Pitfalls of monetary policy under incomplete information: imprecise indicators and real indeterminacy

by Eugenio Gaiotti
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PITFALLS OF MONETARY POLICY UNDER INCOMPLETE INFORMATION: IMPRECISE INDICATORS AND REAL INDETERMINACY

by Eugenio Gaiotti*

Abstract

The paper examines the link between the precision of the available monetary policy indicators and the determinacy of equilibrium in a forward-looking macroeconomic model with partial information and an optimizing central bank. When the information on endogenous variables is not precise enough, the central bank acts too timidly; there is a possibility of self-fulfilling fluctuations in inflation and output. It is argued that, unless they are very precise, projections of output or inflation over the relevant horizon cannot be the only criterion for determining monetary policy actions. Rules which include a sufficient reaction to nominal variables may be necessary to supply an anchor for prices, even when the policymaker intends to consider all relevant information. Appointing a “conservative” central banker may also induce a less timid response to signs of inflation or deflation, even when their interpretation is difficult. In contrast, relying too much on measures of exogenous variables, such as potential output, can be counter-productive, because it could induce an attitude that is not responsive enough to inflation or deflation.

JEL classification: E52, E58.
Keywords: Monetary policy, information variables, incomplete information.

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1. Introduction

The recent literature on optimal monetary policy characterizes the behavior of a central bank in the face of uncertainty as the outcome of an optimal procedure of signal extraction on the state of the economy, given the set of available indicators and their precision. The central bank bases its actions on its forecasts of the target variables, rather than on simpler rules of behavior. In an inflation-forecast targeting framework, the policy instrument is set so that the corresponding conditional inflation forecast is consistent with the inflation target; the relative weight to give observable indicators only depends on their usefulness as an input in conditional inflation projections. Any uncertainty in the forecasts is also factored in the policy process, as the optimal reaction to an indicator variable also inversely depends on the indicator’s precision. Nominal variables do not play a particularly distinct role in a policymaker’s reaction function, unlike real variables, as both are considered only for their information content on the state of the economy. These conclusions have practical implications: the role of money in some central banks’ strategy was criticized, arguing that the relative weight to place on money should depend exclusively on “how useful current money growth is as an input in conditional forecasts of inflation” (Svensson and Woodford, 2003a).

These considerations seem hard to reconcile with some central bankers’ opinions. Doubts on the adoption of a strict forecast targeting strategy were expressed, on the grounds of the limited reliability and precision of projections. As an example, at the time of choosing the policy strategy for the European central bank, the possibility of “projections of inflation playing an important role in guiding policy decisions” (European Monetary Institute, 1997)

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2 See in particular Svensson and Woodford (2003a).

3 Among others, see Svensson and Woodford (2002) and Svensson (2001).

4 This feature is common to the new generation of macroeconomic “forward-looking” models. Woodford (1999) stresses that to solve the problem of price-level indeterminacy in a forward-looking model no feedback to policy from any nominal variable is necessary.
was considered. However, it was decided that a stable relationship between various economic and financial indicators and future inflation could not be taken for granted at that moment for the euro area as a whole. Later, when the Eurosystem’s output and inflation projections were first published, it was stressed that they “play a useful but limited role in the strategy” due to the unavoidable limitations of macroeconomic projections, such as their strong dependence upon highly uncertain external assumptions (ECB, 2000).  

A casual reference to recent history could also raise some puzzling issues. At the beginning of the 1980s, in a number of successful episodes of disinflation, monetary policy was based on relatively simple rules of conduct, rather than on sophisticated signal extraction. In contrast, in the 1970s a delayed or muted reaction to inflation was sometimes justified by a careful – and on the face of it, rational – assessment of the information which could help to disentangle the possible causes of observed inflation. A frequent line of argument was that, as the appropriate policy reaction was dependent on the importance of the causes of inflation (wages, oil prices, demand, profits, temporary factors) and as observed price changes only gave imprecise information on these causes, a moderate reaction was deemed advisable in demand management.

Can excessive concern for signal extraction be counter-productive when facing imprecise indicators? In this paper, I address the issue by studying how the implementability of a policy rule obtained under optimal filtering (real determinacy of the R. E. equilibrium) in a forward-looking model depends on the precision of the indicators, an issue that was raised, but not thoroughly examined, by Aoki (2003). It is well known that some rules of policy behavior may give rise to the possibility of self-fulfilling bursts of inflation and output even in the absence of shocks to fundamentals (Woodford, 1999). This is a result which should be regarded as extremely undesirable by every central banker, as Carlstrom and Fuerst (2000) effectively point out. Clarida, Gali and Gertler (1998) study the indeterminacy of equilibrium stemming from the US policy rule in the 1970s, and consider it

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5 Similar, if more extreme, arguments were also put forward concerning developing countries. Masson, Savastano and Sharma (1997) argue that the conditions for an inflation targeting framework are difficult to satisfy where “the lack of a coherent analytical framework for … forecasting inflation” tends to “impair both the central banks’ capacity to formulate monetary policy and the external observability to assess monetary developments”.

6 Some examples of the Italian policy debate are collected in Balloni (1974).
a case of policy-induced macroeconomic instability. The issue of implementability was extensively addressed and discussed for the models under complete information (Woodford, 1999, Clarida, Gali and Gertler, 1998); the case under partial information was only touched upon by Aoki (2003) for a particular instance, but not addressed further. My aim is to examine it more extensively; I show that, unlike the case studied by Aoki, in a number of circumstances the cautious policy behavior induced by imprecise information may lead to real indeterminacy, unless some additional rule of behavior is adopted.

My strategy is the following. I derive optimal rules (the optimal response of the policy instrument to the behavior of present and past observable variables) in a standard forward-looking macroeconomic model with partial information, following the filtering procedure proposed by Svensson and Woodford (2003a), under different assumption on the parameters, on the information set and on the noise in the indicators. Then, I assume the policy-maker follows this rule and I study the dynamics of the model to see whether the equilibrium is determinate or whether self-fulfilling expectation shocks arise.

A first result is that the implementability of the optimal policy depends on the precision of the indicators of endogenous variables. In a simple model, for a central bank targeting inflation projections the determinacy problem gets worse, the less precise are the available indicators, because the ‘optimal’ reaction to observed endogenous variables gets smaller. If the indicators of inflation are too noisy, sunspots arise.

In this case, considering the set of simple rules for which the equilibrium is determinate, it turns out that in order not to introduce too much noise in the economy they will typically rely somewhat more on a simple rule assigning relatively more weight to observed inflation, and less to the observed output gap, than would be implied by the solution derived from filtering. Intuitively, the policy-maker will want to depart from an information filtering exercise and look for simpler nominal anchors.

