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Reallocation and Learning over the Business Cycle

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REALLOCATION AND LEARNING OVER THE BUSINESS CYCLE

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Abstract

I show how cyclical aggregate shocks can stimulate structural reallocation activities, which in turn amplify the effect of the shock. I emphasize the *informational aspects* related to restructuring activities and their potential interplay with aggregate shocks. Building on work by Caplin and Leahy (1994), I develop a model in which production units are uncertain about the value of staying in the market and learn about it over time in a Bayesian fashion. In addition to their own private assessment, they can also learn from observing other units' decisions. Given that adjusting is costly, each unit has an incentive to delay action and wait for other players to act in order to make a better informed decision. If delay is more costly in a downturn, a negative aggregate shock can break the inertia and induce the most pessimistic agents to exit. The information released by such actions will induce more action, thus generating a burst in restructuring activities that reinforces the initial effect of the aggregate shock. This process of information accumulation and revelation offers both a powerful amplification mechanism of relatively modest aggregate shocks and a potential explanation of why restructuring tends to be concentrated in recessions.

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1. Introduction[1]

A substantial body of recent empirical work has challenged the traditional neoclassical view of the business cycle. On one side, the empirical literature on aggregate fluctuations has been unsuccessful in identifying impulses that can account for the large variations in macroeconomic time series over the cycle (Cochrane, 1994). Figure 1 reports Davis-Haltiwanger's quarterly data for job creation, job destruction and employment growth for the period 1972-1988. The prominent features of the figure are the spikes in job destruction, and the corresponding decrease in employment, that characterize the troughs. The lack of obvious large impulses points to the importance of identifying *amplification mechanisms* that can explain such spikes. On the other side, the body of work initiated by Blanchard and Diamond (1990) and Davis and Haltiwanger (1990), based on the analysis of gross flows of workers in and out of unemployment, has suggested that a substantial part of the job destruction that takes place in a recession is related to reallocation of workers from one production unit to another, rather than to cyclical fluctuations in the demand and supply of labor.[2] This observation suggests a powerful amplification mechanism: relatively small aggregate shocks could trigger a process of reallocation activities during a concentrated period that amplify the overall effects of the initial shock.

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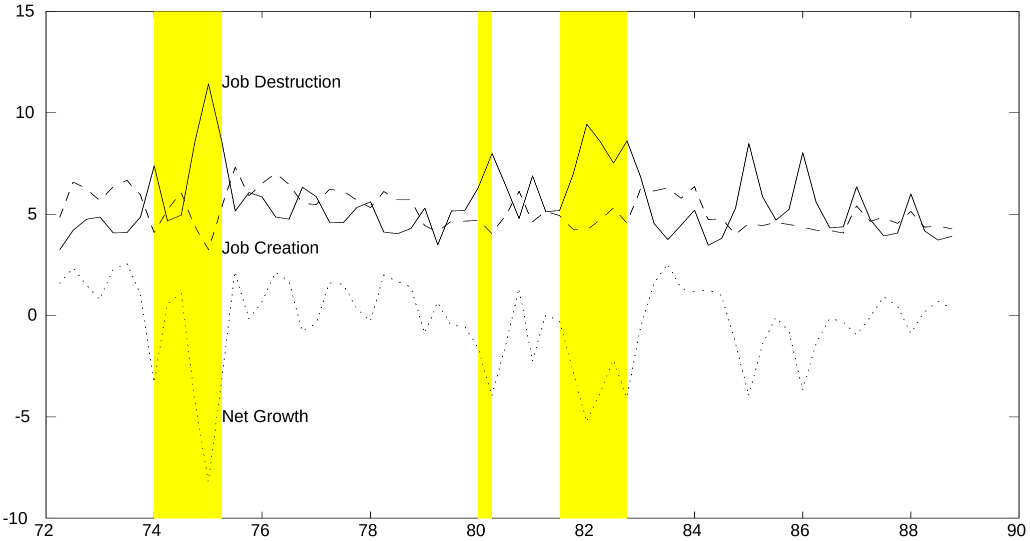
JEL classification numbers: E32, D83, L16, J65, C73.

[2] See Caballero and Hammour (1994) for a model of "creative destruction" along these lines.

Figure 1

NET AND GROSS JOB FLOW RATES IN MANUFACTURING, 1972:1988



Source: LRD data.
Shaded regions represent recessions.

Research on the interrelation between aggregate and allocative shocks,[3] and on their respective roles in generating employment fluctuations, was initiated by Lilien (1982), who showed that during recessions the variance of employment growth rates across sectors increases substantially. Since then, a consensus has emerged that a considerable part of the employment changes taking place in a recession has a structural rather than a cyclical character. Identifying the direction of causality has turned out to be a harder task, as argued, among others, by Loungani (1996): is the reallocation a driving force of aggregate fluctuations or is it that the economy takes advantage of the low level of economic activity to carry out necessary restructuring activities? While this is an important question, an increasing number of empirical studies using micro data show that reallocating workers from one production unit to another takes time and resources.[4] This evidence suggests that, even if aggregate

[3] I refer to allocative shocks as shocks that change the long-run desired allocation of resources in the economy, and to aggregate shocks as having only a temporary and symmetric impact on all production units.

[4] For example, using data from the Displaced Workers Survey and the Panel Study of Income Dynamics, Hall (1995) finds that, after losing a long tenure job, a worker is likely to hold a sequence of short-term jobs. The

shocks were the primary source of aggregate fluctuations, substantial movements of workers across production units would have an important impact on the character and magnitude of the resulting employment fluctuations.

These observations (and the availability of Census data) have led to a renewed interest in the role of allocative shocks in determining employment fluctuations over the business cycle.[5] Although still far from definitive, the main findings can be summarized as follows:

– Allocative shocks have an important role in driving aggregate employment fluctuations.
– The reallocation of workers within sectors is at least as important as that across sectors, even for narrowly defined sectors.
– A large part of job destruction is attributable to plants that reduce employment by 25 percent or more, and the share of such plants increases during recessions.

The characterization of the economy that emerges from this literature is one in which heterogeneous units with changing desired employment follow nonlinear adjustment policies that induce large and infrequent downward adjustments, maybe due to the presence of kinked adjustment costs. It is important then to understand how the timing and the intensity of reallocation activities interact with aggregate shocks to determine the movements in employment over the business cycle.

I build a model of endogenous revelation of information to explain the sudden increase in job destruction that characterizes a recession. I set up an economy in which production is carried out using different technologies. Production units are divided into cohorts, with all units in a given cohort sharing the same technology. They are uncertain about the efficiency of their technology, and consequently about the optimality of remaining in production, and learn about it over time in a Bayesian fashion.[6] Units' assessments are private information, but they can observe the decisions of similar units and infer useful information from that. If shutting

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loss of income resulting from the displacement, due both to the time spent out of work and to the reduction in earnings in the new jobs, is estimated to be about 1.2 years of earnings.

[5] Recent studies that address these issue are Caballero, Engel and Haltiwanger (1997), Campbell and Kuttner (1996) and Davis and Haltiwanger (1996).

[6] Jovanovic (1982) proposes a model in which firms learn about their efficiency over time. The predictions of the model are empirically supported by the findings of Dunne, Roberts and Samuelson (1989).

down is costly, there is an incentive to wait for somebody else to exit in order to make a better informed decision.

With the right restrictions imposed, the model can be solved along the lines of Caplin and Leahy's (1994) model of a multi-stage investment project. The model generates *delay* in adjustments: units tend to postpone costly restructuring activities. In the presence of aggregate shocks, it turns out that the cost of delaying is lower in booms, so that even production units that are quite pessimistic about their prospects might find it preferable to wait; on the contrary, a negative aggregate shock makes delaying more costly, thus prompting the most pessimistic units to act. Once some agents undertake adjustment, the number of liquidations releases information that might induce others to shut down. Aggregate shocks therefore trigger a process of information revelation and actions that speeds up learning and culminates in a large number of units undertaking adjustment in a short period of time. The role of the shocks is not confined to reducing overall productivity: rather, they influence the economy mainly by breaking the inertia that characterizes agents' behavior.

A number of recent papers have studied the problem of information accumulation and endogenous revelation in a strategic contest.[7] By studying the connection between aggregate shocks and exit decision, this paper formally applies the insights of this literature to the explanation of the business cycle. Technically, the model builds on Caplin and Leahy's (1994) model of a multi- stage investment project, from which I borrow the way uncertainty and strategic learning are formalized, as well as the solution technique. The main technical difference is the introduction of an aggregate state to account for aggregate shocks. Without complicating the strategic aspects, I embed the basic model in an environment with entry over time, which allows me to study the time series properties of the economy and the concentration effect on exit induced by aggregate shocks.

The rest of the paper is organized as follows. Section 2 describes the model, and Section 3 solves for the equilibrium. Section 4 illustrates the role of aggregate shocks in determining the timing of reallocation activities, while Section 5 investigates the implications of the interaction between aggregate shocks and reallocative activity. In a simulation exercise,

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[7] For a diverse range of models and applications, see Banerjee (1992), Bikhchandani, Hirshleifer and Welch (1992), Caplin and Leahy (1994,1996), Chamley and Gale (1994), Horvath, Schivardi and Woywode (1997), Rob (1991).

