BANCA D'ITALIA

EUROSISTEMA

Temi di discussione

(Working papers)

Directed matching with endogenous Markov probability: Clients or competitors?

by Emanuela Ciapanna

April 2008

Number 665

DIRECTED MATCHING WITH ENDOGENOUS MARKOV PROBABILITY: CLIENTS OR COMPETITORS?

by Emanuela Ciapanna*

Abstract

We analyze the problem of strategic poaching of consultants by clients with particular reference to the business consulting industry. This article studies the strategic interaction of consulting groups, client firms and consultants, which gives rise to a market equilibrium in a mixed economy. At each date the consulting group faces a new client firm that requires a task to be performed. We show that under very general conditions, when a matching pair of clients and consultants meets, a dominant strategy will be played, where the consultant is captured by the client and the consulting group matches (whenever possible) the client's request. The novelty of this model is that the quality of the consulting services does not depend only on the consulting group's assignment strategy , but also on the capturing behavior of the clients. In this sense, the clients impose a consumption externality on each other, which is a source of inefficiency in this otherwise competitive market.

JEL Classification: D62, J41, L22, L84, M54.
Keywords: strategic poaching, two-sided matching, Nash bargaining, endogenous Markov chain, externality.

Contents

* Bank of Italy, Department for Structural Economic Analysis.

1. Introduction[1]

In an economy characterized by uncertainty about potential employees ability and by labor-contract rigidities, rents can often be extracted by labor intermediaries, such as consulting groups or independent contractors, in return for flexibility and information. These middlemen furnish their clients qualified consultants to perform specific tasks for a set period of time. If the quality of the match between consultants'skills and clients' requests proves to be satisfactory, then clients may find convenient to offer consultants a stable position in their organization, draining human capital from the consulting group. The business-economic literature refers to the practice of raiding key employees from competitors as "strategic poaching". As Peter Cappelli notes, "...Most executives today are *poachers:* they regularly look outside their organization to find key individuals to fill key posts..." (Cappelli 2000). Open competition for other companies' people is now becoming quite widespread. Fast-moving markets require fast-moving organizations that are continually regenerated with new talent and have become adept at outside hiring. Executives tend to be judged on their ability to instill loyalty in their people and the departure of a talented employee can be viewed as a failure; on the other hand, given the tightness of the labor market, it can be very hard and very costly to replace high level human capital.

Though strategic poaching is spreading quite uniformly across industries, it represents a more serious threat to companies, such as consulting groups, that assign their own employees to clients. For them the risk of losing their human capital investment is twofold, coming from both competitors and customers. The latter, in fact, opportunistically exploit the entire duration of the project to test the consultant's ability "on the job". Afterwards, they can use their inside information to decide whether or not to make a job offer to the consultant (human capital is an experience good).

The practice of hiring ex-consultants has proved detrimental to the whole global consulting industry. In a recent article in Fortune, Peter Luiks cites strategic poaching by clients as one of the main factors responsible for the ongoing shrinking of big consulting

[1] I am indebted to Hsueh-Ling Huynh and Michael Manove for their support and guidance. I also thank Bart Lipman, Albert Ma, Marco Li Calzi, Nandini Krishnan, Francesca Lotti, Marco Taboga, two anonimous referees and participants at Boston University and Bank of Italy workshops, SES and International Industrial Organization Conference for helpful suggestions. Any errors are solely mine. The opinions expressed in this paper do not necessarily reflect those of the Bank of Italy. E-mail: emanuela.ciapanna@bancaditalia.it

groups. "Many large companies simply raid the consulting firms for talent. At AOL Time Warner, which is spending 75% less on consulting than it did five years ago, 12 of the 19 people in the company's strategy group are recently hired ex-consultants from McKinsey, Bain, and BCG. American Express, which spends a fraction of what it used to on consulting, has picked up at least fifty people from McKinsey in the past year" (Luiks 2004).

In this paper we analyze the practice of strategic poaching of consultants by clients with particular reference to the business consulting industry. We interpret the market for consulting services as a market for quality goods with externalities. Consulting groups are viewed as middlemen, endowed with both an informative and an allocative role: they observe the requests submitted by their clients as well as the skills available in the pool of their consultants and use this information to help the two sides of the market to meet more efficiently. That is, they gather information that would otherwise be dispersed (production input), they optimally decide an allocation strategy (production technology), in order to provide high quality consultant-client matches (production output). The presence of a middleman is justified by an imperfect market that fails to convey all the information to the agents. Nevertheless, due to transaction costs and capacity constraints, the consulting group cannot always provide a perfect match, which generates distorted incentives among clients, who will poach consultants as soon as somebody with the right profile is assigned to them (a negative consumption externality). Clients "consume" good quality by poaching people, whose skills are known, and consulting firms have to replace them with new hires of unknown type. As a result, the information available to the consulting group gets noisier (inputs worsen), the number of strategic states of nature decreases, and the market probability of creating good matches in the future is further depressed.

The paper has two, analytically independent parts. The first part examines the consulting groups' assignment mechanism as an endogenous Markov process with learning and capacity constraints. Through this process we generate the quality variable Q that will be defined as the probability of a "good" match. This probability is the outcome of consulting firms competing in the market. They hire new consultants and gradually learn their abilities, and they meet clients, who demand different tasks. Once a consultant has worked for a client, his type becomes known to the client as well as to the consulting group. The latter uses this information in deciding future assignments. Meanwhile, the client firms use the same information to decide whether to capture the consultants. When a capture occurs, it is reasonable to assume that the consulting group, which lost its consultant, will suffer a cost -search, training, etc.- before the employee can be replaced.

The second part of the model analyzes the strategic market interaction of the three players: consulting groups, client firms and consultants, who take Q as given. We show that the market for consulting services is a market for quality goods with externalities. Given capacity constraints and replacement costs, consulting groups are not able to guarantee a good match with certainty (so that in equilibrium $Q < 1$), which creates a gap between social and individual optimality. Every time a good match is realized, a bilateral monopoly formes between the consultant and the client, who bargain on a new employment contract. This individual opportunistic behavior creates an externality among the clients: expected match quality and social welfare are further depressed. On the other hand, the need to break even induces the consulting groups to adopt a policy of quality maximization. We show that the game has an equilibrium in dominant strategies, which consists in the consulting group providing the best expected quality given its current inventory and the client poaching the consultant whenever the joint net surplus is positive. On the one hand, this result reinforces the thesis that economic competition provides good incentives to supply high quality products. On the other hand, ironically, the very fact that the middlemen outperform the market in conveying information, allows the clients to capture consultants. This has a negative impact on quality (externality). Without an explicit premium on capture (i.e. pricing the externality), inefficiencies will persist in this market.

The paper is organized as follows: Section 2 reviews the main contributions in the literature; Section 3 presents the model and the equilibrium strategies; Section 4 discusses the main results and concludes. Proofs that are not in the main text are provided in the Appendix.

2. Related literature

The question relates to three strands in the literature: *labor search* (Jovanovic 1979, Mortensen 1985, Acemoglu 1997, Moen 1997, Pissarides 2000, Davis 2001), *matching* (Lu and McAfee 1996, Acemoglu and Shimer 1999, Shimer and Smith 2000, Moscarini 2005), and the *theory of incentives and mechanism design* (Laffont and Martimort 2002). Our analysis shares with the *Search* literature the interpretation of future job offers as an option pricing problem; it uses the link between bargaining and matching typical of *Cooperative Games Models;* and has some interesting features in common with the *Queue Theory* (Huynh and Rosenthal 2000) and *Assignment Mechanism Design* literature (Prescott and Townsend 2003, Hertzberg, Liberti and Paravisini, 2007).

In the classic search models with two-sided market, heterogeneous agents are drawn from a commonly known distribution and assigned to their counterpart. In those models all the strategic interaction between the two sides of the market occurs after a match is exogenously realized. In this paper, we examine more closely a mechanism that generates the consultant-client assignment. This results in an endogenous Markov chain, reflecting the optimizing behavior of the match maker and of the other players that form the matching pair.

While the study of the economic role of intermediaries in both financial and labor markets has been well explored in the literature (Bhattacharya and Yavas 1993, Yavas 1994, Autor 2001, 2005), the economic problem and the modelling strategy that we present here are original and probably share more elements with the finance literature on microstructures than with the micro-labor contributions or with assignment models à la Prescott and Townsend (Prescott and Townsend 2003).

The micro-labor literature explains the existence of middlemen with reference to two features of search markets: uncertainty and externalities. On one side, the search efforts of the agents may not be successful and may not result in a match, on the other the matching technology can be such that the search costs sustained by one agent affect the gains from search realized by his counterpart. These models view the middleman as an institution that can extract a rent by reducing the uncertainty and internalize some externalities. There are essentially two types of intermediary: market makers and match makers. A market maker sets an ask and a bid price at which he sells and buys for his own account (e.g. brokers and dealers operating in the stock market, used car dealers, e-commerce operators and so forth) (Bhattacharya and Yavas 1993, Gehrig 1993). A match maker doesn't trade, but simply matches agents from the two sides: buyers and sellers, as with real estate brokers, or firms and workers, as with temporary employment agencies (Rubinstein and Wolinsky 1987, Yavas 1994, Autor 2005).

Our work considers a special kind of middleman: the consulting group, which operates both as a match maker and as a market maker. In fact, like match makers, it helps demand

and supply to meet more efficiently (acting like a temporary agency). On the other hand, like market makers, it offers human resources to perform specific tasks, charging clients a fee (ask price); and to do so it hires consultants paying them a salary (bid price). It is important to note that the consulting group shares with a market maker the characteristic of buying and selling for its own account. In this sense, it is very different from a temporary agency, which is a pure match maker. The consulting firm does match agents from the two sides of the market, but the main object of the contract subscribed with the client is the service (the end) and not the consultant (the means). This subtle difference must be grasped in order to comprehend the rationale of this work. Here we are not just studying poaching and its implication for any middleman. We propose to analyze a special kind of market, where poaching is a problem rather than part of the ordinary activity. And it is a more sensitive issue than it would be for business generically understood, because the threat comes from clients, not from competitors. In this sense, consulting groups are particularly vulnerable to poaching. They have no incentive to close themselves towards their potential poachers, which makes the trade-off more complex, but also -we think- more interesting.

The existing theoretical models focus on the search behavior of the agents and on the effect that the presence of a middleman can have on the gains from trade and on search intensity. This article focuses on the allocative role of middlemen in a mixed market economy, where consulting firms compete and make zero profits, while clients and consultants, once matched, form bilateral monopolies when bargaining on the new employment contract. Our theoretical framework and notion of equilibrium are conceptually close to the market microstructure approach, (Kyle 1985, Glosten and Milgrom 1985, Veldkamp 2004). For instance, in his rational expectations model, Kyle assumes batch-clearing; that is, all orders are fulfilled simultaneously at the same price; in our model all the consulting firms fulfill their clients' requests at the same market price, which depends on match quality. Kyle assumes that there is a market maker, who sets prices and thus acts as an auctioneer. Moreover, the market maker can take trading positions and has privileged access to information on the order flow. This changes the nature of the pricing rule, because price setting is assigned to a player within the model. The market maker must set prices using only the information available to him, which is determined by the trading protocol. In our model the consulting market as a whole acts in a competitive fashion, implementing a Walrasian mechanism: the simultaneous and dispersed optimizing behavior of rational agents, each elaborating his own available information, is aggregated by the market, which performs both an allocative and an informative role.

3. The directed matching model

We model the market for consulting services as a pair-wise random matching and bargaining process. The basic structure of the model is the following. There are three categories of agents in the economy: consulting groups, consultants and client firms. Consultants' and clients' requests can be only of two types: A or B [2]. A client firm can access a consultant only through a consulting company. That is, we do not allow direct contact between customers and self-employed consultants[3].

We assume an infinite time horizon. The timeline is as follows: at each period there are Y consulting firms operating in the market. Clients' arrival is represented by an exogenous process with arrival rate θ. A client can contact a consulting group, requiring performance of task A or B. Then the consulting group chooses which type of consultant to assign. When submitting the request, the client takes as given both consulting market fees and the quality level of services prevailing in the market, Q. In our model, this quality variable is crucial: it is the probability that the consulting group will implement a good match between the client's requirement and the consultant sent to perform the specific task.

We assume asymmetric information between the consulting group and the client firm at this stage: the client cannot ex ante observe the type of consultant that is sent to him. At the end of the period, after the task has been performed, the quality of the match is observable and verifiable by all the players. Once the quality of the match is revealed, the client can make a "poaching" offer to the consultant. The client-consultant pair will bargain on compensation, solving an optimal stopping problem and deciding whether it is advantageous to exercise the option and subscribe a regular employment contract and cut out the middleman or to remain

---

[2] We assume horizontal differentiation in this model. Consultants' and clients' requests are not ranked according to their level of complexity (from low to high), they pertain to different business areas or different fields (financial services rather than IT). We construct an analogous model with vertical differentiation. The results are analogous, but the algebra is much more tedious, so we chose to focus on the horizontal differentiation case.

[3] This assumpion is consistent with the empirical evidence in many markets, such as security services, business consulting and other activities that are not usually purchased browsing the yellow pages, in a direct way, but through complex organizations.

in the intermediated market. The structure of the game, at this stage, resembles two-sided matching models with heterogeneous agents and endogenous disagreement point (Lu and McAfee 1996, Shimer and Smith 2000 and Moscarini 2005). If the joint surplus, net of the sum of the players' disagreement points, is positive, then the client finds it convenient to make an offer (we call $c > 0$ the client's capturing rate) and in case of acceptance by the counterpart, the spoils generated by the match are shared evenly[4].

Every time a consultant is hired by a client firm, the consulting group is left with a vacancy: it has to replace the lost human resource and to do so it faces a cost C that reflects screening, training investment, search costs and so on. Once the inventory is restored, we are back to the first stage: a new request is made and the game starts over again.

