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Introduction to social choice and welfare

by Kotaro Suzumura

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INTRODUCTION TO SOCIAL CHOICE AND WELFARE

by Kotaro Suzumura *

Abstract

Social choice theory is concerned with the evaluation of alternative methods of collective decision-making, as well as with the logical foundations of welfare economics. In turn, welfare economics is concerned with the critical scrutiny of the performance of actual and/or imaginary economic systems, as well as with the critique, design and implementation of alternative economic policies. The Handbook of Social Choice and Welfare, which is edited by Kenneth Arrow, Amartya Sen and Kotaro Suzumura, presents, in two volumes, essays on past and on-going work in social choice theory and welfare economics. This paper is written as an extensive introduction to the Handbook with the purpose of placing the broad issues examined in the two volumes in better perspective, discussing the historical background of social choice theory, the vistas opened by Arrow’s Social Choice and Individual Values, the famous “socialist planning” controversy, and the theoretical and practical significance of social choice theory.

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* Institute of Economic Research Hitotsubashi University, Tokyo, Japan.
1. Historical Background

Social choice theory is concerned with the evaluation of alternative methods of collective decision-making, as well as with the logical foundations of welfare economics. In turn, welfare economics is concerned with the critical scrutiny of the performance of actual and/or imaginary economic systems, as well as with the critique, design and implementation of alternative economic policies. This being the case, it goes without saying that the origin of social choice theory can be traced back all the way to antiquity. Indeed, as soon as multiple individuals are involved in making decisions for their common cause, one or other method of collective decision-making cannot but be invoked. As a reflection of this obvious fact, there are numerous examples in classic writings on the use and usefulness of alternative methods of collective decision-making. Suffice it to quote Aristotle in ancient Greece, and Kautilya in ancient India; they both lived in the fourth century B.C. and explored several possibilities of collective decision-making in their books entitled, respectively, *Politics* and *Economics*.¹

Likewise, as soon as any collective body designs and implements an economic mechanism and/or an economic policy, paying proper attention to the costs and benefits accruing to its constituent members, one or more social welfare judgements cannot be avoided. In this sense, Joseph Schumpeter (1954, p.1069) was certainly right when he emphasized “the hallowed antiquity of welfare economics.” He observed that “a large part of the work of Carafa and his successors as well as of the work of the scholastic doctors and their successors was welfare economics. We also know that the welfare point of view was much in evidence in the eighteenth century .... For Bentham and the English utilitarians generally this point of view was, of course, an essential element of their creed. Hence, the positive spirit of Ricardian economics notwithstanding, we find it also in the English ‘classics,’ particularly in J. S. Mill. So far as this goes, modern welfare economists merely revive the Benthamite tradition.” It was in similar vein that Paul Samuelson (1947, p.203) began his famous Chapter VIII on Welfare Economics in *Foundations of Economic Analysis* with the following remark: “Beginning as it did in the writings of philosophers, teleologians, pamphleteers, special pleaders, and reformers, economics has always been concerned with problems of public policy and welfare.”

Without contradicting these authoritative verdicts on the long historical background of social choice theory, we may nevertheless claim that the instrumental concern with concrete methods of collective decision-making is one thing, and theoretical investigation into their logical performance is another thing altogether. The former concern may be as old as the origin of human society, but the latter development seems to be of more recent origin. Indeed, it seems fair to say that the real origin of the collective decision-making side of the coin can be attributed to the pioneering contributions by two eminent French precursors around the time of French revolution, viz. Marie-Jean de Condorcet, and Jean-Charles de Borda.² It was in the intellec-
tual atmosphere of the European Enlightenment during the eighteenth century, with its conspicuous concern with human rights and its reasoned design and implementation of rational social order, that Condorcet (1785) addressed the mathematical discipline of collective decision-making in terms of simple majority voting and related procedures. He discovered the paradox of voting, or the Condorcet paradox, to the effect that the method of pairwise simple majority voting may yield a social preference cycle --- a social alternative $A$ defeating another alternative $B$ by a simple majority, $B$ defeating the third alternative $C$ again by a simple majority, and $C$ in its turn defeating $A$ by a simple majority. This paradox sent an unambiguous signal that the logical performance of voting and related procedures for collective decision-making must be the subject of theoretical scrutiny. One of the logical implications of the Condorcet paradox is that, once a simple majority cycle occurs in the set of social alternatives $S = \{A, B, C\}$, there exists no Condorcet winner --- a feasible alternative which is undefeated by any other feasible alternative --- thereby excluding the possibility of basing social choice on the seemingly democratic method of collective decision-making. It is worthwhile to recollect in passing that Condorcet’s first extended illustration of the paradox of voting was taken from voting on economic policy. Indeed, the three policy alternatives were:

$A$ = any restriction placed on commerce is an injustice;
$B$ = only those restrictions placed through general laws can be just;
$C$ = restrictions placed by particular orders can be just.

Condorcet’s contribution seems to have been, at least partly and indirectly, inspired by an earlier work by Borda (1781), who proposed what came to be known as the Borda method of rank-order decision-making. For each voter, this method assigns a score of zero to the last ranked alternative, a score of one to the penultimate alternative, and so on all the way up to the top ranked alternative, which receives a score of $n - 1$ when there are $n$ alternatives altogether. These individual scores are added for each candidate over all voters, and the candidate which earned the largest sum-total becomes the overall winner in the contest. According to Duncan Black (1958, p.180), “[s]oon after hearing Borda’s paper in 1794 the [French] Academy [of Sciences] adopted his method in elections to its membership. It remained in use until 1800, when it was attacked by a new member and was modified soon afterwards. The new member was Napoleon Bonaparte.”

The same rank-order voting procedure was obtained from slightly different premises by Pierre-Simon Laplace (1812). Laplace also acutely observed an obstacle to the use of this procedure to the effect that “its working might be frustrated by electors placing the strongest opponents to their favourite candidates at the foot of their list. This would give a great advan-
tage to candidates of mediocre merit, for while getting few top places they would also get few lowest places [Black (1958, p.182)].” As a matter of fact, the same difficulty was confronted by Borda himself, who, when his procedure was opposed precisely for this reason of strategic vulnerability, had retorted by saying that his scheme is “only intended for honest men [Black (1958, p.182)].” This episode seems to show us unambiguously that the apprehension about the strategic manipulability of voting schemes existed from the formative era of this side of social choice theory.

There was intermittent exploratory work on voting schemes in the nineteenth century, most notably by Charles Lutwidge Dodgson (1873; 1874; 1876), who is better known by his literary pseudonym (Lewis Carroll). His works were circulated only within a limited Oxford circle, and was virtually unknown in the outside world until Black (1958, Appendix) made them widely accessible. Although ample circumstantial evidence [Black (1958, pp.192-194)] exists that Dodgson was acquainted neither with Borda (1781) nor with Condorcet (1785), he was clearly aware of the ubiquity of cyclical majorities as well as of the rank-order method of voting, most probably through Isaac Todhunter (1865, Chapters XVII and XIX), which every late Victorian scholar seems to have known about.7 His major logical concern was to devise a voting procedure which would enable him to choose the Condorcet winner if one exists, and to lexically supplement the simple majority voting if and when the Condorcet winner failed to exist. Black seems certainly right in concluding that “Dodgson had been caught in the grip of the theory of elections and committees and his understanding of the subject was second only to that of Condorcet [Black (1958, p.212)].”

In the last part of the nineteenth century and the first half of the twentieth century, some sporadic contributions such as those by Edward J. Nanson (1882) and Francis Galton (1907) notwithstanding, not much seems to have been done in the theory of collective decisions, the major breakthrough having been accomplished only in the late 1940s by Duncan Black (1948). He found a simple sufficient condition on the profile of voters’ preferences, to be called the assumption of single-peaked preferences, under which simple majority voting will be able to determine a social outcome, since there exists exactly one alternative which will receive a simple majority over any other alternative, provided that the number of voters is odd, and the Black assumption of single-peakedness is satisfied. This assumption has a simple geometric representation to the effect that the utility indicators for the voters’ preferences are such that the social alternatives can be represented by a one-dimensional variable and that each of the graphs of voters’ utility indicators has a single peak. Black’s theorem is the first possibility result of this nature in social choice theory, and it opened up the gate wide towards the modern development of the theory of voting.

