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Tests of seasonal integration and cointegration in multivariate unobserved component models

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TESTS OF SEASONAL INTEGRATION AND COINTEGRATION IN MULTIVARIATE UNOBSERVED COMPONENT MODELS

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Abstract

The paper considers tests of seasonal integration and cointegration for multivariate time series. The locally best invariant (LBI) test of the null hypothesis of a deterministic seasonal pattern against the alternative of seasonal integration is derived for a model with Gaussian i.i.d. disturbances and deterministic trend. A test of seasonal cointegration is then proposed, which parallels the common trend test of Nyblom and Harvey (2000). The tests are subsequently generalized to account for stochastic trends, weakly dependent errors and unattended unit roots. Asymptotic representations and critical values of the tests are provided, while the finite sample performance is evaluated by Monte Carlo simulation experiments. We apply the tests to the indices of industrial production of the four largest countries of the European Monetary Union. We find evidence that Germany does not cointegrate with the other countries, while there seems to exist a common nonstationary seasonal component between France, Italy and Spain.

JEL classification: C12, C32.

Keywords: common components, locally best invariant test, seasonal unit roots.

Contents

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1. Introduction[1]

Economic time series are often characterized by a slowly changing, as opposed to fixed, seasonal pattern. Models with seasonal unit roots, or unit roots at the seasonal frequencies, can account for this kind of behaviour. Statistical tests for the presence of seasonal unit roots in quarterly time series have been proposed by Hylleberg et al. (1990). The tests have been extended to monthly data and seasonal trends in Beaulieu and Miron (1993) and Smith and Taylor (1998), respectively. In a multivariate set-up, Lee (1992) and Johansen and Schaumburg (1999) have proposed likelihood-based tests for the rank of the seasonal cointegration space, which extend the VAR framework of Johansen (1988, 1991, 1995) to seasonal time series. Empirical applications are given in, *inter alia*, Engle et al. (1993), Kunst (1993), Reimers (1997), Huang and Shen (2002). Franses and McAleer (1998) is a comprehensive survey of this literature.

In all those articles the tests are constructed from the autoregressive representation of linear time series. This paper, on the other hand, considers testing for seasonal integration and cointegration within the unobserved component model

$$\mathbf{y}_t = \boldsymbol{\mu}_t + \mathbf{s}_t + \boldsymbol{\varepsilon}_t, \tag{1}$$

where $\mathbf{y}_t = (y_{1t}, ..., y_{Nt})'$ is a $N \times 1$ vector time series, which is made up of a trend $\boldsymbol{\mu}_t$, a seasonal component $\mathbf{s}_t$ and an irregular term $\boldsymbol{\varepsilon}_t$. Specifically, we test for the presence of common non-stationary components in the seasonal patterns $\mathbf{s}_t$; deterministic seasonality will emerge as a special case. The tests are derived in the multivariate LBI framework of Nyblom and Harvey (2000) and may be viewed as a generalization to multivariate models of the CH test of seasonal stability of Canova and Hansen (1995) and subsequent developments by Caner (1998) and Busetti and Harvey (2003).

The main difference between our tests and those of Lee (1992) and Johansen and Schaumburg (1999) is that they reverse the role of the null and the alternative hypotheses, i.e. in our case the model is ”more stationary” under the null hypothesis than under the alternative one. This parallels the difference between Nyblom and Harvey (2000) and the rank tests of

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Johansen (1988, 1991, 1995), and also between the KPSS stationarity test of Kwiatkowski et al. (1992) and the Dickey-Fuller-type unit root tests.

The tests are applied to the series of the index of industrial production of the four largest countries of the European Monetary Union. We find evidence that Germany does not cointegrate with the other countries, while there seems to be a common non-stationary seasonal component between France, Italy and Spain

In summary, the paper proceeds as follows. Section 2 reviews the definition of seasonal integration and cointegration. Section 3 introduces the LBI test of seasonal stability against seasonal integration and section 4 the test of seasonal cointegration when the trend is a deterministic function of time and the disturbances are Gaussian white noise. Section 5 shows how to modify the tests to allow for the presence of stochastic trends and for serial correlation in the error term; we also suggest running the tests on prefiltered data to guard from so-called unattended unit roots, which can potentially vastly reduce the power of the tests in finite samples. The finite sample properties of the tests are evaluated by Monte Carlo simulation experiments in section 6. Section 7 applies the tests to the series of industrial production of European countries and section 8 concludes. The proofs of the propositions are collected in an appendix.

2. Seasonal integration and cointegration

Seasonal integration and cointegration are defined following Hylleberg et al. (1990) and Cubadda (1999). Let $\Delta(\lambda)$ be the difference operator at frequency $\lambda \in [0, \pi]$, that is

$$\Delta(\lambda) = \begin{cases} 1 - \cos\lambda\, L, & \lambda \in \{0, \pi\}, \\ 1 - 2\cos\lambda\, L + L^2, & \lambda \in (0, \pi), \end{cases}$$

where L is the usual lag operator, $L^k x_t = x_{t-k}$, $k = 0, 1, 2, ...$ The operator $\Delta(\lambda)$ is a simple linear filter with zero gain only at the spectral frequency $\lambda \in [0, \pi]$; in other words it removes unit roots at that frequency.

A real-valued vector time series process $\mathbf{y}_t$ is said to be integrated of order d at frequency $\lambda \in [0, \pi]$, denoted $I(d; \lambda)$, if its d-th λ-difference, $\Delta(\lambda)^d \mathbf{y}_t$, is a linear process with a continuous and positive definite spectrum at λ. The process $\mathbf{y}_t$ is said to be (contemporaneously) cointegrated of order d, b at frequency λ, $CI(d, b; \lambda)$, if (i) each component of $\mathbf{y}_t$ is $I(d; \lambda)$ and (ii) there exists a non-zero vector $\boldsymbol{\alpha}$ such that $\boldsymbol{\alpha}'\mathbf{y}_t$ is $I(d - b)$, where $d \geq b > 0$. These definitions generalize to any spectral frequency the concepts of integration and cointegration of Engle and Granger (1987) formulated for frequency zero. Note that, as pointed out by Hylleberg et al. (1990, p.230), a more general statement of cointegration at frequency $\lambda \in (0, \pi)$ is given by replacing the vector $\boldsymbol{\alpha}$ above with a polynomial vector $\boldsymbol{\alpha}(L) = \boldsymbol{\alpha}_0 + \boldsymbol{\alpha}_1 L$; under this latter definition, the restricted case of $\boldsymbol{\alpha}_1 = 0$ is then usually termed that of contemporaneous cointegration. However, since the unobserved components representation of $\mathbf{y}_t$ considered in this paper can only yields seasonal cointegration with $\boldsymbol{\alpha}_1 = 0$, in what follows we will not make further distinctions between the two concepts of contemporaneous and non-contemporaneous cointegration.

In the context of seasonal time series, the interest lies in the seasonal frequencies $\lambda(h) = 2\pi h/s$, $h = 1, ..., [s/2]$, where s is the number of seasons and the notation $[x]$ denotes the biggest integer that is smaller than or equal to x. The period of $\lambda(1)$ is one year. This is denoted as the fundamental frequency, while the other frequencies are called harmonics. For quarterly series, we have $\lambda(1) = \pi/2$ and $\lambda(2) = \pi$, corresponding to one cycle per year and two cycles per year, respectively.

Thus a process is said to be seasonally integrated (cointegrated) if it is $I(d; \lambda(h))$ ($CI(d, b; \lambda(h))$) at one of the seasonal frequencies $\lambda(h)$, $h = 1, ..., [s/2]$. In this paper we concentrate on the cases $d = b = 1$.

A seasonally integrated linear process can be represented in terms of a non-stationary stochastic seasonal component. Likewise, a seasonally cointegrated process implies the existence of common stochastic seasonal components. This can easily be seen by extending the Beveridge-Nelson decomposition from the long-run frequency to the seasonal frequencies, as done in Hylleberg et al. (1990), Cubadda (1999) and Phillips and Solo (1988). In particular, Cubadda (1999) obtains a common trend-common seasonals-common cycle representation for seasonally cointegrated processes.

In the following sections we consider an unobserved component model where the coefficients of an otherwise deterministic seasonal component are stochastic and evolve as random walks. The objective is to make inference on the rank of the disturbances driving those random walks. The case of rank zero corresponds to deterministic seasonality, full rank to seasonal integration, while seasonal cointegration occurs otherwise.

3. The multivariate LBI test against seasonal integration

Let s be the number of seasons and $\lambda(h) = 2\pi h/s$, $h = 1, ..., [s/2]$, be the seasonal frequencies. Denote by $\mathbf{z}_t(h)$, $h = 1, ..., [s/2]$, the spectral indicator variable associated with each of the $\lambda(h)$, that is $\mathbf{z}_t(h) = (\cos\lambda(h)t, \sin\lambda(h)t)'$ for $h < s/2$ and, when s is even, $\mathbf{z}_t(s/2) = \cos\lambda(s/2)t = \cos\pi t$.

We consider a model where each individual series y_{it} is characterized by a deterministic trend $\mathbf{x}_t'\boldsymbol{\beta}_i$, where $\mathbf{x}_t$ is a $k \times 1$ vector of non-stochastic regressors and $\boldsymbol{\beta}_i$ are fixed coefficients, a seasonal component of the form $\sum_{h=1}^{[s/2]} \mathbf{z}_t(h)'\boldsymbol{\gamma}_{it}(h)$, where $\boldsymbol{\gamma}_{it}(h)$ are random walk coefficients[2], and a white noise disturbance term. Note that we work with the trigonometric representation of the seasonal component but an equivalent formulation in terms of the (perhaps more usual) seasonal dummy variables can be obtained; cf. e.g. Harvey (1989, p. 40-43) and Canova and Hansen (1995).

The vector representation of our model is the following:

$$
\begin{aligned}
&(2) \qquad & \mathbf{y}_t &= \boldsymbol{\mu}_t + \mathbf{s}_t + \boldsymbol{\varepsilon}_t, \\
&(3) \qquad & \boldsymbol{\mu}_t &= \mathbf{X}_t\boldsymbol{\beta}, \\
&(4) \qquad & \mathbf{s}_t &= \sum_{h=1}^{[s/2]} \mathbf{Z}_t(h)\boldsymbol{\gamma}_t(h) \\
&(5) \qquad & \boldsymbol{\gamma}_t(h) &= \boldsymbol{\gamma}_{t-1}(h) + \boldsymbol{\eta}_t(h), \quad h = 1, ..., [s/2], \\
&(6) \qquad & \boldsymbol{\eta}_t(h) &\sim IID\left(\mathbf{0}, \boldsymbol{\Sigma}_\eta(h)\right), \quad h = 1, ..., [s/2], \\
&(7) \qquad & \boldsymbol{\varepsilon}_t &\sim IID\left(\mathbf{0}, \boldsymbol{\Sigma}_\varepsilon\right),
\end{aligned}
$$

where $\mathbf{X}_t = (\mathrm{I}_N \otimes \mathbf{x}_t')$, $\boldsymbol{\beta} = (\boldsymbol{\beta}_1', ..., \boldsymbol{\beta}_N')'$, $\mathbf{Z}_t(h) = (\mathrm{I}_N \otimes \mathbf{z}_t(h)')$, $\boldsymbol{\gamma}_t(h) = (\boldsymbol{\gamma}_{1t}(h)', ..., \boldsymbol{\gamma}_{Nt}(h)')'$, $h = 1, ..., [s/2]$. It is also assumed that $\boldsymbol{\eta}_t(h)$ is independent of $\boldsymbol{\eta}_s(l)$ for $h \neq l$, i.e. the seasonal components at different frequencies are orthogonal, and also independent of the irregular disturbance $\boldsymbol{\varepsilon}_s$, for all t, s; $\otimes$ denotes the Kronecker product. The model is just the multivariate analogue of that considered in Canova and Hansen (1995) and Busetti and Harvey (2003).

