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Prices, product differentiation and quality measurement:
a comparison between hedonic and matched model methods

by Gian Maria Tomat

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PRICES, PRODUCT DIFFERENTIATION AND QUALITY MEASUREMENT: A COMPARISON BETWEEN HEDONIC AND MATCHED MODEL METHODS

by Gian Maria Tomat*

Abstract

The paper provides an analysis of the problems of construction of quality-adjusted price indexes within the framework of the theory of product differentiation. In the general case of price-making behaviour on the part of firms, hedonic regressions are defined on the basis of reduced forms of the equation relating equilibrium prices to product characteristics. The paper considers the reduced form given by the marginal cost function and shows that the Laspeyres hedonic price index provides a lower bound to the quality-adjusted rate of price change while the Paasche hedonic price index provides an upper bound to the quality-adjusted rate of price change. The properties of hedonic price indexes are compared with those of matched model indexes. The theory is applied to the study of personal computer prices in Italy during the 1995-2000 period.

JEL classification: C35, C43.
Keywords: discrete choice, price indexes, product quality, personal computer.

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* Bank of Italy, Research Department, Rome Branch.
1. Introduction

The problem of how to account for quality change in the construction of measures of the rate of price change has been for a long time an important subject of research in economic measurement. The subject is of central importance for many fields of economic analysis, ranging from the study of movements of real economic activity to the problem of indexing nominal wages.

The purpose of this paper is to provide an analysis of the problems of construction of quality-adjusted price indexes from the point of view of the theory of product differentiation. The original developments of the theory provided by Becker (1965) and Lancaster (1966) emphasized how it opened a new perspective on the treatment of new goods. Since the consumer choice problem was defined in the space of product characteristics, the introduction of new goods and the disappearance of old ones could be modeled in terms of changes in the consumption possibilities set, simplifying the problem of making welfare comparisons.

Work in this field has subsequently benefited from several contributions from the industrial organization literature. Rosen (1974) provided an analysis of the product differentiation framework in a competitive equilibrium setting, modeling both consumer and firm choices as a function of product characteristics. Bresnahan (1981, 1987) extended the framework to allow for price-making behaviour on the part of firms and showed how the

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2 Much of the recent interest in the problems of price measurement developed after the publication of the Boskin et. al. (1996) report, which analyzed the problems of measurement of the U.S. CPI. Recent reviews of the literature on price measurement include Lebow and Rudd (2003) and Wynne and Rodriguez-Palenzuela (2004). The latter work analyzes the problems of price measurement in the context of the European institutional framework.
model could be used to study the character of industry competition. More recently, Berry, Levinsohn and Pakes (1995, 2004) have provided further extensions of the basic model, drawing, among other subjects, on the field of discrete choice theory.

The implications of the theory of product differentiation for the construction of quality-adjusted price indexes, were analyzed initially by Griliches (1971), who used the theory to provide a rationale for the adoption of hedonic price indexes. Some of the implications of the more recent developments of the theory for the construction of hedonic price indexes are illustrated by Pakes (2003)\textsuperscript{4}.

The analysis of the properties of hedonic price indexes has so far been carried out on the basis of reduced forms of the hedonic function. The reduced form, however, has several problems of identification. In particular, in the reduced form demand and supply parameters are not identified and the unobservable components are generally characterized by cross-section dependence. These two features pose some problems of interpretation of estimates of the coefficients of hedonic regressions that should have implications for the possibility of using hedonic price indexes as measures of the quality-adjusted rate of price change.

In related contributions Pakes, Berry and Levinsohn (1993) and Nevo (2003) consider the problem of making exact welfare comparisons on the basis of the estimated parameters of consumer preferences in a discrete choice model. This approach has been used in these works to study problems such as the effect on consumer welfare of changes in environmental regulation or of the introduction of a new brand of a product. Specifying the consumer’s choice problem in the space of product characteristics, simplifies the analysis in comparison with analogous problems studied on the grounds of traditional demand theory, as is done for example in works by Hausman (1997a, 1997b, 1999). However, the results of the above

\textsuperscript{3} Early versions of the theory of product differentiation were analyzed by Gorman (1956) and Dorfman, Samuelson and Solow (1958) in the form of the model known as the linear characteristics model.

\textsuperscript{4} Seminal works using hedonic price indexes to construct quality-adjusted price measures were provided during the 1960s by Griliches (1961), Triplett (1969) and Chow (1967). Interestingly, the former works were concerned with the measurement of prices of automobiles while the latter was concerned with computer prices and with the implications of price measurement for the study of the diffusion of computers in the U.S. economy. The literature that developed after these works is broad. Collection of studies developed for several sectors of the economy can be found in Jorgenson and Landau (1989), Gordon (1990) and Foss, Manser and Young (1993).
studies show that the approach can at most generate upper and lower bounds to the true price indexes and the empirical evidence shows that these bounds are usually quite wide.

The present paper follows a different line of research, it takes as point of departure the definition of price index provided by the hedonic regression approach and analyzes the properties of hedonic price indexes, compiled on the basis of estimates of the cost function of a partial equilibrium model. In this way the paper provides a partial resolution to the problems of identification that characterize hedonic regression models. The analysis shows that the Laspeyres hedonic price index provides on average a lower bound to the theoretical index while the Paasche hedonic price index provides an upper bound to the theoretical index.

The paper also illustrates how the proposed definitions of Laspeyres and Paasche price indexes can be used to study the behaviour of matched model indexes. It is shown that if matched products in reference and comparison periods are relatively close in characteristics space, the matched model indexes should not be characterized by serious distortions and should therefore be considered a useful alternative to the proposed hedonic indexes.

The analysis is applied to the study of personal computer prices in Italy during the 1995-2000 period. The paper is structured as follows. Section 2 reviews the theoretical framework. Section 3 uses the theory to define hedonic price indexes and analyzes their properties. Section 4 presents the data used for the analysis. Section 5 presents estimates of the partial equilibrium model used for the application with Italian personal computer data. Section 6 presents hedonic price indexes for personal computers in Italy in 1995-2000. Section 7 discusses the implications of the proposed framework for the compilation of matched model indexes and presents matched model indexes for personal computers in Italy for 1995-2000. Section 8 draws some conclusions.

