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Heterogeneity in human capital and economic growth

by Stefania Zotteri

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HETEROGENEITY IN HUMAN CAPITAL AND ECONOMIC GROWTH

by Stefania Zotteri*

Abstract

This paper explores the growth-enhancing role played by heterogeneity in human capital accumulation. It develops a growth model that takes into account more than one mechanism for accumulating human capital; in particular, schooling and on-the-job training are considered. The paper demonstrates that human capital composition, which is generally neglected in growth models, is important in determining the growth rate of the economy and, in particular, that complementarities between different types of human capital investment are important. Moreover, heterogeneity among workers, due to differences in human capital initial endowments, can change the growth rate of the economy.

JEL Classification: J24, O40

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Contents

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* Bank of Italy, Economic Research Department.

1. Introduction[1]

Economic growth theories stress the crucial growth-enhancing role played by human capital.[2] Indeed, the latter can promote growth because "the presence of human capital may relax the constraint of diminishing returns to a broad concept of capital and can lead thereby to long-term per capita growth in the absence of exogenous technological progress" (Barro and Sala-i-Martin, 1995, p. 172). In particular, human capital can stimulate the output dynamic when the latter depends on endogenous technological progress. Moreover, the changes in the utilisation of traditional inputs (land, capital and labour) cannot entirely explain the growth performance observed in many countries. This could indicate that the quality of labour (human capital) plays an important role in determining growth.

Both empirical and theoretical studies of the relationship between human capital and growth basically identify the former with formal education.[3] There are very few exceptions. Some models consider learning by doing as a way of accumulating human capital, so that the accumulation is not the outcome of an investment decision but a by-product of other activities. Others generically refer to human capital without giving a specific definition but still model human capital in a way which leads to identifying it with schooling.[4] This identity between human capital and education is not entirely satisfactory, as human capital is likely to be made up of different components. Accordingly, there are various mechanisms for increasing the individual human capital stock.

The idea that human capital is a heterogeneous aggregate (i.e. that there are different ways of accumulating human capital) arose before endogenous growth theories:[5] it dates back to the early 1960s and stems from the pioneering work of Schultz (1960, 1961)[6]. The

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[2] Human capital is considered as a factor of production; therefore, all aspects related to human capital either as a consumption good or as a pure signalling device are not accounted for. Human capital is knowledge, skills, competence and other attributes that are relevant to economic activity.

[3] "The educational achievement of a society (is) usually referred to as the human capital stock" (Aghion and Howitt, 1998, p. 354).

[4] See Lucas (1988) for the first type of models and Redding (1996) for the second.

[5] The idea that there are many ways to invest in human capital is called the "composite idea of human capital". For further developments of the human capital theories, see also Becker (1964), Ben-Porath (1967, 1970) and Mincer (1974).

[6] Schultz (1960, 1961) considers human resources explicitly as a form of capital. Even if this idea has a long history in economics, the mainstream of economic thought did not generally accept it before Schultz's work. Moreover, Schultz (1960) is the first one to refer to investment in man as "human capital". Schultz stresses that human capital is rather different from other types of capital: it is embodied in people and is rather poor collateral in credit/debt relationships, so that capital market imperfections are more likely to be important for human capital investments than for physical and financial ones (see Aghion and Bolton, 1997; Galor and Zeira, 1993).

human capital investment includes investments in: schooling, on-the-job training, off-the-job training, learning by doing, health, learning general principles. Schultz's analysis mainly focuses on the first two components.

This paper tries to bridge the gap between human capital theories, which develop the composite idea of human capital, and growth theories, which neglect this idea. Bridging the gap means investigating whether the composition as well as the overall level of human capital matters for growth. The tool used is the development of a theoretical growth model in which there is more than one mechanism for accumulating human capital. The interdependence between these ways of accumulating human capital is accounted for and it is important for growth. As in Schultz's work, formal education and on-the-job training are considered. Production depends on the overall level of human capital.

Many features make schooling a different way of investing in human capital than training. Among them are the ones related to: the agent who actually makes the decision, the effect of the investment on the human capital of the following generation, the depreciation rate of human capital and the effects on growth. On the first of these points, either the individuals themselves or their parents are likely to decide about schooling. Their choice depends on their preferences and on the resources that can be devoted to the investment. The decision-making mechanism for training can be very different, because it may be the case that the firm and not the individual decides about the investment. According to Becker (1964), the firm has incentives to invest in training only if training is specific to the firm (it increases the worker's productivity only in the firm that is investing). If training is general (it increases the productivity of the worker in any firm), the firm free rides and therefore the worker has to invest in training.[7] Nevertheless, more recent studies show that the distinction between general and specific training is not exhaustive. Some types of training are neither general nor specific nor the sum of general and specific components (Stevens, 1994a, 1994b, 1994c). Such training, defined as transferable, is "training for skills which are of potential value to at least one other firm in addition to the training firm, without any assumption about the nature of labour market competition" (Stevens, 1996, p. 26). When training is transferable, the firm and the worker both have some incentives to invest in it. Moreover, other studies, such as Katz and Ziderman (1990) and Acemoglu and Pischke (1998) show that, owing to labour market imperfections and asymmetries between firms, firms have incentives to invest in training even if it is general, sharing its costs with the worker. This is also confirmed by some empirical studies.