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[2] For $h < s/2$ the stochastic coefficients $\boldsymbol{\gamma}_{it}(h)$ are 2-dimensional column vectors, while they are scalar when $h = s/2$ for s is even.

The objective of this paper is to test for the presence of unit roots at the seasonal frequencies $\lambda(h)$, $h = 1, ..., [s/2]$. Specifically, if $\mathbf{\Sigma}_\eta(h)$ is of full rank the process displays seasonal integration at frequency $\lambda(h)$; if rank $\mathbf{\Sigma}_\eta(h) = 0$, the seasonal component at that frequency is deterministic; seasonal cointegration occurs otherwise. In this section we focus on testing the null hypothesis of deterministic seasonality against the alternative of seasonal integration; tests of seasonal cointegration are the subject of section 4.

Consider first the case of a model where $\mathbf{\Sigma}_\eta(l) = 0$ for $l \neq h \in \{1, 2, ..., [s/2]\}$, i.e. all seasonal components are deterministic except at the frequency $\lambda(h)$. Using the framework of Nyblom and Harvey (2000) we can obtain an optimal test against the alternative hypothesis of seasonal integration with common signal-to-noise ratio, say q^2, across all series; the test is consistent for any alternative where $\mathbf{\Sigma}_\eta(h)$ has non-zero rank. Specifically, the following proposition provides the locally best invariant (LBI) test of $\mathrm{H}_0 : \mathbf{\Sigma}_\eta(h) = 0$ against $\mathrm{H}_A : \mathbf{\Sigma}_\eta(h) = q^2 \left(\mathbf{\Sigma}_\varepsilon \otimes \mathrm{I}_{a_h}\right)$, where $q^2 > 0$ and $a(h) = 1$ if $h = s/2$ and $a(h) = 2$ otherwise, under the assumption of Gaussianity of the disturbances.

Proposition 1 *Let $\mathbf{y}_t$ be generated from the model (2)-(7) with $\mathbf{\Sigma}_\eta(l) = 0$ for $l \neq h \in \{1, 2, ..., [s/2]\}$, and let $\mathbf{e}_t$ be the OLS residuals from regressing $\mathbf{y}_t$ on $(\mathbf{x}_t', \mathbf{z}_t')'$, $t = 1, ..., T$. Under Gaussianity, the LBI test of $H_0 : \mathbf{\Sigma}_\eta(h) = 0$ against $H_A : \mathbf{\Sigma}_\eta(h) = q^2 \left(\mathbf{\Sigma}_\varepsilon \otimes I_{a_h}\right)$ rejects when*

$$\xi_{0,N}(h) = a(h) trace\left(\widehat{\mathbf{\Sigma}}_\varepsilon^{-1} \mathbf{C}(h)\right) > c \tag{8}$$

where $\widehat{\mathbf{\Sigma}}_\varepsilon = T^{-1}\sum_{t=1}^T \mathbf{e}_t \mathbf{e}_t'$, $\mathbf{C}(h) = T^{-2}\sum_{t=1}^T \left(\mathbf{S}_t^A(h)\mathbf{S}_t^A(h)' + \mathbf{S}_t^B(h)\mathbf{S}_t^B(h)'\right)$, $\mathbf{S}_t^A(h) = \sum_{s=1}^t \mathbf{e}_s \cos\lambda(h)s$, $\mathbf{S}_t^B(h) = \sum_{s=1}^t \mathbf{e}_s \sin\lambda(h)s$, and c is an appropriate critical value.

Remark 1 *When s is even the statistic at the Nyqvist frequency $\lambda(s/2) = \pi$ can be written without the terms $\mathbf{e}_s \sin\lambda(h)s$ as they are identically zero, that is $\mathbf{S}_t^B(s/2) = 0$.*

The test can be viewed as the extension to multivariate series of the CH test of seasonal stability of Canova and Hansen (1995). As Busetti and Harvey (2003) show for CH test, the null limiting distribution of the LBI statistic (8) is independent of the form of the deterministic regressors $\mathbf{x}_t$, as long as they satisfy

$$\lim_{T\to\infty} T^{-1}\sum_{t=1}^T \mathbf{D}_T^{-1}\mathbf{x}_t\mathbf{x}_t'\mathbf{D}_T^{-1} = \mathbf{Q}_x, \tag{9}$$

and, for each $h = 1, ..., [s/2]$,

(10)
$$\lim_{T\to\infty} T^{-1} \sum_{t=1}^{T} \mathbf{D}_T^{-1} \mathbf{x}_t \mathbf{z}_t(h)' = \mathbf{0},$$

where $\mathbf{D}_T$ is a (diagonal) scaling matrix and $\mathbf{Q}_x$ is a positive definite matrix. Note that $\mathbf{x}_t$ may include polynomial trends with possibly level and/or slope shifts. For example, if the regressors are $\mathbf{x}_t = (1, t, d_t(\alpha))$, where $d_t(\alpha)$ is a dummy variable equal to 1, for $t > \alpha T$ with $0 < \alpha < 1$, we have correspondingly $\mathbf{D}_T = diag(1,\ T,\ 1)$.

Proposition 2 *Under* $H_0 : \mathbf{\Sigma}_\eta(h) = 0$, *with* $\mathbf{x}_t$ *satisfying (9)-(10) and with* $\mathbf{\Sigma}_\eta(l) = 0$ *also for* $l \neq h$, *the limiting distribution of* $\xi_{0,N}(h)$ *is Cramér-von Mises with* $a(h)N$ *degrees of freedom,*

(11)
$$\xi_{0,N}(h) \xrightarrow{d} \int_0^1 \mathbf{B}_{a(h)N}(r)'\mathbf{B}_{a(h)N}(r)dr \equiv CvM\left(a(h)N\right),$$

where $\mathbf{B}_k(r) = \mathbf{W}_k(r) - r\mathbf{W}_k(1)$, $r \in [0,1]$, *denotes a* k*-dimensional Brownian bridge process and* $\mathbf{W}_k(r)$ *a* k*-dimensional Brownian motion.*

The test defined by the statistic (8), though locally most powerful against the alternative hypothesis $\mathrm{H}_A : \mathbf{\Sigma}_\eta(h) = q^2 \left(\mathbf{\Sigma}_\varepsilon \otimes \mathrm{I}_{a_h}\right)$, is consistent against any alternative in which $\mathbf{\Sigma}_\eta(h)$ is different from zero; see remark 2 in the next section.

A joint test against seasonal integration at all frequencies is obtained by taking the sum of (8) over h, that is by the statistic

(12)
$$\overline{\xi}_{0,N} = \sum_{h=1}^{[s/2]} \xi_{0,N}(h).$$

Extending the argument in the proof of proposition 1, it is easy to see that $\overline{\xi}_{0,N}$ is the LBI statistic for testing the null hypothesis of stationarity at all frequencies

$$\overline{\mathrm{H}}_0 : \mathbf{\Sigma}_\eta(h) = 0, \qquad h = 1, ..., [s/2],$$

against the alternative of same signal-to-noise ratio q^2 for all frequencies,

$$\overline{\mathrm{H}}_A : \mathbf{\Sigma}_\eta(h) = q^2 \left(\mathbf{\Sigma}_\varepsilon \otimes \mathrm{I}_{a_h}\right), \qquad h = 1, ..., [s/2],$$

From the additivity property of independent Cramér-von Mises random variables (cf. Busetti and Harvey, 2001, p.136), the limiting distribution of (12) under $\overline{\mathrm{H}}_0$ is Cramér-von Mises with $(s-1)N$ degrees of freedom,

$$\overline{\xi}_{0,N} \xrightarrow{d} CvM\left((s-1)N\right).$$

Note that as $T \to \infty$ (12) diverges (and thus the joint test rejects the null hypothesis of deterministic seasonality) if there is a unit root for at least one of the seasonal frequencies.

A test of stability at any subset of the seasonal frequencies can also be constructed in an obvious way, that is by summing over the relevant frequencies, and critical values are obtained from a Cramér-von Mises distribution with the appropriate number of degrees of freedom.

Upper tail percentage points for a $CvM(k)$, for $k \leq 12$, are tabulated in Canova and Hansen (1995). Additional critical values are contained in Table 1 below, in the columns headed $K = 0$. Specifically, the first 6 rows of the table (labelled one frequency) contain the upper tail quantiles of $CvM(2N)$, $N = 1, 2, ..., 6$, the following 6 rows refer to $CvM(3N)$ and the final rows to $CvM(11N)$. The upper tail percentage points have been obtained by direct simulation of the functional (11) for sample sizes of 1000 over 10000 Monte Carlo replications. The random number generator of the matrix programming language Ox 2.20 of Doornik (1998) was used.

For other values of k (that are large enough), the quantiles of a $CvM(k)$ can be obtained by a Gaussian approximation via a standard Central Limit Theorem. As the mean and the variance of a $CvM(1)$ are $1/6$ and $1/45$ respectively, a $CvM(k)$ can be usefully approximated by a $N\left(k/6, k/45\right)$; cf. Hadri (2000) and Harvey (2001). For example, the 0.95 simulated quantile of $CvM(22)$ is 4.907, close to its Gaussian approximation of 4.817.

In a model with fixed seasonal slopes, i.e. where $\boldsymbol{\mu}_t$ of (3) is replaced by

$$\boldsymbol{\mu}_t = \mathbf{X}_t\boldsymbol{\beta} + \sum_{h=1}^{[s/2]} t\mathbf{Z}_t(h)\boldsymbol{\delta}(h), \tag{13}$$

where $\mathbf{X}_t$, $\mathbf{Z}_t(h)$, $\boldsymbol{\beta}$ are defined below (7) and $\boldsymbol{\delta}(h) = (\boldsymbol{\delta}_1(h)', ..., \boldsymbol{\delta}_N(h)')'$, $h = 1, ..., [s/2]$, are corresponding fixed coefficients, the LBI test for testing $\mathrm{H}_0 : \boldsymbol{\Sigma}_\eta(h) = 0$ is obtained from the same statistic as (8) but constructed using the OLS residuals from regressing $\mathbf{y}_t$ on

$(\mathbf{x}_t', \mathbf{z}_t', t\mathbf{z}_t')'$; denote this statistic by $\xi^{\tau}_{0,N}(h)$. Then, a straightforward extension of proposition (3.1) yields that under $\mathrm{H}_0 : \mathbf{\Sigma}_{\eta}(h) = 0$, with $\mathbf{x}_t$ satisfying (9)-(10) and with $\mathbf{\Sigma}_{\eta}(l) = 0$ also for $l \neq h$,

$$\xi^{\tau}_{0,N}(h) \xrightarrow{d} \int_0^1 \mathbf{B}^{\tau}_{a(h)N}(r)'\mathbf{B}^{\tau}_{a(h)N}(r)dr \equiv CvM_2\left(a(h)N\right), \tag{14}$$

where $\mathbf{B}^{\tau}_k(r) = \mathbf{B}_k(r) - 6r(1-r)\int_0^1 \mathbf{B}_k(s)ds$, and $\mathbf{B}_k(r)$ is a k-dimensional standard Brownian bridge process. The process $\mathbf{B}^{\tau}_k(r)$ is sometimes called second level Brownian bridge and the distribution at the right-hand side of (14) second level Cramér-von Mises distribution, CvM_2, with $a(h)N$ degrees of freedom; see McNeill (1978) and Harvey (2001). Then, a joint test against non-stationary seasonal components at all frequencies in the presence of seasonal slopes is provided by the statistic $\overline{\xi}^{\tau}_{0,N} = \sum_{h=1}^{[s/2]} \xi^{\tau}_{0,N}(h)$; under the null hypothesis $\overline{\xi}^{\tau}_{0,N}$ asymptotically follows a second level Cramér-von Mises distribution with $(s-1)N$ degrees of freedom.

