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Endogenous growth and trade liberalization between asymmetric countries

by Daniela Marconi
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ENDOGENOUS GROWTH AND TRADE LIBERALIZATION BETWEEN ASYMMETRIC COUNTRIES

by Daniela Marconi*

Abstract

The paper presents a general equilibrium model of endogenous growth and trade between two countries, an advanced country (A) and a backward country (B). The development stage is summarized by the level of knowledge stock accumulated through R&D investments. Producers of intermediate goods, operating under increasing returns to scale and monopolistic competition, perform R&D investments to obtain process innovations (reduction of production costs) if they are incumbents, or product innovations if they are new entrants. The model shows that convergence in long-run growth rates can be obtained even in the absence of international technology spillover, in which case, under the assumption of no variety overlap, the gain from trade will be only static. Dynamic effects will be delivered instead in the presence of an initial overlap in the varieties produced in the two countries, together with a wide gap in unit production costs. In this case it is shown that the impact of trade liberalization on firms’ profits could generate a cumulative causation process which may lead to a polarization of innovative productions in the advanced country.

JEL Classification: F12, 040.
Keywords: endogenous growth, trade liberalization, scale effect.

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1 Introduction

International trade is one of the major forces driving the allocation of resources within and between countries. Comparative and absolute advantages have static and dynamic effects on output levels and growth rates, as well as relevant welfare implications. In the recent theoretical literature, the effects of trade on growth have been addressed in many frameworks, leading to somewhat opposite results. The present work concentrates on models of endogenous growth. Within this class of models, international trade liberalization has very different implications depending on whether or not technological progress spills over internationally.

In models where symmetric countries are considered, i.e. countries of the same size and technological level (as in Rivera-Batiz and Romer (1991) and Grossman and Helpman (1991) and Peretto (2000)), trade liberalization is usually beneficial in terms of level. Given the symmetry assumption, countries grow at an equal rate before and after trade liberalization and, since long-run growth ultimately depends on market size, perfect international diffusion of technology guarantees that the common rate of growth prevailing after trade integration will be higher.\(^1\)

Under the hypothesis of asymmetric countries, instead, the effects of trade liberalization are somewhat more controversial and again the results depend crucially on the scope of technological spillover. If technological knowledge does not spill over, initial conditions play a major role: differences across countries can lead to large differences in income levels and divergence in the growth rates (Young (1991), Matsuyama(1991), Grossman and Helpman (1991), Feenstra (1996), Redding (1999)). Young (1991), for example, using a learning-by-doing technology, shows that trade might be beneficial in static terms, while it might be harmful or have no effects on backward countries’ growth rates. Redding (1999) argues that in the presence of learning by doing the protection of infant industries with large technological learning potential might be a welfare improving strategy in the long run if it helps to close the gap with the technological leader and reverse the comparative disadvantage. Grossman and Helpman (1991), using a variety expanding model, show that, in the absence of international knowledge spillover, a possible outcome is the concentration of R&D activities in the technological leader countries, with ambiguous welfare effects on the backward ones.

Models predicting convergence of growth rates across countries rely upon the assumption of global knowledge spillover; for example, the North-South

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\(^3\) In the traditional variety expanding and quality ladder models (Rivera-Batiz and Romer (1991) and Grossman Helpman (1991)) the long run growth rate is proportional to the size of the population; this result is called scale effect. In more recent models (Peretto and Smulders (2002)) the long run growth rate does not depend on the scale of the economy, while the size of the market pins down the long-run equilibrium number of firms and varieties. For a stylized description of the different solutions to eliminate scale effects see Jones (1998).
trade literature assumes that southern backward countries have the ability to imitate some of the goods initially invented in the North, the South accumulates knowledge and learns by looking at what the North is producing. Comparative (and absolute) advantages lead the South to produce and sell the imitated goods on the world market. Trade by itself will not be enough to generate convergence, unless it represents the very channel through which backward countries learn to imitate (Grossman and Helpman (1991), van de Klundert and Smulders (1996), Barro and Sala-i-Martin (1997), Connolly (1999), Martin and Ottaviano (1999)).

The aim of this paper is to extend the existing literature by proposing a model that can produce both convergence and divergence in the growth rates in the absence of international spillover. Under this basic assumption, the implications of trade liberalization between asymmetric countries will be further analyzed introducing two novelties: first, scale effects will be eliminated, that is the long-run growth rate will not depend on the size of the population of the economy; and second, the possibility of variety overlaps between the two countries at the moment of trade liberalization will be explicitly considered. This possibility is generally ruled out by assumption, while here I investigate its consequences when unit costs in production are different. These two modifications which, to my knowledge, have never been introduced together in an endogenous growth model with trade between asymmetric countries, are able to deliver new and interesting outcomes. A first result is that, when we impose balanced trade and no production overlaps, the only way in which the two countries can reach a long-run equilibrium with positive production in the innovative sector is when they end up growing at the same rate. This, in turn, implies that if the two countries were already growing at the same rate before trade liberalization they will continue to do so, and the only gain from trade will be static. When we introduce the possibility of production overlaps and a wide gap in unit production costs, if the backward country is a relatively small country, domestic firms’ profits will be negatively affected while foreign firms’ profits will increase. Under these assumptions, a cumulative causation process might be generated and the advanced country’s market share will expand, while that of the backward country will shrink.

The paper is organized as follows. The next section introduces the theoretical framework, develops the model under the assumption that only the final good is internationally traded, and describes consumer preferences, firms’ technology, resource constraints, market clearing conditions, the steady state and the dynamics of the model. Section 2.3 extends the model to the case in which intermediate goods are also traded and analyzes the effects under several possible initial conditions and variety overlap. Section 3 concludes. Two mathematical appendices are added in two final sections (A and B).

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2 Empirical studies on whether spillovers are national or international in scope have produced mixed results; for a comprehensive survey see Keller (2002).
2 The theoretical framework

The goal here is to construct a model of endogenous growth driven by knowledge accumulation and increasing returns to scale. The development stage of a country is summarized by the level of knowledge capital accumulated, which, in turn, affects the productivity of the entire economy. Research and development (R&D) is performed in-house by profit-seeking firms and generates economy-wide externalities in the form of general purpose public knowledge that increases the total factor productivity (TFP) of the entire economy. We consider two countries, a backward country (B) and a more advanced country (A), with three production sectors, the final good sector, the intermediate goods sector and the R&D sector and two factors of production, labour and knowledge capital. The two countries differ in size and research experience. The final good sector produces a traditional good with constant returns to scale. The intermediate goods sector produces a set of differentiated goods under increasing returns to scale and monopolistic competition. Each monopolistic firm already in the market invests in R&D to accumulate knowledge capital. Knowledge capital enters the production function of intermediate goods and, like a process innovation, reduces the unit production cost; also, it generates spillovers increasing the domestic stock of public knowledge, which, in turn, increases the productivity of the entire economy. New entrants must pay an entry cost in order to acquire the state of the art technology to produce and sell a new variety of intermediate good. It is further assumed that knowledge does not spill over internationally, and therefore knowledge accumulation in each country depends only on domestic R&D investments.

