Detecting long memory co-movements in macroeconomic time series

by Gianluca Moretti
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DETECTING LONG MEMORY CO-MOVEMENTS IN MACROECONOMIC TIME SERIES

by Gianluca Moretti*

Abstract

Cointegration analysis tests for the existence of a significant long-run equilibrium among some economic variables. Standard econometric procedures to test for cointegration have proven unreliable when the long-run relation among the variables is characterized by non-linearities and persistent fluctuations around the equilibrium. As a consequence, many intuitive economic relations are empirically rejected. In this paper we propose a simple approach to account for non-linearities in the cointegrating equilibrium and possible long memory fluctuations from such equilibrium. We show that our correction allows us to test robustly for the presence of cointegration both under the null and alternative hypotheses. We apply our procedure to the Johansen-Juselius PPP-UIP database, and unlike the standard case, we do not fail to reject the null of no cointegration.

JEL classification: C22, C51.
Keywords: cointegration analysis, long memory.

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1 Introduction

Since the seminal papers of Granger (1981) and Engle and Granger (1987) the concepts of integration and cointegration have developed in many areas of both econometrics and applied macroeconomics. By a well-known definition two time-series $x_{1,t}$ and $x_{2,t}$ are said to be cointegrated of order $CI(\delta, b)$ if they are individually integrated of order $I(\delta)$ and there exists a linear combination $\varepsilon_t = x_{1,t} - \beta x_{2,t}$ that is integrated of order $I(\delta - b)$.

In recent years, many approaches have been proposed to test for cointegration. In particular, they have been designed for the case when $\delta = 1$ and $b = 1$. Under this assumption, $x_{1,t}$ and $x_{2,t}$ are $I(1)$ variables and they are cointegrated if there exists a linear combination $\varepsilon_t$ that is $I(0)$. This case is very important since it allows us to estimate long-run steady states as linear combinations of non-stationary variables. Furthermore, fluctuations around this steady state equilibrium can be represented using standard ARMA models.

Recently, this notion of cointegration has been criticized by a number of researchers who have asserted that the distinction between $I(0)$ and $I(1)$ is rather arbitrary. They have proposed instead to allow $\varepsilon_t$ to be integrated of order $I(d)$ with $0 \leq d < 1$ (i.e. fractionally integrated) or more generally to belong to the class of long memory processes.

Long memory cointegration implies that although there is an equilibrium between economic variables spanning the long run, these variables can be away from such equilibrium for a very long length of time. Standard cointegration techniques cannot be applied in this context since they cannot distinguish between long memory co-movements and spurious relations. For instance, the two most popular cointegration approaches, the Engle-Granger (E-G) two-step procedure and the Johansen’s full-Information maximum likelihood (FIML), cannot deal with the hypothesis of long memory cointegration. In fact, as recently shown by Diebold and Rudebusch (1991), Hassler and Wolters (1994) and Gonzalo and Lee (1998), they are not robust to dynamic misspecification and are

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1 I would like to thank my supervisor, Gabriel Talmain, Karim Abadir, Huw Dixon and Peter Sinclair for their useful comments. The opinions expressed in this paper do not necessarily reflect those of the Bank of Italy. Any errors and omissions remain my responsibility. Address: via Nazionale 91, 00184 Rome - Italy. E-mail: gianluca.moretti@bancaditalia.it
2 See Watson (1995) for a survey on these approaches.
4 See Johansen (1988).
characterized by low power if the dynamics of the long-run regression residuals \( \varepsilon_t \) depart from the \( I(1) \) assumption. In other words, testing for individual unit root is not enough to reject cointegration if the data generating process has long memory or dynamic behaviors different from those of a unit root process.

This paper presents a methodology to test for the presence of cointegration when two variables are non-stationary and there exists a linear combination that behaves as a long memory process. This approach is based on a modification of the Engle and Granger procedure in order to account for long memory and possible omitted non-linearities in the cointegrating relation.

The contribution of this paper is twofold. First, unlike standard cointegration techniques, our approach is able to detect the presence of long memory co-movements. In this respect, the test we propose is well sized under the null hypothesis (of spurious relation) and characterized by an empirical power close to nominal under the alternative (of long memory relation). Furthermore, unlike the standard E-G approach, our approach is by construction not sensitive to the choice of the number of lags in the ADF regression. Then, it also reduces the small sample bias in the estimation of the long-run relation by more than 37% compared with ordinary least square (OLS). We apply our procedure to the Johansen and Juselius (1992) data base for the UK purchasing power parity (PPP) and uncovered interest rate parity (UIP) and show that the null of no cointegration is rejected at 95% contrary to what was previously shown with standard single equation techniques.\(^5\)

Evidence of long memory in the co-movements of many macroeconomic variables has already been found by Cheung and Lai (1993), Diebold et. al. (1991) and Abadir and Talmain (2005). On one side, Cheung and Lai suggest that deviations from the PPP equilibrium could follow a mean-reverting long memory process. On the other, Abadir and Talmain show that the UIP database is characterized by a high degree of persistence and non-linearities that, if not properly accounted for, can give rise to counterintuitive results.

