The topology of the interbank market: developments in Italy since 1990

by Carmela Iazzetta and Michele Manna
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THE TOPOLOGY OF THE INTERBANK MARKET:
DEVELOPMENTS IN ITALY SINCE 1990

by Carmela Iazzetta* and Michele Manna*

Abstract

When a bank defaults or stops trading in the interbank market, both a liquidity shortage in the market itself and mounting trading losses should be anticipated. To gain more insight into the way a liquidity crisis spreads, we apply network topology techniques to monthly data on deposits exchanged by Italian banks, from 1990 to 2008. Our research yields three main results: first, only a few banks are today pivotal in the redistribution of liquidity across the system, while banks close to, but outside this core circle, weigh less than they used to; secondly, the halt in operations in a second set of banks may cut off some of their counterparts from the rest of the network, with increasingly less negligible effects; finally, only 2-3 banks out of the 10 we identify as most interconnected within the network are currently also among the top 10 banks by volume of traded deposits.

JEL CLASSIFICATION: D4, E5, G2.
Keywords: interbank market, topology, liquidity crisis.

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In 1736 Euler solved the riddle of the Seven Bridges of Königsberg: probably one of the first examples of applied topology.

1. Introduction

A number of well-known features of the banking business – e.g. the maturity mismatch between banks’ assets and liabilities and the sequential constraint banks have to fulfil when serving their customers – make it likely that a system-wide banking crisis materializes first and foremost as a sudden deterioration in trust. The most evident sign of this pattern is the bank run, which may strike both troubled banks as well as banks that would otherwise be sound.

In the market for interbank deposits trust plays a considerable role if one considers that deposits on the leading overnight (1-day) maturity are typically uncollateralized. Furthermore, because of this prevailing maturity, every day the lender faces the decision on whether to extend new credit or not, that is whether to renew its trust in the borrower. Hence, the widely held view that in good times the interbank market provides banks with a form of coinsurance against uncertain liquidity flows, while in bad times it acts as a catalyst in spreading the crisis. Furthermore, the potential for contagion of the crisis from a first bank to the system is magnified by the practice of major players to act simultaneously as lenders and borrowers in this market. As a result, the halt in the operations of one of them triggers two knock-on effects. The banks that have lent to the failed one will recover, at best, only part of their loan; in addition, the banks which used to borrow from the defaulted one will need to seek new funding parties, something which may not be straightforward at times of crisis. Thus, the default of a bank may cause both credit losses and a loss of liquidity for the market as a whole, two patterns also referred to as the exposure contagion channel and the credit line contagion channel.

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1 We wish to thank Roberto Violi, Paolo Emilio Mistrulli, the participants at an internal seminar in the Bank of Italy and two anonymous referees, Cosma Onorio Gelsomino and Emerico Antonio Zautzik encouraged us first to carry out and then complete the research. We are especially indebted to Gerardo Palazzo, who helped us a lot in developing the programming routines, and Alice Chambers, whose accurate editorial work improved the text. Of course, all remaining errors are ours. While the research is the outcome of a joint effort by both authors, Carmela Iazzetta focused in particular on sections 4, 5.1 and 5.2, and Michele Manna on sections 2, 3, 5.3 and 5.4. The views expressed in this paper are those of the authors and do not necessarily reflect those of the Bank of Italy.
In this paper we examine the structure of Italy’s domestic interbank market from January 1990 to June 2008 by means of network topology concepts and measures. Two questions guide our research. First, what is the locus of the banks in the network and which are most central to it? Put differently, which banks play a more important role in the redistribution of liquidity for the system (globally or locally) so that their demise would have the largest knock-on effect. Secondly, has Italy’s interbank market evolved into a network that is more or less resilient to shocks as a result of the massive consolidation process the country banking system underwent during the last two decades?

Answers to these questions may provide insight into the relationship between developments in the structure of the banking system and the likelihood of a system-wide liquidity crisis. If, say, data pointed to an increased likelihood of contagion, then there would be an argument in favour of strengthening the banks’ liquidity buffers across the board. Likewise, a better understanding of the contribution of each bank to the systemic liquidity risk could help to tailor its own liquidity buffers. These insights may be especially valuable when unfortunately it is no longer time for crisis scenarios since actual crisis management is at work. In these circumstances, the identification of the most interconnected banks becomes a key tool in targeting rescue packages and, say, the decision to grant Emergency Liquidity Assistance (ELA).

The pursuit of these goals requires a methodology that works at the level of individual institutions and a dataset on bilateral exposures between banks. As to our methodology, network topology does not explain why a bank is first hit by an idiosyncratic shock. However, and crucially given our objectives, it will shed light on how a shock spreads across a network whose elements are internally connected, even more so when most of the links happen to be indirect, as is often the case in interbank markets (and the Italian one is no exception). Likewise, network topology provides the tools to identify the banks which may act as a trigger point in system-wide liquidity crises or, conversely, as a shock-absorber. Introducing some basic terminology, we will refer to the banks as the nodes and their interbank deposits as the links of the network.

The notion of shock spreading naturally brings to mind that of banks acting as global hub for the system. Their identification would be a relatively straightforward exercise – and accordingly network topology would make a less genuine contribution – if being interconnected and being large were mostly overlapping concepts. In fact, this statement does not hold true even when banks’ size is measured narrowly on the interbank business: our results suggest that in 2007 in Italy only 2-3 out of the top 10 banks by volume of deposits were also among the top 10 by centrality in the network. Hence, the scope for using a methodology specifically geared towards the concepts of centrality and interconnectedness. A further important value-added of topology is its capacity to scan the network to search for “local hubs”, that is banks whose day-to-day functioning is critical to some other banks, even though not necessarily for the system at large. Think of a bank \( p \), which trades deposits with the rest of the banking system only through a bank \( q \). If \( q \) stops operating – say because of a business continuity incident – then \( p \) would be cut off from the rest of system. Of course, bank \( p \) could look for a new counterparty, but this is not likely to be

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2 In a speech delivered on 22 August 2008, the Chairman of the U.S. Federal Reserve, Ben Bernanke, outlined the rationale for the rescue of Bear Stearns: “Although not an extraordinarily large company by many metrics, Bear Stearns was deeply involved in a number of critical markets, including (as I have noted) markets for short-term secured funding”.
achieved swiftly. At the very least, potential new counterparties would take their time to assess the creditworthiness of p itself.

Turning to the dataset, this should exhibit some defining features in order for network topology applications to add meaningful value to the research. If n and m denote the network’s number of nodes and direct links, we would like: (i) n to be reasonably large; (ii) the ratio $m/[n\times(n-1)]$ of active to potential links to be small, (iii) the network to be internally connected, so that for each pair of banks p and q it is always possible to shuttle funds between them, and (iv) the existing active links to be relatively stable over time. Conditions (ii) and (iii) imply that all pairs of nodes in the network are linked but in most cases the link is only indirect, condition (i) argues in favour of a developed methodology and reliable algorithms, while (iv) controls for the risk of random structures in the network and thus an unreliable identification of global and local hubs. Features (i)-(iv) are all but peculiar to the network topology of the interbank market. Albert and Barabási (2002) report some basic statistics of 17 networks studied in the literature (from math to words occurrence and power grids): for the median network, n was close to 22,000 and the ratio $m/n$ was 0.02%, which becomes 1.0% when n<2,000.