Several authors have stressed the essential role of domestic R&D using the concept of absorptive capacity. This concept was first introduced by Cohen and Levinthal (1989) and then extensively studied in the empirical literature. Being involved in innovative activities creates the necessary ability to understand and adopt new technologies even when invented elsewhere (see for example Kinoshita (2000) and Keller (2002)). In this paper, domestic in-house R&D benefits the entire economy, thereby increasing the ability of the final good sector to adopt high-tech intermediates. If, for example, international trade, by decreasing the profitability of the domestic high-tech sector in the backward country, reduces or eliminates the domestic R&D effort, it would have a negative impact on the TFP growth rate of the economy. If the necessary flow of knowledge is not indigenously created, the rate of growth of the economy is reduced.

3In the existing theoretical literature, a concept similar to the absorptive capacity, sometimes called learning-to-learn, is generally introduced only in the intermediate goods sector (see for example Connolly (1999)). Here, instead, as in Romer (1987), the productivity of the final good sector is also positively affected by knowledge accumulation in the intermediate goods sector.

4Ciccone and Matsuyama (1996), using a variety expansion model à la Grossman and Helpman, show the possibility of a vicious circle when the lack of indigenous flow of knowledge force the final good industry to adopt more primitive modes of production, thereby reducing the incentive to start up new firms and to introduce new varieties of intermediate inputs.
In the presence of production overlap and lack of competitiveness, in terms of production costs, an economy might be trapped into a lower level of economic development due to the loss of the necessary incentives to operate in the high-tech sector. A polarization process might then be observed, in which high-tech production and R&D activities would concentrate in the advanced country. This outcome would not be very far from reality; in fact, data on R&D shows that more than 80% of world R&D investments are performed in a few advanced economies and only a few emerging countries devote a significant amount to it (Kumar (1997), Crispolti and Marconi (2005); UNCTAD (2006)). It is interesting, therefore, to analyze the impact of trade on industry dynamics, that is, market shares, entry-exit and R&D investments and try to understand what is the role of initial conditions. Do we observe convergence or divergence? Where do we converge/diverge to? The model that I propose shows that, with endogenous long-run growth and no scale effects, even without international spillovers, when the two countries do not trade in intermediate goods it is possible that they grow at the same rate; however, if they start with different initial stock of knowledge capital, they will not converge in per capita levels. In the absence of trade, the scale of the economy will be relevant only to pin down the equilibrium number of firms and the level of output, while the long-run rate of growth will depend on exogenous parameters. When we allow for trade, two effects will come into the picture. On the one hand, each firm potentially enjoys a larger market but, on the other hand, competition will be higher due to the presence of more firms. Which of the two effects will prevail depends on several factors. It will be shown that, under certain conditions, if there is no overlap in the variety produced in the two countries, the two effects cancel out and the growth rate will not be affected. If there is overlap, then unit production costs become relevant and trade can generate a poverty trap in the backward country.

Overall, the general set up is standard. The innovative sector is a simplified version of that in Peretto (1998, 2000), and Peretto and Smulders (2002), allowing for both product and process innovations; the contemporaneous presence of a vertical dimension of innovation (process innovation) and of a horizontal one (variety expansion) eliminates the scale effect in the growth rate. Here, as in general in this literature, the production of intermediate goods drives the knowledge accumulation process that, in turn, drives output growth, while agents have perfect foresight and markets clear.


\[6\] Although the elasticity of substitution across differentiated goods is constant, the structure of the model allows, as in Peretto (1998, 2000) and Peretto and Smulders (2002), to determine a finite, long-run equilibrium number of firms, which corresponds to the number of differentiated goods.

\[7\] Product innovations introduce new varieties of intermediate goods, leading to what is called the horizontal expansion of the economy; process innovations, instead, reduce the cost of producing existing varieties and can be assimilated to a vertical expansion.
2.1 The model

Firms  There are 3 sectors: The final good sector \((Y)\), the intermediate goods sector \((X)\) and the in-house R&D sector \((H)\) and two production factors: labour \((L)\) and firm-specific stock of knowledge capital \((H)\), which in turn determines the economy-wide general purpose knowledge capital, \(K\). Nations produce in each sector with the same production function, but have different levels of \(H\) and \(L\). In particular, I assume that \(B\)'s initial stock of firm specific, and hence general purpose, knowledge is below \(A\)'s. There are no international knowledge spillovers and labour and financial capital are mobile across sectors but not across countries. Countries are denoted with \(i = A, B\); firms with \(j = 1, 2, ..., N\); at each moment in time the total number of active firms in the world is given by the sum of those producing in country \(A (N_A)\) and in country \(B (N_B)\), \(N_W = N_A + N_B\).

Final good sector  The \(Y\) sector produces a freely traded final good under perfect competition using labour and the whole range of intermediate goods available in the market at each moment \((N)\). Each intermediate good, \(x_{ij}\), enters the production symmetrically with a constant elasticity of substitution \(\alpha\).\(^8\) Also, I assume that the general purpose knowledge of the economy increases the ability of this sector to adopt the technology embodied in the intermediate inputs, thereby increasing total factor productivity (TFP). The production function of this sector is:

\[
Y_i = K_i^\varphi L_i^{1-\alpha} \left[ \sum_{j=1}^{N} (x_{ij})^\alpha \right] \tag{1}
\]

\(i = A, B\), \(0 < \alpha < 1\), \(0 < \varphi < 1\), \(\alpha + \varphi < 1\), \(j = 1, 2, ..., N\)

Intermediate goods sector and corporate R&D  Intermediate goods are produced combining labour and firm-specific knowledge with a constant returns to scale technology with respect to each input (hence overall increasing returns to scale). Each intermediate good producer is a local monopolist; for simplicity, I assume monopolistic competition \(\text{à la} \ Chamberlin:^{9}\)

\[
x_{ij} = H_{ji} L_{xij} \tag{2}
\]

\(^8\)The derivation of the Dixit-Stiglitz demand curves for the differentiated goods used as inputs in a final good production function has been formulated by Ethier (1982).

\(^9\)When firms compete \(\text{à la} \ Chamberlin, they act as if each firm holds a negligible share of the market. This behaviour is plausible when \(N\) is large enough, and I assume that this is the case. Monopoly power derives from the existence of fixed production costs, whereas monopolistic competition is the outcome of the free-entry assumption (see also Tirole (1988)).
where $H_{ij}$ is the firm-specific knowledge that determines the labour productivity in the intermediate goods sector.\(^{10}\) Labour productivity, in the intermediate goods sector, can be improved by employing labour in R&D projects as described by the following production function:

$$\dot{H}_{ij} = \beta K_i L_{H_{ij}}$$  \(3\)

where $\beta$ is a productivity parameter, $L_{H_{ij}}$ is the amount of labour that firm $j$ in country $i$ employs in R&D and $K_i$ is the domestic stock of general purpose knowledge. I assume that this stock is simply given by the arithmetic average of firm specific stocks. That is:

$$K_i = \frac{1}{N_i} \sum_{j=1}^{N_i} H_{ij}, \quad (4)$$

Each individual firm takes $K_i$ as given without internalizing its contribution to it. This means that while firms are producing product-specific knowledge, completely appropriated, to some extent they also produce general purpose knowledge, which takes the form of a public good.\(^{11}\) Devoting more workers to R&D leads to higher rate of production of new knowledge. The larger the total stock of knowledge, $K_i$, the higher the productivity of workers.\(^{12}\) Constant returns to scale with respect to knowledge capital in the R&D production function ensures endogenous growth. To simplify the analysis, we shall assume that firms within a country are symmetric. In a symmetric equilibrium, $H_{ij} = \tilde{H}_i = K_i$. Hence,

$$\gamma_{H_i} = \frac{\dot{H}_i}{H_i} = \beta L_{H_{ij}}.$$  \(5\)

In addition to labour costs for production and R&D, I assume that firms have to incur a fixed, overhead labour requirement $f$.\(^{13}\)

---

\(^{10}\) As pointed out by van de Klundert and Smulders (1995), if we measure $x_{ij}$ in quality units, both forms of innovations, process innovation and quality innovation, can be expressed as an increase in $H_{ij}$.