A second result is that the availability of precise indicators of exogenous state variables, such as potential output, increases the likelihood of indeterminacy, in contrast with the previous case. Intuitively, the policy-maker endowed with good information on the exogenous state will be induced to rely relatively more on it and less on the observations of
endogenous variables like inflation and output; however, too little a response to endogenous variables rules out determinacy.

The possibility that imprecise indicators lead to indeterminacy increases when the economy is hit by both cost-push shocks and demand shocks, which complicates the signal-extraction problem. More interestingly, indeterminacy is less likely to happen, *ceteris paribus*, when the policy-maker’s preferences assign a larger weight to inflation; this result suggests that there may be an advantage in appointing a “conservative” central banker even when a commitment technology is available.

The paper is organized as follows. In section 2, some of the relevant contributions in the literature are summarized. In section 3, assumptions on the available information are added to a standard New Keynesian model and the question is asked whether, in such a model, a rule obtained from the optimal filtering of information is sufficient to guarantee that the rational expectation equilibrium is determinate. The answer is shown to depend on the precision of the available indicators. In section 4, the features of a behavior that ensures determinacy in such a context are studied. In section 5, a general case is addressed by numerical simulation and the effects of changing some parameters are studied. Section 6 summarises the conclusions.

2. Policy rules, determinacy and partial information: related literature

Rudebusch and Svensson (1999) and Svensson (2001a) argue that the decision-making process of central banks is properly modeled as an optimal targeting rule (a commitment to a final target, pursued by making extensive use of all information, like inflation-forecast targeting)\(^7\), on the ground that central banks always consider all available information and do not restrict themselves to behaving mechanically. In contrast, Clarida, Gali and Gertler (1999) and Gali (2000), among others, study simple feedback instrument rules for the

\(^7\) Svensson (2001a) distinguishes a general targeting rule, which takes the form of minimization of an objective function for monetary policy in terms of the final goals, and a specific targeting rule, which takes the form of minimization of a loss function penalizing deviations of projections from the target, which in turn determines the value of the policy instrument. In contrast, an instrument rule directly links the policy instrument to the behavior of some predetermined and observable variables; a forecast-based instrument rule links the interest to the value of the inflation forecasts (the latter is proposed by Batini and Haldane, 1999 and
interest rate, based on the consideration that it is unrealistic to assume that the policy-maker has the information needed to implement optimal policies.\footnote{Rotemberg and Woodford (1999) and Levin et al. (1999) compared empirically the relative performance of simple vs. optimal rules.}

The implications of different policy rules – either optimal or simple ones – for the real determinacy of the rational expectations equilibrium in forward-looking macro models have been extensively addressed.\footnote{The literature on determinacy is extensive and cannot be satisfactorily summarized here. Among others, Carlstrom and Fuerst (2001) show, by means of a discrete-time, money-in-the-utility-function model, that minor differences in trading environment result in large differences in the policy restrictions needed to ensure real determinacy. Christiano and Rostagno (2001) review ways in which a monetary policy characterized by a Taylor rule can induce volatility in the economy, due to the possible indeterminacy of equilibrium, and show a case in which monitoring ranges for the money supply can reduce the problem. Bullard and Mitra (2000) study the stability of rational expectation equilibria under learning dynamics, and provide conditions for unique equilibria. On the implications of learning, see Evans and Honkapohja (2001).} Woodford (1999, 2000b) shows that any targeting strategy that is exclusively forward-looking (or, equivalently, any interest rule that does not include a sufficient reaction to current or past endogenous variables) would not yield a unique equilibrium and would be prone to sunspots. When the nominal rate does not increase sufficiently in response to a rise in inflation, self-fulfilling bursts of inflation and output are possible even without shocks to fundamentals. Woodford (2000a) defines as ‘Taylor’s principle’ the condition that interest rates should increase more than the inflation rate (i. e., real rates must rise with inflation), which is usually sufficient for real determinacy. Clarida, Gali and Gertler (1998 and 1999) also study the consistency of various simple policy rules with the possibility of persistent, self-fulfilling fluctuations in inflation and output. They argue that the observed failure of Taylor’s principle in the US in the 1970s is a case of macroeconomic instability induced by policy.

Gali (2000) and Woodford (2000a) show that an optimal rule obtained as a solution to the monetary policy problem under full information does not necessarily meet the determinacy condition, in which case the optimal allocation is not implementable: i. e., if the rule is followed mechanically, the optimal allocation would not necessarily be attained, due to the possibility of sunspots. They show how appropriate modifications to the rule’s parameters (namely, an increase in the reaction to inflation) make it consistent with Taylor’s principle; under full information, such a modified rule implements the optimal allocation.
In contrast, the implications of partial information for real determinacy have been less extensively addressed in the economic literature. The main exception is Aoki (2003), who shows that the presence of measurement errors in current output and inflation induces a degree of policy cautiousness;\(^\text{10}\) he argues that an indeterminacy problem may then arise when indicators are very noisy, but he rules out that possibility for his model. The issue of determinacy has not been extensively addressed within the general framework introduced by Svensson and Woodford (2003a), which is now the standard tool in the analysis of optimal policy under partial information. This contribution shows how to solve an information variable problem in a model with forward-looking variables. With symmetric information of the public and the central bank,\(^\text{11}\) rational expectations and a quadratic loss function, certainty equivalence and the separation principle apply: optimal policy can first be expressed as a function of the estimated state of the economy, with the same formulation as if the state were observed; the link between the state variables and the observable indicators then depends on the signal-to-noise ratio. As an implication, the relative weight to place on an indicator only depends on how useful it is in forecasting the target variable.

The contribution of this paper is a study of the relation between the noise in the indicators (of either endogenous or exogenous variables) and indeterminacy, in a forward-looking model solved \textit{à la} Svensson-Woodford (2003a).

3. Noisy indicators and self-fulfilling expectations in a simple model

The benchmark model, widely used in the current macroeconomic literature (e. g., see Woodford, 2000 and Gali, 2000), consists of a New Phillips curve and a forward-looking demand curve.

\begin{align*}
\pi_t &= \beta_{\pi} \pi_{t+1} + k (y_t - \bar{y}_t) + \epsilon_t^C \\
y_t &= \gamma_{\pi} \gamma_{y} - \sigma^{-1} (\gamma_{i} - \gamma_{\pi}) + \epsilon_t^D
\end{align*}

\(^\text{10}\) Orphanides (2003a) also stresses that noisy information implies cautious policy, which he considers a desirable outcome as it avoids excessive activism.

