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R&D AND MARKET STRUCTURE IN A HORIZONTAL DIFFERENTIATION FRAMEWORK

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Abstract

This paper examines the dynamic interaction between R&D and market structure in a horizontally differentiated market framework. Firms invest in R&D to modify the level of differentiation of their products, increasing their specialization and their market power. The invested resources in research are declining over time because of decreasing returns from further specialization. Prices, output and short-run profits of the firms producing differentiated products increase and move towards the higher steady state values, while production of the non-differentiated good falls; the number of firms is constant in all periods. The increasing specialization of varieties improves the overall utility of consumers. The comparison with the socially optimal solution shows that firms underinvest in R&D. Firms do not internalize the effects of their research effort on the overall level of substitutability of the other varieties and on the profits of the other firms.

JEL Classification: O3.
Keywords: R&D, market power, horizontal differentiation.

Contents

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1 Introduction[1]

The relationship between technological progress and market structure has been a recurrent element of discussion among economists. In particular, many contributions aimed to understand what are the effects of the different degrees of market power on the incentives to undertake R&D activity. Less attention has been given to the opposite relation, how firms can influence the shape of market competition using research activity.This paper aims to examine a mechanism through which this last relationship can come into effect and how R&D and market structure endogenously interact over time. The relationships between these two variables have important policy implications: policy measures to stimulate R&D indirectly affect competition and, on the other hand, institutional changes to the market structure influence the incentives to research.

We consider a horizontally differentiated framework where firms invest in R&D to increase differentiation between varieties of the same product. We can think of a product as an instrument allowing us to satisfy some needs. In a differentiated market, each variety has different effectiveness in satisfying each need. The consumer chooses the bundle of varieties giving him the highest level of satisfaction.

Firms are able to modify the characteristics of their variety through investments in R&D; they may aim towards a more specialized profile, increasing the level of horizontal differentiation. Doing so, they reduce the degree of substitutability with the other varieties and raise their market power. In the limit, they tend to cut the reciprocal influence between varieties and to transform their products in unrelated ones.

An example of this kind of behaviour can be found among food producers. In the market of biscuits some producers specialized their production over time in low fat products (e.g. Misura) and others in sweet products (e.g. Mulino Bianco).

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Moreover, the movement of firms variety towards areas of specialization not well fulfilled by other varieties raises the overall satisfaction of the consumers. Horizontal differentiation implies a trade-off between level of competition and improvement of consumer welfare, which has been well understood by the antitrust authorities[2]. The introduction of new versions of products whose characteristics damaged competition with other firms has been justified if the innovative characteristics implied a consumers welfare improvement. This has been one of the main discussions around the Kodak vs. Berkey classic case in the 70s; more recently, when Microsoft has been charged by the US Department of Justice (1998), it defended its choice of selling together Windows XP and Internet Explorer saying that an integrated platform simplifies the creation of new applications, with advantages for consumers.

The inclusion of our mechanism in a dynamic framework allows us to determine not only the path of production and R&D, but also the evolution of the market structure over time. Our most important results are that in this environment firms find incentives to invest in R&D to increase their specialization. The quantity of invested resources in research is declining over time, because the returns from further specialization decrease when the firm is more specialized, while prices, output and short-run profits of the firms increase. When we endogenize the number of firms using a zero profit condition, we find that it is constant in all periods.

Moreover, we compare the decentralized outcome and the socially optimal solution. We find that there is a suboptimal investment in R&D. This is because the socially optimal production is larger than the decentralized one; more output taking advantage of research implies more incentives to do research activity. Moreover, the firm does not internalize the benefits of reducing substitutability with the other varieties on the profits of the other producers.

The paper is organized as follows: in the next section, we review the literature on the relationship between research activity and market structure and we highlight connections and differences between our work and the previous ones.

In section 3, we develop the model. Subsection 3.1 examines the market framework, while subsection 3.2 formalizes assumptions and results on the

[2] See Baker (1997) and Weiss (1974) for some considerations about product differentiation and antitrust activity.

R&D activity. In subsection 3.3 we compare the decentralized solution of the model with the social optimum.

In section 4, we examine the consequences of weakening two assumptions of the main model. In subsection 4.1 we endogenize the number of firms entering the market. In subsection 4.2 we examine the optimal decision of the firms about the production of one or more versions of their variety.

In section 5, we conclude and summarize the findings of the paper and further directions of research.

2 Literature review

The first and most influential studies on the relationship between research and market structure are due to Schumpeter (1942) and Arrow (1962). Schumpeter argues that R&D is driven by the attempt to appropriate the monopolistic rents created by innovation. Arrow notices that a competitive market provides more incentives to invest in R&D, because research allows a firm to create advantages over the other competitors escaping the tightness of competition.

Dasgupta and Stiglitz (1980) take into account the endogenous nature of the simultaneous relationship between innovative activity and market structure. They consider the effects of process R&D that reduces the marginal cost of a unit of good in static Cournot oligopoly. The research expenditure is a sunken cost and therefore the optimal choices of the firms determine the barriers to entry and the number of competitors.

Even if they notice that market power is better measured by the charged mark-up than by a concentration index, they use the number of firms as endogenous index of the market structure. Several both theoretical and empirical related works (e.g. Sutton (1998)) do the same. We will see in our model that in a different framework from that developed by Dasgupta and Stiglitz concentration and mark-up have uncorrelated behaviours[3].

The development of the endogenous growth models, in particular the works from Romer (1990), Grossman and Helpman (1991a,b,c) and Aghion and Howitt (1992; 1998a), gives new elements to create theoretical models on the effects of market structure on R&D.

[3]See Vives (2004) for a model where the incentives to R&D negatively depend on the number of firms in the market and positively depend on the the degree of substitutability.

In these papers, firms perform research to increase the ratio utility - cost of the good in vertical differentiation. They emphasize the Schumpeterian point of view of the relationship and imply a negative relationship between competition and research.

However, in the same years several papers (e.g. Geroski (1990), Geroski and Pomroy (1990), Blundell, Griffith and Van Reenen (1995, 1999), Nickell (1996), Rogers (2002)) point out that the empirical evidence seems to be favourable to a positive effect of competition.

The most recent empirical work by Aghion, Bloom, Blundell, Griffith and Howitt (2005) find an inverted-U relationship, where R&D increases in highly competitive industries and falls in more concentrated ones. R&D generates further possibilities of rent extraction and reduces competition.

The attempt of conciliating the theoretical framework with the empirical results follows several lines of research[4]. Peretto (1999) gets results on the same line of the Arrow's argument in an oligopolistic framework with endogenous market structure. An increase in the exogenous level of substitutability between products reduces the equilibrium number of firms and increases the rents from innovation, stimulating R&D.

Aghion, Harris and Vickers (1997), Aghion, Harris, Howitt and Vickers (2001), Aghion, Bloom, Blundell, Griffith and Howitt (2005) use a "step-by-step" model of innovation where oligopolistic firms run a continuous "innovation race". Sometimes one competitor conquests a monopolistic position and other times the competitors share a symmetric Cournot oligopoly.

In this kind of model the Schumpeterian effect is balanced by a "competition escaping" increase in R&D when the firms share the market. The relationship between R&D and market structure is cyclical, in the sense that a successful innovation either increases or reduces the distance between firms in the market and every gain of position in the market structure is temporary until the other firm innovates. This is because firms compete to improve production of the same good in a vertical differentiation framework. Therefore, an innovation reduces the effectiveness of past improvements of the other firms on their own profits.