Percentage points for $CvM_2(k)$, for $k \leq 4$, are tabulated in Nyblom and Harvey (2000). For $k \geq 4$ they are available from myself, on request. A Gaussian approximation can also be used in this case, noticing that the mean and the variance of a $CvM_2(1)$ are given by 1/15 and 11/6300, respectively; cf. Hadri (2000).

4. Tests of seasonal cointegration

Recall from section 2 that the definition of seasonal cointegration of order 1,1 at frequency $\lambda(h)$ implies the existence of some linear combination of the series, say $\boldsymbol{\alpha}'\mathbf{y}_t$, that is $I(0; \lambda(h))$. If $\boldsymbol{\alpha}$ were known a priori, a test for seasonal cointegration would be the (multivariate) LBI test (8) of the previous section applied to $\boldsymbol{\alpha}'\mathbf{y}_t$. The test, however, would not be valid if $\boldsymbol{\alpha}$ is estimated; cf. Nyblom and Harvey (2000) where the same problem, but at frequency zero, is considered. Note that unlike the case of cointegration at frequency zero, where there might be economic reasons for having $\boldsymbol{\alpha}$ known in advance (as for the theories of balanced growth, purchasing power parity, uncovered interest rate parity, among others), treating $\boldsymbol{\alpha}$ as known does not appear convincing in the context of seasonal cointegration.

Following the strategy adopted by Nyblom and Harvey (2000) for testing at frequency zero, we look at the eigenvalues of the statistic $\widehat{\mathbf{\Sigma}}_{\varepsilon}^{-1}\mathbf{C}(h)$, defined in the previous section. While the LBI stability test (8) is based on the trace of $\widehat{\mathbf{\Sigma}}_{\varepsilon}^{-1}\mathbf{C}(h)$, i.e. the sum of the eigenvalues, a test of seasonal cointegration considers only the smallest, say R, of them.

Specifically, we consider the data generating process (2)-(7) under the restriction that the seasonal component at frequency $\lambda(h)$ is driven by reduced rank random walk coefficients, i.e. that

$$\mathbf{\Sigma}_{\eta}(h) = \left(\mathbf{\Sigma}_{\eta}^{*}(h) \otimes I_{a_h}\right)$$

with $rank(\mathbf{\Sigma}_{\eta}^{*}(h)) = K$, $0 \leq K < N$. We take this restriction as the null hypothesis:

$$H_{0,K} : rank(\mathbf{\Sigma}_{\eta}^{*}(h)) = K.$$

It can easily be seen that under $\mathrm{H}_{0,K}$ the vector time series $\mathbf{y}_t$ is seasonally cointegrated at frequency $\lambda(h)$, $CI(1,1;\lambda(h))$, with $R = N - K$ linearly independent cointegrating vectors[3]. The alternative hypothesis is

$$H_{A,K} : rank(\mathbf{\Sigma}_{\eta}^{*}(h)) > K,$$

i.e. that the cointegration space has a lower dimension than under the null hypothesis. As in the previous section, we first maintain that $\mathbf{\Sigma}_{\eta}(l) = 0$ for $l \neq h \in \{1, 2, ..., [s/2]\}$, i.e. that all seasonal components are deterministic except that at the frequency $\lambda(h)$.

The test statistic is the sum of the R smallest eigenvalues of $a(h)\widehat{\mathbf{\Sigma}}_{\varepsilon}{}^{-1}\mathbf{C}(h)$,

$$\xi_{K,N}(h) = \sum_{j=K+1}^{N} \ell_j(h), \tag{15}$$

where $\ell_1(h) \geq \ell_2(h) \geq ... \geq \ell_N(h) \geq 0$ are the N ordered eigenvalues. Notice that $\xi_{0,N}(h)$ is the statistic (8) of the previous section. The following propostion provides the limiting

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[3] Under $\mathrm{H}_{0,K}$ there exists a full rank $N \times K$ matrix $\mathbf{\Theta}$ such that $\mathbf{\Theta}\mathbf{\Theta}' = \mathbf{\Sigma}_{\eta}^{*}$. Let α be a $N \times 1$ vector belonging to the (R-dimensional) left null space of $\mathbf{\Theta}$, i.e. such that $\alpha'\mathbf{\Theta} = \mathbf{0}$. This is a seasonal cointegration vector since it annihilates the stochastic seasonal component at frequency $\lambda(h)$,

$$\boldsymbol{\alpha}' \left(I_N \otimes \mathbf{z}_t'(h)\right) \left(\boldsymbol{\gamma}_t(h) - \boldsymbol{\gamma}_0(h)\right) = 0.$$

In fact, since it holds that $\boldsymbol{\alpha}' \left(I_N \otimes \mathbf{z}_t'(h)\right) = \boldsymbol{\alpha}' \otimes \mathbf{z}_t'(h)$ and we can write $\boldsymbol{\eta}_t(h) = \left(\mathbf{\Theta} \otimes I_{a_h}\right) \boldsymbol{\eta}_t^{*}$ where $\boldsymbol{\eta}_t^{*}$ is $IID\left(0, I_{a_h K}\right)$, we have that

$$\boldsymbol{\alpha}' \left(I_N \otimes \mathbf{z}_t'(h)\right) \left(\boldsymbol{\gamma}_t(h) - \boldsymbol{\gamma}_0(h)\right) = \left(\boldsymbol{\alpha}' \otimes \mathbf{z}_t'(h)\right) \left(\mathbf{\Theta} \otimes I_{a_h}\right) \sum_{j=1}^{t} \boldsymbol{\eta}_j^{*} = \left(\boldsymbol{\alpha}'\mathbf{\Theta} \otimes \mathbf{z}_t'(h)\right) \sum_{j=1}^{t} \boldsymbol{\eta}_j^{*} = 0.$$

distribution of $\xi_{K,N}(h)$ under $H_{0,K}$: $rank(\mathbf{\Sigma}_{\eta}^{*}(h)) = K, 1 \leq K < N$; the case $K = 0$ has been dealt with in the previous section.

Proposition 3 *Under* $H_{0,K} : rank(\mathbf{\Sigma}_{\eta}^{*}(h)) = K, 1 \leq K < N$, *with* $\boldsymbol{x}_t$ *satisfying (9)-(10) and with* $\mathbf{\Sigma}_{\eta}(l) = 0$ *for* $l \neq h$, *and* $h \neq s/2$,

(16) $$\xi_{K,N}(h) \xrightarrow{d} Tr\left(\mathbf{C}_{22}^{*}(h) - \mathbf{C}_{12}^{*}(h)'\mathbf{C}_{11}^{*}(h)^{-1}\mathbf{C}_{12}^{*}(h)\right),$$

with

$$\begin{aligned}
\mathbf{C}_{11}^{*}(h) = & \int_0^1 \left(\int_0^r \overline{\mathbf{W}}_K^A(s)ds\right)\left(\int_0^r \overline{\mathbf{W}}_K^A(s)ds\right)' dr + \int_0^1 \left(\int_0^r \overline{\mathbf{W}}_K^B(s)ds\right)\left(\int_0^r \overline{\mathbf{W}}_K^B(s)ds\right)' dr \\
\mathbf{C}_{12}^{*}(h) = & \int_0^1 \left(\int_0^r \overline{\mathbf{W}}_K^A(s)ds\right)\mathbf{B}_R^A(r)'dr + \int_0^1 \left(\int_0^r \overline{\mathbf{W}}_K^B(s)ds\right)\mathbf{B}_R^B(r)'dr \\
\mathbf{C}_{22}^{*}(h) = & \int_0^1 \mathbf{B}_R^A(r)\mathbf{B}_R^A(r)'dr + \int_0^1 \mathbf{B}_R^B(r)\mathbf{B}_R^B(r)'dr
\end{aligned}$$

where $\overline{\mathbf{W}}_K^A(r)$, $\overline{\mathbf{W}}_K^B(r)$ *are independent* K*-dimensional demeaned Wiener processes and* $\boldsymbol{B}_R^A(r), \boldsymbol{B}_R^B(r)$ *independent* R*-dimensional Brownian bridges.*

Remark 2 *The test is consistent for* $H_{A,K}$: $rank(\mathbf{\Sigma}_{\eta}^{*}(h)) > K$ *as at least one of the eigenvalues in (15) is* $O_p(T)$; *see the proof in the appendix.*

Remark 3 *When* s *in even, the limiting null distribution of* $\xi_{K,N}(s/2)$ *is that of Nyblom and Harvey (2000).*

As in the previous section, a joint test for seasonal cointegration at all frequencies is obtained by taking the sum of (15) over h, that is by the statistic

(17) $$\overline{\xi}_{K,N} = \sum_{h=1}^{[s/2]} \xi_{K,N}(h).$$

As the statistics for each individual frequency are asymptotically independent, the limiting distribution of (17) under the joint null hypothesis $\overline{\mathrm{H}}_{0,K} : rank(\mathbf{\Sigma}_{\eta}^{*}(h)) = K,\ h = 1, ..., [s/2]$, can be obtained by simulating percentage points from the sum, over h, of independent random variables, each with asymptotic representation given by proposition 1 (taking into account of remark 3 which applies for s even).

A non-rejection of the null hypothesis $\overline{\mathrm{H}}_{0,K}$ in the joint test implies seasonal cointegration with $R = N - K$ linearly independent cointegrating vectors at each of the seasonal frequencies. Notice that the cointegrating vectors are allowed to differ across frequencies, that is the linear combination which implies stability at frequency $\lambda(h)$ is in general different from that at frequency $\lambda(l),\, l \neq h.$

Upper tail percentage points for the limiting null distributions of $\xi_{K,N}(h),\, \overline{\xi}_{K,N}$ are provided in Table 1; for the joint statistic we provide values appropriate to quarterly and monthly data. The figures are thus asymptotic critical values for testing at a single frequency (different from the Niqvist frequency π, for which figures are given in Nyblom and Harvey (2000)) and the joint test at all frequencies for quarterly series ($s = 4$) and monthly series ($s = 12$), where $1 \leq N \leq 6$. The columns headed for $K = 0$ correspond to the tests of the previous section where the distribution is a CvM with an appropriate number of degrees of freedom, while those for $1 \leq K < N$ are appropriate for the tests of seasonal cointegration. The quantiles have been obtained by direct simulation of the functional (16) for sample sizes of 1000 over 10000 Monte Carlo replications. The random number generator of the matrix programming language Ox 2.20 of Doornik (1998) was used.

5. Stochastic trends, serial correlation and unattended unit roots

In the previous sections we have considered the multivariate unobserved component model (1) under the assumptions that (i) the trend $\boldsymbol{\mu}_t$ is a deterministic function of time, (ii) the irregular component $\boldsymbol{\varepsilon}_t$ is a white noise, and (iii) the seasonal components at all frequencies except that under test are deterministic. These restrictions are relaxed in each of the following subsections, in turn.

5.1 *Stochastic trends*

In an unobserved component model like (1), in general, the trend is allowed to be stochastic. A flexible form of the trend function which is typically adequate for many economic time series is the local linear trend of Harvey (1989), where both the level and the slope are stochastic and evolve as random walks. In this unrestricted form the trend $\boldsymbol{\mu}_t$ is an $I(2;0)$ process, which becomes $I(1;0)$ if the variance of the slope is kept fixed. The trend will be cointegrated if the variance matrix of the level and/or of the slope disturbance is not of full rank.

Testing seasonal integration and cointegration in a model with a stochastic trend can be carried out by two strategies: either by removing the stochastic trend by appropriate differencing or by estimating a fully parametrized model and constructing the test from the one-step-ahead prediction errors.