\(^{11}\) As in Romer (1990), R&D output is, in part, a non rival good, partially excludable and privately provided. Inputs like labor and the specialized intermediate inputs are rivalrous, that is they can be used only in one sector at time. In contrast, the general purpose knowledge capital, $K$, is non-rival, it can be used in all sectors at the same time. We could also impose that the ability to exploit outside knowledge depends on the firm-specific knowledge stock, in this case we could modify equation (3) introducing $H_{ij}$ among the production factors.

\(^{12}\) As already noted, the accumulation of knowledge increases the TFP of the final sector as well, but the R&D sector does not internalize this contribution.

\(^{13}\) This fixed labor cost can capture costs of management and coordination as in van de Klundert and Smulders (1995).
**Entrants** For simplicity I assume that entrants enter the market with the same level of technology as incumbents in the country and, once they have entered the market, benefit from the same technological spillover.\(^{14}\) Each new firm introduces a new variety of intermediate good. In order to start up a new firm entrants must acquire the state-of-the-art initial stock of firm-specific knowledge, which is a sunk cost. I assume that the entry cost, measured in labour units, is fixed and equal to \(\Psi w_i\).\(^{15}\) Entrants will enter the market if the value of the firm (given by the present discounted value of the flow of profits) is at least equal to the sunk cost, that is: \(V_{ij} \leq \Psi w_i\) and \(N \geq 0\) with at least one equality.

**Consumers** To close the model we need to specify the consumers behaviour. We assume that consumers in both countries have identical logarithmic preferences described by:

\[
U_t = \int_t^\infty e^{-\rho(\tau-t)} \log C(\tau) d\tau, \quad (6)
\]

where \(C\) is consumption of the homogeneous and freely traded final good, which is also the numeraire good, and \(\rho\) is the rate of time preference.

Households in each country maximize their utility subject to a budget constraint stating that the present discounted value of expenditure cannot exceed the present discounted value of labour income \((W(\tau))\) and dividends \((D(\tau))\).\(^{16}\) Formally,

\[
\int_t^\infty e^{-\rho(\tau-t)} C(\tau) d\tau \leq \int_t^\infty e^{-\rho(\tau-t)} [W(\tau) + D(\tau)] d\tau \quad (7)
\]

\(^{14}\)Here entrants contribute to the average stock of knowledge and each firm benefits from it in a symmetric way. In Peretto and Smulders (2002), instead, it is assumed that the intensity of technological spillover is inversely related to the *technological distance* between firms.

\(^{15}\)As will be proven later, in order to have positive long run growth of knowledge capital, the sunk cost faced by an entrant must be higher than the cost of producing a unit of knowledge capital by incumbents; that is, as derived from (2), the cost of producing a unit of knowledge capital is \(w_i \beta K_i\); therefore, in order to acquire \(H_i\) units of knowledge capital an entrant must spend an initial amount higher than \(\frac{1}{\beta} w_i\).

It would be interesting to specify the entry cost as an increasing function of the ratio between the firm specific knowledge that entrants must acquire and the domestic stock of public knowledge. This function can be expressed as \(\Psi \left( \frac{H_i}{K_i} \right)\), with \(\Psi'(\cdot) > 0\) and \(\Psi(0) = 0\).

However, since here, as will be specified later, firms are assumed to be symmetric within a country but not across countries, we need to specify at which technological level firms choose to enter once they compete on the international market and knowledge capital can be bought abroad. At this stage I disregard this further complication, but it is certainly one interesting extension of the model to be explored.

\(^{16}\)The primary factor is available in fixed supply. Therefore, total labor income is: \(W_i(\tau) = w_i L_i\).
where $\bar{r}(\tau - t) = \frac{1}{\tau-t} \int_{\tau-t}^{\tau} r(s) \, ds$ is the average real interest rate. At time $t$ households own the existing firms producing final and intermediate goods and the net revenues are paid to them as dividends. Final goods firms earn zero profits and own no assets and can therefore be ignored in the specification of endowment.

The Euler equation is:

$$\frac{\dot{C}_i}{C_i} = r_i - \rho, \quad i = A, B. \quad (8)$$

Equation (8) implies that, at each moment in time, the rate of growth of consumption is equal to the difference between the instantaneous interest rate and the rate of time preference.

2.2 Equilibrium without intermediate goods trade

First we analyze the properties of the model when only final consumption goods are traded, while intermediate goods are not. This assumption allows us to set a common numeraire between the two countries and at the same time, by the balanced trade assumption, to obtain an equilibrium that is directly comparable to an autarchic one. In fact, in this case, each country produces its final good using only the domestic intermediate inputs.

Given the symmetry across domestic firms and the constant elasticity of substitution, the final good sector in country $i$ will employ the same quantity of each intermediate good $j$, that is, $x_{ij} = x_i$. We can, therefore, simplify (1) in the following way:

$$Y_i = K_i^{\phi_i} N_i^{1-\alpha} (L_{yi})^{1-\alpha} (N_i x_i)^{\alpha}. \quad (9)$$

Equation (9) implies that final good production exhibits constant returns to scale in labour, $L_{yi}$, and the total amount of intermediate inputs employed in production, $N_i x_i$. Overall, there are increasing returns to scale due to knowledge spillover, captured by the term $K_i^{\phi_i}$, and specialized inputs, captured by the term $N_i^{1-\alpha}$.\(^{17}\)

Profit maximization and perfect competition imply that labour will be employed up to the level at which its marginal productivity equals the wage rate and that each variety of intermediate good will be employed up to the level at which its marginal productivity equals its price, that is:

$$w_i = K_i^{\phi_i} (1 - \alpha) L_{yi}^{-\alpha} [N_i (x_i)^{\alpha}] = (1 - \alpha) \frac{Y_i}{L_{yi}}, \quad \text{\,(10)}$$

\(^{17}\)Note that, since I am ruling out by assumption international knowledge spillovers, only the domestic stock of knowledge, $K_i$, affects the domestic TFP.
\[ p_{ij} = p_i = K_i^\alpha L_{yi}^{1-\alpha} (x_{ij})^{\alpha-1}. \] (11)

Equation (11) is the inverse demand function for the \( j \)th variety of intermediate input in country \( i \), which can also be expressed as the direct demand function in the well-known Dixit-Stiglitz form,

\[ x_{ij} = x_i = \left[ \frac{K_i^\alpha L_{yi}^{1-\alpha}}{p_{ij}} \right]^{\frac{1}{\alpha-1}} = \alpha Y_i \frac{p_{ij}^{-\varepsilon}}{\sum_{k=1}^{N} p_{ik}^{1-\varepsilon}}, \] (12)

where \( \varepsilon \equiv \frac{1}{1-\alpha} \) is the elasticity of demand with respect to price, which in turn depends on the degree of substitution between varieties (\( \alpha \)). Equation (12) is the downward sloping demand for each intermediate input \( j \) known to the monopolistic firms.