2. The model

Following Berry (1994) and Berry, Levinsohn and Pakes (BLP) (1995, 2004) we consider a market for a differentiated commodity where there are $M$ consumers and $N$ varieties of the product. Different firms in the market produce different varieties of the product and each variety in the market is identified by a set of performance characteristics.
Consumers and firms observe all the product characteristics relevant for their choices, although some of the characteristics entering both consumer preferences and firm costs are unobserved by the econometrician. Let $x_j \in \mathbb{R}^K$ denote the observable characteristics of product $j$, $\xi_j \in \mathbb{R}$ the unobservable product characteristics influencing consumer choice, $\omega_j \in \mathbb{R}$ the unobservable characteristics entering the firm cost function and $p_j \in \mathbb{R}$ the price of product $j$, for $j \in \{0,1,\ldots,N\}$, where $j=0$ denotes the outside alternative. For convenience, the price and characteristics of the outside alternative are normalized to zero and thus the market characteristics vectors and the market price vector are denoted as $x=(x_1,\ldots,x_N)\in \mathbb{R}^{NK}$, $\xi=(\xi_1,\ldots,\xi_N)\in \mathbb{R}^N$, $\omega=(\omega_1,\ldots,\omega_N)\in \mathbb{R}^N$ and $p=(p_1,\ldots,p_N)\in \mathbb{R}^N$.

Consumers have heterogeneous preferences thus in each time period only a fraction of them will consume each product $j$. The market share function is denoted by $s_j = s_j(x, \xi_j, p, \theta)$, for $j \in \{0,1,\ldots,N\}$, where $\theta$ is a parameter representing consumer preferences. The market share of each product depends on the vectors of prices and characteristics of all products in the market and is derived in relation to the subset of the population of consumers for which consumption of product $j$ is preference maximizing. The preference relation is defined in a way such that, for every vector $(x, \xi, p)\in \mathbb{R}^{N(K+2)}$ of prices and product characteristics, the market share of each product is greater than zero, market shares add up to unity and the market share function is continuously differentiable.$^5$

Given that there are $M$ consumers in the market, the demand for product $j$ is given by:

\[
D_j(x, \xi, p, \theta) = Ms_j(x, \xi_j, p, \theta)
\]

Each product in the market is produced by a different firm. Since the model is used to describe the market for personal computers, which is a relatively young industry with many competing firms, this assumption does not appear to be restrictive in the present context. Marginal costs of production are a function of performance characteristics and the marginal

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$^5$ The distribution of preferences determining the market shares outcome is defined by a distribution of individual characteristics within the population of consumers. Formal derivations of the market share functions can be found in the literature for several specifications of the preference structure. See for example McFadden (1981) or Anderson, de Palma and Thisse (1992). In the more recent BLP (1995, 2004) framework, closed-form solutions of the market share functions cannot be derived and the analysis has to resort to more advanced methods, such as equilibrium analysis and simulation.
cost function is denoted as \( c(x_j, \omega, \gamma) \), where \( \gamma \in \mathbb{R}^K \) is a cost parameter. The marginal cost function is continuous in the characteristics vector \((x_j, \omega) \in \mathbb{R}^{K+1}\) and in the parameter \( \gamma \in \mathbb{R}^K \).

In the general formulation of the model, the equilibrium is characterized by Bertrand pricing, so that firms set prices in order to maximize profits, taking prices of other firms as given. In equilibrium, price is equal to marginal cost plus a mark-up term, which depends on the price elasticity of demand:

\[
p_j = c(x_j, \omega_j, \gamma) + \frac{D_j(x_j, \xi_j, p, \theta)}{\partial p_j}
\]

(2.2)

The pricing equation forms the basis for the analysis of the hedonic price indexes. Since it relates price to product characteristics, it is equivalent to a hedonic regression function. However, in the general case of price-making behaviour on the part of firms, the pricing equation implies that in equilibrium the price of each product depends not only on its own characteristics but also on the price and characteristics of all the other products in the market. In prior research the hedonic regression analysis has been carried out mainly using reduced forms of the pricing equation, giving the price of each product as a function of its own performance characteristics. The next sections propose to use the cost function as a reduced form and analyze the properties of the price and quantity indexes resulting from this choice.

3. Marginal costs and hedonic price indexes

In order to define the hedonic price indexes consider two subsequent periods of time, a comparison period \( t-1 \) and a reference period \( t \), and the question of how to compensate consumers for the price changes that occur between period \( t-1 \) and period \( t \).

Consider initially defining the compensating variation for consumers of period \( t-1 \) product \( j \) as the difference between the price predicted for product \( j \) in period \( t \) on the grounds of period \( t \) cost conditions and its actual period \( t-1 \) price:

\[
CV_j(x_{jt-1}, p_{jt-1}, \gamma_t) = E(c(x_{jt-1}, \omega_{jt-1}, \gamma_t) \mid x_{jt-1}) - p_{jt-1}
\]

(3.1)
In (3.1) \( x_{jt-1} \) and \( p_{jt-1} \) are the observable characteristics and the market price of product \( j \) in period \( t-1 \), \( w_{jt-1} \) is the unobservable cost component in period \( t-1 \) and \( \gamma_t \) is the cost parameter in period \( t \). The price predicted for period \( t-1 \) product \( j \) in period \( t \) is defined as the expected value of the marginal cost of production, given the observable characteristics of the product and the period \( t \) cost parameter. Given the set of expenditures prevailing in period \( t-1 \) and the set of prices prevailing in period \( t \), equation (3.1) provides an estimate of the additional expenditure that consumers of product \( j \) should make in period \( t \) to consume the same product they were consuming in period \( t-1 \). Following Pakes (2003), note that since the set of goods changes between the two periods, if consumers are compensated for the price difference resulting from the changed market conditions, they may be able to make a different choice and improve their personal well-being. This definition of the compensation function can be applied properly only to period \( t-1 \) products that are within period \( t \) goods space, since it is based on the assumption that for each period \( t-1 \) product \( j \) there exists in period \( t \) at least one product with similar characteristics, so that (3.1) defines a meaningful compensation function.

An aggregate price index can be defined by averaging the compensating variations defined by equation (3.1) across the varieties of the differentiated commodity available in the comparison period that are within the reference period goods space. Let \( I_{\leq} \subseteq \{1, \ldots, N\} \) index the relevant comparison period goods space and \( N_{\leq} \leq N \) denote the number of comparison period varieties in the reference period goods space. Giving an equal weight to each variety the Laspeyres hedonic price index is defined as:

\[
(3.2) \quad P_{HL}(x_{t-1}, p_{t-1}, \gamma_t) = \frac{1}{N_{\leq}} \sum_{j \in I_{\leq}} CV_j(x_{jt-1}, p_{jt-1}, \gamma_t)
\]

Since in Bertrand pricing equilibrium the mark-up is in general greater than zero, it can be shown that equation (3.2) defines on average a lower bound to the true theoretical compensation function. Defining the compensating variation as a function of the observable cost component, one abstract from two other components of the true compensating price, an unobservable cost component and the mark-up term. The unobservable cost component is on average equal to zero and thus the mark-up term is what ultimately matters in defining
the compensation error. Since by construction the mark-up is always greater than or equal to zero, so is the average compensation error.

**Proposition 3.1:** In the Bertrand pricing equilibrium defined by equation (2.2), the Laspeyres hedonic price index defined by equation (3.2) is a lower bound to the true price index for the average consumer of product \( j \) in period \( t-1 \).

