The CAPM and the risk appetite index: theoretical differences and empirical similarities

by M. Pericoli and M. Sbracia
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THE CAPM AND THE RISK APPETITE INDEX: THEORETICAL DIFFERENCES AND EMPIRICAL SIMILARITIES

by Marcello Pericoli* and Massimo Sbracia*

Abstract

This paper analyzes the Risk Appetite Index (RAI), a measure of investors’ risk aversion proposed by Kumar and Persaud (2001, 2002). We show that the RAI distinguishes between risk and risk aversion only under theoretically restrictive assumptions on the distribution of returns and the shocks affecting assets’ riskiness. However, by comparing the RAI with a measure of risk aversion derived from the CAPM — a model that does not require those restrictive assumptions — we find that estimates are surprisingly similar. We explain this result by proving that, under a certain condition, the RAI can approximate the risk aversion parameter of a CAPM. This occurs if the ratio between the variance of the returns on assets and the variance of the riskiness of assets is sufficiently small — a condition that is met in our sample.

JEL classification: G11, G12

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1. Introduction

In last few years, the behavior of international asset prices has often been explained on the basis of changes in risk aversion. For instance, according to the IMF, the decline in equity prices recorded between March 2002 and mid-March 2003 followed a sharp increase in investors’ risk aversion connected with the geopolitical tensions that culminated in the war in Iraq (IMF, 2003a). By the same token, both the IMF and the BIS explained the subsequent recovery in equity prices with a decline in risk aversion (see IMF, 2003b, and BIS, 2004). These explanations have been supported empirically with a variety of indicators, usually created by private financial analysts.

The favorable attitude of market analysts towards measuring risk aversion contrasts sharply with the general skepticism that prevails in academic research. For instance, the classical book on the economics of uncertainty by Laffont (1993) has an entire chapter on ‘Measuring Risk Aversion and Risk’, without a single reference to empirical works! In his seminal contribution to the theory of risk-bearing, Arrow (1970) infers the value of risk aversion parameters on the basis of the properties of Von Neumann and Morgenstern utility functions, without attempting any estimate. This skepticism derives not only from the unobservability of individuals’ preferences, but also from the observational equivalence between changes in risk and in risk aversion. This equivalence is due to the fact that an increase (decrease) in either of them causes asset prices to decline (rise) and risk premia to increase (decrease).

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2 Illing and Meyer (2005), Misina (2005), Gai and Vause (2004), and IMF (2002, box 3.1) provide valuable surveys of several atheoretic and model-based measures of risk aversion.

3 In fact, Laffont concludes: “It is of course difficult to obtain sufficient information about an agent’s preferences, to know whether his absolute risk aversion increases or decreases (since this requires information about the third derivative of his utility function).” (Laffont, 1993, p. 24, cited from Hartog et al., 2002).

4 Starting from the boundedness property of utility functions, Arrow (1970) concludes that the relative risk aversion should be approximately equal to one — a condition implying that preferences are represented by a logarithm function, as first suggested by Bernoulli (1738).
Kumar and Persaud (2001, 2002) have recently made an interesting attempt to break this observational equivalence by exploiting a special feature of asset pricing models. According to these authors, standard pricing models are such that changes in risk aversion modify the rank of the expected returns on assets relative to the rank of their riskiness, while changes in the riskiness of assets leave the relative ranks unchanged. Accordingly, they build an indicator of investors’ risk aversion, called the Risk Appetite Index (RAI), given by Spearman’s rank correlation between the expected excess return and the riskiness of a cross-section of assets.

In this work, we examine the RAI both theoretically and empirically. In the theoretical part, we refine a previous analysis of Misina (2003), who gathers the conditions under which the RAI can distinguish between risk and risk aversion into two propositions (Section 2). Next, building on Kumar and Persaud (2002) and Misina (2003), we examine the RAI in the context of the Capital Asset Pricing Model (CAPM) (Section 3). We focus on the CAPM because it is the workhorse of asset pricing theory. Its main prediction, that equilibrium expected returns are proportional to their covariance with the aggregate risk, is shared with virtually every other pricing model that has been taken to the data. We show that the RAI distinguishes between changes in the riskiness of assets and changes in investors’ risk aversion only under very restrictive assumptions. Specifically, we need to assume either that investors hold portfolios with equally weighted assets, or that asset returns are independent, that the shocks affecting riskiness are idiosyncratic and that the number of assets is sufficiently large.

Although these assumptions are theoretically restrictive, we need to verify empirically to what extent they bias the behavior of the RAI. We do so by comparing the RAI with a measure of risk aversion derived from the estimation of a CAPM — a model that does not require either independent returns or specific assumptions on the nature of the shocks. Using monthly data on all the stocks of the Dow Jones Euro Stoxx and those of the Standard & Poor’s 500 from January 1973 to November 2005, we show that the two estimates are surprisingly similar. By focusing on the statistical properties of the RAI, we are able to explain this result proving that, under a certain condition, the RAI can approximate the risk aversion parameter of a CAPM. This condition — which requires the ratio between the variance of asset returns and the variance of asset riskiness to be sufficiently small — is met in our sample (Section 4).

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5 Generally, it is the meaning of aggregate risk that differs across models: in the standard CAPM it is the return of the market portfolio; in Lucas (1978) and Breeden (1979) it is aggregate consumption.
In Section 5 we further discuss the usefulness of the RAI. We conclude that, despite the empirical similarities with the CAPM-based indicator, the theoretical limitations represent a serious shortcoming against the use of the RAI.

2. Theoretical foundations

The RAI is defined as the rank correlation (measured by Spearman’s coefficient) between the expected excess return and the riskiness of a cross-section of assets. The first element needed to build it is the expected excess return (excess return hereafter) on each asset \( i \), that we denote with \( R_{i}^{ex} \), \( i = 1, \ldots, n \). The excess return \( R_{i}^{ex} \) is the difference between the expected return on the risky asset \( i \), \( E(R_i) \), and the return on the risk free asset, \( R_f \).

\[
R_{i}^{ex} = E(R_i) - R_f .
\]

The expected return on asset \( i \), in turn, is defined as the expected price plus the expected dividend, a sum denoted with \( X_i \), over its current price \( P_i \); namely:

\[
E(R_i) = \frac{X_i}{P_i} .
\]

In asset pricing models, a modification in a structural parameter produces a change of current prices; this, in turn, causes changes in expected and excess returns. Thus, excess returns adjust to a new equilibrium through changes — with the opposite sign — in current asset prices \( P_i \).

