Can option smiles forecast changes in interest rates?
An application to the US, the UK and the Euro Area

by Marcello Pericoli
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CAN OPTION SMILES FORECAST CHANGES IN INTEREST RATES?
AN APPLICATION TO THE US, THE UK AND THE EURO AREA

by Marcello Pericoli*

Abstract

This paper evaluates the use of risk-neutral probability density functions implied in 3-month interest-rate futures options to assess market perceptions regarding future monetary policy moves; options allow the information content implied in simpler derivatives to be extended by providing indicators for asymmetry and extreme values. First, a cubic spline is implemented to evaluate the densities. Second, the methodology is applied to quotes on deposits denominated in US dollars, euros and sterling from January 1999 to May 2004; results show that markets correctly forecast the monetary easing of 2001 in the United States in the course of the second half of 2000, but not in the euro area and the United Kingdom. The evidence for the tightening cycle of 1999 is mixed: markets expected an increase in euro area policy rates at the beginning of 1999; expectations were less clear for the United States’ interest-rate increases. In the case of the United Kingdom the increase was not foreseen.

JEL classification: C52, E58, G13, G14, G15

Keywords: risk-neutral density, cubic spline, monetary policy, interest-rate futures options

Contents

1. Introduction .......................................................................................... 7
2. Stylized facts ....................................................................................... 10
3. A review of methodologies ................................................................... 12
   3.1 The parametric approach ............................................................... 15
   3.2 The non-parametric approach ....................................................... 16
   3.3 Non-parametric estimate ............................................................... 18
4. Evaluation of the information content ................................................. 22
   4.1 Two case studies ........................................................................... 22
   4.2 A time series analysis ................................................................... 24
5. Conclusion ............................................................................................ 28
Tables and figures .................................................................................. 29
Appendix ................................................................................................... 38
References ............................................................................................... 40

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1. Introduction

The widespread use and the availability of data of derivatives on short-term interest rates has widened the scope for analyzing the expectations regarding monetary policy decisions prevailing among market participants. This analysis has recently been developed within the research departments of central banks and is extensively used to extract information about market forecasts and market sentiment on future monetary policy changes. The use of a simple futures contract with delivery date around the time of the monetary policy meeting has already shown a persistent bias in predicting future interest rates; furthermore, this kind of derivative instrument can only give point estimates of the prevailing market rate at delivery. Alternatively, options written on short-term interest-rate futures contract (hereafter futures options or, more simply, options) can be used to evaluate market expectations regarding the underlying and also to extract additional information on their dispersion, on the probability attached to the occurrence of extreme events as well as on the relative probability of the occurrence of a decrease compared with an increase in the underlying — in statistical terms these three features are characterized by the implied standard deviation, the kurtosis and the skewness of the densities. These features allow confidence intervals to be constructed for the underlying instrument at different delivery dates.

In order to understand why futures options give the opportunity to obtain confidence intervals it may be useful to consider a simple example. A quote on 14 October 2002 for an American-type call option on the futures contract on the 3-month Euribor with expiration date in March 2003 and discount strike price 95.00, gives the buyer the right to buy the underlying

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3 Even if futures options and futures contracts on 3 month interest rate deposits are widely traded at CME, LIFFE and EUREX, the bulk of derivatives on these instruments is still traded in the over-the-counter market where interest caps and interest floors take the lion’s share. The main difference between an OTC and a regulated market is that in the former derivatives are traded with a custom-made expiry date, while in the latter the expiry date is fixed.
futures at any time before expiration at 95.00 (e.g. 100.0-95.0=5.0 per cent). Usually, every day such options are quoted for several discount strike prices, e.g. 95.125, 95.25, 95.50 etc., and for different expiration dates. The tenet in option pricing for market practitioners is given by the original Black-Scholes-Samuelson formula, which simply assumes that the underlying asset follows a geometric Brownian motion with constant volatility and, hence, the implied probability density function is log-normal. However, even if the Black-Scholes-Samuelson formula is widely used, each currently observed option shows different volatility, which tends to be larger as the strike price moves further away from the underlying spot price (this phenomenon is known as the smile of the option). Thus, the market assigns a higher quote to the option if, for a given volatility of the underlying, it considers the occurrence of the corresponding strike at the expiration date to be more likely. This creates the occurrence of skewed distributions — since markets can assign different probabilities to the occurrence of positive changes in the underlying compared with negative changes — and of fat-tails in the distributions — since extreme events can be considered more likely.

There is a pervasive practice among practitioners to price options in terms of implied Black-Scholes-Samuelson volatility instead of market prices, i.e. units of volatility corresponding to options prices through the Black-Scholes-Samuelson formula. This practice can be regarded as inconsistent since, on the one hand, markets quote options in terms of Black-Scholes-Samuelson implied volatility and, on the other, volatility is not constant across strike prices. Different theoretical models have been proposed to explain the ‘smile effect’ but, more interestingly for policy-makers and asset managers, the estimate of the connected probability density function documents the beliefs of market participants about the underlying data generating process (under the non trivial assumption of risk-neutrality). Then, the shape of the smile provides an evaluation of the probability distribution that market participants assign to the occurrence of a strike price at a certain date given the underlying spot price.

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4 If \( C \) and \( C^{BS} \) are the observed and the Black-Scholes-Samuelson option prices respectively, the implied volatility \( \gamma \) is defined as the solution to \((C^{BS}(S,X,\tau,\gamma,r) = C)\) where the terms in brackets are the underlying price, the strike price, the time to maturity, the volatility and the risk free rate, respectively.

5 Another feature of observed options is that historical volatility is usually lower than the lowest implied volatility (which is usually observed for around at-the-money options). The difference between historical volatility and at-the-money implied volatility is often used as a first tool to assess market sentiment. In this sense, when this difference increases, markets usually show higher kurtosis in their probability density functions and assign a larger probability to the occurrence of extreme events.
When this methodology is applied to derivatives on short-term interest rates it is straightforward to use the ‘smile effect’ as a tool to assess the predictive power of market participants in forecasting changes in policy interest rates at periodical central bank meetings.

The methodology developed here can be used to gauge whether, in the United States, in the euro area and in the United Kingdom, derivatives markets have accurately predicted monetary policy changes. Furthermore, these tools make it possible to test additional hypotheses as to whether policy-makers have driven expectations or, alternatively, have been driven by market sentiment.6

The time series of probability density functions (hereafter pdfs) implied in futures options written onto 3-month interest rates on deposits denominated in US dollars, euros and sterling may therefore provide relevant information about market expectations concerning the moves of the monetary authorities before their periodical meetings. The aim is to assess the degree of market participants’ accuracy in anticipating interest rate changes as well as to gauge the impact of different communication strategies which can differently shape investor sentiment. Statistics extracted from these pdfs are used to evaluate their predictive power of directives expected at the Federal Open Market Committee (FOMC) in the United States, at the Governing Council meeting (GC) in the euro area and at the Monetary Policy Committee (MPC) in the United Kingdom, respectively, from January 1999 to May 2002.7

The novelty of this paper is the use of pdfs to assess market perceptions of monetary policy moves. In fact, the previous literature has only incidentally analyzed the predictive

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6 According to so-called central-bank watchers, a comparative analysis of monetary policies in the United States and in the euro area shows that, since 1999, ECB interest-rate changes have not been completely anticipated by markets, while Federal Reserve changes were fully anticipated until the reversal of monetary stance in January 2001. Since 2001, monetary authorities in the United States, in the euro area and in the United Kingdom have substantially eased their monetary stance, driving short-term rates to historical lows. However, according to market commentary, the moves of the Federal Reserve have been different from those of the ECB since, in the US market, agents appear to have anticipated the decisions of the Federal Reserve and seem to have driven interest rates towards the desired values. This empirical regularity may either reflect a deliberate decision of the Federal Reserve to influence expectations or a conditioning power of markets on the moves of monetary authorities. Even if the direction in this causality relationship is uncertain given the endogeneity of the link between policy and short-term rates, it is interesting to compare the US with the euro area where, conversely, this phenomenon has not been observed, thus drawing criticism of the communication strategy adopted by the ECB.