**Proof:** mathematical appendix p. 33.

Next consider the problem from the perspective of period \( t \) consumers, in this case the compensation function is defined from the equivalent variation:

\[
EV_j(x_{jt}, p_{jt}, \gamma_{t-1}) = p_{jt} - E(c(x_{jt}, \omega_{jt}, \gamma_{t-1} | x_{jt})
\]

where \( x_p \) and \( p_p \) are the observable characteristics and the market price of product \( j \) in period \( t \), \( \omega_p \) is the unobservable cost component in period \( t \) and \( \gamma_{t-1} \) is the cost parameter in period \( t-1 \). The price predicted for period \( t \) product \( j \) in period \( t-1 \) is defined as the expected value of the marginal cost of production, given the observable characteristics of the product and the period \( t-1 \) cost parameter. Given the set of expenditures prevailing in period \( t \) and the set of prices prevailing in period \( t-1 \), equation (3.3) provides an estimate of the change in income that would allow period \( t \) consumers of product \( j \) to consume the same product in period \( t-1 \) and in period \( t \). We note again that since the set of goods changes between the two periods of time, consumers are allowed to make choices that improve their personal well-being in period \( t-1 \). The equivalent variation can be applied properly only to period \( t \) products that are within period \( t-1 \) goods space, since it is based on the assumption that for each period \( t \) product \( j \) there exist in period \( t-1 \) at least one product with similar characteristics.

To define an aggregate price index, let as before \( I_Z \) index the relevant reference period goods space and \( N_Z \) denote the number of reference period varieties in the comparison period goods space. Averaging across varieties in the reference period that are within the comparison period goods space the Paasche price index is defined as:

\[
P_{HP}(x_t, p_t, \gamma_{t-1}) = \frac{1}{N_Z} \sum_{j \in I_Z} EV_j(x_{jt}, p_{jt}, \gamma_{t-1})
\]
Since in Bertrand pricing equilibrium the mark-up is in general greater than zero, equation (3.4) defines on average an upper bound to the true theoretical compensation function.

**Proposition 3.2:** In the Bertrand pricing equilibrium defined by equation (2.2), the Paasche hedonic price index defined by equation (3.4) is an upper bound to the true price index for the average consumer of product $j$ in period $t$.

**Proof:** mathematical appendix p. 35.

Finally, note that despite the bounding properties provided by propositions 3.1 and 3.2 there is in general no relation between the Laspeyres and Paasche hedonic indexes since they are defined with reference to two different sets of goods and two different sets of consumers, the period $t-1$ sets for the Laspeyres index and the period $t$ sets for the Paasche index. However, in the limiting case of an unchanged set of goods between the two periods and if consumer preferences do not change over time it can be easily shown that $P_{HP}(x_t, p_t, g_{t-1}) \geq P_{HL}(x_{t-1}, p_{t-1}, g_t)$. Provided the sets of goods for the two indexes are not too different and if variations in preferences can be neglected this relation should hold more generally.

### 4. The data

Following previous research on computer prices by Aizcorbe, Corrado and Doms (2000), Berndt and Griliches (1993), Berndt, Griliches and Rappaport (1995), Berndt and Rappaport (2001) and Moch (2001) data on prices and characteristics of personal computers traded in Italy during the 1995-2000 period were collected from the monthly issues of the Italian editions of PC Professional and PC Magazine.

Table 1 reports sample averages for price and for some of the main performance characteristics of the PCs included in our sample. The table shows that while prices remained roughly constant over the sample period, the distribution of the main desktop characteristics shifted markedly to the right. This is true for both continuous characteristics such as frequency, RAM, hard disk capacity, video memory and monitor size and for indicator variables of computer accessories such as CD player, DVD player, modem and
audio interface. Moreover, the quality of these devices also increased substantially during the sample period, as is shown by the increase in their measures of speed.

Table 2 presents for each year the distribution of personal computers by processor type. In the first year of the sample period more than half of all computers were already equipped with a Pentium processor while the remaining ones were still using the 486DX2 and 486DX4 processors. In the subsequent years, the latter types of processors gradually disappeared from the market, while the Pentium technology was gaining market shares. Towards the end of the sample period, the share of personal computers equipped with a Pentium processor declined to less than 50 per cent, while the market share of new processor types such as Athlon and Celeron was increasing.

Table 3 reports for each year the distribution of computers by computer brand. The main computer brands we identify in our sample are Acer, Compaq, Dell, Digital Equipment, Hewlett Packard, Ibm and Olivetti. Throughout the sample period, the cumulative share of the main computer brands is of the order of 5 per cent. Most of the personal computers in our sample are thus produced by small to medium sized specialized suppliers, reflecting the structure of the Italian computer industry.

Market shares for each personal computer in the price and characteristics database, were estimated using information on market shares by processor type provided by the Gartner Group corporation. Letting $N$ denote the number of computers in a given year in our sample, $\bar{w}_i$ the sample share of processor $i$ as reported in Table 2, $w_i$ the market share of processor $i$ provided by Gartner Group, the market share of each computer in our sample equipped with processor $i$ was estimated as $(1/N)(w_i/\bar{w}_i)$. The factor $(w_i/\bar{w}_i)$ corrects for over or under sampling of computers with a particular processor type. The estimated shares of each computer $j$ were then multiplied by the share of personal consumption expenditure in computer processing equipment to get an estimate of the shares $s_j$.

5. Estimation results

For estimation we assume that demand takes the form of the nested logit model of McFadden (1978, 1984) and Cardell (1997). This model is based on the assumption that
there are $G$ product groups and that preferences for products belonging to each group are characterized by a common random component, which varies between individuals and groups, in addition to the random component of the simple logit model, which varies between individuals and products. Denoting with $s_{j/g}(x,p,q)$ the within-group market share functions, for $j \in g$ and $g \in \{1,...,G\}$, and provided preferences are linear in prices and product characteristics, the market share functions satisfy the following relation:

\[(5.1) \quad \ln s_j - \ln s_0 = \beta x_j - \alpha p_j + \sigma \ln s_{j/g} + \xi_j\]

where $\alpha > 0$ is a parameter determining the own-price elasticity of demand and $0 \leq \sigma < 1$ is a parameter determining the relative importance of the common and idiosyncratic random components of consumer preferences and shaping the cross-price elasticities.

We also assume that marginal costs are a linear function of performance characteristics:

\[(5.2) \quad c(x_j, \omega_j, \gamma) = \gamma x_j + \omega_j\]

These assumptions imply that the pricing equation takes the following form:

\[(5.3) \quad p_j = \gamma x_j + \left( \frac{1 - \sigma}{\alpha} \frac{1}{1 - \sigma s_{j/g} - (1 - \sigma) s_j} \right) + \omega_j\]

For $\sigma = 0$ the model reduces to the standard logit model of McFadden (1974, 1984), where the common random component is absent, for $\sigma \to 1$ the weight of the idiosyncratic component tends to zero and the correlation between the within-group innovations tends to one\(^6\).

