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(Working Papers)

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THE RISKINESS OF CORPORATE BONDS

by Marco Taboga*

Abstract

When the riskiness of an asset increases, then, arguably, some risk-averse agents that were previously willing to hold on to the asset are no longer willing to do so. Aumann and Serrano (2008) have recently proposed an index of riskiness that helps to make this intuition rigorous. We use their index to analyze the riskiness of corporate bonds and how this can change over time and across rating classes and how it compares to the riskiness of other financial instruments. We find statistically significant evidence that a number of financial and macroeconomic variables can predict time-variation in the riskiness of corporate bonds, including in ways one might not expect. For example, a higher yield-to-maturity lowers riskiness by reducing the frequency and the magnitude of negative holding-period returns.

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1 Introduction

Running a Google search for the phrase "corporate bonds are riskier than government bonds" we found more than 400 documents containing the exact phrase, including newspaper articles, academic books, scholarly articles, university course handouts, educational websites and investment bank brochures. Although such statement appears to be common wisdom, there does not seem to be much consensus on the exact meaning of the adjective "riskier" utilized to compare corporate and government bonds. While some do not clarify what they mean by riskier, others specify that corporate bonds are riskier because there is a greater chance that the underlying loan does not get repaid in full; others explain that, historically, corporate bond investors have experienced more volatile returns; others observe that the prices of corporate bonds are more sensitive to economic activity and interest rate movements; others state that corporate bonds are riskier because they are less liquid than government bonds. These explanations are often complemented by further information: for example, some state that only investors who are more risk tolerant can invest in corporate bonds and others say that corporate bonds are riskier than government bonds, but they are less risky than stocks.

Such wealth of characterizations of the relative riskiness of corporate and government bonds reflects, among other things, the lack of a widely agreed definition of risk. As we will discuss in more detail later, several definitions of risk have been proposed in the literature, but none of them seems exempt from critiques and general enough to gain widespread acceptance. This is at odds with the fact that risk aversion is, instead, a very well-established concept: in fact, most economists are used to think of risk aversion as the concavity of a utility function, defined in a von-Neumann Morgenstern expected utility framework. Hence, trying to link the concept of risk to that of risk aversion seems a very promising avenue to be explored in the search for a widely accepted definition of risk. Recently, Aumann and Serrano (2008) have given an important contribution in this direction. They have provided a characterization of risk that is based on a simple but powerful idea: a gamble is riskier when there are more risk averse agents that do not accept it. Building on this idea, they have proposed an intuitively appealing measure of riskiness, dubbed 'economic index of riskiness', defined as follows: the index of riskiness $R(g)$ of a gamble $g$ is the reciprocal of the absolute risk aversion (ARA) of an individual with constant ARA who is indifferent between taking and not taking the gamble $g$. They prove that the index of riskiness thus defined can be given a rigorous axiomatic characterization and satisfies a number of important properties, among which monotonicity with respect to stochastic dominance and sub-additivity.

We discuss statistical estimation of Aumann and Serrano's (2008) index of riskiness. Thanks to its computational tractability, the index is also very amenable to statistical estimation. We propose a generalized method of moments (GMM) estimator of the index of riskiness, that can be used when we observe multiple draws from the payoff distribution of a gamble. Furthermore, we introduce a conditional version of the index, to accommodate the possibility that the distribution of payoffs be conditionally dependent on some set of predetermined variables. Roughly speaking, we allow the index of riskiness to be a function $R(g; X)$ of a vector of variables $X$ that can be observed prior to the realization of the payoff $g$. The theory of GMM estimation of the index of riskiness we propose is used in this paper to study the riskiness of corporate bonds. However, the theory is fully general and could be used to analyze holding-period returns on any other asset class.

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Estimation of the index of riskiness allows us to give rigorous empirical content to statements like "corporate bonds are riskier than government bonds". More generally, we study the riskiness of corporate bonds, how it varies across rating classes, how it compares to the riskiness of other financial instruments and how it relates to a set of macroeconomic and financial variables. The analysis is focused on the returns obtained in the past by replicating some popular indices of investment grade US corporate bonds, broadly subdivided by categories of rating.

First, we estimate the unconditional riskiness of the bond indices and we make comparisons between corporate bonds belonging to different rating classes, stocks and government bonds. We do not find statistically significant differences between the riskiness of different rating classes, except for the A class, which is significantly riskier than than the AA and AAA classes. The estimated riskiness is not monotonically decreasing with the rating class, as BBB-rated bonds are estimated to be less risky than A-rated bonds. We also test the hypothesis that corporate bonds and government bonds are equally risky and we are not able to reject the hypothesis at a high confidence level, except again for the A class. Stocks are instead significantly riskier than corporate bonds, their index of riskiness being more than ten times higher. Finally, departures from normality increase estimated riskiness, although the increase is not statistically significant in our sample.

In the second part of our empirical analysis, we study the conditional dependence of riskiness on a number of macroeconomic and financial variables. Estimating univariate models, we find statistically significant evidence that the riskiness of corporate bonds varies in tandem with interest rates, the TED spread (a measure of funding liquidity equal to the difference between the LIBOR and the T-Bill rate), industrial production, the past trend and volatility of the stock market, the yield-to-maturity of corporate bonds and an adaptive forecast of the conditional volatility of corporate bond returns. Estimating multivariate models, we encounter statistical problems that resemble those found in the literature on the predictability of stock and bond returns (see e.g. Aït-Sahalia and Brandt - 2001 for a discussion): while several variables individually predict riskiness, it is hard to build statistically sound multivariate models with more than few predictors. Despite these difficulties, a number of parsimonious multivariate models allow to highlight several features of the dynamic behavior of riskiness. As one would expect, riskiness is higher when the conditional volatility of returns is higher. However, volatility alone is not sufficient to capture all the time-variation in riskiness. Other variables, considered together with volatility, are statistically significant predictors of riskiness. One of them is the yield-to-maturity of corporate bonds, which is negatively associated with riskiness. In our sample, a high yield-to-maturity predicts high positive holding period returns and lower riskiness. This finding is worth stressing. Aumann and Serrano’s (2008) index of riskiness has one important merit with respect to more traditional measures of risk such as variance: the economic index of riskiness takes into account the fact that expected return is an important component of riskiness; if, ceteris paribus, you increase the expected return of an asset, its riskiness diminishes, because you are decreasing the probability of facing low-payoff states. Roughly speaking, our empirical findings suggest that a higher yield-to-maturity decreases the probability of disappointing returns on corporate bonds, hence diminishing their riskiness. Besides forecasts of conditional volatility and the yield-to-maturity, some macroeconomic variables provide additional forecasting power, especially for lower-rated bonds. Inflation seems to be the most important: corporate bonds tend to be riskier in periods of high inflation. This is consistent with the evidence provided, inter alia, by Gordon (1982), Wadhwani (1986) and Valckx (2004), who find various channels through which high inflation can increase the probability of corporate defaults and of disappointing returns on corporate bonds.

Predictable changes in the probabilistic distributions of asset returns have been studied by financial economists for many decades. Typically, the focus has been on predicting the first two
conditional moments of the distributions, namely expected returns (e.g.: Ferson and Harvey - 1991 and Cochrane - 1991) and variances (e.g.: Bollerslev -1986, Schwert - 1989, Whitelaw - 1994). Our paper is naturally connected with this literature, but rather than studying the predictability of individual moments, we study the predictability of riskiness, a characteristic of the distribution of asset returns that is closely related to the choices of risk-averse agents and can, in principle, depend on all moments of the distribution. From this point of view, our paper is similar in spirit to a paper by Aït-Sahalia and Brandt (2001): they study the optimal use of information in portfolio allocation decisions and they argue that, rather than predicting individual moments of the distribution of asset returns in order to form optimal portfolio allocations, it is better to directly predict the optimal portfolio allocations themselves, since they may have a non-trivial dependence on many moments of the distribution, as well as on preference parameters.

While corporate bonds have been the subject of many studies, attention has focused mainly on the determinants of corporate bond spreads (e.g.: Kwan - 1996, Duffee - 1998, Collin-Dufresne et al. - 2001, Campbell and Tuokler - 2003) and on expected returns (e.g.: Lamont - 1998, Baker et al. - 2003). We are not aware of any study that specifically dealt with the riskiness of corporate bonds and its predictability. Furthermore, to the best of our knowledge, our paper is the first to deal with statistical estimation of Aumann and Serrano’s (2008) index of riskiness and to use it to conduct an empirical investigation of the riskiness of an asset class.

The rest of the paper is organized as follows: Section 2 introduces Aumann and Serrano’s (2008) index and discusses its interpretation; Section 3 discusses GMM estimation of the index of riskiness; Section 4 presents the empirical analysis; Section 5 discusses directions for further research; Section 6 concludes.

2 Aumann and Serrano’s index of riskiness

Although measuring and understanding financial risk is on top of the agenda of financial economists and policy makers, it is often unclear how risk should be defined and measured and how it could be monitored and predicted.

Since the seminal contributions of Markowitz (1952) and Tobin (1958), variance has been extensively used as a measure of the riskiness of a payoff prospect, to be traded off with expected return, a measure of attractiveness of the payoff itself. Casting the problem of portfolio selection in terms of a trade-off between mean and variance of returns has had an enormous success, due also to the analytical tractability and clear intuitive meaning of this framework. However, many studies (e.g.: Dybvig and Ingersoll - 1982, Bigelow - 1993, Jarrow and Madan - 1997, Maccheroni et al. - 2009) have highlighted the fact that an investor ranking investment prospects based on their variance and expected return may violate non satiation\(^2\), one of the basic tenets of economic rationality. Furthermore, outside a mean-variance framework, using variance alone as a measure of the riskiness of an investment prospect may be very misleading. Consider, for example, two gambles \(g\) and \(h\), such that \(g = h + c\), where \(c\) is a strictly positive constant. Clearly, any rational decision-maker would strictly prefer \(g\) to \(h\), but, according to a ranking based on variance alone, \(g\) and \(h\) are equally risky. This is an important shortfall of using variance to measure riskiness: often, a ranking of riskiness based on variance does not provide any meaningful indication on how rational risk-averse agents actually rank gambles.

Several other measures of riskiness have been proposed (Value-at-Risk, semi-variance, inverse Sharpe ratio, expected shortfall, etc.), but none of them is exempt from critiques and no consensus

\(^2\)The main intuition behind this result is that variance increases not only with negative deviations from the mean, but also with positive deviations; hence, an investor that ranks investment prospects based on variance (and mean) may sometimes prefer less to more, because she dislikes investment prospects that generate positive deviations from the mean.
has been reached yet on a first best. Some authors (e.g.: Artzner et al. - 1999 and Föllmer and Schied - 2002) have proposed sets of desirable properties (so called 'coherency' axioms) that a risk measure should satisfy. However, others (see, again, Aumann and Serrano - 2008 and the references cited therein) have shown that risk measures satisfying such 'coherency' properties may fail to satisfy other equally desirable properties such as monotonicity with respect to first- and second-order stochastic dominance.