7 The FOMC is held eight times a year, although the President of the Federal Reserve system may call for an intra-meeting in order to introduce additional policy-rate changes. The Governing Council of the European Central Bank is held twice a month; since 8 November 2001 decisions on policy rates are taken only at the first meeting of the month. The Monetary Policy Committee of the Bank of England lasts two days and is held every Tuesday-Wednesday or Wednesday-Thursday during the first week of every month.
power of forward-looking securities with respect to interest-rate changes. Söderlind and Svensson (1997) and Fornari and Violi (1998) compare the pdfs of long term government bond yield between different dates to extract also expectations regarding interest-rate changes; Jouandeau and Rockinger (2000) compare different methods of evaluating pdfs of exchange rates at two dates corresponding to tranquil and turbulent periods in the foreign exchange market, respectively. The Bank of England (2004, p. 9) presents charts with time series of the skewness of the 6-month implied skew from interest-rate options for the US dollar, the euro and the pound sterling.

I use a cubic spline to estimate the smile, hereafter defined as the relationship between the volatility and the delta of the options. Pdfs are then computed with the Rubinstein methodology by differentiating twice the estimated call with respect to the strike prices. Estimates use end-of-day settlement data on three regulated markets: 3-month interest-rate futures options are traded at CME, LIFFE and EUREX.

The paper is organized as follows. Section 2 presents the main stylized facts in the money markets of the United States, the euro area and the United Kingdom. Section 3 reviews the methodologies used to estimate pdfs implied in options and the sub-section 3.3 presents the non-parametric method I use. Section 4 shows the results; first a comparison among pdfs at different dates around selected central bank meetings is presented; second, the time series of some relevant pdf statistics are analyzed. Section 5 concludes. A description of the data is presented in the Appendix.

2. Stylized facts

Since 1999 policy and short-term rates in the three areas have moved in synchrony, with the United Kingdom slightly anticipating the other two areas. Central bank decisions for the three areas are presented in Tables 2-4 in the Appendix. In the United States, from July 1999 the Federal Reserve steadily raised the policy rate (federal funds target rate) by 1.75 percentage points up to 6.5 per cent in less than twelve months; rates were left unchanged during the second half of 2000 when doubts arose about the strength of growth of the US economy; from January 2001 until year end the policy rate was cut eleven times, by 4.75 percentage points, down to 1.75 per cent, which is the lowest level since the 1950s. Table 1 also shows the ‘bias’
of the Federal Reserve (or since February 2000 the ‘balance of risks’), that is a forward-looking assessment of the likely future monetary moves, released jointly with the decision on interest-rate changes; it ranges from ‘easy’ to ‘neutral’ and to ‘tighter’. In the euro area, the policy rate (the rate on main refinancing operations) was fairly steady in 1999 and from March 2000 was increased by 1.75 points to 4.75 per cent; from March 2001 the rate was cut four times until year end, by 1.5 points. In the United Kingdom, the policy rate (rate on repurchase agreement operations) was reduced by one point to 5 per cent between January and September 1999; thereafter, the buoyant British business cycle forced the monetary authority to raise the policy rate back to 6 per cent by March 2000, months before the increase implemented in the United States and in the euro area. Between March and December 2001 the rate was cut seven times by a total of 2 percentage points.

Three-month interest rates followed policy rates in the three areas. In the last quarter of 1999, when a quick tightening of monetary stance was in place, the slope of the money market curve — measured by the difference between the 3-month and the policy rate — significantly steepened, signalling expectations of further increases in future policy rates. The slope decreased substantially from January 2000, became flat during 2000 and inverted somewhat at the beginning of the rate cuts. What the money market slope documents then is just a mechanical textbook example: it is steep during phases of tightening and flat or inverted during monetary stance easing.

Similarly, futures contracts on short-term interest-rate deposits in the three areas have not provided good forecasts of future monetary policy moves for any areas.

From 1999 until the end of 2000, market participants assigned to the President of the Federal Reserve a superior information knowledge with respect to that of the European Central Bank Council. According to the financial press and newsletters of the major investment banks, in 1999 and 2000, in the US market the monetary authority drove short-term rates towards the desired values. This phenomenon could be the outcome of the communication strategy adopted by the Federal Reserve and of the transparency perceived by markets through the press communiqués from 1994; on the other hand, this could be the outcome of a leading effect of money markets on the choice of the central bank. However, such a causal relationship is hard to identify owing to endogeneity problems in the link between policy and short-term rates. Conversely, in the euro area, where the judgement of analysts and market participants
regarding monetary policy decisions has been more less favourable, critics have frequently pointed out the failure of the monetary policy communication strategy, which has often tried to surprise the markets with policy-rate changes.

The use of pdf’s implied in futures options may give useful insights for assessing this issue and evaluating the sentiment of market participants about central bank moves and their credibility.

3. A review of methodologies

In this paper I use the following standard terminology:

- \( S \) is the underlying spot price;
- \( X \) the strike price;
- \( \phi(S) \) the risk-neutral pdf, \( \Phi(S) \) the risk-neutral cumulative distribution function (hereafter cdf);
- \( f(S) \) the normal pdf, \( F(S) \) the normal cumulative distribution function;
- \( t, T \) and \( \tau = (T - t)/365 \) are, respectively, the current period, the expiration date and the time to expiration as a percentage of the year;
- \( r \) is the risk-free interest rate and \( e^{-r \tau} \) is the non-stochastic time-homogenous discount factor in continuous time between \( t \) and \( T \);
- \( C \) the current price of a call option and \( P \) the current price of a put option with payoff at time \( T \) given by \((S_T - X)^+ \) and \((X - S_T)^+ \), respectively, where \((y)^+ \) stands for \( \max(0, y) \);
- \( C^{BS} \) the Black-Scholes-Samuelson price of a call option is

\[
C^{BS} = S \cdot F \left( \frac{\ln \frac{S}{X} + \left( r + \frac{\sigma^2}{2} \right) \tau}{\sigma \sqrt{\tau}} \right) - e^{-r \tau} X \cdot F \left( \frac{\ln \frac{S}{X} + \left( r - \frac{\sigma^2}{2} \right) \tau}{\sigma \sqrt{\tau}} \right)
\]

- \( \sigma \) is the implied volatility, e.g. \( \sigma = \arg \min_\sigma \left| (C - C^{BS}) \right| \);
- \( \Delta \) is the delta of the option, i.e. the sensitivity of the call to the underlying \( \partial C/\partial S \), which in a Black-Scholes-Samuelson world is given by \( F(d_1) \), where \( d_1 = \frac{\ln \frac{S}{X} + \left( r + \frac{\sigma^2}{2} \right) \tau}{\sigma \sqrt{\tau}} \);
- \( \nu \) is the vega of the option, i.e. the sensitivity of the call to the volatility \( \partial C/\partial \sigma \), which in a Black-Scholes-Samuelson world is given by \( S \cdot f(d_1) \sqrt{\tau} \).
– I assume that the put-call parity holds, i.e. there are no arbitrage opportunities between a portfolio formed by one call option and $Xe^{-rt}$ of cash and one portfolio formed by one put option and one unity of its underlying, namely $C + Xe^{-rt} = P + S$. Then, put options can be transformed into call options through $C = P + S - Xe^{-rt}$.

During the last ten years the literature on financial econometrics and computational methods in finance has extensively used the analysis of pdf's implied in asset prices to go beyond the simple Black-Scholes-Samuelson option pricing formula, a tenet in option pricing. What the Black-Scholes-Samuelson model assumes is a constant variance for different degree of moneyness. Conversely, it is observed that options traded with different degrees of moneyness have different volatilities and, moreover, that the assumption that the underlying follows a simple or a geometric Brownian motion is too naive and does not give a reasonable description of his distribution. In reality the volatility of the options has a nonlinear relationship with the strike: call options show a negative relationship with their implied volatility, while put options show a positive relationship. Thus, what is actually thought of as a smile effect around the underlying spot price is the left tail of the call option schedule and the right tail of the put option schedule, which correspond to out-of-the-money (OTM) calls and puts, respectively. Implied volatility is as high as the strike is far from the underlying, both OTM calls and OTM puts, and the minimum value is usually observed for OTM calls — see Figure 1.