Our main dataset is compiled using data on the month-end stock of outstanding deposits exchanged between banks located in Italy. The size of the resulting network is in the order of one thousand, its ratio $m/n$ is in the range 1-1.5%, it is internally connected in each of the 222 months of our sample and, finally, its active links display a high degree of persistency. These elements meet the aforementioned desirable features and bode well for an application of network topology. Incidentally, this suitability seems to be found in a number of interbank markets also outside Italy, which may explain the stream of recent papers in this field.

Anticipating the gist of our results, we observe a number of changes in the topology of Italy’s interbank network. Since 1990, fewer banks really act as global hubs within the network, while the gap between these top-notch market players and those one notch below has widened. At the periphery of the network, we observe a thinner structure with considerably less direct links. As a result, smaller players need more intermediaries to reach out to the rest of the system; at the same time the number of local hubs, in the sense defined above, rose strongly. All in all, this could be mixed news for policy makers. On the one hand, having to deal with less and more easily identifiable major players eases the task of targeting the support measures aimed at preventing an eventual system meltdown. On the other hand, the thinner structure at the fringes of the network means that even when major players are up and running, the circulation of funds throughout the whole network could be less than smooth and some pockets of the system could be left out. Hence, the need for the policy maker to keep a close eye not just on the largest players but also on a number of banks with more local yet relevant connections. One practical upshot could be the need to enhance measures for preventing business continuity incidents, including in relatively small banks.

What we do not observe directly is any major break in the interbank network after the onset of the current market turmoil. This should come only as a relative surprise having noted that in mid-2008 the total amount of outstanding interbank deposits was still roughly in line with pre-crisis levels. Nor, is there any sign yet of a break in statistics on the density of the network, on its inner correlation and on persistency in the bilateral relationships. All these patterns stand in sharp contrast with developments in the liquidity of markets, if only
one observes that from June 2007 to September 2008 the volumes exchanged in
the e-MID money market trading platform have shrunk to half. Taken together,
these findings suggest that the role of the market in the redistribution of
liquidity was compressed in favour of more bilateral and behind-the-scenes
activity. This hypothesis suggests an agenda for future research, to which we
will return in section 6.

The rest of the paper is organized as follows: section 2 reviews some
relevant literature; section 3 sets out the algebra of a number of network
topological measures; section 4 describes our dataset and introduces some
descriptive statistics; section 5 presents our main results and section 6
concludes.

2. Some relevant literature

Our research draws on a number of strands of literature. At a more
theoretical level, following the work by Kindleberger (1978) and Bryant (1980),
Diamond and Dybvig (1983) helped devise a model for bank runs due to a
financial shock in a context of limited information, which leads to self-fulfilling
prophecies. This is often referred to as the panic-based view on bank runs (Iyer
and Peydró-Alcalde, 2005). The Diamond and Dybvig model has been
expanded in several directions, e.g. by removing the constraint of limited
information as a precondition for the run (Donaldson, 1992). It was up to
Rochet and Tirole (1996) to develop a formal model of system-wide crises with
n banks enjoying at ‘time 0’ different endowments, deviating from the single
representative bank of previous papers. An alternative non sun-spot view
regards financial crises as an inherent element of the business cycle, which
results from asymmetric information between the banks and their customers
(Jacklin and Bhattacharya, 1988).

Allen and Gale (2000) recognize three types of banking networks: (i) the
complete structure, where each bank is linked to all other banks; (ii) the
incomplete structure, where some of the links are only indirect; and (iii) the
disconnected structure, where at least one pair of banks is not linked (either
directly or indirectly). Their basic argument is that because of transaction and
information costs, banks may refrain from acquiring claims on ‘more distant’
banks. However, while many banks deal directly with only a limited number of
counterparties in the interbank market, the system as a whole is generally fully
connected. The taxonomy defined by (i) to (iii) is relevant in policy terms
because complete structures are usually thought to be more prone to contagion,
but the consequences of the contagion may be less harsh. A more extreme
situation is the money centre model of Freixas, Parigi and Rochet (2000), where
the only existing links in the network are those to/from the money centre bank.

Eisenberg and Noe (2001) set out, using relatively straightforward algebra,
the conditions for the existence and uniqueness of a clearing vector for a
complex financial system. Their work underlies a number of recent papers
simulating the spreading of losses via interbank positions: basically, the
simulation entails the assumption of the demise of a first bank bringing about
losses to its lenders, which in turn could default, and so on and so forth. This

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These authors impose two conditions to define a clearing payment vector: limited liability (total
payments made by a node must never exceed the cash flow available to the node) and absolute priority
(stockholders in the node receive no value until the node is able to completely pay off all of its outstanding
liabilities).
strand of literature includes Sheldon and Maurer (1998), Furfine (2003), Blåvarg and Nimander (2002), Wells (2002), Cifuentes (2004), Upper and Worms (2004), van Lelyveld and Liedorp (2006) and Mistrulli (2007) who work on U.S., Swedish, Chilean, German, Dutch and Italian data. Broadly speaking, these papers end on a positive note, since they judge the default of a single bank, no matter how large, unlikely to cause the failure of large sections of the banking system. A more critical view is offered by Brusco and Castiglionesi (2007), who link the finding of limited potential for system-wide failures to the assumption of a fixed structure in the interbank claims. In their view, the final impact could be much larger if the prices and the balance sheets of the banks were allowed to vary endogenously along with the transmission of the shock. Degryse and Nguyen (2007) add a time dimension to the simulated sequence of defaults. Their main result is that the way interbank linkages may be conducive to contagion does change over time, occasionally it does so swiftly after a major shock such as September 11. A further specific note goes to Müller (2003), who applied Swiss data to expand the basic simulation set-up to embody liquidity rationing along with the ‘more standard’ credit losses.

To various degrees, all these works include some (implicit) reference to the concept of interbank network, where the size and direction of links are crucial to assessing the extent to which the network is prone to contagion. Notwithstanding recent developments, the specialized literature on market topology is still relatively limited. Formal measures of topology can be found in Müller (2003 and 2006), Boss et al. (2004), Soramäki et al. (2006), Cajuiero and Tabak (2007), Iori et al. (2007), von Peter (2007), Becher et al. (2008), Bech and Atalay (2008) and Nier et al. (2008). The last paper offers good hooks to the main results of this literature. Higher levels of the ratio between active and potential links in a network (higher density) are thought to be conducive to swifter shock transmission, although the relationship may happen to be non linear.⁴ For a given level of density, higher concentration in the banking systems leads to more vulnerability to system-wide breakdowns. Finally, it is not always the case that tiered banking systems are more fragile than relatively homogeneous systems, since much depends on the type of links run by the banks at the centre of the network.

