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CREDIT RATINGS IN STRUCTURED FINANCE AND THE ROLE OF SYSTEMIC RISK[#]

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Abstract

This paper explores the implications of systemic risk in Credit Structured Finance (CSF). Risk measurement issues loomed large during the 2007-08 financial crisis, as the massive, unprecedented number of downgrades of AAA senior bond tranches inflicted severe losses on banks, calling into question the credibility of Rating Agencies. I discuss the limits of the standard risk frameworks in CSF (Gaussian, Single Risk Factor Model; GSRFM), popular among market participants. If implemented in a 'static' fashion, GSRFM can substantially underprice risk at times of stress. I introduce a simple 'dynamic' version of GSRFM that captures the impact of large systemic shocks (e.g. financial meltdown) for the value of CSF bonds (ABS, CDO, CLO, etc.). I argue that a proper 'dynamic' modeling of systemic risk is crucial for gauging the exposure to default contagion ('correlation risk'). Two policy implications are drawn from a 'dynamic' GSRFM: (i) when rating CSF deals, Agencies should disclose additional risk information (e.g. the expected losses under stressed scenarios; asset correlation estimates); and (ii) a 'point-in-time' approach to rating CSF bonds is more appropriate than a 'through-the-cycle' approach.

JEL Classification: E44, E65, G01, G12, G13, G14, G18, G21, G24, G28, G34.
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Contents

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1. Introduction

Credit risk measurement issues have loomed very large in the recent Structured Finance (SF) credit rating debacle - e.g. the massive, unprecedented number of downgrades in the AAA senior-tranche segment of the bond market. As discussed at length in Violi (2010), the financial crisis of 2007-08 has shown the importance of model risk and parameter uncertainty in measuring credit risk of (tranched) structured finance products. Unexpectedly, banks have suffered huge (mark-to-market/model) credit losses in the 'safest' segment of the SF markets. The impact of systemic risk changes and its implication in terms of default 'contagion' – including the 'long tail' dimension during times of stress – appear to be largely underestimate by standard market models of credit risk. Several pitfalls regarding the measures of (credit) correlation risk were subject to scrutiny well before the 2007-08 credit crisis, as witnessed by the implied vs. base correlation indicators debate. However, no clear consensus emerged within the profession about the solutions that would avoid these shortcomings. Standard market practice seems to have overlooked the role of systemic risk changes, in that they can have vast implications for the pricing of securitization deals and their associated credit ratings.

With all their imperfections and approximations, we maintain that the basic set of analytical techniques necessary to get reasonably behaved estimates of loan-loss distribution were already available to market participants before the crisis. To substantiate such claim I use a relatively well-known and widely used class of models that antedates the crisis to show that, if properly adapted, it could have indeed yielded sufficient insights regarding the implication of systemic risk changes in SF deals. Unfortunately, standard market practice has often confined this class of models to an essentially static framework, incapable of specifying evolution of parameters to future times under various market conditions. Proper treatment of systemic risk changes over time turns out to be crucial for gaining such insights. One important implication of incorporating systemic risk changes is that it shows how unreliable (as indicators of credit standing) ratings can actually become under stressing conditions. As a result, I argue that credit rating systems based on unconditional - 'through-the-cycle', in the Credit Ratings Agencies (CRAs) jargon, as opposed to conditional ('point-in-time') - valuation are inevitably very fragile when applied to SF deals, in that (conditional) expected loss can vary substantially 'through the cycle' owing to the time-varying nature of systemic risk.

The analytical structure adopted in this paper is primarily focused on a conditional version of the standard Gaussian Copula Credit Risk Factor Model (GCCRF), perhaps the simplest tractable technique for analyzing loan-loss portfolio of assets backing securitization deals, such as Collateralised debt Obligation, CDOs (Li, 2000). The conditional version of the GCCRF model, which requires information regarding the expected changes in the risk factor, stands in sharp contrast with the standard market practice which only exploit the unconditional version. More specifically, standard market practice targets the unconditional loss distribution by integrating out the Gaussian market risk factor. Since the resulting (unconditional) density is not Gaussian (Vasicek, 1991), no simple, closed form solution is available and therefore numerical methods – such as the popular market standard elaborated by Hull and White (2004) – have to be used by performing Monte Carlo simulations.

In addition to the complexity of numerical method and the limitations due to its 'static' nature, it is well acknowledged that the standard, unconditional *GCCRF* model has a number of deficiencies. First of all, there are only two credit risk parameters: the average default probability and a correlation parameter, which is a stylized version of Merton's asset correlation. Secondly, this model is not able to capture all market quotes on the liability side with these two parameters. Therefore each tranche can only be priced with a different correlation, the so-called implied correlation. Thirdly, as the latest credit crises has made clear, the standard *GCCRF* appears to be insufficient to capture the dependence structure between individual obligors backing the credit portfolio of SF deals.

In this paper I argue that the main drawbacks with the unconditional *GCCRF* model are caused by its inherently, purely static nature (one-period model). In practice, portfolio default rates change over time, often in predictable way from period to period, exhibiting well-defined time series properties (for example, defaults clustering during recessions). In times of stress, it is very likely that the 'static' *GCCRF* standard model severely underestimate the actual (systemic) risk exposure of credit portfolios of individual tranches (Bhansali *et al.* 2008). As the conditional *GCCRF* model implemented in this paper shows - more details on this point would be presented later[1] - the impact of systemic risk on the expected loss of a single tranche can vary drastically with its attachment/detachment points (credit seniority). However, the implied base correlation indicator for the unconditional, 'static' model can only reflect the 'average' correlation risk level of the underlying tranches. In this case, we cannot trust the unconditional loss distribution evaluation, if the magnitude of the response to systemic risk changes is bound to be potentially large and highly non-linear across the capital structure and risk levels.

In this paper I adapt the standard GCCRF model to a conditional framework, while preserving its tractable form, in order to obtain a dynamic conditional portfolio loss distribution, propagated in time by the changes in (systemic) risk factor. In so doing, I extend the Vasicek (1991) model, widely employed by academic and industry researchers, to allow for the changes in loan loss distribution driven by the underlying dynamics of the systemic risk factor. Since I maintain a conditional Gaussian risk structure, I am able to get a closed-form implementation for the (conditional) expected loss of individual bond tranches, so that fast and accurate simulations for the entire capital structure of any securitization deal can be performed without resorting to Monte Carlo simulations. Preserving the conditional structure of the loan loss portfolio distribution permits also to highlight the strong dependence of individual tranche 'correlation risk' exposure on the level of systemic risk. As it turn out from my sensitivity analysis, such dependence is of utmost importance for senior bond tranches value. More specifically, as a result of a large systemic shock, they can incur significant losses (provided that default correlation is not immaterial), despite the good quality of their collateral pool in normal times (i.e. low average unconditional default probability). The unconditional ('static') *GCCRF* version cannot reconcile the evidence of large losses for senior bond tranches under stress conditions (e.g. large systemic risk shock) as being consistent with its 'correlation risk' exposure[2].

While keeping my modeling strategy as simple as possible by staying close enough to the standard *GCCRF* model, I draw from the approach of Lamb *et al.* (2008) that have recently shown how an explicit dynamic stochastic process can be introduced to

[1] See section 4.

model systemic risk changes over time (e.g. an autoregressive time series process). I derive a conditionally-evolving dynamic loan loss distribution on a credit portfolio, which I apply to the pricing of (synthetic) CDOs tranches spread in closed form. As the distribution of losses at different future dates is affected by conditioning information, this model can also shed light on the term structure of credit risk.

Lamb *et al.* (2008) fit a similar model to CDX tranches (5, 7 and 10 years tenors and with attachment-detachment points 0-3%, 3-7%, 7-10%, 10-15%, and 15-30%) and the CDX index spread for a sample that includes observations of the contracts weekly from June 2006 to December 2007. However, they do not derive in closed form the value of the bond tranches; they rely on a numerical solution method to derive the (estimated) loan loss distribution. On a similar vein, Sidenius *et al*. (2008) argue that the industry-standard (unconditional) *GCCRF* model specifies a distribution of credit events directly and therefore imposes no (sensible) dynamics on the credit risk (spreads) of the underlying portfolio of names. Their model (called *SPA*, out of the authors' name) is an attempt to provide a novel, tractable framework that extends the current portfolio loss models with realistic dynamic properties. The *SPA* loss distribution is assumed to evolve as a Markov process based on the path of the background process. Unfortunately, its operational implementation is not amenable to a closed form solution (Lamb *et al.*, 2008),. While numerically efficient algorithms can be constructed to value bond tranches linked to the dynamics of loss distributions, these algorithms involve non-linear regression steps to uncover conditional expectations. As the optimal tuning of these

[2] Implied correlation extracted from credit spreads differ across bond tranches (higher for junior and senior, lower for mezzanine tranches; 'correlation smile').

regressions is strongly model specific, they seem to require further investigation before they can be widely used.

The risk characteristics and general behavior of losses in a conditional and unconditional world are very different. These differences have important implications for SF securities rating systems that would need to be discussed[3]. Typically, the rating process for a standard SF deal is staged in two phases. First, the estimation of the loss distribution over a specified horizon and, second, the simulation of the cash flows. The simulations incorporated the CDO waterfall triggers, designed to provide protection to the senior bond tranches in case of bad events, were used to investigate extreme scenarios. The loan loss distribution assumptions allow the determination of the *Credit Enhancement* (*CE*), that is, the amount of loss on the underlying collateral that can be absorbed before the tranche absorbs any loss. If the credit rating is associated with a probability of default, the amount of *CE* is simply the level of loss such that the probability that the loss is higher than *CE* is equal to the probability of default. *CE* is thus equivalent to a standard *Value-at-Risk* type of risk measure[4]. Thus, given the crucial role of credit risk as part of the rating assignment process, the credit risk properties of (plain-vanilla ABS) CDOs notes are investigated using a conditional *GCCRF* model. For the sake of comparability standard assumptions concerning expected loss of the underlying collateral value (default probability and loss given default) are adopted. However, unlike

[3] I am indebted to an anonymous referee for stimulating comments regarding the pros and cons of conditional vs unconditional credit risk measure in SF deals valuation.

[4] In a typical CDO, credit enhancement comes from two sources: "subordination", that is, the par value of the tranches with junior claims to the tranche being rated, and "excess spread" which is the difference between the income and expenses of the credit structure. Over time, the CE, in percentage of the principal outstanding, will increase as prepayments occur and senior securities are paid out. The lower the credit quality of the underlying subprime mortgages in the ABS CDOs, the greater will be credit enhancement, for a given credit rating. Deterioration of credit quality, will lead to a downgrade of the ABS structured credits.

the static treatment of systemic risk exposures routinely used in many applications, we focus our modeling strategy on the fluctuations in market perception regarding the total loss distribution on the collateral value. Changing market perception about the loss distribution is captured by conditioning such distribution upon a (single) macro-risk factor (systemic risk). Since a single macro-risk drives the performance of the underlying collateral value and consequently also the associated tranches, we can trace the allocation of systemic risk to individual tranches. Thus, such modeling strategy sheds light on how systemic risk would impact on the value of (junior/senior) *SF* bond tranches. Also, the macro-risk factor dependency of individual tranches can be investigated and total portfolio losses decomposed in a "two-dimensional way" to states and tranche categories, where states are defined as the macro factor (systemic risk) taking values in certain ranges. In addition, this methodology enables one to analyze the impact of model/estimation risk, represented by differing risk properties of the underlying collateral value (default probability, default correlation, etc.), and to delineate its effect on the risk properties of the various bond tranches.

An outline of this paper is as follows. Section 1 presents a simple model of default risk and expected loss based on the standard mixed binomial model and a common method for modeling the joint incidence of defaults. Section 2 illustrates a basic (yet realistic) synthetic CDO structure with a focus on the mechanics of credit risk evaluation related to securities tranching. Section 3 introduces the *GCCRF* model, distinguishing conditional vs. unconditional approach to portfolio credit risk measurement. Credit risk sensitivity to model parameters and structure are investigated also with the help of some stylized numerical examples. Section 4 takes up the implication of the *GCCRF* model for the tails of the loss distribution. These implications are tested by simulating the bond prices (senior, mezzanine and equity tranches) of a (stylized) CDO backed by a pool of assets. Several types of systemic risk shocks to the credit loss distribution are considered in reporting the expected losses of each bond tranche. In addition to the standard GCCRF framework, we also simulate the impact of systemic shocks under the Student-t distribution assumption, so that we can gauge the impact of fatter tails of the loss distribution on the credit risk of the tranches. Section 5 summarizes the main results of this paper and discusses the implications of switching from 'through-to-cycle' to 'point-in-time' ratings system, in order to take into account the exposure to systemic risk changes.

