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by Fabio Fornari

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# THE SIZE OF THE EQUITY PREMIUM

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## Abstract

Among the many controversial variables in finance, risk premia stand out for their lack of observability. Measuring premia as the difference between realized returns on risky and risk-free assets has not led to unanimous conclusions about their size, which greatly depends on the length of the sample; in addition, investment allocations or inflation expectations are influenced by the ex-ante values of the risk premia and ex-post returns are, if any, rough approximations of these. Many papers have dealt with this issue, from the initial contribution of Mehra and Prescott (1985) to very recent advances within a bayesian framework of Pástor and Stambaugh (2001). This paper uses conditional variance models as approximations of static and intertemporal capital asset pricing models; the size of the equity premium is assessed for the US both at the market level and, through a conditional version of the three-factor model of Fama and French (1993), at a firm-level. The market premium has had large swings with short-lived peaks over the last 75 years, fluctuating around a mean value of 5 per cent on a yearly basis; this value rises to 6.5 percent when time-varying investment opportunities are allowed for. In periods of economic expansion the expected premium on the equity return is nearly half the value expected in recession, 20 percent less if the Great Depression period is excluded; the cross-sectional dispersion of the firm-level premia as a function of firm's size is also influenced by the position of the economy within the business cycle.

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## Contents



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## 1. Introduction[1]

In a recent study, Pástor and Stambaugh (2001) describe the equity premium, i.e. the expected rate of return on the aggregate stock market in excess of the riskless interest rate, as "one of the most important but elusive quantities in finance". The equity premium is indeed very hard to measure because it is not an observable quantity; further it is largely unexplainable, i.e. hard to reconcile with the prediction of a wide range of (consumption-based) theoretical models, as first evidenced in Mehra and Prescott (1985). An additional difficulty arises from the choice of the dataset from which the equity premium should be extracted. While many studies rely on very long excess returns series, with data spanning over a century, other papers employ a much shorter sample, claiming that financial returns' distribution functions are subject either to time variability or to structural breaks. In this case, supposing that a break can be identified to have occurred at time $t$, only the data between time $t$ and the current period are used to estimate the premium, which is likely to make post-break estimates very erratic.

By contrast I estimate the equity premium in the US by adopting a standard reference model, an intertemporal capital asset pricing scheme (henceforth capm), yet transposed to a conditional setting so that the time variability of the mean/variance relation is not precluded, and I choose data spanning a very long period, 75 years of monthly observations. This methodology and the estimates it provides will be compared to those of Pàstor and Stambaugh (2001), possibly the most up to date contribution on the topic (at least as regards the methodology); the setup and the results in Mayfield (1999) and Han (2002) will also be of guidance. I share the view of Pàstor and Stambaugh (2001) that a long return history can be successful in capturing the existence of a positive link between excess returns and volatility, after controlling for the existence of breaks in the relation between the two variables. Adopting a model with structural breaks, however, limits to some extent the frequency upon which the desired inference can be carried out. Suppose that someone is interested in 1-day or 1-

[1] The views and conclusions of this paper are personal and not necessarily shared by Banca d'Italia and the responsibility for any errors rests with the author only. This work tries to put in a consistent setting informal talks and preliminary estimates of the equity premium performed for a working document, for which I am grateful to Andrea Beltratti, Fabio Panetta, Marcello Pericoli and Roberto Violi. Please address any e-mail to: fornari.fabio@insedia.interbusiness.it

week holding period returns, i.e. in very short-term premia; in this case the (un)availability of historical high-frequency data may impose a binding constraint on the analysis, since the number of breakpoints (which in Pàstor and Stambaugh equals 2K+1, i.e. K transition regimes separated by K+1 stable regimes) will hardly exceed one or two working with small sample sizes. There are also a number of economic assumptions incorporated in their analysis that could be difficult to support in a high frequency context. For instance, they assume that changes in equity premia are unlikely to be extreme, which does not necessarily go along with the observed size of price changes at daily (but even monthly) frequency. Seen from a different angle, ruling out extreme changes in the premium requires that the variance of the shifts in its generating process is low, which also enables the pre-break returns to be still informative in the estimation of the post-break association between excess returns and variances; again, this assumption may be at odds with the observed features of excess returns (namely, fast-changing variances). Similarly to Pàstor and Stambaugh, Mayfield (1999) employs a regime-switching model where stock returns move between high- and low-volatility states. In each state compensation is required for both the current-state volatility and the likelihood that the data generating process switches to the competing state. More in line with this paper, Han (2002) estimates the equity premium by letting the conditional volatility of the equity market return be a state variable; his estimates are based on the efficient method of moments applied to a garch-type continuous time scheme.

In estimating risk premia I rely on a *economic* model rather than on a purely statistical framework as in Pàstor and Stambaugh. The intertemporal capm posits a positive link between stock returns and volatility, though the sign of the relation is left to be estimated, while the conditional covariance between stock and bond returns is the second determinant of risk premia. The time variability of such second moments makes risk premia time varying and allows them to share the same statistical properties (for example, mean reversion). In addition to the market level, the analysis also provides an estimate of the risk premium at firm level, through the three-factor model of Fama and French (1993). In this scheme, realized excess returns of specific portfolios of assets are supposed to depend upon the return on the global market portfolio (in excess of the risk-free rate) as well as on the return on two additional portfolios calculated as i) the return on small firms minus the return on large firms (i.e. it is assumed that the investor is *long* on small firms and *short* on large firms, which represents the size factor), and ii) the return on high-performance firms minus the return on low-performance firms (i.e. the investor is *long* on high-performance firms and *short* on low-performance firms, which represents the performance factor). Besides providing time varying estimates of the risk premia at firm level, the conditional setting improves the performance of the Fame and French setup beyond a standard homoskedastic specification.

The estimates of the expected risk premium must be accompanied by a measure of uncertainty. The importance of providing reliable confidence intervals can be grasped very easily by calculating the required change in the equity index level predicted by the so-called Gordon model as a function of the risk premium. According to the Gordon scheme the following equality must hold in equilibrium: $dy = (r + \sigma) - g$, where $dy$ is the dividend yield, $r$ the real interest rate, $g$ the rate of growth of real earnings, $\sigma$ the equity premium. If the current values of $dy, r$ and $g$ were 3, 4 and 3, respectively, and $\sigma$ were $5$, equity prices should fall by 25 percent to reach equilibrium; however, if the estimated value for $\sigma$ happened to be $3$, prices should rise by 25 percent. Owing to the impact of small changes in the estimated premium on the perception of the *equilibrium* equity level, some montecarlo experiments will assess the precision of the reported estimates.

The paper is organized as follows. The next section describes the estimation of risk premia within a conditionally heteroskedastic model, discusses the choice of the factors in cross sections of stock returns and reviews the structure of a general (multi-factor) model with conditional covariances. Section 3 presents the results and some features of the premia at the market and at the firm level as well as across business cycles; estimates are then compared to those reported in other works. Section 4 concludes.

## 2. Measuring risk premia

### 2.1 *A conditional variance setting*

Practitioners typically implement their measure of risk premia by calculating differentials of ex-post (i.e. realized) excess returns. However, the realized values of a stochastic variable are not necessarily in accordance with the predicted values when the conditional distribution is subject to changes over time; apart from purely statistical considerations, economic theory suggests that ex-ante premia are what counts in the formation of inflation expectations, in deciding the optimal allocation of wealth and in selecting investments. Reliable estimates of risk premia can be based on a simple univariate garch-in-mean model, the econometric counterpart of a conditional capm, positing a relation (not necessarily positive) between the first two conditional moments of a return process. Conventional reasoning hypothesizes the existence of a strong link between volatility and excess returns, but contrasts with the results of many empirical papers of the last decade. Fitting capm-like models, these studies have failed to find a significant relation between the two variables; in other cases, as in French et al. (1987), Campbell (1987), Harvey (1989), Turner et al. (1989), Baillie and DeGennaro (1990), Glosten et al. (1993) the relation has been found to be negative. Scruggs (1998) investigates this issue and, recalling that the capm is a partial scheme and as such likely to be misspecified, shows that a positive relation between risk premia and volatility is obtained when full parametrization is adopted. Indeed, under an intertemporal capital asset pricing specification (i-capm), the relation between asset return volatility and risk premia is a partial relation arising within a more complex first order condition of an optimal consumption model. It collapses toward the capm only under (strong) simplifications.

To understand this, let us recall that though the capm is a static model, it is frequently assumed to hold intertemporally. It is not hard to show that if preferences and the set of future investment opportunities are not state-dependent, then the intertemporal portfolio maximization problem can be treated as if the representative investor had a single-period utility function. Merton (1973) was among the first to show that the portfolio behaviour for an intertemporal maximizer is significantly different when he faces changing rather than static investment opportunities. This is easily illustrated by reviewing some standard results. Assume the existence of $K$ consumer/investors acting to maximize the following expression

$$\max E_0 \left[ \int_0^{T^k} U^k(c^k(s), s)ds + B^k(W^k(T^k), T^k) \right] \tag{1}$$

where $k = 1, 2, ..., K$, $c^k(s)$ is consumption as of time $s$, and $W$ is wealth. The accumulation equation for the $k-th$ investor is

$$dW = \sum_1^n w_i W \frac{dP_i}{P_i} + (y - c)dt$$

where $w_i = N_i P_i / W$ is the fraction of total wealth invested in the i-th asset, $N_i$ is the number of shares of the i-th asset and $y$ the wage income. Assuming that the continuous time law of motion for $P_i$ is a geometric Brownian motion,

$$dP_i = \alpha_i P_i dt + \sigma_i P_i dz_i$$

and substituting in the dynamics of $W$ gives:

$$dW = \left[ \sum_1^n w_i(\alpha_i - r) + r \right] W dt + \sum_1^n w_i W \sigma_i dz_i + (y - c)dt.$$

Assume now that individuals derive all their income from capital gains, i.e. that $y = 0$, and vectorise the state variable vector $X$ so as to contain $P$, $\alpha$ and $\sigma$, with the following dynamics

$$dX = F(X)dt + G(X)dQ.$$

The optimality conditions for an investor who follows (1) are

$$\begin{aligned} 0 \quad = \quad & \max_{(c,w)} \Bigg[ U(c,t) + J_t + J_W[(\sum_{i=1}^n w_i(\alpha_i - r) + r)W - c] + \\ & \sum_1^m J_i f_i + 0.5 J_{WW} \sum \sum w_i w_j \sigma_{ij} W^2 + \sum_1^m \sum_1^n J_{iW} w_j W g_i \sigma_j \eta_{ij} + \sum_1^m \sum_1^m J_{ij} g_i g_j \rho_{ij} \Bigg] \end{aligned}$$

subject to $J(W, T, X) = B(W, T)$. The $n+1$ first order conditions for the above maximization problem follow from the definition of the function $\phi(w, C; W, P, t) = U(C, t) + \Im[J]$, where $\Im[J]$ denotes the Dynkin operator over the variables $P$ and $W$ for a given combination of control variables $w$ and $C$, and are:

$$
\begin{aligned}
0 &= \frac{\partial}{\partial C}\phi = U_c(c,t) - J_W(W,t,X) \\
0 &= \frac{\partial}{\partial w_k}\phi = J_W(\alpha_i - r) + J_{WW}\sum_1^n w_j W \sigma_{ij} + \sum_1^m J_{jW} g_j \sigma_i \eta_{ij}
\end{aligned}
$$

Re-writing the second expression as

(2) $$\alpha_i - r = \left[-\frac{J_{WW}}{J_W}W\right]\sum_1^n w_j\sigma_{ij} + \left[-\frac{J_{jW}}{J_W}\right]\sum_1^m g_j\sigma_i\eta_{ij}$$

shows that the excess return on asset $i$ is linearly related to the covariance between asset $i$ and a linear combination of the $n$ assets with weights $w_j$ as well as to the covariance between asset $i$ and the state variables.

The Capm assumption is that there are constant investment opportunities, so that the second term on the right-hand side vanishes, yielding the usual result that the excess return on asset $i$ is linearly related to the covariance between the return on asset $i$ and the return on the market. With this restriction, the theoretical scheme (2), written for the market index, can be made operational according to the following univariate garch-in-mean model (Engle et al., 1987), after placing $\left[-\frac{J_{jW}}{J_W}\right] \equiv 0$ :

(3) $$
\begin{aligned}
r_t - r &= \pi(\sigma_t^2) = \mu + \lambda \cdot \sigma_t^2 + \varepsilon_t \\
\varepsilon_t | I_{t-1} &\sim N(0, \sigma_t^2) \\
\sigma_t^2 &= \omega + \alpha \cdot \varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2
\end{aligned}
$$

where $r_t$ is the market return, $r$ the risk-free rate, $I_t$ the information set dated $t$ and $\alpha$ $(\alpha \geq 0), \beta$ $(\beta \geq 0), \omega$ $(\omega > 0), \mu, \lambda$ are real parameters. The assumption placed on (2) is, however, quite critical; as Merton (1973) remarks, there is at least one element of the opportunity set which is directly observable and stochastic, the nominal interest rate. To understand what the capm simplification implies, one can look at the aggregated (over the $K$ consumers) demand equation for the $i - th$ asset, which is:

$$d_i = \sum_{k=1}^{K} v_{ij}(\alpha_j - r) + \sum_{k=1}^{K} \left( \frac{-\partial c^k / \partial r)}{\partial c^k / \partial W^k} \right) \sum_{i=1}^{n} v_{ij} \sigma_{jr}$$

and collapses to the capm equivalent if $\sum_{k=1}^{K} \left( \frac{-\partial c^k / \partial r)}{\partial c^k / \partial W^k} \right) \equiv 0$ or if $\sigma_{jr} = 0$. The first case is obtained under the Bernoulli logarithmic utility functions, which rules out the importance of changing investment opportunities in equilibrium. For the second case to be obtained, either the interest rate must be non-stochastic, which is not supported by the data, or the correlation between the interest rate and all asset returns must be zero, which would not be an equilibrium. Hence, the introduction of the bond yield into the garch-in-mean framework (3) is likely to induce substantial changes in the estimated premium, and calls for an extension to a bivariate setting, which is the econometric counterpart of an i-capm specification:

$$\begin{array}{rcl} r_t - r & = & \pi(\theta_t) + u_t \equiv \mu + \lambda \cdot \theta_t + \phi \cdot \sigma_{Br,t} + u_t \\ \log \left( \dfrac{r_{B,t}}{r_{B,t-1}} \right) & = & c_B + u_{B,t} \\ \left[ u_t, u_{B,t} \right] | I_{t-1} & \sim & N \left[ \left( \begin{array}{c} 0 \\ 0 \end{array} \right), \left( \begin{array}{cc} \theta_t & \sigma_{Br,t} \\ - & \theta_{B,t} \end{array} \right) \right] \\ \theta_t & = & \omega + \beta \cdot \theta_{t-1} + \alpha \cdot u_{t-1}^2 \\ \theta_{B,t} & = & \omega_B + \eta_B \cdot \theta_{B,t-1} + \phi^B \cdot u_{B,t-1}^2 \\ \sigma_{Br,t} & = & \rho \cdot \sqrt{\theta_{B,t}} \cdot \sqrt{\theta_t} \end{array} \tag{4}$$

where, in addition to the symbols employed so far, $r_B$ is the yield on long-term bonds (in this paper chosen as the redemption yield on US triple-A-rated bonds with 10-year maturity), $\theta_B$ the conditional variance of such yields, $\sigma_{Br}$ the conditional covariance between the bond return and the asset return (the equity return), $\rho$ the (constant) coefficient of conditional correlation between equity and bond returns, $u_B$ is a conditionally gaussian error term and $\omega_B, \eta_B, \phi^B$ are real parameters. Unlike the univariate garch-in-mean model (3), here the conditional mean equation of the equity excess return has an additional term which is the conditional covariance between the bond and the equity return. If the associated coefficient $\phi$ should turn out to be significant, then the introduction of $r_B$ may be strong enough to modify the risk premium estimates obtained within the univariate model.