This stream of applied research has benefited from the increase in and greater access to enhanced computing capabilities. In addition, and perhaps more recently, a trigger for a number of papers is the more widespread understanding that some intermediaries may be “too-interconnected-to-fail” (jointly or alternatively to being “too-big”) and thus ultimately the quest for understanding first, and reducing later, systemic risk in financial markets. The relative novelty of network topology in finance should not mislead anyone. In other fields this methodology is well-established and prides itself on an honourable record.⁵ Possible references include Albert and Barabási (2002) as an example of topology’s application to physics, Watts (2003) to sociology, Bollobás (1998) to mathematics and Albert, Jeong and Barabási (2000) to the internet and World Wide Web. By way of example of an oft-cited result of the literature, many networks are proved to have a low density but nonetheless

⁴ The sign of the relationship between connectivity and contagion may cease to be ambiguous when the ratio is in the region of 20% to 50%. In practice, these levels are seldom found in real-world networks (Albert and Barabási, 2002). A counter-example with a density of 88% is in Becher et al. (2008), who, however, examine a network of only 15 nodes.

⁵ In his survey Newman (2003) cites an impressive number of 429 papers, of which 18 are from the 1970s, 10 from the 1960s, 7 from the 1950s, 2 from the 1930s and 1 from the 1910s.
display the so-called small world pattern, that is most pairs of nodes are not directly linked but they keep being connected through a limited number of intermediaries, often not larger than six in sociological applications. Hence, the well-known law of ‘six degrees of separation’ (Watts and Strogatz, 1998 and Newman, 2003). Moreover, the consensus view is that most real-world networks are highly robust, in the sense that “while key components regularly malfunction, local failures rarely lead to loss of the global information-carrying ability of the network” (Albert and Barabási, 2002).

The resilience of a network to random failures in its nodes is closely linked to the distribution of the number of links across the nodes. When this distribution is “scale-free” – roughly speaking, if \( n \) nodes in a network each hold \( m \) links, then (approximately) \( \alpha n \) nodes will hold \( \beta m \) links where the parameters \( \alpha \) and \( \beta \) (both < 1) do not change with the starting point \( n \) – the literature suggests that the network is highly resilient to random failures. Unfortunately, this ‘virtue’ of scale free networks becomes a ‘vice’ when the failed nodes are the result of a well-targeted attack rather than being the result of some random process. In that case, even a low failure rate may seriously hamper the network.

Even a brief survey on topology cannot omit to deal with terminological and data issues. We introduced above the terms ‘node’ and ‘link’, which we took from the literature on networks in information technology, and more jargon will ensue. Readers should be aware that they may encounter different sets of terms in different contexts: the node becomes a site in physics and an actor in sociology with vertex as a further alternative; again, the link becomes a bond and a tie in physics and sociology respectively. Even the word network itself may be replaced and mathematicians more commonly speak of a graph.

Lack of suitable (bilateral) data often hinders network topology applications in finance. To our knowledge, the best an author can rely on is, as in Mistrulli (2007), a dataset featuring actual gross outstanding deposits reported monthly by each resident bank (or, when datasets at a higher frequency are available, usually coverage is less than full). This is already a great database if, say, Upper and Worms (2004) had to construct their own dataset from a set of plausible assumptions. Less data teething problems occur in analyses of (intraday) payment system flows. The earlier dates of the works by Angelini, Maresca and Russo (1996), and Koponen and Soramäki (1998), who used intraday data for Italy and Finland respectively, should therefore come only as a partial surprise. More recently, Devriese and Mitchell (2005) provided a primer of applied empirical research on Securities Settlement System data, where the financial intermediary is potentially subject to two forms of illiquidity, that of cash and securities. Clearly, the trade-off in using daily data is the associated computational burden, which limits the width of the sample, bearing in mind that network topology is already a computationally demanding technique (\textit{a fortiori}, this holds true with respect to intraday observations).

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6 The law is often associated with the work of Milgram (1967), although the concept had already received attention in earlier times. In his 1909 Nobel Prize speech, Guglielmo Marconi noted that roughly six transmission stations would be needed to cover the inhabited areas on the earth (in his computations, the exact number was 5.83).

7 Albert, Jeong and Barabási (2000) prove that a failure of five percent of randomly chosen nodes, a very high failure rate by most standards, has nearly no impact on communications within the network.
3. Indicators of the topology of the interbank market

We consider an interbank market with \( n \) banks; this number is the size of the network. Each bank represents a node. The outstanding deposit lent by bank \( p \) to bank \( q \) is worth \( w_{p,q} \geq 0 \); the corresponding amount lent by \( q \) to \( p \) is \( w_{q,p} \). Since we deal with data on gross outstanding bilateral exposures, \( w_{p,q} \) and \( w_{q,p} \) may be jointly positive.

There is a direct link between \( p \) and \( q \) if \( w_{p,q} + w_{q,p} > 0 \). If \( w_{p,q} = w_{q,p} = 0 \), nodes \( p \) and \( q \) are not directly linked. Below, the concept of ‘directness’ in a link will become clear when we introduce the distance between two nodes. We define explicitly the link corresponding to \( p = q \); the weight \( w_{p,p} \) is obviously nought. This enables us to compile an \( n \times n \) matrix \( W \), known in the literature as the weighted connectivity matrix, whose generic element of cell \( p,q \) is \( w_{p,q} \).

From \( W \), we derive two \( n \times n \) matrices. The ‘adjacency’ matrix \( S \) flags whether the direct link between two nodes exists or not, irrespective of its weight. Its generic element \( s_{p,q} \) is:

\[
[1] \quad s_{p,q} = \begin{cases} 1 & \text{if } w_{p,q} > 0 \\ 0 & \text{otherwise} \end{cases}
\]

In addition, we refer to a symmetric matrix \( B \) which highlights whether a link between nodes \( p \) and \( q \) exists, irrespective of the direction:

\[
[2] \quad b_{p,q} = \begin{cases} 1 & \text{if } w_{p,q} + w_{q,p} > 0 \\ 0 & \text{otherwise} \end{cases}
\]

We also use an \( n \times 1 \) vector \( i_g \), which takes value 1 in the \( g \)-th cell and 0 otherwise, and an \( n \times 1 \) vector \( u \), filled in with all 1’s. The symbol \( "^T" \) denotes the transpose of a matrix, e.g. \( S^T \) transpose of \( S \). Finally, we occasionally make use of a function \( 1(X=m) \), which applied to the matrix or vector \( X \) returns the number of cells in \( X \) which take value \( m \).

