Estimating expectations of shocks using option prices

by Antonio Di Cesare
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ESTIMATING EXPECTATIONS OF SHOCKS USING OPTION PRICES

by Antonio Di Cesare*

Abstract

The jump-diffusion model introduced by Merton is used to price a cross-section of options at different dates. At any point in time, the parameters of the model are estimated by minimizing the sum of squared implied volatility errors, and their informational content is compared with the widely used Black and Scholes implied volatility, calculated on at-the-money options. While in normal conditions the parameters of Merton’s model do not seem to provide any additional information, in periods of high variability of asset prices the jump-diffusion approach may help to disentangle the cases in which volatility reflects only uncertainty on economic fundamentals from those in which it is fuelled by fears of financial crisis.

JEL classification: G12, G13.

Keywords: jump-diffusion stochastic processes, option pricing, volatility.

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1. Introduction

Measuring agents’ expectations on future values of financial variables such as stock prices, interest rates and foreign exchange rates is of critical interest both for traders, who are interested in correctly pricing and hedging the financial assets they deal with, and for policy-makers, who are mainly interested in agents’ preferences and expectations in order to monitor market conditions.

Nowadays, many methodologies are available to estimate both the historical distribution of asset prices and the full state-price density (SPD) \(^2\), but usually attention is concentrated only on the first two moments of the distribution \(^3\), which have very simple and intuitive meanings. For this purpose, the futures (or forward) markets are typically investigated to recover the expected values of the underlying assets, under the assumption that agents are risk-neutral. Moreover, the implied volatility calculated on at-the-money options, assuming that the model introduced by Black and Scholes (1973) holds, is generally used as a measure of the uncertainty associated with market expectations.

Aït-Sahalia (2001) provided a statistical test showing that some observed option prices are compatible only with underlyings that follow stochastic processes with jumps. His result confirms what was previously found by Bakshi, Cao and Chen (1997) and Bates (2000) about the importance of using models with jumps when pricing stock index options. This kind of model has also been used recently by Pan (2002) and by Beber and Brandt (2003) to study the jump-risk premium implicit in option prices and to assess the effects of macroeconomic news on beliefs.

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\(^2\)The SPD is the continuous-state counterpart to the prices of Arrow-Debreu securities that pay 1 unit of money in a given state of the world and nothing in all other states.

\(^3\)A nice survey of recent methods for extracting market expectations from asset prices for monetary policy purposes is found in Söderlind and Svensson (1997).
and preferences, respectively. Based on these results, our paper focuses on the jump-diffusion model introduced by Merton (1976) in order to gather, in a simple and parsimonious setting, more information on the determinants of volatility than the widely used Black and Scholes model. In fact, Merton’s model is simple enough to allow for closed-form solutions for European option prices and, at the same time, it gives an opportunity to “decompose” the uncertainty into three components: the “pure” volatility (i.e. the stock price variability not due to any particular event), the expected number of jumps and the size of these shocks.

The remainder of the paper is organized as follows. The next Section contains a brief description of the techniques usually used to estimate the volatility, pointing out their main strengths and weaknesses. In Section 3 the pricing model introduced by Merton is described and in Section 4 it is applied to S&P 500 futures options listed at the Chicago Mercantile Exchange. A few final remarks are made in Section 5.

2. Standard measures of volatility

It is useful to distinguish between historical and implied volatility. Within the first approach, $n + 1$ equally spaced historical observations are used to estimate the current level of volatility as

$$
\hat{\sigma} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (u_i - \bar{u})^2}
$$

where $u_i = \log \frac{S_i + D_i}{S_{i-1}}$, $\bar{u} = \frac{1}{n} \sum_{i=1}^{n} u_i$, $S_i$ is the value of the asset at the end of period $i$ and $D_i$ is the dividend paid during period $i$. Many well-known models, such as the ARCH and GARCH models introduced by Engle (1982) and Bollerslev (1986) and the exponentially weighted moving average (EWMA) model used, for instance, in J. P. Morgan (1995), can be seen as an extension of the previous formula\(^4\). The main drawback of historical models is that, by construction, they cannot take into account the current expectations of agents. They just assume that future volatility is a function of its historical value, no matter what agents’ beliefs actually are.

