BANCA D'ITALIA
EUROSISTEMA

Temi di Discussione

(Working Papers)

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RESERVE MANAGEMENT AND SOVEREIGN DEBT COST IN A WORLD WITH LIQUIDITY CRISES

by Flavia Corneli* and Emanuele Tarantino†

Abstract

The accumulation of large amount of sovereign reserves has fuelled an intense debate on the associated costs. In a world with liquidity crises and strategic default, we model a contracting game between international lenders and a country, which delivers the country's optimal portfolio choice and the cost of sovereign debt: at equilibrium, the sovereign allocates the borrowed resources to either liquid reserves or an illiquid and risky production project. We study how the opportunity cost of hoarding reserves is affected by the financial and technological characteristics of the economy. In line with recent empirical evidence, we find two important results: the cost of debt decreases in the level of reserves if the probability of liquidity shocks is high enough; however the cost of debt increases in reserves when the lenders anticipate that the country has an incentive to default after a liquidity shock. Indeed, we show that the country may choose to retain reserves instead of employing them to inject the liquidity needed to bring the production project to maturity.

JEL Classification: F34, F40.
Keywords: sovereign debt, international reserves, liquidity shock, strategic default.

Contents

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1 Introduction[1]

Reinhart and Rogoff (2010) document that after the Great Depression and consequent banking system crisis, a wave of sovereign debt defaults arose during the 1930s. To establish this domino effect as a stylized fact in contemporary history a piece is missing: the failure of major financial institutions in the recent crisis to trigger a series of sovereign debt crises in the years ahead. This threat calls for a reflection on the management of sovereign liquidity and could become a test for the strategy, mainly employed by emerging economies after the experience of the late '90s Asian crisis, of accumulating large amounts of sovereign reserves as buffer stock to face liquidity shocks.

In early 2000 this accumulation seemed to be justified by the self-insurance motive and to be broadly in line with rules of thumb like the "Guidotti-Greenspan rule";[2] however, the recent increase in the resources devoted to reserves has made it urgent to understand the costs associated with this build-up. Rodrik (2006) measures the opportunity cost of accumulating large amounts of liquid assets (in particular US treasury bonds) as the spread between external borrowing costs and reserves' returns. Rodrik (2006) compares this "self-insurance premium" with the expected cost of a financial crisis in terms of reduced output and concludes that in the last decade the amount of reserves accumulated by some countries has grown too much. In order to assess the opportunity cost of holding reserves, we believe that two further elements should be taken into account: the impact of reserves on future domestic output and on the cost of debt.

We analyse the opportunity cost of holding reserves in a world where the sovereign country is subject to liquidity crises and always has the option to default on debt. We develop a model of optimal portfolio choice where a country's resources are determined by a contracting game with international lenders, and characterize the equilibrium level of debt price, sovereign reserves and expected growth. Reserves are the country choice variable. Instead, the debt price is set by the lenders and the country's expected output results from the share of total borrowed resources that is not devoted to reserves.

The strategic interaction between the agents is shaped by two crucial financial frictions: lenders cannot inject the resources needed in the event of a liquidity shock and the country cannot commit not to default on debt.[3] These frictions introduce asset incompleteness into our model, and this is necessary to disentangle the relationship between the country's decision to default and the cost of sovereign debt. Indeed, if assets are not contingent, risk-neutral competitive lenders incorporate the probability of default in the premium set on the debt contract.

The optimal choice of reserves made by the country arises from the equilibrium between two forces: on the one hand, reserves are liquid assets that can be injected in the event of liquidity shocks and therefore help to avoid defaults (*precautionary motive*), on the other hand, they distract resources from a more productive but illiquid project. The country also takes into account the impact of reserves on the cost of debt; there are two conflicting effects at work: the positive one is that, by allowing the country to avoid default in the event of liquidity crises, reserves raise the probability for lenders to be repaid; the negative effect is that reserves are not pledgeable in the event of default and distract resources from the productive activity.

The novelty of our approach is that we do not assess whether the level of reserves is optimal in terms of its opportunity cost *per se*. We take a more general perspective, considering that when the country decides the optimal resource allocation it takes into account not only the opportunity cost of holding reserves in terms of the expected

[1]We thank Giancarlo Corsetti for invaluable discussions and advice. This paper also benefited from comments by Nicola Borri, Fabio Castiglionesi, Andrea Finicelli, Anton Korinek and Wolf Wagner. Finally, we thank the participants at the European University Institute and the Bank of Italy seminars. The views expressed are those of the authors and do not necessarily reflect those of the Bank of Italy. All errors are ours.

[2]To be consistent with this rule, reserves should be equivalent to the country's short-term external debt.

[3]These assumptions are consistent with several papers in the literature and in particular with Holmström and Tirole (1996, 1998), Caballero and Krishnamurty (2002), Caballero and Panageas (2005) and Lorenzoni (2008).

return of an alternative (illiquid) project, but also the effects of its choices on the price set by the lenders. In this way, we can address a number of policy relevant questions regarding the impact of reserves on the cost of sovereign debt and on the expected output.

We establish the following results. First of all, *when there is a high probability of liquidity crises, the cost of sovereign debt may decrease if the level of reserves is large enough*; second, *the country can decide to default at equilibrium if it is hit by an adverse shock and finds it optimal to behave this way.* We also analyse the effect of the chosen level of reserves and the cost of debt on the *sovereign expected output*: *we find that if the probability of a liquidity crisis is high enough, then larger reserves, by insuring the country against a shock, have a positive impact on expected output.*

Our framework allows us to analyse how the sovereign investment choices are affected by the financial and technological characteristics of the economy, namely the degree of riskiness of the production project and the level of the country's capital account openness. More specifically, we show that *the opportunity cost of holding reserves increases with the variance of the productive process* due a *limited liability effect*: when deciding how to invest its resources the country disregards the lower tail of output realization (since in those cases it would default), therefore the return on the productive project is increasing in the risk associated with it. Finally, we show that, *as the degree of sovereign capital account openness increases, larger capital outflows in the event of sudden stops increases the country's incentives to default in such events.*

We contribute to the literature that studies the optimal decision of sovereign countries to accumulate international reserves in the presence of strategic default on sovereign debt, like Alfaro and Kanczuk (2009). Alfaro and Kanczuk (2009) employ a setup in which the sovereign keeps reserves while losing part of the output in the event of strategic default on debt. In their model, the main upside of reserve accumulation is the possibility of employing them to smooth consumption. They introduce the possibility of sovereign liquidity crises in the form of contagion shocks (an abrupt variation in the interest rate) or sudden stops; however, in their setup reserves do not have a particular role in avoiding such crises, instead they are imperfect substitutes of external debt reduction for smoothing consumption. The main result they obtain is that the optimal policy of the country features nil international reserves and the intuition is that the country prefers to smooth consumption by lowering debt exposure instead of accumulating reserves. In our model, we study the effect of the country's willingness-to-repay concerns on the cost of debt by modelling the funding game between lenders and sovereign and obtain the rather different result that the optimal level of accumulated reserves can be positive at equilibrium. Remarkably, this is independent from the working of the precautionary motive. In our setting the sovereign solves the investment game by comparing the opportunity cost of reserves with respect to the productive activity: provided the former is low enough, then hoarding reserves is rational. On top of this, we perform an analysis of the role that reserves have in providing an insurance device against liquidity shocks and study whether self-insurance is preferable to a strategy that does not feature the injection of the necessary liquidity following a shock.

The level of resources available to the country for the investment game is not exogenous, because we endogenously determine the cost of sovereign debt. Instead, we take the level of outstanding liabilities as given and abstract from considerations about the possibility of reducing short-term debt in order to create liquidity and thus lower the impact of a shock. This assumption is consistent with the evidence in Rodrik (2006), which documents that, in recent years, emerging economies have not reduced their exposure to short-term debt, while accumulating large amounts of foreign liquid assets. Finally, we deliver our result by abstracting from the role that reserves have in stabilizing exchange rates and promoting exports (which instead is the main focus of Dooley, Folkerts-Landau and Garber, 2003).

We also contribute to the literature that studies optimal contractual arrangements in the presence of commitment problems and non-contingent contracts, as in Arellano (2008). In analogy to Arellano (2008), we show that default arises at equilibrium after an adverse shock occurs, consistently with the received empirical evidence. However, while Arellano (2008) studies the relationship between default risk and output, consumption and foreign debt, we look at the interaction between default risk and reserves, cost of debt and output.

We map the relationship between the cost of sovereign debt and the level of sovereign reserves: the amount of reserves hoarded by a country may reduce the cost of debt. In particular, this happens if the precautionary motive dominates. However, the opposite can occur: if the rationale behind the accumulation of liquid assets is opportunistic (that is, if the country prefers to keep reserves instead of employing them in the event of a liquidity shock), then an increase in their amount raises the cost of debt. This result, namely that the relationship between reserves and the cost of sovereign debt is not monotonic, is supported by two pieces of empirical evidence. Levy Yeyati (2008) finds in the data that holding reserves reduces the spread on the debt issued over the risk-free return on assets to the extent that reserves lower the probability of a run-induced default; moreover, the empirical analysis in Ruiz-Arranz and Zavadjil (2008) shows that the accumulation of reserves reduces the cost of borrowing only up to a threshold and concludes that the level of reserves observed in Asian countries is in line with an optimal insurance model.

The paper is organized as follows: in the next section we present the main features of the model. In Section 3 we solve for the equilibrium of the game. First, we analyse a benchmark framework without reserves, then we introduce the possibility for the country to (diversify its portfolio and) choose between investing in liquid reserves and an illiquid production project. In Section 4 we analyse the model's equilibrium by undertaking numerical simulations, while two possible extensions of the main setting are presented in Section 5: in the first we model the optimal investment choice of a risk-averse country, in the second we introduce a supranational organization, e.g. the IMF, that can intervene to finance the country in a liquidity crisis. Section 6 concludes.

2 The Model

Consider a sovereign country that needs to borrow D from international lenders. At stage $0a$, the lenders set the rate of return on the resources lent to the country and at stage $0b$, the country decides on the allocation of the same resources.[4] At stage 1, a liquidity shock may take place: if a shock occurs the country has to decide whether to default at stage 2; in the absence of the shock the game proceeds to stage 3. At stage 3 the productivity shock takes place and at stage 4 the sovereign can again choose whether to default on debt. Figures 1 and 2 illustrate, respectively, the timing of the game and the game-tree. We solve the model by backward induction and the equilibrium concept we employ is the Sub-game Perfect Nash Equilibrium (SPNE).

[FIGURES 1 AND 2 ABOUT HERE]

In this Section, we present in detail how we model each relevant node and the main ingredients of the game.

2.1 The Lending Game

Our economy is populated by a continuum of atomistic and *risk-neutral lenders* (indexed by $i \in I$), from which the country can borrow. Each of these lenders is able to lend D, so that the sovereign must borrow from a subset of mass 1 of them. The total mass of lenders is large, ensuring that perfect competition prevails and lenders do not extract any rent; moreover, the lenders are risk-neutral and have unlimited access to fund at the risk-less interest (that we assume to be nil).

[4]For ease of exposition, we present the borrowing decision and the investment decision in sequence. However, if the two games were solved simultaneously we would obtain the same results.

The lending game can be viewed as a general (common agency) contracting game between a principal (the sovereign) and multiple agents (the lenders):[5] as in Bolton and Jeanne (2007), investors participate in a bidding game following the sovereign's announcement of a fund raising goal of D. Lenders move first by each making a bid simultaneously. The sovereign then decides which bids to accept.

The lenders' utility is equal to the value of the repayment D discounted by the probability that the sovereign repays in full. At the bidding stage of the game, then, each lender i makes an offer on the rate of return, $r(i)$, that is required to break even in expectation. A lender i solves a problem of the following sort:

$$\begin{aligned} D &= D(1+r(i))Prob\{\text{The Country is Solvent}\} \iff \\ 1/(1+r(i)) &= Prob\{\text{The Country is Solvent}\} \iff \\ \delta(i) &\equiv Prob\{\text{The Country is Solvent}\}. \end{aligned}$$

Consequently, in the model the contract specifies the discount factor $\delta(i) = 1/(1+r(i))$ that a lender i asks in exchange for a loan D. A Nash equilibrium of the lending game is defined by a set of bids $(\delta(i))_{i\in I}$ such that, for all i, bid $\delta(i)$ maximizes lender i's utility taking all the other bids $\delta(j)$, with $j \neq i$, as given. Clearly, at equilibrium the sovereign squeezes all the surplus from the lending relationship and (randomly) selects among a set of identical bids, $\delta(j) = \delta(i) = \delta$, so *we can focus on a representative sovereign-lender pair.*

2.2 The Investment Game

The *country is risk-neutral* and can invest its funds in public expenditure, g, and/or reserves, R. Therefore, the sovereign feasibility constraint is given by

$$\delta D = g + R \iff g = \delta D - R. \tag{1}$$

When deciding resource allocation, the country maximizes its expected utility, denoted by $E(U(.))$: this is determined by the amount of liquid resources gathered and by the expected value of output. In the model, reserves (R) are a storage and liquid technology that can be carried over from the stage in which they are accumulated to the final stage of the game.

The production activity is the illiquid technology: it requires public expenditure (g) as sole input and is subject to a productivity shock, z. The assumption we make is that the output materializes at stage 4, after the country has decided on the allocation of its resources and uncertainty over the shocks occurs. More specifically, the output is generated by a production function, $Y(z, g)$, that is linearly affected by the productivity shock and that, using (1), can be rewritten as:

$$Y(z, g) = zY(g) = zY(\delta, R).$$

The shock z is such that $z \sim F(z)$, where $F(z)$, the cumulative distribution function, is twice differentiable and continuous and $f(z)$, the probability distribution function, is given by $dF(z)/dz = f(z)$. For the sake of simplicity, we assume that z follows a continuous uniform distribution over $[1-c, 1+c]$; however, our results hold under alternative assumptions on the distribution of z.[6]

Finally, the function $Y(\delta, R)$ is twice differentiable,[7] with $Y'(.) \geq 0$, $Y''(.) < 0$, it is increasing in δ, decreasing in R and satisfies standard Inada Conditions ($\lim_{g\to 0} Y(.) = 0$, $\lim_{g\to 0} Y'(.) = \infty$ and $\lim_{g\to +\infty} Y'(.) = 0$).

[5]See, for example, Bernheim and Whinston (1986a,b), Hart and Tirole (1990).

[6]In particular, Appendix D shows that the results of the model carry over under continuous distributions with exponential density functions, like the normal and log-normal distributions.

[7]We choose this compact notation instead of $Y(\delta D - R)$ to stress that the endogenous variables that affect $Y(.,.)$ are δ and R.

2.3 The Liquidity Shock

We assume that the country incurs in the risk of a liquidity shock at an intermediate stage, before the outcome of the production activity.