Let us now turn to the welfare economics side of the coin. In this arena too, it seems fair to say that the real origin of the critical and systematic approach to the economic mechanism
design and policy evaluation belongs to the relatively recent past, and it may be safely attributed to the work of Jeremy Bentham (1789). He was a contemporary in England of Borda and Condorcet. It is worthwhile to recollect that Condorcet wrote enthusiastically of the new society of the United States that “the spectacle of a great people where the rights of man are respected is useful to all others ... It teaches us that these rights are everywhere the same.” He wrote as well as of the French Revolution that it had “opened up an immense scope to the hopes of the human species ... [T]his revolution is not in a government, it is in opinions and wills.” In sharp contrast, Bentham, a scholar in law and jurisprudence, was a stark critic of the concept of inviolable natural rights. Indeed, it was in his harsh comment on the French “Declaration of the Rights of Man and the Citizen”, which was embodied in the French Constitution of 1791, that he wrote the following famous passage: “[N]atural rights is simple nonsense: natural and imprescriptible rights, rhetorical nonsense, --- nonsense upon stilts [Bentham (1843, p.501)].” Instead of basing the economic policies on the concept of inviolable human rights, Bentham took recourse to the greatest happiness principle, so-called, to the effect that the ultimate criterion for judging the goodness of an economic mechanism and economic policy is that it can bring about the “greatest happiness of the greatest number.” In accordance with this utilitarian view on the goodness of a state of affairs, the legislator’s task is construed to arrange law and other social and economic institutions so that each person in pursuit of his own interest will be led to act so as to bring about the greatest happiness for all persons involved. This utilitarian basis of economic policies permeated the work of John Stuart Mill, Alfred Marshall, Francis Ysidro Edgeworth, and Henry Sidgwick, and it served as a natural basis for the synthesis of this tradition by the hands of Arthur Pigou (1920) in the early twentieth century.

Pigou’s so-called “old” welfare economics, being based on the Benthamite-utilitarian concept of economic welfare, presupposed that the utility of different individuals could be added to, or subtracted from, one another to define the social objective of total utility, viz. the greatest happiness. It was against this epistemological basis of Pigou’s “old” welfare economics that a harsh ordinalist criticism raged in the 1930s, kicked off by a famous essay by Lionel Robbins (1932/1935). Note, however, that Robbins’ criticism boils down to the categorical denial of the possibility of interpersonal comparisons of utility with interobserver validity; careful reading of Robbins (1935, pp.138-140, pp.149-150; 1938, pp.636-637; 1981, p.5) convinces us that he did not reject the possibility of making “subjective” interpersonal comparisons of utility, nor did he claim that economists should not make “subjective” interpersonal comparisons of their own. What he actually asserted is that “subjective” interpersonal comparisons cannot claim any “objective” interpersonal validity.

By the end of the 1930s, it became widely recognized that the foundations of Pigou’s
“old” welfare economics were hopelessly eroded, and new foundations for welfare economics had to be discovered on the basis of ordinal and interpersonally non-comparable utility information, and nothing else, in order to salvage something of substance from the vestige of Pigou’s theoretical superstructure. This is the same informational basis as that of the Borda-Condorcet theory of collective decision-making, which is a slightly ironical fact in view of the sharply contrasting background of the Borda-Condorcet theory on the methods of collective decision-making, on the one hand, and the Bentham-Pigou theory on the enhancement of social welfare, on the other.

The first ordinalist response to this plea was to go back to the ordinalist tradition pioneered by Vilfredo Pareto (1906; 1913), and invoke the seminal concept of the Pareto principle to the effect that a change from one social state to another social state can be judged as socially good if at least one individual is thereby made better off without making anybody else worse off in return. The characterization and implementation of the Pareto efficient resource allocation became the central exercise in this phase of the “new” welfare economics, which may be duly represented by John Hicks (1939). Note, however, that almost every economic policy cannot but favour some individuals at the cost of disfavouring some others, so that there would be almost no situation of real importance where the Pareto principle could claim relevance in isolation.

It was against this background that two distinct approaches were explored to rectify the unsatisfactory state of the post-Pigovian “new” welfare economics. The first approach was the introduction of compensation criteria by Nicholas Kaldor (1939), John Hicks (1940), Tibor Scitovsky (1941) and Paul Samuelson (1950), which endeavoured to expand the applicability of the Pareto principle by introducing hypothetical compensatory payments between gainers and losers from a change in economic policy. According to Johannes de V. Graaff (1957, pp.84-85), “[t]he compensation tests all spring from a desire to see what can be said about social welfare or ‘real national income’ ... without making interpersonal comparisons of well-being ... . They have a common origin in Pareto’s definition of an increase in social welfare ... but they are extended to situations in which some people are made worse off.”

The second approach was the introduction of the concept of a social welfare function by Abram Bergson (1938) and Paul Samuelson (1947, Chapter VIII), which is deeply rooted in the belief that the pursuit of the logical consequences of any value judgements, irrespective of whose ethical beliefs they represent, whether or not they are widely shared in the society, or how they are generated in the first place, is a legitimate task of welfare economics. The social welfare function is meant to be the formal way of encompassing such an ethical belief. It was in terms of this concept of a social welfare function that Bergson and Samuelson tried to separate what belongs to the area of ethics, about which economists qua scientists do not have any qualification to say anything objective whatsoever, from what belongs to the area of welfare
economics, about which economists as scientists have every reason as well as obliga-tion to say something of objective validity.\textsuperscript{13}

Between these two schools of the “new” welfare economics, the former compensationist school met serious logical difficulties. Even before the scaffolds for construction were removed from the construction site, serious logical contradictions in the form of either the lack of asymmetry, or the lack of transitivity, could be found in the social welfare judgements based on the Kaldor-Hicks-Scitovsky compensation criteria by Tibor Scitovsky (1941), William Gorman (1955) and many others, which fatally vitiated the credibility of the “new” welfare economics of the compensationist school. The verdict on the Samuelson compensation principle, which was defined in terms of a uniform outward shift of the utility possibility frontier, is quite different. Indeed, the Samuelson compensation principle can always generate transitive social welfare judgements, so that its logical performance \textit{in isolation} is impeccable. Nevertheless, it may still generate contradictory social welfare judgements \textit{in combination with} the Pareto principle.\textsuperscript{14} On the other hand, the second school of the “new” welfare economics, which is founded on the Bergson-Samuelson social welfare function, has been widely praised as the culmination of the ordinalist “scientific” approach to welfare economics.\textsuperscript{15}

Broadly speaking, this was the intellectual atmosphere surrounding social choice theory when Kenneth Arrow published his Ph. D. Dissertation, \textit{Social Choice and Individual Values}, in 1951. In view of its innovative nature as well as the revolutionary influence it exerted on the whole fields of social choice theory, it will be justifiable to devote the next section in its entirety to this work.

Quite apart from the Robbinsian criticism, which is epistemological in nature, there is a fundamental criticism of, and a proposal for a serious alternative to, the Benthamite utilitarianism by John Rawls (1962; 1963; 1971), which is focused directly on the ethical nature of the Benthamite outcome morality. According to Rawls (1971, p.22), the main idea of classical utilitarianism is that “society is rightly ordered, and therefore just, when its major institutions are arranged so as to achieve the greatest net balance of satisfaction summed over all the individuals belonging to it.” Not only is this classical principle based on \textit{welfarism} to the effect that “[t]he judgment of the relative goodness of alternative states of affairs must be based exclusively on, and taken as an increasing function of, the respective collections of individual utilities in these states”, but also it invokes the aggregation rule of \textit{sum-ranking} to the effect that “[o]ne collection of individual utilities is at least as good as another if and only if it has at least as large a sum total [Sen (1979, p.468)].” Rawls criticises the informational basis of welfarism and proposes the alternative informational basis of \textit{social primary goods}, viz. “things that every rational man is presumed to want,” which “normally have a use whatever a person’s rational plan of life [Rawls (1971, p.62)].” Rawls also criticises the utilitarian aggregation rule of sum-
ranking for its being “indifferent as to how a constant sum of benefits is distributed [Rawls (1971, p.77)].” His proposed alternative to the Benthamite utilitarianism is such that “[a]ll social primary goods --- liberty and opportunity, income and wealth, and the bases of self-respect --- are to be distributed equally unless an unequal distribution of any or all of these goods is to the advantage of the least favored [Rawls (1971, p.303)].” His own justification of this principle of justice makes use of a hypothetical situation called the *original position*, where individuals choose the basic principles of the society behind the *veil of ignorance*, viz. without knowing their own position in the resulting social order as well as being ignorant of their personal identities. In such a situation of primordial equality, Rawls claims that his principles of justice would be generally accepted as a fair agreement in the absence of ethically irrelevant vested interests.\(^{16}\)

The invocation of the logical device of primordial stage of ignorance with the purpose of securing a fair field for designing a set of social rules is not original to Rawls. Other notable examples are William Vickrey (1945; 1960) and John Harsanyi (1953; 1955; 1977), who respectively made use of the same device to find a justification for the Benthamite utilitarianism. Vickrey (1945) gave a brief, yet clear first statement of the original position idea. Harsanyi (1955) proved the following important theorem: Suppose that social preferences as well as individual preferences satisfy the von Neumann-Morgenstern postulates of rationality, and if all individuals being indifferent implies social indifference, then social welfare must be the weighted sum of individual utilities. Under the additional requirement of anonymity, the Harsanyi representation for social welfare boils down to the unweighted sum-total of individual utilities, viz. the classical utilitarianism.\(^{17}\)

2. Social Choice and Individual Values

Without denying the importance of those pioneering contributions made by many precursors, it seems fair to say that Kenneth Arrow’s *Social Choice and Individual Values* elevated social choice theory to a stage which is qualitatively different altogether.