The second element in the RAI is the riskiness of each asset \( i \), that we denote with \( \lambda_i \). In the following sections, the parameter \( \lambda_i \) will be defined precisely according to the asset pricing model considered. We will see that this definition is critical in order to assess the properties of the pricing model and, in turn, the appropriateness of the RAI as a measure of the market’s risk aversion.

As pointed out by Misina (2003), the RAI stems from an important property. We say that a change in a parameter of the asset pricing model that affects the excess returns on assets

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6 Hereafter \( E \) denotes the expectation operator, \( Var \) the variance operator, \( Cov \) the covariance operator and \( Corr \) the correlation operator. All these operators refer to unconditional moments. We also denote by \( sign \) the operator that takes the value 1 if its arguments share the same sign and \(-1\) otherwise.

7 Here we have adopted the standard definition of expected returns (see Cochrane, 2001, Chapter 1). Note that Kumar and Persaud (2001) define \( X_i \) as the long-run price of asset \( i \).
yields a rank effect if the following condition holds:

\[
\begin{align*}
\text{either } & \mathrm{sign} \left[ (\lambda_i - \lambda_j) \left( \Delta R^{ex}_i - \Delta R^{ex}_j \right) \right] = 1 \forall i \neq j \\
\text{or } & \mathrm{sign} \left[ (\lambda_i - \lambda_j) \left( \Delta R^{ex}_i - \Delta R^{ex}_j \right) \right] = -1 \forall i \neq j.
\end{align*}
\]

(3)

In words, Property (3) states that the “rank effect” obtains when a change in a parameter of the pricing model causes changes in the excess returns on each asset that are monotone (either increasing or decreasing) in their riskiness.\(^8\) Definition (3) leaves indeterminate both the sign of the change in expected returns (which can be positive or negative) and the sign of the monotonicity relationship between the change in excess return and the riskiness of each asset (increasing or decreasing). These indeterminacies give rise to four possible cases. We argue below that in all possible cases property (3) affects the rank correlation between excess returns and risks in a cross-section of assets.

In general, there are several measures of rank correlation between two variables (see Stuart and Ord, 1991, Chapter 26, for a brief overview). Following Kumar and Persaud (2001, 2002) in this paper we use Spearman’s measure of rank correlation (denoted with \(\rho^s\)) which takes values in the interval \([-1, 1]\); specifically, \(\rho^s = 1\) (\(\rho^s = -1\)) when the rank of the values of one variable is exactly the same as (inverse of) the rank of the values of the other variable.

In order to understand why the rank effect affects the rank correlation between \(R^{ex}\) and \(\lambda\), suppose that, before the rank effect shows up, the correlation is less than 1. This assumption means that there are at least two assets, say assets \(i\) and \(j\), such that \(\lambda_i > \lambda_j\) and \(R^{ex}_i < R^{ex}_j\). Hence, consider a change in a parameter that causes a rank effect and, for now, assume that this effect gives rise to an increase in excess returns (i.e. \(\Delta R^{ex}_i\) and \(\Delta R^{ex}_j\) are both positive). Suppose also that property (3) holds because we observe increasing changes in excess returns; namely, \(\Delta R^{ex}_i > \Delta R^{ex}_j > 0\). Then, if the increase in the excess return of asset \(i\) is sufficiently larger than the one of asset \(j\), it can reverse the relationship between excess returns into \(R^{ex}_i > R^{ex}_j\), thereby strengthening the rank correlation between excess returns and risks. Analogously, suppose that the rank effect gives rise to a decrease in excess returns (i.e.

\(^8\) The rank effect as defined by property (3) generalizes the one stated by Misina (2003), who considers only changes in excess returns that are increasing in the riskiness of assets. Our definition is more relevant to the analysis of the RAI, because, as we discuss in the rest of this section, we can show that the rank correlation between excess returns and risks is also modified by changes in excess returns which are decreasing in the riskiness of assets.
\(\Delta R^e_i\) and \(\Delta R^e_j\) are both negative) and that property (3) holds with \(\Delta R^e_j < \Delta R^e_i < 0\); then, if the decrease in \(R^e_j\) is sufficiently larger (in absolute terms) than the decrease in \(R^e_i\), the relationship between the excess returns on the two assets can be reversed into \(R^e_i > R^e_j\), and this will strengthen the rank correlation between excess returns and risks.

By the same token, assume that the rank correlation between excess returns and risks is larger than -1, so that there exist at least two assets, say assets \(i\) and \(j\), such that \(\lambda_i > \lambda_j\) and \(R^e_i > R^e_j\). Assume also that the change in the parameter determines a rank effect with decreasing and positive excess returns (i.e. \(\lambda_i > \lambda_j\) and \(\Delta R^e_i < \Delta R^e_j\) with \(\Delta R^e_i, \Delta R^e_j > 0\)). Then, the resulting rank correlation may weaken, as the change in excess returns can turn the relationship between \(R^e_i\) and \(R^e_j\) into \(R^e_i < R^e_j\). The same result obtains if \(\Delta R^e_i < \Delta R^e_j\) and \(\Delta R^e_i\) and \(\Delta R^e_j\) are both negative.

Thus, the rank effect — as defined by condition (3) — tends to modify (either strengthen or weaken) the rank correlation between excess returns and risks.

Now suppose that in an asset pricing model property (3) is fulfilled only by changes in the risk aversion parameter while changes in risk do not fulfill it. Then, we could exploit this property in order to discriminate between changes in risk and changes in risk aversion. Specifically, changes in asset prices that turn out to modify the rank correlation between excess returns and risks will be due to changes in risk aversion; moreover, if changes in asset prices do not modify the rank correlation between excess returns and risks, then they can be attributed to changes in risks.\(^9\)

Following Misina (2003), the conditions under which the RAI can be used to discriminate between changes in risk and changes in risk aversion can be conveniently gathered into two propositions. The first proposition specifies that changes in risk aversion have a rank effect; namely:

Proposition 1 A change in investors’ risk aversion has a rank effect on excess returns across different assets.

