14	99.10	99.02	98.70
-0.2	92.30	99.02	99.28	99.18	99.10	98.94	98.40
-0.15	94.56	99.18	99.16	98.84	98.96	99.06	98.45
-0.1	93.94	98.92	99.12	98.66	98.36	98.54	97.35
-0.05	94.76	98.96	98.58	98.42	98.36	98.38	98.20
0	95.36	99.00	98.48	98.26	98.46	98.26	98.20
0.05	95.70	98.76	98.64	98.52	98.00	97.96	98.55
0.1	94.14	98.80	98.64	98.64	98.60	98.24	98.75
0.15	94.60	98.50	98.78	98.70	98.60	98.54	98.75
0.2	93.76	98.44	98.36	98.68	98.56	98.58	99.05
0.25	92.34	97.90	98.66	98.58	98.68	98.48	98.75
0.3	91.28	96.94	98.64	98.96	98.84	99.02	98.85
0.35	90.22	95.76	98.46	98.68	98.72	98.94	99.55
0.4	87.58	95.48	97.98	98.98	98.92	98.92	99.45
0.45	86.02	93.94	96.98	98.16	98.80	99.18	99.35
0.5	84.50	90.86	96.62	98.04	98.56	99.00	99.20
0.55	81.66	89.66	94.88	96.80	98.06	98.26	99.30
0.6	79.84	86.14	92.24	95.42	96.36	97.48	98.55
0.65	75.38	82.90	89.34	92.90	94.92	95.88	97.95
0.7	72.86	79.06	85.94	89.02	92.16	93.18	94.55
0.75	70.68	74.16	81.28	86.42	88.20	89.04	93.00
0.8	66.80	70.88	75.40	79.62	81.88	83.92	88.60
0.85	62.20	65.88	69.54	73.74	75.30	77.50	83.45
0.9	60.80	61.60	63.82	65.88	68.24	70.12	73.50
0.95	57.28	56.06	58.16	59.94	59.60	61.54	65.00
1	53.14	51.52	49.92	49.90	49.80	52.12	57.05

Table 4: Percentage of Convergence of ML algorithm.

	$n = 50$	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 1000$
-0.99	-1.0057	-0.9723	-1.0000	-1.0000	-1.0000	-1.0000	-1.0000
-0.95	-1.0187	-0.8524	-0.9996	-0.6306	-0.9990	-0.9990	-0.9922
-0.9	-1.0251	-0.9996	-0.9983	-0.9972	-0.9913	-0.9735	-0.9210
-0.85	-0.9555	-0.9990	-0.9959	-0.9777	-0.9401	-0.9045	-0.8582
-0.8	-0.5675	-0.9981	-0.9790	-0.8994	-0.8675	-0.8237	-0.8007
-0.75	-1.0054	-0.9927	-0.9452	-0.8275	-0.7699	-0.7527	-0.7544
-0.7	-1.0020	-0.9936	-0.9179	-0.7279	-0.7260	-0.7129	-0.7043
-0.65	-0.9953	-0.9808	-0.8060	-0.6974	-0.6565	-0.6729	-0.6509
-0.6	-0.7414	-0.9788	-0.6187	-0.6461	-0.5930	-0.6366	-0.6096
-0.55	-0.9039	-0.9626	-0.6829	-0.5213	-0.5769	-0.5454	-0.5643
-0.5	-0.7430	-0.9452	-0.5221	-0.4941	-0.4692	-0.4597	-0.4657
-0.45	-0.2731	-0.8274	-0.4849	-0.4370	-0.4316	-0.4195	-0.4350
-0.4	-0.9533	-0.7771	-0.4366	-0.3919	-0.3899	-0.3791	-0.4011
-0.35	-0.9844	-0.7108	-0.3985	-0.3553	-0.3600	-0.3390	-0.3558
-0.3	-0.9643	-0.5164	-0.2594	-0.2516	-0.3002	-0.3101	-0.2905
-0.25	-0.9336	-0.5935	-0.1766	-0.2209	-0.2690	-0.2247	-0.2471
-0.2	-0.8825	-0.3405	-0.1557	-0.1592	-0.1796	-0.1966	-0.1817
-0.15	0.5661	-0.2211	-0.0991	-0.1096	-0.0969	-0.1445	-0.1520
-0.1	0.9402	-0.0771	-0.0558	-0.0584	-0.0619	-0.0678	-0.1039
-0.05	-1.5720	-0.0219	0.0103	-0.0171	-0.0208	-0.0395	-0.0569
0	-0.3999	0.1318	0.0169	0.0114	-0.0008	-0.0014	-0.0189
0.05	2.0509	0.1705	0.0775	0.0288	0.0163	0.0219	0.0312
0.1	0.0798	0.2129	0.0921	0.0792	0.0869	0.0632	0.0747
0.15	-0.3682	0.3555	0.1470	0.1364	0.1212	0.1488	0.1241
0.2	0.9319	0.4478	0.1887	0.1852	0.1776	0.1955	0.2129
0.25	10.6656	0.6097	0.2752	0.2176	0.1981	0.2209	0.1845
0.3	0.9413	0.6171	0.3740	0.2864	0.2813	0.2859	0.2609
0.35	3.0136	0.6745	0.5400	0.3262	0.2975	0.3411	0.3273
0.4	0.7699	0.6846	0.6469	0.4836	0.3990	0.4229	0.3958
0.45	0.7117	0.7700	0.6521	0.5795	0.5380	0.4536	0.4094
0.5	1.6260	0.7104	0.7940	0.6838	0.5792	0.5810	0.5511
0.55	0.9470	0.6925	0.8015	0.8130	0.7686	0.7118	0.6311
0.6	1.1964	0.6866	0.7402	0.8105	0.8467	0.7810	0.6802
0.65	0.8104	0.6902	0.7436	0.8096	0.8484	0.8559	0.7386
0.7	1.1467	0.6220	0.7474	0.8082	0.8705	0.8702	0.8106
0.75	1.1397	0.6052	0.6648	0.7806	0.8046	0.8784	0.8571
0.8	1.0783	0.5090	0.5985	0.7905	0.8104	0.8776	0.9193
0.85	0.5294	0.4742	0.6113	0.7501	0.7803	0.8561	0.9006
0.9	0.6394	0.4534	0.4635	0.6481	0.6928	0.7987	0.8905
0.95	-2.1563	0.4128	0.3927	0.5838	0.6197	0.7132	0.8679
1	0.7464	0.3564	0.3320	0.4639	0.5618	0.5914	0.7460

Table 5: Estimated Expected Values of MLE for samples when the ML algorithm did not converge.