\(^4\)An introductory description of ARCH, GARCH and EWMA models can be found in Hull (2000).
The implied volatility approach, on the contrary, does not rely on any use of historical data but instead extracts information from current prices. Since prices should reflect agents’ opinions about future asset values, this approach appears to be more suitable for monetary policy purposes. Several methodologies are now available. The most common approach is to model the dynamics of the underlying asset so that the SPD can be written in a parametric form. This approach is heavily model-dependent but the results usually have a natural interpretation, since they are directly linked with the behaviour of the asset. The parametric form of the SPD can also be chosen directly, even if the choice of a particular functional form is often arbitrary and the interpretation of the results is not always clear. A more recent approach, introduced by Ait-Sahalia and Lo (2000), is based on a non-parametric estimation of the pricing function of a call option \( C \), followed by the application of the identity due to Breeden and Litzenberger (1978)

\[
q(x) = e^{rT} \frac{\partial^2 C}{\partial K^2} \bigg|_{K=x}
\]

where \( K \) is the strike price of the option, \( T \) is the maturity date, \( r \) is the risk-free interest rate and \( q(x) \) is the value of the SPD at point \( x \). This method is probably the most general one, but it is also very data intensive, as is usual with non-parametric estimators. For this reason it works well only with large samples and it is necessary to assume that the SPD remains unchanged for a long period of time.

Another indicator frequently used to assess the level of risk implied in the market is the probability that a shock, bigger than a given threshold, occurs in a certain period of time. The Financial Stability Review of the Bank of England, for instance, regularly publishes such an indicator, on the basis of a non-parametric estimate of the PDF. There are two main problems with this approach: i) it requires precise estimates of the tails of the distribution, which is usually a difficult task due to lack of data; ii) it is not clear if the reduction of asset price is going to occur as a result of some kind of shock, i.e. following a single extraordinary event, that hits the market at a given point in time or is, instead, the result of the volatility of the

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5Bank of England (2000) points out a number of important caveats regarding the extent to which PDFs derived from option prices are really correct distributions of market expectations.
process that can bring the quotes down. If the second case is true, it should not be of great concern to agents, since they can hedge their portfolios using standard techniques. Merton’s model, adopted in this paper, allows us to disentangle the two cases.

3. The pricing model

As in Merton (1976), the price $S$ of the asset underlying the option, under the risk-neutral probability measure $Q$, is described by the following stochastic differential equation

$$
\begin{align*}
\left\{ \begin{array}{ll} 
    dS(t) = S(t-) \left[ (r - \delta - \lambda \rho) dt + \sigma dW(t) + \rho dN(t) \right] \\
    S(0) = x
\end{array} \right.
\end{align*}
$$

(2)

where $r$ is the risk-free interest rate, $\delta$ is the asset’s dividend yield, $W$ is a standard Brownian motion and $N$ is a Poisson process, independent of $W$, with intensity $\lambda \geq 0$. As usual, $S(t-) = \lim_{s \to t} S(s)$. All parameters $(r, \delta, \sigma, \rho, \lambda)$ are assumed to be constant over time. Loosely, (2) describes an asset, a stock for instance, whose price fluctuates continuously over time due to the presence of $W$ and which “sometimes” has a jump given to the presence of $N$. The parameter $\sigma$ is a measure of the variability of the process associated with $W$, $\rho > -1$ is the instantaneous percentage change of the price that occurs when there is a shock, and $\lambda$ is the number of expected jumps per unit of time. One can think of the variations of $N$ as a description of the effects on prices of unexpected events.

