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ON VECTOR AUTOREGRESSIVE MODELING IN SPACE AND TIME

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Abstract

Despite the fact that it provides a potentially useful analytical tool, allowing for the joint modeling of dynamic interdependencies within a group of connected areas, until lately the VAR approach had received little attention in regional science and spatial economic analysis. This paper aims to contribute in this field by dealing with the issues of parameter identification and estimation and of structural impulse response analysis. In particular, there is a discussion of the adaptation of the recursive identification scheme (which represents one of the more common approaches in the time series VAR literature) to a space-time environment. Parameter estimation is subsequently based on the Full Information Maximum Likelihood (FIML) method, a standard approach in structural VAR analysis. As a convenient tool to summarize the information conveyed by regional dynamic multipliers with a specific emphasis on the scope of spatial spillover effects, a synthetic space-time impulse response function (STIR) is introduced, portraying average effects as a function of displacement in time and space. Asymptotic confidence bands for the STIR estimates are also derived from bootstrap estimates of the standard errors. Finally, to provide a basic illustration of the methodology, the paper presents an application of a simple bivariate fiscal model fitted to data for Italian NUTS 2 regions.

JEL Classification: C32, C33, R10.
Keywords: Structural VAR model, Spatial econometrics, Identification, Space-time impulse response analysis.

Contents

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1 Introduction[1]

Starting from the seminal article by Sims (1980), the vector autoregressive (VAR) methodology has been applied to a vast range of empirical topics, including monetary and fiscal policy analysis and short-term economic forecasting.

Also in the fields of regional science and spatial economics the scope of issues that could be addressed by means of properly identified structural VARs appears to be wide and includes: the analysis of the regional propagation of demand shocks via trade linkages; the assessment of long-run spatial spillover effects from local public expenditure to private sector performance; and the study of dynamic knowledge externalities linking patenting activity in the business sector to academic research in nearby areas.

However, despite the fact that the VAR approach provides a potentially useful analytical tool allowing for the joint modeling of dynamic interdependencies within a group of connected areas, until lately it has received little attention in the applied spatial economics literature.

This is mainly due to the overparameterization problem encountered when a direct transposition of the standard VAR approach is attempted by simply setting up a system that involves an equation for each endogenous variable and each region in the sample.

At the same time, the identification of structural impulse responses appears to pose specific difficulties, requiring the introduction of correct hypotheses if the bilateral nature of most economic linkages in space is to be properly accounted for.

In this paper, in line with the approach set forth in a number of previous contributions, the inherent overparameterization problem denoting the multi-area VAR model is addressed by imposing a priori restrictions on parameter values, stemming from hypotheses on the spatial decay of interactions across economic agents derived from the spatial econometrics literature.

The main methodological insight lies in the approach to structural parameters identification, where a block triangular scheme is introduced and motivated as a plausible extension to a space-time context of the recursive scheme widely adopted in the empirical time series VAR literature. Apart from structural parameter identification, some new results are also derived in the fields of parameter estimation and of impulse response analysis.

The remainder of the paper is structured as follows. Section 2 briefly reviews the related literature. Model specification and identification issues are then discussed in Section 3. Parameter estimation is dealt with in Section 4. Under the assumptions that the number of locations considered is fixed and a sufficient number of observations is collected over time, estimation is based on the Full Information Maximum Likelihood (FIML) method, a standard choice in structural VAR analysis.

[1] I wish to thank Juri Marcucci, Jesús Mur, the participants at the III Jean Paelink Seminar hosted by Universidad Politécnica de Cartagena and three anonymous referees for their useful comments and suggestions. The views expressed in this paper are my own and do not necessarily reflect those of the Bank of Italy.

The topic of impulse response analysis is dealt with in Section 5, where a synthetic space-time impulse response (STIR) function is introduced as a convenient tool to summarize the information conveyed by individual regional dynamic multipliers. Asymptotic confidence bands for the STIR estimates are also derived from bootstrap estimates of the respective standard errors.

To provide a basic illustration of the methodology and an initial test of the model's empirical performance, the application of a simple bivariate fiscal model estimated on the set of Italian NUTS 2 regions is carried out in Section 6. Lastly, Section 7 summarizes and concludes the paper.

2 Literature review

In this section a few previous contributions extending the VAR methodology to multi-area panel data are briefly reviewed.

Ruling out cross-sectional interactions and assuming fixed coefficients across areas yields a simple panel VAR specification that has received some attention in the panel data literature (Holtz-Eakin et al. 1988), but it is of little interest in regional economic analysis since cross-sectional interdependence is precluded.

In a multi-country set-up, cross-section interactions were more recently dealt with by Pesaran et al. (2004), who introduce the Global VAR specification, where information on trade shares across countries is utilized to specify the channels of transmission of national disturbances across the world economy.

In an intra-national context, Carlino and DeFina (1995) provide a straightforward implementation of the original Sims approach, by fitting a VAR model involving a single endogenous variable (GNP) to the six BEA regions in the US. In this case the limited number of areas (6 regions) and short lag order of the model allows the authors to estimate an unrestricted reduced form VAR specification. They are also among the first to employ impulse response analysis based on VAR estimation to measure the strength of spatial spillover effects across regions. However the identification of structural shocks hinges on the assumption of no contemporaneous spillover effects, a hypothesis that can be overly restrictive in many empirical settings.

Space-time impulse response analysis is also dealt with by Di Giacinto (2006), who implements a VAR approach based on an underlying univariate STARMA (Space-Time ARMA) specification. In this article, a priori information on spatial contiguity is utilized both to place reasonable restrictions on VAR coefficients matrices and to identify structural impulse responses.

Two previous contributions by Lesage and Pan (1995) and Lesage and Krivelyova (2002) introduced information on spatial contiguity to specify the prior distribution of VAR coefficients in a Bayesian univariate regional VAR analysis. However the authors do not deal with the topic of structural form identification, as the methodology is mainly aimed at improving the out–of-sample forecasting precision of standard Bayesian VAR models in a spatio-temporal context.

Remaining within a Bayesian setting, Canova and Ciccarelli (2006) have recently proposed a multi-country panel VAR specification that allows for cross-sectional interdependence in a general framework, solving the incidental parameter problem by imposing standard (i.e. non spatial) prior distributional assumptions. While this specification is potentially appealing in a regional context as well, the lack of a specific reference to the spatial structure of the data sets it aside from the remaining approaches reviewed here.

Di Giacinto (2003) is the first to attempt to apply spatial econometric techniques within the standard multivariate VAR framework provided by a monetary policy model previously fitted to regional and state-level US data by Carlino and DeFina (1998, 1999). Geographical information on the relative locations of the individual states in the US is utilized in a classical rather than a Bayesian framework, providing parameter restrictions that make identification and estimation possible for spatial samples of moderate to large size. However, the need to cope with a non-standard model setting, where variables observed at the intra-national level are included alongside variables that only display variation at the national level, makes the specification utilized in Di Giacinto (2003) somewhat peculiar.

The recent article by Beenstock and Felsenstein (2007) can thus be considered as the first comprehensive treatment of the topic of designing multivariate vector autoregressive models on spatial time series data, by introducing the SpVAR model class. The authors consider a fairly general specification, allowing for contemporaneous and time-lagged spatial interactions and for serially and spatially correlated errors. However, while they deal with the topics of parameter identification and impulse response analysis, they do not introduce parametric restrictions allowing for the identification of the structural shocks in the model, thus making the interpretation of estimated impulse responses somewhat questionable.

3 The spatial VAR model

3.1 Specification

In this section the SpVAR specification is introduced. To state the model formally, let us assume that a K-variate random vector is observed at regular time intervals over a set of N spatial units. Letting y_{ikt} denote the value of the k-th random variable (k=1,2,...,K) recorded on the i-th location (i=1,2,...,N) at time t=1,2,...,T, and stacking observations by location and variable, a standard VAR(p) specification for this data environment can be expressed in the usual form[2] as

$$\mathbf{y}_t = \delta + B_1\mathbf{y}_{t-1} + \ldots + B_p\mathbf{y}_{t-p} + \boldsymbol{\eta}_t \qquad (1)$$

where δ is an NK-dimensional vector of unknown constants, B_h (h=1,..,p) is an unrestricted $NK \times NK$ coefficients matrix and where the following positions are made:

$$\begin{gathered}\mathbf{y}_t = [y_{11t},\ldots,y_{N1t},\ldots,y_{1Kt},\ldots,y_{NKt}]' \\ \boldsymbol{\eta}_t = [\eta_{11t},\ldots,\eta_{N1t},\ldots,\eta_{1Kt},\ldots,\eta_{NKt}]' \\ E(\boldsymbol{\eta}_t) = 0\,,\ E(\boldsymbol{\eta}_t\boldsymbol{\eta}'_t) = \boldsymbol{\Sigma},\ E(\boldsymbol{\eta}_t\boldsymbol{\eta}'_{t-h}) = 0\,,\ h=1,2,\ldots\end{gathered} \qquad (2)$$

[2] While the deterministic part of the model involves only an intercept, the addition of other non-stochastic variables, like trends or seasonals, or the inclusion of strictly exogenous regressors does not alter the main results and is omitted for the sake of brevity.

$\boldsymbol{\Sigma}$ denoting an $NK \times NK$ positive definite covariance matrix.

In the VAR literature, expression (1) is referred to as the unrestricted reduced form, since contemporaneous relations across the endogenous variables are not modeled explicitly but are captured by the instantaneous covariances of the error term.

While the reduced form provides a useful tool to address issues like forecasting and Granger causality analysis, it provides little support when coping with structural types of analyses like impulse response analysis or forecast error decomposition, which only provide useful insights under the assumption that errors are orthogonal across equations.

To overcome this limitation, structural VAR specifications have been introduced that, in general terms, can be stated as:

$$\mathbf{C}_0 \mathbf{y}_t = \alpha + \mathbf{C}_1 \mathbf{y}_{t-1} + \ldots + \mathbf{C}_p \mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t \tag{3}$$

where α is the intercept term, $\mathbf{C}_h$, (h=1,..,p) is, as above, an unrestricted $NK \times NK$ coefficients matrix and where $\boldsymbol{\varepsilon}_t = [\varepsilon_{11t}, \ldots, \varepsilon_{N1t}, \ldots, \varepsilon_{1Kt}, \ldots, \varepsilon_{NKt}]'$ is assumed to have the properties:

$$\begin{gathered} E(\boldsymbol{\varepsilon}_t) = 0, \\ E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_t) = \Omega = diag\{[\omega_{11}, \ldots, \omega_{N1}, \ldots, \omega_{1K}, \ldots, \omega_{NK}]\} \\ E(\boldsymbol{\varepsilon}_t \boldsymbol{\varepsilon}'_{t-h}) = 0, \quad h=1,2,\ldots \end{gathered} \tag{4}$$

The structural VAR specification given by expression (3) is referred in the literature as the A-Model (Amisano and Giannini 1997). When all the coefficients in $\mathbf{C}_0$ are unrestricted, it is widely known that the A-Model is unidentified.