Recently, Aumann and Serrano (2008) have proposed an intuitively appealing measure of riskiness, dubbed 'economic index of riskiness', that enjoys several important properties and is based on the well-established concept of risk aversion, as defined in a von-Neumann Morgenstern expected utility framework. Aumann and Serrano (2008) define the riskiness \( R(g) \) of a gamble\(^3\) \( g \) as the reciprocal of the absolute risk aversion (ARA) of an individual with constant ARA who is indifferent between taking and not taking the gamble \( g \). Their paper contains two main contributions. First, they show that ordering gambles according to \( R(g) \) is rationalizable under a simple and intuitively appealing set of behavioral axioms. Second, they provide a straightforward way of calculating \( R(g) \). Precisely, they prove that the riskiness of a gamble \( g \) is the unique strictly positive number \( R = R(g) \) satisfying:

\[
E \left[ \exp \left( -\frac{g}{R} \right) \right] = 1
\]  

(1)

where \( E \) denotes expected value.

In other words, the riskiness \( R \) is equal to the inverse of the Laplace transform of the distribution of \( g \) evaluated at the point 1.

The index of riskiness thus defined satisfies the two axioms of duality and positive homogeneity, defined below.

It is assumed that agents' preferences are described by von-Neumann Morgenstern utility functions that are strictly monotonic, strictly concave and twice continuously differentiable. An agent with utility function \( u \) accepts gamble \( g \) at wealth \( w \) if:

\[
E[u(w + g)] > u(w)
\]

(2)

An agent \( i \) is said to be uniformly no less risk averse than agent \( j \) if, whenever \( i \) accepts a gamble at some wealth, then \( j \) accepts that gamble at any wealth. An agent \( i \) is said to be uniformly more risk averse than agent \( j \) if \( i \) is uniformly no less risk averse than \( j \) and \( j \) is not uniformly no less risk averse than \( i \).

Let \( g \) and \( h \) be two gambles. An index of riskiness \( R() \) satisfies duality if, whenever \( R(g) > R(h) \), \( i \) is uniformly more risk averse than \( j \) and \( i \) accepts \( g \) at \( w \), then \( j \) accepts \( h \) at \( w \). Stated differently, if a gamble \( g \) is riskier than \( h \) according to the index \( R() \) and agent \( i \) accepts the riskier gamble \( g \), then, a fortiori, agent \( j \), who is less risk averse, accepts the less risky gamble \( h \). An index of riskiness \( R() \) satisfies homogeneity if, for any positive number \( t \) and any gamble \( g \), \( R(tg) = tR(g) \). Roughly speaking, when you double a bet you also double its riskiness.

Aumann and Serrano’s (2008) index of riskiness satisfies the two axioms of duality and positive homogeneity. Most notably, any index of riskiness satisfying these two axioms is a positive multiple of Aumann and Serrano’s (2008) index. This is a remarkable result, because the two axioms are very weak, i.e. they require very little from an index of riskiness. The index also enjoys other important properties. While we refer the reader to the original paper for an exhaustive list, let us mention a very important one: the index is also compatible with an ordering of gambles based on first- or second-order stochastic dominance; if a gamble \( g \) stochastically dominates a

\(^3\)A gamble \( g \) is defined to be a real-valued random variable with positive expectation and some negative values (i.e. \( E[g] > 0 \) and \( P[g < 0] > 0 \)).
gamble \( h \), either under the first- or the second-order criterion, then, according to the index \( R \), \( g \) is less risky than \( h \) (i.e. \( R(g) < R(h) \)).

Hart (2008) has proposed a different set of behavioral axioms, from which he has also been able to derive Aumann and Serrano’s (2008) index of riskiness. He has introduced a very simple “riskier than” order between gambles, based on the intuition that a riskier gamble is rejected more often: given a set of risk-averse agents, a riskier gamble is rejected by a greater number of agents. Hart (2008) has proved that also his "riskier than" order can be represented by Aumann and Serrano’s (2008) index.

The rest of the paper will be concerned with estimating how the riskiness of financial assets changes in time (as predictors of the conditional distribution of returns change) and in space (considering different financial instruments). It is then important to discuss how to interpret changes in the index of riskiness. The first interpretation is ordinal. Suppose you observe an increase in the index of riskiness of an asset. The increase implies that some risk-averse agents that were previously willing to hold the asset, now are no longer willing to hold it. This squares exactly with the idea that risk is what risk-averse agents avert. The second interpretation is cardinal. Suppose the index of riskiness of an asset doubles. Then, if a certain agent was indifferent between investing or not investing 1$ in the asset, now (after the increase) you have to halve the investment to 0.5$ to make him indifferent again. This is a consequence of the positive homogeneity of the index. There are also other more sophisticated interpretations, related to the behavior of Constant Relative Risk Aversion (CRRA) agents. Since these interpretations are not directly relevant to the empirical study presented below, we do not treat them here and refer the reader to the discussion in Aumann and Serrano (2008).

3 GMM estimation of the index of riskiness

3.1 Estimating unconditional riskiness

In this section we discuss GMM estimation of the index of riskiness.

GMM estimation of the index of riskiness \( R \) exploits the population moment condition derived from (1):

\[
E[m(g; R)] = 0
\]

where

\[
m(g; r) = \exp\left(-\frac{g}{r}\right) - 1
\]

The following proposition, proved in the Appendix, presents the GMM estimator of \( R \) and states sufficient conditions for its consistency.

**Proposition 1 (Consistency - Unconditional riskiness)** Let \( \{g_t\}_{t=1,...,T} \) be the first \( T \) elements of an ergodic and stationary sequence of draws from the distribution of \( g \). Let \( E[g] > 0 \) and \( P[g < 0] > 0 \), so that \( g \) has an index of riskiness \( R \in \mathbb{R}_{++} \). Let the support of \( g \) be bounded. Let \( u, l \in \mathbb{R}_{++} \) and \( l < R < u \). Let \( \hat{R} \) be the GMM estimator of \( R \) defined by:

\[
\hat{R} = \arg \min_{r \in [l, u]} [Q_T(r)]^2
\]

where

\[
Q_T(r) = \frac{1}{T} \sum_{t=1}^{T} m(g_t; r)
\]

Then \( \hat{R} \) converges in probability to \( R \) as \( T \) tends to infinity.
Note that the proposition assumes that the parameter space is bounded and has strictly positive elements ($\bar{R} \in [l, u]$). This is necessary because $r = 0$ is a point of discontinuity of $m(g; r)$; furthermore, also $r = \infty$ is always a solution of $E[m(g; r)] = 0$. In practice, the assumption that $\bar{R} \in [l, u]$ is not restrictive, because $l$ and $u$ can be respectively set equal to any arbitrarily small and large positive numbers such that $[l, u]$ certainly includes $\bar{R}$ according to the researcher’s judgement. Note also that, whenever the empirical distribution defined by the $T$ draws $\{g_t\}_{t=1,\ldots,T}$ has positive expectation and negative values with positive probability, there exists a finite and strictly positive solution of the sample moment condition\(^4\):

$$Q_T(r) = 0$$ (7)

In this case, the minimum in (5) is attained at 0 and it is equivalent to solve either (5) or (7).

The following proposition gives a set of sufficient conditions for the asymptotic normality of the estimator $\bar{R}$.

**Proposition 2 (Asymptotic normality - Unconditional riskiness)** Let all the conditions of Proposition 1 be satisfied. Suppose that, as $T$ tends to infinity, $\frac{1}{\sqrt{T}} \sum_{t=1}^{T} m(g_t; R)$ converges in distribution to a multivariate normal distribution with mean zero and strictly positive variance $S$. Suppose also that $D = E \left[ \frac{dm(g_t; R)}{dr} \right] \neq 0$.

Then, $\sqrt{T} \left( \bar{R} - R \right)$ converges in distribution to a normal distribution with mean zero and variance $V = \frac{S}{D^2}$.

Furthermore, if there exists a random variable $\tilde{S}$ converging in probability to $S$ as $T$ tends to infinity, then the random variable $\tilde{V}$ defined by:

$$\tilde{V} = \frac{\tilde{S}}{D^2}$$ (8)

$$\tilde{D} = \frac{1}{T} \sum_{t=1}^{T} \frac{dm(g_t; \bar{R})}{dr}$$ (9)

converges in probability to $V$.

The assumption that $\frac{1}{\sqrt{T}} \sum_{t=1}^{T} m(g_t; R)$ converges in distribution to a multivariate normal distribution with mean zero and strictly positive variance $S$ could be replaced by sets of more primitive assumptions. For example, in the case $\{g_t\}_{t=1,\ldots,T}$ is a sequence of independent draws from the distribution of $g$, the aforementioned assumption could be replaced by the assumptions needed for a Central Limit Theorem for independent draws to hold (e.g.: Hayashi - 2000). However, Proposition 2 allows for greater generality and encompasses, for example, the case of correlated draws that satisfy Gordin’s conditions (see again Hayashi - 2000).

### 3.2 Testing for equal riskiness

In this subsection we discuss a test of the hypothesis that two gambles are equally risky.

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\(^4\)Existence and uniqueness of a finite and strictly positive solution of the sample moment condition (7) is a trivial consequence of the fact that the population moment condition (3) has a unique finite and strictly positive solution for any gamble (Aumann and Serrano - 2008, Theorem A; see also Section 2 above): the empirical distribution defined by the $T$ draws $\{g_t\}_{t=1,\ldots,T}$ is a gamble if it has positive expectation and negative values with positive probability; as a consequence, if these two technical conditions are satisfied, Aumann and Serrano’s theorem readily applies also to the empirical distribution.
To conduct tests of equal riskiness, it is necessary to jointly estimate the indices of riskiness of two different gambles. Extending the results of the previous subsection to the case of joint GMM estimation of two indices is trivial, but the notation becomes cumbersome. Therefore, we just discuss the main features of joint estimation, without going into all the technical details. Once joint estimation is carried out, several asymptotically equivalent tests of the null of equal riskiness can be constructed. Here, we briefly present a $t$-test.

Let \( \{g_t\}_{t=1}^{T} \) and \( \{h_t\}_{t=1}^{T} \) be two jointly ergodic and stationary sequences of draws from the joint distribution of gambles \( g \) and \( h \). Let \( R_g \) and \( R_h \) be their respective indices of riskiness, satisfying the two population moment conditions:

\[
E \left[ m \left( g_t; R_g \right) \right] = 0 \\
E \left[ m \left( h_t; R_h \right) \right] = 0
\]

where \( m \left( g; r \right) \) is defined as in (4).

