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TESTING AGAINST STOCHASTIC TREND AND SEASONALITY IN THE PRESENCE OF UNATTENDED BREAKS AND UNIT ROOTS

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Abstract

This paper considers the problem of testing against stochastic trend and seasonality in the presence of structural breaks and unit roots at frequencies other than those directly under test, which we term unattended breaks and unattended unit roots respectively. We show that under unattended breaks the true size of the Kwiatkowski *et al.* (1992) [KPSS] test at frequency zero and the Canova and Hansen (1995) [CH] test at the seasonal frequencies fall well below the nominal level under the null with an associated, often very dramatic, loss of power under the alternative. We demonstrate that a simple modification of the statistics can recover the usual limiting distribution appropriate to the case where there are no breaks, provided unit roots do not exist at any of the unattended frequencies. Where unattended unit roots occur we show that the above statistics converge in probability to zero under the null. However, computing the KPSS and CH statistics after pre-filtering the data is simultaneously efficacious against both unattended breaks and unattended unit roots, in the sense that the statistics retain their usual pivotal limiting null distributions appropriate to the case where neither occurs. The case where breaks may potentially occur at all frequencies is also discussed. The practical relevance of the theoretical contribution of the paper is illustrated through a number of empirical examples.

JEL classification: C12, C22.

Keywords: stationarity tests, structural breaks, pre-filtering, unattended unit roots.

Contents

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1. Introduction[1]

In a recent paper Canova and Hansen (1995) [CH] have developed score-based tests of the null hypothesis of deterministic seasonality against the alternative of unit roots at some or all of the seasonal, but not zero, spectral frequencies. Their tests are similar in spirit to the those against a zero frequency unit root proposed in Kwiatkowski *et al.* (1992) [KPSS]. The statistics upon which these tests are based retain pivotal Cramér-von Mises limiting null distributions under mixing and seasonally heteroskedastic errors *via* the use of a non-parametric heteroskedasticity and autocorrelation consistent (HAC) covariance matrix estimator. More recently, Taylor (2003a) has amalgamated these cases to develop score-based tests against seasonal and/or zero frequency unit roots.

The above *stationarity* tests can also be derived from the general theory on parameter stability testing in regression models of Nyblom (1989) and Hansen (1992). We provide a review of these testing procedures in Section 2. Specifically, in the context of a regression of the time series variable of interest on a set of zero and seasonal frequency spectral indicator variables, we consider testing the null hypothesis of fixed parameters against the alternative that (at least one of) the parameters on a given subset, say $\Im_1$, of the spectral indicators evolve as random walks, such that the process admits unit root behaviour at (at least one of) those spectral frequencies included in $\Im_1$.

The testing procedures outlined in Section 2 are conducted under the maintained hypothesis, say H_M, that the regressors associated with those spectral frequencies not included in $\Im_1$ must have fixed coefficients. In this paper we focus attention on two particular cases where H_M is violated, both recognised in the literature to be of considerable practical relevance. Firstly we consider the case of *unattended structural breaks*, where some or all of the parameters on the spectral frequency regressors *not* included in $\Im_1$ display a structural break at some known or unknown point in the sample. Secondly, we consider the case of *unattended unit roots*, originally highlighted in Hylleberg (1995) and further developed in Taylor (2003b), where the process admits unit root behaviour at some or all of the spectral frequencies not included in $\Im_1$.

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[1] We thank Andrew Harvey and Cheng Hsiao for helpful comments on an ealier draft. The views expressed here are those of the authors and do not necessarily represent those of the Bank of Italy. Email: busetti.fabio@insedia.interbusiness.it, R.Taylor@bham.ac.uk

Since the work of Perron (1989) on US GNP, it has been known that a process which is stochastically stationary about a deterministic component subject to structural breaks can display properties very similar to a unit root process. Indeed, Perron (1989) demonstrates that the conventional augmented Dickey-Fuller [ADF] tests cannot reject the unit root null hypothesis, even asymptotically, where a broken trend exists. Although consistent against stationary processes about a broken level, Perron (1989) shows that the power of the ADF test is vastly reduced relative to the case where no break occurs. Perron (1989) proposes modifications of the ADF test constructed so as to be invariant to deterministic breaks at a known point while Zivot and Andrews (1992), *inter alia*, allow for an unknown break point. More recently, Busetti and Harvey (2001) have considered the effects of level and trend breaks, of the type discussed in Perron (1989), on the stationarity tests of KPSS. They show that failing to account for these breaks renders the KPSS tests severely over-sized, while KPSS-type tests which explicitly allow for (are invariant to) such breaks will require a different set of critical values.

Empirical results in Ghysels (1990) suggest that seasonal level shifts are a common phenomenon in quarterly US macroeconomic time series and argues that such shifts will have non-trivial consequences on testing for seasonal unit roots. Accordingly, Ghysels (1994), Smith and Otero (1997) and Franses and Vogelslang (1998), *inter alia*, have extended the work of Perron (1989) to the case of testing the null hypothesis of seasonal unit roots in series which undergo seasonal dummy level shifts. Although the standard tests of the seasonal unit root null hypothesis of Hylleberg *et al.* (1990) are shown to be consistent against stochastically stationary processes which are subject to such shifts, their finite sample power is vastly reduced. Indeed, in allowing for seasonal dummy level shifts in quarterly real GDP for fourteen countries, Franses and Vogelslang (1998) find considerably less evidence for seasonal unit roots than from the standard tests. Busetti and Harvey (2003a) consider the effects of structural breaks at the seasonal (but not zero) frequencies on the seasonal frequency stationarity tests of CH. Paralleling the arguments in Busetti and Harvey (2001), they show that the standard CH tests are over-sized in such cases and propose modified statistics which are invariant to such breaks.

In Section 3 of this paper we derive representations for the limiting null distributions of the statistics of Section 2 in cases where there are *unattended breaks*, but not unattended unit roots. We demonstrate that in such cases the true asymptotic size of the tests will fall below the nominal level. This contrasts sharply with cases where some or all of the parameters on the spectral frequency regressors included in $\Im_1$ display a structural break (we shall term these *attended structural breaks* in what follows), considered in Busetti and Harvey (2001, 2003a). Monte Carlo simulations reported in Section 5 demonstrate that the asymptotic theory provides a useful approximation to the behaviour of the statistics in finite-samples and that the under-sizing phenomenon effects an associated, often very dramatic, loss of finite sample power under the alternative.

The assumption that only those spectral frequencies included in $\Im_1$ may display unit root behaviour is clearly untenable in practice, since one cannot know which spectral frequencies admit a unit root. If one did, one would of course have no need for unit root testing. Busetti and Taylor (2002) and Taylor (2003b) demonstrate that the statistics against unit roots in $\Im_1$ converge in probability to zero under the null in such cases. Although the tests based on these statistics are consistent in such cases, their finite sample power is vastly diminished relative to the case where the maintained hypothesis holds; Taylor (2003b) provides considerable numerical and empirical evidence to illustrate this point. Interestingly, this problem does not arise with the tests of Hylleberg *et al.* (1990) since here one may test for the null hypothesis of a unit root at a particular spectral frequency (or frequencies) whilst remaining ambivalent as to the existence or otherwise of unit roots at those frequencies not under test.

From a practical perspective, the impact of unattended breaks and unattended unit roots on the stationarity tests of Section 2 are just as important as those arising from attended breaks. As noted above, the latter effect over-sized tests which diverge, even if there are no unit roots in $\Im_1$. Since these are tests of the null hypothesis of stationarity in $\Im_1$, this is not an unreasonable outcome: it draws the practitioner's attention to non-stationary behaviour in $\Im_1$. However, care must still be taken since routine differencing in response to rejection of the null will yield an over-differenced series if the non-stationarity is due to an attended break. In contrast, unattended breaks and/or unattended unit roots at those frequencies not included in $\Im_2$ vastly diminish the likelihood that we can reject the null of stationarity when analysing series with unit roots in $\Im_1$. The modelling implications of failing to recognise and account for such behaviour is well-known and further analysis of the data could only be expected to yield spurious inferences.

In light of these practical problems, we suggest a variety of remedial actions. In the case where we have unattended structural breaks, but no unattended unit roots, we show in Section 3 that some simple modifications, based on bias-correction, to the existing test statistics can recover the usual limiting null distributions that pertain where there are no breaks. However, where there are unattended unit roots we demonstrate in Section 4 that these statistics will still converge in probability to zero under the null. Pre-filtering the data eliminates potential unattended unit roots and has been shown to be highly effective by Taylor (2003b). We show that pre-filtering simultaneously provides an efficacious remedy for the problem of unattended structural breaks. We therefore recommend the use of pre-filtering as a means of obtaining robust tests in the presence of unattended breaks and/or unattended unit roots.

In Section 6 we discuss two generalisations. The first allows for the case where there are both unattended *and* attended deterministic structural breaks, while the second allows for deterministic trending variables. The former is of considerable empirical importance because seasonal level shifts, of the type discussed in the literature on tests of the null of seasonal unit roots above, may potentially effect both attended and unattended breaks, and hence allow for cases where structural breaks occur at all frequencies. The practical relevance of our theoretical results is illustrated in Section 7 where we apply the existing tests against stochastic trend and seasonality and the modified versions of these tests suggested in this paper to data on U.K. marriages and U.K. (log) consumers' expenditure on tobacco. In the case of UK marriages we show that tests based on the appropriately modified statistics yield considerably more evidence against stationarity than do the existing tests, while the (log) expenditure series illustrates the case where both attended and unattended breaks occur. Section 8 concludes the paper, while an Appendix contains proofs of our main results.

2. Score-Based Stationarity Tests

Consider the scalar process y_t, $t = 1, \ldots, T$, observed with constant seasonal periodicity s, generated according to the model

$$y_t = \bar{D}_t' \gamma_t^* + u_t, \quad u_t \sim NIID(0,\ \sigma^2), \tag{1}$$

where $0 < \sigma^2 < \infty$, and $\bar{D}_t$ is an s-vector of conventional seasonal indicator (dummy) variables with associated, potentially time-varying, parameter vector γ_t^*.

Rather than work directly with the formulation given in (1), it will prove expedient in what follows to adopt the following bijective reparameterization of (1) in terms of the spectral indicator variables of Hannan *et al.* (1970); that is,

$$y_t = Z_t' \gamma_t + u_t, \quad t = 1, \ldots, T. \tag{2}$$

In (2), $Z_t = \left(z_{0,t}, z_{1,t}', \ldots, z_{[s/2],t}' \right)'$ is an s-vector of spectral indicator variables; that is, $z_{0,t} = 1$, $z_{k,t} = (\cos 2\pi kt/s,\ \sin 2\pi kt/s)'$, $k = 1, \ldots, s^*$, where $s^* = s/2 - 1$ if s is even or $[s/2]$ if s is odd, while $z_{s/2,t} = (-1)^t$ if s is even. The first element of Z_t, $z_{0,t}$, corresponds to the zero frequency, $\lambda_0 \equiv 0$, while the kth pair of spectral indicators, $z_{k,t}$, correspond to the kth harmonic seasonal frequency, $\lambda_k \equiv 2\pi k/s$, $k = 1, \ldots, s^*$. Where s is even, the last element of Z_t, $z_{s/2,t}$, corresponds to the Nyquist frequency, $\lambda_{s/2} \equiv \pi$. The exact relationship between γ_t and γ_t^* is given by $\gamma_t = R^{-1}\gamma_t^*$, where the full rank $(s \times s)$ matrix $R = (Z_1, \ldots, Z_s)'$.

Consider now the partition of Z_t into $(Z_{1t}', Z_{2t}')'$ where Z_{1t}, Z_{2t} are disjoint sub-vectors (containing regressors corresponding to disjoint frequencies, but with no particular ordering) of dimension s_1 and s_2 respectively, such that $s_1 + s_2 = s$. Let

$$\Im_i = \left\{ k_j,\ \ j = 1, 2, \ldots, n_i,\ :\ z_{k_j,t} \text{ belongs to } Z_{it} \right\}$$

be the set of indices of the spectral indicator variables corresponding to the frequencies included in Z_{it}, $i = 1, 2$. Using this partition we re-write (2), with an obvious notation, as

$$y_t = Z_{1t}' \gamma_{1t} + Z_{2t}' \gamma_{2t} + u_t. \tag{3}$$

We specify the parameters on Z_{1t} to evolve as a (possibly degenerate) vector random walk; that is,

$$\gamma_{1t} = \gamma_{1,t-1} + \eta_t, \quad \eta_t \sim NIID(0,\ \sigma^2 D_1^*) \tag{4}$$

where $D_1^* = \sigma_\eta^2 D_1$, D_1 a non-null diagonal positive semi-definite matrix of dimension $(s_1 \times s_1)$. We assume that the initial value γ_{10} is fixed, with no loss of generality. We also assume that η_t and u_t are mutually orthogonal.

In this paper our attention focuses on testing the null hypothesis $H_0 : \sigma_\eta^2 = 0$ against the alternative $H_1 : \sigma_\eta^2 > 0$ in (3)-(4). As demonstrated in, *inter alia*, CH under H_1, $\{y_t\}$ admits unit roots at those spectral frequencies associated with the non-zero diagonal elements of D_1. However, the statistical properties of tests of H_0 against H_1 will depend crucially on the specification of γ_{2t}, and it is an exploration of this issue which provides the focus for this paper. In the remainder of this Section we follow the implicit assumption adopted in CH, KPSS and Taylor (2003a) that $\gamma_{2t} = \gamma_{20}$, a fixed s_2-vector of coefficients, for all t. This assumption constitutes the maintained hypothesis, H_M, discussed in Section 1. Recall that H_M imposes the condition that the regressors associated with those spectral frequencies not included in $\Im_1$ must have fixed coefficients, regardless of the behaviour of the coefficient vector γ_{1t}. If both H_0 and H_M hold the entire parameter vector γ_t is fixed and we have what will be termed the *null model*

$$y_t = Z_{1t}'\gamma_{10} + Z_{2t}'\gamma_{20} + u_t, \quad t = 1, ..., T; \tag{5}$$

that is, $\{y_t\}$ is a stochastically stationary process around fixed (deterministic) seasonal effects.

In Section 3 we consider the effects on the tests of Section 2 when we allow some or all of the elements of γ_{2t} to display a deterministic structural break at some known or unknown point in the sample. In Section 4 we subsequently consider the case where γ_{2t} follows a random walk. In Section 6.1 we will also discuss the case where deterministic structural breaks may occur in any of the elements of γ_t, not just in γ_{2t}, thereby allowing for the interesting case where structural breaks may occur at all frequencies. Indeed, notice that deterministic structural breaks in the seasonal dummy parameter vector γ_t^* of (1) will effect structural breaks in either γ_{1t} or γ_{2t}, or both.

Denote by e_t, $t = 1, ..., T$, the Ordinary Least Squares (OLS) residuals from regressing y_t on Z_t and denote by $\widehat{\sigma}^2$ their sample variance, $\widehat{\sigma}^2 = T^{-1}\sum_{t=1}^{T} e_t^2$. As demonstrated in Taylor

(2003a), a locally most powerful invariant (LMPI) test of $H_0 : \sigma^2_\eta = 0$ against $H_1 : \sigma^2_\eta > 0$ in (3)-(4), with H_M maintained, rejects for large values of the statistic

$$(6) \qquad \omega = \sum_{k\in\Im_1} \bar{\omega}_k \equiv \sum_{k\in\Im_1} \left(h_k \frac{\sum_{t=1}^{T} Trace\left(\sum_{j=1}^{t} z_{k,j} e_j \sum_{j=1}^{t} z'_{k,j} e_j\right)}{T^2 \widehat{\sigma}^2} \right),$$

where $h_k = 1$ if $k = 0$ or $k = s/2$, $h_k = 2$ otherwise. Under H_0, ω weakly converges to the right member of (9) below, and is $O_p(T)$ under H_1; see Taylor (2003a). Furthermore, it is not difficult to show that the statistic ω of (6) for testing against all of the spectral frequencies identified by Z_{1t} is precisely the sum of the individual LMPI statistics $\bar{\omega}_k$ for testing at each of these frequencies in turn. As we shall see below, this additive property also holds on the asymptotic null distributions of the tests.

