## BANCA D'ITALIA

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# Optimal Debt Maturity under EMU 

by Raffaela Giordano



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# OPTIMAL DEBT MATURITY UNDER EMU 

by Raffaela Giordano *


#### Abstract

A standard result of optimal debt management models is that in a world of complete markets, where policymakers can make credible announcements, the maturity structure of government debt is totally irrelevant. This paper investigates the role of debt maturity in a very simple context in which policy precommitment is allowed but state-contingent debt cannot be issued and a constraint on the level of deficit is imposed. In line with the optimal taxation approach, in such a context debt maturity has a role to play. In fact, an appropriate choice of the maturity structure may remove the inefficiency introduced by the constraint on excessive deficits, making complete tax smoothing achievable when otherwise it would not be. Optimal maturity is shown to depend on the stochastic structure of the economy. In particular, it lengthens with the volatility of the interest rate and the size of debt; it shortens with the volatility of government spending and in the presence of a negative correlation between changes in interest rates and government financing needs. This suggests that, among the countries joining the European Union, Italy is the one that most needs to lengthen the maturity of its public debt. By looking at the evidence in some EU countries, the paper further investigates whether the introduction of constraints on deficit and debt levels has determined a change in debt management policies. This indirectly provides a test of the optimal taxation argument for public debt design.


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Keywords: debt management, maturity structure, optimal taxation.

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## 1. Introduction ${ }^{1}$

A standard result of optimal debt management models is that in a world of complete markets, where policymakers can make credible announcements, the maturity structure of government debt is totally irrelevant. If either of the two assumptions is relaxed, however, debt maturity has a role to play. ${ }^{2}$

With complete markets for government securities, the optimal path for labor income tax rates across time and states of nature can be achieved by government borrowing in state-contingent securities. The optimal debt contract specifies low returns when revenues from labor income are lower and/or government spending is higher than expected. Because of the costs of financial innovation and moral hazard, however, no debt explicitly contingent on output or government spending is being issued in the real world. So, implicit contingencies must be obtained by an appropriate choice of conventional debt instruments. In particular, if macroeconomic shocks to the budget are associated with changes in the yield curve, the maturity structure can play an important hedging role and limit tax adjustment. Barro (1995) shows, for example, that the maturity structure can be designed to insulate the government's financing costs from shifts in real interest rates. Bohn (1990) tests whether the structure of U.S. debt is consistent with the optimal taxation approach using quarterly data on tax rates and security returns for the period 1954-1987, finding a debt structure that diverges from optimality.

Moreover, public debt induces time inconsistency of fiscal and monetary policy in the presence of distortionary taxation. In fact, once the debt is issued, the least distortionary policy calls for levying taxes on debt or reducing its real value through surprise inflation instead of raising taxes on labor income. In the absence of policy precommitment, debt maturity can be used to affect the incentives of future policymakers and neutralize the time-inconsistency problems. This has been shown by Lucas and Stokey (1983), Persson, Persson and Svensson (1987), Calvo and Guidotti (1990a, 1990b), Missale and Blanchard (1994), Drudi and Giordano (2000) and others. ${ }^{3}$

[^1]In this paper I investigate the role of debt maturity in a very simple context in which policy precommitment is allowed but state-contingent debt cannot be issued and there is a constraint on the level of the deficit. This setup can be said to represent the situation in which countries in the Economic and Monetary European Union are currently working quite well. On the one hand, countries joining the Union agreed to keep inflation below a set threshold. In fact, monetary policy has been delegated to an independent central bank, whose primary objective, as expressely stated in its Statute, is to maintain price stability. Hence, commitment on monetary policy seems a reasonable assumption to make. On the other hand, the Treaty establishing the European Community and the Stability and Growth Pact, signed in Amsterdam in 1997, provide countries in the European Union, and notably those having adopted the euro, with a common code of fiscal conduct that is expected to maintain discipline in the management of government finances. In particular, the current provisions of the Pact specify a ceiling for deficit spending of 3 percent of GDP. A violation of the ceiling will trigger warnings and eventually penalties, unless exceptional circumstance can be invoked.

In the presence of a constraint on deficit the tax smoothing theory of government budget - which implies that deficits and surpluses are used optimally to minimize the distortionary effects of taxation - may not hold, if the deficits resulting by applying such a rule happen to be higher than the maximum level allowed. When the constraint binds, it may no longer be optimal to smooth taxes over periods, since a cost associated with the penalty adds to the cost of distortionary taxation. I show that in such a context an appropriate choice of the maturity structure may remove the inefficiency introduced by the constraint on excessive deficits, making complete tax smoothing achievable when otherwise it would not be. ${ }^{4}$

The existence of an explicit deficit constraint only reinforces the implicit-contingent argument for debt design, so the prescriptions for public debt management that I obtain here are similar to those deriving from the implicit-contingent approach. In particular, optimal maturity depends on the stochastic structure of the economy, lengthening with the volatility of the interest rate and the size of debt and shortening with the volatility

[^2]of government spending and when there is a negative correlation between changes in interest rates and government financing needs.

These findings are tested in the last part of the paper. Because of the commitment on inflation, public debt management policies in the countries joining the Monetary Union should now focus more than in the past on the maturity structure, rather than on indexation features and denomination. This leads me to expect that if a change in debt management policies has occurred under the Union, it has been to assign a more important role to debt maturity. Further, because of the introduction of an explicit deficit constraint, such a choice should now become closer than in the past to the indications of the implicit-contingent approach. Evidence from some EU countries is used to test whether, and how, the introduction of deficit and debt constraints has modified debt management policies. This indirectly provides a test of the optimal taxation argument for public debt design.

## 2. The model

### 2.1 The economy

Agents are risk-neutral and set rates of return on government debt according to the following no-arbitrage conditions:

$$
\begin{gather*}
1+r_{01}=E_{0}\left[1+r_{1}\right],  \tag{1}\\
\left(1+r_{02}\right)^{2}=E_{0}\left[\left(1+r_{1}\right)\left(1+r_{2}\right)\right],  \tag{2}\\
1+r_{12}=E_{1}\left[1+r_{2}\right], \tag{3}
\end{gather*}
$$

where $r_{i j}$ denotes the rate of return on government debt issued in period $i$ with maturity in period $j$, and $r_{t}$ is the interest rate at time $t(t=1,2) .{ }^{5} E_{t}$ is the expectation operator given the information available at time $t$. Equations (1), (2) and (3) can be thought of as first-order conditions of a simple intertemporal optimization problem, in which riskneutral agents choose a path for consumption and saving, given that the rate of return on saving between periods $i$ and $j$ is $r_{i j}$ and their time preference discount rate equals the interest rate.

