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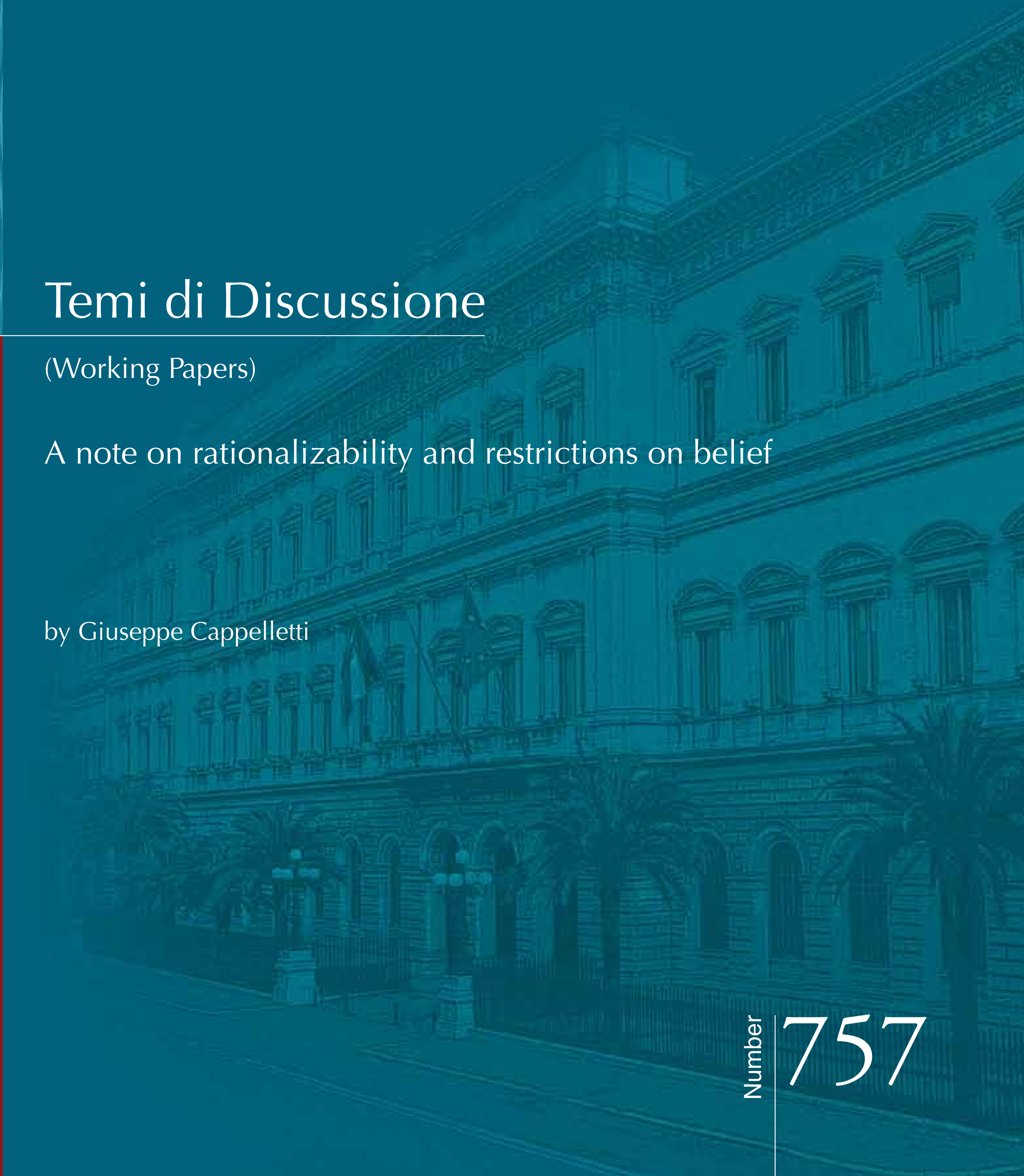

Temi di Discussione

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April 2010

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A NOTE ON RATIONALIZABILITY AND RESTRICTIONS ON BELIEF

by Giuseppe Cappelletti*

Abstract

Rationalizability is a widely accepted solution concept in the study of strategic form game with complete information and is fully characterized in terms of assumptions on the rationality of the players and common certainty of rationality. Battigalli and Siniscalchi extend rationalizability and derive the solution concept called Δ-rationalizability. Their analysis is based on the following assumptions: (a) players are rational; (b) their first-order beliefs satisfy some restrictions; and (c) there is common belief of (a) and (b). In this note I focus on games with complete information and I characterize Δ-rationalizability with a new notion of iterative dominance which is able to capture the additional hypothesis on players' beliefs.