I show that the model generates concentration of reallocation activities that amplifies the effect of the aggregate shocks. Section 6 contains some concluding remarks.

2. The Model

2.1 *The single production unit problem*

I start by describing the single production unit problem and introduce the strategic aspects after that. Time is discrete; a production unit is uncertain about the efficiency of its technology and learns about it over time. The unit has to decide whether to remain active or to exit. In every period the unit receives an idiosyncratic profit realization, drawn from a binary random variable $Z = \{z_g, z_b\}$, with $z_g > z_b$.[8] The production unit is one of two types $\{\theta_l, \theta_h\}$ ("high" and "low"), where the type determines the probability of the realizations of the shock. A θ_h production unit is more efficient in the sense that it is more likely to experience the good realization of the idiosyncratic shock:

$$\Pr\{Z = z_g|\theta_l\} < \Pr\{Z = z_g|\theta_h\}. \tag{1}$$

At time zero the production unit holds prior beliefs and updates them over time in a Bayesian fashion according to the realizations of Z. Given that the unit can be only one of two types, the prior is the probability assigned to being type θ_l: $\lambda_0 = \Pr\{\theta = \theta_l\}$. Given the discrete nature of the prior distribution, the posterior will also be a discrete distribution: in any period, the unit's beliefs are summarized by a value λ representing the probability assigned to the event $\{\theta = \theta_l\}$. In fact, given n_g good and n_b bad realizations of the productivity shock, Bayes rule gives:[9]

$$\begin{aligned} \lambda(n_g, n_b; \lambda_0) \equiv \Pr\{\theta = \theta_l|n_g, n_b; \lambda_0\} = \\ \frac{\Pr\{z_g|\theta_l\}^{n_g}\Pr\{z_b|\theta_l\}^{n_b}\lambda_0}{\Pr\{z_g|\theta_l\}^{n_g}\Pr\{z_b|\theta_l\}^{n_b}\lambda_0 + \Pr\{z_g|\theta_h\}^{n_g}\Pr\{z_b|\theta_h\}^{n_b}(1-\lambda_0)}. \end{aligned} \tag{2}$$

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[8] For example, the shock could be the realization of the production costs, with cost being inversely proportional to the realizations of Z.

[9] Formally, given a random sample from a Bernoulli distribution with unknown parameter, the family of priors from the n-values discrete distribution is (trivially) a conjugate family. Note that Z behaves like a Bernoulli random variable, apart from the fact that the values of the realizations are $\{z_g, z_b\}$ rather then $\{0, 1\}$.

For a given λ, the expected value of profits is:

$$\pi(\lambda) = E(Z|\theta_l)\lambda + E(Z|\theta_h)(1-\lambda) \tag{3}$$

where, for $i = \{l, h\}$,

$$E(Z|\theta_i) = z_b \Pr\{Z = z_b|\theta_i\} + z_g \Pr\{Z = z_g|\theta_i\}. \tag{4}$$

Given the assumption that a θ_l production unit is more likely to receive a bad realization of the shock, we have $E(Z|\theta_l) < E(Z|\theta_h)$, so that $d\pi/d\lambda < 0$: the expected value of profits is decreasing in the probability assigned to being type θ_l.

A production unit starts at time zero with a prior λ_0. Future profits are discounted at rate β. Upon exit, a unit pays a scrapping cost k or receives a salvage value $-k$;[10] exit is an irreversible decision. The exit cost is intended to capture the degree of labor market flexibility. Parameters of the model are selected such that, conditional on types, it is optimal for a low type to exit and for a high type to stay. The unit solves the following dynamic programming problem:

$$v(\lambda) = \max\{-k, \pi(\lambda) + \beta E v(\lambda')\}. \tag{5}$$

Standard arguments ensure existence and uniqueness of a solution. At this point, I do not directly tackle the problem, but note that the solution of similar problems is well known both in discrete and in continuous time.[11] Beliefs constitute a martingale that evolves according to the realizations of Z and the production unit solves an optimal stopping problem; the optimal policy takes the form of a threshold level for beliefs, above which it is optimal to shut down.

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[10] What is essential is the relative value of k compared to the expected profits from remaining in the market. Abusing notation, define $\pi(\theta_i)$ as the expected profit for a unit known to be of type θ_i (so that, for example, $\pi(\theta_l) \equiv \pi(1)$; then, a unit that knows its type will remain in the market if and only if $\pi(\theta_i) \geq (1-\beta)k, i = \{\theta_l, \theta_h\}$, *i.e.* if the expected profits are at least as large as the flow revenue coming from exit.

[11] See for example the literature on Ss adjustment policies (Bertola and Caballero, 1990) and the parallel literature on irreversible investments (Dixit and Pindyck, 1994) for the continuous time case, and Sargent (1987) for the discrete case.

2.2 *The aggregate state*

In each period the economy can be in one of two states $s = \{n, d\}$, where n denotes "normal" and d denotes "downturn". The aggregate state is realized and observed *before* the production unit makes its decision, so that there is no uncertainty about the state of the economy in the current period.[12] The aggregate state influences the average productivity by determining the values that the idiosyncratic shock can take. A downturn lowers both values of the shock: $z_j^d < z_j^n$ for $j = \{g, b\}$; the probability of each realization is independent of the aggregate state.[13] As a consequence, the expected profits are lower in a downturn for all values of λ:

(6) $$\pi(\lambda, n) > \pi(\lambda, d)$$

where

(3′) $$\pi(\lambda, s) = E(Z^s|\theta_l)\lambda + E(Z^s|\theta_h)(1-\lambda)$$

and

(4′) $$E(Z^s|\theta_i) = z_b^s \Pr\{Z = z_b^s|\theta_i\} + z_g^s \Pr\{Z = z_g^s|\theta_i\}.$$

The aggregate state evolves according to a Markov transition matrix. I use the notation $\gamma_t(s'|s)$ to indicate the probability that t periods ahead the state is s', given that the current state is s. I assume that it is more likely that the aggregate state will be n next period if it is n today:

(7) $$\gamma_1(n|n) > \gamma_1(n|d).$$

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[12] The emphasis of the model is on idiosyncratic rather than on aggregate uncertainty. Gonzales (1996) develops a model in which the evolution of the aggregate state is unobservable and agents can costly experiment to obtain information about it.

[13] This assumption makes the pace of learning independent of the aggregate state. An interesting alternative would be one in which the probabilities of receiving a certain shock also change over the business cycle. For example, one could argue that a downturn is more effective at discriminating among efficient and inefficient production units and model the probabilities accordingly.

The introduction of the aggregate state necessitates that the unit's problem be reformulated contingent on the aggregate state itself:

$$(5') \qquad v(\lambda, s) = \max\{-k, \pi(\lambda, s) + \beta E \sum_{s'} v(\lambda, s')\gamma_1(s'|s)\}.$$

The solution is characterized by a couple of state-contingent threshold values for beliefs, $\{\lambda_d^*, \lambda_n^*\}$, above which the unit leaves the market.

2.3 *The game*

Consider now an economy populated by a continuum of units. The problem of the single unit is identical to the one described in the previous section. I make four important assumptions about the interaction among units and about their evolution.

ASSUMPTION 1. There is no interaction at the level of payoff: the profits of one unit do not depend either on the actions or on the number of other units.

ASSUMPTION 2. Signals are private information: a unit can only observe other units' actions.

ASSUMPTION 3. The realizations of signals are independent across production units and over time.

ASSUMPTION 4. In addition to the endogenous exit decision, there is an exogenous probability of death δ.

The first assumption serves to simplify the analysis, and emphasizes the informational aspects of the model. While desirable, the endogenization of profits would make the model intractable. The second assumption is indeed the critical one, which gives rise to the strategic behavior of the units: it generates an incentive to observe other units' actions in order to gain information from them. The last assumption ensures that the economy will be characterized by positive entry and exit in the long run.

The economy has an infinite number of *cohorts*. A cohort is the set of units that enter the market in a given period. Units in a particular cohort share the same technology, but its type is unknown. Given that all units in a cohort have the same characteristics, each one of them can obtain useful information about itself from observing the behavior of the others.[14] In any period, cohorts are named according to their age. Define $x_\tau(t)$ as the mass of units of cohort τ that are in the market at the beginning of period t. The mass of units in each cohort in the market at the beginning of time t is represented by a sequence $\{x_\tau(t)\}_{\tau=1}^{\infty}$. The total mass of incumbents at the beginning of the period is given by:

$$\Phi(t) = \sum_{\tau=1}^{\infty} x_\tau(t).$$

I assume that the there is an upper bound to the measure of units in the economy, which is normalized to 1: $\forall t, \quad \Phi(t) \leq 1$. This is intended to represent the maximum number of "production sites" available, and deviation from the bound will be interpreted as a measure of economic slackness.

The timing of events is the following:

1) the aggregate state is revealed and entry of a new cohort takes place;
2) exit decisions are made;
3) idiosyncratic signals and profits are realized;
4) natural deaths occur.

To complete the description of the environment, the entry process and the strategic aspects of the exit decision must be introduced.