\(^\text{11}\) Svensson and Woodford (2003b) extend the analysis to a case of asymmetric information.
\[
\bar{y}_t = \bar{y}_{t-1} + \varepsilon_{t}^{YBAR}
\]

‘Potential output’ shocks are i.i.d. with zero mean and variance \(\sigma_{YBAR}^2\). ‘Aggregate demand’ shocks \((\varepsilon^D)\) and ‘cost-push’ shocks \((\varepsilon^C)\) have an autoregressive structure:

\[
\varepsilon^D_t = \rho^D \varepsilon^D_{t-1} + \tilde{\varepsilon}^D_t
\]

\[
\varepsilon^C_t = \rho^C \varepsilon^C_{t-1} + \tilde{\varepsilon}^C_t
\]

with \(\rho^D, \rho^C\) non negative and less than 1; innovations are i. i. d. with zero mean and variances \(\sigma^2_D\) and \(\sigma^2_C\).

The monetary authority minimizes the present expected value of a quadratic loss.

\[
\min \sum_{i=0}^{\infty} \beta^i L_{t+i|t}
\]

\[
L_t = (\pi_t^2 + \lambda(y_t - \bar{y}_t)^2)
\]

where \(L_{t+i|t}\) stands for the expected value of \(L_{t+i}\) given information in \(t\). In this section, to get a closed form solution, I introduce simplifying assumptions, i. e. \(\sigma_{YBAR}^2 = \sigma_C^2 = \rho^D = \rho^C = \lambda = 0\), which will be removed later. That is, I rule out cost-push and potential output shocks, as well as persistence in demand shocks. The assumption on \(\lambda\) is equivalent to assuming the unique objective of price stability.\(^{12}\)

\[
\pi_t = 0
\]

Substituting (8) into (1) and (2) and taking expectations, the optimal solution under full information can be written in terms of the underlying shock\(^{13}\).

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\(^{12}\) In absence of ‘cost-push’ shocks, this reproduces the frictionless equilibrium, as shown by Gali 2000. By this assumption, I rule out time-consistency problems for the moment.

\(^{13}\) Here, \(\tilde{\pi}_t = \sigma(\bar{y}_{t+1} - \bar{y}_t)\) is defined as the observable component of the ‘natural’ interest rate. Under the simplifying assumptions above, it would be equal to 0; however, the symbol is maintained through this section for expositional convenience.
Woodford (2000a) and Gali (2000) show that, under (9) one cannot rule out the existence of multiple paths and self-fullfilling fluctuations in expectations. Woodford (1999) shows that in a model of this kind any policy rule which is only a function of exogenous state variables, such as (9), implies that the rational expectations equilibrium is indeterminate. The standard way to study determinacy is by adding to the model (1)-(3) a reaction function which adds the output gap and inflation to (9).

\[ i_t - \bar{i}_t = \sigma e_t^D + \Phi_y (y_t - \bar{y}_t) + \Phi \pi_t \]  

In equilibrium, the two reaction functions (9) and (10) are equivalent, since \( y_t = y_t \), \( \pi_t = 0 \), but the dynamics are different. Substituting (10) into (1) and (2), one obtains the dynamic system:

\[ \begin{pmatrix} \pi_{t+1} \\ y_{t+1} - \bar{y}_{t+1} \end{pmatrix} = (\sigma + k \Phi_p + \Phi \gamma) \left[ \begin{pmatrix} \beta (\sigma + \Phi \gamma) + k & k \sigma \\ 1 - \beta \Phi_p & \sigma \end{pmatrix} \right]^{-1} \begin{pmatrix} \pi_t \\ y_t - \bar{y}_t \end{pmatrix} \]

The necessary and sufficient condition for (11) to have a unique solution is that both roots of the characteristic equation of the system (11) lie outside the unit circle. This amounts to:\[ k (\Phi_p - 1) + (1 - \beta) \Phi \gamma > 0 \]

A large enough reaction of the interest rate to output and inflation is needed to anchor the expectations. A sufficient condition is that the nominal interest rates react more than one for one to inflation (\( \phi_p > 1 \)), so that the real interest rate rises if inflation rises, defined by Woodford (2000a) as “Taylor’s principle”. As Gali (2000) points out, under certainty, any choice of a ‘large’ coefficient on inflation in the reaction function implements the optimal policy, as, in equilibrium, inflation and the output gap are zero, and \textit{ex post} (10) corresponds to (9). The reaction coefficients in (10) are merely a virtual threat to increase the interest rate should inflation deviate from equilibrium, which does not actually need to be exercised.

---

\(^{14}\) A complete derivation is in Bullard and Mitra (2001)
The same, however, is not true if partial information is introduced. I now assume that both the private sector and the central bank have the same partial information, as in Svensson and Woodford (2003a). I assume that the interest rate $i_t$ is set while simultaneously observing (and reacting to) variables at time $t$. The shock $\varepsilon^D_t$ is not observable. For analytical convenience in this example, the equilibrium level of output $\bar{y}_t$ is also assumed to be known. In contrast, current output $y_t$ is only observable with noise:

\begin{equation}
\hat{y}_t = y_t + \varepsilon^Y_t
\end{equation}

Inflation $\pi_t$ is also observable with noise:

\begin{equation}
\hat{\pi}_t = \pi_t + \varepsilon^\pi_t
\end{equation}

The measurement error of output and the measurement error of prices ($\sigma^2_Y, \sigma^2_\pi$) are assumed to be known. Since certainty equivalence holds, the objective of the monetary authority amounts to the attainment of a zero inflation projection in each period:

\begin{equation}
\pi_{t|t} = 0
\end{equation}

which in turn implies the interest rate path:

\begin{equation}
i_t - \bar{i}_t = \sigma^D_{i|t}
\end{equation}

where $\pi_{t|t}$ and $\varepsilon^D_{t|t}$ are the optimal estimates of inflation and the demand shock, given the available information in $t$.