To lend concrete substance to our sweeping assertion, let us start by referring to the pioneering studies of voting schemes by Condorcet, Borda, Dodgson, Black, and many others again. Important though these celebrated works are, it is undeniable that their studies were concerned exclusively with some specified voting schemes such as the method of simple majority voting, the Borda method, the Dodgson method, and so forth. In sharp contrast, Arrow (1950, 1951, 1984) developed an analytical method which allowed him to treat all conceivable voting schemes simultaneously within one unified conceptual framework. To bring the importance of this development into clearer relief, consider the simplest imaginable society with only two individuals, say 1 and 2, and three alternative social states, say \(x, y\) and \(z\). Let us
simplify our arena further by assuming away individual as well as social indifference relations altogether. It is clear, then, that there exist six distinct preference orderings of three social states:¹⁸

\[ \alpha: x, y, z \quad \beta: x, z, y \quad \gamma: y, x, z \quad \delta: y, z, x \quad \epsilon: z, x, y \quad \zeta: z, y, x \]

Each one of these orderings can represent an individual preference ordering for 1 and 2 over three social states. What Arrow christened the social welfare function, or constitution in his more recent terminology, is a function which maps each profile of individual preference orderings into a unique social preference ordering, which is meant to denote the process or rule for aggregating each profile of individual preference orderings into a social preference ordering. In other words, a social welfare function is a mapping defined on the Cartesian product \( \Delta \times \Delta \), where \( \Delta := \{ \alpha, \beta, \gamma, \delta, \epsilon, \zeta \} \), and takes its values on \( \Delta \). Thus, even in our simplest conceivable society, there exist \( 6^{36} \) social welfare functions in the sense of Arrow, which is an astronomically large number indeed (roughly \( 10^{28} \)). It is clearly impossible to check all these Arrovian social welfare functions one by one for their democratic legitimacy, on the one hand, and for informational efficiency, on the other. Instead of attempting to cope with this clearly hopeless task, Arrow pioneered the axiomatic approach in social choice theory, which enabled him to analyse these \( 6^{36} \) Arrovian social welfare functions all at once, by imposing a set of axioms which are deemed necessary for the Arrovian social welfare functions to be reasonable, hence acceptable. It is this novel methodology which enabled him to analyse all the relevant social welfare functions at one stroke, and led him to the celebrated general possibility theorem, or the Arrovian impossibility theorem in the currently prevailing terminology, to the effect that there exists no social welfare function satisfying a set of conditions necessary for democratic legitimacy and informational efficiency.

The novelty of Arrow’s approach is no less conspicuous in the context of the “new” welfare economics as well. For Bergson and Samuelson, their social welfare function was an analytical device for separating what should duly belong to economics from what should duly be relegated to ethics. According to Samuelson (1947, p.220-221), “[i]t is a legitimate exercise of economic analysis to examine the consequences of various value judgments, whether or not they are shared by the theorist, just as the study of comparative ethics is itself a science like any other branch of anthropology.” It was as an analytical vehicle for implementing this “scientific” research program of “new” welfare economics that Samuelson invoked what came to be known as the Bergson-Samuelson social welfare function: “Without inquiring into its origins, we take as a starting point for our discussion a function of all the economic magnitudes of a system which is supposed to characterize some ethical belief --- that of a benevolent despot, or
a complete egoist, or ‘all men of good will,’ a misanthrope, the state, race, or group mind, God, etc. Any possible opinion is admissible .... We only require that the belief be such as to admit of an unequivocal answer as to whether one configuration of the economic system is “better” or “worse” than any other or “indifferent,” and that the relationships are transitive ....

In contrast with the Bergson-Samuelson social welfare function, which Bergson and Samuelson assumed to be given from outside of economics, Arrow was of the conviction that the process or rule through which the social value to be represented by the Bergson-Samuelson social welfare function is formed should also be the subject of logical scrutiny. In other words, in order for the economic analysis not to lose social relevance, it is necessary that the process or rule for constructing the Bergson-Samuelson social welfare function on the basis of individual judgments of the goodness of the social states, viz. the Arrow social welfare function in this arena, must satisfy the minimal requirements of democratic legitimacy and informational efficiency. Interpreted in this new arena, the Arrow impossibility theorem turns out to be a basic criticism against the foundations of “new” welfare economics of the Bergson-Samuelson family. No wonder Arrow’s theorem caused a stir among many reputable economists who created and promoted the “new” welfare economics. For example, Ian Little (1952, pp.423-424) contrasted Bergson’s and Arrow’s social welfare functions with the purpose of criticizing the latter as follows: “Bergson’s welfare function was meant as a ‘process or rule’ which would indicate the best economic state as a function of a changing environment (i.e. changing sets of possibilities defined by different economic transformation functions), the individuals’ tastes being given. ... If tastes change, we must expect a new ordering of all the conceivable states; but we do not require that the difference between the new and the old orderings should bear any particular relation to the changes of taste which have occurred. We have, so to speak, a new world and a new order; and we do not demand correspondence between the change in the world and the change in the order. ... Traditionally, tastes are given; indeed, one might almost say that the given individuals are traditionally defined as the possessors of the given tastes and that no sense is attached to the notion of given individuals with changing tastes.”

Samuelson (1967, p.42), who has always been the most eloquent advocate of the Bergson-Samuelson school of “new” welfare economics, went as far as to declare that “the Arrow result is much more a contribution to the infant discipline of mathematical politics than to the traditional mathematical theory of welfare economics. I export Arrow from economics to politics because I do not believe that he has proved the impossibility of the traditional Bergson welfare function of economics, even though many of his less expert readers seem inevitably drawn into thinking so.”

What, then, are the axioms of democratic legitimacy and informational efficiency which Arrow demonstrated to be logically incompatible? In the 1963 revised version of the theorem [Arrow (1963, pp.96-97; 1987)], there are four transparent axioms altogether. The first axiom
is that each and every individual is free to form and express whatever preference ordering he/she cares to specify, which represents his/her evaluations of the goodness of social states, and the Arrow social welfare function must be robust enough to be able to aggregate the profile of these individual preference orderings into a social preference ordering. The second axiom requires that the Arrow social welfare function must faithfully reflect the unanimous preference expressed by all individuals over a pair of social states, which makes the process or rule of preference aggregation minimally democratic. The third axiom requires that the Arrow social welfare function must be informationally efficient in that, in deciding whether one social state is better than, or worse than, or indifferent to another social state, it is necessary and sufficient to know how individuals rank just these two alternative social states vis-à-vis each other. The fourth and the least controversial axiom requires that there should be no dictator in the society, who can decide a strict social preference for a social state vis-à-vis another social state simply by expressing his personal preference for the former state against the latter.

It is worth emphasizing that these demonstrably contradictory axioms are nothing other than the lineal descendents of what preceded Social Choice and Individual Values. Indeed, in the context of the methods of collective decision-making, the method of simple majority voting satisfies all of the Arrovian conditions except that the generated social preference relation lacks the general assurance of transitivity by virtue of the Condorcet paradox. In the alternative context of the foundations of welfare economics, the “new” welfare economics of the compensationist school of thought, as well as of the Bergson-Samuelson school of thought, is founded squarely on the ordinal and interpersonally non-comparable informational basis; it is also deeply rooted in the Paretian tradition to the effect of requiring social preference to reflect unanimous individual preferences faithfully. Because it respected the preceding tradition, the Arrow impossibility theorem was made not only more relevant, but also a clear indicator of the need of systematic scrutiny in the search for reasonable resolutions of the logical contradiction thereby identified. It is in this sense that the message of Arrow’s general impossibility theorem is clearly positive, rather than negative.