## 2.2 *The cross section of risk premia and the Fama-French model*

The estimate of the equity premium required on the market index may not be a very informative figure for many investors. Not all economic agents buy the market portfolio, and if they decided to allocate their wealth using expected Sharpe ratios based on the market-level premium and riskiness, they could well find their strategy to be largely sub-optimal. To examine the behaviour of risk premia at a firm level, an extension of the bivariate model (4) to a higher dimension would be an ideal working condition. Unfortunately, using conditionally heteroskedastic models of large size is precluded by available optimization routines. Adopting a factor scheme is a valid alternative, as shown by their widespread use in theoretical and applied finance (the simplest factor model in asset pricing is the capm itself). When working with the cross section of equity returns, a large body of evidence suggests that, contrary to the capm assumption, the market portfolio alone cannot provide a reasonable explanation of their cross-sectional variance, especially when assets belong to different industries or countries and when firms have different characteristics, such as size, price/earnings or price/dividend ratios. The view that the traditional market model needs be augmented with additional factors to achieve a reasonable explanation of cross sections of (US) stock returns has been established by Fama and French (1993) who identify two factors, i) the average return on small firms minus the average return on large firms and ii) the average return on high-performing firms minus the average return on low-performing ones, as fundamental to explain, in addition to the market, a cross-section of monthly Nyse, Amex and Nasdaq stock returns observed between 1963 and 1990.[2] Like Engle *et al.* (1990), the Fama-French three-factor model can be cast within an heteroskedastic setting, where all factors have the bivariate garch-in-mean representation (4). Assume that the $k-th$ factor, $f_{k,t}$, is generated according to such bivariate scheme and let the excess returns of a set of stocks behave as:

[2] To save space in the main text, I briefly report here how FF build the factor portfolios. Consider first six portfolios obtained by clustering individual stocks into two size (MV) categories (small and big) and into three performance (MTBV) categories (low, medium, high). The returns on individual assets in the six portfolios are weighted by the market values of individual stocks; portfolios remain unchanged between July of year $t$ and June of year $t+1$, when new book values are released. In the second step of the factors' construction, the influence of, in turn, size and performance are eliminated. The size portfolio is obtained by computing the difference between the equally weighted returns of the three small portfolios (for any performance class, 1, 2 and 3) and the three big portfolios (for any performance), which amounts to eliminating the performance factor. Analogously, the performance portfolio is obtained by computing the difference between the equally weighted return of the two high MTBV portfolios (both small and large as MV is concerned) and the two low MTBV portfolios.

(5) $$r_t = \mu_t + \sum_{k=1}^{K} \beta_k \cdot f_{kt} + \varepsilon_t$$

where $r_t$ are excess returns, $\mu_t$ is a vector of risk premia, $f_{k,t}$ is the value of the $k - th$ factor as of time $t$, $\beta_k$ are real coefficients and $\varepsilon_t$ is a vector of gaussian noises. Suppose also that $R_t^c$, the rate of change of the marginal utility of consumption at time $t$ is given by a similar factor model:

(6) $$R_t^c = \mu_t^c + \sum_{k=1}^{K} b_k \cdot f_{kt} + \varepsilon_t^c$$

where symbols have analogous meaning as in (5). The joint process for $r_t$ and $R_t^c$ can be described as:

(7) $$\begin{bmatrix} r_t \\ R_t^c \end{bmatrix} |F_{t-1} \sim N \begin{bmatrix} \mu_t \\ \mu_t^c \end{bmatrix} ; \begin{bmatrix} \sum_{k=1}^{K} \beta_k \beta_k' \lambda_{kt} + \Omega & \sum_{k=1}^{K} \beta_k b_k \lambda_{kt} \\ \sum_{k=1}^{K} \beta_k' b_k \lambda_{kt} & \sum_{k=1}^{K} b_k^2 \lambda_{kt} + \sigma_c^2 \end{bmatrix}$$

after placing the following hypotheses on the conditional moments:

$$\begin{aligned} \mu_t, \mu_t^c &\subset F_{t-1} \\ E_{t-1}(f_{kt}) &= E_{t-1}(f_{kt} f_{jt}) = 0, \text{ for any } k, j\ (j \neq k), t \\ E_{t-1}(\varepsilon_t) &= E_{t-1}(\varepsilon_t^c) = 0 \\ Var_{t-1}(f_{kt}) &= \lambda_{kt} \\ E_{t-1}(\varepsilon_t \varepsilon_t') &= \Omega; E_{t-1}(\varepsilon_t \cdot \varepsilon_t^c) = 0; Var_{t-1}(\varepsilon_t^c) = \sigma_c^2 \end{aligned}$$

where $F_{t-1}$ denotes the $\sigma$-algebra induced by the sequences $\{r_s\}_{s=1}^{t-1}$ and $\{R_s^c\}_{s=1}^{t-1}$. In consumption beta models, assets' risk premia are required to satisfy (with $\delta_t \equiv \delta$, for any $t$)[3]

[3] The risk aversion parameter can be made time varying quite straightforwardly. Theory, however, does not say much as for how it should vary over time. A typical hypothesis is that it evolves as a random walk plus noise,

$$\mu_t = \delta \cdot cov_{t-1}(r_t, R_t^c)$$

and taking the covariance term from the off-diagonal term of the covariance matrix in (7)

$$\mu_t = \sum_{k=1}^{K} \beta_k (\delta \cdot b_k \cdot \lambda_{kt}) \tag{8}$$

where $\delta$ is a risk aversion parameter. Given that the risk premium on the portfolio representing the k-th factor is given by

$$\Pi_{kt} = a_k' \cdot \mu_t = \delta \cdot b_k \cdot \lambda_{kt}$$

where $a_k$ is a vector of weights orthogonal to $\beta_j$ $(j \neq k)$, with $a_k \cdot b_k = 1$, the portfolio constructed with $a_k$, i.e. $P_{kt} = a_k' \cdot r_t$ has conditional variance $\theta_{kt} = \lambda_{kt} + s_k$, with $s_k$ = $a_k' \Omega a_k$. This structure implies that the conditional variance matrix of the excess returns $r_t$ can be factored as

$$H_t = \sum_{k=1}^{K} \beta_k \beta_k' \theta_{kt} + \Omega^*$$

where $\Omega^* = [\Omega - \sum_{k=1}^{K} \beta_k \beta_k' s_k]$, which in turn requires the conditional variances of the factors $(\theta_{kt})$ to *cause* the conditional variances of the individual assets. The above specification chosen for the asset pricing model has an implication in terms of the beta coefficients which will be shortly made explicit in its econometric counterpart. As in all factor schemes, the estimation starts with a first step in which the dynamics of the $k$ premia required on the factors, $\pi_f(\theta_{ft})$, and their conditional variances, $\theta_{ft}$, are recovered by means of the bivariate garch-in-mean (4). Then, a second step estimation is performed in which the conditional variance of the factors

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$\delta_t = \delta_{t-1} + \varphi_t$. In this case, however, maximum likelihood estimation would be prevented by the latent structure of the risk aversion.

and the risk premia required for the fluctuation of the factors are related, respectively, to the conditional variance and to the excess returns of the $i-th$ portfolio as:

$$
\begin{aligned}
r_{i,t} &= c_i + \sum_{j=1}^{N} \beta_{i,f_j} \cdot \pi_{f_j,t}(\sigma^2_{f_j,t}, \sigma_{fj,B}) \equiv \\
\text{(9)} \qquad & \quad c_i + \sum_{j=1}^{N} \beta_{i,f_j} \cdot (c_{f_j} + \gamma_{f_j} \cdot \sigma^2_{f_j,t} + \gamma_B \sigma_{fj,B,t}) + u_{i,t} \\
\text{(10)} \qquad \sigma^2_{i,t} &= \omega_i + \sum_{j=1}^{N} \beta^2_{i,f_j} \cdot \sigma^2_{f_j,t}
\end{aligned}
$$

where $N$ is the number of factors.[4] Equations (9) and (10) indicate the presence of cross-equation restriction on the $\beta_{i,f_j}$: the level is the risk premium factor loading, the squared value is the loading between the factor's and the portfolio's variance. Following the structure of the FF model, the number of factors is chosen to be $N = 3$.[5]

### 2.3 *Inference in factor arch models*

The estimates of the firm-level premia derived from (9)-(10) deserve great attention as regards their statistical properties. There are indeed many difficulties in performing inference in unobserved heteroskedastic factor models, which are summarized in Sentana and Fiorentini

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[4] Statistical tests for the control of the models are discusses and applied in Section 3.

[5] Strong criticisms about the theoretical structure of the FF model have been raised by Daniel and Titman (1997) and Daniel et al. (2001). These authors are skeptical about the factors chosen by FF being truly *priced* risk factors and claim that the setup lacks power against an alternative hypothesis named *characteristic model*, where asset returns are linked to the characteristics of the firm that they represent (e.g. size or performance) for reasons which may not be related to the covariance structure of the assets themselves and, by means of ad-hoc built portfolios, are led to support their characteristic model against the FF scheme. More supportive of the FF framework are the results in Mian and Teo (2001), who analyze the relation between the Japanese cross section of stock returns and the revision which occurs in the expected rate of growth of firms. They test the hypothesis put forward in La Porta (1996) and La Porta et al. (1997) that the existence of a value premium (i.e. a positive differential between the return of so-called value stocks - firms with low MTBV - and the return of growth stocks - firms with high MTBV) is not related to measures as the MTBV or the price/earnings ratios being priced risk factors; rather, the value premium should come from a substantial (downward) revision to the expected rates of growth of the firms with high MTBV ratios. This idea goes along with the evidence of a significant bias in growth expectations: the large divergence highlighted in the literature between the growth rates of earnings for a large sample of US firms observed between 1951 and 1998 and their short-term expectations is a clear indication of this (Chan et al., 2001). Despite the existence of such a bias in market expectations, Mian and Teo (2001) are not able to show that the largest revision errors occur for firms with higher values of the MTBV or price/earnings ratios, a finding which per se supports the assumption of Fama and French (factor model) over the alternative view of Daniel et al. (characteristic model).

(2001). A lone contribution on inference in conditionally heteroskedastic models of the garch class is Lin (1992) who, among other things, provides a brief assessment of the consistency and asymptotic normality assumptions which are typically placed on the coefficients of a factor garch scheme as (9)-(10). He employs a bivariate structure to assess the empirical performance of four estimators of a one-factor garch model, namely the two-stage univariate garch, the two-stage quasi maximum likelihood (qml), restricted qml and full qml of the system. His simulation scheme is the following:

$$
\begin{aligned}
\varepsilon_t &= H_t^{0.5}\xi_t \\
H_t &= \Omega + gg'(\alpha^2 f'\varepsilon_{t-1}\varepsilon'_{t-1}f + \beta^2 f' H_{t-1} f) \\
\xi_t &\sim N(0, I_2) \\
f'g &= 1 \\
f'\iota &= 1 \\
\eta_t &= f'\varepsilon_t \\
h_t^* &= f' H_t f = w^* + \alpha^2\eta_{t-1}^2 + \beta^2 h_{t-1}^*
\end{aligned}
\tag{11}
$$

where $f_{2\cdot 1}$ are the weights of the two assets in the factor (that is why $f\prime\iota = 1$), $g_{2\cdot 1}$ the factor variance loadings of the two assets, $\xi_t$ is a $2 \cdot 1$ vector of independently and identically distributed zero mean and unit variance noises, $H_t$ and $\Omega$ are $2 \cdot 2$ matrices, $\eta_t$ is the (scalar) shock to the common factor, $h_t^*$ is the conditional variance of the common factor and $\alpha$ and $\beta$ are scalars. In the factor-garch case the model is enlarged with the following equations

$$
\begin{aligned}
y_t^* &= f' \cdot y_t = c^* + \gamma \cdot h_t^* + \eta_t \\
y_t &= c + \gamma \cdot g \cdot h_t^* + \varepsilon_t
\end{aligned}
$$

where $c$ is a $2 \cdot 1$ vector of constants, $\gamma$ a scalar, $y_t^*$ the common factor excess return and $y_t$ a $2 \cdot 1$ vector of asset excess returns. The simulation is performed by assigning initial values, say $H_0$ to $H$, fixing $f$ and $g$ and drawing $\xi_0$; then $\varepsilon_1$, $\eta_1$, $h_1^*, y_1^*$ and $y_1$ are computed and the recursion goes on until time $t$; the $\Omega$ matrix can be computed from the sample cross moments of the two components of $\varepsilon_t$. At this point a garch model is fitted to the simulated factor and the two univariate factor-garch are estimated. At each estimation the values of the relevant parameters are stored and their average over $N$ simulations is calculated. For a factor garch-in-mean the parameters $c = (c_1, c_2), \gamma, \alpha, \beta$ and the factor loadings $g$ are those of interest. In his experiments Lin employs sample sizes of 100 and 500 and persistences of the variance generating process equal to 0.65 and 0.89. He finds that the biases of all estimators are essentially quite small though, with the exception of the full qml estimator, larger for the unconditional part of the covariance matrix than for the other parameters; the coefficients of the conditional mean equation are estimated with less precision than those of the conditional variance equations but the bias does not appear to be a matter of concern. The presence of a bias in the parameters of the conditional mean equations which stems from the analysis of Lin (1992) suggests to report some additional evidence on the properties of the estimators.

