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NETWORKS WITH DECREASING RETURNS TO LINKING

by Filippo Vergara Caffarelli*

Abstract

This paper presents a model of non-cooperative network formation in which the marginal benefit of new links eventually decreases. Agents link with each other to gain information and update their links according to better-reply dynamics. In the long run the system settles to a unique network architecture that consists of a constellation of starred wheels. This is reminiscent of some real-world features. Collections of smaller disjoint networks connecting only a few agents are more common than global networks connecting all the agents in a community. Differences within a connected component such as the centre and the periphery are established.

JEL Classification: C72, D83, D85.
Keywords: networks, coordination, learning dynamics, non-cooperative games.

Contents

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1 Introduction[1]

A network formation game is presented in this paper. This is a non-cooperative game in which agents individually decide whether or not to build links with other agents. Economic investigation into network formation and interaction between networked agents can be divided into two branches. The first takes the network structure as exogenous and studies the interaction of linked agents given the network. Local interaction and peer pressure are examples of agent interaction within given networks. This research agenda on "static networks" was developed both theoretically and empirically.[2] The second branch of investigation focuses on changing networks and adopts a game-theoretic approach to analyse the dynamic process leading to the formation of the actual network. Goyal [23], D'Ignazio and Giovannetti [14], Jackson [35] and [36] and Vega Redondo [59] provide clear and up-to-date overviews of this literature.

Game-theorists initially focused on cooperative network formation.[3] The cooperative feature is the fact that if one agent wants to link to another then the former needs the agreement of the latter, i.e. the two must cooperatively agree on being linked. This literature comprises Dutta and co-authors ([15], [16]), Jackson and co-authors ([2], [37] and [38]), Kannan and Sarangi [39], Slikker and co-authors ([53], [54] and [55]), Tercieux and Vannetelbosch [57] and Watts [61] among others. The cooperative approach is helpful in many contexts in that it is not a limitation to assume that the agent who receives a request to be linked may veto it.

[1] This paper is based on the first chapter of my Ph.D. dissertation at the European University Institute. A preliminary version circulated as [60]. It benefited from helpful comments by Karl Schlag, Steffen Huck, Antonella Ianni and seminar participants at GAMES 2004 (Marseilles), University of Alicante and Bank of Italy. The views in this paper are those of the author and do not necessarily reflect those of the Bank of Italy. The usual disclaimer applies. E-mail: `filippo.vergaracaffarelli@bancaditalia.it`.

[2] Within this literature see for example Calvò-Armengol and Jackson [7], Ellison [18] and Tesfatsion [58] which are theoretical papers and Bertrand *et al.* [5], Case and Katz [8], Ichino and Maggi [33] and Rauch and Casella [51] which are empirical works.

[3] See Myerson ([46], [47] p. 448) and Qin [50].

The stream of literature to which this paper belongs began with Bala and Goyal [1] (henceforth BG) who focus on the importance of non-cooperative incentives for network formation. BG innovate the literature by modelling self-interested boundedly-rational agents who can unilaterally decide whether to build or sever a link. Their predictions depend on the relative cost of a link and on whether information flows in one direction only or in both directions through the links.[4] In every time period BG's agents select the best response given the current network. It is noteworthy that both with one-way and two-way information flow, the BG non-empty steady-state networks connect all the agents in the population.

Goyal and co-authors ([4], [22], [24], [25] and [26]), De Jaegher and Kamphorst [13], Haller and Sarangi [30], Hojman and Szeidl [31] and Larrosa and Tohmé [40] present applications and extensions of this framework. Falk and Kosfeld [20] experimentally test BG's model both with one-way and two-way information flow. The predictions of the one-way flow model can be replicated in the laboratory while those of the two-way flow model cannot.[5] Also Corbae and Duffy [11] examine experiments involving endogenous networks.

Fagiolo [19] and Ehrhardt *et al.* [17] present models of neighbour interaction with endogenous networks. Currarini and Morelli [12] and Mutuswami and Winter [45] develop a mechanism-design approach to characterise the mechanism achieving efficient networks.

Networks, local and group interactions are of interest in all the social sciences. Historians for example use networks to analyse behavioural and power relationships between agents in order to have a better understanding of the micro-

[4] The non-cooperative nature of BG's model allows for the study of the effect of alternative assumptions on the direction of the information flow while in the cooperative approach the focus is on bilateral flows.

[5] Falk and Kosfeld [20] observe that fairness considerations (Fehr and Schmidt [21]) may explain these results. Fairness is defined as inequality aversion, hence BG's two-way information flow equilibria –which are (pay-off) asymmetric– are not fairness-compatible while equilibria with one-way information flow are symmetric and hence fairness-compatible.

determinants of historical events.[6]

The observation that it is extremely rare that real-world networks connect all the individuals in a society motivates this research. In fact there are two main features that arise in the real world. Agents are usually connected locally and not globally with the whole community. Often they are also partitioned in a core-periphery dichotomy where agents in the core are usually better off than those on the periphery.

This paper develops an extension of BG's model with a one-way information flow. The homogeneous agents in the network bear the cost of the links they sponsor. The agents' utility derives from the information received from other players through the network. This is modelled as a concave, non-monotonic function of the number of observed agents, but it does not directly depend on the number of each agent's sponsored links. The shape of the utility function implies that there are decreasing returns to linking. We model congestion or costly network maintenance by decreasing returns where a marginal link ultimately reduces agent utility.

At each stage the agents play the network formation game, each of them with a probability of maintaining the strategy implemented in the previous period. Active agents change their current strategy only if they switch to another strategy that improves their current payoff. So the analysis is based on the better-response dynamics.

The essential difference between this paper and BG is the presence of decreasing returns to linking. This implies that the marginal benefit of every additional (direct or indirect) link is a decreasing function of the total number of observed agents, while the marginal cost is constant. Furthermore, observing too many agents eventually becomes a bad. Hence there exists an optimal number of agents that each player wants to observe. So that while every additional connection will initially increase (at a decreasing rate) each player's payoff, eventually too much information will reduce it.

One-way information flow is a controversial assumption as it is rarely observed

[6] See Padgett and Ansell [49] and Lipp and Krempel [43] among others.

in real-world economic interactions. However, the internet, industrial espionage and scientific literature may provide examples of one-way information flow situations. There is also a stronger motivation for such an assumption that rests in the observation that even in case of mutual information exchange, the flow is often asymmetric with one partner receiving more information than the other. One-way flow can then be seen as the extreme case of such asymmetric information flows which are actually extremely common.

The main results are as follows. The dynamics converges in finite time to a unique limit set. The basic component of the absorbing state architecture is a *starred wheel* where some agents form a wheel[7] and others are linked to the wheel "from outside". Limit networks consist of disjoint components each of which is characterised by the fact that some agents (who are in the wheel at the centre of the network) enjoy a higher payoff than the peripheral ones – in fact the maximum payoff attainable. While all the outside agents observe the central wheel, they do not observe each other.

The results of this paper are in line with the real world features mentioned above: limit networks are local rather than global and, in the absorbing state, agents are partitioned between a centre and the periphery**.** All the starred wheels have the same dimension. This means that the number of agents in the central wheel is the same for all the components of the limit network. The number of peripheral agents connected to each of these wheels may however vary. Social welfare – as measured by the sum of agents' payoffs – increases with the number of disconnected components in the limit network.

A simple comparison between the limit networks obtained here and those of BG shows the impact of the introduction of decreasing returns to linking. As we are considering the one-way information flow, let us recall the two networks that are absorbing states in this case of BG's analysis: the (global) wheel and the empty network.[8] BG's dynamics settles to a wheel for low values of the unitary

[7] Given a (sub)set of agents the wheel is a network that connects all of them, each of whom has one link to one other and is only linked by a third (different) one.

[8] This network is obviously characterised by the absence of any link between any two agents.

cost of a link and to the empty network for high values of the cost, while both networks are absorbing states for intermediate values of the cost of a link. BG's analysis is a special case of the model presented in this paper: there are constant returns to linking. For a small population size BG results hold even in the presence of decreasing returns to linking. In general the bigger the population the larger the number of disconnected components in the limit networks. In addition in BG's limit networks all agents receive the same payoff while central agents in a starred wheel are better off than peripheral ones.

Of particular interest is a comparison of the results of this paper with those of De Jaegher and Kamphorst [13] and Hojman and Szeidl [31]. They present some results which have a similar flavour, building on different assumptions to those used here. Both papers restrict the analysis to the case of two-way information flow and assume that benefits from connections may not be constant. Their findings are that limit networks may be characterised by an insider-outsider dichotomy but connect all the agents of the population. Also Galeotti *et al.* [22] analyse the two-way information flow model and assume that agents are heterogeneous and that their heterogeneity is an observable characteristic. They conclude that limit networks may be either collections of disconnected components or characterised by an insider-outsider dichotomy. These three works reinforce the results of the current paper as they show that absorbing network architectures similar to ours can also be obtained if information flows both ways through the links.

The paper is organised as follows. The next section outlines the model. Section 3 presents the static analysis and section 4 characterises the absorbing state networks. Section 5 concludes. The appendices collect some of the proofs.

2 The Model

There is a population of P agents. With a slight abuse of notation let us indicate with P both the population and its size. Each agent plays the network formation game. This is a non-cooperative game in which each agent decides whether to build or sever a link with another agent. Through the links, information flows to the originating player (the one who built the link) from the target player and from those the latter has a direct or indirect link to. In BG this is the *one-way information-flow model* (with no information decay).

A *strategy* of each agent i indicates for all agents j $j \neq i$ whether i has a direct link to j. It is represented by a vector $g_i = (g_{i,1}, \ldots, g_{i,i-1}, g_{i,i+1}, \ldots, g_{i,P})$ of dimension $P-1$, where each element of the vector takes value 1 if i has one direct link to j and 0 otherwise. We say that agent i *observes* agent j $j \neq i$ if either i built a link to j, i.e. $g_{i,j} = 1$, or there exists a path in the network that goes from i to j, i.e. there exists a set of agents $\{k_1, .., k_n\} \subset P$ such that $g_{i,k_1} = g_{k_1,k_2} = ... = g_{k_m,k_{m+1}} = ... = g_{k_n,j} = 1$. We adopt the convention that each agent always observes himself.