	$n = 50$	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 1000$
-0.99	0.0557	3.5320	0.1801	0.0901	0.0500	0.0001	0.0001
-0.95	0.2267	90.6539	0.0026	288.0079	0.0027	0.0025	0.0021
-0.9	0.3822	0.0100	0.0099	0.0097	0.0091	0.0077	0.0021
-0.85	11.7164	0.0224	0.0218	0.0192	0.0136	0.0065	0.0007
-0.8	615.6928	0.0396	0.0358	0.0217	0.0132	0.0026	0.0007
-0.75	0.1062	0.0607	0.0495	0.0169	0.0041	0.0015	0.0010
-0.7	0.1244	0.0881	0.0622	0.0060	0.0042	0.0029	0.0011
-0.65	0.1228	0.1154	0.0513	0.0137	0.0013	0.0029	0.0011
-0.6	96.8948	0.1510	0.0334	0.0058	0.0043	0.0049	0.0014
-0.55	6.1442	0.1822	0.0586	0.0096	0.0041	0.0075	0.0015
-0.5	64.6074	0.2215	0.0112	0.0096	0.0081	0.0083	0.0039
-0.45	443.4779	0.2670	0.0242	0.0063	0.0040	0.0037	0.0013
-0.4	2.5563	0.2479	0.0167	0.0052	0.0048	0.0039	0.0021
-0.35	0.5843	0.2533	0.0121	0.0102	0.0037	0.0034	0.0025
-0.3	0.6985	0.2358	0.0258	0.0192	0.0067	0.0068	0.0030
-0.25	0.9471	0.3113	0.0280	0.0203	0.0126	0.0111	0.0037
-0.2	4.1507	0.2113	0.0192	0.0141	0.0130	0.0122	0.0030
-0.15	493.6522	0.2141	0.0222	0.0118	0.0114	0.0095	0.0028
-0.1	908.6306	0.1869	0.0163	0.0111	0.0099	0.0072	0.0038
-0.05	297.4151	0.1150	0.0165	0.0094	0.0082	0.0081	0.0044
0	1.9337	0.1474	0.0233	0.0122	0.0073	0.0083	0.0063
0.05	354.5645	0.1849	0.0216	0.0097	0.0116	0.0078	0.0050
0.1	7.9709	0.2191	0.0324	0.0135	0.0115	0.0110	0.0053
0.15	74.6269	0.3010	0.0255	0.0127	0.0155	0.0123	0.0046
0.2	125.9162	0.3016	0.0420	0.0253	0.0173	0.0149	0.0065
0.25	35554.7203	0.3810	0.0701	0.0248	0.0205	0.0143	0.0099
0.3	80.4714	0.3693	0.0932	0.0395	0.0194	0.0132	0.0068
0.35	2097.5776	0.4524	0.1412	0.0487	0.0227	0.0170	0.0118
0.4	20.1237	0.4771	0.1955	0.0717	0.0342	0.0359	0.0107
0.45	14.9122	0.4520	0.2000	0.0862	0.0615	0.0344	0.0102
0.5	417.6718	1.1750	0.2556	0.1616	0.0739	0.0555	0.0183
0.55	64.6936	0.4838	0.2726	0.1374	0.0883	0.0694	0.0303
0.6	26.4655	0.5380	0.3508	0.1633	0.1130	0.0871	0.0184
0.65	454.2022	0.7468	0.3882	0.2365	0.1210	0.0791	0.0181
0.7	369.0362	0.6294	0.4189	0.2678	0.1682	0.1067	0.0252
0.75	99.2059	0.7175	0.5667	0.3594	0.2749	0.1389	0.0225
0.8	316.8554	1.0079	0.6983	0.3813	0.3233	0.1828	0.0391
0.85	451.4573	0.9598	0.7191	0.4905	0.4023	0.2696	0.1098
0.9	1164.7002	1.0332	0.9945	0.6998	0.6076	0.4161	0.1810
0.95	10452.2633	1.3831	1.1687	0.8546	0.7854	0.6099	0.2918
1	163.7763	1.3639	1.3552	1.1351	0.9545	0.8933	0.5765

Table 6: Estimated Mean Square Errors of MLE for samples when the ML algorithm did not converge.