Entry

Entry is modeled in an *ad hoc* way: it is assumed that the mass of entering units is a deterministic function of the difference between the maximum potential measure of the market and the actual measure of incumbents at the beginning of the period.[15] In each period a fraction

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[14] The assumption that units that enter the market in a given period are endowed with the same technology is used, in a different setting, by the literature studying the relation between technological innovation and macroeconomic fluctuations (Campbell, 1997; Caballero and Hammour, 1994). In my context, the essence of the argument relies on the fact that units share some uncertain characteristics with each other and can therefore learn from each other. Linking the common element to the period of entry has some empirical appeal and gives rise to testable restrictions, as will be shown later. The model would work equally well in a setting in which the groups are not dependent on the period of entry but simply on the technology used.

[15] This assumption is made for the sake of simplicity. It would be interesting to extend the model to endogenize entry, for example along the lines of Hopenhayn (1992). This possibility is left to future work.

$\alpha \in (0, 1)$ of the difference is filled.[16]

$$y(t) = \alpha * (1 - \Phi(t)) \tag{8}$$

where $y(t)$ is the measure of entrants at time t and, therefore, the measure of the cohort of age zero at t upon entry. Upon entering, a cohort draws a technology from a known binary distribution. The technology is of type θ_l with probability λ_0 and type θ_h with probability $(1 - \lambda_0)$. Another important assumption characterizes the way in which nature selects technologies.

ASSUMPTION 5. Draws of technologies are independent over time.

This assumption implies that there is no information a unit can get about its type by considering the types of other cohorts.

Exit decision

Exit is determined partially exogenously, as a consequence of natural death, and partially endogenously, as the deliberate choice of incumbent units. The exit choice involves strategic considerations that are at the heart of the model, given that a unit can learn from the behavior of units in the same cohort. I concentrate on the informational aspects involved in the process of discovering types, that is, on how production units learn about their type and how the speed and timing of learning and reallocation interact with the aggregate state. The extensive form of the game is the following. At each point in time a *history* $h^t \in H^t$ is a sequence recording the actions of units currently and previously in the market and the realizations of exogenous events up to the point when the units must act.[17] The action space after history h^t for a unit that has entered the market at or before period t is defined as:

$$A(h^t) = \begin{cases} \{\text{stay,exit}\} & \text{if not previously exited} \\ \emptyset & \text{otherwise} \end{cases}.$$

[16] This assumption is motivated by the large body of literature on matching models of the labor market (Mortensen and Pissarides, 1994), in which it is assumed that in each period a vacancy is filled with a given probability. This representation of the functioning of the labor market is in accordance with many “stylized facts” as presented, for example, in Blanchard and Diamond (1990).

[17] Given the timing convention, a history contains signals, natural deaths and exit decisions up to the previous period, and entry and the aggregate states up to the current one.

Exit constitutes an irreversible decision, and one can think of it as the liquidation of the unit. Given that units that have exited the market have no further role in the model, in what follows I will refer only to incumbent players. The *information partition* is a partition $\mathcal{I}_i$ of H whose elements I_i are the *information sets* for player i. Whereas H is the complete history of the game, I_i represents the information available to player i when making a decision. Given my assumptions on the informational structure, the information sets do not contain all the signals, but only those of player i. The per period payoff function is constituted by the expected profits if the production unit stays in the market and by an exit cost if the unit decides to shut down:

(9)
$$u(a,h) = \begin{cases} \pi(\lambda, s) & \text{if } a = \text{stay} \\ -k & \text{if } a = \text{exit} \end{cases}$$

where I exploit the fact that π depends on history only through the current belief and aggregate state. In case of natural death, it is assumed that the unit pays no exit cost (or gets no scrap value).

A *strategy* for player i is a collection of functions mapping from information sets to probability distributions over actions. Given that there are only two possible actions, I adopt the convention that a strategy at t is the probability assigned to action $\{exit\}$: $\sigma_i^t : \mathcal{I}_i^t \to [0,1]$. Given a strategy profile σ, the expected payoff for any player after history h^t is given by:

(10)
$$U(\sigma, h^t) = E\sum_{i=0}^{\infty} \beta^i (1-\delta)^i u(a_{t+i}, h^{t+i})$$

where the expectation is taken with respect to the probability distribution induced by the strategy σ and by the stochastic evolution of the exogenous variables. The future is discounted for the possibility of natural death.

3. Characterizing the equilibrium

The strategic aspects of the model are complex, entailing the solution of an extensive form game of incomplete information with a continuum of players, who are divided among an infinite number of groups of initially unknown type. However, I show that, by introducing appropriate restrictions on strategies and on the equilibrium concept, a solution can readily be identified. The two fundamental assumptions are that all interaction at the level of payoff has been excluded and that cohorts' types are drawn independently over time. Taken together, they imply that there is no connection, either direct or at an informational level, among different cohorts. This suggests that a natural approach to solving the problem is to consider the evolution of each cohort in isolation and solve the strategic aspects of the game within this narrower environment. Given that all cohorts face the same problem, once the solution for a representative cohort has been found, the economy's evolution can be obtained by applying it to all cohorts and aggregating over them.

In this section I therefore analyze the behavior of a generic cohort that has entered the market at time $\hat{t}$. I use τ to indicate the cohort's tenure: $\tau = t - \hat{t}$. Considering one cohort in isolation formally translates into focusing on a subset of the history, that is, the one describing the history of cohort τ only. Formally, I define a sub-history $\hat{h}^{\tau} \subset h^{\hat{t}+\tau}$ as a sequence recording the actions of the units of cohort τ and the realization of exogenous events (signals, natural deaths and aggregate states) pertaining to this cohort up to time $\hat{t} + \tau$. Strategies are then restricted to map from partitions of such sub-histories: an information set for unit i of cohort τ contains the sequences of player i's signals, of actions by all production units in the cohort and of exogenous events relating to the cohort's evolution.

Another restriction relates to the equilibrium concept: I concentrate on *Symmetric Nash Equilibria*, in which units at the same information set choose the same strategy, and define a symmetric equilibrium as a strategy σ^* such that no production unit can increase its expected payoff by using an alternative strategy $\hat{\sigma}$ when all other units use σ^*. The symmetry restriction implies that the identity of the units taking a given action has no informational content: only their measure is relevant.

Once these restrictions are introduced, the model can be solved along the lines outlined by Caplin and Leahy (1994).[18] The assumption that there is a continuum of agents is the key one. It in fact guarantees that, while the single unit faces uncertainty, at the aggregate level the distribution of signals, conditional on types, is non-stochastic. As a consequence, the measure of players at each information set is a deterministic function of the type. This imposes a very strong restriction on the pace of information revelation: if a strategy prescribes

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[18] Caplin and Leahy (1994) construct a model of a multi-stage investment project, with the initial measure of investors determined by a zero expected profit condition. My solution of the game for a given cohort follows their work closely, differing in the fact that exit is an irreversible decision and that the economy is characterized by the presence of an aggregate state.

exit for different measures of units according to their types, then by observing this measure one can infer the type of the cohort, and uncertainty is immediately resolved. In other words, the continuum of units assumption implies that the process of information revelation has an all-or-nothing character. As long as no unit decides to exit, no information can be obtained by observing actions; but if the strategy prescribes exit at a particular information set, then generally the mass of exiting units will reveal types immediately. I introduce the following definition to formalize this concept.

DEFINITION 1. A strategy is defined to be type-revealing if for any history $\hat{h}$ there exists a τ^* at which the strategy induces different measures of exit for the different types.

The definition establishes that, if players follow a type-revealing strategy, then at some point types will be revealed by the measure of exiting units. Almost any strategy will be type-revealing. An example of a strategy that is not type-revealing is one that prescribes first exit for *all* units (independently of the signals received) at some age τ.

Consider now the following strategy.

DEFINITION 2. A cutoff strategy is a strategy that, when prescribing exit, does so for all units with beliefs exceeding a state contingent threshold $\lambda(s) \in (0, 1), \quad s = \{n, d\}$.

A cutoff strategy is characterized by a vector $\Lambda = \{\lambda(n), \lambda(d)\}$. For cutoff strategies, the following result holds.

PROPOSITION 1. Cutoff strategies are type-revealing if $\lambda_0 < \lambda(s)$, s={n,d}.

PROOF**:** see Appendix I.

