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HOW DEEP ARE THE DEEP PARAMETERS?

by Filippo Altissimo,* Stefano Siviero* and Daniele Terlizzese*

Abstract

Policy evaluation based on the estimation of dynamic stochastic general equilibrium models with aggregate macroeconomic time series rests on the assumption that a representative agent can be identified, whose behavioural parameters are independent of the policy rules. Building on earlier work by Geweke, the main goal of this paper is to show that the representative agent is in general not structural, in the sense that its estimated behavioural parameters are not policy-independent. The paper identifies two different sources of non-structurality. The latter is shown to be a fairly general feature of optimizing representative agent rational expectations models estimated on macroeconomic data.

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1. Introduction[1]

If one were to name the single most entrenched commandment of the young scholar approaching macroeconomic policy evaluation there is little question that the mantra ”Thou shalt beware of the Lucas critique” would spring to mind. Indeed, since Lucas published his rightly famous critique (Lucas, 1976), the usefulness of traditional ”structural” econometric models as a means for macroeconomic policy selection has been seriously questioned. It scarcely needs to be recalled what is the content of the Lucas critique: traditional econometric models — i.e. models embodying decision rules with constant coefficients — fail to recognize that, due to the need to anticipate the future course of policy variables, the coefficients of (rational and forward looking) agents’ decision rules depend on the parameters that govern the policy stochastic process, as well as on the primitive (or ”deep”) parameters that characterize tastes and technology. As a result, traditional ”structural” models do not capture the actual structural parameters and the estimated coefficients are subject to variability in the presence of shifts in the policy rules.

While the stringent logic of the critique undoubtedly contributed to its success,[2] it can be argued that had the critique been merely a negative one it would have been dismissed as paralysing, and consequently neglected or downgraded, as Sims (1982) suggested, to the rank of a cautionary footnote. Instead, much of the strength of the Lucas critique lies in its constructive content, hinted at in Lucas’ original paper and more fully developed in later works by Hansen and Sargent (1980, 1981) and Sargent (1981). Indeed, the critique does not leave the practitioner at sea, as it comes with a ”recipe” for proper (econometric policy evaluation) behaviour: (a) solve the agents’ optimization problems and derive the explicit expressions of their decision rule coefficients as a function of deep and policy parameters; (b) estimate the coefficients of the decision rules together with the coefficients of the policy process, disentangling the dependence on deep parameters from that on policy parameters;

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[2] In truth, the sharp logic of the critique is not as compelling as it looks at first sight. On the one hand, general rational expectation models are plagued by indeterminacy of the equilibria, so that Lucas-proof constant parameter optimal decision rules can be found that do not violate the rationality of expectations (Farmer, 1991). On the other hand, opposite to forward-looking, backward-looking behaviour might prevail in practice, as agents might simply adopt a ”wait, see, react” strategy. Most importantly, whichever behaviour does in fact prevail can be statistically tested, and the Lucas critique is generically refutable (Hendry, 1988; Favero and Hendry, 1992).

(c) recompute the coefficients of the decision rules, taking into account the change in policy parameters, while keeping the deep parameters unchanged. This prescription, as conceptually simple as it is technically demanding, would guarantee that the simulated response to a policy shock takes into account the purposeful response of private agents, thus improving on the naïve prediction of the traditional approach.

Recognizing the importance of the constructive side of the Lucas critique, however, is like being kind to be cruel. Lucas' recipe can be applied by macroeconometricians only if a representative agent (RA henceforth) is warranted, since a single aggregate time series would not allow many idiosyncratic deep parameters to be recovered.[3] But the strength of the recipe — which we shall label as the representative agent *cum* rational expectations ($RARE$) approach — hinges on the possibility of finding an adequate representative agent. Which is where the troubles begin.

In a very remarkable paper Kirman (1992) gave an impressive list of the pitfalls presented by the notion of RA.

First, Debreu, Mantel and Sonneschein's theorem can be taken as an "impossibility theorem": in general equilibrium, the RA does not (in general) exist. This follows from the simple fact that, as aggregate excess demand need only satisfy continuity, homogeneity and Walras' law, it will not in general satisfy the weak axiom of revealed preferences; therefore there will be no utility function that generates the given aggregate excess demand.

Even when confined to the special cases in which an RA can be found, examples can be constructed where the RA does not represent, in welfare terms, the agents whose aggregate actions it reproduces.

Most importantly, the RA might be "non-structural". More explicitly, the RA that is appropriate before a given policy shock could well be different from the RA that will be appropriate after the shock (Geweke, 1985); clearly this would imply that the response of the aggregate economy to the shock would be misrepresented by the response of the RA recovered from data that do not include the shock. But this squarely contradicts the presumption

[3] It should be stressed that this paper is concerned with macroeconomics. Many of the problems that we shall highlight would not arise if the focus of the analysis were the explanation of individual behaviour, using individual rather than aggregate data. A survey of results on aggregation and microfoundations of dynamic macroeconomics can be found in Forni and Lippi (1997).

underlying the $RARE$ approach and implies that it suffers from the same logical difficulty that was originally imputed to the traditional approach. Geweke's point cannot simply be brushed aside: it is a fatal blow.

Yet it has been happily ignored,[4] possibly because the model used to highlight the non-structurality of the RA did not belong to the class of models to which the Lucas critique was originally directed, as it did not require future expectations regarding policy variables. The main goal of this paper is to show that Geweke's result is in fact robust, since the non-structurality of the RA is a fairly general feature of models with heterogeneous agents and rational expectations. We substantiate this claim in two ways.

First, we set up an "experimental world" populated by heterogeneous, infinitely living, rational economic agents, who make their decisions taking into account an exogenous (stochastic) process governing a (pay-off relevant) policy variable, as well as a random idiosyncratic shock (Section 2). Following the spirit of the $RARE$ approach, we then recover from the parameters of an aggregate decision rule obtained either by exact aggregation of deterministic, steady-state values (Section 3), or by constrained maximum likelihood estimation (Section 5) the (supposedly) deep parameters of the agent whose choice would be equal to the average of the individual agents' choices (the representative agent, in other words) and use that knowledge to predict the response of the economy to a policy regime shift. We compare that prediction with the true response, obtained through direct aggregation of the actual individual responses. The comparison shows that the RA response can be very different from the true one. The reason for the difference lies, as anticipated, in the non-structurality of the RA: the deep parameters recovered before the policy shock are different from those recovered afterwards. To put it differently, the deep parameters of the RA are not deep at all, since they are a function of the policy parameters.[5]

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[4] To consider just one representative example, the recent macroeconomic textbook by Turnovsky (1995) - largely centred around the RA device - while quoting Kirman paper as an example of critical reaction to the RA paradigm, does not mention the possible non-structurality of the RA, which is instead very much stressed by Kirman.

[5] There is an additional reason to consider the RA parameters as "shallow", namely the fact that, in general, each of them will be some function of all the deep parameters of all the agents. As a result, what would be called (say) the RA "time discount factor" would in fact be a function of (say) individual technology parameters as well as of individual time discount factors.

It could be objected, however, that the conclusions that can be drawn from our experiment are model-dependent, just as those drawn from Geweke's are. Our immediate reply is that the model underlying our ”experimental world” is the prototype of those supported by the $RARE$ approach. To demonstrate the existence of non-structurality in our model is tantamount to showing a fundamental flaw in that approach. There is a second way in which we corroborate the claim that the RA is in general non-structural. This provides a different and perhaps even stronger reply to the above objection. We investigate analytically the nature of non-structurality (Sections 4 and 5), identifying two separate instances of it. The first prevails when idiosyncratic dynamic variables are aggregated and is a consequence of imposing on the aggregate data a misspecified model, patterned on the model valid for the individual agents (Sections 4.1 and 5.1). The second instance occurs when aggregating common exogenous variables (typically, policy variables) whose coefficients in the individual decision rule are functions which combine deep and policy parameters, and derives from the non-aggregability of those functions. We show that a necessary and sufficient condition for this second kind of non-structurality is that these functions be ”non-separable”, a notion we shall explain below (Section 4.1). We also argue that this condition will be satisfied in almost all the models to which the Lucas critique applies.

A by-product of our analysis is a comparison between the $RARE$ and traditional approaches showing that, contrary to Lucas' claim, the latter is not necessarily worse than the former (Section 4.2). Intuitively, the true response in the coefficients of the aggregate decision rule to a change in the policy parameters might be greater than zero — which is what the traditional approach would predicate — but smaller than that of the RA (recovered using data from a period before the shock occurred), since the true response would be that of a different RA.

2. The set-up of the experiment

In the ideal world where agents take decisions by optimizing over the infinite future the (rationally) expected value of their objective function and where policy regime shifts are publicly announced the Lucas critique unquestionably applies and his ”recipe” promises to be most helpful. It is precisely that ideal world which our experiment replicates.

We consider N heterogeneous agents, namely firms, facing a standard, well known capital accumulation problem subject to idiosyncratic productivity shocks, taking as given the (stochastic) process governing the rental rate (Hansen and Sargent, 1980; see also Ingram, 1995, upon which our discussion of the model properties is largely based). For each agent the decision rule that solves the problem — which the assumptions made guarantee to have a closed form — is simulated, generating a time series of values for the (individual) capital stocks. Those time series are averaged across agents providing, together with the time series for the rental rate, the aggregate data that are the input for the estimation.[6] The latter is carried out following both the $RARE$ and traditional approaches. A shock is then given to the rental rate process parameters and the individual decision rules are recomputed. The true response of the economy to the shock — i.e., the average of the individual time series thus obtained — provides the natural benchmark against which the performance of the two approaches can be assessed.

Before presenting the results of the experiment, and in order to introduce the notation, it is useful to recall briefly the main features of the underlying model. Let firm i, endowed with a linear-quadratic production function and subject to quadratic adjustment costs, choose the capital stock so as to maximize the present discounted value of its profits; i.e., let it solve, under the appropriate transversality condition, the following problem:

(1) $$\max_{k} E_0 \sum_{t=0}^{\infty} \beta_i^t \left[\left(\gamma_i + a_t^i + r_t \right) k_t^i - \frac{1}{2} \vartheta_i (k_t^i)^2 - \frac{1}{2} \delta_i (k_t^i - k_{t-1}^i)^2 \right],$$

where: E_0 is the expectation operator conditional on the initial information;

β, γ, ϑ and δ are the discount factor and the technology parameters, respectively;

r is the rental rate, exogenously given to the firm;

a is a productivity shock.