Instantaneous profits are given by \( \pi_{ij} = x_{ij}p_{ij} - [L_{xij} + L_{Hi} + f]w_i \); in each country each firm \( j \) in the \( X \) sector will maximize the present discounted value of the profits flow,

\[ V_{ij}(t) = \int_t^\infty [x_{ij}(\tau)p_{ij}(\tau) - L_{xij}(\tau)w_i(\tau) - (L_{Hi} + f)w_i(\tau)]e^{-r(\tau-t)}d\tau, \] (13)

subject to technology (2), (3) and demand (12). The current value Hamiltonian (CVH) can be expressed as

\[ CVH_{ij} = \left( p_{xij} - \frac{w_i}{H_i} \right)x_{ij} - (L_{Hi} + f)w_i + q_{ij}\beta K_i L_{Hi}, \] (14)

where \( H_i \) is the state variable, \( p_{xij} \) and \( L_{Hi} \) are the control variables and \( q_{ij} \), the costate variable, is the shadow value of the innovation. The maximization with respect to \( p_{xij} \) yields the optimal pricing rule of a constant mark-up over the marginal cost,

\[ p_{xij} = p_{xi} = \frac{1}{\alpha} \left[ \frac{w_i}{H_i} \right]. \] (15)

Optimizing with respect to labour we get:

\[ q_{ij}\beta K_i = w_i, \] (16)

which states that a firm is willing to invest in R&D up to the point at which the value of the innovation, \( q_{ij} \), is equal to its cost \( \frac{w_i}{\beta K_i} \).
Finally, we have the arbitrage equation which defines the optimal R&D strategy:

\[ r_{R&D_i} = \frac{q_{ij}^*}{q_{ij}} + \frac{1}{q_{ij}} \frac{\partial \pi_{ij}}{\partial H_i}. \] (17)

Equation (17) states that the rate of return of an innovation, \( r_{R&D} \), which is performed by incumbent firms, must be equal to the capital gain or loss plus the net marginal increase of profits due to the innovation. Deriving the instantaneous profit function with respect to \( H_{ij} \) and substituting (16) into (17) it becomes:

\[ r_{R&D_i} = \frac{q_{ij}^*}{q_{ij}} + \beta K_i \frac{L_{xij}}{H_i}; \] (18)

The equilibrium on the capital market requires \( L_{Hij} > 0 \) only if \( r = r_{R&D} \), and since, by the symmetry assumption, \( q_{ij} = q_i = \frac{w_i}{\beta K_i} \Rightarrow \frac{q_{ij}^*}{q_{ij}} = \frac{w_i^*}{w_i} - K_i \equiv \frac{w_i}{w_i} - \gamma_{Hi} \), the arbitrage equation can be written as

\[ r_{R&D_i} = \beta \frac{L_{xij}}{N_i} + \frac{w_i^*}{w_i} - \gamma_{Hi}. \] (19)

The rate of return to R&D is an increasing function of firm size, expressed by the number of workers employed in production, and of the rate of growth of wages (the higher the rate of growth of wages, the more valuable is the reduction of production costs provided by R&D investments); it is a decreasing function of the number of firms and of the rate of growth of knowledge capital, because firms do not internalize their contribution to the public knowledge capital (remember that given (4), \( \gamma_{K_i} = \gamma_{Hi} \)).

Let us consider now the entry process of new firms. Let us assume that entrants need to finance their entry cost by issuing equities. The return on these equities must be given by the usual arbitrage condition:

\[ r_i = \frac{\pi_{ij}}{V_{ij}} + \frac{\dot{V}_{ij}}{V_{ij}}. \] (20)

Using (15), (5) and the symmetry assumption we can rewrite the profit function as

\[ \pi_i = \left[ \frac{1 - \alpha}{\alpha} \frac{L_{xij}}{N_i} - \left( \frac{\gamma_{Hi}}{\beta} + f \right) \right] w_i \] (21)

18Formally, the arbitrage equation is derived by setting \( \frac{\partial CVH}{\partial H} = rq - \dot{q} \).
Also, as noted before, if the rate of entry is positive, then the value of the firm must be equal to the sunk entry cost, i.e. \( V = \Psi \) substituting this condition and (21) into (20) we obtain the rate of return of entrants:

\[
r_{E_i} = \frac{1}{\Psi} \left( \left( 1 - \frac{\alpha}{\alpha} \right) \frac{L_{xi}}{N_i} - \left( \frac{\gamma_{Hi}}{\beta} + f \right) \right) + \frac{w_i}{w_i}; \tag{22}\]

The rate of return to entry is negatively related to the sunk entry cost and to the incumbency costs (the in-house R&D and fixed costs), while, as for incumbents, it is positively correlated with the size of the firm and the growth rate of wages.

In equilibrium, with positive entry and R&D, the rates of return of these two activities must be equal. Since they are both a decreasing function of the rate of growth of knowledge capital, if we plot the two rates in the \( r_i-\gamma_{Hi} \) space (with \( r_i \) on the vertical axis), the existence of such an equilibrium requires that the two functions intersect, and stability requires that above the intersection point the return to R&D be higher than the return to invent a new good. The reason is that only incumbents affect \( \gamma_{Hi} \) and therefore drive \( r_i \) and \( \gamma_{Hi} \) towards the equilibrium point, whereas entrants affect \( r_i \) only by changing the number of firms. The reverse must be true below the intersection point. Therefore, we need two conditions, one on the slope and one on the intercept. The slope condition requires that \( \frac{\delta r_{R&D}}{\delta \gamma_{Hi}} > \frac{\delta r_{E_i}}{\delta \gamma_{Hi}} \), which in turn requires \( \Psi > \frac{1}{\beta} \), that is, the cost of a product innovation must be sufficiently higher than the cost of a process innovation (see note 15, p. 8).

The intercept conditions require that for \( \gamma_{Hi} = 0 \) the return to R&D for incumbents be higher than the return to market a new good for entrants, that is:

\[
L_{yi} + N_i(L_{xi} + f) + \Psi N_i = L_i \tag{23}
\]

Whereas the economy-wide budget constraint can be defined as follows:

\[
C_i + \Psi w_i \dot{N}_i = w_i L_i + N_i \pi_i \tag{24}
\]

Resource constraints and market clearing conditions Let us now define the resource constraints and the market clearing conditions. The labour market clearing condition is the following:

\[
L_{yi} + N_i L_{xi} + N_i(L_{Hi} + f) + \Psi N_i = L_i \tag{23}
\]

Whereas the economy-wide budget constraint can be defined as follows:

\[
C_i + \Psi w_i \dot{N}_i = w_i L_i + N_i \pi_i \tag{24}
\]

19 For a further discussion on the existence and stability of a Nash-equilibrium of this type see Peretto and Smulders (2002).