2. The mixed binomial model: a primer on default risk and expected losses

One common approach in credit-risk analysis is the mixed-binomial model, which is used in a wide class of models analyzing defaults. The key inputs are the default correlations across and within sectors, which determine both the value that is created from pooling assets and the tranching capacity of the pool Mixed Binomials are used in a wide class of models analyzing defaults[5]. We posit that the default probability of a mortgage is a Bernoulli random variable, taking the value of 1 with probability p and 0 with probability $1 - p$. Next, we posit a pool of mortgages in which the default probability of mortgage i is denoted by X_i and is equal to 1 if the mortgage defaults, and 0 otherwise. Each mortgage in the pool is assumed to have a different default probability,

[5] This section draws heavily on Lando (2004).

hence we need to randomize the default probability p. Randomization is achieved using a mixture distribution, which randomizes the default distribution of the binomial model, inducing dependence between different default probabilities. The dependence so generated mimics an environment in which a pools of different mortgages are subject to a common economic risk. Assume that the default parameter

$\tilde{p} \in [0,1]$ is independent of the X_i's and that conditional on $\tilde{p}$ all the X_i's are independent. Denoting the density of $\tilde{p}$ by $f(p)$ we have

$$\bar{p} = E(\tilde{p}) \equiv \int_0^1 pf(p)dp$$

(1a)

Using the law of iterated expectations and variance decomposition, we have

$$E(X_i) = \bar{p}\,; \sigma^2(X_i) = \bar{p}\,(1 - \bar{p}\,); COV(X_i; X_j) = E(\tilde{p}^2) - \bar{p}^2$$

(2a)

We can now express the default correlation as

$$CORR(X_i; X_i) = \frac{E(\tilde{p}^2) - \bar{p}^2}{\bar{p}\,(1 - \bar{p}\,)}$$

(3a)

The default correlation is 0 if $\tilde{p}$ is constant. Moreover, the default correlation in (3a) is always non-negative in this model[6]. The total number of defaults in the pool of mortgages is $D_n = \sum_{i=1}^{n} X_i$ and $E(D_n) = n\bar{p}$. The variance of the total number of defaults in the mortgage pool is:

$$\sigma^2(D_n) = n\bar{p}(1 - \bar{p}) + n(n-1)\left[E(\tilde{p}^2) - \bar{p}^2\right]$$

(4a)

and

$$\sigma^2(D_n/n) = \frac{\bar{p}(1-\bar{p})}{n} + \frac{n(n-1)}{n^2}\left[E(\tilde{p}^2) - \bar{p}^2\right] \rightarrow E(\tilde{p}^2) - \bar{p}^2, \text{as } n \rightarrow \infty$$

(5a)

That is to say, for large enough n, the variance of the average default rate D_n/n is determined by that of the distribution of $\tilde{p}$. Using the fact that when n is large, the realized frequency of defaults is almost identical to the realized value of $\tilde{p}$, the distribution of defaults becomes that of $\tilde{p}$ and hence one can show that:

$$\bar{p} = E(\tilde{p}) \equiv prob(D_n/n \leq \theta) \rightarrow \int_{\theta}^{0} f(p)dp \equiv F(\theta) \text{ as } n \rightarrow \infty$$

(6a)

That is, for a large pool of assets the probability distribution of $\tilde{p}$, $F(\theta)$, determines the risk distribution of the portfolio: the greater the variability in the mixture distribution of $\tilde{p}$, the greater the correlation of defaults, and hence the greater the weight on the tails of the distribution. Increasing the correlation between assets in the collateral pool decreases the value of the most senior tranches, because the likelihood of a large number of defaults increases and more of the junior tranches are likely to be wiped out. On the other hand, as the correlation increases, the value of the junior tranches increases as well, as more weight is being put on the other tail of the distribution – and very few defaults are more likely as well. Hence the mixing distribution in the binomial model is crucial not only for the value of diversification of the collateral pool, but also for the ability to carve out highly rated risk-free tranches. Thus, as noted by Fender and Kiff (2004), given the

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[6] See Lando (2004), p. 217.

diversity of correlation assumptions across rating agencies and CDO methodologies, the ratings that estimated ELs could map into can differ substantially.

3. Collateral pool value and securities tranching

The common method for modeling the joint incidence of defaults is known as the copula method (Schonbucher, 2003). This approach draws a set of n correlated random variables $\{Y_i\ ;\ i=1,n\}$ from a pre-specified distribution and then assumes that a firm defaults if its variable, $Y_i = y_i$, is below the π-th percentile of the corresponding marginal distribution, $F_i(y_i)$. In this framework, by construction, a firm defaults π% of the time and default dependence can be flexibly captured through the proposed joint distribution for $\{Y_i\ ;\ i=1,n\}$. A popular choice for the joint distribution function is the multivariate Gaussian (Vasicek, 2002), in which default correlation is simply controlled by the pairwise correlation of (Y_i, Y_j). Off-the-shelf CDO rating toolkits offered by CRAs, such as Fitch's Default VECTOR models, Moody's CDOROM and Standard and Poor's CDO Evaluator (Standard & Poor's, 2005), all employ versions of this copula model. The numerical simulations reported in section (3)-(4) rely on a simplified version of the copula model that is the industry standard for characterizing portfolio losses.

Consider the simplest types of collateralized debt obligations (CDOs), tranches of structured debt securities backed by pools of bonds (say, ABS). The normal-copula/beta model of bond losses can be used to build up a model of CDO tranche credit losses. More specifically, consider a static CDO deal backed by n bonds. Investments are made at the "deal date" and proceeds are distributed to investors at the maturity date. The value of the collateral pool at the deal date is normalized to one and the value of the collateral pool at the maturity date is denoted V_p. Share k_e of the collateral pool is funded by equity investors at the deal date. The remaining $1-k_e$ of the pool is funded by a continuum of arbitrarily thin debt tranches. Debt tranches are indexed by $k \in [k_e, 1]$, where k is a tranche's attachment point in the CDO capital structure. Thus, higher values of k imply greater seniority. The interest paid to each debt tranche is described by the non-increasing function $r(k)$. At the maturity date, collateral is liquidated and tranche k investors are paid $1+r(k)$, if sufficient funds are available. If V_p is not sufficient to pay all debt investors, tranches are paid according to seniority. If V_p exceeds that needed to pay debt investors, equity investors receive any residual value.

Assuming no credit losses, the total value of all debt tranches senior to tranche k is

$$\overline{V}(k) = (1-k) + R(k) \tag{1b}$$

where $R(k) \equiv \int_k^1 r(s)ds$ is the total interest owed to these tranches. The realized value of tranches senior to k is

$$V(k) = \overline{V}(k) - 1\{V_p \leq \overline{V}(k)\}[\overline{V}(k) - V_p] \tag{1'b}$$

where 1{.} is an indicator function (dummy variable) that is equal to 1 if the inequality in parenthesis is fulfilled and 0 otherwise. The second right-hand term is the value of any realized credit losses for tranches senior to k. Note that the value for a "slice" of the CDO with attachment point k_l and detachment point k_h is $V(k_l) - V(k_h)$. The value of the equity tranche is $V_p - V(k_e)$.

4. Credit model and systemic risk in a Gaussian copula framework

Under the simplest Gaussian copula framework, obligor (say, bond) i defaults during a specified horizon (say, one year) if an unobservable normal latent factor Y_i lies below the default threshold $\Phi^{-1}(\pi)$, where Φ^{-1} is the inverse of the standard normal cumulative density function. The parameter π describes the bond's marginal probability of default.

Consider n bonds (obligors), all of equal size ($1/n$), entering a portfolio and taking the limit $n \to \infty$. This portfolio is referred to as asymptotic or "perfectly-fine-grained". All obligors (issuers) exhibit ex-ante homogeneous credit risk. This is captured by assuming that the value of the assets of each obligor, i, $A_{i,\tau}$, reflects his credit condition and evolves in the following way:

$$Ln\left(A_{i,\tau}\right) = Ln\left(A_{i,\tau-\Delta}\right) + \mu\Delta + \sigma\sqrt{\Delta}Y_{i,\tau} \tag{2b}$$

The drift and the volatility of the asset value are controlled by (constant) positive parameters μ and σ, which are the same for all obligors; Δ denotes the period between two observations (not greater than the specified horizon to default). The riskiness of obligor i is driven by the (latent) credit risk factor, $Y_{i,\tau}$.

$$Y_{i,\tau} = \sqrt{\rho}X_{\tau} + \sqrt{1-\rho}Z_{i,\tau} \tag{3b}$$

where X_τ is a standard normal random factor, $X_\tau \sim N(0,1)$, shared by all obligors, and $Z_{i,\tau}$ is a standard normal idiosyncratic factor, $Z_{i,\tau} \sim N(0,1)$, that is unique for each obligor. In addition, it is postulated that $\{X_\tau, Z_{i,\tau}; i=1,n\}$, are serially uncorrelated. Cross-sectional correlation in defaults across pairs of simple bonds i and j arises from correlation in latent credit factors $Y_{i,\tau}$ and $Y_{j,\tau}$, which is controlled by parameter ρ, lying between zero and one, $\rho \in [0,1]$. This parameter determines the correlation in credit factors between pairs of bonds. Higher values of ρ imply closer correlation between credit factors, and, by extension, between realized defaults. Obligor i defaults on the bond issue if and only if $\mathrm{Ln}(A_{i,\tau})$ is below some non-stochastic (log) debt threshold, $ln(A_i^L)$. Default events are assumed to occur only at the end, $t \in \{1,2,\ldots,T\}$, of a non-overlapping and adjacent time period (say, one year), which may be longer than the periods between two consecutive observations of the obligor's assets (i.e. $1>\Delta$).

As is argued by Adelson (2003), default correlation is a time-varying phenomenon. In addition, the (standard) assumption of constant recovery rate does not conform with the empirical evidence of substantial cyclical variability in recoveries and negative correlation with default probabilities. Thus, the systemic risk located in the tails of pool loss distributions is not adequately accounted for by the (standard) simple constant copula framework.

Assume that bond i is a bullet loan that pays $1 + r_i$ at maturity if the obligor does not default, and $(1 + r_i)(1 - \lambda_i)$ if the obligor defaults. λ_i is a random variable describing the realized loss given default; for simplicity this analysis assumes that λ_i is independent of all other random variables. For corporate bond exposures, this loss rate is often assumed to be drawn from a beta distribution that may or may not depend on the systemic factors that drive asset correlations. The beta distribution is a two-parameter distribution with support on the unit interval that can be fully characterized by a mean parameter μ_λ

and a standard deviation parameter σ_λ. Such parameterization of the beta distribution makes the economic interpretation of model parameters more transparent than the standard two-shape parameters (α, β) characterization found in the statistics literature. It can be shown that

$$\alpha = [\mu_\lambda (1 - \mu_\lambda) - \sigma^2{}_\lambda](\mu_\lambda/\sigma^2{}_\lambda)$$

$$\beta = [(1 - \mu_\lambda)^2 + \sigma^2{}_\lambda](\mu_\lambda/\sigma^2{}_\lambda) -1.$$

(4b)

The payout from a one-dollar investment in bond i at the maturity date is

$$V_i = (1 + r_i)\ [1\text{-}1\{Y_i \leq \Phi^{-1}(\pi)\}\lambda_i]$$

(5b)

The right-most term is the realized contractual loss per dollar invested. Note that when λ_i is large, this loss rate may exceed 100 percent because of accrued but unpaid interest. Given n homogeneous bonds, the joint distribution of [$V_1, V_2, \ldots, V_n$] is fully described by the risk parameter vector $\theta = (\pi, \rho, \mu, \sigma)$.