Arrow (1951, Chapter VII) also made another important contribution by developing a systematic logical method in the analysis of simple majority voting, which enabled him to pursue Black’s geometric idea of single peaked preferences in the general case of any number of alternatives. This neat method of analysis enabled his successors to introduce some other restrictions on the admissible profiles of voters’ preferences under which the method of simple majority voting can escape from the Condorcet paradox. Indeed, it was this method of analysis which eventually led Ken-Ichi Inada (1969), on the one hand, and Sen and Pattanaik (1969), on the other, to discover the necessary and sufficient conditions for this method of collective decision-making to work satisfactorily.
3. “Socialist Planning” Controversy

There is another controversy of historical importance, which was fought mainly in the 1930s. Maurice Dobb (1969, p.183) had the strong opinion that “[t]he old debate about Wirtschaftsrechnung... is nowadays sufficiently familiar... for any suggestion of revisiting it to invite disinclination rather than attention.” Nevertheless, it seems to us that there are several lessons of this harsh controversy with lasting importance in the evolution and orientation of the theory of decentralized planning procedures à la Edmond Malinvaud (1967) and Geoffrey Heal (1973), as well as of the related branch of social choice theory called the implementation theory, or of the theory of mechanism design, à la Leonid Hurwicz (1960; 1972; 1973) and Eric Maskin (1979; 1999).

It was Ludwig von Mises (1920/1935) who kicked off this controversy. In his understanding, rational economic calculation is possible only when monetary prices exist, not only for consumption goods, but also for production goods of any order, since it is monetary calculation which “affords us a guide through the oppressive plentitude of economic potentialities. ... It renders their value capable of computation and thereby gives us the primary basis for all economic operations with goods of a higher order [von Mises (1935, p.101)].” According to von Mises, however, it is impossible to find necessary monetary prices for production goods of a higher order in a socialist state, because no production good will ever become the object of market exchange in a socialist state where, by definition, collective ownership prevails for all means of production.

It is clear that the impossibility thesis à la von Mises holds if and only if there are no prices for production goods in a socialist state with collective ownership of the means of production. It seemed obvious to Oskar Lange (1938, p.61) that the latter thesis was clearly false: “Professor Mises seems to have confused prices in the narrower sense, i.e. the exchange ratios of commodities on a market, with prices in the wider sense of ‘terms on which alternatives are offered.’ ... It is only in the latter sense that ‘prices’ are indispensable for the allocation of resources ....” As Lange correctly pointed out, “prices in the generalized sense,” or “efficiency prices” in the circumlocution of modern economic theory, exist irrespective of the ownership structure of the means of production. This fact alone was enough to eradicate the impossibility thesis à la von Mises.

However, the controversy resurfaced in the hands of Friedrich von Hayek (1935; 1944; 1948), taking a more sophisticated form. Unlike von Mises, von Hayek never denied the theoretical existence of efficiency prices for all goods including the means of production, which, if made available, would enable a socialist state to attain a rational allocation of resources. The problem which von Hayek pointed out, and made the foundations of his impossibility thesis, was how such efficiency prices could be made available in practice: “[T]his is not an impossibility in the sense that it is logically contradictory. But to argue that a
determination of prices ... being logically conceivable in any way invalidates the contention that it is not a possible solution, only proves that the real nature of the problem has not been perceived [von Hayek (1935, pp.207-208)].” To understand why, von Hayek urges us to visualize what the determination of efficiency prices by computational method would imply in practice: “It is clear that any such solution would have to be based on the solution of some such system of equations [for general economic equilibrium] as that developed in [Enrico] Barone’s article [Barone (1908/1935)] ... . [W]hat is practically relevant ... is not the formal structure of this system, but the nature and amount of concrete information required if a numerical solution is to be attempted and the magnitude of the task which this numerical solution must involve ... [von Hayek (1935, p.208)].” To calculate efficiency prices by solving the general equilibrium equations, we must gather information about technology, primary and intermediate resources, and consumers’ preferences, which are widely dispersed and privately owned by numerous economic agents. Given the nature and complexity of these privately held information, it would be prohibitively difficult, if not logically impossible, to motivate numerous private agents to comply with the request from the central planning board and submit these information faithfully for the purpose of computing efficiency prices. Thus, von Hayek concludes, “[i]t is probably evident that the mere assembly of these data is a task beyond human capacity [von Hayek (1935, p.211)].” To make this situation even worse, “[m]ost of [the technical information] consists in a technique of thought which enables the individual engineer to find new solutions rapidly as soon as he is confronted with new constellations of circumstances [von Hayek (1935, pp.210-211)].” This is the essence of the impossibility thesis à la von Hayek.

Once again, Lange was ready to confront von Hayek’s impossibility thesis. Capitalizing and elaborating on the earlier works by Enrico Barone (1908/1935) and Fred M. Taylor (1929/1938), Lange developed a sophisticated trial and error method of price adjustment in a socialist state. To see how he designed this scheme, the so-called *Lange-Lerner market socialism* after Oscar Lange (1936-1937/1938) and Abba Lerner (1944), and how this scheme fares with respect to some performance criteria, is useful in identifying the areas of research called the theory of decentralized planning procedures and the theory of mechanism design.

Lange assumed a socialist state where freedom of choice in consumption and freedom of choice of occupation are guaranteed, and the preferences of consumers are the guiding criteria in production and in the allocation of resources. In this system, there exist market prices for consumption goods and for labour services, but the prices for capital goods and productive resources other than labour are prices in the generalized sense, i.e. mere accounting prices. Some appropriate rules are applied to the distribution of social dividend to the consumers. Subject to these rules of income formation and given market prices, the consumers are free to choose their demand for consumption goods and supply of labour services. Likewise, some
appropriate rules are applied to the production units (in industry with many firms incurring set-up costs) so that average cost of production will be minimized, and marginal cost will be made equal to the price of the product for each and every good produced. The accounting prices for capital goods and productive resources other than labour are formed and adjusted by the Central Planning Board through the instrumental use of the Walrasian tâtonnement process, where the Central Planning Board plays the role of the Walrasian auctioneer. The *modus operandi* of this successive trial and error process is exactly the same as the well-known Walrasian tâtonnement process, and the adjustment of the market price or the accounting price for each good and service are made in accordance with the aggregate excess demand for the good and service in question.

Two properties of this pseudo-Walrasian tâtonnement process deserve particular attention. In the first place, it enables the Central Planning Board to escape from the Hayekian task of gathering dispersed private information for computing accounting prices at the centre, which von Hayek maintained to be practically impossible to perform, since the necessary computation is in effect performed by each and every holder of private information. In the second place, the accounting prices found at the equilibrium of this pseudo-Walrasian tâtonnement process in a socialist state “have quite the same objective character as the market prices in the regime of competition. Any mistake made by the Central Planning Board in fixing prices would announce itself in a very objective way --- by a physical shortage or surplus of the quantity of the commodity or resources in question --- and would have to be corrected in order to keep production running smoothly [Lange (1938, p.82)].” On the basis of these nice properties of his scheme, Lange concluded that “a substitution of planning for the functions of the market is quite possible and workable,” and the immediate successors of the lessons of the controversy gladly concurred. Indeed, “[a]s far as economics profession is concerned,” wrote Paul Sweezy (1949, p.232) in the Economics Handbook Series edited by Seymour Harris, “Lange’s paper may be regarded as having finally removed any doubts about the capacity of socialism to utilize resources rationally.” Upon careful scrutiny, however, this sweeping verdict turns out to be untenable, to say the least.

To begin with, for the quasi-Walrasian tâtonnement process to serve as an algorithm for finding right market prices and accounting prices, it must be guaranteed to converge surely and rapidly to the system of general equilibrium prices. Unless some very special assumptions, such as gross substitutability, or the weak axiom of revealed preference, are imposed on the aggregate excess demand functions, however, there is no guarantee for the global stability of the Lange process of price adjustment.\(^{21}\) In a postscript to the controversy written thirty years later, Lange (1967, p.158) wrote that “[i]t was assumed without question that the tâtonnement process in fact converges to the system of equilibrium prices.” Since there is no general guarantee of such a convergence property, the Lange-Lerner scheme of market socialism offers no
assurance of non-wasteful workability. More remarkably, Lange went on to maintain that “[w]here I to rewrite my essay today my task would be much simpler. My answer to Hayek and Robbins would be: so what’s the trouble? Let us put the simultaneous equations on an electric computer and we shall obtain the solution in less than a second. The market process with its cumbersome tâtonnements appears old-fashioned. Indeed, it may be considered as a computing device of the pre-electronic age.” This statement is truly remarkable, as it “proves that the real nature of the problem has not been perceived.” Recollect that the impossibility thesis à la von Hayek was based not on the limitation of computational capacity on the part of the Central Planning Board, but on the prohibitive difficulty of gathering dispersed and privately owned information for the purpose of central computation. Needless to say, no computer with whatever capacity can work without being provided with the relevant data. Interestingly enough, Abram Bergson (1967, pp.663-664) also posed the possibility of avoiding trial and error procedure by solving pertinent equations by means of mathematical techniques: “[B]oth Lange and [Dickinson] wrote before the age of electronic computers. Given this technology, could not the [Central Planning Board], in performing its cardinal task of fixing prices, confute Hayek after all simply by using mathematical techniques?” However, Bergson was far more careful in trying to answer this question than Lange: “[S]hould the Board seek to employ mathematical procedures in fixing prices comprehensively and in detail, its undertaking surely could become burdensome for managers of production units, who might be called on to predict and articulate in inordinately concrete detail the complex and ever changing constraints and opportunities that confront them, and on this basis to communicate to the Board such data on these matters as the Board would require; and for the Board itself, which promptly would have to digest such information and to communicate the results of its deliberations to the managers. The capacities of managers as well as of the Board to grapple with these tasks might often be enhanced by use of computers, but not always”.