### 2.4 *Relation to recent advances*

I review here the structure of the three alternative approaches to the modeling of risk premia mentioned in the Introduction, which may be useful in interpreting the existence of differences among the estimates that they provide. Implicit in Pàstor and Stambaugh (2001), Mayfield (1999), Han (2002) and in this paper is the idea that the relation between conditional means and conditional variances is subject to changes which allow for the time variation of risk premia. The way in which such differences are introduced is, however, conceptually different in Pàstor and Stambaugh. They postulate the existence of a number of transition regimes separated by stable regimes and further assume that, in the latter, asset excess returns are normally distributed as $r_t \sim N(\mu_i, \sigma_i^2)$, whereas in transitional regimes they come from $r_t \sim N[(\mu_j + \mu_{j+1})/2 + b_j\Delta_j, \sigma^2_{j,j+1}]$. The additional terms appearing in the transition period are the average of the mean excess returns across adjacent states ($j$ and $j+1$) and the returns' jump size $\Delta$ (with $b_j < 0$). These assumptions, which characterize the likelihood of the data, are merged with prior beliefs in the posterior distribution of excess returns. Some of these assumptions are mild and concern the distribution of $b_j$, the distribution of transition regime volatility $\sigma_{j,j+1}$, the transition matrix among states and the duration of the states; others are stronger and concern the premium's association with volatility and the magnitudes of changes in the premium. As for the first, Pàstor and Stambaugh follow Merton (1980) in positing a positive relation between equity premia and asset returns volatility, i.e. $\mu_i = \gamma \cdot \psi_i \cdot \sigma_i^2$ with $\gamma > 0$ and each of the $\psi_i$ drawn from a gamma distribution, whose variance, $\upsilon$, regulates the intensity of the relation; for the second distribution, $p(\mu|\ \bar{\mu})$, they choose a truncated normal (since $\mu > 0$) which has a variance matrix precluding large swings in the equity premium (ad hoc values are therefore assigned to the variances of $\Delta_i$). In this setup the ex-ante risk premium at time $t$ is given by the posterior mean of $p(r|\mu, \psi, \gamma, b, \sigma, q)$, with $q$ specifying the number of lags to be included in the construction of the mean and the standard deviation of such a posterior expectation. Against this setup, the garch-in-mean specification does not allow for the presence of structural breaks. Given the stability of the parameters of the conditional mean equation ($\mu$ and $\lambda$ in (3); $\mu, \lambda$ and $\phi$ in (4)), it will be the presence of large innovations in the conditional mean equation (which can nonetheless be seen, very roughly, as a hint of a break in the generating process) to produce larger and persistent conditional variances and premia. It is also worth noting that the Pàstor and Stambaugh framework is univariate, while the factor garch specification makes it rather easy to provide estimates of ex-ante premia at a firm-level. Also, the factor model does not constrain $\gamma$ to be positive, though the plausibility of negative risk premia remains to be explored (Backus and Gregory, 1993). The setup of Mayfield (1999) lies in between the two other approaches, with asset returns evolving according to a two-regime model, where second moments can switch between low- and high-volatility states, an assumption that is again very close to the garch specification. At each point of time the within-state mean and variance of the excess returns are given by

$$\begin{cases} \mu - r_f = \gamma \cdot \sigma_L^2 - \pi_L \cdot J_L(1+K_L)^{-\gamma} \\ \sigma_t^2 = \sigma_L^2 \end{cases} \text{if } s_t = L$$
$$\begin{cases} \mu - r_f = \gamma \cdot \sigma_H^2 - \pi_H \cdot J_H(1+K_H)^{-\gamma} \\ \sigma_t^2 = \sigma_H^2 \end{cases} \text{if } s_t = H$$

with $J_H = \frac{1}{1+J_L} - 1$, where $J_H$ and $J_L$ are the jumps in the asset returns occurring when the economy switches out of the High (H) or the Low (L) state ($s_t$). The third related paper, due to Han (2002), employs a structure that is very close to (4), though slightly simpler. In his scheme it is the volatility of the stock return that plays the role of the second state variable; thus, while (4) is a bivariate garch scheme, the formulation adopted by Han, equation (12) below, can be cast as a univariate model although the specific continuous time formulation that he proposes cannot be approximated straightforwardly by a simple garch structure. His scheme is:

$$\begin{aligned} d\ln P_t &= (\mu_t - 0.5\cdot\sigma_t^2)dt + \sigma_t\cdot dW_t^{(1)} \\ d\ln\sigma_t^2 &= (\alpha - \beta\cdot\ln\sigma_t^2)dt + \delta\cdot dW_t^{(2)} + \theta\cdot dW_t^{(1)} \\ \mu_t &= \lambda_0 + \lambda_1\cdot\sigma_t^2 + \lambda_2\cdot(\theta\cdot\sigma_t) \end{aligned} \tag{12}$$

where $P_t$ is the stock price, $\sigma_t^2$ its conditional variance, $\mu_t$ the overall risk premium, $dW_t^{(1)}$ and $dW_t^{(2)}$ the increments of two independent Brownian motions, $\alpha, \beta,\ \delta, \lambda_0, \lambda_1$ and $\lambda_2$ real parameters. He estimated the continuous time model (12) with monthly data for the US stock market for the sample January 1926 - December 1998 through the efficient method of moments (EMM) of Gallant and Tauchen (1996), obtaining $\lambda_1 = 1.992$ and $\lambda_2 = 11.55$, both statistically highly significant. Unfortunately, while EMM is a powerful technique to identify the parameters of continuous time models, even when unobservable state variables are present, it is not adequate to filter out, directly, the historical path of both the unobserved and the observed variables; such paths have to be recovered after the estimation, through the so-called reprojection step (Gallant and Tauchen, 1998) [6]. Given the different state variable employed in models (12) and (4) it is interesting to investigate whether the estimates of the risk premium are dependent on the choice of the state variable, i.e. if the information conveyed by the conditional volatility of the stock returns ((in 12)) or by the bond yield (in (4)), happens to be equivalent or instead differs noticeably. While EMM estimation of (12) is easy to perform, I prefer to keep the econometric methodology employed in this paper confined to the garch setting. To do this, instead of estimating (12), consider the following slight modification:

$$dP_t/P_t = \mu\cdot dt + \sigma_t\cdot dW_t^{(1)} \tag{13}$$

$$d\sigma_t^2 = (\omega - \varphi\cdot\sigma_t^2)dt + \lambda\cdot\sigma_t^2\cdot[\rho\cdot dW_t^{(1)} + \sqrt{1-\rho^2}dW_t^{(2)}] \tag{14}$$

which has been shown (Fornari and Mele, 1997) to have the following power arch scheme (Ding et al., 1993) as discrete time counterpart:

---

[6] Very briefly, the latter employs the estimated transitional density to evaluate $E_{t-1}(r_t|r_{t-1};\sigma_{t-1}^2)$ and $E_{t-1}(\sigma_t^2|r_{t-1};\sigma_{t-1}^2)$ in correspondence of each of the realizations of the observable variables.

$$
\begin{aligned}
(15) \qquad r_t &= \mu_1 + \varepsilon_t \\
(16) \qquad \sigma_t^2 &= \omega_1 + \beta \cdot \sigma_{t-1}^2 + \alpha(|\varepsilon_{t-1}| - \tau \cdot \varepsilon_{t-1})^2.
\end{aligned}
$$

This discrete time specification (15)-(16) allows for the well-known leverage effect, i.e. a possibly negative correlation between $\varepsilon_t$ and $\sigma_t^2$. The approximating property of (15)-(16) is important insofar as the continuous time parameters of (13)-(14) can be recovered through closed-form moment conditions from its parameters, although a discretization bias may affect this procedure (Fornari and Mele, 2001). The convergence result naturally extends to the case where the continuous time model is changed to

$$
(17) \qquad
\begin{aligned}
&dP_t/P_t = \mu_t \cdot dt + \sigma_t \cdot dW_t^{(1)} \\
&d\sigma_t^2 = (\omega - \varphi \cdot \sigma_t^2)dt + \lambda \cdot \sigma_t^2 \cdot \left[\rho \cdot dW_t^{(1)} + \sqrt{1-\rho^2}dW_t^{(2)}\right] \\
&\mu_t = \mu + \lambda_1 \cdot \sigma_t^2 + \lambda_2 \cdot [\lambda \cdot \rho \cdot \sigma_t]
\end{aligned}
$$

and, accordingly, the discrete time setup becomes

$$
(18) \qquad
\begin{aligned}
r_t &= \mu + \lambda_1 \cdot \sigma_t^2 + \lambda_3 \cdot \sigma_t + \varepsilon_t \\
\sigma_t^2 &= \omega + \beta \cdot \sigma_{t-1}^2 + \alpha(|\varepsilon_{t-1}| - \tau \cdot \varepsilon_{t-1})^2 \\
\lambda_3 &= \lambda_2 \cdot \lambda \cdot \rho
\end{aligned}
$$

The specification of $\lambda_3$ in the third equation in (18) above follows from the definition of the correlation between $r_t$ and $\sigma_t^2$, after noting that $\lambda$ represents the volatility of the volatility (see the diffusion term in equation (14)). The values of the continuous time parameters $\lambda$ and $\rho$, necessary to recover the partial risk aversion coefficient $\lambda_2$, are obtained from the discrete time parameters through ad-hoc closed formulae provided in Fornari and Mele (2001).

## 3. Estimates of the risk premia

### 3.1 *Garch-based conditional intertemporal capm*

The estimates presented in this paper start from the market premium derived from the conditional capm model; this time series will be compared with the analogous measure derived from the bivariate setting which approximates the i-capm. In the same way univariate and bivariate garch-in-mean models will be estimated for the two factors which in addition to the market return define the Fama and French (1993) setup, i.e. size and performance; the outcome of the three bivariate schemes will be the input for the estimation of risk premia for quintile portfolios (i.e. firm-level premia), according to the factor scheme in (9)-(10). I first provide a brief description of the data. The equity market return is the monthly logarithmic change of the aggregate US index. Firm level returns are the weighted average of the returns on all firms falling within each size and performance quintile. The bond yield is the yield-to-maturity of aaa-rated US bonds converted to a monthly basis. The short-rate refers to the yield of 3-month T-bills on the secondary market. The overall industrial production index is seasonally adjusted and is taken from the Federal Reserve website. Details for the estimation of the univariate factor-garch model (3) are in Table 1. As concerns the market excess return, the intercept ($\mu$) is almost negligible in economic terms, 15 basis points per year (when the quantity of risk, $\sigma_t^2$, is zero, the compensation for risk is approximately zero) and the estimate of risk aversion is significant and positive (1.45). The conditional variance of the market excess return is generated by a persistent process, where the sum of $\alpha$ and $\beta$ equals 0.964; this figure implies that the half-life for the effect of a shock in the expected return process (first equation in (3)) on the conditional variance is 8 months (the shock dies out completely in 1 year and 4 months). The sample average of the market premium is 5.03 percent on an annual basis, with a standard deviation of 6.11 percent. Still within the univariate framework, the estimates of the ex-ante premia required on the two factors of FF have historical means of 1.11 (performance factor) and 4.10 (size factor) with standard deviations of 0.65 and 2.6 percent; the lower mean and standard deviation of the performance factor contrasts with a higher persistence of the variance generating process, 0.97, against 0.85 in the case of the size factor.

The estimates of the i-capm model (4) are reported in Table 2 for the equity market excess return ($r_{m,t}$), the performance factor return ($r_{HML,t}$) and the size factor return ($r_{SMB,t}$). The equity premium and the two premia required on the FF factors bear clear evidence of heteroskedasticity inherited from the conditional second moments of assets' returns (Figure 1); they peaked in correspondence with the Great Depression, World War II and the oil shocks; compared with these turbulences the recent stock market volatility is minor. As to the relation of the estimated premia with *economic phenomena*, the 1930s strongly influence their size, with the market premium peaking at more than 30 percent (annualized), compared with an historical average of just above 4 per cent over the post-Depression sample; between 1939 and 1942 risk premia reached highs of approximately 15 percent but returned below 4 percent faster than in the Great Depression; a fast reversion of the premia to the long-run average was common to all periods: hence, deviations from the central tendency can be characterized as relatively short-lived. As expected, analogous patterns are shown by the conditional volatility series. It is also interesting to note that standard deviations and risk premia are not perfectly correlated across the three factors, which provides indirect support for adopting the FF scheme, evidencing that they are capturing different sources of risk likely to be priced in individual asset returns.

Looking at the estimates of the bivariate garch-in-mean model for the equity return and the bond return, there is a negative, statistically and economically non-negligible ($-0.277$, with Bollerslev-Wooldridge robust $t = -8.63$) correlation (and conditional covariance) between the innovations to the conditional mean of the equity and the bond return; this is coupled with a negative and significant estimate of $\phi$ ($\phi = -1.94$, with $t_{\phi} = -2.74$)) to increase the estimated market risk premium over the single-factor capm prediction. The (partial) risk aversion parameter $\gamma$ becomes smaller (1.31) than the estimate obtained for the univariate model of Table 1 (1.45). The overall effect on the estimated premium is displayed in the second panel of Figure 1: under the two-factor model the market premium is globally higher than that obtained with the one-factor specification and also slightly more erratic. Rather interestingly, the largest deviations between the two estimates of the risk premium occur in two periods, 1978-84 and 1996-2001. The existence of such divergences can be rationalized when one considers that the two periods were characterized by negative, or low, average (ex-post) returns as well as by increasing or high nominal interest rates; hence, in both periods the purchase of equities must have required a *compensation* for the return lost from not buying (or buying less of) the bond. The bivariate setting captures also the same phenomenon for the two FF factors. The coefficients of conditional correlation between these two factor returns and the bond return are negative (-0.42 for size, -0.39 for performance), and so are the partial risk aversion coefficients $\phi$ ($-14.7$ and $-36.5$ for the size and the performance factor, respectively); similarly to the market, then, the bivariate model-based premia are higher than the capm-based counterparts (Figure 1, panels 3 and 4). The mean values and standard deviations of the associated premia over the sample amount to 4.76 and 1.78 percent per year respectively. Overall, the size factor seems to be more important than the performance factor as a perceived source of risk. The premium on the performance factor is very low and the time-variation appears to be significant only in coincidence with the second oil shock and October 1987 (panels 3 and 4 of Figure 1). By contrast, the size factor has a premium which is more or less on the same scale as the premium required for the fluctuations of the market index and reacts rather strongly to the stage of the business cycle. The top panel of Figure 1 shows, for the equity return only, the estimated conditional standard deviations of the bond and the equity returns and their conditional covariance. It is interesting to note that the negativity of $\phi$ implies the positivity of $\left[\frac{J_{jW}}{J_W}\right]$ (see equation 2), hence a positive value for the marginal utility of wealth with respect to the bond price. This also happens for the other two factors and is consistent with Chen et al. (1986) and Scruggs (1998).

### 3.2 *Precision of the estimates*

This section is devoted to judging the precision of the estimated time series of the equity premium as a function of two circumstances: random changes from the distributional assumption placed on the residuals of the two i-capm equations (conditional normality) and random changes in the relevant parameters of the bivariate garch-in-mean models. Concerning the first issue, despite generating unconditional student-t distributed errors, conditional normality may not be fat-tailed enough as a representation of financial returns; many papers have in fact shown that financial returns standardized by the conditional standard deviations are not normal, which contradicts the garch assumption. Nonetheless, conditional normality is the typical choice in multivariate contexts, though other alternatives have been explored at the univariate level. In the present application, the control tests carried out on the factor garch-in-mean models of Section 3.3 (see Table 4) show that the kurtosis of the innovations standardized by the garch conditional standard deviation is much lower than the corresponding values obtained when the residuals are standardized with the unconditional standard deviations, though still not coherent with normality. As for the second issue, the estimated coefficients of the bivariate models (Table 2) are instead all highly statistically significant; in principle, then, losses of efficiency from this particular direction should not constitute a problem (though Section 3.5 is devoted to a more rigorous assessment of this aspect in the factor-garch setting).

To address the first issue I simulate paths of the equity market excess return, of the bond return and of their conditional covariance matrix evaluating recursively the bivariate garch-in-mean model (4) a number $K$ of times. In each of the $K$ replications the parameters are fixed at the values reported in Table 2; within each replication, at each point of time, $t$, the two *true* sources of error $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ are replaced by a draw from their conditional distribution, a normal with standard deviation given by the value of the estimated conditional standard deviation at time $t$. As an example, if the values of the estimated conditional standard deviation of $\varepsilon_1$ were equal to $\sigma_t = 6$ and $\sigma_{t+1} = 6.2$ percent per year at times $t$ and $t + 1$, the values of $\varepsilon_{1,t}$ and $\varepsilon_{1,t+1}$ will be drawn $K$ times from, respectively, a $N(0, 0.06/12)$ and a $N(0, 0.062/12)$; the value of $K$ has been set equal to 5000. The black area reported in Figure 2 shows the confidence interval of the i-capm based equity premium calculated as the mean plus or less two standard deviations; at each time point, $t$, the latter are calculated as the standard deviations of the 5000 simulated premia. The dependence of the risk premium on both the market variance and the covariance between the equity and the bond returns is such that periods of high volatility increase the uncertainty about the true value of the premium. It is interesting to note that the 2.5 percent quintile of the premium almost never goes below 2 percent, a figure which can be taken as a physiological compensation for equity risk. Leaving aside the short-lived periods of extremely high premia, coinciding with the most destabilizing events, a typical confidence interval for the risk premium is between 2 and 6 percent. Compared with the historical experience, the post-1995 period may seem a striking anomaly: the economy was growing fast and yet the risk premium displayed a rising trend; in addition, the central estimate provided by the model is associated with a decreasing precision: the [2-6] range for the expected premium prevailing on January 1996 widens to [3-16] in coincidence with the LTCM collapse and then gradually stabilizes around [3-10]. We may be tempted to judge the risk premium recorded in this period as *high* because of the extraordinary growth of the US economy between 1995 and 1999, high even in an historical perspective. However, this would be misleading since the increasingly high ex-post returns recorded in the first two-thirds of the sample were forecasters of a slowdown of the business cycle (according to wide empirical evidence stock returns tend to anticipate business cycle developments) and ex-ante returns must have been decreasing through the period. This explains the existence of increasing risk premia while the economy was reaching the edge of the expansionary period.