Following Chang and Velasco (2005) we assume that, with probability $\eta \in (0,1)$, the illiquid project needs a further infusion of capital ϵ at stage two in order to be completed.

If a liquidity shock occurs the country can only use the accumulated reserves to inject ϵ, while it cannot dismantle the capital invested in the production process. If it decides not to tackle the shock with the infusion of capital, the country defaults on the process and retains reserves.

2.4 The Default Decisions

There are two stages in the model at which the country may choose to default. The first is after the realization of the liquidity shock at stage two. The second is after the realization of the productivity shock at the final stage (when output is realized). In this way, we introduce the two main frictions of the model: the first is that the lender commits not to inject the resources needed by the country if a liquidity shock occurs. The second is that the country cannot undertake not to default when the realized output is lower than the face value of debt.[8]

In analogy to what is typically assumed in the literature (e.g. Bolton and Jeanne, 2007), the cost of default for the country consists of losing the entire output.[9] However, in the event of default the country keeps the reserves that it has accumulated, while the lender gets nothing.[10]

For simplicity, we distinguish between two plans (that we denote $\mathcal{F}$ and $\mathcal{N}$) depending on the country's choice at stage two to inject the needed liquidity should the shock occur. We find it a useful the distinction as it allows as to easily characterize all possible branches of the game-tree. Indeed, because of the financial friction of no commitment on the country side, at each stage the sovereign can decide to default if it is convenient to do so, i.e. if the expected utility from defaulting is greater that the one from continuation.[11]

[FIGURES 3 ABOUT HERE]

Figure 3 illustrates the branches of the game-tree corresponding to the two plans. Under plan $\mathcal{N}$ the country chooses to default if hit by the *liquidity shock*. Instead, under plan $\mathcal{F}$ the country chooses to face the shock, but for this decision to be credible it must also be feasible and time-consistent. Indeed, if at the investment game the country accumulates a level of reserves lower than ϵ then it cannot inject the needed capital to face the shock (resource constraint). Moreover, a problem of limited commitment may arise: if the country's payoff from the continuation of the game following the shock is less than ϵ then the country will prefer to default after the shock and retain reserves (no-default constraint).

Finally, the country may default after the realization of the *productivity shock*. In this case, the lender can pledge only a fraction of the output generated by the illiquid project, as in the models of funding in the presence of moral hazard (e.g. Holmström and Tirole, 1997).

2.5 A Discussion of Our Assumptions

To keep our model tractable and study the interaction between the effects that crucially determine the country's choices, we make a number of important assumptions.

[8] Or, equivalently, that the lender cannot act as residual claimant if realized output is lower than the face value of debt.

[9] See Alfaro and Kanczuk (2005) for a discussion on output losses in the event of default.

[10] This assumption is consistent with Alfaro and Kanczuk (2009).

[11] Notice, however, that the crucial decision at time zero is how to allocate resources between R and g, and the final equilibrium is always at most one.

First of all, we treat the level of the face value of debt D as an exogenous variable. Alfaro and Kanczuk (2009) show that debt and reserves can both be employed to smooth a country's utility, and therefore their levels are connected. However, Rodrik (2006) finds that emerging economies seem to be more active in deciding how to allocate resources and, in particular, in increasing the level of accumulated reserves; instead, the level of short-term debt has not changed much in time. In this sense, ours can be considered a short-term perspective theory, in which creditors set the debt price, either in the principal or secondary market, by looking at the economic fundamentals of each country and taking the amount of outstanding debt as given. Moreover, the fact that in the model the lender decides the debt discount factor (δ) implies that she influences the total amount of resources available for sovereign investment, and in this sense we develop a model with endogenous country resources.

In the setting presented above there are two shocks; at time two a liquidity shock may hit the country and at time three the productivity process z is realized. We assume that these two shocks are orthogonal to each other and to the main variables of the model (especially reserves). An alternative approach would be to assume that, if a liquidity shock occurs, output would be adversely affected, by the shock itself, but also by a common underlying process that guides the two uncertainties. Moreover, the liquidity and productive shock processes may be linked to the variables set at time zero: for example, empirical studies show that countries with higher reserves performed better during the recent financial crisis, both in terms of output and of lower exchange rate depreciation and capital flight. The analysis of the correlation among the shocks could lead to new dynamics and add interesting results to the model; however, we leave it for future research.

The interest rate on the safe assets bought to build up reserves is set to zero. Also, in the event of default the country loses the entire output. These two assumptions may look somewhat extreme; however, (i) they are not uncommon in the literature and (ii) we do believe that they can be introduced without loss of generality, as a more realistic parameterization would not change the nature of the results we obtain.

3 Solution of the Model

The analysis is conducted under the following simplifying technical restriction:

A1 : $|\epsilon_{Y',R}| > |\epsilon_{\bar{z},R}|$.

Assumption ($A1$) is needed to make sure that the solution to the country's maximization problem is unique and implies that the output is more sensitive to changes in reserves than the expected productivity level in the case of no default. More specifically, $\epsilon_{Y',R}$ represents the elasticity of the derivative of the production function to changes in the reserve level, while $\epsilon_{\bar{z},R}$ is the elasticity to changes in reserves of the default threshold $\bar{z}$, which will be defined in what follows.[12]

In order to better disentangle the impact of reserves in this setting, we first solve the model assuming that reserves cannot be accumulated: clearly, in this benchmark the country can never cover the liquidity shock. Then, we allow the country to decide how to allocate δD between g and R.

3.1 Benchmark without Reserves

If the country is hit by the shock in $t = 1$, then it is not able to repay ϵ and is already in default at $t = 1$. Conversely, if the country is not hit by the shock, then it defaults at the final stage if:

$$0 \geq zY(\delta) - D \iff z \leq \bar{z}(\delta) \equiv \frac{D}{Y(\delta)}.$$

[12] In Section 4, we perform a set of numerical simulations of the model and show that ($A1$) is naturally satisfied by a standard Cobb-Douglas production function.

$\bar{z}$ represents the threshold level of the productive process: if the realized productivity is higher, then the country repays its creditors and keeps the residual, otherwise the country defaults on debt and loses the entire output. This introduces a problem of limited liability on the country side, which influences the outcome of the investment game.

At the investment stage, resources are entirely invested in g. At $t = 0$, the lender sets the optimal δ as to break-even in expectation:

$$\delta D = D(1-\eta)\int_{\bar{z}(\delta)}^{\infty} dF(z) \iff \delta = (1-\eta)[1 - F(\bar{z}(\delta))] \equiv (1-\eta)G(\delta). \quad (2)$$

In the absence of reserves, the lender takes into account that the break-even is attained only if the liquidity shock does not take place, that is with probability $(1 - \eta)$. For convenience, when we solve for the lender's problem we denote by $G(.)$ the survival function and interpret it as the probability that the country is fully solvent after the productivity shock.

Lemma 1.

$$\exists! \quad \delta^* \in (0,1) \quad \text{s.t.} \quad \delta^* = (1-\eta)G(\delta^*).$$

Proof. See Appendix A.

[FIGURE 4(a) ABOUT HERE]

Figure 4(a) illustrates the result of Lemma 1. The deal signed at the initial stage between the lender and the sovereign can be implemented by a debt contract in which the lender earns $(D - \delta^* D)$ (or, equivalently, $Dr^*/(1+r^*)$, with $r/(1 + r) = 1 - \delta$) provided the country repays in full, zero otherwise.

3.2 Framework with Reserves

In what follows, we allow the country to accumulate reserves at the investment game stage.

If the liquidity shock does not occur at $t = 1$, then the model proceeds as in the framework without reserves and at $t = 4$ the country defaults after the realization of the productivity shock if the following condition holds:

$$zY(\delta, R) - D + R - \epsilon \leq R - \epsilon \iff z \leq \bar{z}(\delta, R) \equiv \frac{D}{Y(\delta, R)}.$$

Conversely, if the liquidity shock occurs at $t = 1$, we distinguish between two cases, depending on whether the country decides to face the liquidity shock (plan $\mathcal{F}$) or not (plan $\mathcal{N}$).

Plan $\mathcal{F}$

At $t = 4$ the country defaults after the realization of the productivity shock if the following condition holds:

$$zY(\delta, R) - D + R - \epsilon \leq R - \epsilon \iff z \leq \bar{z}(\delta, R) \equiv \frac{D}{Y(\delta, R)}.$$

At $t = 2$, the country would inject the new capital and go on with the project. Clearly, a necessary condition for the country to do so is that it accumulates enough liquidity. In other words, the outcome of the investment stage must be such that $R \geq \epsilon$ (*resource constraint*).

Moreover, we analyse whether the decision to inject liquidity after the shock is sub-game perfect. Given the choice on R by the country under plan $\mathcal{F}$ and the consequent contract offered at $t = 0$ by the lender, it must be that the sovereign has incentive to continue with the liquid project instead of defaulting strategically at $t = 2$, otherwise the same plan would not be time-consistent. The *no-default constraint* that has to hold follows:

$$\begin{aligned} R - \epsilon + \int_{\bar{z}(\delta,R)}^{\infty} [zY(\delta, R) - D]dF(z) &\geq R \iff \\ \int_{\bar{z}(\delta,R)}^{\infty} [zY(\delta, R) - D]dF(z) &\geq \epsilon. \end{aligned}$$

At $t = 0a$, the lender solves the following zero-profit condition:

$$\delta D = D \int_{\bar{z}(\delta,R)}^{\infty} dF(z) \iff \delta = 1 - F(\bar{z}(\delta, R)) \equiv G(\delta, R).$$

In this case, the formulation of the investor's problem takes into account that the sovereign does not default after a liquidity shock.

At $t = 0b$, the country decides to invest δD in g and/or R by solving the following problem:

$$\max_{R} E(U_F(\delta, R)) = R + \int_{\bar{z}(\delta,R)}^{\infty} [zY(\delta, R) - D]dF(z) - \eta\epsilon, \tag{3}$$

The country is certain to R, because reserves are not lost in the event of default. The second term in (3) is the expected value of the production project net of D: this second term is positive by construction, because the expected value of output is truncated downwards by $\bar{z}$. The third term in expression (3) is the expected value of the liquidity shock.

The maximization problem in (3) is solved under the aforementioned constraints:

$$\int_{\bar{z}(\delta,R)}^{\infty} [zY(\delta, R) - D]dF(z) \geq \epsilon, \tag{4}$$

$$R \geq \epsilon, \tag{5}$$

which make sure that the implementation of plan $\mathcal{F}$ is sub-game perfect.

Plan $\mathcal{N}$

At $t = 2$, if the sovereign chooses to default after the liquidity shock, it loses output but keeps the accumulated liquidity (R).

At $t = 0a$, the lender replies by setting δ as to break-even in expectation:

$$\delta D = D(1 - \eta) \int_{\bar{z}(\delta,R)}^{\infty} dF(z) \iff \delta = (1 - \eta)[1 - F(\bar{z}(\delta, R))] \equiv (1 - \eta)G(\delta, R).$$

In this case, the lender anticipates that the country defaults on the liquidity shock in $t = 2$.

At $t = 0b$, the country's choice of R is derived by solving the following problem:

$$\max_{R \in [0, \delta D]} E(U_N(\delta, R)) = R + (1 - \eta) \int_{\bar{z}(\delta,R)}^{\infty} [zY(\delta, R) - D]dF(z).$$

The decision to default on the liquidity shock implies two things: the first is that the sovereign obtains the net expected payoff from the production project only if exempted from the liquidity shock (with probability $1 - \eta$); the second is that the sovereign does not need to inject ϵ to bring the illiquid project to the end. Finally, the country gets R in all states of the world.

3.3 Equilibrium Definition

The equilibrium is defined by the vector $\{\delta, R\}$ such that the country's behaviour is optimal and the lender breaks even in expectation. Moreover, with liquidity shock coverage (that is, when plan $\mathcal{F}$ is chosen), the country's actions must be sub-game perfect, insofar as both the no-default constraint and the resource constraint must be satisfied.

Definition 1.
Define $\mathcal{F} \equiv \{\delta_F^{**}, R_F^{**}\}$ *as the pair that characterizes the plan in which the country decides to face the liquidity shock. At* $\mathcal{F} \equiv \{\delta_F^{**}, R_F^{**}\}$*, the resource constraint is satisfied*

$$R_F^{**} \geq \epsilon,$$

and the no-default constraint is satisfied

$$\int_{\bar{z}(\delta_F^{**}, R_F^{**})}^{\infty} [zY(\delta_F^{**}, R_F^{**}) - D] dF(z) \geq \epsilon.$$

Define $\mathcal{N} \equiv \{\delta_N^{**}, R_N^{**}\}$ *as the pair that characterizes the plan in which the country decides not to face the liquidity shock.*

The SPNE of the game is given by plan $\mathcal{F}$ *if* $E(U(\delta_F^{**}, R_F^{**})) \geq E(U(\delta_N^{**}, R_N^{**}))$ *and the relevant constraints are satisfied, otherwise plan* $\mathcal{N}$ *is chosen by the country.*

In the following, we first determine $\{\delta_N^{**}, R_N^{**}\}$ and $\{\delta_F^{**}, R_F^{**}\}$. Then, we analyse the country's choice between plan $\mathcal{N}$ and plan $\mathcal{F}$, and the features of the corresponding equilibrium.

We find that when the country accumulates reserves, the discount factor set by the lender is higher than in the benchmark without reserves if the country has an incentive to inject liquidity after the liquidity shock (that is, under plan $\mathcal{F}$) *and* the probability of the liquidity shock (η) is big enough. At the same time, a high enough value of η implies that the country's expected output is greater under plan $\mathcal{F}$ than under plan $\mathcal{N}$ and in the no-reserves case. Finally, we show that the lower the incidence of the liquidity shock (ϵ), the more likely it is that the country will ahve an incentive to face the shock (and that plan $\mathcal{F}$ will arise at equilibrium).

3.4 Equilibrium Analysis

First of all, we analyse country's decision to accumulate reserves and the consequent choice of the discount factor δ taken by the lender under plan $\mathcal{N}$.

Lemma 2.
Assume $(A1)$ holds. If

$$\lim_{R \to 0} 1 + (1 - \eta) \frac{\partial Y(\tilde{\delta}_N, R)}{\partial R} \int_{\bar{z}(\tilde{\delta}_N, R)}^{\infty} z dF(z) > 0 \tag{6}$$

is satisfied, then

$$\begin{aligned} &\exists! \quad \{\delta_N^{**}, R_N^{**}\} \quad \text{s.t.} \\ &R_N^{**} = \arg\max_R E(U_N(\delta_N^{**}, R)), \\ &\delta_N^{**} = (1 - \eta) G(\delta_N^{**}, R_N^{**}). \end{aligned}$$

Proof. See Appendix B.