If we weaken the assumptions on $\{u_t\}$ to allow for the possibility of weak dependence and heterogeneity, we will require a non-parametrically modified version of ω in order to obtain a statistic with a pivotal limiting null distribution. Specific (mixing) conditions on $\{u_t\}$ in this case are quite involved; full details are given in Taylor (2003a) and are not repeated here. We denote by Ω_i the "long run variance" of the process $Z_{it}u_t$, i.e. $\Omega_i \equiv lim_{T\to\infty} T^{-1} E\left((\sum_{t=1}^{T} Z_{it}u_t)(\sum_{t=1}^{T} Z_{it}u_t)'\right)$, $i = 1, 2$, and assume that Ω_1 is positive definite. Then from CH and Taylor (2003a), the modified statistic is given by

$$(7) \qquad \mathcal{L} = T^{-2} Trace\left[\hat{\Omega}_1^{-1} \sum_{t=1}^{T} \left(\sum_{i=1}^{t} Z_{1i} e_i\right) \left(\sum_{i=1}^{t} Z'_{1i} e_i\right)\right],$$

where, following CH, we have defined the HAC estimator of Ω_1 as

$$(8) \qquad \hat{\Omega}_1 = \sum_{j=-T+1}^{T-1} k(j/S_T)\hat{\Gamma}_1(j),$$

where $k(\cdot)$ is a kernel function and $\hat{\Gamma}_1(j) = T^{-1} \sum_{t=j+1}^{T} Z'_{1t} e_t e_{t-j} Z'_{1,t-j}$ is the estimator of the autocovariance of $Z_{1t}u_t$ at lag j.

For $s = 1$, $\mathcal{L}$ of (7) is the η_μ statistic of KPSS (Equation (13), p.165). Furthermore, for s even and $Z_{1t} = (-1)^t$, $\mathcal{L}$ coincides with the $\mathcal{L}_\pi$ statistic of CH (Equation (17), p.6). Moreover, for $Z_{1t} = z_{k,t}$, $k = 1, ..., s^*$, $\mathcal{L}$ is the statistic $\mathcal{L}_{\pi k/s}$ of CH (Equation (17), p.6), while if Z_{1t}

contains all but the first element of Z_t (i.e. $Z_{2t} = 1$), then $\mathcal{L}$ coincides with $\mathcal{L}_f$ of CH (Equation (15), p.5).

Suitable choices for the kernel $k(\cdot)$ may be found in, *inter alia*, Andrews (1991, p.821), while setting the bandwidth parameter S_T such that $S_T \to \infty$ and $S_T/T^{1/2} \to 0$ ensures that $\hat{\Omega}_1 \to^p \Omega_1$ under both the null and local alternatives; see Elliott and Stock (1994). Under the above conditions, Taylor (2003a) establishes the result that under $H_0 : \sigma^2_\eta = 0$

$$\mathcal{L} \Rightarrow \int_0^1 B_{s_1}(r)' B_{s_1}(r) dr, \tag{9}$$

where "$\Rightarrow$" denotes weak convergence and $B_{s_1}(r)$ is a s_1-dimensional standard Brownian bridge process. The right member of (9) is a Cramér-von Mises distribution with s_1 degrees of freedom; see Harvey (2001) for further discussion on the Cramér-von Mises family of distributions. In what follows we will denote this distribution by $CvM(s_1)$, upper tail critical values from which, for $1 \leq s_1 \leq 12$, are provided in Table 1 of CH (page 5).

If we supplant the mixing conditions above by the assumption that $\{u_t\}$ is a linear process, then Ω_1 is a diagonal matrix containing the spectral generating function of u_t, denoted as $g(\lambda)$, evaluated at each of the frequencies $\lambda_k,\ k \in \Im_1$. To be precise, we assume that $u_t = \psi(L)\epsilon_t$, $\{\epsilon_t\}$ a MD sequence satisfying the conditions in Stock (1994, p. 2745), and $\psi(L) \equiv 1 + \sum_{i=1}^{\infty} \psi_i L^i$ a polynomial in L, the conventional lag operator, $L^k y_t \equiv y_{t-k}$, $k = 0, 1, ...$, satisfying: (i) $\psi(\exp\{\pm i2\pi k/s\}) \neq 0$, for all $k \in \Im_1$, and (ii) $\sum_{j=1}^{\infty} j|\psi_j| < \infty$.[2] The first condition rules out a zero in the spectrum of $\{u_t\}$ at any of the frequencies included in $\Im_1$, while the second ensures that poles do not exist in the spectrum of $\{u_t\}$. A leading case which satisfies the above conditions is the class of finite-order stationary and invertible ARMA processes. Following Busetti and Harvey (2003a) we also consider the following alternative non-parametrically modified statistic

$$\overline{\mathcal{L}} = \sum_{k \in \Im_1} \mathcal{L}_k \equiv \sum_{k \in \Im_1} \left(h_k \frac{\sum_{t=1}^{T} Trace\left(\sum_{j=1}^{t} z_{k,j} e_j \sum_{j=1}^{t} z'_{k,j} e_j \right)}{T^2 \widehat{g}(\lambda_k)} \right), \tag{10}$$

[2] Equivalent conditions must also hold in the mixing case. However, it should be noted that the possibility of periodic heteroscedasticity, allowed under the mixing conditions, is not permitted.

where h_k is defined below (6), $\widehat{g}(\lambda_k) = \sum_{j=-T+1}^{T-1} k(j/S_T)\widehat{\gamma}_e(j)\cos\lambda_k j$, and $\widehat{\gamma}_e(j) = T^{-1}\sum_{t=j+1}^{T} e_t e_{t-j}$ is the sample autocovariance of the OLS residuals at lag j. Notice that $\overline{\mathcal{L}}$ and $\mathcal{L}$ of (7) coincide if $s_1 = 1$. The statistic $\overline{\mathcal{L}}$ is intuitively appealing since, unlike $\mathcal{L}$, it retains the additivity property of ω. Under the linear process conditions on $\{u_t\}$, together with the conditions on $k(\cdot)$ and S_T stated below (8), $\overline{\mathcal{L}}$ has the usual $CvM(s_1)$ limiting distribution of (9) under H_0. Under H_1, $\mathcal{L}$ and $\overline{\mathcal{L}}$ are both $O_p(T/S_T)$; see, *inter alia*, CH, KPSS, Taylor (2003a) and Busetti and Harvey (2003a). Finally we note that while ω is an exact LMPI test of $H_0 : \sigma^2_\eta = 0$ against $H_1 : \sigma^2_\eta > 0$ in (3)-(4), neither $\mathcal{L}$ nor $\overline{\mathcal{L}}$ are locally optimal in any formal sense, even asymptotically.

3. Testing with Unattended Structural Breaks

Thus far we have assumed that γ_{2t} of (3) is a fixed s_2-vector of parameters. Suppose now that the process $\{y_t\}$ is generated by (3)-(4) but that γ_{2t} is generated according to

$$\gamma_{2t} = (\gamma_{20} + \theta d_t(\alpha)), \tag{11}$$

where $d_t(\alpha) = 1\,(t \leq [T\alpha],\ \alpha \in (0,1))$ is an indicator variable reflecting the occurrence of a deterministic structural break, and θ is an s_2-vector of coefficients. It is clear that (11) violates H_M because, unless $\theta = 0$, γ_{2t} is not fixed for all t. Notice that the end cases of $\alpha = 0$ and $\alpha = 1$ are excluded since these would not yield a within-sample break in the coefficient vector γ_{2t}. In Section 6.1 we will subsequently discuss the case where deterministic breaks may occur in both γ_{2t} and γ_{1t}.

The DGP (3)-(4)-(11) displays potentially non-stationary stochastic seasonality at the frequencies corresponding to Z_{1t}, exactly as in Section 2, but now also displays deterministic seasonality with a level shift in the seasonal pattern at period $[\alpha T]$ at the frequencies corresponding to Z_{2t}. Observe that the zero frequency regressor $z_{0,t} = 1$ can be in either Z_{1t} or Z_{2t}. Moreover, the components in the vector of level shifts $\theta = (\theta_1, ..., \theta_{s_2})'$ need not be equal and may contain zeros, so that not all frequencies in Z_{2t} need show a break. Moreover, if $\theta = 0$, $\gamma_{2t} = \gamma_{20}$, and so the test based on ω of (6) is a LMPI test for $H_0 : \sigma^2_\eta = 0$ against $H_1 : \sigma^2_\eta > 0$.

Consider now the case where $\theta \neq 0$, such that H_M is violated, and we compute the statistic ω of (6) without taking into account the occurrence of a level shift; that is, there is an unattended structural break. In this case the test based on ω is not LMPI and the asymptotic distribution of ω under H_0 is no longer of the form given in (9). Indeed, as we now show in Proposition 1, it is equal to the right member of (9) multiplied by the factor $m(\alpha,\theta,\sigma^2)$, which arises from an asymptotic bias in the estimator $\widehat{\sigma}^2$ of σ^2. Consequently, the pivotal limiting distribution in (9) can be retained by bias-correcting ω as we shall do in Section 3.1, making use of the work on breakpoint estimation by Bai (1994, 1997).

Proposition 1 *Let y_t be generated by (3),(4) and (11). Then, under $H_0:\sigma^2_\eta=0$,*

$$\omega \;\Rightarrow\; m(\alpha,\theta,\sigma^2)\int_0^1 B_{s_1}(r)'B_{s_1}(r)dr, \tag{12}$$

where $B_{s_1}(r)$ is as defined above, and

$$m(\alpha,\theta,\sigma^2) \;=\; (1+b(\alpha,\theta)/\sigma^2)^{-1}, \tag{13}$$

$$b(\alpha,\theta) \;=\; \lim_{T\to\infty} T^{-1}\sum_{t=1}^{T}(d_t(\alpha)-\alpha)^2\,(Z'_{2t}\theta)^2\,. \tag{14}$$

Remark 1: Notice from (14) that the asymptotic bias in $\widehat{\sigma}^2$, $b(\alpha,\theta)$, and hence $m(\alpha,\theta,\sigma^2)$, are symmetric in α. Consequently, the representation in (12) for α and that for $\bar{\alpha}\equiv(1-\alpha)$ coincide. Where $\theta=0$, the case of no unattended breaks, $b(\alpha,0)=0$ and hence (12) reduces to the right member of (9), as should be expected.

Remark 2: As an example of $b(\alpha,\theta)$, consider the case where either $Z_{2t}=1$ or $Z_{2t}=(-1)^t$. Here the bias is given by $b(\alpha,\theta)=\alpha(1-\alpha)\theta^2$.

The asymptotic bias, $b(\alpha,\theta)$, is strictly positive whenever $\theta\neq 0$, and correspondingly, $m(\alpha,\theta,\sigma^2)$ will therefore always be less than unity. This will clearly yield an undersized test; cf. (9). Notice therefore that unattended breaks in the parameters associated with the spectral regressors in $\Im_2$ have the opposite effects from unattended breaks in the parameters associated with $\Im_1$. In the latter case Busetti and Harvey (2001, 2003a) show that tests against unit roots in $\Im_1$ are *over-sized.* As a simple illustration of Proposition 1, consider the case of a model with two seasons ($s=2$), $Z_t=(1,(-1)^t)'$, where $Z_{1t}=(-1)^t$, corresponds to frequency π, and $Z_{2t}=1$ corresponds to the zero frequency. Suppose that, as a particular case of (11), a shift in γ_{2t}, equal to the standard deviation of the errors, occurs in the middle of the sample, so that $m(0.5,\sigma,\sigma^2)=0.8$. In this case if ω, the test against a unit root at frequency π, is run at the nominal (asymptotic) 5% level, it would display a true asymptotic size of $\Pr\{CvM(1) \geq 0.463/0.8\} \approx 2.5\%$.[3] In the same example, but with $\theta = 2\sigma$ the true asymptotic size would be $\Pr\{CvM(1) \geq 0.463/0.5\} \approx 0.4\%$. Deviations below the nominal level clearly increase as θ is increased. Notice that the same results also apply if we interchange Z_{1t} and Z_{2t}; that is, the case where we are testing against a zero frequency unit root in the presence of an unattended break at frequency π.

Now consider the case of testing against a unit root at frequency zero in a quarterly model ($s = 4$). In this case $Z_{1t} = 1$, corresponds to the zero frequency, and $Z_{2t} = (z'_{1,t},\ (-1)^t)'$, where $z_{1,t} = (\cos \pi t/2,\ \sin \pi t/2)'$, corresponds to the seasonal frequencies. Suppose that there is a neglected break of size $\theta = (\theta^*, \theta^*, \theta^*)'\sigma$ at the seasonal frequencies. Table 3.1 gives the asymptotic biases and true asymptotic size for the test against a zero frequency unit root, ω, run at the nominal 5% level, for various values of the break fraction α and break magnitudes θ^*.

As in the preceding example, we see that the true asymptotic size of ω deviates further below the nominal level as the break magnitude θ^* increases, other things being equal. Moreover, the tabulated results clearly demonstrate that the degree of size distortion is the worse the closer is the break date to the middle of the sample. Recall from Remark 1 that $m(\alpha, \theta, \sigma^2)$ is symmetric in α.

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[3] The asymptotic 5% level upper tail critical value from the $CvM(1)$ distribution is 0.463.

Table 3.1: Asymptotic Bias and Size of ω

		$b(\alpha,\theta)$	$m(\alpha,\theta,\sigma^2)$	$true\ size$
	$\alpha = 1/8$	0.219	0.821	.029
$\theta^* = 1$	$\alpha = 1/4$	0.375	0.727	.019
	$\alpha = 1/2$	0.500	0.667	.013
	$\alpha = 1/8$	0.875	0.533	.004
$\theta^* = 2$	$\alpha = 1/4$	1.500	0.400	.001
	$\alpha = 1/2$	2.000	0.333	.000
	$\alpha = 1/8$	3.500	0.222	.000
$\theta^* = 4$	$\alpha = 1/4$	6.000	0.143	.000
	$\alpha = 1/2$	8.000	0.111	.000

If we switch Z_{1t} and Z_{2t} in the quarterly example considered above, so that our object is to test against unit root behaviour at some or all of the seasonal frequencies, and we have a neglected shift at frequency zero, the true asymptotic size of a test run at the nominal 5% level[4] is obtained from the $CvM(3)$ distribution as $\Pr(CvM(3) \geq 1.010/m(\alpha,\theta,\sigma^2))$, where $m(\alpha,\theta,\sigma^2) = 1/(1+\alpha(1-\alpha)\theta^2/\sigma^2)$. For example, if $\alpha = 1/8$ and $\theta = \sigma$, the true asymptotic size for a nominal 5% level test is approximately 2.5%, while if $\alpha = 1/2$ this drops to 1.8%. A tabulation of asymptotic sizes and biases for a range of values of α and θ in this case is not provided here, as the patterns are qualitatively similar to those seen in Table 3.1. Full details can be obtained from the authors, on request.

3.1 *Bias-Corrected Tests*

The bias in the asymptotic null distribution of ω detailed in Proposition 1 may be easily corrected and this can be done in a number of ways which we detail below. Each modification is shown to deliver a statistic with the usual pivotal $CvM(s_1)$ limiting distribution under H_0 : $\sigma^2_\eta = 0$.

Consider first the case where there is a structural break at an unknown point in the sample, $\alpha_0 \in (0,1)$. An asymptotically unbiased estimator of the error variance is obtained by minimizing the sum of squared residuals over all the possible break dates. Specifically, let $e_t(\alpha)$ denote the OLS residuals from the fitted regression

(15)
$$\hat{y}_t = Z'_{1t}\widehat{\gamma}_1 + Z'_{2t}\widehat{\gamma}_2 + d_t(\alpha)Z'_{2t}\hat{\theta},$$

[4] The 5% level upper tail asymptotic critical value from the $CvM(3)$ distribution is 1.010.

and let $\widehat{\sigma}^2(\alpha) = T^{-1}\sum_{t=1}^{T} e_t(\alpha)^2$ be the sample variance of the residuals. Denote by α^* the argument that minimizes this variance; that is, $\alpha^* = \arg\inf_{\alpha} \widehat{\sigma}^2(\alpha)$.