The set of all strategies for each player is $\mathcal{G}_i = \{0,1\}^{P-1}$ for $i = 1, ..., P$. Each strategy profile translates into a (directed) network. Both a network and the strategy profile that generates it are indicated by $g \in \mathcal{G}_1 \times \ldots \times \mathcal{G}_P$. We write $g = g_i \oplus g_{-i}$ to stress that the network g is made by combining i's strategy with those of his opponents.

Consider the following example: $P = 4$, $g_1 = (0,0,0)$, $g_2 = (0,1,0)$, $g_3 = (1,1,0)$ and $g_4 = (0,0,1)$. Agent 1 has no links. Agents 2 and 4 each have one link only to agent 3. Agent 3 has two links, one with agent 1 and the other with agent 2. This network is depicted in the figure below. The arrows indicate the direction of the information flow (as well as the agent who built the link).

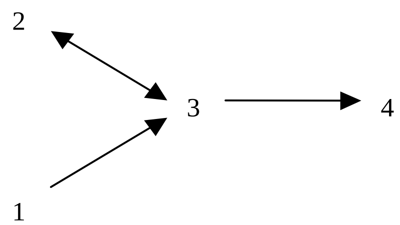

Figure 1: Example of a network

Let us indicate with c the cost building one link, and $\mu_i(g)$ the number of agents that i is directly or indirectly linked with in the network g (including agent i himself) and with $\mu_i^d(g)$ being the number of links set up by agent i. In the example of Figure 1 above $\mu_1 = 1$, $\mu_1^d = 0$, $\mu_2 = 2$, $\mu_2^d = 1$, $\mu_3 = 3$, $\mu_3^d = 2$ and $\mu_4 = 4$, $\mu_4^d = 1$. Note that in Figure 1 the arrows indicate the identity of the agent who is bearing the cost of the link.

The individual payoff function is a function of the network g which player i belongs to:

$$\pi_i(g) = u(\mu_i(g)) - c\,\mu_i^d(g)$$

where $u(\cdot)$ is the utility of observing agents through the network while $c\,\mu_i^d(g)$ is the cost i pays for his links. We normalise $u(1) = 1$ which is the utility of observing oneself only. Assume that $u''(\cdot) < 0$ and $u'(m^*) = 0$ for some m^*, i.e. there are decreasing returns to linking: the marginal utility of observing an additional agent through the network is smaller than that of observing the previous ones (unlike BG). In particular the utility function is bounded from above. In fact, once the utility has reached its maximum it starts decreasing. So observing too many agents through the network is a bad. For instance, this might be due to higher maintenance costs for very large network structures. Alternatively it may be caused by the difficulty of managing too many links simultaneously (e.g. by congestion). As a matter of fact decreasing returns to linking not only imply that the marginal piece of information is less valuable than the previous but also that it eventually reduces the total utility of information received through the links, i.e. too much information eventually becomes a bad.

For the sake of simplicity let $u(\mu_i(g)) = \mu_i(g)[1 - \alpha\,\mu_i(g)]$. Then the payoff function becomes:

$$\pi_i(g_i, g_{-i}) = \mu_i(g) - c\,\mu_i^d(g) - \alpha\,[\mu_i(g)]^2 \tag{1}$$

Parameter values belong to the set $\mathcal{R}$

$$\mathcal{R} = \left\{ (\alpha, c) \in \mathbb{R}^2 \,\middle|\, 0 < \alpha \leq \tfrac{1}{4},\;\; 0 \leq c \leq \tfrac{1}{4\alpha} - 1 \right\} \tag{2}$$

which is needed to guarantee that agents have incentives to connect.[9] In the limit

[9] See Proposition 1 below.

case $\alpha = 0$ the model is the same as that in BG.[10]

Let us define $N(i,g)$ as the set of agents observed by i through the network g. So $\mu_i(g) = \|N(i,g)\|$ i.e. $\mu_i(g)$ is the cardinality of the set $N(i,g)$. Define the geodesic *distance* between agents i and j in a network g $d(i,j;g)$ as the number of links on the shortest path from j to i. If $j \notin N(i,g)$ set $d(i,j;g) = +\infty$. Given an agent i in any network g the agent who is *farthest away* from i among those he observes is $j := \arg\max_{\ell \in N(i,g)} d(i,\ell;g)$. Note that i does not need any of j's links to observe anyone in $N(i,g)$, otherwise j would not be farthest away from i.

For all g in $\mathcal{G}$ we call *architectures* (equivalent to g) all networks that are the same as g with any permutation of the agents. We restrict the analysis to networks that are architecturally equivalent, i.e. identical up to a permutation of the agents' indices. Finally we define a *network component* as a subgraph consisting only of agents that have links to agents belonging to the same component and who are not observed by any other agent. Thus a component is a subset of the network separated from the rest of the network.

Let us now define some special network components. The first component we consider is the wheel: it consists of a subset of agents who are subsequently linked to one other so that each of them observes all the agents in the network component with one link only. Formally:

Definition 1 *A network component is called a wheel of dimension n if there exists $k_1,..,k_n$ with $\{k_1,..,k_n\} \subset P$ such that $g_{k_i k_{i+1}} = 1$ for $i = 1,...,n-1$, $g_{k_n,k_1} = 1$ and $g_{r,s} = 0$ otherwise.*
The set of wheels of dimension n is denoted by $W(n)$.

The figure below depicts a wheel with 6 agents.

[10] Actually BG present their analysis for general payoff functions.

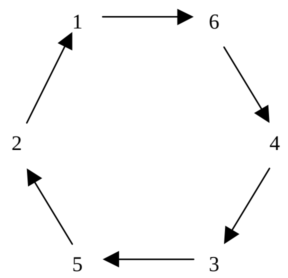

Figure 2: A wheel of dimension 6

In a wheel of dimension n the payoff of each agent belonging to this wheel equals $n - c - \alpha n^2$.

Another important network component for the analysis of this paper is the starred wheel. A starred wheel of dimensions n and m consists of $n + m$ agents, such that n agents are connected in a wheel, with all the further m agents being directly connected to the central wheel. The n agents who form the wheel are called the *central agents* and the other m are the *peripheral agents*.

Definition 2 *A starred wheel of dimensions n and m is a network component connecting $n + m$ agents characterised by the following conditions:*
1. each agent only sponsors one link, i.e. $\mu_i^d(g) = 1$ for all i;
2. there exists a permutation of n agents $k_1, ..., k_n$ such that $g_{k_i k_{i+1}} = 1$ for $i = 1, ..., n-1$ and $g_{k_n, k_1} = 1$;
3. for each $j \notin \{k_1, ..., k_n\}$ there exists $i \in \{1, ..., n\}$ such that $g_{j,k_i} = 1$.
The set of starred wheels of dimensions n and m is denoted by $SW(n, m)$.

We say a *starred wheel of dimension n* (omitting the number of peripheral agents) when it is only important to stress the number of agents forming the central wheel. Figure 3 represents a starred wheel with 4 central agents and 3 peripheral ones.

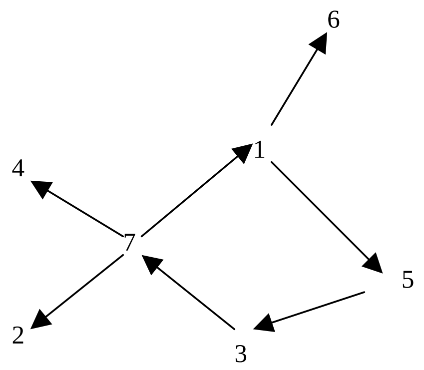

Figure 3: A starred wheel of dimensions 4 and 3

If a network consists of starred wheels all of the same dimension we call it a constellation of starred wheels.

Definition 3 *A constellation of starred wheels of dimension m is a network which can be partitioned into components each of which is a (starred) wheel of dimension m.*

Note that the definition of a constellation of starred wheels of dimension m only indicates the structure of the network components (starred or simple wheels of dimension m) but leaves their number undetermined. Recall that the *floor* of x is indicated with $\lfloor x \rfloor$ and is defined as the largest integer smaller than or equal to x, i.e. $\lfloor x \rfloor = \max\{z \in \mathbb{Z} : z \leq x\}$ for all $x \in \mathbb{R}$. A constellation of starred wheels of dimension m can indeed be made of any number of starred wheels ranging from 1 to $\left\lfloor \frac{P}{m} \right\rfloor$ and is obtainable from any permutation of the agents provided that: i) in the network there are only starred or simple wheels; ii) each wheel has dimension m; and iii) all the P agents are linked. We say the *set of constellations of starred wheels* of dimension m to indicate the set consisting of all possible network architectures that are constellations of starred wheels of dimension m, attainable given a population P of agents.

Not all agents in a network necessarily have links and/or are observed by someone. We define two special roles that an agent can play in a network g: the stand-alone and the terminal. A stand-alone is an agent who does not have any links and is not observed by anyone in the network. Also a terminal has no links, yet he is observed by someone else in the network. Both stand-alones and terminals receive the same payoff: $1-\alpha$.

Definition 4 *Agent i is a <u>stand-alone</u> if $N(i,g)=\{i\}$ and $i \notin N(j,g)$ for all $j \in P \backslash \{i\}$.*
Agent i is a <u>terminal</u> if $N(i,g)=\{i\}$ but i is not a stand-alone, i.e. there exists $k \in P \backslash \{i\}$ such that $i \in N(k,g)$.

Let us now introduce the definition of a Nash network and of a strict Nash network. These are the networks generated by strategy profiles that respectively constitute a Nash equilibrium and a strict Nash equilibrium of the linking game. Formally:

Definition 5 *A network g^* is a <u>Nash network</u> if*

$$\pi_i(g^*) \geq \pi\left(g_i' \oplus g_{-i}^*\right) \tag{3}$$

for all $g_i' \in \mathcal{G}_i$ and all $i \in P$.
A Nash network g^* is a <u>strict Nash network</u> if equality in equation (3) implies $g_i' = g_i^*$ for any agent i in the population.

To illustrate the above definitions consider a population of 9 agents. Set $\alpha = \frac{1}{10}$ and $c = \frac{1}{5}$, so that $(\alpha, c) \in \mathcal{R}$. Then we obtain that $n^* = 5$, $\underline{n} = 2$ and $\overline{n} = 8$. The following figure depicts a Nash equilibrium network which is not a strict Nash network.