Coming to the second issue, changes in the parameters of the conditional mean equation of the equity excess returns (first equation of (4)) do not, as expected, have a large influence on the precision of the time series estimate of the premium. To assess this I performed 5000 simulations of model (4) and, for each of these, values of $\hat{\mu_1}$, $\sqrt{\lambda}\hat{}$ and $\sqrt{\phi}\hat{}$ were drawn from their estimated distributions, i.e. three normals centered on the *true* values $\mu_1$, $\sqrt{\lambda}$ and $\sqrt{\phi}$ and with standard deviations equal to $\sigma_{\mu_1} = 2.35 \cdot 10^{-7}$, $\sigma_{\lambda} = 0.1668$, $\sigma_{\phi} = 0.2598$; the errors $\varepsilon_{1,t}$ and $\varepsilon_{2,t}$ were kept constant at their estimated values for each time point $t$. The bottom panel of Figure 2 shows that the 95 percent confidence interval related to random variations in the relevant parameters is almost indistinguishable from the central estimate; more on this topic is in sub-section 3.5.

### 3.3 *The cross-section of equity premia and control of the estimates*

Based on the estimates of the market premium and on the risk premia required on the size and performance factors obtained within the bivariate i-capm setting, a factor garch-in-mean model is estimated for each of 25 quintile portfolio returns[7]; the associated risk premia are recovered along with a synthetic market premium, calculated as a weighted average of the premia for the five size quintiles. The time series of the risk premia required on the five size quintile portfolios and the five performance quintile portfolios are reported in the two panels of Figure 3. Small and medium-sized firms (second, third and fourth quintiles) require risk premia much higher than the average; as for performance, high premia are paid by firms with the highest market value to book value ratios, which can be thought of as *overvalued* firms. From the inspection of Figure 3, there appears to be a tendency of the cross-sectional dispersion of risk premia to increase with the business cycle development; this is formally tested by regressing the within-size and the within-performance dispersions on the dummy variable capturing whether the economy is in expansion or in recession. Over the whole sample, the cross-sectional spread equals 195 basis points in expansion and 339 in recession for size and passes from 138 to 249 basis points for performance; the t-student of the two coefficients are significant at any reasonable level of confidence. Excluding the Great

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[7] Data are taken from the website of Kenneth French. The 25 quintile portfolio returns are the weighted returns of US firms classified in 5 size classes and 5 performance classes. For details see Fama and French (1993).

Depression and World War II, the difference is much less striking, reaching values of 139 and 157 basis points for performance and 196 and 216 for size, but becomes substantial again between 1973 and 2001 (57 to 103 basis points for performance; 207 to 310 for size). What these figures may suggest is that, under the implicit assumption that economic agents evaluate ex-ante premia according to the model employed in the paper, differences in the perception of risk across firms in the last thirty years have been more closely related to their size (market value) than to their book value, a finding confirmed by other papers investigating the properties of the Fama and French (1993) setup.

So far I have used the conditional setting without reporting any evidence for the existence of conditional heteroskedasticity nor empirical support for the validity of the chosen model. This is the intent of the remainder of this section. Table 3 reports the coefficient of skewness, the kurtosis and the Engle's (1982) $TR^2$ for the 25 quintile portfolios. Though the first two tests are generally synthesized by the Jarque and Bera's normality test, I prefer to report the two statistics separately. The $TR^2$ is a test for the presence of autocorrelated conditional variances and is asymptotically distributed as a chi-squared under the null of homoskedasticity. The evidence is that all of the 25 portfolios returns have time-varying conditional variances, the $\chi^2_{(5)}$ threshold at the 1 percent confidence level being just above 15. With reference to the distribution of such returns, normality would be generally rejected because of a very large excess kurtosis, though in many cases skewness contributes significantly to non-gaussianity. As regards the control of the estimated garch models, I rely on statistics performed on standardized residuals, $z_t$, i.e. on the residuals of the conditional mean equation (9) divided by the time series of the conditional standard deviation (thus $z_t = u_t/\sigma_t$) from (10). According to the hypotheses typical of the garch setup, such residuals should be normally distributed or at least much more normally distributed than the residuals of the conditional mean equation standardized with their unconditional standard deviation ($z_t^* = u_t/\sigma$). Lack of normality could derive from a misspecified dynamics of the conditional variance equation or could be the result of a wrong distributional choice, whereby, for example, conditionally Student's-t errors could provide a better fit for a particularly leptokurtic time series. The existence of possibly misspecified dynamics of the conditional variance equation in a garch(1,1) model can be tested by regressing the squared standardized residuals, $z_t^2$, on the squared residuals of the conditional mean equation ($u_t^2$) lagged from 2 onwards, checking if all these coefficients are indistinguishable from nil.

Table 4 examines the appropriateness of the dynamics chosen for the conditional variance equation. When $z_t^2$ is regressed on the first five lags of $u_{t-j}^2$ excluding the first $(j = 2, ..., 5)$, an F-test for the joint significance of the associated four coefficients shows that a garch(1,1) is a good generating process for the conditional variance in all 25 cases. The table shows that the conditional standard deviation is always able to bring the original kurtosis much more into accordance with the requirement of normality; in 22 cases (out of 25) where the original excess kurtosis is on average above 6, the average kurtosis of the standardized residuals falls to nearly 3 and in all cases the *corrected* skewness is sufficiently close to nil.

It is interesting to judge whether the conditional version of the FF scheme would improve on the standard homoskedastic formulation. A rough estimate of its relative performance can be gathered from the tests reported in Table 5, where the dynamic heteroskedastic estimation of the factor model is compared with a static estimation. The latter is obtained by regressing, in each of the 904 months between July 1926 and October 2001, the excess returns of the 25 quintile portfolios on the three sensitivities against the factor returns (i.e. the market return, the size factor return and the performance factor return), determined in a first stage, then saving the estimated coefficients and evaluating their cross-sectional means and standard deviations. The homoskedastic model produces an estimate of $\gamma$, the loading on the risk premium for the size factor, which is not statistically different from nil; further, the values of $\gamma$ and $\delta$ are negligible, so that only the market return seems to able to explain cross-sectional differences among the quintile returns. In the heteroskedastic approach, on the other hand, all coefficients are significant, with the exception of the intercept $\alpha$. The market premium has a weight equal to 47 percent, the size premium 35 and the performance 18. These weights accord with previous findings that the size factor is more important than the performance factor as a perceived source of risk.

### 3.4 *Risk premia behaviour across business cycles*

The ex-ante values of the risk premia are determined as a combination of the conditional variance of the stock market return and the conditional covariance between the stock return and the bond return. Given that such conditional second moments are influenced by business cycle developments, the estimated premia will also fluctuate significantly in response to the economy being in expansion or in recession. Fama and French (1989) report that the risk premium moves countercyclically, i.e. expected premia during recessions are large relative to premia during expansions. In addition, we should also expect the equity risk premium to co-vary positively with the default risk on bonds[8] and negatively with the short-term rate (a lower equity premium is likely to be accompanied by higher stock valuations and lower interest rates). Some evidence of this relation is reported in Table 6. The equity premia, as measured by the estimate derived from the i-capm scheme, have been regressed on a dummy variable that equals one when the economy is contracting and zero when it is expanding, according to the classification provided by the National Bureau for Economic Research (details for this variable are in Table 6). The difference between the two periods is remarkable: the estimate equals 4.15 in expansion with a standard deviation of 4.9, 9.93 with a standard deviation of 10.6 in recession. In addition to the higher level of the conditional volatility, the estimate of the risk premium in recession is made more volatile (less precisely estimated) by the lower number of occurrences of such events within the sample (26 percent of the 904 analyzed months are recessions, according to the NBER classification of the business cycle commented in Table 6). Throughout the sample there is no relation between the risk premium and the contemporaneous level of the short - term interest rate: the slope of the regression equals -0.05 but it is not distinguishable from nil; on the contrary the premium co-varies positively with the contemporaneous bond yield, with a slope of 0.25 (a rise of 1 percentage point in the long - term yield increases the equity premium by 25 basis points). The equity premium is also very reactive to the defalult premium, measured by the differential between aaa-rated and baa-rated bonds: a 1 percentage point increase in this differential raises the premium by more than 400 basis points. The variability across states is much less evident for the two factors employed in the Fama-French setup: the premium for the size factor is 4.5 in expansion and 5.5 in recession while the premium for the performance factor moves from 1.45 to 2.52. Like the equity premium, these two premia co-vary positively with the bond yield and the default premium, while are not affected by the short rate. This amounts to saying that unless monetary policy transmits along the term structure of interest rates, it does not impact the perceived riskiness of the equity market.

The analysis of the risk premia required on firms with specific characteristics of size and performance confirms that the figures are much larger in recession than in expansion, on average by 40 percent, both across size and performance quintiles. Premia are not

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[8] The return differential between low-rating and high-rating bonds.

monotonically increasing across the two characteristics: the average value of the premium in expansions is 1.4 percent for firms in the first size quintile, 7.7 in the second, 4.9 in the third, 2.6 in the fourth and then rises again to 5.5 in the fifth. The same happens for performance, with the third quintile requiring the highest compensation; overall, firms with high market value to book value ratios are the riskiest. The effect of the bond yield on the risk premium has an average value of 0.22 across size and 0.20 across performance; the short rate has virtually no effect on the premia; there is a high sensitivity of the premium to an increase in the default premium. It is interesting to note that the values of such elasticities are very different over cycles; the short-rate, which has no effect on the risk premium over the whole sample, has no influence in expansions but has a highly significant effect in recessions: on average a (plus) 1 percent move of the short rate raises the equity premium by 59 basis points. The same pattern is detected for the reaction to the level of the bond yield (Table 7). Some additional features of cross-sectional premia can be gathered by comparing Figures 2 and 3. Over the whole sample, small firms tend to require very low premia compared to the overall market premium: the compensation for risk required for firms belonging to, in turn, each of the five size quintiles is below the 5 percent conditional confidence interval for the market premium in 95.2, 92.6, 82.4, 82.9 and 37.9 percent of the (904) cases respectively. Again such percentages change across business cycle regimes: in recession, on average, such occurrencies are five percentage points higher than in expansion (96.1 in expansion against 91.8 in recession for the first size quintile).

In many analyses the equity premium is related to the slope of the term structure of interest rates, measured as the return differential between a long term bond and a short bill, since both variables are ultimately dependent on the business cycle. In one of the typical references, Harvey (1988) finds that the term structure is upward sloping during recessions and especially at the trough of the cycle, and downward sloping in expansions, especially so at the peak of the cycle. It's trivial to show that the equity premium and the slope are both related and dependent on the business cycle. Take the usual asset pricing relation:

$$E_t[m_{t+1} \cdot r_{t+1}] = 1$$

and write it for the slope (the bond return, $r_{t+1}$, over the riskfree rate, $r_{f,t}$) and the equity premium (the stock return, $r_{m,t}$, minus the riskfree rate $r_{f,t}$), i.e.

$$\begin{aligned} E_t[r_{t+1} - r_{f,t}] &= -r_{f,t} \cdot cov[r_{t+1}, m_{t+1}] \\ E_t[r_{m,t+1} - r_{f,t}] &= -r_{f,t} \cdot cov[r_{m,t+1}, m_{t+1}]. \end{aligned}$$

It is then evident that both variables are related to the covariance between their return and the intertemporal rate of substitution which depends itself on the appropriately scaled consumption growth between time $t$ and time $t+1$. The unknown and possibly nonlinear relation between the equity premium ($rp_t$) and the slope of the term structure ($s_t = r_{t+1} - r_{f,t}$), say $f(s_t, \theta)$ can be proxied by a second-order Taylor expansion around zero, leading to:

$$rp_{t+1} = f(s_t, \theta) \simeq \alpha_0 + \alpha_1 \cdot s_t + \alpha_2 \cdot s_t^2 + \varepsilon_{t+1}.$$

To see if the estimated premium bears evidence of this relation, the above equation has been estimated separately for the whole sample January 1927 - October 2001 and for the periods of recession only, with separate intercepts in the two periods. A further splitting based on periods of inverted (i.e. negative) slope of the term structure has not been possible, due to the limited number of such occurrences (Figure 4, top panel). The parabolic relation between the slope of the term structure and the equity premium for the whole sample and the recessions is reported in the bottom panel of Figure 4. While the relation between equity premium and slope is indistinguishable across expansions and recessions when the slope is between .-40 and 40 basis points, recessions are characterized by a much stronger impact when the it is outiside of this interval. With a flat term structure the equity premium has been on average equal to 4 percent; when the slope increased to 100 basis points the premium was 4.3 per cent over the whole sample and 4.8 in recessions; when it reached 200 basis points the premium was approximately 5 percent per year in the full sample and 7.5 in recessions.

### 3.5 *Comparison with alternative formulations of the equity premium*

I now compare the equity premium derived in this paper with the estimated time series reported in Mayfield (1999) and in Pàstor and Stambaugh (2001). Mayfield does not graph his estimates, but panel B of his Figure 2 reports the high-volatility state probability along with the NBER recession phases between 1926 and 1997. The coincidence between such probabilities and the risk premium reported in Figure 1 of this paper is remarkable; both series reach the highest levels during recessions. The estimated premium in Mayfield is on average 7.8 percent, 4.8 in expansion (low-volatility state), 30.3 in recession (high-volatility state). The confidence interval across the whole sample is [3.1-11.9] percent, [1.8-8.1] in expansion. Similarly to the estimates of the present paper, the duration of expansions is much higher than that of recessions, 7.2 versus 1.0 years[9]. A rising trend of the equity premium after 1990 is also found by Mayfield: in his estimates the average risk premium is 8.2 per cent between 1940 and 1989, 10.9 between 1990 and 1997, though the difference between such means, -2.7 percentage points, has a t-ratio of only -0.63.

The estimates reported in Pàstor and Stambaugh span a monthly sample ranging from 1834 to 1999; assuming a simple break in the excess return series in December 1925, the authors estimate that the risk premium was in the [3.64-5.22] percent per year range between 1834 and 1925 and in the [5.22-8.36] range between 1926 and 1999, depending on different values of the jump variance (see sub-section 2.4). Their estimate, on average, decreases after 1926 in the "benchmark" case, from nearly 6 to 4.7 percent; it is a very smooth series with no sign of oscillation, which contrasts sharply with the estimated conditional variances typical of the universal finding of arch effects. Much higher values, similar to those obtained in this paper and in Mayfield (1999) in recessions, are obtained only for very high degrees of the mean-variance association. Hence, though the average estimate of Pàstor and Stambaugh accords with the other two, it is the shape of the time series which does not agree with the reported features of financial returns.

I shed further light on the topic by estimating the time series of the equity premium with annual data spanning two centuries, from 1800 to 2001[10]. These data are end-January figures, so employing average annual figures would result in lower volatility, hence in a lower risk premium, due to time aggregation. Long-term interest rates are available only from 1831, hence the i-capm will be estimated imposing a zero covariance between the equity return and the asset return from 1800 to 1831. What I intend to show is that the expected volatility

[9] This is a direct outcome of the Markov-switching framework in Mayfield. In this paper it can be approximated by the number of times the conditional volatility is above its unconditional value, given by $100 \cdot \sqrt{12}\sqrt{\frac{\omega}{1-\alpha-\beta}}$ per cent per year in (4).