Nickell (1996), Schmidt (1997), Aghion and Howitt (1997; 1998b), Aghion, Dewatripont and Rey (1999) use agency considerations to explain the positive correlation between competition and research: more competition increases

[4] A review of the most important contributions on this question can be found in Aghion and Griffith (2005).

the incentives for managers to maintain a tighter discipline in the firm in order to avoid losses, because the margins of profits are lower in a competitive environment. Therefore, managers work to cut the marginal costs as much as possible and invest in R&D to this aim. Moreover, the introduction of an innovation by one firm increases the incentives to innovate of the other firms, because otherwise they lose their market shares.

Aghion and Howitt (1996; 1998b) separate research from development. An increase in competition raises the speed of adaptability of old production lines to the new standards; through this channel, it increases the development activity and therefore the growth rate of the economy.

Bucci (2003) examines the effects of an exogenous increase of mark-up on aggregate growth in a horizontally differentiated economy where R&D increases the number of available varieties and finds that the shape of the relationship between the two variables depends on the used technology.

Other recent related papers discuss the correlation between process and product R&D in a simplified static framework similar to ours. Lin and Saggi (2002) compare the incentives to the two kinds of research under Bertrand and Cournot duopolistic structures. Product R&D allows the firm to reduce the level of substitutability between the outputs of the two firms, while process R&D allows a reduction of the marginal costs. They find a positive correlation between the two kinds of research and show that Bertrand competition gives more incentives to product differentiation. Rosenkranz (2003), working with a similar framework in a monopolistic competition market, shows that cooperation between firms increases product innovation and that the same happens after an enlargement of the potential market. Weiss (2003) examines how the incentives to product and process R&D change with the degree of substitutability of the products.

A last paper related to our analysis is Bils and Klenow (2001), which examines the expenditure patterns in differentiated and homogeneous products. They find an increase over time of the expenditure in products with increasing differentiation and a fall of the consumption of more static and homogenous products. Our model explains this behaviour.

The main contribution of our work to the literature regarding R&D and market structure is the endogenous development of the relationship in a dynamic horizontal differentiation model, an underexplored framework. Differently from the models based on vertical differentiation, our framework emphasizes that the R&D choices of a firm do not necessarily have negative effects on the strategic environment and on the profits of the other firms.

Moreover, while most of the other models are static, in our case the presence of a time dimension allows the analysis of the transitional dynamics of the firm behaviour in terms of output, prices and research investments.

3 The model

3.1 The market framework

We consider an economy where L (normalized to one) workers/consumers live in continuous time. They inelastically supply their labour and have homogeneous preferences[5].

$N + 1$ goods are produced, using labour as the only production factor. One good is homogeneous and produced under constant returns to scale and perfect competition. The other N goods are differentiated and produced under increasing returns to scale and imperfect competition with strategic interaction between firms. Each firm produces a horizontally differentiated variety of the good. Each variety can be produced in many versions that differ each other in the degree of substitutability with the other varieties. The set of the currently available versions of a variety depends on the past R&D history of a firm.

The resulting framework is a Cournot oligopolistic market with differentiated product, but the model can be developed with similar results under the hypothesis of monopolistic competition[6].

Each good aims to satisfy a subset of needs of the consumer. Different varieties of the same good have slightly different characteristics[7]; therefore, they are comparatively more or less efficient to satisfy each need. Consumers

[5] Homogeneity and quadratic quasilinearity of consumer preferences allow us to obtain a linear inverse demand function after aggregation. If we consider heterogeneous consumers, the resulting demand function is not linear even in the case of quasilinear quadratic utility. In this kind of model, we cannot derive explicit solutions of the equations, but the behaviour of the real variables would be qualitatively the same as in our partial equilibrium economy. Therefore, we can consider our simplified model a good approximation of the most general case with heterogeneous consumers.

[6] The oligopolistic framework seems a better environment because the idea of investing to enhance the idiosyncratic characteristics of the product suggests attention to the other varieties and therefore to the choices of the other firms.

[7] The framework we use here to give an intuition of the meaning of our utility function and of the mechanism of horizontal differentiation is based on the characteristics utility theory developed by Lancaster (1966a-b; 1975; 1979; 1980).

choose a bundle of varieties to satisfy all their necessities, after a comparison of the overall utility they get from the currently produced versions of the different varieties. We capture this kind of environment saying that consumers maximize the following intertemporal quasilinear utility function[8,9]:

$$U=\int_0^{\infty}\{x_0\left(t\right)+\sum_{k=1}^{N}\sum_{i=1}^{m_k}\left[a_k-\sum_{j=1}^{m_k}\frac{b_{ijk}\left(t\right)}{2}x_{jk}\left(t\right)\right]x_{ik}\left(t\right)\}e^{-rt}dt \qquad (1)$$

where $x_0\left(t\right)$ and $x_{ik}\left(t\right)$ are the consumed quantities respectively of the homogeneous good and of the currently produced version of variety i of good k at time t and m_k is the number of firms producing a variety of good k[10]. The current utility derived from each variety of the differentiated goods depends not only on the consumed quantities of that variety, but also on a weight (the term in square brackets), which negatively depends on the consumption of all the different varieties of the good. Therefore, increasing consumption of a variety reduces the marginal utility of additional units not only of the same variety, but also of the others. This is because we suppose there is partial substitutability between varieties: to satisfy its needs, the consumer can substitute one variety with another having similar, but not equal, characteristics, which is therefore only partially suitable to satisfy the

[8] This utility function is an intertemporal generalization of the quadratic partial equilibrium function used in many papers to obtain linear inverse demand functions. See for example Spence (1976), Dixit (1979), Vives (1990), Ottaviano and Thisse (1999). The use of a general equilibrium framework would not change the qualitative behaviour of the real variables, but the model would not be analytically tractable.

[9] The substitutability parameters b_{ijk} have a time index because the firms change the currently produced versions of their variety over time. Newer versions have lower substitutability parameters. A more general formulation of the utility function taking into account all the possible versions of each variety is $U=\int_0^{\infty}\{x_0\left(t\right)+$ $+\sum_{k=1}^{N}\sum_{i=1}^{m_k}\int_0^{b_{iik}}...\int_0^{b_{iik}}\left[a_k-\sum_{j=1}^{m_k}\int_0^{b_{jjk}}...\int_0^{b_{jjk}}\frac{b_{ijk}}{2}x_{jk}\left(t,\{b_{jlk}\}_{l=1}^{m_k}\right)db_{1jk}...db_{m_kjk}\right]*$ $*x_{ik}\left(t,\{b_{ilk}\}_{l=1}^{m_k}\right)db_{i1k}...db_{im_kk}\}e^{-rt}dt$, where different versions of a variety are indexed by the substitution coefficients b_{ijk} and the own effect of a variety i on its price is b_{iik}. We focus on an equilibrium where only the newest version is produced (that is, $x_{ik}(t,\{b_{ijk}\}_{j=1}^{m_k})>0$ only if $b_{ijk}=b_{ijk}(t)\ \forall j$, where the parameter with the time index is the lowest parameter b_{ijk} achievable at time t). Therefore, the utility function can be rewritten as in the main text. When we speak of a variety in the current paragraph we usually mean the currently produced version of that variety.

[10] The number of firms in each market is here assumed to be constant over time.

needs previously satisfied by the other variety. We use a quasilinear specification because it allows aggregation by direct summation of the demand functions of consumers; therefore, we can use a representative consumer approach. Moreover, the separation among the behaviours of the homogeneous "static" good and the differentiated ones follows the empirical results of Bils and Klenow (2001).

The utility maximization problem is subject to the budget constraint of the agent

$$x_0(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} p_{ik}(t)\, x_{ik}(t) \leq w(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} \pi_{ik}(t) \tag{2}$$

where $w(t)$ is the wage and $\pi_{ik}(t)$ are the redistributed profits of the firm producing variety i of good k at time t; the good 0 is the numerary of the economy and its price is normalized to 1.