In the appendix it is shown that, under $H_0 : \sigma^2_\eta = 0, \widehat{\sigma}^2(\alpha^*)$ is an asymptotically unbiased estimator of σ^2. Indeed, for the case $Z_{2t} = 1$, Bai (1994, 1997) shows that, under H_0, α^* is a *superconsistent* estimator of α_0, in the sense that it converges to the true value at rate T. Consequently, and noting the asymptotic invariance of the numerator of ω of (6) to the level shift (see the proof of Proposition 1 in the Appendix), an obvious modified version of ω whose limiting null distribution is $CvM(s_1)$ is given by

(16) $$\omega^* = T^{-2}\sum_{k\in\Im_1} h_k \frac{\sum_{t=1}^{T} Trace\left(\sum_{j=1}^{t} z_{k,j}e_j \sum_{j=1}^{t} z'_{k,j}e_j\right)}{\widehat{\sigma}^2(\alpha^*)},$$

which is obtained by replacing in (6) the biased estimator of the error variance $\widehat{\sigma}^2$ by $\widehat{\sigma}^2(\alpha^*)$.

Notice that in (16) the correction is made only to the denominator of the statistic. One might then conjecture that a test with superior finite sample properties under level shifts could be obtained by modifying the numerator of (16) in a similar fashion to the denominator. Precisely, rather than use the OLS residuals e_t, $t = 1, ..., T$, as in ω and ω^*, one could compute the partial sums involved in the numerator using the residuals $e_t(\alpha^*)$, as defined above. This suggests the statistic

(17) $$\omega^{**} = T^{-2}\sum_{k\in\Im_1} h_k \frac{\sum_{t=1}^{T} Trace\left(\sum_{j=1}^{t} z_{k,j}e_j(\alpha^*) \sum_{j=1}^{t} z'_{k,j}e_j(\alpha^*)\right)}{\widehat{\sigma}^2(\alpha^*)}.$$

Consider now the case where the true breakpoint α_0 is *known*. In this scenario the best option would be to compute (17) with α^* replaced by α_0; we denote this statistic by ω_0. Adapting the results of Busetti and Harvey (2003a), it is easy to show that ω_0 is a LMPI test statistic for $H_0 : \sigma^2_\eta = 0$ against $H_1 : \sigma^2_\eta > 0$, and that its limiting null distribution is $CvM(s_1)$.

Finally, if no break occurs, both ω^* and ω^{**} remain asymptotically unbiased and consistent tests; this follows since adding extra regressors asymptotically uncorrelated with Z_{1t} has no effect on the limiting behaviour of the statistics; cf. Busetti and Harvey (2003a).

The following proposition summarizes the foregoing results.

Proposition 2 *Under the null hypothesis $H_0 : \sigma^2_\eta = 0$, regardless of the existence or timing of a breakpoint, i.e. irrespective of the values of θ and α, the asymptotic distributions of ω^*, ω^{**} are $CvM(s_1)$.*

Remark 3: The same result will clearly apply to the modified statistics ω_0, provided α_0 is indeed the true break date.

Remark 4: Under the fixed alternative, the statistics ω, ω_0, ω^* and ω^{**} all diverge at rate T. Notice that identification of the breakpoint in the coefficients of Z_{2t} is unimportant in this case, as the result is driven by the random walk alternative in the coefficients of Z_{1t}.

Under either the mixing or linear process assumptions on $\{u_t\}$ outlined in Section 2, the non-parametrically modified statistics $\mathcal{L}$ of (7) and $\overline{\mathcal{L}}$ of (10) respectively can be corrected along similar lines as suggested above for ω. This simply amounts to replacing $\widehat{\Omega}_1$ of (7) and $\widehat{g}(\lambda_k)$ of (**??**) with $\widehat{\Omega}_1(\alpha^*)$ and $\widehat{g}(\lambda_k; \alpha^*)$ respectively, constructed using the OLS residuals $e_t(\alpha^*)$, as detailed above. We will denote these statistics with an obvious notation as $\mathcal{L}^*$, $\mathcal{L}^{**}$, $\overline{\mathcal{L}}^*$ and $\overline{\mathcal{L}}^{**}$. The corresponding test statistics derived when the breakpoint is known will be denoted $\mathcal{L}_0$ and $\overline{\mathcal{L}}_0$. The limiting properties of these statistics are as given above for the corresponding bias-corrected variants of ω.

3.2 *Tests based on Pre-filtered Data*

The bias-corrected tests of Section 3.1 finessed the problem of unattended structural breaks by explicitly modelling the breaks. In the case of a known break date a LMPI test can be constructed. Where the break date is unknown, the now standard approach used in time-series econometrics of estimating across all possible break dates and optimising over the resulting sequence was proposed. However, rather than attempting to model the breaks, one might also attack the problem by transforming the data in such a way as to annihilate possible level shifts. This can be achieved simply by running our stability tests on pre-filtered data.

Consider the differencing filter, ∇_2, which reduces, by one, the order of integration at all of the spectral frequencies identified by $\Im_2$. As an example, if $s = 4$ with $Z_{1t} = 1$ and $Z_{2t} = (\cos \pi t/2,\ \sin \pi t/2,\ (-1)^t)'$, as in Table 1 above, then $\nabla_2 \equiv (1 + L)(1 + L^2) = (1 + L + L^2 + L^3)$. We will denote by f the order of this filter; in the above example, $f = 3$.

Notice that $\nabla_2 \equiv \Delta_s/\nabla_1$, where ∇_1 reduces, by one, the order of integration at each frequency identified by $\Im_1$, and $\Delta_s \equiv (1 - L^s)$.

Now consider *pre-filtering* the process $\{y_t\}$ generated by (3)-(4)-(11) by ∇_2, so that

$$\nabla_2 y_t = \nabla_2 Z'_{1t}\gamma_{1t} + \nabla_2 Z'_{2t}\gamma_{2t} + u^{\nabla}_t, \tag{18}$$

$t = 1 + f, \ldots, T$, where $u^{\nabla}_t \equiv \nabla_2 u_t$. The application of ∇_2 to $Z_{2t}\gamma_{2t}$ translates the level shift in the series $\{y_t\}$ into at most f outliers in the series $\{\nabla_2 y_t\}$. For example, suppose $Z_{2t} = 1$, so that $Z'_{2t}\gamma_{2t} = \gamma_{20} + \theta d_t(\alpha)$, where γ_{20} and θ are scalar constants. In such a case $\nabla_2 Z'_{2t}\gamma_{2t} = (1 - L)(\gamma_{20} + \theta d_t(\alpha))$, from which it is clear that $\nabla_2 Z'_{2t}\gamma_{2t} = 0$ for all values of t, except at $t = [T\alpha] + 1$ where $\nabla_2 Z'_{2t}\gamma_{2t} = -\theta$. Although the term $Z'_{1t}\gamma_{1t}$ of (4) has been transformed to $\nabla_2 Z'_{1t}\gamma_{1t}$ in (18), those regressors in Z_{1t} identified with a particular spectral frequency span an identical space to the corresponding regressors in $\nabla_2 Z_{1t}$. Consequently, we may treat $\nabla_2 Z'_{1t}\gamma_{1t}$ exactly as if it were $Z'_{1t}\gamma_{1t}$ in constructing tests against unit root behaviour at any of the spectral frequencies identified by Z_{1t}.

Unfortunately, we cannot apply the statistic ω of (6) to the pre-filtered data and maintain the usual $CvM(s_1)$ limiting distribution under $H_0 : \sigma^2_\eta = 0$. This is because, for ω to have a $CvM(s_1)$ limiting distribution, the error process in the null regression must be serially uncorrelated under H_0. It is clear from (18) that $\{u^{\nabla}_t\}$ is a moving average process of order f, $MA(f)$. Notice that $\{u^{\nabla}_t\}$ is strictly non-invertible at each of the spectral frequencies identified by $\Im_2$ but not at those identified by $\Im_1$; that is, $\Omega^{\nabla}_1 \equiv lim_{T\to\infty} T^{-1} E\left((\sum_{t=f}^{T} Z_{1t}u^{\nabla}_t)(\sum_{t=f}^{T} Z_{1t}u^{\nabla}_t)'\right)$ is positive definite. Consequently, we may consider the non-parametrically modified statistic

$$\mathcal{L}^{\nabla} = T^{-2} Trace\left[\widehat{\Omega}^{\nabla -1}_1 \sum_{t=f}^{T}\left(\sum_{i=f}^{t} Z_{1i}e^{\nabla}_i\right)\left(\sum_{i=f}^{t} Z'_{1i}e^{\nabla}_i\right)\right], \tag{19}$$

where e^{∇}_t, $t = f, \ldots, T$ are the OLS residuals from the regression of $\nabla_2 y_t$ on Z_t, and $\widehat{\Omega}^{\nabla}_1$ is as defined in (8), replacing e_t by e^{∇}_t in the expression for $\hat{\Gamma}_1(j)$. Similarly, $\overline{\mathcal{L}}$ of (10) may be modified in an obvious way to produce the corresponding statistic $\overline{\mathcal{L}}^{\nabla}$.

As discussed above, the application of ∇_2 to $Z_{2t}\gamma_{2t}$ transforms the level shift into at most f outliers. Asymptotically, these are singletons. Consequently, they have no effect on the limiting distribution of $\mathcal{L}^{\nabla}$ of (19). It is then straightforward to show that asymptotically, under H_0, $\mathcal{L}^{\nabla}$ and $\overline{\mathcal{L}}^{\nabla}$ behave exactly as $\mathcal{L}$ and $\overline{\mathcal{L}}$ of Section 2, under the mixing and linear process assumptions, respectively, placed on $\{u_t\}$, and hence $\{u_t^{\nabla}\}$.

In Section 5 we give Monte Carlo evidence into the finite sample properties of the modified statistics developed in this Section. Our results suggest that, for the sample sizes typically seen in practical applications of these statistics, the proposed corrections perform well in practice. In cases where there are no unattended unit roots, the bias-corrected statistic ω^{**} seems, overall, to present the best option in cases where the true break date is unknown, although the non-parametrically modified tests based on the pre-filtered data perform almost as well and have the added advantage that they are also robust to serially correlated errors and unattended unit roots, the latter issue discussed further in the next section.

4. Unattended Unit Roots

In this section we consider the case where the process $\{y_t\}$ is generated by (3)-(4) but where γ_{2t} now follows the vector random walk process,

(20) $$\gamma_{2t} = \gamma_{2,t-1} + \kappa_t, \quad t = 1, ..., T,$$

γ_{20} fixed, with $\kappa_t \sim NIID(0,\ \sigma^2 D_2^*)$, independent of u_t and η_t, and where $D_2^* = \sigma_\kappa^2 D_2$, D_2 a non-null diagonal positive semi-definite matrix of dimension $(s_2 \times s_2)$.

As with the case of the unattended structural break model of Section 3, it is clear that, unless $\sigma_\kappa^2 = 0$, (20) violates the maintained hypothesis, H_M. Specifically, the DGP (3)-(4)-(20) can display unit root behaviour at *any* of the spectral frequencies, regardless of whether the associated spectral indicator variable belongs to $\Im_1$ or $\Im_2$. This is an important empirical issue since in practice one cannot know which of the spectral frequencies admit a unit root. In Busetti and Taylor (2002) and Taylor (2003b) it is demonstrated that if y_t is generated by (3)-(4)-(20) with $\sigma_\kappa^2 > 0$ then under $H_0 : \sigma_\eta^2 = 0$, ω of (6) is $O_p(T^{-1})$ while $\mathcal{L}$ of (7) and $\overline{\mathcal{L}}$ of (10) are $O_p((TS_T)^{-1})$. Consequently, each of these statistics converges in probability to zero under H_0 when there are unattended unit roots. In the following proposition we show that the same result is true for the bias-corrected statistics of Section 3.1.

Proposition 3 *Let y_t be generated by (3)-(4)-(20) with $\sigma_\kappa^2 > 0$. Then, under $H_0 : \sigma_\eta^2 = 0, \omega^*$, ω^{**} and ω_0 are all of $O_p(T^{-1})$, while $\mathcal{L}^*$, $\mathcal{L}^{**}$, $\mathcal{L}_0$ $\overline{\mathcal{L}}^*$, $\overline{\mathcal{L}}^{**}$ and $\overline{\mathcal{L}}_0$ are of $O_p((TS_T)^{-1})$.*

Remark 5: Although the bias-corrected statistics of Section 3.1 were shown to be robust with respect to unattended structural breaks, the results of Proposition 3 show that in the presence of unattended unit roots at any spectral frequency associated with Z_{2t}; that is, where $\sigma_\kappa^2 > 0$, they will converge in probability to zero, under $H_0 : \sigma_\eta^2 = 0$. This occurs because the variance estimators used in constructing these statistics, as do those used in the test statistics of section 2, diverge, albeit at differing rates when $\sigma_\kappa^2 > 0$. Indeed, as is demonstrated in the Appendix, exactly the same result holds if γ_{2t} contains a mixture of random walk elements and determinstically breaking elements, provided at least one of its elements evolves as a random walk.

However, the pre-filtered statistic $\mathcal{L}^\nabla$ of (19) (all of what follows also applies to $\overline{\mathcal{L}}^\nabla$) discussed in Section 3.2 to combat the problem of unattended level shifts is also simultaneously a curative against unattended unit roots; indeed it has been originally suggested in Taylor (2003b) for this purpose. Recall that the pre-filter ∇_2 is such that it reduces, by one, the order of integration at all of the spectral frequencies identified by $\Im_2$. It is therefore clear that the filtered series $\{\nabla_2 y_t\}$ will not admit unit root behaviour at any of the frequencies identified with $\Im_2$. When $\{y_t\}$ is generated according to (3)-(4)-(20) the error process in (18) is $\nabla_2 Z_{2t}'\gamma_{2t} + \nabla_2 u_t$, which we denote by $u_t^{\nabla *}$, which is clearly an $MA(f)$ process. The process $\{u_t^{\nabla *}\}$ is strictly non-invertible at all of the spectral frequencies identified by $\Im_2$ if $\sigma_\kappa^2 = 0$, while, if $\sigma_\kappa^2 > 0$, $\{u_t^{\nabla *}\}$ will be strictly non-invertible at those frequencies associated with zero diagonal elements in D_2; cf. Harvey (1989,pp.54-70) for rigorous discussion on this point. As in Section 3.2, the serial correlation in the error process precludes the use of ω of (6). However, we may directly apply the pre-filtered statistic $\mathcal{L}^\nabla$ of (19) since the matrix $\Omega_1^{\nabla *}$, the long run variance of $Z_{1t}u_t^{\nabla *}$, is positive definite.

The results from Sections 3.2 and above demonstrate that the pre-filtered statistic $\mathcal{L}^\nabla$ of (19) retains the usual $CvM(s_1)$ limiting distribution of (9) under H_0, even if some subset of the parameters on the spectral frequency regressors in $\Im_2$ display *either* level shifts or evolve as random walks. This in contrast to the bias-corrected statistics ω^*, ω^{**}, ω_0, $\mathcal{L}^*$, $\mathcal{L}^{**}$, $\mathcal{L}_0$ $\overline{\mathcal{L}}^*$, $\overline{\mathcal{L}}^{**}$ and $\overline{\mathcal{L}}_0$ of Section 3.1, which all converge in probability to zero in the presence of unattended unit roots. The pre-filtered statistics therefore appear particularly appealing from a practical point of view.

5. Numerical Results

In this section we use Monte Carlo simulations methods to investigate the finite sample properties of the foregoing tests against unit roots in $\Im_1$ in cases where the parameters associated with those spectral regressors not included in $\Im_1$ display deterministic structural breaks. All experiments were programmed using the random number generator of the matrix programming language Ox 2.20 of Doornik (1998), over $50,000$ Monte Carlo replications. All tests were run at the nominal 5% asymptotic level, although other choices of the nominal level did not alter the results qualitatively.