[10] These data are taken from www.globfindata.com

and the compensation demanded by investors change greatly over different horizons; although PS employ monthly data from 1871 to 1998, the mean-variance association is not allowed to reflect its *true* (unobservable) value. Figure 5 shows the two estimates of the equity premium derived, as before, from the conditional capm and the conditional icapm. The capm-based estimate provides a picture of the equity premium which is very close to estimates derived from the Gordon formula mentioned in the Introduction, which can be considered as a long-term equilibrium relation. The premium rises steadily from 1.0 to 2.5 per cent per year in the 1800 - 1870 sample, stays unchanged until 1910 and then gradually increase to 4 percent in 1940. After the war it decreases to 3.0 percent in 2001. The average value of the premium is 2.56 percent, 1.7 between 1800 and 1871, 2.86 between 1871 and 1940, 3.21 afterwards. Looking at the i-capm estimate the picture changes markedly, as the fluctuations of the long - term interest rate were changing the ex-ante compensation for risk. Although in this case the fluctuations of the premium are much more evident than for the capm-based estimates, the values are much lower than those recorded at the 1-month horizon (Figure 1, top panel); the average over the whole sample is 3.77 (nearly 6.5 with monthly data; Table 2), 1.18 between 1800 and 1870, 5.78 between 1870 and 1940 (mostly due to the Great Depression), 4.39 afterwards. Thus, the chosen degree of association for the mean/variance relation ($\psi$ in sub-section 2.4), the fluctuation allowed for the volatility ($\Delta_i$ in sub-section 2.4) and the length of the investment horizon (the holding period) may reconcile figures that one would otherwise judge as very different.

Last, I compare the specification (4) to the alternative scheme, (12), of Han (2002) as approximated by model (17) in continuous time and (18) in discrete time. The maximum likelihood estimation of the power arch-in-mean model (18) for the monthly stock returns, $r_t$, over the period August 1926 - October 2001 produced the following result, with all parameters highly significant:

$$\begin{aligned} r_t &= 1.28 \cdot 10^{-4} + 0.054 \cdot \sigma_t^2 + 0.214 \cdot \sigma_t + \varepsilon_t \\ \sigma_t^2 &= 8.57 \cdot 10^{-5} + 0.645 \cdot \sigma_{t-1}^2 + 0.353 \cdot (|\varepsilon_{t-1}| - 0.09 \cdot \varepsilon_{t-1})^2 \end{aligned}$$

Based on such discrete time parameters, the continuous time parameters $\rho$ and $\lambda$ of model (17) are found to be $-0.194$ and $0.559$ respectively, so that $\lambda_2 = -\frac{0.37}{(0.081 \cdot 0.147)} = -1.78$. The estimates reported in Han, that refer to a slightly shorter sample, evidence a much lower correlation between stock returns and stock returns volatility ($\rho = -0.05$ instead of $-0.194$ in (17)), a smaller volatility of the conditional volatility ($\lambda = 0.33$ instead of $0.56$ in (17)) and a smaller value for $\lambda_2$ ($-0.56$ instead of $-1.78$ in (17)). The approximation to the time series of the equity premium evaluated by Han (2002) and the estimate based on (4) are reported in Figure 6 along with vertical bars identifying the NBER business cycle dating. The two series provide a homogeneous picture for the size of the equity premium, if one excludes a more rapid fall of the estimate based on (4) through the expansionary periods; overall (12) produces a less volatile premium 4.15 per cent per year with a standard deviation of 2.97 per cent in expansion (against 4.15 and 4.93) across the whole sample and a lower premium, 7.67 with standard deviation of 4.16 per cent in recessions (against 9.93 and 10.6); the lower mean of the premium in periods of recessions (7.67 versus 9.93) produced by model (13)-(14) comes entirely from the second oil shock period, when the conditional volatility of the stock return was much less variable than the level of the long-term interest rate.

### 3.6 *Consistency of the factor-garch estimates*

In Section 3.2 I analyzed the effect on the estimated risk premium of random changes in the parameters of the conditional mean equation (9), thus providing an estimated confidence interval for the premium. In the simulation reported there the values of the parameters were sampled from a normal distribution, centered on the (true) estimated value, with standard deviation equal to the estimated standard error, i.e. consistency was considered to hold, as typical of the maximum likelihood estimation. However, in the estimation of the firm-level premia through the conditional factor-garch model, this property was simply assumed to hold owing to the results of the monte carlo experiments of Lin (1992). However, since the properties of the estimators in factor-garch schemes have not been explored in detail and, again according to Lin (1992) some degree of inconsistency is expected, I provide here a brief assessment of the consistency assumption when asset returns and their conditional variances are generated by the following scheme

$$
\begin{cases}
\varepsilon_t = H_t^{0.5} \cdot \xi_t \\
H_t = \begin{bmatrix} 0.1 & 0.1 \\ 0.1 & 0.1 \end{bmatrix} + \begin{bmatrix} 1.2 & 0.8 \end{bmatrix} \begin{bmatrix} 1.2 \\ 0.8 \end{bmatrix} \cdot \\
\left\{ 0.13 \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} & \varepsilon_{2,t-1} \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t-1} \\ \varepsilon_{2,t-1} \end{bmatrix} \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} + 0.81 \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} H_t \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \right\} \\
\eta_t = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix} \\
h_t^* = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} H_t \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \\
y_t^* = 0.2 + 1.15 \cdot h_t^* + \eta_t \\
y_{1t} = 0.2 + 1.2 \cdot h_t^* + \varepsilon_{1t} \\
y_{2t} = 0.2 + 0.8 \cdot h_t^* + \varepsilon_{2t}
\end{cases}
$$

The persistence of the model has been set to reproduce the features of the data, i.e. $0.94$; in another experiment it has been fixed to $0.64$. The factor is built as the simple average of the two assets; the factor loadings in the construction of the conditional variance matrix are $1.2$ and $0.8$ for the two assets respectively. The risk aversion coefficient is $1.15$ (close to our estimates) and the sample size is also close to the dimension of the actual data, i.e. $1000$. The distribution of the relevant parameters is derived from two replications of size $200$ and $1000$, with additional $500$ observations employed as a warm-up of the experiment. The mean values of the parameters of interest are in Table 5, while their distributions, reported for the case in which $\alpha + \beta = 0.94$ and in which there are $200$ replications, are in Figure 7 (top panel). I start from the conditional mean of the factor returns: the estimate of $f' \cdot c$ is $0.233$ with a standard deviation of $0.034$, against a true value of $0.2$. The t-ratio of the difference between the two values is $0.96$. The risk aversion parameter, $\gamma$, is estimated with less precision, the mean value being $0.95$ against a true value of $1.15$; the t-ratio of ($0.95 - 1.15$) is $-0.92$, which again is not significant. The same happens for the conditional mean equations of the two factors: while the constants are estimated with good precision, the two slopes display a sizeable downward bias, the means being $0.93$ and $0.70$ instead of $1.2$ and $0.8$; in both cases the standard deviations are sufficiently high to make the bias not statistically significant. Both intercepts $c(1)$ and $c(2)$ have an upward bias, slightly higher for the first factor than for the second; again, the biases are not significant. In all three cases the parameters of the conditional variance equations are estimated quite precisely. Results are not different when the number of replications is increased to $1000$ (see Table 8). As regards the variability across replications, the shapes of the distributions of the coefficients reported again in Figure 7 (top panel) appear to be rather gaussian for the intercepts of the conditional mean equations (i.e. $f' \cdot c$, $c(1)$,

$c(2)$) and for the coefficients $\alpha$ in the conditional variance equations. The estimates of the risk aversion, the conditional variance factor loadings and the coefficients $\beta$ in the conditional variance equation are strongly negatively skewed. In any case the bias is not statistically relevant, so that the estimates reported throughout the paper can be regarded as sufficiently reliable. The picture changes greatly, especially for efficiency, when the persistence of the variance generating process drops to $0.64$, a value obtained as $(\alpha + \beta) = 0.13 + 0.51$ (Table 8 and bottom panel of Figure 7). This suggests that when the conditional variance is rather noisy, i.e. the relative importance of the shocks ($\varepsilon^2_{t-j}$) tends to dominate the importance of the signal ($\sigma^2_{t-j}$), then the properties of garch as volatility filters are significantly reduced; however, this situation is very far from the features of the excess returns series employed in the paper.

## 4. Conclusions

This paper has provided estimates of the equity premium in the US employing a long time series of excess returns, spanning 75 years of monthly observations. The ex-ante values of the equity premium is extracted from conditional versions of both a capm and an intertemporal capm model, which find empirical counterparts in a univariate and a bivariate garch-in-mean process. The bivariate setup has been employed to estimate the premia required on two additional returns, which represent the factor portfolios of the Fama and French model. These two premia, together with the equity premium, are employed to estimate firm-specific premia, i.e. the premia required on portfolios composed by firms of different size and performance. The equity premium, as estimated within the intertemporal capm specification, has a mean of 6.5 percent between July 1926 and October 2001, reflecting one of 4.15 per cent in periods of expansion and of 9.93 in recessions; the variability of the two series is also very different in the two stages of the business cycle, 3.1 in expansions, 10 in recessions. Time-varying standard errors for the estimated equity premium are obtained by simulating the model and show, among other things, that a 2 percent level can be regarded as a physiological value of the premium. The widest ranges occurred in coincidence with major destabilizing events, most of them recessions (the Great Depression, the Second World War, the oil shocks). A few were time-specific (the Black Friday crash of the US stock market on October 1987 and the collapse of the LTCM in October 1998). The widening confidence interval observed since the beginning of 1996 is peculiar since it happened in a period of sustained economic growth. Throughout the sample, the equity premium is not influenced by the level of the short-term rate, while a positive relation is found in periods of recession; the premium has a positive correlation with the long-term yield and with the default premium, measured by the differential between aaa and baa-rated bonds. The dispersion of the risk premia across firms of different size (market value) and performance (market-to-book equity ratio) is also influenced by business cycle developments. The size factor dominates over the performance factor as a perceived source of risk after 1973. Estimates substantially agree with those reported in a number of recent papers.

# Tables and figures

Table 1

## Univariate garch-in-mean models for the ex-ante risk premium [a]

| | $\mu$ | $\lambda$ | $\omega$ | $\alpha$ | $\beta$ | $E[\mu+\lambda\sigma^2]$ | $st.dev$ |
|---|---|---|---|---|---|---|---|
| market | -0.000152 | 1.45 | $1.02 \cdot 10^{-4}$ | 0.111 | 0.853 | 5.03 | 6.11 |
| size | 0.000770 | 9.11 | $2.70 \cdot 10^{-5}$ | 0.019 | 0.836 | 4.10 | 2.57 |
| performance | 0.000138 | 4.81 | $2.59 \cdot 10^{-5}$ | 0.047 | 0.921 | 1.11 | 0.65 |

$^a$: The risk premium on the three factors is estimated through the following garch-in-mean(1,1) model:

$$r_{i,t} = \mu + \lambda \cdot \sigma^2_{i,t} + \varepsilon_{i,t};\ \varepsilon_{i,t}|\Lambda_{t-1} \sim N(0, \sigma^2_{i,t})$$
$$\sigma^2_{it} = \omega + \alpha \cdot \varepsilon^2_{i,t-1} + \beta \cdot \sigma^2_{i,t-1}.$$

where $r_{i,t}$ is the ex-post excess return on factor i, $\sigma^2_{i,t}$ its conditional variance and $\Lambda_t$ the information set dated t. The expected value of the risk premium, $E[\mu + \lambda\sigma^2]$, and its standard deviation are obtained from the time series (904 monthly observations) of the estimated premium; they are expressed as percent per year. Estimates are based on quasi-maximum likelihood; significance of the parameters is measured with the Bollerslev-Wooldridge (1992) consistent covariance matrix.

Table 2

## Bivariate garch-in-mean models for the ex-ante risk premium[a]

| | $\mu_1$ | $\lambda$ | $\phi$ | $\omega_1$ | $\alpha_1$ | $\beta_1$ | $\mu_2$ | $\omega_2$ | $\alpha_2$ | $\beta_2$ | $\rho$ | $\mu$ | $\sigma$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| market | 0.00082 | 1.31 | $-1.94$ | $1.85 \cdot 10^{-5}$ | 0.095 | 0.70 | $-0.00014$ | $6.69 \cdot 10^{-6}$ | 0.063 | 0.793 | $-0.277$ | $6.50^b$ | 6.55 |
| size | 0.00034 | 0.34 | $-14.66$ | $1.99 \cdot 10^{-5}$ | 0.090 | 0.74 | 0.000229 | $6.15 \cdot 10^{-6}$ | 0.092 | 0.795 | $-0.423$ | 4.76 | 3.41 |
| perf. | $-0.00017$ | 0.21 | $-36.46$ | $3.86 \cdot 10^{-5}$ | 0.061 | 0.74 | $-0.00060$ | $7.37 \cdot 10^{-6}$ | 0.084 | 0.792 | $-0.386$ | 1.78 | 2.47 |

$^a$: The risk premium on the three factors is estimated through the following bivariate garch-in-mean(1,1) model:

$$r_{i,t} = \mu_1 + \lambda \cdot \sigma^2_{i,t} + \phi \cdot \sigma_{ij,t} + \varepsilon_{i,t};\ \varepsilon_{i,t}|\Lambda_{t-1} \sim N(0, \sigma^2_{i,t})$$
$$r_{j,t} = log(r_{B,t}/r_{B,t-1}) = \mu_2$$
$$\sigma^2_{i,t} = \omega_1 + \alpha_1 \cdot \varepsilon^2_{i,t-1} + \beta_1 \cdot \sigma^2_{i,t-1}$$
$$\sigma^2_{j,t} = \omega_2 + \alpha_2 \cdot \varepsilon^2_{j,t-1} + \beta_2 \cdot \sigma^2_{j,t-1}$$
$$\sigma_{ij,t} = \rho \cdot \sigma_{i,t} \cdot \sigma_{j,t}$$

where $r_{i,t}$ is the ex-post excess return on factor i, $\sigma^2_{i,t}$ its conditional variance and $\Lambda_t$ the information set dated t. The expected value of the risk premium, $\bar{\mu} = E(\mu_1 + \lambda\sigma^2_i + \phi\sigma_{ij,t})$, and its standard deviation, $\bar{\sigma}$, are obtained from the time series (904 monthly observations) of the estimated premium; they are expressed as percent per year. The three time series of risk premia are reported in Figure 1. Estimates are based on quasi-maximum likelihood; significance of the parameters is measured with the Bollerslev-Wooldridge (1992) consistent covariance matrix.