Lemma 1 *Maximization of the utility function (1) subject to the budget constraint (2) implies the following linear inverse demand function:*

$$p_{ik}(t) = a_k - \sum_{j=1}^{m_k} b_{ijk}(t)\, x_{jk}(t)\, dj \tag{3}$$

Proof. From the first order conditions of the utility maximization problem. ■

The parameters $b_{ijk}(t)$ are a measure of the influence of consumption of the currently produced version of the variety j on the market of the currently produced version of the variety i; we suppose $b_{iik} = b_{jjk} = b_{0k}\ \forall i, j$ to complete the symmetry between varieties. If $b_{ijk} = b_{iik}\ \forall j$, the effect of consuming one more unit of any variety of the same good on the equilibrium price of variety i of good k is the same. Hence, the resulting market structure is a Cournot oligopoly with homogeneous good. If $b_{ijk} < b_{iik}\ \forall j \neq i$, the equilibrium price of a variety is more sensitive to an increase of the sold quantity of the same variety than to an increase of the sold quantity of another variety and, therefore, the substitutability between varieties is only partial and proportional to the b_{ijk} coefficient.

Let us consider now the production process. We use a simple linear production function only requiring labour, equal for varieties of the same good, but that can differ between goods: if a differentiated producer i wants to produce a quantity $x_{ik}(t)$ of its own variety, he needs

$$l_{ik}(t) = d_k + c_k x_{ik}(t) \tag{4}$$

units of labour.

Given the structure of parameters $b_{ijk}(t)$ of the currently produced versions of a variety i, the price and quantity decisions of the firm do not include any intertemporal element; therefore, they follow the maximization of the current operative profits:

$$\pi_{ik}^{o}(t) = p_{ik}(t) x_{ik}(t) - w l_{ik}(t) \tag{5}$$

subject to the inverse demand function (equation (3)) and the production function (equation (4)). The first order conditions of maximization of the current operative profits imply the following reaction curve:

$$x_{ik}(t) = \frac{a_k - wc_k - \sum_{j=1}^{m_k} b_{ijk}(t) x_{jk}(t)}{b_{0k}} \tag{6}$$

The only parameters depending on the variety index i are the cross-effect coefficients. Therefore, if the b_{ijk} structure is the same $\forall i$, the optimal choice of $x_{ik}(t)$ is the same for all firms producing different varieties of the same product.

Proposition 2 *Given a symmetric parameters structure for all firms producing different varieties of the same good, maximization of the current operative profits (5) subject to the inverse demand function (equation (3)) and the production function (equation (4)) implies a symmetric equilibrium where*

$$x_{ik}(t) = \frac{a_k - wc_k}{b_{0k} + \sum_{j=1}^{m_k} b_{ijk}(t)} = \frac{a_k - wc_k}{b_{0k} + \Gamma_k(t)} \tag{7}$$

and

$$p_{ik}(t) = \frac{a_k b_{0k} + wc_k \Gamma_k(t)}{b_{0k} + \Gamma_k(t)} \tag{8}$$

Proof. From the first order conditions of the current profits maximization problem. ■

Quantities and prices are negatively related to the level of substitutability with the other varieties, defined by the index $\Gamma_k(t)$. We require $w < \frac{a_k}{c_k}$ to rule out corner solutions.

This implies that the operative profits of the producer of variety i of good k are the same for all the producers of the same good:

$$\pi_{ik}^o(t) = \left(\frac{a_k - wc_k}{b_{0k} + \Gamma_k(t)}\right)^2 b_{0k} - wd_k \tag{9}$$

Operative profits negatively depend on the price sensitivity with respect to all varieties too.

Given the overall profit function of firm i in period t

$$\pi_{ik}(t) = p_{ik}(t)\, x_{ik}(t) - wl_{ik}(t) - w\sum_{j=1}^{m_k} R_{ijk}(t)$$

where $R_{ijk}(t)$ is the number of workers employed in R&D by firm i to improve the level of differentiation with variety j[11], we can close the model deriving the demand of the numerary good:

$$x_0 = w - \sum_{k=1}^{N}\sum_{i=1}^{m_k}\left[\left(\frac{a_k - wc_k}{b_{0k} + \Gamma_k(t)}\right) wc_k + wd_k + w\sum_{j=1}^{m_k} R_{ijk}(t)\right] \tag{10}$$

Non negativity requires

$$1 \geqslant \sum_{k=1}^{N}\sum_{i=1}^{m_k}\left[\left(\frac{a_k - wc_k}{b_{0k} + \Gamma_k(t)}\right) c_k + d_k + \sum_{j=1}^{m_k} R_{ijk}(t)\right]$$

Let us suppose that we need c_0 units of labour to produce one unit of homogeneous good. If we assume perfect competition in the homogeneous sector, the zero profit condition determines the equilibrium wage $w = \frac{1}{c_0}$. Coming back to the non negativity condition, we will see at the end of the next paragraph that the amount of labour used in the homogeneous sector is decreasing over time; hence, the non negativity condition is always satisfied on the adjustment path if it is satisfied in the asymptotic steady state, where $R_{ijk} = 0$ and $b_{ijk} = 0\ \forall i \neq j$. Therefore, a necessary and sufficient condition is:

[11] We assume $R_{iik}(t) = 0\ \forall i, k, t$

$$1 \geqslant \sum_{k=1}^{N} m_k \left[\left(\frac{a_k - \frac{c_k}{c_0}}{2b_{0k}} \right) c_k + d_k \right]$$

In the remaining of the text we assume that both this condition and the positivity constraint of the differentiated goods $a_k - \frac{c_k}{c_0} > 0$ are satisfied.

3.2 The innovation activity

We now model how the firm influences the market structure.

The utility of each good for the consumer is determined by its idiosyncratic value in several characteristics. If we associate a numerical value to the consumer evaluation of each characteristic, we can display the position of the good in a characteristics space. Consumers choose their optimal bundle after evaluating the characteristics profiles of the outputs proposed by the firms. From their point of view, spatially nearer characteristics profiles are more substitutable.

A firm add to its feasible set of technologies new positions in the characteristics space through investments in R&D. There is a technological trade-off between characteristics: the development of some of them does not allow or even damages the development of others[12]. The optimal choice of the newly added technological positions implies an increase of the level of specialization in some characteristics of the good.

We call a "variety" the set of all the potential positions on the technological frontier of the same good with the same specialization. For a given variety, a "version" is one of the possible characteristics profile. Different versions show different degrees of specialization, which translate to different levels of substitutability, with effects on the profits of firms.

We can see in figure 1 an example giving the intuition of the ideas[13]: we show the effects of R&D of two firms in a two-dimensional space of characteristics and the link with the b_{ijk} coefficients. The two axes of the graph are the values of two characteristics z_1 and z_2 of a good.

[12] Lancaster (1966a) shows that the technological frontier of the optimally developed combinations of characteristics must be concave and that the optimal behaviour of firms is staying on the frontier.

[13] We assume that the number of potentially exploited characteristics of a product is not smaller than the potential number of firms in the market. This technical assumption is equivalent to say that a sensible entrepreneur is always able to find a new specialization to be exploited.

R&D allows the firms to enlarge the set of feasible technologies on the technology frontier, which includes all the technologically possible z_1/z_2 ratios. In our figure, the level of substitutability between two products (and therefore the value of the b_{ijk} coefficients) is given by the closeness in the z_1/z_2 ratios and by the physical nearness in the Cartesian plane.

Let us suppose that the only available technological position is point A. Both firms must be positioned there and there is perfect substitutability between the produced outputs.

Now the two firms invest in R&D. The farther the produced versions of the varieties are one from the other, the lower is the level of substitutability between them (and the larger are the profits of firms). Therefore, the optimal behaviour of the two firms will be adding new positions on the technological frontier towards the opposite axes, for example towards the points B and C.