An $I(1;0)$ trend is annihilated by applying the standard first difference operator. However, the resulting irregular component is no longer a white noise but a moving average process. The statistics of the previous section are thus no longer appropriate, but the test can be run after a nonparametric modification that allows the irregular component to follow a weakly dependent process. This modification will be the subject of the next subsection. If the data are not differenced the test will be still consistent but suffer from a big loss of power in finite samples for the problem of the unattended unit roots as explained in subsection 5.3. For the case of an $I(2;0)$ stochastic linear trend, as the variance of the slope is tipically small compared with that of the other components, it can be anticipated that it may be adequate to apply the tests to first differenced data. In fact, the Monte Carlo experiments reported in Busetti and Harvey (2003) for the CH test suggest that taking first differences is likely to be a good strategy in practice.

A parametric approach to deal with a stochastic trend can also be employed. In particular, Busetti and Harvey (2003) considers modifications of the CH test to account for the presence of stochastic components, other than the seasonal, by fitting a parametric model to the data. The idea is to put the model in state space form and estimate the nuisance parameters under the alternative hypothesis of seasonal integration or cointegration. Then the Kalman filter will be run under the appropriate null hypothesis and the Kalman filter innovations used to compute the statistic $\xi_{K,N}(h)$ of the previous section. The limiting null distribution will be unchanged; cf. Busetti and Harvey (2003) for more details. In that paper they also produce extensive Monte Carlo experiments for a quarterly (univariate) model made of a stochastic trend component plus a seasonal and a white noise irregular term. Their results show that the parametric test at frequency π (but not that at frequency $\pi/2$) is slightly oversized in a sample of 200 observations, and that oversizing translates to the joint tests. In terms of power, the parametric tests are superior, but not by very much, with respect to running the nonparametric tests based on first differenced data.

5.2 *Nonparametric correction of serial correlation*

If we replace the assumption of IID for the irregular component ε_t of (2) by that of weak dependence, we will require a non-parametrically modified version of the statistics $\xi_{K,N}(h)$ in order to obtain a statistic with a pivotal limiting null distribution. For the case $N = 1$ Busetti and Harvey (2003) suggest a modified statistic where the sample variance of the observations is replaced by a nonparametric estimator of the spectrum of ε_t at frequency $\lambda(h)$. We propose an analogous correction for our multivariate model.

Let $\mathbf{\Omega}(\lambda)$ denote (2π) the multivariate spectral density of $\boldsymbol{\varepsilon}_t$ at frequency λ, $\lambda \in [0, \pi]$. Then in our statistics we replace $\widehat{\mathbf{\Sigma}}_\varepsilon$ by a consistent estimator, say $\widehat{\mathbf{\Omega}}(h)$, of the spectrum at frequency $\lambda(h)$, e.g.

$$\widehat{\mathbf{\Omega}}(h) = \sum_{j=-m}^{m} k(j,m)\widehat{\Gamma}(j)\left(\cos\lambda(h)j - i\sin\lambda(h)j\right)$$

where $k(.,.)$ is a kernel function, e.g. the Newey-West kernel $k(j,m) = 1 - |j|/(m+1)$, $\widehat{\Gamma}(|j|) = T^{-1}\sum_{t=j+1}^{T} e_t e_{t-j}'$ is the sample autocovariance of the OLS residuals and at lag $j \geq 0$, and $\widehat{\Gamma}(-|j|) = \widehat{\Gamma}(|j|)'$. Alternative options for the kernel may be found in, *inter alia*, Priestley(1989) and Andrews (1991). Setting the bandwidth parameter m such that $m \to \infty$ and $m/T^{1/2} \to 0$ as $T \to \infty$ ensures that $\hat{\mathbf{\Omega}}(h) \xrightarrow{p} \mathbf{\Omega}(\lambda(h))$ under the null and remains stochastically bounded under the alternative hypothesis of stochastic seasonality, thereby ensuring consistency of the test; see Stock (1994, p.2797-2799). Note that in general $\widehat{\mathbf{\Omega}}(h)$ is a complex matrix but it can be computed by calculations in the real domain by splitting the real part and the imaginary part.

Thus we have the following *spectral nonparametric statistic* for seasonal integration

$$\xi_{0,N}^{*}(h) = a(h)trace\left(\widehat{\mathbf{\Omega}}(h)^{-1}\mathbf{C}(h)\right) \tag{18}$$

where $a(h)$, $\mathbf{C}(h)$ are defined as in proposition 1. An analogous correction holds for the test of seasonal cointegration,

$$\xi_{K,N}^{*}(h) = \sum_{j=K+1}^{N} \ell_j^{*}(h), \tag{19}$$

where $\ell_1^{*}(h), ..., \ell_N^{*}(h)$ are the N ordered eigenvalues of $a(h)\widehat{\mathbf{\Omega}}(h)^{-1}\mathbf{C}(h)$. Note that, as $\widehat{\mathbf{\Omega}}(h)$ and $\mathbf{C}(h)$ are positive definite hermitian matrices, the eigenvalues of $\widehat{\mathbf{\Omega}}(h)^{-1}\mathbf{C}(h)$ are real and positive; see e.g. Rao (1973).

Testing at all seasonal frequencies can be carried out in an obvious way, by summing the previous statistics over h.

By extending the arguments of Busetti and Harvey (2003) and Nyblom and Harvey (2000) it is straightforward to show that the limiting null distributions of (18), (19) are as given by propositions 3.2 and 4.1 respectively.

An alternative way of allowing for serial correlation in the error term ε_t is to estimate a fully parametric model and compute the test statistics from the Kalman filter innovations as explained in the previous subsection.

5.3 *Unattended unit roots*

Busetti and Taylor (2003) and Taylor (2003) have considered the effect of unit root behaviour at some frequency on the stability tests at other frequencies; this situation is termed "unattended unit roots". They show that the power of the tests is vastly reduced in the presence of unattended unit roots; indeed, under the null hypothesis, the test statistics converge in probability to zero. However, a simple way to avoid this reduction in power is to prefilter the data so as to annihilate any unattended unit roots.

In the context of testing for seasonal integration and cointegration at frequency $\lambda(h)$, $h \in \{1, ..., [s/2]\}$, one may wish to guard against the effects of unit roots at the other seasonal frequencies $\lambda(l)$, $l \neq h$. This is accomplished by computing the tests after the filter $\nabla_s(\lambda(h)) \equiv (1 + L + ... + L^{s-1})/\Delta(\lambda(h))$ has been applied to the data; note that the seasonal sum operator in the numerator is just the product, over frequencies, of the first difference filters $\Delta(\lambda(h))$ of section 2: $\prod_{h=1}^{[s/2]} \Delta(\lambda(h)) = 1+L+...+L^{s-1}$. As an example, the test at frequency π for quarterly data will be computed on the transformed data $(1 + L^2)\,\mathbf{y}_t$.

Since the application of the prefilter $\nabla_s(\lambda(h))$ transforms a white noise into a moving average process, the tests need to be computed with some correction for serial correlation, as in the previous subsection, even if the irregular component is a white noise. The resulting process will be strictly non-invertible at all seasonal frequencies except $\lambda(h)$, that is the spectrum at $\lambda(h)$ is a positive definite matrix.

Consequently, we suggest using the statistics (18), (19) where the OLS residuals are computed from the regression of $\nabla_s(\lambda(h))\mathbf{y}_t$ on $\mathbf{w}_t = (\nabla_s(\lambda(h))\mathbf{x}_t', \mathbf{z}_t')'$; note that the prefiltered regressors $\nabla_s(\lambda(h))\mathbf{z}_t$ span the same space as $\mathbf{z}_t$. If the data generating process also contains a unit root at frequency zero, as when the trend $\boldsymbol{\mu}_t$ is a random walk process, the data should be prefiltered by $(1 - L)\,\nabla_s(\lambda(h))$.

A further advantage of prefiltering is that it makes the tests robust to the presence of structural breaks at the filtered frequencies, as the filter transforms a level shift into at most f outliers, where f is the degree of the filter, that have no effect asymptotically; see Busetti and Taylor (2003).

Alternatively, a parametric approach could also be employed to deal with unattended unit roots. This would require the test to be computed from the Kalman filter innovations, by keeping all the components with unit roots unrestricted except that under test.

6. Monte Carlo results

In this section we use Monte Carlo simulation methods to investigate the finite sample size and power properties of the tests for seasonal integration and cointegration considered in the previous sections.We generate quarterly data from the DGP (2)-(7), setting

$$\mathbf{\Sigma}_{\varepsilon} = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}.$$

We focus on the properties of the tests at the fundamental frequency $\lambda(1) = \pi/2$ and of the joint tests at both seasonal frequencies $\pi/2$ and π. The power of the tests depend on the magnitude of the variance matrices, $\mathbf{\Sigma}_{\eta}(1)$ and $\mathbf{\Sigma}_{\eta}(2)$, driving the non-stationary components at frequencies $\lambda(1) = \pi/2$ and $\lambda(2) = \pi$. As concerns frequency π we set $\mathbf{\Sigma}_{\eta}(2) = q_2^2\mathbf{\Sigma}_{\varepsilon}$, where the square root of the signal-to-noise ratio q_2 varies among $0, 0.1, 0.5$. As our main objective is to study the finite sample behaviour of the tests at frequency $\pi/2$, setting $q_2 > 0$ allows us to see the effect of an unattended unit root.

We consider three cases of the data generating process at frequency $\pi/2$:

(A) $\mathbf{\Sigma}_{\eta}(1) = q_1^2\mathbf{\Sigma}_{\varepsilon}$,

(B) $\mathbf{\Sigma}_{\eta}(1) = q_1^2\mathrm{I}_2$,

(C) $\mathbf{\Sigma}_{\eta}(1) = q_1^2 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$,

where in each case the square root of the signal-to-noise ratio, q_1, varies among $0, 0.025, 0.050, 0.075, 0.1, 0, 5$. The results are reported in Tables 2.A, 2.B, 2.C respectively. Case (A) is the LBI set-up; case (B) departs from the LBI as it does not mantain the same cross correlation in $\mathbf{\Sigma}_{\eta}(1)$ as in $\mathbf{\Sigma}_{\varepsilon}$; case (C) corresponds to a common seasonal component with the same loadings for the two series, that is seasonal cointegration with cointegration vector equal to $(1, -1)$.

The results are for quarterly series of length $T = 100$. For each configuration of the parameters of the data generating process we compute 6 statistics:

(1) $\ell_1(1) + \ell_2(1)$, to test the null hypothesis $K = 0$ at frequency $\pi/2$ with data in levels,

(2) $\ell_1^*(1) + \ell_2^*(1)$, to test the null hypothesis $K = 0$ at frequency $\pi/2$ from prefiltered data,

(3) $\ell_1(1) + \ell_2(1) + \ell_1(2) + \ell_2(2)$, to test the null hypothesis $K = 0$ jointly at frequencies $\pi/2$ and π with data in levels,

(4) $\ell_2(1)$, to test the null hypothesis $K = 1$ at frequency $\pi/2$ with data in levels,

(5) $\ell_2^*(1)$, to test the null hypothesis $K = 1$ at frequency $\pi/2$ from prefiltered data,

(6) $\ell_2(1) + \ell_2(2)$, to test the null hypothesis $K = 1$ jointly at frequencies $\pi/2$ and π with data in levels.

When the prefilter $\nabla_4(\lambda(1)) = 1 + L$ is applied to the data, the eigenvalues $\ell_j^*(h)$, $j, h = 1, 2$, are computed using a spectral estimate $\widehat{\mathbf{\Omega}}(h)$ with a Newey-West kernel with bandwidth $m = 4$.

The empirical rejection frequencies, reported in percentages, are based on 100,000 replications and refer to tests run at the 5% significance level. All experiments were programmed using the random number generator of the matrix programming language Ox 2.20 of Doornik (1998).