It is well-known (see, for instance, Bardhan and Chao, 1993) that the solution of (2) is given by

$$
S(t) = x(1 + \rho)^{N(t)} \exp \left\{ \left( r - \delta - \lambda \rho - \frac{\sigma^2}{2} \right) t + \sigma W(t) \right\}
$$

(3)

and that $e^{-(r-\delta)t}S(t)$ is a martingale under $Q$. It is easy to see that when $\lambda = 0$ (i.e. there are no jumps) or $\rho = 0$ (i.e. the shocks have no effects on $S$) then (3) reduces to the famous model introduced by Black and Scholes (1973).

6Throughout the paper all variables are expressed on an annual basis.
Under suitable hypotheses about the completeness of the market\(^7\), if we use \(C_{BS}(x, r, \delta, T, K, \sigma)\) to denote the Black and Scholes price of a European call option with maturity \(T\) and strike price \(K\) when the price of the underlying is \(x\), the interest rate is \(r\), the dividend yield is \(\delta\) and the volatility is \(\sigma\) \(^8\), then the price \(C(x, r, \delta, T, K, \sigma, \rho, \lambda)\) of a European call option written on the security (3) is given by

\[
C = \sum_{n=0}^{\infty} C_{BS} \left( x e^{-\lambda \rho T} (1 + \rho)^n, r, \delta, T, K, \sigma \right) \frac{e^{-\lambda T} (\lambda T)^n}{n!}.
\]

Again, it can be checked that when \(\lambda = 0\) or \(\rho = 0\), then \(C = C_{BS}\). Since in the empirical implementation we will use options written on futures contracts, we need to modify (4) using the result of Black (1976): a futures option can be seen as an option on a stock paying a continuous dividend yield at rate \(r\). Hence, we can write the price of a European call futures option as

\[
C = \sum_{n=0}^{\infty} C_B \left( x e^{-\lambda \rho T} (1 + \rho)^n, r, T, K, \sigma \right) \frac{e^{-\lambda T} (\lambda T)^n}{n!},
\]

where

\[
C_B(x, r, T, K, \sigma) = e^{-rT} [x N(d_1) - K N(d_2)],
\]

\[
d_1 = \frac{\log(x/K) + \sigma^2 T/2}{\sigma \sqrt{T}},
\]

\[
d_2 = d_1 - \sigma \sqrt{T}.
\]

4. The empirical implementation

4.1 Methodology

Thanks to (5), it would be possible to estimate the parameters \((\sigma, \rho, \lambda)\) using a cross-section of option prices with same time to maturity and different strikes.

\(^7\)It is worth noting that the assumption of continuous trading, which is necessary to guarantee the completeness of the market, is particularly strong when dealing with shocks that could jeopardize the efficiency of the financial markets. Nevertheless, we always assume that all technical conditions required by the model hold (cf. Bardhan and Chao, 1995).

\(^8\)Let us not forget that \(C_{BS}(x, r, \delta, T, K, \sigma) = xe^{-\delta T} N(d_1) - Ke^{-rT} N(d_2)\) where, as usual, \(N(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-y^2/2} dy\), \(d_1 = \frac{\log(x/K) + (r - \delta + \sigma^2/2)T}{\sigma \sqrt{T}}\) and \(d_2 = d_1 - \sigma \sqrt{T}\).
However, at least two problems naturally arise when trying to perform this task. First of all, in order to convey the same information, prices should be observed simultaneously and this is rarely the case, even in a very liquid market. Usually an average of the bid-ask quotes is used. Moreover, one also needs to observe the price of the underlying assets simultaneously with the prices of the options, but sometimes the options and the corresponding underlyings are not even listed in the same market.

Fortunately, there exists a market with liquid options that allows us to circumvent the first problem. The Chicago Mercantile Exchange (CME) manages an electronic platform, named GLOBEX®, where American options on S&P 500 futures contracts are listed. The market is very liquid and is organized in such a way that, every day, at noon, indicative prices are calculated using the bid-ask quotes available at the time. This fact allows us to have all prices calculated at the same time. We were able to download from the CME web site (www.cme.com) a data set covering the period from the last week of August 2000 to the end of July 2002. As a proxy for $r$, and for $\delta$ too, the continuously compounded three-month interest rate on US dollar deposits is used.