Different identification schemes, relying on economic theory and other a priori assumptions regarding the behaviour of the process, have been proposed in the empirical literature (see Hamilton 1994, Section 11.6, for a presentation of the various approaches to SVAR identification). A fairly standard method, yielding an exactly identified specification, assumes a recursive causal ordering of the endogenous variables, in the tradition of the approach originally advocated by Herman Wold. This assumption is formally analogous to the methodology initially proposed by Sims (1980), that derives orthogonal error terms by means of a Choleski triangularization of the covariance matrix of the reduced form residuals (e.g., Hamilton 1994, p. 330).

Once the variables have been numbered according to the desired causal ordering, the recursivity assumption implies the following triangular structure for the matrix of simultaneous interactions:

$$\mathbf{C}_0^{rec} = \begin{bmatrix} c_{11}^{(0)} & 0 & \ldots & 0 \\ c_{21}^{(0)} & c_{22}^{(0)} & \ldots & 0 \\ \ldots. & \ldots & & \ldots. \\ c_{NK-1,1}^{(0)} & c_{NK-1,2}^{(0)} & \ldots & 0 \\ c_{NK,1}^{(0)} & c_{NK,1}^{(0)} & & c_{NK,NK}^{(0)} \end{bmatrix} \tag{5}$$

Two main limitations hinder the direct transposition of the standard recursive VAR approach given by expressions (3) and (5) and assumptions (4) in a space-time data environment:

1. the number of free parameters to be estimated rapidly exhausts degrees of freedom as the number of spatial units N in the sample increases;

2. the causal ordering involves not only the endogenous variables, but also the spatial locations in the sample, a situation that poses specific difficulties, as detailed below.

The first limitation can be tackled by imposing reasonable parameter constraints on VAR coefficients matrices, and in the SpVAR approach set forth in Di Giacinto (2003) and Beenstock and Felsenstein (2007), such constraints are first derived by imposing a spatial structure on coefficients matrices on the R.H.S. of (3), by setting

$$\boldsymbol{C}_h = \begin{bmatrix} \boldsymbol{A}_{11}^{(h)} & \boldsymbol{A}_{12}^{(h)} & \ldots & \boldsymbol{A}_{1K}^{(h)} \\ \boldsymbol{A}_{21}^{(h)} & \boldsymbol{A}_{22}^{(h)} & \ldots & \boldsymbol{A}_{2K}^{(h)} \\ \ldots. & \ldots & & \ldots. \\ \boldsymbol{A}_{K1}^{(h)} & \boldsymbol{A}_{K2}^{(h)} & \ldots & \boldsymbol{A}_{KK}^{(h)} \end{bmatrix} \tag{6}$$

$$h=1,\ldots,p$$

with

$$\mathbf{A}_{kr}^{(h)} = \sum_{l=0}^{s} \mathbf{\Phi}_{kr}^{(hl)} \mathbf{W}_{kr}^{(l)} \tag{7}$$

$$\mathbf{\Phi}_{kr}^{(hl)} = diag\{[\phi_{1kr}^{(hl)},\ldots,\phi_{Nkr}^{(hl)}]'\}$$

$$k,r=1,\ldots,K \qquad h=1,\ldots,p$$

where $\mathbf{W}_{kr}^{(l)}$ denotes the usual $N \times N$ spatial weights matrix of order l, whose elements $w_{kr}^{(l)}(i,j)$ are known a priori and are usually assumed to be non negative and strictly positive if locations i and j are neighbours of order l according to a given spatial ranking (see Anselin and Smirnov 1996, for details on how to define and compute higher order spatial lags).

Aiming at maximum flexibility in model specification, spatial weights matrices are indexed by equation and variable, thus allowing for possibly differing patterns of spatial interaction across the K different variables entering the model. For instance, for a subset of variables contiguity-based weights could provide the preferred choice, while spatial weights specified as an inverse function of distance could provide a more reasonable approach for other variables.

Spatial weights are maintained to be fixed over time, albeit a generalization allowing for time-varying weights appears to be straightforward, as long as the weights are assumed to be known to the researcher at any time horizon.

Autoregressive coefficients are assumed to vary across locations, thus allowing for spatially heterogeneous model dynamics. Nonetheless, a nested spatially homogeneous specification can be immediately derived by setting

$$\mathbf{\Phi}_{kr}^{(hl)} = \phi_{kr}^{(hl)} \mathbf{I}_N \tag{8}$$

$$k,r=1,\ldots,K \qquad h=1,\ldots,p \qquad l=1,\ldots,s$$

where $\phi_{kr}^{(hl)}$ is a scalar.

When coefficient matrices are specified according to (7), the number of free coefficients to be estimated in each $\mathbf{C}_h$ matrix is equal to $NK^2(s+1)$, compared to a number on $(NK)^2$ free coefficients in the corresponding unrestricted VAR specification. This implies linear growth rates in *N*, compared to quadratic growth in the unrestricted VAR.

Turning now to simultaneous interactions, it has to be noted that assuming a strictly lower triangular structure for the $\mathbf{C}_0$ matrix appears to be severely binding in the context of a spatial VAR model, as it would amount to imposing a recursive causal ordering not only on the endogenous variables but also on individual locations. Under this assumption, shocks to variable y_{ikt} would be allowed to affect variable y_{jrt} at all locations if $r > k$ and at locations $j \geq i$ if $r = k$. The latter situation would consequently imply a unilateral causal chain in space, a feature that is quite uncommon in spatial economics where, apart from specific situations pertaining, for instance, to the diffusion of innovations along the urban hierarchy implied by central place theory, bilateral spatial interactions largely prevail.

To overcome this difficulty, the approach advocated in the paper assumes the following block-triangular structure[3] for $\mathbf{C}_0$:

$$\mathbf{C}_0 = \begin{bmatrix} \mathbf{A}_{11}^{(0)} & 0 & ... & 0 \\ \mathbf{A}_{21}^{(0)} & \mathbf{A}_{22}^{(0)} & ... & 0 \\ .... & ... & & .... \\ \mathbf{A}_{K1}^{(0)} & \mathbf{A}_{K2}^{(0)} & ... & \mathbf{A}_{KK}^{(0)} \end{bmatrix} \tag{9}$$

where, in line with the approach outlined above, individual blocks are defined as

$$\begin{gathered} \mathbf{A}_{kr}^{(0)} = \mathbf{I}_N - \sum_{l=1}^{s} \mathbf{\Phi}_{kr}^{(0l)} \mathbf{W}_{kr}^{(l)} \quad \text{if } r = k \\ \mathbf{A}_{kr}^{(0)} = -\sum_{l=0}^{s} \mathbf{\Phi}_{kr}^{(0l)} \mathbf{W}_{kr}^{(s)} \quad \text{if } r < k \\ \mathbf{\Phi}_{kr}^{(0l)} = diag\{[\phi_{1kr}^{(0l)}, ..., \phi_{Nkr}^{(0l)}]'\} \\ k,r = 1, ..., K, \quad r \leq k \end{gathered} \tag{10}$$

The VAR specification given by expression (3) with coefficients matrices as defined in (6) and (10) will be referred to in the following as a structural SpVAR(*p*,*s*) model.

Assuming that proper restrictions on the admissible values of coefficients $\phi_{ikr}^{(0l)}$ are imposed so that the $\boldsymbol{C}_0$ matrix is invertible, the (restricted) reduced form expression of the SpVAR model can be defined in the usual way, by setting

$$\mathbf{y}_t = \mathbf{C}_0^{-1}(\mathbf{C}_1\mathbf{y}_{t-1} + ... + \mathbf{C}_p\mathbf{y}_{t-p} + \boldsymbol{\varepsilon}_t) = \widetilde{\mathbf{C}}_1\mathbf{y}_{t-1} + ... + \widetilde{\mathbf{C}}_p\mathbf{y}_{t-p} + \widetilde{\boldsymbol{\varepsilon}}_t \tag{11}$$

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[3] Block-triangular VAR models are dealt with in Zha (1999) and Lastrapes (2005). In particular, the methodology set forth in the latter, specifically designed to deal with VAR model fitted to panel data involving large cross-sections, appears to be closely related to the approach adopted here, albeit from a non-spatial perspective.

with $\widetilde{\mathbf{C}}_h = \mathbf{C}_0^{-1}\mathbf{C}_h$, h=1,...,p, and $\widetilde{\boldsymbol{\varepsilon}}_t = \mathbf{C}_0^{-1}\boldsymbol{\varepsilon}_t$.

The reduced form can then be utilized, as in the standard VAR case, to compute forecasts on the basis of the conditional expectations given by

$$E[y_{t|t-1}] = \widetilde{\mathbf{C}}_1 y_{t-1} + \ldots + \widetilde{\mathbf{C}}_p y_{t-p} \tag{12}$$

or to derive the coefficients of the Moving Average representation of the model (see Section 4).

3.2 Identification

While a strictly triangular $\mathbf{C}_0$ matrix is known to provide an exactly identified system of equations, provided $\boldsymbol{\Omega}$ is diagonal, the block triangular structure given in expression (9) can result in both under-identified, just identified and over-identified systems.

Under-identification problems can arise when the number of spatial units is very small and the spatial order of the model is high, a situation that is expected to occur only in very specific empirical settings. To check for identifiability in such borderline cases one can rely on the usual order condition for identification via zero restrictions, requiring the number of constraints on off-diagonal terms in the error covariance matrix to be greater or equal to the number of free coefficients in the $\mathbf{C}_0$ matrix (e.g., Hamilton, 1994, p. 332).[4]

In the spatially heterogeneous SpVAR specification, the number of restrictions placed on the error covariance matrix is equal to $NK(NK\text{-}1)/2$, since all covariances are assumed to be nil while variances are unrestricted. Considering that the number of $\phi_{ikr}^{(0l)}$ coefficients to be estimated is equal to $NK(K\text{-}1)/2$ + $sNK(K+1)/2$, the order condition is satisfied if

$$NK(NK\text{-}1) \geq NK(K\text{-}1) + sNK(K+1) \tag{13}$$

which amounts to imposing the following minimum requirement on the number of spatial locations in the sample

$$N \geq s + (s+K)/K \tag{14}$$

that is decreasing with K, since autoregressive coefficients increase linearly with K while the number of restricted parameters is a quadratic function of K. For K=2 (a worst-case scenario, in a multivariate context) the order condition is thus satisfied, given that N has to be an integer, if $N \geq 3$ when s=1 and if $N \geq 4$ when S=2.

Considering that the number of locations in typical applications will exceed the minimum requirement, the SpVAR(p,s) model will actually turn out to be over-identified, as a consequence of having restricted spatial interaction coefficients (i.e. off-diagonal elements in the $\mathbf{A}_{kr}^{(h)}$ blocks (h=0,1,…,p; k,r=1,…,K) composing VAR coefficients matrices) to be proportional to the spatial weights, that are assumed to be known a priori.

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[4] The rank condition for local identification of the A-Model is given in Lütkepohl (2007), Proposition 9.1. However, it can only be checked assuming a specific value for the set of model parameters.