GMM estimators \( \widehat{R}_g \) of \( R_g \) and \( \widehat{R}_h \) of \( R_h \) are defined by:

\[
\left[ \widehat{R}_g, \widehat{R}_h \right] = \arg\min_{r_g \in [l_g,u_g]} \min_{r_h \in [l_h,u_h]} \left[ Q_{g,T} (r_g) \right]^2 + \left[ Q_{h,T} (r_h) \right]^2
\]

where \( l_g < R_g < u_g, \ l_h < R_h < u_h \) and

\[
Q_{g,T} (r_g) = \frac{1}{T} \sum_{t=1}^{T} m \left( g_t; r_g \right)
\]

\[
Q_{h,T} (r_h) = \frac{1}{T} \sum_{t=1}^{T} m \left( h_t; r_h \right)
\]

The two sample moments \( Q_{g,T} (r_g) \) and \( Q_{h,T} (r_h) \) in (12) could be given different weights or, more generally, could be used to construct a quadratic form with a positive definite weighting matrix. We focus here on the special case of an identity weighting matrix to keep the notation simple. Furthermore, in all empirical applications presented below the choice of the weighting matrix was irrelevant, because the sample moment conditions \( Q_{g,T} \left( \widehat{R}_g \right) = 0 \) and \( Q_{h,T} \left( \widehat{R}_h \right) = 0 \) were always satisfied exactly.

The hypothesis that the two gambles are equally risky can be tested by running a simple $t$-test of the null hypothesis that \( R_g - R_h = 0 \). Under the null, \( \widehat{R}_g - \widehat{R}_h \) is asymptotically normal with mean zero and asymptotic variance:

\[
\text{AVar} \left( \widehat{R}_g - \widehat{R}_h \right) = \text{AVar} \left( \widehat{R}_g \right) + \text{AVar} \left( \widehat{R}_h \right) - 2\text{ACov} \left( \widehat{R}_g, \widehat{R}_h \right)
\]

where AVar denotes asymptotic variance and ACov asymptotic covariance. The usual asymptotic theory (see Proposition 6 in the Appendix) can be used to compute a consistent estimator \( \text{AVar} \left( \widehat{R}_g - \widehat{R}_h \right) \) of \( \text{AVar} \left( \widehat{R}_g - \widehat{R}_h \right) \) and the $t$-statistic:

\[
t = \frac{\widehat{R}_g - \widehat{R}_h}{\sqrt{\text{AVar} \left( \widehat{R}_g - \widehat{R}_h \right)}}
\]

If the value of the $t$-statistic lies outside a pre-specified confidence interval, then the null hypothesis of equal riskiness is rejected.
3.3 Estimating conditional riskiness

There is ample empirical evidence that means, variances and higher order moments of asset returns are time-varying and predictable (for a survey, see Aït Sahalia and Brandt - 2001). It is then natural to ask how this predictable time-variation affects the index of riskiness of an asset. In this subsection (and in subsection 4.3) we develop a statistical framework that allows to answer this question.

As in most empirical finance studies, we assume that, at each period, the econometrician observes the realization of a vector of predictor variables $X_t$ (let it be an $N \times 1$ vector), before observing the realization of the asset return $g_t$. Time-variation in the conditional distribution of asset returns is driven by time-variation in $X_t$. More precisely, by conditioning on the information provided by $X_t$, one obtains the conditional distribution of $g_t$ at time $t$. For each realization of $X_t$, $g_t$ has a different conditional distribution, which depends on $X_t$ and is characterized by its own index of riskiness. Denote this conditional index of riskiness by $R(X_t)$. The functional dependence of $R$ on $X_t$ is in general unknown and depends on the joint distribution of $g_t$ and $X_t$. For every $X_t$, $R(X_t)$ satisfies the moment condition:

$$E \left[ \exp \left( -\frac{g_t}{R(X_t)} \right) - 1 \bigg| X_t \right] = 0$$

(17)

We propose a parametric approach to estimation of $R(X_t)$. We assume that there exist a $K \times 1$ ($K \leq N$) vector of parameters $\beta$ and a known function $\xi(X_t; b)$ of $X_t$ and $b$ ($b \in \mathbb{R}^K$), such that:

$$R(X_t) = \xi(X_t; \beta)$$

(18)

$\beta$ is the parameter to be estimated.

By trivial properties of the conditional expectation operator and by the law of iterated expectations, we can obtain $N$ moment conditions:

$$E [m(g_t, X_t; \beta)] = 0$$

(19)

where:

$$m(g_t, X_t; b) = X_t \left( \exp \left( -\frac{g_t}{\xi(X_t; b)} \right) - 1 \right)$$

(20)

The above moment conditions can be used to derive a GMM estimator of $\beta$, as illustrated in the following proposition.

**Proposition 3 (Consistency - Conditional riskiness)** Let $\{w_t\}_{t=1,...,T}$ be the first $T$ elements of an ergodic and stationary sequence of draws from the joint distribution of the $(N+1)$-dimensional random vector $w = [g X_t]^T$, having bounded support. Let $\xi(X_t; b)$ be a scalar function $\xi : \mathbb{R}^N \times \mathbb{R}^K \to \mathbb{R}_+$, continuous in $X_t$ and $b$ and such that $R(X_t) = \xi(X_t; \beta)$. Let $u$ be any positive number strictly larger than $\|\beta\|_1$, the $L^1$-norm of $\beta$. Assume that $b_1, b_2 \in \mathbb{R}^K$ and $b_1 \neq b_2$ imply $P(\xi(X_t; b_1) \neq \xi(X_t; b_2)) > 0$. Let $\hat{\beta}$ be the GMM estimator defined by:

$$\hat{\beta} = \arg \min_{b \in [-u,u]^K} \left[ \frac{1}{T} \sum_{t=1}^T m(g_t, X_t; b) \right]^T \hat{W} \left[ \frac{1}{T} \sum_{t=1}^T m(g_t, X_t; b) \right]$$

(21)

and $\hat{W}$ a $N \times N$ symmetric matrix that converges in probability to a symmetric and positive definite matrix $W$ as $T$ tends to infinity. Then $\hat{\beta}$ is a well-defined random variable and it converges in probability to $\beta$ as $T$ tends to infinity.
As in the unconditional case, the proposition assumes that the parameter space is bounded \((b \in [-u, u]^K)\). This is necessary because the equation \(E_m(g_t, X_t; b) = 0\) may have multiple solutions in \(\mathbb{R}^K\) (the compactification of \(\mathbb{R}^K\)): in fact, the equation is solved by any \(b\) such that \(\forall X_t, \xi(X_t; b) = \infty\). Again, this is not restrictive, because \(u\) can be set equal to any arbitrarily large positive number such that \([-u, u]^K\) certainly includes \(\beta\) according to the researcher’s judgement. The technical conditions imposed on the function \(\xi(X_t; b)\) are quite mild. In the next section we will propose a specification of \(\xi\) that satisfies all such conditions.

The following proposition gives a set of sufficient conditions for the asymptotic normality of the estimator \(b\).

**Proposition 4 (Normality - Conditional riskiness)** Let all the conditions of Proposition 3 be satisfied. Let \(\xi(X_t; b)\) be continuously differentiable in \(b\) for any \(X_t\). Suppose that, as \(T\) tends to infinity, \(\frac{1}{T} \sum_{t=1}^{T} m(g_t, X_t; \beta)\) converges in distribution to a multivariate normal distribution with mean zero and full-rank covariance matrix \(S\). Let \(D = E \left[ \frac{\partial m(g_t, X_t; \beta)}{\partial \beta} \right]\) have rank \(K\).

Then, \(\sqrt{T} \left( \hat{\beta} - \beta \right)\) converges in distribution to a multivariate normal distribution with mean zero and covariance matrix \(V\) equal to:

\[
V = \left( D^T W D \right)^{-1} \left( D^T W S W D \right) \left( D^T W D \right)^{-1}
\]  
(22)

Furthermore, if there exists a matrix \(\hat{S}\) converging in probability to \(S\) as \(T\) tends to infinity, then the matrix \(\hat{V}\) defined by:

\[
\hat{V} = \left( \hat{D}^T \hat{W} \hat{D} \right)^{-1} \left( \hat{D}^T \hat{W} \hat{S} \hat{W} \hat{D} \right) \left( \hat{D}^T \hat{W} \hat{D} \right)^{-1}
\]  
(23)

\[
\hat{D} = \frac{1}{T} \sum_{t=1}^{T} \frac{\partial m(g_t, X_t; \beta)}{\partial \beta}\n\]  
(24)

converges in probability to \(V\).

As in the unconditional case, the assumption that \(\frac{1}{T} \sum_{t=1}^{T} m(g_t, X_t; \beta)\) converges in distribution to a multivariate normal distribution with mean zero and full-rank covariance matrix \(S\) could be replaced by sets of more primitive assumptions. However, such assumption is very general and allows to tackle the case of correlated draws from the joint distribution of \(g_t\) and \(X_t\).

### 4 Empirical evidence

#### 4.1 The data

We use data on a set of popular indices of US corporate bonds constructed by Merril Lynch and available on Bloomberg. The dataset includes a general index, comprising investment grade bonds of all maturities and ratings, and four sub-indices that group bonds belonging to different rating classes: AAA, AA, A and BBB. For each index, we have two daily time series: total daily return on the index and average yield-to-maturity of the bonds belonging to the index. The series start on December 31st, 1988 and end on January 30th, 2009, the day when we last updated our dataset.

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5 For example, if \(X_t \geq 0\), it suffices to set one component of \(b\) equal to \(\infty\).

6 Merril Lynch identifies the indices by the following codes: C0A0 (general index), C0A1 (AAA), C0A2 (AA), C0A3 (A), C0A4 (BBB).
An accurate description of index rules and definitions is reported on Merril Lynch’s website www.mlindex.ml.com. We report here the main features of the indices. The general index (Merrill Lynch US Corporate Index) comprises all US dollar denominated corporate bonds publicly issued in the US domestic market that have an investment grade rating (based on an average of Moody’s, S&P and Fitch), at least one year remaining term to final maturity, a fixed coupon schedule and a minimum amount outstanding of $250 million (as of today). Index constituents are capitalization-weighted based on their current amount outstanding. Intra-month cash flows are reinvested daily, at the beginning-of-month 1-month Libid rate, until the last calendar day of the month, when the index is rebalanced. A sub-index for a specific rating class includes all the securities included in the general index and belonging to that rating class.

Other popular indices of US corporate bonds are those constructed by Barclays and by Citigroup: these indices are available at a daily frequency only for shorter time spans; furthermore, we were not able to find sub-indices of these indices for all the aforementioned rating classes. While the rules and definitions of these indices are slightly different from those of the Merril Lynch indices, their daily returns are highly correlated with the returns on Merril Lynch indices (the correlation coefficients for the general indices are higher than 90 per cent). We limit our analysis to the Merril Lynch indices because they are available for a longer time span and because their construction rules make them highly representative of the universe of liquid US corporate bonds.

In our empirical analysis we also use a Merril Lynch Index of US government bonds (Merrill Lynch US Treasury Index\(^7\)), which tracks the performance of US dollar denominated sovereign debt publicly issued by the US government in its domestic market. The index includes securities that have at least one year remaining term to final maturity, a fixed coupon schedule and a minimum amount outstanding of $1 billion (as of today). Bills, inflation-linked debt and strips are excluded from the index.