Secondly, there is no systemic device in the Lange-Lerner scheme of market socialism to confront the possibility of strategic behaviour by private agents. As Lange (1938, p.81) rightly observed, “[o]n a competitive market the parametric function of prices results from the number of competing individuals being too large to enable any one to influence prices by his own action. In a socialist economy, production and ownership of the productive resource outside of labour being centralized, the managers certainly can and do influence prices by their decisions. Therefore, the parametric function of prices must be imposed on them by the Central Planning Board as an accounting rule. All accounting has to be done as if prices were independent of the decisions taken. For purposes of accounting, prices must be treated as constant, as they are treated by entrepreneurs on a competitive market.” Since there is nothing in the Lange-Lerner scheme to make this accounting rule to be compatible with the private incentives of individual agents, we cannot but conclude that the Lange-Lerner scheme of market
socialism lacks the important property of *incentive compatibility*.

Thirdly, the Lange-Lerner market socialism is designed for the single-minded purpose of enabling a socialist state to use its endowed scarce resources efficiently. As was aptly observed by Sweezy (1949, p.233), “[p]erhaps the most striking feature of Lange’s model is that the function of the Central Planning Board is virtually confined to providing a substitute for the market as the coordinator of the activities of the various plants and industries. The truth is that Lange’s Board is not a *planning* agency at all but a *price-fixing* agency; in his model production decisions are left to a myriad of essentially independent units, just as they are under capitalism.” It is true that achieving the efficient use of scarce resources is a task of no mean difficulty, but “[t]he common features of all collectivist systems may be described ... as the deliberate organisation of the labours of society for a definite social goal. That our present society lacks such ‘conscious’ direction towards a single aim, that its activities are guided by the whims and fancies of irresponsible individuals, has always been one of the main complaints of its socialist critics [von Hayek (1944, p.42)].” If we take this observation at all seriously, we must go beyond mere efficiency and proceed to optimality with reference to the single social goal in order to have a fully-fledged design of a rational collectivist society.

If we retain, as in the Lange-Lerner scheme of market socialism, the crucial value premise of consumers’ sovereignty and want to orient a socialist state towards a definite social goal beyond the mere attainment of efficient allocation of scarce resources, we must find a process or rule to construct a conscious social goal on the basis of individual judgements on what constitutes social goods, since “[t]he effect of the people agreeing that there must be central planning, without agreeing on the ends, will be rather as if a group of people were to commit themselves to take a journey together without agreeing where they want to go [von Hayek (1944, p.46)].” This is precisely the same problem posed and settled in the negative by Arrow in a related but distinct context of collective choice and social welfare. Interestingly enough, von Hayek (1944, p.44) observed that forming “a definite social goal” for its use in orienting central planning “would be impossible for any mind to comprehend the infinite variety of different needs of different people which compete for the available resources and to attach a definite weight to each.” See, also, Leif Johansen (1969), who examined the relevance of Arrow’s impossibility theorem in the context of economic planning.

These negative observations notwithstanding, it should be emphasized that the “socialist planning” controversy, in which both Lange and von Hayek played major roles, was the first serious attempt at designing an alternative economic mechanism with the purpose of satisfying some concrete performance characteristics. In so doing, they became the modern forerunners in the theory of decentralized planning procedures and the theory of mechanism design.
4. Significance of the Subject and Main Lines of Research

Enough has been said so far about the historical background of social choice theory. It remains for us to emphasize the significance of the subject, and identify the major lines of research in this broad and interdisciplinary area.

Ever since the appearance of *Social Choice and Individual Values*, the growth of social choice theory along many distinct lines of research has been quite conspicuous, especially after the 1960s. By now, there is an extensive Social Choice Bibliography prepared and regularly updated by Jerry Kelly (http://www.maxwell.syr.edu/maxpages/faculty/jskelley/biblioho.htm), which is more than 300 pages in single-space printout. Even this extensive and invaluable Kelly Bibliography does not cover some of the issues treated in this *Handbook of Social Choice and Welfare* in full, whereas there are many other issues which are included in the Kelly Bibliography but not in this Handbook. The plan of this Handbook clearly reflects our perception of the special significance of the development along the three lines of research which we have identified in our account of the historical evolution of social choice theory: the methods of collective decision-making, the theoretical foundations of welfare economics, and the theory of incentive compatibility and mechanism design. To explain why we believe these issues to be of special significance, it is useful to go back to *Social Choice and Individual Values* once again.

To begin with, note that Arrow’s theory connected social choice and a social preference ordering, which the Arrow social welfare function associates with each profile of individual preference orderings, through the assumption of collective rationality: Given any set of available social states, the society chooses that available social state which is at least as good as any other available social state, where the judgements of the goodness of social states are performed in terms of the social preference ordering. This crucial assumption has been one of the major targets for critics of the Arrovian framework of social choice theory. Most notable is the criticism by James Buchanan (1954, p.116), according to whom “[t]he mere introduction of the idea of social rationality suggests the fundamental philosophical issues involved. Rationality or irrationality as an attribute of the social group implies the imputation to the group of an organic existence apart from that of its individual components ... . We may adopt the philosophical bases of individualism in which the individual is the only entity possessing ends or values. In this case no question of social or collective rationality may be raised. A social value scale simply does not exist. Alternatively, we may adopt some variant of the organic philosophical assumption in which the collectivity is an independent entity possessing its own value ordering. It is legitimate to test the rationality or irrationality of this entity only against this value ordering.”

Two avenues of research were explored in response to this early criticism, in order to check the robustness of the Arrovian impossibility theorems with respect to the assumption of collective rationality. The first avenue maintained the definition of social choice in terms of the
optimization of the social preference relation, but weakened the required degree of collective rationality. Weakening Arrow’s requirement of completeness as well as transitivity of social preference relation, one may want to discard the exacting requirement of transitivity of the indifference relation, and retain only the more defensible requirement of transitivity of the strict preference relation (to be called quasi-transitivity); one may also go one step further and weaken the requirement of quasi-transitivity, and settle with only the non-existence of any strict preference cycle (to be called acyclicity). The second avenue went further and discarded the assumption of collective rationality altogether; it focused directly on social choice which has no underlying social preference relation, and imposed some choice-consistency property, an important example thereof being path-independence: “the independence of the final choice from the path to it [Arrow (1963, p.120)].” These two avenues were pioneered and vigorously explored by Sen (1969; 1970a, Chapter 4*; 1977a); his leading attempts were followed by Andreu Mas-Colell and Hugo Sonnenschein (1972), Charles Plott (1973), Douglas Blair, Georges Bordes, Jerry Kelly, and Kotaro Suzumura (1976), Suzumura (1983, Chapter 3), and many others. Basically, however, these extensive researches confirmed the robustness of the Arrovian impossibility theorems. As Arrow (1963, p.109) has observed in a related but distinct context, “[t]he para-dox of social choice cannot be so easily exorcised.”

The next crucial step in the search for an escape route from Arrow’s impossibility theorem was to explore the use and usefulness of interpersonal comparisons of utilities, with or without cardinal measurability. The context in which we can meaningfully talk about this potential escape route is one where an ethical observer forms his own subjective interpersonal comparisons of utilities, and makes use of this extended informational basis to define an essentially Arrovian social welfare function. A fruitful and systematic method of analysis was developed mainly in the 1970s by Sen (1970a; 1973; 1977b), Peter Hammond (1976), Claude d’Aspremont and Louis Gevers (1977), and Eric Maskin (1978), among many others, which brought about a neat axiomatization of the Rawlsian difference principle (in its welfaristic version) as well as of the Benthamite principle of utilitarianism. This is a legitimate way out from the Arrovian impossibility theorem in the context of forming someone’s social welfare judgements, but such an escape route is surely not available in the alternative context of collective decision-making. Even in the context of forming social welfare judgements, the phantom of Lionel Robbins cannot be exorcised so easily; if there are multiple ethical observers who form their respective subjective interpersonal comparisons of utilities, their social welfare judgements may well conflict with each other so much so that some variants of the Arrovian impossibility theorems may well come back strenuously, as was demonstrated by Kevin Roberts (1980a, 1980b, 1995) and Suzumura (1996b).