Lemma 2 shows that there exists a unique equilibrium in pure strategies under plan $\mathcal{N}$, provided condition (6) is satisfied. We now turn to the determination of the value of reserves and discount factor under plan $\mathcal{F}$, that is when the country chooses to inject the needed liquidity after the shock ϵ occurs at stage $t = 1$.

Lemma 3.
Assume $(A1)$ holds. If

$$\lim_{R \to \epsilon} 1 + \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z dF(z) > 0 \tag{7}$$

then

$$\exists! \quad \{\delta_F^{**}, R_F^{**}\} \quad \text{with} \quad R_F^{**} \in [\epsilon, \delta_F^{**} D), \tag{8}$$

$$\int_{\bar{z}(\delta_F^{**}, R_F^{**})}^{\infty} [zY(\delta_F^{**}, R_F^{**}) - D] dF(z) > \epsilon, \tag{9}$$

$$\delta_F^{**} = G(\delta_F^{**}, R_F^{**}). \tag{10}$$

Moreover, if, together with (7), the following condition holds

$$\left(1 + \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} dF(z)\right)\Big|_{R = R_F^{**}} > 0, \tag{11}$$

then

$$\exists! \quad \{\delta_F^{**}, R_F^{**}\} \quad \text{such that}$$

$$\int_{\bar{z}(\delta_F^{**}, R_F^{**})}^{\infty} [zY(\delta_F^{**}, R_F^{**}) - D] dF(z) = \epsilon, \tag{12}$$

$$R_F^{**} \in (\epsilon, \delta_F^{**} D), \tag{13}$$

$$\delta_F^{**} = G(\delta_F^{**}, R_F^{**}), \tag{14}$$

Proof. See Appendix C.

Lemma 3 shows that, under the relevant conditions outlined above, there exists a unique equilibrium in pure strategies under plan $\mathcal{F}$ if and only if either the resource constraint or the no-default constraint binds. Instead, there is no Nash equilibrium in pure strategies if both the resource and the no-default constraints are binding.[13]

When deciding the optimal value of R, the country trades-off the marginal return of reserves, equal to 1, with the marginal return of the illiquid project, which is equal to $-\int_{\bar{z}(\delta,R)}^{\infty} z \frac{\partial Y(\delta,R)}{\partial R} dF(z)$ under plan $\mathcal{F}$ and $-(1-\eta)\int_{\bar{z}(\delta,R)}^{\infty} z \frac{\partial Y(\delta,R)}{\partial R} dF(z)$ under plan $\mathcal{N}$. Overall, then, the sovereign accumulates a positive level of reserves provided the marginal return of the illiquid project is low enough. However, the fact that under plan $\mathcal{N}$ the country reaches

[13] The intuition for this outcome is given in the following. If the resource constraint is binding, then the country would like to accumulate a lower level of reserves than the necessary injection, but the constraint satisfaction fixes R exactly at ϵ. In other words, the resource constraint sets a higher R than in an unconstrained optimum. The role of the no-default constraint is the opposite. Since the expected residual output after repaying the creditors (in the event of no default, that is the left-hand-side of the no-default constraint) is decreasing in R, a binding no-default constraint implies that although the country would like to accumulate a higher level of reserves, it has to reduce R in order to satisfy the no-default constraint with an equality. In other words, the impact of the level of reserves in the two constraints goes in two opposite directions: R should increase with respect to the unconstrained optimum in order to satisfy the resource constraint; R should decrease with respect to the unconstrained optimum to satisfy the no-default constraint. Overall, this implies that an equilibrium in which both constraints are contemporaneously binding cannot exist.

the final stage only if it is exempted from the liquidity shock (with probability $1-\eta$) implies that, *ceteris paribus*, the level of accumulated liquidity is higher there.

The country defaults when the value of the productivity shock z falls below $\bar{z}$, that is, it is *limitedly liable in the obligation to repay the lender*. This affects the sovereign decisions at the investment game, because, as will be further explored in the section on comparative statics, the expected return of the output is boosted by a higher volatility of the shock z, at the expense of the investment in the liquid asset (R).

3.4.1 The Impact of Reserves on the Discount Factor set by the Lender

The sovereign choice to inject liquidity after the shock occurs at $t=1$ affects the discount factor set by the lender. If the country decides to default at $t=2$ (as it does under plan $\mathcal{N}$) then the problem solved by the lender is analogous to the one in Lemma 1, with the difference that the value of reserves was nil there. The following lemma compares the value of the discount factor in the benchmark with the one that the country obtains when choosing plan $\mathcal{N}$.

Lemma 4. The optimal value of δ set by the lender in response to plan $\mathcal{N}$, δ_N^{**}, is lower than the one set in the benchmark without reserves, δ^*.

Without the reserves' insurance role, from the point of view of the lender the accumulation of liquid assets diverts resources from the production activity and increases the likelihood of sovereign default after the realization of the productivity shock. Consequently, the amount of resources available to the country to sustain sovereign investment production in plan $\mathcal{N}$ shrinks.[14]

If the country chooses to face the shock at $t=1$ (plan $\mathcal{F}$), then the impact on the problem solved by the lender is ambiguous, and eventually depends on η. With respect to the framework without reserves, on the one hand the accumulation of liquid assets reduces expected output, on the other hand it allows the country to face the liquidity shock successfully.

Below, we compare the discount factor set by the lender in plan $\mathcal{F}$ with the discount factor in the benchmark case without reserves.

Lemma 5.

$$\text{At} \quad \delta=\delta^*, R=R_F^{**}, \quad G(\delta^*, R_F^{**})) \geq (1-\eta)G(\delta^*) \iff$$

$$\eta \geq \eta^{**} \equiv \frac{F(\bar{z}(\delta^*, R_F^{**})) - F(\bar{z}(\delta^*))}{[1-F(\bar{z}(\delta^*))]} \in (0,1).$$

Proof. The proof follows by comparing $G(\delta^*, R_F^{**}))$ with $G(\delta^*)$.

Therefore, if the probability of a shock occurring is greater than η^{**}, the discount factor of debt with reserves is higher than in the case without reserves. Remember that, in the model, there is an inverse relationship between the discount factor δ and the rate of return r, since $\delta = 1/(1+r)$; consequently, if δ increases the rate of return decreases.

[FIGURES 4(b) and 4(c) ABOUT HERE]

[14]Remember that there is an inverse relationship between the discount factor and the rate of return asked by the lender: a higher discount factor is equivalent to a lower rate of return.

Lemma 5 is illustrated in Figure 4(b) and 4(c). More specifically, we show that when the probability attached to the occurrence of the liquidity shock (η) is high enough, then the discount factor is higher when the country accumulates reserves in excess of ϵ and injects liquidity (plan $\mathcal{F}$) than in the benchmark case without reserves (Figure 4c). The lender takes into account that reserves help the country to be solvent when the liquidity shock occurs and "prices" D accordingly. In the complementary case, the lender anticipates that the accumulated reserves are "too high" relative to the real danger of the shock occurring ($\eta < \eta^{**}$) and sets a lower discount factor (δ) under plan $\mathcal{F}$ with respect to the benchmark without reserves (see Figure 4b).

Proposition 1 summarizes the resulting impact of reserves on the lender's decisions under plan $\mathcal{F}$ and plan $\mathcal{N}$ with respect to the case without reserves.

Proposition 1.

- If the country does not face the liquidity shock, then $\delta^* > \delta_N^{**}$.
- If the country faces the liquidity shock, then the impact of the accumulation of reserves on the value of the discount factor δ depends on the probability of the liquidity shock, η:
  - If $\eta < \eta^{**}$ then $\delta^* > \delta_F^{**}$.
  - If $\eta \geq \eta^{**}$ then $\delta^* \leq \delta_F^{**}$.

Proof. See the discussion above.

In the following section, we study the impact of the accumulation of reserves on the country's expected output.

3.4.2 The Impact of Reserves on the Country's Expected Output

The decisions to accumulate reserves and face the liquidity shock affect the country's expected output in a way that is even clearer than in the case of the discount factor.

At equilibrium, the expected value of the production project in the benchmark without reserves is

$$(1-\eta)\int_{\bar{z}(\delta^*)}^{\infty} zY(\delta^*)dF(z).$$

In turn, the expected value of output in plan $\mathcal{N}$ is equal to

$$(1-\eta)\int_{\bar{z}(\delta_N^{**},R_N^{**})}^{\infty} zY(\delta_N^{**},R_N^{**})dF(z).$$

Finally, in the case of plan $\mathcal{F}$ it is given by

$$\int_{\bar{z}(\delta_F^{**},R_F^{**})}^{\infty} zY(\delta_F^{**},R_F^{**})dF(z).$$

First of all, we compare the outcomes of plan $\mathcal{N}$ with the benchmark, in terms of expected output.

Lemma 6. The expected value of output in plan $\mathcal{N}$ is lower than in the benchmark without reserves.

The result in Lemma 6 is a consequence of the result in Lemma 4, in which it is shown that when the country does not inject the capital needed to offset the liquidity shock (plan $\mathcal{N}$), the lender sets a discount factor that is lower than in the benchmark, implying that $\delta^* D > \delta_N^{**} D$. Moreover, in plan $\mathcal{N}$ the country invests in reserves, further reducing the resources allotted to the illiquid project.

In plan $\mathcal{F}$ the country is fully insured against the liquidity shock; instead, in the benchmark the country is certain to default if it is hit by the same shock. This consideration leads us to the following result.

Lemma 7. If $\eta > \hat{\eta}$, the expected value of output in plan $\mathcal{F}$ is larger than in the benchmark without reserves.

Summarizing the results in Lemmata 6 and 7, as η grows to 1 the expected value of production in the case where the country injects liquidity after the shock (plan $\mathcal{F}$) dominates the other two cases. This leads directly to Proposition 2 below.

Proposition 2.

- If the country does not face the liquidity shock, then

$$(1-\eta)\int_{\bar{z}(\delta^*)}^{\infty} zY(\delta^*)dF(z) > (1-\eta)\int_{\bar{z}(\delta_N^{**},R_N^{**})}^{\infty} zY(\delta_N^{**},R_N^{**})dF(z).$$

- If the country faces the liquidity shock, then the impact of the accumulation of reserves on the value of the expected output depends on the probability of the liquidity shock, η:
  - If $\eta < \hat{\eta}$ then $(1-\eta)\int_{\bar{z}(\delta^*)}^{\infty} zY(\delta^*)dF(z) > \int_{\bar{z}(\delta_F^{**},R_F^{**})}^{\infty} zY(\delta_F^{**},R_F^{**})dF(z)$.
  - If $\eta \geq \hat{\eta}$ then $(1-\eta)\int_{\bar{z}(\delta^*)}^{\infty} zY(\delta^*)dF(z) \leq \int_{\bar{z}(\delta_F^{**},R_F^{**})}^{\infty} zY(\delta_F^{**},R_F^{**})dF(z)$.

Proof. See the discussion above.

In Lemmata 2 and 3, we determined the pairs $\{\delta_N^{**}, R_N^{**}\}$ and $\{\delta_F^{**}, R_F^{**}\}$ that solve the country's maximization problem and the lender's zero-profit condition under plan $\mathcal{N}$ and plan $\mathcal{F}$, respectively. In Proposition 1 and Proposition 2 we analyse how the discount factor and the expected output are affected by the probability of a liquidity shock.

In the next section, we continue studying the country's choice at equilibrium by undertaking some comparative statics on the exogenous variables of the model.

3.4.3 Comparative Statics

The goal of the this section is to analyse the impact of other characteristics of the economy that have so far been kept fixed. We investigate the effects on the resource allocation – in reserves or public expenditure – of an increase in the variability of the investment project, and therefore its risk, and of an increase in the dimension of capital outflows in the event of a liquidity crisis, ϵ.

The level of reserves is optimally chosen by looking at the opportunity cost of holding them in terms of the alternative investment. The expected return of the investment in output is given by, for plan $\mathcal{F}$:

$$-\frac{\partial Y(\delta,R)}{\partial R}\int_{\bar{z}(\delta,R)}^{\infty} zdF(z).$$

And for pan $\mathcal{N}$:

$$-(1-\eta)\frac{\partial Y(\delta,R)}{\partial R}\int_{\bar{z}(\delta,R)}^{\infty} zdF(z).$$

In both cases, the opportunity cost has to equal the return on reserves (recall that, for simplicity, we fixed the interest rate on the liquid and safe asset at zero, and therefore the return on reserves is equal to 1).

To assess the impact of the distribution of the shock, and in particular of its variance, on the opportunity cost of holding reserves, we begin by using the assumption that z is distributed as a continuous uniform with support $[1-c, 1+c]$; this implies that:

$$\int_{\bar{z}(\delta,R)}^{\infty} z dF(z) = [(1+c)^2 - (\bar{z}(\delta,R))^2]/4c,$$

and

$$\frac{\partial}{\partial c}\left(\frac{(1+c)^2 - (\bar{z}(\delta,R))^2}{4c}\right) = \frac{(\bar{z}(\delta,R))^2 - (1+c)(1-c)}{16c^2}.$$

Therefore, in the case of a continuous uniform distribution, as c increases above unity the incentive that a country has to invest an additional unit of resources in reserves decreases in both plan $\mathcal{F}$ and plan $\mathcal{N}$.[15] An increase of c also triggers an increase in the variance of the uniform distribution, which is equal to $c^2/3$. Consequently, we can conclude that in the case of the uniform distribution, an increase in the variance can boost the incentive to de-cumulate reserves. This result is even more robust in the case of exponential probability density functions, for which a jump in the variance of the distribution increases the value of $\int_{\bar{z}(\delta,R)}^{\infty} z dF(z)$ and makes hoarding reserves relatively less profitable.[16]

Corollary 1.
An increase in the variance of the productivity shock z increases the opportunity cost of reserves.

In both the distributions considered above, an increase in the variance does not change the expected value of the productivity shock; however, it increases the expected value of z given the possibility of the country defaulting. In other words, Corollary 1 is linked to a *limited liability effect*: the country enjoys the upper tail of the distribution of the realizations of the productivity shock while it disregards the lower tail. Therefore, an increase in the riskiness of the illiquid project raises the opportunity cost of holding reserves and inflates sovereign incentive to invest in g instead of R.[17] We further investigate the impact of the limited liability effect in the section where the optimal portfolio allocation of a risk-averse country is studied in the extension presented below.

We now turn our attention to ϵ, the amount of liquid resources the country needs to inject in the event of a liquidity shock at $t = 1$. This parameter could be interpreted as a proxy for the capital account openness of the country and it therefore represents capital flight in a sudden stop. In line with Obstfeld et al. (2009), ϵ could more generally represent the fraction of the M2 aggregate (a proxy for the size of the banking system) the country needs to inject should a double drain occur.