(i) Degree

The degree is the number of active links originating in node \( g \) or terminating at it. We define an out-degree and an in-degree which, when applied to the interbank market, correspond to the number of banks to which \( g \) lends to and from which it borrows. In many instances, it is also useful to count the total number of banks to which \( g \) is connected, irrespective of the direction of the link.

\[
[3] \quad k_g^{\text{OUT}} = i_g^T S u \in [0, n-1] \quad \text{out-degree of node } g
\]

\[
[4] \quad k_g^{\text{IN}} = i_g^T S^T u \in [0, n-1] \quad \text{in-degree of node } g
\]

\[
[5] \quad k_g = i_g^T B u \in [1, n-1] \quad \text{degree of node } g
\]
Next, we compile the $n \times 1$ vector $k$ whose $g$-th element is $k_g$ and its akin vectors $k_{\text{OUT}}$ and $k_{\text{IN}}$. The ratio of the sum of the in-degrees across all $g$’s, or equivalently of the sum of the out-degrees, divided by $n \times (n-1)$ yields the density (aka connectivity), a basic statistics of any network:

$$ p = \frac{u^T u}{n(n-1)} \in [1/n,1] \quad \text{density (connectivity)} $$

Subsequently, we derive the degree distribution function, by counting how many nodes are of degree 1, 2, .., m, .., n-1:

$$ P_m = 1 \ (k = m) \quad \text{degree distribution function} $$

This function helps to differentiate between two broad classes of networks: the exponential network where most nodes have approximately the same number of links, and the scale-free network, where the majority of nodes hold only a few links but a limited number of nodes boasts a much larger number of links.

Two indices of correlation may yield insights into the structure of the network. The so-called degree correlation gauges the relationship between in- and out-degrees:

$$ \rho^K = \text{correlation} \left( k_{\text{OUT}}, k_{\text{IN}} \right) \quad \text{degree correlation} $$

while the degree of assortativeness measures the relationship between the degree of each bank $g$ and the average degree of the banks directly linked to $g$ itself. To this end, we follow a two-step procedure

$$ \hat{k}_g = \frac{i_s^T B B u}{k_g} \quad \text{average degree of nodes linked to } g $$

$$ \rho^A = \text{correlation} \left( k, \hat{k} \right) \quad \text{degree of assortativeness} $$

where the structure of the two $n \times 1$ vectors $k$ and $\hat{k}$ should by now be obvious. If $\rho^A > 0$ the network is said to be assortative, i.e. nodes usually mate with nodes of similar degree; if $\rho^A < 0$ the network is disassortative.

(ii) Valued degree

This is a set of measures mirroring those of the degree except that they embody the weight of the links. Related formulae are in appendix 1.

(iii) Centrality

Measures of ‘centrality’ describe the position of the node within the network. To this end, a key concept is the distance between the nodes $p$ and $q$, $d_{p,q}$, defined as the shortest path between them:

$$ d_{p,q} = m < n \quad \text{if } \exists m-1 \text{ nodes } a', a'', ..., a^{[m-1]} \text{ s.t. } b_{p,a'} = b_{a',a''} = .. = b_{a^{[m-1]},q} = 1 $$

Nodes $p$ and $q$ are linked directly if $d_{p,q} = 1$; they are linked indirectly if $d_{p,q}$ is finite and greater than one; otherwise they are not linked. If any pair of nodes
in the network is mutually linked, then the network is internally connected. Finally, a bank \( g \) is said to be geodesic as regards banks \( p \) and \( q \) if it stands on the shortest route between them. Algebraically, \( g \) is geodesic to \( p \) and \( q \) if

\[
[11a] \quad d_{p,q} = d_{p,g} + d_{g,q} \quad g, p, q \in [1, n]
\]
against the alternative of

\[
[11b] \quad d_{p,q} < d_{p,g} + d_{g,q} \quad g, p, q \in [1, n]
\]

Further details and the algebra of additional measures of centrality, inter alia the weighted distance and the ergodicity of a node, are in appendix 2.

(iv) Resilience

We adopt two approaches to gauge the resilience of the interbank network: (a) one node at a time is removed and the knock-on effects on other banks in the network are worked out (Furfine, 2003); (b) a less common approach that aims at identifying the banks with highest geodesic frequency (Müller, 2003). The two approaches work quite differently: (a) helps to understand how much thinner the web becomes when a bank defaults and its direct links disappear; (b) looks more at the indirect links as the backbone of the overall circulation of funds throughout the system.

Figure 1 shows three simple networks all of size 7, each populated by a bank \( g \) and six banks called ‘1’ to ‘6’. In view of their small size, the three networks also have similar density, 29% to 38%. Differences arise when \( g \) is removed, so that its links (dotted in the figure) disappear: in the left-hand panel, only bank 1 is cut off from the rest of the system; in the centre panel this happens to banks 1, 2 and 3 while 4, 5 and 6 form a new network; finally, in the panel on the right-hand side each of the six surviving banks is isolated and deposits are no longer exchanged.

Figure 1

When the removal of a bank makes the network disconnected

light / medium   more serious   dramatic

Dotted lines show links which are no longer active after the removal of bank (node) \( g \).

Arguably, moving from left to right we observe more severe forms of disruption, even if the starting conditions are quite close to each other. Even more diverse assessments would be derived if the weights of the nodes were taken into consideration. Consider, for example, when bank ‘1’ in the left-hand panel is smallish or conversely fairly large. The algebra of the two measures of resilience is set out in appendix 3.
4. Our dataset and some descriptive statistics

As anticipated in the introduction, our main dataset is based on month-end gross unconsolidated bilateral interbank deposits reported by Italian banks to the Bank of Italy for supervisory purposes. In particular, we focus on the deposits exchanged with other resident banks. The time series runs from January 1990 to June 2008, for a total of 222 monthly data points. The number of banks covered in the dataset (the size $n$ of the network) is 1,146 at the beginning of the sample, and 790 at the end.

A number of reasons may explain the dominance in the literature of recourse to data on unconsolidated deposits vs. data on banking groups’ net exposures. In cross-border groups, fully centralized management is challenged by the obstacles in the flow of cash and securities across borders identified in BIS (2006). Within the domestic component of the groups, a preliminary data investigation showed that occasionally more banks run simultaneously a significant off-the-group interbank activity, hinting at structures of less than full centralized liquidity management. Moreover, the recourse to consolidated data is tantamount to assuming that intra-group credits enjoy seniority vis-à-vis credit claimed by banks outside the group, which is not the case (Mistrulli, 2008).

In the period under examination, the amount of interbank deposits traded by Italy’s banking system grew considerably. In 1990, Italian banks reported deposits worth €85 billion, comprising €45 billion vis-à-vis other domestic banks and €40 billion of cross-border deposits (average of month-end stocks). Seventeen years later, the corresponding figures were €611, €433 and €179 billion. This gives an average annual growth rate of 12.3% on total interbank deposits, which even adjusting for inflation remains high at 8.5%. The growth rate outpaced developments in the entire banking business: as a result, the weight of such deposits out of the total size of the banking system's balance sheet rose from 9-13% in the early 1990s to 17% in 2008.

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Figure 2

Interbank deposits as a % of total assets of the banking system
(ratio of end-month stocks at systems’ level)

---

10 The share of cross-border positions fell from half (40 billion out of 85) to less than one third (179 out of 611). No hasty conclusions should be drawn from this pattern on the integration of Italian banks in the euro money market, which is verified by the law of one price. If market integration is to be gauged from volumes, this should be based on a criterion of indifference ceteris paribus towards the country of residence of the counterpart, not on the sheer data on volumes (Manna, 2004).
Besides getting bigger, the interbank market also became more concentrated. The market share of the top-50 banks grew from 69% on average in 1990-1992 to 87% in 2005-2007; perhaps more tellingly, the deals where both the lending and the borrowing side was a top-50 bank surged from 47% to 78%. The largest players remain net borrowers from smaller ones, a recurring finding of studies on the microstructure of the interbank market. The gap, however, has almost closed: from 7 percentage points in the first three years of our sample, 26% of gross borrowing minus 19% of lending, to just 0.2 percentage points, (9.1% minus 8.9%).