Thus, if X is the steady state number of clients at any period and Y is the corresponding number of consulting groups in our economy, with $X \leq Y$, then in each period there may be a certain number of consulting firms that are idle, depending on the dynamics that follow from the capturing behavior of the clients. Moreover, given a constant arrival rate θ, we have that in steady state the number of client firms X is endogenously determined and has to satisfy the following:

(1) $$X = [(1 - Q) + Q(1 - c)]\, X + \theta$$

which can be rewritten as:

(2) $$X = \frac{\theta}{Qc}$$

where c is the capture rate and Q the probability of a good match. Therefore, the probability that a consulting group is active (respectively idle) in a generic period in steady state is given by:

(3) $$\zeta = \frac{X}{Y} = \frac{\theta}{QcY}$$

(4) $$(\text{respectively } 1 - \zeta)$$

---

4 We suppose that with a good match the two parties (the client and the consultant) always obtain a greater utility than with a bad match, regardless their internalization strategy, so that capturing in the case of a bad match is always strictly dominated.

We assume that in each period there are only two consultants available in the consulting group's inventory[5]. Their type can be known or unknown to the consulting group, depending on whether they have already worked there for at least one period or have just been hired from among the indifferentiated pool of self-employed consultants. Thus, our state space is given by the Cartesian product between the client's request $\{A, B\}$ and the inventory possibility set, K. The latter is given by six entries, $K := \{NN,\ NA,\ NB,\ AB,\ AA,\ BB\}$ where N stands for "new". Therefore the consulting group can send an A, a B or an N type consultant in response to each client's request.

Let m be the realization of the match quality and suppose, for simplicity, that m is a binary variable: $m = 0$ in case of a bad match and $m = 1$ in case of a good match. When the customer comes to the consulting group, fees are commonly known and contractually established without ambiguity. If the quality of the match is low $(m = 0)$, a low fee, $F_{L,}$ will be charged by the consulting group; if it is high $(m = 1)$, a high fee, F_H. Thus we assume $0 \leq F_L \leq F_H$. Similarly, the consulting salary, earned by consultants while working at the group, is also conditional on the match quality. If the project is a mismatch with respect to their type, then the outcome will be "low quality" and the resulting salary will be the reservation wage, R.[6] In the case of a good match, they may earn a higher salary, W, where $W \geq R$.[7] We suppose that, once revealed, quality is perfectly observable and verifiable to all agents, so a contingent contract can be signed based on the quality of the match.

Our equilibrium concept is a competitive equilibrium in a mixed economy. That is, we have two different market levels in this model. On one side, consulting firms are perfectly competitive: they act in a market (the market for quality goods) with free entry, where they are price-takers and have to break even in expectation. On the other side, each time a good match is realized the client-consultant pair forms a bilateral monopoly. They are now two precise

---

5 This assumption may seem restrictive, but the right way of thinking about it is to focus on the client's request as the event that marks the start of the game. That is, we can imagine that in each period there are several clients contacting the consulting group and making their requests, but we just choose to concentrate on a representative client. In this way it seems reasonable that the group faces an inventory constraint in its human resources: other consultants are working on other projects or at other clients.

6 We can interpret the reservation wage, exogenously given, as what the consultant would earn if he worked as an independent contractor in a non-intermediated market.

7 Here W is endogenous. It seems realistic to assume that the consultant gets a sort of productivity bonus in case of a good match. This is very common in the business and economic consulting industry, where salaries are usually structured as non-linear tariffs. Notice, however that R and W are ranked by weak inequality.

identities, for the moment they are out of the market of consulting services, and they bargain in Nash fashion over the offer (Kyle 1985, Glosten and Milgrom 1985, Veldkamp 2004).

The consulting group faces a trade-off: in the case of a good match, a higher fee can be charged, but capture is more likely; whereas a bad (or uncertain) match reduces the risk of poaching at the cost of a lower fee. An equilibrium strategy is such that the two forces driving the trade-off are balanced.

3.1 *Strategy profiles*

In the market for consulting services our three categories of agents optimally choose a strategy profile that maximizes their expected utilities given the market prices $P = [F_H, F_{L,}W]$, the replacement cost C, the reservation wage R and the market quality variable Q. The latter is the prevailing and commonly known probability of occurrence of a good match.

We restrict our attention to symmetric equilibria. We look for a competitive equilibrium where the consulting group makes zero expected profit and the client-consultant pair, once formed, optimally chooses whether or not to form a stable match.

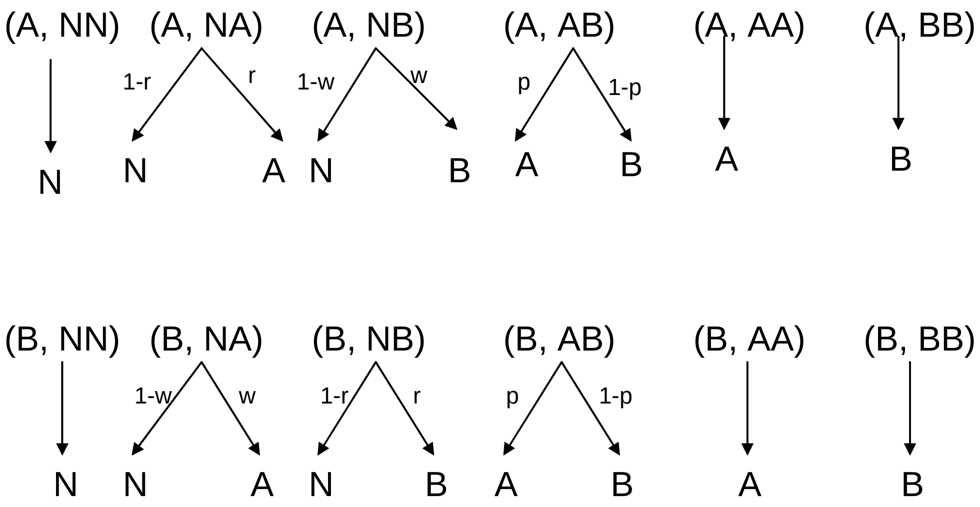
The possible mixed strategies for the consulting group are represented by the following set of conditional probabilities:

$$A_{CG}(K) = \begin{cases} r_A = \Pr\{send\ A\ when\ request = A\ and\ state = NA\} \\ w_A = \Pr\{send\ A\ when\ request = B\ and\ state = NA\} \\ w_B = \Pr\{send\ B\ when\ request = A\ and\ state = NB\} \\ r_B = \Pr\{send\ B\ when\ request = B\ and\ state = NB\} \\ p_A = \Pr\{send\ A\ when\ request = A\ and\ state = AB\} \\ p_B = \Pr\{send\ B\ when\ request = B\ and\ state = AB\} \end{cases}$$

These conditional probabilities represent the mixed strategy space for the consulting group: $\mathcal{A}_{CG}(K)$. The possible strategies for the consulting group are depicted in Fig.1. For simplicity, hereafter we assume that the A and B types are in the same proportion in this market, such that the steady state probability distribution is $\frac{1}{2}$ for each type.[8] A and B cannot be ranked; they only represent two different sectors of consulting or, in general, two independent skills. Therefore, we suppose that prices do not depend on the type of request from the client

---

[8] This assumption is without loss of generality and it is introduced as the consequent symmetric structure is more treatable in derivations.



1.The Tree Structure

and the consulting group is ultimately indifferent between serving one type or the other.[9] Under this hypothesis $r_A = r_B = r,\ w_A = w_B = w$ and the states AA, BB are renamed OO (two oldies), while the states NA, NB are now called NO (a new and an old hire). Therefore, the consulting group's strategy profile reduces to:

$$A_{CG}(K) = \begin{cases} r = \Pr\{\textit{matching client's request when state} = NA\} \\ w = \Pr\{\textit{not matching client's request when state} = NA\} \\ p = \Pr\{\textit{matching client's request when state} = AB\} \end{cases}$$

Let $\Omega := \{\textit{Good Match, Bad Match}\}$ be the state space for the client firm-consultant couple; then the strategies for the client are represented by:

$$A_{CF}(\Omega) = \begin{cases} c = \Pr\{\textit{Capture given good match}\} \\ (1-c) = \Pr\{\textit{Do not capture given good match}\} \\ 0 = \Pr\{\textit{Capture given bad match}\} \\ 1 = \Pr\{\textit{Do not capture given bad match}\} \end{cases}$$

and symmetrically, for the consultant:

$$A_{Co}(\Omega) = \begin{cases} a = \Pr\{\textit{Accept offer given good match and offer}\} \\ (1-a) = \Pr\{\textit{Do not Accept offer given good match and offer}\} \\ 0 = \Pr\{\textit{Accept offer given bad match}\} \\ 1 = \Pr\{\textit{Do not Accept offer given bad match}\} \end{cases}$$

We can see that the strategy "Capture given bad match " is strictly dominated, because it results in a certain loss for the client, so it will never be part of an equilibrium.

[9] What if consulting groups were to specialize? In our construction, due to the clients' poaching behavior, specialization would never be optimal. In fact, assume it was to specialize in type A requests. Then, whenever capturing occurs and replacement is needed, the consulting group fills the vacancy with an N type. With probability $\frac{1}{2}$ N is a B type, therefore if only A type requests are accepted, the consulting group does not exploit the possibility of employing a type B as part of a good match. Recall that clients' requests are for type A or B, with probability $\frac{1}{2}$.

3.2 *The probability of "good match": a market quality measure*

The Markov chain associated with the defined conditional probabilities has the following transition matrix:

$$P = \begin{array}{ccccc} & NN & NO & AB & OO \\ NN & 1-\zeta\frac{(2-c)}{2} & \zeta\frac{(2-c)}{2} & 0 & 0 \\ NO & \zeta\frac{rc}{2} & 1-\zeta\frac{(2rc+(2-c)(2-r-w))}{4} & \zeta\frac{(1-r)+(1-w)(1-c)}{4} & \zeta\frac{(1-w)+(1-r)(1-c)}{4} \\ AB & 0 & \zeta pc & 1-\zeta pc & 0 \\ OO & 0 & \zeta\frac{c}{2} & 0 & 1-\zeta\frac{c}{2} \end{array}$$

Proposition 1 *The Markov chain is irreducible and aperiodic, therefore it admits a unique invariant distribution π* [10].

(5) $$\begin{pmatrix} \pi(NN) \\ \pi(NO) \\ \pi(AB) \\ \pi(OO) \end{pmatrix} = \frac{1}{D(p,c,r,w)} \begin{pmatrix} 4prc^2 \\ 4pc(2-c) \\ (2-c)\left[(1-r)+(1-w)(1-c)\right] \\ 2p(2-c)\left[(1-w)+(1-r)(1-c)\right] \end{pmatrix}$$

where

(6) $$\begin{aligned} D(p,c,r,w) &= 4prc^2 + 4pc(2-c) + (2-c)\left[(1-r)+(1-w)(1-c)\right] + \\ &\quad + 2p(2-c)\left[(1-w)+(1-r)(1-c)\right] \end{aligned}$$

Proof. *See appendix.* ■

Lemma 2 *The invariant distribution π is independent of the probability of being active, ζ, which does not affect the convergence behavior of the Markov chain.*

Lemma 3 *Let $Q \in [0,1]$ be the probability of a good match, that is*

$$Q = \frac{1}{2}\pi(NN) + \frac{1}{2}\left[\frac{1}{2}(1-w) + r + \frac{1}{2}(1-r)\right]\pi(NO) + p\pi(AB) + \frac{1}{2}\pi(OO)$$

---

[10] π satisfies:

$$\underline{0} = \left[I - P'\right]\underline{\pi}$$

and it is unique up to each admissible combination for the parameter value.

This probability is endogenous in the model, obtained by substituting for the expression of the invariant distribution π, *hence:*

$$(7) \qquad Q = \frac{1}{D}\begin{pmatrix} 2prc^2 + p\,(2-c)\,[(1-w)+(1-r)\,(1-c)] + \\ +p\,(2-c)\,[(1-r)+(1-w)\,(1-c)] + pc\,(2-c)\,(2+r-w) \end{pmatrix}$$

where D *is given by* 6.

The Markov probability Q is the key variable in the model. It represents the output of the consulting group "production function" and also a measure of efficiency of the consulting market. In fact, if we were to look at a non-intermediated market, where consultants are freelancers, the probability of a good match would be $Q^D = \frac{1}{2}$, a third best.[11] In this sense, any value of Q higher than this benchmark makes the intermediated market more efficient than the totally decentralized one. We saw how the role of consulting groups is justified by the existence of frictions in the labor market, in particular lack of information and search costs. The intermediary has an advantage in gathering and conveying information. What is the source of this advantage? The consulting group has an inventory of human resources that allows it to act strategically with respect to clients' demand. In an anonymous pool of self-employed consultants, the client finds either an A or a B type with equal probability, and the same is true for his counterpart. A complex and larger corporation, like a consulting group, can partially solve this problem, as it collects and manages many requests and counts on a pool of different types at the same time. Therefore, unless the other agents behave opportunistically, the intermediated market would yield quite an efficient outcome in terms of quality.

It is easy to see that, if poaching does not occur, it would be possible to reach a level of match quality as high as $Q^{SB} = \frac{3}{4}$, well above $\frac{1}{2}$![12] Therefore, the presence of the intermediary reduces inefficiencies, as it is the case in most of results aimed at finding a rationale for middlemen.[13] Nevertheless, due to the presence of inventory constrains and transaction costs,

---

[11] This is because we assume that types A and B have equal shares in the population.

[12] In our construction, due to capacity constraints faced by the consulting group, $Q = 1$ can never be reached, not even in a theoretical equilibrium entailing $c^* = 0$. This is true as long as the inventory of human resources at the consulting group only consists of two consultants per period. In fact, we would always have to assign positive probability to the state OO where the group is stuck with the same profile of consultant and this profile does not meet the request.

A more detailed discussion is given in section 3.6.