In our model, ϵ does not affect the opportunity cost directly; however, it adversely affects country's expected utility of plan $\mathcal{F}$. Moreover, ϵ has an impact on the *no-default constraint* the country has to satisfy in order to credibly adhere to plan $\mathcal{F}$:

$$\int_{\bar{z}(\delta,R)}^{\infty} [zY(\delta,R) - D] dF(z) \geq \epsilon.$$

Intuitively, as the liquidity needs increase, the constraint becomes more binding, and therefore it is more likely that the equilibrium choice of reserves under plan $\mathcal{F}$ is constrained. All these reasons lead us to the conclusion in

[15] Notice that $\partial Y(\delta,R)/\partial R$ is independent from the shape of the distribution of z.

[16] To see this, note that, in the case of exponential probability density functions, were the variance to rise, the value of $\int_{-\infty}^{\infty} z dF(z)$ would stay the same. Instead, as long as $\bar{z}(\delta,R) < \infty$, an increase in the variance implies that the truncated expected value of z weighted by its density $f(z)$ rises.

[17] Another implication of this is that if the country could choose among different projects, it would prefer a riskier one (other things being equal).

the following corollary.

Corollary 2.
An increase in the needed liquidity injection (ϵ) makes the emergence of plan $\mathcal{F}$ less likely at equilibrium.

In the next Section, to gain further insights on the value of the model's key variables (reserves, debt discount factor and public expenditures) at equilibrium, and see under which conditions a country decides to default should the liquidity shock occur, we undertake a numerical analysis.

4 Simulations

Here we present a numerical example that illustrates the results above.[18] More specifically, it is assumed that the production function is a Cobb-Douglas of the following type:

$$Y(z,g) = zY(g) = zY(\delta, R) = z(\delta D - R)^{\alpha}.$$

Also, we maintain the assumption for which the productivity shock is distributed as a uniform random variable:

$$z \sim U(1, c^2/3), z \in [1-c; 1+c].$$

There are five parameters to be set:

α	*income share of capital*	0.3
D	*amount of debt as % of GDP*	40%
ϵ	*capital infusion as % of GDP*	10%
c	*support of the productivity process*	2
η	*probability of the liquidity shock*	15%

The income share of capital is taken from the literature. The face value of debt D is chosen in order to obtain on average an amount of gross debt over GDP of around 40%, in line with the data for 2009 reported by the IMF in *World Economic Outlook* for emerging and developing countries. The needed capital infusion in the event of a liquidity shock is fixed at 10% of the expected GDP, as proposed in Rodrik (2006) and Obstfeld et al. (2009). The variance of the productivity process and the probability of the liquidity shock are arbitrary. We then make them vary to check how those parameters affect the choice variables, δ and R, and to check the robustness of our results. With the proposed parameters we obtain estimates of the value of reserves over expected output of around 15%, slightly below the 20% estimated by Jeanne (2007). Moreover, in our simulations the discount factor generates a rate of return r that oscillates around 1: this is the same rate of return that would be generated by a 10-year bond capitalized at an annual interest rate of about 7%, which is a value of the annual rate in line with the empirical evidence in Borri and Verdelhan (2009).[19]

[FIGURE 5 ABOUT HERE]

Figure 5 reports the main results of the simulation when the probability of the liquidity shock moves from zero to 50%. It shows how the cost of debt, the expected output, the reserve level and the country's expected welfare vary when the probability of the liquidity shock increases.

[18] For the sake of the exposition, we focus on cases in which an equilibrium in pure strategies exists and is well-defined.

[19] Recall that the discount factor δ is equal to $1/(1+r)$. In our model, δ is inversely correlated to the cost of debt: an increase in the value of δ set by the lender reflects an increase in the probability that the lender expects the country to be solvent and stands for a decrease in the cost of debt.

The upper-left panel shows that the cost of debt increases with the increase in the risk of the liquidity shock in two cases, namely in the framework without reserves and when the country chooses not to face the liquidity shock when it occurs (plan $\mathcal{N}$).[20] Instead, if the country decides to face the liquidity shock (plan $\mathcal{F}$) the cost of debt is constant for any value of η, because the country is perfectly insured independently of the probability of the adverse event occurring.

The upper-left panel of Figure 5 also shows the threshold η^{**} (which here corresponds to a probability of the liquidity shock around 4%): if the probability of the liquidity shock is higher than η^{**}, then the discount factor in the framework without reserves (δ^{*}) rises above than the discount factor with reserves when the lender anticipates that the country will face the liquidity shock (δ_{F}^{**}). From this simulation, we also infer that when the country decides not to face the liquidity shock the cost of debt is always higher (δ lower) because this choice makes the lender worse off.

The upper-right panel of Figure 5 displays the path of sovereign expected output. The expected output is higher when the country cannot divert resources from the productive project (that is, in the framework with $R = 0$) if the probability of a liquidity shock is below the threshold $\hat{\eta}$ (which here corresponds to a probability of the liquidity shock of around 30%). When the liquidity shock becomes more likely, the amount of total resources invested under plan $\mathcal{F}$ becomes very large. Moreover in expectation, a larger fraction of the expected amount of output is lost, due to the default in liquidity crises.

The amount of accumulated reserves, reported in the lower-left panel, decreases when η increases with no capital infusion, because the resources available to the country (equal to $\delta_{N}^{**}D$) decrease with η. If, instead, the country decides to face the liquidity shock, should it occur, the level of reserves does not depend on the probability of the shock and the two constraints (resource constraint and no-default constraint) are slack at any level of η.

Finally, the lower-right panel plots the country's welfare in the two plans which the country can choose between: when the country accumulates reserves and faces the liquidity shock (plan $\mathcal{F}$), the welfare is always higher than if it decides not to inject capital (plan $\mathcal{N}$). Moreover, the resource constraint (lower-left panel) and the no-default constraint (not reported in the figure) are always satisfied under plan $\mathcal{F}$, so the country chooses to face the liquidity shock if it occurs.

From this first simulation of the model we can conclude that, under the chosen parameters, the country accumulates reserves and injects capital if the liquidity shock occurs at any value of η. The lender anticipates this choice when setting the discount factor of debt and sets δ_{F}^{**} to satisfy its zero-profit condition.

[FIGURE 6 ABOUT HERE]

The simulations in Figure 6 show how the variables of interest and the final equilibrium of the model vary when the variance of the underlying productivity process (equal to $c^2/3$) moves from 1 to $5\frac{2}{5}$. As mentioned in the section on comparative statics, when the variability of the productivity shock increases, the opportunity cost of holding reserves increases, because of a limited liability effect. Consequently, the country chooses a lower level of reserves and the discount factor on debt decided by the lender in response is lower, because more resources are invested in the productive activity.

Therefore, as the variance rises, the trade-off between accumulating enough reserves to face the shock and putting more resources in the productive project leads to a smaller pile of liquid and safe assets. There is a threshold of the variance for which the country decides to gather $R = \epsilon$ and at which the resource constraint becomes binding

[20]Notice that in the framework in which the country can accumulate a positive amount of reserves, the case "no reserves" cannot be a credible choice since, in the absence of commitment, the country will always prefer a positive amount of reserves at any discount factor offered by the lender as represented by plan $\mathcal{F}$ and plan $\mathcal{N}$. We simulate the equilibrium variables in the framework with no reserves in order to have means of comparison.

under plan $\mathcal{F}$.[21]

The overall impact on expected output and welfare is that they both increase as the sovereign decides to decumulate reserves. This is due to the role of residual claimant of the country if it decides not to default. Finally, when the country decides to face the liquidity shock (plan $\mathcal{F}$), the expected output is higher than when the country chooses not to inject the necessary liquidity (plan $\mathcal{N}$), but for the case of a very high level of variability of the shock. This is due to the fact that under plan $\mathcal{F}$ the level of R cannot get smaller than the needed liquidity infusion.

The equilibrium chosen is plan $\mathcal{N}$ for very low levels of the variance, due to the high cost of facing the shock compared with the opportunity cost of investing in output (weighted by the probability of the liquidity shock not occurring).

For higher levels of the variability, however, the country prefers to accumulate more reserves and face the liquidity shock since the expected output increases. The welfare under plan $\mathcal{F}$ increases more than under plan $\mathcal{N}$ because the underlying cost is fixed in the first case ($\eta\epsilon$) while it increases with the variance in the second case (it is the expected output weighted by the probability of the liquidity shock).

[FIGURE 7 ABOUT HERE]

The last simulation performed shows how the country's choice of equilibrium plan is affected by the importance of the liquidity shock (ϵ). In Figure 7, we let the value of ϵ vary. We change the values of parameters D and c as follows:

α	*income share of capital*	0.3
D	*amount of debt as % of GDP*	60%
c	*support of the productivity process*	$\sqrt{3}$
η	*probability of a liquidity shock*	15%

We increase the value of D, the available liquidity, and decrease the value of c, the variability of the productivity process. In this way we are able to show the influence of the no-default constraint, reported in the lower-left panel. Under plan $\mathcal{F}$, when the no-default constraint becomes binding, the country has to choose a level of reserves lower than the unconstrained optimum; this in turn induces the lender to offer a better discount for the debt (higher δ). The two effects contribute to an increase in the expected output and welfare. As ϵ further increases the level of reserves approaches the resource constraint and therefore, as proved in Appendix C, an equilibrium no longer exists under plan $\mathcal{F}$. The only equilibrium of the game features the choice of plan $\mathcal{N}$.

5 Extensions

In what follows we study two possible extensions of the present analysis. In the first, we introduce risk-aversion in order to show that the optimal level of reserves can be significantly higher if we change the shape of the country's utility function. In the second extension, a third player is introduced: we assume that a supranational organization can provide the liquidity needed in the event of a liquidity crisis. In this way we relax the friction on the financial markets' commitment not to intervene in the event of a liquidity shock.

5.1 Risk-averse utility function

In the main model, we assume that the country is risk-neutral, therefore it takes into account only the expected value of alternative investments either in safe and liquid assets (reserves) or in a risky project (output). In line

[21] In this simulation, the no-default constraint is always verified for plan $\mathcal{F}$ and is not reported in the figure.

with Arellano (2008), in the following we assume that the country is risk-averse, that is, it also looks at the second moment of alternative investment opportunities. We expect to find that, all else being equal, the country prefers to invest more in the safe asset, so that the relative level of reserves chosen should be higher than in the previous analysis with linear utility. Notice that nothing changes for the lender, who is assumed to be risk-neutral and price the debt taking into account the same trade-off induced by the reserves as in the main model: reserves divert resources from the productive investment, but on the other hand they can be employed to face the liquidity crisis, should it occur, and therefore avoid default at time two.

We make use of a standard CRRA utility function with a parameter of risk aversion equal to 1. Following Section 3.2, we re-write the expected output and the related constraints for the two plans ($\mathcal{F}$ and $\mathcal{N}$) by making use of a logarithmic utility function.

Plan $\mathcal{F}$

At $t = 0b$, the country maximizes the following expression:

$$\begin{aligned}\max_R E(U_F(\delta, R)) \quad &= \quad (1-\eta)\left[\int_{\bar{z}(\delta,R)}^{\infty} \log[zY(\delta,R) - D + R]dF(z) + \int_{-\infty}^{\bar{z}(\delta,R)} \log(R)dF(z)\right] + \\ &\quad \eta\left[\int_{\bar{z}(\delta,R)}^{\infty} [zY(\delta,R) - D + R - \epsilon]dF(z) + \int_{-\infty}^{\bar{z}(\delta,R)} \log(R-\epsilon)dF(z)\right],\end{aligned}$$

subject to the no-default and resource constraints:

$$\begin{aligned}&\int_{-\infty}^{\bar{z}(\delta,R)} \log(R-\epsilon)dF(z) + \int_{\bar{z}(\delta,R)}^{\infty} [zY(\delta,R) - D + R - \epsilon]dF(z) \geq \log(R), \\ &R \geq \epsilon.\end{aligned}$$

The resource constraint does not change in the new setup, while the *no-default constraint* incorporates the disutility generated by uncertainty.

Plan $\mathcal{N}$

In this case, at $t = 0b$ the country maximizes the following expression:

$$\max_R E(U_N(\delta, R)) = (1-\eta)\left[\int_{\bar{z}(\delta,R)}^{\infty} \log[zY(\delta,R) - D + R]dF(z) + \int_{-\infty}^{\bar{z}(\delta,R)} \log(R)dF(z)\right] + \eta\log(R).$$

We conduct a numerical simulation to find the equilibrium pair $(\tilde{\delta}, \tilde{R})$. In line with the analysis conducted for the probability of liquidity shock (η) that varies, we set the four parameters as reported in the table below:

α	*income share of capital*	0.3
D	*amount of debt as % of GDP*	40%
ϵ	*capital infusion as % of GDP*	10%
c	*support of the productivity process*	2

[FIGURE 8 ABOUT HERE]

The simulations are reported in Figure 8. In line with the previous analysis, plan $\mathcal{F}$ is the one implemented at any value of η between 0 and 50%. Notice that in this case, even under plan $\mathcal{F}$, the probability of a liquidity crisis affects all the equilibrium variables. In particular, it reduces the expected welfare, induces the country to invest more in reserves, which in turn makes creditors fix a higher debt discount factor. Plan $\mathcal{N}$, with this parametrization, coincides with the case of no-reserves, since the country would like to accumulate a negative amount of reserves, and therefore the constrained equilibrium is at $R = 0$. As a consequence, in this case the country's expected welfare collapses, because of the shape of the utility function, and this plan is never implemented at equilibrium.

In the present simulations the threshold η^{**} is higher than with linear utility and equal to about 10%. The threshold $\hat{\eta}$, instead, is not reported in the second panel. For η between 0 and 50%, the output with no reserves is always higher than under plan $\mathcal{F}$. This happens for two reasons: the first is that a large fraction of the available resources is put in reserves instead of in the risky investment, the second is that in the case of a risk-averse country the expected output decreases with η under plan $\mathcal{F}$.

[FIGURE 9 ABOUT HERE]

In Figure 9, we compare the amount of resources invested in reserves in terms of expected output in the two cases, namely with linear and logarithmic utility. With logarithmic utility, the amount of resources saved in reserves is about 70% of the expected output. It is a very high level, exceeding by far that of reserves in the case of linear utility.

With our analysis, we do not pretend to predict the optimal level of reserves; however, *the results that emerge from this extension allow us to conclude that, by making the hypothesis of linear utility, researchers can obtain only a lower bound for the optimal level of reserves. This lower bound needs to be corrected upwards in the case of perceived risk aversion by the economic authorities.*

5.2 IMF intervention

In this section, we introduce a supranational organization, e.g. the IMF, which can intervene in the interim period and help the country by injecting ϵ should the liquidity shock occur.[22] We assume that the application for IMF intervention is decided by the country before the contracting stage; therefore, at time zero the lender and the country already know whether the IMF will intervene in a liquidity shock. Moreover, the intervention takes place by means of an instrument that covers the entire amount of liquidity needed.

At $t = 4$, if not hit by the liquidity shock, the country defaults if $z \leq \bar{z}(\delta, R) \equiv D/Y(\delta, R)$.