Identification in the block triangular structure given by (9) can be seen to rely on two types of restrictions:

1. standard constraints implied by the recursive ordering of the endogenous variables in an underlying, non-spatial, VAR model, as suggested by the usual theoretical and practical considerations;
2. restrictions on the spatial interactions coefficients, linking the dynamics of endogenous variables observed on different locations, derived from a priori assumptions about the spatial structure of the process and implemented by means of a given series of spatial weights matrices.

Compared to the strictly triangular hypothesis detailed in expression (5), this identification scheme has the feature that, when spatial weights matrices are not triangular, the usual condition of bilateral interactions in space is restored, while maintaining the causal ordering across the endogenous variables.

It has to be noted that, while in principle it could be possible to identify $\mathbf{C}_0$ by exploiting only the second type of restrictions, involving only spatial interactions, this approach is not pursued in this paper.

To illustrate this point, let us consider the expression of the SpVAR model ensuing when all spatial interactions coefficients are set to zero. In this case all $N\times N$ blocks inside the $\mathbf{C}_h$ matrices (h=0,1,…,p) become diagonal and, accordingly, the model expression simplifies to

$$\mathbf{M}_{0,i}\mathbf{y}_{it} = \alpha_i + \mathbf{M}_{1,i}\mathbf{y}_{i,t-1} + \ldots + \mathbf{M}_{p,i}\mathbf{y}_{i,t-p} + \boldsymbol{\varepsilon}_{it} \tag{15}$$

where

$$\mathbf{y}_{it} = [y_{i1t},\ldots,y_{iKt}]', \quad \boldsymbol{\varepsilon}_{it} = [\varepsilon_{i1t},\ldots,\varepsilon_{iKt}]'$$

$$E(\boldsymbol{\varepsilon}_{it}) = 0, \quad E(\boldsymbol{\varepsilon}_{it}\boldsymbol{\varepsilon}'_{it}) = \Omega_i = diag\{[\omega_{i1},\ldots,\omega_{iK}]\} \tag{16}$$
$$E(\boldsymbol{\varepsilon}_{it}\boldsymbol{\varepsilon}'_{i,t-h}) = 0, \; i=1,\ldots, N, \; h=1,2,\ldots$$

and where the $K\times K$ coefficients matrices have structure

$$\mathbf{M}_{0,i} = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ -\phi_{i21}^{(00)} & 1 & \ldots & 0 \\ \ldots. & \ldots & & \ldots. \\ -\phi_{iK1}^{(00)} & -\phi_{iK2}^{(00)} & \ldots & 1 \end{bmatrix} \tag{17}$$

$$\mathbf{M}_{h,i} = \begin{bmatrix} \phi_{i11}^{(h0)} & \phi_{i12}^{(h0)} & \ldots & \phi_{i1K}^{(h0)} \\ \phi_{i21}^{(h0)} & \phi_{i22}^{(h0)} & \ldots & \phi_{i2K}^{(h0)} \\ \ldots. & \ldots & & \ldots. \\ \phi_{iK1}^{(h0)} & \phi_{iK2}^{(h0)} & \ldots & \phi_{iKK}^{(h0)} \end{bmatrix} \tag{18}$$

$$h=1,\ldots, p$$

In what follows, expression (15) will be referred to as the *nested* VAR model, and details what is referred to in the literature as a *panel* VAR specification (e.g., Holtz-Eakin et al. 1988), expressed in structural recursive form.

It is immediately clear that imposing a block triangular structure on $\boldsymbol{C}_0$ automatically preserves the identification of the nested panel VAR even when the SpVAR is actually under-identified. This condition, in the case where the SpVAR is identified, allows for a joint test of the existence of spatial interaction effects in the process by means of a standard test of nested hypotheses, based on the comparison of the performances of the SpVAR and of the nested panel VAR.

Should the SpVAR be identified solely on the basis of restrictions on spatial interaction coefficients, the identification of the nested VAR would be lost, as the latter requires a sufficient number of constraints to be imposed on *local* interaction coefficients as well, i.e. to coefficients linking endogenous variables observed on the same location. This situation is intuitively unappealing and is ruled out in the chosen approach to SpVAR identification.

While the block recursive identification scheme is appealing from both a theoretical and practical point of view, economic theory can suggest others. Although these alternative schemes are not dealt with in this paper, their implementation in the SpVAR context appears to be feasible and is left for future investigation.

In empirical applications, in the presence of over-identifying restrictions, the sample covariance matrix of the structural errors will not be exactly diagonal, as is the case when the structural shocks are identified by means of a Choleski decomposition. This situation by no means prevents the utilization of the model for the usual purposes, provided over-identification restrictions are not rejected on the basis of available data.

However, while in the time series case the reduced form VAR provides a general reference against which over-identified SVAR specifications can be tested, in the case of the SpVAR the unrestricted reduced form, i.e. an ordinary VAR(p) model, usually cannot be estimated, because of the lack of degrees of freedom in standard applications.

Considering such difficulties in relying on standard over-ideintification tests, an alternative empirical approach can be based on the direct check of residual error covariances. The evidence of contemporaneous error correlation in the SpVAR model would in fact offer indirect evidence of a rejection of over-identification restrictions, implying that the model does not adequately portray the spatial interactions across the given set of variables.

While, in principle, single residual covariances can be individually tested for departure from the null hypothesis of a zero value, considering that there are $NK(NK\text{-}1)/2$ such tests to be inspected, a more sensible approach can be devised by relying on the spatial temporal auto and cross-correlation coefficients (Martin and Oeppen 1975, Pfeifer and Deutsch, 1980), evaluated at zero displacement in time.

These statistics, which provide a global assessment of the extent of residual spatial correlation, can subsequently be further complemented by local measures of statistical association, such as the ones proposed in Anselin (1995).

At the present stage it is assumed that, by proper selection of spatial weights matrices and model orders, the amount of residual contemporaneous spatial correlation can actually be made negligible in empirical applications, thus allowing for a structural interpretation of the error term in the model.

4 Maximum likelihood estimation

Parameter estimation in the case of the spatial VAR model can be addressed under two different perspectives. When data are available for panels with a large number of cross-sectional units and a small number of replications over time and VAR coefficients are constant across space, estimation can be carried out by one of the techniques, developed in the panel data literature, designed to yield estimators that are consistent as *N* increases (see, for instance, Baltagi 2005). This is the approach actually adopted in Beenstock and Felsenstein (2007).

However, in its more general form, the proposed SpVAR model has a number of free parameters that grows linearly with the size of the spatial cross-section. In this case, consistent parameter estimation can only be based on an increasing length of individual time series in the panel. At the same time, even when the spatial homogeneity of autoregressive coefficients is assumed, if the cross-sectional dimension of the panel is of moderate size (the maximum possible size being dictated only by computational requirements) and sufficiently long time series are available, inference on model parameters can still be fruitfully based on the methods usually employed in multiple time series analysis, such as maximum likelihood (ML).[5]

In particular, following a standard approach in the structural VAR literature, in this section consistent estimators of model parameters will be derived by applying the FIML method (Full Information Maximum Likelihood: Amisano and Giannini 1997; Lütkepohl 2007, Chapter 9). This method appears to be well suited to deal with a specification involving both a $\mathbf{C}_0$ matrix that is not strictly triangular and a set of parameter constraints on the $\mathbf{C}_h$ matrices (h>0) and the error covariance matrix Ω.

Provided enough restrictions are imposed so that the structural parameters are locally identified, FIML estimators have been proved to be asymptotically normal and unbiased (Lütkepohl 2007, Proposition 9.5). These results, while stated for the SVAR model with unrestricted dynamics, are shown to extend also to the case when a set of constraints is imposed on system coefficient matrices $\mathbf{C}_h$ for h=1,…,p, as is the case of the proposed SpVAR specification.

Under the assumption that $\mathbf{y}_t$ is jointly normal, the distribution of $\mathbf{y}_t$, conditional on past observations $\mathbf{y}_{t-1}$, $\mathbf{y}_{t-2}$,..., will be normal with mean $\hat{\mathbf{y}}_t = \tilde{\mathbf{C}}_1\mathbf{y}_{t-1} + ... + \tilde{\mathbf{C}}_p\mathbf{y}_{t-p}$ and covariance matrix $\tilde{\Omega} = E[\tilde{\boldsymbol{\varepsilon}}_t\tilde{\boldsymbol{\varepsilon}}_{t-h}'] = (\mathbf{C}_0^{-1})\Omega(\mathbf{C}_0^{-1})'$. The log of the conditional distribution will hence be expressed as

$$\log f(\mathbf{y}_t \mid \mathbf{y}_{t-1}, \mathbf{y}_{t-2}, ...) = c - \frac{1}{2}\log(|\tilde{\Omega}|) - \frac{1}{2}(\mathbf{y}_t - \hat{\mathbf{y}}_t)'\tilde{\Omega}^{-1}(\mathbf{y}_t - \hat{\mathbf{y}}_t) =$$

$$= c + \log(|\mathbf{C}_0|) - \frac{1}{2}\log(|\Omega|) - \frac{1}{2}\boldsymbol{\varepsilon}_t'\Omega^{-1}\boldsymbol{\varepsilon}_t \qquad (19)$$

[5] Asymptotic theory for large N and small T in the case of the ML estimation of a static panel spatial autoregressive model with fixed effects and constant coefficient across locations is dealt

where c is a constant and $\boldsymbol{\varepsilon}_t = \mathbf{C}_0\mathbf{y}_t - \boldsymbol{\alpha} - \mathbf{C}_1\mathbf{y}_{t-1} - \ldots - \mathbf{C}_p\mathbf{y}_{t-p}$.

Given the block triangular structure of $\mathbf{C}_0$ it follows that $\left|\mathbf{C}_0\right| = \prod_{r=1}^{K}\left|\mathbf{A}_{rr}^{0}\right|$ and the log of the sample distribution of y_t, conditional on p pre-sample values and assuming T consecutive observations are collected over time, will have the expression

$$\begin{aligned}
&\log f\left(\mathbf{y}_1, \mathbf{y}_2, \ldots, \mathbf{y}_T \mid \mathbf{y}_0, \mathbf{y}_{-1}, \ldots, \mathbf{y}_{-p+1}\right) = \\
&= c + T\log\left(\left|\mathbf{C}_0\right|\right) - \frac{T}{2}\log\left(\left|\Omega\right|\right) - \frac{1}{2}\sum_{t=1}^{T}\boldsymbol{\varepsilon}_t'\Omega^{-1}\boldsymbol{\varepsilon}_t = \\
&= c + T\sum_{r=1}^{K}\log\left(\left|\mathbf{A}_{rr}^{(0)}\right|\right) - \frac{T}{2}\log\left(\left|\Omega\right|\right) - \frac{1}{2}tr\left(\mathbf{E}\,\Omega^{-1}\mathbf{E}'\right)
\end{aligned} \tag{20}$$

with $\mathbf{E} = \left[\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2, \ldots, \boldsymbol{\varepsilon}_T\right]$.