Table 1

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Top 50</td>
<td>47.1</td>
<td>78.4</td>
</tr>
<tr>
<td>Other banks</td>
<td>18.9</td>
<td>8.9</td>
</tr>
<tr>
<td>Total</td>
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<td>87.3</td>
</tr>
<tr>
<td>Top 50</td>
<td>25.6</td>
<td>9.1</td>
</tr>
<tr>
<td>Other banks</td>
<td>8.4</td>
<td>3.6</td>
</tr>
<tr>
<td>Total</td>
<td>34.0</td>
<td>12.7</td>
</tr>
<tr>
<td>Overall market share of top-50 banks</td>
<td>72.7</td>
<td>87.5</td>
</tr>
<tr>
<td>Top 50</td>
<td>69.3</td>
<td>100.0</td>
</tr>
<tr>
<td>Other banks</td>
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<td>12.5</td>
</tr>
<tr>
<td>Total</td>
<td>100.0</td>
<td>100.0</td>
</tr>
</tbody>
</table>

Each cell reports the share of deposits lent by the group of banks reported row-wise to the group of banks reported column-wise. The top 50 banks are identified from their ranking on total (lending + borrowing) deposits.

The process of market concentration developed along the more general consolidation of the Italian banking system. In June 2008 the five largest banking groups accounted for 61% of the total assets of Italy’s banking system, up from 45% at end-1999 and 33% at end-1993. Corresponding shares for the top 5 individual banks were 22% in 1993 and 33% in 2008. The latter milder pattern likely reflects a process of consolidation which took place more often through acquisitions rather than mergers.

Figure 3

Market shares of Italy’s largest banking groups
(end-quarter percentages on total assets)
To disentangle the contribution of the system’s consolidation onto the market enhanced concentration, we compiled an additional dataset covering only the outside-the-group interbank exposures. Having done so for six selected dates, the overall market share for the top 50 banks measured increased from 70.5% in January 1993 to 80.6% in June 2008, while the corresponding figures worked out on the main dataset (which covers both within- and outside-the-group deposits) are 70.5% and 83.3%. We gather that only one fourth of the observed increase in concentration may be directly related to the enhanced system’s consolidation, supporting the view that the concentration mainly took place across and outside groups. In turn, this upholds the hypothesis that some banks are gaining ground as money centres for the whole banking system and not only within the group to which they are affiliated.

Table 2

| Interbank market concentration: the role of group consolidation (percentage values) |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
|———|———|———|———|———|———|———|
| Dataset including both within and outside group deposits | 70.5 | 69.3 | 70.3 | 78.2 | 85.5 | 83.3 |
| Dataset including only outside group deposits | 70.5 | 70.1 | 71.3 | 74.0 | 79.6 | 80.6 |

In annex 4 we describe additional checks on the comparability of our dataset with other datasets studied in the network topology literature, on the persistency of bilateral relationships in interbank trading and on the impact of changes in the banking population.  

---

11 As a rule, we exploit the whole dataset and the full time series in our applications. In two ad hoc exercises we focussed on six dates. Additionally, in computing a measure of resilience to shocks we omitted the “Banche di credito cooperative” (BCC), accounting for one-half of n of the main dataset but only one-tenth of the deposits in value. Both choices were dictated by sheer computational burden. For example, the exclusion of the BCCs reduced the number of triplets to be checked from 1 billion to “only” 120 million.
5. Our results

5.1 Measures of degree

The average Italian bank exchanged deposits with 10 counterparties in 2007, down from 17 in 1990; if this average is adjusted for the changing size of the network, the pattern is less regular although a downward trend emerges clearly anyway (figure 4). We obtain from the outset a picture of a relatively scantly network where only 1-1.5% of the links are active. The variability in individual data is relevant and the standard deviation is twice the mean for the out-degrees and three times for the in-degrees.

Figures 4a-b

To shed light on the patterns prevailing at a micro level, in the following we focus on the bank which each month is ranked first by number of counterparties (degree), as well as those on the 95th and 50th percentile (henceforth, the “top bank”, the “95th pc bank” and the “50th pc bank”; occasionally, we will refer also to a “90th pc bank”, whose meaning is apparent). Starting from the top bank, its in-degree fell from 841 in 1990 to 439 in 2008, its out-degree from 355 to 251. The top bank borrows from a higher number of counterparties than the one it lends to, in a ratio which decreased throughout our sample from 2.4 to 1.7. Again, this supports the view of an increasingly more even dealing behaviour by the largest banks.

Figure 5

The chart shows the maximum in- and out-degree worked out on each month on all banks.
In figure 5 we flag two dates when patterns could have changed swiftly: end-December 1998, the day before the switch from lira to euro, and end-September 2001, the first month-end after the Twin Towers attacks. On both occasions, a simple eye-ball inspection suggests that the reaction of the monthly statistics was virtually muted and the same can be said of most of our statistics. Drawing from other studies which, using daily data, show that the properties of interbank networks did change on September 11 but the impact was short-lived\textsuperscript{12}, an educated guess is that the reaction had disappeared already by the end of the month. This may explain why the reaction did not show up in our series (an alternative explanation could be that the Italian interbank market was not affected at all).

The in-degree of the 95\textsuperscript{th} percentile bank fell from 81 to 34, its out-degree from 68 to 32. This is a more marked decline than the one described above for the top bank. As a result, the ratio between such measures for the two banks increased from 10.4 to 12.9 for the in-degree and from 5.2 to 7.8 for the out-degree. The result is a weakening role by the 95\textsuperscript{th} percentile bank (approximately, ranked 40\textsuperscript{th}). In a nutshell, if before we described the interbank market as a game effectively played by 50-odd banks, we start gathering evidence that within this club the gap between the top and the close-to-the-top banks has widened.

The linear correlation between in- and out-degrees ranges between 0.8-0.9, while the corresponding weighted correlation is often lower and certainly bumpier. The two findings hint at a web of relationships where banks prefer to continue dealing with their usual counterparties, while being less concerned by the variability in the volume of associated deals. The network is also proved to be disassortative, suggesting that active banks frequently deal directly with smaller players, although less today then it used to be in the early 1990s.

\textsuperscript{12} Soramäki \textit{et al.} (2006) note that the density of the network defined by interbank payments over the Fedwire Funds service in the U.S. dropped after the attacks from 3.0\% to 2.6\%, to shoot up again to over 3\% already from the 14th to the 17th.
Figures 7a-b

**Measures of correlation within the network**

<table>
<thead>
<tr>
<th>degree correlation</th>
<th>degree of assortativeness</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.6</td>
</tr>
<tr>
<td>0.6</td>
<td>0.7</td>
</tr>
<tr>
<td>0.7</td>
<td>0.8</td>
</tr>
<tr>
<td>0.8</td>
<td>0.9</td>
</tr>
<tr>
<td>0.9</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The chart on the left shows the linear and weighted correlation between in and out-degrees, using [8] and [A.4]. The chart on the right shows the degree of assortativeness, using [9b].