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8 We define the moneyness as the percentage spread of the strike price, $X$, from the underlying spot price, $S$, e.g. $(S - X)/X$. Usually the moneyness can be measured by the delta ($\Delta$) of the option: in-the-money options have $\Delta \in (0, 0.50)$, at-the-money options have $\Delta = 0.50$ and out-of-the-money options have theoretically $\Delta \in (0.5, \infty)$. In practice, one considers $\Delta = 0.25$ for in-the-money options and $\Delta = 0.75$ for out-of-the-money options, see Malz (1997).
Estimates of pdf's have strong advantages with respect to surveys, market polls and simple futures strip analyses. First, pdf's can be used to give not only point estimates, but also confidence interval estimates for the underlying price. Second, they show how market expectations shift and, through the joint analysis of futures contracts at different expiration dates, provide an evaluation of market sentiment. Last, their higher moments can add further insights about the dispersion of market participants’ beliefs.

However, pdf's are estimated under the assumption of risk-neutrality and, therefore, cannot be directly related to ‘objective market probabilities’, although a precise mathematical relation between them exists. In other words, the probability measure governing the risk-neutral estimates implies that the movements of discounted underlying asset prices are martingales, which is not true under the objective measure. Hence, in this framework the underlying asset price does not depend upon the subjective preferences of market participants. A way to compare the two densities and the risk aversion function is given in Aït-Sahalia and Lo (1998). Analogously, Coutant (1999) establishes a relationship between subjective and risk-neutral pdf using an approximation with Hermite polynomials. What does this imply

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9 Let $M_{t,T}$ be the stochastic discount factor between $t$ and $T$ defined as the ratio between the marginal utilities at period $T$ and $t$, $\Theta$ and $\theta$ the subjective distribution and density functions, respectively, defined over the entire domain of the asset price $S$ and $E_{\Theta}$ the connected expectation operator. The pricing kernel is then given...
for risk-neutral pdf estimates? Approximately, one can say that the true objective pdf will be slightly shifted towards the right with respect to the risk-neutral estimate. Söderlind and Svensson (1997) show that, under the assumption that the asset price and the stochastic discount factor are jointly log-normally distributed, the objective pdf can be obtained by simply shifting to the right the risk-neutral pdf by the amount equal to the covariance between the asset price and the stochastic discount factor. Aït-Sahalia et al. (2001) document how to extract subjective pdfs without introducing any assumption about the risk aversion function.

The literature presents two kinds of approaches in the pdf estimates: the parametric and the non parametric approach.

3.1 The parametric approach: stochastic volatility and mixture of log-normals

The introduction of stochastic volatility in the Black-Scholes-Samuelson framework allows the smile effect to be modelled parametrically. The simplest and tractable version of stochastic volatility assumes the existence of a regime switching between two states of the world which can be thought of as high and low volatility states. Then volatility and the drift terms in the geometric Brownian motion are expected to follow a simple Bernoulli process with probability \( p \). I also assume that there is no correlation between the Brownian motion and the probability of being in a high or low volatility state. This framework allows skewed smiles to be built. Under the risk-neutral probability measure, the asset follows the standard

by

\[
S_t = E_\Theta \left[ M_{t,T} \cdot (S_T - X)^+ \right]
\]

\[
= \int_{S \in \mathbb{S}} M_{t,T} \cdot (S_T - X)^+ d\Theta(S)
\]

and the corresponding risk-neutral pdf by

\[
\phi(S) = \frac{M_{t,T}}{\int_{S \in \mathbb{S}} M_{t,T} \cdot d\Theta(S)} \theta(S)
\]

Then, the risk aversion function and the utility function, which determine the stochastic discount factor, are a key factor in this relationship between subjective and risk neutral. Without introducing ad-hoc assumptions about the form and the distribution of the stochastic discount factor it is not possible to recover a precise link between the two functions.
stochastic differential equation that follows

\[ \frac{dS_t}{S_t} = u_t \cdot dt + \gamma_t \cdot dW_t \]

(1)

\[ (u_t, \gamma_t) = \begin{cases} (u_1, \gamma_1) & \text{with } \Pr = p, \ \forall t \\ (u_2, \gamma_2) & \text{with } \Pr = 1 - p, \ \forall t \end{cases} \]

where \( u_t \) and \( \gamma_t \) are the risk-neutral drift and the variance of the process, respectively, \( dW \) a standard Wiener process. This is basically a Black-Scholes-Samuelson world with two states of nature, and then the Black-Scholes-Samuelson formula applies in each state of nature and the final price of the option is an average of the two prices weighted by the probability of the occurrence of a particular state of nature, i.e. \( C^{BS}(u_t, \gamma_t) = p \cdot C^{BS}(u_1, \gamma_1) + (1 - p) \cdot C^{BS}(u_2, \gamma_2) \). The parameter vector to be estimated is then \( \varrho = (u_1, u_2, \gamma_1, \gamma_2, p) \). The mixture of two log-normal densities can be found through an optimization algorithm by minimizing the error between current and Black-Scholes-Samuelson option prices. This procedure aims to minimize the squared differences between the current price and that implied in its mix-density given by the Black’s approximation, namely

\[ \varepsilon = C - e^{-rT} \sum_{i=1}^{2} p_i \left\{ e^{\ln S_t + \left( u_i - \frac{\nu^2}{2} \right) \cdot r} \cdot F[d_1(i)] - X \cdot F[d_2(i)] \right\}, \]

where \( p_i \in (0, 1), p_1 + p_2 = 1, d_1(i) = \left[ \ln \frac{S_t}{X} + \left( u_i + \frac{\nu^2}{2} \right) \cdot r \right] / \sqrt{\gamma_i \cdot r}, d_2(i) = d_1(i) - \sqrt{\gamma_i \cdot r} \).

Parameters are given by \( \varrho = \arg \min_{\varrho} \sum_{i=1}^{N_O} \varepsilon_i^2 \), where \( N_O \) is the number of options.

Given the estimated parameter values, option price pdf's can be simulated for different strike prices. Applications of this method are used in Bahra (1996), Fornari and Violi (1998), Melick and Thomas (1997) and Pericoli (2000).

3.2 The non-parametric approach

Under the risk-neutral probability measure with pdf \( \phi \) and cdf \( \Phi \), assuming constant interest rate and that \( S \) follows a Markov process, the price at time \( t \) of a call futures option \( C \)
with expiration at $T$ and payoff $(S_T - X)^+$ is

$$C_t = e^{-rt} \cdot E \left\{ (S_T - X)^+ | S = S_t \right\}$$

(2)

$$= e^{-rt} \int_{-\infty}^{\infty} (S - X)^+ \cdot d\Phi(S)$$

$$= e^{-rt} \int_{X}^{\infty} (S - X) \cdot \phi(S)dS$$

analogously the price of a put option can be converted into a call option through the call-put parity and defined in the same way. Then the implied risk-neutral pdf is obtained by double differentiating equation (2) with respect to $X$,\(^{10}\) namely

$$\phi(S_T) = e^{rt} \frac{\partial^2 C}{\partial X^2} \bigg|_{X=S_T}$$

(3)

Hence, numerical differentiation of the price of a call with respect to the strike price gives an estimate of the implied pdf. Methods based on equation (2) are employed by Aït-Sahalia and Lo (1998), Malz (1997) and Neuhaus (1995). Cooper (2000) and Bliss and Panigirtzoglou (2002) show through a Montecarlo simulation the superiority of the non-parametric approach over the parametric one, since the former is more stable and invariant to outliers.