Considered to be a function of the vector of model parameters for a given values of Y, equation (20) defines the conditional log-likelihood function for the SpVAR model parameters. From first order conditions it is immediately clear that the usual expression for the ML estimator of the error covariance matrix ensues

$$\hat{\omega}_{ik} = \frac{1}{T}\sum_{t=1}^{T}\varepsilon_{ikt}^{2}\ ,\ i=1,\ldots,N\ k=1,\ldots,K. \tag{21}$$

Substituting the ML estimators to corresponding parameters in Ω yields the following concentrated likelihood

$$\begin{aligned}
\tilde{L}_{\theta} &= \tilde{c} + T\sum_{r=1}^{K}\log\left(\left|\mathbf{A}_{rr}^{(0)}\right|\right) - \frac{T}{2}\log\left(\left|\hat{\Omega}\right|\right) = \\
&= \tilde{c} + T\sum_{k=1}^{K}\log\left(\left|\mathbf{A}_{rr}^{(0)}\right|\right) - \frac{T}{2}\sum_{k=1}^{K}\sum_{i=1}^{N}\log(\hat{\omega}_{ik}) = \\
&= \sum_{r=1}^{K}\tilde{L}_{\theta_r}
\end{aligned} \tag{22}$$

$$\tilde{L}_{\theta_k} = \tilde{c}_k + T\log\left(\left|\mathbf{A}_{rr}^{(0)}\right|\right) - \frac{T}{2}\sum_{i=1}^{N}\log(\hat{\omega}_{ik}) \tag{23}$$

Expression (23) above shows how the concentrated log-likelihood function, under the assumed block triangular structure for $\boldsymbol{C}_0$, is simply the sum of K unrelated terms, each pertaining to a single endogenous variable. As such, each component can be maximized independently from the others, thus reducing the overall computational burden.

It should be noted how the likelihood function includes a jacobian term involving the determinant of an NxN matrix that can make the optimization process computationally time-consuming or even unfeasible as the spatial sample size increases beyond a given level.

with in Lee and Yu (2008). However, an extension also involving dynamic interactions, that would be relevant for the case of the SpVAR model considered here, has still to be developed.

However, for the moderate spatial samples usually dealt with in empirical regional economic analysis, a FIML estimation of the SpVAR model should be generally feasible on current computers.

To conclude, all statistical estimation and inference is assumed to be carried out for fixed *N*. In this respect, the SpVAR specification, apart from a specific set of parameter constraints, shares the same features of an ordinary SVAR model and, as a consequence, standard asymptotic results for $T \to \infty$ (e.g., Hamilton 1994, Chapter 11) are assumed to apply directly to FIML estimators of the SpVAR parameters and related functions, like impulse response coefficients, under the usual assumptions regarding initial conditions and higher order moments of $\boldsymbol{\varepsilon}_t$.

5 The space-time impulse response function

On the basis of the reduced form expression given by (11), the SpVAR model can be stated in the following infinite order Moving Average (MA) form

$$\mathbf{y}_t = \boldsymbol{\mu} + \sum_{h=1}^{\infty} \widetilde{\Psi}_h \widetilde{\boldsymbol{\varepsilon}}_{t-h} \tag{24}$$

where the coefficients matrices can be obtained from the recursions

$$\widetilde{\Psi}_i = \sum_{h=1}^{i} \widetilde{\Psi}_{i-h} \widetilde{\mathbf{C}}_h \tag{25}$$

and are thus a function of the coefficients of the VAR model (Lütkepohl 2007, Section 3.7).

By setting $\Psi_h = \widetilde{\Psi}_h \mathbf{C}_0^{-1}$, (*h*=0,1,2,..), the MA representation can be expressed in terms of the vector of structural errors

$$\mathbf{y}_t = \boldsymbol{\mu} + \sum_{h=1}^{\infty} \widetilde{\Psi}_h \widetilde{\boldsymbol{\varepsilon}}_{t-h} = \boldsymbol{\mu} + \sum_{h=1}^{\infty} \Psi_h \boldsymbol{\varepsilon}_{t-h} \tag{26}$$

The *NK*×*NK* Ψ_h matrix has the following block structure

$$\Psi_h = \begin{bmatrix} \boldsymbol{\Psi}_{11}^{(h)} & \boldsymbol{\Psi}_{12}^{(h)} & \cdots & \boldsymbol{\Psi}_{1K}^{(h)} \\ \boldsymbol{\Psi}_{21}^{(h)} & \boldsymbol{\Psi}_{22}^{(h)} & \cdots & \boldsymbol{\Psi}_{2K}^{(h)} \\ \cdots\cdot & \cdots & & \cdots\cdot \\ \boldsymbol{\Psi}_{K1}^{(h)} & \boldsymbol{\Psi}_{K2}^{(h)} & \cdots & \boldsymbol{\Psi}_{KK}^{(h)} \end{bmatrix} \tag{27}$$

where each *N*×*N* block has elements

$$\psi_{kr}^{(h)}(i,j) = \frac{\partial y_{ikt+h}}{\partial \varepsilon_{jrt}}, \qquad k,r=1,\dots,K \qquad i,j=1,\dots,N \tag{28}$$

measuring the response of the *k*-th variable observed on location *i* at time *t*+*h* to a one-unit increase in the *r*-th structural error on location *j* and time *t*.

When the number of regions being analysed is larger than a few units - a situation that is likely to occur in most empirical applications - the direct inspection of the impact of a shock to a given variable on the remaining system variables for each couple of spatial locations in the sample and various time horizons rapidly becomes unwieldy.

At the same time, even when the number of regions is small, the researcher could be interested in assessing an overall measure of the strength of spatial spillover effects, especially if a spatially homogenous specification has been fitted to the data, in which case impulse responses should exhibit no spatial variation (apart from that induced by the spatial weighting scheme itself).

In the context of the univariate Space-Time ARMA model, Di Giacinto (2006) proposes a simple synthetic measure of shock responses by introducing the space-time impulse response (STIR) function. A straightforward extension to the context of the SpVAR methodology is set out in this section.

In particular, the average response at time horizon h and spatial lag l for shocks affecting location i, can be measured as

$$\eta_{kr}^{(hl)}(i) = \sum_{j=1}^{N} w_{ij}^{(l)} \psi_{kr}^{(h)}(j,i) \tag{29}$$

an expression that can be referred to as the local *outward* STIR function, and which provides an assessment of the average effect measured after h periods on the value of the k-th endogenous variable recorded on l-th order spatial neighbours of a unit shock to the the r-th variable on location i.

In the same line of reasoning, a second function, referred to as the local *inward* STIR function, assesses the average effect on the k-*th* variable on location i of a contemporaneous unit shock to the r-th variable recorded on its l-th order spatial neighbours

$$\zeta_{kr}^{(hl)}(i) = \sum_{j=1}^{N} w_{ij}^{(l)} \psi_{kr}^{(h)}(i,j) \tag{30}$$

Under spatial homogeneity, both types of synthetic impulse responses can be further summarized with no loss of information by averaging the local STIR function across space, yielding the following expressions

$$\overline{\eta}_{kr}^{(hl)} = N^{-1} \sum_{i=1}^{N} \eta_{kr}^{(h)}(i) \tag{31}$$

$$\overline{\zeta}_{kr}^{(hl)} = N^{-1} \sum_{i=1}^{N} \zeta_{kr}^{(h)}(i) \tag{32}$$

which can be referred to as the global *outward* and *inward* STIR functions.

In empirical applications, where coefficients matrices $\mathbf{C}_h$, (h=0,1,…p), are unknown and have to be estimated from the data, the impulse responses, being a function of autoregressive coefficients, will display sampling variability as well.

Estimated impulse responses, computed by replacing ML estimators for the unknown autoregressive coefficients in (25), have been proved to be asymptotically unbiased and normally distributed (Lütkepohl 1990). Asymptotic confidence intervals for sample estimates can thus be derived on the basis of this result, provided consistent estimators of the standard errors are available. While analytical expressions for the asymptotic standard errors are available for the VAR model, a direct transposition in the context of the SpVAR is complicated by the spatial over-identification restrictions denoting this model class with respect to the usual time series structural VAR models.

It is now quite common in the time series literature to derive confidence intervals for estimated impulse response by means of the bootstrap method, which is expected to provide better approximations in finite samples compared with standard asymptotic results.

The bootstrap approach can be straightforwardly applied to the SpVAR context by setting up a procedure like the one detailed in Benkwitz et al. (1999) on the basis of the VAR expression of the model given in (3), under positions (6) and (9).

The bootstrap procedure involves the following steps:

(1) Estimate the parameters of the SpVAR model by FIML.

(2) Generate bootstrap residuals $\boldsymbol{\varepsilon}_1^*,\ldots,\boldsymbol{\varepsilon}_T^*$ by randomly drawing a sample with replacement from the set of estimated and recentered structural residuals, $\{\hat{\boldsymbol{\varepsilon}}_1-\bar{\boldsymbol{\varepsilon}}_.,\ldots,\hat{\boldsymbol{\varepsilon}}_1-\bar{\boldsymbol{\varepsilon}}_.\}$, where $\hat{\boldsymbol{\varepsilon}}_t=\hat{\mathbf{C}}_0\mathbf{y}_t-\hat{\boldsymbol{\alpha}}-\hat{\mathbf{C}}_1\mathbf{y}_{t-1}-\ldots-\hat{\mathbf{C}}_p\mathbf{y}_{t-p}$, and $\bar{\boldsymbol{\varepsilon}}_.=T^{-1}\sum\hat{\boldsymbol{\varepsilon}}_t$.

(3) Set $\{\mathbf{y}_{1-p+1}^*,\ldots,\mathbf{y}_0^*\}=\{\mathbf{y}_{1-p+1},\ldots,\mathbf{y}_0\}$ and construct bootstrap time series recursively using the restricted reduced form expression given in (11), $\mathbf{y}_t^*=\hat{\mathbf{C}}_0^{-1}(\hat{\mathbf{C}}_1\mathbf{y}_{t-1}^*+\ldots+\hat{\mathbf{C}}_p\mathbf{y}_{t-1}^*+\boldsymbol{\varepsilon}_t^*)$, t=1,…,T.

(4) Re-estimate the parameters of SpVAR model from the generated data.

(5) Calculate a bootstrap version of the impulse responses based on the parameter estimates obtained in Stage (4).

Considering that the bootstrap involves the maximization of the concentrated likelihood for a large number of replications of the process, the computational costs can become quite high as the size of the spatial cross-section increases. However if the objective is restricted to the estimation of standard errors instead of the whole sampling distribution of the estimated STIR function, the number of replications required to obtain reliable results is much lower, thus making the bootstrap approach feasible in a large number of empirical settings.

6 An illustrative application

6.1 Model specification and estimation

In this section the first empirical application of the identified structural SpVAR model defined in Section 2 is undertaken. In particular, an exercise that could be referred to as a spatial Keynesian multiplier analysis is performed, mainly aiming at assessing the direct impact of a local shock to government consumption on output and its spillover effects on output in neighbouring areas.

Since the focus is on providing some insights on the methodology, rather than attempting a fully-fledged empirical analysis of the issue, a basic bivariate set-up is adopted for the purpose. Yearly figures for the logs of gross domestic product (Y) and Government consumption (G) for the 20 Italian NUTS 2 regions over the period 1970-2007 provide the dataset utilized in the analysis.