JEL Classification: C72.
Keywords: rationalizability, strategic form game, complete information.

Contents

* Bank of Italy, Economics, Research and International Relations.

1 Introduction[1]

The solution concept called rationalizability, which was introduced by Bernheim (1984) and Pearce (1984), is widely accepted in the study of strategic form game with complete information and is fully characterized in term of rationality and common belief in rationality. There are settings where it is plausible to assume that players' beliefs satisfy some restrictions that are not implied by assumptions concerning rationality or belief in rationality, or beliefs about such beliefs. Such restrictions may be related to some structural properties of the situation analyzed. For example, in a bargaining situation players can believe that their opponents have some preference for fair division, in an auction bidders can expect positive bids to win with positive probability (Battigalli and Siniscalchi, 2003b) or in a communication game players can believe that their opponents trust their messages (Crawford, 2003). Based on this observation, Battigalli and Siniscalchi (1999) investigate the implications of the following assumptions: (a) players are rational; (b) their first-order beliefs satisfy an exogenous restriction; and (c) there is common belief of (a) and (b). Their analysis extends rationalizability taking as given some exogenous restrictions on players' belief and derives the solution concept called $\Delta-$*rationalizability*, which does not hinge on any assumption on equilibrium or correctness of beliefs. They apply this new solution concept to games with incomplete information (Battigalli, 2003; Battigalli and Siniscalchi, 2003a; Battigalli et al., 2008) and dynamic games (Battigalli, 1997, 2003; Battigalli and Siniscalchi, 2002, 2007).

Despite its great potential $\Delta-$*rationalizability* has not received as many applications as has its unconstrained counterpart. One reason for this lack of attention is that many practitioners find $\Delta-$*rationalizability* difficult to operationalize. In fact, it requires the iterative deletion of strategies that cannot be justified by beliefs consistent with progressively higher degrees of strategic sophistication. This procedure could be analytically cumbersome and numerically intractable.

A connection between $\Delta-$*rationalizability* and dominance would be valuable on both practical and conceptual levels. I introduce a new dominance concept, called $\Delta-$*dominance*, and prove that, under appropriate conditions, rationalizability with exogenous restrictions on players' belief and iterated $\Delta-$*dominance* are equivalent in strategic form games with complete information. This extends the classical iterated dominance characterization of rationalizability and simplifies computation of the $\Delta-$*rationalizable* in this type of games.

2 Rationalizability and restrictions on belief in strategic form games of complete information

To simplify the analysis I focus on strategic form game of complete information, a model of interactive decision-making in which each agent chooses his strategies once and for all, and these choices are made simultaneously. The model is a structure:

$$G = \langle N, \{S_i, u_i\}_{i\in N}\rangle \tag{1}$$

where for each player i, belonging to the set $N = \{1, 2, ..., n\}$, S_i is a finite set of possible strategies. The payoff function u_i is defined on the Cartesian product of players' possible

[1] The views expressed in the article are those of the authors and do not involve the responsibility of the Bank of Italy. I would like to thank Pierpaolo Battigalli, Sandro Brusco, Giorgio Gobbi and two anonymous referees for their helpful comments and suggestions. All remaining errors are my own.

strategies, $\prod_{i \in N} S_i$, and it assumes real values. The set of mixed strategies of player i is denoted as Σ_i and it coincides with the set of all probability measures defined on S_i, a generic element of Σ_i is denoted as σ_i. In order to shorten the notation I denote the opponents of player i with $-i$.