### 2.2 The government

The government is assumed to have a three-period horizon. Government expenditure occurs at two dates, period 1 and period 2.

[^3]In period 0 the government does not levy any tax, and the debt issued is equal to a given initial stock of nominal debt, $D_{0}$. Debt issued in period 0 can be both short-term debt, maturing in period 1 , and long-term debt, maturing in period $2 .{ }^{6}$ In period 1 the government finances primary expenditure, $g_{1}$, pays interest on outstanding debt and repays the maturing principal, by levying distortionary taxes on labor income and issuing new short-term debt. Finally, in period 2 the government finances primary expenditure, $g_{2}$, and repays the debt issued in periods 0 and 1 (interests plus principal) by using the revenue from labor income taxes and by issuing new debt. The amount of outstanding debt left at the end of period 2 cannot exceed a reference value, $D_{2}$. In particular, in the analysis which follows I let $D_{2}=D_{0}$, so that debt in period 2 cannot exceed the initial stock of debt. This assumption, which implies balanced budgets in the medium run, is perfectly consistent with the provisions of the Stability and Growth Pact but not with the Treaty establishing the European Community. As a matter of fact, compliance with the budgetary discipline imposed by the Monetary Union is also examined with respect to the ratio of government debt to GDP, which is expected to be below the reference value of 60 percent or to be diminishing enough to approach that value at a satisfactory pace. In our simplified context, in which GDP growth is zero and no other factor beside the deficit affects the stock of debt, this would imply that countries with government debt above 60 percent are never allowed to run a deficit. For those with initial debt below the reference value, the constraint on debt in period $2\left(D_{2}=D_{0}\right)$ is not necessary to meet the requirements for fiscal discipline. Nonetheless, such an hypothesis allows me to restrict the number of possible situations that may in principle arise (by constraining the deficit to occur in one period only and its size to coincide with that of the surplus in the other period). This simplifies the description of the equilibria without affecting the qualitative features of the results. ${ }^{7}$

The government's budget constraints in the three periods are:

$$
\begin{gather*}
D_{0}=D_{01}+D_{02}  \tag{4}\\
g_{1}+\left(1+r_{01}\right) D_{01}+r_{02} D_{02}=\tau_{1}+D_{12}  \tag{5}\\
g_{2}+\left(1+r_{02}\right) D_{02}+\left(1+r_{12}\right) D_{12}=\tau_{2}+D_{0}, \tag{6}
\end{gather*}
$$

[^4]where $D_{i j}$ denotes the value of public debt issued in period $i$ with maturity in period $j$ and $\tau_{t}$ is the labor income tax in period $t$.

Government expenditure and interest rates are assumed to be exogenous to government policy. ${ }^{8}$ In period 0 government observes interest rate and expenditure in period 1 , but not those in period 2. In particular, I assume

$$
\begin{equation*}
r_{2}=r_{1}+\rho \tag{7}
\end{equation*}
$$

and

$$
\begin{equation*}
g_{2}=g_{1}+\gamma, \tag{8}
\end{equation*}
$$

where $\rho$ and $\gamma$ are random shocks that take values $\bar{\rho}$ and $-\bar{\rho}$ and $\bar{\gamma}$ and $-\bar{\gamma}$, respectively, with probability $1 / 2$.

In period 0 the only decision faced by the government is choosing the maturity structure of the new debt, $D_{0}$. In period 1 the government chooses the amount of maturing debt to be rolled over to period $2, D_{12}$. This determines the sequence of income taxes $\left(\tau_{1}, \tau_{2}\right)$ that must be levied in order to satisfy the budget constraints in periods 1 and 2 .

The government's optimal choice of instruments responds to the objective of minimizing the value of the following intertemporal cost function:

$$
\begin{equation*}
L_{0}^{g}=E_{0}\left[\frac{1}{2} \tau_{1}^{2}+p_{1}+\left(1+r_{2}\right)^{-1}\left(\frac{1}{2} \tau_{2}^{2}+p_{2}\right)\right], \tag{9}
\end{equation*}
$$

where $p_{t}$ denotes the penalty associated with an excessive deficit and the time preference discount is equal to the interest rate.

The government's policy affects welfare via the distortionary costs of taxation. ${ }^{9}$ In addition to this cost, whenever the deficit results to be above the allowed ceiling, $\bar{d}$, the government incurs into a penalty.

Let $d_{t}$ denote the budget deficit in period $t$; that is

$$
\begin{equation*}
d_{1}=g_{1}+r_{01} D_{01}+r_{02} D_{02}-\tau_{1}, \tag{10}
\end{equation*}
$$

[^5]\[

$$
\begin{equation*}
d_{2}=g_{2}+r_{02} D_{02}+r_{12} D_{12}-\tau_{2} \tag{11}
\end{equation*}
$$

\]

By replacing $\tau_{1}$ and $\tau_{2}$, respectively, from budget constraints (5) and (6) we get

$$
\begin{gather*}
d_{1}=D_{12}-D_{01},  \tag{12}\\
d_{2}=D_{0}-D_{02}-D_{12} . \tag{13}
\end{gather*}
$$

Hence,

$$
p_{t}=\left\{\begin{array}{cl}
\alpha & \text { if } d_{t}>\bar{d} \\
0 & \text { otherwise } .
\end{array}\right.
$$

I model the penalty as a constant, for two reasons. First, we can interpret such a cost as associated with the loss of reputation for incurring the excessive deficit procedure, rather than with the monetary sanction itself. Second, the fine is quite small and only up to a point proportional to the amount of excess deficit. ${ }^{10}$

The timing of the game is as follows. In period 0 all agents in the economy observe government expenditure $g_{1}$ and the interest rate $r_{1}$. Given the stock of debt to be rolled over, $D_{0}$, the government issues short-term and long-term debt, $D_{01}$ and $D_{02}$. Private agents set rates of return on government debt according to equations (1) and (2). In period $1 r_{2}$ and $g_{2}$ realize and the goverment chooses income taxes in periods 1 and 2 , $\tau_{1}$ and $\tau_{2}$, and the amount of new short-term debt to be issued, $D_{12}$. Private agents set $r_{12}$ according to (3). ${ }^{11}$ In the next section I consider the unconstrained case, in which no penalty is imposed on goverments running excessive deficits. Under such a setup I investigate the role of debt maturity in minimizing the expected loss to government. I then analyze how the unconstrained "first-best" solution is modified by the introduction of a limit on the allowed level of deficit.

[^6]For all setups, I first solve the problem faced by the government in period 1 ; that is, the choice of income taxes in periods 1 and 2 , given the maturity structure of outstanding debt. Then I move backward to period 0 and solve for the optimal maturity structure, given the incentive compatibility constraints in period 1.