To keep notation simple, this analysis is restricted to CDOs backed by equal-weighted pools of n bonds – all of them of equal size, which is set to $1/n$ – that are homogeneous in the sense that all bonds in the pool share the same risk parameter vector θ and pay the same interest rate r_p. Let

$$D(n) = \sum_{i=1}^{n} 1\left\{Y_i \leq \Phi^{-1}(\pi)\right\}$$

(6b)

be a random variable that describes the number of bonds in the CDO collateral pool that default by the maturity date, and let

$$\overline{\lambda}_D = \frac{1}{D(n)} \sum_{i=1}^{D(n)} \lambda_i \tag{7b}$$

be the average loss given default for those *D(n)* bonds. The value of the collateral pool at the maturity date is

$$V_p = (1 + r_p)\left[1 - \frac{D(n)}{n} \overline{\lambda}_D\right] \tag{8b}$$

The random variables *D(n)* and $\overline{\lambda}_D$ determine V_p. *D(n)* is a draw from a binomial-normal mixture distribution, and, conditional on *D(n)*, $\overline{\lambda}_D$ is an average of *D(n)* independent beta random variables. Neither the marginal distributions of *D(n)* nor the conditional distribution of $\overline{\lambda}_D$, given *D(n)*, can be easily expressed in exact closed form; but both can be computed analytically with high precision. In the next section we will consider a Bernoulli mixture model in which computation can be performed explicitly.

Given the assumption of independence, the product of these two distributions is the joint distribution of *D(n)* and $\overline{\lambda}_D$, which provides all the information necessary to compute the joint distribution of V_p and *V(k)* for all k. The distribution of CDO tranche payouts is fully determined by *n*, r_p, *r(k)*, and the normal copula/beta model parameter vector θ for the collateral pool. *n*, r_p, and *r(k)* are known features of the CDO contract, but market participants cannot directly observe the risk parameter vector θ. Given θ, any number of relevant metrics of the credit risk associated with a CDO note can be computed. The next section examines how three common metrics of credit risk depend on the vector of risk parameters θ.

4.1 Credit risk sensitivity to model parameters and structure.

This analysis considers three standard metrics of credit quality: probability of default, expected loss, and conditional expected loss. Define the expectation operator $E[Z]$ as the expected value of the random variable Z whose distribution is determined by θ. Let V_i be the value of a one-dollar investment in debt security i at the maturity date and let r_i be the contractual interest on that security. The security's probability of default is defined as

$$PD_i = E\,[1\{V_i < (1 + r_i)\}] \tag{9b}$$

PD_i describes the likelihood but not the magnitude of a credit loss. The (unconditional) expected loss

$$EL_i = (1 + r_i) - E[V_i\,] \tag{10b}$$

summarizes the expected likelihood and the magnitude of a credit loss[7]. PD_i and EL_i describe the first moment of a security's loss distribution. In portfolio risk management applications such as economic capital allocation, analysts also require information about a security's marginal contribution to portfolio-wide losses. The literature has proposed a number of risk metrics useful for describing the dependence between an individual exposure's credit losses and those of a broader portfolio. Here, following the treatment of

[7] Note that EL_i may exceed 100 percent because both principal and accrued interest may be lost.

Schonbucher (2003), I consider only one such measure derived from an asymptotic single risk factor approximation. Gordy (2003) shows that if a portfolio is well diversified and its overall loss rate depends on a single systemic factor, *X*, then an exposure's marginal contribution to risk can be determined analytically by calculating the conditional expected loss of the exposure given an adverse draw of the systematic risk factor.

The conditional expected loss associated with a α^{th} percentile portfolio risk measure (CEL_{α}) is

$$CEL_{\alpha} = (1 + r_p) - E[V_p | X = x_{\alpha}] \tag{11b}$$

where x_{α} is the $1 - \alpha^{th}$ percentile of the stochastic systemic risk factor, *X*,

$$x_{\alpha} = \Phi(1-\alpha) \tag{12b}$$

Unlike *PD* and *EL*, which describe the (unconditional) center of the distribution of losses, CEL_{α} describes the conditional losses of this distribution.

Under the normal copula/beta model, *EL*, *PD*, and CEL_{α} for both simple and structured bonds are determined by the parameter vector θ. For plain vanilla bonds, the default probability of obligor *i*, with credit condition regulated by eq. (2b), is given by

$$PD_i \equiv \pi_i = \Phi(Y_i^L), \text{ where, } \mathrm{Y_i^L} = \frac{L_i - Ln\left(A_i^L\right) - \mu_i \Delta}{\sigma_i \sqrt{\Delta}} \tag{13b}$$

where Y_i^L is a critical threshold of credit quality. The expected loss associated with default probability (13b) can be written as

$$EL_i = (1 + r_i)\ \pi_i\, \mu_\lambda.$$

(14b)

Recalling definition (6b), we can get an expression for the conditional expected loss as a function of the fraction of loans that default, L_n

$$\mathrm{CEL}_\alpha = (1 + r_p)\Phi\left[\frac{\Phi^{-1}(\pi_p) - \sqrt{\rho}\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right]\mu_\lambda$$
$$\lim_{n\to\infty} E[L_n|X] = \Phi\left[\frac{\Phi^{-1}(\pi_p) - \sqrt{\rho}\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right] = \Pr ob\left[Y_i \le Y_i^L | X\right]$$
$$L_n \equiv \frac{1}{n}\sum_{i=1}^{n} \mathrm{D_i(n)}$$

(15b)

As the number of credits in our portfolio gets very large ($n \to \infty$), the fraction of defaulted credits in the n obligors' portfolio, L_n, converges to the individual default probability of each individual credit[8]. When $\rho = 0$ defaults are statistically independent, so $L_n = \pi_p$ with probability 1, while when $\rho = 1$ defaults are perfectly correlated, so $L_n = 0$ with probability $1 - \pi_p$, and $L_n = 1$ with probability π_p.

The CEL_α risk measure (15b) assumes crucially that:

1) all obligors' PD_i are identical (e.g. $\pi_i = \pi_p$), hence requiring that $Y_i^L = Y^L$, with

$$\pi_p = \Phi(Y^L)$$

(16b)

2) the exposures of all obligors (i.e. correlation) ρ to the (single) systemic risk factor (underlying the obligor's assets), X, are cross-sectionally identical.

Under these two assumptions the aggregated (total) conditional loss distribution for a large and homogenous (granular) portfolio can be well approximated by applying the central limit theorem[9],

$$
\begin{aligned}
&\mathrm{L}_{\infty} \mid \mathrm{X} \equiv \underset{n\to\infty}{Lim}\, L_{\mathrm{n}} \mid \mathrm{X} \sim \mathrm{N}\left(m_L ; \sigma_L^2\right) \\
&m_L \equiv E\left(L_n \middle| X\right) = \Phi\left[\frac{\Phi^{-1}\left(\pi_p\right) - \sqrt{\rho}\,\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right] \\
&\sigma_L^2 \equiv VAR\left(L_n \middle| X\right) = \frac{1}{n}\left\{\Phi\left[\frac{\Phi^{-1}\left(\pi_p\right) - \sqrt{\rho}\,\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right] - \Phi\left[\frac{\Phi^{-1}\left(\pi_p\right) - \sqrt{\rho}\,\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right]^2\right\}
\end{aligned}
$$

(17b)

Hence, for large portfolios the total loss distribution – conditional on the value of the systemic risk factor, X – is approximately Gaussian, with mean m_L and variance, σ_L^2. However, it is critical to realize that we must assume that the value of the (latent) risk factor, X, is known. In practice this value may be unknown and has to be integrated out. This would generally imply a non-Gaussian distribution for the total loss with generally fat tails, perhaps because of the conditional heteroskedasticity created by the risk factor[10]. Since for very large n, the variance of the loss distribution declines monotonically towards zero,

$$
\underset{n\to\infty}{Lim}\, \sigma_L^2 = \underset{n\to\infty}{Lim} \frac{1}{n}\left\{\Phi\left[\frac{\Phi^{-1}\left(\pi_p\right) - \sqrt{\rho}\,\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right] - \Phi\left[\frac{\Phi^{-1}\left(\pi_p\right) - \sqrt{\rho}\,\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right]^2\right\} = 0
$$

(17c)

---

[8] As Schönbucher (2002) and Vasicek (2002) explain, such convergence is warranted since defaults are independent when conditioned to the realization of the common risk factor, X.

[9] See Foulcher, Gouriéroux and Tiomo (2005) for details. Vasicek (1991) and Schonbucher and Schubert (2001) show that approximation (17b) is quite accurate for the upper tail of the loss distribution even for mid-sized portfolios of about 100 names.

[10] This result is proved by Vasicek (1991); see also Tarashev (2009).

the fraction of defaulted obligors in the collateral pool converges to the its mean, m_L, which coincides with the individual default probability (reported in eq. 15b),

$$\text{Prob}\left\{ \underset{n\to\infty}{Lim}\, L_n = \Phi\left[\frac{\Phi^{-1}(\pi_p) - \sqrt{\rho}\,\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}} \right] = m_L \right\}$$

(17d)

Using this results we obtain the familiar expression for the cumulative loss distribution function, G(L), on a very large portfolio (cf. Vasicek 1991),

$$\text{G(L)} \equiv \text{Prob}\{L_\infty \le L\} = \Phi\left[\frac{\sqrt{1-\rho}\,\Phi^{-1}(L) - \Phi^{-1}(\pi_p)}{\sqrt{\rho}} \right]$$

(17e)

This is a highly skewed distribution with mean, median and mode given by,

$$\text{mean}(L_\infty) = \pi_p \;\; ; \;\; \text{median}(L_\infty) = \Phi\left[\frac{\Phi^{-1}(\pi_p)}{\sqrt{1-\rho}} \right] \; ; \; \text{mode}(L_\infty) = \Phi\left[\frac{\sqrt{1-\rho}}{1-2\rho}\Phi^{-1}(\pi_p) \right] \; \text{for } \rho < \frac{1}{2}$$

(17f)

The probability distribution G(L) can be interpreted as the unconditional version of the conditional loss distribution (17b), which is valid as an approximation only for collateral pools with very low (close to zero) dispersion of losses. This may not be very realistic in many practical applications. However it has the apparent advantage of factoring out the level of systemic risk as a determinant of the loss distribution function. The unconditional ('static') approach in modeling the systemic risk factor within the Gaussian copula model is standard in the literature and is responsible for the theoretical limitation stated above.

Berd *et al.* (2007) and Lamb *et al.* (2008) are important recent contributions that attempt to provide a dynamic formulation of the standard *GCCRF* model by allowing the systemic risk factor to be an auto-regressive time series process with heteroscedastic volatility. Their time series assumption for the systemic risk factor can be extended to the conditional model (17b),

$$X_\tau = \sqrt{\beta} X_{\tau-1} + \sqrt{1-\beta}\upsilon_\tau,\ \upsilon_\tau \sim N(0,1), X_0 \sim N(0,1), \beta \in [0,1]$$

(17g)

with the unconditional variance of the risk factor, X_τ left unchanged (equal to 1; e.g., identical to the variance of X). Without loss of generality, we assume that the initial condition X_0 has the same distribution as the risk factor, X of the standard ('static') GCCRF model. Also, it is easy to show that the steady state (stable) process - $X_0 = 0$ - has zero mean and unit variance. The generalized auto-correlated risk factor model (17g) leaves the loan-loss credit portfolio distribution (17b) virtually unchanged, in that only a new parameter β - controlling for the degree of mean reversion (auto-correlation) in the risk factor process - enters the distribution's moments,

$$m_L \equiv E(L_n|X_{t-1}) = \Phi\left[\frac{\Phi^{-1}(\pi_p) - \sqrt{\rho\beta}\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right]$$

$$\sigma_L^2 \equiv VAR(L_n|X_{t-1}) = \frac{1}{n}\left\{\Phi\left[\frac{\Phi^{-1}(\pi_p) - \sqrt{\rho\beta}\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right] - \Phi\left[\frac{\Phi^{-1}(\pi_p) - \sqrt{\rho\beta}\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right]^2\right\}$$

$$X_{t-1} \equiv \Phi^{-1}(1-\alpha)$$

(17h)

If β=1 (perfect auto-correlation), the credit loss distribution (17h) reverts to (17b), namely to the standard GCCRF model, as the risk process, $X_\tau = X_{\tau-1} = \ldots = X_0$, turns out to be a random walk (actually, constant) process. In this case, the credit loss distribution defined in eq. (17b) can also be made consistent with economic capital regulation, in that the Basel Committee on Banking Supervision (BCBS) suggested a formula for the capital charges that directly replaces the (unknown) risk factor value, X, with a tail approximation (see BCBS, 2004). Such a replacement strategy is used in deriving the credit risk measure adopted in equation (15b).

