Temi di Discussione
(Working Papers)

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by Fabio Busetti and Silvestro di Sanzo
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BOOTSTRAP LR TESTS OF STATIONARITY, COMMON TRENDS AND COINTEGRATION

by Fabio Busetti*, Silvestro di Sanzo+

Abstract

The paper considers likelihood ratio (LR) tests of stationarity, common trends and cointegration for multivariate time series. As the distribution of these tests is not known, a bootstrap version is proposed via a state space representation. The bootstrap samples are obtained from the Kalman filter innovations under the null hypothesis. Monte Carlo simulations for the Gaussian univariate random walk plus noise model show that the bootstrap LR test achieves higher power for medium-sized deviations from the null hypothesis than a locally optimal and one-sided LM test, that has a known asymptotic distribution. The power gains of the bootstrap LR test are significantly larger for testing the hypothesis of common trends and cointegration in multivariate time series, as the alternative asymptotic procedure - obtained as an extension of the LM test of stationarity- does not possess properties of optimality. Finally, it is showed that the (pseudo) LR tests maintain good size and power properties also for non-Gaussian series. As an empirical illustration, we find evidence of two common stochastic trends in the volatility of the US dollar exchange rate against european and asian/pacific currencies.

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1 Introduction

A locally optimal and one-sided $LM$ test for the presence of a random walk component in an otherwise white noise series has been derived by Nyblom and Makelainen (1983) and subsequently extended to the case weakly dependent data by Kwiatkowski et al. (1992) and Leybourne and McCabe (1994). This is commonly known as 'stationarity test' or 'KPSS test'; its limiting representation under the null hypothesis is well-known and it is denoted as Cramer-Von Mises distribution. Nyblom and Harvey (2000) generalize the (locally optimal) stationarity test to multivariate time series and they derive a related statistic for testing the hypothesis of common stochastic trends and cointegration; the asymptotic distributions and critical values are provided.

In this paper we re-consider these issues from a likelihood ratio ($LR$) perspective. The testing problem is non-standard with the parameters of interest lying on the boundary of the parameter space. As the distribution of the $LR$ test is not known, we propose a bootstrap approximation, based on a state space representation where the bootstrap samples are obtained from the Kalman filter innovations under the null hypothesis. Our procedure is based on the parametric bootstrap proposed by Stoffer and Wall (1991, 2004) constructed by resampling from the Kalman filter innovations. Although they do not directly deal with coefficients at the boundary of the parameter space, their approach is suited for tests of the null hypothesis of stationarity, as the underlying model can be easily cast in state space form.

Bootstrap resampling methods are becoming part of the standard toolkit of applied econometricians and practitioners, in all contexts of parameter estimation, testing and forecasting; see Horowitz (1997), Berkowitz and Kilian (2000), Li and Maddala (1996), MacKinnon (2006), Davidson and MacKinnon (2006) for comprehensive surveys with emphasis on econometrics.

Bootstrap tests, similar to the approach taken in this paper, have been proposed by Franco et al. (1999) for the univariate random walk plus noise and the local linear trend model. In contrast, our study is specifically concerned with testing for common trends and cointegration in multivariate time series, instances where the advantage of LR approach is more relevant as the alternative asymptotic procedure does not possess properties of optimality. On the other hand, for a univariate trend plus cycle model, we show good size properties for the bootstrap LR test even for the case of high persistence in the short-term component, when it is known that the KPSS stationarity test runs into difficulties. A recent paper by Morley and Sinclair (2009) also deals with bootstrap stationarity tests, but only for univariate series; their algorithm
however is not apparently obtained within a state space framework.\textsuperscript{1} Note that, as pointed out by Andrews (2000), while the standard bootstrap technique may not be appropriate for estimation of coefficients that lie on the boundary of the parameter space, bootstrap tests are however expected to be consistent and to achieve the correct size.

The use of the likelihood principle seems attractive in non standard situations, e.g. when the asymptotic equivalence among Wald, LM and LR tests fail. In particular, for the null hypothesis of common trends and cointegration, the bootstrap $LR$ test is expected to achieve high power as the competing test of Nyblom and Harvey (2000), with a known limiting representation, is no longer a locally optimal test. Our Monte Carlo results confirm that this is the case; on the other hand, for a simple univariate random walk plus noise model, the advantage of the bootstrap $LR$ test is significant for medium-size deviations from the null hypothesis of stationarity while, in the case of a random walk trend plus $AR(1)$ cycle, the performance of the bootstrap $LR$ test is significantly superior, also in the neighborhood of the null hypothesis, than all other options.

One important feature of the bootstrap $LR$ test is that it requires fitting a parametric model to the data, thus by-passing the delicate issue of bandwidth choice to deal with short range dependence; see e.g. Lee (1996) and Caner and Kilian (2001). One advantage is that it automatically accommodates any specific characteristics of the series like the presence of stochastic cycles and/or seasonal components with unit roots; however this comes to the cost of satisfactory model specification.

Finally, it is showed that the (pseudo) LR tests maintain good size and power properties also for non-Gaussian series. As an empirical illustration, we find evidence of two common stochastic trends in the volatility of the US dollar exchange rates against the euro, the pound, the yen and the Australian dollar.