In order to define the partition of the product space, for each year of the 1995-2000 sample period products are ordered according to their price and the product groups are defined by the quartiles of the price distribution. This definition of the partition can be

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\(^6\) The nested logit model allows for a less restrictive pattern of elasticity than the simple logit model, although it requires the definition of a set of product groups. The more recent BLP (1995, 2004) models provide more general formulations of the consumer preference structure, which avoid the requirement of the definition of a partition of the product space.
justified assuming that consumers with different incomes have different preferences regarding the allocation of expenditure between personal computers and the outside good.

Given the information set, equations (5.1) and (5.3) together with some additional distributional assumptions can be used for estimation. We follow the more recent literature and estimate the model using the generalized method of moments (GMM). This method requires the additional assumptions of orthogonality between the unobservable and the observable characteristics variables. Letting \( \mathbf{u}_j = (\xi_j, \omega_j) \) we thus assume:

\[
(5.4) \quad \mathbb{E}(\mathbf{u}_j \mid x) = 0 \quad j \in \{1, \ldots, N\}
\]

Since the moment conditions are non-linear in the \( \alpha \) and \( \sigma \) parameters, estimation cannot be performed using traditional instrumental variable methods. Moreover, the estimation algorithm cannot be based on traditional gradient methods, such as Newton-Raphson, since these methods cannot in general determine the global optimum of a function characterized by more than one local optima. Preliminary experimentation with the nested logit model using gradient methods shows that, for given different initial conditions within the relevant parameter range, the GMM estimator converges to different optima. We therefore adopt a “concentrated” GMM procedure. The estimation algorithm is based on stochastic search and is articulated in two stages. In the first stage, for each value of the non-linear parameters \( \alpha \) and \( \sigma \), the linear parameters are concentrated out of the criterion function using Hansen’s (1982) two-step heteroskedasticity consistent estimator. In the second stage, the resulting concentrated criterion is optimized on the parameter space for \( \alpha \) and \( \sigma \), using the method of simulated annealing described in Goffe, Ferrier and Rogers (1994)\(^7\).

In model estimation we could not use all the available characteristics variables. Given the number of observations for each year and the character of the estimation procedure the model in this case would be overparametrized. We therefore resort to a specification including continuous variables for frequency, RAM, hard disk capacity and video memory.

\(^7\) Simulated annealing has been found to be superior to other global optimization methods, such as simplex search, for the optimization of functions characterized by multiple optima. Although on conventional
and indicator variables for CD player, DVD player, modem device and audio interface. Moreover, given the limited number of observations available for each year, in order to reduce finite sample distortion we do not use all the available moment conditions. We include in the set of instruments only the characteristics variables used in the specification of the market share and pricing equations and indicator variables for processor type.

Table 4 reports some of the main results of estimation for two empirical specifications of the model. In the base specification, for each observation only the observable characteristic variables of the same observation are used as instruments. In the augmented specification, the characteristics variables of the preceding and subsequent observations in the rank ordering by product price are also used as instruments. In this way we control for the behaviour of the efficiency of the estimator as the number of moment conditions increases. The table shows that in the base specification there is a small number of overidentifying restrictions and the Sargan test rejects the model at a 5 per cent significance level in almost all years. In the augmented specification instead, the number of overidentifying restrictions is more in line with the number of observations and the Sargan test accepts the model in all years. In addition, as the number of moment conditions increases the adjusted $R^2$ of both the market share and the pricing equations typically improves, although by a small amount.

Table 5 reports estimates of the pricing equation implied by the parameter estimates for the augmented specification of the model. For each year, the price and estimates of the mark-up, the marginal cost and the mark-up rates are provided for the personal computers corresponding to the first quartile, the mean and the third quartile of the price distribution.

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8 For computational convenience we do not make any attempt to look for optimal instruments. The criteria used to choose the instruments is that of maximizing their correlation with the independent variables in order to minimize the estimation variance.

9 For ease of exposition we do not report the estimates of the parameters of the market share and pricing equations. We note that most of the coefficients of the characteristics variables turned out significant and with correct signs in both equations in all years. Moreover, the estimated parameters for the characteristics variables remain relatively stable in going from the base to the augmented specification, while their standard errors do not show any sensible improvement. The improvement in efficiency that is implied by the adjusted $R^2$ comparisons seems thus to be due entirely to a better identification of the non-linear parameters and therefore of the mark-up and the demand elasticity.
The price levels and the mark-up rates are reproduced graphically in Figures 1 and 2. The table and Figure 1 show that there was an increase in computer prices following the exchange rate shocks that occurred in 1995. Prices increased sharply in 1996 and then returned to normal levels in the following years. The table and Figure 2 also show that the mark-up is estimated to be close to zero at the beginning of the sample period, to increase substantially in 1997 when mark-up rates rise to more than 20 per cent and to decrease thereafter, returning close to zero at the end of the sample period.

The reported results provide interesting information on the behaviour of the Italian personal computer industry during the 1995-2000 period. In 1995 and 1996 it is likely that the devaluation of the Italian currency, which occurred as Italy was rejoining the European exchange rate mechanism, caused a contraction of mark-up rates for Italian personal computer producers while the impact of higher computer component costs was partly translated into higher prices and lower competition. Similarly, the developments of international financial markets that led to the devaluations of the East Asian currencies in the autumn of 1997 seem to have acted as a favourable cost shock to the personal computers industry in Italy, fostering competition and inducing firms to increase their mark-ups. The model estimates provide evidence of significant market dynamics for 1997 and subsequent years.

6. Hedonic price indexes for PCs

With the background of the data and the analysis presented in the previous sections, we compiled hedonic price indexes for personal computers in Italy for the 1995-2000 period. The first panel of Table 6 reports the rates of change of both Laspeyres and Paasche price indexes, compiled as defined in section 3 of the paper. For each type of index, the table presents both the weighted and the unweighted versions. In the weighted forms, each product is weighted according to its market share instead of receiving an equal weight in the index.

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10 This interpretation is supported by evidence on the behaviour of aggregate prices and mark-ups of the Italian economy over a relatively long period covering the 1980s and the early 1990s. As illustrated by Visco (1994) on the basis of estimates of the Bank of Italy, macroeconomic quarterly model, Italian producers tend to reduce mark-ups and increase prices in response to unfavourable exchange rate shocks and to increase mark-ups in response to favourable exchange rate shocks.
In weighted Laspeyres indexes, period $t-1$ product shares are used as weights and in weighted Paasche indexes, period $t$ market shares are used as weights. In this way we assess the robustness of the results to the sampling variation resulting from the choice of sources for the personal computers data on prices and characteristics. Weighted indexes correct for the finite sample distortion resulting from sampling variation but the asymptotic properties of the indexes are not affected by this choice. The reported indexes are proper, for the Laspeyres price indexes, for each comparison period only products within the sample space defined by the reference period continuous variables used in model estimation are included in the computations and for the Paasche price indexes, for each reference period only products within the goods space defined by the comparison period continuous variables are considered. The match rate for the Laspeyres indexes is relatively good. In most years more than 50 per cent of the products can be included in the computations. For the Paasche price indexes the match rate is generally lower\(^{11}\).