The difference between our approach and the standard cointegration techniques can be understood by analyzing the assumptions that characterize the two approaches. Testing with standard cointegration techniques imposes very strict conditions on the long-run relation among the variables: first, it assumes that the relation is strictly linear; then, the

\(^5\)See, for instance, Harris (1995)
variables must adjust towards this equilibrium at a relatively fast rate ($I(0)$ hypothesis). Therefore, it should not be surprising that such “cointegration” is rejected even when the long-run relation between variables seems economically plausible (as in the PPP-UIP theorem). What is really important when testing for cointegration is not stationarity but mean reversion towards the long-run equilibrium. Strict stationarity is a sufficient but not a necessary condition to have mean reversion. Conversely, the approach we propose is able to test for the existence of a long-run relation, while allowing for possible non-linearities and persistent deviations from its long-run that are not forced to be strictly stationary.

This paper is organized as follows. In the next section, we recall the Engle and Granger approach and describe the moving block and the stationary bootstrap. In section 3 we test for cointegration for the PPP-UIP database using the standard E-G approach. In section 4 we describe our modified testing procedure. Then, in section 5 we apply our procedure to the PPP-UIP to test for the presence of a long memory equilibrium relation. Lastly, in section 6 we run some simulations to calculate the empirical size and power of the ADF test in our modified procedure and we also evaluate the decrease of the small sample bias in the estimation of the cointegrating relation compared with standard E-G approach.

2 Bootstrapping the ADF test

In this section we briefly recall the E-G approach and describe how to calculate stationary bootstrap (SB) and moving block bootstrap (MBB) confidence intervals, under the null hypothesis of no cointegration.

The E-G approach is a very intuitive two-step procedure. Given two time series $\{x_{1,t}\}_{t=1}^T$ and $\{x_{2,t}\}_{t=1}^T$, in the first step we estimate by ordinary least square the relation

$$x_{1,t} = \beta x_{2,t} + \varepsilon_t$$

also called cointegrating vector, while in the second we test whether the regression residuals $\varepsilon_t$ are strictly stationary. To this end, we run the regression

$$\Delta \varepsilon_t = \rho \varepsilon_{t-1} + \rho_1 \Delta \varepsilon_{t-1} + \cdots + \rho_k \Delta \varepsilon_{t-k} + u_t$$

and construct the $t$-value statistic $t_\rho$ for the estimated parameter $\hat{\rho}$, which is called the

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$^6$In the rest of the paper we use the hat $\hat{}$ to indicate to an estimated variable.
ADF statistic of order $k$ or $ADF(k)$. If we reject the hypothesis that $\hat{\rho} = 0$, then $\varepsilon_t$ has an ARMA representation and the variables $x_{1,t}$ and $x_{2,t}$ are cointegrated. Otherwise, if we fail to reject the hypothesis $\hat{\rho} = 0$ then $\varepsilon_t$ is non-stationary and equation 1 is a spurious relation.

It is well known that the ADF statistic $t_{\rho}$ converges, under the null of no-cointegration, to a non-standard distribution. The critical values for this distribution have been derived by Said and Dickey (1984) using simulation. These critical values, as well as those of other cointegration approaches, are justified on an asymptotic ground. It is well known that in a context of cointegration regression models, asymptotic critical values are not very reliable unless the sample size is very large.\(^7\) This implies that testing for cointegration in small samples can give rise to substantial estimation bias as well as size distortion in the associated tests of significance. A solution to this problem has been proposed by Li and Maddala (1997), who suggest using bootstrap methods to reduce, in the E-G context, both estimation bias and size distortions. In particular, they show the superiority of bootstrap-based critical values over asymptotic critical values. On the basis of their results we also use bootstrapped rather than asymptotic critical values in all the ADF tests shown below. For this purpose, in the next few paragraphs we briefly describe the moving block and stationary bootstrap that will be implemented below.

A complete exposition of the statistical properties of the bootstrap can be found in Hall (1992) and Efron and Tibshirani (1993). The idea behind the bootstrap is the following. Let us consider a sample of i.i.d. variables \(\{x_1, \ldots, x_n\}\) with underlying distribution \(F(\theta)\) with the population parameter \(\theta\) on which we want to make inference. Let us define \(\hat{\theta}\) the estimated parameter from \(\{x_1, \ldots, x_n\}\). Then, a bootstrap distribution of \(\hat{\theta}\) can be derived by re-sampling with replacement from \(\{x_1, \ldots, x_n\}\) and by calculating from each re-sample the parameter \(\tilde{\theta}\). This generates a distribution \(\tilde{F}\) of parameters \(\tilde{\theta}\) that provides, under general conditions, an approximation of the true distribution \(F\).

In time series analysis, data are generally not i.i.d., therefore different approaches, such as the moving block bootstrap (MBB) and the stationary bootstrap (SB), have been proposed to capture the dependence structure of the data. The moving block bootstrap was introduced by Carlstein (1986) and further developed by Künsch (1989). In particular, given a time series sample \(\{x_1, \ldots, x_n\}\), Künsch proposes to construct \(n-l+1\) blocks of data of length \(l\), \(B_j = \{x_j, x_{j+1}, \ldots, x_{j+l-1}\}, j = 1, \ldots, n-l+1\) and to resample with replacement

\(^7\)See Li and Maddala (1997).
from those blocks. A different type of block bootstrap is the stationary bootstrap, where the block length \( l \) is sampled from the geometric distribution \( P(l = m) = (1 - p)^{m-1} p \) with \( m = 1, 2, \ldots \) and \( p \in (0, 1) \), while the starting date \( j \) of the first observation of the block is chosen according to a uniform distribution on \([1, n]\). If \( j + l - 1 \) exceeds the index \( n \) of the last observation \( x_n \), then the block is constructed as \( B_j = \{x_j, \ldots, x_n, x_{1-n+j-1}\} \).