As for the second issue, it is possible to infer the price of the underlying contract at exactly the same time when options are evaluated using the model-independent put-call parity for European futures options

$$C + Ke^{-rT} = P + xe^{-rT}, \quad (6)$$

where $C$ and $P$ are, respectively, the prices of a call and a put option written on the same asset, with the same maturity $T$ and the same strike price $K$.

It is worth noting now that all formulas presented so far are suitable for European options only, though GLOBEX® trades American options on S&P 500 futures contracts. Gukhal (2001) provides analytical formulas for pricing American options on jump-diffusion processes but their practical implementation is far from straightforward. On the other hand, Caudhary and Wei (1994) showed that for futures options, independently of the model used, the price of an American call
always lies between a lower and an upper bound, the former being the price $C$ of an equivalent European call option and the latter being equal, at most, to $e^{rT}C$. Hence, especially when interest rates are low and the maturity date is approaching, the premium associated with the early exercise feature of an American futures option is generally small. It turns out that the relative difference between the upper and lower bounds in our sample of options is 0.40 per cent on average and 1.04 per cent at most, which we believe not to be significant for our purposes.

Since options become illiquid when maturity approaches, prices of short-term contracts are not reliable and statistical estimates based on them are not stable. On the other hand, we were most interested in evaluating short-term expectations. We found it was a good compromise to choose in our data set, for every day, the call and put options with the shortest time-to-maturity among those with at least 25 days of residual life. Since not all working days were available in our data set and we excluded days for which less than 5 relevant strikes were reported, we were left with 375 working days and 8,334 option prices, that is about 22 observations per day. A few descriptive statistics of the data are summarized in Table 1. After this first step, we pick the strike price with middle value among the strikes simultaneously available for both call and put options. We consider this strike to be the closest to at-the-moneyness\(^9\). As a matter of fact, at-the-money options are usually the most liquid contracts, so that a reliable value for $x$ can be calculated using (6). Given that in-the-money options are infrequently traded compared with at-the-money and out-of-the-money options, their prices are notoriously unreliable. As suggested by Aït-Sahalia and Lo (2000), the prices of all in-the-money illiquid options are thus replaced with the prices implied by (6) at the relevant strikes. To be more precise, the price of each in-the-money call option $C$ is replaced with $P + xe^{-rT} - Ke^{-rT}$, where $P$ is the price of an out-of-the-money and, thus, liquid put.

\(^9\)Remember that an option is said to be at-the-money whenever $S = K$. When $S > K$ or $S < K$ a call (put) option is said to be in-the-money (out-of-the-money) and out-of-the-money (in-the-money), respectively.
After the steps just described we have reliable prices and it would be possible to estimate, for every day, the parameters \((\hat{\sigma}, \hat{\rho}, \hat{\lambda})\) relative to maturity \(T\) by defining

\[
\hat{C}(K) = \sum_{n=0}^{\bar{n}} C_B \left( x e^{-\hat{\rho}T} (1 + \hat{\rho})^n, r, T, K, \hat{\sigma} \right) \frac{e^{-\hat{\lambda}T} (\hat{\lambda}T)^n}{n!}
\]

for \(\bar{n}\) large enough\(^{10}\) and then solving, numerically,

\[
\arg \min_{\hat{\sigma}, \hat{\rho}, \hat{\lambda}} \sum_{K \in K(T)} \left( \hat{C}(K) - C(K) \right)^2
\]

where \(K(T)\) is the set of strikes available for maturity \(T\) and \(C(K)\) is the market price of a call option with strike \(K\). Note that, even if the parameters of the model are not time-dependent, their estimates actually vary across different maturities. A more complete model should account for this feature.