Turning to the effect on the next generation's human capital, in many growth models the human capital stock of each generation depends on that of the previous one. The question is whether this relationship is likely only for education investments or also for training ones (or vice versa). The answer could be negative if knowledge from on-the-job training is specific (as opposed to that from schooling) and therefore unlikely to be inherited.

Thirdly, as far as the depreciation rate is concerned, if human capital depreciates differently according to the way it is accumulated, it may be likely to depreciate faster when it comes from on-the-job training, owing to its "specificity".

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[7] Oi (1962) first made the distinction between specific and general training.

Finally, schooling and training can have different growth-enhancing effects. Suppose growth depends on innovation/diffusion activities.[8] As education provides more flexible skills, highly educated workers could be more productive in innovating activities, while highly trained ones could be more productive in exploiting the existing technology. If this is the case, the skill composition of the labour force is important for growth, as education and training skills are not perfect substitutes.

The model developed in the paper accounts for some of the differences between education and training. It also differs from the typical growth models because it explicitly considers the role played by firms. In particular, while individuals invest in schooling, firms invest in on-the-job training. Thus, firms play a role not only in demanding human capital but also in investing in it. Analysing the human capital accumulation process within a growth framework is rather difficult; the analysis is even more complicated when firms' role is made explicit. The paper shows that human capital composition, which is generally neglected in growth models, is important in determining the growth rate of the economy and, in particular, that complementarities between different types of human capital investment are important. Moreover, heterogeneity among workers, due to differences in initial human capital endowments, can change the growth rate of the economy.

The paper is organised as follows. The next Section describes the model. Section 3 analyses the equilibrium and Section 4 discusses the growth rate of the economy. Section 5 points out the effects of assuming some heterogeneity among individuals and Section 6 discusses an extension of the model. Section 7 concludes.

2. The model

There is a sequence of non-overlapping generations indexed by t. Each generation, which lives for two periods, is made up of a continuum of workers i and a continuum of entrepreneurs j. Workers earn labour income while they work within firms. Entrepreneurs own the firms, i.e. the technology for producing the final output. They earn a profit.

There is imperfect competition in the labour market: Nash bargaining occurs between the worker and the firm, after they are randomly matched one-to-one for their whole lifetime.[9]

As far as human capital accumulation is concerned, individuals decide about schooling and firms about training. Moreover, individuals benefit from the schooling investment at the end of the schooling period whereas they benefit from the training investment while they are trainees, since they do not work while they study but do work while they train. As pointed out in the introduction, the first assumption does not imply that specific training is considered: in an imperfect labour market, firms pay at least a share of the training cost without any dependency on the training features. With the second assumption, schooling can be represented as the time period during which individuals are not working.

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[8] See Aghion and Howitt (1992).

[9] By assumption, there is no unemployment.

It is an AK-type of growth model: there is only one input (human capital) and there are constant returns to this input.

Workers. – They are risk neutral and they maximise a linear utility function that depends on consumption in the two periods:

(1) $$U_t^i = c_{1t}^i + \delta c_{2t}^i$$

where c_{kt}^i (with k = 1, 2) is consumption in each period for the individual i born at time t, δ is the discount factor.[10]

At birth, individuals are assumed to inherit the human capital stock of their parents ($h_{1t}^i = h_{2,t-1}^i$).[11] They can devote a fraction of their first-period time (v) to formal education ($0 \leq v \leq \bar{v}$, where $\bar{v}$ is the length of the first period) and benefit from their investment when they work within a firm. The education production technology resembles the one considered by Azariadis and Drazen (1990):

(2) $$\mathrm{E}_t^i = (1 + 2\,\gamma\sqrt{v})h_{1t}^i$$

where γ is a positive constant that represents the productivity of education and there are decreasing returns to the investment. Equation (2) only describes the human capital accumulation due to schooling, whereas in Azariadis and Drazen a similar equation describes the overall human capital accumulation.

At birth, individuals decide upon the schooling investment. *Ceteris paribus*, a longer time at school implies a lower first-period income (individuals work only after schooling is over) and a higher second-period income (due to a higher labour productivity). Once the decision concerning education is made, each worker is randomly matched one-to-one with an entrepreneur for the duration of her lifetime. Production takes place during a fraction $(\bar{v} - v)$ of the first period and throughout the second period.