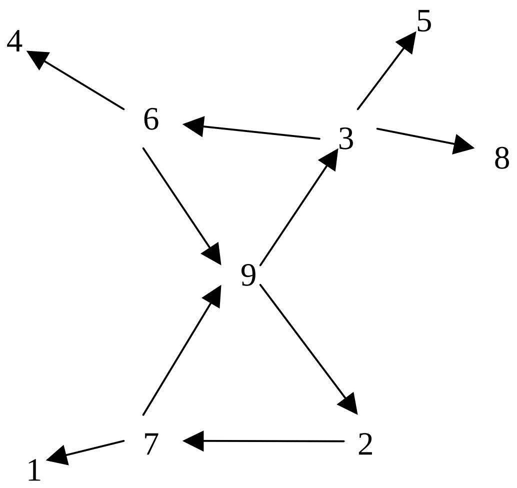

Figure 4: A Nash equilibrium network for $P = 9$, $\alpha = \frac{1}{10}$ and $c = \frac{1}{5}$

For instance, in the network presented in Figure 4 above, agent 2 is indifferent between connecting to 9 (as depicted) or to 6 and player 3 is indifferent between having a link to 9 (as shown) or to 7.

In the rest of the paper we use the following set-wise solution concept that extends the strict Nash equilibrium. It is the restriction to pure strategies of Balkenborg's [6] strict equilibrium set.

Definition 6 *A non-empty set of pure strategy combinations $\mathcal{B} \subseteq \mathcal{G}_1 \times \ldots \times \mathcal{G}_P$ is a Pure-strategy Strict Nash Equilibrium Set (PSNES) if for every strategy profile $\sigma \in \mathcal{B}$, for all $i \in P$ and every $g_i \in \mathcal{G}_i$*

$$\pi_i(\sigma) \geq \pi_i(g_i \oplus \sigma_{-i})$$

where equality implies $(g_i \oplus \sigma_{-i}) \in \mathcal{B}$.

We finally describe the dynamic process. Building on BG and on Ritzberger and Weibull [52], we consider the dynamics induced when, in each round, a single random agent is selected who then chooses from among the strategies that make him better off given that the others do not change their strategies. Formally,

Definition 7 *In a network $g = g_i \oplus g_{-i}$ g'_i is a better response to g_{-i} than g_i for i if $\pi_i(g'_i \oplus g_{-i}) \geq \pi_i(g)$.*
Agent i's better response correspondence to the strategy profile g is indicated by $\beta_i(g)$.

In any time period agents observe the network built in the previous periods. With positive independent probability $\gamma_i > 0$ each agent will exhibit "inertia", i.e. will maintain the strategy played in the previous period. With the complementary probability $1-\gamma_i > 0$ the agent will play a better response to the current network. This induces the better-reply dynamics introduced by Ritzberger and Weibull [52] and defined below

$$g_i^{t+1} = \begin{cases} g'_i \in \beta_i(g^t) & \text{with probability} \quad 1-\gamma_i \\ g_i^t & \text{with probability} \quad \gamma_i \end{cases} \tag{4}$$

for all agents in the population.[11]

A limit network of the better-reply dynamics (4) can be a steady state or belong to an absorbing set. Thus the dynamics can converge either to a single architecture or to a closed set of network architectures.

Definition 8 *A network $\hat{g}$ is steady state of the better-reply dynamics (4) if $g_i^t = \hat{g}_i$ implies that $g_i^{t+1} = \hat{g}_i$ for all $i \in P$.*

Definition 9 *A subset $\mathcal{A} \subset \mathcal{G}_1 \times \ldots \times \mathcal{G}_P$ is an absorbing set of the better-reply dynamics (4) if $g^t \in \mathcal{A}$ implies $g^{t+1} \in \mathcal{A}$.*

3 Static Analysis

We first define three special numbers which will have an important role in the analysis of the linking game. Let us exclude the (zero-measure) case that there

[11] See also Maynard Smith and Price [44].

exists an integer ℓ such that $\alpha = \frac{1}{2\ell+1}$, i.e. α is the inverse of an even number. Then define n^* as the integer that is closest to $\frac{1}{2\alpha}$. Formally $n^* \in \mathbb{N}$ such that

$$\left|n^* - \tfrac{1}{2\alpha}\right| < \tfrac{1}{2}$$

The above restriction on the values of the parameter α guarantees that n^* is unique. Further define

$$\underline{n} := \left\lceil \tfrac{1-\sqrt{1-4\alpha(c+1-\alpha)}}{2\alpha} \right\rceil$$

where $\lceil x \rceil$ is the *ceiling* of x i.e. the smallest integer larger than or equal to x, that is $\lceil x \rceil = \min\{z \in \mathbb{Z} : z \geq x\}$ for all $x \in \mathbb{R}$ and

$$\overline{n} := \left\lfloor \tfrac{1+\sqrt{1-4\alpha(c+1-\alpha)}}{2\alpha} \right\rfloor$$

where $\lfloor x \rfloor$ is the *floor* of x defined earlier.

The following proposition shows the properties of the parameter space $\mathcal{R} := \left\{(\alpha, c) \in \mathbb{R}^2 \left| 0 < \alpha \leq \frac{1}{4}, 0 \leq c \leq \frac{1}{4\alpha} - 1 \right.\right\}$, i.e. that whenever α and c belong to the set $\mathcal{R}$, the integers n^*, $\underline{n}$ and $\overline{n}$ are well defined (Part i) and agents have incentives to link (Parts ii and iii).

Proposition 1 *If $(\alpha, c) \in \mathcal{R}$ the following statements hold:*
i) $\underline{n} < \overline{n}$, $\underline{n} \leq n^*$ and $n^* \leq \overline{n}$.
ii) There exists a network g such that $\pi_i(g) > 1 - \alpha$ for some $i \in P$.
iii) Let $m \in N$, then $m - c - \alpha m^2 \geq 1 - \alpha$ if and only if $m \in \{\underline{n}, ..., \overline{n}\}$.

The proof is in Appendix A. Part ii) compares the payoff of a stand-alone with that of an agent connected in a network establishing the incentives to link: it shows that agents have incentives to connect for parameter values in the set $\mathcal{R}$. Otherwise stand-alones may receive a higher payoff than connected agents. Part iii) offers an intuitive interpretation of the thresholds $\underline{n}$ and $\overline{n}$; $\underline{n}$ and $\overline{n}$ represent respectively the dimension of the smallest (profitable) wheel and of the largest (profitable) wheel which no member has any incentives to break. Part i) ensures that the dimension of the largest profitable wheel is greater than the dimension of the smallest profitable wheel and that n^* lies between them.

Lemma 1 presents an intuitive interpretation of n^*: it indicates the dimension of the payoff-maximising wheel.

Lemma 1 *The payoff of an individual $i \in P$ is maximal if he belongs to a wheel of dimension n^* formally $W(n^*) \subset \{g | \arg\max_g \pi_i(g)\}$.*

Proof. Note that the payoff of each agent is a decreasing function of the number of links he builds. Consider agent i. It is always payoff improving to observe the same number of agents with fewer links as $\pi_i(g) \leq \|N(i,g)\| - c - \alpha \|N(i,g)\|^2$. If agent i has only one link and observes m agents then i's payoff is the one he would get in a wheel of dimension m, i.e. $\pi_i(g) = m - c - \alpha m^2$. So we can restrict our attention to wheel network components. Let us extend the (wheel) payoff function to the real line: i.e. $\varphi(\gamma) := \gamma - c - \alpha\gamma^2$. It can easily be shown that the maximum of $\varphi(\gamma)$ is attained for $\gamma = \gamma^* := \frac{1}{2\alpha}$. The function $\varphi(\gamma)$ is symmetric around the axis $\gamma = \gamma^*$, as $\varphi(\gamma) = \frac{1}{4\alpha} - c - \alpha(\gamma - \gamma^*)^2$. Hence the payoff function (1) is maximised in a wheel of dimension n^* agents where $n^* \in \mathbb{N}$ solves $|n^* - \gamma^*| \leq \frac{1}{2}$. ■

Lemma 1 shows that the decreasing returns to linking equal the marginal benefit of observing one additional agent for $\mu_i(g) = n^*$. Note that the central agents of a starred wheel of dimension n^* enjoy the maximum payoff. The peripheral ones observe n^*+1 agents with one single link: they receive the payoff of an agent who belongs to a $W(n^*+1)$.

For the sake of simplicity, we now restrict ourselves to the case in which agents face strong incentives to connect.

Assumption: *For $(\alpha, c) \in \mathcal{R}$, assume further that $0 < c < 1 - 3\alpha$.*

The parameter space used in the rest of the paper is thus given by

$$\begin{aligned}\mathcal{P} \;=\; &\left\{(\alpha, c) \in \mathbb{R}^2 \,\middle|\, 0 < c \leq 1 - 3\alpha \text{ for } \alpha \in \left(0, \tfrac{1}{6}\right]\right. \\ &\left.\text{and } 0 < c \leq \tfrac{1}{4\alpha} - 1 \text{ for } \alpha \in \left(\tfrac{1}{6}, \tfrac{1}{4}\right)\right\}\end{aligned}$$

Specifically this assumption is used in the following remark.

Remark 1 *If* $(\alpha, c) \in \mathcal{P}$ *then* $\underline{n} = 2$*.*

The proof is given in Appendix A. The remark shows that no agent has an incentive to cut all his links in a network in which he observes 2 agents (at least if he bears the cost of one link only). Combining the previous remark with Proposition 1 we observe that it is never payoff maximising to have no links at all if it is possible to observe no more than $\overline{n}$ agents with one link.

We are now ready to solve the static version of the linking game. In the following we the establish (static) equilibrium result for the case $n^* \leq \overline{n} - 1$ and a large and non-pathological population size. We show that a constellation of starred wheels of dimension n^* is a PSNES of the game. The intuition is as follows. If the network is a constellation of $SW(n^*)$ no central agent in a starred wheel wants to deviate, as he is receiving the maximum payoff. Moreover peripheral agents, who are not earning the top payoff, have no profitable deviation as everyone in the network is either observing n^* or $n^* + 1$ agents. Proposition 3 deals with the special cases.