Player i's first-order beliefs are represented by a probability measure on the set of his opponents' strategies, i.e. a generic first-order belief, μ_i, belongs to the set of probability measures with support contained in $S_{-i} := \prod_{j \neq i} S_j$, the set of first-order beliefs is denoted as $\Delta(S_{-i})$. Players' belief may be assumed to satisfy some restrictions which are justified or related to some structural properties of the game. Let me denote with Δ_i any subset of $\Delta(S_{-i})$ and with Δ the Cartesian product of all the players' restrictions, $\Delta := \Delta_1 \times \Delta_2 \times ... \times \Delta_n$.

Given a belief μ_i and an action s_i let

$$u_i(s_i, \mu_i) := \sum_{s_{-i} \in S_{-i}} u_i(s_i, s_{-i}) \mu_i(s_{-i}) \tag{2}$$

be the expected utility for player i from playing s_i based on his belief μ_i.

Definition 1 *A strategy s_i is rational for player i with respect to μ_i if and only if for every $s_i' \in S_i$ the following inequality is satisfied:*

$$u_i(s_i, \mu_i) \geq u_i(s_i', \mu_i) \tag{3}$$

A strategy s_i is rational for player i if the strategy s_i maximizes his expected utility, that is, s_i is a best response to the belief μ_i. I denote with $\rho_i(\mu_i)$ the set of best responses to μ_i. In other words, a strategy s is rational for a player with first-order beliefs μ, when it is justifiable based on belief μ.

If the set of admissible beliefs is exogenously constrained the set of rational strategies is smaller and this leads to the definition of $\Delta - rationalizability$. A strategy profile (s_i, s_{-i}) is $\Delta - rationalizable$ if and only if for each player i the strategy s_i belongs to $S_i(k, \Delta)$ for any natural number k, where $S_i(k, \Delta)$ is defined as follow: for k equal to 0 the set $S_i(0, \Delta)$ is equal to S_i and for every natural number k strictly greater than 0

$$S_i(k, \Delta) := \{s_i \in S_i(k-1, \Delta) : \exists \mu_i \in \Delta_i \text{ such that } s_i \in \rho_i(\mu_i) \text{ and } \mu_i(S_{-i}(k-1, \Delta)) = 1\} \tag{4}$$

where $S_{-i}(k-1, \Delta)$ is defined as $\prod_{j \neq i} S_j(k-1, \Delta)$.

Definition 2 *Given a strategic form game $G = \langle N, \{S_i, u_i\}_{i \in N} \rangle$ and a set of restrictions on players' beliefs Δ, the strategy s_i is $(k, \Delta) - rationalizable$ if and only if s_i belongs to $S_i(k, \Delta)$. The strategy s_i is $\Delta - rationalizable$ if and only if s_i belongs to $S_i(\infty, \Delta)$ where $S_i(\infty, \Delta) := \bigcap_{k \geq 1} S_i(k, \Delta)$.*

Let me denote the set of strategy profiles that are $(k, \Delta) - rationalizable$ as $S(k, \Delta)$ where $S(k, \Delta)$ is equal to the Cartesian product of $S_i(k, \Delta)$, that is $S(k, \Delta) := \prod_{i \in N} S_i(k, \Delta)$.

2.1 Δ-rationalizability and dominance

A strategy s_i is strictly dominated for player i by a mixed strategy σ_i on a subset of his opponents' strategies $B_{-i} \subseteq S_{-i}$ if and only if for every strategy profile of his opponents s_{-i}

$$u_i(s_i, s_{-i}) < \sum_{s_i' \in S_i} \sigma_i(s_i') \, u_i(s_i', s_{-i}) \tag{5}$$

where $\sigma_i(s_i')$ is the probability assigned to strategy s_i' by mixed strategy σ_i. For a given rectangular subset B of S, let $\mathcal{S}(B)$ be the set of strategy profiles (s_i, s_{-i}) such that, for each player s_i is not strictly dominated on B_{-i} by any mixed strategy σ_i which assigns positive probability only to strategies belonging to B_i.