## 3. The unconstrained case

This section briefly analyzes the first-best solution when no constraints on the level of deficit are imposed. This serves as a benchmark against which to evaluate how the rules for optimal debt management may change when sanctions for excessive deficits are envisaged.

Since period 2 interest rate and expenditure are observed at time 1, the problem of the government in period 1 is under certainty and consists of choosing $D_{12}$ to minimize loss function

$$
\begin{equation*}
L_{1}^{g}=\frac{1}{2} \tau_{1}^{2}+\frac{1}{2\left(1+r_{2}\right)} \tau_{2}^{2} \tag{14}
\end{equation*}
$$

subject to budget constraints (5) and (6), where $r_{01}=r_{1}, r_{02}=r_{1}$ and $r_{12}=r_{2}$.
The first-best choice of the amount of short-term debt to be rolled over to the second period is

$$
\begin{equation*}
D_{12}=\frac{\left(2+r_{1}\right) D_{01}-\gamma}{2+r_{1}+\rho} \tag{15}
\end{equation*}
$$

which depends on the maturity structure of the outstanding debt ( $D_{01}$ ) and on the state of the economy, i.e. on the realizations of $\rho$ and $\gamma$. This in turn implies

$$
\begin{equation*}
\tau_{1}=\tau_{2}=r_{1} D_{0}+g_{1}+\frac{\rho D_{01}+\gamma}{2+r_{1}+\rho} \tag{16}
\end{equation*}
$$

Equation (16) indicates that under the unconstrained regime perfect tax smoothing over time is optimal.

In period 0 the problem is to choose the maturity structure of the given initial stock of public debt $D_{0}$ in order to minimize (9), where $p_{1}=p_{2}=0$ and $\tau_{1}$ and $\tau_{2}$ are given by (16).

By substituting $\tau_{1}$ and $\tau_{2}$ from equation (16) into (9), it is evident that, with no uncertainty over future realizations of the interest rate (i.e. $\rho=0$ ), the problem is entirely independent of the maturity structure of the initial debt; that is, debt maturity is irrelevant. Because of the presence of uncertainty over future rates, however, debt maturity plays a role even in this context. In particular, $D_{02}=0$ solves the government's
problem. In fact, longer maturities are good in the presence of high realizations of the interest rate in the second period but bad with low realizations, since short-term debt will be rolled over at the new rate. As the effect of debt maturity on taxes, and thus on loss, is relatively bigger in the presence of low rates ${ }^{12}$, it is optimal for the government to set the maturity as short as possible. Nonetheless, this is a second-order effect and thus negligible. The important result here is that perfect tax smoothing is achieved for any initial choice of the maturity; this is so even with stochastic interest rates.

## 4. The case with a penalty

In this section I consider the situation in which, if the deficit violates the ceiling, the government is subject to a penalty and bears the associated cost.

Under the hypothesis that debt in period 2 cannot exceed the initial stock, a deficit in one period must be offset by a surplus in the other. Therefore, perfect tax smoothing may either entail fiscal discipline (i.e., a deficit below the ceiling) in both periods or an excessive deficit in one. The first situation will occur when the pattern of expenditures is sufficiently smooth over time; the second when the distribution of expenditures is so uneven that perfect tax smoothing is achieved only by running a large deficit in one period and a large surplus in the other.

When a constraint on the deficit level is imposed, two different situations may actually occur. The maturity structure of outstanding debt and the size of the shocks may be such that the constraint on the deficit is never binding. Or else in some state of the economy perfect tax smoothing can be achieved only at the cost of incurring an excessive deficit.

### 4.1 The equilibrium with no penalty

Let $D_{12}^{*}$ denote the solution to the unconstrained problem, as given by equation (15) and $d_{t}^{*}(t=1,2)$ the associated deficit levels in periods 1 and 2 . That is

$$
\begin{equation*}
d_{1}^{*}=D_{12}^{*}-D_{01}=-\frac{\rho D_{01}+\gamma}{2+r_{1}+\rho} \tag{17}
\end{equation*}
$$

and

$$
\begin{equation*}
d_{2}^{*}=D_{0}-D_{02}-D_{12}^{*}=\frac{\rho D_{01}+\gamma}{2+r_{1}+\rho} . \tag{18}
\end{equation*}
$$

[^7]Notice that, due to the constraint imposed on the stock of debt, $d_{1}=-d_{2}$. The problem can thus be summarized as follows. If

$$
\left|\frac{\rho D_{01}+\gamma}{2+r_{1}+\rho}\right| \leq \bar{d}
$$

then the first-best choice of income taxes also solves problem (9). In this equilibrium the deficit constraint does not bind and complete tax smoothing is optimal.

### 4.2 The equilibrium with risk of penalty

Suppose now that, given the maturity structure of the outstanding debt, in some state of the economy perfect tax smoothing is achieved only at the cost of an excessive deficit in one period. That is, given $D_{01}$, there exist some combinations of $\rho$ and $\gamma$ such that

$$
\left|\frac{\rho D_{01}+\gamma}{2+r_{1}+\rho}\right|>\bar{d}
$$

When this is the case, the government has to choose whether to smooth taxes perfectly over periods and incur the penalty or to allow for different tax levels in the two periods and avoid the sanction. Obviously, the trade-off depends on the relative costs of sanction and tax distortion. Let $L_{1}^{g}\left(\tau_{1}^{*}, \tau_{2}^{*}\right)$ be the government's loss in period 1 in the first-best equilibrium. Then let $D_{12}^{m}$ denote the minimum amount of short-term debt to be rolled over to period 2 compatible with no excessive deficit in period 2 , and $\tau_{t}^{m}$ $(t=1,2)$ denote the associated level of taxes in periods 1 and 2 . Of course, $\tau_{1}^{m} \neq \tau_{2}^{m}$. Such a tax profile minimizes tax distortion, subject to the requirement that the deficit not exceed the ceiling. Analogously, let $D_{12}^{M}$ denote the maximum amount of short-term debt to be rolled over to period 2 compatible with no excessive deficit in period 1 , and $\tau_{t}^{M}(t=1,2)$ the associated levels of taxes in periods 1 and 2. $D_{12}^{m}$ and $D_{12}^{M}$ are obtained simply by solving $D_{01}-D_{12}^{m}=\bar{d}$ and $D_{12}^{M}-D_{01}=\bar{d}$.