Proposition 2 *Assume that* $P > n^*$*, that* $(\alpha, c) \in \mathcal{P} \setminus \left\{ (\alpha, c) : \frac{1}{6} \leq \alpha < \frac{1}{5}, 3 - 15\alpha < c \leq \frac{1}{4\alpha} - 1 \right\}$ *and, in addition, if* $n^* > \frac{1}{2\alpha}$ *that there exists no integer* k *such that* $P = (k+1) n^* - 1$*. Then a constellation of starred wheels of dimension* n^* *is a PSNES.*

Proof. Consider a constellation of starred wheels of dimension n^*. Given the $n^* + m$ agents who form one $SW(n^*, m)$, none of them wants to individually deviate in a way that alters the starred wheel architecture. Consider first the n^* agents who form the $W(n^*)$. They obtain the maximum payoff since they observe n^* agents and only pay for one link. So they have no incentive to deviate. Let us now consider the m peripheral agents who are linked to the wheel. None of them can improve his payoff: if one of them cuts his link and links somewhere else to the wheel neither his payoff nor the architecture change. If he links to someone else who is directly linked to the $W(n^*)$ his payoff is reduced since now this agent observes $n^* + 2$. If a peripheral agent links somewhere outside the starred wheel then the starred wheel still exists. He can only increase his payoff by linking (with only one link) to someone who observes n^*-1. This is impossible since the original network was a constellation of starred wheels of dimension n^*. So a constellation of starred wheels of dimension n^* is a PSNES because every time agents deviate in a way that the resulting architecture is not a constellation of $SW(n^*, m)$, these agents are worse off and agents are indifferent only between strategies that do not alter the architecture. ■

Consider now the special cases of a pathological population size, of $n^* = \overline{n}$ and of a small population.

Proposition 3 *i) Let P be such that there exists an integer k such that $P = (k+1)n^* - 1$ and $n^* > \frac{1}{2\alpha}$. Then a network consisting of k wheels of dimension n^* and one wheel of dimension n^*-1 is a PSNES.*

ii) Let $(\alpha, c) \in \left\{(\alpha, c) : 3 - 15\alpha < c \leq \frac{1}{4\alpha} - 1, \frac{1}{6} \leq \alpha < \frac{1}{5}\right\} \subset \mathcal{P}$. If there exists an integer h such that $P = 3h+1$ then a set of h wheels $W(3)$ and a stand-alone is a PSNES; if $P = 3h+2$ then a set of h wheels $W(3)$ and a $W(2)$ is a PSNES; and if $P = 3h$ a set of h wheels $W(3)$ is a PSNES.

iii) Let $P < n^*$ then one wheel of dimension P is a PSNES.

Proof. Part i) Consider first the agents belonging to the k wheels of dimension n^*. Each of them earns the maximum payoff, so none of them has an incentive to deviate. Consider now the remaining (n^*-1). They can form their own wheel of dimension (n^*-1) or link (from outside) to the existing wheels of dimension n^*. Since $n^* > \frac{1}{2\alpha}$, the payoff of a wheel of dimension (n^*-1) is higher than that one can get by linking to wheel of dimension n^*. Hence if they form a wheel $W(n^*-1)$ they have no profitable deviation.

Part ii) It can easily be verified that the assumptions on α and c are equivalent to the following values for n^*, $\overline{n}$ and $\underline{n}$: $n^* = \overline{n} = 3$ and $\underline{n} = 2$ which is the only possible case in which $n^* = \overline{n}$ for $(\alpha, c) \in \mathcal{P}$. Consider the agents connected in the h wheels of dimension 3. As $n^* = 3$ they are earning the maximum payoff. There will now be either 1 or 2 or no remainders. If there is one remainder only he will become a stand-alone as $\overline{n} = 3$. If there are 2 remainders their best available action is to form a $W(2)$ as $\underline{n} = 2$.

Part iii) As $P < n^*$ the maximum payoff is attained by observing all P agents with one link. Hence no agent belonging to such a $W(P)$ has an incentive to deviate. ■

Indeed if the population has a pathological size and parameter values are such that players prefer to observe $n^* - 1$ agents rather than $n^* + 1$ agents with the same number of links, then a network consisting of $W(n^*)$ and one $W(n^* - 1)$ is an equilibrium (Part i). If parameter values are such that $\underline{n} = 2$ and $n^* = \overline{n} = 3$, then a network consisting of $W(3)$ and, in case, also of one stand-alone or of a wheel of dimension 2 is an equilibrium. Since $n^* = \overline{n}$ no starred wheel can be part of an equilibrium and if there are 2 remainders their best response is to form a $W(2)$. (Part ii) Finally if the population is so small that decreasing returns to linking are never reached then the global wheel is an equilibrium (Part iii).

4 Dynamic Analysis

Let us now analyse the better-reply dynamics. Given the central role wheels play in our analysis as well as in BG's we first show the mechanism of (simple) wheel formation. It is sufficient that one agent is the best off of all those he observes for a wheel to emerge among these agents. Indeed the agent farthest away from the best off has a better response of cutting all his links and linking to him. A wheel is formed by repeating this process until all the agents observed by the original best off[12] move. The following lemma makes this argument formal.

Lemma 2 *Let agent i_1 be such that $\pi_{i_1} \geq \pi_j$ for all $j \in N(i_1, g)$, i.e. i_1 is the best off among those he observes. Let $m := \|N(i_1, g)\|$ so m is the number of agents observed by i_1. If $m > 1$ then in finite time a wheel $W(m)$ is formed by the agents originally in $N(i_1, g)$.*

[12] As we adopted the convention that every agent observes himself, the best off is included in the set of agents who will form the wheel.

Proof. Let $M := N(i_1, g)$. Consider $i_2 \in \arg\max_{\ell \in M} d(i_1, \ell; g)$ so i_2 is farthest away from i_1 among those who are observed by i_1. Note that i_1 does not use any of i_2's links to observe anyone else. Agent i_2 can improve his payoff by cutting all his links and linking to i_1 directly since

$$\pi_{i_2}(g) \leq \pi_{i_1}(g) \leq m - c - \alpha m^2$$

as $i_2 \in N(i_1, g)$. So he does this. We will call this new network $g^{(2)}$.

Note that now i_2 observes all the agents in M (so m) with one single link and his payoff is exactly $m - c - \alpha m^2$ which is the new maximum payoff in M. Moreover all the agents who observe i_2 now observe all the agents in M.

We proceed by induction. Assume we have done the first $\ell - 1$ iterations. Call agent i_ℓ the player who moved in iteration $\ell - 1$ for $\ell > 1$. Let $g^{(\ell)}$ be the network formed at the end of iteration ℓ, i.e. after i_ℓ moved. Notice that all the agents who moved in the previous round are consecutively linked to each other, i.e. i_s has one single link to agent i_{s-1} for $s = 2, \ldots, \ell$. So the distance between i_ℓ and i_1 in the network $g^{(\ell)}$ is $\ell - 1$ for $\ell > 1$. Moreover $\left\|N\left(i_s, g^{(\ell)}\right)\right\| = m$ for $s = 1, \ldots, \ell$ and $\ell > 1$. Consider now $i_{\ell+1} \in \arg\max_{j \in M} d\left(i_\ell, j; g^{(\ell)}\right)$ so $i_{\ell+1}$ is farthest away from i_ℓ. By the reasoning applied to i_2 agent $i_{\ell+1}$ has a better reply of cutting all his links and linking to i_ℓ directly.

We now show that eventually i_1 is selected, so that a wheel $W(m)$ forms. Consider the iterative procedure above. Since $i_h \in M$ and $\left\|N\left(i_h, g^{(h)}\right)\right\| = m$ for all $h > 2$ eventually i_1 is selected as M is finite and the distance between i_h and i_1 increases by 1 in each step. Let i_k be the last agent who moves before i_1 is selected. Notice that i_k observes i_1 through a path consisting of all the agents who moved before him. As i_k observes m agents and i_1 is the farthest away from i_k it follows that $k = m$.

Notice now that i_1 is the only agent in M who can possibly have more than one link. If so then let i_1 play. By the same reasoning as i_2, agent i_1's better response is to cut all his links and link to i_m. Now i_1 observes m agents with one single link and closes a wheel of dimension m among the agents $i_1, \ldots, i_m$ with possibly other players observing them. ■

As we now know how wheels form, we are ready to state the main result of the paper. The following proposition proves that the better-reply dynamics (4) always converges to a constellation of starred wheels of dimension n^* in finite time. Special cases are dealt with by Proposition 5. The key point of the argument concerns the emergence of at least one agent observing n^* agents. Lemmas 3, 4 and 5 in Appendix B analyse some relevant network structures in which the dynamics leads to the emergence of such an agent.

Proposition 4 *Assume that $P > n^*$, that $(\alpha, c) \in \mathcal{P} \setminus \left\{ (\alpha, c) : \frac{1}{6} \leq \alpha < \frac{1}{5}, 3 - 15\alpha < c \leq \frac{1}{4\alpha} - 1 \right\}$ and, in addition, if $n^* > \frac{1}{2\alpha}$ that there exists no integer k such that $P = (k+1)\, n^* - 1$. Then in finite time each agent belongs to a starred wheel of dimension n^*. From then on, while the network might change, it remains a constellation of starred-wheels of dimension n^* in each period.*

The formal proof is given in Appendix B and contains 6 steps. Starting from an arbitrary network we first show that in finite time there will be no connected agents who observe fewer than $\underline{n}$ or more than $\overline{n}$ agents (Step 1). Secondly we prove that all the agents in the network either observe someone or they are stand-alones, so in finite time terminals connect to someone (Step 2). Thirdly also stand-alones have an incentive to join in the network (Step 3). Hence in finite time the network is such that all the agents observe a number of agents between $\underline{n}$ and $\overline{n}$. We then show that starting from such a network in finite time (at least) one agent gets to observe n^* agents (Steps 4). Each time someone observes n^* a starred wheel of dimension n^* emerges (Step 5). The final step of the proof shows that the absorbing set of the better-reply dynamics is a constellation of starred wheels of dimension n^*.

The actual number of starred wheels of dimension n^* that are formed in the limit state of the dynamics is indeterminate. During the dynamic process a peripheral agent of an existing starred wheel might sever his link and join another subset of agents if by so doing he gets to observe n^* agents. Then according to Proposition 4, the process that leads to the formation of a new starred wheel begins. This implies that the better-reply dynamics (4) does not converge to a steady state but that the set of constellations of starred wheels of dimension n^* is an absorbing state of the better-reply dynamics (4). However once the dynamics has set to a constellation consisting of a given number of starred wheels it will rest there: i.e. the number of starred wheels will not change further (by Proposition 2). Moreover, as we show that irrespectively of the initial network the dynamics converges to a constellation of starred wheels of dimension n^* we can state the following uniqueness result.