To predict the riskiness of bond returns we use a number of macroeconomic and financial variables. The macroeconomic variables are a subset of the ‘key country indicators’ collected by Datastream for the US: industrial production, the consumer price index, money supply, the unemployment rate and initial jobless claims. The financial data series are: the 3-month US-dollar LIBOR rate, the constant-maturity yields on 3-month and 10-year US Treasuries and the S&P 500 total return index. In subsection 4.4 we explain in more detail how these data series are used to obtain predictors of riskiness.

4.2 Preliminary analysis\(^8\)

In this subsection, we present some descriptive statistics about the bond return indices and we provide comments about the estimates of the unconditional indices of riskiness (Table 1).

The sample mean and standard deviation of daily returns are very similar across rating classes. The empirical distribution of returns exhibits strong departures from normality, as indicated by large negative values of the skewness and values of the kurtosis much larger than three. The Jarque-Bera statistic rejects the hypothesis of normality at all conventional levels of confidence for all rating classes. The estimate of the unconditional index of riskiness ranges between 1.5 and 1.9 depending on the class of rating. Riskiness does not increase monotonically when the rating class is decreased. The estimated ordering is as follows:

\[
Gov < AAA < AA < BBB < A < SP500
\]  

\(^7\)Merril Lynch identifies the index by the code G0Q0.
\(^8\)To streamline the discussion of our empirical analysis, we gather all technical details about standard errors, long-run covariance matrices and multi-step estimation in subsection 4.6.
where \( A < B \) indicates that \( A \) is less risky than \( B \), \( Gov \) denotes government bonds and \( SP500 \) the Standard & Poor’s 500 stock index. Running two-sided pair-wise tests of equal riskiness at 95 per cent confidence (see section 2), we do not find statistically significant differences between the riskiness of different rating classes, except for the \( A \) class, which is significantly riskier than the \( AA \) and \( AAA \) classes (see Table 2). We also test the hypothesis that corporate bonds and government bonds are equally risky and we are able to reject the hypothesis at 95 per cent confidence only for the \( A \) class. To assess how departures from normality affect the riskiness of corporate bonds, we calculate the index of riskiness for normal distributions having the same mean and variance as the sample distributions of returns (see the last line of Table 1). This is easily done by exploiting the fact that the riskiness of a normal gamble is equal to half its variance divided by its mean (Aumann and Serrano - 2008). We find that for each rating class the actual riskiness is higher than the riskiness of its theoretical normal counterpart. However, such differences are not statistically significant.

4.3 Specification of the functional form of \( R(X_t) \)

As explained in Section 3, the functional dependence of \( R \) on the vector of predetermined variables \( X_t \) is in general unknown and depends on the joint distribution of \( g_t \) and \( X_t \). Taking a parametric approach to the estimation of \( R(X_t) \), we assume that \( R(X_t) \) is a known function \( \xi \) of \( X_t \) and of a vector of parameters \( \beta \) to be estimated:

\[
R(X_t) = \xi(X_t; \beta) \quad (26)
\]

Let \( X_{it} \) denote the \( i \)-th component of \( X_t \) and \( \beta_i \) the \( i \)-th component of \( \beta \). We propose the following log-log specification:

\[
\log(R) = \sum_{i=1}^{N} \beta_i \log(h_i(X_{it})) \quad (27)
\]

where \( h_i \) are continuous functions \( h_i : \mathbb{R} \to \mathbb{R} \) such that \( h_i(X_{1t}, \ldots, X_{Nt}) \in \mathbb{R}_{++} \), \( \forall t \). Note that \( \log(R) \) is well-defined since \( R \) is strictly positive. Hence:

\[
\xi(X_t; \beta) = \exp\left(\sum_{i=1}^{N} \beta_i \log(h_i(\mathbf{X}_{it}))\right) \quad (28)
\]

In the subsequent analysis, we will always assume \( X_{1t} \equiv 1 \) and \( h_1(X_{1t}) = \exp(X_{1t}) \), so that the log-log equation will always include a constant on its right-hand side.

Note that \( \xi(X_t; \beta) \) meets the requirements of strict positivity, continuity and continuous differentiability set forth by proposition 3 and 4.

The above functional form can be motivated as follows. Suppose \( g_t \) is conditionally normal with conditional mean \( \mu_t \) and conditional variance \( \sigma^2_t \). It is possible to prove that the index of riskiness of a normally distributed gamble is half its variance divided by its mean. If you define \( X_{2t} = \sigma_t \) and \( X_{3t} = \mu_t \), then:

\[
\xi(X_t; \beta) = \exp\left(\log(0.5) \log(h_1(X_{1t})) + 2 \log(h_2(X_{2t})) - \log(h_3(X_{3t}))\right) \quad (29)
\]

where \( h_2 \) and \( h_3 \) are identity functions. Hence, the functional form (28) is correctly specified when the gamble \( g_t \) is conditionally normal and the vector of predictors \( X_t \) contains the conditional mean and the conditional variance of \( g_t \). Provided the functions \( h_i() \) are chosen appropriately, (28) is also correctly specified when linear combinations of functions of the predictor variables in \( X_t \) are unbiased predictors of the conditional mean and variance.
In the Appendix we show that, outside the important case of conditional normality, (28) is still general enough to approximate any sufficiently smooth functional form for $R(X_t)$ to any desired degree of accuracy, provided one (eventually) replaces the set of predictors $X_t$ with a larger set containing also non-linear functions of the components of $X_t$.

4.4 Conditioning on a single predetermined variable

A great deal of the empirical finance literature is concerned with identifying the economic and financial variables that best predict time-variation in the distribution of asset returns. As a consequence, an ever-growing set of economic and financial variables has been shown to partly predict the moments of assets returns (for a thorough review see, again, Aït Sahalia and Brandt - 2001). In this subsection we consider a fairly large set of predictor variables, including both the variables that are frequently utilized in the aforementioned literature and other variables that are specific to the corporate bond indices utilized in this study (namely, the average yield-to-maturity and the EWMA volatility of corporate bond returns - see below for more details). We first list these variables and we then discuss the estimates of all the univariate models obtained by considering in turn each variable alone. Whenever we find statistically significant evidence that a variable can predict the time-variation in riskiness, we try to give possible economic explanations for this predictability.

Using the notation previously introduced, we consider the case in which $X_t$ is 2-dimensional: $X_{1t}$ is constant and $X_{2t}$ is the predetermined variable of interest. If $X_{2t} > 0$ the function $h_2()$ is the identity function, otherwise $h_2()$ is chosen so as to ensure that $h_2(X_{2t}) > 0$ (see below).

$h_2()$ is chosen to be the identity function for the following data series, which are strictly positive:

- the unemployment rate (UN);
- the initial jobless claims (IJC);
- the 3-month US-dollar LIBOR rate (LIB3M);
- the constant-maturity yield on 3-month US Treasuries (TRS3M);
- the constant-maturity yield on 10-year US Treasuries (TRS10Y);
- the average yield-to-maturity of the bonds belonging to the corporate bond index under analysis (YTM);
- the TED-spread (TED), defined as the difference between LIB3M and TRS3M, a widely used measure of funding liquidity;
- an adaptive forecast of the conditional volatility of corporate bond returns (VOL), computed using RiskMetrics EWMA methodology\textsuperscript{10};

\textsuperscript{9}See footnote 8.

\textsuperscript{10}Riskmetrics EWMA (RiskMetrics 1996) is an adaptive estimate of conditional volatility. The conditional variance of returns at time $t$ (denote it by $\sigma^2_t$) is recursively computed as:

$$\sigma^2_t = 0.94 \cdot \sigma^2_{t-1} + 0.06 \cdot r^2_{t-1}$$

where $r_{t-1}$ is the return observed at time $t-1$. When $r_t$ has zero mean, the above is equivalent to an IGARCH(1,1) specification for conditional volatility. Setting the decay factor equal to 0.94 is in theory suboptimal with respect to estimating the parameter for each time series. However, such choice has proved to provide excellent out-of-sample performance for a broad range of daily financial time series. Furthermore, estimates of the parameter are known to be quite sensitive to the choice of the statistical loss function used for estimation (González-Rivera et al. - 2007).
an adaptive forecast of the conditional volatility of S&P 500 returns (SPVOL), also computed using RiskMetrics EWMA methodology;

We choose:

\[ h_2(X_{2t}) = 100 + \frac{X_{2t}}{100} \]  \hspace{1cm} (30)

for the following data series, which are not strictly positive:

- inflation (CPI), computed as the 12-month growth rate of the Consumer Price Index;
- growth in industrial production (IP), computed as the 12-month growth rate of the seasonally-adjusted volume of industrial production;
- money growth (M1), computed as the 12-month growth rate of M1;
- the stock market trend (SPTREND), computed as the percentage appreciation of the S&P 500 index over the previous 12 months;
- the slope of the US Treasury yield curve (SLOPE), computed as the constant-maturity yield on 10-year US Treasuries minus the constant-maturity yield on 3-month US Treasuries.

The transformation (30) converts growth rates into growth factors and ensures strict positivity of all transformed data series, since for all series considered above \( X_{2t} > -100 \). Data series whose frequency is lower than daily are transformed to daily series by repeating a figure after its release date on every date preceding the following release.

Table 3 reports the estimates of \( \beta_2 \) for each of the above predictors. According to these estimates, YTM, LIB3M, TRS3M, TRS10Y and VOL are highly significant predictors of riskiness, individually, both for the broad index and for all the sub-indices of bonds.

The higher the average yield to maturity (YTM), the lower predicted riskiness is. We believe that this result deserves some discussion. Note, first, that the total return of a portfolio of bonds is the sum of two components: accrued interests and capital gains (losses). A higher YTM mechanically increases the former, while it has no obvious \textit{a priori} effect on the latter: for example, if YTM has a mean-reverting behavior, a high YTM could anticipate future capital gains; on the other hand, if a high YTM reflects higher default rates, a high YTM could anticipate future capital losses. Hence, it is not clear whether to expect a higher YTM to increase or decrease future expected returns. However, regressing our sample of daily corporate bond returns on YTM, we find a positive and statistically significant coefficient on YTM (the \( p \)-value is 0.002 for the general index). Furthermore, when YTM is low, losses are more frequent and on average larger (see Figure 2). In the previous section (see equation (29)), we have shown that, for normally distributed gambles, a \textit{ceteris paribus} increase in expected returns decreases the riskiness of an asset. In our case, a higher YTM increases expected returns, but the final effect on riskiness is not, in principle, obvious, since YTM could affect other characteristics of the conditional distribution of bond returns and the distribution itself is not normal. However, in our sample, we find that a higher YTM indeed decreases the riskiness of bonds, confirming the intuition developed for normally distributed returns. Discussing the properties of Aumann and Serrano’s (2008) economic index of riskiness, we have highlighted one important merit of it with respect to more traditional measures of risk such as variance: the economic index of riskiness takes into account the fact that expected return is an important component of risk; if, \textit{ceteris paribus}, you increase the expected return of an asset, its risk must diminish, because you are decreasing the probability of facing low-payoff states. Roughly speaking, our empirical findings suggest that a higher YTM decreases the probability of disappointing returns on corporate bonds, hence
diminishing their riskiness. In the next subsection, we will show that this finding is robust to a number of controls: estimating a battery of multivariate models, we find that the relation between YTM and riskiness does not change sign and remains statistically significant also when we consider other predictors simultaneously.