[13] In fact, the micro-labor literature explains the existence of middlemen with reference to two features of

the first best, $Q^{FB} = 1$, cannot be reached, even in presence of middlemen. The motivation is subtle. If $Q^{FB} = 1$ is not implementable, then the clients have an incentive to capture as soon as they observe a joint positive surplus (given market fees and disagreement points). But this opportunistic behavior introduces another externality, a consumption one. Each time a consultant is poached, he must be replaced with an unknown type; this introduces noise in the consulting group's inventory and worsens the overall quality in the market: $Q^* = Q^{TB} < \frac{3}{4}$. Thus, the consulting group finds room in the market for complex services because of market failure in pooling human resources of different types; but once it has entered, it introduces a consumption externality due to poaching. This efficiency analysis and the value of Q in the decentralized framework provides an important benchmark for our results. Is the equilibrium value Q^* higher than Q^D? How much lower is it than Q^{SB}? We can answer these question once we have solved the three-agent model.

3.3 *The client's problem*

From the client's point of view there are two possible states at every period. We call the set of states Ω and we have $\Omega = \{\omega_{1,}\omega_2\}$, where each component represents the characteristics of the consultant:

$$\begin{cases} \omega_1 & := \begin{array}{c} \text{right match} \\ Q \end{array} \\ \omega_2 & := \begin{array}{c} \text{wrong match} \\ 1-Q \end{array} \end{cases}$$

Therefore the value function of the client takes the form $V(\Omega) = (V(\omega_1); V(\omega_2))$ such that:

$$V(\omega_1) = \max_c \left\{(1 - F_H) + c\delta\frac{s}{2} + \delta\left[QV(\omega_1) + (1-Q)V(\omega_2)\right]\right\} \tag{8}$$

$$V(\omega_2) = -F_L + \delta\left[QV(\omega_1) + (1-Q)V(\omega_2)\right] \tag{9}$$

where c is the capturing rate, i.e. the probability of poaching in the event of a good match, $0 \leq \delta \leq 1$ is the discount factor and s is the joint net surplus.

---

search markets: uncertainty and externalities.

3.4 *The consultant's problem*

We analyze the optimization program of a consultant who is working for the consulting group. The possible states in every period are represented by $\Omega = \{\omega_{1,}\omega_2\}$ and because the consultant is called to decide only if the client finds it advantageous to make an offer, only in state ω_1 does the consultant act as a strategic player.

Therefore, the value function for the consultant takes the form:

$$U(\omega_1) = \max_a \left\{ a\delta \frac{s}{2} + W + \delta \left[QU(\omega_1) + (1-Q)\, U(\omega_2) \right] \right\} \tag{10}$$

where a represents the offer acceptance rate.

$$U(\omega_2) = R + \delta \left[QU(\omega_1) + (1-Q)\, U(\omega_2) \right] \tag{11}$$

3.5 *The Nash-bargaining solution*

We assume that whenever a consultant and a client form a good match, they play a bargaining game, and if the net surplus from the match is positive, they choose to become partners in a new employment relationship.

$$s = \frac{1}{(1-\delta)} - \left[(QV(\omega_1) + (1-Q)\, V(\omega_2)) + QU(\omega_1) + (1-Q)\, U(\omega_2) \right] > 0$$

The inequality above represents the condition under which the joint net surplus is positive and the players find it advantageous to form a long run match.[14] The first term, $\frac{1}{(1-\delta)}$,[15] represents the total gross surplus from a good match when they decide to start a permanent labor relationship, while the second term is the sum of the two agents' disagreement points.

---

[14] In the Nash bargaining mechanism the following always holds:

$$a = c$$

Because our players have endogenous disagreement points the spoils they are going to share are represented by the net surplus where it is positive, i.e. whenever matching is positively assortative.

[15] Recall that $m = \begin{cases} 1 & if \quad good\, match \\ 0 & if \quad bad\, match \end{cases}$

Defining

$$\begin{aligned}\overline{V} &:= \left(QV\left(\omega_1\right)+\left(1-Q\right)V\left(\omega_2\right)\right)\\ \overline{U} &:= QU\left(\omega_1\right)+\left(1-Q\right)U\left(\omega_2\right)\end{aligned}$$

we can rewrite:

(12) $$s=\frac{1}{(1-\delta)}-\left(\overline{V}+\overline{U}\right)>0$$

While the salary offer is given by

(13) $$S=\frac{s}{2}+\overline{U}$$

Note that in general, requiring the net surplus to be positive is a stricter condition than requiring a good match. In fact, a match is good or bad depending only on the assignment of types, i.e. on the output quality the consulting group can provide to the market, while the surplus is also influenced by monetary variables, such as salaries and fees. Therefore, "good match" is a necessary, but not a sufficient condition for positive surplus.[16]

Proposition 4 *For any $Q<1$ and any $C>0$, and for any vector of market prices $[W\ F_L\ F_H]$,each time that a good match occurs, the consultant and the client have a dominant strategy that is always to capture ($c=1$). This means that the internalization option is always exercised when a good match occurs, no matter how much it may cost in terms of surplus share.*

Proof. *In appendix.* ■

3.6 *The consulting group's problem*

To study the optimization problem of the consulting group, first note that it can be in one of six states of nature:

NN	
ORN	(the old consultant matches the client's request)
OWN	(the old consultant does not match the client's request)
AB	
OOR	(the old consultant matches the client's request)
OOW	(the old consultant does not match the client's request)

[16] In our model we shall see that it also sufficient, but we think the caveat is needed.

Hence, the value function for the consulting group is:

$$V(K) = (V(NN); V(ORN); V(OWN); V(AB); V(OOR); V(OOW))$$

To simplify the notation let's define

$$V(ON) = \frac{1}{2}V(ORN) + \frac{1}{2}V(OWN) \tag{14}$$

and

$$V(OO) = \frac{1}{2}V(OOR) + \frac{1}{2}V(OOW) \tag{15}$$

Then we have:

$$\begin{aligned} V(NN) &= \frac{1}{2}(F_H - W - cC + c\delta V(NN) + (1-c)\delta V(ON)) + \\ &+ \frac{1}{2}(F_L - R + \delta V(ON)) \end{aligned} \tag{16}$$

$$V(ORN) = \max_r \left[\begin{array}{c} r(F_H - W - cC + c\delta V(NN) + (1-c)\delta V(ON)) + \\ +(1-r)\left(\begin{array}{c} \frac{1}{2}(F_H - W - cC + c\delta V(NN) + (1-c)\delta V(ON)) + \\ +\frac{1}{2}(F_L - R + \delta V(ON)) \end{array} \right) \end{array} \right] \tag{17}$$

$$V(OWN) = \max_w \left[\begin{array}{c} w(F_L - R + \delta V(ON)) + (1-w)\frac{1}{2}(F_H - W - cC + c\delta V(NN)) + \\ +(1-w)\frac{1}{2}((1-c)\delta V(ON)) + \\ +(1-w)\frac{1}{2}(F_L - R + \delta V(ON)) \end{array} \right] \tag{18}$$

$$V(AB) = \max_p \left[\begin{array}{c} p(F_H - W - cC + c\delta V(ON) + (1-c)\delta V(AB)) + \\ +(1-p)(F_L - R + \delta V(AB)) \end{array} \right] \tag{19}$$

$$V(OOR) = F_H - W - cC + c\delta V(ON) + (1-c)\delta V(OO) \tag{20}$$

$$V(OOW) = F_L - R + \delta V(OO) \tag{21}$$

We have assumed that the market for consulting services is competitive; therefore consulting companies break even in expectation. That is, the present discounted value of its expected profit must be equal to zero.

We first show that, in this framework, calculating expected profits as the expectation of the instantaneous profits weighted by transition probabilities is equivalent to calculating the expectation of long run profits with respect to the invariant distribution, the steady state probability.

Lemma 5 *Let $\Pi(s)$ be the long-run expected profit for the consulting group in state s, $\pi(s)$ the instantaneous profit in state s, $\mathbb{P}(s'|s)$ the generic transition probability between states s and s', and $\xi(s)$ the invariant probability distribution associated with state s. Then, in a steady state, the two following conditions are equivalent:*

$$\Pi = \pi(s) + \delta \sum_{s'} \mathbb{P}(s'|s)\, \pi(s'); \tag{22}$$

$$\Pi = \frac{1}{1-\delta} \sum_{s} \Pi(s)\, \xi(s). \tag{23}$$

Proof. See appendix. ■

From the previous lemma we can write the zero profit condition based on the per-period profit and the invariant distribution, as:

$$\begin{array}{rcl} \Pi & = & \left(\frac{1}{2}\pi(NN) + \frac{3}{4}\pi(NO) + \pi(AB) + \frac{1}{2}\pi(OO)\right)(F_H - W - cC) + \\ + & & \left(\frac{1}{2}\pi(NN) + \frac{1}{4}\pi(NO) + 0\pi(AB) + \frac{1}{2}\pi(OO)\right)(F_L - R) = 0 \end{array} \tag{24}$$

that is, substituting for the value of Q,

$$E\Pi = Q(F_H - W - cC) + (1-Q)(F_L - R) = 0 \tag{25}$$

Definition 1 *The cost of replacement of human capital that a consulting group faces each time a consultant is captured by a client is sustainable if, for every $c \in [0,1]$, the following holds:*

$$C \leq \frac{F_H - W}{c} \tag{26}$$

That is, the profit of the consulting group (zero in expectation) is positive in the "good match" state and negative in the "bad match" state.[17]

Proposition 6 *If the replacement cost C is sustainable, then the consulting group has a dominant strategy that consists in always sending the right match when possible, $(r = 1, p = 1, w = 0)$ which is optimal for every c, a chosen by the other players.*

Proof. *See appendix.* ■

Definition 2 *We define prices as sensible iff they are non-negative; therefore we refer to the following:*

$$\begin{cases} 0 \leq R \leq W \\ 0 \leq F_L \leq F_H \\ S \geq 0 \\ C \geq 0 \end{cases} \tag{27}$$

as the sensible price condition.

3.7 *Equilibrium outcome*

We have shown that our market for consulting services is characterized by an *assignment equilibrium in pure dominant strategies,* where the customer always captures when the match is good $(c = 1)$ and the consulting group always assigns the best possible consultant, given the client's request and the state endowment: $(r = p = 1, w = 0)$.

We must now find the other component of this equilibrium. That is, we have to verify that there exists a price vector (F_H, F_L, W) such that the three players are all playing their best response given the opponents' strategy.

[17] If this condition fails to hold, we could have that the consulting group makes negative profits in the good match state and positive profits in the bad match state, to break even in expectation. The equilibrium outcome would change dramatically because in this case the consulting group would have an incentive to produce a mismatch, whenever possible. Even if there are no technical reasons to impose this restriction on C, we don't think this part of the equilibrium strategy is economically sensible. Therefore, we assume sustainable replacement cost and limit our analysis to the case in which the incentives work in the same direction for all the players in the model.

First of all, we compute the invariant probability distribution vector under the assumption $c = 1$ and $p = r = 1$, $w = 0$. In this case:

$$\begin{pmatrix} \pi(NN) \\ \pi(NO) \\ \pi(AB) \\ \pi(OO) \end{pmatrix} = \begin{pmatrix} 0.4 \\ 0.4 \\ 0 \\ 0.2 \end{pmatrix} \tag{28}$$

the associated between-state transition matrix is:

$$P = \begin{array}{c|cccc} & NN & NO & AB & OO \\ NN & \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ NO & \frac{1}{2} & \frac{1}{4} & 0 & \frac{1}{4} \\ AB & 0 & 1 & 0 & 0 \\ OO & 0 & \frac{1}{2} & 0 & \frac{1}{2} \end{array} \tag{29}$$

It is immediate to notice that the state AB has probability zero under this strategy and so becomes a transient state. Intuitively, for this to be an equilibrium strategy the cost of replacement of human capital has to be less than the fee charged by the consulting group for a good match.

Looking at the problem of the client, we first observe that the probability of getting a right consultant in this case, when $c = 1$ and $p = r = 1$, $w = 0$ is given by:

$$Q^* = \frac{3}{5} \tag{30}$$

In correspondence to our equilibrium, $c = 1, p = r = 1, w = 0$, the two components of the value function vectors for the consultant and the client are:

$$\begin{cases} U(\omega_1) &= W + \frac{\delta}{2}s + \delta\overline{U} \\ U(\omega_2) &= R + \delta\overline{U} \\ V(\omega_1) &= (1 - F_H) + \frac{\delta}{2}s + \delta\overline{V} \\ V(\omega_2) &= R + \delta\overline{V} \end{cases} \tag{31}$$

The condition on the net surplus $s > 0$, reflects the optimality of both $c = 1$ and $a = 1$. This is because we are looking for a symmetric equilibrium and the assumption of Nash bargaining between consultant and customer/potential employer rules out the possibility of opposite incentives between these counterparts. In fact the decision criterion is $s > 0$ for both.

Thus, the incentive condition on the net surplus is:

$$\frac{1}{1-\delta}-\left(\left(\frac{3}{5}V\left(\omega_1\right)+\frac{2}{5}V\left(\omega_2\right)\right)+\left(\frac{3}{5}U\left(\omega_1\right)+\frac{2}{5}U\left(\omega_2\right)\right)\right)>0 \tag{32}$$

which after some computations, extensively shown in the appendix, reduces to:

$$s=\frac{2+3C}{5-2\delta}>0 \tag{s+}$$

The previous inequality is always satisfied, as we expected. While the salary offer is given by

$$S=\frac{s}{2}+\overline{U} \tag{33}$$

In this equilibrium we have:

$$\overline{U}=\frac{\left(30-12\delta\right)W+20R+\delta\left(15+4\delta-10\delta^2\right)s}{50\left(1-\delta\right)} \tag{34}$$

$$S=\frac{\left(5-2\delta\right)\left(30-12\delta\right)W+20\left(5-2\delta\right)R+5\left(2+3C\right)\left[5+\left(5-2\delta\right)\left(1+\delta^2\right)\right]}{50\left(1-\delta\right)\left(5-2\delta\right)} \tag{35}$$

which is non-negative for all values of the discount factor and thus satisfies the condition on sensible prices.