This kind of bootstrap was introduced by Politis and Romano (1994) after discovering that the time series generated by the MBB bootstrap may not be stationary even if the original series \( \{x_1, \ldots, x_n\} \) is stationary.

We proceed now with a step-by-step description of the algorithm to calculate bootstrap critical values for the ADF \( t_\rho \)-statistic under the null hypothesis of no cointegration:

1) Estimate the cointegrating vector, \( y_t = \alpha + \beta x_t + \varepsilon_t \), by OLS and get the regression residual \( \hat{\varepsilon}_t = y_t - \hat{\alpha} - \hat{\beta} x_t \)

2) Run the ADF(\( k \)) regression

\[
\Delta \hat{\varepsilon}_t = \rho \Delta \hat{\varepsilon}_{t-1} + \ldots + \rho_k \Delta \hat{\varepsilon}_{t-k} + \epsilon_t
\]  

and calculate the ADF-statistic for the estimated \( \hat{\rho} \), defined as \( t_\rho = \hat{\rho}/SE(\hat{\rho}) \), where \( SE(\hat{\rho}) \) is the standard deviation of \( \hat{\rho} \).

3) Estimate the ADF(\( k \)) regression under the null of no cointegration (i.e. imposing \( \rho = 0 \) ),

\[
\Delta \hat{\varepsilon}_t = \rho_1^{0} \Delta \hat{\varepsilon}_{t-1} + \ldots + \rho_k^{0} \Delta \hat{\varepsilon}_{t-k} + \epsilon_t^{0}
\]  

and calculate regression residuals \( \hat{\epsilon}_t^{0} \).

4) Use the residuals \( \hat{\epsilon}_t^{0} \) to derive a bootstrap distribution for the \( t_\rho \)-statistic under the null hypothesis of no cointegration in the following way. For the stationary bootstrap, choose a value of \( p \) and form \( N \) blocks \( B_i = \{\hat{\epsilon}_{i,j}, \hat{\epsilon}_{i,j-1}, \ldots, \hat{\epsilon}_{i,j-l+1}\}, i = 1, \ldots, N \) where for each \( B_i, l \) is sampled from a geometric distribution and \( t \) from a uniform distribution as described above. For the moving block bootstrap choose a block length \( l \) and construct \( n-l+1 \) blocks \( B_j = \{\hat{\epsilon}_{j,j}, \hat{\epsilon}_{j,j+1}, \ldots, \hat{\epsilon}_{j,j+l-1}\}, j = 1, \ldots, n-l+1 \) and resample with replacement \( N \) times from these blocks.

5) For each resampled block\(^8\) \( B_i = \{\hat{\epsilon}_{i,j}, \hat{\epsilon}_{i,j+1}, \ldots, \hat{\epsilon}_{i,j+l-1}\} \), use the bootstrapped residuals \( \{\hat{\epsilon}_{i,j}\} \) and equation 4 to construct a time-series of residuals \( \{\hat{\epsilon}_{i,t}\}_{t=1} \) under the null of no

\(^8\)Either moving block or stationary bootstrap re-sample.
cointegration.\textsuperscript{9}

6) Calculate for each bootstrapped sample \( \{ \hat{\epsilon}_{i,t} \}_{t=1}^{T} \), \( i = 1, \ldots, N \) the ADF statistic \( \tilde{t}_{i} \) and store it in order to generate an empirical distribution of \( \tilde{t}_{i} \).

7) Finally, define \( \tilde{t}_{L} \) and \( \tilde{t}_{H} \) respectively as the 2.5% lower and upper quantile of the distribution of the \( \tilde{t}_{i} \) and reject the null if \( t_{i} > \tilde{t}_{H} \) or \( t_{i} < \tilde{t}_{L} \).

This is not the only scheme that can be implemented to evaluate bootstrap critical values for the ADF test. Different schemes can be found in Li and Maddala (1997). However, according to their results, the one just described is the most reliable under both the null and the alternative hypothesis.

3 The PPP-UIP cointegration analysis

We reconsider the PPP-UIP data discussed in Johansen and Juselius (1992) and test for cointegration using the E-G approach. Although a more up-to-date database could have been used, the Johansen and Juselius database is a standard database when comparing different cointegration approaches.\textsuperscript{10}

The database\textsuperscript{11} is composed of quarterly, seasonally adjusted, time series from 1972-1 to 1987-2 for the UK wholesale price index \((p_{t}^{uk})\), the UK trade weighted foreign wholesale price index \((p_{t}^{w})\), the three-month UK treasury bill rate \((i_{t}^{uk})\), the three-month Eurodollar interest rate \((i_{t}^{ed})\), and the UK effective exchange rate \((e_{t}^{uk})\). The purpose is to test whether the purchasing power parity and the uncovered interest parity that arise from economic theory hold empirically.

According to this theory, in the long-run internationally produced goods are perfect substitutes for domestic goods (PPP theorem), and the interest rates differential between two countries is equal to the expected change in the spot exchange rates (UIP theorem). In other words, we should expect price differentials between two countries to be equal to the nominal exchange rate differential, and interest rates differentials to be equal to the expected changes in the exchange rate. Following Juselius (1995), a very simple version

\textsuperscript{9}We need to assume that the first \( \hat{\epsilon}_{0}, \ldots, \hat{\epsilon}_{-k+1} \) are equal to zero and \( \hat{\epsilon}_{1} \) is equal to the first observation \( \hat{\epsilon}_{0}^{0} \) of the Bootstrap block.

