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Social mobility and endogenous cycles in redistribution

by Francesco Zollino

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SOCIAL MOBILITY AND ENDOGENOUS CYCLES IN REDISTRIBUTION

by Francesco Zollino*

Abstract

By allowing median voter’s location and preferred policy to change over time, a variety of redistributive policies results in the long-run with no unique relationship to inequality. Single outcome depends on the interaction between the pure economic structure and policy action in determining wealth distribution over time. The standard positive correlation between redistribution and inequality is confirmed when the pattern of social mobility, potentially prevailing in a free market, proves robust to public action. Otherwise the non-linear relationship found in recent literature is confirmed. With balanced intensity of backward and upward mobility in free market, policy cycles endogenously arise, with inequality shrinking and enlarging periodically and counter-cyclically.

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Keywords: social mobility, political cycle, credit rationing, redistributive policy.

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1. **Introduction**

The negative trend in income inequality historically established in industrial countries first showed signs of a slowdown in the 1950s (Paukert, 1973). Since then, growing evidence that the previous trend has halted or even reversed has been spreading across countries, albeit to quite different extents. According to recent data (Gottshalk-Smeeding, 1997), between 1981 and 1992 the increase in income disparities was sharp in the UK, Sweden and the USA (where the Gini coefficient rose, respectively, by around 30, 20 and 15 per cent), less striking in Japan, Denmark, Australia and New Zealand (above 10 per cent), moderate in the Netherlands, Belgium and Norway (around 5 per cent), and negligible in the remaining countries. Interestingly, the result was more pronounced in countries that mostly experienced a resurgence of fiscal conservatism. The clearest example is provided by the USA, where tax reforms endorsed in 1981 and 1986 reduced the top marginal rate from 70 to some 30 per cent while increasing taxation on lowest incomes from 0 to 11 per cent (Pechman, 1987); at the same time, public expenditure control mainly relied on curbing welfare programmes (Economic Report of the President, 1994).

The two pieces of evidence are at odds with the standard prediction of a positive correlation between inequality and redistribution, and have recently been explained in terms of history dependence of political equilibrium due to productive externalities and imperfect democracy (Benabou, 2000 and Saint-Paul, 2001). In this paper we put forward an alternative reading based on a reverse causality argument: the increased inequality recently observed in advanced countries can be traced back to a shift in the political climate back

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2. Preliminary evidence shows that in the USA and UK the increase in inequality has continued in the 1990s, although at a slower rate than in the previous decade (Smeeding, 2000).

3. In Italy, after a noticeable decline in the early 1980s, personal income inequality began to raise again, with an increase in the Gini coefficient slightly less than 10 per cent in the period 1986-95 (Brandolini, 1999).
towards restrictions in the welfare state. In this perspective, in western democracies the very effectiveness of long implemented redistribution in improving social mobility might have gradually reduced the initial consensus for such a policy, eventually leading to an opposite political equilibrium in recent times. Once established, the policy change would have helped to slow down, or even reverse, the negative trend in inequality.

More generally, we address the issue of the long-run sustainability of redistributive policies in connection with changes in inequality they help to cause. In this respect, we wonder whether endogenously determined fluctuations in the extent of redistribution could trap developed democracies in a region where inequality shrinks and enlarges periodically and counter-cyclically. On the theoretical side, we stress that political competition may help to break the virtuous cycle between growth and inequality first addressed by Kuznets and that has recently come under criticism (Aghion et al., 1999). On the empirical side, more emphasis is urged on time series analysis of inequality and redistribution than is usually the case in current literature, which is mostly centred upon international comparisons.


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Among valuable exceptions, Rodriguez (1998) investigates the impact of inequality on the extent of redistribution based on data for 20 OECD countries in the years 1960-1990. In rejecting a positive correlation, he finds that a reverse causality may not be dismissed if inequality is measured with respect to personal incomes.
redistribution through a relaxation of credit constraints on investment by the poor. More recent contributions place the issue in the context of sequential voting and forward-looking behaviour (Saint Paul, 2001, Piketty, 1995, Benabou, 1996b and 2000, Krusell et al., 1997 and Quadrini, 1999). In line with empirical evidence (Perotti, 1996 and references in Benabou, 1996b), the positive correlation between inequality and redistribution previously advocated is questioned due to non-linearities, under deviations from ideal democracy (Saint Paul and Benabou) or imperfect information on economic structure (Piketty and Quadrini).

In the referenced literature the pivotal agent in the political outcome is usually identified once and for all under some rank preserving conditions, and demand for redistribution is invariant with wealth dynamics. An important exception is provided in Benabou (2000), where changes in inequality affect the equilibrium at every election, and a negative relationship between inequality and redistribution is established across multiple steady states. However, in that paper the long-run equilibrium is univocally determined conditional on initial inequality, mainly because the pivotal agent remains wealthier than the median over time. Accordingly, key variables in determining wealth distribution confirm independent of the dynamics of inequality and policy.

As a distinguishing feature of our model, we also take account of potential changes both in the pivotal agent’s location across all social classes and in his preferred policy. This enrichment unveils a more complex relationship between inequality and redistribution than commonly found in the literature, including the possibility of periodical revisions in the political equilibrium and, as a consequence, in the pattern of social mobility. Most

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5 In some cases (Durlauf, 1996, 1993 and Benabou, 1996a, 1993) the analysis of political equilibrium is extended to local public goods, and social segregation arises with local externalities in production and/or in human capital accumulation.

6 Pivotal agent generally identifies the marginal voter in determining political equilibrium. If the median voter theorem applies (see later in the text), he may differ from the median agent when democracy is imperfect, namely when a subset of potential voters does not show up at every election or political influence is increasing with wealth (for USA evidence, cfr. references in Benabou, 2000).

7 As rank-preserving conditions rule out social mobility, its effects on policy are neglected (Krusell-Rios Rull, 2000, Alesina-Rodrick, 1991, Glomm-Ravikumar, 1993 and Fernandez-Rogerson, 1996). In other places the same finding is obtained under non-sequential voting (Bertola, 1993 and Alesina-Rodrick, 1994).

8 The result comes from income distribution remaining log-normal over time even in a stochastic framework, mainly because shocks are assumed to be log-normal as well.
interestingly, it turns out that it is not the size of the inequality but the nature of its ultimate origin that affects redistribution in the long run.

The paper is organized in five sections. The first summarizes the main findings and the macroeconomic links. The second puts forward a theoretical model, which delivers a microfoundation for a Markov chain representation of social mobility in a free market. In the third section the model is extended to include political equilibrium, and the dynamic feedback between social mobility and redistribution is characterized. In the fourth, we prove the main results and suggest a rough taxonomy for the relationship between inequality and redistribution, among other by exploring conditions for endogenous cycles in both macroeconomic and political outcomes. The final section briefly concludes.

2. Main ideas and results

In this section we anticipate the main macroeconomic implications of a stochastic economy with heterogeneous agents and imperfect markets, which will be developed later. We first sketch rules for social mobility under laissez-faire, then with active government, focusing on interactions between inequality and redistribution in the long run.

2.1 Free market economy (laissez-faire)

Under some restrictions on parameters, we obtain that at any date agents are distributed in three classes, whose borders are univocally identified according to initial wealth and social mobility fits a time invariant Markov chain. For the purposes of our analysis, a dynamic economy is then fully represented by a row vector \( \psi_t = (\psi_{1,t}, \psi_{2,t}, \psi_{3,t}) \) where \( \psi_{i,t} \geq 0 \) is the density mass of class \( i \) and the following stochastic matrix

\[
M_F = \begin{bmatrix}
1 - \theta & \theta & 0 \\
1 - q' & 0 & q' \\
0 & 1 - q' & q'
\end{bmatrix}
\]

where \( m_{i,j} \) is the probability that the next generation belongs to class \( j \) when the current one belongs to class \( i \), with class \( i \) poorer than class \( j \) if \( i < j \). The pair \( (\theta, q') \in (0,1) \) identifies the
key structural parameters of our economy, namely the probability of successful investment in two risky projects: $\vartheta$ for the first, which shows low productivity and may be operated by everybody because there is no fixed capital requirement, and $q'$ for the second, whose productivity is higher and which can be operated only by agents who can afford a minimum scale investment under imperfect capital markets. Changes in wealth distribution are simply ruled by $\psi_{t+1} = \psi_t M_F$.

The stochastic matrix $M_F$ is shown to induce a unique stationary distribution, namely $\psi^*$, which proves depending on the structural parameters above mentioned as follows

$$\psi_i^* = \frac{(1-q')^2}{\vartheta + (1-q')^2}; \quad \psi_i^* = \frac{\vartheta (1-q')}{\vartheta + (1-q')^2}; \quad \psi_i^* = \frac{\vartheta q'}{\vartheta + (1-q')^2}$$

Accordingly, a stronger upward mobility in a free market, as identified by high values in the pair $(\vartheta, q')$, implies a larger share of the rich and a smaller share of the poor in the long run; the opposite holds true under a stronger downward mobility. Due to the convergence of $M_F$, these results are independent of initial condition, and trace back to the economic structure ultimately determining inequality at any date.

2.2 Active government economy (redistribution)

Active redistribution changes the picture of social mobility, improving the chance of escaping poverty, on one side, and potentially discouraging effort supply, on the other. For simplicity’s sake, we focus only on the former and adjust the stochastic matrix for active government as follows

$$M_G = \begin{bmatrix} (1-\vartheta)(1-\zeta) & \vartheta + (1-\vartheta)\zeta_i & 0 \\ (1-q')(1-\zeta) & (1-q')\zeta_i & q' \\ 0 & 1-q' & q' \end{bmatrix}$$

where $\zeta_i \in [\underline{\zeta}, \bar{\zeta}]$, with $1 > \underline{\zeta} > \bar{\zeta} > 0$, identifies the extent of pure redistribution (aimed at relaxing the credit constraint on productive investment by the poor), which is endogenously determined by majority voting according to the following.
Proposition 1. At every election, political equilibrium is defined by high redistribution if either the median voter belongs to the poor class or belongs to the middle class and
\[
\Delta_r \equiv \frac{\psi_{3,1}}{\psi_{3,t}} \leq \frac{1 - q'}{1 - \vartheta} \equiv \phi, \text{ by low redistribution otherwise.}
\]

Standard conflict between the rich and the poor arises as the two are respectively net losers and net earners from pure redistribution. As for the middle class, they demand large redistribution only if the benefit they expect in case they become poor exceeds the tax costs of providing assistance to all the other poor. Accordingly, middle class support for a large redistribution declines with \(\Delta_r\), which stands as a proxy for inequality.