Suppose first that perfect tax smoothing implies an excessive deficit in period 2. Then the equilibrium will entail perfect tax smoothing and penalty if

$$
\begin{equation*}
L_{1}^{g}\left(\tau_{1}^{*}, \tau_{2}^{*}\right)+\alpha<L_{1}^{g}\left(\tau_{1}^{m}, \tau_{2}^{m}\right) \tag{19}
\end{equation*}
$$

In this equilibrium the optimal choice of taxes in periods 1 and 2 is still given by $\left(\tau_{1}^{*}, \tau_{2}^{*}\right)$, but a cost $\alpha$ adds to the government's loss under the first-best outcome. If, instead,

$$
\begin{equation*}
L_{1}^{g}\left(\tau_{1}^{*}, \tau_{2}^{*}\right)+\alpha>L_{1}^{g}\left(\tau_{1}^{m}, \tau_{2}^{m}\right) \tag{20}
\end{equation*}
$$

then in equilibrium the government will choose not to get caught in the excessive deficit procedure. In order to do that, however, it must forgo perfect tax smoothing. In this equilibrium the optimal choice of taxes in periods 1 and 2 is $\left(\tau_{1}^{m}, \tau_{2}^{m}\right)$.

Suppose now that perfect tax smoothing implies an excessive deficit in period 1. Then the equilibrium will entail perfect tax smoothing and penalty if

$$
\begin{equation*}
L_{1}^{g}\left(\tau_{1}^{*}, \tau_{2}^{*}\right)+\alpha<L_{1}^{g}\left(\tau_{1}^{M}, \tau_{2}^{M}\right) \tag{21}
\end{equation*}
$$

If, instead,

$$
\begin{equation*}
L_{1}^{g}\left(\tau_{1}^{*}, \tau_{2}^{*}\right)+\alpha>L_{1}^{g}\left(\tau_{1}^{M}, \tau_{2}^{M}\right) \tag{22}
\end{equation*}
$$

then the optimal choice of taxes in periods 1 and 2 is $\left(\tau_{1}^{M}, \tau_{2}^{M}\right)$.
Both equilibria are of course second-best. The question is therefore whether an appropriate choice of the maturity is able to eliminate this inefficiency. Generally it is, the intuition being the following. By lengthening the maturity structure of the initial debt, the government reduces the amount of short-term debt that has to be rolled over to the second period in order to achieve perfect tax smoothing. In the absence of uncertainty over future realizations of the interest rate, such a reduction in $D_{12}$ just offsets the decrease in $D_{01}$, so that the budget outcomes in the two periods remain unchanged (recall that $d_{1}=D_{12}-D_{01}$ and $d_{2}=D_{01}-D_{12}$ ). However, with stochastic interest rates the change in the maturity is not irrelevant. In fact, the tax-smoothing amount of $D_{12}$ does not react to the change in the maturity one-to-one, as the effect is smaller (greater) than one when there is a high (low) realization of the interest rate in period 2. This implies that in the presence of high interest rates in period 2 shorter maturities will be associated with smaller deficits (or larger surpluses) in period 1 and larger ones in period 2. In other words, if the interest rate is high (low), interest payments in period 2 will be lower (higher) the longer the maturity, since a smaller amount of debt must be rolled over at the new rate. This may avoid excessive deficit in period 2 if a high realization of the interest rate occurs (or, analogously, avoid excessive deficit in period 1 for a low interest rate). Formally, this can be seen by differentiating deficits in periods 1 and 2 with respect to $D_{01}$ :

$$
\begin{aligned}
\frac{\partial d_{1}}{\partial D_{01}} & =-\frac{\rho}{2+r_{1}+\rho} \\
\frac{\partial d_{2}}{\partial D_{01}} & =\frac{\rho}{2+r_{1}+\rho}
\end{aligned}
$$

For positive (negative) realizations of $\rho$, larger fractions of long-term debt increase (decrease) the deficit in the first period and decrease (increase) that in the second.

In particular, for given realizations of $\gamma$ and $\rho$, the first best choice of taxes in periods 1 and 2 would be achieved without running an excessive deficit if the amount of shortterm debt satisfied

$$
D_{01}^{*}-\frac{\left(2+r_{1}\right)\left(D_{01}^{*}\right)-\gamma}{2+r_{1}+\rho} \leq \bar{d}
$$

Solving this expression with the equality sign we obtain the value of $D_{01}$ that makes $D_{12}^{*}=D_{12}^{m}$.

In the following section I investigate the optimal choice of the maturity in period 0 , when the realizations of the shocks to primary and interest expenditures in period 2 are still uncertain.

## 5. The optimal choice of maturity

The previous section showed that when there is a deficit constraint the optimal choice of taxes in periods 1 and 2 , given the maturity structure of outstanding debt, may be inefficient. Now let us examine whether an appropriate choice of maturity can overcome the inefficiency.

The government's problem in period 0 is to choose the maturity structure of debt $D_{0}$ to minimize expected loss (9). Given the stochastic properties of the shocks to primary expenditure and interest rates, four states may realize in period 2: (1) high primary and interest expenditure (i.e., $\gamma=\bar{\gamma}$ and $\rho=\bar{\rho}$ ); (2) high primary and low interest expenditure (i.e., $\gamma=\bar{\gamma}$ and $\rho=-\bar{\rho}$ ); (3) low primary and high interest expenditure (i.e., $\gamma=-\bar{\gamma}$ and $\rho=\bar{\rho}$ ); (4) low primary and interest expenditure (i.e., $\gamma=-\bar{\gamma}$ and $\rho=-\bar{\rho})$.

Let $\Pi_{i, j}$ be the probability that the state $i, j$ occurs, where $i=l, h$ refers to primary expenditure and $j=l, h$ refers to interest payments; states $l$ (low) and $h(h i g h)$ occur in the presence of a negative and a positive realization of the shock, respectively. That is, $\Pi_{h, h}=\operatorname{Pr}(\gamma=\bar{\gamma}, \rho=\bar{\rho}), \Pi_{h, l}=\operatorname{Pr}(\gamma=\bar{\gamma}, \rho=-\bar{\rho}), \Pi_{l, h}=\operatorname{Pr}(\gamma=-\bar{\gamma}, \rho=\bar{\rho})$ and $\Pi_{l, l}=\operatorname{Pr}(\gamma=-\bar{\gamma}, \rho=-\bar{\rho})$. In our example we have $\Pi_{h, h}+\Pi_{l, h}=1 / 2$ and $\Pi_{h, h}+\Pi_{h, l}=1 / 2$. This, together with the condition $\Pi_{h, h}+\Pi_{h, l}+\Pi_{l, h}+\Pi_{l, l}=1$, implies that $\Pi_{h, h}=\Pi_{l, l}$ and $\Pi_{l, h}=\Pi_{h, l}$.