According to the first Pearce's lemma (Pearce, 1984), a strategy is strictly dominated if and only if it is not a best response to any conceivable belief. Therefore if $\Delta_i = \Delta(S_{-i})$ for every $i \in N$ the set of $\Delta - \textit{rationalizable}$ strategies coincides with the set of iteratively undominated strategies. If Δ_i is a strict subset of $\Delta(S_{-i})$ the set of $\Delta - \textit{rationalizable}$ and iteratively undominated strategies do not coincide. Hence, if I want to characterize the set of $\Delta - \textit{rationalizable}$ strategies in term of being iteratively undominated I need to generalize the concept of dominance in order to take into account the exogenous restrictions on players' beliefs. Let $p(s_i, s_{-i}; \sigma_i)$ be the set of beliefs that justifies choosing s_i instead of σ_i given that i's opponents choice s_{-i} has a positive probability of being played:

$$p(s_i, s_{-i}; \sigma_i) = \{\mu \in \Delta(S_{-i}) : \mu(s_{-i}) > 0 \text{ and } u_i(s_i, \mu_i) \geq u_i(\sigma_i, \mu_i)\} \tag{6}$$

Now, I can state a definition of dominance that includes restrictions on players' beliefs.

Definition 3 *A strategy s_i is strictly $\Delta - \textit{dominated}$ by σ_i on $B_{-i} \subseteq S_{-i}$ if and only if for every s_{-i} belonging to B_{-i} either $u_i(s_i, s_{-i}) < u_i(\sigma_i, s_{-i})$, or $u_i(s_i, s_{-i}) \geq u_i(\sigma_i, s_{-i})$ implies $p(s_i, s_{-i}; \sigma_i) \cap \{\mu_i \in \Delta_i : \mu_i(B_{-i}) = 1\} = \emptyset$.*

This definition differs from the definition of dominance because a strategy s_i could be justified by some belief μ_i but this belief is not admitted given the restrictions on players' beliefs. As a result the set of strictly $\Delta - \textit{dominated}$ strategies is larger than the set of strictly dominated strategies. Suppose that for some s_{-i} it holds that $u_i(s_i, s_{-i}) > u_i(\sigma_i, s_{-i})$ then it may be the case that $p(s_i, s_{-i}; \sigma_i) \cap \{\mu_i \in \Delta_i : \mu_i(B_{-i}) = 1\} = \emptyset$. Take a strategic form game G with two players labelled 1 and 2. Player 1 has two possible strategies $\{u, d\}$ and player 2 has two possible strategies $\{L, R\}$. The payoffs of player 1 are summarized in the following table.

	L	R
u	2	1
d	1	2

If player 1 is certain that player 2 will choose action R then $u_1(u, L) > u_i(d, L)$ but $p(u, L; d) \cap \{\mu_1 \in \Delta_1 : \mu_1(\{L, R\}) = 1\} = \emptyset$.[2]

Lemma 1 *Given a pure strategy s_i, a mixed strategy σ_i and a subset B_{-i} of S_{-i} the following conditions are equivalent:*

[2] In fact $p(u, L; d) = \{\mu_1 \in \Delta(\{L, R\}) : \mu_1(L) > \frac{1}{2}\}$ and $\{\mu_1 \in \Delta_1 : \mu_1(\{L, R\}) = 1\} = \{\mu_1 : \mu_1(R) = 1\}$.

1. *s_i is strictly $\Delta -$ dominated by σ_i on B_{-i};*

2. *for every $\mu_i \in \{\mu_i \in \Delta_i : \mu_i (B_{-i}) = 1\}$the expected utility associated to strategy s_i is strictly less then the one associated to σ_i (that is $u_i (s_i, \mu_i) < u_i (\sigma_i, \mu_i)$).*

Proof. The proof is by contradiction.

(1) $\Rightarrow$ (2) Suppose that s_i is strictly $\Delta -$ *dominated* by σ_i on B_{-i} and assume that there exists μ_i belonging to $\{\mu_i \in \Delta_i : \mu_i (B_{-i}) = 1\}$ such that $u_i (s_i, \mu_i) \geq u_i (\sigma_i, \mu_i)$, then there must be a strategy profile s_{-i} belonging to B_{-i} such that $u_i (s_i, s_{-i}) \geq u_i (\sigma_i, s_{-i})$ and $\mu (s_{-i}) > 0$. This contradicts the initial assumption that s_i is $\Delta -$ *dominated* by σ_i.