Proposition 1 and the rules for social mobility embodied in matrix \(M_G\) make the analysis of the dynamic feedback between redistribution and inequality analytically treatable. We can identify a variety of equilibria in the long run and characterize their dependence on the relative strength of forces driving social mobility in free market, namely parameters \((\vartheta, q')\), with respect to the policy input \(\zeta_t\).

Proposition 2. A unique steady-state equilibrium is defined by small (large) redistribution if the median voter under the stationary distribution is in the wealthy (poor) class or is in the middle class and \(\Delta'(\zeta) > \phi\) (\(\Delta'(\zeta) \leq \phi\)).

As a special case, high inequality in the long run is associated with large redistribution regardless of initial conditions. A possible reason is that values of the pair \((\vartheta, q')\) are so low that the median voter is definitely attracted by the poor class, whose preferred policy is univocally defined. Symmetric arguments hold true for low redistribution. Accordingly, the proposition confirms the standard prediction of a positive correlation between inequality and redistribution.

Under milder forces for social mobility in a free market, the picture becomes more complex as policy itself delivers a key contribution in its confirmation at every election, so that initial conditions play an important role in determining equilibrium in the long run.
Proposition 3. Multiple equilibrium between redistribution and inequality arise in steady state if social mobility in a free market would invariantly drive the median voter into the middle class and competing policies are different enough that \( \xi < 1 - \frac{q'}{1 - \vartheta} \leq \xi \).

When values of structural parameters imply that the middle class becomes crucial in determining the majority, a large redistribution arises in the long run even under lower inequality, on condition that the same policy has been long implemented, so that the mass of poor is reasonably small. This result is in line with the history dependence found in recent contributions (Saint-Paul, 1994 and 2001, Benabou, 1996b and 2000, Rodriguez, 1998); additionally, it is confirmed even abstracting from imperfect democracy and strictly net welfare gains from redistribution\(^9\).

Finally, we find that the complex feedback between economic structure and policy may hinder the path to steady state, endogenously determining fluctuations in both redistribution and inequality.

Proposition 4. Under any given range of feasible policies, it is possible to identify a subset of pairs \((\vartheta, q')\) such that political equilibrium is periodically revised over time, inducing fluctuations in redistribution and inequality. Sufficient conditions require three joint constraints to be met: a) \( \psi^*_3(\xi) < 0.5 \), b) \( \Delta(\xi) \leq \phi \), c) \( \psi^*_3(\xi) > 0.5 \).

Forces for downward and upward mobility are required to be balanced in a free market, so that their combined effect on inequality under active government is crucially dependent on the extent of redistribution. In this case, as an initially poor median agent eventually becomes rich thanks to large redistribution, he becomes a net contributor to the public budget and starts voting for more restrictive policies. The ensuing revision from high to low redistribution in equilibrium aggravates the forces for backward mobility, and the median voter gradually moves back from the wealthy to lower classes, until initial conditions for large redistribution are restored.

\(^9\) We consider only the case of welfare improving redistribution for simplicity’s sake, as tax distortions on effort supply can easily be taken into account in our framework without affecting the results. Uncertainty at election time is not an essential ingredient either, as is proved in a first version of the paper (Zollino, 1994).
Policy cycles have recently been formalized in Gradstein-Justman (1997) in terms of periodical changes between two radically different education policies. Differently, we restate the result even for minor variations in competing programmes and augment for endogenous fluctuations in policies. Indeed, in our paper they result not only from the median voter moving across classes, but also from his preferred policy potentially changing over time. Finally, the ultimate connection to the economic structure is explicitly made.

3. The model

In a small open economy we assume a continuum of agents with identical preferences and different initial wealth; they live one period, which can be ideally split into three sub-periods. In the first, agents start a risky productive project, in the second they may vote for a redistributive policy, in the third, after productive uncertainty is solved and policy implemented, they realize utility by consuming and leaving bequests to their offspring. Generations succeed ad infinitum without overlapping.\(^{10}\)

3.1 Productive technologies

A sole homogeneous good, suitable for both consumption and investment, can be produced in two sectors in which technologies differ by intensity of effort, fixed capital requirements and expected payoffs. In the first sector, namely the subsistence activity, there is no fixed capital requirement and the technology is a pure chance mechanism whose revenues, provided a minimum effort \(e\) is exerted by the agent, are given by\(^{11}\):

\[
\begin{align*}
    f_{\text{sub}} &= \begin{cases} 
    n & \text{with prob } = \vartheta \\
    0 & \text{with prob } = 1 - \vartheta 
    \end{cases}
\end{align*}
\]

\(^{10}\) This assumption is not crucial for our results which may easily be restated in a traditional OGM. More importantly, we conveniently restrict to imperfect altruism in line with reference literature, since an extension to a dynastic utility would greatly complicate a closed form solution of dynamic equilibrium. More on this issue can be found in Benabou (1996) and Krusell et al. (1997).

\(^{11}\) A rough example of a subsistence activity is a backward agricultural sector, where weather crucially determines an abundant harvest, conditionally on the peasant working the land.
In the second sector, namely the productive activity, technology implies a fixed scale in investment, which we normalize to one; in addition, greater effort by agents can improve the expected payoffs

(4) \[ f_{\text{prod}} = \begin{cases} r & \text{with prob } = p \\ \xi & \text{with prob } = 1 - p \end{cases} \]

with \( p = f(e) \) a real increasing function that maps effort into the probability of success\(^{12}\): Effort is observable only to the agent who exerts it at a disutility cost, measured in monetary metric by a convex increasing function \( h(e) \)\(^{13}\). Accordingly, the agent disutility required to achieve \( p \) is defined by the convex function \( g(p) = h^{-1}(f^{-1}(p)) \).

Aside from operating in one of the two risky sectors, the agent can lend inherited wealth to financial intermediaries in exchange for a riskless interest factor \( A \)\(^{14}\). Under free entry in the financial sector, the latter is exogenously determined in the small open economy we consider. Following Townsend (1978) and Greenwood-Jovanovic (1990), the establishment of trading arrangements costs the lender a fixed commission fee\(^{15}\), equal to \( c \); as a result, lending an amount \( w \) to a financial intermediary proves profitable only if \( wA - c > w \). For smaller amount, it is convenient simply to leave it idle until consumption and bequest decisions are made.

As a characterization of a dynamic economy, we find it reasonable to posit that operating in the productive sector is the dominant option, or that

\[
\begin{align*}
pr + (1 - p)\xi - g(p) + \max\{A(w - 1) - c, w - 1, 0\} - \max\{A(1 - w), 0\} & \geq \\
\vartheta n - h(e) + \max\{Aw - c, w\}
\end{align*}
\]

\(^{12}\) In particular, \( f : [0,1] \to [0,1] \) with \( f(e) = 0 \ \forall e \in [0,\xi] \) and \( f'(e) < 0 \). With \( \xi > 1/2 \) we rule out that the agent works in the two sectors at the same time: a choice where to produce must be made.

\(^{13}\) Convexity of the disutility function stands for the standard concave condition for the consumer's utility maximization programme.

\(^{14}\) Despite the uncertainty in technology outcomes, no default of financial intermediaries results from the law of large number under independent individual risks.

\(^{15}\) Apart from monitoring costs, a positive commission fee may be due to transaction costs coming from book keeping or a minimum share requirement in a mutual fund.
Under perfect capital markets nobody would then invest in the subsistence activity. Indeed, information frictions and a fixed capital requirement cause credit rationing to limit the opportunity of operating the dominant technology. As we show in Appendix B, in equilibrium credit is offered only to agents whose wealth is not lower than a threshold \( w \), which increases with the interest factor and decreases with the financial intermediary’s net expected returns. These are determined within an optimal financial contract, by which the lender is allocated the full property rights on the risky project in exchange for a proper compensation for the effort provided by the borrower/investor.

3.2 Class structures and rules of behaviour

While sharing identical preferences, agents are heterogeneous in inherited wealth \( w_i \). Under imperfect capital markets, they can then be divided into different classes according to the investment opportunities they can afford. For simplicity's sake, we confine our attention to only three classes, namely the poor, the middle and the rich one.

The poor class consists of agents with \( w_i \leq w \) or below the threshold for credit rationing: they cannot afford the minimum investment required in the productive sector and operate the subsistence activity described in (3). In addition, they could lend their inherited wealth to financial intermediaries and earn the interest factor \( A \), should it be profitable after payment of the commission fee. In order to preserve a simple class structure over time we find it useful to rule out this possibility

\[
Assumption 1: c \geq w(A - 1)
\]

The middle class consists of people with \( w_i \in [w, 1] \): they can raise funds on the credit market and operate in the productive sector. Since this is the dominant option, everybody in the middle class is a borrower who solves the following

\[
\max_p pr + (1 - p)\xi - \rho(w_i) - g(p)
\]

where \( \rho(w_i) \) is the cost of the loan needed, in addition to the initial endowment, to meet the fixed capital requirement. As shown in Appendix B, an optimal financial contract is a
payment schedule \( \{0, s(w_i)r\} \), which identifies the compensation offered to the investor respectively in case of failure and success of the risky project. After rearranging, the objective function of the middle class can be restated as

\[
(7) \quad \max_p \, psr - g(p)
\]

with first order conditions requiring

\[
(8) \quad sr = g'(p)
\]

To simplify the calculus and keep the class structure as simple as possible, we model the effort cost function in a piece-wise linear form\(^{16}\):

\[
(9) \quad g(p) = \begin{cases} h(c) + bp & 0 \leq p \leq q' \\ +\infty & p > q' \end{cases} \text{ with } 0 < b < 1.
\]

The equilibrium financial contract and condition (6) imply a stepwise optimal probability of success in the productive project, as from

\[
(10) \quad p^*_s,b = \begin{cases} 0 & \text{if } sr < b \\ p \in [0,q'] & \text{if } sr = b \\ q' & \text{if } sr = b > br \end{cases}
\]

where the index \( s.b. \) labels the solution as second best to take account of moral hazard stemming from unobservable effort.