20 The slope condition and the vertical axis condition are sufficient to satisfy also the horizontal axis condition; in fact: \( \frac{\alpha}{1 - 2\alpha} f < \frac{L_{xi}}{N_i} < \frac{\alpha}{(1 - \alpha) - \alpha \Psi \beta} f \Rightarrow \Psi > \frac{1}{\beta} \).
The worldwide market clearing condition for the consumption good can be specified as follows:

\[ C_A + C_B = C_W = Y_A + Y_B = Y_W \]  \quad (25)

### 2.3 Transitional dynamics and steady state

The transitional dynamics and the long-run growth depend on the behaviour of the intermediate goods sector, and specifically on the value of the monopolistic firms and the entrants’ and incumbents’ behaviour. Two opposite forces are at work. While the entry rate is a positive function of the value of the firm, the increasing number of firms, by reducing the market share of each monopolist, tends to reduce the value of the firm. The interaction between these two effects will determine the equilibrium path. In each country the system of equations that describes the dynamics of the model is the following:

\[ r_{R&D} = \beta \frac{L_x}{N} - \beta \frac{L_H}{N} + \frac{\dot{w}}{w} \]  \quad (26)

\[ r_E = \frac{1}{\Psi} \left[ \left( \frac{1 - \alpha}{\alpha} \right) \frac{L_x}{N} - \left( \frac{L_H}{N} + f \right) \right] + \frac{\dot{w}}{w} \]  \quad (27)

\[ r = \rho + (\alpha + \varphi) \beta \frac{L_H}{N} + \frac{\dot{L_x}}{L_x} + (1 - \alpha) \frac{\dot{N}}{N} \]  \quad (28)

\[ \frac{L}{N} = \frac{\zeta}{\alpha^2} \frac{L_x}{N} + \frac{L_H}{N} + f + \Psi \frac{\dot{N}}{N} \]  \quad (29)

where in (29) \( \zeta = 1 - \alpha + \alpha^2 \) and I made use of the fact that, since the wage rate has to be the same in the final and intermediate goods sector, combining (10) and (12) and (15) together with the symmetry assumption, the constant allocation of workers across the two sectors is equal to:

\[ \frac{L_{xi}}{L_{yi}} = \frac{\alpha^2}{1 - \alpha}. \]  \quad (30)

Before starting to analyze the system dynamics, let us characterize the steady state. In order to find the equilibrium number of firms and the long-run rate of growth of the economy we need to analyze more closely the relationship between these two variables. The first thing to note is that steady state \( \dot{N}_i = 0 \) and, from the Euler equation, the rate of return must equal (28); also, since the labour allocation across sectors is constant, and (5) holds, \( r_i = \rho + (\alpha + \varphi) L_H L_{Hi} = \frac{\dot{w}}{w_i} = \gamma_{yi} = (\alpha + \varphi) \gamma_{Hi}. \)
Let us consider first only the incumbents’ behaviour. Combining (26), (28), (29) and (5) we obtain an inverse relationship between $\gamma_{Hi}$ and $N_i$:

$$\gamma_{SSA}^{Hi} = \frac{\alpha^2 \beta \left( \frac{L_i}{N_i} - f \right) - \zeta \rho}{\phi} \tag{31}$$

where $\phi = 1 - \alpha + 2\alpha^2$.

Setting profits, $\pi_{ij} = \pi_i = \left[ \frac{1 - \alpha}{\alpha} \frac{L_{xi}}{N_i} - \left( \frac{L_{Hi}}{N_i} + f \right) \right] w_i$, equal to zero and again using (29) and (5) we obtain another negative relationship between $\gamma_{Hi}$ and $N_i$ when firms make zero profits:

$$(25)\gamma_{ZPA}^{PA} = \alpha \beta (1 - \alpha) \frac{L_i}{N_i} - \beta f \tag{32}$$

The intersection between (31) and (32) gives $N_{i}^{max}$, that is the number of firms that will drive the incumbents’ profits to zero (see Fig. 2). This equilibrium will be stable only if the SSA curve is flatter than the ZPA curve. In fact, disregarding for a moment the entry cost, to the left of point $Z$, along the SSA curve, profits are positive and firms will tend to enter the market, whereas to the right of $Z$, along the SSA curve, profits are negative and firms will leave the market. At point $Z$, $\gamma_{Hi}^{min}$ and $N_{i}^{max}$ will be given respectively by:

$$\gamma_{Hi}^{min} = \frac{\zeta \left[ \alpha \beta f - (1 - \alpha) \rho \right]}{(1 - \alpha) \phi - \alpha} \tag{33}$$

$$N_{i}^{max} = \frac{\alpha \beta \left[ \phi (1 - \alpha) - \alpha \right]}{\zeta (\beta f - \rho)} L_i \tag{34}$$

We can now find the equilibrium with positive entry, when $r_E = r_{R&D} = r$. By combining (26), (27) and (5) we get:

$$\gamma_{EA}^{Hi} = \frac{\beta}{\Psi \beta - 1} \left[ \frac{\alpha \Psi \beta - (1 - \alpha)}{\alpha} \frac{L_{xi}}{N_i} + f \right] \tag{35}$$

substituting back into $r_{R&D}$ and setting $r_{R&D} = r$, using the fact that $\frac{w}{w} = y = (\alpha + \varphi) \gamma_{H}$, we find:

$$\frac{L_{xi}}{N_i} = \frac{\alpha (\Psi \beta - 1)}{\beta (1 - 2\alpha)} \left( \frac{\beta}{\Psi \beta - 1} f + \rho \right) \tag{36}$$
combining (35) with (29) we get:

$$\frac{L_{xi}}{N_i} = \frac{\alpha^2(\Psi \beta - 1)}{\phi \Psi \beta - 1} \left( \frac{L_i}{N_i} - \frac{\Psi \beta}{\Psi \beta - 1} f \right)$$

(37)

The steady-state number of firms with positive entry and R&D ($N^*$) is then easily found by setting equal (36) and (37) and solving for $\frac{L_i}{N_i}$; the steady-state growth rate ($\gamma_H^*$) is then found by inserting $\left( \frac{L_x}{N^*} \right)^*$ into (35).

From which:

$$N^*_i = \frac{\alpha (1 - 2\alpha)}{f + (\phi \Psi - 1/\beta) \rho} L_i$$

(38)

$$\gamma_H^* = \frac{\alpha \beta f + [\alpha \beta \Psi - (1 - \alpha)] \rho}{(1 - 2\alpha)}$$

(39)

Let us consider (39). When $\Psi \beta > 1 - \frac{\alpha}{\alpha}$, the free-entry equilibrium rate of growth is positively related to the rate of time preference ($\rho$). In fact, in this case the cost of process innovation is sufficiently below the entry cost, and therefore a higher rate of time preference will reduce the incentive to enter the market more than the incentive to invest in R&D, so that the rate of growth of the stock of knowledge increases. Moreover, the rate of growth is positively related to the productivity of the R&D sector ($\beta$), to the elasticity of substitution between differentiated intermediate goods ($\alpha$), to the size of fixed costs ($f$) and to the sunk entry cost ($\Psi$). Turning to (38), in fact, we find that higher values of $\beta$, $\alpha$, $f$, and $\Psi$, make entry less attractive, hence the equilibrium number of firms declines. It is also worth noting that the number of firms is directly proportional to the number of workers in the economy.