In the following, we shall assume δ, the parameter on which adjustment costs depend, to be the same for all firms; the rationale for this assumption will be made clear below. All other parameters and productivity shocks are assumed, in the most general case, to be idiosyncratic

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6 In line with Lucas' prescription, we perform the estimation on data that pertain to the same policy regime.

to firm i.[7] To complete the description of the model, the stochastic processes underlying r and a have to be specified. The rental rate is the policy variable, for which the following stochastic process is assumed:

(2) $$r_t = \mu + \rho r_{t-1} + \varepsilon_t, \text{ with } \varepsilon \sim NID(0, \sigma^2) \text{ and } |\rho| < 1.$$

The process generating the idiosyncratic productivity shocks for firm i is also assumed to be an autoregressive process of order one:

(3) $$a_t^i = \eta_i a_{t-1}^i + \xi_{it}, \text{ with } \xi_{it} \sim NID(0, \tau_i^2) \text{ and } |\eta_i| < 1.$$

Furthermore we assume that all the stochastic components are uncorrelated, both serially and among themselves. Note that the linear-quadratic set-up of the model guarantees that a closed-form expression for the decision rule can be easily computed. Following Ingram (1995), the Euler condition characterising firm i optimal behaviour is:

$$\delta\beta_i E_t k_{t+1}^i - [\delta(1+\beta_i) + \vartheta_i]\, k_t^i + \delta k_{t-1}^i + \gamma_i + a_t^i - r_t = 0$$

and the optimal decision rule for firm i can be written as:

(4) $$\begin{aligned} k_t^i \;=\; & \lambda_i k_{t-1}^i - \frac{\lambda_i}{\delta}\Big(\frac{r_t}{1-\beta_i\lambda_i\rho} - \frac{a_t^i}{1-\beta_i\lambda_i\eta_i} \\ & - \frac{\gamma_i}{1-\beta_i\lambda_i} + \frac{\beta_i\lambda_i\mu}{(1-\beta_i\lambda_i\rho)(1-\beta_i\lambda_i)}\Big), \end{aligned}$$

where λ_i is the stable root of the following equation:

$$(1-\lambda_{1i}L)(1-\lambda_{2i}L) = 1 - \Big(1 + \frac{1}{\beta_i} + \frac{\vartheta_i}{\delta\beta_i}\Big)L + \frac{1}{\beta_i}L^2$$

[7] We have assumed that the only source of heterogeneity is different parameter values. This rules out another potentially very relevant source of heterogeneity, namely the possibility that individuals' payoff functions differ.

and L is the lag operator.[8] Equations (4) and (2), one for each firm, together with equation (3), constitute our data generating process (DGP).

It can easily be checked that the deterministic steady-state for k^i is given by:

$$k^i = \frac{\lambda_i}{\delta\,(1-\lambda_i)\,(1-\beta_i\lambda_i)}\left(\gamma_i - \frac{\mu}{(1-\rho)}\right). \tag{5}$$

To provide a benchmark for the following analysis let us now briefly recall the steps needed to recover the deep parameters. To this end, after dropping the index i (i.e., assuming that all agents are alike), standard algebraic manipulations yield the following VAR:

$$\begin{bmatrix} k_t \\ r_t \end{bmatrix} = \begin{bmatrix} \lambda+\eta & \frac{\lambda(\eta-\rho)}{\delta(1-\beta\lambda\rho)} \\ 0 & \rho \end{bmatrix}\begin{bmatrix} k_{t-1} \\ r_{t-1} \end{bmatrix} + \begin{bmatrix} -\lambda\eta & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} k_{t-2} \\ r_{t-2} \end{bmatrix}$$
$$+ \begin{bmatrix} \frac{\lambda(1-\eta)}{\delta(1-\beta\lambda)}\left(\gamma - \frac{\beta\lambda\mu}{(1-\beta\lambda\rho)}\right) - \frac{\lambda\mu}{\delta(1-\beta\lambda\rho)} \\ \mu \end{bmatrix} + \begin{bmatrix} \frac{\lambda(\eta-\rho)}{\delta(1-\beta\lambda\eta)} & -\frac{\lambda}{\delta(1-\beta\lambda\rho)} \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \xi_t \\ \varepsilon_t \end{bmatrix} \tag{6}$$

More compactly, with obvious redefinition of terms:

$$\begin{bmatrix} k_t \\ r_t \end{bmatrix} = C + A\begin{bmatrix} k_{t-1} \\ r_{t-1} \end{bmatrix} + B\begin{bmatrix} k_{t-2} \\ r_{t-2} \end{bmatrix} + V_t,$$

where: $V_t \sim NID(0,\Omega)$ with:

$$\Omega_{11} = \tau^2\left(\frac{\lambda}{\delta}\frac{(\eta-\rho)}{(1-\beta\lambda\eta)}\right)^2 + \sigma^2\left(\frac{\lambda}{\delta}\frac{1}{(1-\beta\lambda\rho)}\right)^2,$$

$$\Omega_{12} = \Omega_{21} = \sigma^2\left(-\frac{\lambda}{\delta}\frac{1}{(1-\beta\lambda\rho)}\right),$$

$$\Omega_{22} = \sigma^2.$$

Note that A_{11} and B_{11} must satisfy $\eta^2 - \eta A_{11} - B_{11} = 0$, so that, given the error structure, the following restriction — which in the words of Sargent is the hallmark of rational expectations — can be imposed: $\Omega_{22}A_{12} + \Omega_{21}(\eta - A_{22}) = 0$, where η is a solution to the quadratic equation above. This restriction should be taken into account at the estimation stage:

[8] Equation (4) can easily show the main point of the Lucas critique: a change in r_t, if caused by a change in the policy parameter ρ, modifies the parameters of the decision rule, which therefore cannot be considered as structural. If, on the other hand, changes in r_t are caused by a particular realisation of the stochastic disturbance, then no change in the decision rule parameters is expected to take place (on the distinction between these two sources of change in policy see Sims, 1982). The reduced form decision rule is therefore not equipped to make correct inferences concerning the outcome of a change in policy.

this in turn requires adopting a maximum likelihood approach.[9] Note that in this example β, γ, δ and σ^2 are not separately identified, and thus the value of one of these parameters must be specified a priori. As mentioned above, in the following we will assume the parameter δ to be the same across all agents and known exactly a priori.

The structural parameters can be recovered as follows: δ is specified a priori; $\mu = C_2$, $\sigma^2 = \Omega_{22}$, $\rho = A_{22}$,

$$
\begin{aligned}
\eta &= A_{12} - \frac{\Omega_{22}}{\Omega_{21}} A_{22}, \\
\lambda &= A_{11} - \eta, \\
\beta &= \left[1 - \frac{\lambda}{\delta} \frac{(\eta - \rho)}{A_{12}} \right] \frac{1}{\lambda \rho}, \\
\gamma &= \frac{\beta \lambda \mu}{(1 - \beta \lambda \rho)} + \frac{\delta}{\lambda} \frac{(1 - \beta \lambda)}{(1 - \eta)} (C_1 + \frac{\lambda}{\delta} \frac{\mu}{(1 - \beta \lambda \rho)}), \\
\tau^2 &= \frac{\left[\Omega_{11} - \sigma^2 \left(\frac{\lambda}{\delta(1-\beta\lambda\rho)} \right)^2 \right]}{\left(\frac{\lambda}{\delta(1-\beta\lambda\eta)} \right)^2}.
\end{aligned}
$$

With those values, the coefficients of the VAR in (6) can be modified to reflect a policymaker's intervention on (say) ρ. According to the $RARE$ approach the correct prediction of the effects of a change in r_t due to the said intervention would thus obtain.

3. Implementing the $RARE$ approach in a deterministic world

Reintroducing heterogeneity, we now carry out the experiment described in the previous Section, that is we try to recover the "deep" parameters of the agent that represents the (average of the) simulated behaviour of N firms and we then compare the response by that agent (the RA) to a policy regime shift with the true response of the economy. The results are easier

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[9] The approach outlined closely follows the original proposal by Lucas and Sargent and shows quite clearly the need to disentangle deep from policy parameters in estimating decision rules. An alternative — and indeed more frequently adopted — approach would be to estimate directly the Euler condition by GMM, to avoid the structural instability that would affect the decision rule (4) were the policy rule (2) to be modified in the sample period. In this respect note that, as we control the data generating process, we can avoid in-sample policy breaks. It is also important to realise that the problems of non-structurality that we shall be concerned with would appear essentially unchanged under the alternative procedure. In particular, recovering the aggregate decision rule from the aggregation of the individual Euler equations is not equivalent to the straightforward aggregation of the individual decision rules, as a non linear transformation is involved. We will take up this issue again when discussing the sources of non-structurality.

to interpret when the stochastic elements of the model are suspended, mainly because the problems of estimation can be set aside and both the coefficients of the aggregate decision rule and the deep parameters of the RA can be computed explicitly. To this we shall now turn, before presenting the numerical results.

Let us then set both the errors in the policy and in the productivity processes to zero and compute the aggregate stock of capital, k_t, as a simple average of the individual capital stocks.[10] Correspondingly, we can write the aggregate decision rule as a simple average of individual decision rules (first equation in the system (6)):

$$k_t = \left[\frac{1}{N}\sum_{i=1}^{N}(\lambda_i+\eta_i)\frac{k_{t-1}^i}{k_{t-1}}\right]k_{t-1} - \left[\frac{1}{N}\sum_{i=1}^{N}(\lambda_i\eta_i)\frac{k_{t-2}^i}{k_{t-2}}\right]k_{t-2} + \left[\frac{1}{N}\sum_{i=1}^{N}\frac{\lambda_i}{\delta}\frac{(\eta_i-\rho)}{(1-\beta_i\lambda_i\rho)}\right]r_{t-1} + \left[\frac{1}{N}\sum_{i=1}^{N}\frac{\lambda_i}{\delta\,(1-\beta_i\lambda_i)}(\gamma_i\,(1-\eta_i) - \frac{\mu\,(1-\beta_i\lambda_i\eta_i)}{(1-\beta_i\lambda_i\rho)})\right]. \tag{7}$$

Note that out of the steady state, even with all errors set to zero, $\frac{k_t^i}{k_t}$ will not be a constant. Therefore the aggregate decision rule (7) differs from the individual rules in that it does not have constant parameters. Indeed, much richer dynamics would be necessary to generate a constant parameter aggregate equation, it being well known that the appropriate aggregation procedure would first ”solve out” the (own) dynamics in each individual decision rule and then aggregate the common driving variables (see Section 5 for additional discussion of this issue). However the resulting equation could not be interpreted as the decision rule of an RA homologous to the individual agents.