(2) $\Rightarrow$ (1) Assume that $u_i (s_i, \mu_i) < u_i (\sigma_i, \mu_i)$ for every $\mu_i \in \{\mu_i \in \Delta_i : \mu_i (B_{-i}) = 1\}$ and that there exists a strategy profile s_{-i} such that $u_i (s_i, s_{-i}) \geq u_i (\sigma_i, s_{-i})$ and $p (s_i, s_{-i}; \sigma_i) \cap \{\mu_i \in \Delta_i : \mu_i (B_{-i}) = 1\} \neq \emptyset$.
Then there exists μ_i belonging to $\{\mu_i \in \Delta_i : \mu_i (B_{-i}) = 1\}$ such that $u_i (s_i, \mu_i) \geq u_i (\sigma_i, \mu_i)$, this contradicts assumption (2). ■

Definition 4 *A strategy s_i is not strictly $\Delta -$ dominated on B if and only if for every mixed strategy σ_i with support included in B_i there exists $s_{-i} \in B_{-i}$ such that $u_i (s_i, s_{-i}) \geq u_i (\sigma_i, s_{-i})$ and $p (s_i, s_{-i}; \sigma_i) \cap \Delta_i \neq \emptyset$.*

This definition differs from the traditional one because it requires the existence of an acceptable belief that justifies s_i with respect to the any candidate alternative strategy σ_i. That is, a strategy $s_i \in S_i$ is not strictly $\Delta -$ *dominated* by any mixed strategy for player i if and only if for every σ_i there exists $\mu_i \in \Delta_i$ such that $u_i (s_i, \mu_i) \geq u_i (\sigma_i, \mu_i)$. For a given rectangular subset $B \subseteq S$, let $\mathcal{S} (B, \Delta)$ denote the set of strategy profiles $(s_i, s_{-i}) \in S$ such that, for each i, s_i is $\Delta -$ *undominated* on B.

2.2 Main result

It is possible to generalize the first Pearce's lemma characterizing $\Delta -$ *rationalizability* in terms of iterative elimination of strictly $\Delta -$ *dominated* strategies. First, I need a preliminary result which relates strict $\Delta -$ *dominance* and best responses with respect a set of admissible beliefs. Let the set of all the players' restrictions, Δ, be closed and convex if all its components, Δ_i, are closed and convex subsets of $\Delta (S_{-i})$.

Lemma 2 *Let $G = \langle N, \{B_i, u_i\}_{i \in N} \rangle$ be a strategic form game and Δ is a closed and convex set of restrictions on belief, a strategy s_i is not strictly $\Delta -$dominated on B if and only if there exists μ_i belonging to to the set of admissible beliefs $\{\mu \in \Delta_i : \mu (B_{-i}) = 1\}$ and such that $s_i \in \rho_i (\mu_i)$.*

Proof. First I prove by contradiction that being $\Delta -$ *undominated* implies being justifiable by some admissible belief. Assume that s_i is not strictly $\Delta -$ *dominated* and there is no belief $\mu_i \in \Delta_i$ such that $s_i \in \rho_i (\mu_i)$. Then, the following system of inequalities has no solution in $\{\mu \in \Delta_i : \mu (B_{-i}) = 1\}$:[3]

$$\sum_{s_{-i} \in S_{-i}} \mu_i (s_{-i}) \left[u_i (s'_i, s_{-i}) - u_i (s_i, s_{-i}) \right] \leq 0 \text{ for every } s'_i \neq s_i \in S_i \tag{7}$$

[3] Note that $\Delta_i \cap \{\mu \in \Delta (S_{-i}) : \mu (B_{-i}) = 1\} = \{\mu \in \Delta_i : \mu (B_{-i}) = 1\}$.
This implies that if $\{\mu \in \Delta_i : \mu (B_{-i}) = 1\}$ is the intersection of two convex sets then it is convex.