Remark 2 *Under the assumptions of Proposition 4 the set of constellations of $SW(n^*)$ is the unique absorbing set of the better-reply dynamics (4).*

We now define $r, q \in \mathbb{N}$ such that

$$P = q\,n^* + r \qquad \text{for} \quad 0 \leq r < n^* \tag{5}$$

so $q = \left\lfloor \frac{P}{n^*} \right\rfloor$, i.e. q is the maximum number of starred wheel that can be formed in a constellation of $SW(n^*)$ from a population of P agents and r (which is the remainder of the division between P and n^*) is the minimum number of peripheral agents.

We will now define the *aggregate payoff of the population* as the sum of the payoffs of each agent. A network g is a *Pareto efficient architecture* if it is impossible to increase the payoff of any agent without reducing the payoffs of others. The aggregate payoff of the population may be used as measure of *Social Welfare*.

The following corollary relates the number of starred wheels in the limit architecture with the Social Welfare of the population. It shows that aggregate payoff of the population increases strictly with the number of starred wheels and identifies the constellation consisting of the maximum number of $SW(n^*)$ as the Pareto efficient limit network.

Corollary 1 *Assume that $P > n^*$, that $(\alpha, c) \in \mathcal{P} \backslash \left\{(\alpha, c) : \frac{1}{6} \leq \alpha < \frac{1}{5}, 3 - 15\alpha < c \leq \frac{1}{4\alpha} - 1\right\}$ and, in addition, if $n^* > \frac{1}{2\alpha}$ that there exists no integer k such that $P = (k+1)\,n^* - 1$. Consider the absorbing set found in Proposition 4:*

the aggregate payoff of the population increases with the number of starred wheels in the limit architecture. The Pareto efficient architecture *consists of q starred wheels of dimension n^* with the remaining r agents being linked from outside to these wheels, where q and r are defined in equation (5).*

Proof. The first part of the statement is easily verified. The aggregate payoff of the population increases with the number of starred wheels because central agents in a starred wheel of dimension n^* enjoy the maximum payoff and their number increases with the number of starred wheels in the limit architecture.

Let us now prove the second part. Given a constellation of q starred wheels of dimension n^* it is impossible to increase the payoff of anyone agent without reducing that of another. The $n^* \times q$ central agents enjoy the maximum payoff attainable (by Lemma 1) so there is no way to improve it. Peripheral agents observe $n^* + 1$ agents with one single link and receive a lower payoff, that of the members of a $W(n^* + 1)$. The only way they can improve the payoff is to reduce by one unit the number of agents observed. This is impossible because the peripheral players are too few ($r < n^*$ by equation 5) to set up a $W(n^*)$ on their own without reducing the payoff of some central agents. ■

Notice that Pareto efficiency is not guaranteed in Proposition 4. Indeed it is shown that the dynamics (4) converges to a constellation of starred wheels of dimension n^*. Yet the actual number of wheels which the constellation consists of is indeterminate and once convergence is achieved for a given number of starred wheels this number will not change further.

We can now compare our results with BG's. It should be recalled that BG show that for any number of agents and starting from any initial network, the (best-response) dynamic process converges to a wheel or to the empty network with a probability of one, depending on the net benefit of a link. In particular Theorem 3.1(a) in BG[13] shows that the dynamics converges to the global wheel when agents have strong incentives to connect.

[13] BG, Theorem 3.1, Part a), p. 1197.

BG's setting can be easily obtained from this model. For $\alpha = 0$ the payoff function (1) coincides with the linear payoff used as a special case in BG.[14] It should also be noted that the assumption of BG's Theorem 3.1(a) is equivalent to $(\alpha, c) \in \mathcal{P}$ as $c \in [0, 1]$ for $\alpha = 0$. Thus we obtain that $\underline{n} = 2$ and $n^* = +\infty$.

Consequently the dynamics (4) converges to a constellation of starred wheels consisting of one (global) wheel, which is a limit case of the absorbing states found in Proposition 4. The proof is as follows. Take the player who is observing the largest number of agents: either he observes the whole population or there is someone who is not observed by him. In the former case consider the player farthest away from him. He can improve his payoff by cutting (all) his link(s) and linking to the one originally observing the entire population. In the latter case consider a player not observed by the one observing the most: for him it is payoff improving to cut (all) his link(s) and link to the one observing the largest number of agents. Repeat the argument until one agent observes the whole population. In both cases the last agent to move is the best off in the whole population, as by construction he has only one link and observes everyone. Then by Lemma 2 one simple wheel of dimension P is formed.

Consider now the special cases of a pathological population size, of $n^* = \overline{n}$ and of a small population that we left aside in Proposition 4. If $n^* > \frac{1}{2\alpha}$ and there exists an integer k such that $P = (k+1)\,n^* - 1$ or if $(\alpha, c) \in \left\{ (\alpha, c) : 3 - 15\alpha < c \leq \frac{1}{4\alpha} - 1, \frac{1}{6} \leq \alpha < \frac{1}{5} \right\}$ or if $P < n^*$ then the steady-state architectures are as shown by the following proposition.

Proposition 5 *i) Let P be such that there exists an integer k such that $P = (k+1)\,n^* - 1$ and $n^* > \frac{1}{2\alpha}$. Then with positive probability the better-reply dynamics settles in finite time to a network consisting of k wheels of dimension n^* and one wheel of dimension $n^* - 1$, which is a steady state.*

ii) Let $(\alpha, c) \in \left\{(\alpha, c) : 3 - 15\alpha < c \leq \frac{1}{4\alpha} - 1, \frac{1}{6} \leq \alpha < \frac{1}{5}\right\} \subset \mathcal{P}$. If there exists an integer h such that $P = 3h + 1$ then the unique steady state of the better-reply dynamics is a set of h wheels $W(3)$ and the remaining agent is a stand-alone; if

[14] That is equation (2.2) in BG, p. 1190.

$P = 3h + 2$ the unique steady state is a set of h wheels $W(3)$ and the remaining 2 agents form a $W(2)$; and if $P = 3h$ the unique steady state is a set of h wheels $W(3)$.

iii) Let $P < n^*$ then in finite time the agents form one wheel of dimension P which is the unique steady state of the better-reply dynamics.

Proof. Part i) Assume there are k players who observe k disjoint groups of n^* agents. This is an event that happens with positive probability. Then k wheels of dimension n^* surely emerge (applying Step 5.a of the proof of Proposition 4). Assume that the remaining $(n^* - 1)$ agents are linked to each other only. Then they have the choice between forming their own wheel of dimension $(n^* - 1)$ or linking (from outside) to the existing wheels of dimension n^*. Since $n^* > \frac{1}{2\alpha}$, the payoff of a wheel of dimension $(n^* - 1)$ is higher than that one can get by linking to wheel of dimension n^*. Hence they form a wheel $W(n^* - 1)$.

Part ii) As we know the assumptions on α and c are equivalent to $n^* = \overline{n} = 3$ and $\underline{n} = 2$. We can apply Steps 1, 2, 4 and 5 a. of the proof of Proposition 4 to show that at least one $W(3)$ forms. As $\underline{n} = 2$ and $n^* = \overline{n} = 3$, h wheels of dimension 3 form. Now there will be either 1 or 2 or no remainders. If there are 2 remainders they will form a $W(2)$ as $\underline{n} = 2$.

Part iii) Take the agent who is observing the largest number of agents and call him i_1. Now either i_1 observes the whole population or there is someone who is not observed by i_1. In the first case by replacing n^* with P, the argument of Step 5.a of Proposition 4 shows that one simple wheel of dimension P forms. Otherwise take from among the agents not observed by i_1 the one who observes the most agents and call him j. For him it is payoff improving to cut all his link(s) and link to i_1 directly. So j does this and now j is the one who observes most agents in P. Repeat the argument until one agent observes all the P agents. Then apply the argument of Step 5.a. of Proposition 4 (replacing n^* with P). ■

The proposition shows that the PSNES established in Proposition 3 are absorbing states of the better-reply dynamics. In particular the case of Part iii) clarifies that every agent is connected and observes everyone else when the population is small. This means that a small population size prevents the decreasing returns to linking from prevailing over the marginal benefit of observing one additional agent. That is in small populations BG's results hold even in presence of decreasing returns to linking.

5 Conclusions

In this paper the impact of decreasing returns to linking in the process of network formation has been studied as a variant of the model constructed by Bala and Goyal [1]. Decreasing returns to linking affect the payoffs of the agents in a way that dramatically changes the steady-state predictions with respect to the original. The agents who are assumed homogeneous have to trade off the number of links they sponsor with the benefit of observing the others. They also have to consider that the marginal benefit of observing new agents is decreasing. Strategy revision occurs by better response.

In this model, the dynamics converges in finite time. The unique absorbing set is a constellation of disjoint starred wheels, where core agents are linked in the optimally-sized wheel and peripheral agents link to the wheel from outside.

The role of connections between financial institutions has recently attracted attention by both researchers and practitioners.[15] Noticeably core-periphery structures arise in models of financial networks (Castiglionesi and Navarro [9] and Sui [56]) and are also observed in actual interbank markets (Boss *et al.* [3]).

Industrial districts are also a field in which networks are often researched. Guerrieri and Pietrobelli [28] and Lee [41] find similar architectures of links between economic agents in districts both in Italy and in Taiwan. Chetty and Agndal [10] analyse network structure and core-periphery interaction in the boatbuilding district in New Zealand. Gnutzmann [27] studies the importance of connectedness and observes that in the Cambridge Hich-Tech Cluster in England there are apparently non-decreasing returns to linking.

However, examples of disjoint networks are much more common in the real

[15] For instance see Haldane [29], Iazzetta and Manna [32], Leitner [42] and Nier *et al.* [48].

world than global networks. As an illustration consider the textile and apparel industry in Italy: it is spread over in 45 industrial districts which are not all near each other, rather than being agglomerated in one single location. Similarly there are 38 industrial districts in Italy in the mechanics sector and 20 in the leather goods sector (Istat [34]). Our result that social welfare increases with network fragmentation may explain this observation.