Instead, if hit by the shock ϵ, at $t = 4$ the country has to repay the IMF at the market price (recall the risk-free interest rate is nil); however, as in the case of the international lender, the IMF cannot act as a residual claimant and is not repaid in the event of default. More specifically, at the final stage the country decides to default strategically if the following is true:

$$zY(\delta, R) - D + R - \epsilon \leq R \iff z \leq \hat{z}(\delta, R) \equiv \frac{D + \epsilon}{Y(\delta, R)} > \bar{z}(\delta, R) \equiv \frac{D}{Y(\delta, R)}.$$

At $t = 2$, if it is hit by a liquidity shock in $t = 1$, the country's no-default constraint is in what follows:

$$R + \int_{\hat{z}(\delta,R)}^{\infty} [zY(\delta, R) - D - \epsilon]dF(z) \geq R.$$

The condition above is satisfied by construction, implying that the IMF intervention clears the incentive to default on the liquidity shock at $t = 2$. At $t = 0$ the country chooses how to allocate resources between public

[22] This way, we propose a simplified setting for the analysis of Flexible Credit Lines (FCL), a new IMF instrument available without ex-post conditionality for member countries that satisfy pre-determined criteria (for details see: http://www.imf.org/external/np/pdr/fac/2009/032409.htm)

expenditure and reserves by maximizing the following expected utility:

$$\max_{R\in[0,\delta D)} E(U_{IMF}(\delta,R)) = R + (1-\eta)\int_{\bar{z}(p,R)}^{\infty}[zY(\delta,R)-D]dF(z) + \eta\int_{\hat{z}(p,R)}^{\infty}[zY(\delta,R)-D-\epsilon]dF(z).$$

Instead, the lender fixes δ at the level that solves the zero-profit condition:

$$\begin{aligned}
\delta D &= D\eta\int_{\hat{z}(\delta,R)}^{\infty} dF(z) + (1-\eta)D\int_{\bar{z}(\delta,R)}^{\infty} dF(z) \iff \\
\delta &= \eta(1-F(\hat{z}(\delta,R))) + (1-\eta)(1-F(\bar{z}(\delta,R))).
\end{aligned}$$

The intervention of the IMF introduces a trade-off into the model. *On the one hand, the problem of insurance against liquidity shock disappears. On the other hand, the probability of the country's default on debt at* $t = 4$ *is higher than in the case of no liquidity shock, because the outstanding debt is higher in that case. Consequently, the associated threshold below which the country defaults increases in the event of a liquidity shock (with probability* η*).*

The trade-off above implies that the intervention of the IMF impacts on the lender's break-even problem solution in an interesting fashion, especially when compared to the solution of the lender problem in the cases of plan $\mathcal{F}$ and plan $\mathcal{N}$. More specifically, since, with respect to plan $\mathcal{F}$ (see equation 4), the country will default more often on its debt under IMF intervention, the lender will set a lower discount factor δ when the IMF intervenes. However, if we compare the optimal value of δ with the IMF against the one under plan $\mathcal{N}$ (see equation **??**), the contrary is true, so that, everything else being equal, the value of δ under IMF intervention should be higher than in the no injection case ($\mathcal{N}$).

[FIGURE 10 ABOUT HERE]

The simulations are reported in Figure 10. By comparing the levels of expected welfare, it is clear that IMF intervention is preferred to the other plans. Under the IMF scenario, the rate of return (respectively, discount factor) set by the lender is always higher (lower) than the one set under plan $\mathcal{F}$ due to the trade-off discussed above. However, the rate of return under IMF intervention is lower than under plan $\mathcal{N}$, since when the IMF intervenes the lender expects the country not to default should a liquidity crisis occur. Reserves are also slightly lower with the IMF than under plan $\mathcal{F}$, although this difference is not very large.

Overall, this extension predicts that the intervention by an international organization to solve the liquidity crisis does not reduce significantly the will to accumulate reserves. In fact, even though reserves are no longer needed to face the liquidity shock, they are still a form of saving in which the country can invest and retain in the event of default. Consequently, the optimal portfolio allocation still prescribes a positive amount of safe and liquid assets.

Finally, the combined effect of the accumulated level of reserves and the higher probability of default in a liquidity crisis implies that the lender fixes a cost of debt with the IMF that is larger than under plan $\mathcal{F}$. This shows that, per-se, *when liquidity crises can occur IMF intervention does not necessarily ease the pressure on the sovereign debt rate of return. To accomplish this result, the financial instrument proposed by a supranational organization, or simply bilateral agreements, should be designed to limit country's incentive to default on debt.*

6 Concluding Remarks

This paper contributes to the discussion about the management of liquidity and the optimal level of sovereign reserves for countries that are subject to liquidity crises. We are able to model simultaneously endogenous debt prices, optimal reserve accumulation and strategic default decisions. In our setup, the country optimally decides the level of international reserves that maximizes its expected welfare taking into account the probability of a liquidity crisis and the possibility of defaulting (with the cost in terms of output of this action). Competitive international lenders anticipate the country choice and set a rate of return over the lent resources that satisfies the zero-profit condition. Therefore, we deliver a model that abstracts from the role of reserves in managing the exchange rate and that instead draws on the opportunity cost of holding reserves in terms of reduced expected output.

Our results rationalize the self-insurance motive for accumulating reserves and facing liquidity shocks, as well as the limited liability effect due to the impossibility for the country to commit not to default on its debt. The combination of these two forces drives our results: we show that both an equilibrium featuring the country self-insuring against the shock and one with the country not facing the shock can emerge depending on the productivity process, the amount of resources available in the country and the probability and dimension of the liquidity crisis. Finally, we are able to reproduce the empirical evidence of Levy Yeyati (2008) and Ruiz-Arranz and Zavadjil (2008): the level of international reserves reduces the costs of external debt for sovereign countries that face a high probability of being hit by a liquidity crisis.

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A Proof of Lemma 1

The equilibrium value of δ, denoted δ^*, exists and is unique if and only if $G(\delta)$ satisfies the following five conditions.

Condition (i): $\exists \lim_{\delta\to 0} G(.) < \infty$.
Taking limits, it can be shown that this condition holds:

$$\delta \to 0 \Rightarrow \bar{z}(\delta) \to \infty \Rightarrow F(\bar{z}(\delta)) \to 1 \Rightarrow G(\delta) \to 0.$$

Condition (ii): $\lim_{\delta\to 1} G(.) < 1$.
Again, taking limits, one has that also condition (ii) holds:

$$\delta \to 1 \Rightarrow \bar{z}(\delta) \to \frac{D}{Y(1)} \Rightarrow F(\bar{z}(1)) \to k \in (0,1) \Rightarrow G(\delta) \to 1-k < 1.$$

Condition (iii): $dG(\delta)/d\delta > 0$.
Computing the first derivative of $G(\delta)$, one finds that condition (iii) is satisfied:

$$\begin{aligned}\frac{dG(\delta)}{d\delta} &= -\frac{\partial F(\bar{z}(\delta))}{\partial z}\frac{d\bar{z}(\delta)}{d\delta} = \\ &= f(\bar{z}(\delta))\frac{D^2 Y'(\delta)}{(Y(\delta))^2} > 0.\end{aligned}$$

Condition (iv): $d^2G(\delta)/d\delta^2 < 0$.
Condition (iv) is necessary (but not sufficient) for uniqueness and imposes a restriction on the curvature of $G(\delta)$. In particular, it is satisfied if $G(\delta)$ is concave:

$$\frac{d^2G(\delta)}{d\delta^2} = \frac{df(\bar{z}(\delta))}{d\delta}\frac{D^2Y'(\delta)}{(Y(\delta))^2} + D^3 f(\bar{z}(\delta))\left\{\frac{Y''(\delta)}{(Y(\delta))^2} - \frac{2(Y'(\delta))^2}{(Y(\delta))^4}\right\} < 0. \tag{15}$$

The second term of (15) is strictly negative, thus the sign of the inequality depends on the sign of the first term: we then use the assumption of uniform distribution of z, which implies that $df(\bar{z}(\delta))/d\delta = 0$, to get that (15) holds true.

Condition (v): $\lim_{\delta\to 0} dG(\delta)/d\delta > 1$.
This condition is necessary to make sure that an interior solution exists that is different from the trivial one, i.e. $\delta = 0$, to the fixed point problem that we are analysing. Taking limits, one has that in the case of the uniform distribution the following is true:

$$\lim_{\delta\to 0}\frac{dG(\delta)}{d\delta} = \lim_{\delta\to 0}\frac{D^2Y'(\delta)}{(Y(\delta))^2} = \infty. \tag{16}$$

This result uses the assumption of uniform distribution of z, for which $f(\bar{z}(\delta))$ does not depend on δ,[23] and that

[23] Here, we are implicitly restricting our attention to the cases where $\bar{z}(\delta) \in [1-c, 1+c]$, since otherwise the lender would never serve the country and one would have that the solution to the problem is given by $\delta = 0$.

$$Y'(\delta) \rightarrow_{\delta \rightarrow 0} \infty,$$

$$\frac{1}{(Y(\delta))^2} \rightarrow_{\delta \rightarrow 0} \infty.$$

■

B Proof of Lemma 2

The problem solved by the lender under plan $\mathcal{N}$ and the one we solved in Lemma 1 are isomorphic. Therefore, following the steps of the proof in Appendix A with the use of the Envelope Theorem allows us to pin down the unique reaction function $\tilde{\delta}_N = \delta_N(R)$ that solves the lender's zero-profit condition.[24]

Under plan $\mathcal{N}$, the sovereign faces the following problem:

$$\max_{R \in [0, \tilde{\delta}_N D)} E(U_N(\tilde{\delta}_N, R)) = R + (1-\eta) \int_{\bar{z}(\tilde{\delta}_N, R)}^{\infty} [zY(\tilde{\delta}_N, R) - D] dF(z). \tag{17}$$

We now compute the first order condition of the expression above (and employ the Envelope Theorem when disregarding the effect of a change of R on $\tilde{\delta}_N$). After setting the resulting derivative with respect to R to zero, we have:

$$H_N(R) = 1 + (1-\eta) \frac{\partial Y(\tilde{\delta}_N, R)}{\partial R} \int_{\bar{z}(\tilde{\delta}_N, R)}^{\infty} z dF(z) = 0. \tag{18}$$

In order to determine the existence of a solution to our problem we need to show two preliminary results. First, we need to prove that the country does not have an incentive to employ all resources in reserves (i.e. $R_N^{**} \neq \tilde{\delta}_N D$). To see that this is the case, we compute the value of $H_N(R)$ as R approaches $\tilde{\delta}_N D$:

$$\begin{aligned} \lim_{R \to \tilde{\delta}_N D} H_N(R) &= 1 + (1-\eta) \frac{\partial Y(\tilde{\delta}_N, R)}{\partial R}\Big|_{R \to \tilde{\delta}_N D} \int_{\bar{z}(\tilde{\delta}_N, R)|_{R \to \tilde{\delta}_N D}}^{\infty} z dF(z) = \\ &= 1 + (-\infty)(+\infty) = -\infty. \end{aligned}$$

Indeed, from the Inada Conditions we have that:

$$\frac{\partial Y(\tilde{\delta}_N, R)}{\partial R}\Big|_{R \to \tilde{\delta}_N D} = -Y'(\tilde{\delta}_N, R)|_{R \to \tilde{\delta}_N D} \rightarrow -\infty.$$

Moreover,

$$\int_{\bar{z}(\tilde{\delta}_N, R)|_{R \to \tilde{\delta}_N D}}^{\infty} z dF(z) = \frac{E(z | z \geq \bar{z}(\tilde{\delta}_N, R))}{1 - F(\bar{z}(\tilde{\delta}_N, R))} \rightarrow \frac{\infty}{0} = \infty.$$

This is because:

[24] In particular, notice that when evaluating the limit of $G(.,.)$ as δ approaches zero also R tends to zero, because a nil discount factor implies that the country has no resources available at the investment stage.

$$\lim_{R\to\tilde{\delta}_N D} \bar{z}(\tilde{\delta}_N, R) \quad = \quad \lim_{R\to\tilde{\delta}_N D} \frac{D}{Y(\tilde{\delta}_N, R)} = \infty \Rightarrow \lim_{R\to\tilde{\delta}_N D} E(z|z \geq \bar{z}(\tilde{\delta}_N, R)) = \infty$$

and

$$\lim_{R\to\tilde{\delta}_N D} 1 - F(\bar{z}(\tilde{\delta}_N, R)) \quad = \quad 0.$$

Clearly, investing all resources into liquidity is not an optimum for the sovereign.

Now, we need to check that the country does not want to invest the entire allowance in public expenditure (i.e. $R_N^{**} \neq 0$). In particular, the value of $H_N(R)$ as R approaches 0 is equal to:

$$\lim_{R\to 0} H_N(R) = 1 + (1-\eta)\frac{\partial Y(\tilde{\delta}_N, R)}{\partial R}\Big|_{R=0} \int_{\bar{z}(\tilde{\delta}_N, R)|_{R=0}}^{\infty} z dF(z) = H_N(R)|_{R=0}.$$

So, if $H_N(R)|_{R=0} > 0$ there exists a solution to the problem under consideration. For $H_N(R)|_{R=0}$ to be positive it must be that, when all resources are invested in productive inputs, the production function is close to exhausting its returns to scale and its derivative with respect to R is small enough to bring the second term of $H_N(R)|_{R=0}$ to a value smaller than 1 (recall that $\partial Y(.,.)/\partial R = -Y'(.,.) < 0$ and $\partial^2 Y/\partial R^2 = Y'' < 0$).[25]

Finally, the uniqueness of the solution to the problem in (17) is granted if the maximand is concave. More specifically, the derivative of $H_N(R)$ must be negative.

$$\frac{dH_N(R)}{dR} = (1-\eta)\frac{\partial^2 Y(\tilde{\delta}_N, R)}{\partial R^2}\int_{\bar{z}(\tilde{\delta}_N, R)}^{\infty} z dF(z) + (1-\eta)\frac{\partial Y(\tilde{\delta}_N, R)}{\partial R}\Big[- \frac{\partial \bar{z}(\tilde{\delta}_N, R)}{\partial R}\bar{z}(\tilde{\delta}_N, R)\Big] < 0. \tag{19}$$

Where the inequality holds by $(A1)$. Indeed, (19) can be rewritten as:[26]

$$\frac{\partial Y}{\partial R}\Big(- \bar{z}\frac{\partial \bar{z}}{\partial R}\Big) \quad < \quad -\frac{\partial^2 Y}{\partial R^2}\int_{\bar{z}}^{\infty} z dF(z) \iff \tag{20}$$

$$\frac{\frac{\partial Y}{\partial R}}{\frac{\partial^2 Y}{\partial R^2}} \quad < \quad \frac{-\int_{\bar{z}}^{\infty} z dF(z)}{-\bar{z}\frac{\partial \bar{z}}{\partial R}} \tag{21}$$

and

$$-\bar{z}\frac{\partial \bar{z}}{\partial R} = \frac{\partial}{\partial R}\Big(\int_{\bar{z}}^{\infty} z dF(z)\Big) < 0. \tag{22}$$

Where the sign of (22) uses the fact that $\partial \bar{z}/\partial R > 0$. Therefore, (21) can be formulated as:

$$\frac{\frac{\partial Y}{\partial R}}{\frac{\partial^2 Y}{\partial R^2}} \quad < \quad \frac{-\int_{\bar{z}}^{\infty} z dF(z)}{\frac{\partial}{\partial R}\Big(\int_{\bar{z}}^{\infty} z dF(z)\Big)}. \tag{23}$$

[25]For example, take the production function $Y(\tilde{\delta}D - R) = (\delta D - R)^{\alpha}$, with $\alpha < 1$; its first order derivative with respect to R is equal to:

$$\frac{\partial}{\partial R}\Big\{(\delta D - R)^{\alpha}\Big\}\Big|_{R=0} = -\alpha\frac{1}{(\delta D)^{1-\alpha}} = -\alpha\frac{1}{g^{1-\alpha}}.$$

For given $\delta \in (0,1)$ and g, such expression goes to zero if α is small enough.