Also LIB3M, TRS3M and TRS10Y, if considered individually, are significantly and negatively associated with riskiness. Since these variables all provide a measure of the level of prevailing interest rates, this empirical finding seems to suggest that corporate bonds tend to be riskier in a low-interest rate environment. However, these variables are highly correlated with YTM (the correlation coefficient is higher than 70 per cent for all rating classes and all the three variables). As a consequence, their negative association with riskiness in uni-variate models might simply be due to the fact that they are a proxy for YTM and hence for expected returns. In fact, in the next section we will show that, once you control for YTM, the relation can change sign or become statistically insignificant.

VOL, the EWMA estimate of the conditional volatility of bond returns, is significantly and positively associated with riskiness. EWMA estimates of conditional volatility are widely utilized in financial econometrics (e.g.: González-Rivera et al. - 2007), thanks to their ability to capture persistent shifts in the conditional volatility of asset returns (Engle - 1982 and Bollerslev - 1986). In the previous section (see equation (29)), we have shown that the conditional volatility of normally distributed returns is positively associated with riskiness in a bivariate model where conditional volatility and conditional expected return are the two predetermined variables. Hence, if VOL correctly predicts conditional volatility, it is reasonable to conjecture that an increase in VOL increases riskiness. Our empirical findings confirm the intuition.

TED, SPTREND, SPVOL and IP are significantly associated to riskiness for some, but not all classes of rating. Inspecting Table 3, it is possible to observe a general pattern whereby these variables have a greater impact on lower rated bonds.

TED is a widely used measure of funding liquidity (e.g.: Brunnermeier - 2009), the ease with which corporations can find sources of financing for their activity. A higher TED predicts an increase in the riskiness of corporate bonds: a tentative explanation is that, when funding liquidity is scarce (TED is high), corporations may face difficulties in refinancing their debt and therefore their liabilities may become riskier to hold. More financially constrained firms are arguably those more affected by a lack of funding liquidity: this might explain why the effect of an increase in TED is higher for lower rated bonds.

A decline in stock prices over the last 12 months (a decline in SPTREND) predicts an increase in the riskiness of lower-rated bonds. A decline in stock prices may proxy for a decline in the value of corporate assets which decreases the probability that firms be solvent.

Also a rise in the volatility of stock prices (SPVOL) predicts an increase in the riskiness of lower-rated bonds. A rise in SPVOL may reflect an increase in the volatility of corporate assets: as highlighted by popular theoretical frameworks (e.g.: Merton - 1974), this may in turn increase the probability of corporate defaults. Empirically, this has also resulted in a significant correlation between equity volatility and corporate bond yields (Campbell and Taksler - 2003).

Another predictor of heightened riskiness is a decline in industrial production over the last 12 months (a decline in IP). IP is a widely used coincident indicator of the business cycle (e.g.: Pericoli and Taboga - 2008). In phases of economic contraction, firms are likely to be less profitable and their probability of default increases.

Of course, an increase in VOL could also predict an increase in expected returns, dampening the positive association with riskiness; furthermore, departures from normality could make the relation more complex.
4.5 Conditioning on multiple predetermined variables\footnote{See footnote 8.}

While it is commonplace in the empirical finance literature to consider only one predictor at a time (as in the previous section), it is desirable to combine the predictive power of individual variables into multivariate models that are able to summarize in a parsimonious manner most of the information that can usefully be exploited to form conditional distributions of asset returns. Hence, in this subsection we discuss models having more than one predictor of riskiness and we propose a method to select the variables to include in a multivariate model of riskiness. We present only a limited selection of tables obtained from our multivariate analysis and collect all others in a not-to-be-published appendix (NPA) available upon request from the authors. Every time we refer to a table that is not published, this is indicated in parentheses as (NPA).

Estimating multivariate models, we have encountered statistical problems similar to those found in the literature on the predictability of stock and bond returns (see e.g. Aït Sahalia and Brandt - 2001 for a discussion): while several variables individually predict riskiness, it is hard to build statistically sound multivariate models with more than few predictors. The main problem is the fairly high collinearity between predictors (the condition number of the design matrix rises steeply by adding more variables): this is reflected in high standard errors of parameter estimates for richer models. For this reason, and also in consideration of the computational complexities that arise in a non-linear GMM setting, we favour a specific-to-general approach to model selection. We start with a detailed description of the methodology adopted and the results obtained in the case of the general corporate bond index. We then briefly comment the results obtained for the sub-indices, highlighting the main differences with respect to the general index.

Our analysis of the general index starts from a bivariate model where YTM and VOL are the two included predictors. We start with these two variables because they are the only two in the dataset to provide index-specific information about corporate bonds. The estimated coefficients of both variables (Table 4) are highly significant and have the same sign found with uni-variate models: a high YTM decreases riskiness, while a high VOL increases it.

Adding the remaining variables in our dataset one at a time to form three-variable models, we find statistically significant coefficients (with \(p\)-values lower than 10 per cent) only for LIB3M, SLOPE, CPI and TED (NPA). Adding a third variable, VOL always remains highly significant and YTM remains significant in all but two cases (SLOPE and TRS10Y). The latter result might be due to the fact that SLOPE and TRS10Y contain information about long-term rates also contained in YTM.

If we add a fourth variable to the three-variable models thus obtained (YTM and VOL, plus one of LIB3M, SLOPE, CPI and TED, plus one of the remaining variables), we never find a statistically significant coefficient for the fourth variable (Table 4 and NPA). In all the four-variable models we estimate, the signs of YTM and VOL remain the same and in most cases the two variables remain statistically significant (NPA): hence, the finding that a high YTM decreases riskiness, while a high VOL increases it, seems fairly robust.

Since in no case we are able to reject the restriction that the coefficient of the fourth variable is zero, we limit our attention to the four three-variable models (YTM and VOL, plus one of LIB3M, SLOPE, CPI and TED). To select a model out of these four models, we utilize a consistent selection criterion suggested by Andrews and Lu (2001). Consider, for concreteness, the three-variable model including LIB3M as a predictor. We add over-identifying restrictions by imposing orthogonality also to the three variables included in the other models (SLOPE, CPI and TED) and we compute the \(J\)-statistic for testing such over-identifying restrictions (Table 5). We repeat this procedure for all the four models. The best among the three-variable models is
selected according to a ranking based on the $J$-statistic. In our case, a BIC-type criterion or an Hannan-Quinn-type criterion assign the same ranking assigned by the $J$-statistics, because the number of over-identifying restrictions is the same for all four models. The model selected by the criterion (the one having the lowest $J$-statistic) is the one including CPI as the third variable. In this model, as well as in all the four-variable models we have estimated, an increase in CPI increases riskiness.

A number of other papers found that inflation can affect corporate bond returns through various channels. Gordon (1982) argued that higher inflation can lead to both higher leverage and higher bankruptcy rates, because it increases the tax advantage of issuing debt rather than equity. Wadhwani (1986) found that high inflation increases bankruptcy rates because it raises interest payments more than corporate profits. Valckx (2004) found empirical confirmation that when inflation is higher than expected, excess bond returns are lower than expected.

Figure 1 displays the conditional riskiness of the general corporate bond index, estimated with the model selected by Andrews and Lu (2001)'s criterion. According to such estimates, after the year 2000, the riskiness of corporate bonds has been on average much higher than during the previous decade. As volatility has not been dissimilar in the two periods and inflation has on average been lower in the most recent decade, higher predicted riskiness in recent years is mainly accounted for by low corporate bond yields.

We take the same steps described above to build multivariate models for the sub-indices. Again, we start from a bivariate model where YTM and VOL are the two included predictors. Adding a third variable, we find that: 1) no added variable has a statistically significant coefficient for the AAA index; 2) only CPI is significant for the AA index; 3) CPI, LIB3M, SLOPE, TED and UN are significant for the A and BBB index; 4) also IJC and M1 are significant for the BBB index. These findings are consistent with the pattern found with uni-variate models: the riskiness of lower-rated bonds seems to be more responsive to general macroeconomic and financial conditions than the riskiness of higher-rated bonds.

The same model selection procedure adopted for the general index leads to the following selection of predictor variables for the sub-indices (Tables 5 and 6): YTM and VOL for the AAA index; YTM, VOL and CPI for both the AA and the A index; YTM, VOL and UN for the BBB index. Hence, after controlling for YTM and VOL, no other variables in our dataset have significant predictive power for the riskiness of AAA bonds, while the riskiness of AA and A bonds is satisfactorily captured by the same variables selected to model the general index. The best model for the BBB index, instead, includes UN in place of CPI: a rise in unemployment decreases riskiness. The work of Boyd et al. (2005) can help to explain this finding: during economic expansions, which are by far more frequent than recessions, a rise in unemployment is good news for corporations, because it anticipates higher profits and lower inflation and interest rates. Hence, according to this interpretation, it is not surprising that UN appears in place of CPI in a model, because the two variables measure interrelated phenomena.

### 4.6 Standard errors, autocorrelations and other technical details

In this subsection we gather some technical details of estimation that were not discussed in the previous subsections.

First of all, let us address estimation of the long-run covariance matrix $S$, i.e. the asymptotic covariance matrix of the sums $\frac{1}{T} \sum_{t=1}^{T} m(g_t; \tilde{R})$ and $\frac{1}{T} \sum_{t=1}^{T} m(g_t, X_t; \beta)$. One possible source of concern is serial correlation of the sequence $\{m_t\}_{t=1,\ldots, T}$, where either $m_t = m(g_t; \tilde{R})$ or $m_t = m(g_t, X_t; \beta)$. Note, however, that the two-variable model including only YTM and VOL is preferred by the BIC statistics to all the three-variable models (Table 5). On the other hand, according to the Hannan-Quinn statistic the best three-variable model is also preferred to the two-variable one.
\[ m_t = m \left( g_t, X_t; \hat{\beta} \right) \]. If the sequence is serially uncorrelated, then the sample variance of \( m_t \) is a consistent estimator of \( S \). Otherwise, one needs to use estimators, such as Newey-West, that take serial correlation into account. Running various portmanteau tests for serial correlation on \( m_t \), in virtually all cases we were not able to reject the null of serial uncorrelatedness\(^{14}\). Hence we chose to estimate \( S \) with the sample variance of \( m_t \).

Conditional heteroskedasticity is of course found in many time series in our dataset, but it is not a concern, because the estimators proposed in Propositions 1 to 4 are already heteroskedasticity-consistent.

The choice of the weighting matrix \( \hat{W} \) was irrelevant for all just-identified models (\( N = K \)), because all the sample moment conditions were always satisfied exactly. For over-identified models (\( N > K \)), we adopted a multi-step efficient procedure. We started with \( \hat{W} = I \), obtained an estimate \( \hat{S} \) of \( S \), and utilized \( \hat{S}^{-1} \) as the new weighting matrix. We iterated this procedure 10 times.