\(^{16}\) This assumption proves to be less restrictive than at first appears as we content ourselves with working with a limited number of classes. Actually, the linear piece-wise specification as an approximation to the general convex form has the virtue of making agents’ behaviour homogeneous within some range of wealth values. The approximation we suggest can be enriched at will if we are prepared to increase the number of classes we consider. At most, the approximation will converge to the general form, with an infinite number of classes. Since the smaller their number, the simpler the rule for social mobility, as shown later on, we assume the poorest specification for \( g(p) \).
As we show in Appendix B, the intermediary finds it profitable to motivate the investor to the highest effort by offering him a spread \( s(w) \) between the success and failure compensation, which proves increasing with initial wealth, provided it is equal at least to \( w \). As this offer meets the investor’s participation constraint, he actually exerts the effort required for a success probability \( q' \).

The rich class identifies agents with \( w_i \in [1, \infty) \): since their endowments are at least as high as the fixed capital requirement in the productive activity, they operate in this sector without borrowing. Accordingly, no moral hazard arises with people in the rich class, whose rules of behaviour match the solution to the problem

\[
\max_p \, p r + (1 - p) \xi - g(p) \quad \text{with FOC’s requiring } r - \xi = g'(p)
\]

In view of (9), the first best solution for \( p \) is given by

\[
p_{f,b.}^* = \begin{cases} 
0 & \text{if } r - \xi < b \\
p \in [0,q'] & \text{if } r - \xi = b \\
q' & \text{if } r - \xi > b
\end{cases}
\]

Along with investing in the productive sector, the rich lend to the financial intermediaries the residual wealth \( 1 - tw \), provided the gross returns exceed the commission fee. In view of [5], only agents with \( w_i > 1 + c(A - 1)^{-1} \equiv \widetilde{w} \) lend money, while the remaining rich, like the poor, leave their residual wealth idle until the final subperiod.

### 3.3 Social mobility in a free market

In our framework social mobility, namely the individuals’ opportunity to change class across generations, is the combined outcome of a sequence of optimal choices made by agents over the three sub-periods in which their lifetime can be ideally split. In the latest, namely date \( t_2 \), utility is maximized subject to the budget constraint resulting from optimal action in earlier sub-periods. At the initial date \( t_0 \), agents make optimal productive
investment based on their inherited wealth, and only in the case of election at date $t_1$ they optimally vote for redistribution. Then equilibrium may be analyzed by solving a backward strategy, moving from the latest to the earliest date. As the free market is meant to rule out public action, in this section we can skip the sub-period identifying the election time.

At date $t_2$ the agent maximizes utility by deciding how much to consume and to bequest out of the wealth commanded. Under Leontieff preferences, his problem reads

$$
\max_{C_2, B_2} U(C_2, B_2) = \min \{(1 - \delta)C_2, \delta B_2\} \quad \text{such that} \quad C_2 + B_2 \leq M_2
$$

where $C_2$ and $B_2$ are, respectively, consumption and bequests enjoyed at date $t_2$; $M_2$ identifies the resources the agent commands as the result of his own inherited wealth and productive decisions at $t_0$ - whose uncertain payoffs are realized at the beginning of the final sub-period. The standard solutions are $C_2^* = \delta M_2$ and $B_2^* = (1 - \delta)M_2$, with $B_2^*$ identifying the optimal inter-generational transfer which drives social mobility across generations.

At date $t_0$ the agent decides which technology to operate in order to maximize $M_2$, as we have already discussed in the previous section.

In this set up, social mobility is ruled by the following law of motion of $w$

$$
\begin{align*}
(14a) \quad w_{t+1} &= \begin{cases} 
(l - \delta)(n + w_t) & \text{with } \text{prob} = \vartheta \\
(l - \delta)w_t & \text{with } \text{prob} = 1 - \vartheta
\end{cases} \quad \text{for } w_t \in [0, w] \\
(14b) \quad w_{t+1} &= \begin{cases} 
(l - \delta)sr & \text{with } \text{prob} = q' \\
0 & \text{with } \text{prob} = 1 - q'
\end{cases} \quad \text{for } w_t \in [w, l] \\
(14c) \quad w_{t+1} &= \begin{cases} 
(l - \delta)(r + w_t - l) & \text{with } \text{prob} = q' \\
(l - \delta)(\xi + w_t - l) & \text{with } \text{prob} = 1 - q'
\end{cases} \quad \text{for } w_t \in [l, \bar{w}] \\
(14d) \quad w_{t+1} &= \begin{cases} 
(l - \delta)(r + A(w_t - 1) - c) & \text{with } \text{prob} = q' \\
(l - \delta)(\xi + A(w_t - 1) - c) & \text{with } \text{prob} = 1 - q'
\end{cases} \quad \text{for } w_t \in [\bar{w}, \bar{w}]
\end{align*}
$$
where for simplicity’s sake we posit that the provision of effort is the same in both first and second best, so that the probability of success is uniformly equal to \( q' \);
\[
\bar{w} = (1-\delta)(r-A-c)(1-\alpha)^{-1}, \text{ with } \alpha \equiv (1-\delta)A < 1,
\]
identifies the upper bound beyond which individual wealth cannot persist over time, and higher values can be neglected without a significant loss of generality in a long-run analysis\(^{17}\).

System (14) implies that a class structure, albeit initially simple, generally tends to become more and more complex over time, with the number of classes continually increasing. In order to preserve analytical tractability, we identify some restrictions in the parameter space that make clear cut rules for social mobility, in the case of both success and failure in the risky projects. In particular, we assume that in the case of failure the offspring of the wealthy will fall into the middle class irrespective of their parents’ wealth.

(15)  \[
\text{Assumption 2: } w \leq \xi(1-\delta) < 1-(1-\delta)\max\{A(\bar{w}-1)-c, \bar{w}-1\}
\]

Additionally, the offspring of people belonging to the middle and the poor classes are invariably supposed to jump respectively into the rich and the middle class in the event of productive success of their parents. The required of restrictions on the parameters read:

(16)  \[
\text{Assumption 3: } \dot{b} \geq \frac{l}{(1-\delta)}
\]

(17)  \[
\text{Assumption 4: } \frac{w}{1-\delta} \leq n < (1-\delta)^{-1}-w
\]

Interestingly, Assumption 4 is mainly stated to make explicit the way we simplify social mobility as: i) the lower-bound requirement proves redundant under a slightly stricter version of Assumption 2; ii) the upper-bound requirement, namely that the offspring of the poor cannot jump into the rich class, is not binding in the long-run\(^{18}\).

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\(^{17}\) For the very rich, in the indirect utility derived from (13) the value of consumption exceeds the payoff of productive investment and must be partly funded by a depletion of initial wealth, with offspring inheriting less than their parents (Aghion-Bolton, 1997 and Banerjee-Newman, 1991).

\(^{18}\) People in the poor class either come from higher classes, with wealth virtually nil due to failure in the productive sector, or are confirmed poor due to failure in the subsistence sector; in the latter case, inherited (and left idle) wealth converges to zero, at a rate increasing with \( \delta \).
Under the moderate loss of generality implied by previous assumptions, the stochastic process ruling social mobility may be summarized by the low-dimension matrix $M_F$ introduced in section 1. It is an important achievement that the size of long-run inequality induced by (14) and its relation to key parameters may easily be investigated according to the standard theory of Markov chains (Appendix A). Moreover, changes in the location of median voter under social mobility can be tracked at any date.

4. Social mobility and political equilibrium

In this section the endogenous determination of redistribution is analyzed by introducing political competition in the above stochastic framework. As in the standard approach, heterogeneous agents aggregate through majority voting, whereby representatives are appointed to office. Following Alesina (1987), we assume for simplicity that political competition involves two partisan parties, in the sense that they seek to implement a given policy, which they invariably prefer.

4.1 The redistributive programme

The core of political competition is identified by the extent of a purely redistributive programme to be funded by taxation of personal wealth. In our set-up, the rationale of this programme comes from relaxing the credit constraint that restricts investment opportunities by the poor, then allowing for a larger number of agents to operate in the more productive sector. This raises the case for net efficiency gains from redistribution, provided that distortionary effects caused by the related increase in the tax burden prove to be not very strong. In addition, since in our framework everybody has a positive probability of eventually falling into the poor class, redistributive policies can be thought of as

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19 Appendix C shows the coherence of the whole system of constraints and identifies sufficient conditions for its solution, with a moderate loss of generality in the parameter space.

20 The two-party assumption does not involve any loss of generality in our framework, as later analysis shows that classes’ preferences would lead to corner solutions even with a continuum range of feasible policies.

21 Under moral hazard, the gains in efficiency accruing to net recipients should be compared with the loss in efficiency arising for net contributors, namely a reduction in the provision of effort or in $q'$ in our framework. As this issue does not affect our main arguments, we abstract from it for simplicity’s sake. See Benabou (1996b, 2000) and Aghion-Bolton (1997) for an extensive discussion.
supplementing an imperfect insurance market (Loury, 1981). The two factors combine to motivate all agents to support some redistribution, while its actual extent is a matter of political competition.