The paper proceeds as follows. Section 2 reviews the locally optimal test of stationarity and its extension to the hypothesis of common trends and cointegration. Section 3 discusses the bootstrap approximation for the LR test, based on the state space representation. In section 4 the results of Monte Carlo simulation experiments are presented to evaluate the size and power properties of the bootstrap LR test and compare them with those of the locally optimal and related tests. Section 5 provides an empirical illustration. Finally, section 6 concludes hinting at possible

\textsuperscript{1}As regards the related literature on testing for unit roots, bootstrap methods have been investigated in Nankervis and Savin (1996), Ferretti and Romo (1996) and Burridge and Taylor (2004), among others; a common feature with our tests is that resampling is done under the null hypothesis.
extensions. Appendix contains all the tables and figures.

2 The locally optimal test of stationarity and its extension to common trends and cointegration

The basic set up of stationary tests is the following unobserved component model consisting of a random walk plus noise:

\[ y_t = \mu_t + \epsilon_t \quad \epsilon_t \sim NID(0, \Sigma_\epsilon) \]  
\[ \mu_t = \mu_{t-1} + \eta_t, \quad \eta_t \sim NID(0, \Sigma_\eta), \quad t = 1, \ldots, T, \]

(1)
(2)

where \( y_t \) is a vector containing \( N \) time series, which is made up of a trend \( \mu_t \) and an irregular component \( \epsilon_t \). \( \Sigma_\epsilon \) and \( \Sigma_\eta \) are \( N \times N \) positive definite matrix. The model is sometimes known as 'local level model' and it is thoroughly analysed in Harvey (1989) and Durbin and Koopman (2001), inter alia. If \( \Sigma_\eta = 0 \) the series is a white noise that fluctuates around a constant level; otherwise it has a stochastic trend component. Testing the hypothesis of stationarity is non-standard as the parameters lie on the boundary of the parameter space. Nyblom and Harvey (2000, NH henceforth) have provided a locally optimal invariant (LBI) and one-sided LM test of the hypothesis \( H_0 : \Sigma_\eta = 0 \) against the 'homogenous' alternative \( H_1 : \Sigma_\eta = q\Sigma_\epsilon \), where \( q \) is the signal-to-noise ratio; the results for a univariate series, \( N = 1 \), were obtained earlier in Nyblom and Makelainen (1983). The statistic is

\[ \xi_N = tr\left[ S^{-1} C \right], \]

(3)

where \( C = T^{-2} \sum_{t=1}^{T} \left[ \sum_{i=t}^{i=t} (y_t - \bar{y}) \right] \left[ \sum_{t=1}^{i=t} (y_t - \bar{y}) \right]' \) and \( S = T^{-1} \sum_{t=1}^{T} (y_t - \bar{y})(y_t - \bar{y})' \). The test rejects the null hypothesis that \( q = 0 \) for \( \xi_N > \kappa \), where \( \kappa \) is an appropriate critical value. The limiting null distribution of \( \xi_N \) is Cramèr-von-Mises with \( N \) degrees of freedom; the asymptotic critical values are provided. Although the test is derived to maximise the local power against a homogenous alternative, it is consistent for any \( \Sigma_\eta > 0 \).

When \( \epsilon_t \) is a weakly dependent process the statistic can be modified along the lines of Kwiatkowski et al. (1992), the so-called 'KPSS' stationarity test. This is obtained by replacing the sample variance \( S \) with a non-parametric estimate of the long-run variance \( S(m) = \sum_{\tau=-m}^{\tau=m} w_{\tau m} \hat{\Gamma}(\tau) \),

where \( \hat{\Gamma}(\tau) = T^{-1} \sum_{t=\tau+1}^{T} (y_t - \bar{y})(y_{t-\tau} - \bar{y})' \) is the autocovariance matrix at
lag $\tau$ and $w_{\tau m}$ is a weighting function, such as $w_{\tau m} = 1 - \tau/(m + 1)$, $\tau = 1, \ldots, m$. The null limiting distribution and critical values of the modified statistic are unchanged. Alternatively, weak dependence in the data can be accounted for by a parametric correction as suggested, for example, in Leybourne and McCabe (1994).

NH have then extended the locally optimal and $LM$ test of stationarity to the case of cointegration. The null hypothesis is that there are $k < N$ common stochastic trends, $H_0 : \text{rank}(\Sigma_\eta) = k$, against the alternative that the time series has a larger number of non-stationary components $H_1 : \text{rank}(\Sigma_\eta) > k$. The test statistic is simply the sum of the $N - k$ smallest eigenvalues of $S^{-1}C$,

$$\zeta_{k,N} = \sum_{j=k+1}^{N} \lambda_j,$$

where $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_N \geq 0$ are the ordered eigenvalues. Note that $\zeta_{0,N} = \xi_N$. The presence of common stochastic trends implies cointegration, i.e. the existence of stationary linear combinations of the series. In fact, this test can also be viewed as taking the minimum of the stationarity test statistic over all possible $N - k \times N$ cointegration matrices $A$, that is $\zeta_{k,N} = \min_A \text{tr} [(ASA')^{-1} ACA']$. Asymptotic critical values are tabulated in NH against a set of $(k, N)$ pairs. If $k > 0$, $\zeta_{k,N}$ no longer defines a 'locally optimal' test that maximizes the slope of the power function at the null hypothesis.