5 Directions for further research

We believe Aumann and Serrano’s (2008) contribution has opened several new avenues for future research in empirical finance.

First of all, the analysis of riskiness we have proposed for corporate bonds could be replicated for other asset classes, such as stocks, government bonds, currencies and commodities.

Second, within our parametric framework, we have chosen to work with GMM estimators, but other estimators could have been used, such as KLIC (Kitamura and Stutzer - 1997 and Imbens et al. - 1998). Gregory et al. (2002) have shown simulation evidence that KLIC estimators have desirable small-sample properties when used to estimate preference parameters; since the index of riskiness is closely related to preferences, it might be worthwhile to replicate a simulation exercise similar to theirs for the indices of riskiness.

Third, we have chosen a parametric approach to estimating the conditional dependence of the index of riskiness on a set of predetermined variables, but also a non-parametric approach could be undertaken along the following lines: the joint distribution of the returns and the predictor variables could be estimated non-parametrically with a multivariate density estimator (e.g.: Silverman - 1986); then, one could calculate the index of riskiness using the estimated density as if it were the population density, using the population moment condition (17). We have not chosen this approach because it would be much less straightforward to do inference with this two-stage procedure; furthermore, the curse of dimensionality would be even more severe. However, after developing appropriate inferential procedures, a non-parametric approach could be desirable in low-dimensional settings or with larger datasets, because it would be less prone to incur into mis-specification problems.

Finally, performing model selection in the multivariate conditional case is not a trivial task. We have utilized the consistent selection criterion suggested by Andrews and Lu (2001). However, such criterion does not permit comparisons between just-identified models. Furthermore, to our knowledge, there are no other selection criteria that can be readily applied to our estimation framework. Model selection in a general GMM setting is still an open area of research: defining selection criteria to be utilized in the selection of multivariate models of riskiness could be an interesting avenue to explore.

\(^{14}\)The sequences \( \{ g_t \} \), being sequences of asset returns, are, not surprisingly, serially uncorrelated. More surprising might be the fact that also \( m_t = m \left( g_t, X_t; \hat{\beta} \right) \) is serially uncorrelated, despite many instances of high serial correlation in \( X_t \). Intuitively, this is explained by the fact that the sign of \( g_t \) determines whether \( m_t \) is above or below its mean, while \( X_t \) determines only the magnitude of deviations from the mean.
6 Conclusions

Aumann and Serrano (2008) have laid the foundations of a theory of risk measurement that is deeply rooted in decision theory. The index of riskiness they proposed has a straightforward economic interpretation: when the index of riskiness of an asset increases, then some risk-averse agents that were previously willing to hold the asset are no longer willing to hold it. This squares exactly with the idea that risk is what risk-averse agents avert.

This paper has dealt with statistical estimation of Aumann and Serrano’s (2008) index of riskiness. We have proposed a GMM method to estimate the index and a conditional setting that allows to deal with time-variation in the index of riskiness. We have utilized these tools to study the riskiness of corporate bonds.

The statement that "corporate bonds are riskier than government bonds" appears to be common wisdom. However, despite using a long time series of corporate bond returns, we have not been able to reject the hypothesis that corporate bonds and government bonds are equally risky, at least for some rating classes. We have found that a number of financial and macroeconomic variables help to predict changes in the riskiness of corporate bonds. Adaptive forecasts of volatility, widely used to measure financial risk, are not alone sufficient to capture all the time-variation in riskiness. Other variables, considered together with volatility, are statistically significant predictors of riskiness. For example, a high yield-to-maturity tends to reduce riskiness. In our sample, a high yield-to-maturity predicts high positive returns and a reduced probability of facing low-payoff states, hence lowering riskiness. According to our estimates, after the year 2000, low yields have contributed to make the riskiness of corporate bonds on average much higher than during the previous decade.

Overall, bringing Aumann and Serrano’s (2008) theory of risk measurement to the data is relatively straightforward and allows to gain new insights into the riskiness of financial assets. There seems to be much room for further empirical applications of their theory.
7 Appendix
7.1 Proofs of consistency and asymptotic normality

Our proofs of consistency and asymptotic normality are based on the following propositions, adapted from Hayashi (2000 - Proposition 7.7, p. 467, and Proposition 7.10, p. 480):

**Proposition 5 (Consistency - general case)** Let \( \{w_t\}_{t=1,...,T} \) be the first \( T \) elements of an ergodic and stationary sequence of draws from the distribution of a \( d \)-dimensional random vector \( w \). Let \( \Theta \) be a subset of \( \mathbb{R}^p \). Let \( \theta_0 \in \Theta \). Let \( m(w;\theta) \) be a \( k \times 1 \) vector-valued function \( m : \mathbb{R}^d \times \mathbb{R}^p \to \mathbb{R}^k \). Let \( \hat{\theta} \) be the GMM estimator defined by:

\[
\hat{\theta} = \arg \min_{\theta \in \Theta} \left[ \frac{1}{T} \sum_{t=1}^{T} m(w_t;\theta) \right]^T \hat{W} \left[ \frac{1}{T} \sum_{t=1}^{T} m(w_t;\theta) \right]
\]

and \( \hat{W} \) a \( k \times k \) symmetric matrix that converges in probability to a symmetric and positive definite matrix \( W \) as \( T \) tends to infinity. Suppose that:

1. \( E[m(w_t;\theta_0)] = 0 \);
2. \( E[m(w_t;\theta)] \neq 0 \) for all \( \theta \neq \theta_0 \) in \( \Theta \);
3. \( \Theta \) is compact;
4. \( m(w_t;\theta) \) is continuous in \( \theta \) for all \( w_t \);
5. \( m(w_t;\theta) \) is measurable in \( w_t \) for all \( \theta \) in \( \Theta \);
6. \( E[\sup_{\theta \in \Theta} \|m(w_t;\theta)\|] < \infty \).

Then \( \hat{\theta} \) is a well-defined random variable and it converges in probability to \( \theta_0 \) as \( T \) tends to infinity.

**Proposition 6 (Normality - general case)** Let all the conditions of Proposition 5 be satisfied. Suppose also that:

1. \( \theta_0 \) is in the interior of \( \Theta \);
2. \( m(w_t;\theta) \) is continuously differentiable in \( \theta \) for any \( w_t \), with \( k \times p \) derivative \( \frac{\partial m(w_t;\theta)}{\partial \theta} \);
3. as \( T \) tends to infinity, \( \frac{1}{\sqrt{T}} \sum_{t=1}^{T} m(w_t;\theta_0) \) converges in distribution to a multivariate normal distribution with mean zero and full-rank covariance matrix \( S \);
4. there exists a neighborhood \( \Theta_0 \) of \( \theta_0 \) (\( \Theta_0 \subseteq \Theta \)) such that:

\[
E \left[ \sup_{\theta \in \Theta_0} \left\| \frac{\partial m(w_t;\theta)}{\partial \theta} \right\| \right] < \infty
\]

(32)

5. \( D = E \left[ \frac{\partial m(w_t;\theta_0)}{\partial \theta} \right] \) has rank \( p \).
Then, $\sqrt{T}\left(\hat{\theta} - \theta_0\right)$ converges in distribution to a multivariate normal distribution with mean zero and covariance matrix $V$ equal to:

$$V = (D^TWD)^{-1}(D^TWSW^T)(D^TWD)^{-1}$$  \hspace{1cm} (33)

Furthermore, if there exists a matrix $\hat{S}$ converging in probability to $S$ as $T$ tends to infinity, then the matrix $\hat{V}$ defined by:

$$\hat{V} = \left(\hat{D}^T\hat{W}\hat{D}\right)^{-1}\left(\hat{D}^T\hat{W}\hat{S}\hat{W}^T\hat{D}\right)\left(\hat{D}^T\hat{W}\hat{D}\right)^{-1}$$  \hspace{1cm} (34)

$$\hat{D} = \frac{1}{T}\sum_{t=1}^{T} \partial m\left(w_t; \hat{\theta}\right)$$  \hspace{1cm} (35)

converges in probability to $V$.

Thanks to the above results, we can prove the four propositions found in the main text of the paper:

**Proof of Proposition 1.** We have to prove that the conditions in Proposition 5 are satisfied when the conditions in Proposition 1 are met. First of all, let us establish correspondences between the notation in the two propositions: $w_t = g_t; \Theta = [l, u]; \theta_0 = R; \hat{\theta} = 1$. Condition 1 in Proposition 5 is satisfied by the very definition of the index of riskiness (which exists because $E[g] > 0$ and $P[g < 0] > 0$). Since there is a unique strictly positive index of riskiness, by Theorem A in Aumann and Serrano (2008), also Condition 2 is satisfied. $[l, u]$ is compact, hence Condition 3 is satisfied. $m(g_t, r)$ is discontinuous in $r$ on $\mathbb{R}_+$ only at 0, but 0 is not included in $[l, u]$, hence Condition 4 is satisfied. $m(g_t, r)$ is continuous in $g_t$ on all $\mathbb{R}$, hence it is also measurable (Condition 5). $g_t$ has bounded support, $[l, u]$ is compact and $m(g_t, r)$ is continuous in $g_t$ and in $r$, so $m(g_t, r)$ is bounded and Condition 6 trivially holds. Therefore, all conditions are satisfied and $\hat{\theta}$ is a consistent estimator of $R$. \hspace{1cm} ■

**Proof of Proposition 2.** We have to prove that the conditions in Proposition 6 are satisfied when the conditions in Proposition 2 are met. Correspondences between the notation in the two propositions are as in the previous proof: $w_t = g_t; \Theta = [l, u]; \theta_0 = R; \hat{\theta} = 1$. Condition 1 in Proposition 6 is satisfied by the fact that $l < R < u$. $m(g_t, r)$ is non-differentiable in $r$ on $\mathbb{R}_+$ only at 0, but 0 is not included in $[l, u]$, hence Condition 2 is satisfied. Condition 3 is satisfied by requiring that $\frac{1}{\sqrt{T}}\sum_{t=1}^{T} m(g_t; R)$ converges in distribution to a multivariate normal distribution with mean zero and strictly positive variance $S$. Since $m(g_t, r)$ is continuously differentiable in $r$ on the compact $[l, u]$ and $g_t$ is bounded, Condition 4 is trivially satisfied. Condition 5 is satisfied by the requirement that $D \neq 0$. \hspace{1cm} ■

**Proof of Proposition 3.** We have to prove that the conditions in Proposition 5 are satisfied when the conditions in Proposition 3 are met. First of all, let us establish correspondences between the notation in the two propositions: $d = N + 1; p = K; k = N; w_t = [g_t X_t]^{T}; \Theta = [-u, u]^K; \theta_0 = \beta$. Condition 1 in Proposition 5 is satisfied by the assumption that $R(X_t) = \xi(X_t; \beta)$. By the assumption that $P(\xi(X_t; b_1) \neq \xi(X_t; b_2)) > 0$ if $b_1, b_2 \in \mathbb{R}^K$ and $b_1 \neq b_2$, $P(\xi(X_t; b) \neq R(X_t)) > 0$ for all $b \neq \beta$ in $\Theta$. Hence, by uniqueness of the index of riskiness, $E[m(w_t; b)] \neq 0$ for all $b \neq \beta$ in $\Theta$ and Condition 2 is satisfied. $[-u, u]^K$ is compact, hence
Condition 3 is satisfied. Since the only discontinuity of \( f (\xi) = X_t \left( \exp \left( -\frac{\xi^2}{2} \right) - 1 \right) \) is at \( \xi = 0 \) and \( \xi (X_t; b) \) is strictly positive and continuous in \( b \), \( m (g_t, X_t; b) \) is continuous in \( b \), so Condition 4 is satisfied. By the same token, the fact that \( \xi (X_t; b) \) is strictly positive and continuous in \( X_t \) and \( m (g_t, X_t; b) \) is continuous in \( g_t \), implies that \( m (g_t, X_t; b) \) is also measurable in \( w_t = [g_t, X_t]^T \) (Condition 5). \( w_t \) has bounded support, \([-u, u]^K\) is compact and \( m (g_t, X_t; b) \) is continuous in \( g_t, X_t \) and \( b \), so \( m (g_t, X_t; b) \) is bounded and Condition 6 trivially holds. Therefore, all conditions are satisfied and \( \hat{\beta} \) is a consistent estimator of \( \beta \). □

**Proof of Proposition 4.** We have to prove that the conditions in Proposition 6 are satisfied when the conditions in Proposition 4 are met.