Proposition 1 establishes that cutoff strategies are type-revealing unless they might prescribe adjustment only on the basis of prior information, before any signal is realized. For example, if the strategy prescribes adjustment at any time for $\lambda(d) < \lambda_0$, all units will exit simultaneously if at time zero the aggregate state is d and no information will ever be revealed about types. In such a case, there is nothing a unit can learn from the others, and the problem reverts to the one analyzed for the single unit. Outright exit would be an equilibrium if and only if the prior is beyond the threshold value of the single unit problem. I neglect such cases and concentrate on the ones in which units will be willing to stay in the market at the prior beliefs for both normal periods and downturns.[19]

To determine the equilibrium strategy I concentrate on the problem of an individual unit with beliefs λ, who has to decide her action conditional on being at an information set at which she knows that the type will be revealed. The payoff from exit is given by the exit cost:

$$r(\lambda, s) = -k. \tag{11}$$

Given that next period the type will be known, the payoff from staying depends on the continuation value of a unit that finds out that she is a θ_h type in state s. This is the present discounted value of profits for a high type given the aggregate state s:

$$v(s) = \sum_{i=0}^{\infty} \beta^i (1-\delta)^i [\pi(\theta_h, n)\gamma_i(n|s) + \pi(\theta_h, d)\gamma_i(d|s)]. \tag{12}$$

For a θ_h unit the expected value of being in the market tomorrow, given the aggregate state s today, is therefore given by:

$$V(s) = v(n)\gamma_1(n|s) + v(d)\gamma_1(d|s). \tag{13}$$

$V(s)$ plays a fundamental role in determining the equilibrium strategy. It allows us to determine the payoff from staying, given by the expected profits for the current period plus the continuation payoffs:

$$\tilde{r}(\lambda, s) = \pi(\lambda, s) + \beta(1-\delta)[-\lambda k + (1-\lambda)V(s)]. \tag{14}$$

Equation (14) states that the payoffs from waiting, given beliefs λ and aggregate state s, are equal to the current expected profits plus the discounted expected continuation values; these, in turn, are determined by the fact that the production unit expects to be type θ_l with probability λ, in which case she will pay the exit cost and leave, and θ_h with probability $(1-\lambda)$, in which case the expected continuation value is $V(s)$ as defined in equation (13). The condition for λ

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[19] Note that in a model of endogenous entry, with an entry cost the consistency of such strategies would be ruled out: entry would not take place to begin with.

to be such that a production unit would rather shut down is:

$$r(\lambda, s) \geq \tilde{r}(\lambda, s) \tag{15}$$

Equation (15) can be used to determine the equilibrium strategy. By showing that $\tilde{r}(\lambda, s)$ is continuously decreasing in λ and that there is one and only one value for which $r(\lambda, s) = \tilde{r}(\lambda, s)$, the following proposition, characterizing equilibrium strategies, can be established.

PROPOSITION 2. Equilibrium strategies are cutoff strategies.

PROOF: see Appendix I.

Let $\Lambda^* = \{\lambda_n^*, \lambda_d^*\}$ denote the pair of values that satisfies (15) with equality. Λ^* is obtained by solving (15), for each of the aggregate states, as an equality and by applying the definition of $\pi(\lambda, s)$:

$$\lambda_n^* = \frac{k + \pi(\theta_h, n) + \beta(1-\delta)V(n)}{\pi(\theta_h, n) - \pi(\theta_l, n) + \beta(1-\delta)(k + V(n))} \tag{16a}$$

$$\lambda_d^* = \frac{k + \pi(\theta_h, d) + \beta(1-\delta)V(d)}{\pi(\theta_h, d) - \pi(\theta_l, d) + \beta(1-\delta)(k + V(d))}. \tag{16b}$$

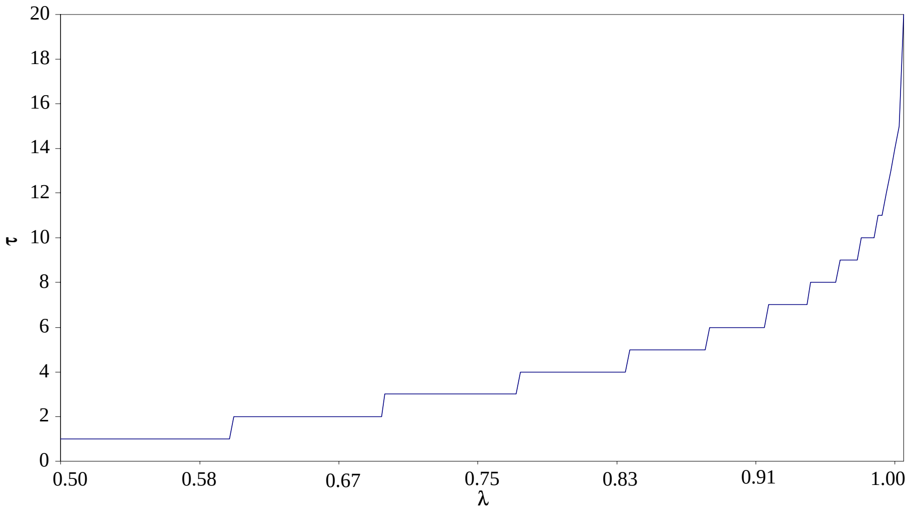
The characterization of the equilibrium strategy in proposition 2 lets us concentrate attention on the most pessimistic production units. Given that there exists a continuum of production units, in each period there will be a nonzero measure of units that has received all possible combinations of signals. In any period, therefore, the beliefs of the most pessimistic production units are the beliefs of those that have received all bad signals. One can then uniquely determine the minimum number of bad signals, and therefore of periods, required for the beliefs of a subset of units to exceed some λ:

$$\tau(\lambda) \equiv \text{Inf}\left\{m \in \mathcal{N}_+ : \frac{\text{Pr}\{z_b|\theta_l\}^m \lambda_0}{\text{Pr}\{z_b|\theta_l\}^m \lambda_0 + \text{Pr}\{z_b|\theta_h\}^m (1-\lambda_0)} \geq \lambda\right\} \tag{17}$$

where $\mathcal{N}_+$ is the set of nonnegative integers. It is easy to show that the fraction in (17) is increasing in m so that $\tau(\lambda)$ is uniquely determined. By equation (17) we can associate with Λ^* a vector $T^* = \{\tau_n^*, \tau_d^*\}$, where, abusing notation, I use τ_s^* for $\tau(\lambda_s^*)$. This vector determines the two *first stopping times*: it indicates the state-contingent minimum number of periods at which the first exit wave could take place in equilibrium. Figure 2 plots $\tau(\lambda)$ for $\lambda \geq 1/2$ for the following parameter values: $\lambda_0 = 1/2, Pr\{Z = z_b|\theta_l\} = .6, Pr\{Z = z_b|\theta_h\} = .4$. Note the discontinuous character of the function $\tau(\lambda)$.

Figure 2

MINIMUM NUMBER OF BAD SIGNALS REQUIRED TO REACH A CERTAIN BELIEFS LEVEL



I have established the existence of cutoff values for beliefs after which a production unit would rather exit than wait. For them to be an equilibrium couple it is also necessary that no production unit would rather exit for less pessimistic beliefs. It could be that a production unit that is sufficiently pessimistic about its type would find it preferable to shut down before reaching the threshold. To eliminate this possibility, it is sufficient to check that the most pessimistic production units are willing to continue for all periods preceding the first exit time: for any $\tau < \text{Max}\{t_n^*, t_d^*\}$, for any s, the value of continuing for the most pessimistic production units, when the equilibrium strategy prescribes so, must be at least as large as the cost of leaving the market:

(18) $$U_\tau(\Lambda^*, s) \geq -k.$$

The complete expression for this value is rather cumbersome, having to take into account the expected payoffs for any possible evolution of the game, and its derivation is confined to Appendix II. Numerical analysis of the game show that existence is indeed a serious issue, and that an equilibrium fails to exist in many instances. The problem is particularly severe for some regions of the parameter values. The existence problem is typical in this class of models with a continuum of agents. I discuss the issue at more length in Appendix II, where I also point to an additional assumption on the structure of exit costs that would ensure existence for any possible configuration of the model without modifying the equilibrium analysis.

Finally, there could also be multiple equilibria, in the sense of more then one couple of first stopping times that satisfy the equilibrium conditions. In all the cases computed I found that a unique equilibrium exists.[20] In case of multiple equilibria, a sensible selection rule would be to pick the smallest first adjustment times T^*.

4. Aggregate shocks and reallocation timing

In this section I analyze more closely the implications of the model, concentrating on the interrelation between aggregate shocks and reallocation activity. We have seen that at the heart of the model is the incentive for production units to free-ride in terms of production of information. As a consequence, even production units that have a high confidence of being of type θ_l might still find it optimal to delay adjustment, in the hope that others will go first. One important question is then how this incentive varies over the business cycle: pessimistic production units might find it relatively cheap to postpone restructuring in normal periods, while this might become more expensive in a downturn. As a consequence, it could be that the equilibrium pessimism level is lower in a downturn than in a normal period. This is indeed the case in the model, as established in the following proposition.

PROPOSITION 3. The equilibrium level of pessimism at which it is optimal to exit is lower in a downturn than in a normal period:

(19) $$\lambda_d^* < \lambda_n^*.$$

[20] Preliminary analytical work indicates that equilibrium is indeed unique.

or, in terms of first adjustment time,

$$\tau_d^* \le \tau_n^*. \qquad (19')$$

PROOF: see Appendix I.

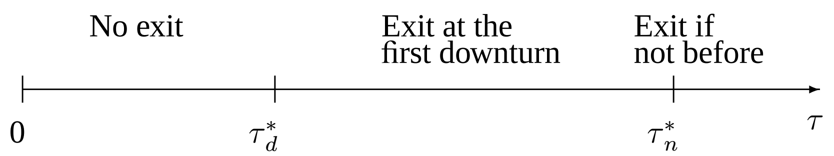
Proposition 2 dictates the following characterization of the evolution of a given cohort:

- for $\tau < \tau_d^*$, no production unit voluntary exits and, therefore, no private information is revealed;
- for $\tau_d^* \le \tau < \tau_n^*$, production units with beliefs greater than or equal to λ_d^* will choose to shut down if in any period the economy is in a downturn;
- for $\tau = \tau_n^*$, exit will take place anyway, if it has not already taken place previously.