In passing, one particular type of interpersonal utility comparison deserves special attention: “People seem prepared to make comparisons of the form: State $x$ is better (or worse)
for me than state y is for you. ... Interpersonal comparisons of the extended sympathy type can be put in operational form; the judgment takes the form: It is better (in my judgment) to be myself in state x than to be you in state y [Arrow (1963, pp.114-115)].”

This is indeed the type of interpersonal utility comparison which formed the informational basis of, e.g. an analysis of economic inequality by Sen (1973), as well as of an axiomatization of the Rawlsian difference principle by Hammond (1976) and Sen (1977b). This is also the informational basis which enables us to extend the celebrated fairness-as-no-envy approach in the theory of resource allocation --- developed most notably by Duncan Foley (1967), Serge-Christophe Kolm (1972) and Hal Varian (1974) --- to the theory of social choice, which was initiated by Suzumura (1981a, 1981b).

Still centering around the original Arrow impossibility theorem itself, one may try to see how tight this remarkable theorem in fact is by carefully checking whether or not any one of the constituting axioms can be weakened without upsetting the validity of the theorem. One may also try to see the trade-off relationship which may hold between different axioms, keeping the essential validity of the theorem intact. These ideas have been pursued, e.g., by Julian Blau (1979) and Robert Wilson (1972), on the one hand, and by Donald Campbell and Jerry Kelly (1994), on the other.

All the lines of research mentioned so far are, to a great extent, correctly describable as being the lineal descendants of Arrow’s seminal work. There are some other lines of research which were mentioned, but not explored, in Social Choice and Individual Values. One salient example is the strategic aspects of collective decision-making, which we have briefly mentioned in the context of the Borda-Laplace rank-order method of collective decision-making. Arrow (1951, p.7) was careful enough to point out that “once a machinery for making social choices from individual tastes is established, individuals will find it profitable, from a rational point of view, to misrepresent their tastes by their actions, either because such misrepresentation is somehow directly profitable or, more usually, because some other individual will be made so much better off by the first individual’s misrepresentation that he could compensate the first individual in such a way that both are better off than if everyone really acted in direct accordance with his tastes.” As a matter of fact, Samuelson (1954, pp.388-389) pointed out the ubiquity of strategic misrepresentation of preferences in the specific context of the efficient provision of public goods: “[I]t is in the selfish interest of each person to give false signals, to pretend to have less interest in a given collective consumption activity than he really has, etc.” This free-rider problem, so-called, can be traced back much further to Knut Wicksell (1896; 1958, p.81): “If the individual is to spend his money for private and public uses so that his satisfaction is maximized, he will obviously pay nothing whatsoever for public purposes (at least if we disregard fees and similar charges). Whether he pays much or little will affect the scope of public service so slightly, that for all practical purposes, he himself will not notice it at
all. Of course, if everyone were to do the same, the State would soon cease to function.” In
the context of social choice theory, however, the first general treatment of the strategic mis-
representation issue, of which Arrow was aware from the inception of social choice theory, but
left unexplored, had to wait until 1970s when Allan Gibbard (1973) and Mark Satterthwaite
(1975) came up with a general theorem on the manipulability of voting schemes. Recollect
that a voting scheme is a social choice mechanism which assigns a single outcome to each and
every profile of voters’ preference orderings over outcomes. As long as there are at least three
alternative outcomes and at least two voters, there exists no non-dictatorial voting scheme
which is free from strategic misrepresentation of preferences by individuals. It is worthwhile to
point out that the Arrow theorem is closely related to the Gibbard-Satterthwaite theorem in the
sense that the former theorem can provide the crucial step in proving the latter theorem. Given
the validity of the basic Gibbard-Satterthwaite theorem on the ubiquity of strategic manipulation
of voting schemes, it is natural that a huge literature was created in the search for either the
escape route from the Gibbard-Satterthwaite impossibility theorem, or directions in which their
theorem may be generalized.

Since the strategic misrepresentation of preferences is demonstrably ubiquitous, there is
a further problem to be tackled with: “Even in a case where it is possible to construct a proce-
dure showing how to aggregate individual tastes into a consistent social preference pattern,
there still remains the problem of devising rules of the game so that individuals will actually
express their true tastes even when they are acting rationally [Arrow (1951, p.7)].” It was pre-
cisely in response to this plea that a fruitful area of research, to be called the implementation
theory, or the theory of mechanism design, was created at the hands of Leonid Hurwicz (1960;
1972; 1973), Partha Dasgupta, Peter Hammond and Eric Maskin (1979) and Eric Maskin
(1979; 1999). A mechanism is a game form, which is designed and managed by the helmsman
of the economy, so that it can attain the social objective at the equilibrium of the game by
assigning to each individual agent an appropriate set of admissible strategies and a payoff
function. In view of the Gibbard-Satterthwaite theorem and Hurwicz’s (1972) theorem to the
same effect in economic environments, the constructed game forms are such that the set of
admissible strategies cannot be that of individual preference orderings, but that of much wider
nature. Although the public objective, which the helmsman tries to opti-mize, is typically
dependent on the private information, it need not be concordant with the private incentives of
individual agents. It follows that the requirement to the effect that individual agents within the
designed mechanism should be so induced as to bring about the social objective optimization at
equilibrium, cannot but impose a constraint on the mechanisms to be designed and on the
public objectives to be implemented.

Another game-theoretic background of social choice theory deserves to be mentioned,
which can be traced back all the way to the cooperative game theory of John von Neumann and
Oscar Morgenstern (1944/1947). Notable cooperative solution concepts to the axiomatic bargaining problem due to John Nash (1950) such as the Nash bargaining solution, or the Kalai-Smorodinsky (1975) solution, as well as to the games of characteristic function forms such as the Shapley value, the core, or the nucleolus, provide social choice theory with a rich class of reasonable (fair) compromises in the situation which mixes cooperation and competition among individual agents.

Not only Arrow’s social choice theory, but also the Gibbard-Satterthwaite theorem on the non-manipulability of voting schemes, as well as the Hurwicz-Maskin theory on implementation, and the cooperative game-theoretic approach to fair compromises, all make extensive use of axiomatic methods. Many of the strengths and weaknesses of these theories hinge squarely on this common analytical character. As was observed by Arrow (1951, p.87), “[o]ne of the great advantages of abstract postulational methods is the fact that the same system may be given several different interpretations.” In exchange for this great merit of interpretational versatility, however, the axiomatic methods tend to be plagued with the potential weakness of a formal neglect of substantial issues. A good case in point is a warning by Leif Johansen (1977) to the effect that the theoretically undeniable ubiquity of “playing down one’s preferences for a public good in order to get a lower share in the costs of providing the good” does not seem “likely to succeed in an open political decision-making process involving elected representatives.” According to Johansen, “the two-tier system of electors and representatives tends to diminish the significance and relevance of the theoretical problem of unwillingness to reveal preferences for public goods.” This warning seems to urge us to examine in concrete details the institutional structures of the society, political as well as economic, in search of the empirical relevance of purely theoretical results obtained in a general axiomatic framework. This is an interesting step to take if one wants to verify that the paradox of voting is not just a theoretical curiosity, but a phenomenon of substantial empirical relevance; it also motivates us to analyse the logical performance of representative democracy vis-à-vis direct democracy. Furthermore, instead of merging “voting, typically used to make ‘political’ decisions, and the market mechanism, typically used to make ‘economic’ decisions [Arrow (1951, p.1)]” into one and the same axiomatic system, it may prove useful to develop an idiosyncratic model of social choice in economic environments, along with developing a separate model of political decision-making. All these steps have been taken vigorously in the social choice literature with rich ramifications of specific results.