If, however, we limit the analysis to steady states, it will be true that the aggregate data are (exactly) represented by a constant parameter decision rule, similar to the individual ones, of the form:

$$k_t = \alpha_0 + \alpha_1 k_{t-1} + \alpha_2 k_{t-2} + \alpha_3 r_{t-1}, \tag{8}$$

[10] While different aggregation criteria could be considered that avoid some of the problems we shall highlight, aggregation by sum or average characterizes national account time series.

where the parameters of (8) are computed as functions of individual deep parameters and of policy parameters according to the following expressions:

(9) $$\alpha_0 = \left[\frac{1}{N}\sum_{i=1}^{n}\frac{\lambda_i}{\delta\left(1-\beta_i\lambda_i\right)}(\gamma_i\left(1-\eta_i\right)-\frac{\mu\left(1-\beta_i\lambda_i\eta_i\right)}{\left(1-\beta_i\lambda_i\rho\right)})\right]$$

(10) $$\alpha_1 = \left[\frac{1}{N}\sum_{i=1}^{N}(\lambda_i+\eta_i)\frac{k^i_{t-1}}{k_{t-1}}\right]$$

(11) $$\alpha_2 = -\left[\frac{1}{N}\sum_{i=1}^{N}(\lambda_i\eta_i)\frac{k^i_{t-2}}{k_{t-2}}\right]$$

(12) $$\alpha_3 = \left[\frac{1}{N}\sum_{i=1}^{N}\frac{\lambda_i}{\delta}\frac{\left(\eta_i-\rho\right)}{\left(1-\beta_i\lambda_i\rho\right)}\right]$$

with the steady-state ratio $\frac{k^i_t}{k_t}$ given, according to equation (5), by:

(13) $$\frac{k^i}{k} = \frac{\frac{\lambda_i}{\delta(1-\lambda_i)(1-\beta_i\lambda_i)}\left(\gamma_i-\frac{\mu}{(1-\rho)}\right)}{\left(\frac{1}{N}\sum\frac{\lambda_i}{\delta(1-\lambda_i)(1-\beta_i\lambda_i)}\left(\gamma_i-\frac{\mu}{(1-\rho)}\right)\right)}.$$

From the computed αs we can finally recover the ”deep” parameters of the RA, i.e., the value of the parameters that, when assigned to a given agent, and under an unchanged policy regime, would yield a decision rule whose coefficients would be equal to the αs in eqs. (9)-(12).[11] More explicitly, given a value for the policy parameters, we could recover the ”deep” parameters of the RA, $(\lambda^{RA},\eta^{RA},\beta^{RA},\gamma^{RA},\mu^{RA})$, by solving the following system:

(14) $$\alpha_0 = \frac{\lambda^{RA}}{\delta\left(1-\beta^{RA}\lambda^{RA}\right)}(\gamma^{RA}\left(1-\eta^{RA}\right)-\frac{\mu^{RA}\left(1-\beta^{RA}\lambda^{RA}\eta^{RA}\right)}{\left(1-\beta^{RA}\lambda^{RA}\rho\right)})$$

(15) $$\alpha_1 = \lambda^{RA}+\eta^{RA}$$

[11] The RA will only represent the aggregate economy in steady state. Were a temporary shock to disturb the equilibrium, the short-run response of the aggregate would differ from the short-run response of the RA.

(16)
$$\alpha_2 = -\lambda^{RA}\eta^{RA}$$

(17)
$$\alpha_3 = \frac{\lambda^{RA}}{\delta}\frac{\left(\eta^{RA}-\rho\right)}{\left(1-\beta^{RA}\lambda^{RA}\rho\right)},$$

where the αs are given by equations (9)-(12). Note that the system can be solved in a somewhat recursive fashion: first solve (15) and (16) simultaneously for λ^{RA} and η^{RA},[12] then solve (17) for β^{RA} and finally solve (14) for γ^{RA}. For future reference, it is worth opening a brief digression on the over-identifying constraints that are common in this kind of problem. In fact, for a given value of δ, the RA parameters are exactly identified in the system (14)-(17). The cross equation restriction that was recalled in Section 2 is lost here, as a consequence of our neglecting the stochastic component. Given that we are here abstracting from estimation problems by actually computing the exact (steady state) coefficients, the loss of efficiency arising from the neglect of the restriction is immaterial.

It must however be remarked that an alternative strategy could have been followed to recover the deep parameters of the RA. We could have started from a structural representation of the individual decision rule, obtained by multiplying both sides of equation (4) by $(1-\eta L)$, substituting ξ_t for $(1-\eta L)a_t$ and finally, in line with the assumption maintained in this Section, setting ξ_t to zero:

(18)
$$\begin{aligned} k_t &= (\lambda+\eta)k_{t-1} - \lambda\eta k_{t-2} + \frac{\lambda}{\delta\left(1-\beta\lambda\rho\right)}r_t \\ &+\frac{\lambda\eta}{\delta\left(1-\beta\lambda\rho\right)}r_{t-1} + \frac{(1-\eta)\lambda}{\delta\left(1-\beta\lambda\right)}(\gamma - \frac{\beta\lambda\mu}{(1-\beta\lambda\rho)}). \end{aligned}$$

Then, as before, we could have computed the aggregate, steady-state counterpart of (18) and solved for the RA deep parameters that, appropriately combined with the (known) policy parameters, yield the computed coefficients. It is apparent from (18) that the (absolute value) of the ratio between the coefficient on r_{t-1} and that on r_t would provide the value of η^{RA}, which subtracted from the coefficient on k_{t-1} would give λ^{RA}. The product of these two values would give then the coefficient on k_{t-2} which, however, is also given independently. This would then provide a constraint that should be imposed if we were to estimate the coefficients. Given that

[12] It is worth remarking that, given the symmetry of the system (14)-(16), the method used to match the two roots of the system with the two parameters η^{RA} and λ^{RA} is arbitrary. In the example below, we choose the solutions in such a way that the values for the remaining parameters β^{RA} and γ^{RA} fall within the feasible set.

we compute them by exact aggregation, this strategy cannot be pursued. Rather, it provides a measure of the possible violation of the constraint in the aggregate data. We shall return to this issue later.

For now, let us close the digression and suppose we have determined the values of the deep parameters of the RA and want to predict the effect of a policy shock to the aggregate economy by following the $RARE$ approach. Even before we look at the numerical results, we can anticipate that a number of problems would arise in carrying out this task.

First, the coefficients α_1 and α_2 , from which the values of the λ and η parameters of the RA (eqs. (9)-(12)) should be recovered, are not functions only of the individual λs and ηs. From (9)-(12) and (14)-(17) it is clear that in general α_1 and α_2 will also depend on βs, γs, δ, μ and ρ (through the "weights" k^i/k).[13] Therefore:
(a) the estimate of λ^{RA} and η^{RA} cannot be given any behavioural interpretation, as these parameters would in fact also be a mixture also of all the other parameters of the agents. Given that the estimate of the other deep parameters of the RA are conditional on the values obtained for λ^{RA} and η^{RA}, the same difficulty with the behavioural interpretation holds for them as well.
(b) more importantly, the estimated λ^{RA} and η^{RA} would not be invariant vis-à-vis a policy shift: if we introduce a change in ρ or in μ, generate new individual data and aggregate, the new steady state aggregate behaviour of the economy would be represented by a decision rule similar in form to (8) but with different values for those coefficients — α_1 and α_2 — that in the decision rule of the RA would be taken as invariant vis-à-vis the change in the policy. Hence, the RA will not do its job, as it will misrepresent the (steady state) behaviour of the economy. To put it differently, after the policy shift the data would lead to a different estimate for the (supposedly) deep parameters λ^{RA} and η^{RA}, so that the economy would be represented by a different RA: λ^{RA} and η^{RA} are not deep. Given that, as argued above, the estimate of the other deep parameters of the RA is conditional on the values obtained for λ^{RA} and η^{RA}, none of the parameters is in fact deep.

Secondly, suppose that the individual γs where the same. It follows immediately from (13) that the steady state value of the ratio k_i/k would be independent of the policy parameters

[13] Returning to the Euler equation, we can note that the aggregation of individual first order conditions would involve weighting individual deep parameters with terms of the form k^i/k. Therefore, the problems highlighted in the text for the decision rule also apply to the Euler equation.

ρ and μ and, though the problem listed under (a) above would still be relevant, λ^{RA} and η^{RA} would at least be invariant in relation to the policy. As a result, the α_1 and α_2 coefficients in the RA decision rule would correctly be taken as unchanged. This would not, however, guarantee that the RA response to the policy shift correctly represents the aggregate response. In other words, even if λ^{RA} and η^{RA} are invariant vis-à-vis policy shifts, the remaining parameters are not guaranteed to be deep. Consider a change from ρ_0 to ρ_1 (a change in μ would have similar implications). In the decision rule of the RA the coefficient on the policy variable will be changed from $\alpha_3^{'} = \left[\frac{\lambda^{RA}\left(\eta^{RA}-\rho_0\right)}{\delta\left(1-\beta^{RA}\lambda^{RA}\rho_0\right)}\right]$ to $\alpha_3^{''} = \left[\frac{\lambda^{RA}\left(\eta^{RA}-\rho_1\right)}{\delta\left(1-\beta^{RA}\lambda^{RA}\rho_1\right)}\right]$. The true (post-shock) coefficient will be given by $\alpha_3^{'''} = \left[\frac{1}{n}\sum_{i=1}^{N}\frac{\lambda_i(\eta-\rho_1)}{\delta(1-\beta_i\lambda_i\rho_1)}\right]$. The numerical results, to which we now turn, will show that $\alpha_3^{''}$ is different from $\alpha_3^{'''}$, so that β^{RA} is not deep and the response of the RA to the policy shift does not match the aggregate response of the economy.

3.1 *The numerical results*

To exemplify the above analysis we now consider ten different firms with heterogeneous parameters. In Table 1 the assumed parameters are listed, together with their means and standard deviations. Assuming first a policy regime characterised by ρ and μ equal to 0.55 and 1 respectively, we let each of the firms compute its optimal investment decision according to equation (6) — with the error terms set to zero. We then aggregate individual data and use equations (9)-(12) to compute the coefficients of the steady state, aggregate decision rule. These are shown in the first row of Table 2. The supposedly deep parameters of the representative agent are computed by solving the system (14)-(17) and are shown in the first row of Table 3. The procedure is repeated in a new policy regime, characterized by ρ equal to 0.605 (a 10 per cent shock). The second rows of Tables 2 and 3 present the corresponding ”true” decision rule coefficients and ”deep” parameters, that is the coefficients (and associated behavioural parameters) that reproduce the actual aggregate, post-shock data. It must be stressed that recomputing coefficients — and therefore ”deep” parameters — with data relative to the new policy regime would obviously not be a viable procedure in real life. By comparison, the third row of Table 2 gives the coefficients of the decision rule that would be inferred were one to follow the $RARE$ approach — i.e., if the RA parameters were deemed to be structural.

The numerical results — which confirm our previous analysis — show that the true (post-shock) coefficients of lagged capital A_{11} and B_{11} (second row of Table 2) are different from those inferred under the $RARE$ approach (third row of Table 2), the latter being incorrectly assumed to be independent of ρ. The effects of the policy shift on the other decision rule coefficients (second row of Table 2) are also different from those predicated by the $RARE$ approach (third row of Table 2). This failure to predict correctly the changes in the decision rule coefficients in response to the policy shift is mirrored in the lack of structurality of the behavioural parameters observed by comparing the first and second rows of Table 3. As mentioned in the previous Section, non-structurality would occur also if the individual parameters were modified so as to eliminate the dependence on ρ of the coefficients representing the dynamic component of the decision rule. Tables 2.1 and 3.1 — obtained following the same procedure adopted in Tables 2 and 3 but with the assumption that all individual γs are the same — confirm this assertion. To stress further the non-structurality of the RA parameters Figure 1 plots the functions relating the deep parameters to ρ, in the general case in which the γs are different among agents.