5.2 Measures of distance and degree distribution

This analysis of the interbank market microstructure may be enriched using the degree distribution function. Much like a number of other networks, ours displays a smooth scale-free function with a large asymmetry on the left. This supports the view of our network as more robust to random failures than to targeted attacks. On average over the whole time span, 88 banks held five links, but only 25 banks held 10 links and 2 reached 50 links. That is, only a handful of Italian banks boasts a large number of counterparties in the interbank market; a similar result is in Boss et al. (2004) for the Austrian market. Figure 8b breaks down the function by subperiods (we cut the x-axis to 25 as most of the variability is on the lower degrees). Over time, the mode of the distribution shifts to the left and, on the vertical of 10 links, it moves down offering further evidence of a diminished role in the market of the relatively large players.

Figures 8a-b

**Degree distribution function**

- whole sample, 1999-2008
- selected sub-periods

The chart on the left counts how many banks hold one link, two links .. on average over the whole sample. The chart on the right does the same with reference to four selected sub-periods. The circle highlights the area of shift to the left of the curves, along the vertical of 10 links.
Through this set of results we also derive the mass distance function. In the early 1990s, the distance was 2 in more than 80% of the pairs of banks and 3 in 10% of them. Proportions began to change in the late 1990s and, by June 2008, they had become respectively 54% and 43%. In total, at the latter date we counted 56% of pairs within 2 links and 99% within 3 links, reasonably close to the 41% and 95% obtained by Soramäki et al. (2006) for the US Fedwire.

The chart shows the percentage of pairs of banks whose distance is, on each month, 2 and 3.

The end result is an increase in the average path length, from 2.1 to 2.4. The eccentricity (maximum value) of the distance went up from 3-4 to 5: if the rule of thumb described by the law of 'six degrees of separation' is adapted to a network of relatively small size, this finding provides further evidence of a network that is well behaved as far as its topology goes.

If the average linear distance increases, the corresponding weighted measure does so much more rapidly. The drift away from the centre of the smallish banks located at the fringes of the network may not suffice to explain this finding, because of the associated low weight. Rather, we should more reasonably search for an explanation in the drift away of the large-but-not-top banks, whose weight is more relevant.

The chart on the left shows the average linear (unweighted) distance, result [A.6]. The chart on the right shows values of the median version of [A.6] and [A.9].
5.3 Measures of resilience

A first approach to measure the resiliency of the network of interbank deposits insists on removing each bank one at a time and identifying which of them make, as a result of the removal, the network disconnected. The number of banks boasting this ‘disconnecting skill’ is on the rise: from 5 or less in the early 1990s to around 15 in recent years.

Figure 11

Number of banks which, if removed, make the network disconnected

![Graph showing number of banks which, if removed, make the network disconnected](image)

We argued above that in a disconnected network the split may be more or less severe. To work out a more precise measure of this intuition, first we identified the “disconnecting banks”. Second, we simulated their removal and measured the area of the network, which remained cut off from the core (using [A.13]). Third, we ranked the derived measures in descending order and binned into a higher, medium and low third. Fourth and finally, we calculated the average for each of the three groups. The procedure was repeated for six dates. We already knew that the number of the “disconnecting banks” had increased. Through this procedure, we observe a heterogeneous impact on the network of their removal, across banks and over time. For the banks in the higher third, the measure of the disconnected area more than doubled comparing the results for January 1990 and June 1994 to those of later dates (from less than 2% to 4-5%). Conversely, the impact is more limited and notably there is no visible trend when we turn to the banks grouped in the medium and lower third. One interpretation of these results is that while “global hubs” have increased in role (when assessed against their potential for contagion), “local hubs” have increased in number.

Figure 12

Measures of the disconnected networks once banks are removed

![Graph showing measures of the disconnected networks once banks are removed](image)

Average of [A.13] for the “disconnecting banks” at each specified date and third.
The fact that the demise of a bank is able to cut off a non-marginal portion of the network may depend on the size of the bank’s interbank exposures (as lender and borrower) but also on the number of its direct counterparts, for given size. Above, we gathered evidence on the enhanced market concentration; here we examine the latter profile through measures of geodesic frequency. Having worked out the measure of such frequency for each bank and each month, the maximum monthly frequency is proven to fluctuate at around 60%, with no visible pattern. That is, the observed average reduced in density (number of links) described in figure 6a did not extend to the bank most central to the network.

The two results are reconciled by the pattern followed by the geodesic frequency of the 95th pc bank, which declined from 20-30% in the early 1990s to 7% in the most recent observations. The locus of this bank in the network has shifted away from the centre. A simple way of measuring this shift is through the ratios between the two frequencies now described, a ratio which went up from 3-4 in most of the 1990s to 6 in more recent years (figure 13b). At the same time, this shift was not the outcome of a parallel expansion across the whole network, as suggested by the convergence to one of the ratio between the geodesic frequency of the 95th pc and 90th pc banks.

**Figures 13a-b**

**Geodesic frequencies of selected banks**

<table>
<thead>
<tr>
<th>Absolute values</th>
<th>Ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="chart.jpg" alt="Chart" /></td>
<td><img src="chart2.jpg" alt="Chart" /></td>
</tr>
</tbody>
</table>

The chart on the left shows the geodesic frequency of the “top bank”, “95th pc bank” and “90th pc bank”, calculated using [A.14]. The chart on the right shows two derived ratios.

Further evidence can be collected by counting the number of banks with geodesic frequency above a given threshold, set equal to one half and one third in the results shown in figure 14 (overleaf). Until 1997, 15 odd banks passed the threshold of one third and 6-8 that of one half; after a rapid drop around the turn of the millennium and some limited recovery, the corresponding statistics are respectively of 3-5 and 1 in 2008.
5.4 Does topology matter after all?

By and large, so far we have tried to identify which banks are more pivotal in the transfer of deposits across the banking system (more central to the network). One possible conclusion is that in this context the use of topology helps identify the big banks, even if through a different and less common perspective. In fact, such a common wisdom interpretation appears to hold true only if we attribute a loose meaning to the adjective 'big'. Figure 15 shows the number of banks which are both in the top 50 list by geodesic frequency and in the top 50 list by total interbank deposits, along with the number of overlaps for the two corresponding top 10 lists. In the mid 1990s some 40 banks were in both top-50 lists, a number which fell to 30 in the mid 2000s. If we restrict the analysis to the top 10 lists, the number of 'dual' banks goes down from 6-7 banks to 2-3: the overlap halves and hits much lower values.