We may conclude that, in this model, as long as the two countries have the same parameters, the steady-state rate of growth will be the same, whereas the equilibrium number of firms will depend on the size of the economy.

We can now turn to analyze the system dynamics. The dynamics can be described by two differential equations (for the derivation see Appendix A),

$$\frac{\dot{v}_i}{v_i} = \frac{1}{\Psi B - 1} \left[ (\Psi \beta - 1) \rho + \beta f - \frac{\alpha \Psi \beta (1 - 2\alpha)}{v_i N_i} \right]$$

(40)

$$N_i = \begin{cases} 
\geq 0 & \text{if } V_{ij} \geq \Psi w_i \\
= 0 & \text{if } V_{ij} < \Psi w_i \text{ and } N < N_{\text{max}} 
\end{cases}$$

(41)

where $v_i = \frac{V_i}{Y_i}$.

**Proposition 1** Assume (a) $\alpha < 1/2$ and (b) $\Psi \beta > 1 - \frac{\alpha}{\alpha}$. Then,
1. There is a unique free-entry steady state with positive growth and a positive number of firms, \( N_i = N^*_i > 0 \) and \( \gamma^{SSA}_H = \gamma^{EA}_H = \gamma^*_H > 0 \);

2. There is a unique perfect foresight dynamic general equilibrium:

   - if \( N_i < N^*_i \), the economy jumps on the saddle path and converges over time to the free-entry steady state;
   - if \( N^*_i < N_i < N^\max_i \), the economy enters immediately a steady state with no entry.
   - If \( N_i > N^\max_i \), the economy jumps to the no-entry steady state corresponding to \( N^\max_i \).

Let us analyze the plausibility of the assumptions. \( \alpha \) represents the share of output that will be absorbed by the intermediate inputs as production factors. If we consider the set of intermediate inputs as physical capital, then from the standard Cobb-Douglas function estimates we generally obtain that this share is less than 1/2, and therefore the necessary condition seems plausible. The second restriction requires that the cost of improving on an existing good, \( \frac{1}{\beta} \), is sufficiently below the cost of introducing a new good, \( \Psi \), which also seems a plausible assumption.

The dynamics of the system is described in Fig.1. Along the VV locus \( \dot{N}_i = 0 \) and along the NN locus \( \dot{V}_i = 0 \). There are three different regions in which the behaviour of the system differs. If the economy starts out with a number of firms \( N_i(0) < N^*_i \) then it will jump on the saddle path and move along it until it reaches the equilibrium E. After \( N^*_i \) is reached there is no more entry. If \( N^*_i < N_i(0) < N^\max_i \), then the economy will be in steady state at that point along the VV locus. That is, the economy shows hysteresis because there is asymmetry between incumbents and entrants. The presence of overhead fixed costs implies that there will be a maximum number of firms above which profits become negative. When the value of the firm lies below the sunk entry cost (between E and Z on the VV locus) we would not observe entry. But, once in the market, a firm would not decide to exit unless profits were negative. At \( N^\max_i \) profits are zero and above \( N^\max_i \) profits are negative. Therefore, after \( N^\max_i \) we are in the region where \( \dot{N}_i < 0 \). If the economy starts with \( N_i(0) > N^\max_i \) it moves back to Z along the saddle path to the right of the \( N^\max_i \) region. Any other trajectory can be ruled out because it violates either the economy resource constraint or the rational expectation hypothesis.

The long-run growth rate of per capita output can derived taking the log and time derivative of (9) and (2) and making use of the result that in steady state

\[ v_i = \frac{\Psi L_{yi}}{bL_i}, \text{ with } c = \frac{\Psi}{b}. \]
state labour allocation is constant across sectors:\textsuperscript{22,23}

\[\gamma_{SSA}^{yi} = \varphi \gamma_{Hi}^{SSA} + \alpha \gamma_{xi}^{SSA} = (\varphi + \alpha) \gamma_{Hi} \]  

(42)

It is interesting to note that in the absence of international technology spillover and intermediate goods trade the two economies can grow at the same rate only if they both perform indigenous R&D. In this model, if a country does not innovate, the only way to obtain positive long-run growth is by importing a growing number of varieties; in fact, as shown in (9), increased specialization of intermediate inputs increases labour productivity; however, its growth rate would always be lower than that of the innovating country: the growth differential would be stable, whereas output levels would diverge.

2.4 Equilibrium with intermediate goods trade

In this section I extend the model allowing international trade of intermediate goods. I need, therefore, to change some of the equations. First of all, when intermediate goods are traded, each producer sells at home and abroad, therefore facing domestic and foreign competition. Let us assume for the moment that there is no overlap between the intermediate goods produced in the two countries. Firms will be symmetric within a country and will differ across countries if their unit cost of production differs. The total demand that each producer faces in country \(i\) will be now given by:

\[
\bar{x}_i = \alpha Y_w \frac{P_i^{1-\varepsilon}}{N_i P_i^{1-\varepsilon} + N_j P_j^{1-\varepsilon}}
\]  

(43)

Let us define \(s_y \equiv \frac{Y_B}{Y_w}\) as the share of country B in world production of good \(Y\), and

\[
s_x \equiv \frac{N_B P_B^{1-\varepsilon}}{N_B P_B^{1-\varepsilon} + N_A P_A^{1-\varepsilon}}
\]  

(44)

\textsuperscript{22}The constant allocation of labor across sectors in the long run can be easily verified from the labor market clearing condition:

\[L_y + \frac{\alpha^2}{1-\alpha} L_y + \frac{\gamma_H}{B} N_f + \dot{N} = L\]

in the long run, \(\gamma_H\) is constant, \(N\) is constant, and therefore \(\dot{N} = 0\), hence:

\[L_y = \frac{1-\alpha}{1-\alpha + \alpha^2} (L_i - \frac{\gamma_H}{B} + N_f)\]

which is a constant as well, given the assumption of a constant labor force.

\textsuperscript{23}If the labor force growth rate were positive \((\gamma_L > 0)\), we would subtract \(\alpha \gamma_L\) from the expression.
as the world share of country B in the intermediate goods production, with
\( \varepsilon = \frac{1}{1-\alpha} \) being the price elasticity of demand for intermediate goods. Then,
the balance of trade equilibrium reads:

\[
(Y_A - C_A) + \alpha (1 - s_x) s_y Y_w = (Y_B - C_B) + \alpha s_x (1 - s_y) Y_w \quad (45)
\]

Using the definitions of \( s_y \) and \( s_x \) and the market clearing conditions for
final and intermediate goods, the labour market constraint can be expressed as follows:\[24\]

\[
\frac{\alpha^2 s_x}{w_i N_i} Y_w + \frac{(1 - \alpha) s_y}{w_i N_i} Y_w + \left( \frac{L H_i}{N_i} + f \right) + \Psi \frac{N_i}{N_i} = L_i \quad (46)
\]

The rate of change of the country’s market share for the high-tech inputs \[25\] is given by:

\[
\frac{\dot{s}_x}{s_x} = (1 - s_x) \left[ \left( \frac{\dot{N}_i}{N_i} - \frac{\dot{N}_j}{N_j} \right) + (1 - \varepsilon) \left( \frac{\dot{w}_i}{w_i} - \frac{\dot{w}_j}{w_j} \right) - (\gamma_{H_i} - \gamma_{H_j}) \right] \quad (47)
\]