[26]Notice that, for ease of exposition, in the following we are dropping functional forms.

Now, by using the definition of elasticity, one can re-write (23) as

$$\frac{1}{\epsilon_{Y',R}} \quad < \quad \frac{1}{\epsilon_{\bar{z},R}} \iff |\epsilon_{\bar{z},R}| < |\epsilon_{Y',R}|. \tag{24}$$

Which is equivalent to $(A1)$.

To close the proof, the value of R_N^{**} that solves (18) needs to be plugged into $\tilde{\delta}_N$ to compute the optimal discount factor set by creditors under plan $\mathcal{N}$. ■

C Proof of Lemma 3

Also in this case, the problem solved by the lender under plan $\mathcal{F}$ and the one we solved in Lemma 1 are isomorphic, so we follow the steps of the proof in Appendix A with the use of the Envelope Theorem to pin down the reaction function $\tilde{\delta}_F = \delta_F(R)$ that solves the lender's participation constraint.

Under plan $\mathcal{F}$, the sovereign faces the following maximization problem:

$$\max_R E(U_F(\tilde{\delta}_F, R)) = R + \int_{\bar{z}(\tilde{\delta}_F,R)}^{\infty} [zY(\tilde{\delta}_F, R) - D]dF(z) - \eta\epsilon. \tag{25}$$

Under the resource and the no-default constraints:

$$\begin{aligned} \int_{\bar{z}(\tilde{\delta}_F,R)}^{\infty} [zY(\tilde{\delta}_F, R) - D]dF(z) \quad &\geq \quad \epsilon, \\ R \quad &\geq \quad \epsilon. \end{aligned}$$

We solve the problem by maximizing the following expression, where we denote the Lagrange multipliers by λ and μ:[27]

$$\mathcal{L}(R, \lambda, \mu) = R + \int_{\bar{z}(\tilde{\delta}_F,R)}^{\infty} [zY(\tilde{\delta}_F, R) - D]dF(z) - \eta\epsilon + \lambda\Big\{ \int_{\bar{z}(\tilde{\delta}_F,R)}^{\infty} [zY(\tilde{\delta}_F, R) - D]dF(z) - \epsilon \Big\} - \mu(R - \epsilon).$$

The relevant constraints are given below:

$$\frac{\partial \mathcal{L}(R, \lambda, \mu)}{\partial R} = 1 + (1 + \lambda) \int_{\bar{z}(\tilde{\delta}_F,R)}^{\infty} z\frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} dF(z) - \mu = 0 \tag{26}$$

$$\frac{\partial \mathcal{L}(R, \lambda, \mu)}{\partial \lambda} = \int_{\bar{z}(\tilde{\delta}_F,R)}^{\infty} [zY(\tilde{\delta}_F, R) - D]dF(z) - \epsilon = 0 \tag{27}$$

$$\frac{\partial \mathcal{L}(R, \lambda, \mu)}{\partial \mu} = R - \epsilon = 0 \tag{28}$$

$$\lambda\Big\{ \int_{\bar{z}(\tilde{\delta}_F,R)}^{\infty} [zY(\tilde{\delta}_F, R) - D]dF(z) - \epsilon \Big\} = 0 \tag{29}$$

$$\mu\Big(R - \epsilon\Big) = 0 \tag{30}$$

$$\mu \geq 0, \lambda \geq 0 \tag{31}$$

There are four cases to be discussed.

[27] Note that, as shown in the proof of Lemma 2, the maximand is concave under $(A1)$.

Case I: $\lambda = \mu = 0$.

In this case, the two constraints are both slack. Then, the optimal level of reserves is determined by the following condition:

$$1 + \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} dF(z) = 0. \tag{32}$$

We know from Lemma 2 that the left-hand-side of the equality above tends to $(-\infty)$ as R tends to $\tilde{\delta}_F D$. Thus, one has that if the following holds,

$$\lim_{R \to \epsilon} H_F(R)|_{R=\epsilon} \equiv 1 + \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R}\Big|_{R=\epsilon} \int_{\bar{z}(\tilde{\delta}_F, R)|_{R=\epsilon}}^{\infty} z dF(z) > 0, \tag{33}$$

the solution to the problem is given by $R_F^{**} \in (\epsilon, \tilde{\delta}_F D)$.

Case II: $\lambda > 0$, $\mu = 0$.

In this case, the no-default constraint is binding, while the resource constraint is slack (meaning that $R > \epsilon$ at this candidate equilibrium). The optimal level of reserves and the Lagrange multiplier λ are determined by the following conditions:

$$1 + (1 + \lambda) \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} dF(z) = 0,$$
$$\int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} [zY(\tilde{\delta}_F, R) - D] dF(z) = \epsilon.$$

Therefore, the optimal value of reserves R_F^{**} is computed by solving the no-default constraint for R and the value of the multiplier is equal to:

$$\lambda^{**} = \frac{1 + \int_{\bar{z}(\tilde{\delta}_F, R_F^{**})}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R_F^{**})}{\partial R} dF(z)}{-\int_{\bar{z}(\tilde{\delta}_F, R_F^{**})}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R_F^{**})}{\partial R} dF(z)}.$$

Note that for λ^{**} to be strictly positive it has to be that at $R_F^{**} > \epsilon$ the following condition is satisfied:

$$\left(1 + \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} dF(z)\right)\Big|_{R=R_F^{**}} > 0. \tag{34}$$

This condition holds if the value of R that solves the no-default condition in this sub-case is lower than the one given in *case II*, because there the no-default constraint is slack at the equilibrium and the same no-default constraint is decreasing in R.[28] Therefore, for $R_F^{**} > \epsilon$ to be an equilibrium in this case two conditions must hold; namely, (32) and (34).

[28] For completeness, notice that

$$(-\partial Y(\tilde{\delta}_F, R)/\partial R) \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z dF(z)|_{R=R_F^{**}} > 0 \tag{35}$$

at any finite value of R in $(\epsilon, \tilde{\delta}_F D)$.

Case III: $\lambda = 0$, $\mu > 0$.

In this case, the no-default constraint is slack. Instead, the resource constraint is binding. Then, the optimal level of reserves and the Lagrange multiplier μ are determined by the following conditions:

$$
\begin{aligned}
& 1 + \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} dF(z) = \mu, \\
& R = \epsilon.
\end{aligned}
$$

More specifically, $R_F^{**} = \epsilon$ is an equilibrium if

$$
\lim_{R \to \epsilon} H_F(R)|_{R=\epsilon} \equiv 1 + \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R}\Big|_{R=\epsilon} \int_{\bar{z}(\tilde{\delta}_F, R)|_{R=\epsilon}}^{\infty} z dF(z) > 0,
$$

which is equivalent to condition (33) above and implies that $\mu^{**} > 0$.

Case IV: $\lambda > 0$, $\mu > 0$.

In this case, both the no-default constraint and the resource constraint are binding. The equilibrium would be determined by:

$$
\begin{aligned}
& 1 + (1 + \lambda) \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} z \frac{\partial Y(\tilde{\delta}_F, R)}{\partial R} dF(z) = \mu, \\
& \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} [zY(\tilde{\delta}_F, R) - D] dF(z) = \epsilon, \\
& R = \epsilon.
\end{aligned}
$$

However, notice that

$$
\lim_{R \to \epsilon} \int_{\bar{z}(\tilde{\delta}_F, R)}^{\infty} [zY(\tilde{\delta}_F, R) - D] dF(z) > \epsilon \quad \forall \epsilon \in [0, \tilde{\delta}_F D).
$$

Indeed, if $\epsilon = 0$, then, by construction, $\int_{\bar{z}(\tilde{\delta}_F, \epsilon)}^{\infty} [zY(\tilde{\delta}_F, \epsilon) - D] dF(z) > 0$. Moreover, given that $\partial Y(.,.)/\partial R < 0$, if ϵ increases to $\tilde{\delta}_F D$ then $\int_{\bar{z}(\tilde{\delta}_F, \epsilon)}^{\infty} [zY(\tilde{\delta}_F, \epsilon) - D] dF(z)$ goes to 0. However, the country cannot deplete available resources in reserves, because the lender would reply by setting a nil value of the discount factor. Hence, this case cannot be an equilibrium of the maximization problem.

Finally, to get the equilibrium value of δ one has to use the optimal level of R obtained in each case above and plug it into $\tilde{\delta}_F$.■

D Extension with Distributions with Exponential Density Functions

In this extension, we relax the assumption of uniform distribution to show under which conditions the results of the paper carry over in a setting with exponential density functions.

We use the assumption of uniform distribution to derive the fixed point that solves the lender's break-even condition. More specifically, we employ the assumption on the shape of z to show that condition (iv) and condition (v) hold true in the proof of Lemma 1.

More specifically, condition (iv) is satisfied if $G(\delta)$ is concave, that is if:

$$\frac{d^2G(\delta)}{d\delta^2} = \frac{df(\bar{z}(\delta))}{d\delta}\frac{D^2Y'(\delta)}{(Y(\delta))^2} + D^3 f(\bar{z}(\delta))\Big\{\frac{Y''(\delta)}{(Y(\delta))^2} - \frac{2(Y'(\delta))^2}{(Y(\delta))^4}\Big\} < 0. \tag{36}$$

In the case of uniform distributions (36) is satisfied, because $f(\bar{z}(.))$ is constant in δ (the second term is negative independently of the type of distribution assumed). In the case of exponential distributions, a sufficient condition for (36) to hold is that $df(\bar{z}(\delta))/d\delta \geq 0$ is negative. More specifically, it has to be that:

$$\begin{aligned} \frac{df(\bar{z}(\delta))}{d\delta} &= \frac{\partial f(\bar{z}(\delta))}{\partial z}\frac{d\bar{z}(\delta)}{d\delta} = \\ &= -\frac{\partial f(\bar{z}(\delta))}{\partial z}\Big(\frac{Y'D^2}{(Y(\delta))^2}\Big) \leq 0. \end{aligned} \tag{37}$$

(37) is negative if $\partial f(\bar{z}(\delta))/\partial z \geq 0$ and this is the case if the following assumption holds:

A2 : $i.$ $z \sim F(z)$ s.t. $\arg\max_z f(z) = E(z) = 1$.
$ii.$ $E(z)Y(\delta, R) - D = Y(\delta, R) - D > 0$.

The implications of $(A2)$ are two: the first is that $f(z)$ reaches its maximum at $E(z) = 1$ and the second is that the expected value of the production plan is ex-ante profitable, so that $\bar{z}(\delta) < 1$. Consequently, $f(z)$ is increasing in z when it is evaluated in $\bar{z}(\delta)$.

Condition (v) is satisfied if $\lim_{\delta\to 0} dG(\delta)/d\delta > 1$. Taking limits, one has that:

$$\lim_{\delta\to 0}\frac{dG(\delta)}{d\delta} = \lim_{\delta\to 0}\frac{f(\bar{z}(\delta))}{(Y(\delta))^2}Y'(\delta)D^2 = \infty. \tag{38}$$

In the proof of Lemma 1, the result above follows from

$$Y'(\delta) \to_{\delta\to 0} \infty$$

and from the fact that for a uniform distribution the value of $f(\bar{z}(\delta))$ is a constant in δ, hence

$$\frac{1}{(Y(\delta))^2} \to_{\delta\to 0} \infty.$$

When analysing the case of exponential distributions, one needs to show that

$$\frac{f(\bar{z}(\delta))}{(Y(\delta))^2} \to_{\delta\to 0} \infty.$$

We can prove it by making use of the following condition:

A3 : $\lim_{\delta\to 0} Y^2(\delta) < \lim_{\delta\to 0} f(\bar{z}(\delta))$.