\textsuperscript{10}See, for instance, Boswijk and Doornik (2005), Rahbek and Mosconi (1999) and Harris (1995).

\textsuperscript{11}A full description of this data, and the source, goes beyond the scope of this paper and it can be found in the given references.
of the purchasing power parity can be defined as

$$p_t^{uk} = e_t^{uk} + p_t^w$$  \hspace{1cm} (5)$$

and the uncovered interest rate parity as

$$i_t^{uk} = i_t^{ed} + E_t(e_{t+1}^{uk}) - e_t^{uk}$$  \hspace{1cm} (6)$$

where $E_t(\cdot)$ represent the expectation at time $t$.

Now, if the markets are efficient, it is reasonable to assume that the expected exchange rate is affected by the price index differential between the two countries. More specifically, if we assume that the relation between the expected exchange rate and the price index differential is given by

$$E_t(e_{t+1}^{uk}) = p_t^{uk} - p_t^w$$

we can create a link between the capital and the goods markets and combine the PPP and the UIP relations in the following way:

$$i_t^{uk} - i_t^{ed} = p_t^{uk} - p_t^w - e_t^{uk}$$  \hspace{1cm} (7)$$

We can estimate this long-run relation\(^{12}\) in equation 7 by running the regression

$$p_t^{uk} = \alpha_0 p_t^w + \alpha_2 e_t^{uk} + \alpha_3 i_t^{uk} + \alpha_4 i_t^{ed} + \varepsilon_t$$  \hspace{1cm} (8)$$

The OLS estimates\(^ {13}\) together with some diagnostics\(^ {14}\) are shown in Table 1.

\(^{12}\)In order to account for the possibility that the prices are $I(2)$, we also used the different specification suggested in Rahbek and Mosconi (1999)

$$p_t^{uk} - p_t^w = \alpha_0 + \alpha_1 \Delta p_t^w + \alpha_2 e_t^{uk} + \alpha_3 i_t^{uk} + \alpha_4 i_t^{ed} + \varepsilon_t$$

However, since we failed to reject the null of no cointegration we report the cointegration results only for the standard case.

\(^{13}\)Heteroskedastic and autocorrelation-consistent t-ratios are displayed in parentheses.

\(^{14}\)D-W represent the Durbin-Watson statistics, L-B is the Ljung-Box test for autocorrelated residuals, and Reset is the test for omitted non-linearities.
Estimated long-run relation
\[ p_t^{uk} = 1.44 p_t^{w} + 0.47 e_t^{uk} + 1.08 i_t^{ek} - 0.98 i_t^{ed} + \varepsilon_t \]
\[ R^2 = 0.983 \quad \sigma^2 = 0.005 \quad D - W = 0.22 \quad L - B = 86.2 \quad \text{RESET} = 20.81 \]

<table>
<thead>
<tr>
<th>ADF(k) statistic and 95% critical values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{\rho} )-statistic</td>
</tr>
<tr>
<td>---------------------------</td>
</tr>
<tr>
<td>-1.8542</td>
</tr>
<tr>
<td>-2.3458</td>
</tr>
</tbody>
</table>

Table 1: Long-run UIP-PPP regression and ADF(k) statistic: standard E-G approach.

It can be readily seen that the residuals diagnostic reveals the presence of omitted non-linearities and highly autocorrelated residuals. Furthermore, a very low D-W statistic together with a very high \( R^2 \) could be a sign that the above relation is a spurious regression.

In the lower part of Table 1, we report the ADF\(^{15}\) statistic for \( \varepsilon_t \) together with its asymptotic\(^{16}\) critical values and both the stationary and moving block bootstrap 95\% critical values, named respectively ACI, SBCI and MBBCI.

The null of no cointegration cannot be rejected at 95\% and therefore we should conclude that the relation we found is spurious. This result is not a novelty, since the empirical evidence of the PPP-UIP conjecture has been generally very poor.

Many economists\(^{17}\) have tried to justify this counter-intuitive result and the issue is still controversial.\(^{18}\) Some of the reasons for such failure are related to trade barriers, pricing to market, international trade costs\(^{19}\) (such as transport costs), product heterogeneity and indirect tax differences.\(^{20}\)

\(^{15}\)We run the ADF test up to five lags in the ADF regression. Since we fail to reject the null hypothesis in all cases, we report the results only when one lag is considered.

\(^{16}\)The asymptotic critical values are taken from Said and Dickey (1984).

\(^{17}\)A good survey on the causes of the PPP failure can be found in Obstfeld and Rogoff (2000), while a survey on the UIP failure in Lewis (1995).

\(^{18}\)Abadir and Talmain (2005) recently solved the UIP puzzle with a similar approach although they did not test explicitly for cointegration.

\(^{19}\)The presence of omitted non-linearities in equation 8 is empirically consistent with the hypothesis of trade costs in international markets (see Micheal et al. (1997) and Taylor (2001)).