The first graphical inspection of the two series shows how both are clearly trending over time (Figs. 1a and 1b). To check for non-stationarity, unit roots tests were performed both at the individual regional level and considering the panel jointly, and the results are displayed in Table 1. Both individual ADF tests and the *t*-bar panel tests proposed by Im, Pesaran and Shin (2003) and Pesaran (2006), the latter allowing for cross-section dependence in the data,[6] fail to reject the null hypothesis that the series are integrated.

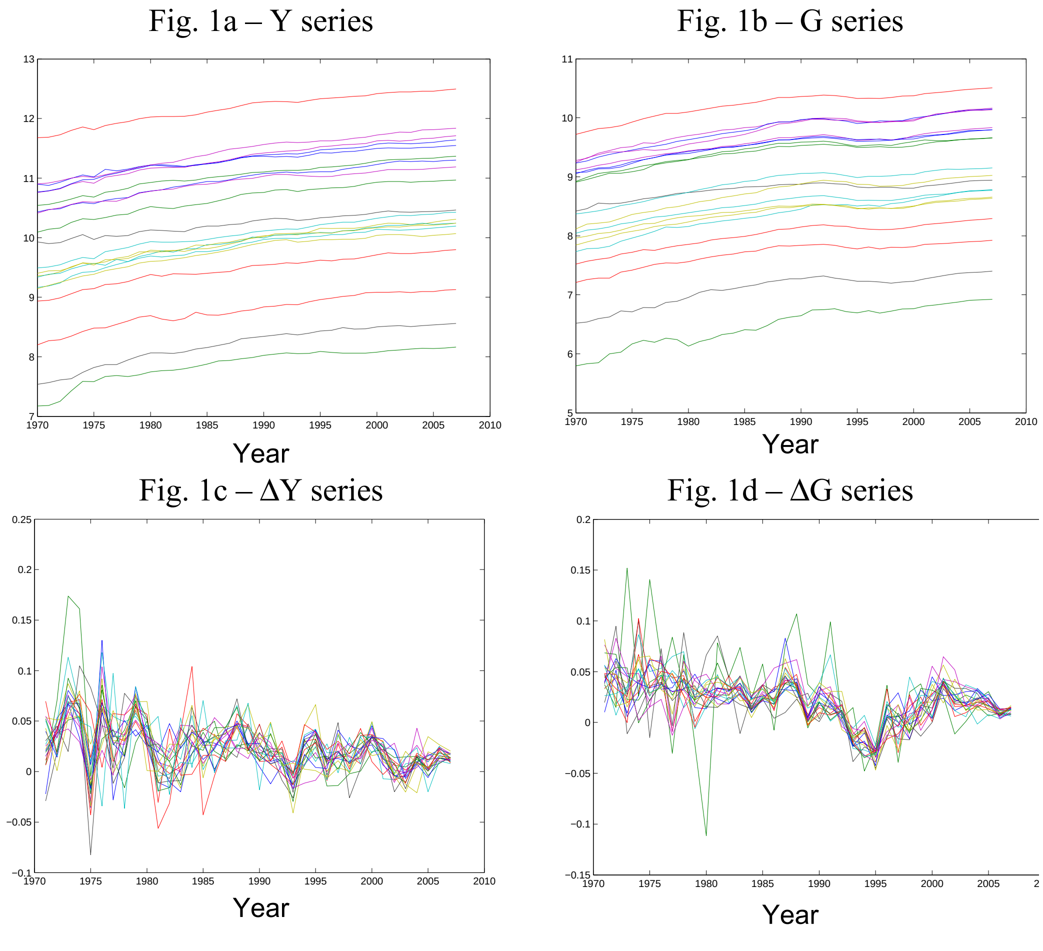

Fig. 1: Plot of the regional time series in levels and first differences
(Source: Prometeia regional database).

At the same time, the results of individual cointegrating ADF tests, a simple test procedure that performs well in the simple bivariate case considered here and

[6] The type of cross-section dependence considered in Pesaran's CADF approach derives from an underlying common factor hypothesis and, as such, is different from the case considered in Baltagi et al (2007), which analysed the performance of standard panel unit roots tests when dependence is due to spatial spillovers of local shocks.

the panel cointegration test recently developed by Westerlund (2007) appear to rule out the existence of stationary linear combinations of the two variables.

Table 1. Unit root and cointegration tests for the single Italian regions and the whole panel (1)

Regions	(Y)	(G)	(Y;G)
	ADF	ADF	Coint. ADF
Piedmont	-2.983	-3.140	-3.257
Valle d'Aosta	-2.976	-2.064	-2.751
Lombardy	-1.413	-2.718	-2.594
Trentino-Alto Adige	-1.882	-3.518	-2.421
Veneto	-1.472	-2.168	-1.816
Friuli-Venezia Giulia	-1.811	-2.817	-2.283
Liguria	-2.177	-2.631	-2.928
Emilia-Romagna	-2.687	-2.954	-2.924
Tuscany	-2.397	-2.105	-2.455
Umbria	-3.356 *	-2.494	-2.667
Marche	-2.693	-2.431	-2.495
Lazio	-0.684	-1.977	-1.253
Abruzzo	-1.136	-2.291	-1.043
Molise	-2.660	-2.242	-1.978
Campania	-0.596	-1.747	-1.418
Apulia	-0.711	-1.338	-2.304
Basilicata	-2.618	-3.241 *	-2.620
Calabria	-0.825	-2.132	-2.351
Sicily	-1.100	-1.489	-2.617
Sardinia	-1.545	-2.544	-2.925
Panel tests (2)			
	t-bar	*t*-bar	$P\tau$
Im, Pesaran and Shin	-2.344 (0.116)	-1.940 (0.832)	-
Pesaran	-2.523 (0.176)	-2.155 (0.826)	-
Westerlund (3)	-	-	-6.987 (0.920)

(1) All tests are carried out allowing for a linear trend in the data and lagged differences up to order 2. *, ** and *** denote statistical significance at the 10, 5 and 1 per cent level, respectively. (2) P-values in brackets. (3) P-value based on bootstrapped standard errors to allow for cross-section dependence.

Based on this evidence the SpVAR is subsequently specified on first-differenced data, whose plots are displayed in Figs. 1c and 1d.

An SpVAR specification is only motivated under the assumption that there is significant spatial dependence in the data. Consequently, as a further preliminary analysis, the space-time auto and cross-correlograms for the two indicators were computed and the results are displayed on Table 2. Estimates show positive and significant correlation among own and cross spatially lagged values of the two variables. In both cases spatial correlation appears to be highly persistent in space, showing only a small decrease when moving from 1st to 4th order neighbours.

Correlation with time-lagged values appears to be relatively smaller in size, although it is rather persistent, especially in the case of the public consumption indicator.

Table 2. Space-Time Auto and Cross-correlation Function for the Regional GDP and Government Consumption Series (*growth rates*)

Temporal lag	*Spatial lag* 0	1	2	3	4
			ΔG-ΔG		
0	1.000	0.658 ***	0.650 ***	0.641 ***	0.618 ***
1	0.349 ***	0.460 ***	0.453 ***	0.459 ***	0.450 ***
2	0.316 ***	0.382 ***	0.386 ***	0.384 ***	0.372 ***
3	0.259 ***	0.286 ***	0.288 ***	0.271 ***	0.287 ***
4	0.172 ***	0.199 ***	0.221 ***	0.201 ***	0.226 ***
5	0.058	0.126 ***	0.134 ***	0.125 ***	0.140 ***
			ΔY-ΔY		
0	1.000	0.654 ***	0.695 ***	0.625 ***	0.511 ***
1	0.097 **	0.128 ***	0.115 ***	0.170 ***	0.212 ***
2	0.068 *	0.063 **	0.017	0.077 ***	0.064 **
3	0.065 *	0.110 ***	0.097 ***	0.159 ***	0.152 ***
4	0.090 *	0.066 **	0.044 *	0.018	0.002
5	-0.009	0.054 *	0.047 **	0.084 ***	0.086 ***
			ΔG-ΔY		
0	0.225 ***	0.173 ***	0.196 ***	0.230 ***	0.235 ***
1	0.221 ***	0.229 ***	0.210 ***	0.256 ***	0.252 ***
2	0.316 ***	0.343 ***	0.329 ***	0.320 ***	0.320 ***
3	0.152 ***	0.206 ***	0.182 ***	0.191 ***	0.164 ***
4	0.121 ***	0.173 ***	0.156 ***	0.134 ***	0.144 ***
5	0.019	0.050 *	0.043 *	0.054 **	0.043 *
			ΔY-ΔG		
0	0.225 ***	0.179 ***	0.190 ***	0.217 ***	0.219 ***
1	0.224 ***	0.243 ***	0.230 ***	0.238 ***	0.227 ***
2	0.175 ***	0.177 ***	0.188 ***	0.179 ***	0.199 ***
3	0.123 **	0.180 ***	0.148 ***	0.178 ***	0.169 ***
4	0.095 *	0.096 ***	0.148 ***	0.142 ***	0.128 ***
5	0.021	0.089 ***	0.090 ***	0.106 ***	0.077 ***

*, ** and *** denote statistical significance at the 10, 5 and 1 per cent level, respectively.

Apart from a true spatial spillover mechanism propagating local disturbances, a highly persistent spatial correlation pattern is consistent with the existence of common macro shocks driving the local dynamics of the series. To identify the local component of the series in this case a straightforward empirical strategy could be based on a two stage procedure, where the common component is filtered out in a preliminary stage (i.e. by centering on cross-sectional means) and then the SpVAR specification and estimation is carried out on adjusted data. However if this procedure is not carefully designed, it is likely to remove or adversely affect the informative part of the observed spatial correlation pattern as well, i.e. the part that is induced by the spillover mechanism.

In finite samples the effect of common disturbances may actually turn out to be impossible to differentiate from the effect of highly spatially persistent local shocks and, as such, it may prove difficult to remove this effect without setting up a complex statistical model that explicitly allows for the identification and estimation of both the common and local components in the data. Considering that the development of such a model is clearly outside the scope of the present contribution, and given the illustrative nature of this empirical exercise, the application was run on unfiltered data. Nonetheless, to make some allowance for the impact of common shocks on SpVAR estimates, individual model equations were augmented by introducing the lagged values of the General Government deficit and of aggregate GDP growth.[7] At the same time, dummy variables for the years 1993-1995 were also included, since the aggregate figures for the ΔG series displayed unusually low values in this period, in conjunction with the process of fiscal adjustment that the country was undergoing prior to joining the EMU.

The next modeling steps involve the choice of the spatial weighting scheme and the selection of lag orders in the SpVAR specification.

Two alternative definitions of spatial weights, selected among the most common choices in the spatial econometric literature, were considered. The first scheme is based on binary weights, first order contiguity being defined by the existence of a common border between two regions and with higher order contiguity derived in the usual fashion. In the second scheme weights are taken to be equal to the inverse of the distance between the regions, measured by the simple average of the distance between the main municipalities located in the two areas. In this case higher order spatial lags are not defined, since all possible spatial interactions are already allowed for in a single weights matrix. All spatial weights matrices are subsequently row-normalized.