There is yet another crucial point of departure from Arrow’s original formulation of social choice theory. Not only the traditional welfare economics, “old” as well as “new”, but also the Arrovian social choice theory itself, are deeply rooted in the philosophical approach of welfarist-consequentialism in that they are based on the assessment of the goodness of states of affairs in terms of individual utilities obtained from these states of affairs. It was Sen’s (1970a,
impossibility of a Paretian liberal which casted a serious doubt on this long tradition by establishing an impossibility theorem to the effect that the weak welfaristic requirement of the Pareto principle cannot but conflict with the non-welfaristic requirement of the respect for minimal individual liberty. Sen’s seminal analysis can be traced back to the problem which John Stuart Mill (1859/1977; 1861) had to face in his simultaneous belief in the utilitarian outcome morality, on the one hand, and in the sanctity of libertarian rights, on the other. In view of the remarkable pervasiveness of welfarist-consequentialism in the whole spectrum of normative economics, it is natural to find many attempts in the literature to try to find an escape route from Sen’s impossibility theorem, e.g. Gibbard (1974), Blau (1975), Sen (1976a, Sections III-XI), and Suzumura (1978/1979); to gauge the robustness of Sen’s liberal paradox, so-called, e.g. David Kelsey (1985; 1988), and Sen (1976a, Section II and Appendix A2); and to examine critically Sen’s original articulation of individual liberty, e.g. Peter Gärdnfor (1981), Robert Sugden (1985), Peter Hammond (1986), Wulf Gaertner, Prasanta Pattanaik and Kotaro Suzumura (1992), and Pattanaik and Suzumura (1994; 1996). The implications and relevance of these works on the impossibility of a Paretian liberal are critically evaluated by Suzumura (1996a) who distinguished the three related but distinct issues in the social choice-theoretic analysis of welfare and rights: the issue of the analytical articulation of rights, the issue of the realization of rights, and the issue of the initial conferment of rights. There are also many criticisms of welfarist-consequentialism in terms of the counter-intuitive implications of this informational constraint in some paradigmatic cases, e.g. Ronald Dworkin (1981a), Amartya Sen and Bernard Williams (1982), Jon Elster (1983), Amartya Sen (1985), and many others.

Once Pandora’s box is opened, and we are given a glimpse of the possibilities which lie beyond the narrow confines of welfarist-consequentialism, nothing prevents us from asking questions which can be properly posed only when we are ready to go beyond the traditional informational basis of welfarist-consequentialism. In the analysis of individual well-being, for example, we need not necessarily analyse it only through the looking glass of individual welfares. Alternative articulations of individual advantages have been proposed, which have opened new possibilities in welfare economics in general, and social choice theory in particular. Representative proposals to this effect include social primary goods in Rawls’ (1971) theory of justice, resources in Dworkin’s (1981b; 2000) theory of equality, and capabilities in Sen’s (1985; 1999b) theory of well-being. The new vistas thereby opened have far-reaching implications with innovative perspectives on the theory and policy of economic development, as expounded in Sen (1999b). We may even proceed beyond consequentialism as such, and pose some questions such as the intrinsic value of opportunities to choose and/or the intrinsic value of procedures for choice, along with their instrumental values. Indeed, it is only with these new developments in clear perspective that we can gauge the true usefulness and limitations of
the traditional informational basis of welfarist-consequentialism. Some of these new vistas opened in this direction are expounded in Sen (2001), Suzumura (1999a; 2000; 2001), Reiko Gotoh and Kotaro Suzumura (2001), and Kotaro Suzumura and Yongsheng Xu (2001a, 2001b).

Overlapping partly with this trend to go beyond welfarist-consequentialism as the informational basis of social welfare analysis, there were conspicuous developments in the theory of how to measure economic well-being. It was Serge-Christophe Kolm (1969) and Anthony Atkinson (1970) who kicked off the modern resurgence of interest in the measurement of income inequality. Soon afterwards, Sen (1976b) axiomatized a new measure of income poverty, which went substantially beyond the crude traditional measure such as the head count ratio, and incorporated a new distributional dimension into the measurement of poverty. More recently, Prasanta Pattanaik and Yongsheng Xu (1990) started a new area of research concerning how to measure freedom of choice. Each one of these seminal works generated substantial follow-up works of their own, which are enriching our theoretical tool box for the measurement of well-being.

This *Handbook of Social Choice and Welfare* is a systematic attempt to provide, in two volumes, an up-to-date overview of the current state of the art in social choice theory and welfare economics, encompassing all these issues we have so far identified and even more. Plenty of dishes are on the table. It is our sincere hope that the readers will enjoy them and be motivated to participate in the vigorous research activities which are currently taking place.

5. A Disclaimer

It has been said that social choice theory is “a science of the impossible.” This statement contains an element of the truth only to the limited extent that the development of modern social choice theory received strong momentum from many impossibility theorems. Arrow’s monumental theorem on the impossibility of democratic and informationally efficient preference aggregation procedures, Sen’s theorem on the impossibility of a Paretian liberal, and the Gibbard-Satterthwaite theorem on the impossibility of non-manipulable and non-dictatorial voting schemes, to cite only a few most salient examples, have served us positively by sending an unambiguous signal that there are logical problems which await our careful scrutiny and serious attempt for resolution. In the process of understanding these impossibility theorems, we are brought to the far deeper perception of what underlies social conflicts of important values than ever. Likewise, in the process of finding some meaningful escape routes from these logical impasses, we are brought to much richer understanding on what makes several social values mutually compatible than otherwise. In this sense, there is nothing intrinsically negative about social choice theory in general, and impossibility theorems in particular.
It has also been said that welfare economics is plagued with elegance nihilism. In this context, it is worthwhile to recollect that Pigou’s “old” welfare economics started with the following manifest: “The complicated analyses which economists endeavour to carry through are not mere gymnastic. They are instruments for the bettering of human life. The misery and squalor that surround us, the dying fire of hope in many millions of European homes, the injurious luxury of some wealthy families, the terrible uncertainty overshadowing many families of the poor --- these are evils too plain to be ignored. By the knowledge that our science seeks it is possible that they may be restrained [Pigou (1920, p.vii)].” Forty years later, however, Edward Mishan (1960, p.197) commenced his survey of welfare economics over the period 1939-1959 with the following remark: “While it continues to fascinate many, welfare economics does not appear at any time to have wholly engaged the labours of any one economist. It is a subject which, apparently, one dabbles in for a while, leaves and, perhaps, returns to later in response to troubled conscience ... .” Since Mishan’s survey covered the period over which the “new” welfare economics was created so as to replace the crumbling “old” welfare economics only to receive harsh criticisms on their logical foundations even before the scaffolds of construction were removed from their construction sites, Mishan’s cynicism may be understandable at least to some extent. But the cynicism persisted ever since, and Atkinson (2001) felt it necessary to talk about “The Strange Disappearance of Welfare Economics” from the mainstream economics. However, as we have observed at the beginning of this Introduction, “as soon as any collective body designs and implements an economic mechanism and/or an economic policy, paying proper attention to the costs and benefits accruing to its constituent members, one or more social welfare judgements cannot be avoided.” Since social choice theory is partly concerned with the logical foundations of welfare economics, we cannot but maintain that the study of social choice theory and welfare economics is indispensable as long as one is interested in the problem of any economic policy, be that macroeconomic or microeconomic in nature. Pigou thought that welfare economics was a potent instrument for the bettering of human life. The same can be said of social choice theory.
Footnotes

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1 See Sen (1999a, p.350).

2 Iain McLean and John London (1990, p.107) maintained convincingly that they found “two medieval thinkers, hitherto unknown to historians of social choice [viz. Ramon Lull (c. 1235-1315), who proposed the Condorcet method of pairwise comparisons, and Nicolas Cusanus (1401-1464), who proposed the Borda method of rank-order comparisons], who anticipated the work of Condorcet, Borda and Dodgson by over 500 years.” They aptly added, however, that “[n]either writer gives a mathematical or logical justification for his scheme: such justifications had to await Condorcet and Borda [McLean and London (1990, p.106)].” It was for this reason that McLean (1995) later christened the period over which Borda, Condorcet, and their contemporaries worked on the theoretical performance of voting schemes “the first golden age of social choice.”

3 It is worthwhile to recollect that “Condorcet’s work on social choice (1785-94) spans the most active constitution-making era in Western history until then, and the most active ever until 1989. Constitutions for the United States, Poland, and France were written, and Condorcet was connected with all three. ... In 1792, Condorcet was made the chairman of a committee to draw up a Constitution for France. ... After the Jacobin coup d’état of June 1793, Condorcet was out of power. His constitution was dumped in favor of one drawn up in great haste by Robespierre, who dropped all Condorcet’s voting schemes [McLean (1995, pp.23-26)].” Condorcet’s work on the theory of voting is translated into English by Iain McLean and Fiona
Hewitt (1994).


5 Borda’s rank-order method was first proposed orally at the French Academy of Science in 1770, which remained unpublished until 1784. Condorcet was well aware of this method, and immediately recognized it to be an important challenge to his own pairwise comparison method. He stated in Condorcet (1785, Discours préliminaire, p.clxxix) that he had heard of Borda’s method orally, but that it was not published until after his own work was in press. According to McLean (1995, p.16), however, it was actually Condorcet himself who published Borda’s work.