\(^9\) In order to identify correctly changes in risk aversion with changes in the rank correlation, it is also important that changes in risk aversion of opposite signs have opposite effects on the rank correlation. If this condition did not hold, in fact, the rank correlation, as measured by Spearman’s coefficient, would eventually go to one of its borders (1 or -1). On the contrary, when this condition holds we can identify an increase in risk aversion with, for instance, an increase in the rank correlation and vice versa. We thank a referee for pointing out the importance of this further condition.
Of course, this proposition cannot be verified empirically, since risk aversion is an unobservable parameter. Moreover, we cannot use its consequences on the rank correlation between excess returns and risks to detect changes in risk aversion because, in principle, a rank effect can show up for reasons other than changes in risk aversion. Therefore, we can introduce a second proposition that addresses both issues. Specifically, we can assume that only changes in risk aversion have a rank effect, or, equivalently, if we assume that risk and risk aversion are the sole parameters of the pricing model that can change over time, then the second proposition states that changes in risk do not have a rank effect:

Proposition 2. *A change in the riskiness of assets does not have a rank effect on excess returns across different assets.*

Thus, when both propositions hold, the rank effect can be used to break the observational equivalence between risk and risk aversion. Specifically, we can use the rank correlation between excess returns and risks to detect changes in investors’ risk aversion.

In the following section we will examine whether Propositions 1 and 2 can be proved in the context of a standard asset pricing model such as the CAPM.

3. The Risk Appetite Index and the CAPM

In Kumar and Persaud (2002), the authors motivate the RAI by considering changes in risk and in risk aversion in the CAPM. However, they focus only on the induced changes in the excess return on the market portfolio (i.e. on the optimal portfolio of the representative investor) and do not calculate explicitly the effect of the parameter changes on the excess return on each risky asset. The latter calculations are critical to verify whether Propositions 1 and 2 hold and, therefore, to establish whether the RAI can be a measure of changes in risk aversion.

Kumar and Persaud (2002) start their analysis with a well-known relationship between the excess return and the variance of the market portfolio. For the sake of simplicity, suppose that there are only risky assets in the market. If investors prefer frontier portfolios — defined as the portfolios with the minimum variance in the class of the portfolios with the same expected rate of return — then the following relationship holds (see Huang and Litzenberger, 1988,
Chapter 3):

\[
\sigma_m^2 = a \left[ E (R_m) \right]^2 + b E (R_m) + c ,
\]

where \( R_m \) is the stochastic return on the market portfolio, \( \sigma_m^2 \) denotes its variance, and \( a, b \) and \( c \) are constants which depend on the expected returns on each risky asset and the variance-covariance matrix of asset returns. Equation (4) defines the portfolio frontier — i.e. the locus of all frontier portfolios — which is a parabola in the \( \sigma_m^2 - E (R_m) \) space (the Risk-Return space).

![Figure 1: Effect of a change in the risk aversion](image)

The slope of the curve (4) is the investors’ risk aversion (see Kumar and Persaud, 2002, or Cochrane, 2001, Chapter 5). Hence, changes in risk aversion determine a shift of the optimal portfolio that modifies both the expected return and the variance of the market portfolio, as illustrated in Figure 1.

Kumar and Persaud (2002) also consider an alternative scenario in which a simultaneous change in the riskiness of all assets occurs. However, they focus on a very specific change: namely, one that gives rise only to a change in a single parameter of equation (4), that is the parameter \( c \) of the parabola. If \( c \) changes, say it goes from \( c \) to \( c' > c \), it modifies only the riskiness of the market portfolio, without changing its expected return. Figure 2 illustrates this effect.
By comparing the different consequences of these two scenarios, the authors conclude that changes in risk aversion modify the rank correlation between expected returns and risks, while a simultaneous increase in the riskiness of all assets does not affect it. This claim motivates the RAI as a measure of the market’s risk aversion.

![Figure 2: Effect of a simultaneous change in the riskiness of all assets](image)

By focusing only on the implications for the market portfolio of parameter changes, however, Kumar and Persaud neglect the implications for the excess returns of each risky asset, which are essential for the validity of the RAI. For instance, in the case represented in Figure 1, a change in the return of the market portfolio does not necessarily imply that the excess returns on assets change monotonically in their riskiness, as Proposition 1 requires. Similarly, the case represented in Figure 2 does not exclude that the change in the excess returns on assets yields a rank effect, as Proposition 2 establishes.\(^\text{10}\)

Thus, even within the very specific change in the riskiness of assets considered by Kumar and Persaud (2002), the validity of the RAI remains dubious. Therefore, we now turn to examine the consequences of changes in risk and risk aversion for the riskiness of each asset to verify whether Propositions 1 and 2 hold.

\(^{10}\) Similarly, Pericoli and Sbracia (2004) show that the RAI cannot distinguish, in general, between changes in risk and changes in risk aversion even in the ad hoc asset pricing model proposed by Kumar and Persaud (2001).
3.1 *The CAPM with exponential utility and normal returns*

Following Cochrane (2001, Chapter 9) and Misina (2003), and in order to set the notation, in this section we consider a standard CAPM with multivariate normal returns and identical investors with Constant Absolute Risk Aversion (CARA) preferences. These assumptions are by no means necessary for our conclusions on the RAI. In fact, Appendix A shows that if we focus on an appropriate measure of risk aversion, then a more general CAPM with heterogeneous agents and risk averse preferences would yield the same conclusions as those obtained with the simpler CAPM analyzed here.

Consider a single consumer, interpreted as a representative agent of a large number of identical consumers, with preferences given by the CARA utility:

\[ u(C) = -e^{-\gamma C}, \]

where \( \gamma \) is the Arrow-Pratt coefficient of absolute risk aversion.

This representative investor has initial wealth \( W \), which can be split between a risk free asset paying \( R_f \) and a set of \( n \) risky assets paying a stochastic return \( R = (R_1, ..., R_n) \). Let \( a = (a_1, ..., a_n) \) denote the amount of wealth invested in each asset \( i \) with \( a_i \in \mathbb{R}, \forall i = 1, ..., n \). The budget constraint then implies:

\[ W = a_f + \sum_{i=1}^{n} a_i, \]

while consumption is given by: \( C = a_f R_f + \sum_{i=1}^{n} a_i R_i. \)

We also assume that asset returns are multivariate normally distributed with mean \( E(R) = (E(R_1), ..., E(R_n)) \) and variance-covariance matrix \( \Sigma \):

\[ R \sim N(E(R), \Sigma). \]

---

\[ ^{11} \] We are implicitly assuming a two-period framework where agents invest in the first period, and, in the second period, returns are distributed and consumption occurs.
The hypothesis (7) implies that consumption, which is an affine transformation of multivariate normal returns, will be normally distributed:

\[ C \sim N(\mu_C, \sigma^2_C), \]

with \( \mu_C = a_f R_f + a' E(R) \) and \( \sigma^2_C = a' \Sigma a \). Hence, using a property of normal distributions we can write:

\[ E[u(C)] = E \left( -e^{-\gamma C} \right) = -e^{-\gamma \mu_C + \frac{\gamma^2}{2} \sigma^2_C}. \]