The main problem with (7) is that, as Bakshi, Cao and Chen (1997) also observed, it implicitly gives more weight to relatively expensive options. The most valuable options are those in-the-money whose prices, in the present case, are indirectly determined, as described before, using put prices. This would give too much weight to prices that, on one side, are not directly observed in the market and, on the other, refer to options that are not very liquid. In order to deal with this feature of (7), we adopt an approach similar to that of Shimko (1993). By defining \(\sigma(C(K))\) to be the implied volatility of a call option with strike price \(K\) when the Black and Scholes model is used\(^{11}\), it is possible to project the space of option prices on the space of volatilities. Since an option price might change just because different strikes are considered, measuring everything in terms of volatility is particularly useful because it gives a more precise meaning to the relative value of the option\(^{12}\). In order to estimate \((\hat{\sigma}, \hat{\rho}, \hat{\lambda})\), the objective function (7) is thus

\(^{10}\)We used \(\bar{n} = 20\) even if our results show that the terms of the summation are usually negligible for \(n\) greater than 5.

\(^{11}\)It can be shown that \(C_{BS}(x, r, \delta, T, K, \sigma)\) is strictly increasing in \(\sigma\) for any choice of \((x, r, \delta, T, K)\). Thus, \(\sigma(C(K))\) can easily be recovered with standard numerical procedures.

\(^{12}\)Actually, professional traders conventionally quote options by referring to their Black and Scholes implied volatilities, no matter what model they use to price them.
modified to

\[
\arg\min_{\delta, \hat{\phi}, \lambda} \sum_{K \in K(T)} \left( \sigma(\hat{C}(K)) - \sigma(C(K)) \right)^2.
\]

Since (8) is not globally convex\(^{13}\), the algorithm of Maranas et al. (1997) is applied to find the global minimum.

Finally, it should be noted that, even if Merton’s model can be viewed as an extension of the model by Black and Scholes, it is itself a special case of a more general class of models, introduced by Bakshi, Cao and Chen (1997), where volatility, interest rates and jumps are stochastic. In Merton’s model, in particular, since the size of the jump is assumed to be constant (it can only be either positive or negative, but not both), the shock can hit the market in one direction only. This causes the so-called "implied volatility smile" generated by the model to actually become an "implied volatility smirk", that is a monotonic function, for all reasonable values of the parameters\(^{14}\). In other words, if shocks can only be negative, options with low strikes are worth relatively more because they provide insurance exactly against this kind of event and the implied volatility curve is downward sloping. The opposite happens if the direction of the jumps is positive. For this reason, Merton’s model cannot be used, for instance, for currency options\(^{15}\), in which both out-of-the-money and in-the-money options are worth relatively more than the corresponding at-the-money contracts and more general models are needed (cfr. Bates, 1996). Nevertheless, the simple Merton’s model still worked well in our case.

\(^{13}\)For instance, it can be numerically shown that \(\sigma(C(K))\) is indeed a concave function of \(\lambda\) for many reasonable values of the other parameters.

\(^{14}\)In fact, the implied volatility \(\hat{\sigma}(C(x, r, \delta, T, K, \sigma, \rho, \lambda))\) is implicitly defined by the equality \(C_{BS}(x, r, \delta, T, K, \sigma) = C(x, r, \delta, T, K, \sigma, \rho, \lambda)\). Hence \(\frac{\partial C_{BS}}{\partial K} + \frac{\partial C_{BS}}{\partial \hat{\sigma}} \frac{\partial \hat{\sigma}}{\partial K} = \frac{\partial C}{\partial K}\). Since \(\frac{\partial C_{BS}}{\partial \hat{\sigma}} > 0\) and, moreover, it can be numerically proved that \(\rho < 0 \Rightarrow \frac{\partial C_{BS}}{\partial \hat{\sigma}} > \frac{\partial C}{\partial \hat{\sigma}}\), then \(\frac{\partial C}{\partial \hat{\sigma}} < 0\). The opposite result holds when \(\rho > 0\).