It is important to note that when the risk factor value is treated as fixed (deterministic; with a risk level α), the implied capital risk charge does not correspond to the usual tail Value-at-Risk, and it likely underestimates the risk on the total loss, since it neglects a large part of the random risk factor realizations. However, we can still take advantage of the approximated loss probability distribution to measure the tail risk associated with the anticipated level of the risk factor, *X*. As we will see in the next section, full knowledge of the loss probability distribution is crucial to assessing the value of the various bond tranches in any SF deal.

Under the assumption of independent defaults – $\rho = 0$ – the CEL_α risk measure coincides with the EL risk measure for the portfolio,

$$CEL_\alpha = EL_p = (1 + r_p)\, \pi_p\, \mu_\lambda \tag{18b}$$

For plain-vanilla bonds, *PD* is determined by the normal copula marginal default probability parameter π_i, *EL* depends on both π_i and the expected loss-given-default parameter μ_λ, and CEL_α is a function of π_p, μ_λ and ρ, the asset value correlation parameter. For structured bonds, there are no simple analytic formulas for *PD*, *EL*, and CEL_α, but these risk metrics can be computed numerically for any value of θ. In contrast to the case for simple bonds, *PD*, *EL*, and CEL_α for structured bonds each depend on all four elements of the risk parameters vector θ. Thus estimates of θ are important determinants of the quality of credit risk management system.

Some stylized numerical examples are provided in Tables 1A, 1B and 1C under some simplifying assumptions – constant (non-stochastic) unitary LGD function (e.g. zero recovery rate), zero interest rate and 1-year horizon,

$$r_p = 0,\ \mu_\lambda = 1\ (e.g.\ \overline{\lambda}_D = 0) \tag{19b}$$

Under the assumptions (19b), the CEL_α defined in eq. (15b) can be directly related to the portfolio (conditional) loss distribution mean parameter

$$\mathrm{CEL}_\alpha = m_L = \Phi\left[\frac{\Phi^{-1}(\pi_p) - \sqrt{\rho\beta}\,\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right] \tag{20b}$$

The factor model (3b) also allows us to derive a measure of default correlation, the true measure of the dependence between the default indicators of two obligors. The relationship between asset correlation and default correlation depends on the probability of default. It can be shown that the default correlation is a convex increasing function of the asset correlation parameter, ρ, taking value 0 at 0 and 1 at 1. In particular the default correlation is always smaller than the asset correlation and can be computed as

$$\textit{Default Correlation} = \frac{\Psi\left[\Phi^{-1}(\pi_p), \Phi^{-1}(\pi_p), \rho\right] - \pi_p{}^2}{\pi_p(1-\pi_p)} \tag{21b}$$

where $\Psi[.]$ is the bivariate standard normal distribution function.

We can now proceed to quantify portfolio credit risk expected loss using eq. (20b). Our approach is consistent with the Basel II model for capital charges in a highly stylized empirical framework. This standard framework, notwithstanding its simplifying assumptions, yields some significant insights. To the extent that it can document fairly large ranges of estimated credit risk measures, it offers evidence that market participants should have been aware of the uncertainty surrounding credit risk assessment in the presence of systemic (default correlation) risk (cf. Heitfield, 2008).

A fixed range of *PD*,

[0.5 ; 1.0 ; 2.5 ; 5.0] percent

is selected along with a grid of reasonable degrees of exposure (e.g. correlation parameter) to the systemic default risk factor, ρ

[0 ; 5 ; 10 ;15 ; 20 ; 25 ; 30 ; 35 ; 40 ; 50] percent

To establish this idea, we assume that the investor is interested in a portfolio of assets (collateral pool) whose value is expected to be subject to a systemic risk factor shock with 99.9% confidence level (e.g. $CEL_{99.9\%}$); also we assume, β=1 (perfect auto-correlation). Due to the simplifying assumptions adopted here, estimation risk (and/or noise) is completely ignored, that is, parameter estimates of θ are treated as if they are known with certainty and are not subject to any estimation/model error or risk adjustment. This is not an innocuous assumption, in light of the fairly wide range of outcomes that we are going to get out of the computed credit-risk measures.

Using equation (20b) to compute the $CEL_{99.9\%}$ risk measure we get 14.55 cents on the euro, with PD = 1% and ρ = 20% (default correlation equal to an appreciable 2.41 percent). Lowering the correlation parameter to 5% – default correlation now drops to very modest level, 0.41 percent – would drastically reduce the computed credit loss measure, as $CEL_{99.9\%}$ declines to 4.67 cents. In the absence of default correlation (e.g. with correlation parameter ρ = 0) – i.e. following an apparently almost insignificant reduction in the measure of the default correlation – $CEL_{99.9\%}$ drops to only 1 cent on the euro (i.e. the average default probability of the portfolio). Raising the correlation parameter to somewhere in the higher range of the spectrum – e.g. ρ = 0.40 (default correlation equal to 7.73 percent) – the conditional expected credit loss jumps to 31.56 cents on the euro. Thus, even simplistic credit risk models – such as the static one-factor Gaussian copula – can generate fairly large expected loss when systemic risk shocks are great enough and the default correlation rate is significant.

Such fairly drastic changes persist, although marginally smaller, also at higher levels of default probability. For default probability below 1% (say, 0.5%) correlation sensitivity is slightly higher. While we acknowledge that the value of the risk factor at the 99.9% confidence level implies a very large deviation from its average level – a very large systemic risk shock (increase) hitting the credit loss distribution – similar results are obtained if we lower the confidence level to 99% or 95% percent (as reported in Tables 1B and 1C), albeit with lower conditional expected credit losses.

Thus, it should not come as a surprise that parameter uncertainty can have a substantial impact on the credit-risk measure. In particular, it is most important in gauging the level of systemic risk shock to which the investor is conditioning credit risk assessment. While a user can probably identify a reasonable range of values for the relevant model parameters, it is far from obvious how to calibrate them to specific point value and future systemic risk shock realizations. For example, estimating the default correlation even under the simple assumptions of the factor model (3b), requires mapping asset correlation to observable variables such as stock market prices. A great deal of accurate information on covariance/volatility (cross-section and over time) structure – including perhaps appropriate modeling assumptions – is likely to be needed in order to measure the dependence of a varying asset pool on the systemic risk factor.

5. Calibrating a stylized SF deal: implications for bond tranche credit risk

As mentioned above, knowing the entire credit loss distribution of the underlying collateral would be of great importance in assessing bond tranche credit risk in any SF deal. So far we have only tested the impact of various assumptions on average default probability and default correlation on the expected credit loss. We still need to check the implication of these assumptions for the tails of the portfolio credit loss distribution. It is plausible that, inasmuch as they were relevant for the first moment of the loss distribution (e.g. expected loss), higher moments of the credit loss distribution will not be immune from the uncertainty regarding the values of the model's parameters. This conjecture will now be taken directly into the SF credit risk evaluation domain, testing the dependence of the credit loss distribution tails on the parameters and structure of the model.

To proceed with the testing we consider a stylized SF deal (say, a simple ABS CDO structure) backed by a pool of assets – a portfolio of ABS – with known loss distribution function – as described by the factor model (3b) – with the tranches of subordinated bonds defined according to the following attachment/detachment points:

1) equity tranche: 0-3 percent losses

2) mezzanine tranche: 3-6 percent losses

3) senior tranche: 6-13 percent losses

4) super-senior tranche: 13-100 percent losses.

We assume that the credit loss distribution of the collateral pool (ABS portfolio) is consistent with the moments specified in eq. (17b) - adjusted for auto-correlation in eq. (17h) - i.e. with the assumption of approximation to a large and homogeneous portfolio. For the sake of simplicity, we also assume β=1 (perfect auto-correlation). To simplify the notation I omit the subscript *n* in denoting the loss ncurred by the underlying pool of assets (ABS portfolio), as the number of obligors entering the collateral pool is now going to be fixed in advanced. Given the level of systemic risk, X, the conditional credit loss on the collateral pool has following distribution,

$$\mathrm{L}|\mathrm{X} \sim \mathrm{N}\left[\mathrm{m_L}(X); \sigma_L^2(X)\right]$$

where its mean and variance expression are laid out in eq. (17h). The derived conditional credit loss function for each bond tranche – with attachment/detachment points defined above – is given by

$$\mathrm{L}\left(\mathrm{k_j};\mathrm{k_{j-1}}|X\right) = \begin{cases} 0 & \Leftrightarrow \mathrm{L} \le \mathrm{k_{j-1}} \\ \mathrm{L}|\mathrm{X} - \mathrm{k_{j-1}} & \Leftrightarrow \mathrm{k_{j-1}} \le \mathrm{L} \le \mathrm{k_j} \\ \mathrm{k_j} - \mathrm{k_{j-1}} & \Leftrightarrow \mathrm{k_j} \le \mathrm{L} \end{cases} \qquad j = 1, J$$

(22b)

where $L(k_j,k_{j-1})$ indicates the amount lost by each tranche with attachment/detachment points, (k_{j-1} , k_j), as a result of the loss, $L|X$, caused by defaults in the collateral pool. The attachment/detachment points associated with each tranche (k_j = 0%, 3%,6%,13%,100%, with the number of tranches, *J-1*, equal to 4) allocate the total loss of the collateral pool.

It is very convenient to rewrite the credit loss function of each tranche, $L(k_j,k_{j-1}|X)$, in (22b) as the payoff of a call option spread, written on the underlying credit loss $L|X$ of the collateral pool and with strike prices set according to the attachment/detachment points, (k_j,k_{j-1}),

$$L(k_j;k_{j-1}|X) = MAX[L|X - k_{j-1};0] - MAX[L|X - k_j;0] \qquad j = 1, J \tag{23b}$$

Also, it is convenient to measure the credit loss of each tranche as a fraction of the total tranche. For this purpose we define the following ratio,

$$L(k_j;k_{j-1}|X)/(k_j - k_{j-1}) \qquad j = 1, J \tag{24b}$$

We can appraise the value of the expected credit loss for each bond tranche – i.e. eqs.(23b) and (24b) – by pricing the call option spread payoff indicated in eq. (23b):

$$E[L(k_j;k_{j-1}|X)] = Call[k_{j-1}|m_L(X);\sigma_L^2(X)] - Call[k_j|m_L(X);\sigma_L^2(X)] \qquad j = 1,J \tag{25b}$$

Each call option in (25b) can be priced by computing the following expression

$$Call[k_j|m_L(X);\sigma_L^2(X)] = (1 - F_L(k_j|X))\left[\frac{1}{(1 - F_L(k_j|X))}\int_{k_j}^{\infty} L dF_L(L|X) - k_j\right] \qquad j = 1, J \tag{26b}$$

where $F_L(L|X)$ indicates the probability distribution of the credit losses given X, L|X. The first term in square brackets coincides with the Expected-Shortfall (ES) risk measure, evaluated at the detachment point, k_j. In pricing the call option (26b) we ignore, for the sake of simplicity, the time–value of the option payoff (i.e. we set the risk-free interest rate to zero).

To compute the ES value component in (26b), I employ two specifications of credit loss distribution, $F_L(L|X)$; namely,

1) Gaussian distribution function (asymptotic approximation), as laid out in eq. (17b),

$$F_L(L|X) \sim N[m_L(X),\sigma_L^2(X)] \tag{27b}$$

2) Student-t distribution, which exhibits more fat-tailed behavior than the Gaussian distribution.

$$F_L(L) \sim t[m_L(X),\sigma_L^2(X), \upsilon] \tag{28b}$$

where parameter υ indicates the degrees of freedom of the Student-t distribution.

The second specification, (28b), acknowledges the limitations implied by the "fine-graining" of the portfolio composition underlying the validity of the Gaussian approximation, (27b). This assumption might badly underestimate the magnitude of tail-risk of actual collateral pool losses[11].