The basic difference between the two approaches is given by the assumption about the distribution of the short-term interest rate given by equation (1). Model (1) is unrealistic for interest rates; more reasonable models which encompass mean reverting properties or the possibility of modelling the term structure and the volatility term structure in a Heath-}

\(^{10}\) By applying the Leibniz-Newton rule to equation 2 , the first derivative is

$$\frac{\partial}{\partial X} \left( e^{-rt} \int_{X}^{\infty} (S - X) \cdot \phi(S)dS \right) \bigg|_{X=S_T} = e^{-rt} \left( [S - X] \cdot \phi(S) \bigg|_{X=S_T} + \int_{X}^{\infty} \phi(S)dS \right)$$

and the second

$$\frac{\partial}{\partial X} \left( e^{-rt} \int_{X}^{\infty} \phi(S)dS \right) \bigg|_{X=S_T} = e^{-rt} \left( \phi(S) - \int_{X}^{\infty} \frac{\partial \phi(S)}{\partial X}dS \right)$$
Jarrow-Morton framework would be more interesting. However, when one departs from the simplest models, estimates become very complicated and the larger structure does not allow much flexibility in the pdf shape, which sometimes becomes unrealistic. On the other hand, the parametric approach provides estimates of the parameters which are necessary when one is pricing the option through a Feynman-Kac partial differential equation. However, when the pdf is uniquely intended as a tool to convey information on market expectations, the non-parametric approach can be more useful and more flexible. In this work the focus is on the information content of the pdfs and I use the latter approach to exploit its greater flexibility.

3.3 Non-parametric estimate

My methodology departs somewhat from the standard non-parametric methods in order to gain stability in its estimates: I move from the \([\sigma, X]\) space to the \([\sigma, \Delta]\) space since in the former the smile is very sensitive to deep ITM options. The rationale behind this choice can be seen in Cooper (2000). Estimates are done through the following steps:

A. Quotes of put options written on interest-rate futures contracts are taken every day for a given time horizon. In order to use the most liquid contracts only in-the-money (ITM) and at-the-money (ATM) option quotes are used since they are more liquid; moreover, only put options are used since, given the discount quotes, they are equivalent to call options, which are more liquid in this type of market, i.e. a put on a futures with discount strike 96.50 is equivalent to a call on an interest-rate futures with strike 3.50 per cent. Note that for futures options the Black-Scholes-Samuelson formula for a call is given by (see Hull 2000)

\[
C^{BS} = e^{-rT} \left[ S \cdot F \left( \frac{\ln(S/X)}{\sigma \sqrt{T}} + \frac{\sigma \sqrt{T}}{2} \right) - X \cdot F \left( \frac{\ln(S/X)}{\sigma \sqrt{T}} - \frac{\sigma \sqrt{T}}{2} \right) \right]
\]

where \(S\) and \(X\) are redefined as 100 minus the futures contract and the strike, respectively.

11 The simplest extension of equation (1) is given by the Ho-Lee model which encompasses the basic features of the Heath-Jarrow-Morton framework. Let \(f(t, T)\) be the instantaneous forward interest rate between \(t\) and \(T\), then the stochastic differential equation for the short term rate is given by

\[
dr_t = \left( \frac{\partial f(t, T)}{\partial T} \right) dt + \gamma dW_t.
\]

Note that when one considers only the pdf for one interest rate at a given point in the future, \(t\) becomes a constant and the only difference from equation (1) is given by the slope of the forward curve between 0 and \(t\). For an extensive application of parametric estimates in the Heath-Jarrow-Morton framework see Amin and Ng (1997).
B. The implied volatility, \( \sigma \), for each option is found by minimizing through a Newton Rapson method the module of the distance between the call price given by the Black-Scholes-Samuelson formula and the current call price weighted by its vega, i.e. \( \sigma = \arg \min \left| \frac{(C^{BS} - C) / \mathcal{V}}{\mathcal{V}} \right| \).\(^{12}\) American-type options are corrected with the Barone-Adesi-Whaley methodology to apply the Black-Scholes-Samuelson formula. I then obtain the volatility smile, i.e. the volatility as a function of the strike, \( \sigma = g(X) \).

C. I pass from the \([\sigma, X]\) space to the \([\sigma, \Delta]\) space by considering the approximation \( \Delta = F(d_1) \). Put options with \( \Delta \) larger than 0.9 and smaller than 0.1 are not considered since very deep-ITM and deep-OTM futures options are very illiquid and can unnaturally stress the skewness of the pdf. The passage from the \([\sigma, X]\) space to the \([\sigma, \Delta]\) space is motivated by its stability with respect to the former.\(^{13}\) Figure 2 suggests that an evaluation of the smile through the strike/volatility (or also the strike/moneyness) schedule would overweight the tails of the distributions, signalling the existence of humps for very far from the money quotes. Alternatively, the delta/volatility schedule, cleaned of extreme values, would give more ‘reasonable’ shapes for the pdf.

D. The schedule between \( \sigma \) and \( \Delta = \delta(\sigma, X) \) is built by means of numerical interpolation. The volatility is given by

\[
\hat{\sigma} = \arg \min \left[ \eta(\sigma, \delta(\sigma, X)) - \sigma \right]^2
\]

where \( \eta \) is a cubic spline which gives a smooth piecewise polynomial interpolation between \( \sigma \) and \( \Delta = \delta(\sigma, X) \) weighted by their vega \( \mathcal{V}(X, \sigma) \) with 1,000 points.

E. Estimated volatility is plugged into the Black-Scholes-Samuelson formula to obtain the option price, namely \( \hat{C} = C^{BS}(X, \hat{\sigma}) \).

F. The central second order derivatives of \( \hat{C}(X) \) give the risk-neutral pdf, namely \( e^{\gamma r} \cdot \frac{\partial^2 \hat{C}}{\partial X^2} \bigg|_{X=S_r} = \phi(S) \).\(^{14}\)

\(^{12}\) Similarly, the implied volatility given by the squared difference between the current and the BS price, \( \sigma = \arg \min \left[ \left( \frac{(C^{BS} - C)}{\mathcal{V}} \right)^2 \right] \), gives similar results.

\(^{13}\) Some authors consider the \([\sigma, X/S]\) space where \( X/S \) is the distance from its strike, i.e. its moneyness, instead of the \([\sigma, \Delta]\) space.

\(^{14}\) The central second order derivative is defined as \( \frac{\partial^2 C}{\partial X^2} = \frac{C_{i+1} - 2C_i + C_{i-1}}{(\text{step})^2} \), where \( \text{step} \) is the partition of the exercise price scale. This approximation is of order \( O \left[ (\text{step})^2 \right] \).
Figure 2

SMILE AND DENSITY FUNCTION

![Graphs showing the relationship between volatility and other variables.](image-url)
The time series of the statistics of the pdfs can be used to analyze changes in monetary policy stance in the three areas.15

A further caveat is in order. The Black-Scholes-Samuelson option formula is supposed to hold for European-type options while options on 3-month interest-rate futures are of the American type, i.e. they can be exercised at any time before expiration. A correction must then be carried out to eliminate this bias. In this paper the Barone, Adesi and Whaley (1987) correction is implemented. Alternatively, Fornari and Violi (1998) and Malz (1997) show Black-Scholes-Samuelson estimates of American-type option pdfs where a lower bound is given for European-type pdf estimates.