3 The state-space representation and the bootstrap LR tests

The random walk plus noise data generating process (1)-(2) underlying the tests of stationarity, common trends and cointegration is a simple case of linear Gaussian state-space model. A more general model may include deterministic terms (like a fixed slope of the trend) and additional unobserved components, e.g. representing business cycle or seasonal fluctuations. The state space representation and the Kalman filter are the basis for the statistical treatment of linear time series models defined in terms of unobserved components; see e.g. Harvey (1989), Hamilton (1994), Durbin and Koopman (2001).

A state-space model is defined by the equations, $t = 1, 2, \ldots, n$,

$$s_t = Fs_{t-1} + w_t$$

$$y_t = Hs_t + v_t$$

where $y_t$ is a $N$-dimensional observation vector and $s_t$ is a $p$-dimensional state vector with a given (fixed or random) initial condition $s_0$. The
constant matrices $F$ and $H$ represent the model coefficients of dimension compatible with the matrix operations required in (5)-(6). Equation (5) is known as the transition equation, while (6) is the measurement equation. The $(p \times 1)$ vector $w_t$ and the $(N \times 1)$ vector $v_t$ are white noises with

\[ E(w_t w'_{\tau}) = Q \quad \text{for } t = \tau \text{ and } 0 \text{ otherwise,} \quad (7) \]
\[ E(v_t v'_{\tau}) = R \quad \text{for } t = \tau \text{ and } 0 \text{ otherwise,} \quad (8) \]

where $Q$ and $R$ are $(p \times p)$ and $(N \times N)$ matrices, respectively. The disturbances are assumed to be uncorrelated at all lags, $E(w_t w'_{\tau}) = 0$ for all $t$ and $\tau$. The model coefficients and the correlation structure are assumed to be uniquely parametrized by a $K \times 1$ vector $\Theta$; thus, $F = F(\Theta), H = H(\Theta), Q = Q(\Theta), R = R(\Theta)$. The $\Theta$ is assumed to be an element of some compact space, $\Psi$, usually a subset of $R^K$.

For the local level model (1)-(2) we have $F = H = I_N$, the identity matrix, $R = \Sigma_\varepsilon, Q = \Sigma_\eta, \Theta = (\Sigma_\varepsilon, \Sigma_\eta)$. The parameters under the null hypothesis are $\Theta_0 = (\Sigma_\varepsilon, 0)$ for the case of stationarity and $\Theta_0 = (\Sigma_\varepsilon, \Sigma_\eta)$ for the case of common trends/cointegration where $\Sigma_\eta$ is a matrix of rank $k, 0 < k < N$. Under the alternative hypothesis the model is estimated leaving the rank of $\Sigma_\eta$ unrestricted.

The Kalman filter is given by the following equations

\[ \epsilon_t = y_t - Hs_{t|t-1} \quad (9) \]
\[ \Sigma_t = HP_{t|t-1}H' + R \quad (10) \]
\[ K_t = P_{t|t-1}H'\Sigma_t^{-1} \quad (11) \]
\[ P_{t|t-1} = FP_{t-1|t-1}F' + Q \quad (12) \]
\[ s_{t|t-1} = Fs_{t-1|t-1} \quad (13) \]
\[ s_{t|t} = s_{t|t-1} + K\epsilon_t \quad (14) \]
\[ P_{t|t} = P_{t|t-1} - P_{t|t-1}H'\Sigma_t^{-1}HP_{t|t-1} \quad (15) \]

From the Kalman filter recursions one also obtains the innovation form representation of the model,

\[ s_{t+1|t} = Fs_{t|t-1} + FK_t \epsilon_t, \quad (16) \]
\[ y_t = Hs_{t|t-1} + \epsilon_t, \quad (17) \]

that will be used to construct the bootstrap data set.

For stationary $s_t$, the unconditional mean and covariance matrix of $s_t$ may be employed as the initial values, $s_{1|0}$ and $P_{1|0}$. When the transition equation is not stationary, the unconditional distribution of the state
vector is not defined. In this case, unless genuine prior information is available, the initial distribution of $s_t$ must be specified in terms of a diffuse or non-informative prior. If we write $P_{1|0} = kI$ where $k$ is a positive scalar, the diffuse prior is obtained as $k \to \infty$. Setting $k$ equal to a large but finite number a good approximation can be obtained. If some elements of state vector are stationary and some non-stationary, the stationary part of the model is initialised in the Kalman filter by its unconditional mean and covariance matrix, while the non-stationary part is initialised with a diffuse prior; see e.g. Harvey (1989).

Estimation of the model parameters is accomplished by maximizing the (Gaussian) likelihood function. The log-likelihood, written in terms of the Kalman filter innovations $\epsilon_t$, is

\[
L_Y(\Theta) = -\frac{1}{2} \sum_{t=1}^{n} (\ln |\Sigma_t| + \epsilon'_t \Sigma_t^{-1} \epsilon_t),
\]

which, in general, is maximized over $\Theta$ by numerical and iterative methods.