The optimal estimate of the demand shock is a projection on the observables $\hat{y}_t, \bar{y}_t, \hat{\pi}_t$. The coefficients of this conditional projection depend on the variances of the unobserved disturbances.\footnote{A derivation is in Appendix A.}

\begin{equation}
\varepsilon^D_{t|t} = \frac{\sigma^2_Y}{\sigma^2_Y} (\hat{y}_t - \bar{y}_t) + \frac{\sigma^2_\pi}{\sigma^2_\pi} (k \hat{\pi}_t)
\end{equation}
The solution for the interest rate is then obtained by substituting (17) into (16):

\[
(18) \quad i_t = \tilde{i}_t + \vartheta^*_i (\hat{y}_t - \bar{y}_t) + \vartheta^*_p \hat{\pi}_t
\]

with

\[
(19) \quad \vartheta^*_i = \sigma \frac{\sigma^2_D}{\sigma^2_{\hat{y}}} ; \quad \vartheta^*_p = \sigma \frac{\sigma^2_D}{\sigma^2_p} k
\]

In (18) the interest rate optimally reacts to the observed output gap and headline inflation. As implied by certainty equivalence, the coefficient on the estimated demand shock \( \varepsilon^D_{\Pi t} \) in (16) does not depend on the precision of the signals, while the coefficients linking this estimated value to the observables \( (\hat{\pi}, \hat{y}) \) in (17) do. As a consequence, the reaction to the observed output gap and to observed headline inflation in (19) also depends on the variances of the demand shock, the measurement error in the gap and the erratic component of inflation. The relative weight of inflation \textit{vis-à-vis} the output gap in the reaction function is proportional to its role as an input to forecast the state variable. If the signal-to-noise ratio is large, a large reaction to output and inflation is warranted. In contrast, if the signal-to-noise ratio is small, the central bank, as in Aoki (2003), will not move much in reaction to current signals.

Unlike under full information, the solution (18) already embodies a feedback from endogenous variables, which is a necessary condition for determinacy.\(^\text{16}\) To see whether the sufficient condition for determinacy is also met, I rewrite the deterministic part of the system including the reaction function (18) and check the eigenvalue condition. It is immediately evident that this amounts to substituting the coefficients obtained from (19) in place of \( \Phi_P, \Phi_Y \) into (12). The resulting expression states that the equilibrium is determinate if and only if:

\[
(20) \quad \left[ \frac{(1 - \beta)}{k} \right] \frac{1}{\sigma_{\hat{y}}} + \frac{1}{\sigma_p} > \frac{\sigma^{-1}}{\sigma_D}
\]

\(^{16}\) As mentioned, Woodford (1999) shows that a policy rule as (9) results in indeterminacy, because it is only a function of exogenous states. However, this is not necessarily true of a rule such as (16), as \textit{expected} states are partly endogenous, because their estimate depends on endogenous variables.
PRECISION OF THE INDICATORS AND DETERMINACY OF EQUILIBRIUM

Condition (20) shows that, under partial information, the determinacy of equilibrium depends on the quality of the information available to the policy-maker; determinacy fails if the quality is not good enough. The expression is a function of the precisions of the indicators (times the variance of the demand shock: \( \sigma_D^2 / \sigma_Y^2, \sigma_D^2 / \sigma_P^2 \)) and of the parameters of the model; it is not met if the indicators available to the central bank are very noisy (\( \sigma_Y^2, \sigma_P^2 \), are large).

Fig. 1 shows the region of determinacy of equilibrium as a function of the signal-to-noise ratios, in the case \( \sigma=1 \) and \( k=0.3 \). When the signal-to-noise ratio is below the threshold, uniqueness of equilibria is not guaranteed and self-fulfilling shocks to expectations may arise.
How restrictive this condition is depends on the parameters of the model. Some examples are in Tab. 1. For various values of $\sigma^2_D / \sigma^2_Y$ and of the model’s parameters,\(^{17}\) the variance in the noisy component of inflation must be between 3 or 250 times smaller than the variance of the aggregate demand shock. The size of this interval suggests that there is at least a range of parameters for which the problem is relevant\(^ {18}\), contrary to the model by Aoki (2003).

<table>
<thead>
<tr>
<th>$\sigma$</th>
<th>$k$</th>
<th>$\beta$</th>
<th>$\sigma^2_D / \sigma^2_Y$</th>
<th>$\sigma^2_D / \sigma^2_P$</th>
</tr>
</thead>
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<tr>
<td>1</td>
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<td>0.99</td>
<td>0.1</td>
<td>3.3</td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
<td>0.99</td>
<td>0.5</td>
<td>3.3</td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
<td>0.99</td>
<td>1.0</td>
<td>3.2</td>
</tr>
<tr>
<td>1</td>
<td>0.3</td>
<td>0.99</td>
<td>2.0</td>
<td>3.1</td>
</tr>
<tr>
<td>1</td>
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<td>0.99</td>
<td>0.1</td>
<td>19.6</td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.99</td>
<td>0.5</td>
<td>18.0</td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.99</td>
<td>1.0</td>
<td>16.0</td>
</tr>
<tr>
<td>1</td>
<td>0.05</td>
<td>0.99</td>
<td>2.0</td>
<td>12.0</td>
</tr>
<tr>
<td>0.157</td>
<td>0.024</td>
<td>0.99</td>
<td>0.1</td>
<td>263.7</td>
</tr>
<tr>
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<td>0.024</td>
<td>0.99</td>
<td>0.5</td>
<td>256.7</td>
</tr>
<tr>
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<td>0.99</td>
<td>1.0</td>
<td>248.0</td>
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<tr>
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<td>0.024</td>
<td>0.99</td>
<td>2.0</td>
<td>230.7</td>
</tr>
</tbody>
</table>

While imprecise signals on endogenous variables imply over-cautious behavior, the opposite applies when direct indicators of the exogenous shock $\epsilon^{D}_{t|t}$ are available: more precise direct information on the exogenous shock results in less reaction to inflation and output and in real indeterminacy. This may be shown by assuming that the policy-maker also observes a direct, but noisy, signal of the demand shock:

\[
\hat{S} = \epsilon^{D}_{t} + \epsilon^{S}_{t}
\]

\(^{17}\) Parameters are taken from Bullard and Mitra, 2001 and Clarida et al 1999.