The hedonic price indexes show that during the sample period there have been large decreases in PC prices on a quality-adjusted basis. The average yearly rate of price change over the sampling period is equal to -0.354 for the unweighted Laspeyres price index and to -0.355 for the weighted index. The differences in the yearly rates of change of the weighted and unweighted indexes appear to be small. The Paasche price indexes show higher rates of price change than the Laspeyres indexes, the average yearly rate of price change for the unweighted index is -0.119 while the average rate of change for the weighted index is -0.131. For the Paasche price indexes in some years the distortion of the unweighted index appears to be larger. Nevertheless the rates of change of the Paasche indexes are higher than the rates of change of the corresponding Laspeyres indexes in all years.

The second panel of Table 6 reports adjusted hedonic price indexes. The compensating and equivalent variations for these indexes are computed by adding to the estimated observed cost components the estimated unobserved cost components. For the Laspeyres price indexes the unobserved cost components refer to period $t-1$ and for the Paasche price

\(^{11}\) We do not impose sample space constraints for indicator variables since in some cases these may be too binding. For example, consumers may substitute a CD player with a DVD player or a modem with an audio interface. Requiring perfect matches on these devices would therefore substantially reduce the match rate, introducing a sample selection distortion in the price indexes.
indexes to period $t$. In this way the adjusted indexes remove the finite sample distortion arising from the unobservable cost components. Adjusted hedonic indexes tend to show lower rates of price change than hedonic ones, the average annual rate of price change is -0.370 for the unweighted adjusted hedonic Laspeyres index and -0.369 for the weighted index. The yearly rates of change are lower than the rates of change of the corresponding hedonic indexes in almost all years. The effect of weighting is relatively small. The average rate of price change is equal to -0.215 for the unweighted Paasche index and to -0.197 for the weighted Paasche index and the yearly rates of change are lower than the corresponding hedonic Paasche indexes in almost all years. The rates of change of the adjusted Paasche indexes are higher than the rates of change of the adjusted Laspeyres indexes in all years.

7. Matched model indexes

In order to provide an assessment of the results of the hedonic regression analysis, in this section we use the price and characteristics data to compile matched model indexes. To provide a definition of a matched model index, we note that since the set of available goods changes over time and in each period the characteristics of the available goods include many observable and unobservable components, we can proceed by assuming that the set of goods available in period $t$ is completely different from the set available in period $t-1$.

Considering the problem of measurement from the point of view of period $t-1$ consumers, provided we restrict the analysis to the set of period $t-1$ products that are within the period $t$ goods space, we could compile a price index by matching each period $t-1$ product with the period $t$ product with the closest characteristics. The Laspeyres matched model index could thus be defined as:

$$P_{ML}(\hat{p}_t, p_{t-1}) = \frac{1}{N_Z} \sum_{j \in I_Z} \left( \hat{p}_{jt} - p_{jt-1} \right)$$

where $\hat{p}_{jt}$ is the price of the product in period $t$ goods space that is closest in observable characteristics to the period $t-1$ product $j$ and $\hat{p}_t = (\hat{p}_{1t}, ..., \hat{p}_{Nt})$.

Similarly, for period $t$ consumers and restricting the analysis to the set of period $t$ products that are within the period $t-1$ goods space, we could compile a price index by
matching each period $t$ product with the period $t-1$ product with the closest characteristics. The Paasche matched model index could then be defined as:

$$P_{MP}(p_t, \hat{p}_{t-1}) = \frac{1}{N_Z} \sum_{j \in I_Z} \left( p_{jt} - \hat{p}_{jt-1} \right)$$

where $\hat{p}_{jt-1}$ is the price of the product in period $t-1$ goods space that is closest in observable characteristics to the period $t$ product $j$ and $\hat{p}_{t-1} = (\hat{p}_{1t-1}, ..., \hat{p}_{Nt-1})$.

Matched model indexes are characterized by distortions of a different nature from the ones characterizing hedonic price indexes, the distortions depend on the rate of technological change. In general, products in the comparison period tend to be matched with reference period products with higher characteristics and products in the reference period tend to be matched with products of lower characteristics. Therefore both Laspeyres and Paasche indexes have a tendency to show an upward bias, which is an increasing function of the rate of product innovation. However, note that since matched model indexes are compiled by sampling prices directly rather than by estimating them as in hedonic indexes, they are not characterized by the distortions characterizing the latter ones. Provided a good quality of the matches between one period and the next is ensured, they should therefore provide a useful alternative to hedonic price indexes.

The matched model indexes are reported in the third panel of Table 6, both the Laspeyres and the Paasche indexes are compiled with reference to the same sample spaces used in the compilation of the hedonic price indexes. The panel shows that over the 1995-2000 period, both the weighted and the unweighted Laspeyres matched model indexes decline at an annual average rate of -0.252, the Paasche matched model index declines at an average rate of -0.122 and the Paasche matched model weighted index declines at an average annual rate of -0.145. Similarly to the hedonic indexes, the Paasche indexes are more sensitive to the choice of weights than the Laspeyres indexes. Comparing the matched model indexes with the hedonic price indexes, we note that the rates of change of the Laspeyres indexes are higher than the rates of change of the corresponding hedonic and adjusted hedonic indexes in almost all years. The rates of change of the Paasche price index instead tend to be lower than the rates of change of the corresponding hedonic index and higher than
the rates of change of the corresponding adjusted hedonic indexes. This holds for both weighted and unweighted indexes.

It is interesting to compare these indexes with the official index for information processing equipment produced by Istat for the compilation of the CPI. The official index is compiled using a matched model method and during the 1995-2000 period declines at an average rate of \(-0.077\). This is much lower than the average rates of the indexes compiled in the present paper. The distortion partly reflects a distortion of the index for the years 1995-98. During this period the index was compiled using a fixed base approach and declined at an average rate of \(-0.010\). In 1998 Istat adopted a chain index system for the compilation of the CPI and the index displays a rate of price decline equal to \(-0.144\) in 1998-99 and to \(-0.195\) in 1999-2000. The bias of the official index is thus mainly the result of a failure of the fixed base approach to correct properly for quality change.