The LR test requires estimating the model under both the null hypothesis of stationarity (or cointegration) and the alternative one where there are no restrictions on the variance of random walk component. The LR statistic is

\[
LR = -2 \left[ L_Y(\hat{\Theta}_0) - L_Y(\hat{\Theta}_1) \right],
\]

where $\hat{\Theta}_0$ and $\hat{\Theta}_1$ denote maximum likelihood estimates of the parameters under, respectively, the null and the alternative hypotheses. If the model disturbances are non-Gaussian, then the statistic defines a Pseudo Likelihood Ratio test (PLR).

The bootstrap algorithm for approximating the distribution of (18) and computing the p-value of the test consists of the following steps:

**Step 1** Construct the standardized Kalman filter innovations under $H_0$,

\[ e_t = \hat{\Sigma}_t^{-1/2} \epsilon_t, \]

where $\epsilon_t$, $\hat{\Sigma}_t$ are obtained by evaluating the Kalman filter recursions (9)-(15) at $\hat{\Theta}_0$. $\hat{\Sigma}_t^{-1/2}$ denotes the inverse of the square root matrix of $\hat{\Sigma}_t$ defined by $\hat{\Sigma}_t^{1/2} \hat{\Sigma}_t^{1/2'} = \hat{\Sigma}_t$.

**Step 2** Obtain the bootstrap errors $\{ \epsilon^*_t, t = 1, ..., T \}$ by sampling, with replacement, from the set of standardized innovations $\{ e_t, t = 1, ..., T \}$.

**Step 3** Construct the bootstrap data set under $H_0$, $\{ y^*_t, t = 1, ..., T \}$, by plugging in the bootstrap errors in the innovation form representation.
\begin{align*}
  s^*_{t+1|t} &= \hat{F} s^*_{t|t-1} + \hat{F} \hat{K}_t \hat{\Sigma}_t^{1/2} e^*_t, \\
  y^*_t &= \hat{H} s^*_{t|t-1} + \hat{\Sigma}_t^{1/2} e^*_t,
\end{align*}

where \( \hat{F} \), \( \hat{H} \), \( \hat{K}_t \) are obtained by the Kalman filter at \( \hat{\Theta}_0 \). The initial condition \( s^*_{1|0} \) is kept fixed throughout the bootstrap replications.

\textbf{(Step 4)} Compute the LR statistic, \( LR^b \), using the bootstrap data \( \{y^*_t, t = 1, \ldots, T\} \)

\textbf{(Step 5)} Repeating steps 2 through 4 for \( b = 1, \ldots, B \), gives a set of values \( \{LR^b : b = 1, \ldots, B\} \) that mimics a random sample of draws from the distribution of \( LR \) under \( H_0 \). The bootstrap \( p \)-value of the LR test is therefore \( p_B = \text{card}(LR^b \geq LR)/B \), that is the fraction of \( LR^b \) values that are greater than the observed value \( LR \).

\section{Monte Carlo evaluation of the properties of the tests}

This section evaluates the properties of the bootstrap LR test of stationarity, common trends and cointegration and compares them with those of the locally optimal and related tests through a series of Monte Carlo experiments. The basic set-up is the data generating process (1)-(2). A simple trend plus cycle model where the short run component an AR(1) is also considered. Finally, we analyze the behavior of the tests for non-Gaussian series with thick tails and skewed distributions. The results are obtained with 1000 bootstrap replications over 10000 Monte Carlo simulations. All the procedures for estimating the models described in this section were written in GAUSS programming language and maximization was carried out by BFGS algorithm.

\subsection{Univariate models}

We first consider the univariate local level models for a range of values of signal-to-noise ratio \( q^2 = \frac{c^2}{T} \), with \( c = 0, 2.5, 5, 10, 25 \); the case \( c = 0 \) corresponds to the null hypothesis, while \( c > 0 \) allows to evaluate the power under the (local) alternative. The variance of the noise is set to \( \sigma^2 = 1 \).

Table 1 contains the percentage rejection frequencies of the locally optimal (\( LBI \)) test, the bootstrap \( LR \) test and a bootstrap version of the \( LBI \) test (obtained from the bootstrap distribution of the \( \xi_1 \) statistic (3)) for a sample of \( T = 25, 50, 100 \) observations and for a nominal size of \( \alpha = 0.05, 0.10 \). Consider first the results for \( T = 25 \). Notwithstanding we are using the asymptotic critical values, the size of the \( LBI \) test is very
near the nominal one while the power closely corresponds to the limiting power function as computed by Tanaka (1996). Overall the performance of the bootstrap $LR$ test seems superior: though slightly oversized, it displays non-negligible power gains for medium sized deviations from the null ($c \geq 10$), while the rejection frequencies are comparable with those of the $LBI$ test for lower $c$. It is also interesting to see that the bootstrap distribution of the $LBI$ test closely replicates the limiting true distribution, which can be seen as an indirect confirmation of the correctness of the bootstrap procedure. The results for bigger sample sizes $T = 50$ and $T = 100$ are very similar to those for $T = 25$; this is clearly seen as we are presenting simulated rejection frequencies under the local alternative hypothesis, with the signal-to-noise ratio being a function of the sample size. Note however that if we keep the signal-to-noise ratio fixed the power increases with the sample size, which is a reflection of the consistency of the tests; for example if $\sigma^2_\epsilon = 0.01$ the power of the bootstrap $LR$ test is $20.4, 38.4, 72.4$ for $T = 25, 50, 100$ respectively.