The representative consumer maximizes his expected utility (8) under the budget constraint (6). The first order conditions for this problem yield the following solution:

\[ \overline{\alpha} = \overline{\Sigma}^{-1} \frac{E(R) - 1 R_f}{\gamma}. \]

where \( \overline{\alpha} \) is the vector whose elements \( \alpha_i \) are the optimal amounts of wealth invested in each risky asset \( i \), and where \( \overline{1} \) is a vector of ones. Of course, the budget constraint (6) implies that the optimal amount of wealth invested in the risk free asset is: \( \overline{\alpha}_f = W - \overline{1}' \overline{\alpha} \). It is important to note that: (i) the strict concavity of the utility function (5) implies that the solution (9) is unique; (ii) each parameter \( \alpha_i \) is a solution of the problem for given parameter values \( R_f, \gamma \) and \( \Sigma \), and for given expected returns \( E(R) \).

Thus, the total return on the investor’s portfolio is \( \overline{\alpha}_f R_f + \overline{\alpha}' R \), where the latter addendum is the return on the risky portfolio, which we denote by \( R_m \). Note that the assumption of CARA preferences implies that the amount invested in each risky asset is independent of wealth. Hence, if investors were heterogeneous in their level of wealth, they would buy the same amounts of risky assets and different amounts of the risk free asset, the latter depending on investors’ level of wealth.

In order to obtain the standard formulation of the CAPM, note that: \( \text{Cov} (R, R_m) = \text{Cov} (R, \overline{\alpha}' R) = \Sigma \overline{\alpha} \). Denote with \( R^{ex} \) the vector of the excess returns on each asset, i.e. \( R^{ex} = E(R) - 1 R_f \). Then, rearranging expression (9), we obtain:

\[ R^{ex} = \gamma \cdot \text{Cov} (R, R_m). \]
3.2 *The rank effect*

We can now verify whether Propositions 1 and 2 hold; i.e. whether changes in the risk aversion parameter yield a rank effect (Proposition 1), while changes in risk do not cause a rank effect (Proposition 2).

We have noted above that the optimal coefficients $\pi$ defined by (9), from which the CAPM (10) originates, represent the unique solution of the consumer’s problem for given parameters $R_f$, $\gamma$ and $\Sigma$, and for given expected returns. Now suppose that the representative consumer holds the optimal portfolio $(\pi_f, \pi')$ with rate of return $\pi_f R_f + \pi' R$. We can ask what happens when one parameter changes. In equation (9), optimal quantities are obtained for given prices. Then, in this model only one variable between prices and quantities can change. Of course, it is reasonable to assume that, for a given optimal allocation $(\pi_f, \pi)$, the adjustment after a change in a parameter will occur through prices — i.e. through the excess returns $R_{ei}^e$. Recall, also, that for each asset $i$ an increase (decrease) in $R_{ei}^e$ occurs through a decrease (increase) in the asset price $P_i$. In other words, after a parameter change quantities remain fixed, equal to $\pi$, and prices, i.e. excess returns, adjust.

A preliminary step to verify whether Propositions 1 and 2 hold concerns the definition of the riskiness of each asset $i$. In the CAPM (see Cochrane, 2001, Chapter 1, or Misina, 2003), this is defined as:

\[
\lambda_i = Cov(R_i, R_m).
\]

Given (11), we can rewrite equation (10) as:

\[
R_{ei}^e = \gamma \lambda_i.
\]

Hence, consider a change in the risk aversion parameter $\gamma$:

\[
\frac{\partial R_{ei}^e}{\partial \gamma} = \lambda_i.
\]

Then, Proposition 1 is established in this model.
Now consider a change in the riskiness $\lambda_i$:

$$\frac{\partial R_{xi}^{ex}}{\partial \lambda_i} = \gamma$$

It would seem that Proposition 2 is established, because the derivatives $\partial R_{xi}^{ex} / \partial \lambda_i$ are constant for any $i$; then, a simultaneous increase in the riskiness of all assets does not seem to yield a rank effect. However, as Misina (2003) points out, the result that the derivatives of the excess returns on each asset with respect to its riskiness are constant does not necessarily establish Proposition 2: one has to consider explicitly which parameter has caused the increase in riskiness.

In order to prove that Proposition 2 does not hold, we just need to provide a counter example, which we borrow from Misina (2003).

**Example.** Assume that there are only two assets, denoted with $i$ and $j$, with variances $\sigma_i^2$ and $\sigma_j^2$ and covariance $\sigma_{ij}$. The CAPM is:

$$R_{xi}^{ex} = \gamma \left( \bar{\alpha}_i \sigma_i^2 + \bar{\alpha}_j \sigma_{ij} \right)$$

$$R_{xj}^{ex} = \gamma \left( \bar{\alpha}_j \sigma_j^2 + \bar{\alpha}_i \sigma_{ij} \right).$$

Suppose that the riskiness of asset $i$ changes because the covariance of asset $i$ with asset $j$ increases; then:

$$\frac{\partial R_{xi}^{ex}}{\partial \sigma_{ij}} = \gamma \bar{\alpha}_j.$$

Clearly, the change in covariance will also affect the riskiness of asset $j$; therefore:

$$\frac{\partial R_{xj}^{ex}}{\partial \sigma_{ij}} = \gamma \bar{\alpha}_i.$$

Now if $\bar{\alpha}_i \neq \bar{\alpha}_j$, then $\frac{\partial R_{xi}^{ex}}{\partial \sigma_{ij}} \neq \frac{\partial R_{xj}^{ex}}{\partial \sigma_{ij}}$, which gives rise to a rank effect.

This example highlights a general problem (see Misina, 2003): changes in the riskiness of one asset will affect expected returns also on other assets and will, in turn, be affected by changes in the riskiness of other assets. The previous example shows that this type of
dependence may give rise to a rank effect, unless all assets are equally weighted \((a_i = a_j \forall (i, j))\).