Following McNeil, Frey, Embrechts (2005; pp.45-46), we compute the ES measure component (eq. 26b) for both credit loss distribution assumptions. For the Gaussian distribution function, (27b), we have,

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[11] "Granularity adjustments" (due to departures from the asymptotic distribution assumption) for the credit risk measures are well known issues in the risk-management literature (see Gordy 2003). These adjustments can constitute a substantial portion of the total value of the "adjusted" credit risk measure, particularly for portfolios of high credit quality.

$$ES(k_j|X) \equiv \frac{1}{\left(1-F_L\left(k_j|X\right)\right)}\int_{k_j}^{\infty} LdF_L\left(L|X\right) = m_L\left(X\right)+\sigma_L\left(X\right)\frac{z_j\left(X\right)}{\left(1-\Phi_{Gaussian}\left(z_j\left(X\right)\right)\right)};\Phi_{Gaussian} \sim \mathrm{N}\left(0,1\right),$$

$$\phi_{Gaussian} \equiv \frac{\partial \Phi_{Gaussian}}{\partial z}, z_j\left(X\right) \equiv \frac{k_j - m_L\left(X\right)}{\sigma_L\left(X\right)}$$

(29b)

and for the Student-t loss distribution, (28b),

$$ES(k_j) \equiv \frac{1}{\left(1-F_L\left(k_j|X\right)\right)}\int_{k_j}^{\infty} LdF_L\left(L|X\right) = m_L\left(X\right)+\sigma_L\left(X\right)\frac{\phi_{Student-t}\left(z_j\left(X\right)\right)}{\left(1-\Phi_{Student-t}\left(z_j\left(X\right)\right)\right)}\left[\frac{\nu+\left(z_j\left(X\right)\right)^2}{\nu-1}\right];$$

$$\Phi_{Student-t} \sim \mathrm{t}\left[m_L\left(X\right),\sigma_L\left(X\right),\upsilon\right];\phi_{Student-t} = \Phi'_{Student-t} \quad z_j\left(X\right) \equiv \frac{k_j - m_L\left(X\right)}{\sigma_L\left(X\right)}$$

(30b)

Substituting (29b) and (30b) into eq. (26b) and subsequently into (25b) we get the value of the call option spread for each bond tranche, namely its conditional expected (credit) loss measure.

$$CEL_{\alpha(X)}\left(k_j;k_{j\text{-}1}\right) = E\left[L\left(k_j;k_{j\text{-}1}\right)\middle|X\right] \quad j=1,J$$

(31b)

It is important to notice in eq. (31b) that the conditional expected loss of each tranche depends on the parameter values, $\theta=(\pi_p, \rho, \alpha)$, of the asset pool – default probability, correlation parameter, systemic risk level – not only through the mean of the loss distribution, $m_L(X)$, but also through its variance, $\sigma_L^2(X)$, as the tails of the distribution come into play in determining the call spread value. For the t-distribution assumption, the magnitude of the tails is also regulated by the choice of the degrees-of-freedom parameter, υ (smaller values are associated with fatter tails; large values imply little deviation from the Gaussian benchmark).

We obtain the value of the Gaussian call options by substituting eq. (29b) into eq. (26b),

$$Call(k_j|Gaussian) = (m_L - k_j)\left[1 - \Phi_{Gaussian}\left(\frac{k_j - m_L}{\sigma_L}\right)\right] + \sigma_L \phi_{Gaussian}\left(\frac{k_j - m_L}{\sigma_L}\right)$$

(32b)

Notice that the call option value (32b) is a monotonic decreasing function of the strike price, k_j (as in the classic Black-Scholes option pricing formula), so that the call spread measure (25b) is always positive. Thus, the credit risk measure (31b) – e.g. the economic value of each bond tranche – is positive as well. This property can be checked by differentiating (32b) with respect to k_j,

$$\frac{\partial Call(k_j|Gaussian)}{\partial k_j} = \Phi_{Gaussian}\left(\frac{k_j - m_L}{\sigma_L}\right) - 1 \leq 0$$

(33b)

since the following condition is fulfilled by the Gaussian density function,

$$\frac{\partial \phi_{Gaussian}(z)}{\partial z} = -z\phi_{Gaussian}(z), \quad z \equiv \frac{k_j - m_L}{\sigma_L}$$

(34b)

A similar check can be done for the value of the Student-t call option,

$$Call(k_j|Student-t) = (m_L - k_j)\left[1 - \Phi_{Student-t}\left(\frac{k_j - m_L}{\sigma_L}\right)\right] + \sigma_L \phi_{Student-t}\left(\frac{k_j - m_L}{\sigma_L}\right)\left[\frac{\upsilon + \left(\frac{k_j - m_L}{\sigma_L}\right)^2}{\upsilon - 1}\right]$$

(35b)

obtained by substituting (30b) into (26b). Differentiating (35b) with respect to k_j yields

$$\frac{\partial Call(k_j \mid Student-t)}{\partial k_j} = \Phi_{Student-t}\left(\frac{k_j - m_L}{\sigma_L}\right) - 1 \leq 0$$

(36b)

since the following condition is fulfilled by the Student-t density function,

$$(\upsilon+1) z \phi_{Student-t}(z) + \frac{\partial \phi_{Student-t}(z)}{\partial z}\left(\upsilon + z^2\right) = 0, \qquad z \equiv \frac{k_j - m_L}{\sigma_L}$$

(37b)

5.1 Systemic risk and expected losses on bond tranches

In Tables 2A and 2B we report the value of the expected losses of each bond tranche under the Gaussian distribution assumption. We consider the following grid of parameter correlation values.

[0 ; 5 ; 10 ;15 ; 20 ; 25 ; 30 ; 35 ; 40 ; 50] percent.

We limit the results reported here to two possible values for the average default probability of the asset pool: 1% (Table 2A) and 2.5% (Table 2B). Also, we focus primarily on the implications of a very large (extreme) systemic risk shock to the credit loss distribution – that is, we assume that the value of the systemic risk factor, *X*, is associated with a 99.9% significance level, *α* (i.e. 0.1% probability of shock). Similarly, we report the expected losses of each bond tranche computed under the Student-t distribution assumption, in order to gauge the impact of fatter tails on the credit risk of the deal (Tables 3A and 3B). To avoid a proliferation of tables, the results are reported for a concentrated portfolio only. For a given risk budget, considering more diversified portfolios would further magnify the impact of systemic shocks on correlation risk[12].

Predictably, the sheer size of the systemic risk shock has great impact on the junior tranches. Under the Gaussian distribution assumption, even at relatively low correlation levels the destruction of value is significant. At correlation parameters equal to .05 and .10), respectively, 56 percent and 60 percent of the equity tranche is expected to be wiped out (Table 2A, last column, denominated "call-spread ratio"). Perhaps more interestingly, mezzanine and senior tranches do not perform much better, as they are expected to lose roughly half of their value (55 and 47 percent, respectively). Due to the very large magnitude of the systemic shock under consideration, defaults peak in the assets pool at very high levels – over 14% of the pool (with parameter correlation set at 0.2) is expected to be wiped out – spilling over to more senior tranches. Only the super-senior tranche appears to be relatively sheltered from the systemic shock, as it is expected to lose less than 10 percent of its value.

By increasing the default correlation rate of the asset pool, the expected losses of each bond tranche always rise monotonically. Thus, all tranches are <u>always short (default) correlation risk</u> – that is, as default correlation increases a bond tranche becomes less valuable (the expected loss is now larger). For example, setting the correlation parameter at 0.40, the junior and mezzanine tranches would be expected to lose over 70 percent of their value and the senior tranche 68 percent. As a result of the sheer size of

[12] The number of securities included in the collateral pool (portfolio concentration) affects the (approximated) credit loss distribution only by changing its dispersion parameter (increasing portfolio diversification reduces the credit loss variance of the collateral pool; see eq. 17b). Our reported simulation results are based on the assumption of a perfectly homogeneous collateral pool (n=1). Similar results are obtained by considering more diversified portfolios (n>1; these are available upon request from the author).

the systemic shock, even the super-senior would be hard hit and lose about a third of its value (Table 2A). Defaults are expected to wipe out almost a third of the collateral pool.

Degrading the credit quality of the asset pool – i.e. increasing its expected (unconditional) default frequency to 2.5 percent – worsens monotonically, albeit not uniformly, the expected credit losses of all bond tranches (Table 2B).

Moving on to the Student-t distribution assumption results (Tables 3A and 3B) – i.e. incorporating fatter-tail behavior – there are not many changes in the pattern of expected credit losses for the bond tranches under scrutiny. To gauge the deviation from the Gaussian case, we have set the degrees of freedom parameter, υ, to low levels (e.g., 5), so that we can clearly appreciate the impact of fatter tails on the credit risk of the tranches. This calibration of the Student-t distribution implies that the tail risk of the asset pool as measured by the ES risk measure increases, with respect to the Gaussian distribution case, by more than 60 percent for low correlation levels and more than 50 percent for high levels (Table 4)[13]. Thus, under the Student–t distribution assumption unexpected losses for the assets pool are significantly greater than in the Gaussian case. As a result, senior tranches are now more likely to be hit by defaults.

Simulating the call spread values under the Student-t credit loss distribution – i.e. using eq. (32b) to compute the option value – the results for the expected losses are as follows. All bond tranches are <u>short correlation risk as in the Gaussian case</u>. The equity tranche is slightly less risky than the Gaussian case (Table 3A vs. Table 2A). Considering bond tranches of increasing seniority, credit losses decrease but at a lower pace than with the Gaussian distribution. Fatter tails of the loss distribution tend to magnify losses on the senior tranches (and symmetrically reduce them for the junior tranches) vis-à-vis the

Gaussian distribution. As a result, while the mezzanine tranche is still marginally less risky than with the Gaussian distribution, the senior tranche becomes uniformly (marginally) riskier. The super-senior tranche would also now become more risky. However, credit loss differences remain modest –between 1 and 2 percentage points – despite the much fatter tail of the Student-t loss distribution. Similar results are obtained if we lower the credit quality of the asset pool, from 1% default probability to 2.5% (see Tables 3B and 3B).

All in all, fattening the tails of the loss distribution does not bring about drastic changes in the credit risk of structured finance deals. Thus, the Gaussian approximation provides a reasonable benchmark for assessing bond tranche credit risk. To test the robustness of this conclusion, we also check the implications of far smaller systemic risk shocks. Table 5 reports the expected credit loss of each bond tranche for various realizations of systemic risk shock. Inspecting the benchmark of macroeconomic risk factor set at its mean level –parameter α equal to 0.50 – we notice that the shape of the correlation risk is reversed with respect to the case of large systemic shock. That is, <u>all bond tranches are now long correlation risk</u> (as default correlation rises, bond value increases as expected credit losses decline). The same pattern is displayed by the Gaussian and the Student-t loss distributions alike. As the size of the systemic shock increases (say, α is raised to 0.75), correlation risk becomes inverse U-shaped; that is, each bond tranche is long correlation risk for low levels of default correlation and short correlation risk for relatively higher levels. The same correlation risk pattern (inverse U-shape) persists for greater deviation (shock) of the risk factor from its mean level (parameter α equal to 0.90), namely for shocks with frequency of between every five and

[13] The ES risk measure of the loss distribution is measured at the 99% confidence level.

every ten years. For larger (less frequent) shocks, bond tranches tend gradually to revert to be short correlation risk (α equal to 0.95 and above). All in all, the size of the (anticipated) systemic shock is crucial in shaping correlation risk. However, for a given size of anticipated systemic shock, the qualitative shape of correlation risk is constant across tranches and does not change with respect to the fattening of the credit loss tail. In addition, a fatter tail makes relatively senior tranches always more risky, irrespective of the size of shock, whereas the equity tranche would be marginally less risky only for some (positive) deviation from the mean of the risk factor (for α equal to or greater than 0.75). All in all, a fatter tail would augment individual bond tranche credit risk only modestly; in relative terms, credit risk changes would be of some importance only for the super senior tranche, whose losses are expected to be quite contained for systemic shocks that are not too large. Moreover, in absolute terms credit risk increases across tranches are just barely sensitive to systemic risk shocks. Again, in relative terms, credit risk worsening is really material only for the super senior tranche.