The inequality above uses a specific infinitesimal order that is satisfied by a production function that meets Inada Conditions and exponential density functions. An example may provide a better understanding of why this is the case. Assume $z \sim N(0, 1)$ and define $(1/Y(\delta)) = x^{\alpha}$, then

$$f(\bar{z}(\delta)) = f\Big(\frac{D}{Y(\delta)}\Big) \approx f(x) \to_{\delta\to 0} e^{x}.$$

Indeed, as δ goes to zero, $1/x^\alpha$ goes to infinity, like e^x does. Therefore,

$$\lim_{\delta \to 0} \frac{f(x)}{x^{-2\alpha}} \approx \lim_{\delta \to 0} \frac{e^x}{x^{-2\alpha}} = \infty,$$

And the last result follows from:

$$ord_\infty e^x \geq ord_\infty x^a.$$

■

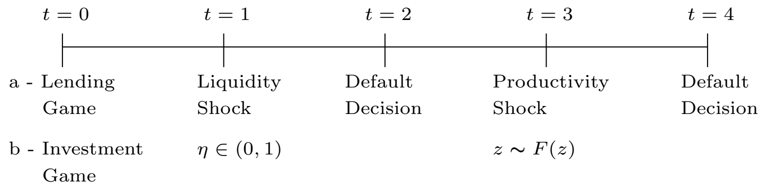

Figure 1: Timeline

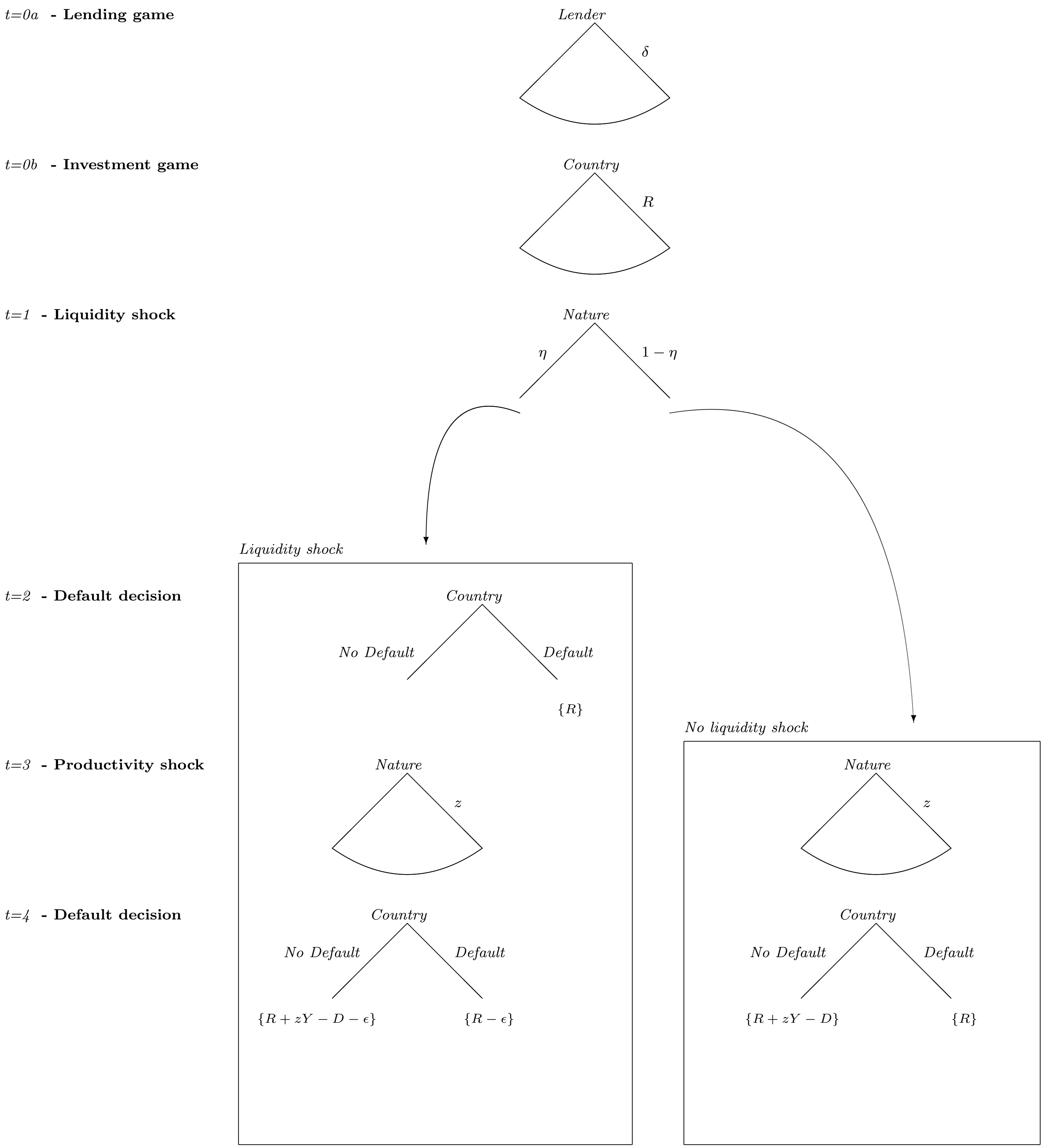

Figure 2: Game-tree with the country's payoffs

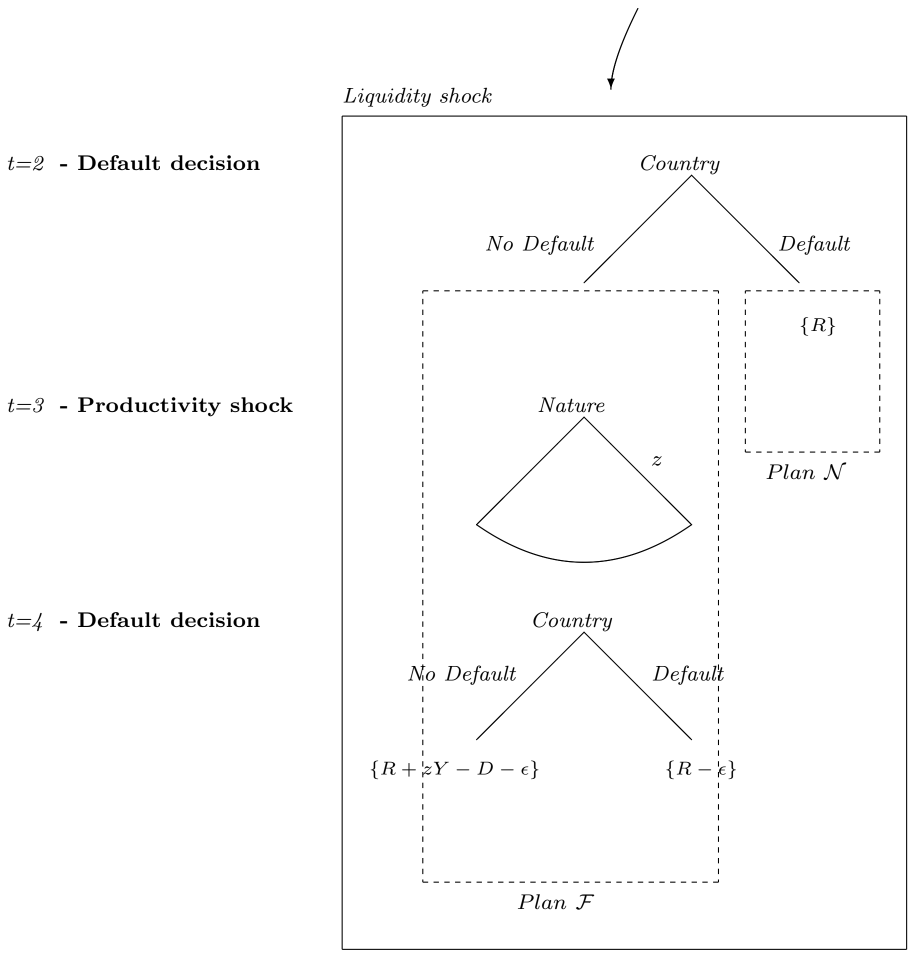

Figure 3: Liquidity shock case, plan $\mathcal{F}$ and plan $\mathcal{N}$

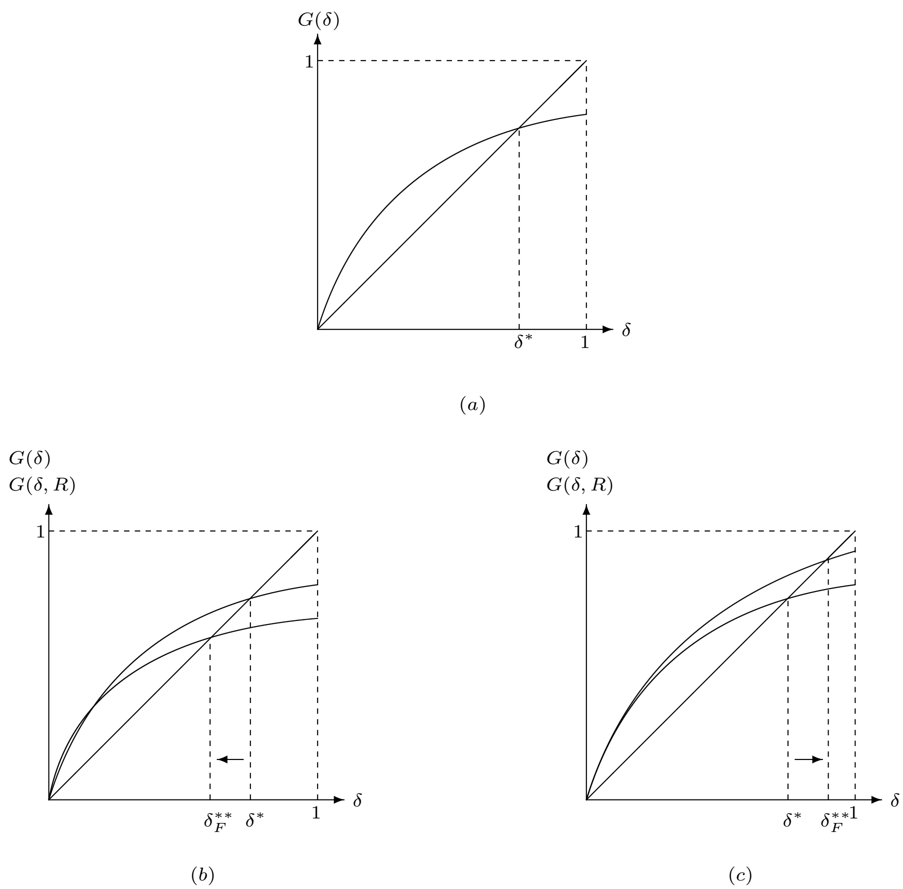

Figure 4: Equilibrium discount factors

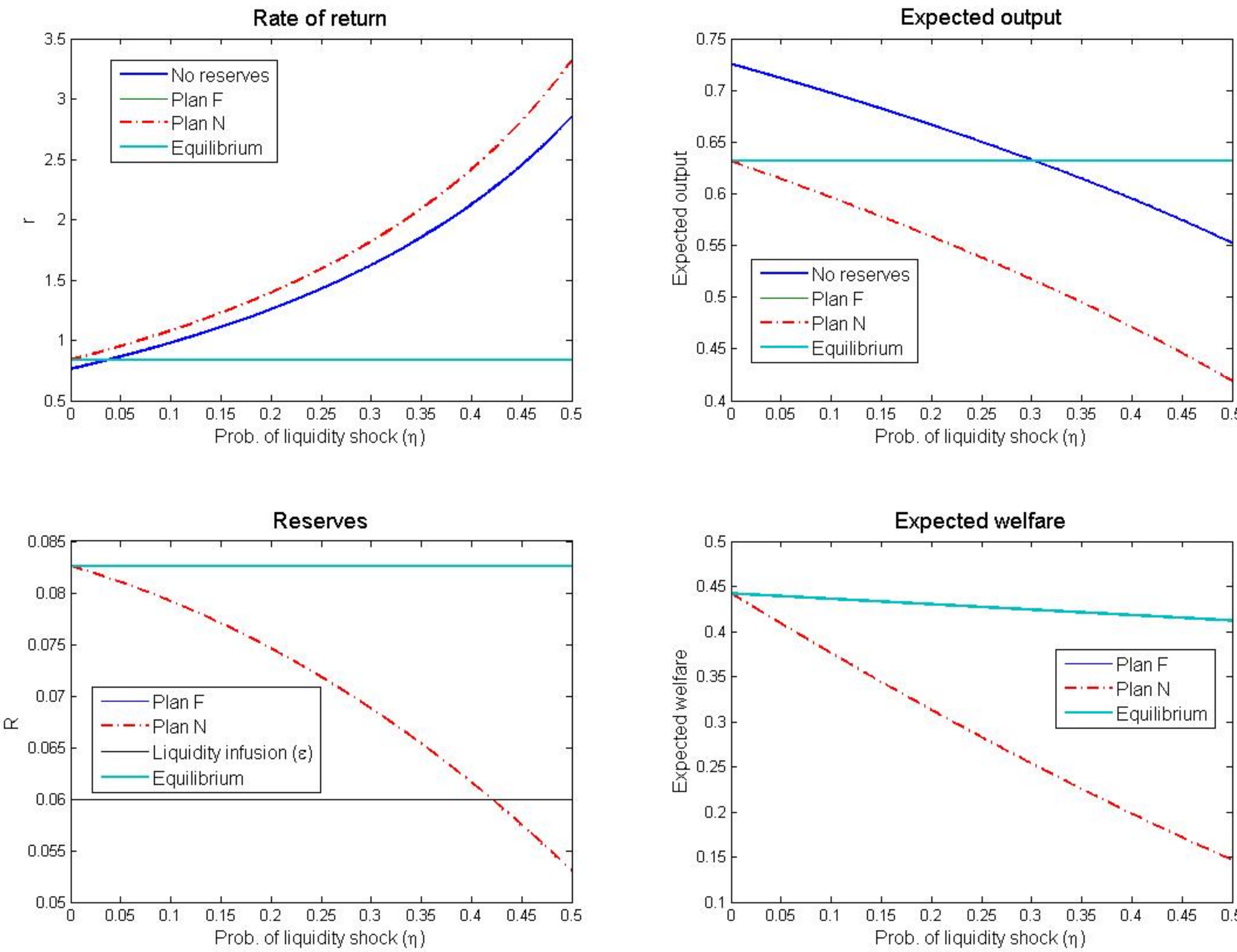

Figure 5: Numerical example - liquidity shock likelihood (η)

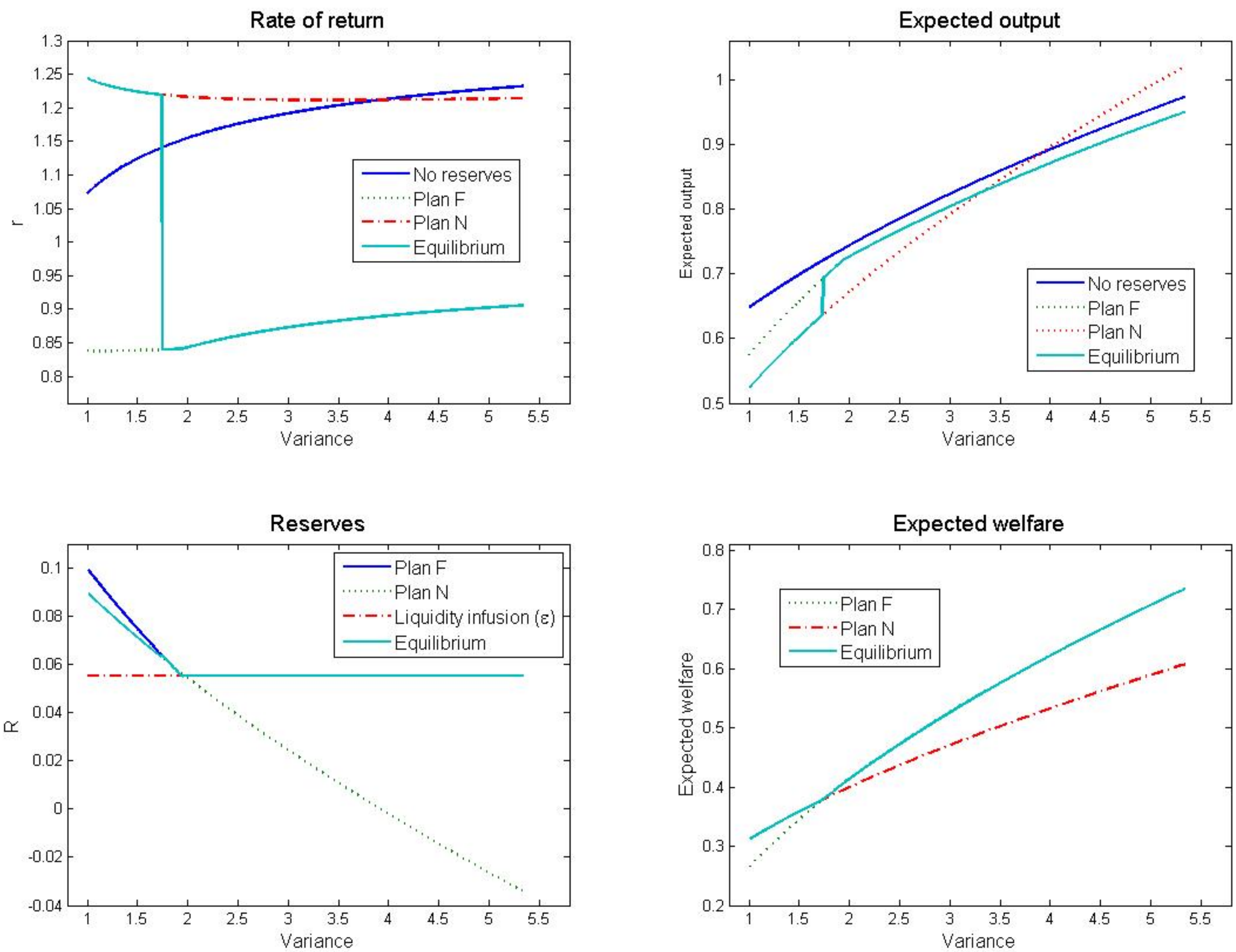

Figure 6: Numerical example - productivity shock variance ($c^2/3$)