6 For Laplace’s theory of elections, those who are interested should refer to Isaac Todhunter (1865, pp.546-548) and Duncan Black (1958, pp.180-183).

7 Although Black (1958, p.193) went as far as to deny even the indirect influence of Borda and Condorcet on Dodgson’s theory of committees and elections through Todhunter’s (1865) authoritative account of Borda’s and Condorcet’s contributions, which “every mathematical lecturer in the country ought to have studied” in Black’s own admission, I found his argument less than persuasive.

8 John Hicks (1975, p.307) was certainly right when he asserted that “[the] ‘official’ history [of welfare economics] begins with [Arthur Pigou’s] *The Economics of Welfare* (1920). For it was certainly Pigou who gave its name to the subject. If it existed before Pigou, it must then have been called something else.” However, the consideration of nomenclature alone should not vitiate the substantial fact that Pigou’s welfare economics is nothing other than the lineal descendent of the long tradition of the Bentham-Mill-Marshall-Edgeworth-Sidgwick utilitarian calculus.

9 Both citations from Condorcet are due to Rothschild (2001, p.6).

10 For Bentham, the only category of rights, whose existence he could recognize at all, were those which depended on law and legislation; a natural right was for him nothing other than a contradiction in terms: “[T]here are no such things as natural rights --- no such things as rights anterior to the establishment of government --- no such things as natural rights opposed to, in contradistinction to, legal; [T]he expression is merely figurative: [W]hen used, in the moment
you attempt to give it a literal meaning it leads to error, and to that sort of error that leads to mischief --- to the extremity of mischief (Bentham (1843, p.500)).

11 At this juncture, two remarks seem to be in order. In the first place, while Pigou in principle subscribed to the utilitarian viewpoint, careful reading of The Economics of Welfare reveals how discriminatingly was the use he actually made of it. Having said this, however, it should be pointed out that Pigou’s discussions of tax-subsidy policies related to externalities, with which he is much associated, were directly derived through a utilitarian way of reasoning. It is true that Pigou’s use of the utilitarian principle is not as conspicuous in reference to income distribution as was the case with Edgeworth, but it was in fact Pigou who inspired Hugh Dalton’s (1920) famous utilitarian measure of inequality. In the second place, unlike Bentham, who was strongly and outspokenly against the idea of natural rights, which goes squarely against the foundations of utilitarianism, Pigou (1920; 1952 edition, p.759) made an early use of the non-welfarist notion of individual rights when he discussed people’s claim to “minimum standard of real income,” which “must be conceived, not as a subjective minimum of satisfaction, but as an objective minimum of conditions.” Pigou’s characterization of “an objective minimum of conditions” is close to what we now call the “basic needs,” which consist of “some defined quantity and quality of house accommodation, of medical care, of education, of food, of leisure, of the apparatus of sanitary convenience and safety where work is carried on ... .” Pigou might have thought that such rights could be justified on utilitarian grounds in the Benthamite tradition of regarding rights as intrinsically unimportant, but instrumentally crucial, but The Economics of Welfare is completely reticent concerning the utilitarian justification of these rights.

12 According to John Chipman and James Moore (1978, p.548, footnote 2), Enrico Barone (1908/1935) had developed the compensation principle much earlier than Kaldor and Hicks, “who mentioned it no less than four times.” Barone’s pioneering contribution was left unnoticed among English speaking economists, however, even after the Italian original was translated into English in von Hayek (1935).

13 The genesis of the Bergson-Samuelson social welfare function was traced as far back as to Pareto (1913) by Chipman (1976) and Chipman and Moore (1978). True enough, Pareto was remarkably ahead of his time, and sympathetic eyes may catch the glimpse of social welfare function in Pareto’s early writings. Nevertheless, it seems fair to say that, without Bergson (1938) and Samuelson (1947, Chapter VIII), the concept of social welfare function could not have established itself as the central piece of modern welfare economics. It is in this sense that
Samuelson (1981, p.248) is absolutely right when he wrote in a related context that “[a]fter, and only after, you have worked out a clear understanding of this subject are you able to recognize the bits of the puzzle that Pareto had already discerned.”

Let $P_p$, $P_s$ and $P$ stand, respectively, for the Pareto superiority relation, the Samuelson superiority relation, and the social preference relation. The social preference relation is said to respect the Pareto superiority relation as well as the Samuelson superiority relation if and only if it satisfies $P_p \subseteq P$ and $P_s \subseteq P$. It was shown by Suzumura (1980; 1999b) that there exists a situation, which is not concocted at all, where we have four social states, say $x$, $y$, $z$ and $w$, such that $x P_p y$, $z P_p w$, $y P_s z$ and $w P_s x$ hold. If the social preference relation respects the Pareto superiority relation as well as the Samuelson superiority relation, then we cannot but obtain $x P y$, $y P z$, $z P w$ and $w P x$, which clearly vindicate the social preference cycle.

Thus, Samuelson (1981, p.223) could assert without any reservation the following: “As I write, the new welfare economics is just over four decades old. This subject, in its essentials as we know it today, was born when the 24-year-old Abram Bergson --- then still a Harvard graduate student --- wrote his classic 1938 *Quarterly Journal of Economics* article. To one like myself, who before 1938 knew all the relevant literature on welfare economics and just could not make coherent sense of it, Bergson’s work came like a flash of lightning, describable only in the words of the pontifical poet:

> Nature and Nature’s laws lay hid in night:
> God said, Let Newton be! and all was light.”

Rawls’ theory of “justice as fairness” exerted a strong influence on the contemporary welfare economics in general, and social choice theory in particular. But it is predominantly, if not exclusively, in the modified welfaristic version in which the Rawlsian concern with the well-being of the least favoured individual is expressed with reference to the individuals’ welfare levels, which are assumed to be interpersonally comparable. Needless to say, Rawls’ own “difference principle” focuses directly on the minimal availability of “social primary goods,” and not on the minimal individual welfare.

However, as Sen (1976c) acutely pointed out, utility is only used to represent preferences in the theorem of Harsanyi (1955). Thus, there is an ample room for reservation on the claim that Harsanyi’s argument can be interpreted as being an argument in support of utilitarianism. See, also, Prasanta Pattanaik (1968).
Alternatives are arranged horizontally, the more preferred alternative being to the left of the less preferred. Thus, the preference ordering $\alpha$ means that $x$ is preferred to $y$, $y$ is preferred to $z$, hence $x$ is preferred to $z$.

Little’s criticism to this effect was strongly supported by Samuelson (1967, pp.48-49): “For Bergson, one and only one of the ... possible patterns of individuals’ orderings is needed. It could be any one, but it is only one. From it (not from each of them all) comes a social ordering. ... The only Axiom restricting a Bergson Social Welfare Function (of individualistic type) is a ‘tree’ property of Pareto-optimality type.” It is this sharp contrast between the Arrow social welfare function and the Bergson social welfare function that created the widespread perception that the Arrow impossibility theorem, which requires the full force of multiple profiles of individual preference orderings, does not apply to the Bergson social welfare function which is rooted in the single profile framework.

To keep the record straight, let us emphasize that the Arrovian impossibility theorem is not a theorem which negates the existence of the Bergson-Samuelson social welfare ordering; it is a theorem which negates the existence of a “reasonable” process or rule which can associate a Bergson-Samuelson social welfare ordering with each profile of individual preference orderings. See Suzumura (1976, 1987) and Arrow (1983).

Herbert Scarf (1960) constructed an explicit example where the competitive equilibrium is globally unstable. See also Takashi Negishi (1962).

As far as the relative performance of the competitive market economy and the Lange-Lerner scheme of market socialism is concerned, this objection is a double-edged sword; it applies not only to the Lange-Lerner scheme of market socialism, but also to the competitive market economy. But the basic fact remains that the Lange-Lerner scheme is not successful as a decentralized algorithm for computing a general equilibrium solution in a socialist state, as it was originally meant to be.

Note, in passing, that cardinality of individual utilities without interpersonal comparability does not provide us with any escape route from the Arrovian impossibility theorems. Indeed, it was shown by Sen (1970a, Theorem 8*2) that there exists no social welfare functional --- which is “a mechanism that specifies one and only one social ordering given a set of individual welfare functions, one function for each individual [Sen (1970a, pp.123-124)]” --- satisfying the following conditions: unrestricted domain, independence of irrelevant alternatives, non-
dictatorship, weak Pareto principle, cardinality, and non-comparability.

24 The interpersonal comparisons of the extended sympathy type was first formulated with rich applications by Patrick Suppes (1966).


26 See, also, Arrow, Sen and Suzumura (1996-1997).
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