An obvious way to preclude the possibility of these patterns is to assume that asset returns are independent. In this case, the CAPM becomes:

\[
R_{ex}^i = \gamma \bar{a}_i \sigma_i^2 ,
\]

where \(\sigma_i^2 = Var(R_i)\). In this model, \(\lambda_i = \bar{a}_i \sigma_i^2\). It is immediately clear that a change in risk aversion yields a rank effect:

\[
\frac{\partial R_{ex}^i}{\partial \gamma} = \lambda_i .
\]

Consider, however, a change in \(\sigma_i^2\):

\[
\frac{\partial R_{ex}^i}{\partial \sigma_i^2} = \gamma \bar{a}_i ;
\]

this change, then, may still yield a rank effect. For instance, if there are only two assets, say assets \(i\) and \(j\), then a simultaneous change in their riskiness yields a rank effect as long as \(\bar{a}_i \neq \bar{a}_j\). Thus, even with independent returns, the RAI cannot discriminate between a change in risk aversion and a simultaneous change in the riskiness of all assets, unless \(a_i = a_j \forall (i, j)\).

Another possibility is to consider independent returns and an idiosyncratic shock, i.e. a change in the riskiness of only one asset, instead of a simultaneous increase in the riskiness of all assets. We can show that the rank correlation could be affected in this case as well. However, if the cross-section of assets is sufficiently large (as it should be in the CAPM), it is reasonable to presume that the change in the rank correlation following the change in the riskiness of one asset is small. In fact, consider an idiosyncratic shock to a single asset, say asset \(i\), and suppose that assets \(i\) and \(j\) are such that \(\lambda_i > \lambda_j\) while \(R_{ex}^i < R_{ex}^j\). If \(\bar{a}_i > 0\), an increase in \(\sigma_i^2\) causes an increase in \(\lambda_i\) which, therefore, does not change the inequality \(\lambda_i > \lambda_j\); in addition, however, it causes an increase in \(R_{ex}^i\) that could reverse the relationship with \(R_{ex}^j\) into \(R_{ex}^i > R_{ex}^j\), thereby increasing the rank correlation. However, since asset \(i\) is the only asset for which we observe some change (in both its riskiness and return), and given the independence of the returns on assets, we can expect that, if the cross-section of assets is large, the change in the rank correlation will be rather small.
3.3 When is the RAI a good measure of risk aversion?

The previous analysis has focused on the properties of the RAI in the context of the standard CAPM. This analysis shows that one can prove that the RAI distinguishes between changes in risk and changes in risk aversion only under very restrictive assumptions.

First, drawing on Misina (2003) we have shown that if asset returns covariate and the assets in the market portfolio are not equally weighted, the rank correlation between risks and returns is affected not only by changes in risk aversion but also by changes in the riskiness of all assets; therefore, the RAI cannot distinguish between a change in risk aversion and a simultaneous change in the riskiness of all assets. Second, we have shown that, with unequally weighted assets, the same conclusion applies even if returns are independent. However, with independent returns and unequally weighted assets, the RAI can distinguish between changes in risk aversion and idiosyncratic shocks to the riskiness of single assets, provided that the cross-section of assets is sufficiently large.

Thus, proving Propositions 1 and 2 in the context of a CAPM requires either equally weighted assets, or independent returns, idiosyncratic shocks to the riskiness, and a large cross-section of assets.

4. An application

Theoretically, the assumptions identified above under which the RAI can correctly measure risk aversion are very restrictive. Nonetheless, we should verify empirically to what extent departures from them bias the measure of risk aversion provided by the RAI. In principle, this task can be accomplished in two ways. One approach would consist in estimating, through Montecarlo simulations, the effects of departing from each assumption. This methodology, however, would be very complicated. In fact, one would need to take into account not only all the assumptions (e.g. number of assets, joint distribution of their returns and of the shocks that affect their riskiness, etc.), but also the several ways in which they may not be satisfied. Think, for example, of the hypothesis of independence of asset returns when there is a large number of different assets. Moreover, this effort may be not rewarding, since the assumptions that we have identified (either equally weighted assets, or independent returns, idiosyncratic shocks to the riskiness and a large cross-section of assets)
are only sufficient conditions for the RAI to work well. Hence, we cannot exclude that other sufficient conditions provide the same result.

An alternative method would consist in looking for an indicator that does not suffer from theoretical shortcomings, and comparing it with the RAI. This method, albeit less precise in theory, is more relevant for empirical applications. In this section we pursue this strategy and compare the RAI with a measure of risk aversion derived from the estimation of a CAPM.

To estimate the risk aversion parameter from a standard CAPM we use the classical methodology introduced by Fama and MacBeth (1973). In line with the tradition that estimates risk aversion on a cross-section of asset returns referred to long time periods (generally from 5 to 10 years), here, for each month \( t \), we run our estimates on a cross-section of monthly returns referred to the 5 years ending in \( t \).\(^{12}\) Thus, we obtain a measure of risk aversion that we can compare with the RAI. Our results will show that, in our sample, the dynamics of these estimates and that of the RAI are almost equivalent.

4.1 Methodology

In order to obtain an estimate of the risk aversion parameter \( \gamma_t \), we need to estimate the model:

\[
R_{i,t}^{ex} = \gamma_t \lambda_{i,t}
\]

(12)

on a cross-section of assets \( i, i = 1, ..., n \), at each time \( t \).\(^{13}\) The assets included in our analysis are all the stocks of the Dow Jones Euro Stoxx and those of the Standard & Poor’s 500; time periods are calendar months from January 1973 to November 2005; data is end-of-month; the source is Thomson Financial Datastream. To estimate equation (12) we need the excess returns \( R_{i,t}^{ex} \) and the covariance of asset \( i \) with the market portfolio at time \( t, \lambda_{i,t} \).

In order to determine the excess returns \( R_{i,t}^{ex} \) (which, we recall, are equal to \( E(R_{i,t}) - R_f \)) we have to find the expected returns \( E(R_{i,t}) \). A standard practice followed by the literature

\(^{12}\) For instance, in their classical studies, Black et al. (1972) consider non-overlapping 5-year periods; Fama and MacBeth (1973) use overlapping periods from 5 to 8 years; Sharpe (1965) uses a single 10-year period.