Without loss of generality, the variety of firm 1 is z_1 intensive and that one of firm 2 is z_2 intensive. Firm 1 (2) learnt how to produce all versions of its variety between A and B (C), but finds optimal to produce variety B (C) only. The two firms increase the level of specialization of their varieties and move towards the two opposite axes.

Let us go back to the formalization of this situation in our model.

We formally define the dynamics of the lower bound of the achievable substitutability coefficients between the newest versions of two varieties i and j with the following equation:

$$\dot{b}_{ijk}(t) = -\gamma b_{ijk}(t) \left[\phi\left(R_{ijk}\left(t\right)\right) + \phi\left(R_{jik}\left(t\right)\right)\right] \tag{11}$$

where $R_{ijk}\left(t\right)$ is the number of workers employed in R&D by firm i to reduce substitutability with variety j.

We suppose that the dynamics depends on the efforts of the two interested firms in that direction. R&D is increasingly difficult to be efficiently organized and consequently there are decreasing returns to scale when the firm increases the employed quantity of resources. This fact is captured by the function ϕ, which is increasing and concave ($\phi' > 0$ and $\phi'' < 0$) and $\phi\left(0\right) = 0$. We assume that the more diversified is the product, the more difficult is finding new useful characteristics to be developed without damaging the efficiency of past specialization. If we consider the limit variety, which is completely unrelated to the others, the development of new specialized features does not change the level of substitutability; therefore, it is useless from the point of view of the firm. Hence, we suppose that a given effort in

R&D has the same relative, and not absolute, result on market power.

The research process is completely deterministic to keep a symmetric simplified outcome, not possible in presence of uncertainty. Moreover, we assume that the firm only produces the most differentiated version of its variety (that is, the version with the lowest values of b_{ijk})[14].

Last, we assume that, because of patent protection or industrial secrecy, no firm can copy the newly developed version of a variety. Including the ability to imitate some (but not all) characteristics of the new version would weaken the effects of R&D and slow the speed of movement towards the steady state, but would not change the qualitative results.

The R&D choices are an intertemporal decision. Therefore, firm i considers the effects on the discounted value of future profits:

$$\Pi_{ik}(t) = \int_t^{\infty} \left[p_{ik}(s)\, x_{ik}(s) - w l_{ik}(s) - w \sum_{j=1}^{m_k} R_{ijk}(s) \right] e^{-rs} ds \qquad (12)$$

We assume that production and R&D workers are perfectly substitutable.

The optimal choice of prices and quantities for a given symmetric structure of b_{ijk} still follows the analysis of the previous section.

We examine now the optimal R&D path.

Proposition 3 *The solution of the optimal control problem where the firm maximizes its intertemporal profits (12) subject to the demand function (3), to the production function (4) and to the dynamics of the lowest achievable values of the b_{ijk} coefficients (11) imply the following growth rate of $R_{ijk}(t)$:*

$$\frac{\dot{R}_{ijk}(t)}{R_{ijk}(t)} = \frac{1}{\eta_{\phi' R}(R_{ijk}(t))} \left[r - \gamma c_0 \left(a_k - \frac{c_k}{c_0} \right)^2 \frac{b_{ijk}(t)\, \phi'(R_{ijk}(t))}{(b_{0k} + \Gamma_k(t))^2} \right] \qquad (13)$$

where $\eta_{\phi' R}(R_{ijk}(t))$ is the elasticity of the $\phi'(R_{ijk}(t))$ function with respect to $R_{ijk}(t)$.

[14] We show in subsection 4.2 that the optimal choice of the firm is the production of the most differentiated version only, if the fixed cost are high enough or if there are three firms or more. Otherwise, the optimal choice could be the production of both the most differentiated and the non-differentiated versions, but not of the intermediate versions. We consider the first case in the main model, but the second case can be easily accommodated in the model.

Proof. From the first order conditions of the optimal control problem. ■

The two differential equations (11) and (13) imply that the variation rates of $b_{ijk}(t)$ and $R_{ijk}(t)$ are the same for all varieties, given a common initial value $b_{ijk}(0)$ and a common choice of $R_{ijk}(t)$ for some value of t.

In the case $\phi(R_{ijk}(t)) = \frac{R_{ijk}(t)^{1-\eta}}{1-\eta}$ with $0 < \eta < 1$, which implies a constant elasticity $\eta_{\phi' R}(R_{ijk}(t)) = \eta$, the differential equations imply that there exists an asymptotic steady state where $b_{ijk}(\infty) = 0$ and $R_{ijk}(\infty) = 0$ $\forall i, j$.

Because in this case the end point of the path of R_{ijk} and its variations in each period are fixed and common for all the varieties, the full path is backwardly determined. We can qualitatively display the steady state values and the transitional dynamics in figure 2: under the hypotheses of symmetric behaviour of firms and constant elasticity of the ϕ' function, the locus of points where $\dot{b}_{ijk}(t) = 0$ in the (b_{ijk}, R_{ijk}) space is defined by

$$R_{ijk}(t)\,|_{\dot{b}=0} = 0$$

and

$$b_{ijk}(t)\,|_{\dot{b}=0} = 0$$

while the set of points where $\dot{R}_{ijk}(t) = 0$ is described by

$$R_{ijk}(t)\,|_{\dot{R}=0} = 0$$

and

$$R_{ijk}(t)\,|_{\dot{R}=0} = \left[\frac{\gamma c_0 \left(a_k - \frac{c_k}{c_0}\right)^2 b_{ijk}(t)}{r\,(b_{0k} + \Gamma_k(t))^2}\right]^{\frac{1}{\eta}}$$

There are multiple candidate equilibrium behaviours of the firm; any path converging to a steady state with $R_{ijk}(\infty) = 0$ and $0 \leqslant b_{ijk}(\infty) \leqslant b_{0k}$ can be an equilibrium. The saddlepath converging to $R_{ijk}(\infty) = 0$ and $b_{ijk}(\infty) = 0$ dominates all the other equilibriums: given the concentrated profit function (with $p_{ik}(t)$ and $x_{ik}(t)$ already at their optimal value) in absence of R&D, one infinitesimal unit of R&D yields infinite returns if $b_{ijk}(\infty) > 0$:

$$\frac{\partial \Pi_{ik}(t)}{\partial R_{ijk}(t)}\,|_{R_{ijk}(t)=0} = \infty$$

This means that the other candidate behaviours converge to local minima of the profit function, which cannot be optimal. Therefore, all firms choose a positive level of R&D in equilibrium and follow the saddlepath towards $b_{ijk}(\infty) = 0$. They gradually reduce the quantity of invested resources in research and move towards the steady state situation, where there is complete differentiation and, therefore, no incentives to invest in R&D. The level of research in each period is the same for all firms producing different varieties of the same good and, therefore, $b_{ijk}(t)$ follows the same path $\forall i \neq j$.

Let us consider now what are the consequences of the implied dynamics on quantities, prices and profits. We assume that firms begin with a perfectly substitutable version of the product ($b_{ijk}(0) = b_{0k}\ \forall i, j$). They feel the pressure of competition. Therefore, they choose to specialize their variety. They invest a positive initial level of resources in R&D, which consequently moves b_{ijk} down towards the lower steady state level. The lower substitutability level of the produced variety reduces the pressure of competition and, therefore, the incentive to invest in R&D.

The equilibrium levels of quantities, prices and operative profits are the static ones for the current b_{ijk} configuration. A review of equations (7), (8) and (9) tells us that they increase during the transitional dynamics and asymptotically tend to the higher steady state levels. This is because the demand function is less sensitive to the level of output of the other firms when there is more differentiation. Therefore, the residual demand function, which is the space where the firm maximizes its own profits, has a higher intercept. A larger quantity is produced for a given price. Moreover, the firm can better exploit the new residual demand function to charge a higher price for its output.