A final observation regards the order of magnitude of the (simulated) expected loss of the bond tranches. Such expected loss from defaults would need to be offset, under the assumption of a well functioning capital market, by the yield spread in excess of some risk-free interest rate (say, government bond yield to maturity) promised to the bondholder. Comparing bond tranches' expected losses with this spread implies that the credit loss distribution should be adjusted for risk, i.e. that the actual ("historical") loss distribution should be properly risk-neutralized by a suitable stochastic discount factor to get an equivalent martingale (risk-neutral) measure. A non-zero risk premium on default risk is equivalent to a risk-neutral credit loss probability being higher than that of its actual counterpart. This is consistent with the evidence reported in Brigo *et al.* (2007) that the risk-neutral loss distribution privileges large realizations of the loss with respect to the objective distribution, thus confirming the presence of a risk premium on standardized CDO tranches based on the ITRAXX indices obligors' pool.

Focusing on the super-senior tranche, for relatively moderate systemic risk shock – say, α of at most 0.75 – the magnitude of the expected loss would appear to be broadly compatible with the actual bond spreads for such tranches as priced by the capital market before the outbreak of the SF crisis (below 100bps[14]; see D'Amato and Remolona, 2005). For greater systemic risk shock – assuming a non-negligible default correlation rate – simulated expected credit losses for the most senior tranche appear to be far greater than would be implied by their observed market spreads. This conclusion implies that if our (admittedly very stylized) credit risk model is broadly correct, the pre-crisis bond market prices of the most senior tranches may well have been "underpricing" the possibility of a significant rise in systemic risk.

6. Conclusions

This paper explores the implications of systemic risk in credit SF deals using a generalized version of the standard credit risk model widely used by market participants (Gaussian Copula Credit Risk Factor Model; GCCRF). Despite its widely known limitations, we provide a framework for supporting the conclusion that such model - that

[14] Notice that our expected credit loss simulations assume zero recovery (given default) and ignore the liquidity risk of issued bond tranches. Taken together these assumptions tend (at least in part) to offset each other. Upgrading the quality of the asset pool (say, reducing its average default probability from 1% to 0.5%) would lower our simulated expected losses somewhat but would not affect our main qualitative conclusions. The lower default probability estimate would broadly correspond to the examples reported in

antedates the crisis in its broad structure - could have indeed yielded sufficient insights regarding the risk underlying Structured Finance (SF) deals, if due attention and care were paid to modeling systemic risk. Simulations show that our generalized standard model, which stresses the advantages of maintaining a conditional structure in modeling risk factors, can capture the implications of a large systemic shock on bond senior tranches. Unlike the 'static' treatment of systemic risk exposures routinely used in many standard applications, our modeling strategy emphasizes the fluctuations in market perception regarding the total loss distribution on the collateral value. This methodology allows to analyze also the impact of model risk, represented by differing risk properties of the underlying collateral value, and to delineate its effect on the risk properties of the various bond tranches in SF deals.

A set of (stylized) CDO (backed by a pool of assets) valuations were performed taking into consideration several types of systemic risk shocks to the credit loss distribution. It is shown that AAA super-senior bond tranches can incur significant losses (provided that default correlation across the assets of the backing pool is not immaterial), despite the good quality of their collateral pool in normal times (i.e. low average unconditional default probability) in times of stress. In addition to the simulations carried out in a standard Gaussian framework, systemic shocks under the Student-t distribution assumption are evaluated. It is shown that fatter tails of the loss distribution, albeit of sizeable magnitude, do not bring about so drastic changes on the credit risk of CDO tranches. Fatter tails of the loss distribution tend to magnify losses on the senior tranches

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D'Amato and Remolona (2005), Table 2, suggesting an actual default intensity of 0.7% per year and a recovery rate of 41% (in this case, the expected loss will amount to 40 basis points in annual terms).

(and symmetrically reduce them for the junior tranches) vis-à-vis the Gaussian distribution.

Unlike the implications of the standard 'static' (unconditional) GCCRF model, the generalized version adopted in this paper shows that changes in systemic risk level are crucial in shaping the profile of 'correlation risk'. Bond value exposure to default correlation changes can be positive or negative, depending upon the level of systemic risk. More specifically, the shape of the correlation risk is reversed as we move from small to large systemic shock. That is, all bond tranches are long correlation risk - as default correlation rises, bond value increases because expected credit losses decline - for low level of systemic risk. The same pattern is displayed by the Gaussian and the Student-t loss distributions alike. As the size of the systemic shock increases, correlation risk becomes inverse U-shaped; that is, each bond tranche is long correlation risk for low levels of default correlation and short correlation risk for relatively higher levels. For large systemic risk shocks correlation risk turns negative ('short'; as default correlation rises, bond value declines because expected credit losses increase).

Under the generalized GCCRF model simulated in this paper, correlation risk becomes a non linear function of systemic risk shocks. This is a challenge to traditional credit risk modeling, in that it does not allow for a proper conditional loan-loss distribution framework that would permit to incorporate systemic risk dynamics. Unlike the standard model, the order of magnitude of expected loss implied by the generalized GCCRF model can be sufficiently high to match the credit spread data of senior bond tranches observed during the 2007-08 credit crisis. As a result, the standard (unconditional) GCCRF model may cause, especially in times of stress, very serious under-pricing of risk as well as credit hedging strategy failures ('correlation risk' changes not tracked closely enough by credit hedges).

As conditional expected losses can be computed for different values of the exposure to systemic risk, each structured security has a set of conditional expected losses associated to it. Each expected loss is a realization over time of a systemic risk factor. Standard credit ratings systems, taking a traditional 'through-the-cycle' (unconditional) view of default probability, average out large fluctuations in the tails of the loss distributions. This is a recipe for massive underpricing of credit risk in SF deals, especially damaging for lower risk (e.g. higher rating grades) bond tranches, as changes in systemic risk can have a huge impact on their expected losses. Switching to a 'point-in-time' (conditional) concept of rating grade is probably the only sensible way to remedy the shortcomings of traditional 'through-the-cycle' (unconditional) credit risk measures[15].

However, a conditional measure of rating grade would make individual security rating relatively more (sometimes extremely) volatile compared to a traditional (unconditional) measure, if the association of individual rating grade (say, AAA) to a given range of expected losses is kept unchanged over time. In addition, we need to be aware that measurement errors in modeling/assessing systemic risk dynamics, as implied by a conditional approach to ratings, may inject new noise – e.g. undesirable additional volatility - in credit-risk valuation. Ratings 'instability', at least for investors' categories relying on mandated investment, should be avoided, as dictated by the need to limit agency (delegation) problems between fund managers (agent) and investors (principal).

[15] I owe to an anonymous referee the interesting question regarding the practical implications of introducing conditional risk measure within the current format adopted by credit rating systems.

To limit instability of ratings in SF deals, a reasonable case for adjusting over time the rating scale can be made. Namely, the implied credit risk content of each grade would be open to fine-tuning adjustment over time by changing the grid of (conditional) expected losses (taking into account the exposure to systemic risk) associated to a (fixed) rating scale. Reviews of the credit risk content of the rating scale should take place only at fixed dates (well publicized in advance; say, on a quarterly basis) and any change would need to be widely disseminated and thoroughly explained by the CRAs. It is likely that a broadly correct timing of (systemic) risk – as mirrored in the credit risk content adjustment on the ratings scale - would also limit 'ratings pro-cyclicality', as in recession times downgrading (upgrading) prospects would be contained (enhanced), while exactly the opposite would be true in expansionary times. To avoid (wasteful) allegations of ratings manipulation, along with rating grades the CRAs should start publishing on a regular basis the (estimated) expected loss for all individual security tranche of any (rated) SF deal. At a minimum, information on expected loss should match the rating scale review frequency (e.g. quarterly frequency; monthly would probably be better).

Changes in the credit risk content of ratings scale could have the undesirable implication of reducing the value of information that can be inferred from intertemporal analysis of ratings transition. However, such negative implication would be largely mitigated by the availability of expected loss estimates. That is, current rating grade of individual security – with its expected loss estimate associated - could always be converted into its 'equivalent rating' under a new classification system. Such 'equivalent' rating would be imputed by finding the class of credit risk in the new classification system matching the security expected loss estimate. In practice, such rating imputation mechanism has strong similarities with the so called ‘market implied’ rating measures, a service already well marketed by CRAs. That is, a ‘notional’ rating is assigned to individual security based on market price information (for example, bond credit spread) using the current CRAs ratings scale. Expected loss indicator could play the same role as market price information for imputing rating grades under a time-varying ratings system.

TABLES

Table 1A	PD	Correlation Parameter	Default Correlation	Quantile (Risk-Factor)	Conditional Expected Loss
	0.50%	0.00	0.00%	0.1%	0.50%
	0.50%	0.05	0.25%	0.1%	2.66%
	0.50%	0.10	0.58%	0.1%	4.60%
	0.50%	0.15	1.03%	0.1%	6.74%
	0.50%	0.20	1.60%	0.1%	9.10%
	0.50%	0.30	3.24%	0.1%	14.56%
	0.50%	0.35	4.37%	0.1%	17.69%
	0.50%	0.40	5.76%	0.1%	21.12%
	0.50%	0.50	9.45%	0.1%	29.03%
	1.00%	0.00	0.00%	0.1%	1.00%
	1.00%	**0.05**	**0.41%**	**0.1%**	**4.67%**
	1.00%	0.10	0.94%	0.1%	7.75%
	1.00%	0.15	1.60%	0.1%	11.03%
	1.00%	**0.20**	**2.41%**	**0.1%**	**14.55%**
	1.00%	0.30	4.61%	0.1%	22.44%
	1.00%	0.35	6.04%	0.1%	26.83%
	1.00%	**0.40**	**7.73%**	**0.1%**	**31.56%**
	1.00%	0.50	12.05%	0.1%	42.08%
	2.50%	0.00	0.00%	0.1%	2.50%
	2.50%	0.05	0.77%	0.1%	9.65%
	2.50%	0.10	1.69%	0.1%	15.01%
	2.50%	0.15	2.77%	0.1%	20.39%
	2.50%	0.20	4.03%	0.1%	25.91%
	2.50%	0.30	7.16%	0.1%	37.46%
	2.50%	0.35	9.06%	0.1%	43.51%
	2.50%	0.40	11.21%	0.1%	49.72%
	2.50%	0.50	16.39%	0.1%	62.49%
	5.00%	0.00	0.00%	0.1%	5.00%
	5.00%	0.05	1.20%	0.1%	16.39%
	5.00%	0.10	2.55%	0.1%	24.08%
	5.00%	0.15	4.08%	0.1%	31.35%
	5.00%	0.20	5.78%	0.1%	38.44%
	5.00%	0.30	9.75%	0.1%	52.27%
	5.00%	0.35	12.05%	0.1%	59.00%
	5.00%	0.40	14.58%	0.1%	65.53%
	5.00%	0.50	20.39%	0.1%	77.76%

Table 1B	PD	Correlation parameter	Default Correlation	Quantile (Risk-Factor)	Conditional Expected Loss
	0.50%	0.00	0.00%	1.0%	0.50%
	0.50%	0.05	0.25%	1.0%	1.75%
	0.50%	0.10	0.58%	1.0%	2.62%
	0.50%	0.15	1.03%	1.0%	3.46%
	0.50%	0.20	1.60%	1.0%	4.30%
	0.50%	0.30	3.24%	1.0%	5.99%
	0.50%	0.35	4.37%	1.0%	6.84%
	0.50%	0.40	5.76%	1.0%	7.69%
	0.50%	0.50	9.45%	1.0%	9.40%
	1.00%	0.00	0.00%	1.0%	1.00%
	1.00%	**0.05**	**0.41%**	**1.0%**	**3.19%**
	1.00%	0.10	0.94%	1.0%	4.68%
	1.00%	0.15	1.60%	1.0%	6.11%
	1.00%	**0.20**	**2.41%**	**1.0%**	**7.53%**
	1.00%	0.30	4.61%	1.0%	10.43%
	1.00%	0.35	6.04%	1.0%	11.93%
	1.00%	**0.40**	**7.73%**	**1.0%**	**13.48%**
	1.00%	0.50	12.05%	1.0%	16.76%
	2.50%	0.00	0.00%	1.0%	2.50%
	2.50%	0.05	0.77%	1.0%	6.98%
	2.50%	0.10	1.69%	1.0%	9.84%
	2.50%	0.15	2.77%	1.0%	12.54%
	2.50%	0.20	4.03%	1.0%	15.19%
	2.50%	0.30	7.16%	1.0%	20.62%
	2.50%	0.35	9.06%	1.0%	23.45%
	2.50%	0.40	11.21%	1.0%	26.41%
	2.50%	0.50	16.39%	1.0%	32.80%
	5.00%	0.00	0.00%	1.0%	5.00%
	5.00%	0.05	1.20%	1.0%	12.43%
	5.00%	0.10	2.55%	1.0%	16.89%
	5.00%	0.15	4.08%	1.0%	20.99%
	5.00%	0.20	5.78%	1.0%	24.96%
	5.00%	0.30	9.75%	1.0%	32.89%
	5.00%	0.35	12.05%	1.0%	36.95%
	5.00%	0.40	14.58%	1.0%	41.14%
	5.00%	0.50	20.39%	1.0%	50.01%