Entrepreneurs. – As workers, they maximise a linear utility function that depends on consumption. They finance their consumption through profits (Π_{kt}^j); accordingly, they maximise:

(3) $$\Pi_t^j(\cdot) = \Pi_{1t}^j(\cdot) + \delta\,\Pi_{2t}^j(\cdot)$$

Each of the entrepreneurs produces a homogeneous good y with the following technology:

(4) $$y_{i,t\,k}^j = A \cdot h_{tk}^i$$

where j refers to the j-th firm, i to the i-th worker belonging to generation t; k = 1, 2 and, in particular, if k = 1 the production function refers to the first period (production does not yet

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10 There is no saving in this model, so that consumption coincides with income. Individuals can change the time path of consumption by changing the allocation of time between schooling and production.

11 The population growth rate is zero (each parent has a child).

benefit from the schooling investment), whereas if $k = 2$ the production function refers to the second period (production benefits from the schooling investment). This means that at the beginning of the worker-firm relationship the worker is not able to exploit all her skills (schooling skills do not matter until the beginning of the second period).

Entrepreneurs maximise profits with respect to the investment in on-the-job training, which takes place during the second period of production[12]. This investment makes the worker more productive, but investing is costly. On-the-job training is represented by the non-negative variable ξ which stands for the resources the firm devotes to the training investment, including not only financial resources but also the time devoted to on-the-job training rather than to full production. In order to take into account the time component of the training investment (i.e. there is a limited period of time that can be devoted to training), ξ has to have an upper bound, as v does: $0 \le \xi \le \bar{\xi}$.[13]

The training investment increases the worker's productivity. The training contribution to human capital accumulation is proportional to the human capital accumulated via schooling. So, taking into account both schooling and on-the-job training, the overall dynamic of human capital is:

$$h_{2t}^{i} = (1+\xi)(1+2\,\gamma\,\sqrt{v})\,h_{1t}^{i} \tag{5}$$

Therefore, the production function (equation (4)) can be also written as follows:

$$y_{i,t\,k}^{j} = A \cdot h_{tk}^{i} = \begin{cases} A(\bar{v}-v)h_{1t}^{i} & \text{in period } 1 \ (k=1) \\ A(1+\xi)(1+2\gamma\sqrt{v})h_{1t}^{i} & \text{in period } 2 \ (k=2) \end{cases} \tag{4'}$$

Wages and profits. – As in Acemoglu (1994), at the beginning of the production period workers and entrepreneurs are randomly matched one-to-one so that all workers are employed. Then each entrepreneur bargains with the worker over the sharing of the output surplus. The entrepreneur invests in human capital and shares the output surplus with the worker. The output surplus is defined as the value of production minus training costs and disagreement payoffs. In what follows the variables that determine the output surplus are considered.

As far as the value of production is concerned, the firm owns the technology for producing the final output y, whose price is normalised to one. The production technology is linear in the unique input that is human capital (see equations (4) and (4')).

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[12] The length of the second period is normalised to one.

[13] The restriction $\bar{\xi} = \bar{v}$ is not imposed in order to take into account that any training period can benefit from some financial resources. However, for the sake of simplicity, it is assumed that the maximum training investment is reached when the individual spends all the first period at school. This means that the steady-state is reached at a point where $\xi = \bar{\xi}$ and $v = \bar{v}$.

The training costs are given by a quadratic function of the training investment:

$$c(\xi) = (a/2)\,\xi^2 \tag{6}$$

where a > 0. Given that ξ is non negative, this function is always upward sloping. Stevens (1994b) finds empirical support for assuming a quadratic training cost function. The firm and the worker share the training costs, as the output surplus they share excludes them.

The disagreement payoffs are zero for both the worker and the firm; the firm cannot produce without the worker and the individual cannot produce without the production technology.

Therefore, during the first period the output surplus is simply the value of first-period production (see equation (4')): $(\bar{v} - v)h_{1t}^{i} A$. The second-period output surplus is given by the value of second-period production (see equation (4')) net of the training costs: $(1+\xi)(1+2\,\gamma\sqrt{v})h_{1t}^{i} A$ - $(a/2)\xi^2$. The bargaining outcome makes the firm gain a fraction $(1-\beta)$ of the output surplus.

The equilibrium concept. – The model is solved for a Nash equilibrium. This is reasonable as long as it is assumed that the firm somehow commits itself to the investment in the first period, even if this is not explicitly modelled.[14] It would be interesting, as a subject for future research, to solve the model for the Stackelberg equilibrium concept where the worker acts as the leader. Under full information, the choice of a different solution concept should not alter the main conclusions and, in particular, should not change the effects of the complementary relationship between different ways of investing in human capital.[15]

The optimisation problems. – Given the equilibrium concept and the objective functions, the optimisation problems can be considered.[16]

The worker solves the following utility maximisation problem:[17]

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[14] This could be modelled by having firms bearing fixed costs during the first period of production.

[15] If the full-information assumption does not hold, the framework of the model will be different than the one presented in what follows. See Part I of the Appendix for some notes on the model without full information.

[16] In order to keep notation as simple as possible, whenever it is unambiguous that reference is made to a specific worker and to a specific firm, the indexes *i* and *j* are skipped.