\(^{20}\)Other explanations that have been put forward to justify the PPP empirical failure refer to differences in the price index weight, in the productivity growths and in the proportion of tradeable to non-tradeable goods.
Below, we give a new insight into this result by showing that it is an artifact caused by the non-linearities and the long memory of the residuals, and by the inability of standard cointegration approaches to account for such a degree of persistence. In fact, these approaches would find evidence of a long-run relation in the PPP-UIP database only if this relation were strictly linear and the variables converged towards their equilibrium values at a relatively fast rate. In economic terms, these conditions would require perfect competition in the (foreign goods and exchange rate) markets, which is rejected by empirical evidence. On the other hand, if we take into account all the market failures just mentioned, it is reasonable to expect the variables to adjust very slowly towards parities. The possibility that deviations from the UIP and PPP equilibrium could follow a mean reverting long memory process was already suggested by Cheung and Lai (1993), Abuaf and Jorian (1990), Imbs et al. (2005) and Abadir and Talmain (2005). Our results also confirm the presence of such long memory. A hint of such persistence and non-linearities can be found by an inspection of Figure 1, where the autocorrelation function (ACF) of $\hat{e}_t$ is plotted. We see that it does not converge towards zero exponentially, as implied by the I(0) assumption, but it clearly does not support evidence of a possible unit-root either. Its slow rate of decay could therefore suggest the presence of long memory.

4 A modified Engle-Granger approach for long memory cointegration

In this section we present our approach to test for the existence of a long memory co-movements. As mentioned above, testing for cointegration in the E-G context is equivalent to testing whether the long-run regression residuals $\epsilon_t$ in equation 1 are an I(1) or I(0) process. However, in both cases we are assuming that deviations from the long-run equilibrium evolve according to an ARIMA model.

Recently, some researchers have started to doubt the ability of ARIMA process to fit the dynamics of many economic variables, in favor of the more general class of long memory processes.\textsuperscript{21} These processes are characterized by a very slow decaying autocorrelation function, but unlike unit root processes they are mean-reverting. This strong autocorre-

\textsuperscript{21}Considerable evidence of long memory in macroeconomic times series has been found in the works of Sowell (1992), Diebold and Rudebusch (1989), Baillie et al. (1996), Crato and Rothman (1994), Hassler and Wolters (1995) and very recently Abadir et al. (2006).
lation means that inaccurate approximations of its dynamics are likely to lead to spurious results and rejections of plausible long-run economic relations. A way to deal with such persistent dynamics has been proposed in a couple of papers by Abadir and Talmain (2002, 2005). In particular, they have shown that the dynamics of most macroeconomic variables follow a new type of mean-reverting long memory process. This process is characterized by a very slow decay of the ACF, whose leading term can be represented by the functional form\(^{22}\)

\[
\rho(\tau) \approx \frac{1 - a [1 - \cos(\omega \tau)]}{1 + b \tau^c}
\]  

where \(a, \omega, b, c\) are parameters to be estimated. They show that it is possible to use this functional form to disentangle co-movements between variables from the effects of their own persistence. Specifically, they fit the functional form in equation 9 to the ACF of the data and construct a GLS procedure to estimate consistently a spurious relation between the variables. Starting from their results, in this section we extend their approach to the Engle and Granger approach\(^{23}\) to test explicitly for a long memory equilibrium between the variables. If we define the autocovariance matrix of \(\hat{\varepsilon}_t = y_t - \hat{\alpha} - \hat{\beta} x_t\) as

\[
R \equiv E\left(\hat{\varepsilon}_t \hat{\varepsilon}_t^\prime\right) = \rho_0 \begin{pmatrix}
1 & \rho_1 & \cdots & \rho_{T-2} & \rho_{T-1} \\
\rho_1 & 1 & \cdots & \rho_{T-2} & \rho_{T-1} \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
\rho_{T-2} & \vdots & \ddots & 1 & \rho_1 \\
\rho_{T-1} & \rho_{T-2} & \cdots & \rho_1 & 1
\end{pmatrix}
\]  

we can account for long memory dynamics in the following way:

1) Estimate the long-run relation \(y_t = \alpha + \beta x_t + \varepsilon_t\) by maximizing the likelihood

\[
ML(\alpha, \beta) = (2\pi)^{-\frac{1}{2}} |R|^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(y_t - \alpha - \beta x_t)^\prime R^{-1}(y_t - \alpha - \beta x_t)\right)
\]

with respect to \(\alpha\) and \(\beta\) and the parameters \((a, b, c, \omega)\) of the functional form.

\(^{22}\)This functional form is the leading term of an asymptotic expansion for the ACF of a process that is generated by the aggregation of geometric ARMA process. See Abadir and Talmain (2002) for more details.

\(^{23}\)Gil-Alana (2003) has also proposed a two-step procedure based on Robinson univariate tests for the case of fractional cointegration.
\[ \rho_e(\tau) = \frac{1 - a \left[ 1 - \cos(\omega \tau) \right]}{1 + b \tau^c} \]

This can be done recursively given some starting values for \( a, b, c \) and \( \omega \).

2) Calculate the regression residuals \( \hat{\varepsilon}_t = y_t - \hat{\alpha} - \hat{\beta} x_t \) and estimate the ADF regression

\[ \Delta \hat{\varepsilon}_t = \phi \hat{\varepsilon}_{t-1} + u_t \quad (11) \]

maximizing the likelihood functions

\[ ML(\rho, \rho_1, ..., \rho_k) = (2\pi)^{-\frac{1}{2}} |\hat{\Omega}|^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (\Delta \hat{\varepsilon}_t - \phi \hat{\varepsilon}_{t-1})' \hat{\Omega}^{-1} (\Delta \hat{\varepsilon}_t - \phi \hat{\varepsilon}_{t-1}) \right\} \quad (12) \]

where

\[ \hat{\Omega} = \{ \hat{\omega}_{i,j} : \hat{\omega}_{i,j} = \hat{\rho}_u(\tau) ; \tau = |i - j| ; \ i = 1, ..., T - 1 ; \ j = 1, ..., T - 1 \} \]

\[ \hat{\rho}_u(\tau) \approx \frac{1 - a_u \left[ 1 - \cos(\omega_u \tau) \right]}{1 + b_u \tau^{c_u}} \]

with respect to \( \phi \) and the parameters of the functional form \( \hat{\rho}_u(\tau) \) fitted to the ACF of \( u_t \).