On the other hand, the produced quantity of homogeneous good (equation (10)) falls because now the raw utility of one unit of differentiated product is higher (the penalty to the utility for each unit of the other varieties is lower) and, therefore, the differentiated products are preferred[15]. A consequence of this fact is that the benefits of the successful research activity are not limited to the firms: consumers prefer the bundle of the newly developed varieties, where they obtain a larger quantity of more diversified goods and a smaller one of homogeneous good.

[15] This dynamics explains the empirical patterns reported by Bils and Klenow (2001) where consumption of the "static" homogeneous good falls and expenditure in the varieties of the differentiated ones increases over time.

3.3 Comparison between the social optimum and the decentralized economy solution

Now, let us consider the comparison between the social optimum and the solution of the decentralized economy problem.

We suppose that there is a benevolent planner choosing the allocations of the real variables $x_{ik}(t)$, $l_{ik}(t)$, $x_0(t)$, $R_{ijk}(t)$ and $m_k(t)$ to maximize the present value of the utility of the consumers. We will see that the socially optimal number of varieties $m_k(t)$ is not constant over time. Therefore, to allow a comparison between the two cases we start by determining the socially optimal $x_{ik}(t)$ and $R_{ijk}(t)$ for a given m_k and then we discuss the $m_k(t)$ behaviour.

The benevolent planner maximizes the utility function (1) subject to the production functions (4) for differentiated products, the production function for the homogeneous product $l_0(t) = c_0 x_0(t)$, the full employment condition $l_0(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} l_{ik}(t) + \sum_{k=1}^{N} \sum_{i=1}^{m_k} \sum_{j=1}^{m_k} R_{ijk}(t) = 1$ and the differential equations (11) determining the $b_{ijk}(t)$ of the currently produced versions of the varieties.

Lemma 4 *For a given number of firms m_k, the chosen quantities of the socially optimal solution $\forall i, k$ are given in each period by*

$$x_{ik}^{SO}(t) = \frac{a_k - \frac{c_k}{c_0}}{\sum_{j=1}^{m_k} b_{ijk}(t)\, dj} = \frac{a_k - \frac{c_k}{c_0}}{\Gamma_k(t)} > x_{ik}^{D}(t)$$

Proof. From the first order conditions of the benevolent planner's maximization problem. ■

Here, we can see a first distortion: the socially optimal production is larger than the decentralized output. This is because in the decentralized outcome firms choose quantities to equate marginal cost and marginal revenue, while the socially optimal production equates implicit price[16] and marginal cost. The socially optimal level of production cannot be implemented in the decentralized economy because it would imply a loss for the firms due to fixed costs.

[16] That is the price that would prevail in a decentralized framework where firms produce the socially optimal quantities.

Proposition 5 *There is a second distortion in the competitive equilibrium, which affects the other firms: when taking the decision of investing in R&D the firm does not internalize the benefits of reducing substitutability with the other varieties on the profits of the other producers.*

There are two sides of this fact: on one hand, the firm does not internalize the positive effect of the R&D of the other firms on the substitutability coefficients of the currently chosen version of its variety. On the other hand, it does not internalize the effect of its own research on the level of substitutability of the currently chosen versions of the other varieties. The two sides have opposite effects[17].

Moreover, the above mentioned distortion in quantities has negative effects on the optimal R&D because it reduces the production taking advantage of research and therefore its returns.

The overall effect of the externalities is such that the decentralized level of R&D is lower than the socially optimal one.

Proof. The solution of the socially optimal maximization problem implies that the R&D path must satisfy

$$\frac{\dot{R}_{ijk}(t)}{R_{ijk}(t)} = \frac{1}{\eta_{\phi' R}(R_{ijk}(t))}\left[r - \gamma c_0 \left(a_k - \frac{c_k}{c_0}\right)^2 \frac{b_{ijk}(t)\,\phi'(R_{ijk}(t))}{\Gamma_k^2(t)}\right]$$

If we consider the difference in the slope of the paths for a given $R_{ijk}(t)$, We see that

$$\frac{\partial R_{ijk}}{\partial b_{ijk}}|_{SO} - \frac{\partial R_{ijk}}{\partial b_{ijk}}|_{D} = -\frac{R_{ijk}\gamma c_0 \left(a_k - \frac{c_k}{c_0}\right)^2 b_{0k} b_{ijk} \phi'(R_{ijk})}{\eta_{\phi' R}\dot{b}_{ijk}\Gamma_k^2 (b_{0k} + \Gamma_k)^2}(b_{0k} + 2\Gamma_k) > 0$$

The first factor is always positive because in the model $\dot{b}_{ijk}(t) < 0$. The second factor is always positive too. Therefore, we can conclude that the R&D paths in the socially optimal solution are always steeper than in the decentralized case. Hence, if we consider a point on the decentralized saddlepath,

[17] We can show that when we consider the socially optimal level of production, the overall effect of the two sides of the externality is null. Instead, if we consider another level of production (for example, if we implement the decentralized solution quantities or more in general if there are sources of distortions), this is not true anymore. We can show that with a smaller output than the optimal one the overall distortion due to this externality is negative.

the associated R&D path in the social optimum case crosses the horizontal axis, which is not the optimal path, as shown in subsection 3.2. The R&D level on all the points of the socially optimal saddlepath must therefore be larger than in the decentralized case. We can graphically see the comparison between the two cases in figure 3. ■

Let us consider now what happens to the socially optimal number of varieties $m_k(t)$ if it is allowed to change over time. In this case, the formal analysis becomes quite complicated, because the optimal number of varieties is not constant and the currently produced versions of different varieties have now different substitution indexes $\Gamma_{ik}(t)$, depending on the period they entered the market. The optimal real variables are now asymmetric and we can have multiple solutions, where the produced quantities are given by the solutions of the first order conditions with respect to $x_{ik}(t)$:

$$\sum_{j=1}^{m_k(t)} b_{ijk}(t)\, x_{jk}(t) = a_k - \frac{c_k}{c_0} \quad \forall i$$

The socially optimal R&D decision is symmetric among firms because of the decreasing efficiency of the ϕ function ($R_{ijk}(t) = R_{jik}(t)$): the path depends on the chosen quantities and on the value of the $b_{ijk}(t)$ coefficients of the currently produced versions

$$\frac{\dot{R}_{ijk}(t)}{R_{ijk}(t)} = \frac{1}{\eta_{\phi' R}(R_{ijk}(t))}\left[r - c_0\gamma b_{ijk}(t)\, x_{ik}^{*SO}(t)\, x_{jk}^{*SO}(t)\, \phi'(R_{ijk}(t))\right] \quad (14)$$

Proposition 6 *Under the hypothesis of constant elasticity of the ϕ' function, the first order condition with respect to the number of varieties implies the following condition, which equates the fixed cost of one more variety with the future gain in terms of substitutability due to R&D:*

$$d_k = \frac{2\eta}{(1-\eta)} \sum_{j=1}^{m_k(t)} R_{mjk}(t) \quad (15)$$

where the m index is referred to the marginal variety, which is either the last produced or the last abandoned.

Proof. From the first order conditions of the benevolent planner's maximization problem. ■

We cannot have a solution where the number of varieties is decreasing: in this case, the solution would be symmetric because, given a symmetric initial situation, the first order conditions are symmetric too. Therefore, all the decisions are always the same for all the varieties. This implies that the R&D and production paths should be positive also for the varieties to be abandoned, which contradicts our assumption of decreasing number of varieties.