	$n = 50$	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 1000$
-0.99	-0.9971	-0.9899	-0.9830	-0.9814	-0.9784	-0.9755	-0.9719
-0.95	-0.9979	-0.9781	-0.9738	-0.9658	-0.9662	-0.9538	-0.9685
-0.9	-0.9711	-0.9518	-0.9481	-0.9374	-0.9368	-0.9230	-0.9113
-0.85	-0.9431	-0.9238	-0.9135	-0.9017	-0.8986	-0.8917	-0.8711
-0.8	-0.9257	-0.8928	-0.8726	-0.8338	-0.8552	-0.8217	-0.8105
-0.75	-0.8956	-0.8655	-0.8177	-0.8141	-0.7535	-0.7607	-0.7474
-0.7	-0.8691	-0.8383	-0.8016	-0.7160	-0.7179	-0.7117	-0.7032
-0.65	-0.8430	-0.8036	-0.7460	-0.6798	-0.6572	-0.6699	-0.6411
-0.6	-0.8112	-0.7736	-0.5731	-0.6439	-0.5944	-0.6294	-0.6084
-0.55	-0.7891	-0.7558	-0.6213	-0.5046	-0.5695	-0.5316	-0.5528
-0.5	-0.7643	-0.7263	-0.5093	-0.4838	-0.4614	-0.4501	-0.4544
-0.45	-0.7502	-0.6461	-0.4509	-0.4251	-0.4223	-0.4119	-0.4263
-0.4	-0.7148	-0.5872	-0.4266	-0.3801	-0.3817	-0.3705	-0.3923
-0.35	-0.7000	-0.5229	-0.3903	-0.3508	-0.3471	-0.3299	-0.3496
-0.3	-0.6679	-0.3877	-0.2470	-0.2411	-0.2926	-0.3029	-0.2835
-0.25	-0.6250	-0.4126	-0.1620	-0.2159	-0.2643	-0.2211	-0.2398
-0.2	-0.5506	-0.2229	-0.1497	-0.1482	-0.1780	-0.1888	-0.1684
-0.15	-0.4936	-0.1879	-0.0861	-0.1039	-0.1008	-0.1333	-0.1490
-0.1	-0.4204	-0.0727	-0.0385	-0.0527	-0.0629	-0.0599	-0.1001
-0.05	-0.3191	-0.0139	0.0172	-0.0148	-0.0238	-0.0418	-0.0550
0	-0.2256	0.1183	0.0113	0.0195	0.0042	-0.0030	-0.0199
0.05	-0.1189	0.1375	0.0704	0.0299	0.0151	0.0215	0.0332
0.1	0.0841	0.2079	0.0956	0.0800	0.0817	0.0576	0.0729
0.15	0.1661	0.2494	0.1404	0.1292	0.1221	0.1391	0.1150
0.2	0.3212	0.3155	0.1874	0.1649	0.1586	0.1802	0.1965
0.25	0.4140	0.4936	0.2411	0.2026	0.1918	0.2160	0.1900
0.3	0.5224	0.5686	0.3309	0.2729	0.2788	0.2754	0.2563
0.35	0.6338	0.5856	0.4904	0.2863	0.2749	0.3180	0.3026
0.4	0.6880	0.6931	0.5791	0.4672	0.3812	0.3857	0.3520
0.45	0.7503	0.7437	0.6071	0.5110	0.4942	0.4256	0.3888
0.5	0.7376	0.7833	0.7186	0.6817	0.5395	0.5380	0.5294
0.55	0.7878	0.8247	0.7975	0.7286	0.7318	0.6641	0.6156
0.6	0.8066	0.8600	0.8306	0.7880	0.8127	0.7294	0.6487
0.65	0.8616	0.8921	0.8538	0.8350	0.8288	0.8181	0.7540
0.7	0.8555	0.9240	0.9133	0.8815	0.8773	0.8509	0.7913
0.75	0.8910	0.9371	0.9533	0.9252	0.9009	0.8885	0.8308
0.8	0.9121	0.9647	0.9588	0.9470	0.9439	0.9300	0.9026
0.85	0.9183	0.9799	0.9949	0.9799	0.9764	0.9565	0.9380
0.9	0.9365	0.9157	0.9019	0.8988	0.9070	0.9016	0.9021
0.95	0.9454	0.9713	0.9670	0.9701	0.9601	0.9542	0.9521
1	0.9896	1.0416	1.0425	1.0613	1.0462	1.0500	1.0401

Table 7: Estimated Expected Values of $f\left(\widehat{\beta}_3\right)$ for samples when the ML algorithm did not converge.

	$n=50$	$n=100$	$n=200$	$n=300$	$n=400$	$n=500$	$n=1000$
-0.99	0.0476	0.0214	0.0100	0.0064	0.0045	0.0038	0.0021
-0.95	0.0476	0.0221	0.0101	0.0060	0.0045	0.0038	0.0025
-0.9	0.0509	0.0236	0.0114	0.0069	0.0055	0.0035	0.0004
-0.85	0.0522	0.0244	0.0121	0.0080	0.0079	0.0052	0.0009
-0.8	0.0559	0.0274	0.0138	0.0091	0.0081	0.0023	0.0022
-0.75	0.0608	0.0310	0.0140	0.0104	0.0039	0.0031	0.0014
-0.7	0.0649	0.0359	0.0172	0.0040	0.0037	0.0037	0.0018
-0.65	0.0722	0.0390	0.0196	0.0078	0.0021	0.0028	0.0010
-0.6	0.0803	0.0455	0.0191	0.0058	0.0054	0.0061	0.0015
-0.55	0.0920	0.0571	0.0213	0.0104	0.0041	0.0072	0.0019
-0.5	0.1049	0.0676	0.0091	0.0098	0.0101	0.0106	0.0049
-0.45	0.1251	0.1019	0.0098	0.0068	0.0056	0.0041	0.0014
-0.4	0.1414	0.0860	0.0141	0.0061	0.0048	0.0044	0.0021
-0.35	0.1671	0.0841	0.0116	0.0095	0.0036	0.0039	0.0024
-0.3	0.1799	0.0970	0.0227	0.0216	0.0060	0.0058	0.0033
-0.25	0.1935	0.1124	0.0334	0.0198	0.0116	0.0109	0.0033
-0.2	0.2316	0.0952	0.0181	0.0155	0.0128	0.0116	0.0036
-0.15	0.2987	0.1044	0.0242	0.0125	0.0112	0.0100	0.0025
-0.1	0.3028	0.0870	0.0188	0.0127	0.0098	0.0083	0.0037
-0.05	0.3375	0.0740	0.0180	0.0110	0.0088	0.0087	0.0046
0	0.3885	0.0806	0.0193	0.0163	0.0077	0.0083	0.0056
0.05	0.3655	0.1246	0.0225	0.0128	0.0124	0.0083	0.0079
0.1	0.4236	0.1393	0.0374	0.0163	0.0122	0.0142	0.0062
0.15	0.4046	0.1247	0.0254	0.0130	0.0162	0.0135	0.0051
0.2	0.4065	0.1137	0.0462	0.0305	0.0168	0.0166	0.0063
0.25	0.3447	0.1964	0.0520	0.0259	0.0210	0.0171	0.0130
0.3	0.3922	0.2152	0.0715	0.0417	0.0283	0.0190	0.0074
0.35	0.3618	0.1880	0.1044	0.0426	0.0259	0.0229	0.0142
0.4	0.3368	0.2215	0.1390	0.0640	0.0424	0.0317	0.0099
0.45	0.3037	0.1907	0.1219	0.0686	0.0550	0.0375	0.0151
0.5	0.2791	0.1818	0.1362	0.0946	0.0604	0.0460	0.0257
0.55	0.2742	0.1930	0.1321	0.0904	0.0757	0.0472	0.0272
0.6	0.2496	0.1681	0.1227	0.0900	0.0971	0.0561	0.0185
0.65	0.2464	0.1632	0.1031	0.0859	0.0804	0.0668	0.0286
0.7	0.2299	0.1537	0.1122	0.0773	0.0683	0.0541	0.0310
0.75	0.2327	0.1434	0.1047	0.0738	0.0619	0.0518	0.0266
0.8	0.2213	0.1375	0.0923	0.0701	0.0618	0.0501	0.0300
0.85	0.2184	0.1243	0.0867	0.0680	0.0556	0.0459	0.0261
0.9	0.2119	0.1164	0.0791	0.0643	0.0520	0.0416	0.0274
0.95	0.2121	0.1171	0.0760	0.0536	0.0469	0.0428	0.0262
1	0.2113	0.1215	0.0718	0.0587	0.0484	0.0395	0.0258

Table 8: Estimated Mean Square Errors of $f\left(\widehat{\beta}_3\right)$ for samples when the ML algorithm did not converge.