Consider first the results of Table 2.A for the case of no unattended unit root, $q_2 = 0$. The LBI test (1) of $K = 0$ at frequency $\pi/2$ appears slightly oversized; in fact, the prefiltered test (2) computed with bandwidth $m = 4$ has a size closer to the nominal 5% and, although prefiltering is not advisable as $q_2 = 0$, it does not suffer from a significant power loss with respect to (1). As expected, the power of the joint test (3) is lower than that of the LBI test at the single frequency $\pi/2$.

The tests for seasonal cointegration (4)-(6) display much lower power than (1)-(3), but they are consistent since the smallest eigenvalues $\ell_2(1)$, $\ell_2^*(1)$ are $O_p(T)$. Thus, in finite samples one might find spurious evidence for seasonal cointegration when in fact the series are seasonally integrated without common components. However, in an additional simulation (not reported in the table), where the sample size has been enlarged to $T = 200$, we have obtained 25.7, 49.1, 66.3, 99.7 as rejection frequencies for the test (4) when $q_1 = 0.050$, 0.075, 0.100, 0.500 respectively; thus, for moderately large samples, the hypothesis of seasonal cointegration is much more likely to be rejected.

It is interesting to examine the effect of the unattended unit root at frequency π. When $q_2 = 0.5$ the power of test (1) in the levels is very low; on the other hand, the rejection frequencies of the prefiltered test (2) are largely comparable to those of the LBI test where $q_2 = 0$. Here the joint test in levels (3) has almost unit rejection probability, being driven by the unit root at frequency π. Analogous effects apply to the tests (4)-(6) of seasonal cointegration.

Table 2.B has been included mainly to demonstrate that the power of the tests is not much influenced by the cross correlation structure of $\mathbf{\Sigma}_\eta(1)$. The figures in Table 2.B are broadly comparable with those in Table 2.A; similar figures would also apply for moderate negative cross correlation.

The case of a data generating process with perfect correlation in $\mathbf{\Sigma}_\eta(1)$, that is with seasonal cointegration, is examined in Table 2.C. Consider first the case $q_2 = 0$. Even for large values of q_1 the rejection frequencies of the seasonal cointegration tests (4)-(6) never exceed 5.1%; that is, the empirical size of the test, defined as maximum probability of rejecting the null hypothesis when it is true, turns out to be close to the nominal size even in a sample of $T = 100$. Note that the finite sample power of the seasonal cointegration test is in the figures of Table 2.A-B and thus has already been discussed above. As concerns the power of the tests (1)-(3) of the null hypothesis $K = 0$, it is somehow lower than the corresponding figures in Table 2.A-B but higher than the power of the seasonal cointegration tests (4)-(6) of the same tables. Finally, the unattended unit root, $q_2 > 0$, has the effect of reducing power in a qualitatively similar way to that of the previous cases.

7. Application: industrial production in the euro area

Figures 1A-1D show the logarithm of the monthly index of industrial production in the four largest countries of the European Monetary Union: Germany, France, Italy and Spain. The data refer to the period 1985M1-2001M12; the source is Eurostat.

All series are characterized by large seasonal swings and it also appears, from visual inspection, that the seasonal patterns are not constant over time. The main questions we want to address are whether the seasonality of industrial production is deterministic and whether there are co-movements at the seasonal frequencies.

We apply the tests of seasonal integration and cointegration to each combination of the four countries, allowing for serial correlation in the error term as explained in subsection 5.2. The results are displayed in Tables 3.A-B, for values of the bandwidth parameter $m = 5, 10, 15$; these values are compatible the number of observations 204. The choice of m reflects the usual trade-off between size and power of the tests; see e.g. Kwiatkowski et al. (1992). For our case, a good compromise between correct size and good power could be $m = 10$. The shaded figures indicate rejection at 5% significance level of the null hypothesis that there are K non-stationary seasonal components, that is seasonal cointegration with $R = N - K$ cointegrating vectors. In Table 3.A the tests are applied to first differenced data to account for a stochastic trend; in Table 3.B the data have also been prefiltered to guard from unattended unit roots, as explained in section 5.3 (that is the tests at frequency $\lambda(h)$ are carried out on the filtered observation $(1 - L)\,\nabla(\lambda(h))\mathbf{y}_t$).

The rows of the tables indicate to which subset of the four countries, Germany, France, Italy and Spain, the tests are applied; the columns indicate the seasonal frequency to which the figures refer. The last 3 columns contain the joint test at all seasonal frequencies.

Consider the last 4 rows of Table 3.A, which apply the tests to the 4-dimensional vector of the series of industrial production. The results for each single seasonal frequency $\lambda(h) = 2\pi h/s$, $h = 1, ..., [s/2]$, maybe with the exception of $\lambda(2) = \pi/3$, seem to indicate one common seasonal component across the four countries; the joint test, however, supports the view of 2 non-stationary seasonal components. If we consider the same results for the prefiltered data, in Table 3.B, the situation changes substantially. Here the tests strongly point to rejection of the hypothesis of one common seasonal component ($K = 1$) for each of the frequencies $\lambda(1), \lambda(2)$, $\lambda(3)$, $\lambda(4)$; there also appears to be a rejection of $K = 2$ when the bandwidth parameter m is set equal to 5. Again, combining the results for each frequency in the joint test gives a stronger indication of non-stationarity. In the light of subsection 5.3 it is no surprise that the tests based on prefiltered data provide less evidence of seasonal cointegration as they do not suffer from the reduction of power due to unattended unit roots.

Given that the four countries seem to be characterized by two common seasonal components at most frequencies, it is interesting to figure out whether there is any country whose seasonal pattern does not cointegrate with the others. In fact this seems to be the case of Germany. The pairwise analysis contained in the first 6 rows of Table 3.B shows that Germany does not cointegrate with any of the other European countries at most of the seasonal frequencies. On the other hand, the results of the tests with $m = 10$ and 15 on the trivariate series of French, Italian and Spanish industrial production provide evidence for a single non-stationary component at each frequency except $\pi/3$ (the rejection of $K = 1$ in the joint test is also influenced by the outcome at $\pi/3)$. This view is also supported by looking at the pairwise analyses of France-Italy, France-Spain and Italy-Spain. Note that the corresponding results for non-prefiltered data in Table 3.A show much more evidence of seasonal cointegration; however, as previously explained, these outcomes are likely to be an artifact due to the presence of unattended unit roots.

Finally, we also present the results for the Nyblom-Harvey tests at frequency zero in order to understand whether there are also co-movements in the stochastic trend components of the series. The results are contained in Table 4 for $m = 3, 6, 9, 12, 15$; the prefiltered data are obtained by applying the seasonal sum filter $1 + L + L^2 + ... + L^{11}$. It is interesting to see that there is less evidence of cointegration at frequency zero than at the seasonal frequencies. In fact, the results for the prefiltered data with $m = 9$ indicates a 5% rejection of the null hypothesis of two non-stationary trends among the four series, which would imply the presence of a single cointegration vector. In particular, it is worth noticing that the three countries that seemed characterized by co-movements at the seasonal frequencies, France, Italy and Spain, appear, on the other hand, to have their own idiosyncratic trends.

8. Concluding remarks

The paper has proposed tests of seasonal integration and cointegration in the framework of multivariate unobserved component models. The tests have been derived under the assumption of Gaussian white noise disturbances and then extended to models with stochastic trends, weakly dependent errors and unattended unit roots. The finite sample properties of the tests have been investigated by Monte Carlo simulation experiments. The Monte Carlo results point to the practical advice of prefiltering the data to avoid large power reduction due to unattended unit roots, and confirm analogous findings for univariate series in Busetti and Taylor (2003) and Taylor (2003). Prefiltering seems particularly relevant for the seasonal cointegration test, whose power is not so large in a sample of 100 observations.

The application of the tests to the prefiltered series of industrial production across the main countries of the European Monetary Union has provided evidence of seasonal cointegration with a single common component for France, Italy and Spain. Germany, on the other hand, seems to be characterized by its own idiosyncratic seasonal pattern. Much more evidence of cointegration has emerged from the same tests but without prefiltering the data; however, in the light of the theoretical arguments and simulation results on the effect of unattended unit roots, that evidence is likely to be related to the limited power of the tests in this context.

Appendix: proofs of the propositions

Proof of proposition 1: The proof amounts to first showing that (2)-(7) belongs to the class of models considered by Nyblom and Harvey (2000) and then applying their theorem A.1.

Let $\mathbf{x}_t^* = (\mathbf{x}_t', \mathbf{z}_t(1)', ..., \mathbf{z}_t([s/2])')'$, $t = 1, ..., T$, be the $p^* \times 1$ augmented vector of regressors, with $p^* = p + s - 1$, and $\boldsymbol{\beta}_i^* = (\boldsymbol{\beta}_i', \boldsymbol{\gamma}_{i0}(1)', ..., \boldsymbol{\gamma}_{i0}([s/2])')'$, $i = 1, ..., N$, the corresponding $p^* \times 1$ vector of coefficients. Write (2)-(7) as

(20)
$$\underset{(T \times N)}{\mathbf{Y}} = \underset{(T \times p^*)}{\mathbf{X}^*} \underset{(p^* \times N)}{\mathbf{B}} + \underset{(T \times N)}{\mathbf{U}}$$

where $\mathbf{Y} = (\mathbf{y}_1, ..., \mathbf{y}_T)'$, $\mathbf{X}^* = (\mathbf{x}_1^*, ..., \mathbf{x}_T^*)'$, $\mathbf{B} = (\boldsymbol{\beta}_1^*, ..., \boldsymbol{\beta}_N^*)$, $\mathbf{U} = (\mathbf{u}_1, ..., \mathbf{u}_T)'$ with, for $t = 1, ..., T$, $\mathbf{u}_t = \sum_{h=1}^{[s/2]} \mathbf{Z}_t(h)(\boldsymbol{\gamma}_t(h) - \boldsymbol{\gamma}_0(h)) + \boldsymbol{\varepsilon}_t$. Clearly,

$$E(\mathbf{u}_t\mathbf{u}_s') = \sum_{h=1}^{[s/2]} \mathbf{Z}_t(h)\boldsymbol{\Sigma}_\eta(h)\mathbf{Z}_s(h)' \min(t,s) + 1(t = s)\boldsymbol{\Sigma}_\varepsilon, t, s = 1, ..., T.$$

Then, under $\mathrm{H}_A : \boldsymbol{\Sigma}_\eta(h) = q^2(\boldsymbol{\Sigma}_\varepsilon \otimes \mathrm{I}_{a_h})$ and assuming $\boldsymbol{\Sigma}_\eta(l) = 0$ for $l \neq h$, the covariance matrix of $vec(\mathbf{U})$ is given by

(21)
$$\boldsymbol{\Sigma}_\varepsilon \otimes \left(\mathrm{I}_T + q^2\mathbf{G}(h)\right),$$

where $\mathbf{G}(h)$ is defined by

$$[\mathbf{G}(h)]_{ts} = \min(t,s)\cos(\lambda(h)(t-s)), \qquad t, s = 1, ..., T.$$

Under Gaussianity, theorem A.1 of Nyblom and Harvey (2000) can be applied to the model (20) with covariance structure (21). This immediately yields the LBI statistic

(22)
$$trace\left\{(\mathbf{E}'\mathbf{E})^{-1}(\mathbf{E}'\mathbf{G}(h)\mathbf{E})\right\},$$

where $\mathbf{E} = (\mathbf{e}_1, ..., \mathbf{e}_T)' = \mathbf{Y} - \mathbf{X}^*(\mathbf{X}^{*\prime}\mathbf{X}^*)^{-1}\mathbf{X}^{*\prime}\mathbf{Y}$ are the OLS residuals from the multivariate regression (20). The test is locally most powerful and invariant under the group of affine linear transformations

$$\mathbf{Y} \longmapsto \mathbf{YP} + \mathbf{X}^*\mathbf{A},$$

where **P** is an arbitrary non-singular $N \times N$ matrix and **A** is an arbitrary $p^* \times N$ matrix.