We have a collection of closed proper convex (linear) functions on $\mathbb{R}^{|S_{-i}|}$ indexed by s_i', that is for each s_i' in S_i we have a linear function $\mu_i \to \sum\limits_{s_{-i}\in S_{-i}} \mu_i(s_{-i})[u_i(s_i', s_{-i}) - u_i(s_i, s_{-i})]$. Δ_i is a non-empty closed, convex set in $\mathbb{R}^{|S_{-i}|}$ and since Δ_i is bounded it has no direction of recession[4] (see Rockafellar, 1996). Hence, the linear functions $\mu_i \to \sum\limits_{s_{-i}\in S_{-i}} \mu_i(s_{-i})[u_i(s_i', s_{-i}) - u_i(s_i, s_{-i})]$ have no common direction of recession which is also direction of recession of Δ_i. Based on these consideration, I can apply Theorem 21.3 in Rockafellar (1996) which states that if system (7) has no solution then there exists a non-negative real vector λ, belonging to $\mathbb{R}^{|S_i|-1}$, and $\varepsilon > 0$ such that

$$\sum_{s_i'\in S_i\setminus\{s_i\}} \sum_{s_{-i}\in S_{-i}} \lambda(s_i') \left[u_i(s_i', s_{-i}) - u_i(s_i, s_{-i})\right] \mu_i(s_{-i}) \geq \varepsilon \tag{8}$$

for every $\mu_i \in \Delta_i$. Therefore,

$$\sum_{s_i'\in S_i\setminus\{s_i\}} \sum_{s_{-i}\in S_{-i}} \frac{\lambda(s_i')}{\sum\limits_{s_i'\in S_i\setminus\{s_i\}} \lambda(s_i')} \left[u_i(s_i', s_{-i}) - u_i(s_i, s_{-i})\right] \mu_i(s_{-i}) \geq \frac{\varepsilon}{\sum\limits_{s_i'\in S_i\setminus\{s_i\}} \lambda(s_i')}$$

or equivalently,

$$\sum_{s_{-i}\in S_{-i}} [u_i(\sigma_i, s_{-i}) - u_i(s_i, s_{-i})] \mu_i(s_{-i}) \geq \varepsilon' \tag{9}$$

where $\varepsilon' := \frac{\varepsilon}{\sum\limits_{s_i'\in S_i\setminus\{s_i\}} \lambda(s_i')} > 0$ and σ_i is a mixed strategy assigning to each strategy $s_i' \in S_i\setminus\{s_i\}$ probability equal to $\frac{\lambda(s_i')}{\sum\limits_{s_i'\in S_i\setminus\{s_i\}} \lambda(s_i')}$. Inequality (9) states that σ_i is strictly better than s_i for every conjecture μ_i in Δ_i, contradicting the initial assumption that s_i is not strictly $\Delta -$ *dominated* (see Lemma 1).

In order to prove the opposite it is sufficient to notice that if a $\mu_i \in \Delta_i$ exists such that $s_i \in \rho(\mu_i)$ then s_i is not strictly $\Delta -$ *dominated* by definition. ∎

In order to relate $\Delta -$ *rationalizability* and $\Delta -$ *dominance* I have to consider that the set of feasible strategy and the set of admissible beliefs changes along the iterative procedure that define $\Delta -$ *rationalizability*. For an arbitrary natural number k the set of admissible strategy is $S(k-1,\Delta)$ and the set of relevant restrictions on beliefs is the projection of Δ on $S(k-1,\Delta)$, therefore the set of not strictly $\Delta -$ *dominated* strategies has to be computed on $S(k-1,\Delta)$ taking as relevant restrictions Δ^k defined as the Cartesian product of $\Delta_i^k := \{\mu \in \Delta_i : \mu(S_{-i}(k-1,\Delta)) = 1\}$.

Lemma 3 *Let $G = \langle N, \{S_i, u_i\}_{i\in N}\rangle$ be a strategic form game and Δ is a closed and convex set of restrictions on belief, for every natural number $k \geq 1$, the set of $(k,\Delta)-$ rationalizable strategy profiles coincides with the set of not strictly $\Delta-$dominated strategies on B^k, $S(k,\Delta) = \mathcal{S}(B^k, \Delta^k)$, where $B^k := S(k-1,\Delta)$ and the set of restrictions is $\Delta^k = \prod\limits_{i\in N} \Delta_i^k$ with $\Delta_i^k := \{\mu \in \Delta_i : \mu(S_{-i}(k-1,\Delta)) = 1\}$.*

[4]Let Δ be a non-empty convex set in $\mathbb{R}^n$. Δ recedes in the direction d if and only if Δ includes all the half-lines in the direction d which start at points of Δ. In other words, Δ recedes in the direction d, where $d \neq 0$, if and only if $x + \lambda d \in \Delta$ fir every $\lambda \geq 0$ and $x \in \Delta$.