\(^{15}\)Dumas et al. (1995) show that if both domestic and foreign investors make use of the Merton jump-diffusion model, they get different option values, thus violating the law of one price. However, the bias introduced by mixing the domestic and foreign versions of the jump-diffusion formula is usually small.
4.2 Results

Merton’s model proved to be very good at pricing the options included in our sample. The average absolute error, for both volatilities and prices, was only 0.2 per cent. In more than 95 (90) per cent of the cases the absolute error was less than 0.5 per cent in terms of volatility (price) and for 99.7 (97.9) per cent of the options the error was less than 1 per cent. Figures 1 and 2 plot, respectively, the behaviour of the S&P 500 index during the sample period and the corresponding values of the estimated parameters ($\hat{\sigma}, \hat{\rho}, \hat{\lambda}$), as well as the values of $\hat{\sigma}_{BS}$, that is the implied volatility of at-the-money call options calculated using the Black and Scholes model. Let us remember that the latest parameter, or some moving average of it, is the one usually watched by financial institutions to monitor the level of uncertainty in the market.

Looking at Table 2 it can be noted that $\hat{\sigma}_{BS}$ is, on average, 6.5 percentage points greater $\hat{\sigma}$. Moreover, Table 3 shows that the correlation between the two coefficients is very high: about 94 per cent. This can be interpreted as the fact that usually only 70 per cent of the uncertainty in the market is explained by market volatility (Figure 3) and that the rest is a sort of premium required for the possible occurrence of a significant unknown event that hits the market.

Figure 2 shows that volatility peaked in September 2001 and in July 2002. It was very high in March and April 2001, too. Although all these events are associated with a strong and sudden depreciation of the S&P 500 index, the factors underlying these episodes are probably very different. In March 2001 the economy in the United States was showing the first signs of what has been subsequently recognized as a recession, whereas in September 2001 the main source of concern was the possibility of further terrorist attacks. In July 2002, finally, a few cases of accounting manipulations raised new doubts about the reliability of corporate accounts. In other words, while in March 2001 uncertainty was probably mainly due to the difficulty of assessing the outlook of the economy, in September 2001 it
was mainly linked to extraordinary geo-political concerns. In July 2002 as well, the uncertainty was mostly driven by non-economic factors, i.e. by accounting frauds, but the general mood was that even if other enterprises revealed “problems” in their accounts the market as a whole had to be considered sound. Estimated parameters seem not to contradict this interpretation. In March 2001, the high level of $\hat{\sigma}_{BS}$ mostly reflected an increase in the level of $\hat{\sigma}$ whereas $\hat{\lambda}$ and $\hat{\rho}$ remained almost stable. The market was thus dealing with a higher level of “ordinary” volatility but people were not expecting any extraordinary movement (or at least this expectation was not substantially greater than in preceding months). Things changed substantially after September 11th. During the days immediately following, the high level of uncertainty (as measured by $\hat{\sigma}_{BS}$) was associated with a surge in the (absolute) value of $\hat{\rho}$ that reached the astonishing level of 36 per cent on September 25th. Hence, the market was mainly concerned with the possibility of a huge shock in the near future (like, for instance, that induced by panic selling or liquidity crises), even if the probability associated with such an event was sometimes very low. Overall, the proportion of uncertainty not measured by $\hat{\sigma}_{BS}$ remained around the sample mean, as shown by Figure 3. A few days later, when trading activity resumed completely and asset prices began to recoup quickly, the risk of a deeper slowdown of the economy and the possibility of further terrorist attacks were the main concerns for people. In that period, the size of expected shocks diminished considerably but remained high. Moreover, the substantial reduction of $\hat{\sigma}$ put the ratio between the volatility implied by the models of Merton and Black and Scholes at less than 60 per cent. In January 2002 all the parameters went back to their sample means, indicating that at the beginning of 2002 agents did not have abnormal expectations of sudden, large drops in equity prices. The level of uncertainty was significantly high again in July when a few American firms revealed that their accounts were faked. In this case, as argued above, the market as a whole was considered pretty sound or, in other words, the risks associated with firms’ accounts had to be considered non-systematic and hence manageable in a well-diversified portfolio. In fact, the proportion of uncertainty not measured by pure market volatility was not particularly low.
5. Conclusions