3) Evaluate the \( t \)-statistic \( t_\phi \) for the parameter estimated \( \hat{\phi} \) in equation 11.

4) Calculate the bootstrap critical values \( \tilde{t}_L^\rho \) and \( \tilde{t}_H^\rho \) for \( t_\rho \) using the approach described in section 2.

5) Reject the null hypothesis of no cointegration if either \( t_\rho < \tilde{t}_L^\rho \) or \( t_\rho > \tilde{t}_H^\rho \).

A notable feature of this approach is in its simplicity, which allows us to extend the E-G approach to the case of long memory co-movements. However, despite its simplicity, it is characterized by high empirical size and power unlike the standard approach, as we show in the next section. Another noticeable feature is that we need not be concerned about the number of lags to include in the ADF regression in 11. A well-known drawback of the E-G approach is that the results are sensitive to the number of lags chosen in the ADF regression. In particular, too many lags can reduce the power of the ADF test, while too few can bias the estimation results. Since in step 2 we account by construction for possible omitted autocorrelation in the residuals \( u_t \), this drawback does not apply in our
5 The PPP-UIP cointegration test revised

In this section we apply our procedure to the PPP-UIP database and show that it detects a long memory equilibrium between the variables. We start by fitting the functional form of equation 9 to the ACF of \( \hat{\varepsilon}_t \), which leads to the estimated ACF

\[
\hat{\rho}(\tau) \simeq \frac{1 - 1.047 \left[ 1 - \cos(0.28762 \tau) \right]}{1 + 0.3225 \tau^{-0.17045}}
\] (13)

As shown in Figure 1, equation 13 reveals a striking accuracy of the functional form in fitting the ACF of \( \hat{\varepsilon}_t \). We then estimate the PPP-UIP relation in equation 8 and then test for long memory cointegration using the approach described in the previous section. The estimation results and the ADF \( t_{\rho} \)-statistic together with the SB and MBB critical value are shown in table 2

<table>
<thead>
<tr>
<th>Estimated long-run relation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_t^{uk} = 1.41 p_t^{w} + 0.41 \varepsilon_t^{uk} + 0.06 i_t^{uk} - 0.93 i_t^{ed} + \varepsilon_t )</td>
</tr>
<tr>
<td>( R^2 = 0.998 )</td>
</tr>
<tr>
<td>( \sigma^2 = 0.0046 )</td>
</tr>
<tr>
<td>( D - W = 1.90 )</td>
</tr>
<tr>
<td>( L - B = 1.66 )</td>
</tr>
<tr>
<td>( \text{RESET} = 0.69 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DF statistic and 95% critical values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_{\rho} )-statistic ( \text{-2.8012} )</td>
</tr>
<tr>
<td>95% MBBCI ( \text{-2.6028} )</td>
</tr>
<tr>
<td>95% SBCI ( \text{-2.416} )</td>
</tr>
</tbody>
</table>

Table 2: Log-run UIP-PPP regression and DF statistic: modified E-G approach.

---

\(^{24}\) This result was confirmed by the simulations, since the size and power of the ADF test were independent of the number of lags chosen.

\(^{25}\) The \( R^2 \) is higher then 0.98.

\(^{26}\) To get the bootstrapped critical value we choose a number of draws equal to 1000. For the moving block bootstrap we set a block length equal to 15, while for the stationary bootstrap we set a value of \( p \) (the parameter of the geometric distribution) equal to 0.05, which gives an average sample length of 20.
First, it can be clearly seen that all the problems that emerged with the standard E-G approach, specifically autocorrelated residuals and omitted non-linearities, have disappeared. Furthermore, a high $R^2$ together with a D-W close to 2 eliminates any possibility of a spurious regression.

Finally, we reject the null hypothesis of no cointegration. This result indicates that there exists a long memory cointegrating relation among the PPP and UIP variables. Although such relation is not strictly stationary, as required by standard cointegration, it is still mean-reverting. This very slow adjustment is not completely unrealistic; as already mentioned, it is consistent with the assumption that the foreign goods and exchange rate markets are characterized by market failures. Therefore, as anticipated above, it is possible to detect a long-run stable relation between the variables by allowing for possible non-linearities and deviations from the long-run equilibrium, that are strongly persistent. In the next section, through simulation we give more support to this intuition.

6 Simulations

In this section we run some simulations to compare the standard E-G cointegration approach with the approach proposed in section 4. Specifically, we evaluate the empirical size and power\(^{27}\) of the ADF test in the E-G original framework and in our modified procedure. Furthermore, we calculate the difference in the small-sample estimation bias between the two approaches. Finally, we also evaluate the performance of our approach in the case of fractional cointegration.