Specifically, we now investigate the finite sample properties of the tests based on the non-bias-corrected statistic ω, the bias-corrected statistics ω^*, ω^{**} and ω_0, and the statistic based on pre-filtered data, $\mathcal{L}^{\nabla}$, against data generated by (3)-(4)-(11). In the context of this DGP we considered a range of possible breakpoint magnitudes $\theta,$ breakpoint locations α and signal-to-noise ratios σ_{η}. We considered samples of size $T = 100$ and $T = 200$ observations.

In the context of quarterly time series data, we have focused attention on two cases which we consider to be of most relevance for applied workers, each of which subsequently arises in the empirical examples of Section 6: (i) testing against a unit root at frequency zero when there is a break equal to $\theta = (\theta^*, \theta^*, \theta^*)'\sigma$ at the seasonal frequencies; (ii) testing jointly against unit root behaviour at the seasonal frequencies when there is a break equal to $\theta^*\sigma$ in the underlying level of the process. Other scenarios are clearly possible, for example the case where breaks may occur at all frequencies. We will discuss such possibilities further in Section 6.1. Simulation results for the bias-corrected tests are summarised in Tables 5.1a and 5.1b for case (i), and Tables 5.2a and 5.2b for case (ii). Corresponding results for the tests based on pre-filtered data are reported in Table 5.3a for case (i) and Table 5.3b for case (ii). For the non-parametric test on pre-filtered data, $\mathcal{L}^{\nabla}$, we have followed the recommendation given on page 12 of CH and used a Newey and West (1987) HAC estimator employing a Bartlett kernel with lag truncation $S_T = 3, 4, 5, 6$ for the case of the zero frequency test and $S_T = 0, 1, 2, 3$ for the test against stochastic seasonality. Results for the tests based on pre-filtered data are reported only for $T = 100$. Results for $T = 200$ are omitted as they parallel the progression from $T = 100$ to $T = 200$ seen in the bias-corrected tests; that is, power under the alternative is improved and rejection frequencies under the null are much closer to nominal levels. These results are available from the authors on request.

Consider first Table 5.1a, which reports results for case (i) above with $T = 100$. The columns labelled $\sigma_\eta = 0$ contain the empirical rejection frequencies (empirical size) of the tests, using the asymptotic 5% critical values, under the null hypothesis of a constant level; $H_0 : \sigma_\eta^2 = 0$. In the first row where $\theta^* = 0$, the maintained hypothesis H_M holds and there is no break in the seasonal, while for the remaining rows H_M is violated by a variety of seasonal level shifts. In the absence of a break, $\theta^* = 0$, the results in Table 5.1a demonstrate that the finite sample rejection frequency of ω under H_0 is slightly above that predicted by the asymptotic theory. As shown in Proposition 1, the presence of a level shift biases the asymptotic null distribution of ω below the nominal level. In finite samples the bias in ω appears to be slightly smaller than that predicted by Proposition 1, although this is somewhat artifactual given the slight over-sizing seen for $\theta^* = 0$. As an example, for $\theta^* = 1$ and $\alpha = 1/2$ the empirical rejection frequency under H_0 for a sample of 100 observations is seen from Table 5.1a to be 1.62%, while the asymptotic size in this case is approximately 1.3%, the latter obtained from the tables of the $CvM(1)$ distribution, and reported in Table 3.1. The bias-corrected statistics ω^*, ω^{**} and ω_0 all seem to work quite well in practice. Where the true breakpoint is known, ω_0 generally outperforms the other bias-corrected tests and is unaffected by the value of θ^*, as should be expected given that it is a LMPI test in this scenario. Where the true breakpoint is unknown, ω^{**} seems to be preferable to ω^*, at least in terms of their relative properties under H_0. Interestingly, in all but the case of a massive shift in the seasonal pattern (e.g. 8 times the standard deviation of the error, unlikely to remain unnoticed in the data), knowledge of the existence and the timing of the break does not seem to provide any great advantage in terms of the properties of the tests under H_0. Indeed, where $\theta^* = 0.8$ both ω^* and ω^{**} are seen to be rather over-sized. This appears to be a small sample effect, attributable to the estimation of θ^* in (15), since these distortions are vastly diminished for $T = 200$; cf. Table 5.1b.

All three bias-corrected tests appear to display quite reasonable power[5] properties under $H_1 : \sigma_\eta^2 > 0$. In cases where the θ, α combination does not yield an over-sized bias-corrected test, power is largely comparable, for a given value of σ_η, with that of the LMPI statistic ω when $\theta^* = 0$. Where the bias-corrected tests were over-sized, one sees correspondingly higher power and that should of course be borne in mind when assessing the results in the Tables.

[5] Strictly speaking, what is reported is not finite sample power because the tests are constructed using the asymptotic, rather than exact, 5% critical values.

Notice, however, that, as predicted by Remark 4, this effect is wiped out as σ_η increases, and has all but vanished for $\sigma_\eta = 0.5$. The three bias-corrected tests display very similar power properties under H_1, again remaining close to that observed for these tests in the absence of a seasonal shift. In contrast, the power of the biased test ω depends very heavily on the values of θ^* and α, even for large values of σ_η. For example, where $\sigma_\eta = 0.1$ and no break occurs, ω has power of 59.44%, but this falls below 0.00 % for $\theta^* = 0.8$, $\alpha = 0.5$. Notice that, as predicted by Proposition 1, the loss of power in ω becomes more pronounced as the breakpoint approaches the middle of the sample; that is, as α approaches 0.5.

The results for case (i) and $T = 200$ are displayed in Table 5.1b. The power of all of the bias-corrected tests is improved, relative to $T = 100$, as would be expected from the asymptotic theory in Section 3. Moreover, the properties of the bias-corrected tests under the null are much closer to the nominal level than for $T = 100$. In turn the effect of θ, α on the power of the bias-corrected tests is further diminished, with little or no impact seen for $\sigma_\eta \geq 0.05$. When $\theta^* \leq 4$ it appears that all of the bias-corrected tests display power comparable with that appropriate for the LMPI test in the absence of shifts. The finite sample size of the biased test ω also gives a closer match with the asymptotic size obtained from the quantiles of the $CvM(1)$ distribution, reported in Table 3.1. The effects of θ, α on the power of ω are also diminished relative to $T = 100$, but remain considerable.

Tables 5.2a and 5.2b contain the results for the case of testing against unit roots at the seasonal frequencies when there is a break in the underlying level of the process. The properties of the tests are broadly in line with those seen for case (i) in Tables 5.1a and 5.1b. For $T = 100$, however, the bias-corrected tests display less reliable size properties for $\theta^* \geq 4$. Interestingly, ω^* outperforms ω^{**} for $\alpha = 1/8$ and $\alpha = 1/2$, with this pattern being reversed when $\alpha = 1/4$. As with case (i), these finite-sample effects are much diminished when T is increased to 200. Notice that, for a given value of σ_η, the empirical powers observed in Tables 5.2a-5.2b are generally higher than the corresponding entries in Tables 5.1a-5.1b. This is due to the fact that under the alternative hypothesis there are two non-stationary cycles at the seasonal frequencies, as opposed to the previous case where only one non-stationary component was present under the alternative.

Consider now the properties of the pre-filtered test $\mathcal{L}^\nabla$ for case (i), or pre-filtered KPSS test, reported in Table 5.3a for $T = 100$. The Monte Carlo results here are directly comparable with those in Table 5.1a, as the same set of random numbers (and, hence, the same artificially generated observations) were used. It clearly emerges that the outcome of the pre-filtered test is largely unaffected by the existence, location or magnitude of a structural break. The size of the pre-filtered test clearly depends on the lag truncation parameter, S_T. For smaller values of S_T the test is rather over-sized; this is expected given that the error term is now an $MA(3)$ process; see Section 3.2. However, for $S_T \geq 4$ the pre-filtered test displays size properties which are largely comparable with those of the bias-corrected tests ω^* and ω^{**}, yet avoids the rather bad over-sizing problem noted above for those tests when $\theta^* = 8$. For $S_T = 6$, the size properties of the pre-filtered test are largely comparable with those of the test based on knowledge of the true breakpoint, ω_0, regardless of θ^*. Other things being equal, test power declines as S_T is increased, again as would be expected. In practice all of the tests will need to correct for serially correlated innovations and so this observation will not be confined to the pre-filtered tests. However, as with ω_0, the power of the pre-filtered test for a given value of S_T is little affected by the values of θ^* and α.

The results for case (ii), the CH test applied to data pre-filtered by $\nabla_2 \equiv (1 - L)$, are reported in Table 5.3b for $T = 100$. In this case, pre-filtering the data appears to have very little effect on the properties of the test. The size of the pre-filtered test in this case is very close to the nominal level, even for $S_T = 1$, with power only slightly below that of the LMPI test ω_0. For example, when $\sigma_\eta = 0.1$, $\theta^* = 2$, $\alpha = 1/2$ and $S_T = 1$, the power of $\mathcal{L}^\nabla$ is 77.23% against 80.41% for ω_0. Increasing S_T tends, other things being equal, to reduce the empirical rejection frequency of the pre-filtered test as would be expected.

Although not reported here we also investigated the effects of unattended unit roots on the finite sample properties of the bias-corrected tests, ω^*, ω^{**} and ω_0 of Section 3.1. Under $H_0 : \sigma^2_\eta = 0$ all of the tests were severely under-sized with an associated dramatic loss of power under $H_1 : \sigma^2_\eta > 0$, relative to the results reported in Tables 5.1a-5.2b. Similar patterns were observed for the $\mathcal{L}^*$, $\mathcal{L}^{**}$, $\mathcal{L}_0$ $\overline{\mathcal{L}}^*$, $\overline{\mathcal{L}}^{**}$ and $\overline{\mathcal{L}}_0$ tests, whose behaviour closely paralleled those reported for the $\mathcal{L}$ test under unattended unit roots in Taylor (2003b). In contrast, the pre-filtered tests, $\mathcal{L}^\nabla$ and $\overline{\mathcal{L}}^\nabla$, behaved very similarly to the results reported in Tables 5.3a and 5.3b.

To summarize, the reported simulation evidence has shown that in the case of unattended structural breaks, by using the bias-corrected tests ω^*, ω^{**} even in finite samples one can achieve almost the same size and power properties as the LMPI test ω_0, based on the knowledge of the breakpoint, provided the magnitude of the break is not too large ($\theta^* \geq 4$), and providing there are no unattended unit roots. Similar findings apply to the test $\mathcal{L}^{\nabla}$, based on pre-filtered data, which has the added advantage of simultaneously proving an effective remedy for the problem of unattended unit roots. Pre-filtering was found to be particularly effective when testing against non-stationarity at the seasonal frequencies, where the pre-filter is the simple first difference operator. We therefore recommend the use of pre-filtering as a means of obtaining robust tests in the presence of unattended breaks and/or unattended unit roots.

6. Generalisations

6.1 *Attended and Unattended Structural Breaks*

As explained in Busetti and Harvey (2001, 2003a), the KPSS and CH stationarity tests of Section 2 are likely to be severely oversized if there are (attended) structural breaks in the parameters associated with $\Im_1$. Indeed, it is known from the work of Nyblom (1989) that these statistics will diverge in such cases. Where the breakpoint is known and all attended breaks occur at the same point in the sample, Busetti and Harvey (2001, 2003a) demonstrate that constructing the KPSS and CH statistics as in Section 2 but replacing e_t, $t = 1, ..., T$, by the OLS residuals from the regression of y_t on Z_t and $d_t(\alpha)Z_{1t}$, where $d_t(\alpha)$ is as defined below (11), renders these statistics exact invariant to the attended breaks, with a further modification employed to remove the dependence of the limiting null distribution of the statistics on the break location. However, these modifications change the limiting null distribution of the statistics from $CvM(s_1)$ to $CvM(2s_1)$; see Busetti and Harvey (2001, 2003a) for full details. In what follows, we will refer generically to the tests based on the modified statistics of Busetti and Harvey (2001, 2003a) as BH tests. When the breakpoint is not known Busetti and Harvey (2003b) suggest estimating the break by minimizing, over all possible break-dates, the error variance from the regression of y_t on Z_t and $d_t(\alpha)Z_{1t}$, and then use the resulting estimate, $\hat{\alpha}$ say, in constructing the BH statistic. Notice that this parallels our construction of bias-corrected tests in Section 3.1.

It is straightforward to show that the BH tests outlined above will suffer from the problems of unattended breaks and unattended unit roots in exactly the same way as did the tests of Section 2; that is, under $H_0 : \sigma^2_\eta = 0$ they will be under-sized in the presence of unattended breaks and will converge in probability to zero under unattended unit roots. However, running the BH tests on appropriately pre-filtered data, in exactly the same way as was proposed in Sections 3.2 and 4 in the context of the tests of Section 2, will simultaneously treat the problems of both attended and unattended breaks (including the possibility of breaks at all frequencies) and will also be robust against unattended unit roots. If there are no unattended unit roots and both the attended and unattended breaks occur at the same point in the sample, an alternative strategy is to compute the BH statistic using the residuals from the regression of y_t on Z_t and $d_t(\alpha)Z_t$, where α may be known or estimated. The $CvM(2s_1)$ limiting null distribution remains appropriate under each of these approaches. However, the BH tests based on pre-filtering have a further advantage in that they will also be robust to cases where the attended and unattended breaks are located in different points in the sample.

6.2 *Trending Variables*

The inclusion of a full set of spectral time trends $Z_{1t}t$ at the frequencies $\lambda \in \Im_1$ has the effect of changing the asymptotic null distributions of the statistics presented in the previous sections from standard Cramér-von Mises distributions into second level Cramér-von Mises distributions with s_1 degrees of freedom; again see Harvey (2001) for representations of these distributions. Appropriate critical values are tabulated in Taylor (2003a) and Nyblom and Harvey (2000). In particular, an unattended structural break will imply the same asymptotic bias factor $m(\alpha, \theta, \sigma^2)$ as in Proposition 1 but applied to this new distribution; again the bias will disappear if the bias-corrected statistics of Section 3.1 or the pre-filtered statistic of Section 3.2 are adopted.

Busetti and Harvey (2003a) and Taylor (2003a) have shown that a modelled time trend at frequency zero does not affect the asymptotic null distribution of the CH statistic for stochastic seasonality. By extending their argument to our framework, it follows that modelled time trends at the frequencies $\lambda \in \Im_2$ will have no effect under the maintained hypothesis H_M. However, the presence of a neglected level shift in this case will change the bias term derived in Proposition 1, but again the same asymptotic critical values will apply for the appropriate bias-corrected and pre-filtered tests. Note that not only will the pre-filtered test statistics of Section 3.2 be asymptotically unaffected by the presence of spectral time trends and level shifts at the frequencies $\lambda \in \Im_2$, but as pre-filtering reduces spectral time trends to spectral indicators, the spectral trends need not be included in the regression model for the pre-filtered data. Finally, a neglected slope change at the frequencies $\lambda \in \Im_2$ can be dealt with either by computing bias-corrected statistics in the same spirit of Section 3.1 or by double pre-filtering, i.e. by applying the operator ∇_2 of Section 3.2 twice.

7. Empirical Examples

As a first example we consider testing at frequency zero for a well-known series characterized by a structural break in the seasonal patterns. Figure 7.1 shows the quarterly, seasonally unadjusted, series of marriages registered in the UK from 1958q1 to 1982q4; the source of the data is the U.K. Monthly Digest of Statistics. As emphasised in Busetti and Harvey (2003a), there is a structural break in the seasonal pattern in this series, occurring in the first quarter of 1969. This is seen very clearly in Figure 7.1 and was due to a change in U.K. taxation laws, which effected a switch from winter marriages to marriages in the spring quarter.