It is worth mentioning that the value of the RA parameters need not be a convex combination of the individual deep parameters. Moreover, it is very simple to select individual parameter configurations so that some of the RA parameters are outside the range of admissible values. Yet these RA parameters are the only true ones, as they are the only ones that, when assigned to a hypothetical agent, would give rise to a decision rule that (in steady state and within a given policy regime) yields the true aggregate value. In these circumstances, in spite of the linearity of the individual decision rules, there would be no representative agent.[14]

The numerical example allows us to highlight a final important point: the simulated response of the economy to a policy shift obtained through the $RARE$ approach is not necessarily closer to the true response than the response resulting from the admittedly naïve traditional approach, according to which all the coefficients of the decision rule would remain

[14] As mentioned before, the cross equations constraint that characterises the problem is lost in our deterministic set-up. If however we were to try to recover the RA parameters from a structural representation of the aggregate decision rule following the procedure outlined in the previous section, we would discover that in general there is no set of ”representative” parameters such that the aggregate coefficients are obtained from them through equations (6).

unchanged.[15] In Figure 2 we show these three responses (the data refer to the parameter configuration underlying Tables 1-3, but qualitatively similar results would be obtained with the parameters underlying Tables 2.1-3.1). The traditional approach outperforms the $RARE$ approach period by period. While we do not claim that this ranking is a general result — indeed, it is easy to show parameter configurations yielding the opposite ranking — we believe it is remarkable that, even if the setting of the experiment is the most favourable to the $RARE$ approach one could think of — short of assuming homogeneous agents, that is — the latter is nonetheless outperformed by its much despised contender.

It is worth mentioning that the traditional approach, which does not constrain the model dynamics, could more easily cope with the (dynamic) misspecification problem. Also, by extending the conditioning set, an approximation of the change of coefficients due to a policy regime shift could be obtained. For the sake of comparability, we shall refrain from allowing the traditional approach these ”degrees of freedom” and shall therefore be (purposely) unfair to it. More generally, as the results we present below are critical of the $RARE$ approach, we shall adopt the methodological principle of choosing the ”battleground” most favourable to the target of our criticism.

4. Interpreting the results

The numerical results clearly show that, in spite of the ideal conditions in which our experiment is conducted — all firms are rational, forward-looking, optimize over the infinite future subject to production functions belonging to the same parametric class, discount the future in the same qualitative way, are all simultaneously and completely informed of the policy regime shift — the non-structurality of the RA is not a far-fetched possibility, with relevant quantitative implications for the performance of the $RARE$ approach, both in absolute terms and in relation to the traditional approach. We believe that, given the almost paradigmatic nature of the example considered, the non-existence of deep parameters for the

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[15] Since in recovering the RA parameters we are assuming knowledge of the true values of ρ and μ, a simulation conditional on the actual values of the interest rate — as it is the use in the traditional approach — has no advantage over the unconditional simulation required by the $RARE$ approach. In fact, the latter would yield exactly the same results if a structural decision rule — obtained from the recovered ”deep” parameters — were simulated.

RA cannot be shrugged off as a theoretical *curiosum*: it is a serious logical and practical problem at the root of the $RARE$ approach.

Yet an example, albeit prototypical, is still an example. Moreover, the origin of the non-structurality is not immediately apparent from the example.[16] In fact, by exploring the nature and possible sources of non-structurality of the RA we shall very naturally show it to be a fairly general feature of models with rational expectations, thus strengthening the lesson that can be drawn from our example.

To start our enquiry into the nature and causes of non-structurality, let us note that in the example above we implicitly identified two different ways in which the non-structurality of the RA can show up, corresponding to the two ways in which the coefficients of the aggregate decision rule (8) might differ from the homologous coefficients in the individual decision rules: the possibility that α_1 and α_2, the coefficients of the (own) dynamic component of the decision rule (8) — whose individual counterparts are independent of the policy parameters (μ and/or ρ) — turn out to be a function of those parameters; the possibility that the functional dependence on the policy parameters of α_3 and α_0, the coefficients of the exogenous driving forces of the decision rule, may be different from that of their individual counterparts so that, for example, $\alpha_3^{'}$ is different from $\alpha_3^{''}$.[17] It is worth remarking that the possibility of seeing these two instances of non-structurality separately at work — as we did in Section 3.1 — is only open in a deterministic world: as will be shown in Section 5, when the stochastic nature of individual decision rules is taken into account the equality of the γs is no longer sufficient to isolate the impact of the policy shift on α_3 and α_0 (that is on β^{RA} and γ^{RA}).[18] The two instances of non-structurality can be seen to stem from two logically independent sources. The first is the imposition on the aggregate data of a dynamically misspecified model, patterned on the individual decision rule. As shown above, the requirement that the aggregate decision rule (8) have the same dynamic structure as the individual rules (first equation of the system (6))

[16] Indeed, a failure to identify the causes of the non structurality is perhaps the major weakness of the otherwise very insightful paper by Geweke (1985), a weakness that might have contributed to the neglect with which the profession welcomed it.

[17] There is indeed a third way in which the parameters of the RA might fail to be deep, noted in Section 3 above. This would occur when the aggregate coefficients depend on deep parameters other than those appearing in the expression of the homologous coefficients in the individual decision rules. We will keep the expression ”non-structural ” to refer to the dependence of the RA ”deep” parameters on the policy parameters, mainly neglecting this other form of potential ”shallowness”.

[18] This indeed is one of the reasons why it is useful to consider the deterministic case separately.

implies that the coefficients of the dynamics, α_1 and α_2, are a function of the ”weights” k^i/k (see equation (7)), which in turn are a function of policy parameters. No terms of the form k^i/k would be necessary if richer dynamics had been allowed for in the aggregation, as mentioned above. As this issue will prove to have an inherent effect on the estimation step of the $RARE$ approach, a more detailed discussion is deferred to Section 5.1.

The second source, reminiscent of Jensen’s inequality, is the ”non-separability” — in a sense to be specified below — of the ”mixture” of deep and policy parameters that appears in the individual decision rule coefficients pertaining to the exogenous driving forces. Under non-separability, aggregation of the *individual* ”mixtures” will inextricably bind together deep and policy parameters. We shall now turn to this source of non-structurality .

4.1 *Generalizing the non-structurality of the* RA

As already argued, proponents of the $RARE$ approach should be somewhat disturbed by the possibility that the RA is non-structural precisely in the model they favour to illustrate its ”mechanics”. In fact, we intend to show that non-structurality essentially plagues all rational expectation models. There is a preliminary issue that we need to get out of the way, namely the question as to whether results concerning the non-structurality of the RA in a linear framework are of any general relevance and, more specifically, are robust to non-linear generalizations. There are a number of reasons to conclude that they are indeed both.

First and most importantly, if there are aggregation problems in a linear framework, they can only become worse in a non-linear one, according to the most obvious of Jensen’s inequality-type of arguments: as the sum over i of a non-linear function of a variable x_i, $i = 1, 2...N$, is different from the same non-linear function of the sum over i of the variables x_i, there is no way at the aggregate level — that is for the RA — to preserve the same functional form valid at the individual level.[19] In fact, moving away from a linear-quadratic setting, the agents’ optimization problem would in general not even have a closed form solution. In these circumstances, either we linearize the original problem around the steady state — thereby returning the linear-quadratic world — or we try to estimate — by a generalized method of

[19] There is, of course, the possibility of approximate aggregation. It might be argued that the RA is indeed non-structural, but only slightly so. Given that the RA can be grossly non-structural in a linear framework, we find this argument unconvincing.

moments or by simulation[20] — Euler conditions that, being non-linear, suffer from problem of the Jensen's inequality type mentioned above.

Secondly, the linear-quadratic setting encompasses some interesting models that have been proposed to study consumption and permanent income, the dynamic demand for factors of production and many other issues. Even if issues involving risk aversion cannot be seriously addressed in that framework, there is nevertheless a large class of problems that can be explored fruitfully under the simplifying assumption of linearity.

Thirdly, in keeping with the methodological principle of choosing the "battleground" most favourable to the target of our criticisms, the linear-quadratic framework is the obvious choice, given the extensive use that contributors to the $RARE$ approach have made of it.

In sum, we believe that there are enough reasons to stick with linear decision rules. In any event, this is what we shall do. Moving now to the core of our argument, let us first note that the distinction we were able to draw in our example between two different forms of non-structurality — one related to the aggregation of idiosyncratic dynamics and a second one arising from the aggregation of common exogenous variables — can be maintained in a large class of linear-quadratic, rational expectation problems. It can be easily verified that in the class of so-called "augmented linear regulator problems"[21] (Hansen and Sargent, 1981), the coefficients of the dynamic terms in the decision rule are functions only of the individual parameters while the coefficient of the exogenous (policy) variable is a (non-linear) function of the individual and policy parameters, as in our example. The latter conclusion — crucial to our purpose — is of a very general nature. Indeed, to the extent that we can refer to "coefficients" in non-linear decision rules, it is obvious that, even beyond the linear-quadratic setting, at least some of the coefficients of the agents decision rule must be a mixture of deep and policy parameters for the Lucas critique to apply.

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[20] It is possible to estimate the deep and policy parameters of stochastic general equilibrium models by a generalized method of moments or by simulation, using either the simulated MLE or simulated method of moments or indirect inference. The GMM method exploits orthogonality conditions originating from the maximization problem of the agent to create a quadratic function which has to be minimized with respect to the parameters. The estimation by simulation is based on the possibility of simulating the desired model, so as to use the simulated data to generate proper moment conditions to be used in the estimation A general survey of the results of estimation by simulation is given by Gourieroux and Monfort (1996).

[21] More explicitly, this is the class of optimal control problems with quadratic objective function and linear constraints in the presence of exogenous policy variables.

Suppose then that one of those coefficients is given by $f(\beta_i, \rho)$, where β_i is the (individual) deep parameter and ρ is the policy parameter and where, for the sake of simplicity, we are considering the special case in which β_i is a scalar. Note also that, maintaining the assumption made in the previous section, we shall side-step estimation problems and recover the (supposedly) deep parameter in question directly by "inverting" the function f, given a value for the policy parameter ρ. More explicitly, the RA parameter β^{RA} can be found as the solution to

(19) $$f\left(\beta^{RA}, \rho\right) = \frac{1}{N}\sum_{i=1}^{N} f(\beta_i, \rho),$$

for a given value of ρ. Here and in the following we shall assume that the hypotheses of the implicit function theorem are satisfied, so that implicit equations like (19) above always have a solution. An example of equation (19) is given by equation (16) above.

Once a value β^{RA} has been determined, according to the $RARE$ approach we could predict how the economy would react to a policy shift from ρ_0 to ρ_1 (say) by changing the appropriate coefficient in the RA decision rule from $f\left(\beta^{RA}, \rho_0\right)$ to $f\left(\beta^{RA}, \rho_1\right)$. For this change in the value of the coefficient to be correct it must be that the value of β^{RA} that solves equation (19) for ρ_0 also solves it for ρ_1 or, more generally, that β^{RA} is not a function of ρ. Equivalently, it must be that β^{RA} is also a solution to:

(20) $$f_\rho\left(\beta^{RA}, \rho\right) = \frac{1}{N}\sum_{i=1}^{N} f_\rho(\beta_i, \rho),$$

where f_ρ is the partial derivative with respect to ρ. Indeed, if β^{RA} is not a function of ρ we can simply take a derivative with respect to ρ on both sides of (19) to get (20). On the other hand, by applying the implicit function theorem to (19) and using (20) we immediately obtain that the derivative of β^{RA} with respect to ρ is identically zero.

However, it is readily apparent that the system (19) and (20) will in general have no solution, since it is a system of two equations and one unknown. More precisely, the system (19) and (20) will have a unique solution for every ρ if and only if the function f_ρ is an affine transformation of the function f, that is if and only if we can write:

(21) $$f_\rho\left(\beta,\rho\right)=\pi_0(\rho)+\pi_1(\rho)f\left(\beta,\rho\right),$$

for an appropriate choice of the ”weights” $\pi_0(\rho)$ and $\pi_1(\rho)$.