Tables 2-4 present results for a simple stochastic trend plus cycle model, where the data generating process is obtained by replacing the white noise disturbance $\epsilon_t$ in (1)-(2) with the AR(1) process, $u_t = \rho u_{t-1} + \omega_t$, $\omega_t \sim NID(0, \sigma^2_\omega)$. The data are simulated for a range of values of signal-to-noise ratio $\frac{\sigma^2_\omega}{\sigma^2_\epsilon} = \frac{c^2}{T^2}$, with $c = 0, 2.5, 5, 10, 25, 50$, and for autoregressive parameter $\rho \in (0.5, 0.8, -0.5)$. Percentage rejection frequencies of tests run at nominal significance level $\alpha = 10\%$ are computed for: (a) the $KPSS(m)$ stationarity test where $m$ is the number of autocovariances used to compute the non-parametric correction for serial correction, (b) a parametric variant of the $LBI$ test constructed from the Kalman filter innovations of a fitted trend plus cycle model (as suggested in Busetti and Harvey, 2003, in the context of testing for seasonal stability), (c) the bootstrap $LR$ test, (d) a bootstrap version of the $LBI$ test, obtained from the bootstrap distribution of the statistic $\xi_1$ (3). The $KPSS(m)$ statistics are computed across four values of the lag truncation parameter $m(0)$, $m(4)$, $m(8)$ and $m(12)$, where, following Kwiatkowski et al. (1992), $m(x)$ is given by the formula $m(x) = \text{integer}(x(T/100)^{1/4})$; note that $KPSS(m(0))$ corresponds to the $LBI$ statistic $\xi_1$. The choice of lag truncation parameter reflects a trade-off between size and power; in general, higher $m$ corresponds to better size properties but lower power for the test.

Consider first Table 2 where $\rho = 0.5$. The $KPSS$ test is strongly

\footnote{As the $LBI$ test maximizes the slope of the power function at the null hypothesis, no gains were expected for the bootstrap $LR$ test when $c$ is small.}
over-sized for $m(x) \leq m(4)$ while the parametric $LBI$ test appears undersized in the small samples of $T = 25$ and $T = 50$ observations, i.e. the limiting null distribution does not appear to provide an adequate approximation in these cases. On the other hand, the bootstrap tests appear to control size rather well, except for some undersizing in the small sample of $T = 25$ observations. More interesting are the power properties of the tests. Near the null hypothesis, e.g. when $c \leq 5$, the $KPSS(m(12))$, the bootstrap $LBI$ and the parametric $LBI$ tests behave rather similarly, even though the latter is somehow penalized by being undersized. Large gains in terms of power can be achieved with the bootstrap $LR$ test: e.g. for $c = 2.5$ and $T = 25$, the power of $LR$ is 32% against around 13% for the $KPSS(m(12))$ and 8% for the parametric $LBI$ test. Farther away from the null hypothesis the bootstrap $LR$ test confirms its neat advantage over the others, with the parametric $LBI$ test ranking second. It is also interesting to notice that, in contrast with the local level model, the simulated small sample power of $LR$ does not provide a good approximation to the limiting power function: for $c > 10$ the power figures are quite different for the three cases of $T = 25, 50, 100$.

As the degree of serial correlation gets higher ($\rho = 0.8$, cf. table 3) the $KPSS$ test becomes less reliable; see Caner and Kilian (2001) among else. On the other hand, the bootstrap $LR$ test still works well and it is overall preferable to the parametric $LBI$ test, although both are somehow undersized. Finally, table 4 contains the results for $\rho = -0.5$. The size of the $LR$ test is very close to the nominal 10% also for a sample of $T = 25$ observations, while the $KPSS$ test now suffers from undersizing. As concerns power, the relative ranking of $LR$, parametric $LBI$ and $KPSS$ is maintained as in the previous experiments.

Finally, we evaluate the properties of the bootstrap $PLR$ test for non-Gaussian series. For the case of thick tails, table 5 reports the results for a random walk plus noise models where the errors are generated by a $t$-distributions with 5 degrees of freedom. The figures can be compared directly with the basic case of table 1, as the $t(5)$ errors are rescaled such that the signal-to-noise ratio is $q^2 = \frac{c^2}{T}$, with $c = 0, 2.5, 5, 10, 25$. The advantage of the bootstrap $PLR$ test over the $LBI$ test is greater than in the Gaussian case; for example for $\alpha = 5\%$, $c = 25$ and $T = 100$ the simulated rejection probabilities are 94% for the $PLR$ test (very close to the figure in table 1) and 70% for the $LBI$ test (against 88% obtained under Gaussianity). Table 6 considers instead the case of skewness in the noise component, which is generated from a skewed-Normal distribution with two different values of the shape parameter, $\lambda = 2$ and $\lambda = 5$ where higher $\lambda$ implies more asimmetry ($\lambda = 0$ is the Gaussian case); see Azzalini (1985). The results, reported for a nominal size of 5%, indicate
again a clear advantage for the bootstrap PLR test; as expected, higher skewness corresponds to lower power of the tests.