A solution where the number of varieties is constant is possible, but it is unlikely, because equation (15) implies that the overall R&D of the incumbent should be the same $\forall t$, which requires the product $b_{ijk}(t)x_{ik}(t)\,x_{mk}(t)$ to be constant over time.

Instead, the usually verified solution requires an increasing number of varieties. The R&D path of equation (14) for the marginal variety is concave over time, which implies that the product $b_{ijk}(t)x_{ik}(t)\,x_{mk}(t)$ must decrease. In this case, the economy asymptotically moves towards a situation where the homogeneous good is not produced any longer and all the products are differentiated. The overall number of varieties, in the simplified symmetric case where $a_k = a$, $b_{0k} = b$, $c_k = c$, $d_k = d\ \forall k$, is given by $m_k = \left\{N\left[d + \frac{c}{b_0}\left(a - \frac{c}{c_0}\right)\right]\right\}^{-1}\ \forall k$.

In fact, while the decentralized number of firms is determined by the zero profit condition, the socially optimal one depends on the comparison between the marginal utility of a new variety and the marginal utility of the old one. Because the produced quantity of the old varieties is increasing, their marginal utility is decreasing over time; therefore, the consumer is better off by introducing new varieties. Increasing the number of varieties reduces the marginal utility of an additional one (because it increases the number of b_{ijk} terms in the demand function). Hence, a situation with increasing quantities and number of varieties is compatible with the first order conditions of the social optimum problem.

When we compare the number of firms in the decentralized solution (given by equation (16) in the subsection 4.1) and in the social optimum, we see that the former depends on parameters that are not relevant in the steady state behaviour of the latter, like the intertemporal discount parameter r.

Examining equation (15), we can easily see that if r is high enough, the social optimum steady state number of varieties always exceeds the number of varieties in the decentralized case (because R&D will be near 0, which implies a large socially optimal number of varieties). The comparison in the short run depends on the size of the R&D distortions. With small distortions, the socially optimal number is always larger, while this is not always true when distortions are more relevant.

At the opposite, if r is low enough the optimal R&D level is high and, therefore, there are more varieties in the decentralized solution than in the social optimum $\forall t$.

4 Extensions of the model

4.1 Endogenization of the firm number m_k

The previous analysis considered an exogenous number of varieties m_k. We try now to endogenize this variable. The results depend on the market entry conditions of the new firms. In particular, they depend on the initial level of differentiation of the variety produced by the newcomer.

Let us suppose that a new firm can enter the market with a perfectly substitutable version of the product. Therefore, if we call $b_{ijk}^{inc}(t)$ the value of the substitutability parameter reached by the already established firms, the newcomer i will be initially characterized by a value of

$$b_{ijk}^{ent}(t) = b_{0k} - \frac{b_{0k} - b_{ijk}^{inc}(t)}{2} = \frac{b_{0k} + b_{ijk}^{inc}(t)}{2} \; \forall j$$

The new firm takes advantage of the R&D previously performed by the other firms to differentiate their varieties. Therefore, the initial value of b_{ijk} of the entrant will be lower than b_{0k}. Because the expenditure of all firms in the past was symmetric and each parameter b_{ijk} depends on the R&D of two firms, the substitutability parameters of the entrant are symmetric and take advantage of half the improvement achieved by the other firms.

Proposition 7 *When we endogenize the number of firms $m_k(t)$ in the previously described framework, we find that at time 0 new firms enter the market until the discounted value of expected profits is zero:*

$$m_k(0)\,|\Pi_{ik}(0) = \int_0^\infty [p_{ik}^*(t)\, x_{ik}^*(t) - w l_{ik}^*(t) - w \sum_{j=1}^{m_k(0)} R_{ijk}^*(t)\,] e^{-rt} dt = 0 \quad (16)$$

where the starred variables are the optimal values given by the previous analysis as functions of m_k. In the following periods, no firm has incentives to enter or exit the market.

Proof. Let us consider what happens if a firm tries to enter the market in a period $t > 0$. We split the analysis in two steps. In the first one, we examine what happens to the static operative profits in a given period, while in the second one we analyse the consequences of the changes in the R&D path and the dynamic effects.

We saw that the equilibrium prices and quantities negatively depend on the parameter structure of the currently produced versions $b_{ijk}(t)$. Given the R&D path of the old firms, we saw in the previous subsection that in each period after 0 the equilibrium quantities of the old firms will be larger than those produced at time 0. Therefore, inspection of equations (3) and (6) shows that the newcomer price and quantity will be lower, because the old firms are able to exploit their previous research to get an advantage in production. Increasing returns to scale imply that the average cost of the product will be higher and the operative profits of the entrant immediately after the entry in the market will be lower than those achieved by the other firms at time 0.

Let us examine now the research decisions. The R&D path (equation (13)) in the case of asymmetric solution is a complicated function of the $b_{ijk}(t)$ structure. The path of the entrant can be either steeper or flatter than that one of the average firm when we do not have entrants and the comparison of the investments path in R&D is uncertain. In case of stronger investments in R&D, the temporal profile of profits will be steeper, but the initial investment (and, therefore, the initial reduction of profits) will be larger than for the incumbent firms, while the opposite happens in the case of weaker investments. The optimal choice of R&D of the entrant implies a dynamics of $b_{ijk}^{ent}(t)$ such that $b_{ijk}^{ent}(t) > b_{ijk}^{inc}(t)$ $\forall t$. This fact implies that the profits of the entrant will always be less than those experienced by an incumbent firm in a market without entrants at the time it achieved the same improvements in the b_{ijk} coefficients through its own R&D. Therefore[18], the

[18] We do not take into account that the R&D schedule of the entrant at the time of entry $t > 0$ is different from that one of the incumbent (in a market without entrants) at time 0. This means that the profits of the entrant and of the incumbent for a given b_{ijk} will be discounted for a different number of periods. Simulations showed that taking into account this fact does not change our conclusions.

overall profits of the entrant will always be dominated by those expected by an incumbent firm at time 0, which are zero because of the free entry hypothesis and the consequent zero profit condition.

Let us consider now the possibility that a firm exits. If we examine the path of profits over time, we see that

$$\dot{\Pi}_{ik}(t) = -\frac{\left(a_k - \frac{c_k}{c_0}\right)^2 b_{0k}}{\left(b_{0k} + \Gamma_k(t)\right)^3} \sum_{j=1}^{m_k} \dot{b}_{ijk}(t) - \frac{1}{c_0} \sum_{j=1}^{m_k} \dot{R}_{ijk}(t) > 0$$

because both $\dot{b}_{ijk}(t)$ and $\dot{R}_{ijk}(t)$ are negative. Consequently, the discounted value of the expected profits is increasing over time and no firm finds optimal to leave after $t = 0$. ■

A consequence of the fact that profits are increasing over time and the overall expected profits are zero is that firms bear negative profits at the beginning and positive ones in the steady state. Therefore, a necessary condition for an equilibrium is $\Pi_{ik}(\infty) > 0$, which implies $\frac{\left(a_k - \frac{c_k}{c_0}\right)^2 c_0}{4b_{0k}} - d_k > 0$.

4.2 Endogenous choice of the produced versions

We examine here the conditions under which the optimal behaviour of the firm is the production of the newest version and what happens when these conditions are not satisfied. We find that the case where the only produced version is the one with the lowest b_{ijk} coefficients, examined in the main model, is the right one for most values of the parameters. Moreover, we find that the model can be easily extended to tackle with the other case, where the optimal behaviour of a subset of firms is the production of both the most differentiated and the perfectly substitutable versions of their variety.