The simulation has been conducted as follows. For the sake of comparison with similar works (Engle and Granger (1987), Cheung and Lai (1993) and Gil-Alana (2003) for instance) we use artificial data $x_{1,t}$ and $x_{2,t}$ generated by the bivariate system

\[
\begin{align*}
x_{1,t} + x_{2,t} &= u_t \\
2x_{1,t} + x_{2,t} &= v_t
\end{align*}
\]

\(^{27}\)We recall that the size of a test is the probability of rejecting the null hypothesis when this is true while the power is the probability of rejecting the null when the alternative is true.
We consider two different data-generating processes (DGP). In the first, DGP1, we assume no cointegration and define \((1 - L) u_t = \xi_t\) and \((1 - L) v_t = \eta_t\) where both \(\xi_t\) and \(\eta_t\) are \(IN(0,1)\) variables. Given this data-generating process, we evaluate the size of the ADF test for both the standard E-G approach and our modified procedure.

In the second, DGP2, we assume that \(v_t\) is a long memory process with the same ACF structure as \(\hat{\varepsilon}_t\), the equilibrium error from the PPP-UIP relation, and evaluate the power of the ADF test for the two approaches. Under this assumption, \(x_{1,t}\) and \(x_{2,t}\) are by construction non stationary variables but they are linked together by the cointegrating vector \([1, -0.5]\) that describes the long memory equilibrium between the two variables.

The artificial data for \(v_t\) is constructed in the following way. First, starting from the estimated functional form in equation 13, we construct the variance-covariance matrix \(\hat{R}\) of \(\hat{\varepsilon}_t\), as defined in 10. Then, using the Cholesky factorization we decompose \(\hat{R} = \Gamma\Gamma'\) where \(\Gamma\) is lower triangular. Finally, given the sequence \(\{\eta_t\}_{t=1}^T\) of \(IN(0,1)\) variables we construct \(v_t\) as

\[
v_t = \Gamma \eta_t
\]

This transformation generates in \(v_t\) the same autocorrelation structure as \(\hat{\varepsilon}_t\).

In the first simulation we evaluate the empirical size and the power of the standard E-G cointegration approach. We have set the number of replications to 1000. For each replication, we apply the original E-G approach, as described in previous section, and evaluate the ADF \(t_\rho\)-statistic and its bootstrapped 2.5% lower and upper quantiles \(\hat{\tau}_L^\rho\) and \(\hat{\tau}_H^\rho\). When \(x_1\) and \(x_2\) are generated according to DGP1, we calculate the percentage of times that the null hypothesis is rejected when it is true (size of the test). Conversely, when the artificial data is generated according to DGP2, we calculate the percentage of times that the test rejects the false null hypothesis of no cointegration (power of the test).

In the upper part of Table 3 we report the empirical size and power of the ADF test\(^{28}\) for the standard E-G approach, calculated respectively using moving block (MBB) and stationary (SB) bootstrap for a sample length respectively equal to 100 and 200.

\(^{28}\)It needs to be mentioned that when the null is true we have set \(k\), the number of lags in the ADF-regression, equal to 1 (i.e. the true data-generating process under the null). On the other hand, when the alternative is true we have chosen a number of lags equal to three in order to remove all the autocorrelations from the residuals. Using the same number of lags for both cases would have reduced the power and the size of the ADF-test even more. It is important to note that the modified approach is not sensitive to the choice of \(k\), since the procedure is designed to account for any autocorrelations in the residuals.
First, using bootstrapped critical values the ADF test has a size that is close to the nominal, especially for a sample length of 200 observations. This result confirms the findings in Li and Maddala (1997) that bootstrap critical values improve the size of the ADF test under the null hypothesis. Then, most importantly, when the alternative hypothesis of long memory cointegration is true, the ADF test has very low power. In fact, it rejects the null hypothesis at most 14% of times when the alternative is true. This means that the ADF test, in the standard E-G context, is unable to distinguish between long memory and unit root even in fairly large samples. This result has an important implication for the macroeconomist. It shows that rejection of a long-run equilibrium by the ADF test does not represent conclusive evidence for excluding any relations between economic data. In fact, the E-G approach would lead to the conclusion that no equilibrium relation exists between the variables any time that this is not strictly $I(0)$.

In the lower part of Table 3 we report the empirical size and power of the ADF test in our modified approach. The rejection frequencies under the null do not present any significant difference to the standard case; in fact, we can reject at most 10% of times the null of no cointegration when it is true. On the other side, the power of the test shows a striking improvement. In fact, the rejection frequency is very high and close to its...
nominal values even for short samples. Already with a sample length of 100 observations we are able to reject 90% of times the null of no cointegration. Therefore, on one hand our approach is as reliable as the standard approach when there is no relation among the variables; on the other, it is able to detect a long-run equilibrium when the fluctuations from such equilibrium are not strictly stationary. Thus, by allowing (and accounting) for possible non-linearities and long memory it is possible to detect the true cointegrating relation and long memory fluctuations around the long-run equilibrium.