Proof. Lemma (2) implies that the set of $(1,\Delta)-$*rationalizable* strategy profiles is equal to the set of not strictly $\Delta-$*dominated* strategy profiles, namely $S(1,\Delta)$=$\mathcal{S}(S,\Delta)$. For every natural number k let me consider the strategic form game $G^k := \left\langle N, \left\{B_i^k, u_i\right\}_{i\in N}\right\rangle$ where the set of strategy for player i is defined as $B_i^k := S_i(B_{k-1},\Delta)$. The set of restrictions on beliefs for each player is the projection of the initial restriction on the set of $(k-1,\Delta)-$ *rationalizable* strategy profiles, formally it is the set Δ_i^k defined as $\Delta_i^k := \{\mu \in \Delta_i : \mu(S_{-i}(k-1,\Delta)) = 1\}$. Since Δ_i^k is the intersection of two closed and convex sets, it is a closed and convex set. Given this observation, I can apply Lemma (2) and conclude that a strategy s_i is not strictly $\Delta-$ *dominated* on B_{-i}^k with respect to B_i^k if and only if there exists μ_i belonging to $\left\{\mu \in \Delta_i^k : \mu(B_{-i}) = 1\right\}$ and such that $s_i \in \rho_i(\mu_i)$. This means that $S(k,\Delta) = \mathcal{S}\left(B^k, \Delta^k\right)$. ■

The previous lemma states that a strategy s_i is $(k,\Delta)-$ *rationalizable* if and only if it survives k step of iterative elimination of strictly $\Delta-$ *dominated* strategies. As a result I have a full characterization of $\Delta-$ *rationalizability*.

The following example shows that the requirement for Δ to be convex is necessary in order for Lemma 2 to hold. Take a strategic form game G with two players labelled 1 and 2. Player 1 has three possible strategies $\{u,m,d\}$ and player 2 has two possible strategies $\{L,R\}$. The payoffs of Player 1 are summarized by the following table.

	L	R
u	2	0
m	1.5	1.5
d	0	2

Suppose that player 1 has just two admissible beliefs about his opponent's choice, labelled μ_1' and μ_1''. In particular, he believes that player 2 chooses L either with probability $\frac{4}{5}$ (μ_1') or with probability $\frac{1}{5}$ (μ_1'').[5] Let me consider the set of $(1,\Delta)-$*rationalizable* strategy for player 1, that is the set of all strategies that are the best response to one of the two acceptable beliefs that is the set of $(1,\Delta)-$ *rationalizable* strategies for player 1 is

$$S_1(1,\Delta) = \bigcup_{\mu_1\in\{\mu_1',\mu_1''\}} \rho(\mu_1) = \{u,d\}$$

Now, let me focus on the set of strategies that are not $\Delta-$ *dominated* for player 1. If u is the best response to μ_1' and d is the best response to μ_1'' then u and d are not $\Delta-$*dominated.* A mixed strategy σ_1 $\Delta-$*dominates* strategy m if and only if it satisfies the following inequalities:

$$\mu_1'(L)\left[2\sigma + (1-\sigma-\lambda)\frac{3}{2}\right] + \mu_1'(R)\left[2\lambda + (1-\sigma-\lambda)\frac{3}{2}\right] > \frac{3}{2} \tag{10}$$

$$\mu_1''(L)\left[2\sigma + (1-\sigma-\lambda)\frac{3}{2}\right] + \mu_1''(R)\left[2\lambda + (1-\sigma-\lambda)\frac{3}{2}\right] > \frac{3}{2} \tag{11}$$

where σ is the probability that the mixed strategy σ_1 assigns to strategy u and λ is the probability that the mixed strategy σ_1 assigns to strategy d. Substituting the probability

[5] Formally, I assume that the set of feasible beliefs has just two elements μ_1' and μ_1'' which are such that $\mu_1'(L) = \frac{4}{5}$ and $\mu_1''(L) = \frac{1}{5}$.