We identify a redistributive policy in terms of $\zeta_i$, namely the proportion of poor to whom the party $i$ grants a fixed subsidy to enhance their mobility into higher classes. By imposing a balanced public budget constraint, the programme can be represented as follows

\[
\tau_i^M \int_0^\infty dF_i(w) + \tau_i^R \int_0^\infty dF_i(w) = \zeta_i \gamma \int_0^\infty dF_i(w) \quad \text{with } i = 1, 2
\]

where $\gamma \geq \frac{w}{1 - \delta}$ is a fixed subsidy which allows the poor to bequest so much that their offspring can be offered credit, then operate in the productive sector; $F_i(w)$ is wealth distribution at date $t_i$, after realization of uncertain technologies; $\tau_i^M, \tau_i^R$ is the fiscal burden levied on the middle and the rich class, respectively, and taxation of the poor is normalized to zero to reinforce the equity target $\zeta_i$.

Some unusual features in the design of redistribution deserve additional comments.

In the first place, redistribution is meant to benefit exclusively the poor class and is not lump-sum across all agents. This highlights the contribution of public policies in relaxing the credit constraint, with their positive effects on social mobility showing very clearly. Interestingly, although the action programme enforces a pure redistribution, it allows poor parents to leave their offspring an improved ability to earn, like inherited wealth under imperfect capital markets. In this respect, (19) may be alternatively meant as subsidizing education investment by parents to augment the inter-generational transfer of human capital. As a technical point, a fixed subsidy gives every poor agent a chance to end up in the middle class regardless of his own initial wealth, so that we may disregard its evolution over time.

In the second place, the policy design leaves room for progressive taxation. As it stands, taxation is independent of wealth within each class, in line with our basic approach

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22 The measure $\zeta_i$ could be alternatively defined as the fraction of after-tax poor to be helped.
to approximate a continuum of classes by a discrete representation \(^{23}\). It is worth mentioning that taxation on both the middle and the rich classes could be adjusted over time to preserve a balanced public budget against changes in wealth distribution. However, since adjustment in \(\tau_i^M\) may augment the inflows from the middle to the poor class - which would counteract the original equity target- we assume that both parties tax the middle class at a fixed and, without loss of generality, equal rate, with \(\tau_{i,t}^M = \tau^M\) \(^{24,25}\).

A related issue regards the impact of taxation on the investor participation constraint on the financial market. In this regard, we greatly simplify our analysis by ruling out distortionary taxation by assuming that \(\tau^R \leq \tau^M + \frac{(1-q')\varphi}{q'} \) (cfr. Appendix D) \(^{26}\).

In the third place, since policy is implemented in the subperiod following the election, voting is affected by wealth distribution at \(t_1\) and that expected for \(t_2\). As the latter depends on the pattern of social mobility, which includes past redistribution, it turns out that a full appraisal of the feedback between economic and political equilibria must take into account both the distinction of ex ante from ex post inequality and how much it is affected by the kind of policy implemented in previous periods.

\(^{23}\) Technically, the evolution of political equilibrium may be checked by tracking only the mass of agents in each class and ignoring the respective average wealth, with gains in tractability. While this last task is very hard to manage analytically, the first may be dealt with easily in our framework owing to the recursive property of a Markov chain.

\(^{24}\) While abstracting from substantial variation across parties in the middle class taxation may resemble a realistic ingredient of political competition, leaving it unchanged over time sounds more restrictive. We could alternatively assume that parties differ in this respect, too, and reasonably require that \(\frac{d\tau_i^M}{dt} \leq \frac{d\tau_i^r}{dt}\); voting behaviour would prove unaffected for all classes but the poor, who would then demand a large taxation only under a high enough ratio of rich to middle class agents. While the conclusions of our analysis are confirmed, the calculus would be unnecessarily more complicated.

\(^{25}\) With reference to restrictions on the parameters underlying the stochastic matrix \(M_{ij}\), a fixed \(\tau^M\) delivers the advantage that sufficient conditions identified in Appendix C for a free market economy directly apply with active government, after rescaling by a fiscal constant.

\(^{26}\) These conditions are to be considered sufficient, since any combination of stricter ranges of admissible values identified in Appendix C for a subset of parameters may still comply with the rationality constraints for the relatively poor within the middle class even when \(\tau^R > \tau\).
The role of social mobility in political competition has recently been addressed in two strands of explanations why an agent’s voting may deviate from maximizing the payoffs of his current class. In the first (Pickett, 1997 and Quadrini, 1999), it is imperfect information about the true rules for social mobility that motivates people to vote for large redistribution even if they are currently net contributors to the public budget. This argument does not hold true in our framework, where everybody knows the true process driving social mobility. In the second approach (Hirshman, 1973 and Ok-Benabou, 2001), even a very poor median voter may not demand prohibitive taxation on the rich in view of the prospect of his own upward mobility. In this case, we share the emphasis on ex post inequality in ruling out extreme political outcomes, such as either expropriation of the rich or no redistribution at all.

In other words, in our model a non-zero probability for everybody of moving into either the rich or the poor class in some future time delivers the rationale for unanimous consensus for both a floor in the assistance to the needy, namely $\bar{\xi} \geq \xi_1 \geq \xi_2 \geq \underline{\xi}$, and a ceiling in the taxation of the wealthy, $\tau^R \leq \bar{\tau}$. Aside from this argument, the “prospect of upward mobility” does not play a crucial role in explaining our outcomes. As shown in the following section, the preferred policies of the ex ante poor and rich classes show at any time a standard conflict about the demand for redistribution. As for the ex ante middle class, whose voting is actually affected by the prospects of social mobility, it is both upward and backward transition rules that play a key role.

Finally, the programme [19] defines a pair-wise political choice, since at any date it implicitly identifies a one-to-one relationship between $\tau^R_i$ and $\xi_i$ for any given wealth distribution and taxation on the middle class. People are then called to vote on the size of redistribution while knowing its marginal tax cost, as defined by

$$\frac{\partial \tau^R}{\partial \xi} = \frac{\gamma}{\int_0^\infty \gamma w dF(w)}$$

27 Our results are basically confirmed even abstracting from the role of the middle class, for example by slightly changing the policy design so that this class does not show up at the election (Zollino, 1994).
As later shown, political preferences on $\zeta$ are single peaked, and the median voter theorem applies in determining the political equilibrium (Atkinson-Stiglitz, 1980).

### 4.2 Social mobility with active government

Taking account of political competition significantly changes the stage for social mobility, owing to the feedback between economic and political equilibrium in shaping wealth dynamics. To identify equilibrium, we replicate under active government the same backward strategy as in a free market.

In the first place, no change occurs for optimal choices at date $t_2$: everybody consumes and bequeaths constant shares of disposable wealth, the latter now being a function of both the productive and voting decisions taken in the past two sub-periods. The result immediately follows from maximizing (13) under a resource constraint, which now includes the effects of policy

\[
C_2 + B_2 \leq M_2(\tau, \zeta)
\]

where $\tau$ and $\zeta$ identify the political programme voted at date $t_1$ and enforced at $t_2$.

At date $t_1$, agents vote for redistribution on the basis of the expected payoff from the productive investment made in the previous sub-period and taking account of [20]. Under linear indirect utility at date $t_2$, the agent’s problem at date $t_1$ is

\[
\max_{\zeta_1(\tau_1)} \text{ such that } \zeta_1(\tau_1) \in \left\{ \zeta_1(\tau_1), \zeta_2(\tau_2) \right\}
\]

with

\[
M_2 = \begin{cases} 
 w_0 + \vartheta(n - \tau^M) + (1 - \vartheta)\zeta_i & \text{if } 0 \leq w_0 \leq w \\
 q'(r - \tau^R) + (1 - q')\zeta_i & \text{if } w < w_0 \leq 1 \\
 q'(r - \tau^R) + (1 - q')(\zeta - \tau^M) + \max\left\{(w_0 - 1)A - c, w_0 - 1\right\} & \text{if } 1 < w_0 \leq \bar{w}
\end{cases}
\]

where $\zeta_1(\cdot) > \zeta_2(\cdot)$ identify the two competing parties at date $t_1$, with party 1 supporting the larger redistribution; $M_2$ is the expected wealth for period $t_2$, when the voted policies will
be enforced and the technological uncertainty solved; \(w_0\) is the initial wealth, which affects productive decisions at \(t_0\) due to imperfect capital markets.

The policy design implies conflicting preferences for the poor and the rich class, with the former voting for the largest redistribution (party 1) and the latter for the smallest (party 2)\(^{28}\). As to the middle class, the preferred policy is univocally identified at any date, for any given inequality, but may change over time with the fiscal cost of redistribution. In particular, for this class the first order conditions for maximum utility imply that the largest redistribution is preferred if \((1 - q')\int_0^w dF_1/w \geq q'\int_0^w dF_1/w\), otherwise the competing programme is voted. By rearranging in view of (20), the conditions for the middle class to support party 1 require\(^ {29}\)

\[
\int_0^w dF_0(w) \quad \frac{1 - q'}{1 - \vartheta} \equiv \phi 
\]

Two factors help determine the net gains that the middle class expects from redistribution, then its voting behaviour: \(i)\) the \textit{ex ante} relative mass of poor compared with rich people, as it affects the expected tax cost for the middle class; \(ii)\) the ratio of the probability of the middle and poor classes proving \textit{ex post} eligible for assistance, as this affects the expected benefits to the middle class from redistribution. As stressed in the next sections, condition (23) enriches the dynamic feedback between economic and political equilibria in determining both social mobility and policy sustainability in the long run. The latter may indeed prove questionable owing to the following: \(i)\) movements of the median

\(^{28}\) It comes immediately from an increasing (decreasing) \(M_\zeta\) with \(\zeta\) within the poor (rich) class.

\(^{29}\) It follows from substituting for \(\frac{q_0}{\vartheta} dF_0(w) = (1 - \vartheta) \int_0^w dF_0(w) + (1 - q') \frac{1}{\vartheta} dF_0(w)\) and for

\[
\int_0^w dF_1(w) = q' \left( \frac{1}{\vartheta} dF_0(w) + \frac{q}{\vartheta} dF_0(w) \right) \quad \text{in} \quad (1 - q') \int_0^w dF_1(w) \geq q' \frac{1}{\vartheta} dF_1(w), \quad \text{and rearranging.}
voter across classes; \( ii \) switches in the middle class’s preferred policy as a result of changes in inequality partly induced by redistribution itself.