\(^{13}\) The parameter \( \gamma_t \) is the *Arrow-Pratt coefficient of absolute risk aversion* in the context of the theoretical model (10); it can be interpreted as an *aggregate relative risk aversion of the economy* in the context of the more general model (19) (see Appendix A).
is to use ex post returns, hereafter denoted, for the sake of simplicity, with $R_{i,t}$. By rational expectations, we can assume that:

$$R_{i,t} = E(R_{i,t}) + \varepsilon_{i,t},$$

where $\varepsilon_{i,t}$ is a white noise. Therefore, model (12) becomes:

$$R_{i,t} - R_{f,t} = \gamma_t \lambda_{i,t} + \varepsilon_{i,t},$$

where $R_{i,t} - R_{f,t} = R_{i,t}^{ex}$ is the (ex post) excess return, and $R_{f,t}$ is approximated with the 1-month interest rates for DM-denominated (euro-denominated from January 1999) deposits when we consider European stocks and dollar-denominated deposits when we consider US stocks.\(^{14}\)

In order to obtain the regressors $\lambda_{i,t}$ we use the first step of the “two-pass” procedure of Fama and MacBeth (1973). Hence, for each asset $i$ we estimate, using OLS, the following rolling regression:

$$R_{i,k}^{ex} = \alpha_i + \beta_{i,t}^{ex} R_{m,k}^{ex} + \eta_{i,k},$$

on a time-series of $h$ periods (i.e. with $k$ that goes from $t - h + 1$ to $t$), where $\alpha_i$ is an asset-specific constant, $\beta_{i,t}^{ex}$ is the asset “beta” (namely, it is the covariance between the return on asset $i$ and the return on market portfolio divided by the variance of the market portfolio), $R_{m,k}^{ex}$ is the excess return on the market portfolio, $\eta_{i,k}$ is the residual;\(^{15}\) in our benchmark regression we set $h = 60$ months. The outcome of this regression is a point estimate of the

\(^{14}\) To avoid introducing a more cumbersome notation we are using the same symbol $R_{i,t}^{ex}$ to denote the true and measured (i.e. ex post) excess returns.

\(^{15}\) We use the stock market indices (Dow Jones Euro Stoxx for the euro area and Standard & Poor’s 500 for the United States) as the market portfolios. Note that our application only includes stock prices. On this point, the Roll critique (Roll, 1977) pointed out that the validity of the model may depend on the assets included in the portfolio: the CAPM, in fact, should include all assets, tradable and non-tradable, tangible or intangible, that add to world wealth. However, Stambaugh (1982) builds a number of market portfolios, which included also government bonds, corporate bonds, Treasury bills, real estate and consumer durables, and finds that even when stocks represent only 10 per cent of the market portfolio, inferences about the model are the same as those obtained with a stocks-only index.
parameter $\hat{\beta}_{i,t}$ for each asset $i$ and time $t$.\textsuperscript{16} The product between the variance of $R_m$ in the $h$ months before period $t$ and $\hat{\beta}_{i,t}$ provides an estimate of $\lambda_{i,t}$, denoted with $\hat{\lambda}_{i,t}$.

We are now ready to estimate equation (13) on a cross-section of assets using GLS. Hence, for each time period $t$ we estimate:

$$R_{it}^{ex} = k_t + \gamma_t \hat{\lambda}_{i,t} + \varepsilon_{i,t},$$

where $\gamma_t$ is our measure of risk aversion, $k_t$ is a constant representing the difference between the true and the measured risk-free rate, and $\varepsilon_{i,t}$ is the residual.\textsuperscript{17} To address the issues of serial correlation and heteroskedasticity, we use the Newey-West estimator with the Bartlett window.

The estimated risk aversion $\hat{\gamma}_t$ has to be compared with the RAI. The latter is given, for each time $t$, by the rank correlation between risks and returns. Following Kumar and Persaud (2001, 2002), we measure the rank correlation with the Spearman’s coefficient, which we denote with $\rho_s$. Therefore, we consider the following measure of the RAI: $\rho_s \left( R_t, \hat{\lambda}_t \right)$, where we write $R_t$ and $\hat{\lambda}_t$ instead of $R_{i,t}$ and $\hat{\lambda}_{i,t}$ to underscore that the correlation is measured for a cross-section of returns and risks, for each time $t$. Consistently with the CAPM estimates, we consider returns and risks in the 60 months before $t$.\textsuperscript{18}

4.2 Results

Our estimates of $\hat{\gamma}_t$ and $\rho_s \left( R_t, \hat{\lambda}_t \right)$ for the euro area and the United States are illustrated in Figure 3, together with their confidence intervals.\textsuperscript{19} For the euro area, both indicators show

\textsuperscript{16} The proper specification of the CAPM requires that asset weights in the market portfolio do not change over time. In the stock market index that we have adopted as a proxy for the market portfolio, weights do change. However, given the large number of assets contained in that index, this is usually considered a good working approximation (see Ferson et al., 1987, for further discussions of this issue).

\textsuperscript{17} Recall that according to the Sharpe-Lintner version of the CAPM, $k_t$ should be zero. The Black version, instead, allows for $k_t \neq 0$; in this case, $k_t + R_{f,t}$ is the return on the zero-covariance portfolio.

\textsuperscript{18} Our application departs from the case study of Kumar and Persaud (2002) in three main respects: (i) we consider monthly stock returns instead of daily exchange-rate returns; (ii) we measure returns and risks over the same time period (while they consider non-overlapping time periods and obtain risks from the period that precedes the one used to compute returns); (iii) we measure risk with a covariance and not with a variance. Points (ii) and (iii), in particular, follow directly from the CAPM and the standard finance literature (see, Cochrane, 2001).

\textsuperscript{19} Confidence intervals are obtained from the GLS estimates for the CAPM risk aversion and from bootstrapping for the RAI. For the latter indicator, the bootstrapped standard errors are almost identical to the asymptotic standard errors of the linear correlation coefficient ($1/\sqrt{n-3}$, where $n$ is the sample size).
a steady decrease from 1982 to 1997, an increase until 2000, corresponding to the peak in world stock markets, and a renewed decline from then until the end of 2005; the average of the CAPM risk aversion coefficient is equal to 2.1. For the United States, results are somewhat different: the two indicators steadily increase from 1984 to 1997 and decrease thereafter, reaching negative values; the CAPM risk aversion coefficient has an average of 4.5.

The results on the average level of the CAPM risk aversion in the euro area and in the United States are in line with those of the literature (see the classic Friend and Blume, 1975, or, for a survey, Cochrane, 2001, Chapter 21 and the references therein). Instead, the values taken by the RAI, which is a correlation coefficient, are different, and are always included in the interval \([-1, 1]\). Recall from Section 2, however, that the RAI can only detect changes in risk aversion. Therefore, its level is not very informative. On the other hand, its dynamics are mostly relevant and, as Figure 3 shows, this turns out to be very similar to that of the CAPM-based indicator.