4.2 Multivariate models: null hypothesis of stationarity

Here we consider testing the null hypothesis of stationarity with the LBI and the bootstrap LR tests in the multivariate local level model (1)-(2) with $\sum_\eta = q \sum_\varepsilon$ for a range of values of signal-to-noise ratio $q^2 = \frac{\sigma^2}{\psi^2}$, $c = 0, 1, 2.5, 5, 10, 25$; given the invariance properties of the tests with respect to non-singular transformations, we set $\sum_\varepsilon$ as the identity matrix. In table 7 we present results for $N = 1, 2, 3, 4$ and for a sample size of $T = 100$ observations; the significance level is 5%.

Both tests are able to control size well for each $N$ when $T = 100$; unreported simulations show a slight oversizing of the bootstrap LR test for smaller samples. The findings of the univariate model are confirmed also as regards to power: the tests behave broadly the same for small values of $c$ while the bootstrap LR test has a non-negligible advantage for medium-sized deviations from the null hypothesis (e.g. when $c = 10$). Note that -as in the univariate case- the LBI test is locally optimal for this data generating process. As expected, the power of the test increases when $N$ is higher; for example if $c = 5$ the rejection frequencies of the tests are equal to about 30% for $N = 1$ and to 70% for $N = 4$.

4.3 Multivariate models: null hypothesis of common trends and cointegration

We compare the properties of the test (4), denoted as $NH$, and the bootstrap $LR$ test for the null hypothesis of $k$ common trends in the context of "balanced growth" cointegration for the multivariate local level model. More specifically, under the null hypothesis we assume $r = N - k$ cointegration relations with unit coefficients between the 1st and the 2nd, the first and the 3rd, ..., the first and the $r$-th series; for example for $k = 2$ and $N = 3$ we assume two cointegrating vectors equal to $(1, -1, 0)'$ and $(1, 0, -1)'$, which we collect in a $r \times N$ matrix $A$.

Allowing for a scale variance parameter $q$, under the null hypothesis the variance matrix of the disturbance driving the random walk component is therefore $\sum_\eta = q11'$, where $1$ is a $N \times r$ matrix of 1's; note that $A \sum_\eta = 0$; the elements on the main diagonal of the variance matrix of the noise $\sum_\varepsilon$ are set equal to 1, while the off-diagonal elements are drawn from a beta distribution.

Table 8 reports the results for the size of the tests run at 5% significance level, where the data generating process is simulated under
the null hypothesis of balanced growth, with \( k = 1, q = c^2/T^2, \ c = 0, 2.5, 5, 10, 25, \) and \( T = 100. \) The size properties of the NH and bootstrap LR tests are very similar. In particular, the rejection frequencies of the tests do not exceed the nominal level of 5\% even for relatively large values of the signal-to-noise ratio.

More interesting are the results in terms of power, that are contained in table 9 for \( N = 2, 3, 4 \) and \( 1 \leq k \leq N - 1. \) Here the data generating process is the same as in the previous sub-section, i.e. a full rank variance matrix of the disturbance driving the random walk component \( \sum_{\eta} = q \sum_e \) with \( q^2 = \frac{c^2}{T^2}. \) First note that the power of the tests of a given number \( k \) of common trends (or a given number \( r = N - k \) cointegrating relations) increases with \( N; \) similarly, for a given \( N, \) the power is lower for higher \( k. \) The important result is that the bootstrap LR test is now significantly more powerful than its competitor even near the null hypothesis; for example if \( c = 5, k = 1 \) and \( N = 4 \) the rejection frequencies are equal to about 40\% for the bootstrap LR and 24\% for the NH test, which is no longer locally optimal for testing common trends and cointegration.

5 Empirical illustration

As an illustration, we consider testing for common stochastic trends in the volatility of the daily exchange rates of the US dollar against the euro, the British pound, the yen and the Australian dollar over the period 2007-2009. Let \( x_{it}, i = 1, 2, 3, 4, \) be the exchange rate of the dollar against those four currencies. Figure 1 plots the logarithm of squared returns of the series, \( \log(y_{it}^2) \) where \( y_{it} = \log(x_{it}) - \log(x_{i,t-1}) \), that is the data transform\(^3\) for estimating stochastic variance models; see e.g. Harvey et al. (1994). There are clear comovements in the series, as seen in the graphed trends obtained by Quasi Maximum Likelihood estimation of the unrestricted multivariate local level model (1)-(2): in fact, the off-diagonal entries in the correlation matrix of the level disturbances take values ranging from 0.83 to 0.97.