Apart from the definition of the spatial weights, the specification of an identified SpVAR model requires the choice of a recursive ordering of the variables. In this case only two sequences are possible and, considering that public expenditure is largely predetermined during a given year both because it depends on budget decisions taken in the previous year and because most of the expenditure (compensation of employees) is highly rigid in the short run, the G series was ordered first. This allows shocks to G to affect local output in the current year, while the effect of local shocks on GDP are allowed to affect local public spending with at least a one-year delay.

Table 3. Information criteria for alternative SpVAR(*p*,*s*) specifications (1)

Model order	*LOGL*	*AIC*	*BIC*
	Spatial weights: contiguity		
p=1 ; S=1	4,790.5	-9,541.0	-9,450.0
p=1 ; S=2	4,858.8	-9,663.6	-9,540.8
p=2 ; S=1	4,808.5	-9,561.0	-9,433.6
p=2 ; S=2	4,878.0	-9,677.9	-9,500.4
	Spatial weights: inverse distance		
p=1 ; S=1	4,872.6	-9,705.3	-9,614.2
p=2 ; S=1	4,884.1	-9,712.2	-9,584.8

(1) All specifications were estimated on the same sample, including observations from 1972 to 2007.

[7] Lagged values were utilized to prevent the obvious simultaneity problem that would arise if current values of the indicators were included. This choice can be further motivated if common shocks to local public expenditure reflect decisions taken at the central level on the basis of past aggregate budget conditions, rather than current ones.

Based on this identifying assumption, and for given spatial weights and lag orders, the model can be estimated by the FIML method.

A spatially homogeneous specification was considered first, a reasonable choice when the research aims mainly at assessing the overall dynamics of spatial linkages in a given area. The temporal and spatial orders of the model were then selected by estimating a number of relevant alternatives and subsequently evaluating the results of the standard information criteria. In this case AIC and BIC provide different indications. Based on BIC, that has better large sample properties, an SpVAR(1,2) appears to be the preferred choice when spatial weights are based on contiguity, while an SpVAR(1,1) is identified with distance based weights (Table 3).

Table 4. FIML estimation results for the spatially homogeneous SpVAR(1,1) specification (*p-values in brackets*)

Variables and statistics	ΔG equation				ΔY equation			
	Coefficient		Coefficient		Coefficient		Coefficient	
Spatial-temporal lags:								
ΔG_t	–		–		0.1628	(0.000)	0.1638	(0.000)
$L\Delta G_t$	0.6026	(0.000)	0.6049	(0.000)	-0.1556	(0.038)	-0.1659	(0.025)
$L\Delta Y_t$	–		–		0.7475	(0.000)	0.7487	(0.000)
ΔG_{t-1}	-0.0621	(0.097)	-0.0620	(0.097)	0.0840	(0.038)	0.0872	(0.031)
$L\Delta G_{t-1}$	0.2322	(0.000)	0.2369	(0.000)	0.0169	(0.794)	0.0246	(0.702)
ΔY_{t-1}	0.0924	(0.008)	0.0918	(0.008)	-0.0750	(0.046)	-0.0720	(0.057)
$L\Delta Y_{t-1}$	-0.0258	(0.832)	-0.0567	(0.258)	-0.0192	(0.885)	0.1078	(0.048)
Constant	0.0044	(0.002)	0.0048	(0.001)	0.0012	(0.548)	0.0024	(0.156)
Dummy year 1993	-0.0131	(0.003)	-0.0128	(0.003)	-0.0055	(0.276)	-0.0062	(0.210)
Dummy year 1994	-0.0091	(0.049)	-0.0086	(0.057)	0.0038	(0.466)	0.0043	(0.394)
Dummy year 1995	-0.0157	(0.001)	-0.0153	(0.001)	0.0046	(0.411)	0.0046	(0.394)
National ΔY_{t-1}	-0.0087	(0.655)	–		-0.0187	(0.372	–	
National $PUBDEF_{t-1}$	-0.0347	(0.772)	–		0.1332	(0.305)	–	
R-squared (1)	0.5015		0.5015		0.4950		0.4938	
Error variance (ω)	0.0003		0.0003		0.0004		0.0004	
AIC	-5,048		-5,052		-4,914		-4,916	
BIC	-4,998		-5,011		-4,855		-4,866	
Observations	720		720		720		720	

The prefix *L* refers to the spatial lag operator of order 1, as defined, e.g., in Pfeifer and Deutsch (1980). (1) Provides only an approximate measure of goodness-of-fit, since standard variance decomposition does not apply.

An inspection of the space-time correlogram of model residuals showed how the specification utilizing weights based on the inverse distance did better in tracking the highly persistent spatial autocorrelation in the data and, consequently, the specification employing these spatial weights turned out to be the preferred choice.[8]

FIML estimates of the final SpVAR specification are detailed in Table 4 separately for the two variables. Initially, the two macro control variables were included, but these were subsequently dropped as neither are statistically significant and information criteria improve when the two indicators are removed. A high value for the coefficient of the simultaneous own spatial lag is estimated for both G and Y, but other spatial interactions coefficients are also significant.

Residual space-time correlation, displayed in Table 5, shows how the model is able to capture most of the correlation displayed by the observed series.

Table 5. Space-Time residual auto and cross-correlation function for the spatially homogeneous SpVAR(1,1) specification (spatial weights based on inverse distance)

Temporal lag	*Spatial lag*				
	0	1	2	3	4
			ΔG-ΔG		
0	1.000	-0.061 **	-0.052 **	0.036 *	0.002
1	-0.012	-0.011	-0.097 ***	0.002	0.002
2	0.021	0.009	0.006 *	0.023	0.014
3	0.078 *	0.012	-0.024	-0.029	0.031
4	0.011	-0.039 *	0.009 *	-0.045 *	0.031
5	-0.072 *	0.035	-0.010	0.018	0.050 *
			ΔY-ΔY		
0	1.000	-0.012	0.161 ***	0.072 ***	-0.021
1	0.001	-0.010	-0.090 ***	0.001	0.036
2	0.026	0.037	-0.086 ***	0.010	-0.017
3	-0.057	-0.009	-0.068 **	0.046 *	0.041 *
4	0.074 *	0.013	-0.013	-0.027	-0.012
5	-0.065 *	0.024	-0.036	0.015	0.006
			ΔG-ΔY		
0	0.000	-0.051 *	-0.009	-0.005	0.034
1	0.017	0.002	-0.103 ***	0.041 *	0.025
2	0.123 ***	0.094 ***	0.052 **	-0.013	0.047 *
3	-0.020	0.050 *	-0.001	0.042 *	0.000
4	0.027	0.058 *	0.033 *	-0.016	-0.034
5	-0.036	0.035	0.000	0.018	-0.011
			ΔY-ΔG		
0	0.000	-0.037	0.011	0.018	0.023
1	0.004	0.018	0.029 *	0.009	0.016
2	0.029	-0.039 *	-0.009	-0.060 **	0.012
3	-0.047	0.096 ***	-0.039 *	0.037 *	0.016
4	-0.001	-0.052 *	0.085 ***	0.014	0.013
5	-0.102 **	0.030	0.029 *	0.032	-0.040 *

*, ** and *** denote statistical significance at the 10, 5 and 1 per cent level, respectively.

Spatial auto and cross-correlations between contemporaneous residual values, apart from a significant but rather small value at spatial lag=2 for the residual autocorrelation in the ΔY equation, are mostly negligible, providing some broad evidence in favour of the validity of the over-identifying restriction imposed in the

[8] To save space, only results for the specification involving distance-based weights are reported in Table 5. Residual correlations for the specification utilizing contiguity-based weights are available from the author upon request.

considered structural SpVAR specification.[9] At this stage, to uncover possible local differences in the features of the process, a spatially heterogeneous specification was finally considered, by letting all model parameters in the SpVAR(1,1) model vary unrestrictedly at the level of the individual regions.

Estimation results are given in Table 6, where basic summary statistics provide an assessment of the regional variation in model parameters. The average values of autoregressive coefficients are in line with the estimates obtained assuming constant parameters across space, but the range of variation appears to be wide, thus providing evidence of extensive heterogeneity in the underlying spatial process. The model fit increases considerably in comparison with results obtained with the spatially homogeneous specification. Considering that estimated parameters increase by a factor of 20, information criteria that penalize model fit on the basis of the number of unrestricted parameters provide a better way to assess model performance. In this case there is no clear consensus - while the AIC provides evidence in favour of the heterogeneous SpVAR specification, the more restrictive BIC is optimized under the spatial homogeneity assumption.

Table 6. FIML estimation results for the spatially heterogeneous SpVAR(1,1) specification (*p-values in brackets*)

Variables and statistics	ΔG equation				ΔY equation			
	Coefficient estimates				Coefficient estimates			
	Mean	Stand. Dev.	Min	Max	Mean	Stand. Dev.	Min	Max
ΔG_t	–	–	–	–	0.1037	0.2317	-0.3709	0.6393
$L\Delta G_t$	0.6349	0.355	-0.7029	0.9329	-0.0562	0.3511	-0.7201	0.6234
$L\Delta Y_t$	–	–	–	–	0.7841	0.3645	0.0637	1.3248
ΔG_{t-1}	-0.0562	0.1701	-0.3808	0.2013	0.0219	0.2383	-0.3648	0.4147
$L\Delta G_{t-1}$	0.2146	0.3764	-0.1866	1.5142	0.0337	0.2815	-0.4476	0.574
ΔY_{t-1}	0.0433	0.2821	-0.304	0.9993	-0.0385	0.2605	-0.6975	0.5645
$L\Delta Y_{t-1}$	-0.0005	0.4206	-1.5043	0.5051	0.0628	0.3531	-0.3334	1.152
Constant	0.0037	0.0076	-0.0036	0.0300	0.0020	0.0053	-0.0082	0.0127
Error variance (ω_i)	0.0002	0.0004	0.0001	0.0017	0.0002	0.0002	0.0001	0.0007
R-squared (1)	0.6147				0.6864			
AIC	-5,327				-5,074			
BIC	-4,764				-4,327			
Observations	720				720			

Regressors also include time dummies for the years 1993-95. The prefix *L* refers to the spatial lag operator of order 1, as defined in Pfeifer and Deutsch (1980). (1) Provides only an approximate measure of goodness-of-fit, since standard variance decomposition does not apply.

[9] A further improvement in model fit could have been achieved by allowing for different spatial weights by equation and variable, as allowed by the general SpVAR specification given in Section 2, but this further refinement was not pursued considering the limited scope of the present application.

The residual correlograms, displayed in Table 7, show no further improvement with respect to the SpVAR model with constant parameters across space, a feature that is not surprising since space-time correlation coefficients provide global indicators, possibly masking locally different correlation patterns that are captured by the SpVAR model when coefficients are allowed to vary across locations.