Using standard trigonometric identities, it can be verified that (22) can be rewritten as

$$Ta(h)\, trace \left\{ \widehat{\boldsymbol{\Sigma}}_{\varepsilon}^{-1} \mathbf{C}(h) \right\},$$

where $\widehat{\boldsymbol{\Sigma}}_{\varepsilon} = T^{-1} \sum_{t=1}^{T} \mathbf{e}_t \mathbf{e}_t'$, $\mathbf{C}(h) = T^{-2} \sum_{t=1}^{T} \left(\mathbf{S}_t^A(h) \mathbf{S}_t^A(h)' + \mathbf{S}_t^B(h) \mathbf{S}_t^B(h)' \right)$, $\mathbf{S}_t^A(h) = \sum_{s=1}^{t} \mathbf{e}_s \cos \lambda(h)s$, $\mathbf{S}_t^B(h) = \sum_{s=1}^{t} \mathbf{e}_s \sin \lambda(h)s$. On dividing by T, we then obtain the expression given in the proposition.

Proof of proposition 2: By the functional central limit theorem of Chan and Wei (1988) and using the same arguments as proposition 1.1 of Busetti and Harvey (2003), under H_0 $T^{-\frac{1}{2}} \mathbf{S}_{[Tr]}^A(h) \Rightarrow \mathbf{W}_N^A(r; \boldsymbol{\Sigma}_{\varepsilon})$ and, for $h \neq s/2$, $T^{-\frac{1}{2}} \mathbf{S}_{[Tr]}^B(h) \Rightarrow \mathbf{W}_N^B(r; \boldsymbol{\Sigma}_{\varepsilon})$, where the notation $\Rightarrow$ indicates weak convergence of the associated probability measure and $\mathbf{W}_N^A(r; \boldsymbol{\Sigma}_{\varepsilon})$, $\mathbf{W}_N^B(r; \boldsymbol{\Sigma}_{\varepsilon})$ are independent N-dimensional Wiener processes with variance $\boldsymbol{\Sigma}_{\varepsilon}$. As under the null hypothesis $\widehat{\boldsymbol{\Sigma}}_{\varepsilon} \xrightarrow{p} \boldsymbol{\Sigma}_{\varepsilon}$, the proposition follows by an application of the Continuous Mapping Theorem and using the result that the sum of two independent random variables with a Cramér-von Mises distribution with a and b degrees of freedom respectively follows a Cramér-von Mises distribution with $a + b$ degrees of freedom (see Busetti and Harvey, 2001, p.136).

Proof of proposition 3: Since there exists a non-singular matrix **P** such that $\mathbf{P}\boldsymbol{\Sigma}_{\varepsilon}\mathbf{P}' = I$ and $\mathbf{P}\boldsymbol{\Sigma}_{\eta}(h)\mathbf{P}' = diag(q_1, ..., q_N)$, see Rao(1973, p.41), and the test is invariant to premultiplication of the observations by an arbitrary $N \times N$ matrix, without loss of generality we can consider, throughout this proof, the case $\boldsymbol{\Sigma}_{\varepsilon} = \mathbf{I}$ and $\boldsymbol{\Sigma}_{\eta}(h) = diag(q_1, ..., q_N)$.

Partition the $N \times 1$ vector $\mathbf{e}_t$ as $\mathbf{e}_t = (\mathbf{e}_{1t}', \mathbf{e}_{2t}')'$, where the two components have dimensions $K \times 1$ and $(N - K) \times 1$ respectively. Without loss of generality, under $H_{0,K}$: $rank(\boldsymbol{\Sigma}_{\eta}(h)) = K$, we can assume that $\boldsymbol{\Sigma}_{\eta}(h) = diag(q_1, ..., q_K, 0, ..., 0)$, i.e. that the first

K components of the observation vector $\mathbf{y}_t$ have non-stationary stochastic seasonality and that the remaining $N-K$ components have deterministic seasonality.

From a simple extension of the arguments of Theorem B1 of Nyblom and Harvey (2000), it follows that, under $\mathrm{H}_{0,K}$, the K largest eigenvalues of $a(h)\widehat{\boldsymbol{\Sigma}}_{\varepsilon}{}^{-1}\mathbf{C}(h)$ are $O_p(T)$ while the sum of the $N-K$ smallest eigenvalues, $\xi_{K,N}(h)$, is $O_p(1)$ and asymptotically equivalent to

$$a(h)Tr\left(\mathbf{C}_{22}(h)-\mathbf{C}_{21}(h)\mathbf{C}_{11}(h)^{-1}\mathbf{C}_{12}(h)\right),$$

where

$$\mathbf{C}_{ij}=\sum_{t=1}^{T}\left(\mathbf{S}_{it}^{A}(h)\mathbf{S}_{jt}^{A}(h)'+\mathbf{S}_{it}^{B}(h)\mathbf{S}_{jt}^{B}(h)'\right),\quad i,j=1,2,$$

and $\mathbf{S}_{it}^{A}(h)=\sum_{s=1}^{t}\mathbf{e}_{is}\cos\lambda(h)s$, $\mathbf{S}_{it}^{B}(h)=\sum_{s=1}^{t}\mathbf{e}_{is}\sin\lambda(h)s$, $i=1,2.$

By the functional central limit theorem of Chan and Wei (1988) and the continuous mapping theorem, we have the following asymptotic results:

$$\begin{aligned}
T^{-\frac{3}{2}}\sqrt{a(h)}\mathbf{Q}^{-\frac{1}{2}}\mathbf{S}_{1[Tr]}^{A}(h) &\Rightarrow \int_0^r \overline{\mathbf{W}}_K^{A}(s)ds,\\
T^{-\frac{3}{2}}\sqrt{a(h)}\mathbf{Q}^{-\frac{1}{2}}\mathbf{S}_{1[Tr]}^{B}(h) &\Rightarrow \int_0^r \overline{\mathbf{W}}_K^{B}(s)ds,\quad h\neq s/2,\\
T^{-\frac{1}{2}}\sqrt{a(h)}\mathbf{S}_{2[Tr]}^{A}(h) &\Rightarrow \mathbf{B}_R^{A}(r),\\
T^{-\frac{1}{2}}\sqrt{a(h)}\mathbf{S}_{2[Tr]}^{B}(h) &\Rightarrow \mathbf{B}_R^{B}(r),\quad h\neq s/2,
\end{aligned}$$

where, for $i=A,B$, $\overline{\mathbf{W}}_K^{i}(r)=\mathbf{W}_K^{i}(r)-\int_0^1\mathbf{W}_K^{i}(r)dr$, $\mathbf{W}_K^{i}(r)$ are indepemdent K-dimensional Wiener processes and $\mathbf{B}_R^{i}(r)$ independent R-dimensional Brownian bridges, and $\mathbf{Q}=diag(q_1,...,q_K)$. Then the proposition follows by an application of the Continuous Mapping Theorem.

		K=0			K=1			K=2			K=3			K=4			K=5		
	N	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99	0.90	0.95	0.99
one frequency (≠π for s even)	1	0.608	0.749	1.074															
	2	1.065	1.223	1.603	0.447	0.567	0.858												
	3	1.479	1.683	2.082	0.795	0.932	1.244	0.296	0.377	0.603									
	4	1.885	2.111	2.560	1.103	1.262	1.604	0.540	0.656	0.912	0.215	0.264	0.409						
	5	2.270	2.510	2.992	1.416	1.588	1.948	0.768	0.901	1.203	0.388	0.452	0.649	0.165	0.197	0.286			
	6	2.683	2.949	3.439	1.709	1.899	2.266	0.989	1.135	1.493	0.553	0.644	0.843	0.300	0.346	0.453	0.132	0.157	0.224
all frequencies (s=4)	1	0.846	1.000	1.351															
	2	1.479	1.683	2.082	0.561	0.675	0.978												
	3	2.102	2.325	2.800	1.006	1.159	1.517	0.363	0.447	0.659									
	4	2.683	2.949	3.439	1.412	1.596	1.979	0.663	0.778	1.053	0.255	0.308	0.454						
	5	3.255	3.543	4.066	1.811	2.018	2.468	0.947	1.082	1.381	0.473	0.544	0.739	0.196	0.229	0.323			
	6	3.846	4.144	4.689	2.218	2.432	2.884	1.218	1.385	1.747	0.675	0.761	0.972	0.364	0.410	0.523	0.158	0.184	0.248
all frequencies (s=12)	1	2.484	2.718	3.242															
	2	4.594	4.907	5.534	1.796	1.995	2.423												
	3	6.619	7.009	7.770	3.329	3.578	4.086	1.241	1.382	1.702									
	4	8.642	9.088	9.911	4.800	5.093	5.678	2.322	2.516	2.891	0.904	0.996	1.217						
	5	10.647	11.111	12.020	6.235	6.575	7.280	3.372	3.587	4.009	1.696	1.821	2.114	0.695	0.754	0.912			
	6	12.611	13.142	14.064	7.695	8.073	8.843	4.384	4.645	5.127	2.469	2.630	2.982	1.316	1.403	1.607	0.564	0.609	0.714

Table 1. Asymptotic critical values for the tests of seasonal integration and cointegration.

		q_1					
		0	0.025	0.050	0.075	0.100	0.500
q_2=0	(1) $\ell_1(1)+\ell_2(1)$, level	5.94	13.80	40.11	67.41	83.85	99.99
	(2) $\ell_1(1)+\ell_2(1)$, prefiltered	5.35	12.19	35.63	61.80	78.92	99.77
	(3) $\ell_1(1)+\ell_2(1)+\ell_1(2)+\ell_2(2)$, level	5.63	11.55	33.63	60.53	78.67	99.97
	(4) $\ell_2(1)$, level	0.30	0.87	4.56	12.97	24.12	94.20
	(5) $\ell_2(1)$, prefiltered	0.23	0.69	3.41	9.99	18.86	76.81
	(6) $\ell_2(1)+\ell_2(2)$, level	0.19	0.57	3.43	10.54	20.45	91.33
q_2=0.1	(1) $\ell_1(1)+\ell_2(1)$, level	3.21	8.64	30.70	58.97	78.21	99.99
	(2) $\ell_1(1)+\ell_2(1)$, prefiltered	5.38	12.18	35.43	61.66	78.75	99.77
	(3) $\ell_1(1)+\ell_2(1)+\ell_1(2)+\ell_2(2)$, level	69.67	73.53	82.73	90.73	95.28	99.99
	(4) $\ell_2(1)$, level	0.14	0.47	2.88	9.42	19.09	92.76
	(5) $\ell_2(1)$, prefiltered	0.22	0.66	3.38	9.96	18.74	76.78
	(6) $\ell_2(1)+\ell_2(2)$, level	1.20	1.77	4.65	11.37	20.77	90.36
q_2=0.5	(1) $\ell_1(1)+\ell_2(1)$, level	0.04	0.16	1.73	8.61	22.19	99.11
	(2) $\ell_1(1)+\ell_2(1)$, prefiltered	5.36	11.29	32.30	57.80	75.77	99.74
	(3) $\ell_1(1)+\ell_2(1)+\ell_1(2)+\ell_2(2)$, level	99.03	99.13	99.41	99.66	99.81	100.00
	(4) $\ell_2(1)$, level	0.00	0.00	0.03	0.23	1.07	63.99
	(5) $\ell_2(1)$, prefiltered	0.22	0.59	2.90	8.64	16.69	75.84
	(6) $\ell_2(1)+\ell_2(2)$, level	45.83	45.98	46.69	47.93	49.85	78.56

Table 2.A. Percentage rejection frequencies under DGP (A).