Summarizing the results obtained so far, note that consistently with the predictions of the theory, the Paasche hedonic indexes tend to show higher rates of price change than the Laspeyres hedonic indexes. This is true for both the weighted and the unweighted versions. The same result holds for adjusted hedonic indexes. Moreover, the Laspeyres matched model indexes show higher rates of change than the corresponding hedonic indexes while the Paasche matched model indexes show lower rates of change than the hedonic Paasche indexes and higher rates of change than the adjusted hedonic price indexes. These results are also broadly in line with the theory. Note that both hedonic and matched model Paasche indexes seem to display a poorer performance compared with the corresponding Laspeyres indexes. In particular, the finite sample corrections for product weighting and for the presence of unobservables seem to have a greater influence on the behaviour of the indexes. This result could be explained by the lower match rate that is observed for the Paasche indexes since the effect of introducing finite sample corrections has a tendency to be stronger for years where the match rate is lower. A low match rate implies that the indexes are characterized by greater sample selection and by greater uncertainty, making them less interpretable. Finally, looking closely at the data, we find that for both Laspeyres and Paasche matched model indexes product matchings are usually made between goods that are very close in characteristics space. This implies that the indexes should not be characterized by a substantial innovation bias.
8. Conclusions

The previous sections provide an analysis of the problems of construction of quality-adjusted price indexes using the theory of product differentiation. The analysis focuses on the recent developments of the theory, which extend prior attempts to model a partial equilibrium with product differentiation.

The general model defines consumer and firm choices in the space of product characteristics and allows for price-making behaviour on the part of firms. The industry equilibrium is stated in terms of a system of simultaneous equations, giving the market share and the equilibrium price of each product as a function of performance characteristics. The pricing equation of the partial equilibrium model forms the basis for the hedonic regression analysis. In the general case of Bertrand pricing, the equilibrium price of each product depends both on its own performance characteristics and on the prices and performance characteristics of all other products in the market.

The approach followed most often in the quality change literature has been to define an hedonic regression function using reduced forms of the pricing equation. The paper proposes to use as a reduced form the marginal cost function of the partial equilibrium model and analyzes the properties of the resulting hedonic price indexes. The Laspeyres hedonic price index is shown to provide a lower bound to the true rate of price change and the Paasche price index is shown to provide an upper bound to the true rate of price change. The paper also provides definitions for Laspeyres and Paasche matched model indexes. These are given according to the theoretical concepts for the quality-adjusted rate of price change implied by the model. Since the set of products available in the market usually improves in quality over time, matched model indexes tend to be characterized by an upward bias. However, provided matches are made between products that are close in characteristics space, the matched model indexes should provide a useful alternative measure of the rate of price change.

The paper contains an application of the theory to the study of personal computer prices in Italy during the period 1995-2000. The application is developed on the basis of data on prices, characteristics and market shares of personal computers available in the Italian market during the sample period, collected from personal computer magazines and private research institutions. The data show that the computer market was very dynamic during the
period, personal computers experienced substantial increases in performance characteristics while prices remained roughly constant.

The data are used to estimate a specification of the partial equilibrium model based on nested logit preferences. Following the more recent research, estimation is based on the generalized method of moments. The non-linear character of the model leads to the adoption of a concentrated version of the generalized method of moments procedure, based on the global optimization algorithm of simulated annealing. Estimation leads to satisfactory results, the coefficients of the share and pricing equations are usually significant and have the correct sign and the estimates of the pricing equation reflect in an interpretable way the history of the shocks affecting the Italian personal computer industry during the period.

The estimated nested logit model and the data are then used to compile both Laspeyres and Paasche hedonic price indexes. The hedonic price indexes show that the rate of price decline for personal computers in Italy during the sample period was considerable. The results are consistent with findings for other countries and the indexes seem to conform to the properties implied by the theory. The paper also presents Laspeyres and Paasche matched model indexes compiled with the same data used in the hedonic regression analysis. Conforming to prior expectations, the Laspeyres matched model indexes show rates of price decline that are usually lower than the corresponding hedonic indexes and the Paasche matched model indexes show rates of price decline that are higher than the corresponding hedonic indexes, though for the Paasche indexes the results are less clear cut due to the lower match rate which is observed for these indexes. Moreover, the matched model indexes do not seem to be characterized by any substantial innovation bias.

The empirical results suggest that hedonic and matched model methods should in general be thought of as complementary alternatives. While hedonic price indexes are characterized by distortions depending on the properties of the reduced forms of the hedonic function, matched model indexes are characterized by the classical distortions arising from the requirement of making appropriate sample replacements. There is no prior reason why one method should perform better than the other, rather the theory suggests that the choice between the two methods is essentially an empirical matter.
## Tables and figures

### Table 1

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<td>4'223</td>
<td>4'165</td>
<td>4'130</td>
<td>4'319</td>
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<td>0.000</td>
<td>0.027</td>
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<td>0.690</td>
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<td>0</td>
<td>0</td>
<td>1</td>
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<td>8</td>
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<tr>
<td>Video memory</td>
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<td>2</td>
<td>3</td>
<td>5</td>
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<td>15.3</td>
<td>15.9</td>
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<td>16.7</td>
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<td>0.549</td>
<td>0.798</td>
<td>0.910</td>
<td>0.981</td>
<td>0.952</td>
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<td>Observations</td>
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<td>91</td>
<td>99</td>
<td>111</td>
<td>107</td>
<td>84</td>
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</table>

(1) Prices inclusive of the monitor price and of value added tax in thousands of lira, frequency in Mhz, RAM in Mbytes, hard disk capacity in Gbytes, video memory in Mbytes, monitor size in inches. The variables CD, DVD, modem and audio are indicator variables. CD and DVD speed in standard units of reading speed, each unit of reading speed is equal to 150 Kbytes per second. Modem speed in Kbytes per second.
### Table 2

**DISTRIBUTION OF PERSONAL COMPUTERS BY PROCESSOR TYPE**

<table>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
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<td>486DX4</td>
<td>0.213</td>
<td>0.011</td>
<td>0.000</td>
<td>0.000</td>
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</tr>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.019</td>
<td>0.369</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.117</td>
<td>0.140</td>
<td>0.119</td>
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<td>Duron</td>
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<td>0.000</td>
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<td>0.000</td>
<td>0.048</td>
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<td>K6-3</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.019</td>
<td>0.000</td>
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<td>Pentium</td>
<td>0.596</td>
<td>0.978</td>
<td>0.283</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>Pentium II</td>
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<td>0.000</td>
<td>0.030</td>
<td>0.820</td>
<td>0.262</td>
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<td>Pentium III</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.561</td>
<td>0.464</td>
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<tr>
<td>Pentium MMX</td>
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<td>0.000</td>
<td>0.687</td>
<td>0.063</td>
<td>0.000</td>
<td>0.000</td>
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<tr>
<td>Pentium Pro</td>
<td>0.000</td>
<td>0.011</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