Proposition 7 *The pure strategy equilibrium of this market is given by a price vector* $P=[F_H,F_{L,}W]$*, and an allocation rule of human resources, summarized by the probability vector* $[r,w,p,c]=[1,0,1,1]$*, such that all the following conditions are simultaneously satisfied:*

$$\Pi=0 \tag{36}$$

The zero profit condition for consulting group:

$$\Pi=\frac{3}{5}\left(F_H-W-C\right)+\frac{2}{5}\left(F_L-R\right)=0 \tag{37}$$

The incentive constraint (IC) of the consulting group: $p=1$*:*

$$F_H-W-C\geq\frac{10-\delta-11\delta^2}{\left(1-\delta\right)\left(10-4\delta-\delta^2\right)}\left(F_L-R\right) \tag{38}$$

The incentive constraint (IC) of the consulting group: $r = 1 \Leftrightarrow w = 0$*:*

$$F_H - W - C \geq \frac{10\,(2-\delta)}{\left(30 - 13\delta - \delta^2\right)}\,(F_L - R) \tag{39}$$

The participation constraint (PC) of the consultant:

$$\overline{U} \geq 0 \tag{40}$$

The participation constraint (PC) of the client:

$$\overline{V} \geq 0 \tag{41}$$

The positive net surplus condition:

$$s = \frac{(4\,(1-\delta) + 6\,(1-\delta)\,(F_H - W) + 4\,(1-\delta)\,(F_L - R))}{2\,(1-\delta)\,(5-2\delta)} > 0 \tag{42}$$

The sensible prices condition:

$$\begin{cases} 0 \leq R \leq W \\ 0 \leq F_L \leq F_H \\ S \geq 0 \\ C \geq 0 \end{cases} \tag{43}$$

Proof. See Appendix. ■

We have assumed throughout that the consulting group does not price the real option; that is, we do not allow for premiums on capturing. This choice is not random. When we began to inquire into strategic poaching by clients in business consulting, we conducted interviews with partners and CEOs in the sector in order to assess perceptions of the importance of this opportunistic practice and to discover the most common retaining policies, if any, that consulting firms employ to prevent or to react to poaching.

We found that consulting groups tend to put the relationship with clients above the need to retain their consultants. Accordingly, they always seek to form the best possible match for the customer's request. In other words, they prefer to count on the long run advantages that may ensue, such as network and reputation effects, rather than actively fight poaching. In fact, the degree of competition in this market makes it difficult indeed to follow any alternative strategy: if losing the investment in human capital is costly, losing a client because of a bad signalling policy (the imposition of tedious no hire covenants) or suboptimal assignment behavior is seen as irreversible.

As to possible retention strategies, legal remedies may be either non-competition or no-hire covenants, depending on which contract they refer to (the employment contract or the consulting contract). From our interviews we gleaned that the weak point of such contract clauses is lack in enforceability, i.e. the extent to which these restrictions are applicable once they are brought to the attention of a court. In the non-competition case Omniplex World Services Corp. vs. US Investigation Services Inc. et al, the US Court of Appeals found that the restrictive covenant at issue was overly broad because it had no geographic specification, (it was in effect a worldwide covenant), it had no duration specification, and it failed to specify a function scope. In the preliminary discussion of the judicial case it emerged that the standards usually applied in reviewing a covenant not to compete in a poaching case are well established. "A non-competition agreement between an employer and an employee will be enforced if the contract is narrowly drawn to protect the employer's legitimate business interest, is not unduly burdensome on the employee's ability to earn a living, and is not against public policy."[18]

3.8 *Limiting behavior*

Our model shows that it is impossible, without imposing a price on the real option, to avoid or even to limit capturing behavior by clients. We also find that this strategy profile further depresses the quality output provided by the market. In fact, customers hiring their experimented consultants end up introducing a consumption externality in the industry: the consulting group loses known human resources and has to replace them with new employees, whose type is *a priori* unknown. This circumstance induces a lower steady state probability of providing a good match (Q is lower), as the level of uncertainty in the market increases. With particular reference to this outcome, it is worth asking what the determinants are and how we coul get a different result. First, notice that the only way to eliminate the consumption externality is to induce an equilibrium outcome in which $c = 0$ is the client's best response. This would be possible if and only if the net surplus was non-positive: $s \leq 0$. In this case, the client wouldn't capture the consultant and would go back to the consulting group in the

[18] Modern Events Inc. vs Stinnett, 263 Va. 491, 493, 561 S.E. 2d 694, 695 (2002); Simmons vs Miller, 261 Va. 561, 580, 581, 544 S.E. 2d 666, 678 (2001).

following period. To make the surplus non-positive, the market should always be able to offer the good match, so that the client is at least indifferent between capturing or not. Therefore, if the replacement cost C is negligible (that is, if $C \rightarrow 0$), then $Q = 1$ is the only quality level that can sustain such an outcome. But we have seen that in our model, due to capacity constraints on the consulting group, the maximum value for Q is $Q^{\max} = \frac{3}{4}$. Thus, $Q = 1$ can never be reached, not even in a theoretical equilibrium entailing $c^* = 0$. This holds as long as the human resources inventory of the consulting group consists of only two consultants per period. In fact, we would always have to assign positive probability to the state OO, where the group is stuck with the same consultant profile and this profile doesn't match the request. If we relax this assumption and we allow for larger inventory, then, at the limit, we can reach a situation with $Q \rightarrow 1$. A very big consulting firm can count on a number of consultants that is large enough to make the probability of the non-strategic states negligible. In this case, if $\Pr(OO....O) \rightarrow 0$, then the AB state becomes the only absorbing state in the Markov process. The corresponding invariant distribution becomes:

$$\begin{pmatrix} \pi(NN) \\ \pi(NO) \\ \pi(AB) \\ \pi(OO) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \tag{44}$$

Given the expression for the probability of good match

$$Q = \frac{1}{2}\pi(NN) + \frac{1}{2}\left[\frac{1}{2}(1-w) + r + \frac{1}{2}(1-r)\right]\pi(NO) + p\pi(AB) + \frac{1}{2}\pi(OO) \tag{45}$$

we can see that at the limit:

$$Q \rightarrow p \tag{46}$$

Hence, the following holds:

Proposition 8 *If the number of consultants available at the consulting group in each period is large enough and the replacement cost is negligible, then*

A. $\Pr(OO....O) \rightarrow 0$;

B. $\begin{pmatrix} \pi(NN) \\ \pi(NO) \\ \pi(AB) \\ \pi(OO) \end{pmatrix} \rightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$

C. $Q \rightarrow p$

Moreover, it is easy to show that the limiting equilibrium in this case is given by a pair of probabilities $(p^*, c^*) = (1, 0)$. Hence, our quality variable Q approaches the unity: $Q^* \to 1$.

Proof. See appendix. ■

According to the result of Proposition 11, it is possible to reach the first best and to avoid capturing, if the variety of human capital available at the consulting group is great enough to exclude, in the limiting case, the non-strategic states. If this is so, then the middleman encounters no obstacles in designing its optimal assignment strategy. This result is appealing when we look at a stylized fact regarding the evolution of consulting services' market structure. In a recent sector analysis by the Bureau of Labor Statistics, shows a tendency to concentration, i.e. a smaller number of larger consulting companies. Intuitively, if consulting firms gained market power, they could charge higher fees (the zero profit constraint wouldn't bind) and they could use their extra margins to offer their consultants monetary incentives ("golden handcuffs"), up to the point where the net surplus becomes negative and poaching ceases.

It would be interesting to investigate whether the intention to overcome inventory capacity constraints, as part of a human capital retention strategy, may be one of the determinants of concentration. The relation between market structure developments and consulting firms' strategic behavior goes beyond the scope of this paper, but it would be an interesting topic for future research.

4. Concluding remarks

Clemons and Hitt define poaching as "the risk that in any contractual relationship, information that is transferred between parties for purposes specified in the contract will deliberately be used by the receiving party for purposes outside the contract, to its own economic benefit and to the detriment of the party that provided the information" (Clemons and Hitt, 2001). In this paper we have analyzed the phenomenon of poaching in the consulting industry, where customers may have an incentive to hire consultants, once they have tested their ability on the job.

In our framework, consulting services are traded in a complex market, where the two-sided matching process is not random but directed by a strategic player, the consulting group. This middleman acts as a match maker and strategically assigns consultants to clients, given current and future requests, their available human capital endowment (what types of consultant are in its inventory) and the vector of market prices.

We modelled the consulting groups' assignment mechanism as an endogenous Markov process with learning and capacity constraints. Through this process we generated the quality variable $Q,$ the probability of a "good" match. Next, we examined the strategic market interaction of the three players: consulting groups, client firms and consultants taking Q as given. We have explained how the market for consulting services is a market for quality goods with externalities. Given technological constraints faced by the consulting groups, such as limited inventory and replacement costs, they cannot guarantee a good match, (so that in equilibrium $Q < 1$), which makes it impossible to conceal social and individual optimality. Each time a good match is realized, the consultant and the client form a bilateral monopoly, who bargain on a new employment contract. We proved that under the assumption of sustainable replacement costs, if no price is set on the capturing option, the dominant strategy for the consulting group is to provide the best expected quality, given its current inventory and market fees, and for the client to capture the consultant whenever a good match is sent. The ultimate outcome is, accordingly, a negative impact on the equilibrium market quality, due to consumption externality. As a consequence even when the consulting group is present to serve as an intermediary, inefficiencies will persist in this market.

Appendix

Definition 3 *Let X_k with $k \in N$, denote a Markov chain with transition function P, and let ω_1, ω_2 denote some arbitrary pair of states in Ω.*

A. The state ω_1 has access to the state ω_2, if

$$P[X(m) = \omega_2 | X(0) = \omega_1] > 0 \tag{1}$$

for some m in N that possibly depends on ω_1 and ω_2. In other words, it is possible to move (in m steps) from ω_1 to ω_2 with positive probability.

B. The states ω_1 and ω_2 communicate, if ω_1 has access to ω_2 and ω_2 has access to ω_1.

C. The Markov chain (equivalently its transition function) is said to be irreducible, if all pairs of states communicate.[19]

Definition 4 *An irreducible Markov chain X_k with $k \in N$ is aperiodic, if there exists some state ω in Ω such that*

$$P[X(1) = \omega | X(0) = \omega] > 0$$

This means that a sufficient condition for a chain to be aperiodic is that there is at least one state that has positive probability of transiting to itself.

Proof. [Proposition 1] We first show that the Markov Chain is irreducible. We know from Definition 4, that a Markov process is irreducible if all of the states can be reached starting from any other state with positive probability, even not in one step. The sufficient condition is that the transition matrix doesn't have an empty line, i.e. there be no state that can be reached only starting from some specific other state. The matrix T is such that for every row index i and column index j, there exist at least an element $t_{ij} > 0$. Therefore in a finite number of steps it is possible to reach every state from every other state

To prove that our Markov chain is aperiodic we refer to the Corollary of Wilhelm Huisinga, and Eike Meerbach, which provides a sufficient condition. To satisfy this condition we need that there is at least one state that has positive probability of transiting to itself, and we satisfy this condition.

---

[19] For the proof we refer to An Introduction to the theory of discrete time Markov chains on countable state spaces by Wilhelm Huisinga, & Eike Meerbach.

Therefore our chain admits a unique invariant distribution.[20]

$$\begin{pmatrix}0\\0\\0\\0\end{pmatrix}=\left[\begin{pmatrix}\zeta\frac{2-c}{2} & -\zeta\frac{rc}{2} & 0 & 0\\ -\zeta\frac{2-c}{2} & \frac{\zeta}{4}\left(2rc+(2-c)(2-r-w)\right) & -\zeta pc & -\zeta\frac{c}{2}\\ 0 & -\zeta\frac{(1-r)+(1-w)(1-c)}{4} & \zeta pc & 0\\ 0 & -\zeta\frac{(1-w)+(1-r)(1-c)}{4} & 0 & \zeta\frac{c}{2}\end{pmatrix}\right]\begin{pmatrix}\pi(NN)\\ \pi(NO)\\ \pi(AB)\\ \pi(OO)\end{pmatrix}$$

$$\begin{pmatrix}0\\0\\0\\0\end{pmatrix}=\zeta\begin{pmatrix}\frac{2-c}{2}\pi(NN)-\frac{rc}{2}\pi(NO)\\ -\frac{2-c}{2}\pi(NN)+\frac{2rc+(2-c)(2-r-w)}{4}\pi(NO)-pc\pi(AB)-\frac{c}{2}\pi(OO)\\ -\frac{(1-r)+(1-w)(1-c)}{4}\pi(NO)+pc\pi(AB)\\ -\frac{(1-w)+(1-r)(1-c)}{4}\pi(NO)+\frac{c}{2}\pi(OO)\end{pmatrix}$$

Because $\zeta > 0$, for the above equality to hold we need:

$$\begin{pmatrix}0\\0\\0\\0\end{pmatrix}=\zeta\begin{pmatrix}\frac{2-c}{2}\pi(NN)-\frac{rc}{2}\pi(NO)\\ -\frac{2-c}{2}\pi(NN)+\frac{2rc+(2-c)(2-r-w)}{4}\pi(NO)-pc\pi(AB)-\frac{c}{2}\pi(OO)\\ -\frac{(1-r)+(1-w)(1-c)}{4}\pi(NO)+pc\pi(AB)\\ -\frac{(1-w)+(1-r)(1-c)}{4}\pi(NO)+\frac{c}{2}\pi(OO)\end{pmatrix}$$

Hence, we find

$$\begin{pmatrix}\pi(NN)\\ \pi(NO)\\ \pi(AB)\\ \pi(OO)\end{pmatrix}=\frac{1}{D(p,c,r,w)}\begin{pmatrix}4prc^2\\ 4pc(2-c)\\ (2-c)\left[(1-r)+(1-w)(1-c)\right]\\ 2p(2-c)\left[(1-w)+(1-r)(1-c)\right]\end{pmatrix}$$

where

$$D(p,c,r,w)=4prc^2+4pc(2-c)+(2-c)\left[(1-r)+(1-w)(1-c)\right]+2p(2-c)\left[(1-w)+(1-r)(1-c)\right]$$

■

Proof. [Proposition 3] We want to show that $c=1$ is a dominant strategy for the match. Let $c^{\#}$ be the social capturing rate, prevailing in the economy. We want to prove that an individual match always has an incentive to capture, because no matter the value of $c^{\#}$, the joint net surplus from a good match is always positive.