As described in section 3, the PLR test is obtained comparing the Gaussian log-likelihood of the unrestricted model with the one imposing \( k \) common stochastic trends, \( k = 0, 1, 2, 3. \) Using our bootstrap procedure we strongly reject the null hypotheses \( k = 0 \) and \( k = 1 \) while we obtain a \( p \)-value of 0.563 for \( k = 2. \) The evidence thus suggests the presence of two common trends, perhaps associated with movements against european vs. asian/pacific currencies. Consistently with the

\(^3\)To be precise we adopt the slightly different transform \( \log(y_{it}^2 + .02s_t^2) - .02s_t^2 / (y_{it}^2 + .02s_t^2), \) where \( s_t^2 \) is the unconditional variance of \( y_{it}^2, i = 1, 2, 3, 4; \) see Fuller (1996, p. 494-497).
Monte Carlo simulations, less evidence against the null hypothesis is found using the NH test: the test statistic $\zeta_{1.4}$ (4) does not reject the null hypothesis $k = 1$ even at the 10% level of significance.

6 Concluding remarks

In this paper a bootstrap version of the LR test of stationarity, common trends and cointegration is proposed, based on fitting a state space model to the data. Monte Carlo simulations show that substantial power gains can be achieved with our test, in particular in the context of multivariate models and for data generating processes that are more elaborate than a Gaussian random walk plus noise model. The bootstrap LR test requires some model building effort compared with the KPSS-type tests where the dynamic properties of the series are handled non-parametrically; this effort is however compensated by good size properties and higher power.

It is straightforward to extend the bootstrap LR approach to testing stability at the seasonal frequencies. It just requires to estimate a model with a stochastic seasonal component and to obtain the bootstrap samples from the Kalman filter innovations under the null hypothesis of seasonal stability or seasonal cointegration; compare with Canova and Hansen (1995) and Busetti (2006), *inter alia*. Testing for stability of regression coefficients can also be framed in a state space framework, where time varying parameters may follow a random walk or an AR(1) model. A bootstrap LR test could therefore be devised and its properties evaluated against the standard tests of parameter stability, such as Nyblom (1989) and Andrews (1993). We leave these issues to future research.
References


### Tables and figures

#### Tab. 1. Monte Carlo rejection frequencies for the LBI, bootstrap LBI and the bootstrap LR tests of stationarity.

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#### Tab. 2. Monte Carlo rejection frequencies for the KPSS, the parametric LBI, the bootstrap LBI and the bootstrap LR tests of stationarity, ρ=0.5; the significance level is 10%.

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Tab. 1. Monte Carlo rejection frequencies for the LBI, bootstrap LBI and the bootstrap LR tests of stationarity.

Tab. 2. Monte Carlo rejection frequencies for the KPSS, the parametric LBI, the bootstrap LBI and the bootstrap LR tests of stationarity, ρ=0.5; the significance level is 10%. 
### Tab. 3. Monte Carlo rejection frequencies for the KPSS, the parametric LBI, the bootstrap LBI and the bootstrap LR tests of stationarity, $\rho=0.8$; the significance level is 10%.

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<td>c=2.5</td>
<td>15.86</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c=5</td>
<td>29.72</td>
<td>c=5</td>
<td>30.10</td>
<td>c=5</td>
<td>30.42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c=10</td>
<td>54.10</td>
<td>c=10</td>
<td>59.24</td>
<td>c=10</td>
<td>59.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c=25</td>
<td>69.31</td>
<td>c=25</td>
<td>75.16</td>
<td>c=25</td>
<td>77.18</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tab. 5. Monte Carlo rejection frequencies for the LBI, bootstrap LBI and the bootstrap LR tests of stationarity, t-distributions with 5 degrees of freedom.

<table>
<thead>
<tr>
<th></th>
<th>T=25</th>
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<th>T=50</th>
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<th>T=100</th>
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<tbody>
<tr>
<td>c=0</td>
<td>5.21</td>
<td>c=0</td>
<td>5.10</td>
<td>c=0</td>
<td>5.62</td>
</tr>
<tr>
<td>c=2.5</td>
<td>8.45</td>
<td>c=2.5</td>
<td>8.72</td>
<td>c=2.5</td>
<td>8.59</td>
</tr>
<tr>
<td>c=5</td>
<td>20.32</td>
<td>c=5</td>
<td>22.12</td>
<td>c=5</td>
<td>22.03</td>
</tr>
<tr>
<td>c=10</td>
<td>43.17</td>
<td>c=10</td>
<td>49.21</td>
<td>c=10</td>
<td>51.10</td>
</tr>
<tr>
<td>c=25</td>
<td>60.78</td>
<td>c=25</td>
<td>68.51</td>
<td>c=25</td>
<td>71.19</td>
</tr>
</tbody>
</table>