In view of the net expected benefits from redistribution to each class, voting preferences prove to be single-peaked and monotone in agents’ own endowment under any given inequality. In equilibrium, the extent of redistribution at any time \( t \) is then determined by the median voter’s preferred policy, as identified by

\[
\zeta_t = \begin{cases} 
\gamma_1 & \text{if } w_{m,t} \in [0, \bar{w}] \\
\gamma_2 & \text{if } w_{m,t} \in [\bar{w}, w]\n\end{cases}
\]

(24)

with \( w_{m,t} \) being the median wealth at a given point in time.

Finally, under non-distortionary taxation productive decisions at \( t_0 \) are taken according to the same rules as in a free market, with everybody operating in the productive sector but the \textit{ex ante} poor.

As a combined result of optimal investment, voting and consumption over each agent’s lifetime, the inter-generational dynamics of personal wealth is ruled by the following

\[
w_{t+1} = \begin{cases} 
(1 - \delta)(n + w_t - \tau^M) & \text{with } prob = \vartheta \\
(1 - \delta)(y + w_t - \tau^M) & \text{with } prob = (1 - \vartheta)\zeta_t, \quad \text{for } w_t \in [0, \bar{w}] \\
(1 - \delta)w_t & \text{with } prob = (1 - \vartheta)(1 - \zeta_t)
\end{cases}
\]

(25a)

\[
w_{t+1} = \begin{cases} 
(1 - \delta)(s + w_t - \tau^M) & \text{with } prob = \vartheta \\
(1 - \delta)(\bar{w} + w_t - \tau^M) & \text{with } prob = (1 - \vartheta)\zeta_t, \quad \text{for } w_t \in [\bar{w}, 1] \\
0 & \text{with } prob = (1 - \vartheta)(1 - \zeta_t)
\end{cases}
\]

(25b)
(25c) \[
\begin{aligned}
  w_{i+1} = &\begin{cases} 
    (1-\delta)(r + w_i - 1 - \tau_i^R) & \text{with } \text{prob } q' \\
    (1-\delta)(\xi + w_i - 1 - \tau_i^U) & \text{with } \text{prob } 1-q'
  \end{cases} \\
  &\text{for } w_i \in [1, \bar{w})
\end{aligned}
\]

(25d) \[
\begin{aligned}
  w_{i+1} = &\begin{cases} 
    (1-\delta)(r + A(w_i - 1) - c + \tau_i^R) & \text{with } \text{prob } q' \\
    (1-\delta)(\xi + A(w_i - 1) - c + \tau_i^U) & \text{with } \text{prob } 1-q'
  \end{cases} \\
  &\text{for } w_i \in [\bar{w}, \bar{w}] 
\end{aligned}
\]

Under (25) the stochastic process driving social mobility with active government basically preserves the low-dimension Markov chain featured by the matrix \( M_G \) introduced in section 2.1. Based on this result, we can easily investigate both the location of the median voter and his preferred policy at every election.

5. Social mobility and political equilibrium in the long run

The simplified model outlined in the previous sections makes it possible to analytically treatable the analysis of long-run equilibria induced by the dynamic feedback between economic and political equilibria. It allows identifying conditions for steady state versus cycles in policy and inequality based on a straightforward application of standard theorems for the convergence of Markov chains. In this respect, our dynamic economy summarizes in the two laws of motion

(26) \[
\begin{aligned}
  \xi_t &= Z(\psi_t, \phi) \\
  \psi_{t+1} &= \psi_t M_G(\xi_t)
\end{aligned}
\]

with \(Z\) implicitly defining the political equilibrium in terms of the density measure \(\psi\) and the structural parameter \(\phi \equiv (1-q')/(1-\hat{\theta})\).

Accordingly, a redistributive policy voted at date \(t\) remains in place in the long run if it affects the transition matrix \(M_G\) in a way that the induced stationary wealth distribution
complies with the median voter to keep voting for the same policy. Then, a steady state political equilibrium can be defined as

\[ \zeta^* = Z[\psi^*(\zeta^*), \phi] \]

and the induced stationary distribution of wealth can be characterized by exploiting the standard properties of Markov chains (Appendix A). As opposed to steady states, in the long run policy and inequality may periodically fluctuate, as implied by

\[
\begin{align*}
\bar{\psi} &= \psi M_G \left[ \psi \right] \\
\underline{\psi} &= \psi M_G \left[ \underline{\psi} \right]
\end{align*}
\]

with \( \bar{\psi} \) and \( \underline{\psi} \) identifying the invariant measures induced by \( M_G(\cdot) \) and supporting, respectively, high and low redistribution in equilibrium. We show that sufficient conditions for policy cycles and the key factors explaining them can be analytically identified, too.

Interestingly, our analysis allows us to disentangle the input from the economic structure and from the policy itself, thus unveiling a richer pattern in the relationship between inequality and redistribution than commonly found in recent literature. In this section, we first deal analytically with sufficient conditions for the variety of outcomes that may occur in the long run, then we resort to a numerical simulation to illustrate the dynamics of the adjustment process in selected examples.

We find it useful to think about the political equilibrium in the long run in a notional dimension of time by positing that the next election will be called after convergence to the stationary distribution induced by current policy has taken place. This is equivalent to convergence of \( \psi(\zeta_i) \) to \( \psi^*(\zeta_i) \) materializing in one period of time. Although we then fail to consider the effects of changes in the political equilibrium which might occur at a higher frequency of elections, we are still in a position to identify sufficient conditions that if a policy remains unchanged over our notional time, it will be the same at every possible date (and all the same in case of policy fluctuations). What we miss in this way is the ability to
figure out the path of political equilibrium election by election, which is not at the core of our analysis; in this, however, numerical simulation may help.

5.1 Characterizing equilibrium in the long run: analytical results.

Starting from any redistribution $\xi_t$ implemented at time $t$, the long-run equilibrium can be analyzed by checking which policy would gain majority voting under the stationary wealth distribution $\psi^*(\xi_t)$, to which the Markov chain $M_t(\xi_t)$ converges at time $T(\xi_t)$. A variety of possible equilibria may be identified in the long run, which differ in either the extent of the redistribution or the main factors explaining it. In the first place, Proposition 2 in section 2.1 directly results from optimal voting. By exploiting conditions for the convergence of Markov chains outlined in Appendix A, we can restate as follows:

Proposition 2a. Sufficient conditions for low redistribution as a unique equilibrium in steady state require either of the two:

a1) parameters $(\vartheta, q')$ are so high that in the long run the median voter belongs to the rich class regardless of the extent of redistribution:

$$
\psi^*(\xi_2) = \frac{q'[\vartheta(1-\xi_2)] + \xi_2}{(1-\xi_2)[\vartheta + (1-q')^2] + \xi_2} > \frac{1}{2};
$$

a2) parameters $(\vartheta, q')$ are high enough that in the long run the median voter does not belong in the poor class and, under any policy, the mass of poor is relatively large:

$$
\psi^*(\xi_2) = \frac{(1-q')^2(1-\xi_2)}{(1-\xi_2)[\vartheta + (1-q')^2] + \xi_2} < \frac{1}{2} \quad \text{and} \quad \frac{\psi^*(\xi_1)}{\psi^*(\xi_2)} > \frac{1-q'}{1-\vartheta}.
$$

Proof. Low redistribution results in the long run if: i) $w_{n,rsj} \geq 1$ at any $j > \bar{r} \geq 0$; ii) $w_{n,rsj} \geq w$ and $\Delta_{rsj} > \phi$ at any $j > \bar{r} \geq 0$. As for case i), it is easy to show that the mass of rich agents at any date increases with redistribution, thus if in the long run the median voter remains rich class under invariably low redistribution (condition a1), a fortiori it does so when large redistribution occasionally occurs at some elections. As for case ii), we consider the following: i) since the relative
Proposition 2b. Sufficient conditions for high redistribution as a unique equilibrium in steady state require either of the two:

b1) parameters \( (\theta, q') \) are so low that they eventually push the median voter into the poor class regardless of the extent of redistribution:

\[
\psi_1^*(\zeta_1) = \frac{(1-q')^2(1-\zeta_1)}{(1-\zeta_1)[\theta+(1-q')^2]} + \zeta_1 > \frac{1}{2};
\]

b2) parameters \( (\theta, q') \) are low enough that in the long run the median voter does not belong to the rich class and, under any policy, the mass of poor is relatively small:

\[
\psi_2^*(\zeta_1) = \frac{q'[(1-\zeta_1)+\zeta_1]}{(1-\zeta_1)[\theta+(1-q')^2]} < \frac{1}{2} \quad \text{and} \quad \psi_2^*(\zeta_2) = \frac{(1-q')^2(1-\zeta_2)}{q'[\theta(1-\zeta_2)+\zeta_2]} \leq \frac{1-q'}{1-\theta}
\]

Proof. Arguments for low redistribution hold symmetrically true for large redistribution.

Sufficient conditions for a unique equilibrium in steady state ultimately rule out a significant impact of policy itself in affecting the demand for redistribution. The latter is then primarily determined by social mobility as driven by the purely economic structure. However, when policy significantly contributes to determine majority at every election, the long-run equilibrium proves to be not univocally determined any more.