The sufficiency of condition (21) for the solution of the system is obvious. To prove necessity, first of all note that (21) is equivalent to the existence of a constant c, possibly dependent on ρ, such that for all i and j the following holds:

(22) $$\frac{f_\rho\left(\beta_i,\rho\right)+c}{f_\rho\left(\beta_j,\rho\right)+c}=\frac{f\left(\beta_i,\rho\right)}{f\left(\beta_j,\rho\right)}.$$

Consider now the system (19) - (20) in the simplest case $N=2$, and assume it has a solution. By adding an arbitrary constant c to both sides of (20) and rearranging the terms we can combine the two equations to read:

(23) $$\sum_{i=1}^{2}\left[\frac{f_\rho\left(\beta_i,\rho\right)+c}{f_\rho\left(\beta^{RA},\rho\right)+c}-\frac{f\left(\beta_i,\rho\right)}{f\left(\beta^{RA},\rho\right)}\right]=0.$$

It is clear that we can choose the constant c in such a way that the expression in square brackets is zero for each i. Hence, in light of (22), (21) holds. This conclusion can then be extended to any N by induction.

Summing up thus far, there will be a structural RA , i.e., independent of the policy regime, if and only if the function f is such that condition (21) is satisfied. It can easily be checked that the condition is not satisfied for the function implicitly defining β^{RA} in (17). We can provide a more transparent characterization of the class of functions which allow the possibility of recovering a structural RA parameter. First a definition and an auxiliary technical result are needed. We shall call a function $f(\beta,\rho)$ *separable* if we can write it as follows:[22]

(24) $$f\left(\beta,\rho\right)=h_1(\rho)+h_2(\rho)g\left(\beta\right)$$

for some functions h_1, h_2 and g. We can then prove the following auxiliary result:

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[22] Note that our definition of separability is not symmetrical since it assigns different roles to policy and deep parameters.

A function $f(\beta,\rho)$ is *separable* if and only if the function $\frac{f_{\beta\beta}(\beta,\rho)}{f_{\beta}(\beta,\rho)}$ does not depend on ρ, where subscripts denote partial derivatives.

Proof. The sufficiency is obvious: simply take the required derivatives of the RHS of (24) and compute their ratio. As for necessity, let

(25) $$\frac{f_{\beta\beta}(\beta,\rho)}{f_{\beta}(\beta,\rho)} = \phi(\beta),$$

independent of ρ, and assume, without loss of generality, that $\phi(\beta) = \frac{g^{''}(\beta)}{g^{'}(\beta)}$, where the primes, as usual, indicates derivatives. Now integrate both sides of (25) with respect to β, to obtain:

(26) $$log(f_{\beta}(\beta,\rho)) = log(g^{'}(\beta)) + \varphi(\rho),$$

where $\varphi(\rho)$ is the constant of integration that we take, in full generality, to be a function of ρ. Moreover, we can write, again without loss of generality, that $\varphi(\rho) = \log(h_2(\rho))$. Therefore, taking the anti-log in (26), we have:

(27) $$f_{\beta}(\beta,\rho) = g^{'}(\beta)h_2(\rho).$$

Integrate now both sides of (27) with respect to β to get:

(28) $$f(\beta,\rho) = g(\beta)h_2(\rho) + h_1(\rho),$$

where $h_1(\rho)$ is the constant of integration. ■

We are now ready to prove the following claim:

The system (19)-(20) admits a solution β^{RA} - or, equivalently, it is possible to recover from equation (19) a value for the behavioural parameter of the RA that is independent of the policy — if and only if the function $f(\beta,\rho)$ is separable.

Proof. We need to show that condition (21) is equivalent to separability of the f, that is to condition (24). That separability should imply (21) is fairly obvious, as it only requires taking the appropriate derivative of the f, under (24), and rearranging the terms. Indeed, the sufficiency of separability for the structurality of the solution β^{RA} can be established directly by checking that, in (19), the solution for β^{RA} would not involve terms in ρ. As for necessity, the auxiliary result previously proved implies that separability of the f is equivalent to the following condition: $\frac{d}{d\rho}\left[\frac{f_{\beta\beta}(\beta,\rho)}{f_{\beta}(\beta,\rho)}\right] = 0$. Equivalently, separability holds iff:

(29)
$$f_{\beta\beta\rho}(\beta,\rho)f_{\beta}(\beta,\rho) - f_{\beta\rho}(\beta,\rho)f_{\beta}(\beta,\rho) = 0.$$

Assume now (21) and compute the derivatives appearing in (29). We have:

(30)
$$f_{\beta\rho}(\beta,\rho) = \pi_1(\rho)f_{\beta}(\beta,\rho)$$

(31)
$$f_{\beta\beta\rho}(\beta,\rho) = \pi_1(\rho)f_{\beta\beta}(\beta,\rho).$$

Hence:

(32)
$$\begin{gathered} f_{\beta\beta\rho}(\beta,\rho)f_{\beta}(\beta,\rho) - f_{\beta\rho}(\beta,\rho)f_{\beta}(\beta,\rho) = \\ \pi_1(\rho)(f_{\beta\beta}(\beta,\rho)f_{\beta}(\beta,\rho) - f_{\beta\beta}(\beta,\rho)f_{\beta}(\beta,\rho)) = 0 \end{gathered}$$

Therefore, (21) implies separability. ■

Summing up thus far, we have shown that — even if the dependence on the policy parameter brought about by the dynamics is neglected — the RA parameter will be structural only when the coefficient on the policy variable in the individual decision rule mixes deep and policy parameters in a ”separable” way. Given that any aggregate decision rule which is potentially subject to the Lucas critique must involve at least one coefficient that can be expressed as a function of deep and policy parameters, our result provides a general necessary and sufficient condition for the viability of the $RARE$ approach. To put it differently, our result characterizes in a fairly general way the conditions under which the non-structurality of the RA, first pointed out by Geweke, occurs.

Two questions naturally arise: does the result hinge on the simplifying assumption whereby only a scalar deep parameter has been considered? how likely will separability be?

As to the first, let us simply note that the above arguments can be extended with only minor changes to the case in which the RA ”deep” parameter is a vector and, correspondingly, there is a certain number of coefficients in the decision rule[23] — bearing in mind that the presence of idiosyncratic dynamic terms will be a source of non-structurality *per se*. Turning now to the likelihood of separability, we can note that in all linear-quadratic problems we would end up solving the Euler equation by eliminating the future expected values of the policy variable. This in turn involves inextricably bounding together the subjective evaluation of the future and the objective law of motion of the policy variable, so that the non-separability of deep and policy parameters will result. To take just one example, if the Euler equation involves a term like $\sum_{t=0} \beta^t E_0(r_t)$ where β is the subjective discount factor, and if r_t follows an AR(1) process with autoregressive parameter $\rho < 1$, we would end up with a term $\frac{1}{1-\beta\rho}$ multiplying r_t, clearly non-separable. A similar conclusion would hold for any AR(n) process. Given our result, this means that the non-structurality of the RA, far from being a non -generic *curiosum*, is an almost sure consequence of heterogeneity.

4.2 *Is the $RARE$ approach necessarily better than the traditional one?*

We showed, in Section 3.1, that the $RARE$ approach can be dominated by the traditional one. This clearly follows from the non-structurality of the RA, which induces a misrepresentation of the true aggregate response. Since the traditional approach also misrepresents the true response (as it does not take into account at all the response of the coefficients in the aggregate decision rule), we are left with the comparison — *a priori* ambiguous — between two approximate representations of the truth. Our ambition is to say something more than this essentially agnostic conclusion. Before becoming involved in more formal arguments, it is useful to spell out the simple intuition underlying them: if aggregation produces attenuation of the response to a given shock, then the response of the RA will be ”excessive” and a dampened response (such as that provided by the traditional approach) might be closer to the true one.

To flesh out that intuitive explanation, let us start by noting that the policy parameters show up in the coefficients of the RA decision rule to take care, as it were, of (the anticipation of) the future levels of the policy variable. In particular, the (absolute value of the) response to

[23] The strategy of proof followed can be kept essentially unchanged, with separability defined as in (24) for each of the functions defining the coefficients of the decision rule and allowing β to be a vector. The only modifications required are: (1) in the proof of the auxiliary result the condition that the ratio of derivaties is independent of ρ for all i and j has to be imposed, generalising the condition required for the scalar case; (2) in the proof of the claim, all the cross-derivatives have to be computed.

a current policy regime shift would in general be greater than the mere response to a change in the level of the current policy variable. The latter, instead, is the response of the traditional approach, which would only account for the effect of the policy parameters on the current level of the policy variable. Therefore it seems reasonable to assume that, in comparison with the $RARE$ approach, the traditional one ”underreacts”.[24]

If that is the case, a necessary (though not sufficient) condition for the traditional approach to be closer than the $RARE$ approach to the actual response of the aggregate is that, with respect to the true aggregate response, the $RARE$ approach ”overreacts”. To clarify when the latter condition can be satisfied, let us note that, without loss of generality — equation (5) providing just one example — the deterministic steady state value of the individual decision variable can be written as:

$$k^i = h(\beta_i, \rho), \tag{33}$$

where for simplicity we consider only one idiosyncratic deep parameter and only one policy parameter. The steady state value of the aggregate decision, $k^{AG} = \frac{1}{N} \sum k^i$, is then:

$$k^{AG} = h(\beta\left(\rho\right), \rho), \tag{34}$$

where, for each ρ, $h(\beta\left(\rho\right), \rho) = \frac{1}{N} \sum\limits_{i=1}^{N} h(\beta_i, \rho)$. Note that the ”deep” parameter is explicitly considered a function of the policy parameter, following our previous analysis.

The steady state value of the RA decision is given by:

$$k^{RA} = h(\beta^{RA}, \rho), \tag{35}$$

[24] The claim in the text needs two qualifying remarks. The first is that it assumes that the expected future changes in the level of the policy variable are of the same sign as the initial one. This will often be the case, however, given that the policy regime shift is engineered precisely to produce those changes. Therefore it seems very likely that the change in the decision rule coefficient would buttress the reaction to the current policy variable level. The second remark is that the claim neglects the possibility that in deriving the RA decision rule a constraint is imposed which is not in fact satisfied by the aggregate data. This would make the comparison between the (absolute value of the) reaction under the $RARE$ and the traditional approaches a priori ambiguous.

where $\beta^{RA} = \beta(\rho)$ for the particular value of ρ that generated the data used in computing the coefficients of the aggregate decision rule. For that value of ρ, then, (34) and (35) are numerically identical.