	$n = 50$	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 1000$
-0.99	-0.7237	-0.8734	-0.9431	-0.9603	-0.9716	-0.9768	-0.9871
-0.95	-0.7486	-0.8805	-0.9356	-0.9483	-0.9521	-0.9531	-0.9539
-0.9	-0.7486	-0.8677	-0.9019	-0.9085	-0.9090	-0.9072	-0.9042
-0.85	-0.7468	-0.8360	-0.8627	-0.8628	-0.8596	-0.8572	-0.8534
-0.8	-0.7193	-0.8022	-0.8176	-0.8130	-0.8100	-0.8086	-0.8035
-0.75	-0.6976	-0.7599	-0.7667	-0.7636	-0.7592	-0.7582	-0.7548
-0.7	-0.6681	-0.7164	-0.7196	-0.7132	-0.7090	-0.7066	-0.7041
-0.65	-0.6225	-0.6749	-0.6682	-0.6622	-0.6596	-0.6588	-0.6546
-0.6	-0.5851	-0.6313	-0.6215	-0.6122	-0.6087	-0.6080	-0.6046
-0.55	-0.5506	-0.5801	-0.5698	-0.5620	-0.5599	-0.5571	-0.5544
-0.5	-0.5091	-0.5370	-0.5183	-0.5125	-0.5091	-0.5079	-0.5029
-0.45	-0.4731	-0.4875	-0.4688	-0.4618	-0.4623	-0.4569	-0.4526
-0.4	-0.4237	-0.4344	-0.4182	-0.4135	-0.4087	-0.4061	-0.4030
-0.35	-0.3750	-0.3821	-0.3686	-0.3637	-0.3585	-0.3589	-0.3549
-0.3	-0.3335	-0.3351	-0.3196	-0.3120	-0.3113	-0.3086	-0.3038
-0.25	-0.2843	-0.2858	-0.2666	-0.2605	-0.2573	-0.2567	-0.2543
-0.2	-0.2344	-0.2273	-0.2156	-0.2125	-0.2078	-0.2061	-0.2050
-0.15	-0.1966	-0.1890	-0.1625	-0.1624	-0.1598	-0.1564	-0.1516
-0.1	-0.1494	-0.1364	-0.1133	-0.1087	-0.1100	-0.1069	-0.1029
-0.05	-0.1086	-0.0802	-0.0683	-0.0616	-0.0586	-0.0573	-0.0534
0	-0.0586	-0.0312	-0.0164	-0.0140	-0.0082	-0.0077	-0.0044
0.05	-0.0175	0.0214	0.0360	0.0431	0.0440	0.0433	0.0465
0.1	0.0369	0.0699	0.0855	0.0898	0.0966	0.0974	0.1006
0.15	0.0730	0.1197	0.1393	0.1412	0.1400	0.1435	0.1458
0.2	0.1185	0.1680	0.1853	0.1894	0.1946	0.1970	0.1978
0.25	0.1629	0.2147	0.2410	0.2452	0.2473	0.2464	0.2464
0.3	0.1971	0.2586	0.2934	0.2901	0.2905	0.2933	0.2976
0.35	0.2279	0.2968	0.3405	0.3422	0.3453	0.3494	0.3468
0.4	0.2603	0.3518	0.3832	0.3926	0.3982	0.3953	0.3979
0.45	0.2932	0.3919	0.4302	0.4389	0.4431	0.4462	0.4466
0.5	0.3271	0.4332	0.4790	0.4874	0.4939	0.4913	0.4961
0.55	0.3510	0.4683	0.5191	0.5363	0.5408	0.5436	0.5429
0.6	0.3989	0.5072	0.5627	0.5803	0.5863	0.5933	0.5958
0.65	0.4311	0.5330	0.6005	0.6217	0.6309	0.6374	0.6453
0.7	0.4398	0.5643	0.6413	0.6651	0.6777	0.6817	0.6967
0.75	0.4847	0.5969	0.6735	0.6993	0.7138	0.7230	0.7405
0.8	0.4998	0.6281	0.7062	0.7345	0.7492	0.7645	0.7874
0.85	0.5204	0.6445	0.7355	0.7653	0.7910	0.8023	0.8352
0.9	0.5371	0.6731	0.7531	0.7960	0.8113	0.8347	0.8690
0.95	0.5545	0.6953	0.7844	0.8224	0.8513	0.8669	0.9094
1	0.5953	0.7053	0.8032	0.8503	0.8794	0.8976	0.9436

Table 9: Estimated Expected Values of MLE for samples when the ML algorithm converged.