Table 3

**Tests on portfolios returns**

US, monthly data: January 1926 - October 2001

| quintiles: size,perf. | skewness | kurtosis-3 | T$\cdot R^2$ |
|---|---|---|---|
| 1,1 | 2.84 | 1.41 | 76.99 |
| 1,2 | 3.99 | 2.47 | 70.03 |
| 1,3 | -0.24 | 1.88 | 77.73 |
| 1,4 | -0.56 | 4.50 | 70.73 |
| 1,5 | -0.81 | 7.03 | 43.59 |
| 2,1 | -0.24 | 5.34 | 66.29 |
| 2,2 | -0.63 | 10.98 | 50.29 |
| 2,3 | -0.38 | 5.97 | 56.29 |
| 2,4 | -0.49 | 5.84 | 62.77 |
| 2,5 | -0.19 | 5.10 | 54.12 |
| 3,1 | 0.00 | 1.39 | 82.10 |
| 3,2 | -0.29 | 2.61 | 62.26 |
| 3,3 | -0.41 | 3.45 | 82.13 |
| 3,4 | -0.33 | 2.97 | 80.05 |
| 3,5 | -0.51 | 5.80 | 54.80 |
| 4,1 | -0.81 | 8.88 | 37.51 |
| 4,2 | -0.45 | 8.54 | 36.98 |
| 4,3 | -0.64 | 4.11 | 69.96 |
| 4,4 | -0.58 | 4.66 | 85.16 |
| 4,5 | -0.17 | 3.50 | 73.03 |
| 5,1 | -0.29 | 1.65 | 83.23 |
| 5,2 | -0.02 | 1.22 | 79.46 |
| 5,3 | -0.32 | 2.61 | 67.04 |
| 5,4 | -0.95 | 5.49 | 56.35 |
| 5,5 | -0.86 | 2.86 | 110.19 |

The $TR^2$ is the product of the sample size (T) times the coefficient of determination ($R^2$) of a regression of the squared demeaned portfolio returns on a constant and their five lags. It is asymptotically distributed as a chi-squared with five degrees of freedom.

Table 4

## Control of the estimated garch models

| quintiles: size,perf. | skewness[a] | skewness[b] | kurtosis[a] | kurtosis[b] | volatility dynamics[c] |
|---|---|---|---|---|---|
| 1,1 | 2.71 | 0.92 | 28.06 | 7.69 | 0.72 |
| 1,2 | 3.88 | 0.29 | 49.04 | 3.97 | 0.29 |
| 1,3 | 1.98 | 0.29 | 16.24 | 4.36 | 0.90 |
| 1,4 | 2.63 | 0.05 | 28.28 | 3.33 | 0.36 |
| 1,5 | 3.04 | 0.80 | 28.69 | 8.32 | 0.07 |
| 2,1 | 0.35 | -0.29 | 5.25 | 2.50 | 0.29 |
| 2,2 | 1.70 | -0.36 | 18.99 | 2.96 | 0.17 |
| 2,3 | 2.14 | -0.39 | 22.73 | 3.03 | 0.18 |
| 2,4 | 1.67 | -0.27 | 17.58 | 2.36 | 0.76 |
| 2,5 | 1.55 | -0.03 | 15.01 | 3.19 | 0.93 |
| 3,1 | 0.97 | -0.49 | 10.09 | 2.05 | 0.28 |
| 3,2 | 0.27 | -0.26 | 6.87 | 2.56 | 0.51 |
| 3,3 | 0.90 | -0.43 | 12.42 | 2.56 | 0.99 |
| 3,4 | 1.15 | -0.37 | 13.24 | 1.87 | 0.56 |
| 3,5 | 1.81 | -0.06 | 18.37 | 2.76 | 1.32 |
| 4,1 | -0.27 | -0.27 | 3.71 | 1.92 | 0.12 |
| 4,2 | 0.98 | -0.50 | 13.08 | 3.10 | 0.43 |
| 4,3 | 1.00 | -0.50 | 14.52 | 2.15 | 0.21 |
| 4,4 | 1.87 | -0.11 | 20.81 | 2.14 | 1.18 |
| 4,5 | 2.00 | 0.06 | 20.85 | 2.96 | 1.37 |
| 5,1 | -0.07 | -0.05 | 5.38 | 1.70 | 0.42 |
| 5,2 | -0.10 | -0.09 | 5.29 | 2.03 | 0.51 |
| 5,3 | 0.72 | -0.28 | 13.32 | 2.46 | 0.63 |
| 5,4 | 1.73 | 0.06 | 20.56 | 2.19 | 1.55 |
| 5,5 | -3.92 | -0.72 | 31.42 | 4.59 | 2.12 |

The table reports the skewness and the kurtosis of the residuals of the conditional mean equation (9) standardized with the unconditional, case [a], and the conditional, case [b], standard deviation (from 10) in turn. The correct specification of the volatility dynamics is tested by checking that the coefficients $c_2 - c_5$ in the regression

$$\left[\frac{u_t}{\sigma_t}\right]^2 = c_1 + c_2 \cdot u_{t-2}^2 + c_3 \cdot u_{t-3}^2 + \ldots + c_5 \cdot u_{t-5}^2 + \varepsilon_t$$

are nil, where $u_t$ is the residual from the mean equation (9) of the factor garch model and $\sigma_t$ is the estimated conditional standard deviation of (10). It is asymptotically F-distributed with 4 and 904 degrees of freedom.

## Cross-sectional regression of expected returns on betas

unconditional FF model: July 1926 - October 2001

| | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ |
|---|---|---|---|---|
| mean[1] | -0.0016 | 0.977 | 0.004 | 0.0036 |
| t-ratio | 0.29 | 3.82 | 0.86 | 3.02 |

1. The coefficient are obtained by first estimating the Fama-French model for each of the 25 quintile portfolios. Then the cross-sectional returns are regressed on the cross-sectional values of $\alpha, \beta, \gamma, \delta$ for each of the 904 months. The time series thus obtained of length 904 for $\alpha, \beta, \gamma$ and $\delta$ are regressed on a constant and this constant is the figure reported in the table. The coefficient $\beta$ refers to the market premium, $\gamma$ to the size factor premium, $\delta$ to the performance factor premium.

conditional FF model: July 1926 - October 2001

| | $\alpha$ | $\beta$ | $\gamma$ | $\delta$ |
|---|---|---|---|---|
| mean[2] | -0.0071 | 0.57 | 0.43 | 0.22 |
| t-ratio | -1.34 | 6.76 | 3.82 | 1.90 |

2. The coefficient are obtained by first estimating the following factor garch-in-mean model for the 25 cross-sectional returns:

$$r_{it} = \alpha + \beta \cdot \pi_{m,t} + \gamma \cdot \pi_{SMB,t} + \delta \cdot \pi_{HML,t} + \varepsilon_{i,t}$$
$$\sigma^2_{i,t} = \beta^2 \cdot \sigma^2_{m,t} + \gamma^2 \cdot \sigma^2_{SMB,t} + \delta^2 \cdot \sigma^2_{HML,t}$$

and then collecting the 25 values of $\alpha, \beta, \gamma$ and $\delta$ and regressing them on a constant; the value of such a constant and its Student's-t (for the null that it equals zero) is reported in the table.

**Relation between the risk premium and some economic variables**

| | Expansion[1] | Recession[1] | x = Short-rate[2] | x = Long-rate[2] | x = Default premium[2] |
|---|---|---|---|---|---|
| market[3] | $3.32$ | $8.74$ | $6.62 - 0.43 \cdot x$ | $5.98 - 0.17 \cdot x$ | $0.86 + 0.82 \cdot x$ |
| market[4] | $4.15$ | $9.93$ | $6.69 - 0.05^5 \cdot x$ | $5.02 + 0.25 \cdot x$ | $1.30 + 4.16 \cdot x$ |
| smb[4] | $4.47$ | $5.47$ | $4.22 + 0.14 \cdot x$ | $3.17 + 0.27 \cdot x$ | $2.31 + 2.14 \cdot x$ |
| hml[4] | $1.45$ | $2.52$ | $1.76 + 0.00^5 \cdot x$ | $1.12 + 0.12 \cdot x$ | $-0.53 + 2.01 \cdot x$ |
| size: 1 | $1.42$ | $2.25$ | $1.61 + 0.02^5 \cdot x$ | $1.18 + 0.08 \cdot x$ | $0.19 + 1.29 \cdot x$ |
| size: 2 | $7.73$ | $12.31$ | $8.80 + 0.08^5 \cdot x$ | $6.48 + 0.44 \cdot x$ | $1.06 + 7.05 \cdot x$ |
| size: 3 | $4.94$ | $8.37$ | $5.88 + 0.03^5 \cdot x$ | $4.33 + 0.28 \cdot x$ | $0.19 + 5.06 \cdot x$ |
| size: 4 | $2.64$ | $4.35$ | $3.08 + 0.02^5 \cdot x$ | $2.23 + 0.15 \cdot x$ | $0.12 + 2.65 \cdot x$ |
| size 5: | $4.55$ | $9.66$ | $6.74 + 0.02^5 \cdot x$ | $4.86 + 0.32 \cdot x$ | $-0.30^5 + 2.22 \cdot x$ |
| market[6] | $4.22$ | $9.03$ | $6.31 + 0.02^5 \cdot x$ | $4.56 + 0.30 \cdot x$ | $-0.20^5 + 5.75 \cdot x$ |
| mtbv: 1 | $1.67$ | $2.55$ | $1.85 + 0.02^5 \cdot x$ | $1.35 + 0.10 \cdot x$ | $0.32 + 1.44 \cdot x$ |
| mtbv: 2 | $1.62$ | $2.56$ | $1.84 + 0.02^5 \cdot x$ | $1.34 + 0.10 \cdot x$ | $0.19 + 1.50 \cdot x$ |
| mtbv: 3 | $10.65$ | $17.47$ | $12.38 + 0.09^5 \cdot x$ | $9.12 + 0.60 \cdot x$ | $1.00 + 10.25 \cdot x$ |
| mtbv: 4 | $5.96$ | $10.12$ | $7.11 + 0.03^5 \cdot x$ | $5.21 + 0.34 \cdot x$ | $0.15 + 6.18 \cdot x$ |
| mtbv: 5 | $2.37$ | $4.19$ | $2.94 + 0.00^5 \cdot x$ | $2.06 + 0.15 \cdot x$ | $-0.40 + 2.91 \cdot x$ |

1. The definition of expansion and recession is based on the usual NBER chronology. It is a dummy variable, $I_t$, equalling zero in expansion and one in recession. The coefficient reported for the expansion period is the $\alpha$ from the following regression: $r_t=\alpha+\beta\cdot I$, while the coefficient reported for the recession period is $(\alpha+\beta)$. 2. The following regression has been run: $r_t=\alpha+\beta\cdot x_t+\varepsilon_t$. 3. The series of the market premium is estimated from a univariate garch-in-mean (1,1) process (see Table 1). 4. The time series of the market premium is estimated from a bivariate garch-in-mean process where the conditional market variance and the conditional covariance between the equity return and the stock return concur to determine the ex-ante equity premium (see Table 2). 5. Not significant at the 5 percent level. 6. This time series of the market premium is the market-value weighted average of the five size quintiles' risk premia.

Table 7

**Relation between risk premium and interest rates**

| | Short rate[1] | | Bond rate[1] | |
|---|---|---|---|---|
| | expansion | recession | expansion | recession |
| size: 1 | $0.00^5$ | $0.13$ | $0.06$ | $0.22$ |
| size: 2 | $-0.03^5$ | $0.72$ | $0.33$ | $1.19$ |
| size: 3 | $-0.06^5$ | $0.49$ | $0.19$ | $0.82$ |
| size: 4 | $-0.02^5$ | $0.27$ | $0.11$ | $0.43$ |
| size 5: | $-0.08^5$ | $0.64$ | $0.23$ | $0.98$ |
| market[6] | $-0.07^5$ | $0.59$ | $0.21$ | $0.91$ |
| mtbv: 1 | $0.00^5$ | $0.14$ | $0.08$ | $0.25$ |
| mtbv: 2 | $0.00^5$ | $0.15$ | $0.07$ | $0.25$ |
| mtbv: 3 | $0.00^5$ | $1.10$ | $0.44$ | $1.71$ |
| mtbv: 4 | $-0.07^5$ | $0.59$ | $0.24$ | $1.01$ |
| mtbv: 5 | $-0.04^5$ | $0.24$ | $0.11$ | $0.45$ |

1. The slope of the following regression is reported in the table: $r_t = \alpha + \beta \cdot x_t + \varepsilon_t$ ,where $r_t$ is the estimated risk premium according to the i-capm scheme (Table 2) and $x_t$ is the short rate or the bond rate.

Table 8

**Monte Carlo experiment on the factor Garch model**

mean values of the parameters[1]

| | $f' \cdot c$ | $c(1)$ | $c(2)$ | $\gamma$ | $\alpha$ | $\beta$ | $g(1)$ | $g(2)$ |
|---|---|---|---|---|---|---|---|---|
| $N$=200; $\alpha+\beta$=0.64 | 0.204 | 0.208 | 0.196 | 1.02 | 0.127 | 0.460 | 0.93 | 0.96 |
| $N$=1000;$\alpha+\beta$=0.64 | 0.213 | 0.220 | 0.206 | 0.97 | 0.126 | 0.479 | 0.95 | 0.68 |
| $N$=200; $\alpha+\beta$=0.94 | 0.233 | 0.239 | 0.216 | 0.927 | 0.138 | 0.790 | 0.928 | 0.696 |
| $N$=1000; $\alpha+\beta$=0.94 | 0.231 | 0.238 | 0.216 | 0.932 | 0.140 | 0.790 | 0.930 | 0.699 |
| $True\ values$ | 0.2 | 0.2 | 0.2 | 1.15 | 0.13 | 0.52;0.81 | 1.2 | 0.8 |

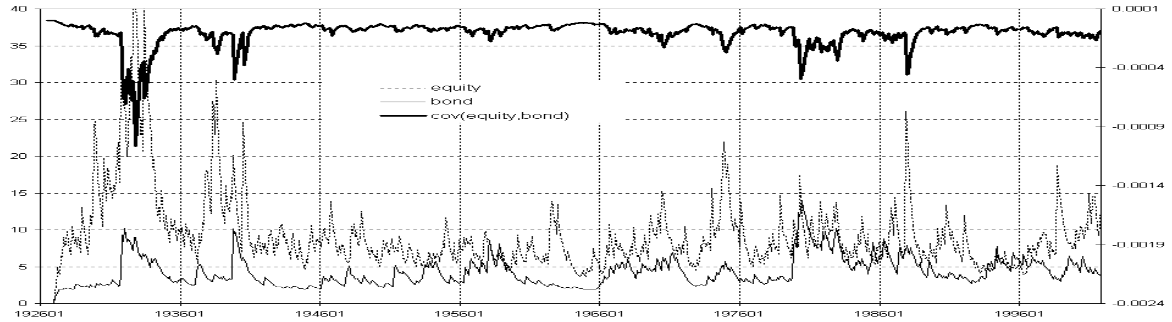
student's-t of the difference between the mean values and the true values of the parameters

| | $f' \cdot c$ | $c(1)$ | $c(2)$ | $\gamma$ | $\alpha$ | $\beta$ | $g(1)$ | $g(2)$ |
|---|---|---|---|---|---|---|---|---|
| $N$=200;$\alpha+\beta$=0.64 | 0.06 | 0.13 | 0.06 | $-0.05$ | $-0.06$ | $-0.24$ | $-0.12$ | 0.06 |
| $N$=1000;$\alpha+\beta$=0.64 | 0.18 | 0.32 | 0.05 | $-0.18$ | $-0.08$ | $-0.14$ | $-0.32$ | $-0.49$ |
| $N$=200;$\alpha+\beta$=0.94 | 0.96 | 0.96 | 0.55 | $-0.92$ | 0.26 | $-0.30$ | $-0.95$ | $-0.50$ |
| $N$=1000;$\alpha+\beta$=0.94 | 0.93 | 0.95 | 0.55 | $-0.92$ | 0.34 | $-0.37$ | $-0.95$ | $-0.55$ |

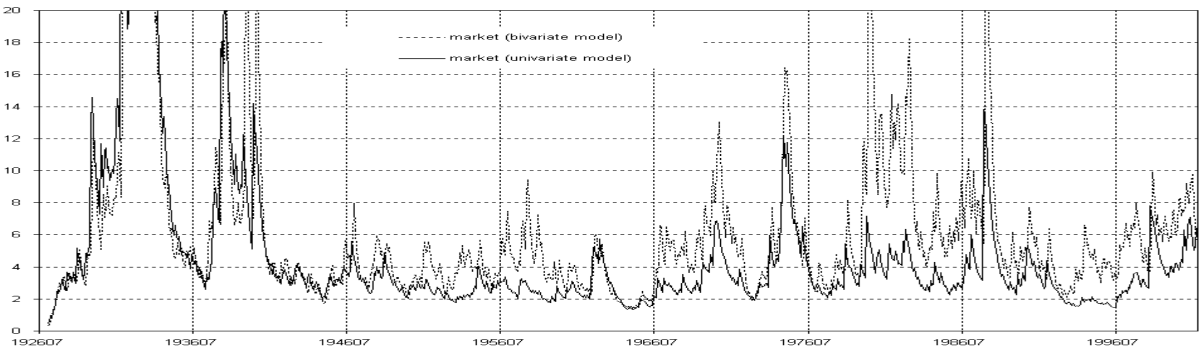
1. The parameters belong to the simulation scheme reported in sub-section 3.5. N is the number of replications, $\alpha+\beta$ the persistence of the conditional variance generating process.