First we have to compute the value functions of the two agents in the two states:

$$\begin{cases} U(\omega_1) &= W+c^{\#}\delta\frac{s}{2}+\delta\overline{U}\\ U(\omega_2) &= R+\delta\overline{U}\\ V(\omega_1) &= 1-F_H+c^{\#}\delta\frac{s}{2}+\delta\overline{V}\\ V(\omega_2) &= -F_L+\delta\overline{V}\end{cases}$$

then, given that

$$\begin{cases}\overline{U} &= QU(\omega_1)+(1-Q)U(\omega_2)\\ \overline{V} &= QV(\omega_1)+(1-Q)V(\omega_2)\end{cases}$$

[20] Obviously this invariant distribution is unique up to each admissible combination for the parameter value.

and recalling the definition of net surplus,

$$s = \frac{1}{(1-\delta)} - \left(\overline{V} + \overline{U}\right)$$

we can write:

$$(1-\delta)\left(\overline{V} + \overline{U}\right) = Q\left(1 + W - F_H\right) + (1-Q)\left(R - F_L\right) + Q\delta c^{\#} s$$

recalling that

$$\Pi = Q\left(F_H - W - cC\right) + (1-Q)\left(F_L - R\right)$$

in equilibrium, when the expected long-run profit is equal to zero, we have:

$$s = \frac{1 - Q\left(1 - c^{\#}C\right)}{1 - \delta + \delta Q c^{\#}}$$

The denominator of this expression is always positive, for every $\delta \in (0,1)$, and every c and Q in $[0,1]$. Thus the sign of the fraction is the same as the sign of the numerator

$$s > 0 \Leftrightarrow 1 - Q\left(1 - c^{\#}C\right) > 0$$

that is iff

$$(1-Q) + Qc^{\#}C > 0$$

but this is always true, for every value of the parameters. So the net surplus is always positive and the clients will always have an incentive to capture the consultant in case of a good match.

It interesting to notice how, if the replacement cost was null, $C = 0$, a probability of good match equal to one could make us able to reach indifference between capturing or not, while as long as we have a positive replacement cost the incentive is always stronger, no matter how good is the quality that this market is able to guarantee and this looks quite interesting if we think that the cost is paid only because of capturing. ■

Proof. [Lemma 4] I want to show $\Pi = \pi(s) + \delta \sum_{s'} \mathbb{P}(s' \mid s)\, \Pi(s') = \sum_{s} \pi(s)\, \xi(s)$. The invariant distribution for state s' is given by

(2) $$\sum_{s} \mathbb{P}(s' \mid s)\, \xi(s) = \xi(s')$$

Then $\Pi = \sum_{s} \Pi(s)\, \xi(s) = \sum_{s} \pi(s)\, \xi(s) + \delta \sum_{s} \left(\sum_{s'} \mathbb{P}(s' \mid s)\, \Pi(s') \right) \xi(s)$. Exchanging the order of summation $\sum_{s} \Pi(s)\, \xi(s) = \sum_{s} \pi(s)\, \xi(s) + \delta \sum_{s'} \left(\sum_{s} \mathbb{P}(s' \mid s)\, \xi(s) \right) \Pi(s')$ and by 2 I can rewrite the previous expression as $\sum_{s} \pi(s)\, \xi(s) + \delta \sum_{s'} \left(\sum_{s} \mathbb{P}(s' \mid s)\, \xi(s) \right) \Pi(s') = \sum_{s} \pi(s)\, \xi(s) + \delta \sum_{s'} \xi(s')\, \Pi(s') = \sum_{s} \pi(s)\, \xi(s) + \delta \Pi$. Therefore I have shown that

(3) $$22 \Longleftrightarrow 23.$$

■

Proof. [Proposition 5] Let's rewrite the value function of the consulting group that is given by:

$$V(S) = \left(V(NN); V(ORN); V(OWN); V(AB); V(OOR); V(OOW)\right)$$

To simplify the notation define

$$V(ON) = \frac{1}{2}V(ORN) + \frac{1}{2}V(OWN)$$

and define

$$V(OO) = \frac{1}{2}V(OOR) + \frac{1}{2}V(OOW)$$

then we have:

$$V(NN) = \frac{1}{2}\left(F_H - W - cC + c\delta V(NN) + (1-c)\,\delta V(ON) + F_L - R + \delta V(ON)\right)$$

$$V(ORN) = \max_r \left[\begin{array}{c} r\left(F_H - W - cC + c\delta V(NN) + (1-c)\,\delta V(ON)\right) + \\ +(1-r)\left(\frac{1}{2}\left(F_H - W - cC + c\delta V(NN) + (1-c)\,\delta V(ON)\right) + \frac{1}{2}\left(F_L - R + \delta V(ON)\right)\right) \end{array} \right]$$

$$V(OWN) = \max_w \left[\begin{array}{c} w\left(F_L - R + \delta V(ON)\right) + (1-w)\,\frac{1}{2}\left(F_H - W - cC + c\delta V(NN) + (1-c)\,\delta V(ON)\right) + \\ +(1-w)\,\frac{1}{2}\left(F_L - R + \delta V(ON)\right) \end{array} \right]$$

$$V(AB) = \max_p \left[p\left(F_H - W - cC + c\delta V(ON) + (1-c)\,\delta V(AB)\right) + (1-p)\left(F_L - R + \delta V(AB)\right)\right]$$

$$V(OOR) = F_H - W - cC + c\delta V(ON) + (1-c)\,\delta V(OO)$$

$$V(OOW) = F_L - R + \delta V(OO)$$

We can focus the attention on the three strategic states, because in the other states, NN, OOR, OOW the control only enters indirectly as they are monotone transformations of the $V(\cdot)$ function in the strategic states. Then once the optimal values are found for states ORN, OWN, AB they are optimal also in the other states.

Remark 1 *First observe that in every state the value function is linear in the control, therefore we end up with a bang-bang control. This means that we expect a probability vector of zeros and ones as a solution to our optimization problem. In other words, if a dominant strategy exists, then it has to be a pure strategy.*

Remark 2 *By the Bellman Principle, the vector of degenerate probabilities* $[r^*, w^*, p^*] = [1, 0, 1]$ *is the solution of our optimization problem iff it satisfies the following system of inequalities:*

$$\left\{ \begin{array}{rcl} (F_H - W - cC) - (F_L - R) - \delta c\left(V(ON) - V(NN)\right) & \geq & 0 \\ (F_H - W - cC) - (F_L - R) - \delta c\left(V(ON) - V(NN)\right) + 2\delta V(ON) & \geq & 0 \\ (F_H - W - cC) - (F_L - R) - \delta c\left(V(ON) - V(AB)\right) & \geq & 0 \end{array} \right.$$

In matrix form the system of inequalities is:

$$\begin{pmatrix} -\delta c & \delta c & 0 \\ -\delta c & -\delta(2-c) & 0 \\ 0 & \delta c & -\delta c \end{pmatrix} \begin{pmatrix} V(NN) \\ V(ON) \\ V(AB) \end{pmatrix} \leq \begin{pmatrix} 1 & -1 \\ 1 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} (F_H - W - cC) \\ (F_L - R) \end{pmatrix}$$

We can now simplify and rewrite our value function in the states of interest as a linear system of three equations in three unknowns:

$$\left\{\begin{array}{lcl} V\left(NN\right) & = & \frac{\left(8-2\delta-4\delta c+4\delta^2 c-3\delta^2 c^2\right)\left(F_H-W-cC\right)+\left(8-6\delta+2\delta^2 c-\delta^2 c^2\right)\left(F_L-R\right)}{\left(2-\delta c\right)\left[8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)\right]} \\ V\left(ON\right) & = & \frac{6\left(F_H-W-cC\right)+2\left(1-\delta c\right)\left(F_L-R\right)}{\left[8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)\right]} \\ V\left(AB\right) & = & \frac{\left(8-\left(1-\delta\left(1-c\right)\right)+\delta^2 c\left(4-3c\right)\right)\left(F_H-W-cC\right)+\left(2\delta c\left(1-\delta c\right)\right)\left(F_L-R\right)}{\left(1-\delta\left(1-c\right)\right)\left[8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)\right]} \end{array}\right.$$

In matrix form:

$$\begin{aligned} \left(\begin{array}{c} V\left(NN\right) \\ V\left(ON\right) \\ V\left(AB\right) \end{array}\right) &= \frac{1}{\left[8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)\right]} \asymp \left(\begin{array}{cc} \frac{\left(8-2\delta-4\delta c+4\delta^2 c-3\delta^2 c^2\right)}{\left(2-\delta c\right)} & \frac{\left(8-6\delta+2\delta^2 c-\delta^2 c^2\right)}{\left(2-\delta c\right)} \\ 6 & 2\left(1-\delta c\right) \\ \frac{\left(8-\left(1-\delta\left(1-c\right)\right)+\delta^2 c\left(4-3c\right)\right)}{\left(1-\delta\left(1-c\right)\right)} & \frac{\left(2\delta c\left(1-\delta c\right)\right)}{\left(1-\delta\left(1-c\right)\right)} \end{array}\right) \asymp \\ &\asymp \left(\begin{array}{c} \left(F_H-W-cC\right) \\ \left(F_L-R\right) \end{array}\right) \end{aligned}$$

Hence, the system of inequalities can be rewritten as follows:

$$\begin{aligned} \left(\begin{array}{ccc} -\delta c & \delta c & 0 \\ -\delta c & -\delta\left(2-c\right) & 0 \\ 0 & \delta c & -\delta c \end{array}\right) &\asymp \left(\begin{array}{cc} \frac{\left(8-2\delta-4\delta c+4\delta^2 c-3\delta^2 c^2\right)}{\left(8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)\right)\left(2-\delta c\right)} & \frac{\left(8-6\delta+2\delta^2 c-\delta^2 c^2\right)}{\left(8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)\right)\left(2-\delta c\right)} \\ \frac{6}{\left(8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)\right)} & \frac{2\left(1-\delta c\right)}{\left(8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)\right)} \\ \frac{\left(8-\left(1-\delta\left(1-c\right)\right)+\delta^2 c\left(4-3c\right)\right)}{\left(8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)\right)\left(1-\delta\left(1-c\right)\right)} & \frac{\left(2\delta c\left(1-\delta c\right)\right)}{\left(8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)\right)\left(1-\delta\left(1-c\right)\right)} \end{array}\right) \\ &\asymp \left(\begin{array}{c} \left(F_H-W-cC\right) \\ \left(F_L-R\right) \end{array}\right) \leq \left(\begin{array}{cc} 1 & -1 \\ 1 & -1 \\ 1 & -1 \end{array}\right) \asymp \left(\begin{array}{c} \left(F_H-W-cC\right) \\ \left(F_L-R\right) \end{array}\right) \end{aligned}$$

Solving for all the coefficients we obtain:

$$\left\{\begin{array}{cr} \begin{array}{c} \left[\left(2-\delta c\right)\left[8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)\right]-\delta c\left(4+2\delta-2\delta c-4\delta^2 c-3\delta^2 c^2\right)\right]\left(F_H-W-cC\right)- \\ -\left[\left(2-\delta c\right)\left[8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)\right]-\delta c\left(4-6\delta+6\delta c+2\delta^2 c-3\delta^2 c^2\right)\right]\left(F_L-R\right) \end{array} & \geq 0 \\ \begin{array}{c} \left[2\left(8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)+6\delta\left(2-c\right)\right)+\delta c\left(8-2\delta-4\delta c+4\delta^2 c-3\delta^2 c^2\right)\right]\left(F_H-W-cC\right)- \\ -\left[\left(2-\delta c\right)\left(8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)-2\delta\left(2-2\delta c-c+\delta c^2\right)\right)-\delta c\left(8-6\delta+2\delta^2 c-\delta^2 c^2\right)\right]\left(F_L-R\right) \end{array} & \geq 0 \\ \begin{array}{c} \left[\left(1-\delta\left(1-c\right)\right)\left[8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)\right]-\delta c\left(2-2\delta+2\delta c+4\delta^2 c-3\delta^2 c^2\right)\right]\left(F_H-W-cC\right)- \\ -\left[\left(1-\delta\left(1-c\right)\right)\left[8\left(1-\delta\right)+\delta c\left(2+4\delta-3\delta c\right)\right]-2\delta c\left(1-\delta\left(1-c\right)+\delta c\left(1-\delta c\right)\right)\right]\left(F_L-R\right) \end{array} & \geq 0 \end{array}\right.$$

Given our assumption of sustainable replacement cost

$$F_H - W - cC \geq 0 \tag{4}$$

and the zero expected profit condition

$$Q\left(F_H-W-cC\right)+\left(1-Q\right)\left(F_L-R\right)=0$$

that together imply

$$\left(F_L-R\right)\leq 0$$

the first inequality is always satisfied for any combination of values of the parameters $c \in [0,1]$ and $\delta \in (0,1)$. In fact the coefficient of $\left(F_H-W-cC\right)$ is always positive and the coefficient that multiplies $-\left(F_L-R\right)$, that is a nonnegative term, is nonnegative as well. Thus the first inequality is satisfied. Looking at the second inequality we observe that it is implied by the first inequality iff

$$V\left(ON\right)\geq 0$$

So this offers us a sufficient condition to show that the second condition is also satisfied.

$$V(ON) = \frac{6(F_H - W - cC) + 2(1-\delta c)(F_L - R)}{[8(1-\delta) + \delta c(2 + 4\delta - 3\delta c)]} \geq 0$$