\(^{18}\) His result hinges on simplifying assumptions which guarantee that the optimal policy has always a sufficient feedback from past output or inflation. The previous period’s output and inflation are assumed to be
where $\varepsilon_t$ is i.i.d. with zero mean and variance $\sigma^2_S$. In this case, it may be shown that the expression for the optimal predictor of the demand shock becomes:

$$e_t^D = \frac{\sigma_D^2}{\sigma_D^2 + \sigma_D^2} (\hat{\pi}_t - \bar{\pi}_t) + \frac{\sigma_D^2}{\sigma_D^2 + \sigma_D^2} (k\hat{\pi}_t) + \frac{\sigma_D^2}{\sigma_D^2 + \sigma_D^2} \hat{S}_t$$

Substituting into (16), one obtains the interest rate rule:

$$i_t = \hat{i}_t + \frac{\sigma_D^2}{\sigma_D^2 + \sigma_D^2} \sigma(\hat{\pi}_t - \bar{\pi}_t) + \frac{\sigma_D^2}{\sigma_D^2 + \sigma_D^2} \sigma(k\hat{\pi}_t) + \frac{\sigma_D^2}{\sigma_D^2 + \sigma_D^2} \alpha\hat{S}_t$$

As $\sigma^2_S$ tends to zero, the coefficients on the observed output gap and inflation also tend to zero and the interest rate only reacts to the exogenous variable $\hat{S}_t$, which violates (12). This is a consequence of the general principle that a policy rule which does not include a feedback from the endogenous variables, but only reacts to exogenous states, leads to indeterminacy. In this context, the policy-maker would not pay much attention to the behavior of endogenous variables if he already had good information on the underlying state of the economy. This result has some interesting implications for the role that should be assigned to “precise” direct measures of potential in the conduct of monetary policy; these implications are discussed in the extended version of the model in section 5 below.

4. Policy implementation when information is imprecise: the need for nominal anchors

What if the determinacy condition is not met? The case of partial information has one fundamental difference with respect to the full information case. Due to the noise component, observed inflation and output ($\hat{\pi}_t, \hat{y}_t$) are not zero in equilibrium; a threat to react to these variables if they deviate from target values has to be exercised, in which case the central bank would also be reacting to noise, not only to changes in fundamentals.\footnote{This is also argued by Woodford (2000a): “complete reliance upon the threat of extreme responses of inflation and output gap variations is not obviously the most desirable approach” to guarantee a determinate equilibrium, as “the random noise in the particular measure to which the central bank responds might require violent adjustments of interest rates, that in turn create havoc in the economy”. Similar considerations are expressed by Galì (2000).} Hence, observed with absolute precision, giving information on the (unobserved) natural interest rate.
implementability comes at a cost, as it adds volatility to output and inflation. As a consequence, not all choices of large $\vartheta_Y, \vartheta_P$ are equally efficient. In general, once noise is taken into account, it will no longer be optimal to respond to the indicators in proportion to their information content for the state variables.

An intuition can be obtained by examining a policy rule whose coefficients are large enough to satisfy the determinacy constraint but at the same time small enough to minimize the noise which is introduced in the economy by reacting to imprecise indicators. Under the simplifying assumptions adopted in the previous section, such a rule is found by assuming a generic reaction function in terms of the observable variables $(\hat{y}_t - \bar{y}_t), \hat{\pi}_t$ and finding the value of the coefficients in that function that minimize the period loss function, subject to an inequality constraint. The reaction function is assumed to be as:

\[(24) \quad i_t = \bar{i}_t + \vartheta_Y (\hat{y}_t - \bar{y}_t) + \vartheta_P \hat{\pi}_t,\]

and the coefficients are constrained to satisfy the condition:

\[(25) \quad k(\vartheta_P - 1) + (1 - \beta)\vartheta_Y \geq \zeta,\]

where $\zeta$ is a small positive number, arbitrarily close to zero. The model augmented by (24) admits the solution where $\pi_{t+i|t} = 0$, $y_{t+i|t} = \bar{y}_{t+i}$. Deriving the reduced form for $\pi_t$ (as a function of $\varepsilon^D_t, \varepsilon^P_t, \varepsilon^Y_t$) and substituting in the period loss function, one obtains:

\[(26) \quad E(\pi_t)^2 = \frac{k^2}{(\sigma + \vartheta_Y + k\vartheta_P)^2}(\sigma^2 \sigma_D^2 + \vartheta_Y^2 \sigma_Y^2 + \vartheta_P^2 \sigma_P^2)\]

Minimizing (26) subject to the inequality constraint (25), the first-order conditions (see Appendix B) are:

\[(27) \quad \begin{cases} \vartheta_Y = \vartheta^*_Y \\ \vartheta_P = \vartheta^*_P \end{cases} \quad \text{if } h \equiv \frac{\sigma^{-1}}{\sigma_D^2} (1 + \frac{\zeta}{k}) - \frac{1}{\sigma_P^2 k} - \frac{1}{\sigma_Y^2 k} \geq 0\]

\[(28) \quad \begin{cases} \vartheta_Y = \vartheta^*_Y [1 + h\Delta(1 - \beta) + k + \zeta] \\ \vartheta_P = \vartheta^*_P [1 + h\Delta(\sigma + k + \zeta)] \end{cases} \quad \text{if } h \equiv \frac{\sigma^{-1}}{\sigma_D^2} (1 + \frac{\zeta}{k}) - \frac{1}{\sigma_P^2 k} - \frac{1}{\sigma_Y^2 k} < 0\]

where $\vartheta^*_Y, \vartheta^*_P$ are the same as in (19) and $\Delta > 0$. 
If the variance of the measurement errors is small enough, the constraint is not binding \( (h \leq 0) \) and the solution (27) applies, which coincides with the one previously obtained. If the constraint is binding as in (28) \( (h \geq 0) \), both coefficients must be larger than in the unconstrained solution. Moreover, since \( 0 < \beta < 1 \), the proportional increase in the reaction coefficient on headline inflation \( (\vartheta_P) \) is always larger than the proportional increase in the coefficient on the output gap \( (\vartheta_Y) \). The behavior of the ratio \( \vartheta_P / \vartheta_Y \), as a function of \( h \), is shown in Fig. 2. The larger \( h \) is, the larger is the relative weight to be placed on the inflation term.

Fig. 2

RELATIVE REACTION TO INFLATION AND TO THE OUTPUT GAP

Hence, imprecise indicators call for a relatively larger reaction to inflation than to the gap, all other things being equal. While a large enough reaction to the output gap may in principle deliver price level determinacy (as argued by Woodford, 1999), using the gap as an anchor would be quite undesirable. Since the coefficient associated with \( \vartheta_Y \) in the constraint (25) is small, a large reaction would be needed, which would result in a large noise being transmitted through policy.
Equations (27) and (28) show that, in order to rule out self-fulfilling bursts of inflation, the policy-maker should combine the efficient use of all available information (implicit in the derivation of \( \varphi^*_Y, \varphi^*_P \)) with simpler rules of conduct based on nominal variables, such as the constraint that real rates will in any case not decrease in response to observed inflation (Taylor’s principle). This may be interpreted as an insurance against the risk of being too cautious when the information is imprecise. From this perspective, the fact that simple rules of behavior played a crucial role in the disinflation at the end of the 1970s turns out not to be inconsistent with the claim that central banks try to make efficient use of all available information, as argued by Rudebusch and Svensson (1999).