Figure 1

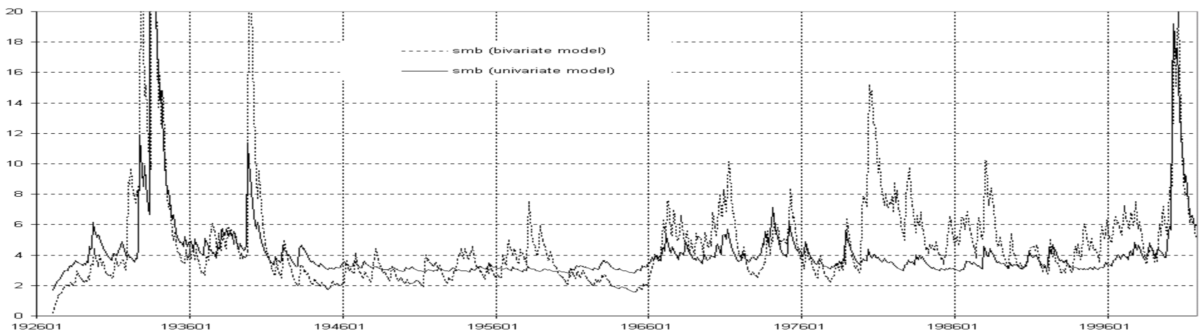
**Standard deviation of stocks and aaa-rated bonds (% per year) and conditional covariance**



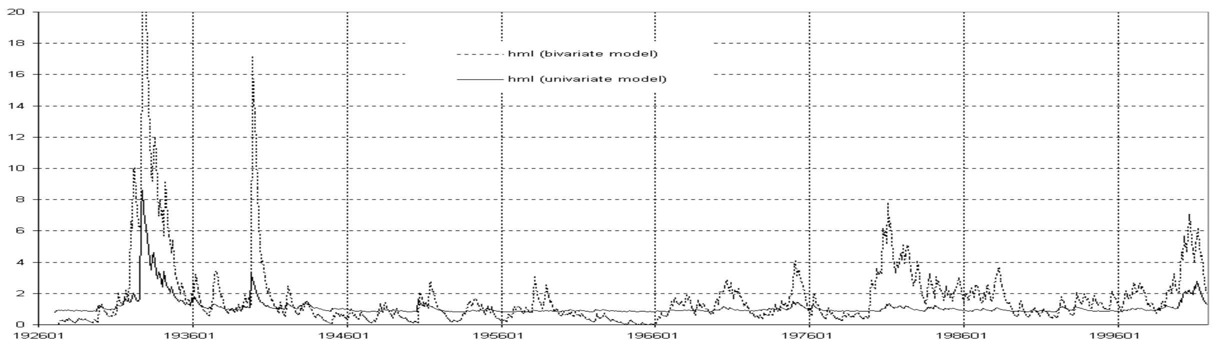

**Risk premium on the market factor**




**Risk premium on the smb factor**




**Risk premium on the hml factor**

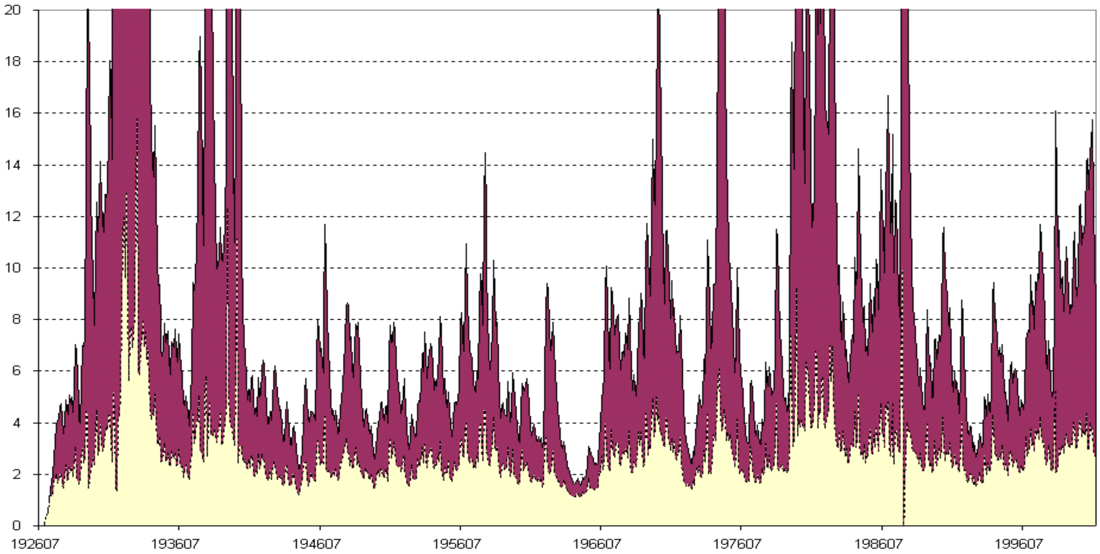



Conditional covariances and risk premia come from the estimation of the bivariate garch-in-mean model (4). The premia reported in the second, third and fourth panel, are compared to the estimate derived from model (3). Data are monthly and expressed as percent per year.

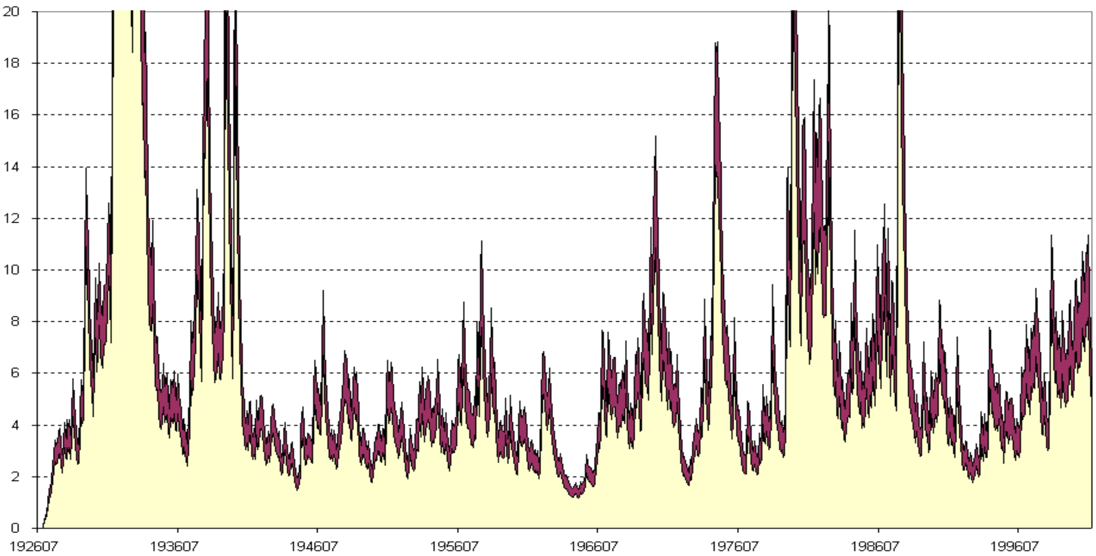
Figure 2

**Monte Carlo - based 95% confidence intervals for the equity premium**[a]

due to random changes in the conditional distribution of the errors



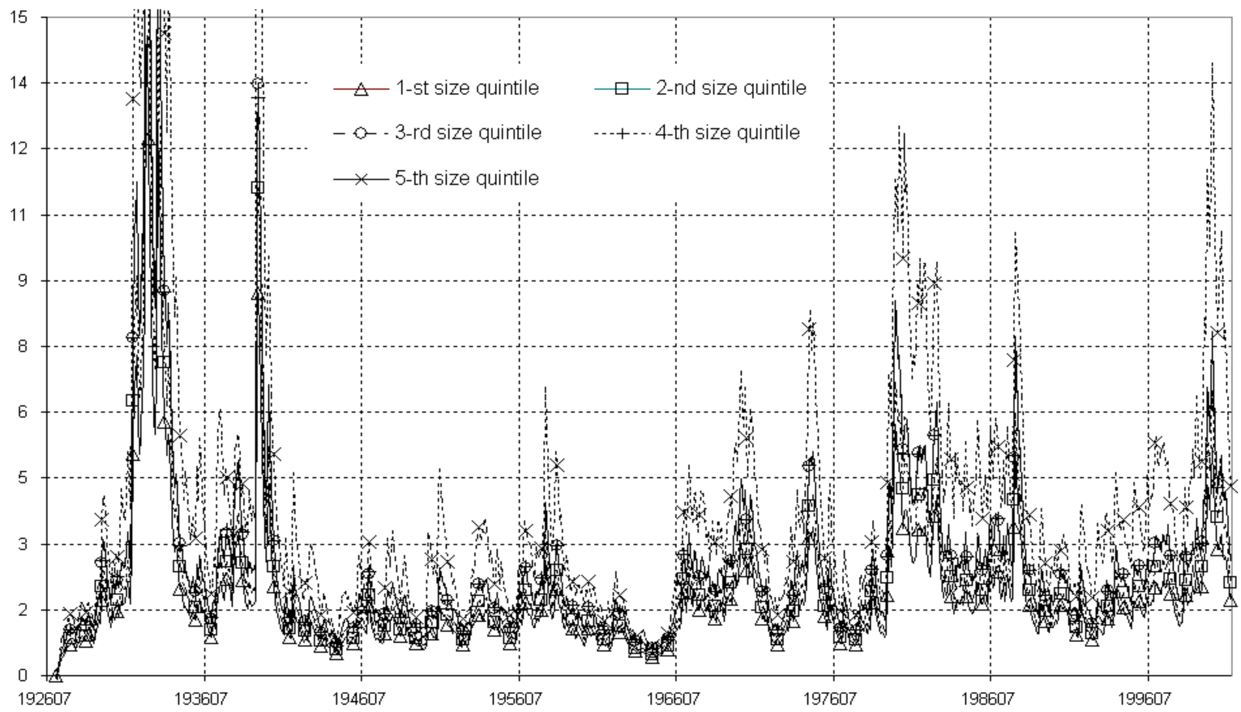

due to random changes in the values of the estimated parameters



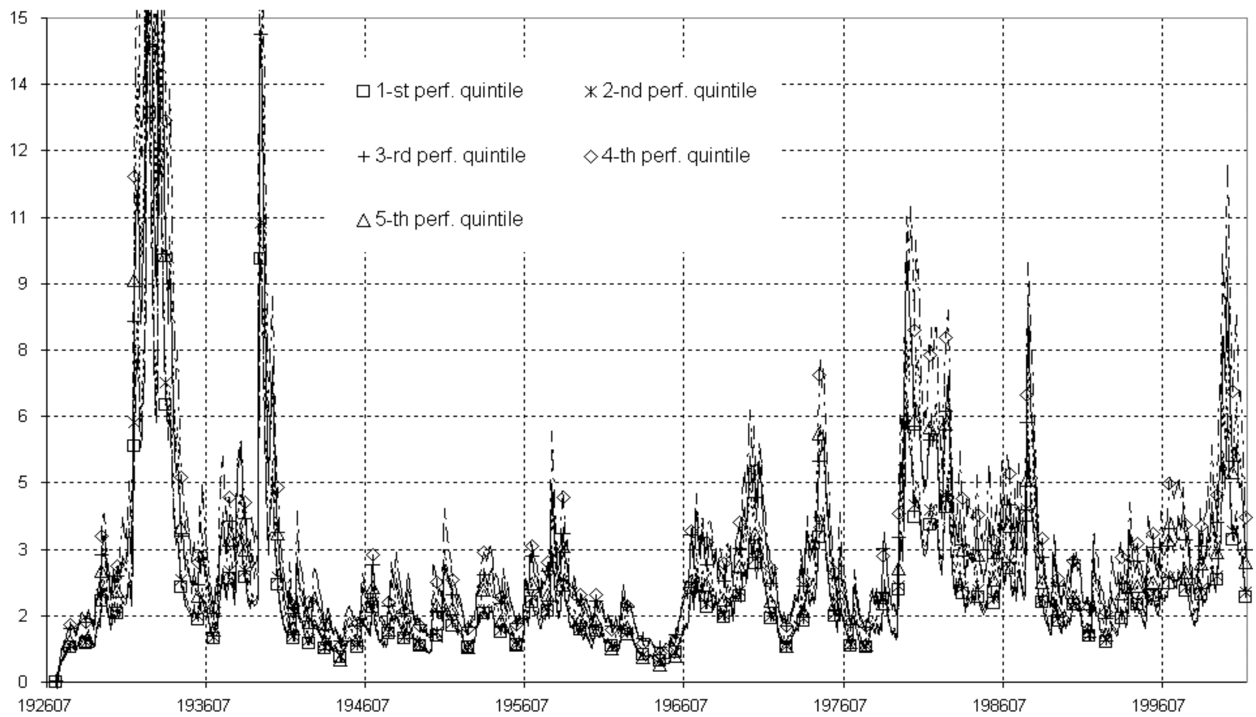

a: The black area in the two panels of this figure is the 95 percent confidence interval for the monthly values of the equity premium estimated from July 1926 to October 2001. The estimate of the risk premium comes from an intertemporal capm scheme (Table 2). The confidence interval in the first panel is calculated as the risk premium plus or less two standard deviations. The standard deviations is obtained by simulating 5000 times the intertemporal capm (4), drawing the time-t error from its time-(t-1) conditional distribution. The confidence interval in the second panel is calculated by simulating 5000 times the intertemporal capm scheme (4) drawing the parameters from their unconditional distribution.