In the analysis that follows I let $\Pi_{l, h}=\Pi_{h, l}=0$, to address the case of perfectly positively correlated shocks, and $\Pi_{h, h}=\Pi_{l, l}=0$, to address the case of negatively correlated shocks. Finally I allow all probabilities to be positive; in particular, $\Pi_{h, h}=$
$\Pi_{l, l}=\Pi_{l, h}=\Pi_{h, l}=1 / 4$ makes the correlation between the shocks equal to zero.
As we know from equations (17) and (18), perfect tax smoothing induces a deficit in one period equal to

$$
\frac{\rho D_{01}+\gamma}{2+r_{1}+\rho}
$$

From this expression it is evident that a constant pattern of primary and interest expenditures (i.e., $\bar{\rho}=0$ and $\bar{\gamma}=0$ ) implies balanced budgets in both periods. Moreover, for $\bar{\rho}=0$ the deficit is independent of $D_{01}$. That is, debt maturity has a role to play only in the presence of volatile interest rate processes, as only in this circumstance does it affect interest payments and hence deficits. I thus assume $\bar{\rho}>0$ and $\bar{\gamma} \geq 0$.

In our context, a maturity will be optimal if it minimizes the expected number of states in which an excessive deficit is incurred.

Proposition 1: Let $\Pi_{l, h}=\Pi_{h, l}=0$. Then the optimal choice of the maturity satisfies

$$
D_{01}^{*} \leq \frac{\left(2+r_{1}-\bar{\rho}\right) \bar{d}-\bar{\gamma}}{\bar{\rho}}
$$

With such a choice of the maturity and perfectly positively correlated shocks, the deficit never exceeds the ceiling and the first-best outcome is achieved. But notice that if the right-hand side of the inequality is lower than zero, the deficit remains above the allowed level even setting $D_{01}=0$. In this situation, which is likely to occur for large enough values of $\bar{\gamma}$ and thus in the presence of highly volatile primary expenditure, an appropriate choice of maturity minimizes the deficit. But it cannot remove the inefficiency associated with the deficit constraint, which remains binding.

Proposition 2: Let $\Pi_{h, h}=\Pi_{l, l}=0$. Then the optimal choice of the maturity satisfies

$$
\frac{\bar{\gamma}-\left(2+r_{1}-\bar{\rho}\right) \bar{d}}{\bar{\rho}} \leq D_{01}^{*} \leq \frac{\bar{\gamma}+\left(2+r_{1}-\bar{\rho}\right) \bar{d}}{\bar{\rho}}
$$

That is, in the presence of perfectly negatively correlated shocks, short enough maturities can eliminate the risk of incurring excessive deficits. Notice that the higher $\bar{\gamma}$, the larger the fraction of short-term debt that should be issued. In fact, shocks to primary expenditure are compensated here by opposite movements in interest payments. For these to be large enough, the maturity has to be short enough. Maturity ought not to be too short, though, as the reaction of interest payments to rate shocks should not exceed the changes in primary expenditure induced by fluctuations in $\gamma$ by too much.

Proposition 3: Let both $\Pi_{h, h}>0$ and $\Pi_{l, h}>0$; by symmetry, we will also have $\Pi_{l, l}>0$ and $\Pi_{h, l}>0$. Shocks to primary and interest expenditures will be uncorrelated if $\Pi_{i, j}=1 / 4$ for all combinations of $i$ and $j$. Then if

$$
\frac{\gamma}{2+r_{1}-\bar{\rho}} \leq \bar{d}
$$

that is if the shock to primary expenditure is not large enough to generate an excessive deficit by itself, the optimal choice of the maturity satisfies

$$
D_{01}^{*} \leq \frac{\left(2+r_{1}-\bar{\rho}\right) \bar{d}-\gamma}{\bar{\rho}} .
$$

If instead

$$
\frac{\gamma}{2+r_{1}-\bar{\rho}}>\bar{d}
$$

a value of $D_{01}$ that eliminates the risk of an excessive deficit does not exist. In such circumstances the best the government can do is to set the maturity short enough to prevent the emergence of excessive deficits, at least in the presence of realizations of the shocks with opposite sign. This implies

$$
\frac{\gamma-\left(2+r_{1}-\bar{\rho}\right) \bar{d}}{\bar{\rho}} \leq D_{01}^{*} \leq \frac{\gamma+\left(2+r_{1}-\bar{\rho}\right) \bar{d}}{\bar{\rho}}
$$

In the case of positively correlated shocks, the deficit decreases with maturity. The intuition is straightforward: longer maturities make interest expenditure less sensitive to changes in interest rates. Since shocks have the same sign, longer maturities also make total expenditure less volatile, hence closer in both periods to the first-best level of taxes.

When shocks occur with opposite sign, an increase in $D_{01}$ generally lowers the deficit and therefore reduces the risk of incurring an excessive deficit.

When all probabilities $\Pi_{i, j}$ are positive, whether the risk of incurring an excessive deficit is reduced or increased by lenghtening the maturity depends on the relative sizes of $\bar{\rho} D_{01}$ and $\bar{\gamma}$. In particular, given the maturity structure of outstanding debt, let $\bar{\gamma}$ be large enough to generate an excessive deficit under both realizations of the interest rate. Then shorter maturity may keep the deficit constraint from binding in at least one state of the economy. The problem is analogous to that which arises in the presence of negatively correlated shocks, as the objective here is to bring the deficit below the ceiling only when shocks have opposite signs. In this circumstance, therefore, shortening the maturity is optimal. In contrast, for sufficiently small $\bar{\gamma}$ a longer maturity reduces the risk.

## 6. Some evidence

We have seen that the optimal maturity of government debt depends on the stochastic structure of the economy. In particular, it lengthens with the volatility of interest rates $(\bar{\rho})$ and shortens with that of primary expenditure $(\bar{\gamma})$. The budgetary effect of interest rate shocks is augmented when the stock of debt is large. Hence, optimal maturity lengthens with the amount of debt. ${ }^{13}$ A positive (negative) correlation between interest rates and government expenditure decreases (increases) the fraction of short-term paper that should be issued.

In the sections that follow I perform two separate tests: one on a panel of European Union countries, the second a specific analysis of the Italian case.

### 6.1 A test on some European Union countries

The sample used here covers of 11 EU countries (Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Netherlands, Spain, Sweden and United Kingdom) for which data on the relevant variables are available for a reasonable number of years.

Data on debt maturity are from Missale (1999). All other figures are based on Bank of Italy, European Commission and Eurostat data. Debt maturity is the average residual life, corrected where possible for the presence of variable-rate bonds. Government spending and debt are expressed as ratios to GDP. Data on primary expenditure refer to current expenditure only. In fact, this is the budget item most likely subject to shocks. However, very similar results are obtained when total primary expenditure is used. Interest rates are 3 -month interbank rates for all countries but Italy, for which I use the average rate on 3 -month, 6 -month and 1-year Treasury bills. Data on interest rates are monthly. Nominal interest rates are deflated by the change in the price level occurred in the previous 12 months. Volatility and correlation are measured by the standard deviation and the correlation coefficient.