Proposition 3a. Sufficient conditions for multiple equilibrium in steady state (history dependence of redistribution) require that
c1) social mobility in a free market is strong enough to leave the median voter in the middle class regardless of the extent of redistribution, as from the requirement identified by the first part sub a2) and b2);

c2) competing policies are different enough that

\[
\frac{q'}{1-\theta} \leq \xi_1
\]

Proof. Arguments previously made imply that when large [small] redistribution self-reinforces owing to median voter remaining poor [rich], in the long run it appears even if redistribution is initially small [large]; then condition c1) identifies a first requirement for history dependence. In the second place, c2) results from rearranging

\[
\Delta_j(\xi) = \frac{(1-q')(1-\xi)}{q'(\theta(1-\xi_1) + \xi)} \leq \frac{1-q'}{1-\theta} < \frac{(1-q')(1-\xi_2)}{q'(\theta(1-\xi_2) + \xi_2)} = \Delta_j(\xi_2),
\]

with \(\Delta_j(\xi_j) = \frac{\psi_{j+1}(\xi_j)}{\psi_{j+1}(\xi_j)}\), implying that large [small] redistribution occurs in the long run only if it is voted at every election. Consider large [small] redistribution: when the median voter remains in the middle class, policy self-reinforces over time provided that

\[
(1-q')(1-\xi_j) \leq \phi \forall t \quad [\Delta_j(\xi_j) > \phi \forall t].
\]

From \(\psi_{j+1} = (1-\xi_j)(1-\nu_j) + (1-q')\nu_j\) and \(\psi_{j+1} = q'(\psi_j + \nu_j)\), it follows that \(\Delta_j(\xi_j) = \frac{\psi_{j+1}(\xi_{j+1})}{\psi_{j+1}(\xi_j)} = \frac{(1-q')(1-\xi_j)}{q'(1-\nu_j)} \left[\psi_j \left(1 + \frac{\Delta_j - \phi}{\nu_j}\right) + \nu_j\right] = \frac{(1-q')(1-\xi_j)}{q'(1-\nu_j)} \left[1 + \frac{\Delta_j - \phi}{\nu_j}\right] + \nu_j > 0 \quad \text{[<0] when } \xi_{j+1} = \xi_2 \quad [\xi_j = \xi_1].\]

As small redistribution self-reinforces when

\[
\frac{(1-q')(1-\xi_j)}{q'} + \nu_j(\xi_{j+1}) > \phi \quad \text{or} \quad (1-\xi_j) > \frac{q'}{1-q'}[\phi - \nu_j(\xi_{j+1})],
\]

by substituting for \(\phi = \frac{1-q'}{1-\theta}\) and rearranging, we obtain the requirement that

\[
\xi_{j+1} < 1 - \frac{q'}{1-\theta} + \frac{q'}{1-q'}\nu_j(\xi_{j+1}), \quad \text{with } \nu_j(\xi_{j+1}) > 0. \quad \text{A symmetric argument holds true for large redistribution.}
\]

When the middle class is pivotal in political equilibrium, long-run policy might depend, ceteris paribus, on initial conditions. Noticeably, history dependence is also obtained in Benabou (2000) and Saint-Paul (2001), where imperfect democracy plays a key role. In restating the same argument, even abstracting from such restriction, we cast multiple equilibria as particular cases out of a variety of feasible outcomes in the long run.
Proposition 4. No invariant equilibrium arises in the long run, with inequality and redistribution periodically fluctuating, when forces for downward and upward mobility in free market are balanced enough that they can be offset by high and low redistribution, respectively. Sufficient conditions identify the joint requirements:

\[ d1) \text{ high redistribution is not confirmed in the long run, namely the violation of either } a1) \text{ or } a2); \]

\[ d2) \text{ low redistribution is not confirmed in the long run, namely the violation of either } b1) \text{ or } b2). \]

Proof. It follows from opposite arguments to those proving steady state equilibria.

To our knowledge, a case for endogenous policy cycles has been previously made only in Gradstein-Justman (1997) in terms of a periodical switch between radically different political regimes, ultimately driven by random extraction by nature of individual innate ability. By contrast, we find a similar result in a much more general framework, in which political cycles may arise even among reasonably similar competing policies and for quite a large set of admissible values for parameter characterizing the economic process. However, an additional innovation is the identification of the contribution made by the feedback between the economic structure and political equilibrium.

5.2 Exploring the short-run dynamics: a numerical simulation

In our stochastic framework, no standard measure of probability would be preserved over time. This raises the need for a numerical simulation in order to shed light on the evolution of inequality and redistribution election by election. In particular, it allows us to explore the direction of changes in the median voter’s location and/or preferred policy and the way they are affected by both economic and political inputs.

In view of the recursive structure of a Markov chain, our numerical simulation simply consists of the following steps: \( i) \) assuming a set of initial values for the parameters \( \xi, q, \theta \) and wealth distribution \( \psi_0 \), which in turn determines which party is initially in office; \( ii) \) detecting the median voter's location at the first electoral test as it is implied by


\[ \phi_i = \phi_0 M_G(\zeta_0) ; \]

iii) finding the ensuing equilibrium value of \( \zeta_i \); if it is different from \( \zeta_0 \), substituting for it in matrix \( M_G \), otherwise leaving \( M_G \) unchanged; iv) searching for the median voter's location at the following electoral test as follows from \( \phi_1 = \phi_0 M_G(\zeta_1) \); v) starting again from iii) by updating the parameters where required.

In validating the variety in the long-run relationship between inequality and redistribution previously identified on analytical grounds, the numerical exercise addresses the key role of the pure economic structure versus policy in affecting short-run dynamics. By focusing attention on more informative examples, the standard prediction of positive correlation between redistribution and inequality is confirmed when factors which would drive backward/upward mobility in a free market are too strong to be offset by public action (Fig. 1.A and 1.B)\(^{30}\). Taking as a proxy of inequality the mass of poor agents, under strong upward mobility this measure proves invariably very low in steady state and small redistribution is confirmed over time (Fig. 2.A). Even if the median agent would initially vote for a more active policy, he will very soon move into the upper class and will stay there forever, causing a once-for-all change in political regime. The opposite is symmetrically true with strong backward mobility (Fig. 2B).

In the second place, under milder forces for social mobility in a free market, both the short and long-run equilibrium crucially depend on the way those forces interact with policy in determining wealth distribution at any point in time, and the ensuing net gains from redistribution expected by the middle class. This can be traced back to two factors: a) changes in the parameters \( \vartheta \) and \( q' \), which in our model summarize the economic structure, affect the threshold level \( \phi = \frac{1-q'}{1-\vartheta} \) for the ratio \( \Delta \) of the mass of poor to rich people; b) at every election, the actual value of this ratio responds to the joint effect on social mobility of the economic structure itself and the extent of redistribution enforced in previous periods. Depending on the way the two factors combine to determine social mobility under active government, the preferred policy of the median voter may change either because he jumps

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\(^{30}\) In the numerical simulation, the initial distribution is arbitrarily chosen as pointless since we skip the case of history dependence in order to save space. A larger than reported set of results is available from the author on request.
across classes or, when remaining in the middle class, the sign of \((\Delta - \phi)\) reverts. As a possible result, small redistribution follows either from low inequality (Fig. 2C, with poor people in the specific example below 10 per cent in steady state) or from high inequality (Fig. 2D, with poor people above 40 per cent). And, under minor variations in the economic structure underlying inequality, even an initially large unanimous consensus for a given policy, which would otherwise confirm over time, may be reverted very quickly (Fig. 2E and 2F).

An interesting contribution of the numerical exercise is to clarify the case for endogenous political cycles. As shown analytically, they occur when opposite forces of attraction towards the rich and the poor classes under a free market are balanced enough that their combined effect on social mobility can be offset by redistribution. In the numerical exercise, political cycles typically occur when upward (backward) mobility in a free market is not so strong that the median agent can jump into the rich (poor) class regardless of the extent of redistribution. For instance (Figs. 3.A-B), after a large redistribution has taken place for some time, the median voter might eventually move into the rich class; this means that the middle has been voting for high redistribution for a number of periods, when the rich were not the majority.

Once the median agent becomes rich, he starts voting for lower redistribution, and the intensity of upward mobility diminishes; this could occur to such an extent that consensus for the newly established policy is gradually eroded as the mass of rich people shrinks. At the same time, output per capita falls below the initial level, when redistribution was large, since the change to low redistribution augments the net inflow of agents from the middle to the poor class, then the share of agents producing in the subsistence sector. At some point in time, the median voter moves back from the rich to the middle class, and conditions can gradually resume for his preferred policy returning a large redistribution; when it eventually happens, per capita output will again rise since the new policy effectively relaxes the credit constraint that limits the opportunity to invest in the productive sector.
6. Concluding remarks

By modelling social mobility as a stochastic process in discrete space, we have explored the feedback between economic and political factors in determining the extent and the invariance of redistributive policies in the long run. When both the location and the preferred policy of the median voter are allowed to change endogenously over time, a large variety of political equilibria may result in steady state, with no clear-cut relationship with inequality. Actual outcomes depend on the way structural economic parameters, which would drive social mobility under no public action, combine with the policy input in determining the change in wealth distribution over time. It turns out that the standard prediction of a positive correlation between redistribution and inequality is confirmed when the pattern of social mobility that would prevail in a free market proves robust to public action. Otherwise the dynamic feedback between economic and political equilibrium leads to a non-linear relationship between inequality and the extent of redistribution, which proves much richer than commonly found in recent literature. In this respect, political cycles may endogenously arise when the free market rules for backward and upward mobility are similar enough that their overall effect on inequality may be offset by a redistributive policy.
Social mobility in free market

With $\psi_0 = [20, 60, 20]$; percentage share of total population

A. With $\vartheta = 0.8; q' = 0.8$

B. With $\vartheta = 0.2; q' = 0.2$

C. With $\vartheta = 0.7; q' = 0.7$

D. With $\vartheta = 0.4; q' = 0.4$

E. With $\vartheta = 0.8; q' = 0.2$

F. With $\vartheta = 0.7; q' = 0.2$

- mass in poor class
- mass in middle class
- mass in rich class
Social mobility with redistribution

With $\psi_0 = [20, 60, 20]$ and $\zeta_1 = 0.2$ $\zeta_2 = 0.1$; percentage shares of total population