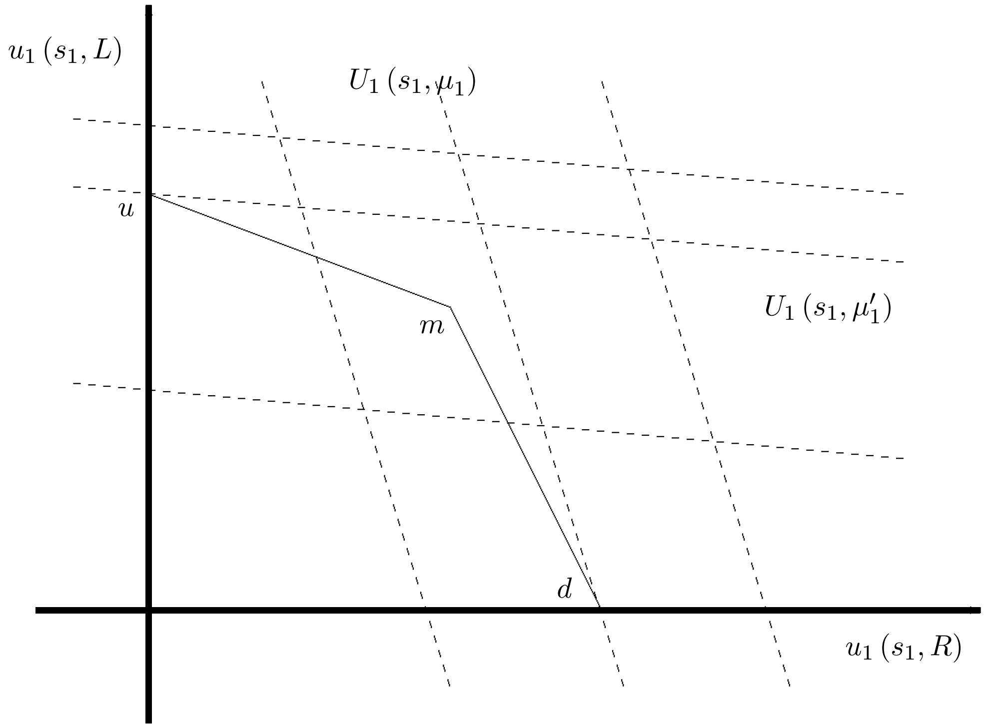
assigned by the two acceptable beliefs:

$$\sigma > 11\lambda \quad (12)$$

$$\lambda > 11\sigma \quad (13)$$

which are mutually incompatible. Then, there is no mixed strategy that $\Delta - dominates$ m and the set of $\Delta - undominated$ strategies for Player 1, denoted as $\mathcal{S}_1(S, \Delta)$, is equal to $\{u, m, d\}$ and is different from $S_1(1, \Delta)$.

Figure 1: An example of the necessity of the convexity assumption



3 Conclusion

There are situation where it is plausible to assume that players' beliefs satisfy some restrictions that are not implied by assumptions concerning rationality or belief in rationality, or beliefs about such beliefs. Based on this observation, Battigalli and Siniscalchi (2003a) introduce a new solution concept, called $\Delta - rationalizability$, based on the assumptions that agents are rational, players' beliefs satisfy some exogenous restrictions and there is common belief of the previous two hypothesis.

I characterize $\Delta - rationalizability$ in term of iterated $\Delta - dominance$, which generalizes the well-known relationship between rationalizability and iterated dominance in standard settings. $\Delta - dominance$ differs from the traditional definition of dominance, because a strategy could be justified by some beliefs but this belief is not admissible given the assumed restrictions on players beliefs. This characterization simplifies the application of $\Delta - rationalizability$ and broadens my understanding of this solution concept.

As a result, this research can facilitate the use of this kind of non-equilibrium analysis that could shed new light on economic behavior. This characterization can offer some clarification of the concept of rationalizability for those interested in the foundation of game theory (Harsanyi, 1967; Mertens and Zamir, 1985; Brandenburger and Dekel, 1987; Bergemann and Morris, 2005, 2007; Ely and Peski, 2006). It would then be possible to generalize Lemma 2 to games with incomplete information (Battigalli and Siniscalchi, 2003; Dekel, Fudenberg, and Morris, 2005) and dynamic games (Shimoji and Watson, 1998; Battigalli, 2003).

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