Table 1 reports the average debt, the average debt maturity, the standard deviations of primary expenditure and the real short-term interest rate and the correlation coefficient between them. I also report the volatility of interest expenditure, which can be read as summarizing the interest rate volatility and the size and the maturity of the public debt. Naturally, a large stock of debt or a short maturity will make interest payments

[^8]highly reactive to changes in interest rates. All statistics refer to 1980-1998, with the exception of the average maturity which is computed for 1980-1995. ${ }^{14}$

A striking case is that of Italy, which according to the model is the country that most needs to lengthen the maturity of its public debt. The volatility of its primary expenditure is significantly lower than the average while that of interest rates is just below the average. This, associated with its large debt and short maturity, makes the standard deviation of interest payments the highest in the sample. Interest expenditure varied substantially during the sample period owing to the sharp decline in Italian interest rates ${ }^{15}$, as well as the size and the maturity structure of the debt. But primary expenditure in Italy does not typically react significantly to changes in economic activity. This may explain the relatively low variance. Moreover, the correlation between primary expenditure and real short-term interest rates seems to be positive and high.

The objective of the empirical analysis is twofold. First I ask whether debt management policies actually follow the indications suggested by the model. Second, I test whether the introduction of constraints on deficit and debt levels produced a change in debt management policies in the countries adopting the single currency. As such constraints reduce the scope for deficit financing to smooth taxes across time and states of the economy, this is an indirect test of the optimal taxation argument for public debt design. To address the first question I look at the entire panel of data, using all information available for each country. ${ }^{16}$ For the second issue, this approach would introduce a bias, since some countries would be absent in one of the sub-periods, making the comparison meaningless. Thus the second analysis is performed on a sub-sample of countries for which data are available from at least 1981: Belgium, Germany, Ireland, Italy, the Netherlands, Spain and the UK.

The volatility of the real interest rate is measured by the standard deviation of monthly observations within each year. The standard deviation of primary expenditure and the correlation between the yearly average of the interest rate and government expenditure are computed using 5 -period centered moving averages. Including more than

[^9]5 periods in the moving averages reduces the number of observations without significantly affecting the results.

I test the following relationship:

$$
\begin{equation*}
M A T=\alpha_{0}+\alpha_{1} D E B T+\alpha_{2} C O R R+\alpha_{3} S T D R A T E+\alpha_{4} S T D G \tag{23}
\end{equation*}
$$

where MAT denotes the average maturity and $D E B T$ the debt/GDP ratio; CORR is the correlation coefficient between the real short-term interest rate and government expenditure; STDRATE and STDG denote the standard deviations of real interest rates and government expenditure.

My argument suggests that optimal maturity lengthens with the volatility of the interest rate and the size of debt and shortens with the volatility of government spending and with a negative correlation between changes in interest rates and government financing needs. Therefore, positive estimated coefficients for $\alpha_{1}, \alpha_{2}$ and $\alpha_{3}$ and a negative estimated coefficient for $\alpha_{4}$ support the prediction of the theory.

I test the model over the whole period and in two sub-periods, namely up to 1989 and from 1990 on. The choice of the sub-periods should allow us to capture the effect of the monetary union on debt management policies. I use random-effects GLS estimation in order to take into account the presence of specific cross-sectional effects.

The results of the regressions are shown in table 2.
Over the entire sample period, maturity is positively and significantly linked to the size of debt, the volatility of the interest rate and the correlation between the interest rate and government expenditure, as suggested by the theory. However, the volatility of goverment expenditure has the wrong sign. Using our two sub-periods should provide information about the effects of EMU on debt management policies. In fact, the fit improves in the second sub-period, suggesting that explicit constraints on deficit levels may have given governments stronger incentives to manage their public debts appropriately. Before 1990, maturity shortens with the size of debt, possibly signalling an inability to issue long-term paper in the presence of a large stock of debt; the other regressors are either not significant or wrongly signed. In contrast, in the second sub-period the relationship between maturity and each of the explicative variables takes the sign expected; the size of debt and the correlation coefficient are highly significant.

The concern for proprer public debt management should increase with the size of debt. Accordingly, I seek to confirm the empirical findings excluding Belgium and Italy, the countries with the highest debt levels. And indeed, the data reject the theory for
this subset in the overall period and in the two sub-periods. In the subset of high-debt countries, conversely, the fit improves. In particular, the relationship between maturity and debt is positive, significant and robust to changes in the sample period. Interestingly, from 1990 onwards the volatility of primary expenditure is strongly and negatively linked to the maturity. In fact, whereas tying maturity both to interest rate volatility and the correlation between changes in interest rates and primary expenditure is always advisable to minimize budget risk, the volatility of government expenditures is relevant to optimal maturity only because there is an explicit deficit constraint. As we have seen, in the presence of highly volatile primary expenditure, issuing short-term debt may reduce the number of cases in which excessive deficits are incurred. This is so when the government budget is more sensitive to shocks in primary expenditure than to changes in interest rates (e.g., because of a small stock of debt). In these circumstances, the combination of bad shocks to primary expenditure and low reactivity of interest payments to changes in rates (due to a long maturity) may generate an excessive deficit, which is avoided when maturity is short and interest rates are low.

### 6.2 A test on Italian data

Now we turn to the relationship between the maturity structure of the public debt and the relevant statistics spotlighted by the theoretical model for Italy over the period 1981-1998. The data on debt maturity here are from the Bank of Italy. They are monthly and adjusted for the presence of long-term papers whose returns are indexed to short-term interest rates (Treasury credit certificates). All other data are as described in the previous section. In particular, the real short-term interest rate is the average for the three maturities ( 3,6 and 12 months) of Treasury bills, deflated by the CPI over the preceding 12 -month period. Standard deviations of the real interest rate and government expenditure, as well as the correlation between them, are computed using 5 -period centered moving averages.

The specification of the model is as in equation (23). The test is run both for the entire period and for the two sub-periods, 1981-1989 and 1990-1998. The results of the regressions are shown in table 3.

For the overall period, only the size of debt and the correlation between interest rate and expenditures are correlated with the average residual maturity of the public debt significantly and with the expected sign. The volatility of the interest rate and the volatility of government expenditure both have the wrong sign. In the first sub-period, the debt ratio and the standard deviation of the real interest rate are both negatively
related to maturity, suggesting that increasing stock of debt and volatile interest rates (reflecting high and volatile inflation expectations) made it too costly for the government to issue long-term paper. Debt management strategy was directed mainly to minimizing cost rather than budget risk. In the 1990s, by contrast, the model does explain the choice of the maturity structure when fiscal discipline began to be imposed on countries joining the monetary union. In this sub-period the average residual life of the public debt lengthens with the stock of debt and with the correlation between changes in interest rates and primary expenditure; it shortens with the volatility of government spending. However, the correlation with the volatility of the interest rate is negative and significant, contradicting the requirement for optimality.