A. With $\vartheta = 0.8$; $q' = 0.8$

B. With $\vartheta = 0.2$; $q' = 0.2$

C. With $\vartheta = 0.7$; $q' = 0.7$

D. With $\vartheta = 0.4$; $q' = 0.4$

E. With $\vartheta = 0.8$; $q' = 0.2$

F. With $\vartheta = 0.7$; $q' = 0.2$

mass voting for large redistribution – mass in middle class’ – mass in rich class
Social mobility and political cycles
With \( \psi_0 = [20, 60, 20] \) and \( \zeta_1 = 0.2 \) \( \zeta_2 = 0.1 \); percentage shares of total population

A. With \( \vartheta = 0.6 \), \( q' = 0.6 \)

B. With \( \vartheta = 0.3 \), \( q' = 0.3 \)

Social mobility in free market
With \( \psi_0 = [20, 60, 20] \); percentage shares of total population

C. With \( \vartheta = 0.6 \), \( q' = 0.6 \)

D. With \( \vartheta = 0.3 \), \( q' = 0.3 \)
Appendix A

A1. Existence and parameter dependency of stationary distribution. As from the received theory of stochastic processes, a time-invariant Markov chain admits a unique invariant (or stationary) measure \( \pi \) if \( \lim_{n \to \infty} \pi^n = \pi \), where \( \pi \) is a column vector representing the probability of each of the \( m \) possible states and \( Q^n \) is a matrix representing the transition probability from one state to another in \( n \) periods time (Stockey-Lucas, 1989). Conditions for stationarity of a Markov chain can easily be checked on the basis of the transition matrix, since they require that each element of \( Q^n \) proves strictly positive for some value of \( n \geq 1 \). With reference to matrices \( M_F \) and \( M_G \) introduced in the text, it can be immediately verified that these conditions are met with \( n=2 \); we can then exploit the proof of stationarity to characterize the induced invariant measures in the terms of the key parameters.

By transposing \( \pi'Q = \pi' \) in \( Q'\pi = \pi \), the invariant measure \( \pi \) can be interpreted as the eigen-vector associated with the unit eigen-value of \( Q' \) and normalized to satisfy \( \Sigma \pi_i = 1 \) (Ljungqvist-Sargent, 2000). Representing for simplicity’s sake both \( M_F \) and \( M_G \) by a generic matrix \( A \), with

\[
A = \begin{bmatrix}
\alpha & \beta & 0 \\
\gamma & \delta & \epsilon \\
0 & \nu & \epsilon
\end{bmatrix}
\quad \text{and} \quad
A' = \begin{bmatrix}
\alpha & \gamma & 0 \\
\beta & \delta & \nu \\
0 & \epsilon & \epsilon
\end{bmatrix}
\]

the invariant measure \( \psi^* \) referred to in the text can be obtained by solving the system of three homogeneous equations given by \( [A'-I] \psi^* = 0 \), which is equivalent to

\[
\begin{align*}
(\alpha-1)\psi^*_1 + \gamma\psi^*_2 &= 0 \\
\beta\psi^*_1 + (\delta-1)\psi^*_2 + \nu\psi^*_3 &= 0 \\
\epsilon\psi^*_2 + (\nu-1)\psi^*_3 &= 0
\end{align*}
\quad \text{or, in view of } \Sigma \psi^*_i = 1,
\begin{align*}
(\alpha-1)\psi^*_1 + \gamma(1-\psi^*_1-\psi^*_3) &= 0 \\
\epsilon(1-\psi^*_1-\psi^*_3) + (\nu-1)\psi^*_3 &= 0 \\
\psi^*_2 &= 1-\psi^*_1-\psi^*_3
\end{align*}
\]

Solutions in terms of parameters \( \theta \) and \( \zeta \) can eventually be retrieved after proper substitutions in the expressions.
\[ \psi_1^* = \frac{\gamma (1 - \epsilon)}{(1 - \alpha) + \gamma (1 - \epsilon)}, \quad \psi_2^* = \frac{(1 - \alpha)(1 - \epsilon)}{(1 - \alpha) + \gamma (1 - \epsilon)}, \quad \psi_3^* = \frac{(1 - \alpha)\epsilon}{(1 - \alpha) + \gamma (1 - \epsilon)}. \]

**A2. Conditions for a stable low redistribution.** In view of the characterization of a stationary distribution, the conditions identified in the text can be rearranged to require either

i) \( q' [\theta (1 - \zeta) + \zeta] > 0.5 * \Xi(\zeta) \) or

ii) \( (1 - q')^2 (1 - \zeta) < 0.5 * \Xi(\zeta) \) and \( (1 - \theta)(1 - q')(1 - \zeta) > q'[\theta (1 - \zeta) + \zeta] \)

with \( \Xi(\zeta) = (1 - \zeta)[\theta + (1 - q')^2] + \zeta \).

**A3. Conditions for a stable high redistribution.** As before, they can be rearranged to require either

iii) \( (1 - q')^2 (1 - \zeta) > 0.5 * \Xi(\zeta) \) or

iv) \( q'[\theta (1 - \zeta) + \zeta] < 0.5 * \Xi(\zeta) \) and \( (1 - \zeta)(1 - \theta)(1 - q') \leq q'[\theta (1 - \zeta) + \zeta] \)

with \( \Xi(\zeta) = (1 - \zeta)[\theta + (1 - q')^2] + \zeta \).

**A4. Conditions for political cycles.** They can be split into two sets to be satisfied jointly. The first deals with revisions of initial high redistribution and the second with revisions of initial low redistribution. The former, which represent a combination of complements to conditions A3, requires either

v) \( q'[\theta (1 - \zeta) + \zeta] > 0.5 * \Xi(\zeta) \) or

vi) \( (1 - q')^2 (1 - \zeta) < 0.5 * \Xi(\zeta) \) and \( (1 - \zeta)(1 - \theta)(1 - q') > q'[\theta (1 - \zeta) + \zeta] \).

The latter, which combines complements to conditions A2, requires either

vii) \( (1 - q')^2 (1 - \zeta) > 0.5 * \Xi(\zeta) \) or
ivii) \( q'(\vartheta (1-\xi)+\xi) \leq 0.5^* \Xi(\xi) \) and \( (1-\vartheta)(1-q')(1-\xi) \leq q' \vartheta (1-\xi)+\xi \).

Any combination of conditions v) and vi) on one side and vii) and viii) on the other identifies a subset of parameter values, which induces cyclical revisions in redistributive policies.

**Appendix B**

**Equilibrium conditions in the financial sector.** Asymmetric information about the effort provided by borrowers in the risky project points to both superiority of indirect versus direct lending and efficiency equivalence between an incentive compatible contract and a standard debt one, with bankruptcy provision. In this respect, the arguments put forward by Diamond (1984) and Williamson (1986) may be incorporated easily in our framework by stressing the role of intermediation in minimizing the costs of the information frictions. For simplicity’s sake, we skip a formal proof of this tenet, which would require adding costly monitoring in our formal setting\(^{31}\), and we provide a formal derivation of both an optimal financial contract and credit rationing in equilibrium.

Under the conditions we are going to identify, a possible equilibrium is that the financial intermediary: i) raises the funds required for investing in the risky project by paying lenders the interest factor net of commission, namely \( A-c \); ii) takes over the full property rights on the risky project by properly compensating the effort exerted by the investor/borrower. Accordingly, the financial contract between the latter and the intermediary identifies a pair of compensations, one in the case of success and one in the case of failure, which meet the individual rationality constraints of both parties and the investor's limited liability constraint. Following Grossman-Hart (1983)\(^{32}\), the compensation schedule matches a simple form, consisting of a common basis augmented by some bonus in the case of success. We define the common basis as \( y = \pi + i_{f} \), with \( \pi = \max\{w, Aw-c\} \) the borrower's opportunity cost to

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\(^{31}\) Following Freeman (1986), a further heuristic argument for indirect lending would come from the locally increasing returns characterizing the risky investment in the model, motivating lenders to pool.

\(^{32}\) The variation of risk neutrality we introduce does not change their efficiency results but drops the uniqueness ones.
invest his initial endowment in the risky project and $i_f$ a quantity to be properly determined; in the case of success an additional bonus $i_s = sr$ is offered to the investor. Accordingly, the non-negative profit condition for the financial intermediary requires

\[(1A) \quad pr + (1-p)\xi - A(l-w) \geq y + pi_i = \pi + i_f + pi_i,\]

where the left-hand side measures the expected returns on the risky project, net of the cost of raising the required funds on the credit market, and the right-hand side shows the total compensation the intermediary expects to pay the borrower/investor in exchange for the full property rights on the project. Since the left-hand side increases with the endowment of investors, the constraint is binding for the intermediary dealing with the poorest of them (thereafter the marginal investors). Should the compensation schedules be valuable all the same, regardless of the initial wealth of the investors, the financial intermediary would gain positive profits by targeting the richest of them. Free entry in the financial sector, however, leads to a sort of “Ricardian rent” accruing to the richer agents than the marginal investor in the middle class, as the former gain a more valuable compensation schedules. As a result in competitive equilibrium (1A) is binding for all intermediaries. In characterizing equilibrium conditions we first identify the compensation schedule offered to the marginal investors, then that offered to the relatively richer ones.

From the intermediary's standpoint, an optimal strategy is to offer the marginal investors the compensation spread that motivates them to the highest effort and then minimize the cost of the failure compensation\(^{33}\). In view of expression (10) in the main text, setting

\[s = \frac{b}{r} = \inf \left\{ x \in (0, l) \text{ s.t. } x > \frac{b}{r} \right\} \]

implies that the marginal borrower will exert the effort required for a success probability $q'$ and the optimal failure compensation follows from:

\[(2A) \quad \max_{i_f} q' \left( r - \frac{b}{r} \right) + (1-q')\xi - A(l-w) - i_f - \pi \]

\(^{33}\) Indeed the intermediary would like to extract the largest effort provided that $pr(1-s) - \xi \geq 0$; it is easy to check that this condition is always met under the non negative profit condition (11) for lenders dealing with marginal investors.
such that

(2Aa) \[ \pi + i_f \geq 0 \]

(2Ab) \[ q'(r - b) + (1 - q')\xi - A(1 - w) \geq \pi + i_f \]

(2Ac) \[ q'\tilde{b} + i_f - g(q') \geq \pi + \vartheta n - b(\xi) \]

where the first inequality identifies the investor’s limited liability constraint and the remaining two the participation constraints of the intermediary and the borrower, respectively. It is immediately apparent that the intermediary’s optimal choice is to set a penalty \( i_f = -\pi \) should it satisfy the second and third constraint.