The denominator is always positive, then we focus on the numerator:

$$6(F_H - W - cC) + 2(1-\delta c)(F_L - R) \geq 0$$

in equilibrium the zero expected profit condition holds, so we can replace it to obtain:

$$Q \leq \frac{3}{4 - \delta c}$$

and we observe that

$$\frac{3}{4} \leq \frac{3}{4-\delta c} \leq 1$$

and the maximum possible value that the social quality can take is exactly $Q^{\max} = \frac{3}{4}$. Thus the second condition holds in an equilibrium as well. In the third inequality it's easy to see that the coefficient of $(F_H - W - cC)$ is positive for any c, δ and the same it's true also for the one associated to $\left[-\left(F_L - R\right)\right]$. Therefore we can conclude that all the three inequalities hold and our vector is the unique solution to the system.

$$[r^*, w^*, p^*] = [1, 0, 1]$$

for every value of the other players' strategy. ■

Proof. [Proposition 6]

The two parties' value functions in the two states under the equilibrium condition $c = 1$ are

$$\left\{ : \begin{array}{lcc} U(\omega_1) & = & W + \frac{\delta}{2}s + \delta\overline{U} \\ U(\omega_2) & = & R + \delta\overline{U} \\ V(\omega_1) & = & (1 - F_H) + \frac{\delta}{2}s + \delta\overline{V} \\ V(\omega_2) & = & R + \delta\overline{V} \end{array} \right.$$

The condition on the net surplus $s > 0$, reflects the optimality of both $c = 1$ and $a = 1$. This is due to the fact that I am looking for a symmetric equilibrium and the assumption of Nash bargaining between consultant and customer/potential employer rules out the possibility of opposite incentives between these counterparts. In fact the decision criterion is $s > 0$ for both. Thus the incentive condition on the net surplus is:

$$\frac{1}{1-\delta} - \left(\left(\frac{3}{5}V(\omega_1) + \frac{2}{5}V(\omega_2)\right) + \left(\frac{3}{5}U(\omega_1) + \frac{2}{5}U(\omega_2)\right)\right) > 0$$

First let's find $\overline{U} + \overline{V}$, that is, substituting for $U(\omega_1), U(\omega_2), V(\omega_1)$ and $V(\omega_2)$ in

$$\left\{ \begin{array}{lcc} \overline{U} & = & \frac{3}{5}U(\omega_1) + \frac{2}{5}U(\omega_2) \\ \overline{V} & = & \frac{3}{5}V(\omega_1) + \frac{2}{5}V(\omega_2) \end{array} \right.$$

we have:

$$\begin{array}{lcl} \overline{U} & = & \frac{1}{2(1-\delta)}\left[2Q(1-\delta(1-Q))W + 2(1-Q)R + \left[Q\delta[1-\delta(1-Q)] + \delta^2(1-\delta)(1-Q)\right]s\right] \\ \overline{V} & = & \frac{1}{2(1-\delta)}\left[2Q(1-\delta(1-Q))(1-F_H) - 2(1-Q)F_L + \left[Q\delta[1-\delta(1-Q)] + \delta^2(1-\delta)(1-Q)\right]s\right] \end{array}$$

Then, in general, the net surplus is

$$s = \frac{1}{2(1-\delta)(5-2\delta)}\left(4(1-\delta)+6(1-\delta)(F_H-W)+4(1-\delta)(F_L-R)\right)$$

Under the zero profit condition we can substitute the following conditions:

$$\begin{cases} \overline{U} & = & \frac{1}{10-7\delta}\left(6W+4R+3\frac{\delta}{1-\delta}-3\delta\overline{V}\right) \\ \overline{V} & = & \frac{1}{10-7\delta}\left(6-6F_H-4F_L+3\frac{\delta}{1-\delta}-3\delta\overline{U}\right) \end{cases}$$

$$F_H - W = C + \frac{2}{3}R - \frac{2}{3}F_L$$

Hence

$$\overline{U}+\overline{V} = \frac{1}{2(5-2\delta)}\left(6(1-F_H+W)+4(R-F_L)+6\frac{\delta}{1-\delta}\right)$$

then we obtain:by the zero profit condition it's easy to see how the left hand side is always nonnegative while the right hand side is always non positive, thus:

$$F_H - W - C > F_L - R$$

1. First of all by the zero profit condition it's easy to see how the left hand side is always nonnegative while the right hand side is always non positive, thus:

$$F_H - W - C > F_L - R$$

From the zero profit condition we obtain:

$$(F_L - R) = -\frac{3}{2}(F_H - W - C)$$

Under our initial assumption of sustainable costs, this expression tells us that in case of good match the Consulting firm makes positive profits while in case a bad match occurs it will turn out in a loss.

2. The incentive constraints $r = 1$ and $w = 0$ yield:

$$\frac{(F_H-W-C)3(1-\delta)\left(4-\delta+4\delta^2\right)+(F_L-R)[(1-\delta)(4-\delta)+\delta]}{4(1-\delta)+3\delta} \geq$$
$$\geq \frac{(F_H-W-C)(1-\delta)\left(4-\delta+4\delta^2\right)+(F_L-R)[3(1-\delta)(4-\delta)+\delta]}{4(1-\delta)+\delta}$$

from which

$$(8-5\delta)(F_H-W-C) - \left[8(1-\delta)(4-\delta)+2\delta^2\right](F_L-R) > 0$$

always satisfied.

3. The incentive constraint $p = 1$ yields:

$$\frac{(F_H-W-C)\left((1-\delta)(4-\delta)[4(1-\delta)+3\delta]+3(1-\delta)\left(4-\delta+4\delta^2\right)\right)+(F_L-R)\delta[(1-\delta)(4-\delta)+\delta]}{(4-\delta)(1-\delta)[4(1-\delta)+3\delta]} \geq \frac{(F_L-R)}{(1-\delta)}$$

that is obviously always true as the LHS is positive and the RHS is always negative.

4. Let's turn the attention to the individual rationality constraints for the consultant, $\overline{U} \geq 0$. This constraint is:

$$\overline{U} = \frac{(30-12\delta)W+(20-8\delta)R+\delta\left(15+4\delta-10\delta^2\right)s}{50(1-\delta)} \geq 0$$

and the inequality is always satisfied. Hence, as we were expecting, the consultant has always incentive to participate in this market.

5. For the client, we turn the attention to the individual rationality constraint of the client firm: $\overline{V} \geq 0$, that is

$$\overline{V} = \frac{(30-12\delta)(1-F_H)-(20-8\delta)F_L+\delta\left(15+4\delta-10\delta^2\right)s}{50(1-\delta)} \geq 0$$

that implies:

$$W \leq 1 - C - \frac{2}{3}R + \frac{\delta\left(15+4\delta-10\delta^2\right)}{6(5-2\delta)}s$$

that provides an upper bound for W. Call

$$\overline{W} := 1 - C - \frac{2}{3}R + \frac{\delta\left(15+4\delta-10\delta^2\right)}{6(5-2\delta)}s$$

then, this condition implicitly states another constraint on the replacement cost C and in the reservation wage R. In fact for $\overline{W}$ to be a sensible upper bound we need

$$\overline{W} \geq R$$

as the sensible prices constraint requires $W \geq R$, then

$$R \leq \frac{6(5-2\delta)+2\delta\left(15+4\delta-10\delta^2\right)+3C\left[\delta\left(15+4\delta-10\delta^2\right)-2(5-2\delta)\right]}{10(5-2\delta)}$$

that must be nonnegative implying

$$C \leq \frac{30+18\delta+8\delta^2-20\delta^3}{30-27\delta-12\delta^2+30\delta^3}$$

that is always positive.

6. The positive net surplus condition

$$s = \frac{1}{2(1-\delta)(5-2\delta)}\left(4(1-\delta)+6(1-\delta)(F_H-W)+4(1-\delta)(F_L-R)\right) > 0$$

yields, as we know

$$\frac{2+3C}{5-2\delta} \geq 0$$

that is always satisfied $\forall\, C \geq 0$.

7. From the condition $0 \leq F_L \leq F_H$, that is from assuming that prices are well defined we get:

$$F_H - W - C \leq \frac{2}{3}R$$

that is already implied by the zero profit condition and

$$F_H \geq \frac{3}{5}W + \frac{3}{5}C + \frac{2}{5}R$$

Thus taking the two conditions together we have

$$F_H \in \left[\frac{3}{5}W + \frac{3}{5}C + \frac{2}{5}R; W + C + \frac{2}{3}R\right]$$

■

Example 1 *Suppose* $\delta = 0.9$, $C = \frac{3}{2}$ *and* $R = \frac{1}{2}$. *In this case we have that*

$$F_H \in \left[\frac{3}{5}W + \frac{11}{10}; W + \frac{11}{6}\right]$$

and the zero profit condition yields:

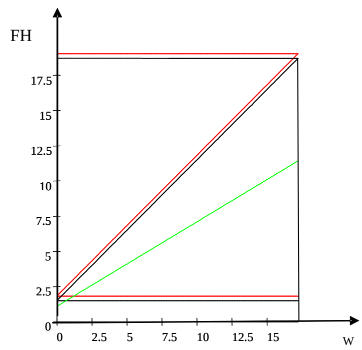
$$F_H = W + \frac{11}{6} - \frac{2}{3}F_L$$

$$F_H = \frac{11}{6} + W$$

if $F_L = 0$ *and*

$$F_H = \frac{3}{2} + W$$

if $F_L = R$, *that is the maximum value for the low fee. It is interesting to notice how the isoprofit lines are parallel lines: they do not have any intersection. This tells us that the first Welfare Theorem holds in our model, as it should be in a competitive equilibrium market economy.*



2.The isoprofit lines between $F_L = 0$ and $F_L = R$.

Proof. [Proposition 7] When we ask for the inventory of the consulting group to be large enough, we are asking that for every client's request A or B there exists at least a pair of consultants of known and distinct type. In this case the recurrent state can be summarized by state AB. The probability of being stuck with the same type goes to zero and the invariant distribution tends to its limit:

$$\begin{pmatrix} \pi(NN) \\ \pi(NO) \\ \pi(AB) \\ \pi(OO) \end{pmatrix} \to \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

It is also straightforward to see that the probability of good match now coincides with the probability p of matching the client's request when the state is AB.

$$Q \to p$$

We are left to check that if $C \longrightarrow 0$, then the limiting equilibrium is given by a pair of probabilities $(p^*, c^*) = (1, 0)$.

1.The matching pair's problem

We want to show that $c = 0$ can be part of an equilibrium strategy for the match. Let $c^{\#}$ be the social capturing rate, prevailing in the economy. We want to prove that an individual match has always incentive to capture, because no matter the value of $c^{\#}$, the joint net surplus from a good match is always nonnegative.

First we have to compute the value functions of the two agents in the two states:

$$\begin{cases} U(\omega_1) &= W + c^{\#}\delta\frac{s}{2} + \delta\overline{U} \\ U(\omega_2) &= R + \delta\overline{U} \\ V(\omega_1) &= 1 - F_H + c^{\#}\delta\frac{s}{2} + \delta\overline{V} \\ V(\omega_2) &= -F_L + \delta\overline{V} \end{cases}$$

then, given that

$$\begin{cases} \overline{U} &= QU(\omega_1) + (1-Q)\,U(\omega_2) \\ \overline{V} &= QV(\omega_1) + (1-Q)\,V(\omega_2) \end{cases}$$

and recalling the definition of net surplus,

$$s = \frac{1}{(1-\delta)} - \left(\overline{V} + \overline{U}\right)$$

we can write:

$$(1-\delta)\left(\overline{V} + \overline{U}\right) = Q\,(1 + W - F_H) + (1-Q)\,(R - F_L) + Q\delta c^{\#} s$$

recalling that

$$\Pi = Q\,(F_H - W) + (1-Q)\,(F_L - R)$$

in equilibrium, when the expected long run profit is equal to zero, we have:

$$s = \frac{1-Q}{1-\delta+\delta Q c^{\#}}$$

The denominator of this expression is always positive, for every $\delta \in (0,1)$, and every c and Q in $[0,1]$. Thus the sign of the fraction is the same sign of the numerator that is

$$s \leq 0 \Leftrightarrow 1 - Q \leq 0$$

that is iff

$$\begin{aligned} 1 - Q &= 0 \\ Q^* &= 1 \end{aligned}$$

We have to check whether or not $Q = p = 1$ is a best response for the cobnsulting group.

2.The consulting group's problem.

Let's rewrite the value function of the consulting group that is given by:

$$V(S) = V(AB)$$

$$V(AB) = \max_p \left[p(F_H - W) + (1-p)(F_L - R) + \delta V(AB) \right]$$

Is $p = 1$ optimal? It is trivial to see that, given that $F_H - W \geq F_L - R$, then the consulting group has always incentive to send the right consultant and

$$p^* = 1$$

Thus, our quality variable Q approaches the unity: $Q^* \to 1$. ■

References

Acemoglu, D. (1997). Training and innovation in an imperfect labor markets. *The Review of Economic Studies 64*(3), 445–464.

Acemoglu, D. and R. Shimer (1999). Holdups and efficiency with search frictions. *International Economic Review 40*(4, Special Issue on Search, Matching and Related Topics), 827–849.

Andrew Hertzberg, J. M. L. and D. Paravisini (2007). Information and incentives inside the firm: Evidence from loan officers' turnover. *Mimeo*.

Autor, D. H. (2001). Why do temporary help firrms provide free general skills training? *The Quarterly Journal of Economics*, 1409–1448.

Autor, D. H. (2005). Do temporary help jobs improve labor market outcomes for low-skilled workers? evidence from random assignments. *Mimeo*.

Bhattacharya, U. and A. Yavas (1993). In search of the right middleman. *Economics Letters 42*, 341–347.

Cappelli, P. (2000). A market-driven approach to retaining talent. *Harvard Business Review*, 103–111.

Clemons, E. K. and L. M. Hitt (2001). Poaching and the misappropriation of information: An analysis of relationship risks in information-intensive production. *mimeo*.

Davis, S. J. (2001). The quality distribution of jobs and the structure of wages in search equilibrium. *NBER Working Paper Series* (8434).

Gehrig, T. (1993). Intermediation in search markets. *Journal of Economics and Management Strategy 2*, 97–120.

Glosten, L. and P. Milgrom (1985). Bid, ask, and transaction prices in a specialist market with heterogenously informed traders. *Journal of Financial Economics 13*, 71–100.