Let us suppose that we are in the short run equilibrium described in the main text and one firm (which we suppose is producing variety i) deviates producing both the newest version of its variety and an older version. We can restrict our proof to this case: if the introduction of a second version is not optimal, production of more than two versions will be suboptimal a fortiori. This is because increasing returns to scale imply that differential profits from one additional version are increasing in the produced output of that version and, therefore, decreasing in the number of produced versions[19]. We show that this deviation is never profitable, but in one case, where our main model can be easily extended. Because the choice of the produced versions is a pure choice of production and does not require intertemporal elements, we omit the time dimension. Our reasoning can be repeated in each period t.

Given the newest version of a variety, we index all the previously developed versions of the variety using a variable h, which measures the relative distance between the average level of substitutability of the newest and of an older version, calculated at the time of development of the newest version:

$$h = \frac{\frac{1}{m_k-1}\sum_{j\neq i} b_{ijk}^{old} - \frac{1}{m_k-1}\sum_{j\neq i} b_{ijk}^{new}}{b_{0k} - \frac{1}{m_k-1}\sum_{j\neq i} b_{ijk}^{new}} = \frac{b_{ijk}^{old} - b_{ijk}^{new}}{b_0 - b_{ijk}^{new}}$$

where the last equality holds because in equilibrium we have symmetry in the b_{ijk} coefficients. The index h is equal to 1 if we consider the perfectly substitutable version of the good ($b_{ijk}^{old} = b_{0k}$), while it tends to 0 if we approach the newest version of the variety.

In the main text, we defined the substitutability level between two varieties, but we did not consider that one between versions of the same variety. We will examine now a reasonable assumption to define substitutability of an old version of a variety with the other varieties and with other versions of the same variety. Let us consider the two extreme cases of $h = 1$ (perfectly substitutable version of the product) and $h = 0$ (a second copy of the newest version of the variety).

In the former case, the substitutability level of the perfectly substitutable version does not benefit at all of the direct past efforts in R&D of the firm i, but only of the effort of the other firms to differentiate their variety from the others. In equilibrium, R&D is symmetric for all firms. Therefore, when considering substitutability with another variety, the perfectly substitutable version of variety i benefits of half the current maximum progress on differentiation (that is all the progress attributed to investments on the other varieties). The same is true when we consider substitutability with the newest version of the variety of the same firm. We will call $b_{iik}^{o,n}(h)$ the substitutability parameter between an older (with index h) and the newest version of the same variety i and $b_{ijk}^{o,n}(h)$ the substitutability parameter between an older

[19] In the case the production of two versions is preferred to the production of the newest version only, we can show using the same methodology of this paragraph that the introduction of a third version is never profitable.

version of the variety i and the newest version of the variety j. Therefore, we have that

$$b_{iik}^{o,n}(1) = b_{ijk}^{o,n}(1) = b_{0k} - \frac{b_{0k} - b_{ijk}^{new}}{2} = \frac{b_{0k} + b_{ijk}^{new}}{2} \forall j$$

On the other hand, if we produce a second copy of the newest version of the variety i, it is perfectly substitutable with the other copy of the variety i and has the lowest available level of substitutability with the other varieties. Therefore, we have that

$$b_{iik}^{o,n}(0) = b_{0k} \text{ and } b_{ijk}^{o,n}(0) = b_{ijk}^{new}$$

The level of substitutability between an older version of variety i and the newest version of another variety linearly depends on h by definition of this parameter. If we suppose that this is also true for the substitutability level between different versions of the same variety, we obtain these two expressions of $b_{iik}^{o,n}(h)$ and $b_{ijk}^{o,n}(h)$:

$$b_{iik}^{o,n}(h) = b_{0k} + h\left(\frac{b_{0k} + b_{ijk}^{new}}{2} - b_{0k}\right) = b_{0k} - h\frac{b_{0k} - b_{ijk}^{new}}{2}$$

$$b_{ijk}^{o,n}(h) = b_{ijk}^{new} + h\left(\frac{b_{0k} + b_{ijk}^{new}}{2} - b_{ijk}^{new}\right) = b_{ijk} + h\frac{b_{0k} - b_{ijk}^{new}}{2}$$

Firm i now maximizes the sum of the operative profits due to the newest and to the older versions of its variety:

$$\pi_{ik}^{(2)}(h) = p_{ik}^{new} x_{ik}^{new} + p_{ik}^{old} x_{ik}^{old} - w\left(l_{ik}^{new} + l_{ik}^{old}\right)$$

where the indexes *new* and *old* define the variables referred respectively to the newest and the older versions of the variety; p_{ik}^{new} and p_{ik}^{old} are the prices implied by the demand function (3), remembering that we now have $m_k + 1$ different versions of the good. The operative profits function of the other firms follows equation (5) as before.

Proposition 8 *In the equilibrium described in section 3, let us assume that substitutability among different versions of the same variety is linear in h.*

If $m_k \geqslant 3$, a deviation from the equilibrium where the firm produces two or more versions of its variety is never profitable.

If $m_k < 3$ and $\frac{d_k}{c_0}$ is larger than a threshold, a deviation is never profitable too.

If $m_k < 3$ and $\frac{d_k}{c_0}$ is smaller than a threshold, a deviation can be profitable.

Proof. In the period of deviation, maximization of profits implies the following equilibrium quantities (we call x_{jk}^{new} the quantities produced by the other firms):

$$x_{ik}^{new}(h) = \frac{\chi\left\{2(m_k+1)(2b_{0k}-b_{ijk}) - 3h(m_k-1)(b_{0k}-b_{ijk})\right\}}{2}$$

$$x_{ik}^{old}(h) = \chi(2b_{0k}-b_{ijk})(3-m_k) \tag{17}$$

$$x_{jk}^{new}\Big|_{j\neq i}(h) = \chi\left[4(2b_{0k}-b_{ijk}) - 3h(b_{0k}-b_{ijk})\right]$$

where $\chi = \frac{a_k - \frac{c_k}{c_0}}{\left\{b_{ijk}b_{0k}[8(m_k-2)-3h(m_k-3)] - b_{0k}^2[h(m_k+3)-16] + b_{ijk}^2[h(4m_k-6)-4(m_k-1)]\right\}}$.

We are interested in equilibriums where $x_{ik}^{old}(h) > 0$, otherwise the model collapses to the main text structure. This implies that $x_{ik}^{new}(h)$ must be greater than zero too, because the residual demand when only the older version of the variety is produced has a lower intercept and the same slope as in the situation where only the newest version is produced. $x_{ik}^{new}(h) > 0$ implies that the denominator is always positive. Moreover, $a_k > \frac{c_k}{c_0}$ by assumption and $b_{0k} > b_{ijk}$ by construction.

Therefore, equation (17) implies that the older version of variety i is only produced if the number of firms m_k is less than three. Otherwise, the optimal production of $x_{ik}^{old}(h)$ is 0.

With $m_k \geqslant 3$, the competition is tight. The negative effects of the introduction of another version on demand are so strong that a positive production of $x_{ik}^{old}(h)$ yields negative effects on profits, whatever are the fixed costs.

We continue our analysis examining the effects on profits in the case we have less than three firms in the market of good k.

Profits depend on the chosen version h of the variety. There is a trade-off between a high and a low h. If h is high, the version is more substitutable with the other varieties, but less substitutable with the newest version of the same variety. The opposite is true when h is low. The choice of the firm depends on the relative weight of these two effects.

Let us consider the comparison between profits when firm i produces the latest version only of the good ($\pi_{ik}^{(1)}$) and when it produces both the latest and an older version:

$$\Delta\pi_{ik}(h) = \pi_{ik}^{(1)} - \pi_{ik}^{(2)}(h)$$

If we maximize this function with respect to h, we obtain the most positive variation of profits achievable by exploiting the trade-off in the production of two versions of the same variety. The optimal value of h is always $h^* = 1$. The deviating firm maximizes its variation of profits from deviation producing the perfectly substitutable version of the good together with the newest version of its variety. The deviation is the best behaviour if $\Delta\pi_{ik}(h^*) < 0$. This inequality implies that producing the newest version only of the variety is the optimal behaviour if $\frac{d_k}{c_0}$ is larger than a threshold defined by the parameters.