[17] This objective function derives from the Nash bargaining assumption. The general Nash bargaining solution is the outcome of the following problem: $\max_{L,w} \Pi(L,w)^{1-\beta}(w-\overline{w})^{\beta}$, where Π stands for profit, L for labour, w and $\overline{w}$ stand, respectively, for wage and disagreement wage. Given that only one worker is employed in each firm (L = 1), given individual risk neutrality and after considering logarithms and computing the first order condition, the following equation holds: $\frac{\beta}{w-\overline{w}} = \frac{1-\beta}{\Pi}$. Since $\Pi = Y - w$, the previous equation can be rewritten as: $w = \beta(Y-\overline{w}) + \overline{w}$. Since in the model $\overline{w} = 0$, then $w = \beta Y$, which is the expression of the worker's second-period income in equation (7). As a consequence, the firm's payoff is given by: $\Pi = Y - w = (1-\beta)Y$, as in equation (9).

$$\max_{v} U(v) = \beta(\bar{v} - v)h_{1t}A + \beta\delta[(1+\xi)(1+2\,\gamma\sqrt{v})h_{1t}A - (a/2)\xi^2] \tag{7}$$

From the first order conditions, it follows that the individual's reaction function is:

$$v^*(\xi) = \min\{[\delta\gamma(1+\xi)]^2; \bar{v}\} \tag{8}$$

Equation (8) implies that, for any interior solution, the schooling period is an increasing function of the training investment: when firms do invest in on-the-job training, individuals have more incentives to invest in formal education because its returns are higher.

As is intuitive, according to equation (8) the optimal level of education is an increasing function of the discount factor δ and of the productivity of the investment γ.

The entrepreneur solves the following maximisation problem:[18]

$$\max_{\xi} \Pi(\xi) = (1-\beta)(\bar{v} - v)\,h_{1t}A + (1-\beta)\delta[(1+\xi)(1+2\,\gamma\sqrt{v})h_{1t}A - (a/2)\xi^2] \tag{9}$$

From the first order conditions, it follows that the firm's reaction function is:

$$\xi^*(v) = \min\{(1+2\gamma\sqrt{v})\frac{h_{1t}A}{a}; \bar{\xi}\} \tag{10}$$

The most important implication of equation (10) is related to the human capital accumulation. In particular, there are two aspects: the role of the initial human capital endowment (see Section 5) and that of the schooling investment. The optimal training investment is an increasing function of both. Since the initial endowment is not a choice variable, here the focus is on the second aspect: from equation (10) it follows that firms invest more when they hire better educated workers because the returns to training are higher.

Equation (10) also implies that the firm's optimal choice is an increasing function of the technological level A; as the technology is more productive, teaching how to exploit it to the worker is more valuable to the firm.

3. The equilibrium analysis

Before studying the characteristics of the equilibrium, the features of the two reaction functions (equations (8) and (10)) can be first considered.

As far as the worker's reaction function $v^*(\xi)$ is concerned, for any interior solution, $v^*(\xi)$ is a monotonically increasing and convex function of the training investment. This means that the incentives to invest in schooling increase at an increasing rate until the time constraint is binding. Moreover, if the firm does not invest in training ($\xi = 0$), it is optimal

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[18] See Part II of the Appendix for continuous discounting in the second period of production.

for the worker to invest in schooling; even if the two investments are complementary, education by itself can lead to a positive return during the worker's second period of life.

As far as the firm's reaction function $\xi^*(v)$ is concerned, for any interior solution, $\xi^*(v)$ is a monotonically increasing and concave function of the education investment; this means that the incentives to invest in training increase at a decreasing rate. Moreover, if the individual does not invest in formal education ($v = 0$), then the optimal training investment is still positive.

The equilibrium values of the schooling and of the training investment can be determined by solving the system of two equations into two unknowns given by equations (8) and (10). As the entrepreneur's reaction function $\xi^*(v)$ is linear in $\sqrt{v}$ and the worker's reaction function $v^*(\xi)$ is a quadratic function of ξ, the system can be rewritten as a linear system in $\sqrt{v}$ and ξ, where the variable representing the schooling investment v can be renamed as $V = \sqrt{v}$. Since the reaction functions are linear in V and ξ and both choice variables have an upper bound, a unique solution always exists.

Solving the system, the Nash equilibrium is as follows:

$$V^* = \begin{cases} \dfrac{a + \mathrm{h}_{1t} A}{a - 2\mathrm{h}_{1t} A\delta\gamma^2}\delta\gamma & \text{if } \mathrm{h}_{1t} < \dfrac{a}{2A\delta\gamma^2} \\ \overline{V} & \text{otherwise} \end{cases}$$

(11)

$$\xi^* = \begin{cases} \dfrac{\mathrm{h}_{1t} A + 2\mathrm{h}_{1t} A\delta\gamma^2}{a - 2\mathrm{h}_{1t} A\delta\gamma^2} & \text{if } \mathrm{h}_{1t} < \dfrac{a}{2A\delta\gamma^2} \\ \overline{\xi} & \text{otherwise} \end{cases}$$

where $\overline{V} = \sqrt{\overline{v}}$.