## 7. Conclusions

I have explored the optimal maturity structure of public debt in a context in which policy precommitment is allowed but a constraint on the level of the deficit is imposed. I showed that optimal maturity lengthens with the volatility of the interest rate and the size of debt; it shortens with the volatility of government spending and in the presence of a negative correlation between changes in interest rates and the borrowing requirement. Evidence from some EU countries was used to test whether the introduction of constraints on deficit and debt levels induced a change in debt management policies in the countries joining the Monetary Union. As such constraints reduce the scope for deficit financing to smooth taxes across time and states of nature, this provides an indirect test of the optimal taxation argument for public debt design. The results indicate debt management policies adhering to the prescriptions of the theory only in the high debt countries and, generally, only after they joined EMU.

Table 1

## DESCRIPTIVE STATISTICS

| Country | Average <br> debt <br> as percentage <br> of GDP) | Average <br> debt <br> maturity | Primary <br> expend. <br> standard <br> deviation | Interest <br> rate <br> standard <br> deviation | Correlation <br> between <br> prim. exp. <br> and int.rate | Interest <br> expend. <br> standard <br> deviation |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Belgium | 120.2 | 3.8 | 2.58 | 1.95 | 0.18 | 1.37 |
| Denmark | 64.4 | 2.7 | 3.37 | 2.58 | -0.09 | 1.47 |
| Finland | 30.0 | 3.5 | 7.05 | 2.51 | -0.30 | 1.75 |
| France | 37.9 | 6.2 | 1.97 | 2.24 | 0.17 | 0.63 |
| Germany | 45.6 | 4.9 | 1.11 | 1.55 | -0.50 | 0.48 |
| Ireland | 88.6 | 3.8 | 4.04 | 3.70 | -0.15 | 1.95 |
| Italy | 96.0 | 1.2 | 1.73 | 2.01 | 0.69 | 1.98 |
| Netherlands | 71.3 | 5.9 | 2.47 | 1.86 | 0.37 | 0.71 |
| Spain | 46.0 | 2.1 | 2.31 | 2.89 | 0.37 | 1.55 |
| Sweden | 62.1 | 3.9 | 2.72 | 2.36 | -0.17 | 1.09 |
| United Kingdom | 48.9 | 10.5 | 1.91 | 2.07 | -0.37 | 0.71 |
| Unweighted |  |  |  |  |  |  |
| average | 64.7 | 2.84 | 2.34 | 0.02 | 1.24 |  |

Table 2

## REGRESSION RESULTS: POOLED DATA

|  | Entire sample |  |  | High-debt countries excluded |  |  | High-debt countries |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | overall period | $\begin{gathered} \hline \text { before } \\ 1990 \end{gathered}$ | $\begin{aligned} & \text { from } \\ & 1990 \end{aligned}$ | overall period | $\begin{gathered} \hline \text { before } \\ 1990 \end{gathered}$ | $\begin{aligned} & \text { from } \\ & 1990 \end{aligned}$ | overall period | $\begin{gathered} \hline \text { before } \\ 1990 \end{gathered}$ | $\begin{gathered} \text { from } \\ 1990 \end{gathered}$ |
| DEBT | $\begin{gathered} 0.010 \\ (2.222) \end{gathered}$ | $\begin{gathered} -0.017 \\ (-2.168) \end{gathered}$ | $\begin{gathered} 0.031 \\ (3.192) \end{gathered}$ | $\begin{gathered} 0.009 \\ (1.388) \end{gathered}$ | $\begin{gathered} -0.009 \\ (-0.459) \end{gathered}$ | $\begin{gathered} -0.030 \\ (-0.965) \end{gathered}$ | $\begin{gathered} 0.048 \\ (4.954) \end{gathered}$ | $\begin{gathered} 0.038 \\ (2.073) \end{gathered}$ | $\begin{gathered} 0.074 \\ (8.697) \end{gathered}$ |
| CORR | $\begin{gathered} 0.545 \\ (5.106) \end{gathered}$ | $\begin{gathered} 0.285 \\ (1.871) \end{gathered}$ | $\begin{gathered} 0.501 \\ (3.550) \end{gathered}$ | $\begin{gathered} 0.584 \\ (4.830) \end{gathered}$ | $\begin{gathered} 0.616 \\ (0.727) \end{gathered}$ | $\begin{gathered} -0.399 \\ (-0.419) \end{gathered}$ | $\begin{gathered} -0.887 \\ (-2.363) \end{gathered}$ | $\begin{gathered} -0.652 \\ (-1.903) \end{gathered}$ | $\begin{gathered} -0.082 \\ (-0.309) \end{gathered}$ |
| STDRATE | $\begin{gathered} 0.197 \\ (2.284) \end{gathered}$ | $\begin{gathered} 0.194 \\ (1.586) \end{gathered}$ | $\begin{gathered} 0.093 \\ (0.745) \end{gathered}$ | $\begin{gathered} 0.192 \\ (2.029) \end{gathered}$ | $\begin{gathered} -0.862 \\ (-1.391) \end{gathered}$ | $\begin{gathered} -0.978 \\ (-1.223) \end{gathered}$ | $\begin{gathered} -0.068 \\ (-0.154) \end{gathered}$ | $\begin{gathered} 1.156 \\ (1.866) \end{gathered}$ | $\begin{gathered} -0.639 \\ (-2.880) \end{gathered}$ |
| STDG | $\begin{gathered} 0.171 \\ (2.136) \end{gathered}$ | $\begin{gathered} 0.552 \\ (4.727) \end{gathered}$ | $\begin{gathered} -0.053 \\ (-0.309) \end{gathered}$ | $\begin{gathered} 0.160 \\ (1.744) \end{gathered}$ | $\begin{gathered} 0.666 \\ (1.122) \end{gathered}$ | $\begin{gathered} 0.146 \\ (0.150) \end{gathered}$ | $\begin{gathered} -0.138 \\ (-0.338) \end{gathered}$ | $\begin{gathered} 0.567 \\ (0.976) \end{gathered}$ | $\begin{gathered} -1.451 \\ (-5.601) \end{gathered}$ |
| Constant | $\begin{gathered} 3.282 \\ (4.168) \end{gathered}$ | $\begin{gathered} 5.008 \\ (3.585) \end{gathered}$ | $\begin{gathered} 2.147 \\ (1.739) \end{gathered}$ | $\begin{gathered} 3.922 \\ (3.985) \end{gathered}$ | $\begin{gathered} 6.467 \\ (4.896) \end{gathered}$ | $\begin{gathered} 7.845 \\ (2.932) \end{gathered}$ | $\begin{aligned} & -2.519 \\ & (-2.257) \end{aligned}$ | $\begin{gathered} -2.824 \\ (-1.593) \end{gathered}$ | $\begin{gathered} -4.458 \\ (-3.649) \end{gathered}$ |
| $R^{2}$ (overall) | 0.51 | 0.11 | 0.24 | 0.03 | 0.07 | 0.14 | 0.68 | 0.89 | 0.97 |
| Sample size | 144 | 68 | 43 | 116 | 52 | 31 | 28 | 16 | 12 |

T-statistics in parentheses.