Busetti and Harvey (2003a) considered testing against stochastic seasonality for this series, taking the known seasonal break into account. They found some evidence for non-stationary stochastic seasonality. In the context of this paper we are interested in testing against stochastic non-stationarity at frequency zero, given that there may be a seasonal structural break and/or non-stationary seasonal cycles present in the data. Regressing the first difference of the series on the complete set of spectral indicators and the break dummies at the seasonal frequencies yielded a regression standard error $\widehat{\sigma} = 8.12$, a long run variance estimate, computed using the first 8 residual autocovariances, $\widehat{\sigma}_{LR}(8) = 7.96$ and a break magnitude $\widehat{\theta} = (-39.25, -10.34, 21.69)'$, with associated t-ratios $(-16.96, -4.41, 13.16)'$. The structural break then appears sufficiently big to affect the behaviour of the test at frequency zero if computed without either bias-correcting or pre-filtering the data by $\nabla_2 \equiv (1 + L + L^2 + L^3)$.

Table 7.1: Tests at frequency zero for UK marriages

	S_T=0	S_T=1	S_T=2	S_T=3	S_T=4	S_T=5	S_T=8	10%	5%	1%
$\mathcal{L}$	0.534	0.523	0.577	0.569	0.428	0.364	0.279	0.347	0.463	0.739
$\mathcal{L}_0$	1.608	1.037	0.758	0.598	0.488	0.413	0.294	0.347	0.463	0.739
$\mathcal{L}^*$	1.535	0.989	0.731	0.577	0.472	0.399	0.285	0.347	0.463	0.739
$\mathcal{L}^{**}$	1.600	1.031	0.762	0.601	0.492	0.416	0.297	0.347	0.463	0.739
$\mathcal{L}^{\nabla}$	2.349	1.212	0.823	0.628	0.511	0.434	0.280	0.347	0.463	0.739

Setting $\Im_1 = \{0\}$, Table 7.1 reports the values taken by the KPSS-type statistic $\mathcal{L}$ and the modifications thereof proposed in Section 3: $\mathcal{L}_0$, $\mathcal{L}^*$, $\mathcal{L}^{**}$ and $\mathcal{L}^{\nabla}$. These statistics were constructed using the OLS residuals from the regression of the data on Z_t. As in Section 5.2, we have used a Newey and West (1987) HAC estimator with a Bartlett kernel with lag truncation parameter S_T. The 10%, 5%, 1% asymptotic critical values appropriate for each reported test are provided in the last three columns of Table 7.1.

Given the empirical results in Busetti and Harvey (2003a) and the simulation results of Section 5 one would anticipate that the KPSS-type statistic $\mathcal{L}$ will provide less significant outcomes than bias-corrected and pre-filtered versions of $\mathcal{L}$. From the first row of Table 7.1 we observe that the null hypothesis of stationarity around fixed seasonal effects is not rejected using the $\mathcal{L}$ statistic at the 1% significance level for any value of S_T and not rejected at the 5% level for $S_T \geq 4$. On the other hand, the non-parametric bias-corrected statistic $\mathcal{L}_0$, which uses knowledge of the breakpoint in the seasonal, rejects the null at the 5 % level for all $S_T \leq 4$ and at the 1 % level for $S_T \leq 2$. The same inferences are drawn using the $\mathcal{L}^{**}$ statistic which does not assume knowledge or existence of the breakpoint. The $\mathcal{L}^*$ statistic behaves similarly but rejects the null only at the 5 % level for $S_T = 2$. The pre-filtered statistic $\mathcal{L}^{\nabla}$ also provides stronger evidence than $\mathcal{L}$ against the null hypothesis; here a value of $S_T \geq 3$ needs to be chosen as pre-filtering, in this case, would turn an otherwise white noise error into an MA(3) process. Overall, the bias-corrected and pre-filtered statistics yield more significant outcomes than the KPSS-type statistic, $\mathcal{L}$.

As a second example, which entails testing at the seasonal frequencies, consider Figure 7.2 which graphs the logarithm of real quarterly seasonally unadjusted U.K. consumers' expenditure on tobacco goods for the period 1975q1 1996q1, the data obtained from the U.K. O.N.S. macroeconomic database. Fitting the Basic Structural Model (BSM) of Harvey (1989, p.47) to these data yielded the following values of the hyperparameters[6] (standard deviations of the level, slope, seasonal and irregular components respectively): 1.48, 0.02, 0.36, 0, each multiplied by 10^{-2}. Notice, in particular that the hyperparameter associated with the level is approximately four times as large as that associated with the seasonal component. The Box-Ljung statistic with 9 lags and 6 degrees of freedom, $Q(9, 6)$, was 7.39, with p-value 0.29. The extracted seasonal component is depicted in Figure 7.3. It is quite clear from both Figures 7.3 and 7.2 that the seasonal pattern in the data is not fixed through time. In particular, there is a clear structural break in the seasonal pattern around 1980-1981.

Table 7.2 reports the values taken by the pre-filtered CH-type seasonal stability statistics, $\mathcal{L}^{\nabla}$ and $\overline{\mathcal{L}}^{\nabla}$, together with the corresponding BH-type statistics, denoted by $\mathcal{BH}$ and $\overline{\mathcal{BH}}$, discussed in Section 6.1. The former allow for unattended unit roots and unattended breaks, while the latter also control for the possibility of attended breaks. It is clear from Figures 7.2 and 7.3 that attended breaks will be an important issue in this application. By setting $\Im_1 = \{1, 2\}$, we have considered joint tests against non-stationary stochastic seasonality at either the harmonic seasonal frequency, $\lambda_1 \equiv \pi/2$, or the Nyquist frequency, $\lambda_2 \equiv \pi$, or both. We also report the corresponding individual frequency tests against unit roots at the harmonic and Nyquist frequencies which obtain on setting $\Im_1 = \{1\}$ and $\Im_1 = \{2\}$, repectively.

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[6] Imposing the restriction of a fixed slope did not alter the estimates of the remaining hyperparameters.

Table 7.2: Tests at the seasonal frequencies for tobacco expenditure

$\Im_1$		$S_T = 0$	$S_T = 1$	$S_T = 2$	$S_T = 3$	$S_T = 5$	$S_T = 8$	10%	5%	1%
$\{1, 2\}$	$\mathcal{L}^{\nabla}$	1.941	1.551	1.411	1.377	0.973	0.783	0.846	1.010	1.350
	$\overline{\mathcal{L}}^{\nabla}$	3.672	3.194	3.091	2.940	2.020	1.553	0.846	1.010	1.350
	$\mathcal{BH}^{\nabla}$	2.015	2.038	2.031	2.001	1.857	1.870	1.490	1.680	2.120
	$\overline{\mathcal{BH}}^{\nabla}$	2.011	1.972	1.946	1.928	1.887	1.904	1.490	1.680	2.120
$\{1\}$	$\mathcal{L}^{\nabla}$	3.270	2.145	1.547	1.295	0.949	0.722	0.610	0.749	1.070
	$\overline{\mathcal{L}}^{\nabla}$	3.324	3.324	2.343	2.041	1.485	1.093	0.610	0.749	1.070
	$\mathcal{BH}^{\nabla}$	0.898	0.904	0.841	0.795	0.747	0.779	1.070	1.240	1.600
	$\overline{\mathcal{BH}}^{\nabla}$	0.951	0.951	0.887	0.858	0.814	0.845	1.070	1.240	1.600
$\{2\}$	$\mathcal{L}^{\nabla}$	3.279	1.841	1.263	0.981	0.697	0.509	0.347	0.463	0.739
	$\mathcal{BH}^{\nabla}$	1.198	1.211	1.008	0.919	0.799	0.717	0.610	0.749	1.070

The above statistics were constructed using the OLS residuals from the regression of the appropriately pre-filtered data on Z_t.[7] In the case of the joint seasonal statistic the appropriate pre-filter is $\nabla_2 \equiv (1 - L)$, since $\Im_2 = \{0\}$. For $\Im_1 = \{1\}$, $\nabla_2 \equiv (1 - L^2)$ while $\nabla_2 \equiv (1 - L)(1 + L^2)$ for $\Im_1 = \{2\}$. Graphs of the three pre-filtered series are provided in Figures 7.4-7.6. Again we have used a Newey and West (1987) HAC estimator with a Bartlett kernel and lag truncation parameter S_T. The 10%, 5%, 1% asymptotic critical values appropriate for each reported test are provided in the last three columns of Table 7.2. Due to the sizeable random walk component estimated in the BSM, we will only consider pre-filtered tests against seasonal unit roots in this application. The bias-corrected tests of Section 3.1 are clearly inappropriate.

The pre-filtered CH-type statistics reported in Table 7.2, which do not allow for attended breaks at the seasonal frequencies, all provide a consistent body of evidence against the null: there is only one case reported where the null cannot be rejected at the 10 % level, and in most cases the null can be rejected at the 1 % level. In the case of the joint seasonal tests and the tests against unit root behaviour at the harmonic seasonal frequency, the evidence from the $\overline{\mathcal{L}}^{\nabla}$ statistics are stronger that from the $\mathcal{L}^{\nabla}$ statistic. Overall, application of the pre-filtered CH tests strongly indicates the presence of non-stationary behaviour at both the harmonic and Nyquist seasonal frequencies.

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[7] The reported statistics were constructed from a regression without a linear time trend since, as was noted in Section 6.2, the filter ∇_2 will remove a linear time trend from the data. Although not reported here, we also computed the pre-filtered tests with an unnecessary time trend fitted. This yielded much less evidence against the null, reflecting the usual argument on the reduction of the power of tests in the presence of additional nuisance parameters.

The BH-type statistics, $\mathcal{BH}$ and $\overline{\mathcal{BH}}$, reported in Table 7.2 allow us to investigate whether the evidence found against seasonal stability provided by the $\mathcal{L}^{\nabla}$ and $\overline{\mathcal{L}}^{\nabla}$ statistics in the series of (log) tobacco expenditure is attributable to seasonal unit root behaviour or purely to the deterministic break in the seasonal pattern. In computing these series we took the breakpoint as unknown. In each case the breakpoint was estimated as discussed in Section 6.1, and was identified at 1980q4 for the joint test and at 1981q1 and 1980q3 for the tests at frequencies $\pi/2$ and π, respectively.

The joint BH-type test against unit roots at frequencies π and $\pi/2$ again provides consistent evidence against the null of stochastic seasonal stationarity, allowing for seasonal level shifts: both variants of the test reject at the 5 % level for all values of S_T reported. Neither variant of the BH-type test against unit roots at the harmonic seasonal frequency $\pi/2$ is able to reject the null, even at the 10% level, regardless of the value of S_T. In contrast, the BH-type test against unit roots at frequency π points strongly towards seasonal unit root behaviour at frequency π, even on allowing for seasonal level shifts. Consequently, the tests which explicitly allow for unattended unit roots and both attended and unattended breaks provide a strong indication of unit root behaviour at the Nyquist frequency but no indication of unit root behaviour at the harmonic seasonal frequency. In contrast, the standard tests would have led the practitioner to conclude that unit roots were present at all of the seasonal frequencies.

8. Conclusions

In this paper we have investigated the effects of structural breaks on tests against stochastic trend and seasonality. In contrast to the existing literature we have considered the effects of structural breaks at frequencies *other* than those being subject to stability testing. We have shown that such breaks alter both the large sample and finite sample distributions of the stationarity statistics, effecting a severe under-sizing problem in the tests with an associated, often very dramatic, loss of power under the alternative. This effect coincides with what is seen when there are unattended unit roots, but is in contrast to the case where there is a structural break at the frequencies under test, where an over-sizing problem occurs.

We have suggested two methods of modifying the stationarity tests to remove these problems. The first, appropriate, for the case of structural breaks, involves bias-correcting the original stability test. The second, effective against both structural breaks and unattended unit roots, is achieved by running the stability tests on pre-filtered data. Both are shown to recover the usual limiting null distribution, appropriate to the case where there are no breaks or unattended unit roots. We have reported simulation evidence suggestive that the corrected statistics perform well in practice, adequately correcting test size under the null, without sacrificing test power under the alternative. Applications of the original and modified tests to data on U.K. marriages and U.K. consumers' expenditure on tobacco were considered. These demonstrated the practical relevance of the problems discussed in this paper and the usefulness of the remedial suggestions which we have made.

Appendix: proofs

Proof of Proposition 1: Let $\widehat{\gamma}_1,\ \widehat{\gamma}_2$ be the OLS estimators of the regression coefficients for $Z_{1t},\ Z_{2t}$ from (5). Using standard arguments it is not difficult to show that, under $H_0 : \sigma_\eta^2 = 0$,

$$T^{\frac{1}{2}} \begin{pmatrix} \widehat{\gamma}_1 - \gamma_{10} \\ \widehat{\gamma}_2 - \gamma_{20} - \alpha\theta \end{pmatrix} \Rightarrow N \left(\begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} \sigma^2 S_1^{-1} & 0 \\ 0 & \sigma^2 S_2^{-1} \end{pmatrix} \right),$$

where $S_i = \lim_{T\to\infty} T^{-1} \sum_{t=1}^T Z_{it} Z_{it}',\ i = 1, 2$. Note that $S_1,\ S_2$ are diagonal matrices with either 1 or $\frac{1}{2}$ on the main diagonal; precisely, they contain h_k^{-1} for the element(s) corresponding to the k-th spectral regressor.

Then we can show that the partial sum process $\sum_{j=1}^t Z_{1j} e_j$ converges weakly to a Brownian bridge, on scaling. Write the OLS residuals as

$$e_t = u_t + Z_{2t}'\theta d_t(\alpha) - Z_{1t}' (\widehat{\gamma}_1 - \gamma_{10}) - Z_{2t}' (\widehat{\gamma}_2 - \gamma_{20}). \tag{21}$$

Since $\widehat{\gamma}_2 - \gamma_{20} \xrightarrow{p} \alpha\theta$ and from the orthogonality relations between each pair of spectral indicators we have from (21) that

$$\begin{aligned} \sigma^{-1} S_1^{-\frac{1}{2}} T^{-\frac{1}{2}} \sum_{t=1}^{[Tr]} Z_{1t} e_t &= \sigma^{-1} S_1^{-\frac{1}{2}} T^{-\frac{1}{2}} \sum_{t=1}^{[Tr]} Z_{1t} u_t - \sigma^{-1} S_1^{-\frac{1}{2}} T^{-1} \sum_{t=1}^{[Tr]} Z_{1t} Z_{1t}' T^{\frac{1}{2}} (\widehat{\gamma}_1 - \gamma_{10}) + o_p(1) \\ \text{(22)} \qquad &\Rightarrow W_{s_1}(r) - r W_{s_1}(1) \equiv B_{s_1}(r), \end{aligned}$$

using the Functional Central Limit Theorem of Chan and Wei (1988). It then follows directly from (22) and an application of the Continuous Mapping Theorem (CMT) that

$$\sigma^{-2} T^{-2} \sum_{k\in\Im_1} h_k \sum_{t=1}^T Trace \left(\sum_{j=1}^t z_{k,j} e_j \sum_{j=1}^t z_{k,j}' e_j \right) \Rightarrow \int_0^1 B_{s_1}(r)' B_{s_1}(r) dr, \tag{23}$$

i.e. the numerator of the statistic ω is asymptotically unaffected by the level shift.

The factor $m(\alpha, \theta, \sigma^2)$ in the asymptotic null distribution of ω is due to a bias in estimating σ^2. After taking the square of (21), it is easy to show that

$$\widehat{\sigma}^2 \xrightarrow{p} \sigma^2 + b(\alpha, \theta),$$

where

$$b(\alpha,\theta) = \lim_{T\to\infty} T^{-1}\sum_{t=1}^{T}\left(d_t(\alpha)-\alpha\right)^2\left(Z_{2t}'\theta\right)^2.$$

The stated result then follows directly, using an application of the CMT.