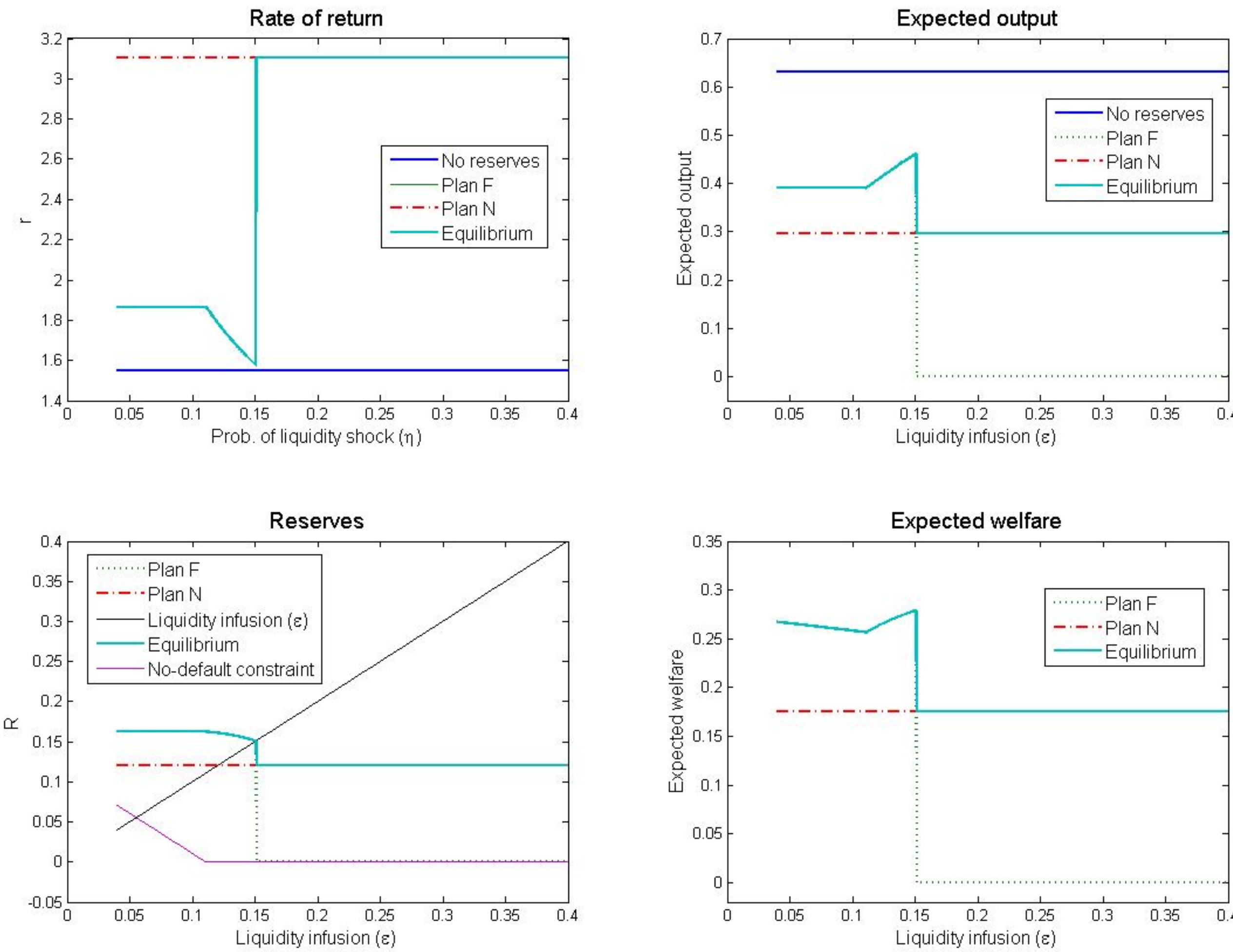

Figure 7: Numerical example - liquidity injection (ϵ)

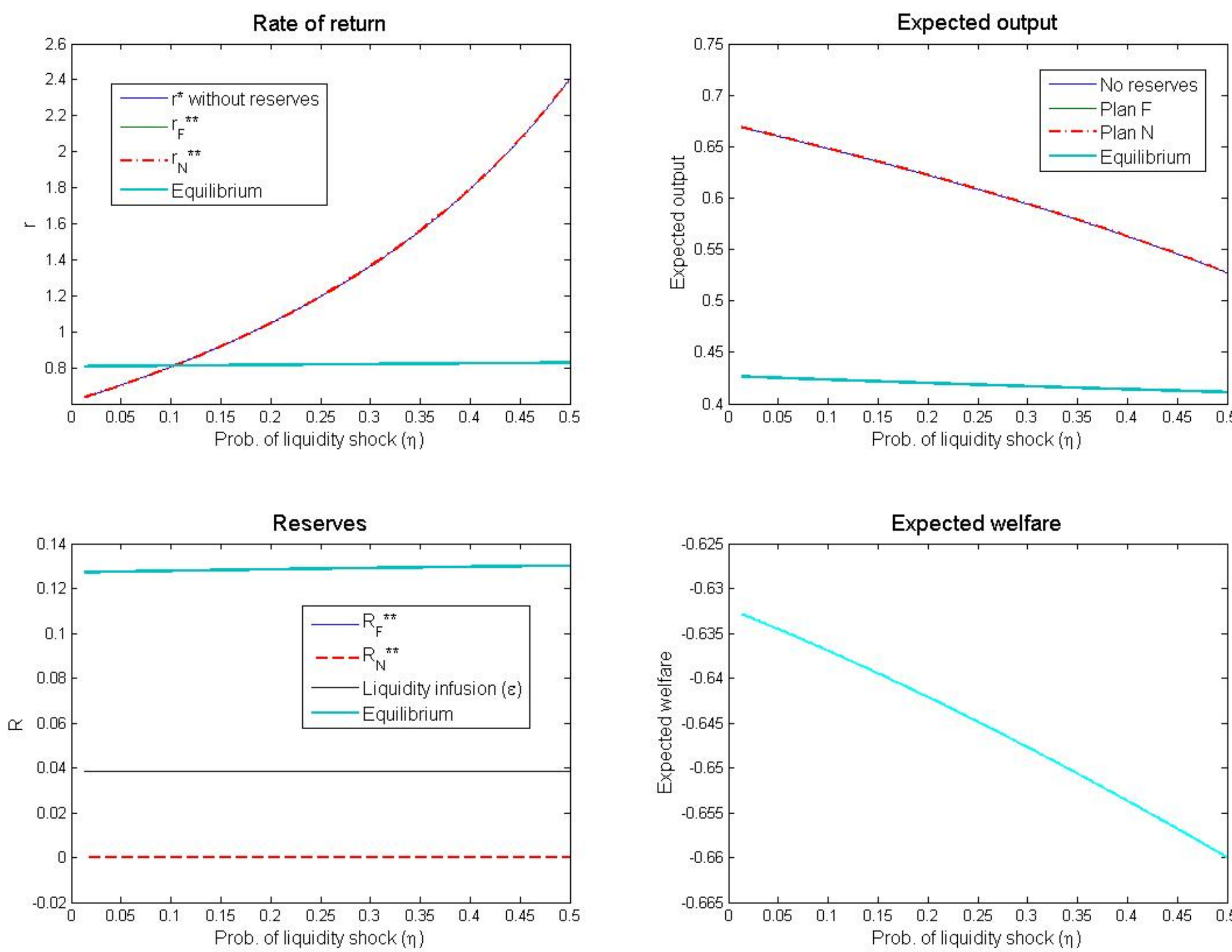

Figure 8: Risk-averse country

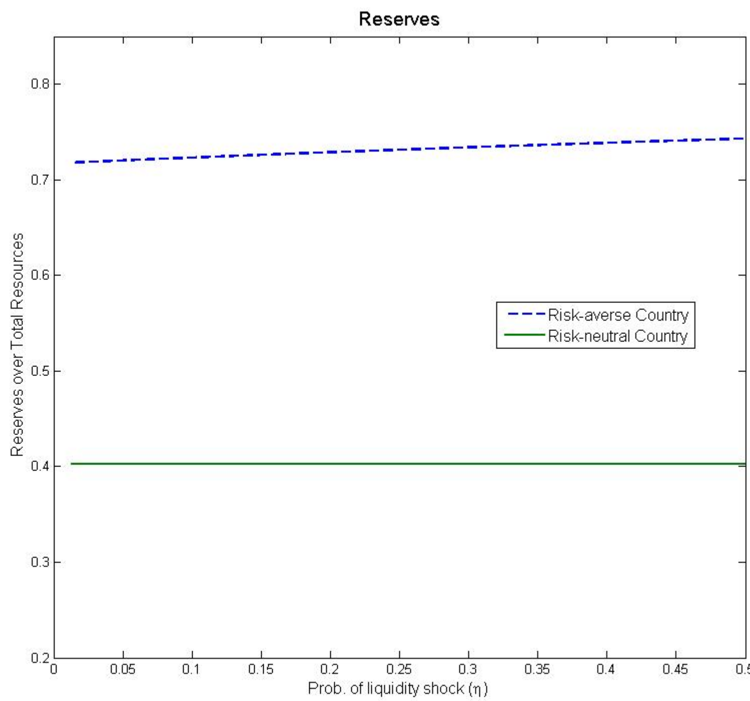

Figure 9: Comparison, risk-neutral v. risk-averse country

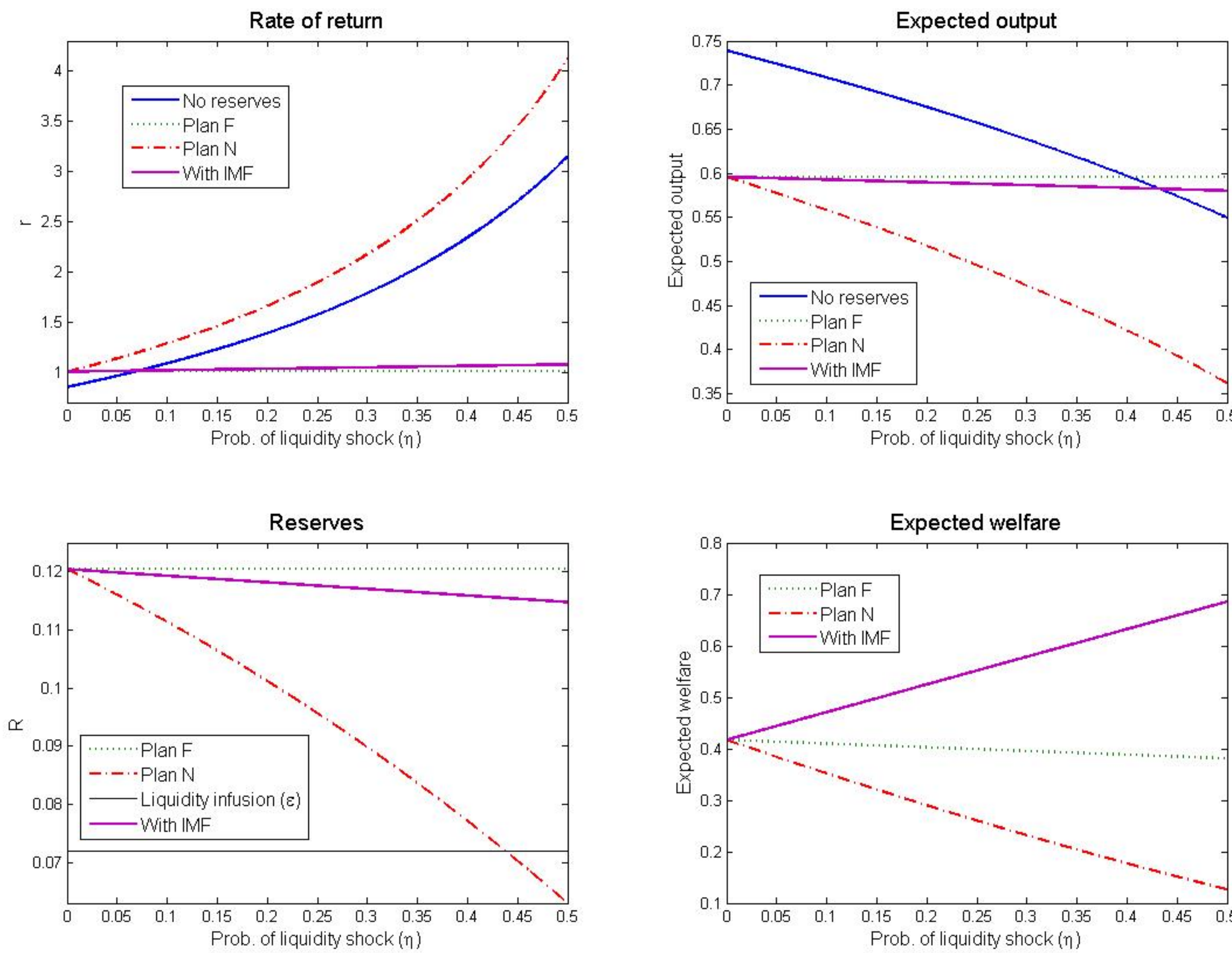

Figure 10: IMF intervention

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