	$n=50$	$n=100$	$n=200$	$n=300$	$n=400$	$n=500$	$n=1000$
-0.99	0.1027	0.0187	0.0032	0.0014	0.0006	0.0003	0.0001
-0.95	0.0658	0.0094	0.0011	0.0006	0.0004	0.0004	0.0002
-0.9	0.0466	0.0056	0.0019	0.0014	0.0011	0.0009	0.0004
-0.85	0.0320	0.0065	0.0030	0.0022	0.0016	0.0013	0.0006
-0.8	0.0282	0.0078	0.0047	0.0030	0.0022	0.0017	0.0007
-0.75	0.0258	0.0105	0.0057	0.0038	0.0026	0.0021	0.0009
-0.7	0.0259	0.0140	0.0071	0.0044	0.0031	0.0025	0.0012
-0.65	0.0358	0.0177	0.0081	0.0051	0.0039	0.0028	0.0013
-0.6	0.0427	0.0207	0.0096	0.0058	0.0042	0.0033	0.0016
-0.55	0.0464	0.0239	0.0104	0.0067	0.0050	0.0038	0.0018
-0.5	0.0498	0.0274	0.0116	0.0079	0.0054	0.0044	0.0021
-0.45	0.0584	0.0297	0.0137	0.0084	0.0061	0.0048	0.0024
-0.4	0.0645	0.0344	0.0143	0.0094	0.0068	0.0053	0.0028
-0.35	0.0751	0.0360	0.0162	0.0104	0.0076	0.0058	0.0029
-0.3	0.0810	0.0384	0.0174	0.0111	0.0083	0.0063	0.0033
-0.25	0.0926	0.0424	0.0196	0.0123	0.0090	0.0073	0.0036
-0.2	0.1006	0.0478	0.0203	0.0131	0.0097	0.0077	0.0039
-0.15	0.1069	0.0489	0.0223	0.0143	0.0105	0.0083	0.0041
-0.1	0.1121	0.0508	0.0231	0.0155	0.0115	0.0088	0.0044
-0.05	0.1202	0.0578	0.0261	0.0169	0.0120	0.0094	0.0047
0	0.1232	0.0594	0.0272	0.0178	0.0131	0.0104	0.0047
0.05	0.1321	0.0630	0.0295	0.0189	0.0141	0.0108	0.0056
0.1	0.1388	0.0711	0.0311	0.0210	0.0147	0.0115	0.0061
0.15	0.1465	0.0709	0.0326	0.0214	0.0156	0.0127	0.0064
0.2	0.1459	0.0702	0.0360	0.0230	0.0165	0.0134	0.0067
0.25	0.1542	0.0785	0.0369	0.0246	0.0184	0.0135	0.0066
0.3	0.1626	0.0763	0.0386	0.0259	0.0190	0.0153	0.0075
0.35	0.1636	0.0817	0.0399	0.0266	0.0199	0.0153	0.0081
0.4	0.1719	0.0837	0.0426	0.0283	0.0202	0.0166	0.0085
0.45	0.1720	0.0826	0.0430	0.0299	0.0220	0.0168	0.0081
0.5	0.1723	0.0837	0.0430	0.0309	0.0222	0.0180	0.0090
0.55	0.1891	0.0825	0.0432	0.0313	0.0239	0.0185	0.0094
0.6	0.1967	0.0855	0.0431	0.0312	0.0230	0.0195	0.0094
0.65	0.1994	0.0893	0.0441	0.0309	0.0237	0.0192	0.0096
0.7	0.2220	0.0941	0.0451	0.0306	0.0229	0.0191	0.0106
0.75	0.2243	0.0952	0.0440	0.0318	0.0227	0.0185	0.0107
0.8	0.2409	0.0961	0.0462	0.0329	0.0245	0.0196	0.0104
0.85	0.2583	0.1105	0.0506	0.0338	0.0241	0.0191	0.0102
0.9	0.2825	0.1190	0.0567	0.0368	0.0286	0.0216	0.0111
0.95	0.3093	0.1288	0.0633	0.0408	0.0306	0.0244	0.0116
1	0.3384	0.1520	0.0737	0.0479	0.0346	0.0281	0.0135

Table 10: Estimated Mean Square Errors of MLE for samples when the ML algorithm converged.

	$n = 50$	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 1000$
-0.99	-0.7298	-0.8865	-0.9515	-0.9557	-0.9658	-0.9685	-0.9694
-0.95	-0.7493	-0.8896	-0.9443	-0.9539	-0.9550	-0.9532	-0.9488
-0.9	-0.7605	-0.8854	-0.9127	-0.9157	-0.9156	-0.9133	-0.9088
-0.85	-0.7550	-0.8502	-0.8715	-0.8682	-0.8667	-0.8647	-0.8585
-0.8	-0.7231	-0.8037	-0.8201	-0.8162	-0.8164	-0.8145	-0.8091
-0.75	-0.6919	-0.7589	-0.7644	-0.7642	-0.7615	-0.7608	-0.7567
-0.7	-0.6578	-0.7071	-0.7146	-0.7125	-0.7093	-0.7055	-0.7031
-0.65	-0.6134	-0.6612	-0.6614	-0.6577	-0.6559	-0.6556	-0.6506
-0.6	-0.5713	-0.6147	-0.6120	-0.6045	-0.6026	-0.6020	-0.5982
-0.55	-0.5362	-0.5623	-0.5593	-0.5524	-0.5521	-0.5495	-0.5469
-0.5	-0.4905	-0.5173	-0.5062	-0.5028	-0.5000	-0.4995	-0.4947
-0.45	-0.4509	-0.4670	-0.4565	-0.4512	-0.4527	-0.4477	-0.4434
-0.4	-0.4014	-0.4152	-0.4066	-0.4032	-0.3992	-0.3966	-0.3942
-0.35	-0.3520	-0.3649	-0.3570	-0.3542	-0.3495	-0.3501	-0.3465
-0.3	-0.3138	-0.3198	-0.3087	-0.3032	-0.3027	-0.3003	-0.2961
-0.25	-0.2659	-0.2717	-0.2575	-0.2526	-0.2499	-0.2498	-0.2478
-0.2	-0.2219	-0.2183	-0.2082	-0.2055	-0.2015	-0.2002	-0.1998
-0.15	-0.1842	-0.1806	-0.1564	-0.1576	-0.1562	-0.1519	-0.1477
-0.1	-0.1408	-0.1317	-0.1103	-0.1048	-0.1065	-0.1040	-0.0999
-0.05	-0.1022	-0.0824	-0.0687	-0.0614	-0.0571	-0.0563	-0.0518
0	-0.0580	-0.0352	-0.0177	-0.0154	-0.0087	-0.0084	-0.0046
0.05	-0.0180	0.0138	0.0322	0.0398	0.0421	0.0431	0.0453
0.1	0.0254	0.0592	0.0794	0.0848	0.0916	0.0947	0.0988
0.15	0.0475	0.1045	0.1307	0.1330	0.1334	0.1384	0.1408
0.2	0.1011	0.1493	0.1719	0.1785	0.1874	0.1899	0.1922
0.25	0.1344	0.1906	0.2236	0.2330	0.2362	0.2378	0.2420
0.3	0.1596	0.2305	0.2737	0.2760	0.2799	0.2833	0.2923
0.35	0.1902	0.2641	0.3201	0.3293	0.3316	0.3404	0.3416
0.4	0.2154	0.3130	0.3573	0.3726	0.3835	0.3862	0.3891
0.45	0.2424	0.3476	0.4027	0.4172	0.4274	0.4327	0.4361
0.5	0.2677	0.3846	0.4478	0.4650	0.4744	0.4761	0.4870
0.55	0.2930	0.4153	0.4849	0.5063	0.5203	0.5263	0.5345
0.6	0.3296	0.4470	0.5247	0.5512	0.5635	0.5750	0.5862
0.65	0.3547	0.4759	0.5576	0.5902	0.6074	0.6157	0.6366
0.7	0.3613	0.4965	0.5983	0.6306	0.6512	0.6594	0.6860
0.75	0.3886	0.5305	0.6253	0.6629	0.6870	0.7004	0.7253
0.8	0.4075	0.5599	0.6555	0.6997	0.7190	0.7364	0.7777
0.85	0.4228	0.5669	0.6839	0.7235	0.7518	0.7772	0.8220
0.9	0.4275	0.5917	0.6939	0.7520	0.7775	0.8050	0.8535
0.95	0.4436	0.6076	0.7248	0.7801	0.8092	0.8305	0.8943
1	0.4637	0.6129	0.7449	0.8061	0.8335	0.8589	0.9235