Figure 1 represents the equilibrium (point E) by plotting together the reaction functions (in particular, Figure 1 describes an interior solution).

Within the model it is possible to consider the effects of an increase in the discount factor δ, which implies that second-period income is more important within the objective functions, and in the education productivity parameter γ.

A larger discount factor δ implies an increase in both the intercept and the slope of the $V^*(\xi)$ schedule, while the $\xi^*(v)$ curve is unaffected. Thus, both schooling and training increase in equilibrium. This outcome is based on the complementary relationship between the two ways of investing in human capital: the change in the discount factor directly affects only the decision of a worker who has more incentives to invest in education because the second-period income is more important; nevertheless, the training investment increases as an indirect effect of the worker's decision.

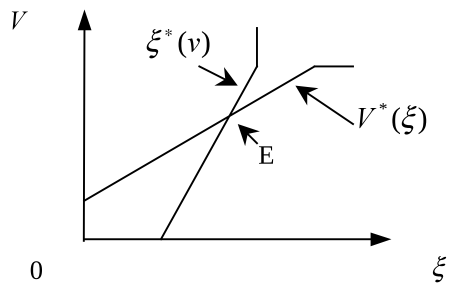

Figure 1

An increase in γ leads to an increase in both the intercept and the slope in the $V^*(\xi)$ schedule and to an increase in the slope of $\xi^*(v)$ (its intercept does not change). Therefore, an improvement in the technology of education investment leads to more education and more training.

4. The growth rate

From the production function (equation (4)), it follows that the output growth rate is given by the growth rate of human capital, that is:

(12) $$g = \frac{h_{2t}}{h_{1t}} - 1 = \frac{(1+\xi^*)(1+2\gamma V^*)h_{1t}}{h_{1t}} - 1 = \xi^* + 2\gamma V^* + 2\gamma\xi^* V^*$$

Thus the growth rate is made up of three components: the direct effect of schooling, the direct effect of training and the indirect effect of each of these variables via the other. All these effects are positive: an increase in any of the choice variables leads to an increase in the economy's growth rate.

Since both the investment made by the individual and that by the firm are always positive, human capital always increases over time. Therefore, the growth rate is always positive. This is a typical feature within the endogenous growth literature. *A fortiori*, this also holds in this model, where human capital does not stand only for schooling.

During the transition to the steady state, the growth rate is given by equation (12) where both v^* and ξ^* are strictly positive and have not yet reached their upper bound values.

In the steady state, both types of human capital investment reach their respective upper bounds. Therefore, the growth rate reaches its maximum value and the economy keeps growing at a constant and positive rate $(\overline{g})$:

(13) $$g = \overline{g} = \overline{\xi} + 2\gamma\overline{V} + 2\gamma\overline{\xi}\,\overline{V}$$

Figure 2 summarises the two phases of growth; it qualitatively plots the logarithm of output against time: first, the economy grows at an increasing exponential rate, then it grows at a constant exponential rate.

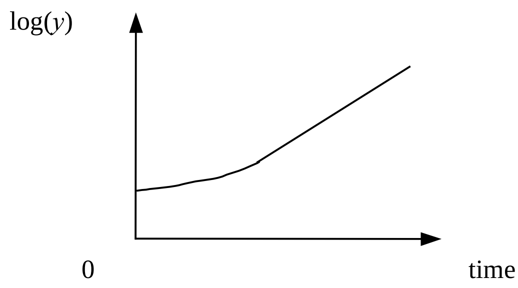

Figure 2

5. Heterogeneity among workers

In this section some degree of heterogeneity among individuals, due to differences in initial endowments of human capital, is introduced. This opens the path to the discussion of some distributive issues: differences in human capital are nothing but income differences.

The optimisation problem that the individual solves does not change even if now it depends on the specific level of human capital of the individual herself; *a fortiori*, the firm's behaviour does not change. However, the heterogeneity assumption allows, *ceteris paribus*, the comparison between the equilibria corresponding to different initial human capital endowments.