Correspondences between the notation in the two propositions are as in the previous proof: \( d = N + 1 \); \( p = K \); \( k = N \); \( w_t = [g_t, X_t]^T \); \( \Theta = [-u, u]^K \); \( \theta_0 = \beta \). Condition 1 in Proposition 6 is satisfied by the fact that \( u \) is strictly larger than \( \|\beta\|_1 \). \( f (\xi) = X_t \left( \exp \left( -\frac{\xi^2}{2} \right) - 1 \right) \) is non-differentiable only at \( \xi = 0 \), but \( \xi (X_t; b) \) is strictly positive and continuously differentiable in \( b \) for any \( X_t \), hence \( m (g_t, X_t; b) \) is continuously differentiable in \( b \) and Condition 2 is satisfied. Condition 3 is satisfied by requiring that \( \frac{1}{\sqrt{T}} \sum_{t=1}^{T} m (g_t, X_t; \beta) \) converges in distribution to a multivariate normal distribution with mean zero and full-rank covariance matrix \( S \). Since \( m (g_t, X_t; b) \) is continuously differentiable in \( b \) on the compact \([-u, u]^K\) and \( g_t \) and \( X_t \) are bounded, Condition 4 is trivially satisfied. Condition 5 is satisfied by the requirement that \( D = E \left[ \frac{\partial m(g_t, X_t; \beta)}{\partial \beta} \right] \) has rank \( K \). □

### 7.2 Approximations of \( R (X_t) \)

Outside the case of conditional normality, the specification

\[
\xi (X_t; \beta) = \exp \left( \sum_{i=1}^{N} \beta_i \log (h_i (X_{it})) \right)
\]

(36)

can approximate any sufficiently smooth functional form for \( R(X_t) \) to any desired degree of accuracy. For example, provided some technical conditions are satisfied (see e.g. Judd – 1998), the function:

\[
\rho (X_t) = \log \left( R (X_t) \right)
\]

(37)

can be approximated to any degree of accuracy by tensor product bases. In fact, let \( \Phi = \{ \phi_k (x_i) \}_{k=1}^{\infty} \) be an orthogonal basis for real valued functions of one variable and \( \Psi \) a tensor product basis for real valued functions of \( N \) variables:

\[
\Psi = \left\{ \psi (x) \mid \psi (x) = \prod_{i=1}^{N} \phi_{k_i} (x_i) , \{k_i\}_{i=1}^{N} \in C_N^N \right\}
\]

(38)

where \( C_N^N \) is the set of combinations of \( N \) natural numbers. Then, \( \rho (X_t) \) can be approximated to any desired degree of accuracy by a linear combination of \( N^* \) functions \( \{ \psi_i (x) \}_{i=1}^{N^*} \) belonging to \( \Psi \):

\[
\rho (X_t) \simeq \sum_{i=1}^{N^*} \beta_i \psi_i (X_t)
\]

(39)
Let $Z_t$ be an $N^*$-dimensional vector such $Z_{it} = \psi_i (X_t)$ ($Z_{it}$ is the $i$-th component of $Z_t$). Then:

$$R (X_t) \simeq \exp \left( \sum_{i=1}^{N^*} \beta_i \log (h_i (Z_{it})) \right)$$

(40)

where $h_i (Z_{it}) = \exp (Z_{it})$. Hence, the functional form (28) is approximately valid also for non-normal gambles, provided one replaces the set of predictors $X_t$ with a larger set $Z_t$ containing non-linear functions of the components of $X_t$. 

26
References


[38] Wadhwani, S. B. (1986) "Inflation, bankruptcy, default premia and the stock market", 

8 Tables and Figures

Table 1 - Summary statistics and estimates of unconditional riskiness

<table>
<thead>
<tr>
<th></th>
<th>General</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>Gov.</th>
<th>S&amp;P 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0272</td>
<td>0.0284</td>
<td>0.0278</td>
<td>0.0268</td>
<td>0.0267</td>
<td>0.0284</td>
<td>0.0353</td>
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<tr>
<td>St. dev.</td>
<td>0.2985</td>
<td>0.2949</td>
<td>0.2931</td>
<td>0.3079</td>
<td>0.2994</td>
<td>0.2835</td>
<td>1.1196</td>
</tr>
<tr>
<td>Skew.</td>
<td>-0.476</td>
<td>-0.481</td>
<td>-0.482</td>
<td>-0.555</td>
<td>-0.556</td>
<td>-0.270</td>
<td>-0.071</td>
</tr>
<tr>
<td>JB</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>$\hat{R}$</td>
<td>1.606</td>
<td>1.590</td>
<td>1.610</td>
<td>1.843</td>
<td>1.744</td>
<td>1.453</td>
<td>17.84</td>
</tr>
<tr>
<td>$\hat{R}$ (st. dev.)</td>
<td>0.268</td>
<td>0.239</td>
<td>0.249</td>
<td>0.307</td>
<td>0.282</td>
<td>0.208</td>
<td>7.902</td>
</tr>
<tr>
<td>$R$ (normal)</td>
<td>1.637</td>
<td>1.529</td>
<td>1.544</td>
<td>1.770</td>
<td>1.676</td>
<td>1.417</td>
<td>17.76</td>
</tr>
</tbody>
</table>

The table reports descriptive statistics of the daily returns on the general corporate bond index (General), its four sub-indices (AAA, AA, A, BBB), the government bond index (Gov.) and the S&P 500 total return index. JB is the p-value of the Jarque-Bera statistic. $\hat{R}$ is the estimated index of riskiness, $\hat{R}$ (st. dev.) is the standard deviation of the estimate and $R$ (normal) is the riskiness of a theoretical normal distribution of returns having the same mean and variance as the real one. The sample period is from December 31st, 1988 to January 30th, 2009, for a total of 5200 daily observations.
### Table 2 - Pair-wise tests of equal riskiness (p-values)

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>Gov</th>
<th>S&amp;P 500</th>
</tr>
</thead>
<tbody>
<tr>
<td>General</td>
<td>0.201</td>
<td>0.276</td>
<td>0.021</td>
<td>0.515</td>
<td>0.069</td>
<td>0.041</td>
</tr>
<tr>
<td>AAA</td>
<td>0.774</td>
<td>0.031</td>
<td>0.237</td>
<td>0.257</td>
<td>0.040</td>
<td></td>
</tr>
<tr>
<td>AA</td>
<td>0.034</td>
<td>0.325</td>
<td>0.240</td>
<td>0.040</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>0.406</td>
<td>0.027</td>
<td>0.043</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>BBB</td>
<td></td>
<td>0.052</td>
<td>0.042</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table reports the $p$-values of the test statistics obtained testing the hypothesis that two distributions of asset returns are characterized by the same index of riskiness. The pair-wise tests are conducted on the following indices: the general corporate bond index (General), its four sub-indices (AAA, AA, A, BBB), the government bond index (Gov.) and the S&P 500 total return index. The sample period is from December 31st, 1988 to January 30th, 2009, for a total of 5200 daily observations.
Table 3 - Univariate models

<table>
<thead>
<tr>
<th></th>
<th>General</th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(0.935)</td>
<td>(0.613)</td>
<td>(0.840)</td>
<td>(1.001)</td>
<td>(0.940)</td>
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<td>1.677</td>
<td>2.197</td>
<td>2.412</td>
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<td></td>
<td>(0.332)</td>
<td>(0.377)</td>
<td>(0.382)</td>
<td>(0.391)</td>
<td>(0.303)</td>
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<td>-1.267</td>
<td>-1.346</td>
<td>-1.579</td>
<td>-1.732</td>
</tr>
<tr>
<td></td>
<td>(0.574)</td>
<td>(0.550)</td>
<td>(0.556)</td>
<td>(0.559)</td>
<td>(0.597)</td>
</tr>
<tr>
<td>TRS3M</td>
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<td>-1.090</td>
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<td>-1.182</td>
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<tr>
<td></td>
<td>(0.457)</td>
<td>(0.426)</td>
<td>(0.462)</td>
<td>(0.417)</td>
<td>(0.440)</td>
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<tr>
<td>TRS10Y</td>
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<tr>
<td></td>
<td>(0.498)</td>
<td>(0.473)</td>
<td>(0.535)</td>
<td>(0.477)</td>
<td>(0.454)</td>
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<tr>
<td>SLOPE</td>
<td>14.98</td>
<td>19.56</td>
<td>22.48</td>
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<td>(19.68)</td>
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<td>(15.46)</td>
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<tr>
<td>TED</td>
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<td>0.251</td>
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<td></td>
<td>(0.153)</td>
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<td>(0.179)</td>
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<tr>
<td>SPTREND</td>
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<td></td>
<td>(0.776)</td>
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<td>(0.833)</td>
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<td>(0.724)</td>
</tr>
<tr>
<td>SPVOL</td>
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</tr>
<tr>
<td></td>
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<td>(0.589)</td>
<td>(0.517)</td>
<td>(0.535)</td>
<td>(0.458)</td>
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<tr>
<td></td>
<td>(4.327)</td>
<td>(3.799)</td>
<td>(4.086)</td>
<td>(4.431)</td>
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<td>UN</td>
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</tr>
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<td></td>
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<td>(1.535)</td>
<td>(2.265)</td>
<td>(4.698)</td>
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<td>0.602</td>
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<tr>
<td></td>
<td>(0.895)</td>
<td>(0.857)</td>
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<td>(1.886)</td>
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<tr>
<td>CPI</td>
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<td>0.231</td>
<td>10.56</td>
<td>2.077</td>
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<tr>
<td></td>
<td>(12.52)</td>
<td>(10.87)</td>
<td>(12.43)</td>
<td>(11.84)</td>
<td>(12.81)</td>
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<td>M1</td>
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<td>1.227</td>
<td>1.307</td>
<td>-0.177</td>
<td>-3.021</td>
</tr>
<tr>
<td></td>
<td>(6.154)</td>
<td>(4.540)</td>
<td>(4.687)</td>
<td>(7.133)</td>
<td>(9.023)</td>
</tr>
</tbody>
</table>