Table 3

## REGRESSION RESULTS: ITALIAN DATA

|  |  |  |  |
| :--- | :---: | :---: | :---: |
|  | $1981-1998$ | $1981-1989$ | $1990-1998$ |
| DEBT | 0.024 | -0.018 | 0.096 |
| CORR | $(5.395)$ | $(-10.751)$ | $(10.092)$ |
| STDRATE | 0.372 | 0.175 | 0.723 |
|  | $(2.365)$ | $(4.815)$ | $(2.074)$ |
| STDG | -0.410 | -0.035 | -0.541 |
|  | $(-2.651)$ | $(-0.584)$ | $(-2.919)$ |
| Constant | 0.519 | 0.271 | -1.317 |
|  | $(2.933)$ | $(4.246)$ | $(-3.819)$ |
| Adjusted $R^{2}$ | -1.119 | 2.559 | -7.351 |
| Sample size | $(-2.838)$ | $(15.732)$ | $(-7.914)$ |

T-statistics in parentheses.

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[^1]:    ${ }^{1}$ I wish to thank Fabrizio Balassone and Daniele Franco for useful comments and discussions; I also thank Roberto Violi for providing me with data on debt maturity. The views expressed in this paper are those of the author and do not imply any responsability of the Bank of Italy. E-mail: giordano.raffaela@insedia.interbusiness.it.
    ${ }^{2}$ These results refer to the optimal taxation approach to public debt management. Under this approach, the design of public debt should support optimal taxation and ensure its time consistency. The optimal taxation approach is reviewed by Missale (1997).
    ${ }^{3}$ The works by Calvo and Guidotti and of Missale and Blanchard focus on the government's incentive to use opportunistic inflation to reduce the ex-post real value of its nominal obligations; Drudi and Giordano extend the analysis to the case in which both inflation and default risks affect government

[^2]:    liabilities and explore the role of debt maturity in managing such risks.
    ${ }^{4}$ Lucas and Stokey (1983) show that contingent debt allows for a smoother consumption path that would be possible by deficit financing alone. In fact, with non-contingent debt the present value of taxes increases after the realization of a bad shock, and conversely. While deficit financing allows the distribution of tax distortion over all future periods, it cannot eliminate the need for a change in tax rates in some future period and, hence, a change in work effort and consumption. However, in the absence of contingent debt, deficit financing remains a useful instrument to minimize tax distortions.

[^3]:    ${ }^{5}$ Given that no inflation is considered in this model, these rates are both nominal and real.

[^4]:    ${ }^{6}$ In our context short-term debt is equivalent to debt indexed to short-term interest rates.
    ${ }^{7}$ In fact, whether the constraint on deficit or that on debt is more binding is irrelevant for my purposes. Both limit the possibility of using deficit financing to minimize the distortionary effects of taxation and provide debt maturity with the same role. Here I simply neglect the constraint on debt, by assuming that that on deficit is always more binding. This is possible if one allows for variations in GDP and/or other factors besides the deficit that affect the level of debt.

[^5]:    ${ }^{8}$ In contrast, income taxes are assumed to be entirely under the goverment's control. Therefore, the analysis that follows neglects any randomness in revenue. A simple way to introduce it into the model is to think of a shock hitting not only expenditures, $g$, but the overall primary balance, $g-\tau$. The results would then remain unaffected, once of course all considerations about expenditures have been reinterpreted in terms of primary balance.
    ${ }^{9}$ Assuming a quadratic cost of taxation is quite standard. A simple way to originate it is to assume an underlying production technology, increasing and concave in labor. This makes the government loss function consistent with individual preferences, which are decreasing and convex in labor and linear in consumption. This last feature assures risk neutrality on the part of private agents, as postulated in equations (1), (2) and (3).

[^6]:    ${ }^{10}$ According to the Stability and Growth Pact, should the Member State's government deficit be considered excessive, the ECOFIN Council will formulate recommendations for the correction of this budgetary imbalance. Effective measures have to be taken by the Member State within four months. If the ECOFIN Council judges that such effective action has not been taken, it can impose sanctions. These initially take the form of a non-interest-bearing deposit; the amount of the first deposit comprises a fixed component equal to 0.2 percent of GDP and a variable component equal to one-tenth of the difference between the deficit as a percentage of GDP in the preceding year and the reference value of 3 percent of GDP. In each following year, if a decision is taken in favor of an additional deposit, it will be equal to one-tenth of the difference between the deficit as a percentage of GDP in the preceding year and the reference value of 3 percent of GDP, without exceeding the upper limit of 0.5 percent of GDP. The deposit will, as a rule, be converted into a fine if, two years after the decision requiring action to be taken, the excessive deficit has not been corrected.
    ${ }^{11}$ The game may then repeat as from period 0 , with the government in period 2 choosing the optimal maturity structure of the new stock of debt. The problem would be exactly the same as in period 0 .

[^7]:    ${ }^{12}$ This can be seen simply by differentiating (16) with respect to $D_{01}$.

[^8]:    ${ }^{13}$ This results simply by replacing, in the conditions stated in propositions 1,2 and $3, D_{01}$ with $s D_{0}$, where $s$ denotes the fraction of short-term debt in total debt. The conditions will then be expressed in terms of $s$ instead of $D_{01}$.

[^9]:    ${ }^{14}$ For Denmark, Finland and France, data on debt maturity are available since 1982, 1985 and 1990, respectively. Figures for Finland and Sweden concerning short-term interest rates refer to 1987-1998.
    ${ }^{15}$ The convergence of interest rates realized among countries joining the Monetary Union produced the greatest benefits in Italy; the long-term rate differential with respect to the EU countries with the lowest yield (Belgium, Germany, France, the Netherlands and Austria)has been virtually eliminated; at the beginning of 1990 it was about 5 percentage points.
    ${ }^{16}$ This implies that data start from 1979 for Netherlands, 1977 for Spain, 1979 for Belgium, 1980 for Germany, Ireland and UK, 1981 for Italy, 1982 for Denmark, 1987 for Finland and Sweden, 1990 for France.

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