Optimal choices of different individuals. – For the sake of simplicity, assume that there are only two types of individuals: *H*-type ones who have a high initial endowment and *L*-type ones who have a low endowment. As in the preceding sections, first there is random matching, then bargaining and finally the firm chooses the training investment. While bargaining, the firm can observe the worker's characteristics (her endowment).[19] The reaction functions are still given by equations (8) and (10); nevertheless equation (8) depends on h_{1t}^{x}, where $x = H, L$. In equilibrium, different individuals can benefit not only from different training levels but also from different schooling investments. Even if the individual's reaction function does not depend on the initial human capital level, its equilibrium value does (as the firm's reaction function depends on that endowment). It

[19] It is assumed that *a priori* the firm does not know the characteristics of the worker it will be matched with, but after the matching occurs, the firm can learn about the worker's type before investing in training. If the firm were not to observe the worker's characteristics at all, it would not be possible to represent the equilibrium in the way it is represented in this section. In fact, there would be two different equilibria. One equilibrium would be characterised by the training investment that would be optimal if all workers were *H*-type, the other by the training investment that would be optimal if all workers were *L*-type. In the latter case, the training investment would lead to an externality similar to the so-called "poaching externality" (Stevens, 1996). See Part I of the Appendix for the main features of a framework within which it is possible to represent firms' and workers' decisions under asymmetric information.

follows from equations (11) that better endowed individuals not only invest more in schooling but also benefit from a larger training investment: the initial human capital distribution affects both the individual's and the firm's behaviour (Figure 3).

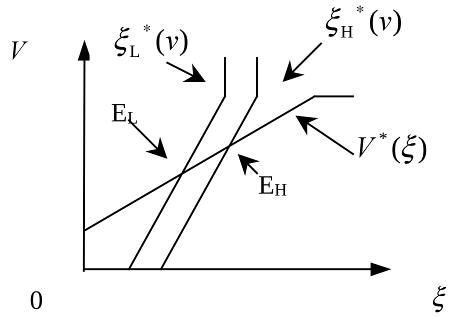

Figure 3

This outcome is confirmed by some empirical analyses. According to Blundell, Dearen and Meghir (1996), both men and women with higher education qualifications have a greater probability of benefiting from on-the-job training. Moreover, the OECD underlines that "the European countries showing low levels of participation in job-related training […] are also countries with relatively low levels of educational attainment in the adult population" (1998, p. 44).

The human capital dynamics. – In the model human capital always increases over time because both human capital investments are always positive. Consequently, the dynamic equation for human capital given by equation (5) has no fixed points (with the exception of the origin). Nevertheless, something can be said about the dynamics of human capital distribution (or inequality in human capital) over time.

It has just been shown that the equilibrium values V^* and ξ^* increase with the initial human capital level; moreover, in Section 4 it has been shown that the economy's growth rate increases with education and training investments. Therefore, during the transition to the steady state, different dynasties (i.e. sequences of generations characterised by the same index *i*) are characterised by different growth rates and, in particular, better-endowed dynasties have a higher human capital growth rate. Thus, inequality in human capital increases over time. This outcome depends on the assumption about the human capital inheritance mechanism (individuals inherit their parents' human capital; $h_{1t}^i = h_{2,t-1}^i$), which is not completely satisfactory even if generally used in the literature. Once both dynasties are in the steady state, inequality in human capital persists over time.

Suppose that there is a sort of intergeneration depreciation of human capital, that is to say a share ρ (where $0 \le \rho \le 1$) of the human capital stock is lost when moving from generation *t* to generation *t*+1 (for any *t*). Therefore, human capital changes over time as follows:

(14) $$h_{t+1}^i = (1-\rho)(1+\xi^*(h_t^i))(1+2\gamma V^*(h_t^i))\, h_t^i$$

This is a non-linear function of h_t^i until the upper bounds for both V and ξ are reached, as both ξ^* and V^* depend on the initial human capital endowment. When the upper bounds are reached, it becomes linear with a slope greater than one.[20] Two cases can be distinguished: (1) ρ is so small that human capital always increases over time for any initial human capital level; (2) ρ is high enough so that for low level of the initial human capital endowment human capital does not increase over time. In the latter case the human capital dynamics equation has one more fixed point other than the origin (call this fixed point $\hat{h}_t$); therefore, the population can be divided into two groups: (a) the poorly-endowed individuals, whose human capital initial endowment is less than $\hat{h}_t$, for whom human capital decreases over time; and (b) the well-endowed individuals, whose human capital always grows over time.

There are some policy measures that could affect inequality in human capital distribution and enhance growth: a transfer from the public sector to firms, earmarked for on-the-job training and financed by non-distortionary taxes, would lower the training costs borne by firms and thus, for any given level of initial human capital and schooling investment, increase on-the-job training, and the growth rate of the economy. This measure would reduce the population belonging to group (a) and, correspondingly, increase that of group (b).

6. An extension: a more general training cost function

Among possible extensions of the model, this section considers a more general quadratic training cost function than the one considered up to now (i.e. $c(\xi) = (a/2)\,\xi^2$, where a > 0). In particular, a quadratic cost function with a positive linear component is considered:[21] $C(\xi) = (a/2)\,\xi^2 + b\xi$ where $a, b > 0$. The introduction of a linear term in the cost function has two implications: the cost corresponding to any level of the training investment is higher, and the training cost increases faster for any given change in the training investment. Overall, the modified cost function can tell us something about how changes in the training cost affect the behaviour of the firm.

In particular, the more general cost function significantly changes the reaction function of the firm. In fact, owing to the higher training cost, the firm is no longer always willing to invest in training; it invests in training only when the initial human capital endowment of the worker is high enough (i.e. when the return on the training investment is high enough to compensate for its cost).