The evolution is graphically represented in the following Figure 3.

Figure 3

EVOLUTION OF A GIVEN COHORT: FIRST EXIT TIMES



The mass of units shutting down during the first exit wave is small. For example, if the downturn hits exactly at τ_d^*, then the percentage of units in the cohort that will exit equals $Pr\{z_b|\theta_l\}^{\tau_d^*}$ if the cohort is θ_l and $Pr\{z_b|\theta_h\}^{\tau_d^*}$ if it is θ_h. This is in fact the probability of receiving all bad signals conditional on types. For values of τ_d^* sufficiently high, these numbers are small. However, the exit induced by the downturn has the important effect of revealing types. If the cohort is θ_l, then in the following period all units exit, with exit taking place independently of the aggregate state.[21] It could well be that the largest share of job destruction takes place in a period in which the state has reverted to normal. If the exit wave is large, however, the economy will suffer a recession. A recession is therefore a joint consequence of cyclical and structural events. The effect of a downturn is primarily that of breaking the inertia induced by the joint presence of microeconomic (unit level) uncertainty and unrecoverable costs: it influences the economy mostly through the informational changes that it induces. This view of aggregate shocks overcomes the difficulties of the traditional one, in which the shocks influence the economy only through their direct effect on productivity, and can reconcile both the need for amplification mechanisms and the extent of reallocation activities that characterize recessions. Note that I obtain this result without going as far as the literature on sunspots (Farmer, 1993), in which aggregate shocks only have a role as coordination devices: in my model, the shocks do have a concrete informational effect.

It is important to understand what determines the difference between the two threshold values, defined as $\Delta\Lambda^* \equiv \lambda_n^* - \lambda_d^*$. The higher this difference, the more likely that the reallocation activity takes place following a negative aggregate shock. To analyze this point, I refer to equation (14), which determines the payoffs from waiting at a particular belief and state, given that some units will exit today. The difference in the threshold values depends on two elements: the continuation value and current profits. Lemma 2 in the appendix shows that $v(n) > v(d)$, that is, the expected continuation payoff for a θ_h unit is higher in a normal period than in a downturn. This, together with the assumptions about the structure of the Markov chain governing the evolution of the aggregate state, implies that if the state today is n, then the continuation value conditional on being θ_h is higher than if the state is d, that is $V(n) > V(d)$. This means that a mistake (exiting when type θ_h) is more costly in a normal period, thus increasing the equilibrium level of pessimism in normal times. In the same way, given that $\pi(\lambda, d) < \pi(\lambda, n) \;\; \forall \, \lambda \in [0, 1]$, the lower expected revenue in a downturn makes it more costly to delay adjustment, thus inducing production units to act sooner.

A larger difference between current expected profits in the two states, as well as between continuations, implies a larger $\Delta\Lambda^*$. Equation (3') shows that when a production unit places a high probability on being type θ_l, the expected value of its profits depends in large measure on

[21] Recall that by construction it is optimal for a θ_l unit to exit irrespective of the aggregate state. Of course, this need not be necessarily the case.

$E(Z^s|\theta_l)$, as defined in equation (4'). The model predicts larger differences in the threshold beliefs if θ_l production units experience a large drop of productivity in downturns, that is if $\pi(\theta_l, n) - \pi(\theta_l, d)$ is large. This feature suggests a characterization of the economy as a system with heterogeneous production units with different efficiency levels, in which inefficient production units might be able to remain in production in normal periods, but are more adversely affected by cyclical downturns than efficient ones.

While the threshold levels are important for understanding the functioning of the model, in terms of describing the evolution of the economy we need to consider the first exit times τ_d^*, τ_n^* and their difference ΔT^*. I have shown that, for $s = \{n, d\}$, τ_s^* is monotonically increasing in λ_s^*, so that all the considerations of the previous paragraph apply to the analysis of the first exit times. However, in determining the first exit time the informativeness of the signals plays a fundamental role: the more informative the signals, the shorter the time needed to reach a given threshold of beliefs. Given that the first exit time is determined by production units that have received only bad signals, I define the *informational content* in terms of such signals:

DEFINITION 3. The informational content of the signals is defined as the difference between the probability of receiving a bad signal conditional on types:

(20) $$\eta \equiv \Pr\{z_b|\theta_l\} - \Pr\{z_b|\theta_h\}.$$

When η is low, a realization of z_b is not much more likely for a θ_l production unit than for a θ_h, so that the informational content is low. A given number of bad signals will generate more pessimistic posterior beliefs the higher η: put differently, a given threshold value for beliefs will be reached with fewer signals.[22] As a consequence, a given gap between λ_n^* and λ_d^* will induce a higher difference between the two first exit times when signals are less informative:

(21) $$\left.\frac{\partial \Delta T^*}{\partial \eta}\right|_{\Delta\Lambda^*} < 0.$$

[22] In terms of Figure 2, this means that the steps become longer as η increases.

Less informative signals therefore imply a higher probability that the reallocation process occurs after an aggregate downturn.

Finally, we note that an increase in k increases both threshold values. This means that an economy with higher adjustment costs will be characterized by an average lower turnover rate. The consequences of this fact reach beyond the scope of this paper, which focuses on the cyclical aspects of production units' turnover. This would be an interesting direction in which to extend the analysis.

5. The concentration of reallocation activities

After the discussion of the single cohort problem, I revert to the whole economy and analyze the implications of aggregate downturns for the process of aggregate entry and exit, which are intended to be proxies for job creation and job destruction. Recall that the composition of units in the economy at the beginning of time t is represented by a sequence $\{x_\tau(t)\}_{\tau=1}^{\infty}$, where $x_i(t)$ is the mass of cohort of age i in the market at time t. Such a description contains redundant information. From the equilibrium characterization it is known that, for any cohort, the type will be revealed at age τ_n^* at the latest, so that cohorts $\tau_n^*+1, \tau_n^*+2, ...$ are of known type. But once the type is revealed, there is no need to keep track of each individual cohort's evolution: if the type is θ_l, all units will exit and will play no further role in the economy; if the type is θ_h, all units will stay until natural death occurs, with the probability of death being independent of age. We can therefore aggregate all units that are known to be type θ_h at time t and denote their mass by $x_{\theta_h}(t)$. Then, we only need to keep track of the mass and type of cohorts of age τ_n^* and younger, whose type might still be unknown. A more parsimonious representation of the market composition is therefore constituted by a (τ_n^*+1) vector $X(t) = \{x_1(t), ..., x_{\tau_n^*}(t); x_{\theta_h}(t)\}$. In addition to this vector, I define another vector $\Theta(t) = \{\theta_1(t), ..., \theta_{\tau_n^*}(t)\}$ recording the types of cohorts $1, ..., \tau_n^*$. These variables, together with the current aggregate state, allow the determination of entry and of both voluntary and involuntary exit, so that they are sufficient to determine the evolution of the economy.

The above economy has a natural interpretation in terms of empirical counterparts. There is a mass $x_{\theta_h}(t)$ of mature units that have found they are θ_h and whose probability of death is therefore low, and a mass of young units, represented by the vector $\{x_\tau(t)\}_{\tau=1}^{\tau_n^*}$, that are still uncertain about their type and therefore might shut down within a short period of time. I name the first group *stable units* and the second group *fragile units*. While it is a crude description of reality, excluding the possibility of partial expansion and contraction of employment, such a characterization finds strong empirical support in a number of studies. Dunne, Roberts and Samuelson (1989), using data from the US manufacturing sector for over 200,000 plants in the 1967-1977 period, show that the variance of the growth rate of plants declines with age. More importantly, the failure rate also declines with age, with the probability of exiting within five years of entry being equal to approximately 41 percent. Horvath, Schivardi and Woywode (1997), in their study of the US beer brewing industry, show that the life-cycle of cohorts that entered at different points in time is remarkably similar. In particular, the hazard rates are very high in the first years after entry and tend to stabilize around relatively low values after that. Similar conclusions are reached by Davis, Haltiwanger and Schuh (1996) in their study of the relation between plant age and patterns of job creation and destruction. All these studies point to the description of the life cycle of a unit as characterized by an initial turbulent phase, in which the unit is learning about its long-term prospects and the probability of exit is remarkably high, and a mature, more stable phase.[23]

Consider now the *concentration effect* of aggregate shocks. Borrowing the terminology from Hall (1997), I define the cohorts within the age interval $[\tau_d^*, \tau_n^*]$ as *vulnerable* to aggregate downturns, in the sense that an aggregate downturn will induce type-revealing actions, potentially inducing exit of such cohorts. If the economy is in a normal period, then only the cohort of age τ_n^* will undertake an adjustment. If the aggregate state switches to a downturn, however, all the vulnerable cohorts will have their types revealed. As a consequence, the following period will on average be characterized by a high mass of units exiting, thus inducing a concentration of restructuring activities within one period.