Noticeably, in our results, the group of banks whose halt in business (their removal in our simulations) may render the network disconnected turns out to be a further distinct group, since it generally does not overlap with the banks with a high geodesic frequency, or with those running large volumes of interbank deposits. This suggests that while troubles with “disconnecting banks” are less likely to cause system-wide consequences, they may nonetheless have non-marginal consequences for some of their direct counterparts.
6. Concluding remarks

In this paper we examine the structure of Italy's domestic interbank market, applying a network topology approach to a monthly series of stock data from January 1990 to June 2008. Our research set out to answer two major questions. First, how central are banks in the network and how has their positioning changed over time? Second, has the network become more or less resilient to shocks, also in the light of the parallel process of consolidation of the country banking system?

The country's interbank market has the following main topological features. First, it displays a low and decreasing density. If, in the early 1990s the average bank exchanged deposits with 1.5% of other resident banks, towards the end of the sample this percentage was close to 1.0%. Second, the network remains connected in each of the 222 months we covered. This means that it is always possible to find a route, on already existing links, to shuttle funds between any pair of banks. Third, the distance between the nodes – the number of intermediaries required to shuttle the funds – is gradually but steadily rising. We can think of an expanding network, with the important qualification that the speed of expansion is not homogeneous across the network. A number of findings show that the ring of nodes in the network, which stand just a notch outside the core – what we have nicknamed the close-to-the-top banks, whose behaviour can be epitomized by the 40th-odd bank by number of counterparties – is drifting away more rapidly. This makes the role of such banks in the market more similar to that of less important players while its gap from the core – which is less populated in 2008 than it used to be in the early 1990s – has widened. Fourth, the network is found to be disassortative, albeit increasingly less so, where this technical term signals that major players often deal directly also with smaller ones. Fifth, the estimated degree distribution function for Italy's domestic interbank market is highly smoothed, closely resembles the textbook scale-free type and, again, offers evidence of a diminishing role by the relatively large players.

A number of these findings, notably the low density, the interconnectedness and the scale-free distribution, suggest that our network is well behaved as far as its topology goes. Hence, inferring from the results of a rich and well-established empirical literature (in finance as well as in several other fields), one could, for example, view the interbank market in Italy as highly resilient to shocks that randomly hit individual banks.

The patterns observed in several of our statistics point to a transition, which is under way since roughly the turn of the millennium or just before it. The model, which prevailed until the early 1990s – whereby large banks used to be net borrowers from the rest of the system, and dealt directly with a number of small players at the fringes of the network, while the close-to-the-top banks were not so distant from the really top ones – is on the wane. At the same time, an alternative model of a tiered banking system, where an elite of well-identified leading players is largely segmented from smaller players, appears to have not yet been fully accomplished. For example, while the declining degree of disassortativeness speaks in favour of this transition, we would wait for the degree to change sign (the system turning assortative) before calling the case closed.

A thinner and expanding network is also more subject to being frayed, especially at the edges. More precisely, it is possible to identify the banks whose halt in the operations, due to their default but also to less extreme events,
such as a technical incident which temporarily knocks out their IT centre, cuts one or more of the other banks off from the rest of the system. The number of such “disconnecting” banks is clearly on the rise, from less than 5 in the early 1990s to almost 20 in more recent years. Because in most cases the size of the network becoming disconnected is relatively limited (less than one per cent), we may think of such banks as local hubs in the system.

At the same time, perhaps as a result of the already mentioned reduced role of an intermediate layer of banks which contributed to keep the network connected, the demise of one of the largest players would have twice as large an impact in terms of disruption to the circulation of funds than it had in the early 1990s.

Such a diverse set of results implies that any assessment of changes in the network shock resilience need to be qualified. We said earlier that fewer banks populate the very centre of the network and really act today as its global hubs. Hence, we are confronted with a smaller number of banks with a large potential for spreading a crisis throughout the system. However, if these banks are hit, the impact can be greater than it used to be. In addition, more banks can trigger disruptions in liquidity distributions, where this disruption may be small on aggregated terms but highly relevant for specific pockets of the banking system.

From the point of view of the policy maker, these developments could be mixed news overall. On the one hand, a more precise identification of the banks acting as global hubs in the process of redistribution of liquidity should ease the targeting of support measures to systematically relevant institutions. On the other hand, the potential for damage brought about by the demise of these banks has increased over time. Moreover, due to the rise in the number of local hubs, the fact that major market players are up and running is no longer a sufficient condition to ensure the smooth circulation of funds across the whole banking system.

Our results suggest that the overlap between the cluster of big banks and that of most interconnected ones in the interbank market is limited, even when banks' size is measured narrowly on the volume of its interbank deposits. This finding suggests that crisis management could gain from a deeper understanding of the structure of this market, as network topology may provide.

It is beyond the scope of our paper to examine how customer relationships are established in the interbank market and why ultimately bank $p$ chooses to deal with bank $q$ and not with $g$. As part of a broader data investigation there remains, however, the empirical finding of a high persistence in the bilateral relationships between banks. This suggests that the search for a new counterpart may not be fulfilled swiftly and is all but left to the invisible hand of an auctioneer. This finding is likely to hold even more when a crisis has broken out.

A paper such as ours could not fail to tackle, or at least touch on, a number of data issues, which may also form the object of future research. To mention one possible item for this agenda, the demand for and supply of funds at the group level could be investigated further and with it the governance of the inter-group liquidity redistribution. Daily data could be examined to zoom in on the market patterns along with a business case approach, e.g. from September to November 2008 when the process of liquidity distribution was especially impaired and certainly not only in Italy. Whether network topology is the right approach for this additional line of research depends on the data set used and the features (i) to (iv) we mentioned in the introduction of this paper could provide guidance. One would also very much like to broaden the analysis to an
international basis, say encompassing the whole euro area interbank market. Unfortunately, right now there does not seem to be any suitable fully fledged data set, covering all bilateral exposures among banks resident in the area.
APPENDIX 1

Measures of valued degree

This is a set of measures that mirrors those relating to the degree except that we consider the weight of the link. In this case it better serves the intuition of the resulting statistics to rely on relative measures, rather than absolute ones as in [3]-[5]:

[A.1] \[ v_{g}^{\text{OUT}} = \frac{i_{g}^{T} W u}{u^{T} W u} \in [0,1] \] valued out-degree of node g

[A.2] \[ v_{g}^{\text{IN}} = \frac{i_{g}^{T} W^{T} u}{u^{T} W u} \in [0,1] \] valued in-degree of node g

[A.3] \[ v_{g} = \frac{i_{g}^{T} (W + W^{T}) u}{u^{T} W u} \in (0,1] \] valued degree of node g

[A.4] \[ \rho^{V} = \text{correlation}(v_{g}^{\text{OUT}}, v_{g}^{\text{IN}}) \] valued degree correlation

where \( v_{g}^{\text{OUT}} \) and \( v_{g}^{\text{IN}} \) are \( n \times 1 \) vectors which take on the g-th cell the value of \( v_{g}^{\text{OUT}} \) and \( v_{g}^{\text{IN}} \) respectively.