With a small $\frac{d_k}{c_0}$, given a situation where all the other firms produce the newest version of the good, firm i finds optimal a deviation where it produces a positive quantity of both the newest version and the perfectly substitutable version of its variety. ■

We can easily extend our main model to take into account a situation where a firm produces both the newest version and the perfectly substitutable version of its variety. In the new situation, a subset of firms chooses to produce both the most differentiated and the perfectly substitutable versions of their variety, while the remaining ones produce the differentiated version only. The share of firms producing both versions of their variety is pinned down by the equality of profits of the two types of firms.

While the time path of production of the most differentiated versions of the differentiated good is increasing as in the main model for both types of firms, the firms producing the perfectly substitutable versions of the differentiated good continuously reduce the perfectly substitutable output.

Because the other results about the R&D choices of the firms, our main aim, do not qualitatively change, we do not explicitly derive the new version of the model, which is quite straightforward, given the previous analysis.

5 Conclusions

In this paper, we examined the relationship between R&D and the evolution of market structure over time.

We developed a mechanism of interaction between R&D and market structure based on the idea that firms can invest in research to increase the level of horizontal differentiation between their product and the others. Producers try to modify the characteristics of their output to better satisfy needs of consumers that are not fully fulfilled by the other varieties. Doing so, they are able to increase the level of specialization of their product and, therefore, to reduce substitutability with the other varieties. We develop a dynamic framework, which allows us to see how the interaction between market structure and incentives to research changes over time.

Moreover, we compare the decentralized equilibrium with the socially optimal solution and we find that firms subinvest in R&D.

The developed analysis is a good starting point for further extensions: introducing uncertainty in the model would allow a greater realism, but afterwards the simplifying hypothesis of symmetry cannot be maintained and the complexity of the model substantially increases. The inclusion of capital as a production factor could be interesting, because adjustment costs when converting from one variety to another can influence development costs and profits and therefore incentives to research.

6 Figures

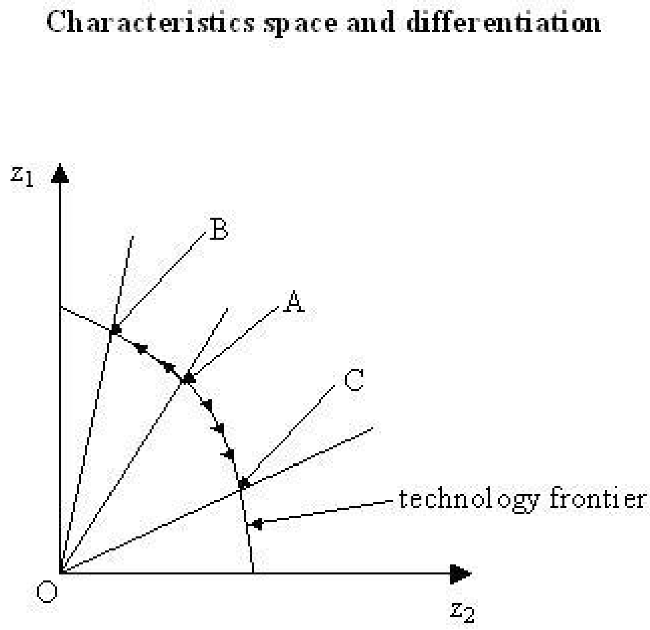

Figure 1

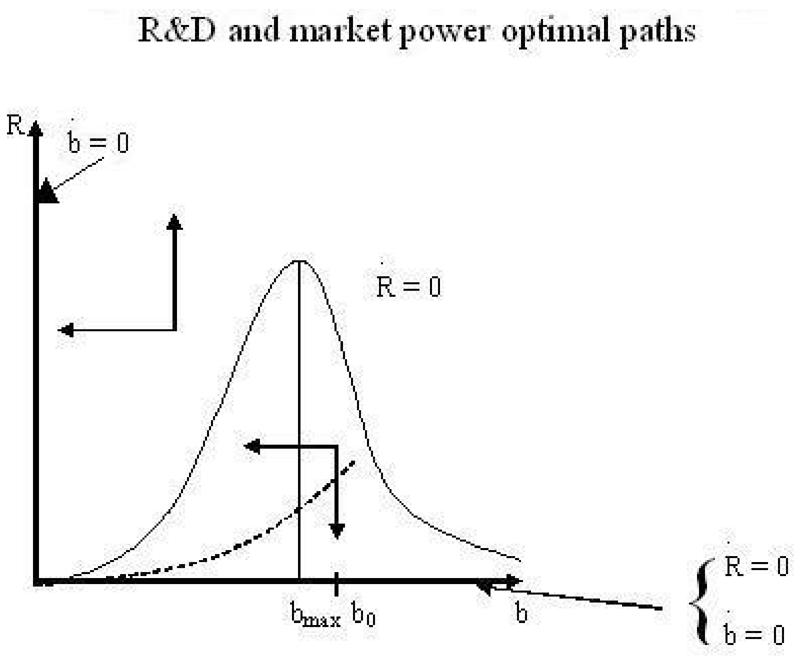
R&D and market power optimal paths
R
$\dot{b} = 0$
$\dot{R} = 0$
b_{max} b_0
b
$\dot{R} = 0$
$\dot{b} = 0$

Figure 2

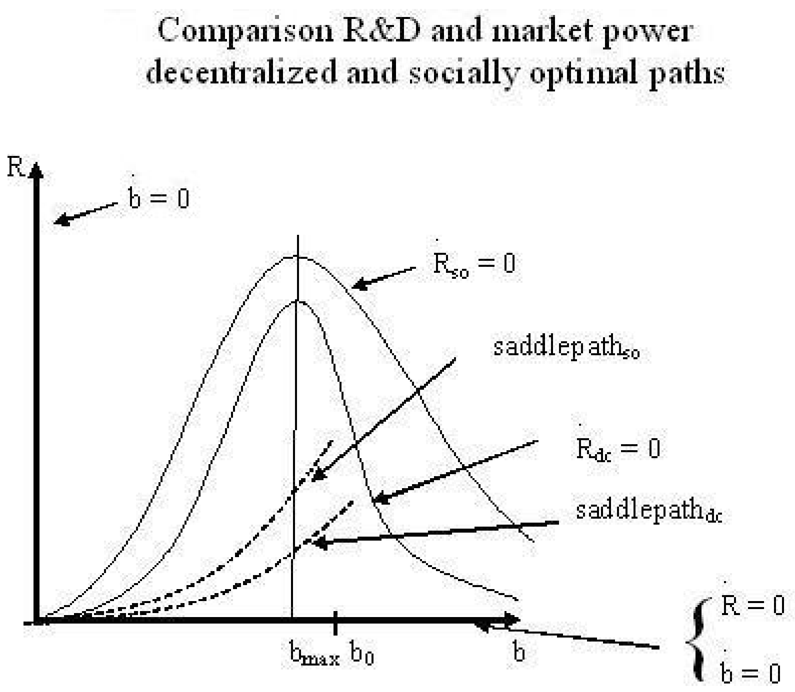
Comparison R&D and market power
decentralized and socially optimal paths
R
$\dot{b} = 0$
$\dot{R}_{so} = 0$
saddlepath$_{so}$
$\dot{R}_{dc} = 0$
saddlepath$_{dc}$
$\dot{R} = 0$
$\dot{b} = 0$
b_{max} b_0
b

Figure 3

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