A Preliminary Results

Proposition 1 *If* $(\alpha, c) \in \mathcal{R}$ *the following statements hold:*
i) $\underline{n} < \overline{n}$, $\underline{n} \leq n^*$ *and* $n^* \leq \overline{n}$.
ii) There exists a network g *such that* $\pi_i(g) > 1 - \alpha$ *for some* $i \in P$.
iii) Let $m \in N$, *then* $m - c - \alpha m^2 \geq 1 - \alpha$ *if and only if* $m \in \{\underline{n}, ..., \overline{n}\}$.

Proof. Part i) Recall that n^* is defined as the integer that is closest to $\frac{1}{2\alpha}$. Formally, $n^* \in \mathbb{N}$ such that $\left|n^* - \frac{1}{2\alpha}\right| < \frac{1}{2}$ which is positive and well-defined for all positive α given that we excluded the zero-measure case where there exists an integer ℓ such that $\alpha = \frac{1}{2\ell+1}$, i.e. that α is the inverse of an even number. Moreover recall that $\underline{n} := \left\lceil \frac{1-\sqrt{1-4\alpha(c+1-\alpha)}}{2\alpha} \right\rceil$ and $\overline{n} := \left\lfloor \frac{1+\sqrt{1-4\alpha(c+1-\alpha)}}{2\alpha} \right\rfloor$. The thresholds $\underline{n}$ and $\overline{n}$ are well-defined for $\alpha > 0$ and $c \in \left[0, \frac{1}{4\alpha} + \alpha - 1\right]$. Also note that both $\underline{n}$ and $\overline{n}$ are non-negative, as $\frac{1-\sqrt{1-4\alpha(c+1-\alpha)}}{2\alpha} > 0$ if and only if $c+1 > \alpha$ – which is always true – and also $\frac{1+\sqrt{1-4\alpha(c+1-\alpha)}}{2\alpha} > 0$. Finally $\underline{n} < \overline{n}$, $\underline{n} \leq n^*$ and $n^* \leq \overline{n}$ is guaranteed by $\frac{1+\sqrt{1-4\alpha(c+1-\alpha)}}{2\alpha} - \frac{1-\sqrt{1-4\alpha(c+1-\alpha)}}{2\alpha} \geq 2$ which holds as $(\alpha, c) \in \mathcal{R}$.

Part ii) Let a network g contain a wheel of dimension m, i.e. in g there exists a subset M of the population such that the agents in M form a wheel $W(m)$. Consider an agent i who belongs to this wheel. Agent i receives a payoff $\pi_i(g) = m - c - \alpha m^2$ in the network g. Recall that stand-alones receive a payoff equal to $1 - \alpha$. It can be shown that $-m^2\alpha + m + \alpha - c - 1 > 0$ for some m if $(\alpha, c) \in \mathcal{R}$. Let $\phi(\gamma) := -\alpha\gamma^2 + \gamma + \alpha - 1 - c$. Consider the equation $\phi(\gamma) = 0$. Its roots are $\gamma_{1,2} := \frac{1\mp\sqrt{1-4\alpha(c+1-\alpha)}}{2\alpha}$. The roots γ_1 and γ_2 are real number only if $1 - 4\alpha(c + 1 - \alpha) \geq 0$, i.e. $c \leq \frac{1}{4\alpha} - 1 + \alpha$. If $1 - 4\alpha(c + 1 - \alpha) = 0$ then $\phi(\gamma) = 0$ for $\gamma = \frac{1}{2\alpha}$ and $\phi(\gamma) < 0$ otherwise. So $1 - 4\alpha(c + 1 - \alpha) > 0$ which means that the parameters belong to the set $\mathcal{R}$. The statement follows as $\alpha > 0$ and hence $-m^2\alpha + m - c > 1 - \alpha$ if, and only if, $\gamma_1 < m < \gamma_2$. Notice that Part i) above guarantees the existence of such m.

Part iii) Recall that $\underline{n} = \lceil\gamma_1\rceil$ and $\overline{n} = \lfloor\gamma_2\rfloor$ and that γ_1 and γ_2 are the solutions of the equation $-\alpha\gamma^2+\gamma+\alpha-1-c=0$. So $m-c+\alpha m^2 \geq 1-\alpha$ for all $m=\underline{n},...,\overline{n}$ and $m-c+\alpha m^2<1-\alpha$ for $m<\underline{n}$ and $m>\overline{n}$. This concludes the proof of the proposition. ■

Remark 1 *If* $(\alpha, c) \in \mathcal{P}$ *then* $\underline{n}=2$ *which means that no agent has an incentive to cut all his links in a network in which he observes 2 agents.*

Proof. Recall the definition of the set $\mathcal{P}=\left\{(\alpha,c)\in\mathbb{R}^2 \mid 0<c\leq 1-3\alpha \text{ for } \alpha\in\left(0,\frac{1}{6}\right] \text{ and } 0<c\leq\frac{1}{4\alpha}-1 \text{ for } \alpha\in\left(\frac{1}{6},\frac{1}{4}\right)\right\}$. Note that $\frac{1}{4\alpha}-1<1-3\alpha$ for $\alpha\in\left(\frac{1}{6},\frac{1}{4}\right)$. Figure A1 below plots the set $\mathcal{P}$.

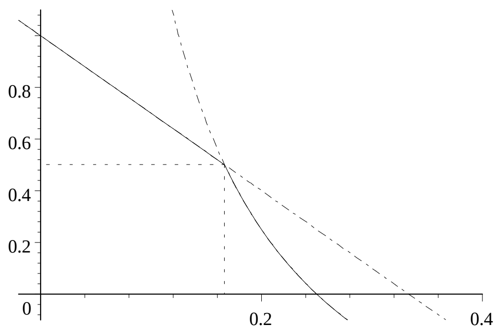

Figure A1: The parameter space $\mathcal{P}$

Recall that $\underline{n}=\lceil\gamma_1\rceil$. So it is enough to verify that $1<\gamma_1\leq 2$. Note that $\gamma_1=\frac{1-\sqrt{1-4\alpha(c+1-\alpha)}}{2\alpha}>1$ as $(1-2\alpha)^2>1-4\alpha+4\alpha^2-4\alpha c$ since $c>0$. Now verify that γ_1 is smaller than 2: $\gamma_1=\frac{1-\sqrt{1-4\alpha(c+1-\alpha)}}{2\alpha}\leq 2$ if, and only if, $1-4\alpha<\sqrt{1-4\alpha\,(c+1-\alpha)}$. As $\alpha<\frac{1}{4}$ then $1-4\alpha>0$. So $\gamma_1\leq 2$ if, and only if, $(1-4\alpha)^2\leq 1-4\alpha\,(c+1-\alpha)$, i.e. $c\leq 1-3\alpha$ which holds for all $\alpha,c\in\mathcal{P}$. So $\underline{n}=2$ for all $\alpha,c\in\mathcal{P}$. ■

B Proof of the Main Results

To prove Proposition 4 we need to establish some preliminary results. The Lemmas in the next subsection present some relevant cases in which the better-response dynamics (4) leads to the emergence of an agent who observes n^* players. Together with Lemma 2 these are building blocks for the proof of the proposition. We will show that every time an agent observes n^* players a starred wheel of dimension n^* is formed.

B.1 Preparatory Lemmas

In the proof of Proposition 4 we find a path of the better-reply dynamics that leads to the formation of a constellation of starred wheels when at least one agent in the population observes exactly n^* players. The next Lemmas consider first the case of a highly fragmented network and then that of a highly connected one. In particular Lemma 3 assumes that there exist more than n^* agents who observe fewer than n^* and shows that one of them will observe n^* agents in finite time.

Lemma 3 *If there exist more than n^* agents each of whom observes between $\underline{n}$ and $n^* - 1$ agents then in finite time there will be at least one agent who observes exactly n^* agents, i.e. the better-response dynamics g^t is such that for some finite t' there exists an agent j for whom $\left\| N\left(j, g^{t'}\right)\right\| = n^*$.*

Proof. Call H the set of agents each of whom observes between $\underline{n}$ and $n^* - 1$ agents in the network g. Notice that $\|H\| \geq n^* + 1$. Define $\hat{P}$ as the subset of the population consisting of all the agents observed by some $h \in H$, formally $\hat{P} = \bigcup_{h \in H} N(h, g)$. Among the agents belonging to $\hat{P}$ let i_1 be an agent who observes the most agents, i.e. $i_1 \in \arg\max_{j \in \hat{P}} \|N(j, g)\|$. Let $m_1 := \|N(i_1, g)\|$. Note that $m_1 \leq n^* - 1$.

Consider $i_2 \in \arg\max_{\ell \in N(i_1, g)} d(i_1, \ell; g)$ so i_2 is farthest away from i_1 among those who are observed by i_1. Recall that i_1 does not use any of i_2's links to observe anyone else. Agent i_2 can improve his payoff by cutting all his links and linking to i_1 directly since $\pi_{i_2}(g) \leq m_1 - c - \alpha m_1^2$ as $i_2 \in N(i_1, g)$. So he does this and receives a payoff exactly equal to $m_1 - c - \alpha m_1^2$.

Call this new network g^0. Note that $N(i_2, g^0) = N(i_1, g)$ and that i_2 is the best off among the agents he observes. So by Lemma 2 the agents observed by i_2 form a $W(m_1)$.

Call this new network g'. Assume there exists a $k \in \hat{P} \backslash N(i_j, g')$ for an agent i_j in the $W(m_1)$ such that $d(k, i_j; g') = 1$, i.e. agent k is directly linked to the wheel without belonging to it. As $m_1 \leq n^* - 1$, agent i_{j+1} in the $W(m_1)$ whose only link is to i_j has a better response of cutting his link and linking to k. This makes a wheel of dimension $m_1 + 1$. Repeat until an agent gets to observe n^* (which completes the proof) or no such k outside the wheel exists anymore.

In the latter case call the new network $\tilde{g}$ and m_2 the dimension of the (enlarged) wheel. So $m_2 \geq m_1$. Note that all the agents in $\hat{P}$ either belong to the wheel $W(m_2)$ or are not in the wheel and hence observe fewer than or equal to m_1 agents. Call L the subset of $\hat{P}$ of agents who do not belong to the wheel.