In order to compare the statistics, estimated pdfs should be re-scaled to a common expiration date. In fact, the shape of the pdf for options traded on 15 May 2002 with expiration date 16 September 2002 — with \( \tau \) equal to 0.34 — is different from that of options traded on 29 May 2002 with the same expiration date — with \( \tau \) equal 0.30; more precisely, longer dated options tend to be more dispersed around the mean and have larger kurtosis. The Bank of England (2001) adjusts the estimated pdfs by interpolating the volatilities along the maturity spectrum of the options. In this paper a simpler correction is implemented; the implied volatility is divided by \( \sqrt{\tau} \) at point 4 above. It can be shown that this method is empirically equivalent to the one employed by the Bank of England (2001).16

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15 The statistics commonly used are: i) the mean which is the expected value of the distribution and by construction coincides with the futures current price, ii) the mode which is the most likely outcome, iii) the median which is the value that assigns a 50 per cent probability to the occurrence of that outcome, iv) the skewness which characterizes the distribution of probability on either side of the mean (a positively/negatively skewed distribution is one for which there is more/less probability attached to outcomes higher than the mean than to outcomes below the mean; a normal distribution has a skew of zero), v) the Pearson skewness which is given by (mean-median)/standard deviation and has a similar interpretation to the simple skewness coefficient, vi) the kurtosis which measures the fat tails or alternatively the peak of a distribution (it also measures the likelihood of extreme outcomes: the greater the likelihood of extreme outcomes, the fatter the tails of the distribution and the more peaked it is around the mean; the normal distribution has a kurtosis equal to three). Note that kurtosis can also be defined by \( \frac{0.5 \cdot (\sigma_{0.25} + \sigma_{0.75}) - \sigma_{0.50}}{\sigma_{0.50}} \) and skewness by \( \frac{(\sigma_{0.25} - \sigma_{0.75})}{\sigma_{0.50}} \), where \( \sigma_i \) stands for the implied volatility computed for an option with \( \Delta \) equal to \( i \). See Malz (1996 and 1997) for an application to currency options.

16 In the example above, the volatilities of the first options are divided by \( \sqrt{0.34} \), that of the second options by \( \sqrt{0.30} \).
Following Aït-Sahalia and Lo (1998) the relationship between $\Delta$ and $\sigma$ has been estimated through a Epanetchev-kernel with optimal bandwidth; call options have also been introduced in addition to put options; estimates for different time horizons have been performed. Results do not differ.

4. Evaluation of the information content

4.1 Two case studies

As a case study I compare the pdf’s for the 3-month interbank eurodeposit interest rate denominated in dollars, euros and sterling around the start of an inversion of the monetary policy stance. In particular, the tightening cycle started at the end of June 1999 in the United States, on 4 November 1999 in the euro area and on 8 September 1999 in the United Kingdom; the easing cycle started on 3 January 2001 in the United States, on 10 May 2001 in the euro area and on 8 March 2001 in the United Kingdom. For three dates around these episodes (two before and one after) pdf’s are estimated with a constant 90-day horizon – see Figures 3, 4, 5.

Phases of monetary tightening

In the United States the market was taken completely by surprise by the increase decided at the FOMC on 29 June 1999, which, furthermore, in the following days was perceived as isolated and not as the first of many — see the top panel of Figure 3. On 1 June 1999, the 3-month eurodollar spot rate was at 5 per cent and the futures contract with delivery at September 1999 quoted at around $5\frac{3}{4}$; moreover, the pdf did show a negative skewness. Two weeks later the shift in the pdf mean and the decrease in the skewness show that markets were expecting an increase in short-term rates that was not signalled by the spot short rate quotes, which remained fairly steady. Two weeks after the increase of 0.25 percentage points market sentiment was slightly oriented towards additional increases (as shown by the shape of the pdf on 12 July and by its skewness) but the subsequent 1.5 point rise which took place by May 2000 was not expected. The additional five increases in federal funds target rates neither shifted the pdf’s nor affected their skewness in the following month (not shown).
In the euro area the increase on 4 November 1999 was widely anticipated by agents — see the top panel of Figure 4. The pdf of 1 October 1999 had shown a large positive skewness slightly below 1.0 since the end of the summer. On 20 October the market perceptions were for a 3-month rate of around 3.75 per cent; one week after the increase perceptions were stable, if less dispersed around the mean. The skewness continued to remain large and pointed to additional rate increases.

In the United Kingdom the increase in the repo rate on 8 September 1999 was also anticipated by the markets: on 2 August 1999 the pdf was skewed to the left, assigning a larger probability to the occurrence of an increase in short-term rates — see the top panel of Figure 5. On 25 August the pdf shifted to the left, thus signalling an increase in the uncertainty of agents. One week after the rate increase the distribution moved markedly to the right, recording an additional increase in its skewness. Markets assigned a large likelihood to the occurrence of additional rate increases.

**Phases of monetary easing**

The starting date of cuts in US policy rates (FOMC on 3 January 2001) was partially anticipated by market participants. The pdf’s started to show a negative skewness (lower rates are deemed more probable) from the beginning of the last quarter of 2000 — see the bottom panel of Figure 3. Thus, the direction of short-term rates was expected even if the size of the decrease was not. On 15 December 2000, about three weeks before the Federal Reserve decision, the expected 3-month rate was below 6 per cent and the skewness was even more negative. After the decision had been taken, on 16 January 2001, market perception was of a substantial additional cut in policy rates (shown by a decrease in skewness as well as by the expected 3-month rate).

In the euro area, conversely, the monetary easing cycle on 10 May 2001 seems not to have been anticipated by the markets — see the bottom panel of Figure 4. In the course of the month preceding the GC meeting, from 4 to 25 April 2001, the pdf shifted to the right, even if the skewness decreased somewhat below zero. Moreover, one week after the decision had been taken, perceptions were almost unchanged.

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17 In this analysis I do not consider the possibility that the increase in the US policy rate could have affected the decisions in the euro area and the United Kingdom.
In the United Kingdom the decrease in the repo rate decided on 8 March 2001 was unexpected — see the bottom panel of Figure 5. The two pdf’s of 1 and 23 February 2001 are identical; after the cut the dispersion increased substantially but the skewness remained steady around nil, signalling that the market was assigning the same probability to either move around the mean.

Results show that market participants did not expect the tightening monetary cycle in the United States and the additional policy-rate moves in 1999. The easing cycle of 2001 was largely anticipated and markets may have played a role in ‘driving’ US central bank decisions, since the distributions rapidly shifted to the left, showing that a business cycle slowdown was widely expected. In the euro area, agents had less clearer perceptions; the tightening cycle of 1999 was partly anticipated even if its duration was not expected; the easing cycle of 2001 was completely unexpected and, conversely, an increase seemed to be expected after the first rate cut. The uncertainty about the recovery in the euro area business cycle could have also contributed to this misperception. In the United Kingdom the rate increases of 1999 were not expected, the cuts only modified the uncertainty of market participants about successive monetary moves.

4.2 A time series analysis

In order to go beyond the simple comparison of pdf’s at two dates around the central bank meeting, the time series of the pdf statistics have also been evaluated. These time series allow the dynamics of market sentiment during the sample period to be evaluated. I calculate the time series for the implied ATM volatility, the skewness, the kurtosis, the probability of a decrease of over 0.25 percentage points in the interest rate and that of an increase of over 0.25 percentage points, for the three areas.18 The skewness and the probability of an upward/downward move give the most straightforward intuition of the link between rate

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18 The skewness is defined as the ratio between the third central moment and the standard deviation to the power of three, i.e. $\mu_3/\sigma^3$, the kurtosis as $\mu_4/\sigma^4$. The probability of a decrease in the interest rate by over 0.5 percentage points is defined as $\Pr(x - \bar{x} < -0.5) = \Phi(\bar{x} - 0.5)$, where $x$ is the interest rate and $\bar{x}$ is its average. Analogously, the probability of an increase of over 0.5 percentage points is defined as $\Pr(x - \bar{x} > 0.5) = 1 - \Phi(\bar{x} + 0.5)$. 
changes and market expectations; the kurtosis consistently shows a strong co-movement with the skewness.