Although has been mentioned several times the concept of long memory cointegration, no explicit reference has been made to the case of fractional cointegration so far. In the last few years some interesting work has been done in cointegration analysis to test under the alternative hypothesis of fractional cointegration (see for instance Gil-Alana (2003), Dolado, Gonzalo and Mayoral (2002), and Baillie and Bollerslev (1994)). Our procedure can be applied to the case when the cointegrating relation evolves as a fractionally integrated ARMA process but in general it works with more general class of long memory processes. This point can be clarified in the following way. As can be seen from the functional form in equation 9, by setting the parameter $a$ equal to zero and $b$ equal to one we get a rate of decay which is asymptotically equivalent to the rate of decay of the ACF of a fractionally integrated process.\footnote{We recall that the ACF of a fractionally integrated process decays at a rate given by $\rho(\tau) \simeq A_0 \tau^{2d-1}$ where $A_0$ is a constant term.} In other words, under certain condition, the ACF of a fractionally integrated process is a special case of the ACF patterns produced by the functional form in equation 9.\footnote{It can be shown that, under certain conditions, the leading terms of the ACF of a fractionally integrated process and Abadir and Talmain’s process coincide (see Moretti (2006)).} We give more support to this point using simulation. Specifically, we repeat the same exercise above but this time the variable $\varepsilon_t$ is generated according to the fractionally integrated noise

$$(1 - L)^d \varepsilon_t = u_t$$

with $d \in (0, 1)$ and $u_t \sim IN(0, \sigma^2)$.

In the first part of Table 4 we show the empirical power of our modified procedure for different values of $d$ and a sample size $T$ equal to 100. The difference in power between
the standard E-G approach and our modified procedure is pronounced for all the orders of integration considered. In particular, our approach seems to perform quite well for values of \( d \) smaller than 0.7 but, to in light of the small sample size, this result shows, for the case of fractional cointegration, a significant increase of power of the ADF test in our approach over the standard approach.

<table>
<thead>
<tr>
<th>( d )</th>
<th>0.35</th>
<th>0.45</th>
<th>0.6</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>MBB</td>
<td>0.877</td>
<td>0.773</td>
<td>0.514</td>
<td>0.366</td>
<td>0.291</td>
<td>0.226</td>
</tr>
<tr>
<td>SB</td>
<td>0.881</td>
<td>0.757</td>
<td>0.458</td>
<td>0.3</td>
<td>0.197</td>
<td>0.151</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sample size</th>
<th>( B(\beta^{QML}) )</th>
<th>( B(\beta^{ols}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T = 100 )</td>
<td>0.0106</td>
<td>0.0169</td>
</tr>
<tr>
<td>( T = 200 )</td>
<td>0.0067</td>
<td>0.0083</td>
</tr>
</tbody>
</table>

Table 4: Empirical power of the ADF test against fractional alternatives and small sample estimation bias in the standard and modified E-G approach.

In the last simulation we evaluate the small-sample estimation bias for both the approaches and show that our modified procedure leads to a substantial improvement.

We call the OLS estimate of the cointegrating parameter in the standard E-G approach \( \beta^{SEG} \) (equal to -0.5 in the data-generating process) and the estimate from our approach \( \beta^{MEG} \), and evaluate the mean of the small-sample bias for both approaches, which is

---

31 The results for the standard ADF test are taken from Bisaglia and Procidano (2002). They are based on 1000 replications and obtained using sieve bootstrap. Differently from those in Cheung and Lai, who fixed in their simulation the number of lags in the ADF regression, these are obtained using the AIC criteria to select the number of lags in the ADF regression.
defined as

\[ B(\beta^{SEG}) = \frac{\sum_{i=1}^{N} (\beta_{i}^{SEG} + 0.5)}{N} \]

\[ B(\beta^{MEG}) = \frac{\sum_{i=1}^{N} (\beta_{i}^{QEG} + 0.5)}{N} \]  

(14)

where \( N \) is the number of replications in the simulation.

In lower part of Table 4 we report the small-sample bias, for the estimation of the cointegrating vector respectively for our approach and the standard estimation. In both cases our procedure reduces significantly the estimation bias compared to standard OLS, which is usually implemented in the E-G approach. For a sample size of 100 observation, the small-sample bias for the estimator \( \beta^{SEG} \) is about 0.017 while the bias for estimator \( \beta^{MEG} \) is 0.011, which is 37% smaller. When the number of observations doubles, the bias decreases for both estimators but the \( \beta^{SEG} \) is still significantly more biased than \( \beta^{MEG} \).

This improvement can be justified considering that with the QML procedure we account for possible omitted non-linearities and strong autocorrelations in the regression residuals.

In the light of the results presented in this section, we can conclude that the potential advantage in terms of power and consistency of our modified procedure is quite substantial when testing for a cointegrating equilibrium characterized by non-linearities and persistent dynamics.

7 Conclusion

Standard cointegration analysis techniques such as the E-G approach and the FIML impose very stringent restrictions of the data-generating process. In fact, in order to find an equilibrium relation among economic variables, this should be strictly linear and characterized by fast convergence rate towards the long run equilibrium. It might be plausible that these conditions do not hold empirically and therefore it is very likely that cointegration is rejected even when economically plausible. In this work we show that once these two assumptions are relieved, we can find the existence of a long memory equilibrium among the variables. To this end we present a methodology to test for the presence of cointegration when the variables are non-stationary and there exists a linear combination
that is characterized as long memory process rather than as ARMA process. We show
that, unlike standard cointegration technique, our approach is able to detect the presence
of a long memory co-movements. Furthermore, we report for this test high size under the
null hypothesis and high power under the alternative. Our approach also reduce the small-
sample bias in the estimation of the cointegrating vector by more than 41% compared to
standard ordinary least square estimate. We applied our procedure to the data base for
the UK purchasing power parity and uncovered interest rate parity and reject the null
hypothesis of no cointegration, in contrast with what was previously shown with standard
single equation technique.

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8 Figures

Figure 1: Autocorrelation function of the PPP-UIP regression residuals and estimated functional form
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