So $\hat{P}$ is partitioned into the agents belonging to the wheel $W(m_2)$ and those in L. Each agent $\ell \in L$ has a better response of cutting all his links and linking directly to someone who belongs to the wheel as $\|N(\ell, \tilde{g})\| \leq m_2 \leq n^* - 1$. By so doing all the agents in L directly link to the wheel without belonging to it. So we re-apply the argument developed above for an agent k who is linked to the wheel directly without belonging to it. Note that at some point an agent observes n^* players as $\left\|\hat{P}\right\| > n^*$ by assumption. This completes the proof. ■

In the next Lemma we consider the case of a network which is so highly connected that there is a subset of the population such that all the agents in this subset observe more than n^* agents and all the agents observed by them also observe more than n^*. We then show that in this case an agent observing exactly n^* will emerge.

Lemma 4 *Assume there exists an agent k such that $n^* + 1 \leq \|N(\ell, g)\| \leq \overline{n}$ for all $\ell \in N(k, g)$, i.e. such that all the agents observed by him observe between $n^* + 1$ and $\overline{n}$ agents. Then in finite time there will be at least one agent who observes exactly n^* agents.*

Proof. Let $\hat{P} = \bigcup_{\ell \in N(k,g)} N(\ell, g)$. Note that $\left\|\hat{P}\right\| \geq n^* + 1$ since $\|N(i, g)\| \geq n^* + 1$ for all $i \in \hat{P}$. Consider now the agent who is best off in $\hat{P}$ and call him i_1, so $\pi_{i'}(g) \leq \pi_{i_1}(g)$ for all $i' \in N(i_1, g)$. Let $m_1 = \|N(i_1, g)\|$ so $m_1 \geq n^* + 1$. By Lemma 2 the agents in $N(i, g)$ form a wheel $W(m_1)$.

Call this new network g'. Now take an agent j belonging to this wheel $W(m_1)$. Agent j has a better response (in fact it is his best response) of cutting his link and linking to agent r in the wheel such that $d(r, j; g') = n^*$. So he does so and observes exactly n^* agents improving his payoff. This completes the proof ■

The following lemma considers another highly-connected network situation in which there is an agent j_1 who observes more than n^* agents. It assumes further that among the agents he observes all those who observe fewer than n^* players observe so few that they have a better response to cut all their links and link to j_1 directly. Once all of them are linked to j_1, no-one observed by j_1 observes fewer than n^*. Then by Lemma 4 there will be one agent who observes n^* players in finite time.

Lemma 5 *Assume there exists an agent j_1 such that $n^*+1 \leq \|N(j_1, g)\| \leq \overline{n}$ and for each agent $\ell \in N(j_1, g)$ such that $\|N(\ell, g)\| \leq n^* - 1$ we have $\|N(j_1, g)\| + \|N(\ell, g)\| + 1 < \frac{1}{\alpha}$. Then in finite time there will be at least one agent who observes exactly n^* agents.*

Proof. Let $m_1 := \|N(j_1, g)\|$, so m_1 is the number of agents observed by j_1. Consider $j_2 \in \arg\max_{\ell \in \{\ell \in N(j_1, g) :\, \|N(\ell, g)\| < n^*\}} \|N(\ell, g)\|$ so j_2 is the agent who observes the maximum number of agents among those observed by j_1 who observe fewer than n^*. Agent j_2 exists by assumption. Consider an agent $\hat{\jmath} \in \arg\max_{\ell \in N(j_2, g)} d(j_2, \ell; g)$ so $\hat{\jmath}$ is farthest away from j_2 among those who are observed by j_2. So $\|N(\hat{\jmath}, g)\| \leq \|N(j_2, g)\| \leq n^* - 1$ and $\pi_{\hat{\jmath}}(g) \leq \|N(\hat{\jmath}, g)\| - c - \alpha \|N(\hat{\jmath}, g)\|^2$. Notice that $m_1 + \|N(\hat{\jmath}, g)\| + 1 < \frac{1}{\alpha}$ implies that $\|N(\hat{\jmath}, g)\| - c - \alpha \|N(\hat{\jmath}, g)\|^2 < m_1 - c - \alpha m_1^2$. Hence it is a better response for $\hat{\jmath}$ to cut all his links and link to j_1 getting to observe m_1 agents. So he does this and receives a payoff equal to $m_1 - c - \alpha m_1^2$.

Call this new network g'. Notice that $\hat{\jmath}$ and all the agents in $N(j_1, g)$ who observed $\hat{\jmath}$ in g now observe exactly $m_1 \geq n^* + 1$ agents in g' as they were all observed by j_1 in g. However nothing has changed for the other agents observed by j_1. So g' is identical to g with the exception of the move made by $\hat{j}$. In particular for each agent $\ell \in N(j_1, g')$ such that $\|N(\ell, g')\| \leq n^* - 1$ it is still true that $\|N(j_1, g')\| + \|N(\ell, g')\| + 1 < \frac{1}{\alpha}$. We can then replicate this argument until no agent observes fewer than n^*. Now by Lemma 4 at least one player who observes exactly n^* agents emerges. This completes the proof. ■

We are now ready to prove the main result of the paper.

B.2 Proof of Proposition 4

Proposition 4 *Assume that* $P > n^*$, *that* $(\alpha, c) \in \mathcal{P} \setminus \left\{(\alpha, c) : \frac{1}{6} \leq \alpha < \frac{1}{5}, 3 - 15\alpha < c \leq \frac{1}{4\alpha} - 1\right\}$ *and in addition if* $n^* > \frac{1}{2\alpha}$ *that there exists no integer* k *such that* $P = (k+1)\, n^* - 1$. *Then in finite time each agent belongs to a starred wheel of dimension* n^*. *From then on, while the network might change, it remains a constellation of starred-wheels of dimension* n^* *in each period.*

Proof. We find a path of the better-reply dynamics (4) such that starting from an arbitrary network it leads in finite time to a constellation of starred wheels of dimension n^* through the following steps:

1. From an arbitrary network in finite time we eliminate all agents whose payoff is less that $1 - \alpha$. In the resulting network there are stand-alones or terminals or agents who do not observe fewer than $\underline{n}$ or more than $\overline{n}$ agents.

Proof. Take an arbitrary network g. Consider the set of the agents $Q(g) = \{i \in P |\, \pi_i(g) < 1 - \alpha\}$. Take the agent with the highest index in $Q(g)$ and call this agent j. So j's better response is to cut all his links. Thus j becomes a stand-alone (or a terminal in the case he was observed by someone else) and receives $1 - \alpha$.

Call this new network g'. For every agent $i \notin Q(g)$ such that $j \in N(i, g)$ if j cuts all his links then $\pi_i(g') \geq 1 - \alpha$ since $\underline{n} = 2$. So $\|Q(g')\| \leq \|Q(g)\| - 1$. Replicate this argument until all the agents receive a payoff greater or equal to $1 - \alpha$. Then by Proposition 1, part ii) all the agents who are neither stand-alones nor terminals observe between $\underline{n}$ and $\overline{n}$. ■

2. Let us eliminate the terminals from the network in finite time.

Proof. Consider a terminal agent i. By definition there exists an agent $j \in P \backslash \{i\}$ who observes i, i.e. $i \in N(j,g)$. We show that if i links to j then i is better off. Call the new network g'. When i links to j then $N(i,g') = N(j,g)$ as i was a terminal in g. Since i has only one link in g', $\|N(i,g')\| = \|N(j,g)\| \leq \overline{n}$ and $\pi_j(g) \geq 1-\alpha$ we find $\pi_i(g') \geq 1-\alpha$. Replicate this argument until all terminals connect. ∎

3. We now show that in finite time stand-alones connect.

a. *If there exists an agent $i \in g$ such that $\underline{n} \leq \|N(i,g)\| \leq \overline{n}-1$, i.e. if there exists an agent i who observes no more than $\overline{n}-1$ and no fewer than $\underline{n}$ then we eliminate all the stand-alones in finite time.*

Proof. Any stand-alone has a better response of linking to i. By so doing he observes $\|N(i,g)\|+1 \leq \overline{n}$ and receives a payoff greater than or equal to $1-\alpha$. Notice that agent i still observes the same agents as in g. ∎

b. *If $\|N(j,g)\| = \overline{n}$ holds for all $j \in g$ such that $\|N(j,g)\| > 1$ we eliminate all the stand-alones in finite time.*

Proof. The assumption $(\alpha,c) \in \mathcal{P} \setminus \left\{(\alpha,c) : \frac{1}{6} \leq \alpha < \frac{1}{5}, 3-15\alpha < c \leq \right.$ $\left. \leq \frac{1}{4\alpha}-1\right\}$ is equivalent to $n^* \leq \overline{n}-1$ and $\underline{n}=2$. Let us divide the proof into two cases.

First, let us assume that there exist an agent i such that $\mu_i^d(g) = 1$, i.e. i has only one link. Therefore i is the best off among all the agents he observes. So by Lemma 2 a wheel $W(\overline{n})$ forms among all the agents in $N(i,g)$. Call this new network g'. Take now an agent j belonging to this wheel $W(\overline{n})$. As $n^* \leq \overline{n}-1$ agent j in the wheel has a better response of cutting his link and linking to agent r in the wheel such that $d(r,j;g') = \overline{n}-1$. So he does this and observes exactly $\overline{n}-1$ agents improving his payoff. Now we are back to the case considered in part a) of this Step and the proof is complete.

Second assume that $\mu_i^d(g) \neq 1$ for all i. Fix one agent and call him i_1. Consider an agent $i_2 \in \arg\max_{\ell \in N(i_1,g)} d(i_1, \ell; g)$ so i_2 is farthest away from i_1 among those who are observed by i_1. Note that i_1 does not use any of i_2's links to observe anyone else. Agent i_2 improves his payoff by cutting all his links (which are more than one) and linking to i_1 directly. Now agent i_2 has one link only and we are back in the previous case of part b). ■

4. Now all the players in the network only observe a number of agents between $\underline{n}$ and $\overline{n}$. From any such network in finite time a player that observes n^* agents emerges.