In order to understand the dynamics of the system it is important to under-
stand when and where it is possible to converge in the long run. We need,
therefore, to analyze how the rate of return to R&D and entry together with
the zero profit condition change with free trade in both countries. To this end
I will use one of the results of the model stated in the following proposition:\[26\]

**Proposition 2** If \( 0 < s_x < 1 \) balanced trade implies \( s_x = s_y \).

In steady state the number of firms must be constant in both countries and
the wage rates must grow at the rate of world output, that is:

\[
\frac{\dot{w}_i}{w_i} = \frac{\dot{w}_j}{w_j} = \gamma_y \gamma_w = [s_x \phi + (1 - s_x) \alpha] \gamma_{H_i} + [(1 - s_x) \varphi + s_x \alpha] \gamma_{H_j}, \quad (48)
\]

Inserting these two conditions into (47) I can state the following proposition:

---

\[24\] See appendix B for the derivation.
\[25\] See appendix B for the derivation.
\[26\] The proof is reported in appendix B.
Proposition 3 In the long run \( s_x \) tends to a positive constant if and only if the two countries grow at the same rate.

We know that in steady state, with positive R&D, \( r_{R&D} = \rho + \gamma_y \) must hold in both countries. By the same procedure we applied in the previous section we can derive \( \gamma^{SSFT}_{Hi} \), where FT means free trade. It is easy to verify that under these conditions \( \gamma^{SSFT}_{Hi} = \gamma^{SSA}_{Hi} \), which also implies that \( \gamma^{FT*}_{Hi} = \gamma^{A*}_{Hi} \). Moreover, since the zero profit condition under free trade is equivalent to that under autarchy, the free trade minimum rate of growth and maximum number of firms will also be the same as under autarchy in both economies. We can therefore conclude that when there is no production overlap, in order to converge to a steady state in which both countries produce a positive amount of intermediate goods and perform R&D they need to converge towards the same long-run rate of growth.

As to the dynamics, it is easy to verify from equations (12) and (43) that under the assumption of balanced trade and no variety overlap, since proposition 2 holds, intermediate goods firms face the same demand curve they faced in autarchy. Therefore, we would not observe any transition dynamics. Nonetheless, allowing trade in intermediate goods would have a positive effect on GDP levels, in that the number of intermediate goods that it is possible to employ in production would expand, the country that can access the greater number of varieties compared with autarchy would experience the greatest jump in GDP levels, but the two countries would never converge in levels.

If there is overlap in the range of intermediate goods produced by the two countries, when they start trading, by the assumption of Bertrand’s competition and the form of the demand function, only the lower-cost producer will survive. If, due to the gap in firm-specific (and hence average) knowledge between the two countries, the unit cost of intermediate goods is greater in \( B \) than in \( A \); then \( A \)’s firms producing in the same product lines as \( B \)’s firms can drive those firms’ sales to zero. In order for this to happen, the knowledge gap must be high enough.\(^{27}\) Combining (10) and (15) for the two countries, the unit cost will be higher in \( B \) if

\[
\frac{K_A}{K_B} > \left( \frac{N_A}{N_B} \right)^{\frac{1}{1 - \alpha - \phi}}
\]

We can further assume that switching from a production line to another implies the need to build firm-specific knowledge from the beginning; that is, once production is shut down, the firm has to pay the sunk entry cost again. Once trade is opened there is incentive for firms to specialize in different designs. Therefore, \( B \) firms will be forced to enter the production of new varieties. To understand the consequences of shutting down some monopolistic firms we need to check what is the effect on incumbents’ profits in the two countries. Let us suppose that at the time of opening up to trade the number of firms changes

\[^{27}\text{From (10) we know that the relative wage is given by } \frac{w_{1B}}{w_{1A}} = \left( \frac{K_B}{K_A} \right)^{\gamma} \frac{N_B}{N_A}.\]
in country $i$. By deriving the profit function of a monopolistic firm in country $i$ we get

$$\frac{\partial \pi_i}{\partial N_i} = \left[ \alpha(1-\alpha) \frac{s_{xi}Y_i}{N_i} \left( \frac{1-2s_{xi}}{N_i} - \frac{\gamma H_i + f}{\beta} \right) \frac{1-s_{xi}}{N_i} \right] w_i \leq 0 \quad (50)$$

The sign of equation (50) is not determined, but we can say that if country $i$ is big enough, that is if $s_{xi} > 1/2$, profits are negatively affected by an increase in the number of firms in the same country. When $N_i$ increases, the market share of each monopolistic firm decreases (negative effect on profits) but at the same time the demand for labour increases, thereby increasing the domestic wage rate and mark-up (positive effect on profits); moreover, when the number of firms increases, the number of specialized inputs also increases, which in turn increases the final good sector TFP with a positive feedback on intermediate input demand (positive effect on profits). The total effect on profits will depend on the relative strength of these forces. The bigger is the country’s world market share, the higher is the negative effect coming from the increased competition.

As far as concerns the other country, we can say that profits are negatively related to the number of firms in country $i$, that is

$$\frac{\partial \pi_j}{\partial N_i} = -\left( \frac{\gamma H_j + f}{\beta} \right) w_i \frac{s_{xi}}{N_i} < 0. \quad (51)$$

This result allows us to conclude that, when there exist overlaps in production, the backward country’s firms which have higher production costs must exit the market. The impact on A’s firms’ profits is positive; therefore, in this country there will be incentive to enter new production lines. In B the effect is uncertain. If B is large enough, incumbents’ profits will be positively affected and firms will enter new production lines; by contrast, if B is a small country, then profits might be negatively affected and the incentive to enter will be reduced and, due to the presence of fixed costs, we might even observe other firms exiting the market. A polarization effect could emerge, and a cumulative causation process could be generated. The pressure of new entrants in A will push wages up; in the short run wages will grow at a higher rate and the return to R&amp;D will also be pushed up. Equation (47) implies that the market share of A will grow larger and larger while the share of B will shrink. Still, if the variety overlap is not complete, there will be a positive jump in GDP levels in both countries.

To summarize, I find that in this model the relative size of a country is a significant feature in evaluating its ability to face increased international competition because country size affects the incentive that innovative firms face in the global market. In the absence of technological spillovers, large countries can adjust better than small countries to increased international competition.
3 Concluding remarks

This paper explores the consequences of trade liberalization between countries at different development levels and in the presence of increasing returns to scale in a model of endogenous growth with two types of innovations: cost-reducing innovation and product innovation. Here capital and labour are assumed to be immobile across countries and technology spillovers are only national in scope.

A first result that this framework, based on a stylized industrial organization model, delivers is that when we impose balanced trade and no production overlap the only way in which the two countries can reach a long-run equilibrium with positive production in the innovative sector is when they end up growing at the same rate. This, in turn, implies that if the two countries were already growing at the same rate before trade liberalization they will continue to do so and the only gain from trade will be static. When we introduce the possibility of production overlap and a wide gap in unit production costs, if the backward country is a relatively small country, domestic firms’ profits will be negatively affected, while foreign firms’ profits increase. Under these assumptions, a cumulative causation process might be generated and the advanced country’s market share expands while that of the backward country shrinks.