5. A more general case

In this section, I relax the simplifying assumptions made above. I allow for cost-push shocks, shocks to potential output, a policy-maker also caring about output \((\lambda \neq 0)\), autocorrelation in the cost-push and demand shocks and shocks to potential output. I assume a direct, but noisy, observation of potential, adding the equation \( \hat{Y}_t = \bar{Y}_t + \tilde{\varepsilon}^T_{t+1} \). I solve the model numerically for both discretion and commitment.

The whole model may be written in the format of Svensson and Woodford (2003a) as:

\[
\begin{bmatrix}
\bar{Y}_{t+1} \\
\varepsilon^C_{t+1} \\
\varepsilon^D_{t+1} \\
Y_{t+1} \\
\pi_{t+1}
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & \rho^C & 0 & 0 & 0 \\
0 & 0 & \rho^D & 0 & 0 \\
-k(\beta\sigma)^{-1} & (\beta\sigma)^{-1} & -1 & k(\beta\sigma)^{-1} + 1 & -(\beta\sigma)^{-1} \\
-k(\beta)^{-1} & -\beta^{-1} & 0 & -k(\beta)^{-1} & \beta^{-1}
\end{bmatrix}
\begin{bmatrix}
\bar{Y}_t \\
\varepsilon^C_t \\
\varepsilon^D_t \\
Y_t \\
\pi_t
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0 \\
\sigma^{-1} \\
0
\end{bmatrix} \cdot \begin{bmatrix}
i_t \\
0 \\
0
\end{bmatrix}
\]

Similarly, the period loss function may be written as:

\[
L_i = Y_t W Y_t; \quad Y_t = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 \\
-1 & 0 & 0 & 0 & 1
\end{bmatrix} \begin{bmatrix}
\bar{Y}_t \\
\varepsilon^C_t \\
\varepsilon^D_t \\
Y_t \\
\pi_t
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
0
\end{bmatrix} \cdot \begin{bmatrix}
i_t \\
0 \\
0
\end{bmatrix}; \quad W = \begin{bmatrix}
1 & 0 \\
0 & \lambda
\end{bmatrix}
\]
and the measurement equation as:

\[
\begin{bmatrix}
\tilde{\gamma}_t \\
\tilde{\delta}_t \\
\hat{\pi}_t
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\gamma'_t \\
\delta'_t \\
\pi'_t
\end{bmatrix} +
\begin{bmatrix}
\gamma'_{\text{BAR}}_t \\
\delta'_{\text{BAR}}_t \\
\pi'_{\text{BAR}}_t
\end{bmatrix}
\]

I define the state vector \( X_t = (\tilde{\gamma}_t, \tilde{\delta}_t, \hat{\pi}_t)^T \) and the vector of indicators \( Z_t = (\tilde{\gamma}_t, \tilde{\delta}_t, \hat{\pi}_t)^T \). By applying the Kalman filter, as in Svensson and Woodford (2003a), one obtains the recursive expression for the expected states in terms of the observables.

\[
X_{t|t} = X_{t|t-1} + U[Z_t - Z_{t|t-1}]
\]

The expression for the matrix \( U \), which depends on \( A_1, B, C_1, C_i, W, D_i \), and the covariance matrices of \( u_t \) and \( \nu_t \), is derived in Svensson and Woodford (2003a). The optimal reaction function can be written in terms of the expected states:

\[
i_t = Y_1 i_{t-1} + Y_2 X_{t|t} + Y_3 X_{t-1|t-1}
\]

where the expression for the matrices \( Y_1, Y_2, Y_3 \) is reported in Gerali and Lippi (2003). They show that, under commitment, the reaction function (33) is dependent on the past history of expected states, while under discretion it is \( Y_1 = Y_2 = 0 \). By recursive substitution, using the transition equation (29) and the measurement equation (31), \( X_{t|t-1} \) and \( Z_{t|t-1} \) in (32) can be expressed as a distributed lag of \( X_{t-1|t-1} \). Consequently, \( X_{t|t} \) can then be written as a distributed lag of the indicators \( Z_t \):

\[
X_{t|t} = E^{-1}(L)Z_t
\]

where \( E(L) \) is a matrix polynomial in the lag operator. \( X_{t|t} \) can then be substituted in the reaction function.

I obtain the optimal policy as a function of the expected states \( (\tilde{\gamma}_{t|t}, \tilde{\delta}_{t|t}, \hat{\pi}_{t|t}) \) implementing the Kalman filtering approach by Svensson and Woodford (2003a) with the Mat-lab package by Gerali and Lippi (2003). For a given linear forward-looking model and a set of assumptions on the information set, the package computes and simulates the optimal
policy (33) as a function of the expected states, both under discretion and commitment. I then rewrite the policy rule as an optimal instrument rule, substituting the expected states with an appropriate number of lags of the observable indicators \((\pi_t, \hat{y}_t, \hat{y}_t)\) and of the interest rate itself.\(^{20}\) I then consider the dynamics of the whole model, computing the eigenvalues of its deterministic part. To have determinacy, two eigenvalues must lie outside the unit circle.\(^{21}\)

The charts show the second largest eigenvalue as a function of the noise in each signal (respectively, the measure of output, inflation and potential output) and as a function of the preferences of the central banker (the weight of output in the loss function). The result is reported both under discretion (continuous line) and under commitment (dotted line).\(^{22}\)

An increase in the noise in the inflation observation, or in the noise in the output observation (Fig. 3 and 4), causes the second largest eigenvalue to decrease, which confirms the finding of the simple model of the previous sections. When the observation of output is less precise, it is more difficult to interpret the movements in observed inflation and to attribute them to the underlying cost and demand shock. As a consequence, the reaction to inflation is more muted and could be insufficient to ensure determinacy. This holds both under discretion and under commitment. To be sure, commitment implies more history-dependence, which favors determinacy; in each of the above charts the dotted line (obtained under commitment) is above the continuous line (obtained under discretion). However, even under commitment, the second largest eigenvalue is not necessarily outside the unit circle.

---

\(^{20}\) The policy rule is obtained by regressing the simulated policy instrument from the Gerali and Lippi package on the set of simulated current and lagged observables and on its own lags. By construction, the projection holds exactly, provided the lag length is appropriate.

\(^{21}\) The eigenvalues are computed by writing the dynamic system in matrix form, considering the vector of the two forward-looking variables (output and inflation) and the predetermined variables which enter the policy rule (i.e., lagged output, inflation and interest rate, with the lag depending on the form of the rule).