Indeed, (2Ab) implies that the intermediary participation constraint is met iff \( w \geq w \) with

\[
(3A) \quad \bar{w} = l - \frac{q'(r - b) + (1 - q')\xi}{A}
\]

Accordingly, people with a smaller endowment are restricted to stay out of the credit market and cannot invest in the risky project, although more productive, because of imperfect information about the effort they provide. Indeed, credit rationing occurs in equilibrium for two reasons: a) the poorer the borrower, the lower the failure penalty he can afford to pay under the limited liability constraint; b) lending to the poorer is more costly to the intermediary owing to the larger amount of funds to be raised on the credit market to meet the fixed capital investment requirement. Therefore, credit rationing does not hurt at random as in Stiglitz-Weiss (1981) and Williamson (1983) and, as in Aghion-Bolton (1997), people to be rationed are fully identifiable with the poor. In our model \( \bar{w} \) identifies the marginal investors and, according to the rules of behaviour, the lower bound of the middle class.

With reference to the marginal investor participation constraint (2Ac), in view of (7) and Assumption 1 in the text it may be rearranged as

\[
(4A) \quad q'\tilde{b} \geq \bar{w} + \vartheta n + bq'
\]
Since (3) in the main text implies that \( w + \epsilon n + bq' \leq q'r + (1-q')\xi - A(l-w) \), a sufficient condition for (4A) to hold is that \( q'r + (1-q')\xi \geq A(l-w) \), namely that the intermediary participation constraint is not violated. As a result, the borrower finds it profitable to agree upon the financial contract he is offered: by exchanging the full property rights on the risky project for a compensation equal to zero in the case of failure and to \( b \) in the case of success, he will be happy to exert the effort required to achieve a success probability equal to \( q' \).

Turning to richer borrowers, should they be offered the same contract as the marginal ones, financial intermediaries would gain positive profits in dealing with them and free entry would lead the infra-marginal borrowers to gain a positive rent. Since under risk neutrality both investors and intermediaries are indifferent whether this rent is the result of higher compensations in the event of either success or failure, we focus attention on the first case: intermediaries compete by bidding a higher spread \( s \) to the investors who require fewer funds to be raised on the credit market. Under free entry, competition for attracting the relatively richer customers would raise the spread to the highest level allowed by the intermediary zero profit condition

\[
q'r[l - s(w)] + (1-q')\xi = A(l-w)
\]

Accordingly, the equilibrium financial contract consists of a pair of compensations \((s(w),0)\), with \( s(w) = l + \frac{1}{q'r}[(1-q')\xi - A(l-w)] \) identifying the success bonus, which increases with the investors’ initial endowment. It is easy to prove that this contract, although not the only one that may occur in equilibrium in view of risk neutrality, meets the participation constraint of the richer borrowers/investors than the marginal ones, too.

**Appendix C**

In order to preserve a simple social structure over time, in the main text six restrictions on the parameter space have been identified, leading to the following system of equations:
In this section we check for the coherence of the system as a whole and identify the set of sufficient conditions for its solution. We focus on sufficient conditions both to simplify the algebra and to gauge the minimum admissible variability in parameter values as a proxy of the highest loss of generality implied by the restrictions. Then, conditions could be relaxed to some extent without necessarily violating the requirements for a low dimension stochastic matrix, as discussed in the text. In this respect, the number of equations is easily reduced to five for the following reasons: since (5) in the main text implies that $\xi \geq \vartheta$, the constraint 4a [2a] proves redundant under $n \leq \xi \leq \vartheta$. In this section we proceed by taking out constraints 4a, with the results being confirmed, *mutatis mutandis*, if constraint 2a is alternatively dismissed. Moreover, since setting $c$ at its lower bound $c$ identified by 1 makes stricter restriction 2b, which would *a fortiori* be satisfied under more general conditions, the system (5A) simplifies to

(5A.a)

\begin{align*}
1. & \quad c \geq w(A-1) \\
2a. & \quad (1-\delta)\xi \geq w \\
2b. & \quad (1-\delta)\xi < 1 - (1-\delta)(d_1 - c) \\
3. & \quad b(1-\delta) > 1 \\
4a. & \quad n(1-\delta) \geq w \\
4b. & \quad (n + w)(1-\delta) < 1
\end{align*}

**Sufficient conditions for 2b.** From substituting for $w = (1-\delta)(r-A-c)[1-\alpha]^1$, under $\alpha \equiv (1-\delta)A < 1$ and rearranging the algebra, we get $A[w-1] - c = [\alpha r - A - c][1-\alpha]^1$. By plugging in the stricter than required condition that $c = w(A-1)$, substituting for the expression of $w$ and rearranging, $A[w-1] - c = (1 + \alpha r - 2A + (1-A^{-1}))[q'(r-b) - (1-q')\xi])$ $(1-\alpha)^{-1}$ or, from the equilibrium conditions on the financial markets (cfr. Appendix B), $A[w-1] - c = (1 + \alpha r - 2A + (1-A^{-1}))A(1-w)(1-\alpha)^{-1} = (1 + \alpha r - 2A + (A-1)(1-w))(1-\alpha)^{-1}$. If this entity is not greater than unity, a sufficient condition for the constraint 2b is that
\[ \xi < \delta (1 - \delta)^{-1} \equiv \bar{\delta}. \]

For that to be the case, the strictest requirement reads
\[ \{1 + \alpha r - 2A + (A - 1)(1 - w)\} \leq 1 - \alpha \]
or, taking the extreme case of \( w \) close to zero and rearranging, \( 1 - \delta \leq (1 + A)(1 + Ar)^{-1} \equiv \bar{\Delta}. \)

**Sufficient conditions for Ia.** From substituting for \( w = 1 - [q'(r - b' + (1-q')\xi)]A^{-1} \) and rearranging the algebra, the requirement immediately reads as
\[ \xi \geq [A - q'(r - b)]A(1 - \delta) + (1 - q')]^{-1}. \]

Since equilibrium on the financial market implies that \( q'(r - b) = A(1 - w) - (1 - q')\xi \), by taking the strictest case with \( w \) close to 1 and substituting for the ensuing expression in the previous constraint, we get \( \xi \geq A[\{A(1 - \delta) - 2(1 - q')\}]^{-1}, \)
which is always satisfied for non-zero values of \( \xi \) under a negative denominator or \( (1 - \delta) < 2(1 - q') A^{-1} \equiv \bar{\Delta}. \)

**Sufficient conditions for II.** Since \( \bar{b} \equiv b'r \) with \( b' > b \), the constraint may simply be restated as \( \xi \geq [b(1 - \delta)]^{-1} \equiv \bar{r}. \)

**Sufficient conditions for III.** From \( (1 - \delta) < (n + w)^{-1} = A[1 + nA - (1 - w)]^{-1} \), taking the strictest case of \( w = 1 \), we obtain \( (1 - \delta) < A[1 + nA]^{-1} \equiv \bar{\Delta}. \)

In sum, the system (5A) proves to be coherent as a whole, with sufficient conditions for its solution requiring i) \( \xi \in \left(0, \delta [1 - \delta]^{-1}\right) \); ii) \( (1 - \delta) \in \left(0, \min\left\{\Delta, \bar{\Delta}, \Delta \right\}\right); \) iii) \( r \geq \bar{r}. \)

Noticeably, the possibility of reducing the variety of constraints to a three-equation system with six unknowns leaves the parameter space reasonably undetermined despite the assumed restrictions.

**Appendix D**

With active government, the investor participation constraint on the financial market requires that \( q'(sr - \tau_\alpha) + (1-q')\xi_s - g(q') \geq \delta n + \max\{w, Aw - c\} - h(\xi) - T_{1,\alpha} \) or, provided that this condition is met in a free market,
(6A) \[ (1 - q') \gamma_{j} \geq q' r_{i,j}^{w} - T_{i,j} \]

where \( T_{i,j} \) identifies the expected taxation on the reservation payoffs. From the optimal rules for investment outlined in the text, it easy to show that within the middle class \( T_{i,j} \) follows the step-wise function

(7A) \[
T_{i,j} = \begin{cases} 
\tau_{i,j}^{M} & \text{if } \max\{w, A w - c\} < 1 - n \\
t_{i,j}^{w} + (1 - \vartheta) \tau_{i,j}^{M} & \text{if } 1 - n \leq \max\{w, A w - c\} < 1 \\
\tau_{i,j}^{w} & \text{if } \max\{w, A w - c\} \geq 1 
\end{cases}
\]

In view of (7A), condition (6A) may be rearranged as follows

\[
\begin{cases} 
\tau_{i,j}^{w} \leq \frac{\tau_{i,j}^{M} + (1 - q') \gamma_{j}}{q'} & \text{if } \max\{w, A w - c\} < 1 - n \\
\tau_{i,j}^{w} \leq \frac{(1 - \vartheta) \tau_{i,j}^{M} + (1 - q') \gamma_{j}}{q' - \vartheta} & \text{if } 1 - n \leq \max\{w, A w - c\} < 1 \\
\tau_{i,j}^{w} \geq - \gamma_{j} & \text{if } \max\{w, A w - c\} \geq 1 
\end{cases}
\]

It turns out that the establishment of a redistributive programme does not at all affect the original incentive to invest in the upper bound of the middle class and, a fortiori, of the rich class as a whole. It could exert a negative impact on agents in the middle range and, to higher extent, in the lower bound of the middle class. In order to rule out at all distortionary effects of taxation, we need conditions to hold for \( \tau^{w} \leq \frac{- \tau^{M} + (1 - q') \gamma_{j}}{q'} \).
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