These results prove quite robust to several changes in the estimation strategy. For both stock indices, the behavior of risk aversion estimated with the two indicators has remained essentially the same after any change performed in our sensitivity analysis. First, we have focused on the assets included in the regressions. In fact, plugging directly an estimate of \(\lambda_{i,t}\) into equation (13) causes an errors-in-variable problem, which is usually addressed by gathering stocks into portfolios in order to increase the precision of the betas (Campbell et al., 1997, Chapter 5). Hence, along the lines of Fama and French (1992) we have considered: (i) 20 value-weighted portfolios obtained by ranking the stocks by size; and (ii) 25 portfolios obtained by ranking stock returns in 5 percentiles by size and 5 by book-to-market-value and considering their intersection.

Second, as an alternative method of addressing the problems of heteroskedasticity and correlation of the residuals, we have included in the regression two other explanatory variables; namely, the logarithm of market capitalization (Schwert, 1983) and the “systematic” skewness (Kraus and Litzenberger, 1976).\(^{20}\)

\(^{20}\) The “systematic” skewness is introduced in order to account for the possible effect of higher order moments of the utility function of the representative investor. Following Kraus and Litzenberger (1976), we compute it as: 

\[
E[(R_t - E(R_t))(R_{mt} - E(R_{mt}))^2]/E[(R_{mt} - E(R_{mt}))^3].
\]
Third, we have also experimented with different indices representing the market portfolio, such as indicators with equally-weighted stocks and indices with a subset of value-weighted stocks. Similarly, we have performed regressions using as assets only the stocks that were always listed in the two indices during the entire sample period.

Fourth, we experimented different sizes of the moving window in the time-series regressions, using $h = 36, 24$ and $12$ months. This set of robustness tests is the only one that has produced significant differences with respect to the benchmark regression. Specifically,
by shrinking the size of the moving window we have obtained, not surprisingly, more volatile
dynamics of the risk aversion.

Overall, the most striking and robust result is the strong correlation between the RAI and
the CAPM-based measure of risk aversion. The correlation between the two indicators over
the entire sample period is 0.88 for the euro area, and 0.75 for the United States. In addition,
both correlations are very large in any sub-period of our sample and have remained strong in
all our robustness checks; as a matter of fact, seldom have we obtained a correlation smaller
than 0.7. Thus, despite the theoretical differences discussed in Section 4, the RAI and the
CAPM-based measure of risk aversion provide essentially the same results. We devote the
next section to the explanation of this puzzle.

4.3 Explaining the similarities

To explain why the dynamics of $\hat{\gamma}_t$ are similar to those of $\rho^s \left( R_t, \hat{\lambda}_t \right)$ we need to focus
on the statistical properties of both indicators.

First, recall that Spearman’s rank correlation $\rho^s$ is a robust statistic for the linear
correlation $\rho$ (see Huber, 1981, Chapter 8, and Stuart and Ord, 1991, Chapter 26). Thus,
not surprisingly, it is $\rho^s \left( R_t, \hat{\lambda}_t \right) \simeq \rho \left( R_t, \hat{\lambda}_t \right)$ for any $t$.\footnote{For both samples, the correlation between $\rho$ and $\rho^s$ is about 0.97 in our benchmark, and remains very high in all the robustness tests performed.} Hence, our problem becomes that of explaining why the dynamics of $\hat{\gamma}_t$ are similar to those of $\rho \left( R_t, \hat{\lambda}_t \right)$.

Second, the OLS estimate of $\gamma_t$ in the cross-section of model (13) is:

$$\hat{\gamma}_t = \frac{\text{Cov}(R_t, \hat{\lambda}_t)}{\text{Var}(\hat{\lambda}_t)} ,$$

that we can rewrite as:

$$\hat{\gamma}_t = \rho \left( R_t, \hat{\lambda}_t \right) \cdot c_t , \quad (14)$$
where $\rho(R_t, \hat{\lambda}_t)$ denotes the linear correlation between $R_t$ and $\hat{\lambda}_t$ for the cross-section of assets at time $t$, and:

$$c_t = \left[ \frac{\text{Var}(R_t)}{\text{Var}(\hat{\lambda}_t)} \right]^{1/2}.$$ 

Equation (14) implies that if the variability of $c_t$ is sufficiently low with respect to that of $\rho(R_t, \hat{\lambda}_t)$, then the behavior of $\hat{\gamma}_t$ over time is dominated by that of $\rho(R_t, \hat{\lambda}_t)$. In other words, if $\text{Var}(c_t)$ is sufficiently low, then the dynamics of $\hat{\gamma}_t$ are similar to those of $\rho(R_t, \hat{\lambda}_t)$, which is the result we were looking for. The required condition is largely met in our two samples, as in both of them $\text{Var}(c_t)$ is about $10^4$ times smaller than $\text{Var}[\rho(R_t, \hat{\lambda}_t)]$.

To sum up, although the theoretical analysis performed in Sections 3 and 4 suggested that one needs very restrictive assumptions to prove that the RAI is an indicator of risk aversion, the evidence illustrated in Figure 3 shows that the RAI could be a proxy of a standard CAPM-based risk aversion indicator. The similarity between $\rho^* (R_t, \hat{\lambda}_t)$ and $\hat{\gamma}_t$ stems from two main elements: (i) the rank correlation is approximately equal to the linear correlation; (ii) provided that a certain variance is sufficiently low, the dynamics of the regression coefficient in the equation that explains returns with risks are approximately equal to those of the linear correlation between returns and risks.

5. Conclusion

This paper provides a theoretical and an empirical analysis of the Risk Appetite Index of Kumar and Persaud (2001, 2002). The theoretical analysis shows that, in general, the RAI cannot correctly identify risk aversion in a standard CAPM. In fact, if the assets in the market portfolio are not equally weighted, the RAI requires very restrictive assumptions, such as independent asset returns, idiosyncratic shocks to riskiness and a sufficiently large cross-section of assets.

Although these assumptions are theoretically restrictive, it is necessary to evaluate empirically the extent to which they tend to bias the RAI. A simple way to accomplish this task is to compare the RAI with a measure of risk aversion derived from the estimation of a CAPM. Hence, we consider as a case study the stocks of the Dow Jones Euro Stoxx and those
of the Standard & Poor’s 500 from January 1973 to November 2005. This comparison shows that the behavior of these two indicators is surprisingly similar.