\(^{22}\) All charts refer to a benchmark specification with \(\lambda=0.25\), a unit standard deviation of the demand shock, the cost-push shock and the noise in observed output, and a standard deviation of observed inflation equal to 0.25.
NOISE IN THE INFLATION OBSERVATION AND DETERMINACY OF EQUILIBRIUM

Fig. 3

NOISE IN THE OUTPUT OBSERVATION AND DETERMINACY OF EQUILIBRIUM

Fig. 4
CONSERVATIVENESS OF THE CENTRAL BANKER AND DETERMINACY OF EQUILIBRIUM

Fig. 5

NOISE IN THE POTENTIAL OUTPUT OBSERVATION AND DETERMINACY OF EQUILIBRIUM

Fig. 6
The second largest eigenvalue is also negatively related to the weight of output in the objective function (Fig. 5): a conservative central banker is less likely to run into problems of sunspots due to imprecise signals. The intuition is the following. When the weight of output ($\lambda$) is larger, the optimal reaction to observed inflation depends on the identification of the source of the underlying shocks (cost-push or demand shocks). Therefore, with imprecise information, the response to inflation is more muted. This is less true for a conservative central banker (small $\lambda$), who, ceteris paribus, would react more to observed inflation, no matter whether it is demand-driven or cost-determined. The result also holds under commitment. An important conclusion is that, in this framework, some degree of conservativeness may be beneficial even when a commitment technology is available; conservativeness induces the policy-maker to be less hesitant in reacting to inflation in the face of uncertainty.

Fig. 6 shows that the less precise is the direct measure of potential, the larger is the second eigenvalue. In this case a precise direct observation of an exogenous variable, potential output, induces less reaction to observed inflation; symmetrically, more noise in the measure of potential would favor a more reactive policy. In the latter case observed inflation provides information on the - poorly observed - output gap and calls for adjusting policy aggressively. Too much direct information on potential “hurts”, as the policy-maker relies on it and reacts too little to observed inflation.

Interestingly, the result somewhat recalls the argument of Orphanides (2003b), that an excessive reliance by the Fed on a potential output measure prompted the accommodation of inflation in the 1970s. However, the framework is quite different. The argument of Orphanides centers on the claim that at the time potential was overestimated by the Fed, which in turn produced a systematic bias in policy (a systematically too low level of the policy rate). In contrast, the model in this section allows for no systematic bias in the potential output measure; the problem does not lie in a wrong level of the interest rate, but in an insufficient threat to react to possible future acceleration of prices, thus accommodating self-fulfilling inflation expectations. Here, good information on potential (therefore, on the ‘right’ level of interest rates) makes the policy-maker shy in reacting to imprecisely observed inflation.
Tab. 2

**OPTIMAL RULES AND DETERMINACY OF EQUILIBRIUM**

*a) discretion*

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<tr>
<th>$\lambda$</th>
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<th>autocorrelation of innovations:</th>
<th>standard deviation of measurement error:</th>
<th>optimal rule coefficients:</th>
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*b) commitment*

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<td>(7)</td>
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Common parameters: $k=0.05$, $\sigma=1$, $\beta=0.99$. The indicator of potential output also enters the optimal policy rule but it is not reported in the table.

Table 2 provides some numerical examples of the optimal rule obtained under different – and rather arbitrary – assumptions on the parameters, under discretion and commitment. In each panel, rows 1 to 3 show that the introduction of a non-zero standard deviation of the cost-push shock (increased from 0 to 1) and a larger noise in the output indicator (increased from 1.5 to 10) make it more likely that the “Taylor principle” will fail; in the case under discretion, the inflation coefficient in the policy rule decreases from 0.8 to 0.09. Rows 4 and 5 show that more noise in the potential output indicator (increased from 1 to 10), yields a policy that is more persistent and more responsive to observed inflation, which satisfies the determinacy condition (the second eigenvalue moves outside the unit circle). Rows 6 and 7 show that adding persistence in the shocks (the auto-correlation terms are increased to either 0.25 or 0.8) yields a more history-dependent policy and a larger response to inflation; however, in this example, the determinacy condition is satisfied only for high values of the auto-correlation parameter.
6. Conclusions

When agents are forward-looking, policy-makers should react in a sufficiently decisive manner to the behavior of inflation. If the indicators of endogenous variables over the relevant horizon are not very precise, policy-makers move less, as Aoki (2003) and Orphanides (2003a) have shown. This paper shows that it is possible that they move too little: below a threshold of minimal precision of the indicators, the solution of an optimal signal extraction problem does not yield a viable rule of behavior and may induce self-fulfilling fluctuations of inflation and output. It may be argued that such a ‘rational shyness’ in the face of uncertainty prevailed at some point in the 1970s, contributing to Taylor’s principle not being respected and inflation expectations picking up.

The exercise sheds light on some characteristics of a desirable monetary policy in this context. Projections of output and inflation over the relevant horizon cannot be the only criterion for determining monetary policy actions unless they are very precise, nor should the weight assigned to the various indicators necessarily be proportional to their information content for the final targets. Simple rules, which include a reaction to nominal variables, may at times be necessary to supply an anchor for prices, even when the policy-maker still intends to consider all relevant information. Appointing a “conservative” central banker, who will act less timidly in response to signs of inflation or deflation even when the difficulty in interpreting these signs would otherwise call for cautiousness, may be an appropriate choice even without time consistency problems. Relying too much on measures of potential output unrelated to the observed behavior of prices could also induce an attitude that is not responsive enough to inflation or deflation.
Appendix

Appendix A

The model to solve includes equations (1) (with \(\epsilon_t^C \equiv 0\)), (2), (13) and (14). The optimal policy is \(\pi_{t+i|t} = 0\), for \(i \geq 0\). By (1) and (2), this also implies \(y_{t+i|t} = \bar{y}_{t+i}\), for \(i \geq 0\). By substituting into (2), the behavior of the interest rate implied by the optimal policy is obtained as (16).