Huynh, H.-L. and R. Rosenthal (2000). Waiting for the good doctor: Information in queues. *Mimeo*.

Jovanovic, B. (1979). Firm-specific capital and turnover. *The Journal of Political Economy 87*, 1246–1260.

Kyle, A. S. (1985). Continuous auctions and insider trading. *Econometrica 53*(6), 1315–1335.

Laffont, J. J. and D. Martimort (2002). *The Theory of Incentives: The Principal-Agent Model*. Princeton.

Lu, X. and R. P. McAfee (1996). Matching and expectations in a market with heterogeneous agents. *Advances in Applied Microeconomics 6*, 121–156.

Luiks, P. (2004). Global management consulting in change: The incredible shrinking global consultant. *Fortune Magazine*, 1–3.

Moen, E. R. (1997). Competitive search equilibrium. *Journal of Political Economy 105*(2), 385–411.

Mortensen, D. (1985). *Job Search and Labor Market Analysis.* New York: North Holland.

Moscarini, G. (2005). Job matching and the wage distribution. *Econometrica 73*(2), 481–516.

Pissarides, C. (2000). *Equilibrium Unemployment Theory*. MIT Press.

Prescott, E. S. and R. M. Townsend (2003). Mechanism design and assignment models. *Mimeo*.

Rubinstein, A. and A. Wolinsky (1987). Middlemen. *The Quarterly Journal of Economics 102*, 581–593.

Shimer, R. and L. Smith (2000). Assortative matching and search. *Econometrica 68*(2), 343–369.

Veldkamp, L. L. (2004). Slow boom, sudden crash. *Journal of Economic Theory 124*, 230–257.

Yavas, A. (1994). Middlemen in bilateral search markets. *Journal of Labor Economics 12*(3), 406–429.

RECENTLY PUBLISHED "TEMI" (*)

N. 638 – *Comparative advantage patterns and domestic determinants in emerging countries: An analysis with a focus on technology*, by Daniela Marconi and Valeria Rolli (September 2007).

N. 639 – *The generation gap: Relative earnings of young and old workers in Italy*, by Alfonso Rosolia and Roberto Torrini (September 2007).

N. 640 – *The financing of small innovative firms: The Italian case*, by Silvia Magri (September 2007).

N. 641 – *Assessing financial contagion in the interbank market: Maximum entropy versus observed interbank lending patterns*, by Paolo Emilio Mistrulli (September 2007).

N. 642 – *Detecting long memory co-movements in macroeconomic time series, by* Gianluca Moretti (September 2007).

N. 643 – *The producer service sector in Italy: Long-term growth and its local determinants*, by Valter Di Giacinto and Giacinto Micucci (September 2007).

N. 644 – *Aggregazioni bancarie e specializzazione nel credito alle PMI: peculiarità per area geografica*, by Enrico Beretta and Silvia Del Prete (November 2007).

N. 645 – *Costs and benefits of creditor concentration: An empirical approach*, by Amanda Carmignani and Massimo Omiccioli (November 2007).

N. 646 – *Does the underground economy hold back financial deepening? Evidence from the Italian credit market*, by Giorgio Gobbi and Roberta Zizza (November 2007).

N. 647 – *Optimal monetary policy under low trend inflation*, by Guido Ascari and Tiziano Ropele (November 2007).

N. 648 – *Indici di bilancio e rendimenti di borsa: un'analisi per le banche italiane*, by Angela Romagnoli (November 2007).

N. 649 – *Bank profitability and taxation,* by Ugo Albertazzi and Leonardo Gambacorta (November 2007).

N. 650 – *Modelling bank lending in the euro area: A non-linear approach,* by Leonardo Gambacorta and Carlotta Rossi (November 2007).

N. 651 – *Revisiting poverty and welfare dominance,* by Gian Maria Tomat (November 2007).

N. 652 – *The general equilibrium effects of fiscal policy: Estimates for the euro area,* by Lorenzo Forni, Libero Monteforte and Luca Sessa (November 2007).

N. 653 – *Securitisation and the bank lending channel,* by Yener Altunbas, Leonardo Gambacorta and David Marqués (November 2007).

N. 654 – *The cyclical response of fiscal policies in the euro area. Why do results of empirical research differ so strongly?*, by Roberto Golinelli and Sandro Momigliano (January 2008).

N. 655 – *What's behind "inflation perceptions"? A survey-based analysis of Italian consumers*, by Paolo Del Giovane, Silvia Fabiani and Roberto Sabbatini (January 2008).

N. 656 – *The effects of fiscal policy in Italy: Evidence from a VAR model*, by Raffaela Giordano, Sandro Momigliano, Stefano Neri and Roberto Perotti (January 2008).

N. 657 – *Excess money growth and inflation dynamics*, by Barbara Roffia and Andrea Zaghini (January 2008).

N. 658 – *R&D and market structure in a horizontal differentiation framework*, by Davide Fantino (January 2008).

N. 659 – *Housing market spillovers: Evidence from an estimated DSGE model*, by Matteo Iacoviello and Stefano Neri (January 2008).

N. 660 – *Real exchange rate volatility and disconnect: An empirical investigation*, by Riccardo Cristadoro, Andrea Gerali, Stefano Neri and Massimiliano Pisani (April 2008).

N. 661 – *The effect of investment tax credit: Evidence from an atypical programme in Italy*, by Raffaello Bronzini, Guido de Blasio, Guido Pellegrini and Alessandro Scognamiglio (April 2008).

N. 662 – *Accounting for sampling design in the SHIW*, by Ivan Faiella (April 2008).

N. 663 – *Delayed privatization*, by Bernardo Bortolotti and Paolo Pinotti (April 2008).

N. 664 – *Portfolio selection with mononotone mean-variance preferences*, by Fabio Maccheroni, Massimo Marinacci, Aldo Rustichini and Marco Taboga (April 2008).

(*) Requests for copies should be sent to:
Banca d'Italia – Servizio Studi di struttura economica e finanziaria – Divisione Biblioteca e Archivio storico – Via Nazionale, 91 – 00184 Rome – (fax 0039 06 47922059). They are available on the Internet www.bancaditalia.it.

2005

L. DEDOLA and F. LIPPI, *The monetary transmission mechanism: Evidence from the industries of 5 OECD countries,* European Economic Review, 2005, Vol. 49, 6, pp. 1543-1569, **TD No. 389 (December 2000)**.

D. Jr. MARCHETTI and F. NUCCI, *Price stickiness and the contractionary effects of technology shocks*. European Economic Review, Vol. 49, 5, pp. 1137-1164, **TD No. 392 (February 2001)**.

G. CORSETTI, M. PERICOLI and M. SBRACIA, *Some contagion, some interdependence: More pitfalls in tests of financial contagion,* Journal of International Money and Finance, Vol. 24, 8, pp. 1177-1199, **TD No. 408 (June 2001)**.

GUISO L., L. PISTAFERRI and F. SCHIVARDI, *Insurance within the firm*. Journal of Political Economy, Vol. 113, 5, pp. 1054-1087, **TD No. 414** (**August 2001**)

R. CRISTADORO, M. FORNI, L. REICHLIN and G. VERONESE, *A core inflation indicator for the euro area,* Journal of Money, Credit, and Banking, Vol. 37, 3, pp. 539-560, **TD No. 435 (December 2001)**.

F. ALTISSIMO, E. GAIOTTI and A. LOCARNO, *Is money informative? Evidence from a large model used for policy analysis,* Economic & Financial Modelling, Vol. 22, 2, pp. 285-304, **TD No. 445 (July 2002)**.

G. DE BLASIO and S. DI ADDARIO, *Do workers benefit from industrial agglomeration?* Journal of regional Science, Vol. 45, (4), pp. 797-827, **TD No. 453 (October 2002).**

G. DE BLASIO and S. DI ADDARIO, *Salari, imprenditorialità e mobilità nei distretti industriali italiani,* in L. F. Signorini, M. Omiccioli (eds.), *Economie locali e competizione globale: il localismo industriale italiano di fronte a nuove sfide*, Bologna, il Mulino, **TD No. 453 (October 2002).**

R. TORRINI, *Cross-country differences in self-employment rates: The role of institutions*, Labour Economics, Vol. 12, 5, pp. 661-683, **TD No. 459 (December 2002)**.

A. CUKIERMAN and F. LIPPI, *Endogenous monetary policy with unobserved potential output,* Journal of Economic Dynamics and Control, Vol. 29, 11, pp. 1951-1983, **TD No. 493 (June 2004)**.

M. OMICCIOLI, *Il credito commerciale: problemi e teorie*, in L. Cannari, S. Chiri e M. Omiccioli (eds.), *Imprese o intermediari? Aspetti finanziari e commerciali del credito tra imprese in Italia,* Bologna, Il Mulino, **TD No. 494 (June 2004)**.

L. CANNARI, S. CHIRI and M. OMICCIOLI, *Condizioni di pagamento e differenziazione della clientela,* in L. Cannari, S. Chiri e M. Omiccioli (eds.), *Imprese o intermediari? Aspetti finanziari e commerciali del credito tra imprese in Italia*, Bologna, Il Mulino, **TD No. 495 (June 2004)**.

P. FINALDI RUSSO and L. LEVA, *Il debito commerciale in Italia: quanto contano le motivazioni finanziarie?,* in L. Cannari, S. Chiri e M. Omiccioli (eds.), *Imprese o intermediari? Aspetti finanziari e commerciali del credito tra imprese in Italia*, Bologna, Il Mulino, **TD No. 496 (June 2004)**.

A. CARMIGNANI, *Funzionamento della giustizia civile e struttura finanziaria delle imprese: il ruolo del credito commerciale,* in L. Cannari, S. Chiri e M. Omiccioli (eds.), *Imprese o intermediari? Aspetti finanziari e commerciali del credito tra imprese in Italia,* Bologna, Il Mulino, **TD No. 497 (June 2004)**.

G. DE BLASIO, *Credito commerciale e politica monetaria: una verifica basata sull'investimento in scorte,* in L. Cannari, S. Chiri e M. Omiccioli (eds.), *Imprese o intermediari? Aspetti finanziari e commerciali del credito tra imprese in Italia,* Bologna, Il Mulino, **TD No. 498 (June 2004)**.

G. DE BLASIO, *Does trade credit substitute bank credit? Evidence from firm-level data.* Economic notes, Vol. 34, 1, pp. 85-112, **TD No. 498 (June 2004).**

A. DI CESARE, *Estimating expectations of shocks using option prices,* The ICFAI Journal of Derivatives Markets, Vol. 2, 1, pp. 42-53, **TD No. 506 (July 2004).**

M. BENVENUTI and M. GALLO, *Il ricorso al "factoring" da parte delle imprese italiane,* in L. Cannari, S. Chiri e M. Omiccioli (eds.), *Imprese o intermediari? Aspetti finanziari e commerciali del credito tra imprese in Italia*, Bologna, Il Mulino, **TD No. 518 (October 2004)**.

L. CASOLARO and L. GAMBACORTA, *Redditività bancaria e ciclo economico,* Bancaria, Vol. 61, 3, pp. 19-27, **TD No. 519 (October 2004)**.

F. PANETTA, F. SCHIVARDI and M. SHUM, *Do mergers improve information? Evidence from the loan market*, CEPR Discussion Paper, 4961, **TD No. 521 (October 2004)**.

P. DEL GIOVANE and R. SABBATINI, *La divergenza tra inflazione rilevata e percepita in Italia,* in P. Del Giovane, F. Lippi e R. Sabbatini (eds.), *L'euro e l'inflazione: percezioni, fatti e analisi,* Bologna, Il Mulino, **TD No. 532 (December 2004).**

R. TORRINI, *Quota dei profitti e redditività del capitale in Italia: un tentativo di interpretazione*, Politica economica, Vol. 21, 1, pp. 7-41, **TD No. 551 (June 2005)**.

M. OMICCIOLI, *Il credito commerciale come "collateral"*, in L. Cannari, S. Chiri, M. Omiccioli (eds.), Imprese o intermediari? Aspetti finanziari e commerciali del credito tra imprese in Italia, Bologna, il Mulino, **TD No. 553 (June 2005)**.

L. CASOLARO, L. GAMBACORTA and L. GUISO, *Regulation, formal and informal enforcement and the development of the household loan market. Lessons from Italy*, in Bertola G., Grant C. and Disney R. (eds.) The Economics of Consumer Credit: European Experience and Lessons from the US, Boston, MIT Press, **TD No. 560 (September 2005)**.

S. DI ADDARIO and E. PATACCHINI, *Lavorare in una grande città paga, ma poco*, in Brucchi Luchino (ed.), *Per un'analisi critica del mercato del lavoro*, Bologna , Il Mulino, **TD No. 570 (January 2006)**.

P. ANGELINI and F. LIPPI, *Did inflation really soar after the euro changeover? Indirect evidence from ATM withdrawals,* CEPR Discussion Paper, 4950, **TD No. 581 (March 2006)**.

S. FEDERICO, *Internazionalizzazione produttiva, distretti industriali e investimenti diretti all'estero,* in L. F. Signorini, M. Omiccioli (eds.), *Economie locali e competizione globale: il localismo industriale italiano di fronte a nuove sfide*, Bologna, il Mulino, **TD No. 592 (October 2002).**

S. DI ADDARIO, *Job search in thick markets: Evidence from Italy*, Oxford Discussion Paper 235, Department of Economics Series, **TD No. 605 (December 2006)**.

2006

F. BUSETTI, *Tests of seasonal integration and cointegration in multivariate unobserved component models,* Journal of Applied Econometrics, Vol. 21, 4, pp. 419-438, **TD No. 476 (June 2003).**

C. BIANCOTTI, *A polarization of inequality? The distribution of national Gini coefficients 1970-1996*, Journal of Economic Inequality, Vol. 4, 1, pp. 1-32, **TD No. 487 (March 2004)**.