The difference between these two decision rules appears when we try to use them to predict the response of the economy to a policy regime shift, namely to a change in ρ. Intuitively, the $RARE$ approach would account for the effect of ρ on the current level of r as well as considering how the RA, anticipating that the change in ρ would affect future values of r, would modify its reaction to the current level of r. The true aggregate would, in addition, take into account the change that the aggregation process induces in the "identity" of the RA. More formally, let us compute, in the two cases, the derivative of the steady state k with respect to ρ. We have:

(36) $$\frac{dk^{AG}}{d\rho} = h_\beta(\beta,\rho)\frac{d\beta(\rho)}{d\rho} + h_\rho(\beta,\rho)$$

(37) $$\frac{dk^{RA}}{d\rho} = h_\rho(\beta,\rho),$$

where the functions are all evaluated at the same point (β^{RA},ρ), with $\beta^{RA} = \beta(\rho)$. Combining the two equations we then have:

(38) $$\frac{dk^{AG}}{d\rho} = h_\beta(\beta,\rho)\frac{d\beta(\rho)}{d\rho} + \frac{dk^{RA}}{d\rho}.$$

Equation (38) makes it clear that there will be an overreaction whenever the change in the "identity" of the RA ($\frac{d\beta(\rho)}{d\rho}$) has an impact on the aggregate decision ($h_\beta(\beta,\rho)\frac{d\beta(\rho)}{d\rho}$) which (partially) offsets the straightforward response of the RA ($h_\rho(\beta^{RA},\rho)$). More formally, we can say that there will be overreaction if:

(39) $$h_\rho(\beta^{RA},\rho)h_\beta(\beta^{RA},\rho)\frac{d\beta(\rho)}{d\rho} < 0.$$

Condition (39) involves explicit reference to the aggregation process, as it is a function of the change in the RA that is brought about by the policy shift. It is possible, however, to reformulate that condition so that it only involves properties of the function h.[25] The

[25] It can easily be shown, by reproducing the Arrow-Pratt derivation of the (approximate) expression for the

aggregation process implies that there is not a clear cut answer to the question and the conditions in (39) leave open the possibility that a traditional rule (fixed coefficients) would be a better approximation of the true response.

5. Implementing the $RARE$ approach in a stochastic world

The assumption that all error terms are zero — which proved useful to highlight the different sources of non-structurality and to simplify the derivation of the analytical results — might be considered unnatural for the analysis of an approach that stresses sophisticated estimation techniques. More importantly, it could be argued that our numerical or analytical results would be changed in a stochastic world.

To the contrary, we shall now show that, if anything, the $RARE$ approach faces even deeper problems when that assumption is dropped. More explicitly, we shall present numerical results that confirm the non-structurality of the estimated "deep" parameters of the RA; we shall also show that the misspecification of the estimated (aggregate) model implies unavoidable non structurality, which combines with the one arising from non separability, as in the deterministic case; finally, we shall point out the aggravating role of imposing a cross equation constraint which, though valid for each firm, is not satisfied by the aggregate data.

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risk premium, that:

$$\beta(\rho) \cong \overline{\beta} + \frac{h_{\beta\beta}\left(\overline{\beta},\rho\right)}{h_{\beta}\left(\overline{\beta},\rho\right)}\sigma_{\beta}^{2},$$

where $\overline{\beta}$ is the average of the βs and σ_{β}^{2} is their variance. Therefore, we have approximately:

$$\frac{d\beta(\rho)}{d\rho} \cong \frac{h_{\beta\beta\rho}h_{\beta} - h_{\beta\rho}h_{\beta\beta}}{(h_{\beta})^{2}}\sigma_{\beta}^{2}.$$

Substituting this in the top equation, and assuming that the various derivatives of the function h involved do not change sign in their range of definition, we can reformulate (39) as a condition on the sign of an expression only involving derivatives of the h :

$$h_{\rho}\left(\frac{h_{\beta\beta\rho}h_{\beta} - h_{\beta\rho}h_{\beta\beta}}{h_{\beta}}\right) < 0.$$

As recalled in Section 2, in the case of a linear-quadratic decision process — the one we are considering in our experiment — the estimation can be performed by maximum likelihood on the reduced form of the solution of the model, imposing the relevant non-linear cross-equation restrictions. This is the approach that we shall follow to recover the supposedly deep parameters of the RA.

5.1 *The numerical results*

The set-up of the experiment we performed is identical to that described in Section 3.1, both for the values of the individual deep parameters and for the shock given to the policy process. The only difference is that — in the ante-shock regime — each firm computes its optimal capital stock in a stochastic environment, with shocks to both the idiosyncratic productivity and the interest rate processes.[26] As a result, it is no longer possible to compute by exact aggregation a constant-coefficients, aggregate decision rule — not even in a steady state — and we need to resort to estimation. As mentioned above, we recover the RA parameters from a constrained maximum likelihood estimate of the coefficients of the aggregate decision rule, using data from the ante-shock regime. We use a sample of 90000 observations essentially to eliminate sample variability and small sample bias which can induce spurious dependence of the ”deep” parameters from the policy one. The first rows of Tables 4 and 5 present the estimated coefficients and the implied ”deep” parameters. Differently from the deterministic case, we cannot claim these coefficients (and implied RA parameters) to be strictly ”true”, as they cannot reproduce exactly the aggregate capital stock. Given the size of the sample they are, however, a close approximation to the probability limit of the — possibly misspecified — estimated model under the true DGP — that is, they can be interpreted as pseudo-true values of the corresponding population parameters. The second rows of the two Tables present the corresponding values obtained by repeating the procedure using an equally long sample drawn entirely from the post-shock regime. Given the nature of pseudo-true values of these estimates, the logic of the $RARE$ approach would be seriously called into question were the parameters to change, as they would no longer have any claim to being deep or structural. Equivalently, serious troubles for the $RARE$ approach would be signalled by sizeable differences between these ”actual” post-shock decision rule coefficients and the coefficients — shown in the third

[26] The variance of the both idiosyncratic and policy innovations were set equal to 0.01.

row of Table 4 — recomputed by taking into account the policy change and assuming all RA parameters to be unchanged. As it is apparent from the Tables, neither the "inferred" decision rule coefficients (third row of Table 4) nor the ante-shock RA parameters (first row of Table 5) are equal to, respectively, the actual decision rule coefficients (second row of Table 4) and the post-shock RA parameters (second row of Table 5).

It is interesting to note that, differently from the deterministic case, the equality of the individual γs is not sufficient to guarantee that the coefficients on the lagged capital stocks in the aggregate decision rule (A_{11} and B_{11}) are independent of policy changes. This is shown in Table 4.1, where the procedure leading to Table 4 is repeated after the individual deep parameters have been modified accordingly. In a stochastic environment, then, the two sources of non-structurality previously identified cannot be disentangled. The non-structurality of the RA parameters is graphically exemplified in Figure 3, where the functions relating the pseudo-true value of the deep parameters to ρ are plotted, with each corresponding parameter being the estimate computed using data generated consistently with the corresponding value[27] of ρ. The graph highlights the non-linear dependency of the supposedly deep parameters on the policy parameter.

In order to check the estimation results and the appropriatedness of the sample size, we performed the same experiment considering a single agent economy with parameter values equal to the averages used in the heterogenous case and with the same sample size. The variability of the deep parameters in this case is insignificant and of the order of magnitude of one percent of that present in the heterogenous case.

5.2 *Interpreting and generalizing the results*

As shown in Section 4, in the context of a deterministic DGP, a source of non-structurality of the RA can arise if the dynamics of the aggregate model are required to be equal to the dynamics of individual decision rules, thus forcing a misspecified model onto aggregate data. The misspecification would disappear, however, in a deterministic steady state. We now analyze the issue in greater detail, showing that, once the stochastic nature of the decision rules is taken into account, the attempt to estimate the same model that is appropriate for individual

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[27] To draw the graphs, we simulated and estimated the model for forty different values of the policy parameter ρ on the interval $(0.51, 0.83)$ using the same underlying set of innovations.

agents, using aggregate data, is bound to generate dynamic misspecification. In turn, the latter will induce a dependence of the estimated parameters of the misspecified model on all the parameters of the true underlying model; in particular, on the policy parameters.

Generalizing our example, while retaining linearity of the decision rule and simplifying to a scalar deep parameter, we can generically write the result of the intertemporal maximizing behavior of the agent as an ARMAX model:

$$\Phi(L)(\beta_i, \rho) k_t^i = \Psi(L)(\beta_i, \rho) \varepsilon_t^i + \Theta(L)(\beta_i, \rho) r_{t-1}, \tag{40}$$

where, in keeping with the notation of the example, k_t^i is the action of agent i in time t, β_i is his deep parameter, ε_t^i is the idiosyncratic shock component, r is the policy variable, ρ is the policy parameter[28] and Φ, Ψ and Θ are lag polynomials, respectively of (finite) order p, q and l[29]. Note that equation (40) implicitly defines a set non linear cross equation restriction as the same deep and policy parameters appear as arguments of all lag polynomials.

If however agents' parameters differ, then the DGP that generates the aggregate data can differ quite dramatically from all individual decision rules, as the sum of ARMAX processes is still an ARMAX but of different order lag; in particular, if the individual decision rule is given by (40) then the aggregate (by average) over N agents is:

$$\begin{aligned} &\prod_{i=1}^{N} \Phi(L)(\beta_i, \rho) k_t \\ = &\ \frac{1}{N} \sum_{i=1}^{N} \prod_{j \neq i} \Phi(L)\left(\beta_j, \rho\right) \left(\Psi(L)(\beta_i, \rho) \varepsilon_{it} + \Theta(L)(\beta_i, \rho) r_{t-1}\right), \end{aligned} \tag{41}$$

which is still an ARMAX but of order $(N \times p, (N-1) \times p + q, (N-1) \times p + l)$. Note that if all the roots of the polynomials Φ and Ψ are equal across agents and the aggregate model

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[28] The idea is that the DGP of the policy variable is a function of some stochastic components and some parameters ρ. The presence of both the parameters and the policy variable derives from the fact that in most cases (40) will be the solution to a rational expectation problem in which the agent's action today is a function of the expected value of the future policy and to compute those the agent uses the knowledge of the DGP of the policy variable. More precisely, we assume that different agents will discount differently the future policy and this is one of the origin of the non-linear mixing between policy and individual taste parameters.

[29] The finite order of the polynomials is not a restriction given that every linear process can be approximated to any degree of precision by a finite lag polynomial representation.

has the same ARMA structure as the agents' and, on top of that, the coefficients of Θ have the separability property as given in Section (4.1), then (41) is nested into (40) and the model (40) is not misspecified for the aggregate data. Naturally, this is the case if all parameters (including those determining the stochastic properties of the idiosyncratic disturbances) are the same across agents, in which case the aggregate data will be consistent with the same DGP underlying each individual's behaviour and a perfect aggregation can be carried out.

As an example, consider the special case in which the single agent's decision rule is specified as follows:

$$k_t^i = \beta_i k_{t-1}^i + \rho r_{t-1} + \varepsilon_t^i,\ \varepsilon_t^i \sim NID(0, \sigma^2) \tag{42}$$

and assume that in this economy there are only two agents, $i = 1, 2$ and the policy variable r follows an AR(1) process with coefficient ρ and innovation $u_t \sim NID(0, \sigma_u^2)$; the aggregate DGP is obtained by solving (42) for k in function only of the exogenous variable and the innovation and by then summing over i:

$$\begin{aligned} k_t &= (\beta_1 + \beta_2)k_{t-1} - (\beta_1\beta_2)k_{t-2} + \rho r_{t-1} - \frac{1}{2}\rho(\beta_1 + \beta_2)r_{t-2} + \\ &\quad + \frac{1}{2}(1 - \beta_2 L)\varepsilon_t^1 + \frac{1}{2}(1 - \beta_1 L)\varepsilon_t^1, \end{aligned} \tag{43}$$

so that while the original model was an ARMAX(1,0,0) the aggregate one is an ARMAX(2,1,1).