APPENDIX 2

Measures of centrality

A few notes on [10]: (i) the distance is 1 if the two nodes are linked directly, i.e. if \( b_{p,q} = 1 \); (ii) the distance from \( p \) to \( q \) is the same as the one from \( q \) to \( p \) because it hinges on the symmetric matrix \( B \); (iii) there may be more than one set of values \( (a', a'', \ldots, a^{[m-1]}) \) which links \( p \) and \( q \), but (iv) there is no set formed by less than \( m-1 \) nodes, otherwise we would scale down \( d_{p,q} \) accordingly (this follows directly from the definition of distance as shortest path); (v) there may be no suitable set of \( a \)'s, in which case the distance from \( p \) to \( q \) cannot be worked out or, alternatively, we say that such distance tends to infinite.

In parallel to the unweighted measure of the distance defined by [10], we can derive a weighted distance:

[A.5] \[ d_{p,q}^{W} = d_{p,q} \times \left[ \frac{4}{n} \left( \frac{i_{p}^{T} + i_{q}^{T}}{u^{T} W u} \right) \left( W + W^{T} \right) \left( i_{p} + i_{q} \right) \right] \]

The loading factor between square parentheses is one (less than one/more than one) if the sum of the interbank deposits borrowed from and lent to both \( p \) and \( q \) is equal to (more/less than) \( 4/n \) times the sum of all interbank deposits binned in \( W \). For example, the weighted distance is calculated shorter than its unweighted measure if \( p \) and \( q \) run relatively large amounts of interbank deposits.
A number of statistics can be derived from the matrices $D$ and $D^W$ which have as generic elements $d_{p,q}$ and $d^w_{p,q}$ respectively:

[A.6] \[ h_g = \frac{i^T_g D u}{n-1} \] average distance of node $g$

[A.7] \[ h_g^w = \frac{i^T_g D^W u}{n-1} \] weighted average distance of node $g$

[A.8] \[ h = \frac{u^T D u}{n(n-1)} \] average distance of the network

[A.9] \[ h^w = \frac{u^T D^W u}{n(n-1)} \] weighted average distance of the network

[A.10] \[ e_g = \max \left( i^T_g D \right) \] eccentricity of node $g$ (maximum distance)

[A.11] \[ e = \max (e_g) \] eccentricity (diameter) of the network

[A.12] \[ F_m = \frac{1(D = m)}{(n(n-1))} \] mass distance function

$h_g$ is the average distance from node $g$ to the other $n-1$ nodes; $h$ is the average across all $g$'s; $e_g$ is the maximum distance from bank $g$; $F$ counts (in terms of relative frequency) the number of pairs whose distance is $m \in [1,n-1]$. $h_g^w$, $h^w$ are the weighted versions of $h_g$ and $h$, while $e$ is the aggregated measure of $e_g$. If the system is disconnected, $h$ tends to $\infty$ and measures [A.6]-[A.11] cannot be derived ([A.12] only up to a tail). Of course, the option remains to work them out on the resulting connected sub-sets established within the broader disconnected network. An alternative solution is to rely on harmonic measures, derived as functions of the inverse of the distance where the ratio between 1 and $\infty$ is set equal to zero (of course, this is not strictly correct in the finite domain), so that pairs of nodes which are not linked give no contribution to the resulting sum (Newman, 2003).

APPENDIX 3

Measures of resilience

To measure the shape of the split and, jointly, the size of the involved banks, our approach is (i) to identify the largest connected sub-network within the $(n-1)$-sized disconnected network; (ii) to derive as a residual the more isolated banks; (iii) to assess the weight of their deposits against the total deposits of the banks in the $(n-1)$-sized network. The formula we used is:

[A.13] \[
\tau_w^{[-g]} = 1 - \frac{(u_{n-1}^{\partial,\text{MAX}})^T W^{-g} u_{n-1}^{\text{w,\partial,\text{MAX}}}}{(u_{n-1}^T)^T W^{-g} u_{n-1}}
\]
where $u_{n-1}$ and $u_{n-1}^{MAX}$ are n-1 vectors, the former filled in with all 1’s while the latter takes value 1 only corresponding to the banks belonging to the largest connected sub-network (zero elsewhere); $W^g$ is derived from W, after having eliminated its g-th row and column.

To measure the geodesic frequency associated with each bank, we apply the following procedure. First, we compile all possible combinations of nodes $p,q,g$ out of the $n$ nodes. Then, for each triplet we verify whether any of the three nodes is geodesic to the other two. Once the check is run on all triplets, we compile a geodesic score for each node. Precisely, we are interested in the relative measure of this score, which we call the geodesic frequency. In algebraic terms and as regards node $g$, we use the formula:

$$f_g = \frac{1}{(n-1)(n-2)/2} \left( d_{p,q} + d_{p,g} + d_{g,q} \right)$$

geodesic frequency of bank $g$

APPENDIX 4

Additional checks on our dataset

Comparability of our dataset with other datasets studied in network topology

Figure A.1 shows a simple comparison of the network topology properties of our main dataset vis-à-vis other datasets examined in the specialized literature. On the x-axis we report the log size and on the y-axis the average path length. Data for the other networks are taken from Albert and Barabási (2002). Our dataset lies very close to the trend line and, based on this simple test, it would not seem to embody elements of peculiarity as far as its topology goes.

Figure A.1

The Italian interbank market vs. other datasets used in topology

Data for the interbank market are a snapshot of September 2007. Data for other networks are from Albert and Barabási (2002).

Persistency in bilateral trading

In figure [A.2a] overleaf we set equal to 100 the number of bilateral deposits outstanding at the end of each month $t$ and we show the number of those deposits that existed already at the end of month $t-1$. In figure [A2b], we present a measure of correlation between the deposits outstanding at the end of the two consecutive months, where this time the measure reflects not only where a given bilateral deposit existed but also its size. By and large, we observe a high
degree of persistency, with a correlation coefficient mostly above 0.8 in the 1990s (with a few downward spikes) and virtually equal to 1 since 2003. The finding of a high degree of persistency between consecutive month-end stocks should also suggest that had comparable intra-month observations been available, they would not have painted a very different picture.

Changes in population

Over the almost 20 years covered by our sample, there were a number of changes in the population of banks resident in Italy. As a result, the patterns followed by a number of our statistics may, in principle, owe to a mixture of developments in the underlying economic phenomenon and to population changes.

This is a common and well-known problem in statistics, typically dealt with by introducing the concept of “flow” next to that of “difference in stocks”. The first differs from the second precisely because “flows” adjust for population changes (other adjustments are also common, e.g. the impact of exchange rate changes in the valuation of stocks, but are not relevant in our case). The next step is the derivation of an index series based at the start of the sample. The procedure is described in more detail on the first page of the technical notes in the Statistics Section of the ECB monthly bulletin. We show below the results of an exercise along these lines on the total amount of interbank deposits outstanding at end June from 1990 to 2008. For ease of comparison, stocks are rebased to 100 in June 1990. Until 2002, changes in the population had virtually no impact. Thereafter some gap opened, of which roughly half can be attributed to one
observation (June 2003). In any case the slope of the series and its smoothness
do not seem to be affected.

**Raw and adjusted stocks**
*(base 1990=100)*

Figures A.3
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