Figure 3

**Risk premia by size quintile (monthly figures expressed as percent per year)**



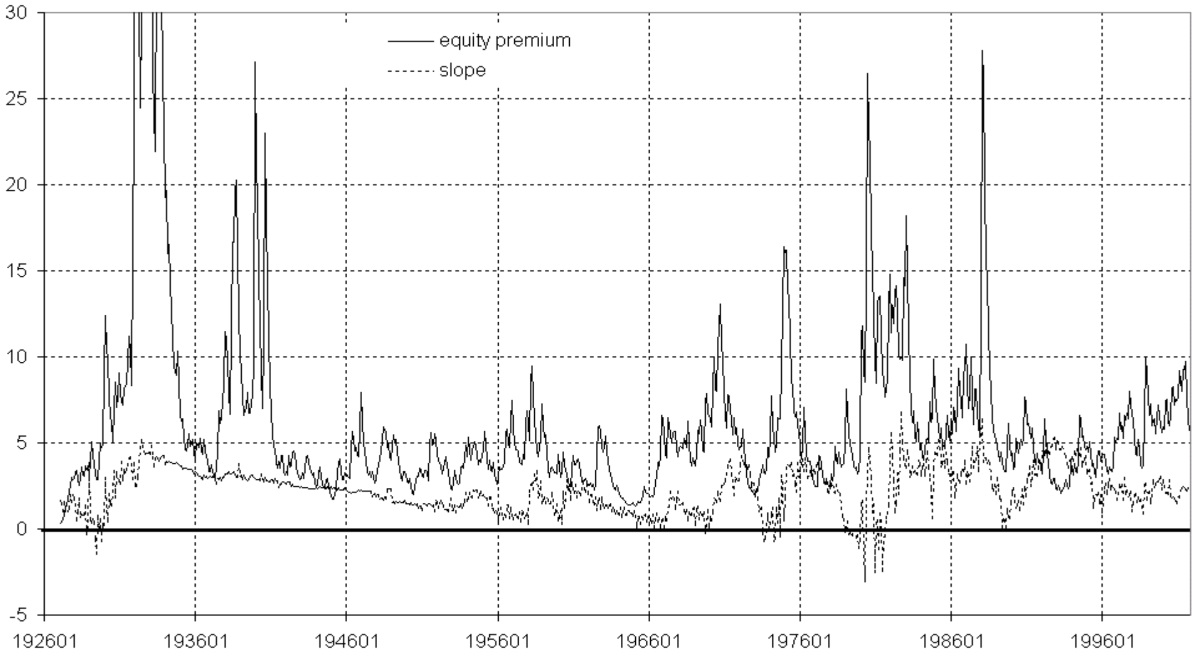

**Risk premia by performance quintile (monthly figures expressed as percent per year)**




Risk premia for quintile portfolios are estimated by means of the factor-garch-in-mean model (9)-(10). Size is measured by the market value of a firm; performance by the ratio of its market equity to its book equity.

Figure 4

**Equity premium and slope of the term structure (percent per year)**




**Relation between the slope of the term structure and the equity premium**

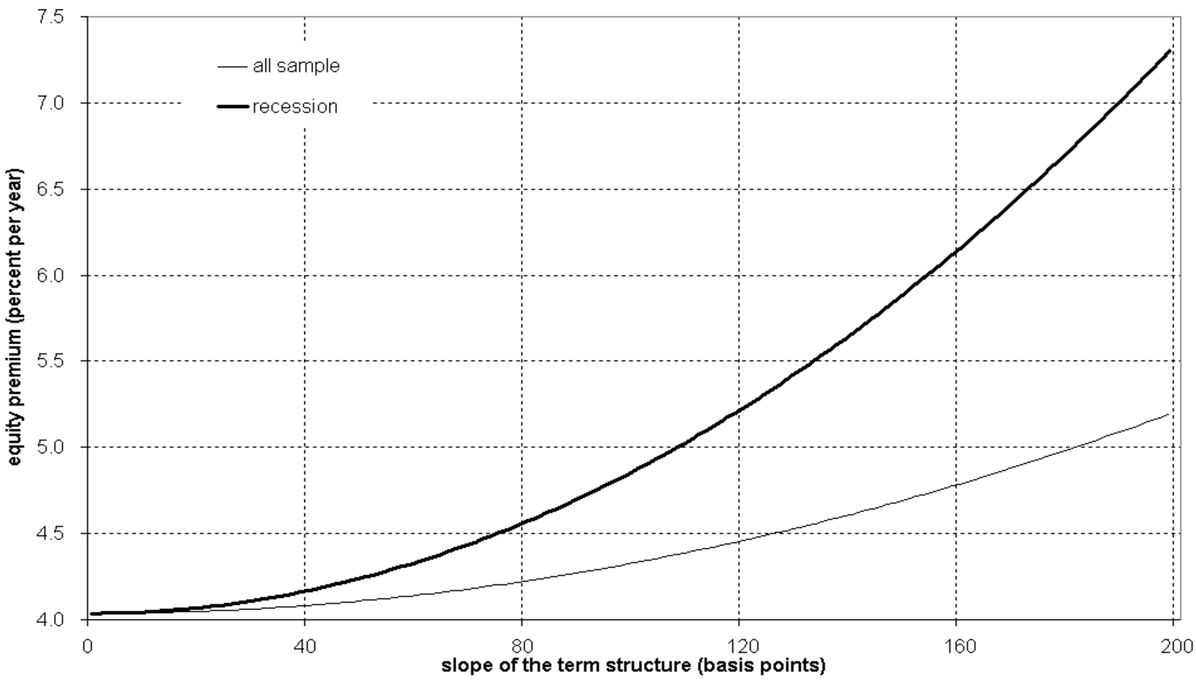
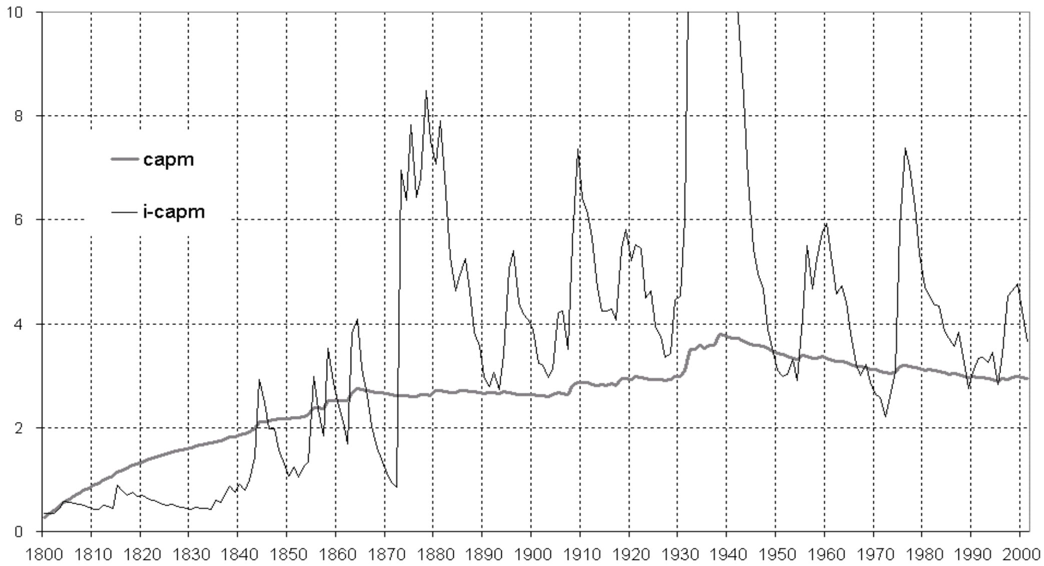


Figure 5

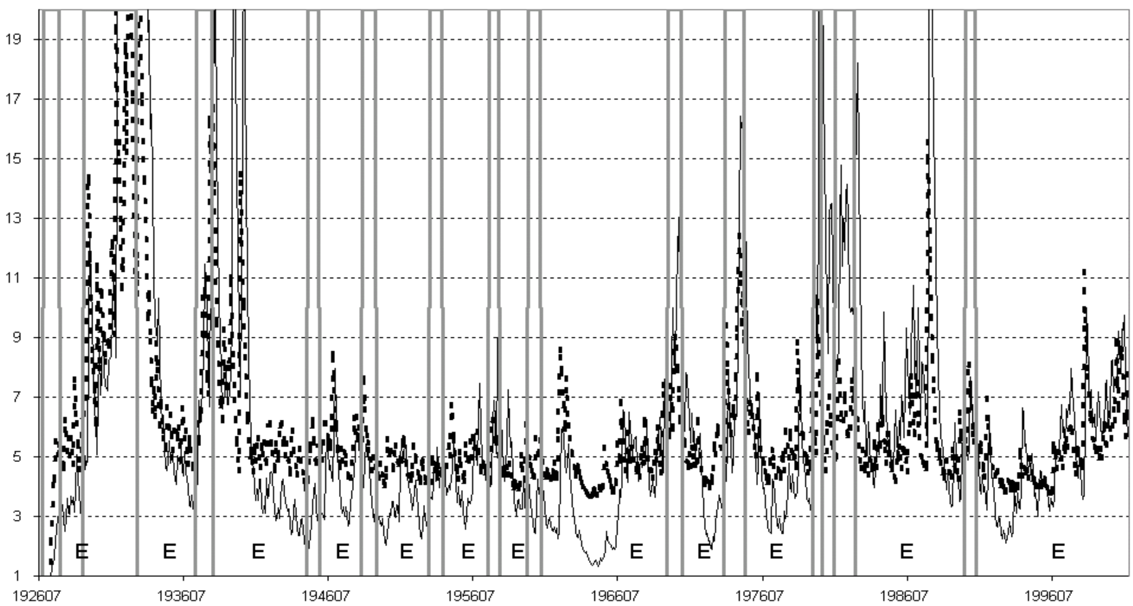
**Long term estimate of the equity premium (annual data; percent per year)**




These estimates of the equity premium come from the capm scheme (3) and the i-capm (4). Estimation relies on the values of the equity index observed monthly at the end of January of each year between 1800 and 2001; the bond yield is available from 1831 only. In the estimation of (4) the covariance between the equity and the bond return is set equal to zero between 1800 and 1831.

Figure 6

**Equity premium (monthly data; percent per year)**

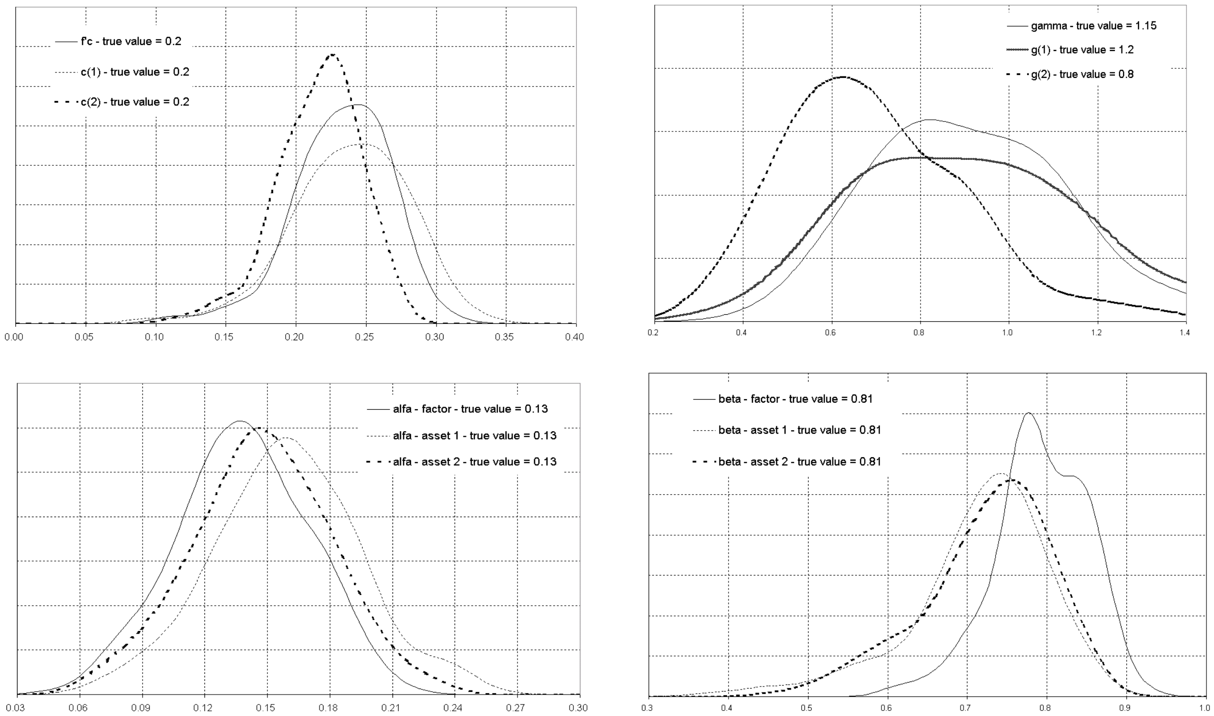



This figure compares the equity premium derived from model (4) - continuous line - with the premium derived from model (12) - dotted and bold line. Vertical lines identify peaks and troughs of the US business cycle as defined by the NBER. Areas market with 'E' are periods of expansion.

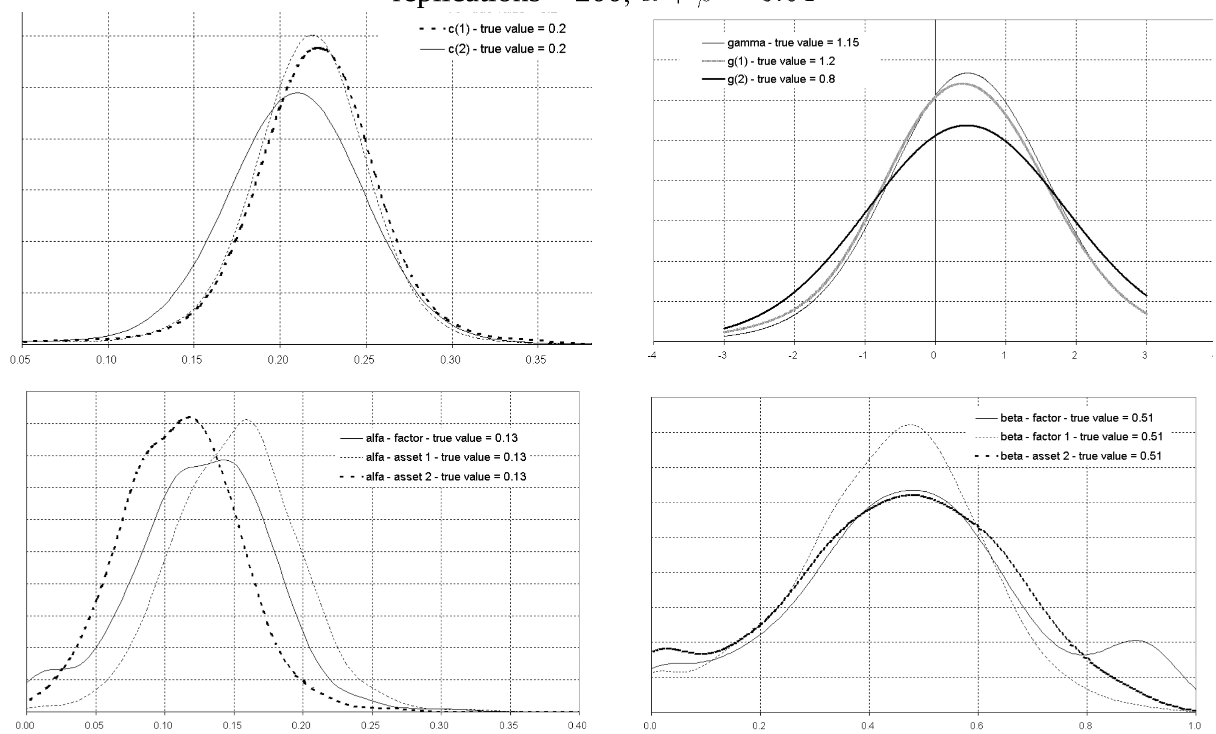
Figure 7

**Monte carlo distribution of the parameters of a bivariate one-factor-garch-in-mean model**

replications = 200; $\alpha + \beta = 0.94$




replications = 200; $\alpha + \beta = 0.64$




The distributions of the parameters are recovered simulating the bivariate one-factor garch-in-mean model of sub-section 3.5. For the distributions reported in this figure, the number of replications was 200; the persistence of the conditional variance generating process amounts to 0.94, in the first case, to 0.64 in the second. The length of the sample is 1000.

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