Proof of Proposition 2: Let $\alpha_0 \in (0,1)$ be the true breakpoint location parameter in the model (3)-(11). As we want to compute regressions for the whole range of breakpoints, it is useful to express the null model in terms of a breakpoint parameter α not necessarily equal to the true value:

$$y_t = Z_{1t}'\gamma_{10} + Z_{2t}'\gamma_{20} + d_t(\alpha)Z_{2t}'\theta + u_t^*, \quad (24)$$

$$u_t^* = \left(u_t + \left(d_t(\alpha_0) - d_t(\alpha)\right)Z_{2t}'\theta\right). \quad (25)$$

Let $e_t(\alpha)$ be the OLS residuals from regression (24) and denote by $\widehat{\sigma}^2(\alpha) = T^{-1}\sum_{t=1}^{T} e_t(\alpha)^2$ their sample variance. Note that (24) can be viewed as a regression with omitted variables. Then, it is a standard result that $\widehat{\sigma}^2(\alpha) \geq \widehat{\sigma}^2(\alpha_0)$, and thus, under $H_0 : \sigma_\eta^2 = 0$, $\widehat{\sigma}^2(\alpha^*) \equiv \inf_\alpha \widehat{\sigma}^2(\alpha) \xrightarrow{p} \sigma^2$.

Then we have that under $H_0 : \sigma_\eta^2 = 0$ and for every $\alpha \in (0,1)$

$$\sigma^{-1}S_1^{-\frac{1}{2}}T^{-\frac{1}{2}}\sum_{t=1}^{[Tr]} Z_{1t}e_t(\alpha) \Rightarrow B_{s_1}(r); \quad (26)$$

this follows from extending the result of Busetti and Harvey (2003a) on the null distribution of ω in the presence of modelled breaks in the trend (i.e. at frequency zero) together with the argument of Proposition 1 on the presence of neglected level shifts, which in this case would correspond to the term $\left(d_t(\alpha_0) - d_t(\alpha)\right)Z_{2t}'\theta$.

It therefore follows directly from (26) and an application of the CMT that

$$\sigma^{-2}T^{-2}\sum_{k\in\Im_1} h_k \sum_{t=1}^{T} Trace\left(\sum_{j=1}^{t} z_{k,j}e_j(\alpha)\sum_{j=1}^{t} z_{k,j}'e_j(\alpha)\right) \Rightarrow \int_0^1 B_{s_1}(r)'B_{s_1}(r)dr \equiv CvM(s_1), \quad (27)$$

independently of α. Using the previous results for $\widehat{\sigma}^2(\alpha^*)$ and by applying the CMT, we obtain that the right member of (27) is the asymptotic null distribution for both ω^* of (16) and ω^{**} of (17).

If no breakpoint actually occurs, under H_0, $\widehat{\sigma}^2(\alpha^*)$ is still asymptotically unbiased and thus, by the result of Busetti and Harvey (2003a), the limiting null distributions of ω^* and ω^{**} are $CvM(s_1)$ as before.

Proof of Proposition 3: Under the conditions of Proposition 3, (24)-(25) is appropriate on substituting u_t^* in (24) by $(u_t + Z_{2t}'(\gamma_{2t} - \gamma_{20}) + (d_t(\alpha_0) - d_t(\alpha))\, Z_{2t}'\theta)$. Again denote the OLS residuals from (24) by $e_t(\alpha)$, which therefore satisfy

$$\begin{aligned} e_t(\alpha) &= u_t^* - Z_{1t}'\left(\widehat{\gamma}_1 - \gamma_{10}\right) - Z_{2t}'\left(\widehat{\gamma}_2 - \gamma_{20}\right) - d_t(\alpha) Z_{2t}'\left(\hat{\theta} - \theta\right) \\ &= u_t + d_t(\alpha_0) Z_{2t}'\theta - d_t(\alpha) Z_{2t}'\hat{\theta} - Z_{1t}'\left(\widehat{\gamma}_1 - \gamma_{10}\right) - Z_{2t}'\left(\widehat{\gamma}_2 - \gamma_{2t}\right), \end{aligned} \tag{28}$$

where $\widehat{\gamma}_1, \widehat{\gamma}_2$ and $\hat{\theta}$ denote the OLS estimators of the regression coefficients for Z_{1t}, Z_{2t} and $d_t(\alpha)Z_{2t}$ respectively from (24). Routine algebra establishes the result that

$$\sigma^{-1} S_1^{-\frac{1}{2}} T^{-\frac{1}{2}} \sum_{t=1}^{[Tr]} Z_{1t} e_t(\alpha) \Rightarrow B_{s_1}(r) + O_p(1), \tag{29}$$

since, independently of α, $T^{-1/2}\sum_{t=1}^{[Tr]} Z_{1t}Z_{2t}'\left(\widehat{\gamma}_2 - \gamma_{2t}\right)$ and $T^{-1/2}\sum_{t=1}^{[Tr]} Z_{1t}(d_t(\alpha)Z_{2t}'\hat{\theta})$ are both $O_p(1)$ while $T^{-1/2}\sum_{t=1}^{[Tr]} Z_{1t}(d_t(\alpha_0)Z_{2t}'\theta) = o_p(1)$. Notice that it was not necessary to impose the condition that $\theta = 0$ in deriving (29); cf. Remark 5. Consequently,

$$\sigma^{-2} T^{-2} \sum_{k \in \Im_1} h_k \sum_{t=1}^{T} Trace\left(\sum_{j=1}^{t} z_{k,j} e_j(\alpha) \sum_{j=1}^{t} z_{k,j}' e_j(\alpha) \right) \Rightarrow \int_0^1 B_{s_1}(r)' B_{s_1}(r) dr + O_p(1), \tag{30}$$

i.e. the numerator of (6) remains $O_p(T^2)$, but when appropriately scaled differs from the $CvM(s_1)$ distribution by the presence of an additional $O_p(1)$ random variable. Using the same development as in KPSS (p.168), it is straightforward to demonstrate that the long-run variance estimators $\widehat{\Omega}_1(\alpha)$ and $\widehat{g}(\lambda_k; \alpha)$ both diverge at rate $O_p(TS_T)$ for all α, and hence for $\alpha = \alpha^*$, when $\sigma_\kappa^2 > 0$, regardless of θ. The OLS variance estimator $\widehat{\sigma}^2(\alpha)$ is a special case of the long-run estimator $\widehat{g}(0; \alpha)$ for $S_T = 0$ and, hence, is $O_p(T)$ for all α. Consequently, ω^* of (16) and ω^{**} of (17) are both $O_p(T^{-1})$ while $\mathcal{L}^*$, $\mathcal{L}^{**}$, $\overline{\mathcal{L}}^*$ and $\overline{\mathcal{L}}^{**}$ are all $O_p((TS_T)^{-1})$, their respective orders in probability differing through the respective divergence rates of the variance estimators used in their construction.

Table 5.1a: Empirical rejection probabilities for the bias-corrected tests of stationarity at frequency zero in the presence of an unattended break at the seasonal frequencies.* The sample size is 100.

		$\sigma_\eta=0$				$\sigma_\eta=0.01$				$\sigma_\eta=0.05$				$\sigma_\eta=0.1$				$\sigma_\eta=0.5$			
		ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0
$\theta^*=0$		5.69	7.24	6.80	N.A	7.00	8.60	8.01	N.A	31.48	34.46	33.45	N.A	59.44	61.98	61.01	N.A	96.66	97.14	96.85	N.A
	$\alpha=1/8$	3.20	6.47	6.07	6.14	4.06	7.86	7.29	7.51	25.83	33.20	32.00	32.37	54.40	60.99	59.88	60.31	95.93	96.91	96.61	96.73
$\theta^*=1$	$\alpha=1/4$	2.09	6.64	6.12	6.15	2.82	7.81	7.34	7.62	22.34	33.18	31.99	32.46	50.98	60.87	59.81	60.16	95.34	96.86	96.59	96.73
	$\alpha=1/2$	1.62	6.68	6.11	6.16	2.03	7.95	7.40	7.58	19.69	33.21	32.30	32.37	48.48	60.91	59.95	60.34	94.95	96.89	96.54	96.79
	$\alpha=1/8$	0.69	6.45	6.04	6.14	0.92	7.73	7.23	7.51	14.51	33.09	31.90	32.37	42.45	60.89	59.75	60.31	93.91	96.84	96.59	96.73
$\theta^*=2$	$\alpha=1/4$	0.19	6.68	6.24	6.15	0.23	8.00	7.42	7.62	8.52	33.42	32.28	32.46	34.11	60.95	59.93	60.16	91.90	96.93	96.67	96.73
	$\alpha=1/2$	0.04	6.89	6.33	6.16	0.08	8.11	7.56	7.58	5.43	33.58	32.51	32.37	29.06	61.22	60.06	60.34	90.22	96.90	96.62	96.79
	$\alpha=1/8$	0.00	6.94	6.57	6.14	0.00	8.48	7.82	7.51	1.84	34.03	33.07	32.37	19.06	61.64	60.53	60.31	86.57	96.97	96.73	96.73
$\theta^*=4$	$\alpha=1/4$	0.00	7.74	7.16	6.15	0.00	9.15	8.54	7.62	0.23	35.13	34.08	32.46	9.80	62.34	61.22	60.16	80.23	97.11	96.82	96.73
	$\alpha=1/2$	0.00	8.36	7.85	6.16	0.00	9.85	8.99	7.58	0.07	36.28	35.21	32.37	5.37	62.99	62.08	60.34	75.16	97.04	96.78	96.79
	$\alpha=1/8$	0.00	10.05	9.32	6.14	0.00	11.36	10.62	7.51	0.00	38.44	37.58	32.37	1.09	65.25	64.14	60.31	65.60	97.35	97.13	96.73
$\theta^*=8$	$\alpha=1/4$	0.00	14.30	13.92	6.15	0.00	16.13	15.28	7.62	0.00	43.94	42.77	32.46	0.11	68.92	67.88	60.16	53.48	97.55	97.34	96.73
	$\alpha=1/2$	0.00	20.74	20.03	6.16	0.00	22.58	21.48	7.58	0.00	49.88	48.66	32.37	0.00	72.56	72.02	60.34	45.83	97.61	97.35	96.79

* The DGP corresponds to equations (2.1)-(2.2)-(3.1) for quarterly data, $\mathfrak{F}_1=\{0\}$.

Table 5.1b: Empirical rejection probabilities for the bias-corrected tests of stationarity at frequency zero in the presence of an unattended break at the seasonal frequencies.* The sample size is 200.

		$\sigma_\eta=0$				$\sigma_\eta=0.01$				$\sigma_\eta=0.05$				$\sigma_\eta=0.1$				$\sigma_\eta=0.5$			
		ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0
$\theta^*=0$		5.55	6.07	5.93	N.A	10.06	10.99	10.65	N.A	60.47	62.01	61.41	N.A	85.33	86.01	85.65	N.A	99.74	99.80	99.78	N.A
	$\alpha=1/8$	2.97	5.77	5.67	5.70	6.53	10.55	10.17	10.33	55.09	61.18	60.64	60.91	82.14	85.65	85.24	85.52	99.64	99.76	99.76	99.76
$\theta^*=1$	$\alpha=1/4$	1.78	5.80	5.64	5.71	4.75	10.53	10.17	10.39	51.58	61.20	60.53	60.93	80.00	85.68	85.15	85.58	99.58	99.76	99.75	99.76
	$\alpha=1/2$	1.27	5.82	5.66	5.72	3.70	10.65	10.19	10.41	48.82	61.37	60.65	60.97	78.33	85.70	85.19	85.51	99.54	99.75	99.74	99.75
	$\alpha=1/8$	0.44	5.74	5.65	5.70	1.77	10.52	10.09	10.33	42.61	61.19	60.54	60.91	74.09	85.60	85.18	85.52	99.42	99.76	99.75	99.76
$\theta^*=2$	$\alpha=1/4$	0.10	5.86	5.66	5.71	0.64	10.62	10.20	10.39	34.69	61.30	60.63	60.93	67.68	85.67	85.20	85.58	99.12	99.74	99.74	99.76
	$\alpha=1/2$	0.01	5.89	5.70	5.72	0.24	10.73	10.38	10.41	29.57	61.53	60.76	60.97	62.63	85.70	85.22	85.51	98.82	99.76	99.73	99.75
	$\alpha=1/8$	0.01	5.93	5.77	5.70	0.02	10.84	10.39	10.33	19.28	61.70	60.90	60.91	53.16	85.79	85.42	85.52	97.90	99.76	99.75	99.76
$\theta^*=4$	$\alpha=1/4$	0.00	6.18	6.02	5.71	0.00	11.13	10.86	10.39	9.61	62.11	61.47	60.93	41.03	86.05	85.62	85.58	95.90	99.75	99.75	99.76
	$\alpha=1/2$	0.00	6.63	6.38	5.72	0.00	11.69	11.18	10.41	5.54	62.54	61.98	60.97	33.60	86.23	85.72	85.51	94.12	99.75	99.75	99.75
	$\alpha=1/8$	0.00	7.04	6.70	5.70	0.00	12.33	11.78	10.33	1.49	63.33	62.71	60.91	21.44	86.67	86.43	85.52	89.85	99.77	99.76	99.76
$\theta^*=8$	$\alpha=1/4$	0.00	8.21	7.92	5.71	0.00	13.95	13.40	10.39	0.14	65.13	64.60	60.93	10.39	87.35	87.08	85.58	83.35	99.80	99.78	99.76
	$\alpha=1/2$	0.00	9.66	9.36	5.72	0.00	16.09	15.71	10.41	0.03	66.81	66.35	60.97	5.74	87.97	87.64	85.51	78.14	99.80	99.80	99.75

* The DGP corresponds to equations (2.1)-(2.2)-(3.1) for quarterly data, $\mathfrak{F}_1=\{0\}$.

Table 5.2a: Empirical rejection probabilities for the bias-corrected tests of stationarity at the seasonal frequencies in the presence of an unattended break at frequency zero.* The sample size is 100.

		$\sigma_\eta=0$				$\sigma_\eta=0.01$				$\sigma_\eta=0.05$				$\sigma_\eta=0.1$				$\sigma_\eta=0.5$			
		ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0
$\theta^*=0$		5.23	6.61	6.03	N.A	6.27	7.84	7.18	N.A	39.59	42.93	40.84	N.A	80.14	81.70	80.48	N.A	99.81	99.84	99.77	N.A
	$\alpha=1/8$	3.29	6.09	6.17	5.52	4.05	7.30	7.13	6.58	33.90	41.96	40.30	40.29	76.76	81.39	80.17	80.40	99.78	99.82	99.78	99.82
$\theta^*=1$	$\alpha=1/4$	2.43	6.46	5.82	5.50	2.93	7.78	6.80	6.60	29.96	42.36	39.93	40.15	73.99	81.41	79.90	80.42	99.67	99.83	99.75	99.82
	$\alpha=1/2$	1.73	6.14	5.68	5.57	2.11	7.44	6.87	6.62	27.00	41.88	39.77	40.20	71.82	81.29	80.09	80.41	99.67	99.83	99.75	99.82
	$\alpha=1/8$	0.87	5.97	7.26	5.52	1.21	7.17	8.46	6.58	20.18	41.76	41.42	40.29	66.76	81.19	80.21	80.40	99.59	99.82	99.77	99.82
$\theta^*=2$	$\alpha=1/4$	0.22	7.00	5.89	5.50	0.28	8.33	6.97	6.60	12.07	43.26	40.33	40.15	56.76	81.41	80.01	80.42	99.46	99.80	99.75	99.82
	$\alpha=1/2$	0.03	6.28	6.14	5.57	0.05	7.43	7.22	6.62	7.69	42.25	40.53	40.20	49.48	81.24	80.14	80.41	99.35	99.84	99.78	99.82
	$\alpha=1/8$	0.00	6.71	14.37	5.52	0.00	8.01	15.92	6.58	2.53	43.25	48.11	40.29	33.95	81.76	82.19	80.40	98.86	99.84	99.79	99.82
$\theta^*=4$	$\alpha=1/4$	0.00	10.73	7.43	5.50	0.00	12.29	8.63	6.60	0.32	47.35	43.66	40.15	16.26	82.88	81.40	80.42	97.80	99.78	99.78	99.82
	$\alpha=1/2$	0.00	7.81	9.22	5.57	0.00	9.36	10.50	6.62	0.05	46.13	45.08	40.20	9.00	82.74	82.07	80.41	96.84	99.84	99.76	99.82
	$\alpha=1/8$	0.00	12.67	54.50	5.52	0.00	14.49	55.37	6.58	0.00	53.45	71.71	40.29	1.88	85.82	88.82	80.40	93.59	99.87	99.80	99.82
$\theta^*=8$	$\alpha=1/4$	0.00	35.70	21.50	5.50	0.00	37.41	23.86	6.60	0.00	66.21	60.89	40.15	0.12	89.28	88.62	80.42	86.01	99.76	99.83	99.82
	$\alpha=1/2$	0.00	23.06	32.40	5.57	0.00	25.42	34.70	6.62	0.00	64.17	65.71	40.20	0.00	89.95	89.53	80.41	79.71	99.86	99.80	99.82

* The DGP corresponds to equations (2.1)-(2.2)-(3.1) for quarterly data, $\mathfrak{F}_1=\{1,2\}$.