Table 1C	PD	Correlation parameter	Default Correlation	Quantile (Risk-Factor)	Conditional Expected Loss
	0.50%	0.00	0.00%	5.0%	0.50%
	0.50%	0.05	0.25%	5.0%	1.17%
	0.50%	0.10	0.58%	5.0%	1.51%
	0.50%	0.15	1.03%	5.0%	1.77%
	0.50%	0.20	1.60%	5.0%	1.98%
	0.50%	0.30	3.24%	5.0%	2.26%
	0.50%	0.35	4.37%	5.0%	2.34%
	0.50%	0.40	5.76%	5.0%	2.37%
	0.50%	0.50	9.45%	5.0%	2.29%
	1.00%	0.00	0.00%	5.0%	1.00%
	1.00%	**0.05**	**0.41%**	**5.0%**	**2.22%**
	1.00%	0.10	0.94%	5.0%	2.85%
	1.00%	0.15	1.60%	5.0%	3.35%
	1.00%	**0.20**	**2.41%**	**5.0%**	**3.77%**
	1.00%	0.30	4.61%	5.0%	4.42%
	1.00%	0.35	6.04%	5.0%	4.66%
	1.00%	**0.40**	**7.73%**	**5.0%**	**4.84%**
	1.00%	0.50	12.05%	5.0%	5.00%
	2.50%	0.00	0.00%	5.0%	2.50%
	2.50%	0.05	0.77%	5.0%	5.12%
	2.50%	0.10	1.69%	5.0%	6.45%
	2.50%	0.15	2.77%	5.0%	7.57%
	2.50%	0.20	4.03%	5.0%	8.55%
	2.50%	**0.30**	**7.16%**	**5.0%**	**10.28%**
	2.50%	0.35	9.06%	5.0%	11.05%
	2.50%	0.40	11.21%	5.0%	11.76%
	2.50%	0.50	16.39%	5.0%	12.99%
	5.00%	0.00	0.00%	5.0%	5.00%
	5.00%	0.05	1.20%	5.0%	9.51%
	5.00%	0.10	2.55%	5.0%	11.79%
	5.00%	0.15	4.08%	5.0%	13.72%
	5.00%	0.20	5.78%	5.0%	15.47%
	5.00%	**0.30**	**9.75%**	**5.0%**	**18.70%**
	5.00%	0.35	12.05%	5.0%	20.24%
	5.00%	0.40	14.58%	5.0%	21.76%
	5.00%	0.50	20.39%	5.0%	24.78%

Table 2.A	PD	Correlation parameter	Quantile (Risk-Factor)	Conditional Expected Loss	Call-spread	Call-spread ratio
EQUITY TRANCHE						
	1.00%	0.00	0.10%	1.00%	1.44%	48.0%
	1.00%	**0.05**	**0.10%**	**4.67%**	**1.68%**	**56.0%**
	1.00%	0.10	0.10%	7.75%	1.78%	59.2%
	1.00%	0.15	0.10%	11.03%	1.86%	61.9%
	1.00%	**0.20**	**0.10%**	**14.55%**	**1.93%**	**64.4%**
	1.00%	0.30	0.10%	22.44%	2.08%	69.2%
	1.00%	0.35	0.10%	26.83%	2.15%	71.6%
	1.00%	**0.40**	**0.10%**	**31.56%**	**2.22%**	**74.1%**
	1.00%	0.50	0.10%	42.09%	2.38%	79.4%
MEZZANINE TRANCHE						
	1.00%	0.00	0.10%	1.00%	1.09%	36.3%
	1.00%	**0.05**	**0.10%**	**4.67%**	**1.51%**	**50.3%**
	1.00%	0.10	0.10%	7.75%	1.65%	54.8%
	1.00%	0.15	0.10%	11.03%	1.75%	58.2%
	1.00%	**0.20**	**0.10%**	**14.55%**	**1.84%**	**61.2%**
	1.00%	0.30	0.10%	22.44%	2.00%	66.6%
	1.00%	0.35	0.10%	26.83%	2.08%	69.3%
	1.00%	**0.40**	**0.10%**	**31.56%**	**2.16%**	**72.0%**
	1.00%	0.50	0.10%	42.08%	2.33%	77.7%
SENIOR TRANCHE						
	1.00%	0.00	0.10%	1.00%	1.41%	20.1%
	1.00%	**0.05**	**0.10%**	**4.67%**	**2.87%**	**41.0%**
	1.00%	0.10	0.10%	7.75%	3.32%	47.4%
	1.00%	0.15	0.10%	11.03%	3.64%	51.9%
	1.00%	**0.20**	**0.10%**	**14.55%**	**3.90%**	**55.7%**
	1.00%	0.30	0.10%	22.44%	4.35%	62.2%
	1.00%	0.35	0.10%	26.83%	4.56%	65.2%
	1.00%	**0.40**	**0.10%**	**31.56%**	**4.78%**	**68.2%**
	1.00%	0.50	0.10%	42.08%	5.22%	74.5%
SUPER-SENIOR TRANCHE						
	1.00%	0.00	0.10%	1.00%	0.55%	0.6%
	1.00%	**0.05**	**0.10%**	**4.67%**	**4.90%**	**5.6%**
	1.00%	0.10	0.10%	7.75%	8.24%	9.5%
	1.00%	0.15	0.10%	11.03%	11.51%	13.2%
	1.00%	**0.20**	**0.10%**	**14.55%**	**14.77%**	**17.0%**
	1.00%	0.30	0.10%	22.44%	21.27%	24.5%
	1.00%	0.35	0.10%	26.83%	24.54%	28.2%
	1.00%	**0.40**	**0.10%**	**31.56%**	**27.83%**	**32.0%**
	1.00%	0.50	0.10%	42.08%	34.63%	39.8%

Table 2.B	PD	Correlation parameter	Quantile (Risk-Factor)	Conditional Expected Loss	Call-spread	Call-spread ratio
EQUITY TRANCHE						
	2.50%	0.00	0.10%	2.50%	1.58%	52.5%
	2.50%	0.05	0.10%	9.65%	1.83%	60.9%
	2.50%	0.10	0.10%	15.01%	1.94%	64.7%
	2.50%	0.15	0.10%	20.39%	2.04%	68.0%
	2.50%	0.20	0.10%	25.91%	2.13%	71.1%
	2.50%	0.30	0.10%	37.47%	2.31%	77.1%
	2.50%	0.35	0.10%	43.51%	2.40%	80.2%
	2.50%	0.40	0.10%	49.72%	2.50%	83.3%
	2.50%	0.50	0.10%	62.49%	2.69%	89.6%
MEZZANINE TRANCHE						
	2.50%	0.00	0.10%	2.50%	1.35%	44.9%
	2.50%	0.05	0.10%	9.65%	1.71%	56.9%
	2.50%	0.10	0.10%	15.01%	1.85%	61.6%
	2.50%	0.15	0.10%	20.39%	1.96%	65.3%
	2.50%	0.20	0.10%	25.91%	2.06%	68.7%
	2.50%	0.30	0.10%	37.46%	2.26%	75.2%
	2.50%	0.35	0.10%	43.51%	2.35%	78.4%
	2.50%	0.40	0.10%	49.72%	2.45%	81.7%
	2.50%	0.50	0.10%	62.49%	2.65%	88.4%
SENIOR TRANCHE						
	2.50%	0.00	0.10%	2.50%	2.30%	32.8%
	2.50%	0.05	0.10%	9.65%	3.51%	50.2%
	2.50%	0.10	0.10%	15.01%	3.93%	56.1%
	2.50%	0.15	0.10%	20.39%	4.24%	60.6%
	2.50%	0.20	0.10%	25.91%	4.52%	64.6%
	2.50%	0.30	0.10%	37.46%	5.03%	71.8%
	2.50%	0.35	0.10%	43.51%	5.27%	75.3%
	2.50%	0.40	0.10%	49.72%	5.52%	78.9%
	2.50%	0.50	0.10%	62.49%	6.04%	86.3%
SUPER-SENIOR TRANCHE						
	2.50%	0.00	0.10%	2.50%	2.34%	2.7%
	2.50%	0.05	0.10%	9.65%	10.17%	11.7%
	2.50%	0.10	0.10%	15.01%	15.18%	17.4%
	2.50%	0.15	0.10%	20.39%	19.67%	22.6%
	2.50%	0.20	0.10%	25.91%	23.87%	27.4%
	2.50%	0.30	0.10%	37.46%	31.72%	36.5%
	2.50%	0.35	0.10%	43.51%	35.52%	40.8%
	2.50%	0.40	0.10%	49.72%	39.33%	45.2%
	2.50%	0.50	0.10%	62.49%	47.27%	54.3%

Table 3.A (DF Student-T=5)	PD	Correlation parameter	Quantile (Risk-Factor)	Conditional Expected Loss	Call-spread	Call-spread ratio
EQUITY TRANCHE						
	1.00%	0.00	0.10%	1.00%	1.44%	48.1%
	1.00%	**0.05**	**0.10%**	**4.67%**	**1.67%**	**55.7%**
	1.00%	0.10	0.10%	7.75%	1.76%	58.8%
	1.00%	0.15	0.10%	11.03%	1.84%	61.3%
	1.00%	**0.20**	**0.10%**	**14.55%**	**1.91%**	**63.7%**
	1.00%	0.30	0.10%	22.44%	2.04%	68.1%
	1.00%	0.35	0.10%	26.83%	2.11%	70.4%
	1.00%	**0.40**	**0.10%**	**31.56%**	**2.18%**	**72.7%**
	1.00%	0.50	0.10%	42.09%	2.33%	77.6%
MEZZANINE TRANCHE						
	1.00%	0.00	0.10%	1.00%	1.11%	37.0%
	1.00%	**0.05**	**0.10%**	**4.67%**	**1.51%**	**50.3%**
	1.00%	0.10	0.10%	7.75%	1.64%	54.6%
	1.00%	0.15	0.10%	11.03%	1.74%	57.8%
	1.00%	**0.20**	**0.10%**	**14.55%**	**1.82%**	**60.6%**
	1.00%	0.30	0.10%	22.44%	1.97%	65.7%
	1.00%	0.35	0.10%	26.83%	2.05%	68.2%
	1.00%	**0.40**	**0.10%**	**31.56%**	**2.12%**	**70.7%**
	1.00%	0.50	0.10%	42.08%	2.28%	76.0%
SENIOR TRANCHE						
	1.00%	0.00	0.10%	1.00%	1.54%	22.1%
	1.00%	**0.05**	**0.10%**	**4.67%**	**2.90%**	**41.4%**
	1.00%	0.10	0.10%	7.75%	3.33%	47.5%
	1.00%	0.15	0.10%	11.03%	3.63%	51.8%
	1.00%	**0.20**	**0.10%**	**14.55%**	**3.88%**	**55.4%**
	1.00%	0.30	0.10%	22.44%	4.31%	61.5%
	1.00%	0.35	0.10%	26.83%	4.51%	64.4%
	1.00%	**0.40**	**0.10%**	**31.56%**	**4.71%**	**67.2%**
	1.00%	0.50	0.10%	42.08%	5.11%	73.1%
SUPER-SENIOR TRANCHE						
	1.00%	0.00	0.10%	1.00%	1.14%	1.3%
	1.00%	**0.05**	**0.10%**	**4.67%**	**6.37%**	**7.3%**
	1.00%	0.10	0.10%	7.75%	9.99%	11.5%
	1.00%	0.15	0.10%	11.03%	13.34%	15.3%
	1.00%	**0.20**	**0.10%**	**14.55%**	**16.54%**	**19.0%**
	1.00%	0.30	0.10%	22.44%	22.73%	26.1%
	1.00%	0.35	0.10%	26.83%	25.79%	29.6%
	1.00%	**0.40**	**0.10%**	**31.56%**	**28.86%**	**33.2%**
	1.00%	0.50	0.10%	42.08%	35.21%	40.5%