To provide a quantitative assessment of the "pooling" of reallocation activities induced by aggregate downturns, I simulate the model numerically, after having obtained the

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[23] In the model I have assumed that all units in a given cohort share the same type. This implies that, once the type is revealed, the cohort will experience either total or null exit. It is easy to reconcile such predictions with empirical evidence. First, one can think of cohorts in the model as defined for a high frequency (semi-annual for example), while those defined in the empirical studies are for medium to low frequencies. Horvath, Schivardi and Woywode (1997), for example, use five years spans to define cohorts. Alternatively, an easy modification would entail assuming that each cohort is composed of two groups, one θ_l and the other θ_h, with uncertainty about which group is the high type. In this case, the model would imply that only the θ_l types will exit after the first adjustment, thus delivering mortality rates for each cohort similar to those observed in the data. In the simulations that follow, a similar result is obtained by constructing a multi-sector economy.

equilibrium first adjustment times for the set of parameter values reported in Table 1. For these parameters, there exists an equilibrium with first adjustment times $\tau_n^* = 8$ and $\tau_d^* = 4$. One additional problem needs to be tackled: a one-sector economy would depend critically on the particular realizations of types. For example, a sequence of consecutive high types would imply that there is no secondary wave of exit after a downturn, given that all cohorts find out they are viable. Given that in this paper we are not interested in the variability introduced by this aspect of the model, but rather in the average evolution of the economy, it is important to net this effect out. The easiest way to do this is to construct a multi-sector economy, with each sector identical to the single sector described above but characterized by its own independently drawn sequence of types. The evolution of the economy is then obtained by averaging over all the sectors: in this way, as the number of sectors grows, the law of large numbers will ensure that we will indeed obtain the average path for the endogenous variables.

The results reported in Figure 4 are based on an economy comprising 50 sectors. To allow for comparisons, I also construct an economy without an aggregate state. In such an economy, the equilibrium strategy is described by a single threshold value, which I obtain as a simple average of the economy with the aggregate state.

Table 1

PARAMETER VALUES

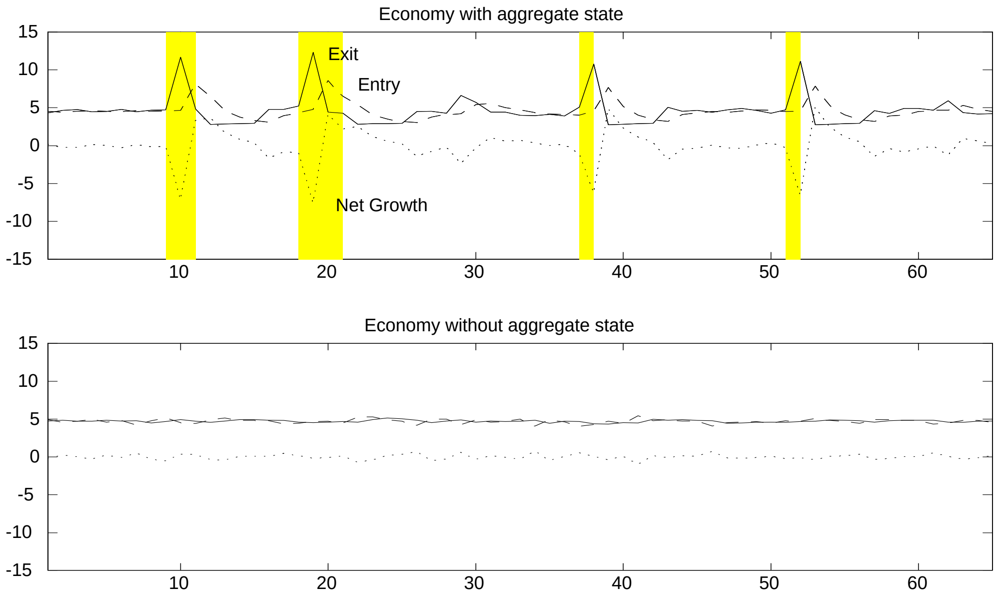
λ_0	k	β	δ	$\Pr\{z_b\vert\theta_l\}$		$\Pr\{z_b\vert\theta_h\}$
.5	3.10	.98	.03	.6		.25
$\gamma_1(n\vert n)$	$\gamma_1(d\vert d)$	z_b^n	z_b^d	z_g^n	z_g^d	α
.93	.3	-.6	-1	.5	.1	.5

The simulation is carried out by fixing an initial value for the state variables, generating a sequence of values for the aggregate state, drawing types of entering cohorts and computing the corresponding evolution for the economy. I let the model run for 2000 periods to eliminate the effects of the initial conditions. Figure 4 plots the paths for entry, for exit and for the net flow. The aim is to compare the model economy with the real one shown in Figure 1. The y-axis indicates values as a percentage of the total size of the economy at its full employment level, where each sector has a total mass of incumbents of measure one.[24]

[24] Given the high level of stylization of the model, no attempt is made to carry out a more careful calibration exercise. The aim is rather to asses the capacity of the model to account for some qualitative features of the data.

Figure 4

NET AND GROSS JOB FLOW RATES FOR THE MODEL ECONOMIES



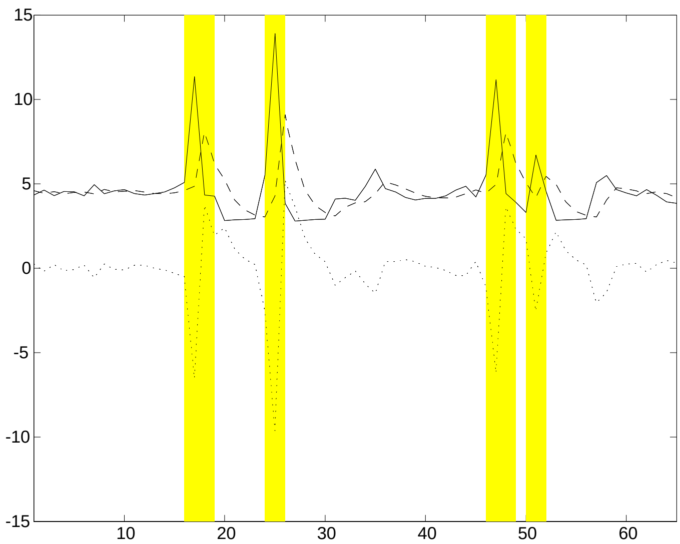
Shaded regions represent downturns (not necessarily recessions).

The series in the upper panel of Figure 4 show that the presence of an aggregate state induces the concentration of reallocation activities within a short period of time. When the aggregate state switches to a downturn, all the vulnerable cohorts will undertake the adjustment, so in the following period there will be a spike in exit at the aggregate level, inducing the spike in job destruction that, as shown in Figure 1, characterizes recessions in real economies. Note that in the period in which the exit rate reaches its peak the aggregate state might have reverted to normal: this is the sense in which downturns and recessions are distinct concepts in the model. In terms of comparison, the lower panel of the figure shows that, without the concentration effects induced by the switches of the aggregate state, the economy tends to be characterized by stable flows of entry and exit that offset each other, without the peaks of the upper panel: without the concentration effects of aggregate shocks, reallocation activities are spread over time and cannot account for the burst in job destruction characterizing the series in Figure 1.

The simulations also point to other interesting implications of the model. To explore these further, I carry out an experiment in which I choose a particular series for the aggregate state rather than randomly generating it. The series has an initial long sequence of normal periods, followed by a combination of downturns and normal periods. The behavior of the economy in the second phase is reported in Figure 5. First, I stress the cleansing effect of downturns: a downturn induces a period of intense reallocation activity, during which all vulnerable units discover their type and act accordingly. This implies that in the next few periods the mass of vulnerable units will be low.[25] Therefore, a downturn closely following another one will not induce high exit, as the two close downturns in periods 46-52 in the figure show.

Figure 5

EXPERIMENTS



The next point relates to the *intensity* of the effects of a downturn: should exit be higher when a downturn hits the economy after a prolonged period of expansion? Consider the first thirty periods in Figure 5. With no downturns, the economy behaves in a fashion that closely matches that of the economy without the aggregate state, as the first 15 periods in the figure show. The effects of the first downturn are represented by the first spike in exit. I then inflict a second downturn 9 periods later. The figure shows that the second recession is indeed deeper than the first one, with a higher spike in job destruction. This result is due to the fact that the exit wave following the first downturn induces some periods of high entry as the economy

25 In terms of the state vector $X(t)$, the revelation of types for all units of age τ_d^* and older implies that at the end of next period all the cohorts of age $\tau_d^* + 1$ or more will be empty.