Proof. Take the agent who observes the maximum number of agents $i_1 \in \arg\max_{j \in P} \|N(j,g)\|$. If $\|N(i_1,g)\| = n^*$ the proof is complete. Take $i_2 \in \arg\max_{j \in \{\ell \in P:\, \|N(\ell,g)\| < n^*\}} \|N(j,g)\|$, so i_2 is the agent who observes the maximum number of agents among those who observe fewer than n^*. If i_2 does not exist then by Lemma 4 at least one player who observes exactly n^* agents will emerge. Let $m_1 := \|N(i_1,g)\|$ and $m_2 := \|N(i_2,g)\|$ whenever it exists.
Let us consider the following 3 cases.

a. *Assume that $m_1 = m_2$. In finite time one player observing n^* emerges.*

Proof. If $m_1 = m_2$ then all the players observe fewer than n^* agents. Hence by Lemma 3 at least one player observing exactly n^* agents emerges. ■

b. *Assume that $m_1 + m_2 + 1 < \frac{1}{\alpha}$ and $m_1 \geq n^* + 1$. Then in finite time one agent observes exactly n^* agents.*

Proof. Assume first that some agents who observe fewer than n^* are observed by i_1. Note that for all agents j with $\|N(j,g)\| \leq n^* - 1$ we have $\|N(j,g)\| \leq m_2$. In particular for all $\hat{\jmath} \in N(i_1,g)$ such that $\|N(\hat{\jmath},g)\| \leq n^* - 1$ we have $m_1 + \|N(\hat{\jmath},g)\| + 1 < \frac{1}{\alpha}$. So by Lemma 5 in finite time at least one player observing exactly n^* agents emerges.

If instead for all j such that $\|N(j,g)\| \leq n^* - 1$ we have $j \notin N(i_1,g)$ then $\|N(k,g)\| \geq n^* + 1$ for all agents $k \in N(i_1,g)$ and hence by Lemma 4 at least one player who observes exactly n^* agents will emerge. ■

c. *Assume* $m_1 + m_2 + 1 \geq \frac{1}{\alpha}$ *and* $m_1 \geq n^* + 1$. *To show: one agent that observes* n^* *players will emerge in finite time.*

Proof. Let $\hat{P} := N(i_1, g) \cup N(i_2, g)$. Note that $\left\|\hat{P}\right\| \geq m_1 > n^*$. Also note that $m_1 + m_2 + 1 \geq \frac{1}{\alpha}$ implies $\pi_{i_1}(g) \leq m_2 + 1 - c - \alpha(m_2 + 1)^2$. So agent i_1's better response is to link to i_2 cutting all his original links. Let him do it and call the new network g'.

Note that $N(i_1, g') = \{i_1\} \cup N(i_2, g)$ and $\|N(i_1, g')\| = m_2 + 1 \leq n^*$. If $\|N(i_1, g')\| = n^*$ then the proof is complete. If $\|N(i_1, g')\| \leq n^* - 1$ then i_1 is best off among the agents in $N(i_1, g')$. So by Lemma 2 a wheel $W(m_2 + 1)$ forms possibly with other agents observing it. Call this new network g''.

We now show that the wheel expands so that no agent in $\hat{P}$ can be directly linked to the (enlarged) wheel and observe fewer than n^* overall. Take any such agent and call him player i. By definition of i there exists an agent i' in the wheel such that $d(i, i'; g) = 1$. As $m_2 + 1 \leq n^* - 1$ agent i'' in the wheel whose only link is to i' has a better response of cutting his link and linking to i. So the wheel expands by 1. Call this new network g'''. Assume further that i observes some agents who do not belong to the $W(m_2 + 1)$. As $\|N(i, g'')\| \leq n^* - 1$ also $\|N(i, g''')\| \leq n^* - 1$. Notice that for all j in the wheel $N(j, g''') = N(i, g''')$. In particular agent i' who belongs to the wheel $W(m_2 + 1)$ is best off among the agents in $N(i', g''')$ so by Lemma 2 a wheel of dimension $\|N(i', g''')\|$ forms. Repeat until an agent gets to observe n^* (which completes the proof) or no such i, that is linked to the wheel and observes fewer than n^* agents overall, exists anymore. Call this new network $\check{g}$ and the wheel dimension $\check{m} := \|N(i, g''')\|$.

We now enlarge the wheel further so as to partition the agents in $\hat{P}$ into those who belong to the wheel and observe fewer than n^* players and those who observe more than n^* players. So we eliminate all the agents observing fewer than n^* agents who do not belong to the wheel. Note that in $\check{g}$ if agent j does not belong to the $W(\check{m})$ then either $\|N(j, \check{g})\| \geq n^* +$

1 or $\|N(j,\breve{g})\| \leq m_2$ by definition of m_2. So $\|N(j,\breve{g})\| \leq \breve{m}$ whenever $\|N(j,\breve{g})\| \leq n^*-1$ for all $j \in \hat{P}$. Take agent h_1 who does not belong to the $W(\breve{m})$ such that $\|N(h_1,\breve{g})\| \leq n^*-1$. As $\|N(h_1,\breve{g})\| < \breve{m}$ and $\breve{m} \leq n^*-1$ then h_1 has a better response of cutting all his links and linking to the $W(\breve{m})$ (from outside). So he does this and observes $\breve{m}+1$ agents.

If $\breve{m}+1 = n^*$ then the proof is complete. Otherwise take h_2 among the agents not observed by h_1 in the new network such that h_2 observes fewer than n^*. By the same reasoning applied to h_1, agent h_2 improves his payoff by cutting all his links and linking to h_1. Repeat this argument until either some agents observe n^* or there exist no agents in the network who do not belong to the wheel and observe fewer than n^* players. If some agents observe n^* then the proof is complete. Otherwise call $\breve{h}$ the last agent who moved and note that he is best off among all those he observes as he observes the most and still observes fewer than n^* players with one link only. So by Lemma 2 a wheel forms among the agents observed by $\breve{h}$. Call this new network $\hat{g}$ and the (new) wheel dimension $\hat{m} \leq n^*-1$.

Note that in $\hat{g}$ all the agents in $\hat{P}$ either observe more than n^* agents or belong to the wheel $W(\hat{m})$ and thus observe $\hat{m} \leq n^*-1$. Take $j_0 = \arg\min_{h \in \left\{\ell \in \hat{P}:\, \|N(\ell,\hat{g})\| > n^*\right\}} \|N(h,\hat{g})\|$. Let $m_0 := \|N(j_0,\hat{g})\|$.

Assume first $\hat{m}+m_0+1 < \frac{1}{\alpha}$ then i_1 who belongs to the wheel $W(\hat{m})$ has a better response of cutting his only link and linking to j_0. By so doing i_1 breaks the wheel. As in $\hat{g}$ everyone who observed fewer than n^* agents belonged to the wheel $W(\hat{m})$ now there exists no agent $\ell \in \hat{P}$ such that $\|N(\ell,\hat{g})\| < n^*$. So by Lemma 4 one agent observing n^* agents surely emerges.

Assume instead $\hat{m}+m_0+1 \geq \frac{1}{\alpha}$ then j_0 cuts all his links and links to the wheel. By definition of m_0 for all $j' \in \hat{P}$ with $\|N(j',\hat{g})\| \geq n^*$ we have $\hat{m}+\|N(j',\hat{g})\|+1 \geq \frac{1}{\alpha}$. So all $j' \in \hat{P}$ with $\|N(j',\hat{g})\| \geq n^*$ have the same better response and link to the wheel. As $\hat{m} \leq n^*-1$ and $\left\|\hat{P}\right\| > n^*$ the wheel enlarges to $n^*-\hat{m}-1$ (peripheral) agents and an agent observing n^*

players will emerge. ■

5. Now in the network at least one agent observes n^* players. In finite time all the agents in the network observing n^* players belong to a $SW(n^*, m)$.

Proof. The proof is divided in two parts.

a. *If a player observes n^* agents then this player belongs to a $W(n^*)$ in finite time.*

Proof. By assumption there exists a player who observes n^* agents. Take an agent who observes n^* agents and call him i_1^*, that is $\|N(i_1^*, g)\| = n^*$. Consider $i_2^* \in \arg\max_{\ell \in N(i_1^*, g)} d(i_1^*, \ell; g)$, i.e. consider an agent who is farthest away from i_1^*. We already know that then i_1^* does not use any of i_2^*'s links to observe anyone else. Agent i_2^* improves his payoff (in fact it is his best response) by cutting all his links and linking to i_1^* directly. So he does this.

Call this new network g'. Now i_2^* enjoys the maximum payoff attainable: he observes n^* agents paying the cost of one single link. Notice that i_2^* is best off in $N(i_2^*, g')$ and that $\|N(i_2^*, g')\| = n^*$ so by Lemma 2 a wheel $W(n^*)$ forms. Replicate this argument until there exist no agents observing n^* players who do not belong to a wheel $W(n^*)$. ■

b. *If all the agents who observe n^* players belong to a wheel $W(n^*)$ and there exists an agent who does not belong to a $W(n^*)$ then a starred wheel of dimension n^* emerges in finite time.*

Proof. Call i the agent who does not belong to a $W(n^*)$. Let $m_i := \|N(i, g)\|$. Note that $m_i \neq n^*$.

We claim that agent i has a better response of cutting all his links and linking to the wheel from outside since by assumption of the Proposition if $n^* > \frac{1}{2\alpha}$ then there exists no integer d such that $P = (d+1)n^* - 1$.

Assume first that either $m_i > n^*$ or $m_i < n^* - 1$ then $\pi_i(g) \leq n^* + 1 - c - \alpha(n^* + 1)^2$. So i's better response is to cut all his links and to link to a wheel of dimension n^* forming a starred wheel of the same dimension.

Assume instead that $m_i = n^* - 1$. Note that $n^* - 1 - c - \alpha(n^* - 1)^2 \gtreqless n^* + 1 - c - \alpha(n^* + 1)^2$ is equivalent to $n^* \gtreqless \frac{1}{2\alpha}$. So if $n^* \leq \frac{1}{2\alpha}$ any agent observing $n^* - 1$ has a better response of linking to a $W(n^*)$ forming a starred wheel of the same dimension. Assume now that $n^* > \frac{1}{2\alpha}$. If there exists another agent j who does not observe n^* then j's better response is to cut all the links and link to i getting to observe n^* agents. Then a new wheel will emerge by part a) of this Step. If no such agent j exists then there exists an integer d such that $P = (d+1)n^* - 1$ which is a contradiction. ■

6. In finite time the better-reply dynamics converges to a constellation of h starred wheels of dimension n^*, $h = 1, ..., \left\lfloor \frac{P}{n^*} \right\rfloor$.

Proof. By Proposition 2 a constellation of starred wheels of dimension n^* is a PSNES which is absorbing for the better-reply dynamics (4). ■

This concludes the proof of the Proposition. ■

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