Thus, there is a no-training range within the firm's optimal choices set. This implies two different growth phases within the transition to the steady state: one where growth is driven only by the schooling investment (which corresponds to the no-training range) and

[20] The upper bounds for both types of the human capital investment are positive.

[21] Considering a constant in this set up is not intuitively appealing, as the training cost would not be zero at the no-investment point.

one where growth is driven by both the schooling and the training investment (which corresponds to the transition of the baseline model).

The presence of a no-training range also has implications for human capital inequality. Suppose there are two individuals characterised by two different initial endowments and that the difference between the two initial endowments is such that only one benefits from training. As in the baseline model, inequality in human capital increases over time because of differences in the overall human capital investment; however, as long as the poorly-endowed dynasty does not reach a human capital equal to $\hat{h}_t$, the difference between the two dynasties grows even larger, because only one individual benefits from the training investment.

Hence, during the first phase of the transition to the steady state, a policy measure that requires firms to provide a minimum level of training could both enhance growth and lessen the degree of human capital inequality.

Alternatively, a policy measure that lowers the minimum human capital level firms require for investing in training could reduce the differences between the two dynasties. This minimum level depends on the firm's training costs: it depends on the parameter *b*, which is the marginal cost of training at the no-training point. Therefore, any transfer aimed at reducing firms' training costs can lower the minimum initial human capital level firms require in order to invest in training[22]. However, this second type of policy measure may imply incentives and monitoring problems.

7. Conclusions and future developments

The purpose of this paper is to contribute to the analysis of human capital as a growth-enhancing factor of production. In particular, human capital is a very broad concept and includes many different components: education, training, learning by doing, health and so on. While heterogeneity is apparent in the real world, growth theories have neglected it and have usually simply identified human capital with education. This means that while these theories stress the importance of human capital, they do not properly account for the presence of more than one mechanism for accumulating it. This has important policy implications: most growth models' policy prescriptions refer only to education, and this is a rather biased view of policy measures affecting human capital.

By emphasising the importance of the first human capital theories of the 1960s, this work brings together the "traditional literature" about human capital and the more recent endogenous growth theories. This can give some insights, which the model developed in the paper explores: (1) the composite idea of human capital, developed in the literature of the 1960s, can be relevant in determining growth; (2) accounting for the demand side of human capital in the labour market (firms' behaviour) can be important for the analysis of its growth-enhancing role.

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[22] It is assumed that this measure is financed out of non-distortionary taxes.

The model takes account of two mechanisms for accumulating human capital: investing in education and investing in on-the-job training. Firms' behaviour in both demanding and providing human capital is modelled and the effects of human capital composition on growth are considered. Production depends on the overall human capital level.

Two results hold. First, when workers invest in education and firms in training, the complementary relationship between the two human capital investments affects the growth rate. The latter turns out to depend not only positively and directly on each type of human capital investment, but also on their interdependence.

Any policy measure aimed at enhancing growth should not ignore such a composite idea of human capital; a policy focusing only on education, as the ones most endogenous growth models suggest, would be to some extent misleading.

Second, the distribution of human capital is important, as the firms' incentives to invest in training depend on the individual initial human capital endowment. In this sense the distribution of human capital, which is closely tied to that of income, can affect growth. In particular, there can be room for a policy measure that can both affect the degree of human capital inequality and enhance growth.

In particular, a transfer from the public sector to firms, earmarked for on-the-job training and financed by non-distortionary taxes, would lower the training costs borne by firms and, for any given level of initial human capital and schooling investment, increase on-the-job training and hence the growth rate of the economy. This measure would allow a certain group of the population to benefit from more training, and thus change its relative position in the income distribution.

Formalising the policy implications of the model and explicitly considering both the revenue and the expenditure effects of policy measures can be an interesting issue for further investigation.

Many differences between education and on-the-job training are qualitatively considered in the paper, although only some of them are formalised. Dealing with those not formally considered can be part of the agenda for future research. In particular, the difference between the growth-enhancing role of education and that of training, when growth is driven by innovation/diffusion, is of the greatest interest. While education skills are more flexible and can be better used in innovating, training skills can be better for exploiting a given technology. As a new innovation enters, the training skills may be "lost", in the sense that retraining may be necessary. It may be the case that there are too many innovations. In this framework the importance of discussing human capital composition is crucial.

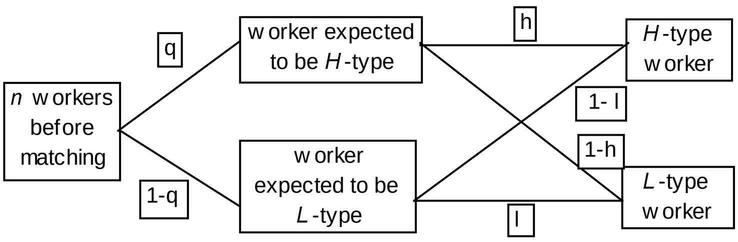
Another possible extension concerns the type of education investment rather than just its amount. In particular, the problem of mismatching between the type of "education human capital" a firm is looking for and the type the individual actually has can be considered. This underlines the importance of firms in demanding human capital in the labour market.