		q_1					
		0	0.025	0.050	0.075	0.100	0.500
$q_2=0$	(1) $\ell_1(1)+\ell_2(1)$, level	5.94	13.86	38.96	64.96	81.40	99.99
	(2) $\ell_1(1)+\ell_2(1)$, prefiltered	5.35	12.21	34.51	59.08	75.70	99.65
	(3) $\ell_1(1)+\ell_2(1)+\ell_1(2)+\ell_2(2)$, level	5.63	11.55	32.91	58.15	75.88	99.94
	(4) $\ell_2(1)$, level	0.30	0.86	3.70	9.58	17.99	90.77
	(5) $\ell_2(1)$, prefiltered	0.23	0.63	2.73	7.36	14.03	73.05
	(6) $\ell_2(1)+\ell_2(2)$, level	0.19	0.56	2.62	7.51	14.83	87.17
$q_2=0.1$	(1) $\ell_1(1)+\ell_2(1)$, level	3.21	8.73	30.15	56.56	75.29	99.98
	(2) $\ell_1(1)+\ell_2(1)$, prefiltered	5.38	12.14	34.37	58.91	75.56	99.65
	(3) $\ell_1(1)+\ell_2(1)+\ell_1(2)+\ell_2(2)$, level	69.67	73.55	82.34	89.99	94.52	99.99
	(4) $\ell_2(1)$, level	0.14	0.43	2.24	6.67	13.68	88.56
	(5) $\ell_2(1)$, prefiltered	0.22	0.63	2.71	7.32	13.96	73.01
	(6) $\ell_2(1)+\ell_2(2)$, level	1.20	1.67	3.84	8.46	15.35	85.67
$q_2=0.5$	(1) $\ell_1(1)+\ell_2(1)$, level	0.04	0.18	1.90	8.94	21.73	98.72
	(2) $\ell_1(1)+\ell_2(1)$, prefiltered	5.36	11.19	31.43	55.17	72.57	99.62
	(3) $\ell_1(1)+\ell_2(1)+\ell_1(2)+\ell_2(2)$, level	99.03	99.14	99.41	99.64	99.79	100.00
	(4) $\ell_2(1)$, level	0.00	0.00	0.01	0.13	0.54	53.63
	(5) $\ell_2(1)$, prefiltered	0.22	0.57	2.32	6.33	12.29	71.75
	(6) $\ell_2(1)+\ell_2(2)$, level	45.83	45.98	46.46	47.20	48.24	70.88

Table 2.B. Percentage rejection frequencies under DGP (B).

		q_1					
		0	0.025	0.050	0.075	0.100	0.500
$q_2=0$	(1) $\ell_1(1)+\ell_2(1)$, level	5.94	11.14	26.44	41.96	53.78	92.29
	(2) $\ell_1(1)+\ell_2(1)$, prefiltered	5.35	9.61	22.56	36.32	46.89	78.85
	(3) $\ell_1(1)+\ell_2(1)+\ell_1(2)+\ell_2(2)$, level	5.63	9.48	22.65	37.25	48.80	88.86
	(4) $\ell_2(1)$, level	0.30	0.61	1.35	2.04	2.58	5.08
	(5) $\ell_2(1)$, prefiltered	0.23	0.46	0.99	1.62	2.06	3.85
	(6) $\ell_2(1)+\ell_2(2)$, level	0.19	0.39	0.91	1.42	1.81	3.22
$q_2=0.1$	(1) $\ell_1(1)+\ell_2(1)$, level	3.21	6.86	19.91	35.08	47.22	90.88
	(2) $\ell_1(1)+\ell_2(1)$, prefiltered	5.38	9.59	22.48	36.22	46.82	78.87
	(3) $\ell_1(1)+\ell_2(1)+\ell_1(2)+\ell_2(2)$, level	69.67	72.21	77.70	82.64	86.15	96.78
	(4) $\ell_2(1)$, level	0.14	0.31	0.71	1.15	1.51	3.32
	(5) $\ell_2(1)$, prefiltered	0.22	0.47	1.00	1.59	2.02	3.81
	(6) $\ell_2(1)+\ell_2(2)$, level	1.20	1.49	2.02	2.49	2.83	2.83
$q_2=0.5$	(1) $\ell_1(1)+\ell_2(1)$, level	0.04	0.15	1.54	6.10	13.08	74.73
	(2) $\ell_1(1)+\ell_2(1)$, prefiltered	5.36	9.02	20.72	34.13	44.80	78.50
	(3) $\ell_1(1)+\ell_2(1)+\ell_1(2)+\ell_2(2)$, level	99.03	99.11	99.28	99.42	99.53	99.91
	(4) $\ell_2(1)$, level	0.00	0.00	0.00	0.01	0.02	0.08
	(5) $\ell_2(1)$, prefiltered	0.22	0.44	0.92	1.45	1.89	3.66
	(6) $\ell_2(1)+\ell_2(2)$, level	45.83	45.90	45.95	45.67	45.10	25.02

Table 2.C. Percentage rejection frequencies under DGP (C).

		$\lambda=\pi/6$			$\lambda=\pi/3$			$\lambda=\pi/2$			$\lambda=2\pi/3$			$\lambda=5\pi/6$			$\lambda=\pi$			all λ's		
		m=5	m=10	m=15	m=5	m=10	m=15	m=5	m=10	m=15	m=5	m=10	m=15	m=5	m=10	m=15	m=5	m=10	m=15	m=5	m=10	m=15
GE-FR	K=0	2.861	3.144	2.822	2.795	2.778	2.413	2.395	3.237	2.602	4.023	2.931	2.360	1.552	1.813	1.503	2.398	1.540	1.140	16.024	15.443	12.839
	K=1	0.222	0.390	0.437	0.733	0.669	0.587	0.347	0.634	0.490	0.534	0.561	0.502	0.131	0.314	0.251	0.057	0.081	0.074	2.023	2.650	2.341
GE-IT	K=0	3.036	3.378	3.209	3.384	3.925	3.841	2.853	2.555	2.093	3.111	2.754	2.263	2.420	2.762	2.285	2.574	1.682	1.224	17.378	17.056	14.915
	K=1	0.461	0.608	0.629	1.172	1.097	0.891	0.230	0.306	0.289	0.473	0.541	0.504	0.283	0.519	0.377	0.120	0.111	0.089	2.738	3.182	2.779
GE-SP	K=0	3.328	3.387	3.333	2.871	3.077	2.888	3.206	3.610	3.256	2.028	2.305	1.883	4.644	3.453	2.595	2.125	1.568	1.151	18.201	17.399	15.107
	K=1	0.401	0.470	0.543	0.495	0.551	0.538	0.654	0.748	0.648	0.450	0.551	0.468	0.278	0.404	0.323	0.101	0.192	0.143	2.379	2.916	2.663
FR-IT	K=0	2.176	2.688	2.583	2.164	2.617	2.309	2.147	2.512	1.875	3.774	2.945	2.240	2.031	2.517	2.049	1.049	0.896	0.702	13.341	14.174	11.759
	K=1	0.352	0.475	0.487	0.369	0.427	0.421	0.213	0.287	0.249	0.348	0.285	0.240	0.202	0.302	0.287	0.045	0.046	0.041	1.529	1.821	1.725
FR-SP	K=0	2.650	2.669	2.450	2.024	2.351	1.957	2.230	2.443	1.849	3.388	2.830	2.131	4.423	3.226	2.447	0.619	0.695	0.586	15.334	14.215	11.419
	K=1	0.112	0.182	0.233	0.153	0.154	0.161	0.085	0.178	0.145	0.422	0.509	0.394	0.185	0.300	0.272	0.101	0.191	0.143	1.058	1.514	1.349
IT-SP	K=0	2.756	2.681	2.463	2.465	3.292	3.151	2.671	2.599	2.038	2.396	2.541	1.985	4.516	3.320	2.477	1.182	0.954	0.730	15.987	15.388	12.843
	K=1	0.245	0.271	0.273	0.533	0.696	0.687	0.353	0.329	0.289	0.221	0.293	0.258	0.123	0.185	0.170	0.100	0.187	0.140	1.575	1.961	1.818
GE-FR-IT	K=0	3.456	3.846	3.716	3.773	4.360	4.266	3.239	3.752	2.926	4.922	3.865	3.097	2.660	3.076	2.529	2.704	1.752	1.286	20.754	20.652	17.820
	K=1	0.641	0.904	0.952	1.370	1.298	1.093	0.501	0.834	0.670	0.869	0.849	0.763	0.421	0.702	0.564	0.163	0.163	0.138	3.965	4.750	4.181
	K=2	0.167	0.293	0.318	0.162	0.173	0.167	0.154	0.180	0.173	0.316	0.275	0.232	0.082	0.172	0.168	0.041	0.046	0.041	0.921	1.139	1.098
GE-FR-SP	K=0	3.528	3.682	3.664	3.274	3.459	3.176	3.449	4.155	3.712	4.601	3.874	3.118	4.821	3.718	2.832	2.514	1.734	1.287	22.188	20.622	17.790
	K=1	0.505	0.668	0.777	0.838	0.801	0.727	0.742	0.925	0.776	1.002	1.122	0.992	0.453	0.662	0.555	0.152	0.251	0.198	3.692	4.429	4.025
	K=2	0.103	0.179	0.226	0.083	0.092	0.102	0.070	0.158	0.124	0.385	0.443	0.338	0.100	0.245	0.213	0.050	0.058	0.053	0.790	1.174	1.056
FR-IT-SP	K=0	3.053	3.174	3.067	2.682	3.546	3.363	2.834	3.037	2.343	4.097	3.458	2.619	4.740	3.660	2.800	1.331	1.112	0.872	18.737	17.986	15.063
	K=1	0.437	0.600	0.658	0.679	0.849	0.845	0.454	0.522	0.450	0.636	0.675	0.546	0.327	0.514	0.482	0.150	0.240	0.185	2.682	3.400	3.165
	K=2	0.081	0.114	0.133	0.128	0.126	0.130	0.080	0.169	0.138	0.149	0.159	0.145	0.119	0.177	0.161	0.044	0.044	0.040	0.602	0.789	0.747
GE-FR-IT-SP	K=0	3.946	4.257	4.545	4.161	5.007	5.174	4.102	4.686	4.111	5.320	4.570	3.798	5.126	4.232	3.221	2.846	1.944	1.431	25.501	24.696	22.281
	K=1	0.810	1.086	1.191	1.462	1.398	1.203	1.053	1.174	0.991	1.177	1.261	1.138	0.578	0.906	0.747	0.258	0.338	0.265	5.338	6.164	5.535
	K=2	0.271	0.394	0.438	0.224	0.249	0.256	0.225	0.329	0.289	0.540	0.582	0.471	0.194	0.376	0.349	0.136	0.126	0.107	1.589	2.055	1.911
	K=3	0.080	0.101	0.115	0.059	0.075	0.088	0.066	0.131	0.106	0.124	0.135	0.130	0.069	0.097	0.098	0.037	0.042	0.039	0.435	0.580	0.575

Table 3.A. Results of the tests for non prefiltered data. Shaded figures indicate rejection at 5% significance level.