Information in \(t\) includes:

\[I_t = [\hat{y}_t, \bar{y}_t, \bar{y}_{t+1}, i_t, \bar{i}_t, \bar{\pi}_t, \sigma_D^2, \sigma_y^2, \sigma_p^2, \sigma, \beta, k] \]

The estimator \(\epsilon^D_{t|t}\) is found by projecting \(\epsilon^D_t\) on the vector

\[
Z_t \equiv \begin{pmatrix} (\hat{y}_t - \bar{y}_t) + \frac{1}{\sigma}(i_t - \bar{i}_t) \\ k^{-1}\hat{\pi}_t + \frac{1}{\sigma}(i_t - \bar{i}_t) \end{pmatrix} \equiv \begin{pmatrix} \epsilon^D_t + \epsilon^Y_t \\ \epsilon^D_t - k^{-1}\epsilon^P_t \end{pmatrix}
\]

(the last term is obtained after substituting \(\pi_{t+i|t} = 0\), \(y_{t+i|t} = \bar{y}_{t+1}\)):

\[
\epsilon^D_{t|t} = \left[ Var(Z_t) \right]^{-1} Cov(Z_t, \epsilon^D_t) Z_t = \left( \frac{k^2}{\sigma_y^2} + \frac{1}{\sigma_y^2} + \frac{1}{\sigma_d^2} \right)^{-1} \left( \frac{1}{\sigma_y^2} \right)^T \left( \hat{y}_t - \bar{y}_t \right) + \frac{1}{\sigma} (i_t - \bar{i}_t)
\]

Substituting (16) into (36) and solving, one finds the solution:

\[
\epsilon^D_{t|t} = \frac{\sigma_D^2}{\sigma_y^2} (\hat{y}_t - \bar{y}_t) + \frac{\sigma_D^2}{\sigma_p^2} (k\hat{\pi}_t)
\]

---

23 In this simple case this is equivalent to applying the Svensson and Woodford (2003a) Kalman filter formulae.
Appendix B

I substitute $\pi_{t+i|t} = 0$, $y_{t+i|t} = \overline{y}_{t+i}$ into (1) (with $\epsilon_i^C = 0$) and (2). I also substitute (24) in (2) and solve for $\pi_i$, obtaining:

$$(\sigma + \vartheta_i + k\vartheta_p)\pi_i = k(\sigma \epsilon_i^D + \vartheta_i \epsilon_i^Y + \vartheta_p \epsilon_i^P)$$

The problem to solve is:

$$\left\{ \begin{array}{l}
\min_{\vartheta_i, \vartheta_p} L(\vartheta_i, \vartheta_p) \equiv \frac{k^2}{(\sigma + \vartheta_i + k\vartheta_p)^2} (\sigma^2 \sigma_r^2 + \vartheta_i^2 \sigma_r^2 + \vartheta_p^2 \sigma_r^2) \\
\text{sub} \ g(\vartheta_i, \vartheta_p) \equiv k(\vartheta_p - 1) + (1 - \beta)\vartheta_i \geq \zeta
\end{array} \right.$$ 

The Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial \vartheta_i} - \lambda \frac{\partial g}{\partial \vartheta_i} = 0 \quad (i = Y, P)$$

$$g \geq 0$$

$$\lambda \geq 0$$

$$g \lambda = 0$$

When the constraint is not binding ($g \geq 0, \lambda = 0$), the value of $\vartheta_i, \vartheta_p$ is given by $\partial L / \partial \vartheta_i = 0$:

$$\left\{ \begin{array}{l}
\vartheta_i = \sigma \frac{\sigma_r^2}{\sigma_y^2} \\
\vartheta_p = \sigma \frac{\sigma_r^2}{\sigma_p^2} k
\end{array} \right.$$ 

Substituting (41) into $g$, it is also:

$$\left( \frac{1}{\sigma_r^2} (1 + \frac{\zeta}{k}) - \frac{1}{\sigma_p^2} k - \frac{1 - \beta}{k} \right) \leq 0$$

When the constraint is binding ($g = 0, \lambda = 0$), the value of $\vartheta_i, \vartheta_p$ can be obtained from the F.O.C.s $\partial L / \partial \vartheta_i - \lambda \partial g / \partial \vartheta_i = 0$ and $g = 0$: 
\[
2k^2 \frac{\vartheta_s \sigma_p^2 [\sigma + \vartheta_t + k \vartheta_p]}{[\sigma + \vartheta_t + k \vartheta_p]^4} - \left[ \sigma + \vartheta_t + k \vartheta_p \right] \lambda \left( \frac{\vartheta_t^2 \sigma_p^2 + \sigma_p^2 + \sigma_D^2}{\sigma + \vartheta_t + k \vartheta_p} \right) - k \lambda = 0
\]
\[
2k^2 \frac{\vartheta_s \sigma_p^2 [\sigma + \vartheta_t + k \vartheta_p]}{[\sigma + \vartheta_t + k \vartheta_p]^4} - \left[ \sigma + \vartheta_t + k \vartheta_p \right] \lambda \left( \frac{\vartheta_t^2 \sigma_p^2 + \sigma_p^2 + \sigma_D^2}{\sigma + \vartheta_t + k \vartheta_p} \right) - (1 - \beta) \lambda = 0
\]
\[
k \vartheta_t + (1 - \beta) \vartheta_p - k = \zeta
\]

Rearranging the terms, after some algebra, one eventually gets:

\[
\vartheta_t = \sigma \frac{\sigma_D^2}{\sigma_y^2} \left\{ 1 + \Delta \left[ \frac{\sigma^{-1}}{\sigma_D^2} \left( 1 + \frac{\zeta}{k} \right) - \frac{1}{\sigma_p^2} k - \frac{1 - \beta}{\sigma_y^2} k \right] (\sigma(1 - \beta) + k + \zeta) \right\}
\]
\[
\vartheta_p = \sigma k \frac{\sigma_D^2}{\sigma_y^2} \left\{ 1 + \Delta \left[ \frac{\sigma^{-1}}{\sigma_D^2} \left( 1 + \frac{\zeta}{k} \right) - \frac{1}{\sigma_p^2} k - \frac{1 - \beta}{\sigma_y^2} k \right] (\sigma + k + \zeta) \right\}
\]
\[
\lambda = 2k^2 \frac{(\vartheta_p \sigma_p^2 - k \vartheta_t \sigma_y^2)}{\beta(\sigma + k \vartheta_p + \vartheta_t)^2}
\]

where \( \Delta \equiv \left[ \frac{1}{\sigma_p^2} k(\sigma + k + \zeta) + \frac{1 - \beta}{\sigma_y^2} k (\sigma(1 - \beta) + k + \zeta) \right]^{-1} > 0 \)

Moreover, from the condition \( \lambda \geq 0 \) one derives:

\[
\left[ \frac{\sigma^{-1}}{\sigma_D^2} \left( 1 + \frac{\zeta}{k} \right) - \frac{1}{\sigma_p^2} k - \frac{1 - \beta}{\sigma_y^2} k \right] \geq 0
\]
References


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