Moreover, human capital, which firms look for, can also depend on technological innovation. What if the technology the "average firm" is adopting when the individual begins to invest in a certain type of education is very different from the one it is using when the education investment is over? This is likely to have implications for both income distribution and growth.

Assuming a specific human capital distribution function can further develop the human capital dynamics analysis and even can enrich the growth implications of the human capital distribution itself. Finally, more technical extensions are possible. For instance more general objective functions can be considered in developing the model presented in this paper.

Appendix

Part I. – A useful framework within which it is possible to represent firms' and workers' decisions under asymmetric information is the following. Suppose that there are *n* workers and that each firm expects to be matched with an *H*-type worker with probability *q* and thus with a *L*-type worker with probability (1- *q*) (see the chart below). Moreover, *h* is the probability of a worker believed to be *H*-type actually being of *H*-type; *l* is the probability of a worker believed to be *L*-type actually being of *L*-type.



Let *p* be the probability that an individual is *H*-type and let *CA* be the firm's analysis capability (its ability to recognise the characteristics of the workers it is matched with; $0 \le CA \le 1$), the following equalities must hold:

$$p = qh + (1\text{-}q)\,(1\text{-}l)$$

$$h = p + (1\text{-}p)\,CA$$

As *CA* increases (less important information asymmetries are), *h* increases and the firm invests more in training. This occurs not only because the firm can better recognise the worker's features, but also because the worker invests more in schooling, as the firm will recognise her investment. Obviously, this is only a framework within which the decision mechanisms can be modelled under information asymmetries.

Part II. – As explained in Section 2, the firm faces training costs and benefits during the second period. These are therefore discounted according to the discount factor δ. However, it may be more likely to assume that while the costs are borne at the beginning of the period, the flow of benefits occurs over the whole period. Thus, having a continuous discounting within the second period could give a more accurate description. The benefits of training could be discounted according to the discount factor *R* so that the firm would have the following objective function (here each period's length is normalised to one):

$$\Pi(\xi) = (1-\beta)(1-v)h_{1t}A + (1-\beta)\delta\left\{-\frac{a}{2}\xi^2 + \int_T^{T+1} e^{-Rz}\,[(1+\xi)(1+2\gamma\sqrt{v})h_{1t}A - h_{1t}B]\,dz\right\}$$

Since all variables within the integral do not depend on the time period from T to T+1, the firm's objective function can be written as follows:

$$\Pi(\xi) = (1-\beta)(1-v)h_{1t}A + (1-\beta)\delta\left\{-\frac{a}{2}\xi^2 + \left[(1+\xi)(1+2\gamma\sqrt{v})h_{1t}A - h_{1t}B\right]\int_T^{T+1} e^{-Rz}dz\right\}$$

If T = 0 and the integral is solved, the following equation is obtained:

$$\Pi(\xi) = (1-\beta)(1-v)h_{1t}A + (1-\beta)\delta\left\{-\frac{a}{2}\xi^2 + \left[(1+\xi)(1+2\gamma\sqrt{v})h_{1t}A - h_{1t}B\right]\left[\frac{1-e^{-R}}{R}\right]\right\}$$

Thus, the first order condition for the firm is as follows:

$$\xi^*(v) = \min\left\{(1+2\gamma\sqrt{v})h_{1t}\,\frac{A}{a}\left[\frac{1-e^{-R}}{R}\right];\bar{\xi}\right\}$$

This equation differs from equation (10) only because of the square brackets. The latter term is always positive so that ξ^* still has the correct sign. The dependence of ξ^* on the discount rate R can be studied; the sign of the first derivative is given by the following:

$$\text{sign}\left(\frac{\partial\xi^*}{\partial R}\right) = \text{sign}\,\frac{\partial\left[{(1-e^{-R})}/{R}\right]}{\partial R} = \text{sign}\left(\frac{\mathrm{Re}^{-\mathrm{R}} - (1-e^{-R})}{R^2}\right) = \text{sign}\left(\mathrm{e}^{-\mathrm{R}}(R+1)-1\right) = \text{sign}\left(\frac{(R+1)}{\mathrm{e}^{\mathrm{R}}}-1\right)$$

As (R+1) is tangent to the e^R schedule, the last expression is always non positive so that $\frac{\partial\xi^*}{\partial R} \leq 0$ (where the equality holds only if R = 0). Therefore, the intuitive result, that as the firm discounts the training benefits more heavily, it will invest fewer resources, is confirmed. This more accurate analysis leads to less analytical tractability without substantially changing the analysis, which is why continuous discounting within the second period is not explicitly modelled in Section 2.

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