We applied the jump-diffusion model introduced by Merton to S&P 500 futures options listed at the Chicago Mercantile Exchange. The model proved able to reflect actual option prices very accurately. Moreover, and more importantly from the point of view of the policy-maker, we showed that the parameters of Merton’s model can be used to “decompose” the widely used Black and Scholes implied volatility ($\sigma_{BS}$) into three parts: the “genuine” volatility ($\sigma$), the expected number of large shocks ($\lambda$) and the size of these big jumps ($\rho$). While in normal conditions the knowledge of ($\sigma, \rho, \lambda$) does not substantially improve the information provided by $\sigma_{BS}$, in periods of high volatility it can help to clarify whether the upsurge in volatility is due to greater uncertainty about the outlook for the economy or to greater risks of unexpected extreme events.
Tables and figures

Table 1
Data set statistics

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>StD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days to maturity</td>
<td>40.81</td>
<td>9.08</td>
<td>25</td>
<td>59</td>
</tr>
<tr>
<td>Strikes per day</td>
<td>22.22</td>
<td>3.19</td>
<td>5</td>
<td>25</td>
</tr>
<tr>
<td>Risk-free interest rate</td>
<td>3.53</td>
<td>1.76</td>
<td>1.62</td>
<td>6.49</td>
</tr>
</tbody>
</table>

Notes: The risk-free interest rate is in percentage points. It refers to the continuously compounded three-month interest rate on US dollar deposits.

Table 2
Statistics of the estimated parameters

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>StD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\sigma}_{BS}$</td>
<td>22.35</td>
<td>4.03</td>
<td>16.46</td>
<td>39.89</td>
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<tr>
<td>$\hat{\sigma}$</td>
<td>15.83</td>
<td>2.71</td>
<td>10.96</td>
<td>28.06</td>
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<tr>
<td>$\hat{\lambda}$</td>
<td>1.81</td>
<td>0.56</td>
<td>0.56</td>
<td>4.51</td>
</tr>
<tr>
<td>$\hat{\rho}$</td>
<td>-12.80</td>
<td>2.82</td>
<td>-36.00</td>
<td>-7.66</td>
</tr>
</tbody>
</table>

Notes: $\hat{\sigma}_{BS}$, $\hat{\sigma}$ and $\hat{\rho}$ are in percentage points. $\hat{\sigma}_{BS}$ is the implied volatility of the at-the-money call option calculated using the Black and Scholes model. $\hat{\sigma}, \hat{\rho}, \hat{\lambda}$ are computed using Merton’s model and represent, respectively, the volatility of the asset, the percentage change of the asset price when a jump occurs and the number of expected jumps per unit of time (which is one year).

Table 3
Correlations of the estimated parameters

<table>
<thead>
<tr>
<th>$\hat{\sigma}_{BS}$</th>
<th>$\hat{\sigma}$</th>
<th>$\hat{\lambda}$</th>
<th>$\hat{\rho}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\sigma}_{BS}$</td>
<td>1.00</td>
<td>0.94</td>
<td>0.26</td>
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<tr>
<td>$\hat{\sigma}$</td>
<td>0.94</td>
<td>1.00</td>
<td>0.17</td>
</tr>
<tr>
<td>$\hat{\lambda}$</td>
<td>0.26</td>
<td>0.17</td>
<td>1.00</td>
</tr>
<tr>
<td>$\hat{\rho}$</td>
<td>-0.74</td>
<td>-0.72</td>
<td>0.34</td>
</tr>
</tbody>
</table>
S&P 500 Index

Fig. 1

Estimated Parameters

Fig. 2

Notes: (1) Percentage points; (2) Absolute values; (3) Right-hand scale.
Note: The mean and the confidence bounds of the series are also shown.
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