L. CANNARI and S. CHIRI, *La bilancia dei pagamenti di parte corrente Nord-Sud (1998-2000),* in L. Cannari, F. Panetta (a cura di), Il sistema finanziario e il Mezzogiorno: squilibri strutturali e divari finanziari, Bari, Cacucci, **TD No. 490 (March 2004).**

M. BOFONDI and G. GOBBI, *Information barriers to entry into credit markets,* Review of Finance, Vol. 10, 1, pp. 39-67, **TD No. 509 (July 2004).**

FUCHS W. and LIPPI F., *Monetary union with voluntary participation,* Review of Economic Studies, Vol. 73, pp. 437-457 **TD No. 512 (July 2004).**

GAIOTTI E. and A. SECCHI, *Is there a cost channel of monetary transmission? An investigation into the pricing behaviour of 2000 firms*, Journal of Money, Credit and Banking, Vol. 38, 8, pp. 2013-2038 **TD No. 525 (December 2004)**.

A. BRANDOLINI, P. CIPOLLONE and E. VIVIANO, *Does the ILO definition capture all unemployment?,* Journal of the European Economic Association, Vol. 4, 1, pp. 153-179, **TD No. 529 (December 2004)**.

A. BRANDOLINI, L. CANNARI, G. D'ALESSIO and I. FAIELLA, *Household wealth distribution in Italy in the 1990s,* in E. N. Wolff (ed.) International Perspectives on Household Wealth, Cheltenham, Edward Elgar, **TD No. 530 (December 2004)**.

P. DEL GIOVANE and R. SABBATINI, *Perceived and measured inflation after the launch of the Euro: Explaining the gap in Italy,* Giornale degli economisti e annali di economia, Vol. 65, 2 , pp. 155-192, **TD No. 532 (December 2004).**

M. CARUSO, *Monetary policy impulses, local output and the transmission mechanism*, Giornale degli economisti e annali di economia, Vol. 65, 1, pp. 1-30, **TD No. 537 (December 2004).**

A. NOBILI, *Assessing the predictive power of financial spreads in the euro area: does parameters instability matter?,* Empirical Economics, Vol. 31, 1, pp. 177-195, **TD No. 544 (February 2005)**.

L. GUISO and M. PAIELLA, *The role of risk aversion in predicting individual behavior, In P. A. Chiappori e C. Gollier (eds.) Competitive Failures in Insurance Markets: Theory and Policy Implications, Monaco, CESifo*, **TD No. 546 (February 2005).**

G. M. TOMAT, *Prices product differentiation and quality measurement: A comparison between hedonic and matched model methods*, Research in Economics, Vol. 60, 1, pp. 54-68, **TD No. 547 (February 2005)**.

F. LOTTI, E. SANTARELLI and M. VIVARELLI, *Gibrat's law in a medium-technology industry: Empirical evidence for Italy*, in E. Santarelli (ed.), Entrepreneurship, Growth, and Innovation: the Dynamics of Firms and Industries, New York, Springer, **TD No. 555 (June 2005).**

F. BUSETTI, S. FABIANI and A. HARVEY, *Convergence of prices and rates of inflation*, Oxford Bulletin of Economics and Statistics, Vol. 68, 1, pp. 863-878, **TD No. 575 (February 2006).**

M. CARUSO, *Stock market fluctuations and money demand in Italy, 1913 - 2003*, Economic Notes, Vol. 35, 1, pp. 1-47, **TD No. 576 (February 2006)**.

S. IRANZO, F. SCHIVARDI and E. TOSETTI, *Skill dispersion and productivity: An analysis with matched data*, CEPR Discussion Paper, 5539, **TD No. 577 (February 2006).**

R. BRONZINI and G. DE BLASIO, *Evaluating the impact of investment incentives: The case of Italy's Law 488/92.* Journal of Urban Economics, Vol. 60, 2, pp. 327-349, **TD No. 582 (March 2006).**

R. BRONZINI and G. DE BLASIO, *Una valutazione degli incentivi pubblici agli investimenti*, Rivista Italiana degli Economisti , Vol. 11, 3, pp. 331-362, **TD No. 582 (March 2006).**

A. DI CESARE, *Do market-based indicators anticipate rating agencies? Evidence for international banks*, Economic Notes, Vol. 35, pp. 121-150, **TD No. 593 (May 2006).**

L. DEDOLA and S. NERI, *What does a technology shock do? A VAR analysis with model-based sign restrictions*, Journal of Monetary Economics, Vol. 54, 2, pp. 512-549, **TD No. 607 (December 2006)**.

R. GOLINELLI and S. MOMIGLIANO, *Real-time determinants of fiscal policies in the euro area*, Journal of Policy Modeling, Vol. 28, 9, pp. 943-964, **TD No. 609 (December 2006).**

P. ANGELINI, S. GERLACH, G. GRANDE, A. LEVY, F. PANETTA, R. PERLI,S. RAMASWAMY, M. SCATIGNA and P. YESIN, *The recent behaviour of financial market volatility*, BIS Papers, 29, **QEF No. 2 (August 2006)**.

2007

L. CASOLARO. and G. GOBBI, *Information technology and productivity changes in the banking industry*, Economic Notes, Vol. 36, 1, pp. 43-76, **TD No. 489 (March 2004)**.

M. PAIELLA, *Does wealth affect consumption? Evidence for Italy*, Journal of Macroeconomics, Vol. 29, 1, pp. 189-205, **TD No. 510 (July 2004).**

F. LIPPI. and S. NERI, *Information variables for monetary policy in a small structural model of the euro area*, Journal of Monetary Economics, Vol. 54, 4, pp. 1256-1270, **TD No. 511 (July 2004)**.

A. ANZUINI and A. LEVY, *Monetary policy shocks in the new EU members: A VAR approach*, Applied Economics, Vol. 39, 9, pp. 1147-1161, **TD No. 514 (July 2004)**.

R. BRONZINI, *FDI Inflows, agglomeration and host country firms' size: Evidence from Italy*, Regional Studies, Vol. 41, 7, pp. 963-978, **TD No. 526 (December 2004).**

L. MONTEFORTE, *Aggregation bias in macro models: Does it matter for the euro area?*, Economic Modelling, 24, pp. 236-261, **TD No. 534 (December 2004)**.

A. DALMAZZO and G. DE BLASIO, *Production and consumption externalities of human capital: An empirical study for Italy*, Journal of Population Economics, Vol. 20, 2, pp. 359-382, **TD No. 554 (June 2005).**

M. BUGAMELLI and R. TEDESCHI, *Le strategie di prezzo delle imprese esportatrici italiane,* Politica Economica, v. 3, pp. 321-350, **TD No. 563 (November 2005)**.

L. GAMBACORTA and S. IANNOTTI, *Are there asymmetries in the response of bank interest rates to monetary shocks?*, Applied Economics, v. 39, 19, pp. 2503-2517, **TD No. 566 (November 2005).**

S. DI ADDARIO and E. PATACCHINI, *Wages and the city. Evidence from Italy*, Development Studies Working Papers 231, Centro Studi Luca d'Agliano, **TD No. 570 (January 2006)**.

P. ANGELINI and F. LIPPI, *Did prices really soar after the euro cash changeover? Evidence from ATM withdrawals*, International Journal of Central Banking, Vol. 3, 4, pp. 1-22, **TD No. 581 (March 2006)**.

A. LOCARNO, *Imperfect knowledge, adaptive learning and the bias against activist monetary policies*, International Journal of Central Banking, v. 3, 3, pp. 47-85, **TD No. 590 (May 2006)**.

F. LOTTI and J. MARCUCCI, *Revisiting the empirical evidence on firms' money demand*, Journal of Economics and Business, Vol. 59, 1, pp. 51-73, **TD No. 595 (May 2006)**.

P. CIPOLLONE and A. ROSOLIA, *Social interactions in high school: Lessons from an earthquake*, American Economic Review, Vol. 97, 3, pp. 948-965, **TD No. 596 (September 2006).**

A. BRANDOLINI, *Measurement of income distribution in supranational entities: The case of the European Union*, in S. P. Jenkins e J. Micklewright (eds.), Inequality and Poverty Re-examined, Oxford, Oxford University Press, **TD No. 623 (April 2007).**

M. PAIELLA, *The foregone gains of incomplete portfolios*, Review of Financial Studies, Vol. 20, 5, pp. 1623-1646, **TD No. 625 (April 2007).**

K. BEHRENS, A. R. LAMORGESE, G.I.P. OTTAVIANO and T. TABUCHI, *Changes in transport and non transport costs: local vs. global impacts in a spatial network*, Regional Science and Urban Economics, Vol. 37, 6, pp. 625-648, **TD No. 628 (April 2007).**

G. ASCARI and T. ROPELE, *Optimal monetary policy under low trend inflation*, Journal of Monetary Economics, v. 54, 8, pp. 2568-2583, **TD No. 647 (November 2007).**

R. GIORDANO, S. MOMIGLIANO, S. NERI and R. PEROTTI, *The Effects of Fiscal Policy in Italy: Evidence from a VAR Model*, European Journal of Political Economy, Vol. 23, 3, pp. 707-733, **TD No. 656 (December 2007).**

2008

S. MOMIGLIANO, J. Henry and P. Hernández de Cos, *The impact of government budget on prices: Evidence from macroeconometric models*, Journal of Policy Modelling, v. 30, 1, pp. 123-143 **TD No. 523 (October 2004).**

P. DEL GIOVANE, S. FABIANI and R. SABATINI, *What's behind "inflation perceptions"? A survey-based analysis of Italian consumers, i*n P. Del Giovane e R. Sabbatini (eds.), *The Euro Inflation and Consumers' Perceptions. Lessons from Italy*, Berlin-Heidelberg, Springer, **TD No. 655 (January 2008).**

FORTHCOMING

S. SIVIERO and D. TERLIZZESE, *Macroeconomic forecasting: Debunking a few old wives' tales*, Journal of Business Cycle Measurement and Analysis, **TD No. 395 (February 2001)**.

P. ANGELINI, *Liquidity and announcement effects in the euro area*, Giornale degli economisti e annali di economia, **TD No. 451 (October 2002).**

S. MAGRI, *Italian households' debt: The participation to the debt market and the size of the loan*, Empirical Economics, **TD No. 454 (October 2002)**.

P. ANGELINI, P. DEL GIOVANE, S. SIVIERO and D. TERLIZZESE, *Monetary policy in a monetary union: What role for regional information?*, International Journal of Central Banking, **TD No. 457 (December 2002)**.

L. MONTEFORTE and S. SIVIERO, *The Economic Consequences of Euro Area Modelling Shortcuts*, Applied Economics, **TD No. 458 (December 2002).**

L. GUISO and M. PAIELLA,, *Risk aversion, wealth and background risk*, Journal of the European Economic Association, **TD No. 483 (September 2003).**

G. FERRERO, *Monetary policy, learning and the speed of convergence*, Journal of Economic Dynamics and Control, **TD No. 499 (June 2004).**

F. SCHIVARDI e R. TORRINI, *Identifying the effects of firing restrictions through size-contingent Differences in regulation*, Labour Economics, **TD No. 504 (giugno 2004)**.

C. BIANCOTTI, G. D'ALESSIO and A. NERI, *Measurement errors in the Bank of Italy's survey of household income and wealth*, Review of Income and Wealth, **TD No. 520 (October 2004)**.

D. Jr. MARCHETTI and F. Nucci, *Pricing behavior and the response of hours to productivity shocks,* Journal of Money Credit and Banking, **TD No. 524 (December 2004).**

L. GAMBACORTA, *How do banks set interest rates?,* European Economic Review, **TD No. 542 (February 2005).**

P. ANGELINI and A. Generale, *On the evolution of firm size distributions*, American Economic Review, **TD No. 549 (June 2005).**

R. FELICI and M. PAGNINI,, *Distance, bank heterogeneity and entry in local banking markets*, The Journal of Industrial Economics, **TD No. 557 (June 2005).**

M. BUGAMELLI and R. TEDESCHI, *Le strategie di prezzo delle imprese esportatrici italiane*, Politica Economica, **TD No. 563 (November 2005).**

S. DI ADDARIO and E. PATACCHINI, *Wages and the city. Evidence from Italy*, Labour Economics, **TD No. 570 (January 2006)**.

M. BUGAMELLI and A. ROSOLIA, *Produttività e concorrenza estera*, Rivista di politica economica, **TD No. 578 (February 2006).**

PERICOLI M. and M. TABOGA, *Canonical term-structure models with observable factors and the dynamics of bond risk premia*, **TD No. 580 (February 2006).**

E. VIVIANO, *Entry regulations and labour market outcomes. Evidence from the Italian retail trade sector*, Labour Economics, **TD No. 594 (May 2006)**.

S. FEDERICO and G. A. MINERVA, *Outward FDI and local employment growth in Italy*, Review of World Economics, Journal of Money, Credit and Banking, **TD No. 613 (February 2007).**

F. BUSETTI and A. HARVEY, *Testing for trend*, Econometric Theory **TD No. 614 (February 2007).**

V. CESTARI, P. DEL GIOVANE and C. ROSSI-ARNAUD, *Memory for Prices and the Euro Cash Changeover: An Analysis for Cinema Prices in Italy*, In P. Del Giovane e R. Sabbatini (eds.), The Euro Inflation and Consumers' Perceptions. Lessons from Italy, Berlin-Heidelberg, Springer, **TD No. 619 (February 2007).**

B. ROFFIA and A. ZAGHINI, *Excess money growth and inflation dynamics*, International Finance, **TD No. 629 (June 2007)**.

M. DEL GATTO, GIANMARCO I. P. OTTAVIANO and M. PAGNINI, *Openness to trade and industry cost dispersion: Evidence from a panel of Italian firms*, Journal of Regional Science, **TD No. 635 (June 2007)**.

A. CIARLONE, P. PISELLI and G. TREBESCHI, *Emerging Markets' Spreads and Global Financial Conditions*, Journal of International Financial Markets, Institutions & Money, **TD No. 637 (June 2007).**

S. MAGRI, *The financing of small innovative firms: The Italian case*, Economics of Innovation and New Technology, **TD No. 640 (September 2007)**.