Table 5.2b: Empirical rejection probabilities for the bias-corrected tests of stationarity at the seasonal frequencies in the presence of an unattended break at frequency zero.* The sample size is 200.

		$\sigma_\eta=0$				$\sigma_\eta=0.01$				$\sigma_\eta=0.05$				$\sigma_\eta=0.1$				$\sigma_\eta=0.5$			
		ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0	ω	ω^*	ω^{**}	ω_0
$\theta^*=0$		5.30	5.87	5.57	N.A	10.61	11.84	11.50	N.A	81.37	82.30	81.69	N.A	97.96	98.06	98.00	N.A	100.00	100.00	100.00	N.A
	$\alpha=1/8$	3.55	5.70	5.40	5.38	7.64	11.41	11.05	10.87	77.91	81.83	81.40	81.57	97.36	98.06	97.94	97.98	100.00	100.00	100.00	100.00
$\theta^*=1$	$\alpha=1/4$	2.44	5.69	5.54	5.43	5.88	11.32	11.23	10.92	75.39	81.83	81.50	81.58	96.90	98.08	97.89	97.97	100.00	100.00	100.00	100.00
	$\alpha=1/2$	1.77	5.63	5.52	5.44	4.82	11.33	11.18	10.94	73.23	82.07	81.55	81.52	96.58	98.03	97.88	97.96	100.00	100.00	100.00	100.00
	$\alpha=1/8$	0.92	6.20	5.35	5.38	2.76	11.89	11.10	10.87	67.19	81.92	81.26	81.57	95.20	98.10	97.93	97.98	100.00	100.00	100.00	100.00
$\theta^*=2$	$\alpha=1/4$	0.27	6.15	6.00	5.43	0.99	11.75	11.51	10.92	57.68	81.98	81.65	81.58	92.78	98.08	97.86	97.97	100.00	100.00	100.00	100.00
	$\alpha=1/2$	0.05	5.87	5.98	5.44	0.42	11.53	11.47	10.94	50.77	82.21	81.80	81.52	90.61	98.03	97.85	97.96	100.00	100.00	100.00	100.00
	$\alpha=1/8$	0.02	8.56	5.69	5.38	0.09	14.90	11.56	10.87	35.43	82.56	81.65	81.57	84.28	98.15	97.94	97.98	100.00	100.00	100.00	100.00
$\theta^*=4$	$\alpha=1/4$	0.00	8.10	7.73	5.43	0.00	13.85	13.82	10.92	19.31	82.92	82.51	81.58	72.81	98.12	98.01	97.97	99.99	100.00	100.00	100.00
	$\alpha=1/2$	0.00	7.04	7.58	5.44	0.00	13.20	13.78	10.94	10.92	83.23	82.79	81.52	64.17	98.09	97.89	97.96	99.96	100.00	100.00	100.00
	$\alpha=1/8$	0.00	21.43	7.80	5.38	0.00	28.53	14.08	10.87	2.89	85.84	83.88	81.57	42.59	98.55	98.26	97.98	99.83	100.00	100.00	100.00
$\theta^*=8$	$\alpha=1/4$	0.00	18.11	17.63	5.43	0.00	25.45	25.39	10.92	0.23	86.23	85.83	81.58	21.28	98.52	98.35	97.97	99.27	100.00	100.00	100.00
	$\alpha=1/2$	0.00	14.81	16.93	5.44	0.00	22.82	25.06	10.94	0.02	88.10	87.45	81.52	11.99	98.69	98.56	97.96	98.50	100.00	100.00	100.00

* The DGP corresponds to equations (2.1)-(2.2)-(3.1) for quarterly data, $\mathfrak{F}_1=\{1,2\}$.

Table 5.3a: Empirical rejection probabilities for the pre-filtered KPSS test in the presence of an unattended break at the seasonal frequencies.* The sample size is 100.

		σ_η=0				σ_η=0.01				σ_η=0.05				σ_η=0.1				σ_η=0.5			
		S_T=3	S_T=4	S_T=5	S_T=6	S_T=3	S_T=4	S_T=5	S_T=6	S_T=3	S_T=4	S_T=5	S_T=6	S_T=3	S_T=4	S_T=5	S_T=6	S_T=3	S_T=4	S_T=5	S_T=6
θ^*=0		10.62	8.44	7.06	6.16	11.66	9.41	8.09	7.06	35.96	32.27	29.40	27.33	58.44	54.47	51.18	48.39	84.61	78.74	74.38	70.79
	α=1/8	10.67	8.26	6.98	6.14	11.80	9.65	8.12	7.11	35.91	32.29	29.43	27.30	58.59	54.38	51.05	48.35	84.64	78.72	74.38	70.71
θ^*=1	α=1/4	10.56	8.34	7.00	6.20	11.77	9.50	8.08	7.01	35.85	32.23	29.55	27.26	58.53	54.41	51.01	48.22	84.71	78.74	74.41	70.73
	α=1/2	10.55	8.40	7.15	6.21	11.56	9.42	8.00	7.09	36.02	32.06	29.42	27.26	58.50	54.33	51.15	48.41	84.63	78.71	74.40	70.83
	α=1/8	10.60	8.24	6.93	6.11	11.81	9.51	8.16	7.19	35.67	32.23	29.60	27.48	58.34	54.20	50.92	48.13	84.49	78.81	74.44	70.71
θ^*=2	α=1/4	10.37	8.22	6.85	6.15	11.77	9.35	8.01	7.02	35.55	31.96	29.48	27.13	58.29	54.16	50.72	48.04	84.64	78.73	74.39	70.70
	α=1/2	10.35	8.20	7.00	6.04	11.33	9.22	7.92	6.94	35.75	31.87	29.20	27.03	58.38	54.05	51.06	48.23	84.60	78.76	74.40	70.84
	α=1/8	10.46	8.37	6.99	6.09	11.67	9.43	8.13	7.14	35.19	31.61	29.33	26.93	57.62	53.55	50.35	47.72	84.55	78.72	74.49	70.65
θ^*=4	α=1/4	9.91	7.98	6.83	6.00	11.30	9.14	7.93	6.89	34.82	31.19	28.53	26.46	57.43	53.42	49.97	47.47	84.45	78.60	74.46	70.63
	α=1/2	9.51	7.49	6.38	5.48	10.57	8.47	7.29	6.40	34.44	30.95	28.28	26.07	57.55	53.49	50.39	47.63	84.38	78.71	74.29	70.70
	α=1/8	10.25	8.67	7.68	6.74	11.67	9.88	8.42	7.48	32.72	29.75	27.37	25.69	54.73	51.29	48.32	46.08	83.79	78.46	74.17	70.47
θ^*=8	α=1/4	8.86	7.21	6.16	5.42	9.86	8.41	7.17	6.33	31.42	28.09	25.68	24.09	54.10	50.60	47.54	45.06	84.04	78.45	73.96	70.18
	α=1/2	6.58	5.25	4.53	3.98	7.54	6.13	5.23	4.46	30.32	26.99	24.64	22.89	54.32	50.60	47.72	45.07	83.96	78.47	74.09	70.46

* The DGP corresponds to equations (2.1)-(2.2)-(4.1) for quarterly data, $\mathfrak{F}_1$={0}.

Table 5.3b: Empirical rejection probabilities for the pre-filtered (joint) CH test in the presence of an unattended break at frequency zero.* The sample size is 100.

		σ_η=0				σ_η=0.01				σ_η=0.05				σ_η=0.1				σ_η=0.5			
		S_T=0	S_T=1	S_T=2	S_T=3	S_T=0	S_T=1	S_T=2	S_T=3	S_T=0	S_T=1	S_T=2	S_T=3	S_T=0	S_T=1	S_T=2	S_T=3	S_T=0	S_T=1	S_T=2	S_T=3
θ^*=0		13.53	6.08	4.26	3.29	15.21	7.36	5.38	4.06	53.93	39.51	34.70	30.99	85.38	77.55	74.40	71.94	99.70	98.91	98.84	98.69
	α=1/8	13.57	6.02	4.32	3.26	15.26	7.43	5.28	4.06	53.68	39.04	34.34	30.81	85.35	77.42	74.41	71.97	99.71	98.92	98.85	98.72
θ^*=1	α=1/4	13.39	6.19	4.35	3.26	15.30	7.43	5.45	4.13	53.57	39.52	34.61	30.99	85.39	77.46	74.38	71.85	99.71	98.91	98.82	98.69
	α=1/2	13.43	6.15	4.23	3.25	15.13	7.31	5.30	4.10	53.60	39.38	34.42	30.74	85.26	77.43	74.33	71.82	99.71	98.90	98.81	98.68
	α=1/8	13.51	6.17	4.47	3.53	15.33	7.37	5.34	4.26	53.18	38.83	34.34	30.77	84.92	77.27	74.04	71.67	99.72	98.91	98.84	98.68
θ^*=2	α=1/4	13.18	6.08	4.24	3.29	15.15	7.41	5.44	4.06	53.18	39.08	34.39	30.55	85.04	77.24	74.11	71.62	99.72	98.92	98.76	98.70
	α=1/2	13.14	5.91	4.16	3.14	14.72	7.08	5.04	3.98	53.07	38.73	33.84	30.31	84.98	77.23	73.99	71.48	99.71	98.88	98.77	98.70
	α=1/8	13.53	6.44	4.80	3.82	15.59	7.58	5.64	4.63	52.10	38.43	33.53	30.05	84.13	76.15	73.06	70.66	99.70	98.93	98.79	98.66
θ^*=4	α=1/4	12.52	5.79	4.11	3.10	14.43	7.12	5.16	3.91	51.34	37.88	33.10	29.50	83.87	76.35	73.14	70.26	99.69	98.87	98.71	98.57
	α=1/2	11.72	5.19	3.74	2.87	13.15	6.26	4.47	3.51	50.69	36.96	32.02	28.80	84.01	76.07	72.73	70.10	99.67	98.86	98.75	98.65
	α=1/8	13.49	6.67	4.86	3.86	15.48	7.75	5.85	4.69	49.95	37.09	32.40	28.74	81.68	73.66	70.66	67.89	99.68	98.87	98.69	98.56
θ^*=8	α=1/4	10.89	5.22	3.61	2.74	12.74	6.24	4.54	3.35	47.09	34.46	29.68	26.10	80.84	72.76	69.34	66.66	99.58	98.78	98.51	98.38
	α=1/2	9.12	3.91	2.92	2.14	10.34	4.87	3.42	2.67	45.21	32.31	28.00	24.90	80.36	72.64	69.40	66.37	99.62	98.76	98.60	98.42

* The DGP corresponds to equations (2.1)-(2.2)-(4.1) for quarterly data, $\mathfrak{F}_1$={1,2}.

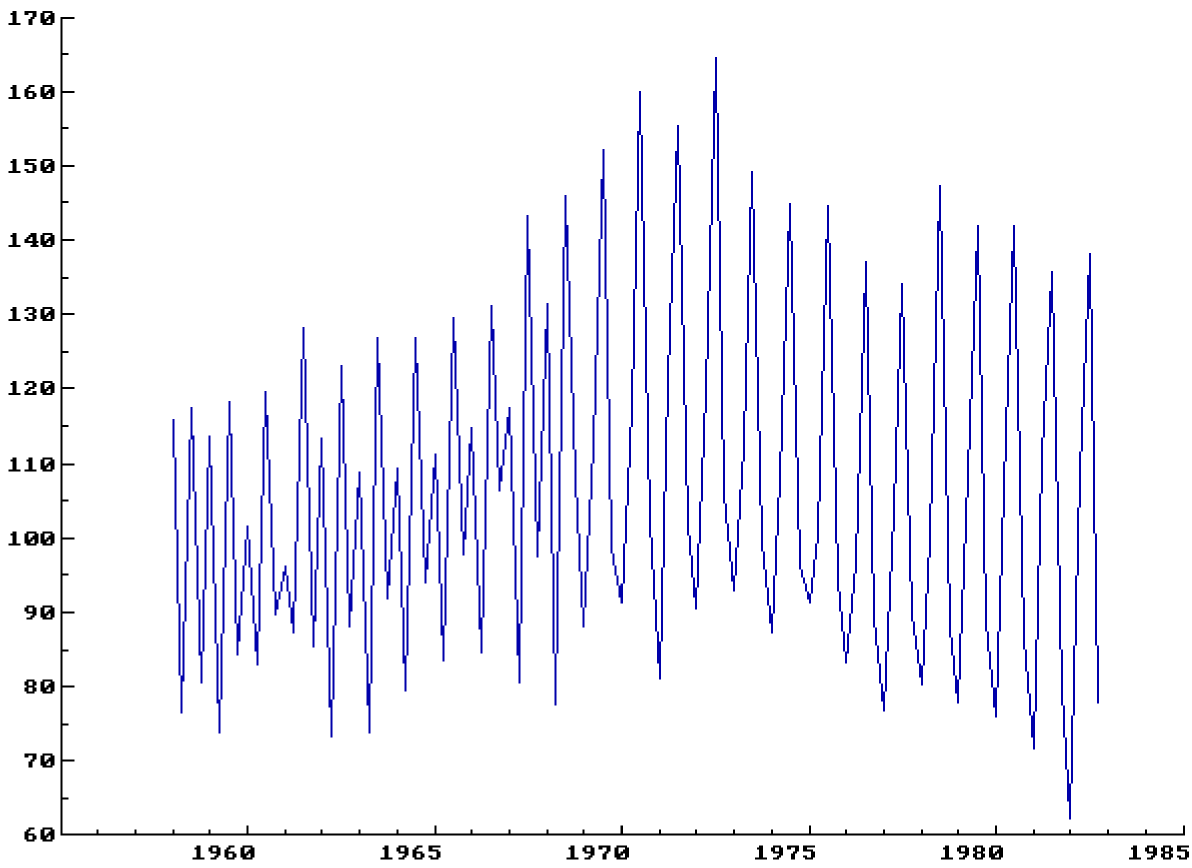

Figure 7.1: Number of Marriages (x1000) registered in UK, 1958Q1-1982Q4. Source: UK Monthly Digest of Statistics.