Table 11: Estimated Expected Values of $f\left(\widehat{\beta}_3\right)$ for samples when the ML algorithm converged.

	$n=50$	$n=100$	$n=200$	$n=300$	$n=400$	$n=500$	$n=1000$
-0.99	0.1132	0.0329	0.0100	0.0075	0.0052	0.0040	0.0021
-0.95	0.0851	0.0263	0.0100	0.0064	0.0045	0.0034	0.0017
-0.9	0.0655	0.0223	0.0099	0.0064	0.0045	0.0035	0.0017
-0.85	0.0553	0.0216	0.0100	0.0063	0.0046	0.0037	0.0017
-0.8	0.0501	0.0204	0.0100	0.0062	0.0046	0.0035	0.0018
-0.75	0.0455	0.0214	0.0096	0.0064	0.0045	0.0036	0.0016
-0.7	0.0419	0.0212	0.0100	0.0065	0.0048	0.0037	0.0017
-0.65	0.0479	0.0231	0.0103	0.0066	0.0050	0.0037	0.0018
-0.6	0.0498	0.0231	0.0108	0.0068	0.0051	0.0040	0.0018
-0.55	0.0508	0.0249	0.0112	0.0073	0.0054	0.0042	0.0021
-0.5	0.0516	0.0264	0.0116	0.0081	0.0056	0.0045	0.0022
-0.45	0.0567	0.0273	0.0133	0.0083	0.0061	0.0048	0.0025
-0.4	0.0590	0.0300	0.0135	0.0090	0.0066	0.0053	0.0028
-0.35	0.0629	0.0317	0.0150	0.0098	0.0072	0.0056	0.0028
-0.3	0.0679	0.0333	0.0163	0.0106	0.0080	0.0061	0.0033
-0.25	0.0739	0.0368	0.0182	0.0116	0.0088	0.0070	0.0036
-0.2	0.0816	0.0411	0.0187	0.0127	0.0094	0.0076	0.0038
-0.15	0.0871	0.0423	0.0212	0.0138	0.0102	0.0083	0.0041
-0.1	0.0895	0.0446	0.0222	0.0151	0.0116	0.0089	0.0047
-0.05	0.0994	0.0503	0.0252	0.0168	0.0123	0.0097	0.0051
0	0.1049	0.0536	0.0272	0.0181	0.0138	0.0110	0.0053
0.05	0.1133	0.0578	0.0303	0.0199	0.0152	0.0122	0.0064
0.1	0.1170	0.0673	0.0321	0.0224	0.0162	0.0129	0.0071
0.15	0.1271	0.0678	0.0345	0.0239	0.0175	0.0148	0.0076
0.2	0.1316	0.0715	0.0393	0.0256	0.0193	0.0159	0.0082
0.25	0.1414	0.0779	0.0410	0.0276	0.0220	0.0167	0.0088
0.3	0.1471	0.0805	0.0432	0.0306	0.0235	0.0192	0.0104
0.35	0.1584	0.0881	0.0473	0.0332	0.0250	0.0202	0.0109
0.4	0.1738	0.0911	0.0508	0.0342	0.0267	0.0218	0.0122
0.45	0.1758	0.0950	0.0534	0.0385	0.0297	0.0240	0.0122
0.5	0.1873	0.0990	0.0551	0.0412	0.0305	0.0252	0.0134
0.55	0.2075	0.1038	0.0585	0.0432	0.0338	0.0276	0.0145
0.6	0.2237	0.1129	0.0619	0.0455	0.0341	0.0292	0.0162
0.65	0.2350	0.1215	0.0655	0.0473	0.0366	0.0308	0.0161
0.7	0.2615	0.1312	0.0702	0.0487	0.0377	0.0316	0.0176
0.75	0.2767	0.1408	0.0737	0.0534	0.0396	0.0325	0.0191
0.8	0.2979	0.1456	0.0798	0.0562	0.0438	0.0357	0.0193
0.85	0.3330	0.1728	0.0864	0.0605	0.0453	0.0363	0.0203
0.9	0.3686	0.1881	0.1018	0.0674	0.0530	0.0438	0.0228
0.95	0.4025	0.2117	0.1133	0.0755	0.0590	0.0484	0.0224
1	0.4310	0.2401	0.1274	0.0848	0.0668	0.0547	0.0273

Table 12: Estimated Mean Square Errors of $f\left(\widehat{\beta}_3\right)$ for samples when the ML algorithm converged.