		$\lambda=\pi/6$			$\lambda=\pi/3$			$\lambda=\pi/2$			$\lambda=2\pi/3$			$\lambda=5\pi/6$			$\lambda=\pi$			all λ's		
		m=5	m=10	m=15	m=5	m=10	m=15	m=5	m=10	m=15	m=5	m=10	m=15	m=5	m=10	m=15	m=5	m=10	m=15	m=5	m=10	m=15
GE-FR	K=0	3.932	3.543	3.046	6.691	4.128	3.341	6.994	4.459	3.585	5.238	3.131	2.451	3.509	2.232	1.802	2.836	1.615	1.167	29.200	19.108	15.391
	K=1	0.857	0.781	0.653	1.560	0.936	0.709	1.323	0.823	0.647	0.984	0.641	0.522	0.611	0.417	0.360	0.201	0.134	0.114	5.536	3.732	3.006
GE-IT	K=0	3.715	3.566	3.242	11.203	7.177	6.123	4.794	2.905	2.222	5.587	3.422	2.656	5.384	3.482	2.777	3.017	1.705	1.224	33.701	22.257	18.244
	K=1	1.051	0.937	0.783	2.288	1.334	0.982	0.575	0.367	0.301	1.030	0.677	0.554	0.997	0.632	0.501	0.182	0.110	0.086	6.123	4.057	3.206
GE-SP	K=0	3.685	3.681	3.647	7.778	4.852	4.066	8.804	5.737	4.861	4.317	2.662	2.072	6.339	3.746	2.798	2.960	1.697	1.239	33.883	22.375	18.684
	K=1	0.724	0.820	0.722	1.423	0.895	0.710	1.742	1.082	0.838	0.930	0.597	0.482	0.791	0.517	0.424	0.325	0.204	0.165	5.936	4.116	3.339
FR-IT	K=0	3.865	3.268	2.756	6.428	4.051	3.282	4.926	2.958	2.246	5.617	3.249	2.399	4.909	3.061	2.366	1.644	0.993	0.763	27.390	17.580	13.813
	K=1	0.755	0.609	0.534	1.124	0.731	0.598	0.615	0.390	0.314	0.502	0.308	0.246	0.686	0.458	0.368	0.125	0.079	0.064	3.807	2.574	2.124
FR-SP	K=0	3.539	3.013	2.552	5.437	3.233	2.446	5.371	3.271	2.541	5.291	3.109	2.340	5.930	3.460	2.536	1.381	0.879	0.709	26.949	16.965	13.125
	K=1	0.375	0.291	0.285	0.407	0.272	0.236	0.512	0.356	0.305	0.911	0.546	0.420	0.596	0.379	0.301	0.336	0.212	0.171	3.137	2.057	1.719
IT-SP	K=0	3.701	3.197	2.653	9.915	6.862	6.104	5.545	3.308	2.516	5.018	2.989	2.261	6.042	3.531	2.592	1.676	1.004	0.772	31.897	20.890	16.897
	K=1	0.477	0.363	0.327	1.831	1.161	0.906	0.813	0.500	0.397	0.529	0.347	0.289	0.343	0.240	0.207	0.336	0.212	0.172	4.328	2.824	2.299
GE-FR-IT	K=0	4.769	4.354	3.858	12.867	8.457	7.549	7.811	4.929	3.946	7.371	4.433	3.435	6.034	3.874	3.106	3.182	1.816	1.318	42.034	27.863	23.211
	K=1	1.554	1.357	1.146	2.802	1.666	1.252	1.733	1.084	0.860	1.542	0.994	0.815	1.385	0.902	0.730	0.343	0.218	0.179	9.359	6.221	4.983
	K=2	0.500	0.399	0.350	0.435	0.282	0.232	0.361	0.231	0.189	0.498	0.305	0.243	0.352	0.252	0.224	0.119	0.073	0.058	2.265	1.542	1.295
GE-FR-SP	K=0	4.502	4.267	4.094	8.706	5.373	4.435	10.434	7.126	6.538	7.120	4.296	3.363	6.861	4.101	3.112	3.211	1.853	1.360	40.835	27.015	22.901
	K=1	1.230	1.169	1.027	1.869	1.146	0.896	2.104	1.330	1.049	2.067	1.335	1.110	1.283	0.844	0.703	0.459	0.295	0.243	9.013	6.119	5.028
	K=2	0.364	0.291	0.284	0.224	0.157	0.144	0.360	0.245	0.207	0.737	0.445	0.339	0.462	0.306	0.253	0.110	0.073	0.062	2.258	1.517	1.289
FR-IT-SP	K=0	4.379	3.784	3.269	10.393	7.177	6.374	6.707	4.105	3.232	6.488	3.773	2.802	6.820	4.048	3.009	1.996	1.216	0.944	36.782	24.103	19.631
	K=1	1.062	0.849	0.765	2.202	1.406	1.117	1.283	0.832	0.688	1.131	0.691	0.545	1.051	0.703	0.575	0.424	0.269	0.219	7.153	4.750	3.908
	K=2	0.268	0.201	0.193	0.338	0.225	0.194	0.420	0.286	0.239	0.216	0.143	0.123	0.303	0.206	0.175	0.088	0.056	0.046	1.634	1.117	0.970
GE-FR-IT-SP	K=0	5.232	4.972	4.792	16.172	10.983	10.094	11.905	8.170	7.677	8.840	5.454	4.387	7.903	4.776	3.644	3.514	2.026	1.488	53.565	36.381	32.084
	K=1	1.807	1.617	1.386	3.006	1.796	1.363	2.598	1.631	1.293	2.377	1.570	1.328	1.737	1.140	0.938	0.609	0.386	0.315	12.134	8.140	6.623
	K=2	0.741	0.568	0.512	0.558	0.372	0.320	0.702	0.465	0.388	0.978	0.598	0.466	0.703	0.489	0.430	0.238	0.148	0.119	3.920	2.641	2.235
	K=3	0.143	0.121	0.119	0.121	0.089	0.087	0.264	0.168	0.136	0.215	0.143	0.122	0.178	0.129	0.115	0.081	0.053	0.045	1.001	0.703	0.623

Table 3.B. Results of the tests for prefiltered data. Shaded figures indicate rejection at 5% significance level.

		standard					prefiltered				
		m=3	m=6	m=9	m=12	m=15	m=3	m=6	m=9	m=12	m=15
GE-FR	K=0	1.202	0.741	0.546	0.436	0.366	1.378	0.799	0.571	0.451	0.378
	K=1	0.427	0.256	0.188	0.150	0.127	0.469	0.273	0.196	0.156	0.132
GE-IT	K=0	0.663	0.406	0.301	0.244	0.209	0.735	0.430	0.311	0.249	0.212
	K=1	0.208	0.135	0.103	0.085	0.074	0.243	0.144	0.105	0.085	0.074
GE-SP	K=0	1.152	0.697	0.508	0.405	0.341	1.308	0.759	0.543	0.429	0.360
	K=1	0.442	0.264	0.192	0.154	0.130	0.478	0.279	0.200	0.159	0.134
FR-IT	K=0	1.041	0.642	0.475	0.381	0.323	1.102	0.643	0.463	0.367	0.310
	K=1	0.230	0.149	0.114	0.095	0.083	0.255	0.151	0.111	0.090	0.079
FR-SP	K=0	0.942	0.602	0.454	0.371	0.318	1.191	0.698	0.506	0.405	0.343
	K=1	0.288	0.204	0.163	0.138	0.123	0.437	0.261	0.192	0.157	0.135
IT-SP	K=0	1.146	0.715	0.532	0.429	0.364	1.266	0.739	0.531	0.422	0.355
	K=1	0.235	0.152	0.116	0.097	0.084	0.267	0.158	0.116	0.095	0.082
GE-FR-IT	K=0	1.635	1.015	0.750	0.602	0.508	1.804	1.050	0.753	0.596	0.500
	K=1	0.634	0.390	0.290	0.235	0.201	0.709	0.415	0.300	0.241	0.206
	K=2	0.207	0.133	0.102	0.084	0.073	0.240	0.142	0.104	0.084	0.073
GE-FR-SP	K=0	1.290	0.806	0.607	0.497	0.430	1.504	0.881	0.638	0.512	0.438
	K=1	0.510	0.321	0.245	0.204	0.181	0.589	0.349	0.257	0.210	0.184
	K=2	0.055	0.048	0.046	0.045	0.046	0.095	0.061	0.049	0.045	0.045
FR-IT-SP	K=0	1.446	0.928	0.705	0.577	0.497	1.726	1.014	0.735	0.589	0.500
	K=1	0.534	0.365	0.287	0.241	0.212	0.709	0.421	0.310	0.252	0.217
	K=2	0.203	0.132	0.100	0.083	0.073	0.237	0.140	0.103	0.084	0.073
GE-FR-IT-SP	K=0	1.742	1.076	0.799	0.647	0.555	1.903	1.112	0.802	0.641	0.545
	K=1	0.697	0.438	0.334	0.278	0.247	0.797	0.472	0.347	0.284	0.249
	K=2	0.241	0.164	0.132	0.115	0.107	0.299	0.180	0.136	0.115	0.105
	K=3	0.031	0.029	0.029	0.030	0.033	0.059	0.038	0.032	0.031	0.032

Table 4. Results of the Nyblom-Harvey test (at frequency zero). Shaded figures indicate rejection at 5% significance level.

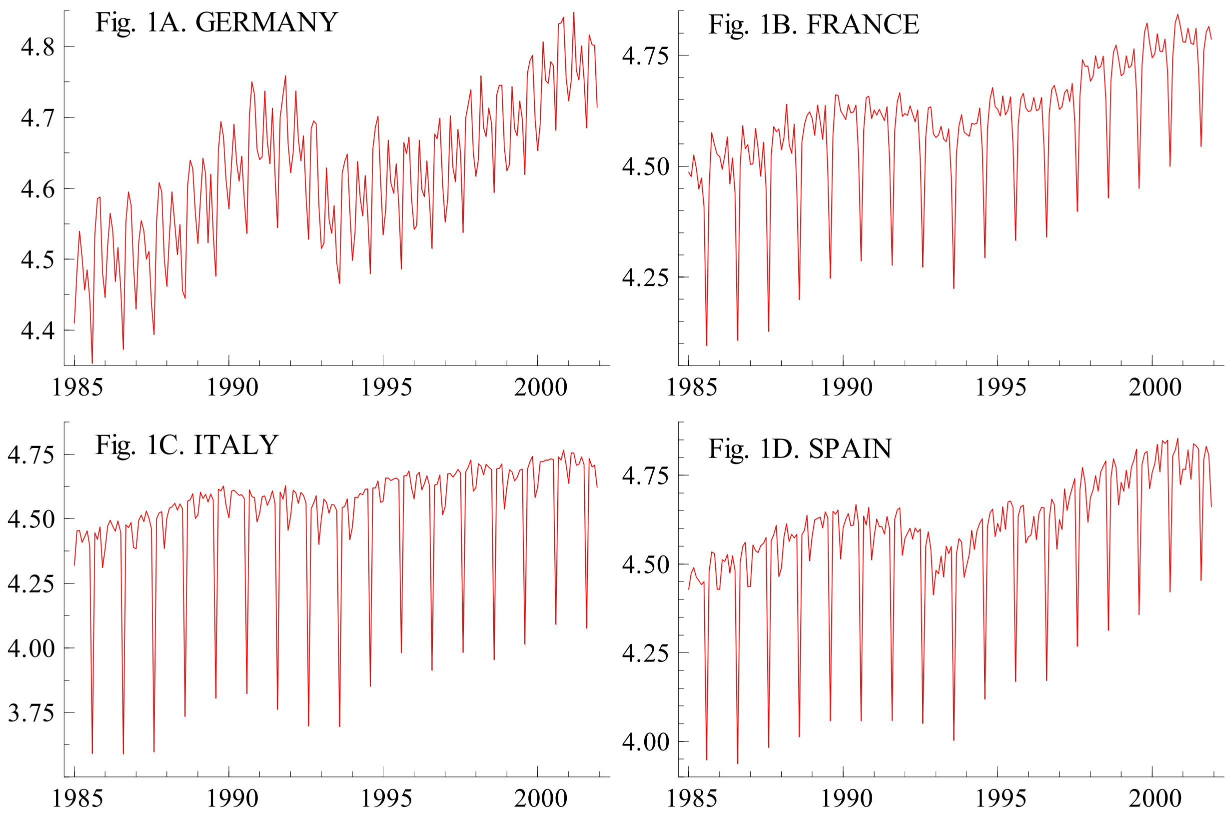

Fig. 1. Logarithm of the index of Industrial Production for Germany, France, Italy and Spain. 1985M1-2001M12. Source: Eurostat.

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