Table 7. Space-Time residual auto and cross-correlation function for the spatially heterogeneous SpVAR(1,1) specification

Temporal lag	*Spatial lag*				
	0	1	2	3	4
			ΔG-ΔG		
0	1,000	-0,112***	-0,090***	0,019	-0,013
1	0,012	0,024	-0,082***	-0,006	-0,031
2	-0,041	0,010	0,017	0,039*	0,012
3	0,006	0,018	0,024	-0,025	0,01
4	-0,116	0,001	0,021	-0,041*	0,053*
5	-0,079	0,015	0,046*	-0,010	0,077***
			ΔY-ΔY		
0	1,000	-0,066**	0,197***	-0,029	0,004
1	0,043	-0,010	0,045*	0,020	-0,002
2	-0,143***	0,036	-0,118***	0,016	-0,018
3	-0,069*	0,018	-0,053**	0,005	0,013
4	-0,014	0,053*	0,035*	-0,047*	-0,028
5	-0,028	0,059**	0,037*	0,061**	-0,001
			ΔG-ΔY		
0	-0,002	-0,066**	-0,016	0,010	-0,025
1	-0,028	-0,043*	-0,044*	0,022	0,038*
2	0,025	0,058*	0,053**	-0,039*	0,038
3	-0,028	0,088***	0,032*	0,023	0,009
4	-0,005	0,044*	0,047*	-0,009	-0,013
5	-0,005	-0,014	0,003	-0,021	-0,041*
			ΔY-ΔG		
0	-0,002	-0,035	0,014	0,006	-0,034
1	0,015	-0,035	0,053**	-0,015	-0,006
2	0,040	-0,006	0,044*	-0,005	0,021
3	-0,035	0,048*	-0,053**	0,003	-0,031
4	-0,031	0,017	0,058**	0,045*	0,012
5	-0,010	0,011	0,028	0,022	-0,081***

*, ** and *** denote statistical significance at the 10, 5 and 1 per cent level, respectively.

6.2 Structural impulse response analysis

Given the complex feedback effects allowed for by the SpVAR model, dynamic causal effects within the system are better evaluated by inspecting the impulse response functions rather then model coefficients.

Based on the global *outward* STIR definition,[10] accumulated structural responses to a one-shot increase of local public consumption and output growth rates are displayed in Figs. 2a-2d, together with 95 per cent (two standard errors) asymptotic confidence bands.

[10] Under spatial homogeneity, the outward and inward STIR definitions can be shown to yield the same results on regular spatial lattices (apart from border distortions). This feature holds approximately for irregular spatial configurations.

The main interest in this simplified empirical application lies in the evaluation of the impact of local public expenditure in the own area and on remaining regions. The first plot in Fig. 2b shows how the effect of a 1 per cent increase in public consumption on local GDP is positive, as expected,[11] and equal to about 0.15 in the first year.

The effect tends to accumulate over the following years, attaining a long-run elasticity of about 1/3. All these effects are statistically significant at the 5 per cent level.

While in the current year a shock to G has no impact on output in neighbouring regions, the effect tends to cumulate over time up to a value of about 0.13 at spatial lag 1 (i.e on 1st order neighbours), showing only a mild tendency to decline as spatial lag increases.[12] Also in this case the effects, although not very pronounced, are statistically significant.

A structural shock to local GDP growth has, by assumption, no contemporaneous effect on public consumption. Positive feedback effects are found, however, with an elasticity of about 0.12 in the long run, showing how public consumption is not strictly exogenous in the regional sample analysed (Fig. 2d). Public consumption in neighbouring regions is affected by output disturbances only to a minor extent, the positive impact being only statistically significant in the long run.

The empirical STIR estimates also allow us to evaluate the extent of the spatial spillover effects of structural shocks to G and Y on the level of the same variables.

As depicted in Fig. 2c, a positive shock to local GDP is estimated to induce positive spillover effects, the long-run elasticity being equal to about 0.2 at spatial lag 1 and declining slowly as the spatial lag increases. This evidence is consistent with the existence of robust trade linkages across Italian regions, propagating local shocks to income.

An idiosyncratic 1 per cent increase of public consumption expenditure is estimated to affect the level of the same variable in neighbouring regions with a positive and significant impact (Fig. 1a). The cumulative long run effect is more than double (0.24 per cent) and also in this case the effects appear quite persistent in space and statistically significant.

To provide an economic interpretation of this evidence, it should be noted that, in the VAR approach, impulse responses reflect the combination of both direct and indirect effects and, while a direct effect of local public spending can be motivated on the basis of some type of coordinated action across local public authorities, the observed spillover effects are likely to derive mostly from indirect effects, in the long run in any case. A positive shock to local public consumption has been found to increase GDP in neighbouring areas, which can lead to an increase of fiscal revenues eventually fostering public expenditure by releasing the budget constraint.

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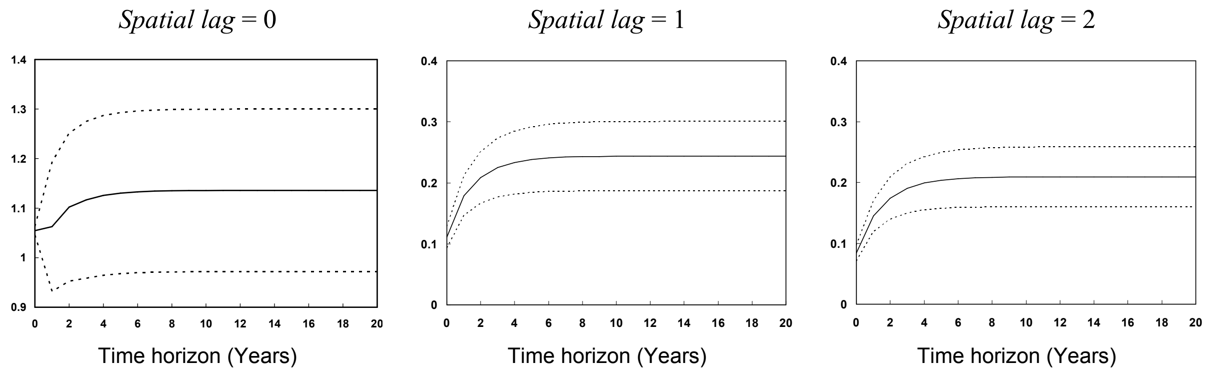
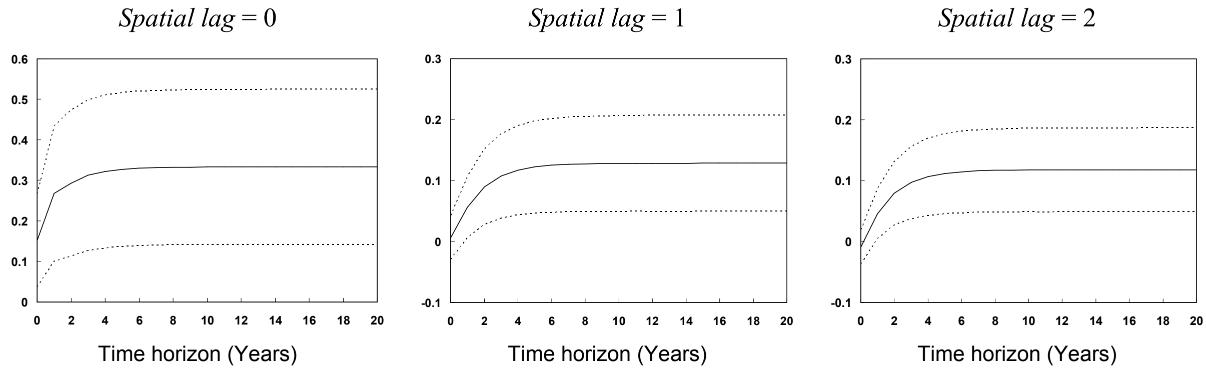
[11] The effect is at least partly mechanical, as the GDP figures utilized in the analysis include value added from public entities, that is known to be largely imputed to public sector consumption itself. Spillover effects across time and space, however, are not a simple statistical artefact and are worth analysing also in this highly simplified set-up.

[12] The slow spatial decline of estimated impulse responses is a model feature related to the inverse distance-based spatial weighting scheme, that allows for long-range spatial interactions.

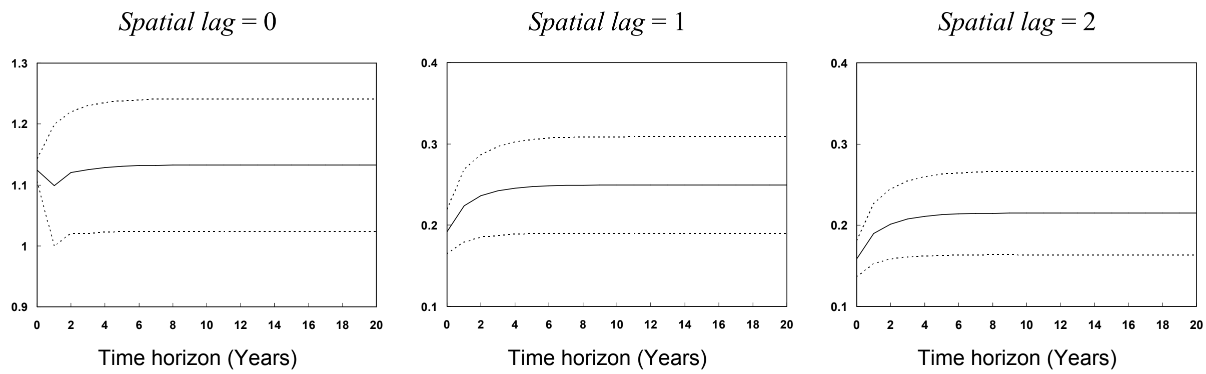
(a) **Response of G to a unit shock to G**



(b) **Response of Y to a unit shock to G**



(c) **Response of Y to a unit shock to Y**



(d) **Response of G to a unit shock to Y**

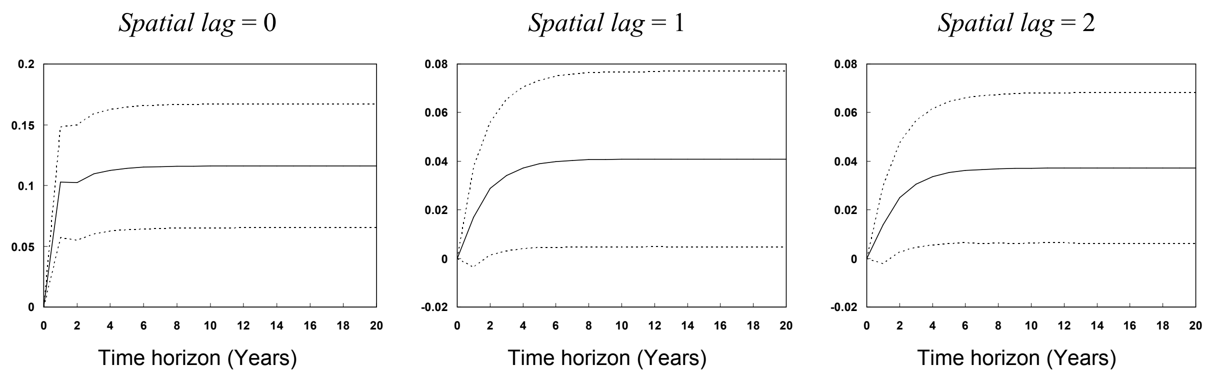


Fig 2: Estimated impulse responses (The graphs refer to the *outward* STIR definition. Dotted lines represent ±2 standard errors confidence bands, using bootstrap estimates of the standard errors obtained by resampling 100 replications of the process).