	$n = 50$	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 1000$
-0.99	-0.9886	-0.9818	-0.9804	-0.9790	-0.9769	-0.9744	-0.9710
-0.95	-0.9741	-0.9635	-0.9628	-0.9589	-0.9580	-0.9533	-0.9491
-0.9	-0.9358	-0.9269	-0.9223	-0.9179	-0.9165	-0.9134	-0.9088
-0.85	-0.8955	-0.8812	-0.8764	-0.8691	-0.8671	-0.8648	-0.8585
-0.8	-0.8524	-0.8298	-0.8224	-0.8165	-0.8167	-0.8145	-0.8091
-0.75	-0.7996	-0.7774	-0.7656	-0.7647	-0.7614	-0.7608	-0.7566
-0.7	-0.7507	-0.7227	-0.7158	-0.7125	-0.7093	-0.7055	-0.7031
-0.65	-0.6980	-0.6713	-0.6620	-0.6578	-0.6559	-0.6556	-0.6506
-0.6	-0.6429	-0.6229	-0.6118	-0.6046	-0.6025	-0.6021	-0.5982
-0.55	-0.5964	-0.5697	-0.5595	-0.5522	-0.5522	-0.5494	-0.5469
-0.5	-0.5478	-0.5219	-0.5062	-0.5026	-0.4998	-0.4993	-0.4946
-0.45	-0.5037	-0.4701	-0.4565	-0.4510	-0.4526	-0.4475	-0.4433
-0.4	-0.4425	-0.4174	-0.4066	-0.4031	-0.3990	-0.3964	-0.3942
-0.35	-0.3913	-0.3667	-0.3572	-0.3542	-0.3495	-0.3499	-0.3465
-0.3	-0.3485	-0.3203	-0.3084	-0.3028	-0.3026	-0.3003	-0.2959
-0.25	-0.2955	-0.2733	-0.2570	-0.2523	-0.2500	-0.2495	-0.2477
-0.2	-0.2472	-0.2183	-0.2078	-0.2050	-0.2013	-0.2001	-0.1993
-0.15	-0.2011	-0.1806	-0.1558	-0.1569	-0.1556	-0.1517	-0.1478
-0.1	-0.1577	-0.1310	-0.1096	-0.1041	-0.1058	-0.1034	-0.1000
-0.05	-0.1135	-0.0817	-0.0675	-0.0607	-0.0566	-0.0561	-0.0519
0	-0.0658	-0.0337	-0.0173	-0.0148	-0.0085	-0.0083	-0.0049
0.05	-0.0224	0.0154	0.0327	0.0397	0.0416	0.0426	0.0451
0.1	0.0288	0.0610	0.0797	0.0847	0.0915	0.0940	0.0985
0.15	0.0539	0.1067	0.1308	0.1329	0.1333	0.1384	0.1405
0.2	0.1148	0.1519	0.1722	0.1783	0.1870	0.1898	0.1922
0.25	0.1558	0.1970	0.2239	0.2326	0.2356	0.2375	0.2413
0.3	0.1913	0.2409	0.2745	0.2760	0.2799	0.2832	0.2919
0.35	0.2336	0.2777	0.3227	0.3287	0.3309	0.3402	0.3414
0.4	0.2741	0.3302	0.3618	0.3735	0.3835	0.3862	0.3889
0.45	0.3134	0.3716	0.4089	0.4189	0.4282	0.4326	0.4358
0.5	0.3405	0.4211	0.4569	0.4692	0.4753	0.4767	0.4874
0.55	0.3837	0.4576	0.5009	0.5134	0.5244	0.5287	0.5351
0.6	0.4258	0.5042	0.5485	0.5620	0.5726	0.5789	0.5871
0.65	0.4795	0.5471	0.5892	0.6076	0.6186	0.6241	0.6390
0.7	0.4954	0.5860	0.6426	0.6582	0.6689	0.6725	0.6917
0.75	0.5359	0.6355	0.6867	0.6985	0.7122	0.7210	0.7326
0.8	0.5750	0.6778	0.7301	0.7501	0.7597	0.7675	0.7919
0.85	0.6101	0.7078	0.7786	0.7909	0.8073	0.8175	0.8412
0.9	0.6270	0.7468	0.8086	0.8397	0.8504	0.8608	0.8849
0.95	0.6580	0.7893	0.8512	0.8773	0.8957	0.9046	0.9346
1	0.7101	0.8207	0.8940	0.9340	0.9403	0.9504	0.9736

Table 13: Estimated Expected Values of $f\left(\widehat{\beta}_3\right)$, whole simulated samples.

	$n = 50$	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 1000$
-0.99	0.0519	0.0223	0.0100	0.0065	0.0046	0.0038	0.0021
-0.95	0.0512	0.0228	0.0101	0.0062	0.0045	0.0035	0.0017
-0.9	0.0533	0.0231	0.0103	0.0065	0.0046	0.0035	0.0017
-0.85	0.0530	0.0228	0.0102	0.0064	0.0047	0.0037	0.0017
-0.8	0.0538	0.0224	0.0101	0.0062	0.0046	0.0035	0.0018
-0.75	0.0536	0.0231	0.0097	0.0064	0.0045	0.0036	0.0016
-0.7	0.0520	0.0230	0.0101	0.0065	0.0047	0.0037	0.0017
-0.65	0.0568	0.0243	0.0104	0.0066	0.0049	0.0037	0.0018
-0.6	0.0589	0.0243	0.0109	0.0068	0.0051	0.0040	0.0018
-0.55	0.0606	0.0261	0.0112	0.0073	0.0054	0.0043	0.0021
-0.5	0.0628	0.0273	0.0116	0.0081	0.0056	0.0046	0.0022
-0.45	0.0688	0.0286	0.0133	0.0083	0.0060	0.0048	0.0025
-0.4	0.0698	0.0307	0.0135	0.0090	0.0066	0.0053	0.0028
-0.35	0.0747	0.0323	0.0149	0.0098	0.0072	0.0056	0.0028
-0.3	0.0789	0.0338	0.0163	0.0107	0.0080	0.0061	0.0033
-0.25	0.0838	0.0377	0.0183	0.0117	0.0088	0.0071	0.0036
-0.2	0.0932	0.0416	0.0187	0.0127	0.0095	0.0076	0.0038
-0.15	0.0986	0.0429	0.0212	0.0138	0.0102	0.0083	0.0041
-0.1	0.1025	0.0451	0.0221	0.0151	0.0116	0.0089	0.0047
-0.05	0.1119	0.0506	0.0251	0.0167	0.0123	0.0097	0.0051
0	0.1181	0.0538	0.0270	0.0180	0.0137	0.0110	0.0053
0.05	0.1242	0.0586	0.0302	0.0198	0.0152	0.0121	0.0065
0.1	0.1350	0.0682	0.0322	0.0223	0.0162	0.0130	0.0071
0.15	0.1421	0.0687	0.0344	0.0237	0.0175	0.0148	0.0075
0.2	0.1487	0.0722	0.0394	0.0257	0.0192	0.0159	0.0082
0.25	0.1570	0.0804	0.0411	0.0276	0.0220	0.0167	0.0088
0.3	0.1685	0.0847	0.0436	0.0307	0.0236	0.0192	0.0104
0.35	0.1783	0.0923	0.0481	0.0333	0.0250	0.0202	0.0109
0.4	0.1940	0.0970	0.0526	0.0345	0.0268	0.0219	0.0122
0.45	0.1937	0.1008	0.0555	0.0391	0.0300	0.0241	0.0122
0.5	0.2016	0.1066	0.0579	0.0423	0.0309	0.0254	0.0135
0.55	0.2198	0.1130	0.0622	0.0447	0.0346	0.0279	0.0146
0.6	0.2289	0.1205	0.0666	0.0476	0.0364	0.0299	0.0162
0.65	0.2378	0.1286	0.0695	0.0500	0.0388	0.0323	0.0163
0.7	0.2529	0.1360	0.0761	0.0518	0.0401	0.0331	0.0184
0.75	0.2638	0.1414	0.0795	0.0562	0.0422	0.0346	0.0196
0.8	0.2725	0.1433	0.0829	0.0590	0.0470	0.0380	0.0205
0.85	0.2897	0.1562	0.0865	0.0625	0.0479	0.0385	0.0213
0.9	0.3072	0.1605	0.0936	0.0663	0.0527	0.0431	0.0240
0.95	0.3212	0.1701	0.0977	0.0667	0.0541	0.0462	0.0237
1	0.3280	0.1826	0.0995	0.0717	0.0576	0.0474	0.0267