It is worth stressing that the difficulty with aggregation lies in the presence of idiosyncratic dynamic terms. These, however, will almost invariably be part of the decision rule of forward-looking rational agents, at least as long as the problem is genuinely intertemporal — i.e., it cannot be reduced to a sequence of unrelated static problems — which in turn is a precondition for the future to overshadow the present and for the parameters of the policy process to show up in the coefficients of the decision rule: in a word, for the Lucas critique to apply. We can then conclude that whenever a representative agent decision rule mirroring the individual one is estimated on the aggregate data, the resulting model will be misspecified.

The misspecification implies that the RA is non-structural. More precisely, the estimated parameters of equation (40) will converge to their pseudo-true values under the actual DPG, given by equation (41). As a result, the pseudo-true values of the estimate will be a function of all the parameters of the aggregate DGP (41). Hence, the estimated parameters that the approach identifies as structural are not independent of policy shifts: once again, the deep parameters are in fact not deep at all, so that any prediction about the aggregate reaction to a change in policy is bound to be biased.[30] To illustrate this result let us consider equations (42) and (43). By imposing on the aggregate DGP (43) a model of the same form as the individual DGPs, (42) amounts to omitting two variables from the estimate and neglecting the moving average structure of the disturbances. The estimates of the ”deep” parameters will thus be a function of everything that has been omitted. This can be seen from the form of the probability limit of the parameter of the lagged endogenous variable, β, when equation (42) is estimated by conditional maximum likelihood under the hypothesis that the true DGP is (43):

$$p\lim\widehat{\beta} = \frac{\beta_1+\beta_2}{1+\beta_1\beta_2} + \frac{\beta_1\beta_2(\rho^2-1)\frac{\rho^2\sigma_u^2}{4(1-\rho^2)}\left(\frac{2-\beta_1-\beta_2}{(1-\beta_1\rho)(1-\beta_2\rho)}\right)^2 - \frac{\left(\beta_1\sigma_2^2+\sigma_1^2\beta_2\right)}{2(1+\beta_1\beta_2)}}{\left(\gamma_k\left(0\right) - \frac{\rho^2\sigma_u^2}{4(1-\rho^2)}\left(\frac{2-\beta_1-\beta_2}{(1-\beta_1\rho)(1-\beta_2\rho)}\right)^2\right)},$$

where $\gamma_k\left(0\right)$ is the unconditional variance of k. Even if the actual correlation has not been explicitly written, it is immediately evident that in the numerator of the second term on the RHS of the above expression ρ does not cancel out, so that the probability limit of β will be function of ρ.[31] The instance of non-structurality that the above result highlights is similar to the first of the two identified in Section 4 — where the aggregate decision rule coefficients were obtained by explicit aggregation — as both stem from the misspecification of the model imposed on aggregate data. There is a difference, however. While it can be easily verified that, paralleling the analysis in Section 3, a deterministic steady state, aggregate decision rule

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[30] It is possible to show that in the general case of linear decision rule, with a large number of agents, the pseudo-true value of the estimates of the representative agent model are a function both of the policy parameters and of a measure of the heterogeneity of the agents, given by the variance of the distribution of the taste parameters across agents.

[31] It is worth pointing out that the dependence of the pseudotrue value of the coefficient in the aggregate model from the policy parameter would arise even were ρ not present in the individual decision rule, as the estimated model is misspecified. As a result, the estimated equation is missing some lags of the policy variable - which are indeed a function of the policy parameter ρ - and it is through that channel that the pseudotrue value of the taste coefficient ends up being a function of ρ.

could be obtained from (42) where the coefficient of the lagged dependent variable does not depend on ρ, the limit in probability of its estimate was shown to depend on ρ.[32] On the other hand, it is worth noting that these differences are more apparent than real, as in a stochastic environment the steady state is never attained and the coefficient computed — period by period — by aggregation would end up being a function of ρ as well.[33]

The non-structurality of the RA induced by misspecification is then a pervasive characteristic of the $RARE$ approach when implemented in "real life" circumstances, as it neglects the fact that the aggregate DGP does not mimic any individual decision rule. In addition it should not be forgotten, of course, that the second cause of non-structurality — which was shown in Section 4 to be the non-separability of the function of deep and policy parameters representing (at least one of) the coefficients of the individual decision rule — is still at work, even in a stochastic world. Differently from the deterministic case, in which we were able to isolate it, that source of non-structurality will always be mixed with the first, since — as argued above — individual decision rules will almost invariably possess idiosyncratic dynamics, which in turn is sufficient to produce non-structurality of the first kind.

To tie up two loose ends of Section 3 (and 5), the issues of the cross equation constraint and of "bizarre" parameters estimates need to be addressed. As we verified in Section 4, heterogeneity of agents implies in general that the non-linear cross equation restrictions that hold at the micro level are not necessarily consistent with the aggregate data. If, nevertheless, we impose these constrains in estimation we introduce a further source of misspecification, which is added to the previous one. More explicitly, imposing a representative model implies that we are restricting the analysis to a class of models that does not include the data generating process of the aggregate data; if we also impose cross equation restrictions which are not actually satisfied, this will narrow the search to an even smaller class of model, which is, in a way, more misspecified than the true model.

Finally it is worthy to mention that — in spite of the large sample used — very frequently the estimates of the aggregate decision rule coefficients implied extreme values for some of

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32 The numerical results in Section 5.1 show that the estimates of the dynamic terms are non-structural even in those circumstances - when the γs are all equal - in which the deterministically computed coefficients would be independent of ρ.

33 Indeed, in a stochastic environment there would be no exact, constant parameter aggregate decision rule.

the ”deep” parameters, sometimes outside the admissible range. Indeed, a qualitatively similar phenomenon was seen to occur even in the most favourable case in which the coefficients of the aggregate decision rule could be computed by exact aggregation. The presence of non-linear cross equation restrictions implies that the deep parameters are recovered through non-linear transformations of the reduced form coefficient estimates. The non-linear transformation can easily induce large changes in the variance of the estimated deep parameters. It is therefore possible that even if the econometrician is able to recover quite precise estimates of the reduce form parameters, the non-linear transformation of these can present a very large variance. So it is not infrequent to recover fairly bizarre deep parameters.

6. Conclusions

The device of interpreting macroeconomic phenomena as corresponding to the optimizing behaviour of a (large) RA is — as a cursory look at the most recent macro textbooks will confirm — a corner-stone of modern, micro-founded macroeconomic theory. From an empirical point of view, the RA device finds its support in the possibility of recovering (aggregate) deep parameters from aggregate data.[34]

Our results, which expound and strengthen arguments outlined in Geweke (1985) and Kirman (1992), clearly imply that that research program is logically faulty, as there is no reason to expect that the aggregate economy behaves as a single optimizing agent, suitably chosen, even in the most favourable circumstance in which each and every agent in the economy behaves in that way.

This leaves economists in a very uncomfortable position. If we cannot interpret aggregate data with the conceptual framework provided by microeconomic theory, should we limit ourselves to take note of broad aggregate correlations and hope that they will persist?

As is often the case in economics, a ”corner” solution is not the optimal one. Neither ascribing ”*legitimacy only to models that are exact aggregation of agents who optimize subject*

[34] Critics of this program are usually not taken very seriously. To quote just one example, while Kirman (1992) original wording was hardly ambiguos ”*it is clear that the representative agent deserves a decent burial, as an approach to economic analysis that is not only primitive, but fundamentally erroneous,”* Turnovsky (1995) refers to it by writing: ”*Kirman seems to suggest that we should [*abandon the representative agent model*]”,* and hastens to add: *”although that view seems extreme*.”

to constraints" (Hahn and Solow, 1995), nor resorting to pure time series analysis is likely to be the appropriate way out. Rather, to quote again Hahn and Solow, we should "*pay attention to micro-foundations in the sense that [our macro models] are suggested by or analogous to or loosely abstracted from the micro models.*" As a consequence "*econometricians who are working in view of studying macroeconomic policy must be satisfied with impure procedures: they are evidently not justified in deforming the reality of complex phenomena in order to force it to fit into overly simplified specifications*" (Malinvaud, 1981).

Table 1

ASSUMPTIONS ON THE PARAMETERS

	η	β	γ	λ	δ
Firm 1	0.610	0.950	5.50	0.724	0.50
Firm 2	0.650	0.900	8.00	0.693	0.50
Firm 3	0.690	0.850	10.50	0.660	0.50
Firm 4	0.730	0.800	13.00	0.626	0.50
Firm 5	0.770	0.750	15.50	0.591	0.50
Firm 6	0.810	0.700	18.00	0.556	0.50
Firm 7	0.850	0.650	20.50	0.519	0.50
Firm 8	0.890	0.600	23.00	0.481	0.50
Firm 9	0.930	0.550	25.50	0.443	0.50
Firm 10	0.970	0.500	28.00	0.405	0.50
Average	0.790	0.725	16.75	0.570	0.50
Standard Dev.	0.121	0.151	7.56	0.108	0.00

Table 2

AGGREGATE REDUCED FORM COEFFICIENTS

BEFORE AND AFTER THE POLICY SHOCK

(ρ raised from 0.55 to 0.605)

	A_{11}	A_{12}	B_{11}	C_1
Before the shock	1.3586	0.3182	-0.4413	4.3317
After the shock	1.3589	0.2397	-0.4411	4.2452
After the shock, using the estimates of deep parameters obtained before the shock	1.3586	0.2561	-0.4413	4.3205

Table 3

AGGREGATE DEEP PARAMETERS

BEFORE AND AFTER THE POLICY SHOCK

(ρ raised from 0.55 to 0.605)

	β	λ	η	γ
Before the shock	0.2835	0.5372	0.8213	24.4720
After the shock	0.0807	0.5362	0.8226	26.9380

Table 2.1

AGGREGATE REDUCED FORM COEFFICIENTS

BEFORE AND AFTER THE POLICY SHOCK WITH γs EQUAL

(ρ raised from 0.55 to 0.605)

	A_{11}	A_{12}	B_{11}	C_1
Before the shock	1.3502	0.3182	-0.4455	9.4502
After the shock	1.3502	0.2397	-0.4455	9.3637
After the shock, using the estimates of deep parameters obtained before the shock	1.3502	0.2463	-0.4455	9.4139

Table 3.1

AGGREGATE DEEP PARAMETERS

BEFORE AND AFTER THE POLICY SHOCK WITH γs EQUAL

(ρ raised from 0.55 to 0.605)

	β	λ	η	γ
Before the shock	0.5843	0.5741	0.7759	28.459
After the shock	0.5206	0.5741	0.7759	29.704

Table 4

AGGREGATE REDUCED FORM COEFFICIENTS

BEFORE AND AFTER THE POLICY SHOCK

(ρ raised from 0.55 to 0.605)