Finally, to offer a term of comparison, STIR functions were also computed on the basis of the estimated spatially heterogeneous specification. Focusing on long-run spillover effects, Fig. 3 portrays the regional cross-section of the responses of the two endogenous variables to identified structural shocks measured at spatial lag equal to one.[13] Both the outward and inward STIR functions are plotted, as these no longer provide the same information under spatial heterogeneity.

The analysis, apart from the presence of a few outliers, uncovers some interesting spatial patterns. In particular, the spatial transmission to GDP of local shocks to public consumption appears to be broadly increasing along the North-South direction, while spatial spillover effects of local shocks to GDP show an opposite, and more precise, trend. Considering that more developed regions in Italy are located in the Centre-North area, this pattern of responses appears to show how the more industrialized northern regions benefit more from positive output shocks originating from the private sector, while in the southern regions spillover effects from public expenditure are relatively more important.

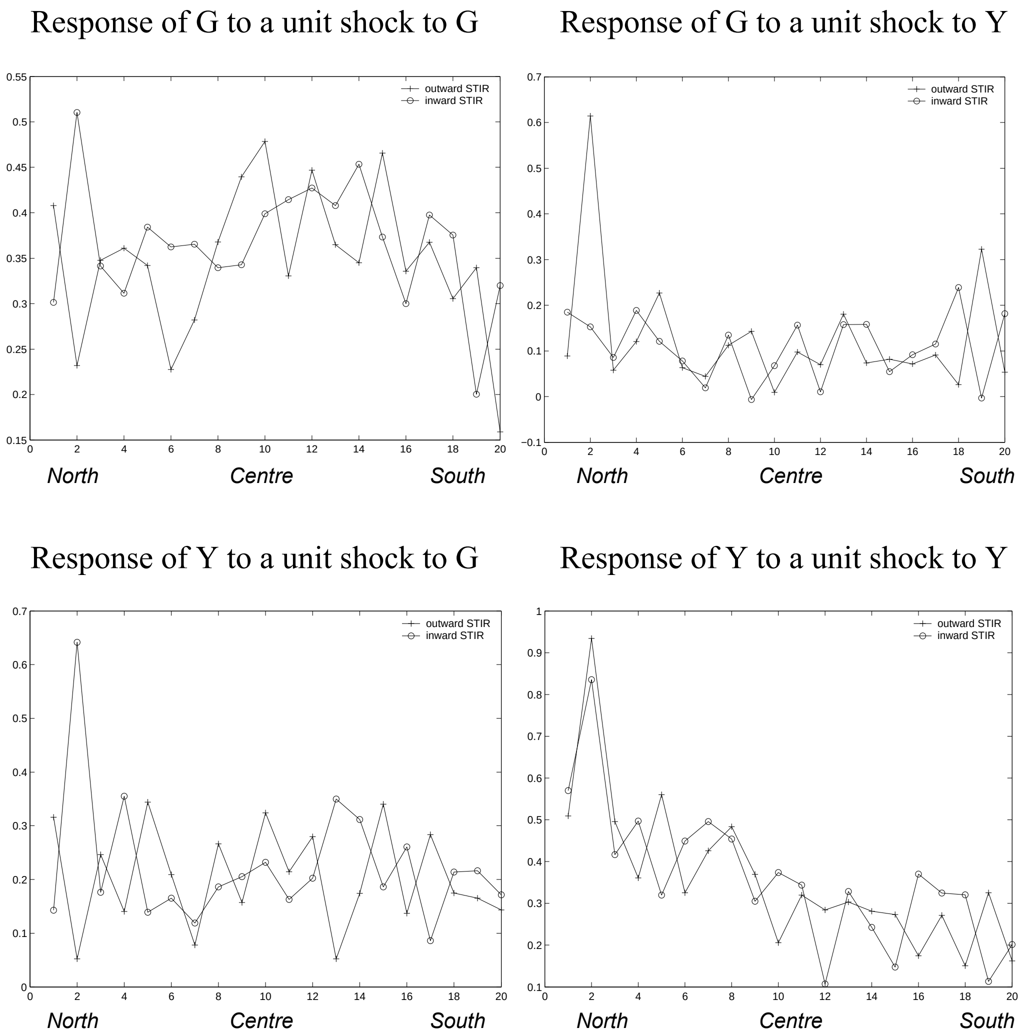
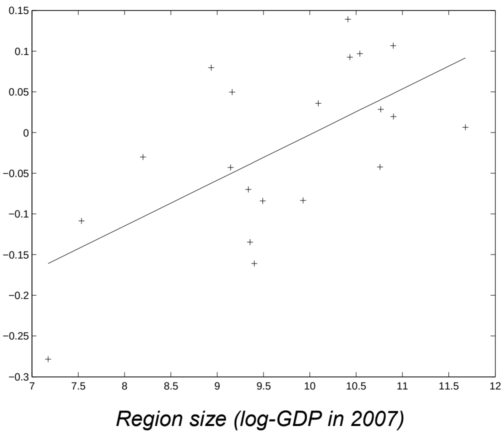


Fig. 3: Estimated long-run impulse responses at spatial lag 1 for the individual regions: spatially heterogeneous SpVAR specification (Long-run responses are defined as the accumulated responses at a time horizon of 20 years).

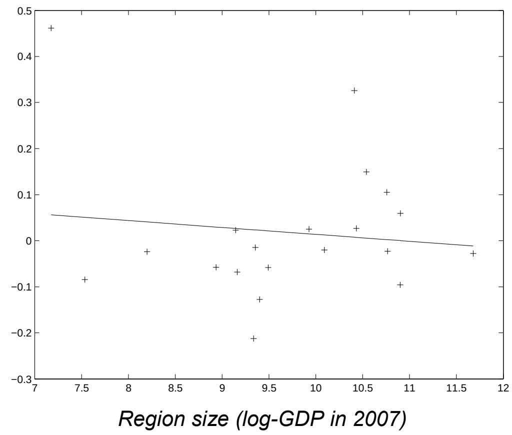
[13] Impulse responses at higher spatial lags broadly share the same features, displaying the same slow spatial decay found in the case of the SpVAR with constant coefficients across space.

Upon closer inspection, outward and inward responses appear to differ in a rather systematic fashion. In particular, as displayed in Fig. 4, the difference between the two STIR coefficients is positively related (with the exception of the response of G to Y) to the size of the region measured by GDP in the initial sample period. There is, in other terms, a tendency of the outward STIR to exceed the corresponding inward STIR value as the size of the region increases, providing some initial evidence that the largest and more densely populated Italian regions could have a greater influence as local sources of economic disturbances than they have as collectors of impulses originating in contiguous areas.

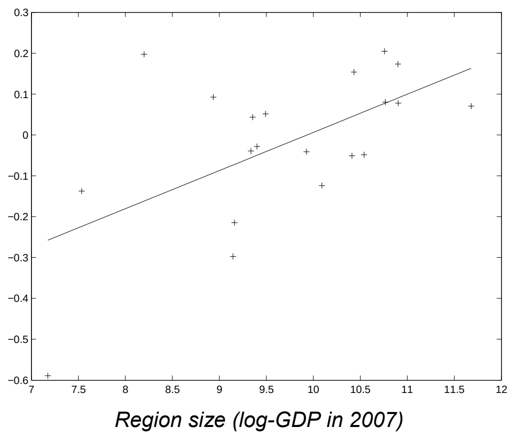
Response of G to a unit shock to G



Response of G to a unit shock to Y



Response of Y to a unit shock to G



Response of Y to a unit shock to Y

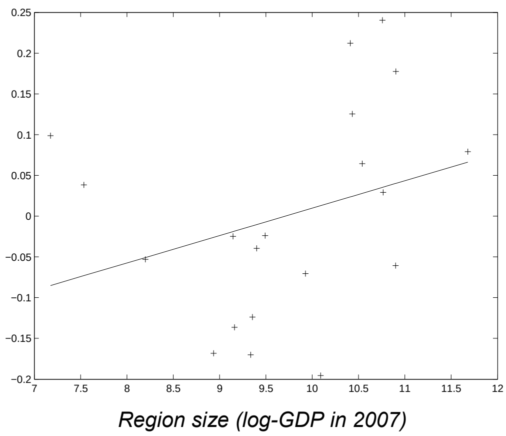


Fig. 4: Outward–Inward STIR differentials and region size (Long run impulse responses at spatial lag 1. The solid line represents the OLS regression line).

7 Final remarks

In this paper the issue of parameter identification and estimation is discussed for the SpVAR class of multivariate space-time models that have recently been introduced in the literature.

A structural specification of the model was introduced by assuming a block-triangular structure for the matrix of simultaneous interactions, which provides an adaptation in a spatio-temporal environment of the recursive identification scheme utilized in a large number of empirical VAR analyses. Identification of the structural SpVAR model thus specified was then discussed, showing how the order condition imposes only mild requirements on the minimal size of the spatial cross-section. For spatial samples of the usual size, the model will imply a set of over-identifying restrictions, whose validity has to be checked in empirical applications, although recourse to standard over-identification tests is mostly ruled out by data limitations.

Parameter estimation was then dealt with by implementing the FIML approach, a standard reference in the time series SVAR literature, yielding consistent and asymptotically normal estimators as the length of the time series in the panel diverges. The estimation procedure is shown to simplify considerably under the proposed recursive causal scheme, since in this case it can be carried out equation by equation. However, iterative optimization routines are still required when the model includes simultaneous spatial lags of the endogenous variables, a standard feature of ML estimators in spatial econometrics.

Having defined the STIR function as a convenient way to organize individual impulse response coefficients in a spatial environment, the final part of the paper has been devoted to an empirical application of the proposed SpVAR modeling approach. A basic bivariate fiscal model is fitted to data for the 20 Italian NUTS 2 regions, aiming at providing an illustration of main model features in an applied context.

Positive and sizeable spatial spillover effects were found for identified shocks to both public expenditure and GDP. In all cases dynamic responses tend to cumulate over time and appear to be highly persistent in space. Asymptotic confidence bands for the estimated impulse responses were also computed based on bootstrap estimates of the standard errors, showing how the effects are generally statistically significant.

Some interesting regional patterns in the spatial propagation of local disturbances are finally uncovered upon fitting an SpVAR specification allowing for different parameters across regions.

Overall, the SpVAR methodology appears to be quite promising for a range of potentially interesting empirical analyses, although some effort appears to required in properly selecting the specification that best fits the observed data.

In conclusion, it should be noted that the estimated spatial spillover effects only identify a true spatial propagation mechanism under the additional assumption that no common macroeconomic disturbances affect the observed set of local economies or, more realistically, that the influence of common shocks is properly controlled for in the empirical specification, e.g. by augmenting the SpVAR model with a set of exogenous macroeconomic indicators.

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