Table 3.B (DF Student-T=5)	PD	Correlation parameter	Quantile (Risk-Factor)	Conditional Expected Loss	Call-spread	Call-spread ratio
EQUITY TRANCHE						
	2.50%	0.00	0.10%	2.50%	1.57%	52.4%
	2.50%	**0.05**	**0.10%**	**9.65%**	**1.81%**	**60.3%**
	2.50%	0.10	0.10%	15.01%	1.92%	64.0%
	2.50%	0.15	0.10%	20.39%	2.01%	67.1%
	2.50%	**0.20**	**0.10%**	**25.91%**	**2.10%**	**69.9%**
	2.50%	0.30	0.10%	37.47%	2.26%	75.5%
	2.50%	0.35	0.10%	43.51%	2.35%	78.2%
	2.50%	**0.40**	**0.10%**	**49.72%**	**2.43%**	**81.0%**
	2.50%	0.50	0.10%	62.49%	2.60%	86.8%
MEZZANINE TRANCHE						
	2.50%	0.00	0.10%	2.50%	1.35%	45.2%
	2.50%	**0.05**	**0.10%**	**9.65%**	**1.70%**	**56.6%**
	2.50%	0.10	0.10%	15.01%	1.83%	61.0%
	2.50%	0.15	0.10%	20.39%	1.94%	64.5%
	2.50%	**0.20**	**0.10%**	**25.91%**	**2.03%**	**67.7%**
	2.50%	0.30	0.10%	37.47%	2.21%	73.7%
	2.50%	0.35	0.10%	43.51%	2.30%	76.6%
	2.50%	**0.40**	**0.10%**	**49.72%**	**2.39%**	**79.6%**
	2.50%	0.50	0.10%	62.49%	2.57%	85.8%
SENIOR TRANCHE						
	2.50%	0.00	0.10%	2.50%	2.36%	33.8%
	2.50%	**0.05**	**0.10%**	**9.65%**	**3.51%**	**50.2%**
	2.50%	0.10	0.10%	15.01%	3.91%	55.8%
	2.50%	0.15	0.10%	20.39%	4.21%	60.1%
	2.50%	**0.20**	**0.10%**	**25.91%**	**4.47%**	**63.8%**
	2.50%	0.30	0.10%	37.47%	4.94%	70.6%
	2.50%	0.35	0.10%	43.51%	5.17%	73.8%
	2.50%	**0.40**	**0.10%**	**49.72%**	**5.40%**	**77.1%**
	2.50%	0.50	0.10%	62.49%	5.87%	83.8%
SUPER-SENIOR TRANCHE						
	2.50%	0.00	0.10%	2.50%	3.42%	3.9%
	2.50%	**0.05**	**0.10%**	**9.65%**	**11.98%**	**13.8%**
	2.50%	0.10	0.10%	15.01%	16.94%	19.5%
	2.50%	0.15	0.10%	20.39%	21.23%	24.4%
	2.50%	**0.20**	**0.10%**	**25.91%**	**25.16%**	**28.9%**
	2.50%	0.30	0.10%	37.47%	32.49%	37.3%
	2.50%	0.35	0.10%	43.51%	36.04%	41.4%
	2.50%	**0.40**	**0.10%**	**49.72%**	**39.60%**	**45.5%**
	2.50%	0.50	0.10%	62.49%	47.03%	54.1%

Table 4 (DF = 5)

PD	Correlation parameter	Quantile (Risk-Factor)	CONDITIONAL EXPECTED LOSS	Significance Level (Tail-Risk)	EXPECTED SHORTFALL (Gauss)	VALUE-AT-RISK (Gauss)	EXPECTED-SHORTFALL (Student-T)	VALUE-AT-RISK (Student-T)	CONDITIONAL STANDARD DEVIATION	VaR Ratio (Student-t/Gauss)
1.00%	0.00	0.10%	1.00%	99.0%	27.5%	24.1%	45.3%	41.1%	9.95%	1.65
1.00%	**0.05**	**0.10%**	**4.67%**	**99.0%**	**60.9%**	**53.7%**	**98.6%**	**89.7%**	**21.10%**	**1.62**
1.00%	0.10	0.10%	7.75%	99.0%	79.0%	70.0%	126.8%	115.6%	26.74%	1.60
1.00%	0.15	0.10%	11.03%	99.0%	94.5%	83.9%	150.5%	137.3%	31.32%	1.59
1.00%	**0.20**	**0.10%**	**14.55%**	**99.0%**	**108.5%**	**96.6%**	**171.6%**	**156.7%**	**35.26%**	**1.58**
1.00%	0.30	0.10%	22.44%	99.0%	133.6%	119.5%	208.2%	190.6%	41.72%	1.56
1.00%	0.35	0.10%	26.83%	99.0%	144.9%	129.9%	224.1%	205.5%	44.31%	1.55
1.00%	**0.40**	**0.10%**	**31.56%**	**99.0%**	**155.4%**	**139.7%**	**238.5%**	**218.9%**	**46.47%**	**1.53**
1.00%	0.50	0.10%	42.09%	99.0%	173.7%	156.9%	261.9%	241.2%	49.37%	1.51
2.50%	0.00	0.10%	2.50%	99.0%	44.1%	38.8%	72.0%	65.5%	15.61%	1.63
2.50%	**0.05**	**0.10%**	**9.65%**	**99.0%**	**88.3%**	**78.3%**	**141.1%**	**128.7%**	**29.52%**	**1.60**
2.50%	0.10	0.10%	15.01%	99.0%	110.2%	98.1%	174.0%	159.0%	35.72%	1.58
2.50%	0.15	0.10%	20.39%	99.0%	127.8%	114.1%	199.8%	182.8%	40.29%	1.56
2.50%	**0.20**	**0.10%**	**25.91%**	**99.0%**	**142.7%**	**127.8%**	**221.0%**	**202.6%**	**43.81%**	**1.55**
2.50%	0.30	0.10%	37.47%	99.0%	166.5%	150.1%	253.0%	232.6%	48.40%	1.52
2.50%	0.35	0.10%	43.51%	99.0%	175.6%	158.8%	264.2%	243.4%	49.58%	1.50
2.50%	**0.40**	**0.10%**	**49.72%**	**99.0%**	**183.0%**	**166.0%**	**272.3%**	**251.3%**	**50.00%**	**1.49**
2.50%	0.50	0.10%	62.49%	99.0%	191.5%	175.1%	278.1%	257.7%	48.41%	1.45
5.00%	0.00	0.10%	5.00%	99.0%	63.1%	55.7%	102.0%	92.9%	21.8%	1.62
5.00%	0.05	0.10%	16.39%	99.0%	115.0%	102.5%	181.2%	165.6%	37.0%	1.58
5.00%	0.10	0.10%	24.08%	99.0%	138.0%	123.5%	214.4%	196.5%	42.8%	1.55
5.00%	0.15	0.10%	31.35%	99.0%	155.0%	139.3%	237.9%	218.4%	46.4%	1.53
5.00%	0.20	0.10%	38.44%	99.0%	168.1%	151.6%	255.0%	234.6%	48.6%	1.52
5.00%	0.30	0.10%	52.28%	99.0%	185.4%	168.5%	274.7%	253.7%	49.9%	1.48
5.00%	0.35	0.10%	59.00%	99.0%	190.1%	173.4%	278.0%	257.3%	49.2%	1.46
5.00%	0.40	0.10%	65.53%	99.0%	192.2%	176.1%	277.1%	257.2%	47.5%	1.44
5.00%	0.50	0.10%	77.76%	99.0%	188.6%	174.5%	262.9%	245.4%	41.6%	1.39

Table 5

Call-Spread Ratio:	Gauss Distribution: alpha=0.95				Student-t Distribution: alpha=0.95			
Correlation parameter	Equity	Mezzanine	Senior	Super-Senior	Equity	Mezzanine	Senior	Super-Senior
0.00	48.00%	36.30%	20.13%	0.63%	48.10%	37.02%	22.06%	1.31%
0.05	51.96%	43.88%	31.25%	2.30%	51.86%	44.18%	32.31%	3.47%
0.10	53.22%	46.04%	34.56%	3.17%	53.06%	46.24%	35.40%	4.51%
0.15	54.08%	47.44%	36.69%	3.86%	53.88%	47.57%	37.39%	5.31%
0.20	54.73%	48.46%	38.23%	4.43%	54.50%	48.54%	38.84%	5.97%
0.30	55.65%	49.85%	40.29%	5.31%	55.37%	49.86%	40.79%	6.96%
0.35	55.96%	50.31%	40.97%	5.62%	55.66%	50.29%	41.43%	7.32%
0.40	56.18%	50.64%	41.45%	5.86%	55.88%	50.61%	41.88%	7.58%
0.50	56.37%	50.91%	41.85%	6.06%	56.06%	50.87%	42.26%	7.80%
	Gauss Distribution: alpha=0.90				**Student-t Distribution: alpha=0.90**			
0.00	48.00%	36.30%	20.13%	0.63%	48.10%	37.02%	22.06%	1.31%
0.05	50.95%	42.06%	28.49%	1.73%	50.90%	42.46%	29.74%	2.76%
0.10	51.77%	43.55%	30.75%	2.18%	51.68%	43.87%	31.84%	3.33%
0.15	52.25%	44.39%	32.04%	2.49%	52.14%	44.67%	33.04%	3.69%
0.20	52.55%	44.91%	32.82%	2.68%	52.42%	45.16%	33.77%	3.93%
0.30	52.77%	45.29%	33.40%	2.84%	52.63%	45.52%	34.31%	4.12%
0.35	52.73%	45.22%	33.29%	2.81%	52.59%	45.45%	34.21%	4.08%
0.40	52.59%	44.97%	32.92%	2.71%	52.46%	45.22%	33.87%	3.96%
0.50	51.97%	43.90%	31.29%	2.31%	51.87%	44.20%	32.34%	3.48%
	Gauss Distribution: alpha=0.75				**Student-t Distribution: alpha=0.75**			
0.00	48.00%	36.30%	20.13%	0.63%	48.10%	37.02%	22.06%	1.31%
0.05	49.22%	38.76%	23.59%	0.99%	49.26%	39.34%	25.23%	1.81%
0.10	49.28%	38.88%	23.76%	1.01%	49.32%	39.45%	25.39%	1.84%
0.15	49.12%	38.57%	23.31%	0.96%	49.17%	39.15%	24.97%	1.76%
0.20	48.82%	37.96%	22.44%	0.86%	48.88%	38.58%	24.17%	1.63%
0.30	47.84%	35.96%	19.66%	0.59%	47.94%	36.70%	21.63%	1.25%
0.35	47.14%	34.50%	17.74%	0.45%	47.28%	35.33%	19.87%	1.03%
0.40	46.27%	32.66%	15.43%	0.31%	46.45%	33.61%	17.76%	0.81%
0.50	43.76%	27.30%	9.67%	0.09%	44.08%	28.64%	12.41%	0.39%
	Gauss Distribution: alpha=0.50				**Student-t Distribution: alpha=0.50**			
0.00	48.00%	36.30%	20.13%	0.63%	48.10%	37.02%	22.06%	1.31%
0.05	47.19%	34.61%	17.88%	0.46%	47.33%	35.44%	20.00%	1.05%
0.10	46.27%	32.67%	15.44%	0.31%	46.46%	33.62%	17.77%	0.81%
0.15	45.22%	30.43%	12.85%	0.19%	45.46%	31.53%	15.38%	0.60%
0.20	44.00%	27.81%	10.16%	0.10%	44.30%	29.11%	12.87%	0.42%
0.30	40.79%	21.13%	4.91%	0.01%	41.27%	22.97%	7.69%	0.16%
0.35	38.60%	16.94%	2.75%	0.00%	39.21%	19.14%	5.27%	0.09%
0.40	35.80%	12.24%	1.19%	0.00%	36.59%	14.83%	3.19%	0.04%
0.50	27.33%	3.18%	0.04%	0.00%	28.77%	5.85%	0.64%	0.00%

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