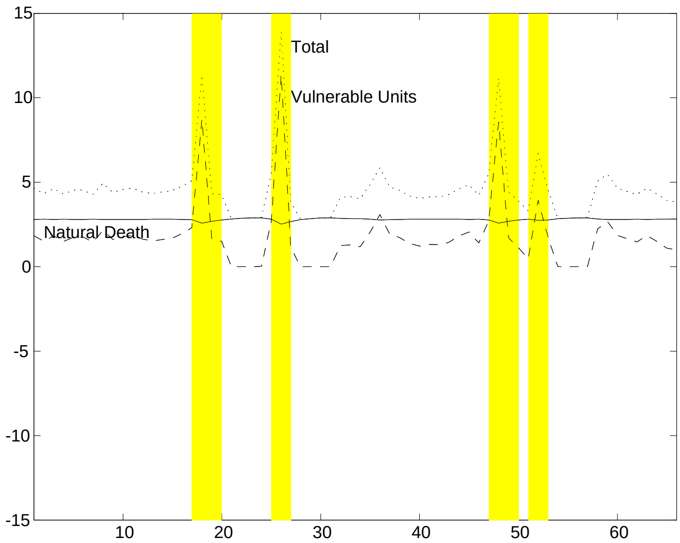
fills up again, so that the cohorts that enter after the recession will be larger than average. Consequently, after a surge in exit, as the economy fills up again the mass of fragile units increases; the second downturn hits when such large cohorts are vulnerable, thus inducing a particularly high level of exit.[26] This feature of the model accords with the particular severity of the 1981-82 recession as illustrated in Figure 1: this recession was in fact preceded by a short and sharp one at the beginning of 1980. Many observers claim that the recession of 1981-82 was characterized by a high level of restructuring activities. According to the model, much of such restructuring can be attributed to units that entered during the recovery following the previous recession.

The previous observation leads to one final point, which relates aggregate downturns to the composition of exit. A downturn induces a surge in voluntary exit by fragile units, but not a change in natural death. This implies that the ratio of fragile to mature units exiting is noticeably higher than average after a downturn. To make this point clear, Figure 6 plots the decomposition of the exit flow of Figure 5. Recessions are induced by a surge in exit of vulnerable units, with the ratio of voluntary to natural exit going from less then one for the initial period to approximately four in the period immediately following a downturn. This observation could be the starting point for the empirical assessment of the model's predictions.

[26] This is also apparent in the echo effects generated by recessions: a prolonged series of normal periods will induce a surge in exit some periods after the downturn, when the large cohorts that entered immediately after the recession undertake adjustment.

Figure 6

DECOMPOSITION OF EXIT FLOW



6. Conclusions

I have studied the effects of aggregate shocks on the level of restructuring activities, showing how modest aggregate shocks can induce a burst in relocation activities that magnify the response of the economy to the shock. The model stresses the informational aspects of restructuring activities. Aggregate shocks trigger an endogenous increase in the amount of information available to decision makers, which stimulates reallocation. The model offers both an amplification mechanism and an explanation of why restructuring activities tend to be concentrated in recessions.

While the extreme level of stylization leaves room for generalizations, such as endogenizing entry and studying the welfare implications of the pace of restructuring activities, it will be essential to assess the empirical validity of the model. This can be done at two levels. The first is to consider the relation between aggregate shocks and restructuring activities, and analyze how the level and pace of the restructuring activities vary over the business cycle. This is an area of increasing interest in the empirical analysis of the business cycle (Davis and Haltiwanger, 1996; Campbell and Kuttner, 1996; Caballero, Engel and Haltiwanger, 1997).

While the model does have distinctive restrictions, such as the amplification of shocks and the age composition of exit over the cycle, some of its predictions would be shared by a traditional Ss model without learning. A direct test of the learning mechanism is then warranted. This is a challenging task, and the most promising way to tackle it might be to consider case studies of specific episodes of massive restructuring, such as the one of the US steel industry in the 1981-82 recession as documented by Barnett and Crandall (1986) and popularized by Davis, Haltiwanger and Schuh (1996).

Appendix I

Proofs

I begin with a lemma that will be used to prove proposition 1.

LEMMA 1. Consider two discrete probability functions f and g, with corresponding cumulative distributions F and G, defined over a common support $X = \{x_1, x_2, ..., x_n\}$, where $x_1 < x_2 < ... < x_n$. Assume that there exists an $\bar{\imath}$ such that $f(x_i) > g(x_i)$ for $i < \bar{\imath}$, and $f(x_i) < g(x_i)$ for $i > \bar{\imath}$. Then, $\forall\ i < n,\ F(x_i) > G(x_i)$.

PROOF. First, for any $i < \bar{\imath}$, $f(x_i) > g(x_i)$ implies that $\sum_{j=1}^{i} f(x_j) > \sum_{j=1}^{i} g(x_j)$, or $F(x_i) > G(x_i)$. For $\bar{\imath} < i < n$, $f(x_i) < g(x_i)$ implies $\sum_{j=i}^{n-1} f(x_j) < \sum_{j=i}^{n-1} g(x_j)$ or $1 - F(x_i) < 1 - G(x_i)$. Finally, for $i = \bar{\imath}$, if $f(x_{\bar{\imath}}) \neq g(x_{\bar{\imath}})$, the same argument can be extended to such a point, while if $f(x_{\bar{\imath}}) = g(x_{\bar{\imath}})$, then, given that $F(x_{\bar{\imath}-1}) > G(x_{\bar{\imath}-1})$, it follows that $F(x_{\bar{\imath}}) = F(x_{\bar{\imath}-1}) + f(x_{\bar{\imath}}) > G(x_{\bar{\imath}-1}) + g(x_{\bar{\imath}}) = G(x_{\bar{\imath}})$. ■

PROOF OF PROPOSITION 1. If $\lambda(s) > \lambda_0$ for $s = \{n, d\}$, then all units will stay at least one period, so that each of them receives a signal. Consider now a generic period $\tau > 0$ before which no voluntary exit has taken place. Given the continuum of units assumption, there will be a non-zero mass of units that will have received all possible combinations of signals. For Bayesian updating, we only need the total number of bad and good signals, given that the order in which they are received does not matter. There will be $\tau+1$ points for beliefs with a non-zero mass of units, corresponding to having received $0, 1, ..., \tau$ bad signals out of τ total signals. For n_b bad signals, the value of the posterior is $\lambda(n_b) \equiv \lambda(\tau - n_b, n_b; \lambda_0)$ as calculated according to Bayes rule in (2). Clearly, in any period τ, $\lambda(0) < \lambda_0 < \lambda(\tau)$. Define $f_\tau(n_b|\theta)$ as the discrete density function of the *share* of units at belief $\lambda(n_b)$, and $F_\tau(n_b|\theta)$ as the cumulative density function. Then, we want to show that $\forall\ n_b < n_\tau,\ F_\tau(n_b|\theta_h) > F_\tau(\lambda_{n_b}|\theta_l)$. If this is the case, the mass of units at or above a given beliefs level will be different for the two types, which is enough to prove the proposition. To ease notation, define $p_h \equiv \Pr\{z_b|\theta_h\}$ and $p_l \equiv \Pr\{z_b|\theta_l\}$. Given that there is a continuum of units, the share of units with belief $\lambda(n_b)$ is the probability of n_b successes in τ trials for a binomial distribution:

(1Aa) $$f_\tau(n_b|\theta_h) = \binom{\tau}{n_b} p_h^{n_b}(1-p_h)^{\tau-n_b}$$

(1Ab) $$f_\tau(n_b|\theta_l) = \binom{\tau}{n_b} p_l^{n_b}(1-p_l)^{\tau-n_b}.$$

Consider the inequality $f_\tau(n_b|\theta_h) > f_\tau(n_b|\theta_l)$. Taking the logarithm of both sides and rearranging, we get:

(2A) $$dlf_\tau(n_b) \equiv \tau log\left(\frac{1-p_h}{1-p_l}\right) + n_b log\left(\frac{p_h}{1-p_h}\frac{1-p_l}{p_l}\right).$$

For $n_b = 0$, $dlf_\tau(0) = \tau log\left(\frac{1-p_h}{1-p_l}\right) > 0$ given that $p_h < p_l$. For $n_b = \tau$, $dlf_\tau(\tau) = \tau log\left(\frac{p_h}{p_l}\right) < 0$. Finally, given that $\frac{p_h}{1-p_h}\frac{1-p_l}{p_l} < 1$ it follows that:

(3A) $$log\left(\frac{p_h}{1-p_h}\frac{1-p_l}{p_l}\right) < 0.$$

This implies that $dlf_\tau(n_b)$ is monotonically decreasing in n_b, from which it follows that $f_\tau(n_b|\theta_h)$ and $f_\tau(n_b|\theta_l)$ satisfy the condition of lemma 1, so that $F_\tau(n_b|\theta_h) > F_\tau(n_b|\theta_l)$ $\forall\ \lambda < \lambda_\tau$. Finally, given that $\lambda(0) < \lambda_0 < \lambda(s)$, $\quad s = \{n, d\}$, it follows that $\forall\ \tau > 0$, there is a non-zero mass of units below the exit cutoff, which excludes the possibility that the full mass of units undertake exit simultaneously. ■

PROOF OF PROPOSITION 2. I show that any best response strategy is a cutoff strategy. Consider $\tilde{r}(\lambda, s)$ as defined in equation (14). First, note that the assumption that it is optimal for a θ_l unit to leave the market implies that $\pi(\theta_l, s) < -(1-\beta(1-\delta))k$, which in turn implies that $\tilde{r}(1, s) = \pi(\theta_l, s) - \beta(1-\delta)k < -k$. Second, given that it is optimal for a θ_h unit to stay, we have $\tilde{r}(0, s) = v(s) > -k$. If we show that $\tilde{r}(\lambda, s)$ is monotonically decreasing and continuous in λ, then there exists a $\bar{\lambda}(s)$ such that:

(4A) $$r(\bar{\lambda}, s) = \tilde{r}(\bar{\lambda}, s)$$

and

(5A) $$r(\lambda, s) > \tilde{r}(\lambda, s) \qquad \forall\ \lambda > \bar{\lambda}.$$

From equation (63′) it is immediate that $\pi(\lambda, s)$ is continuous and decreasing in λ. Moreover, given that $v(s) > -k$ for $s = \{n, d\}$ and that $\sum_{s'} \gamma_1(s', s) = 1$, it follows that $V(s) > -k$, which implies that the second term on the right hand side of (14) is decreasing and continuous. Therefore, for all $\lambda > \bar{\lambda}$ it is optimal to exit even if the type will be revealed next period. This means that the best response is in cutoff strategies. Given that, under the condition discussed above, cutoff strategies are type revealing, the equilibrium must be in cutoff strategies. ■

The proof of proposition 3 will follow immediately from this rather obvious lemma. To ease notation, define $a \equiv \gamma_1(n|n)$ and $b \equiv \gamma_1(d|d)$. Note that $\gamma_1(d|n) = 1 - a$ and $\gamma_1(n|d) = 1 - b$.