Therefore, we have found that even within our framework, as in Grossman Helpman (1991; chapter 8), country size matters in an open economy and yet both frameworks seem too rigid, needing many restrictive assumptions in order to get some kind of close-form solution that allows autarchy to be compared with free trade. The assumptions of capital and labour immobility across countries are very restrictive and not very realistic; the usual full employment and perfect mobility of labour across sectors are also highly unrealistic assumptions, yet many of the results are driven by them in this literature of trade and endogenous growth. As far as technology spillovers are concerned, by assuming their national scope, I wanted here to stress the beneficial role of indigenous R&D effort in the vertical dimension of innovation, that is the cost reduction innovations performed by incumbents, while the role of horizontal innovations, that is the variety expanding innovations introduced by new entrants, can and will exploited through international trade liberalization. If process innovation technological spillovers were international in scope, there would no longer be a need to talk of backward and advanced countries, and the share of a country in the world market would be simply determined by its size, which does not seem a realistic outcome either.

As far as possible extensions are concerned, first one could introduce labour market segmentation and distinguish between skilled and unskilled workers, with skilled workers employed in the R&D sector. In this case, the equilibrium number of firms will not be a function of the entire work-force but of the skilled workers employed in the economy. It would be also interesting to consider the case in which skilled workers are mobile across countries.\(^\text{28}\) This would certainly introduce a wage rigidity and cross-country resource reallocation that could be

\(^{28}\)On this last feature, in particular, recent studies (see for example Carrington and De-
interesting to explore in order to describe possible polarization processes of R&D investments. The other important extension would be to drop the assumption of balanced trade and consider international mobility of financial capital. This would certainly change dramatically the incentive to invest in one location or the other. The specification of entry costs could also be improved to let them differ once trade is opened and firms can choose the level of technology with which to enter the market.

tragiache (1998); Docquier (2006); Bugamelli and Marconi (2006)) show that skilled workers tend to be quite mobile world-wide and to show greater probability to emigrate compared with unskilled workers. In relative terms, the outflow of highly educated individuals is particularly significant for the most backward countries, such as in Central America and Africa.
4 Appendix A: derivation of the dynamics without trade

Equation (40) is derived from the arbitrage condition for entrants given by (20) and under the assumption that \( r_i = r_{R&D} = r_E \).

Profits for firm \( j \) in country \( i \) are given by:

\[
\pi_{ij} = \left[ 1 - \frac{\alpha}{\alpha} \frac{L_{eij}}{N_i} \right] \left( \frac{L_{Hi}}{N_i} + f \right) w_i \quad (A1)
\]

From the final good production function, the goods market clearing condition and the symmetry assumption we get

\[
x_{ij} = \alpha \frac{Y_i}{P_i N_i} = \frac{\alpha^2 Y_i}{w_i N_i} H_i \Rightarrow L_{eij} = \frac{\alpha^2 Y_i}{w_i N_i} H_i = L_{xij} \quad (A2)
\]

Inserting (A2) and (5) into (A1) and dividing by \( V_{ij} \) we get:

\[
\frac{\pi_{ij}}{V_{ij}} = \alpha (1 - \alpha) \frac{Y_i}{N_i} - \frac{1}{\Psi} \left( \frac{\gamma H_f}{\beta} + f \right) \quad (A3)
\]

Let’s define a new variable \( v_i = \frac{V_{ij}}{Y_i} \), then \( \frac{\dot{v}_i}{v_i} = \frac{\dot{V}_i}{V_i} - \gamma_y \), then by setting \( r_E = r_i \), (20) can be expressed as:

\[
\rho = \frac{\pi_{ij}}{V_{ij}} + \frac{\dot{v}_i}{v_i} \quad (A4)
\]

Substituting (A3) and (35) into (A4), (40) is obtained.

5 Appendix B: derivation of the dynamics with trade

Equation (46) is derived from the labour market clearing condition (23) using: the demand function that each domestic producer faces, given by equation (43), the definition of \( s_x \), given by (44), equation (10), which still holds when intermediate inputs are traded, and the fact that \( Y_i \), by definition, is equal to \( s_y Y_w \).
Equation (47) is derived by taking the time derivatives of (44):

\[
\dot{s}_x = \frac{\dot{N}_i P_i^{1-\varepsilon} + (1 - \varepsilon) N_i P_i^{1-\varepsilon} \dot{P}_i}{N_i P_i^{1-\varepsilon} + N_j P_j^{1-\varepsilon}} - \left\{ \frac{\dot{s}_x N_i P_i^{1-\varepsilon} + (1 - \varepsilon) N_i P_i^{1-\varepsilon} \dot{P}_i + \dot{N}_j P_j^{1-\varepsilon} + (1 - \varepsilon) N_j P_j^{1-\varepsilon} \dot{P}_j}{N_i P_i^{1-\varepsilon} + N_j P_j^{1-\varepsilon}} \right\}
\]

(B1)

Also, since

\[
\frac{1}{N_i P_i^{1-\varepsilon} + N_j P_j^{1-\varepsilon}} = \frac{s_x}{N_i P_i^{1-\varepsilon}} = \frac{1 - s_x}{N_j P_j^{1-\varepsilon}}.
\]

(B2)

substituting (B2) into (B1) and dividing left-hand and right-hand side by \( s_x \) we get equation (47).

\textbf{Proof of proposition 3.}

To prove proposition 3 we use the balanced trade condition given by equation (45) and the equilibrium condition given by equation (24). Combining those two we get:

\[
Y_i - (w_i L_i + \pi_{ij} N_i) + \alpha (1 - s_y) Y_w s_x = Y_j - (w_j L_j + \pi_{jj} N_j) + \alpha s_y Y_w (1 - s_x).
\]

(B3)

Let us consider the left-hand side of equation (B3). If we substitute in it the expression for profits and the labour market clearing condition we get:

\[
Y_i - (w_i L_i + \pi_{ij} N_i) - w_i \Psi \dot{N}_i + \alpha (1 - s_y) Y_w s_x = Y_i - \left( L_{yi} + \frac{1}{\alpha} L_{xi} + \Psi \dot{N}_i \right) w_i + \alpha (1 - s_y) Y_w s_x.
\]

(B4)

Using the fact that \( L_{yi} = \frac{(1 - \alpha) Y_i}{w_i} = \frac{1 - \alpha}{w_i} s_y Y_w \) and \( L_{xi} = \frac{\alpha^2}{w_i} s_y Y_w \) we get

\[
Y_i - (w_i L_i + \pi_{ij} N_i) - w_i \Psi \dot{N}_i + \alpha (1 - s_y) Y_w s_x =
\]

\[
s_y Y_w - \alpha s_x Y_w - (1 - \alpha) s_y Y_w + \alpha (1 - s_y) Y_w s_x =
\]

\[
\alpha s_y Y_w - \alpha s_x s_y Y_w
\]

(B5)

Plugging (B5) back into (B3) and using the fact that the right-hand side is going to be symmetric to the left-hand side:
\[ \alpha s_y (1 - s_x) Y_w = \alpha (1 - s_y) s_x Y_w \] (B6)

(B6) is satisfied if and only if \( s_x = s_y \).
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Figure 1:
Figure 2:
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