**Observations**

|        | 47 | 91 | 99 | 111 | 107 | 84 |

### Table 3

**DISTRIBUTION OF PERSONAL COMPUTERS BY BRAND(1)**

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<td>Acer</td>
<td>0.000</td>
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<td>0.010</td>
<td>0.000</td>
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<tr>
<td>Compaq</td>
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<td>0.000</td>
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<td>0.010</td>
<td>0.024</td>
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<td>Dell</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.010</td>
<td>0.010</td>
<td>0.012</td>
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<td>Digital Equipment</td>
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<td>0.000</td>
<td>0.010</td>
<td>0.000</td>
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**Observations**

|        | 47 | 91 | 99 | 111 | 107 | 84 |

(1) Main personal computer brands.
Table 4

SUMMARY STATISTICS FOR THE NESTED LOGIT MODEL

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<td>Demand adj. $R^2$</td>
<td>0.748</td>
<td>0.024</td>
<td>0.172</td>
<td>0.035</td>
<td>0.107</td>
<td>0.017</td>
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<td>0.204</td>
<td>0.199</td>
<td>0.258</td>
<td>0.661</td>
<td>0.640</td>
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<td>6</td>
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<td>9</td>
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<td>11</td>
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<tr>
<td>Sargan test (P-value)</td>
<td>0.104</td>
<td>0.026</td>
<td>0.009</td>
<td>0.000</td>
<td>0.003</td>
<td>0.047</td>
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<td>89</td>
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<td>Demand adj. $R^2$</td>
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<td>0.401</td>
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<td>0.103</td>
<td>0.315</td>
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<td>87</td>
<td>106</td>
<td>105</td>
<td>54</td>
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</table>

(1) Mean utility levels and marginal cost functions are linear in the performance characteristics. Characteristics variables are frequency, RAM, hard disk capacity, video memory, CD, DVD, modem and audio. Instrumental variables include indicator variables for processor type. Base specification for each observation uses as instruments only variables of the same observations. Augmented specification uses also variables of the preceding and succeeding observations in the ordering by product price.
**Table 5**

**PRICING EQUATIONS AND MARK-UP RATES\(^{(1)}\)**

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<td>3’180</td>
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<td>2’951</td>
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<td>Mark-up</td>
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<td>295</td>
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<td>Marginal cost</td>
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<td>0.000</td>
<td>0.287</td>
<td>0.102</td>
<td>0.009</td>
<td>0.000</td>
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</table>

|        |      |      |      |      |      |      |
| **Mean** |      |      |      |      |      |      |
| Price     | 4’203 | 5’242 | 4’223 | 4’165 | 4’130 | 4’319 |
| Mark-up   | 0     | 0    | 784   | 295   | 28    | 0    |
| Marginal cost | 4’203 | 5’242 | 3’439 | 3’870 | 4’102 | 4’319 |
| Mark-up rate | 0.000 | 0.000 | 0.228 | 0.076 | 0.007 | 0.000 |

|        |      |      |      |      |      |      |
| **3rd qrt.** |      |      |      |      |      |      |
| Price     | 4’582 | 6’057 | 4’690 | 5’058 | 5’508 | 5’148 |
| Mark-up   | 0     | 0    | 784   | 295   | 28    | 0    |
| Marginal cost | 4’581 | 6’057 | 3’906 | 4’763 | 5’480 | 5’148 |
| Mark-up rate | 0.000 | 0.000 | 0.201 | 0.062 | 0.005 | 0.000 |

(1) Prices, mark-ups and marginal costs in thousands of lira. Personal computers at the first quartile, the mean and the third quartile of the price distribution.
Figure 1

**PRICES OF PERSONAL COMPUTERS\(^{(1)}\)**

![Graph of prices of personal computers from 1995 to 2000 showing the first quartile, mean, and third quartile.](image)

(1) Prices in thousands of lira. Personal computers at the first quartile, the mean and the third quartile of the price distribution.

Figure 2

**MARK-UP RATES\(^{(1)}\)**

![Graph of mark-up rates from 1995 to 2000 showing the first quartile, mean, and third quartile.](image)

(1) Personal computers at the first quartile, the mean and the third quartile of the price distribution.
### Table 6

**PRICE INDEXES FOR PERSONAL COMPUTERS**

<table>
<thead>
<tr>
<th></th>
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<td>Laspeyres</td>
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<td><strong>Adjusted hedonic indexes</strong></td>
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<tr>
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<td>Paasche</td>
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<td>Paasche</td>
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<td>72</td>
<td>43</td>
<td>27</td>
<td>7</td>
<td></td>
</tr>
</tbody>
</table>

(1) Annual rates of change, compiled using the ratio of arithmetic means formula. Laspeyres indexes use comparison period products, Paasche indexes use reference period products. Weighted indexes use market shares to weight each product in the sample. Adjusted hedonic indexes remove the finite sample distortion arising from the unobservable cost components.
Appendix I

Mathematical Appendix

We provide here a formal derivation of the propositions made in the text. In order to establish notation and some preliminary results let $X_s=(X_{1s},\ldots,X_{Ns})\in\mathbb{R}^{NK}$, $Z_s=(Z_{1s},\ldots,Z_{Ns})\in\mathbb{R}^{N}$, $\Omega_s=(\Omega_{1s},\ldots,\Omega_{Ns})\in\mathbb{R}^{N}$, and $P_s=(P_{1s},\ldots,P_{Ns})\in\mathbb{R}^{N}$ denote respectively the random vector corresponding to the sets of observable characteristics, unobservable preference characteristics, unobservable cost characteristics and to the set of prices of the goods available in period $s$. The observables are $K$-dimensional so that $X_{js}\in\mathbb{R}^{K}$ for all $j\in\{1,\ldots,N\}$ and $s\in\{t-1,t\}$. We follow the convention of denoting realizations of random vectors by lower case letters.

Laspeyres and Paasche price indexes have been defined in the main text with reference to products belonging to subsets of the relevant product space. To study the properties of the indexes it is therefore necessary to establish the sampling properties of products drawn in subsets of the product space.

Let $F_{js}(x_{js})$ denote the distribution function of the random vector $X_{js}$, for $j\in\{1,\ldots,N\}$ and $s\in\{t-1,t\}$, and for a subset $Z_{j}\subseteq\mathbb{R}^{K}$ of the characteristics space let $P(x_{js}\in Z_{j})=\int_{Z_{j}}dF_{js}(x_{js})$ denote the probability that $x_{js}$ belongs to $Z_{j}$. Similarly, let $F_{s}(x_{s})$ be the distribution function of the random vector $X_{s}$, for $s\in\{t-1,t\}$, and for a subset $Z_{s}\subseteq\mathbb{R}^{NK}$ of the product space let $P(x_{s}\in Z)=\int_{Z}dF_{s}(x_{s})$ denote the probability that $x_{s}$ belongs to $Z$. The probability and distribution functions for the random variables $Z_{js}$ and $\Omega_{js}$ are defined in a similar way.

We make the following assumption about the distribution of the random variables $X_{js}$, $Z_{js}$ and $\Omega_{js}$.