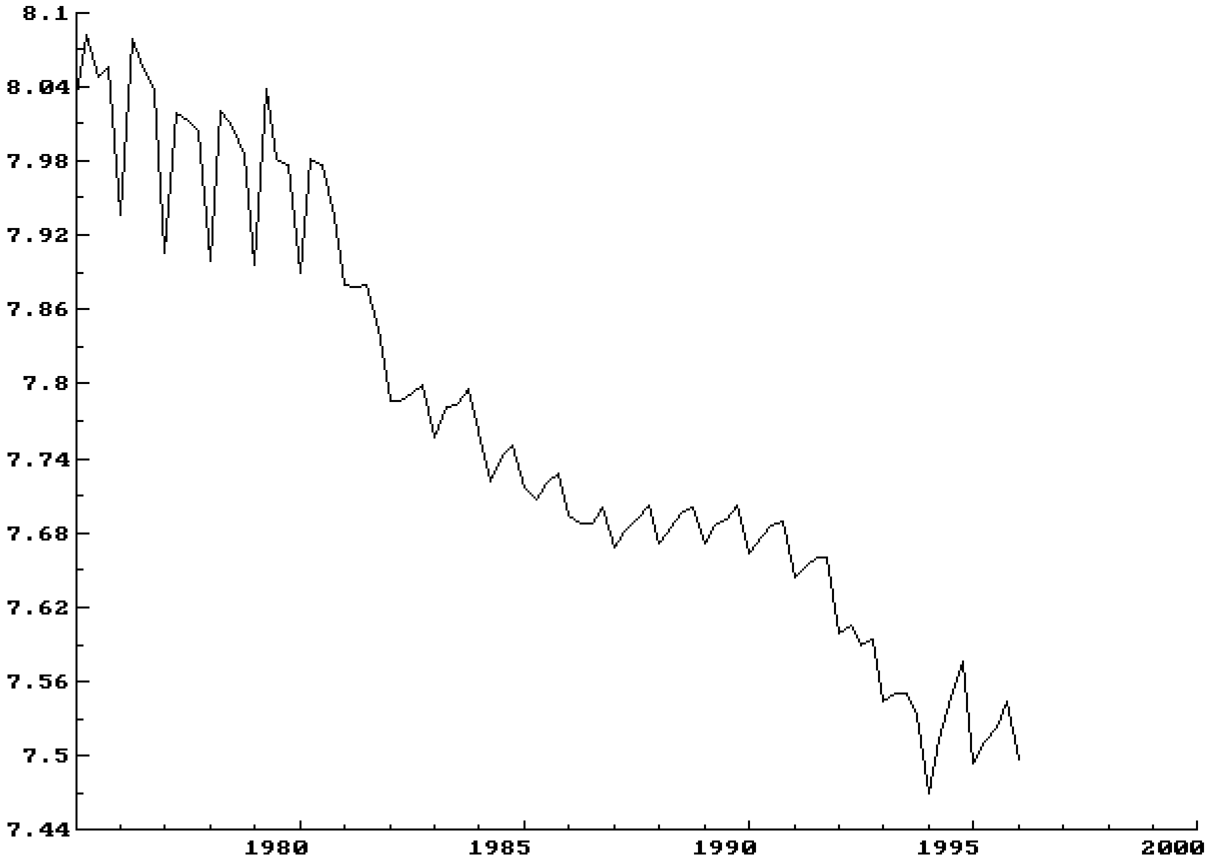

Figure 7.2: Logarithm of UK consumers' expenditure on tobacco products, 1975Q1-1996Q1. Source: UK ONS macroeconomic data-base.

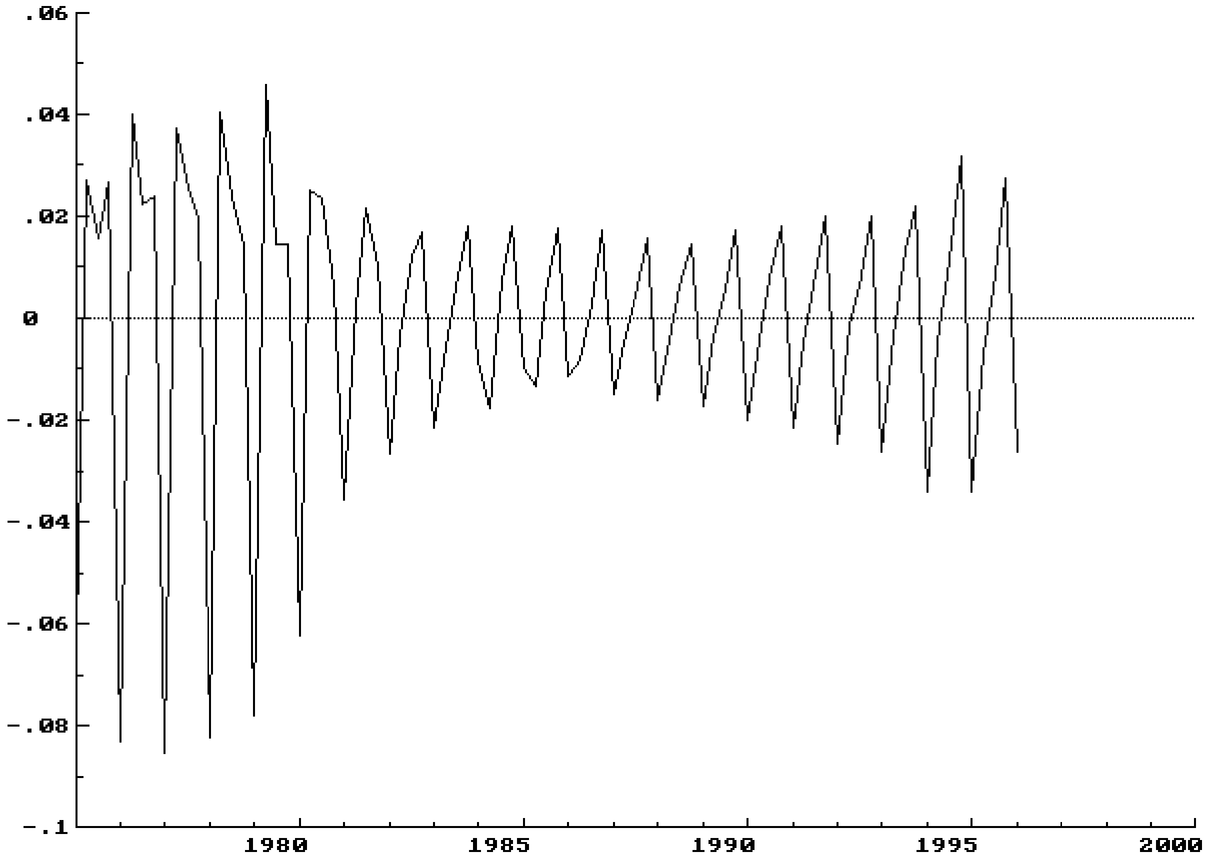

Figure 7.3: Extracted seasonal component from a Basic Structural Model (BSM) of the series of the logarithm of UK expenditure on tobacco products.

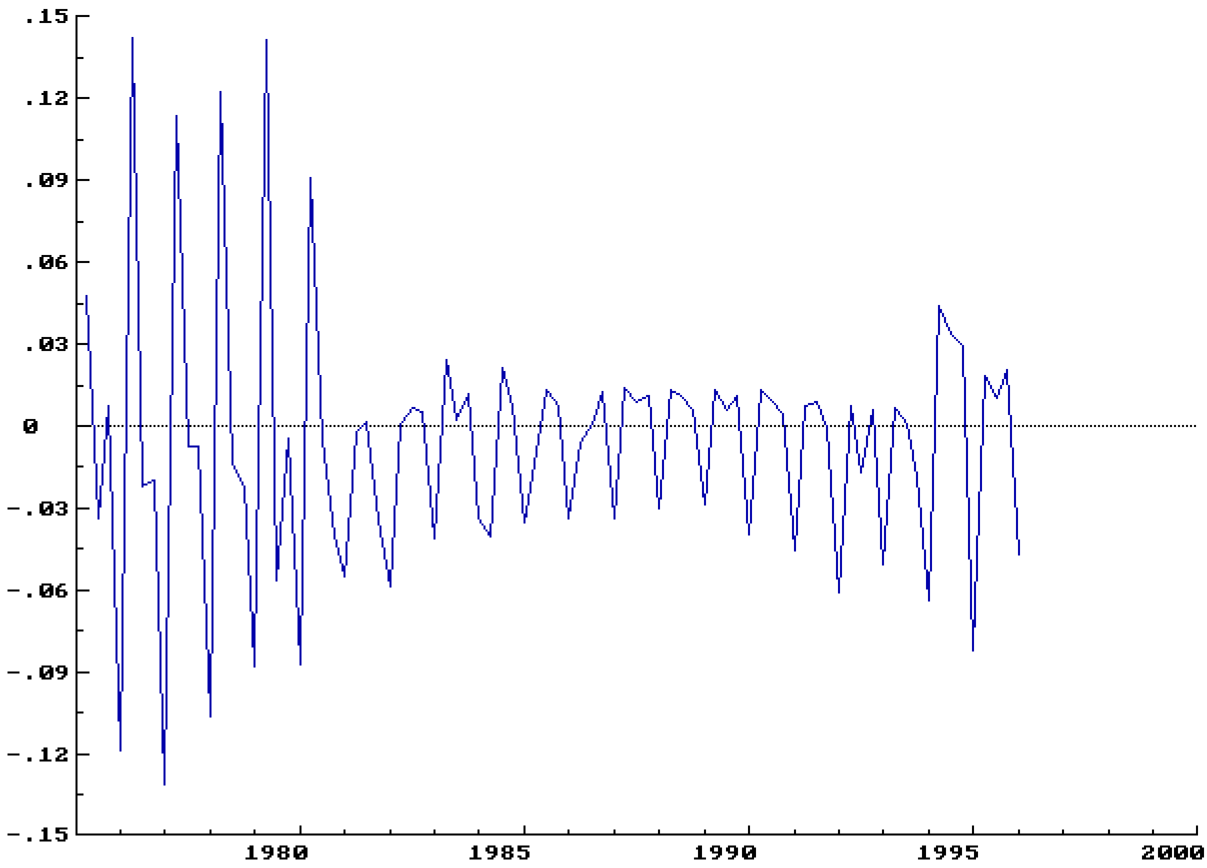

Figure 7.4: Logarithm of UK consumers' expenditure on tobacco products prefiltered at frequency zero (i.e. first differences).

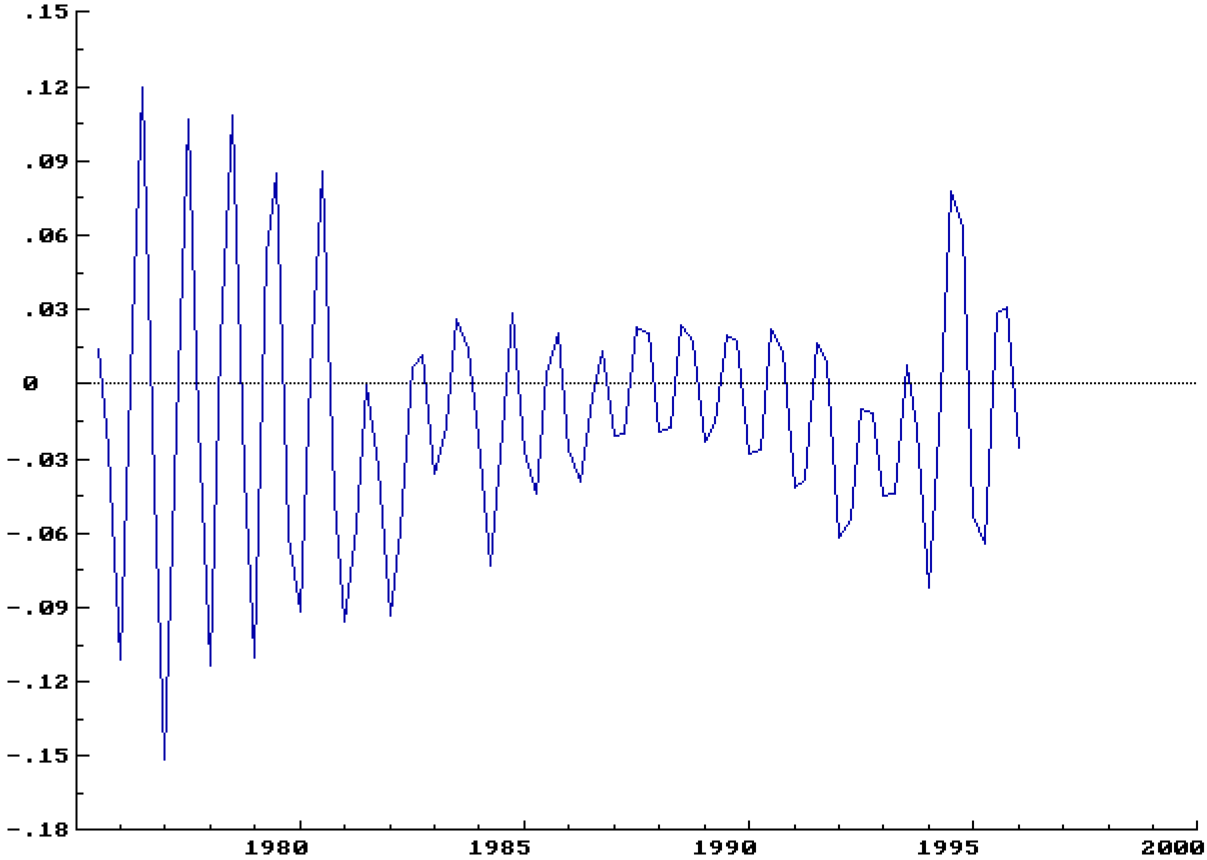

Figure 7.5: Logarithm of UK consumers' expenditure on tobacco products prefiltered at the frequencies zero and π.

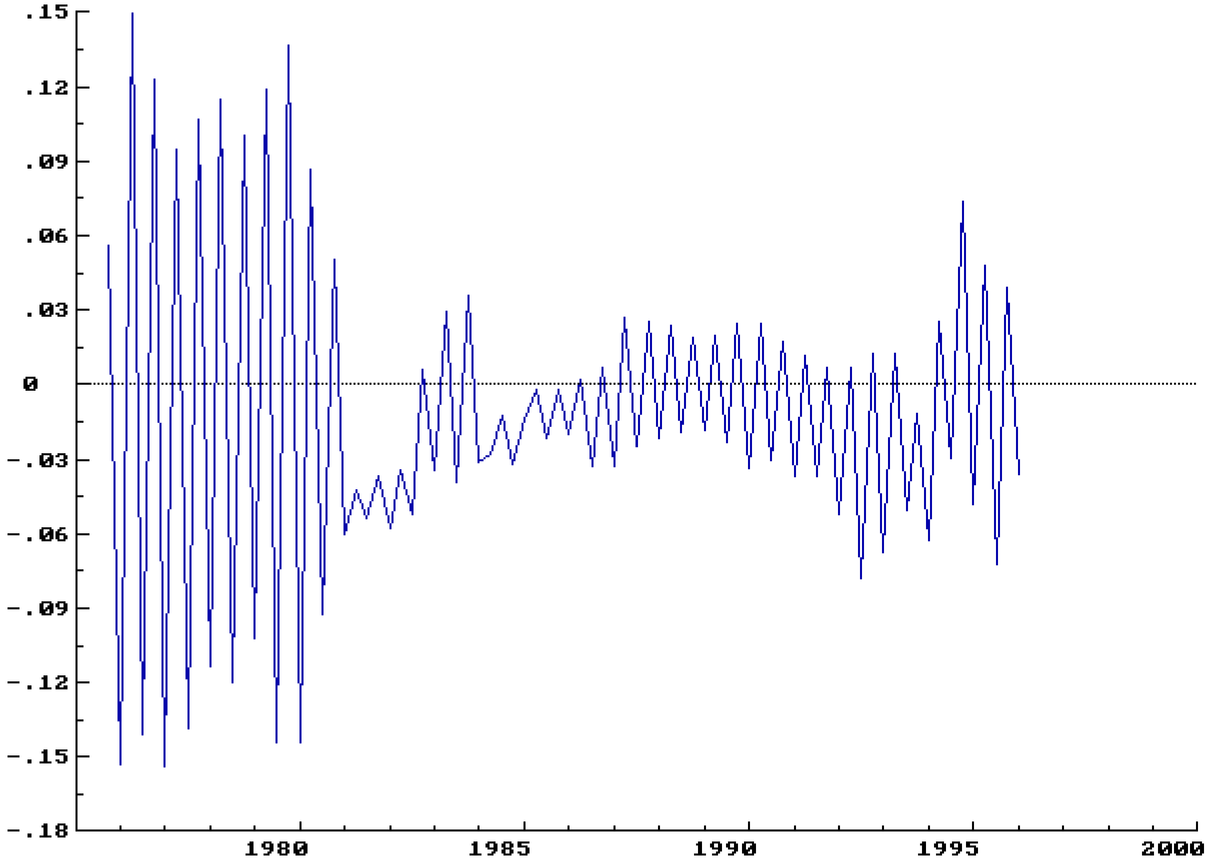

Figure 7.6: Logarithm of UK consumers' expenditure on tobacco products prefiltered at the frequencies zero and $\pi/2$.

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