For the sake of brevity, I report the policy rate and the skewness in the three areas in Figure 6; the other statistics also document interesting features of pdf's. In the United States — top panel — the skewness gently increased by the second quarter of 1999, signalling that markets were expecting a monetary tightening; during the third quarter of 2000 the skewness rapidly moved downwards, anticipating the start of the monetary easing cycle. In the euro area — mid panel — the change in the skewness during 1999 were much more dramatic: it increased from −0.2 to 1.0 in two months, signalling that markets were unanimously expecting a policy-rate increase. The euro area easing cycle of 2001 seems not to have been anticipated by markets, which continued to show a zero skewness until January 2002. In the United Kingdom — bottom panel — the rate increase of 1999 was not anticipated, as shown by the skewness which lagged behind the policy-rate moves; conversely, the rapid decrease in skewness during the second half of 2000 shows that the tightening was largely expected.

The information content of the pdf statistics is evaluated by GMM-regressing the monthly change in the 3-month interest rate on a set of variables, namely

\[ 3M_t - 3M_{t-1} = \alpha_0 + \alpha_1 \cdot \text{slope}_{t-k} + \alpha_2 \cdot (3M - fut)_{t-k} + \alpha_3 \cdot (3M - 1M)_{t-k} \]

\[ + \alpha_4 \cdot \text{skew}_{t-k} + \alpha_5 \cdot \text{vol}_{t-k} + \varepsilon_t \]  

for \( k = 1, 2, 3, 4 \), where 3\( M \) the 3-month rate, \text{slope} is difference between the 10-year government bond yield and the 3-month rate, 1\( M \) is the 1-month interest rate, \text{fut} is the futures contract written on the 3-month rate with the closest expiration date, \text{skew} the skewness of the pdf, \text{vol} the ATM implied volatility.

Results (coefficients and t-statistics are presented in Table 1) document that the slope of the yield curve is never significant, except in the euro area; in the euro area and in the UK the difference between the spot and the futures (3\( M \) − \text{fut}) is the most significant variable in explaining changes in short-term rates; the slope of the term structure at the very short-term (3\( M \) − 1\( M \)) is significant in all of the three markets to different degrees. Among the statistics obtained from the pdfs, the skewness is very significant in the euro area and in the UK, the volatility only in the euro area; neither in the US. The test on the joint significance of the skewness and of the volatility is never rejected at any lag in the euro area and in the UK.
ESTIMATES OF EQUATION (4)

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Monthly data; coefficients and t-statistics (in brackets) of GMM estimates for the equation $3M_t - 3M_{t-1} = \alpha_0 + \alpha_1\text{slope} + \alpha_2(3M - fut)_{t-k} + \alpha_3(3M - 1M)_{t-k} + \alpha_4\text{skew}_{t-k} + \alpha_5\text{vol}_{t-k} + \epsilon_t$ for $k = 1, 2, 3, 4,$ where $3M$ is the 3-month rate, $\text{slope}$ the difference between the 10-year government bond yield and the 3-month rate, $1M$ the 1-month rate, $\text{fut}$ the futures contract on the 3-month rate with the closest expiration date, $\text{skew}$ the skewness of the pdf, $\text{vol}$ the ATM implied volatility. Instruments are a constant and the same regressors with $t - k$ and $t - k - 1$. I use the Hansen method to take into account overlapping observations. The column under the header Wald reports the p-value for the Wald test that $\alpha_4 = \alpha_5 = 0$.

Despite the small cross-section sample I also investigate the time series extracted from the pdfs around the monetary policy meeting. Figure 7 reports the cross-sectional average
and median skewness from 40 business days before to 40 business days after the meetings of the monetary authorities in the three areas, gathered according to the directive announced; in what follows, a tightening/steady/easing meeting is defined as one where an increase/no-change/decrease is made. In the United States, the skewness around a tightening move is steady around 0.5 and does not show a particular trend; the average around a meeting where no change is made is steady before the meeting and slightly decreasing afterwards; the skewness around an easing move is increasing until the meeting and accelerates thereafter. The only clear trend is shown by the skewness around an easing meeting, which supports the hypothesis of a mean-reverting process for market expectations. In the euro area, there is also no trend in the skewness. The main difference with the US is that the skewness around an easing meeting has negative values of around \(-0.1\). In the United Kingdom the skewness around an easing meeting shows an increasing trend after the meeting as in the United States; no trend appears in the other two lines.

In the three areas the skewness around a tightening meeting is largely positive, signalling that markets, on average, correctly predict an increase in short-term rates; it is either negative or nil around an easing meeting but reverts soon after. In order to assess market expectation movements around a meeting, I regress the cross-sectional average and median on a constant and a trend; results are shown in Table 5. The coefficients of the trend document that around a meeting where a decrease is made the skewness tends to move upwards, while the trend is uncertain around a tightening meeting; for a neutral meeting the positive value of the coefficient could be due to the extremely low level of interest rates during the sample period, which made it reasonable to assume an upward move in the foreseeable future.

Table 5 also reports the p-value for a test of equality of the means before and after monetary meetings of mean and median; results show that there is a significant difference between means in almost all of the cases.

A caveat is in order. The averages are calculated on very small samples (for example the increasing meetings for the United States are only six, the decreasing meetings eleven, the steady meetings thirteen) and confidence intervals are quite large. Only the confidence interval of average skewness — computed as plus and minus two standard errors — around a tightening meeting does not contain the zero level in all the three areas.\[^{19}\]

\[^{19}\] The asymptotic distribution for the skewness and the kurtosis are, respectively, \(\sqrt{n} \cdot \text{skewness} \sim N(0, 6)\)
5. Conclusion

The use of pdf’s implied in futures options is extensively used by central banks to assess market perceptions on future monetary moves. The analysis of the pdf makes it possible to go beyond the simple point estimates provided by the futures contract for the chosen delivery date since it provides confidence intervals characterized by their skewness and probability of extreme values. This paper aims to evaluate whether this application has a solid rationale.

The methodology applied to the monetary policy decisions taken in the United States, the euro area and the United Kingdom from January 1999 to May 2002 shows that markets correctly forecasted the monetary easing of 2001 in the United States in the course of the second half of 2000, but not for the euro area and the United Kingdom. The evidence for the tightening cycle of 1999 is mixed: markets expected an increase in euro area policy rates at the beginning of 1999; expectations were less clear for the United States’ increases. In the case of the United Kingdom the increase was not forecasted.

A cross-sectional analysis might have wider possibility of application but it is still at an early stage given the small sample. However, preliminary results show that the time series statistics extracted from the pdf’s have different levels and slopes according to the expected monetary policy decision.

\[ \sqrt{n} \cdot kurtosis \overset{d}{\sim} N(3, 24), \text{ where } n \text{ is the sample population.} \]
### ECB DIRECTIVES AND POLICY INTEREST RATES

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<td>Increase</td>
<td>3.75</td>
<td>–</td>
<td>4.75</td>
</tr>
<tr>
<td>September 1, 2000</td>
<td>Increase</td>
<td>3.50</td>
<td>–</td>
<td>4.50</td>
</tr>
<tr>
<td>June 28 (announced on 8 June*), 2000</td>
<td>Maintain</td>
<td>3.25</td>
<td>4.25</td>
<td>–</td>
</tr>
<tr>
<td>June 9, 2000</td>
<td>Increase</td>
<td>3.25</td>
<td>4.25</td>
<td>–</td>
</tr>
<tr>
<td>April 28, 2000</td>
<td>Increase</td>
<td>2.75</td>
<td>3.75</td>
<td>–</td>
</tr>
<tr>
<td>March 17, 2000</td>
<td>Increase</td>
<td>2.50</td>
<td>3.50</td>
<td>–</td>
</tr>
<tr>
<td>February 4, 2000</td>
<td>Increase</td>
<td>2.25</td>
<td>3.25</td>
<td>–</td>
</tr>
<tr>
<td>November 5, 1999</td>
<td>Increase</td>
<td>2.00</td>
<td>3.00</td>
<td>–</td>
</tr>
<tr>
<td>April 9, 1999</td>
<td>Decrease</td>
<td>1.50</td>
<td>2.50</td>
<td>–</td>
</tr>
<tr>
<td>January 22, 1999</td>
<td>Decrease</td>
<td>2.00</td>
<td>3.00</td>
<td>–</td>
</tr>
<tr>
<td>January 4, 1999</td>
<td>Maintain</td>
<td>2.75</td>
<td>3.00</td>
<td>–</td>
</tr>
<tr>
<td>January 1, 1999</td>
<td>Maintain</td>
<td>2.00</td>
<td>3.00</td>
<td>–</td>
</tr>
</tbody>
</table>