Tab. 6. Monte Carlo rejection frequencies for the LBI, bootstrap LBI and the bootstrap LR tests of stationarity, skewed normal distributions; the significance level is 5%.
<table>
<thead>
<tr>
<th>N=1</th>
<th>c=0</th>
<th>c=1</th>
<th>c=2.5</th>
<th>c=5</th>
<th>c=10</th>
<th>c=25</th>
</tr>
</thead>
<tbody>
<tr>
<td>LBI</td>
<td>5.46</td>
<td>8.04</td>
<td>12.62</td>
<td>29.62</td>
<td>58.71</td>
<td>87.93</td>
</tr>
<tr>
<td>LR/boot</td>
<td>5.53</td>
<td>8.72</td>
<td>13.08</td>
<td>31.40</td>
<td>65.20</td>
<td>95.59</td>
</tr>
<tr>
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<td>c=5</td>
<td>c=10</td>
<td>c=25</td>
<td></td>
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<tr>
<td>LBI</td>
<td>5.49</td>
<td>10.13</td>
<td>17.72</td>
<td>42.24</td>
<td>86.21</td>
<td>92.13</td>
</tr>
<tr>
<td>LR/boot</td>
<td>5.57</td>
<td>10.62</td>
<td>18.32</td>
<td>44.15</td>
<td>94.44</td>
<td>99.18</td>
</tr>
<tr>
<td>N=3</td>
<td>N=2</td>
<td>c=0</td>
<td>c=2.5</td>
<td>c=5</td>
<td>c=10</td>
<td>c=25</td>
</tr>
<tr>
<td>LBI</td>
<td>5.51</td>
<td>13.20</td>
<td>29.40</td>
<td>57.20</td>
<td>90.00</td>
<td>96.40</td>
</tr>
<tr>
<td>LR/boot</td>
<td>5.60</td>
<td>13.96</td>
<td>30.55</td>
<td>59.38</td>
<td>97.20</td>
<td>1.00</td>
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<td>c=5</td>
<td>c=10</td>
<td>c=25</td>
</tr>
<tr>
<td>LBI</td>
<td>5.53</td>
<td>17.18</td>
<td>37.23</td>
<td>68.10</td>
<td>93.30</td>
<td>99.81</td>
</tr>
<tr>
<td>LR/boot</td>
<td>5.62</td>
<td>17.77</td>
<td>38.04</td>
<td>70.02</td>
<td>99.04</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Tab. 7. Monte Carlo rejection frequencies for the LBI and the bootstrap LR tests of stationarity; the significance level is 5%.

<table>
<thead>
<tr>
<th>N=1</th>
<th>c=0</th>
<th>c=2.5</th>
<th>c=5</th>
<th>c=10</th>
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</thead>
<tbody>
<tr>
<td>NH</td>
<td>0.41</td>
<td>0.81</td>
<td>1.69</td>
<td>2.99</td>
</tr>
<tr>
<td>LR/boot</td>
<td>0.45</td>
<td>0.93</td>
<td>1.84</td>
<td>3.21</td>
</tr>
<tr>
<td>N=2</td>
<td>N=1</td>
<td>c=0</td>
<td>c=2.5</td>
<td>c=5</td>
</tr>
<tr>
<td>NH</td>
<td>0.69</td>
<td>1.08</td>
<td>1.94</td>
<td>3.25</td>
</tr>
<tr>
<td>LR/boot</td>
<td>0.74</td>
<td>1.19</td>
<td>2.03</td>
<td>3.42</td>
</tr>
<tr>
<td>N=3</td>
<td>N=2</td>
<td>c=0</td>
<td>c=2.5</td>
<td>c=5</td>
</tr>
<tr>
<td>NH</td>
<td>0.84</td>
<td>1.22</td>
<td>2.11</td>
<td>3.44</td>
</tr>
<tr>
<td>LR/boot</td>
<td>0.93</td>
<td>1.34</td>
<td>2.25</td>
<td>3.56</td>
</tr>
</tbody>
</table>

Tab. 8. Monte Carlo size for the NH and the bootstrap LR tests of cointegration, k=1; the significance level is 5%.

<table>
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<tr>
<th>N=1</th>
<th>c=5</th>
<th>c=10</th>
<th>c=20</th>
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<td>k=1</td>
<td>N=1</td>
<td>c=5</td>
<td>c=10</td>
<td>c=20</td>
</tr>
<tr>
<td>NH</td>
<td>5.05</td>
<td>25.22</td>
<td>70.82</td>
<td>90.74</td>
</tr>
<tr>
<td>LR/boot</td>
<td>7.74</td>
<td>36.97</td>
<td>84.29</td>
<td>96.44</td>
</tr>
<tr>
<td>N=2</td>
<td>k=1</td>
<td>N=1</td>
<td>c=5</td>
<td>c=10</td>
</tr>
<tr>
<td>NH</td>
<td>15.40</td>
<td>41.30</td>
<td>82.60</td>
<td>93.50</td>
</tr>
<tr>
<td>LR/boot</td>
<td>28.90</td>
<td>59.34</td>
<td>93.11</td>
<td>99.30</td>
</tr>
<tr>
<td>N=3</td>
<td>k=2</td>
<td>N=1</td>
<td>c=5</td>
<td>c=10</td>
</tr>
<tr>
<td>NH</td>
<td>24.45</td>
<td>57.70</td>
<td>90.48</td>
<td>98.66</td>
</tr>
<tr>
<td>LR/boot</td>
<td>39.70</td>
<td>73.29</td>
<td>98.70</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Tab. 9. Monte Carlo power for the NH and the bootstrap LR tests of cointegration; the significance level is 5%.
Fig. 1. Stochastic volatility transform of daily exchange rates of the US dollar against the Japanese yen, the Australian dollar, the euro, the British pound over the three years period 2007-2009.
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