	A_{11}	A_{12}	B_{11}	C_1
Before the shock	1.3818	0.3525	−0.4690	4.537
After the shock	1.3827	0.2751	−0.4701	4.464
After the shock, using the estimates of deep parameters obtained before the shock	1.3818	0.2753	−0.4690	4.4865

Table 5

AGGREGATE DEEP PARAMETERS

BEFORE AND AFTER THE POLICY SHOCK

(ρ raised from 0.55 to 0.605)

	β	λ	η	γ
Before the shock	0.6628	0.5997	0.7819	14.477
After the shock	0.6693	0.6025	0.7798	14.126

Table 4.1

AGGREGATE REDUCED FORM COEFFICIENTS

BEFORE AND AFTER THE POLICY SHOCK WITH γs EQUAL

(ρ raised from 0.55 to 0.605)

	A_{11}	A_{12}	B_{11}	C_1
Before the shock	1.3865	0.3664	−0.4716	7.0152
After the shock	1.3875	0.2891	−0.4728	6.9527
After the shock, using the estimates of deep parameters obtained before the shock	1.3865	0.2903	−0.4716	6.9644

Table 5.1

AGGREGATE DEEP PARAMETERS

BEFORE AND AFTER THE POLICY SHOCK WITH γs EQUAL

(ρ raised from 0.55 to 0.605)

	β	λ	η	γ
Before the shock	0.6684	0.5987	0.7877	20.703
After the shock	0.6734	0.6020	0.7853	20.213

Figure 1

PARAMETERS CALCULATED AS FUNCTIONS OF THE POLICY

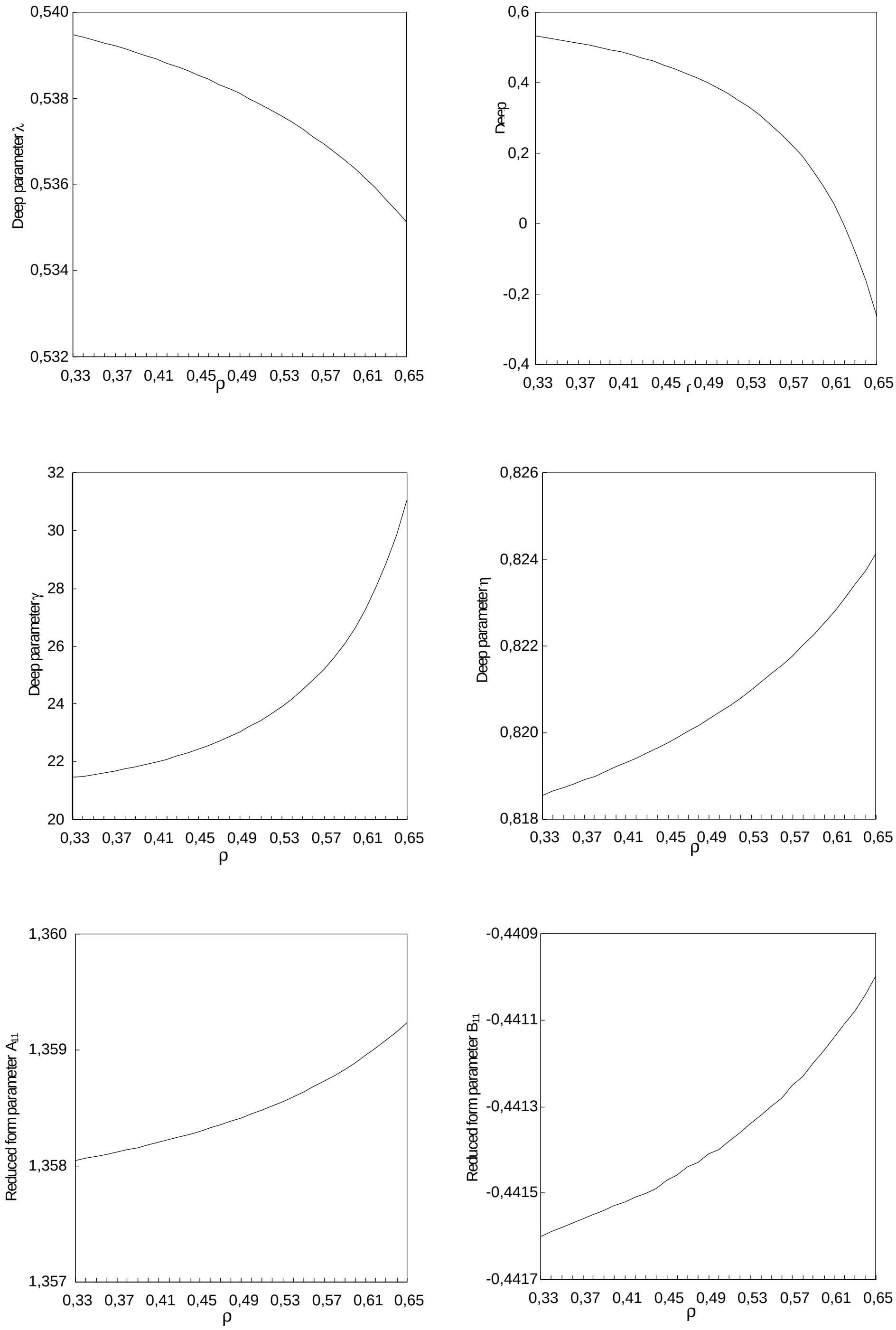
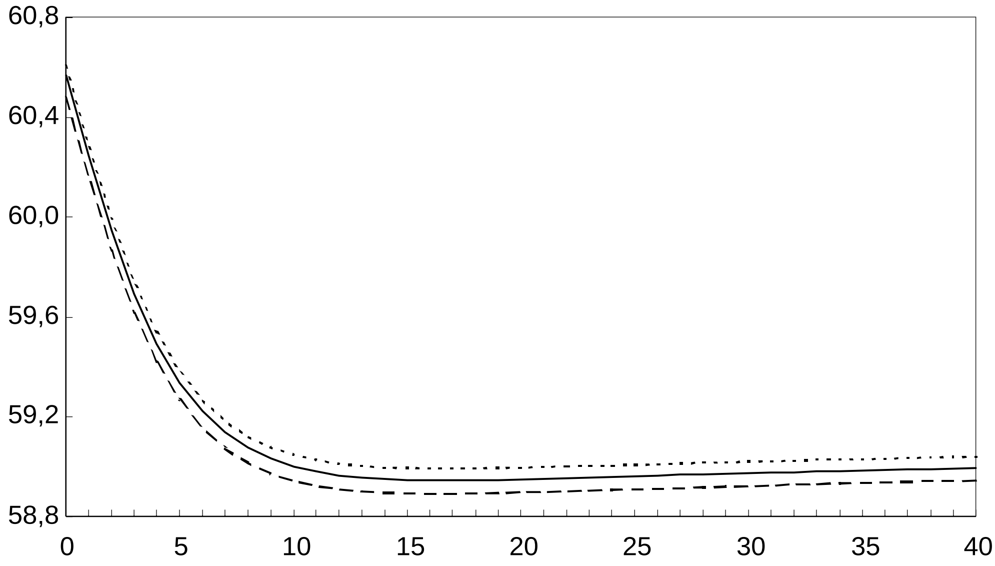


Figure 2

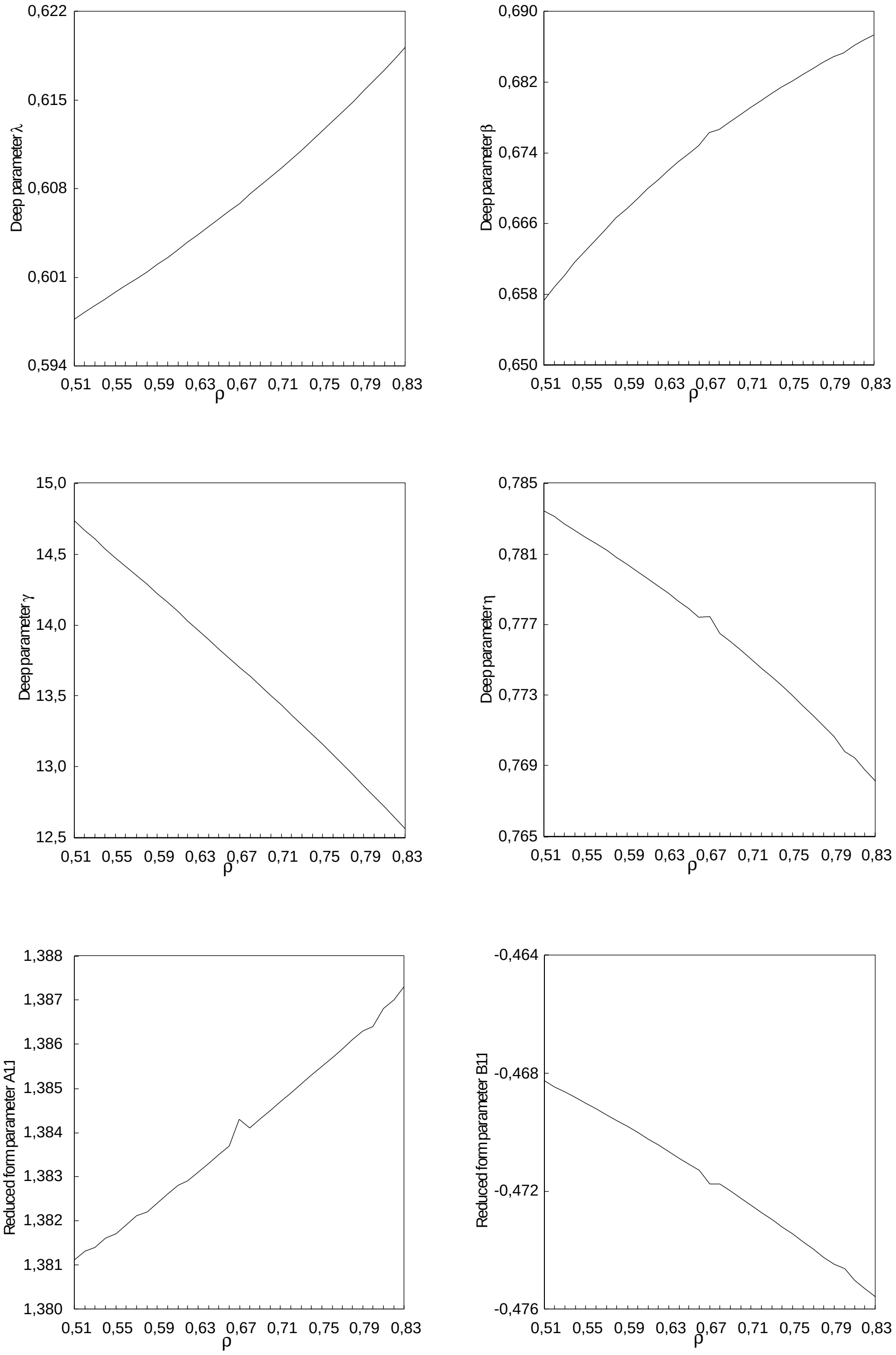
SIMULATED RESPONSES OF THE AGGREGATE CAPITAL
TO THE POLICY SHOCK



- - - - $RARE$ approach, ——— Traditional approach and – – True reaction

Figure 3

PARAMETERS ESTIMATED AS FUNCTION OF THE POLICY



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