* Announcement of change from fixed to variable rate from the 28 June.
### TABLE 3

<table>
<thead>
<tr>
<th>Date</th>
<th>Directive</th>
<th>Funds rate</th>
<th>Discount rate</th>
<th>Official bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>June 25, 2003</td>
<td>Decrease</td>
<td>1.00</td>
<td>0.50</td>
<td>Easier</td>
</tr>
<tr>
<td>November 6, 2002</td>
<td>Decrease</td>
<td>1.25</td>
<td>0.75</td>
<td>Easier</td>
</tr>
<tr>
<td>March 19, 2002</td>
<td>Maintain</td>
<td>1.75</td>
<td>1.25</td>
<td>Neutral</td>
</tr>
<tr>
<td>January 29-30, 2002</td>
<td>Maintain</td>
<td>1.75</td>
<td>1.25</td>
<td>Easier</td>
</tr>
<tr>
<td>December 11, 2001</td>
<td>Decrease</td>
<td>1.75</td>
<td>1.25</td>
<td>Easier</td>
</tr>
<tr>
<td>November 6, 2001</td>
<td>Decrease</td>
<td>2.00</td>
<td>1.50</td>
<td>Easier</td>
</tr>
<tr>
<td>October 2, 2001</td>
<td>Decrease</td>
<td>2.50</td>
<td>2.00</td>
<td>Easier</td>
</tr>
<tr>
<td>September 17*, 2001</td>
<td>Decrease</td>
<td>3.00</td>
<td>2.50</td>
<td>Easier</td>
</tr>
<tr>
<td>August 21, 2001</td>
<td>Decrease</td>
<td>3.50</td>
<td>3.00</td>
<td>Easier</td>
</tr>
<tr>
<td>June 27, 2001</td>
<td>Decrease</td>
<td>3.75</td>
<td>3.25</td>
<td>Easier</td>
</tr>
<tr>
<td>May 15, 2001</td>
<td>Decrease</td>
<td>4.00</td>
<td>3.50</td>
<td>Easier</td>
</tr>
<tr>
<td>April 18*, 2001</td>
<td>Decrease</td>
<td>4.50</td>
<td>4.00</td>
<td>Easier</td>
</tr>
<tr>
<td>March 20, 2001</td>
<td>Decrease</td>
<td>5.00</td>
<td>4.50</td>
<td>Easier</td>
</tr>
<tr>
<td>January 31, 2001</td>
<td>Decrease</td>
<td>5.50</td>
<td>5.00</td>
<td>Easier</td>
</tr>
<tr>
<td>January 3-4*, 2001</td>
<td>Decrease</td>
<td>6.00</td>
<td>5.50</td>
<td>Easier</td>
</tr>
<tr>
<td>December 19, 2000</td>
<td>Maintain</td>
<td>6.50</td>
<td>6.00</td>
<td>Easier</td>
</tr>
<tr>
<td>June 27, 2000</td>
<td>Maintain</td>
<td>6.50</td>
<td>6.00</td>
<td>Inflationary</td>
</tr>
<tr>
<td>May 16, 2000</td>
<td>Increase</td>
<td>6.50</td>
<td>6.00</td>
<td>Inflationary</td>
</tr>
<tr>
<td>March 21, 2000</td>
<td>Increase</td>
<td>6.00</td>
<td>5.50</td>
<td>Inflationary</td>
</tr>
<tr>
<td>February 2, 2000</td>
<td>Increase</td>
<td>5.75</td>
<td>5.25</td>
<td>Tighter</td>
</tr>
<tr>
<td>December 21, 1999</td>
<td>Maintain</td>
<td>5.50</td>
<td>5.00</td>
<td>Neutral</td>
</tr>
<tr>
<td>November 16, 1999</td>
<td>Increase</td>
<td>5.50</td>
<td>5.00</td>
<td>Neutral</td>
</tr>
<tr>
<td>October 5, 1999</td>
<td>Maintain</td>
<td>5.25</td>
<td>4.75</td>
<td>Tighter</td>
</tr>
<tr>
<td>August 24, 1999</td>
<td>Increase</td>
<td>5.25</td>
<td>4.75</td>
<td>Neutral</td>
</tr>
<tr>
<td>June 29-30, 1999</td>
<td>Increase</td>
<td>5.00</td>
<td>4.50</td>
<td>Neutral</td>
</tr>
<tr>
<td>May 18, 1999</td>
<td>Maintain</td>
<td>4.75</td>
<td>4.50</td>
<td>Tighter</td>
</tr>
<tr>
<td>February 2-3, 1999</td>
<td>Maintain</td>
<td>4.75</td>
<td>4.50</td>
<td>Tighter</td>
</tr>
</tbody>
</table>

* Denotes policy change outside of scheduled meeting
TABLE 4

BANK OF ENGLAND DIRECTIVES AND POLICY INTEREST RATES

<table>
<thead>
<tr>
<th>Date</th>
<th>Directive</th>
<th>repo rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 5-6, 2004</td>
<td>Increase</td>
<td>4.00</td>
</tr>
<tr>
<td>November 5-6, 2003</td>
<td>Increase</td>
<td>3.75</td>
</tr>
<tr>
<td>July 9-10, 2003</td>
<td>Decrease</td>
<td>3.50</td>
</tr>
<tr>
<td>February 5-6, 2003</td>
<td>Decrease</td>
<td>3.75</td>
</tr>
<tr>
<td>November 7-8, 2001</td>
<td>Decrease</td>
<td>4.00</td>
</tr>
<tr>
<td>October 3-4, 2001</td>
<td>Decrease</td>
<td>4.50</td>
</tr>
<tr>
<td>September 18, 2001</td>
<td>Decrease</td>
<td>4.75</td>
</tr>
<tr>
<td>August 1-2, 2001</td>
<td>Decrease</td>
<td>5.00</td>
</tr>
<tr>
<td>May 9-10, 2001</td>
<td>Decrease</td>
<td>5.25</td>
</tr>
<tr>
<td>April 4-5, 2001</td>
<td>Decrease</td>
<td>5.50</td>
</tr>
<tr>
<td>March 7-8, 2001</td>
<td>Decrease</td>
<td>5.75</td>
</tr>
<tr>
<td>February 9-10, 2000</td>
<td>Increase</td>
<td>6.00</td>
</tr>
<tr>
<td>January 12-13, 2000</td>
<td>Increase</td>
<td>5.75</td>
</tr>
<tr>
<td>November 3-4, 1999</td>
<td>Increase</td>
<td>5.50</td>
</tr>
<tr>
<td>September 7-8, 1999</td>
<td>Increase</td>
<td>5.25</td>
</tr>
<tr>
<td>June 9-10, 1999</td>
<td>Decrease</td>
<td>5.00</td>
</tr>
<tr>
<td>April 7-8, 1999</td>
<td>Decrease</td>
<td>5.25</td>
</tr>
<tr>
<td>February 3-4, 1999</td>
<td>Decrease</td>
<td>5.50</td>
</tr>
<tr>
<td>January 6-7, 1999</td>
<td>Decrease</td>
<td>6.00</td>
</tr>
</tbody>
</table>
Around the start of US monetary tightening (FOMC of 29/30 June 1999)

To the extent that agents are risk averse their true pdf may differ from those shown and may be shifted to the right.

The pdf indicates the likelihood of a particular event occurring.