Table 14: Estimated Mean Square Errors of $f\left(\widehat{\beta}_3\right)$, whole simulated samples.

	$n = 50$	$n = 100$	$n = 200$	$n = 300$	$n = 400$	$n = 500$	$n = 1000$
-0.9900	0.0519	0.0222	0.0099	0.0064	0.0044	0.0036	0.0017
-0.9500	0.0506	0.0226	0.0099	0.0061	0.0044	0.0035	0.0017
-0.9000	0.0520	0.0224	0.0098	0.0062	0.0043	0.0033	0.0016
-0.8500	0.0509	0.0218	0.0095	0.0060	0.0044	0.0035	0.0016
-0.8000	0.0511	0.0215	0.0096	0.0059	0.0043	0.0033	0.0017
-0.7500	0.0511	0.0223	0.0095	0.0062	0.0044	0.0035	0.0016
-0.7000	0.0494	0.0225	0.0099	0.0063	0.0046	0.0037	0.0017
-0.6500	0.0545	0.0238	0.0103	0.0065	0.0049	0.0037	0.0018
-0.6000	0.0571	0.0238	0.0108	0.0068	0.0051	0.0040	0.0018
-0.5500	0.0584	0.0257	0.0111	0.0073	0.0054	0.0043	0.0021
-0.5000	0.0605	0.0268	0.0116	0.0081	0.0056	0.0046	0.0022
-0.4500	0.0659	0.0282	0.0133	0.0083	0.0060	0.0048	0.0025
-0.4000	0.0680	0.0304	0.0135	0.0090	0.0066	0.0053	0.0028
-0.3500	0.0730	0.0320	0.0148	0.0098	0.0072	0.0056	0.0028
-0.3000	0.0765	0.0334	0.0162	0.0107	0.0080	0.0061	0.0033
-0.2500	0.0817	0.0372	0.0183	0.0117	0.0088	0.0071	0.0036
-0.2000	0.0910	0.0413	0.0186	0.0127	0.0095	0.0076	0.0038
-0.1500	0.0960	0.0420	0.0212	0.0138	0.0102	0.0083	0.0041
-0.1000	0.0992	0.0441	0.0220	0.0151	0.0116	0.0089	0.0047
-0.0500	0.1079	0.0496	0.0248	0.0166	0.0123	0.0097	0.0051
0.0000	0.1138	0.0527	0.0267	0.0178	0.0136	0.0109	0.0053
0.0500	0.1190	0.0574	0.0299	0.0197	0.0151	0.0120	0.0065
0.1000	0.1299	0.0667	0.0318	0.0221	0.0161	0.0130	0.0071
0.1500	0.1329	0.0668	0.0340	0.0234	0.0172	0.0147	0.0074
0.2000	0.1414	0.0699	0.0386	0.0252	0.0190	0.0158	0.0081
0.2500	0.1481	0.0776	0.0404	0.0273	0.0218	0.0165	0.0087
0.3000	0.1567	0.0812	0.0429	0.0301	0.0232	0.0189	0.0103
0.3500	0.1648	0.0871	0.0474	0.0328	0.0246	0.0201	0.0108
0.4000	0.1781	0.0921	0.0511	0.0338	0.0265	0.0217	0.0121
0.4500	0.1750	0.0947	0.0538	0.0381	0.0295	0.0238	0.0120
0.5000	0.1762	0.1004	0.0560	0.0414	0.0303	0.0249	0.0133
0.5500	0.1921	0.1045	0.0598	0.0434	0.0339	0.0274	0.0144
0.6000	0.1986	0.1113	0.0639	0.0462	0.0356	0.0295	0.0160
0.6500	0.2087	0.1180	0.0658	0.0482	0.0378	0.0316	0.0162
0.7000	0.2110	0.1230	0.0728	0.0501	0.0391	0.0323	0.0183
0.7500	0.2180	0.1283	0.0755	0.0535	0.0408	0.0338	0.0193
0.8000	0.2219	0.1284	0.0780	0.0565	0.0454	0.0369	0.0204
0.8500	0.2321	0.1360	0.0814	0.0590	0.0461	0.0374	0.0212
0.9000	0.2327	0.1370	0.0852	0.0627	0.0502	0.0416	0.0238
0.9500	0.2359	0.1443	0.0879	0.0614	0.0512	0.0441	0.0235
1.0000	0.2440	0.1505	0.0883	0.0673	0.0540	0.0449	0.0260

Table 15: Estimated Variance of $f\left(\widehat{\beta}_3\right)$, whole simulated samples.

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