By focusing on the statistical properties of the RAI, we prove that, under a certain condition, this indicator may provide a good approximation of the risk aversion parameter of a CAPM. This condition requires the ratio between the variance of the return on assets and the variance of their riskiness to be approximately constant.

Thus, despite the sharp theoretical differences, the RAI shares strong empirical similarities with the CAPM indicator. It is worth noticing, however, that the use of the RAI does not yield any real benefit while, at the same time, it entails several shortcomings. First, with respect to a CAPM indicator, it is not computationally easier to calculate. Second, it is not derived from the estimation of an equation and, hence, it is not possible to evaluate the goodness of fit or assess the reliability of the underlying model. On the other hand, from the analysis performed in Section 4 it follows that phases of sharp increase in the ratio between the cross-sectional variance of the returns on assets and the cross-sectional variance of their riskiness would distort the RAI. Therefore, one should always be careful and preliminarily check whether the variance ratio is sufficiently low over the entire sample period. Another drawback is given by the fact that the RAI can only detect changes in risk aversion, while other available measures provide an assessment of its very level, a more comprehensive piece of information.
Appendix: Heterogenous agents with risk averse preferences

Section 4 was based on the restrictive assumptions that agents are identical and that their preferences are given by CARA utility functions. Following Huang and Litzenberger (1988), here we discuss a more general setting where agents are heterogeneous in both preferences and wealth. In this more general model our conclusions on the RAI will remain essentially unchanged once we focus on an appropriately specified global risk aversion parameter. This is because this setting leads to a version of the CAPM that has the same functional form as equation (10).

We assume that preferences are represented by increasing and concave utility functions. With respect to the previous section, we will maintain the assumption that asset returns are multivariate normally distributed. In addition, we will redefine the problem in terms of shares of wealth rather than in value terms. Namely, denote by $a_{i,h}$ the amount of wealth invested in asset $i$ by investor $h$; then, the share of investor $h$’s wealth, $W_h$, invested in that asset is:

$$w_{i,h} = \frac{a_{i,h}}{W_h}$$

The total wealth of the $N$ investors in the economy is $W_m = \sum_{h=1}^{N} W_h$. In equilibrium, the total wealth $W_m$ is equal to the total value of the assets. We denote with $w_{i,m}$ the portfolio weight of asset $i$ in the market portfolio, namely:

$$w_{i,m} = \sum_{h=1}^{N} \frac{W_h}{W_m} \cdot w_{i,h}$$

Using the budget constraint (6), we can rewrite the consumption $C_h = a_{f,h}R_f + \sum_{i=1}^{n} a_{i,h}R_i$ of the investor $h$ endowed with wealth $W_h$ as:

$$C_h = \left( W_h - \sum_{i=1}^{n} w_{i,h}W_h \right) R_f + \sum_{i=1}^{n} w_{i,h}W_hR_i = W_h \left[ R_f + \sum_{i=1}^{n} w_{i,h}(R_i - R_f) \right]$$  (15)
The maximization problem of this investor, whose preferences are represented by the increasing and concave function $u_h \in \mathbb{C}^2$, becomes:

$$
\max_{a'} \left\{ E \left[ u_h \left( W_h R_f + \sum_{i=1}^{n} w_{i,h} W_h (R_i - R_f) \right) \right]\right\}.
$$

(16)

Let us assume that a solution to (16) exists. Since $u_h$ is concave, the first order (necessary) conditions are also sufficient and are:

$$
E \left[ u'_h \left( W_h R_f + \sum_{i=1}^{n} w_{i,h} W_h (R_i - R_f) \right) \cdot (R_i - R_f) \right] = 0 \ \forall i = 1, ..., n,
$$

(17)

where the coefficients $w_{i,h}$ are the optimal shares of wealth invested in asset $i$ by individual $h$. The optimal consumption of this investor then is:

$$
\bar{C}_h = W_h R_f + \sum_{i=1}^{n} w_{i,h} W_h (R_i - R_f).
$$

Using the definition of covariance, we can rewrite condition (17) as:

$$
E \left[ u'_h (\bar{C}_h) \right] \cdot E (R_i - R_f) + Cov \left[ u'_h (\bar{C}_h), R_i \right] = 0.
$$

In addition, using Stein’s lemma we find:

$$
Cov \left[ u'_h (\bar{C}_h), R_i \right] = E \left[ u''_h (\bar{C}_h) \right] \cdot Cov (\bar{C}_h, R_i)
$$

which we can substitute back into the previous expression and, recalling that $E (R_i - R_f) = R_i^{ex}$, we have:

$$
E \left[ u'_h (\bar{C}_h) \right] \cdot R_i^{ex} = -E \left[ u''_h (\bar{C}_h) \right] \cdot Cov (\bar{C}_h, R_i).
$$

(18)

We can now define the global absolute risk aversion of the investor $h$ as:

$$
\Gamma_h = -\frac{E \left[ u''_h (\bar{C}_h) \right]}{E \left[ u'_h (\bar{C}_h) \right]}.
$$
Dividing both terms of (18) by $E\left[u''_h(C_h)\right]$, summing across the $N$ investors and rearranging we obtain:

$$R^e_{i} = \left(\sum_{h=1}^{N} \Gamma^{-1}_h\right)^{-1} \cdot \sum_{h=1}^{N} Cov\left(C_h, R_i\right).$$

Note that the first term in brackets on the right-hand side is the harmonic mean of the investors’ global absolute risk aversions. Moreover, we can write:

$$\sum_{h=1}^{N} Cov\left(C_h, R_i\right) = Cov\left(\sum_{h=1}^{N} C_h, R_i\right) = Cov\left(\sum_{i=1}^{n} \sum_{h=1}^{N} w_{i,h} W_h (R_i - R_f), R_i\right) = Cov\left(\sum_{i=1}^{n} \sum_{h=1}^{N} w_{i,h} W_h R_i, R_i\right) = W_m Cov\left(R_m, R_i\right).$$

Substituting into the previous expression and putting

$$\Gamma = W_m \left(\sum_{h=1}^{N} \Gamma^{-1}_h\right)^{-1},$$

which represent a sort of aggregate relative risk aversion of the economy, we get:

$$R^e_{i} = \Gamma \cdot Cov\left(R_i, R_m\right),$$

which has exactly the same functional form as (10).

Equation (10) shows that our conclusions on the risk appetite index do not depend on the specific hypothesis made for the investors’ preferences: as long as asset returns are multivariate normally distributed (a hypothesis that cannot be rejected for monthly and quarterly data) they hold for any non-satiated and risk averse preferences.
References


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