The probability of the 3-month eurodollar deposit rate being $X \pm 0.125$ ticks is given by the area under the curve between $X + 0.125$ and $X - 0.125$. The area under the whole curve is always 100%.
Figure 4

CONSTANT 90-DAY HORIZON PDFs
FOR EURO AREA SHORT-TERM RATES

Around the start of euro area monetary tightening (GC of 4 November 1999)

Around the start of euro area monetary easing (GC of 10 May 2001)

To the extent that agents are risk averse their true pdf may differ from those shown and may be shifted to the right.

The pdf indicates the likelihood of a particular event occurring.

The probability of the 3-month eurodollar deposit rate being $X \pm 0.125$ ticks is given by the area under the curve between $X+0.125$ and $X-0.125$. The area under the whole curve is always 100%.
To the extent that agents are risk averse their true pdf may differ from those shown and may be shifted to the right.

The pdf indicates the likelihood of a particular event occurring.

The probability of the 3-month eurodollar deposit rate being $X \pm 0.125$ ticks is given by the area under the curve between $X+0.125$ and $X-0.125$. The area under the whole curve is always 100%.
Figure 6

SKEWNESS AND POLICY RATES

USA

3 month rate  skewness (RHS)

Euro area

3 month rate  skewness (RHS)

UK

3 month rate  skewness (RHS)
The three lines show the mean and the median of the skewness 40 business days before and 40 after monetary policy meetings held at $t = 0$ according to the chosen directive; the dashed line presents the mean/median skewness around a meeting when an increase has been announced, the dotted line around a meeting when a decrease has been announced, the continuous line around a meeting when no change in the policy rate has been made.
### CROSS-SECTIONAL REGRESSION AND TEST OF EQUALITY

OF THE MEANS BEFORE AND AFTER THE MONETARY MEETING

<table>
<thead>
<tr>
<th></th>
<th>median</th>
<th>mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>easing steady tightening</td>
<td>easing steady tightening</td>
</tr>
<tr>
<td>USA</td>
<td>constant <strong>-0.17</strong> 0.38** 0.48**</td>
<td><strong>0.07</strong> 0.62** 0.36**</td>
</tr>
<tr>
<td></td>
<td>trend <strong>0.00</strong> 0.00** -0.00</td>
<td><strong>0.00</strong> 0.00 0.00</td>
</tr>
<tr>
<td></td>
<td>test p-value 0.00 0.03 0.09</td>
<td>0.00 0.35 0.27</td>
</tr>
<tr>
<td>euro area</td>
<td>constant <strong>-0.10</strong> 0.51** 0.53**</td>
<td><strong>-0.04</strong> 0.15** 0.57**</td>
</tr>
<tr>
<td></td>
<td>trend <strong>0.00</strong> 0.00** -0.00**</td>
<td>0.00 0.00** -0.00**</td>
</tr>
<tr>
<td></td>
<td>test p-value 0.34 0.00 0.00</td>
<td>0.67 0.00 0.00</td>
</tr>
<tr>
<td>UK</td>
<td>constant 0.01 0.35** 0.41**</td>
<td>0.047** 0.22** 0.40**</td>
</tr>
<tr>
<td></td>
<td>trend <strong>0.00</strong> 0.00** -0.00</td>
<td><strong>0.00</strong> -0.00** 0.00</td>
</tr>
<tr>
<td></td>
<td>test p-value 0.00 0.01 0.96</td>
<td>0.00 0.01 0.35</td>
</tr>
</tbody>
</table>

The table reports coefficients and t-statistics of the regression of the cross sectional mean and median of Figure 7 on a constant and a trend. * (**) indicate that the coefficient is significant at the 5 (1) per cent level. The rows with the ‘test p-value’ report a test of equality of the mean: it is made by classifying the sample before and after the monetary meeting and is based on a single-factor, between-subjects analysis of variance (ANOVA); the row reports the p-value of a t-test with 79 degrees of freedom: the null hypothesis that the sample means are equal is rejected at α per cent for a p-value smaller than α.
Appendix

The data

Futures options are traded on the same exchanges as the underlying futures contract and are executed and cleared with similar procedures as the underlying, the main differences being the margining and resettlement practices. A distinction can be drawn between pure and conventional futures options (Duffie 1989). The former give the buyer daily any change in the futures option price in order to mark the buyer’s margin account. The latter require the payment of the option premium when it is purchased and at exercise pays the buyer the difference between the underlying and the strike price. Thus, a pure futures option is not an option at all, but rather a futures contract that delivers the corresponding conventional option on expiration. The futures options traded at the London International Financial Futures and Options Exchange (LIFFE) have rulings that make them more like pure than conventional options. Futures options traded at CME are hybrids of these two categories. Thus, the pure futures option price is the forward price of the underlying option. If $O(t, T, \ldots)$ is the European type conventional futures option price at time $t$ with expiration at $T$, its pure futures option equivalent price is $O^F(t, T, \ldots) = e^{-r(T-t)} \cdot O(t, T, \ldots)$, where $r$ is the risk-free rate.

I collect daily quotes of end-of-day settlement prices of futures options written on 3-month euro-currency interbank deposits denominated in US dollars, euros and sterling from January 1999 to May 2004. US dollar 3-month interest-rate futures options are traded at the Chicago Mercantile Exchange (CME), while euro and sterling 3-month interest-rate futures options are traded at the LIFFE. Both datasets are available on the Internet at the addresses \texttt{www.cme.com} and \texttt{www.liffe.com}, as well as from Thomson Financial Datastream.

At LIFFE, the American-type options are written on 3-month interest-rate futures contracts on euro and sterling interbank deposits and have delivery months in March, June, September, December — quarterly expiry months — and two serial months, so that ten expiry months for the euro and six for the sterling are available for trading, with the nearest three expiry months being consecutive calendar months. The last trading day is two business days prior to the third Wednesday of the expiry month for both serial expiry months and quarterly expiry months. The minimum price movement is 0.005 and the associated tick sizes and values are euro 12.50 and £6.25 for the euro and sterling deposits, respectively. Strike price
intervals are 0.25 but 0.125 for the first four quarterly expiry months and for the serial expiry months. As regards futures contracts, they are written onto the European Bankers Federations’ Euribor Offered Rate (EBF Euribor) for 3-month euro deposits and onto the British Bankers’ Association London Interbank Offered Rate (BBA LIBOR) for 3-month sterling deposits; the futures contract delivery month associated with each option expiry month is March in respect of the January, February and March expiry months; June in respect of the April, May and June expiry months; September in respect of the July, August and September expiry months; December in respect of the October, November and December expiry months.

At CME, the American-type options are written on the IMM index for the 3-month ‘Eurodollar Time Deposit’ futures contracts and have delivery months in March, June, September, December — March quarterly cycle — and two serial months, so that eight expiry months area available for trading, with the nearest three expiry months being consecutive calendar months. The last trading day is two business days prior to the third Wednesday of the expiry month at 7.00 on the day of exercise for both quarterly cycle and serial expiry months. The strike price interval is 0.25 and 0.125 for some particular expiry months. There are three minimum price movements, depending on the expiry month, 0.0025, 0.005 and 0.01 (one tick) whose associated tick sizes and values are $6.25, $12.50 and $25.00, respectively. The IMM index for 3-month ‘Eurodollar Time Deposit’ futures contracts underlying is the 3-month BBA LIBOR on eurodollar deposits, with a principal value of $1,000,000, forty delivery months in the March quarterly cycle, and the four nearest serial contract months.

In what follows we refer to policy rates in the three areas to the target federal funds rate in the US, the rate on main refinancing operations in the euro area, the repurchase agreement rate in the UK. The 3-month interest rates are the 3-month EBF Euribor for euro-denominated interbank deposits, the three 3 BBA LIBOR for dollar and sterling-denominated deposits. policy-rate target changes are determined eight times per year by the Federal Open Market Committee (FOMC) in the United States, and every month by the General Council Meeting in the euro area,20 every month by the Monetary Policy Committee (MPC) in the UK. Central bank directives and policy rates are shown in Tables 1-3.

20 Since 8 November the General Council meeting where policy changes are decided has a four week frequency.
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1999


2000


2001


2003


2004


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