The table reports coefficient estimates (t-statistics in parentheses) obtained from univariate models of riskiness. All models are estimated both for the general corporate bond index (General) and for its four sub-indices (AAA, AA, A, BBB). For example, the coefficient -2.247 found at the intersection of the YTM row and the AAA column indicates that the riskiness of AAA bonds is predicted to be lower when the yield-to-maturity (YTM) is higher. Each univariate model also includes a constant, whose estimate is not reported. *** indicates a p-value below 1 per cent, ** below 5 per cent and * below 10 per cent. The meaning of the acronyms in the first column is reported in section 3.4. The sample period is from December 31st, 1988 to January 30th, 2009, for a total of 5200 daily observations.
<table>
<thead>
<tr>
<th></th>
<th>YTM</th>
<th>VOL</th>
<th>CPI</th>
<th>Coef. Var.</th>
<th>Var.</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.988</td>
<td>-1.385</td>
<td>1.515</td>
<td>35.19</td>
<td>(15.79)**</td>
<td></td>
</tr>
<tr>
<td>7.577</td>
<td>-3.182</td>
<td>1.572</td>
<td>35.05</td>
<td>(15.23)**</td>
<td></td>
</tr>
<tr>
<td>7.899</td>
<td>-3.396</td>
<td>1.975</td>
<td>19.22</td>
<td>(16.95)</td>
<td>LIB3M</td>
</tr>
<tr>
<td>7.448</td>
<td>-3.060</td>
<td>2.462</td>
<td>27.91</td>
<td>(11.69)**</td>
<td>TRS3M</td>
</tr>
<tr>
<td>7.591</td>
<td>-3.217</td>
<td>1.587</td>
<td>35.05</td>
<td>(15.23)**</td>
<td></td>
</tr>
<tr>
<td>7.523</td>
<td>-2.726</td>
<td>1.826</td>
<td>27.51</td>
<td>(18.00)</td>
<td>SLOPE</td>
</tr>
<tr>
<td>7.843</td>
<td>-3.250</td>
<td>1.467</td>
<td>29.94</td>
<td>(22.81)</td>
<td>TED</td>
</tr>
<tr>
<td>7.759</td>
<td>-3.333</td>
<td>1.478</td>
<td>35.90</td>
<td>(17.53)**</td>
<td>SPTREND</td>
</tr>
<tr>
<td>7.546</td>
<td>-3.147</td>
<td>1.539</td>
<td>35.96</td>
<td>(16.88)**</td>
<td></td>
</tr>
<tr>
<td>7.580</td>
<td>-3.184</td>
<td>1.571</td>
<td>35.18</td>
<td>(15.83)**</td>
<td></td>
</tr>
<tr>
<td>13.62</td>
<td>-2.976</td>
<td>1.681</td>
<td>32.36</td>
<td>(12.70)**</td>
<td></td>
</tr>
<tr>
<td>7.157</td>
<td>-2.789</td>
<td>1.677</td>
<td>30.67</td>
<td>(16.37)*</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-2.511 (4.488)</td>
<td>M1</td>
</tr>
</tbody>
</table>

Table 4 - Multivariate models - General index

The table reports coefficient estimates (t-statistics in parentheses) obtained from multivariate models of the riskiness of the general corporate bond index. Each row refers to a different model. All models (except the first one) include YTM, VOL and CPI as predictors, plus a fourth variable, indicated in the last column. The penultimate column reports the coefficient estimate for the fourth variable. Each multivariate model also includes a constant (C). *** indicates a p-value below 1 per cent, ** below 5 per cent and * below 10 per cent. The meaning of the acronyms in the first row and last column is reported in section 3.4. The sample period is from December 31st, 1988 to January 30th, 2009, for a total of 5200 daily observations.
### General index

<table>
<thead>
<tr>
<th>Non-zero</th>
<th>Zero</th>
<th>J-stat (p-val)</th>
<th>MMSC-BIC</th>
<th>MMSC-HQIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>C,YTM,VOL, CPI, LIB3M, TED, SLOPE</td>
<td>9.506 (0.002)</td>
<td>-24.719</td>
<td>-8.5261</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, CPI, TED, SLOPE</td>
<td>1.952 (0.162)</td>
<td>-23.717</td>
<td>-11.5721</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, CPI, LIB3M, SLOPE</td>
<td>3.848 (0.049)</td>
<td>-21.821</td>
<td>-9.6726</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, TED, SLOPE</td>
<td>2.496 (0.114)</td>
<td>-23.173</td>
<td>-11.028</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, SLOPE, CPI, LIB3M</td>
<td>3.082 (0.079)</td>
<td>-22.587</td>
<td>-10.442</td>
<td></td>
</tr>
</tbody>
</table>

### A index

<table>
<thead>
<tr>
<th>Non-zero</th>
<th>Zero</th>
<th>J-stat (p-val)</th>
<th>MMSC-BIC</th>
<th>MMSC-HQIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>C,YTM,VOL, CPI, LIB3M, TED, SLOPE, UN</td>
<td>10.42 (0.001)</td>
<td>-32.362</td>
<td>-12.120</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, CPI, LIB3M, TED, SLOPE, UN</td>
<td>2.650 (0.103)</td>
<td>-31.575</td>
<td>-19.890</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, LIB3M, CPI, TED, SLOPE, UN</td>
<td>4.714 (0.029)</td>
<td>-29.511</td>
<td>-17.826</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, TED, CPI, LIB3M, SLOPE, UN</td>
<td>4.220 (0.039)</td>
<td>-30.005</td>
<td>-18.320</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, SLOPE, CPI, LIB3M, TED, UN</td>
<td>2.887 (0.089)</td>
<td>-31.338</td>
<td>-19.653</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, UN, CPI, LIB3M, SLOPE, TED</td>
<td>6.050 (0.013)</td>
<td>-28.176</td>
<td>-16.490</td>
<td></td>
</tr>
</tbody>
</table>

### BBB index

<table>
<thead>
<tr>
<th>Non-zero</th>
<th>Zero</th>
<th>J-stat (p-val)</th>
<th>MMSC-BIC</th>
<th>MMSC-HQIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>C,YTM,VOL, CPI, LIB3M, TED, SLOPE, UN, IJC, M1</td>
<td>19.90 (0.000)</td>
<td>-39.994</td>
<td>-11.656</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, CPI, LIB3M, TED, SLOPE, UN, IJC, M1</td>
<td>12.01 (0.001)</td>
<td>-39.328</td>
<td>-15.038</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, LIB3M, CPI, TED, SLOPE, UN, IJC, M1</td>
<td>9.627 (0.002)</td>
<td>-41.711</td>
<td>-17.421</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, TED, CPI, LIB3M, SLOPE, UN, IJC, M1</td>
<td>10.66 (0.001)</td>
<td>-40.678</td>
<td>-16.388</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, SLOPE, CPI, LIB3M, TED, UN, IJC, M1</td>
<td>10.53 (0.001)</td>
<td>-40.808</td>
<td>-16.518</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, UN, CPI, LIB3M, TED, SLOPE, IJC, M1</td>
<td>8.987 (0.003)</td>
<td>-42.351</td>
<td>-18.061</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, IJC, CPI, LIB3M, TED, SLOPE, IJC, M1, UN</td>
<td>13.09 (0.000)</td>
<td>-38.248</td>
<td>-13.958</td>
<td></td>
</tr>
<tr>
<td>C,YTM,VOL, M1, CPI, LIB3M, TED, SLOPE, IJC, M1, UN</td>
<td>15.24 (0.000)</td>
<td>-36.098</td>
<td>-11.808</td>
<td></td>
</tr>
</tbody>
</table>

The tables report the $J$-statistics obtained in the tests for over-identifying restrictions, used to perform model selection with Andrews and Lu's (2001) criterion. Each row refers to a different model. The first column reports, for each model, the variables that have a non-zero coefficient, while the second lists the variables that have a zero coefficient but are used to form the over-identifying orthogonality restrictions. The last two columns report the values of the Bayesian (BIC) and Hannan-Quinn (HQ) information criteria. The meaning of the acronyms in the first and second columns is reported in section 3.4. The sample period is from December 31st, 1988 to January 30th, 2009, for a total of 5200 daily observations.
<table>
<thead>
<tr>
<th>Third variable Coefficient</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>General 7.577 (2.040)***</td>
<td>-3.182 (1.235)***</td>
<td>1.572 (0.452)**</td>
</tr>
<tr>
<td>AAA 4.257 (0.857)***</td>
<td>-1.037 (0.534)*</td>
<td>1.572 (0.457)***</td>
</tr>
<tr>
<td>AA 5.834 (1.544)***</td>
<td>-2.281 (0.949)**</td>
<td>1.650 (0.516)***</td>
</tr>
<tr>
<td>A 7.593 (2.204)***</td>
<td>-3.231 (1.406)**</td>
<td>1.663 (0.487)***</td>
</tr>
<tr>
<td>BBB 10.07 (3.606)***</td>
<td>-1.431 (1.039)</td>
<td>1.776 (0.402)***</td>
</tr>
</tbody>
</table>

The table reports, for each index, the coefficient estimates ($t$-statistics in parentheses) of the multivariate models selected by Andrews and Lu's (2001) criterion. Each row refers to a different index. All models (except the first one) include YTM and VOL as predictors, plus (eventually) a third variable, indicated in the penultimate column. The last column reports the coefficient estimate for the third variable. Each multivariate model also includes a constant ($C$). *** indicates a p-value below 1 per cent, ** below 5 per cent and * below 10 per cent. The meaning of the acronyms in the first row is reported in section 3.4. The sample period is from December 31st, 1988 to January 30th, 2009, for a total of 5200 daily observations.
The figure plots the conditional variance of returns and the conditional riskiness of the general corporate bond index, estimated with a multivariate model selected by Andrews and Lu’s (2001) criterion and containing three predictors: the average yield-to-maturity of the corporate bonds, an adaptive forecast of the conditional volatility and CPI inflation. Values of riskiness are on the left-hand axis and values of variance are on the right-hand axis. The sample period is from December 31st, 1988 to January 30th, 2009, for a total of 5200 daily observations.
The figure plots the sample cumulative distributions of returns on the general corporate bond index, conditional on the yield-to maturity (YTM) being above or below its median. Returns are on the abscissae and cumulative probabilities are on the ordinates. The sample period is from December 31st, 1988 to January 30th, 2009, for a total of 5200 daily observations.
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