**Assumption I.1:** In each period of time $s\in\{t-1,t\}$, the random variables $X_{js}$, $Z_{js}$ and $\Omega_{js}$ are identically and independently distributed for all $j\in\{1,\ldots,N\}$. 
From assumption I.1 it follows that $F_{js}(x_{js}) = F_{ks}(x_{ks})$ for all $j, k \in \{1, \ldots, N\}$, $s \in \{t-1, t\}$ and $x_{js}=x_{ks}$ and $F_{s}(x_{s}) = \prod_{j=1}^{N} F_{j}(x_{js})$ for $s \in \{t-1, t\}$. Given subsets of the characteristics space $Z_{j} \subseteq \mathbb{R}^{K_{j}}$ for $j \in \{1, \ldots, N\}$, let the cartesian product $Z = Z_{1} \times \cdots \times Z_{N} \subseteq \mathbb{R}^{NK}$ be the corresponding subset of the product space. The following result concerning the conditional properties of the random variables $X_{js}$, $\Xi_{j}$ and $\Omega_{j}$ holds.

**Lemma I.1**: In each period of time $s \in \{t-1, t\}$, for the set of period $s$ products $j \in \{1, \ldots, N\}$ with observable characteristics $x_{js}$ belonging to the subset of the characteristics space $Z_{j}$, the random variables $X_{js}$, $X_{j}$ and $W_{js}$ are identically and independently distributed.

**Proof of Lemma I.1**: To prove the lemma, define the conditional distribution functions $F_{js|Z_{j}} = F_{js}(x_{js}) / P(x_{js} \in Z_{j})$ for $j \in \{1, \ldots, N\}$ and $x_{js} \in Z_{j}$ and the conditional distribution function $F_{s|Z} = F_{s}(x_{s}) / P(x_{s} \in Z)$ for $x_{s} \in Z$ (Billingsley (1986), pp. 448 ss.). Since $x_{s} \in Z$ if and only if $x_{js} \in Z_{j}$ for all $j \in \{1, \ldots, N\}$ and by the assumption of identical and independent distributions the conditioning probabilities can be factorized as:

(I.1) \[ P(x_{s} \in Z) = \left\lfloor_{Z} \prod_{j=1}^{N} F_{js}(x_{js}) \right\rfloor_{Z_{j}} \prod_{j=1}^{N} P(x_{js} \in Z_{j}) \]

the following factorization of the conditional distributions holds:

(I.2) \[ F_{s|Z}(x_{s}) = \prod_{j=1}^{N} F_{js|Z_{j}}(x_{js}) \]

Similar derivations could be provided for the random variables $\Xi_{j}$ and $\Omega_{j}$ hence the result follows.

Lemma I.1 establishes that even after conditioning on appropriate subsets of the product space, the random variables $X_{js}$, $\Xi_{j}$ and $\Omega_{j}$ remain independent and therefore conventional asymptotic theory can be applied to transformations of these random variables to study their properties. We can use this result to discuss the propositions establishing the asymptotic properties of the proposed hedonic price indexes.

**Proof of Proposition 2.1**: For each consumer, denote $p_{j}$ the price that would be predicted by the model for period $t-1$ product $j$ in period $t$ on the basis of the period $t$ market conditions. This price includes a provision for the mark-up that would be charged for period
\( t-1 \) product \( j \) in period \( t \). The compensation error for each consumer is given by the difference between the compensation that would be granted on the basis of the price \( p_j \) and the compensation defined by the Laspeyres price index:

\[
\varepsilon_{jt-1} = (p_j - p_{jt-1}) - P_{HL}(x_{t-1}, p_{t-1}, \gamma_t)
\]

Substituting in (I.3) the definition of the Laspeyres price index given in equation (3.2) then yields:

\[
\varepsilon_{jt-1} = \left( p_j - \frac{1}{N_Z} \sum_{j \in I_Z} E(c(x_{jt-1}, \omega_{jt-1}, \gamma_t) \mid x_{jt-1}) \right) - \left( p_{jt-1} - \frac{1}{N_Z} \sum_{j \in I_Z} p_{jt-1} \right)
\]

Averaging over products and considering that in this way the second term on the right hand side of (I.4) vanishes, we can express the average compensation error as the average of the difference between the theoretical price \( p_j \) and the price predicted on the basis of the cost function alone:

\[
\frac{1}{N_Z} \sum_{j \in I_Z} \varepsilon_{jt-1} = \frac{1}{N_Z} \sum_{j \in I_Z} \left( p_j - E(c(x_{jt-1}, \omega_{jt-1}, \gamma_t) \mid x_{jt-1}) \right)
\]

Each term in the right hand side of (I.5) can be interpreted as the compensation error for consumers of product \( j \) that would result by applying the compensation function (3.1) instead of the average compensation defined by the Laspeyres price index. While for each \( j \) this compensation error is different from \( \varepsilon_{jt-1} \), their sample averages are equal by definition.

Now note that since the theoretical price \( p_j \) is equal to the sum between predicted marginal costs \( c(x_{jt-1}, \omega_{jt-1}, \gamma_t) \) and the mark-up term \( m_j(x_{jt-1}, \xi_{jt-1}, p, \theta_{t-1}) \), where \( p \) is the vector of period \( t-1 \) product prices predicted by the model for period \( t-1 \) consumers given the period \( t \) cost function, each term on the right hand side of (I.5) is equal to the sum of an unobservable cost component \( \zeta_{jt-1} = c(x_{jt-1}, \omega_{jt-1}, \gamma_t) - E(c(x_{jt-1}, \omega_{jt-1}, \gamma_t \mid x_{jt-1}) \) and the mark-up term \( m_j(x_{jt-1}, \xi_{jt-1}, p, \theta_{t-1}) \). It follows that the average compensation error can be expressed as the sum of the sample average of the unobserved cost component and the sample average of the mark-up term:
The assumption of continuity of the marginal cost function implies that the cost function is measurable, its conditional expected value by construction is also measurable and therefore $\zeta_{p,i}$ is a well defined random variable that satisfies usual orthogonality conditions. From assumption I.1 and lemma I.1 these regularity conditions imply that $\zeta_{p,i}$ is identically and independently distributed across products with mean equal to zero in each time period and therefore its sample average converges almost surely to zero as $N \to +\infty$ by the strong law of large numbers (Billingsley 1986, Theorem 22.1; White 1984, Theorem 3.1). In turn, this implies that the average compensation error converges almost surely to the average mark-up, which is greater than or equal to zero for all characteristics vectors $(x_{t-1}, \xi_{t-1}) \in \mathbb{R}^{K+i}$ by construction:

\[(I.7) \quad \frac{1}{N} \sum_{j \in I} \varepsilon_{jt-1} \xrightarrow{a.s.} \frac{1}{N} \sum_{j \in I} m_j(x_{t-1}, \xi_{t-1}, p, \Theta_{t-1}) \geq 0\]

Proof of Proposition 2.2: Follows along the same lines as the proof of proposition 2.1 by suitable changes in notation.


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