LEMMA 2. The value of being a θ_h type is higher in a normal period than in a downturn: $v(n) > v(d)$.

PROOF. First, note that $\{v(n), v(d)\}$ must satisfy the following system of equations:

(6Aa) $$v(n) = \pi(\theta_h, n) + \beta(1-\delta)[av(n) + (1-a)v(d)]$$

(6Ab) $$v(d) = \pi(\theta_h, d) + \beta(1-\delta)[(1-b)v(n) + bv(d)].$$

Solving this system, the implied values for $v(n), v(d)$ are:

(7Aa) $$v(n) = \frac{(1 - b\beta(1-\delta))\pi(\theta_h, n) + (1-a)\beta(1-\delta)\pi(\theta_h, d)}{(1 - a\beta(1-\delta))(1 - b\beta(1-\delta)) - (1-a)(1-b)(\beta(1-\delta))^2}$$

(7Ab) $$v(d) = \frac{(1 - a\beta(1-\delta))\pi(\theta_h, d) + (1-b)\beta(1-\delta)\pi(\theta_h, n)}{(1 - a\beta(1-\delta))(1 - b\beta(1-\delta)) - (1-a)(1-b)(\beta(1-\delta))^2}.$$

Then, given that $\gamma_1(s'|s) \in (0,1) \quad \forall s, s'$, and that $\beta(1-\delta) < 1$, the denominator of the expression is positive. Comparing the numerators, after collecting terms we obtain that $v(n) > v(d)$ if and only if $[\pi(\theta_h, n) - \pi(\theta_h, d)](1 - \beta(1-\delta)) > 0$, or equivalently if $\pi(\theta_h, n) > \pi(\theta_h, d)$. ■

PROOF OF PROPOSITION 3. It is easier to resort to $\tilde{r}(\lambda, s)$ rather that using directly the values of $\{\lambda_n^*, \lambda_d^*\}$ as obtained in equation (16). First, note that given the assumption that $\gamma_1(n|n) > \gamma_1(n|d)$ and the result established in lemma 2, it is immediate to show that $V(n) > V(d)$, with $V(s)$ defined in equation (13). In addition, I have shown that $\pi(\lambda, d) < \pi(\lambda, n)$, so that

(8A) $$\tilde{r}(\lambda, n) > \tilde{r}(\lambda, d) \ \ \forall \lambda \in [0, 1]$$

Consider than λ_d^*. Given what established in equation (8A), and given that $\tilde{r}(\lambda_d^*, d) = -k$, it must be that:

(9A) $$\tilde{r}(\lambda_d^*, n) > -k.$$

Therefore, given that $\tilde{r}(\lambda, n)$ is decreasing in λ, it follows that $\lambda_n^* > \lambda_d^*$. ■

Appendix II

Existence discussion and no-deviation condition

In terms of existence, the numerical solution of the model shows that it is indeed an issue. In particular, the cost of exit needs to be relatively large for a pessimistic unit to be willing to wait for the first exit times. The problem is particularly severe when η is low, arguably because in that case, for given Λ^*, a unit needs to wait longer. For higher values of η equilibria exist for a large selection of parameter values.

The existence issue is a very important problem for the model. However, as already noted by Caplin and Leahy (1994), it seems more a technical issue than a substantive one. The problem arises because of the continuum of units assumption, which implies that the information cannot be realized at any rate other than "all" or "nothing". With a discrete number of units, it would be possible to choose (mixed) strategies that control the amount of information being released and can therefore keep pessimistic units from exiting.[27] While it would be interesting to pursue a formulation of the model along these lines, there seems to be no easy way to tackle the problem once we dismiss the continuum of units assumption.

A drastic way to solve the existence issue is to increase the cost of exit in periods when no other unit voluntarily exits. Formally, if we define the mass of voluntary exit from the cohort in period t with e_t, then we impose:

$$k(e_t) = \begin{cases} k & \text{if } e_t > 0 \\ k' & \text{otherwise} \end{cases} \tag{10A}$$

This assumption does not modify the previous analysis. Then, for suitable values of k', such as for all $k' > \pi(\theta_l, d)$, it is easy to show that an equilibrium exists.

For the no deviation condition, I only sketch the derivation[28] of the condition for $\tau < \tau_d^*$. The one for $\tau_d^* \leq \tau < \tau_n^*$ follows the same logic. Consider a generic period $\tau_0 < \tau_d^*$. I have argued in the text that we only need to worry about the value of continuing for the

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[27] Models with a discrete number of agents can be found in Horvath, Schivardi and Woywode (1997) and Chamley and Gale (1994). Those models are however simpler in that, in addition to not having an aggregate state, either there is no private information (the former) or there is no arrival of new information over time (the latter).

[28] Detailed calculations are available upon request.

most pessimistic units, that is, for those that have received all bad signals. Therefore, all the expectations in the following derivation are conditional on current beliefs $\lambda_{\tau_0} = \lambda(0, \tau_0; \lambda_0)$. The conditioning is not explicitly reported to ease the notation. By linearity of $\pi(\lambda, s)$ in λ and by the martingale property of the beliefs, we have:

$$\text{(11A)} \qquad E(\pi(\lambda_\tau, s)) = \pi(\lambda_{\tau_0}, s)$$

where λ_τ is the belief at τ. Define the following truncated expectations and probabilities:

$$\text{(12A)} \qquad \lambda_\tau^s = E(\lambda_\tau | \lambda_\tau < \lambda_s^*)$$

$$\text{(13A)} \qquad P_\tau^s = Pr\{\lambda_\tau \geq \lambda_s^*\}.$$

λ_τ^s is the expected value for beliefs at time τ conditional on the fact that beliefs are below the equilibrium threshold, and P_τ^s the probability that beliefs are above the threshold.

Define $\Gamma_\tau(s'|s)$ as the probability that the first adjustment takes place at time τ in state s' given that the state at τ_0 is s. For example, for $s = d, \tau_d^* < \tau < \tau_n^*$, we have:

$$\text{(14A)} \qquad \Gamma_\tau(d|s) = \gamma_{\tau_d^* - \tau_0}(n|s)\gamma_1(n|n)^{\tau - \tau_d^* - 1}\gamma_1(d|n).$$

$\Gamma_\tau(d|s)$ is the probability that the first downturn in the interval $[\tau_d^*, \tau]$ hits the economy at τ. Then, we obtain:

$$\begin{aligned}
\text{(15A)} \quad U_{\tau_0}(\Lambda^*, s) = & \sum_{\tau=0}^{\tau_d^*-1} \{(\beta(1-\delta))^{\tau-\tau_0} \textstyle\sum_{s'} \pi(\lambda_{\tau_0}, s')\gamma_{\tau-\tau_0}(s'|s)\} + \\
& \sum_{\tau=\tau_d^*}^{\tau_n^*} (\beta(1-\delta))^{\tau-\tau_0} \pi(\lambda_{\tau_0}, n)\gamma_{\tau-\tau_0}(n|s)\gamma_1(n|n)^{\tau-\tau_d^*} + \\
& \sum_{\tau=\tau_d^*}^{\tau_n^*} (\beta(1-\delta))^{\tau-\tau_0} \Gamma_\tau(d|s)\{-kP_\tau^d + \\
& (1 - P_\tau^d)[\pi(\lambda_\tau^d, d) + (\beta(1-\delta))(-k\lambda_\tau^d + (1-\lambda_\tau^d)V(d))]\} + \\
& (\beta(1-\delta))^{\tau_n^*-\tau_0} \Gamma_{\tau_n^*}(n|s)\{-kP_\tau^n + \\
& (1 - P_\tau^n)[\pi(\lambda_{\tau_n^*}^n, n) + (\beta(1-\delta))(-k\lambda_{\tau_n^*}^n + (1-\lambda_{\tau_n^*}^n)V(n))]\}.
\end{aligned}$$

The first two lines represent the expected payoff from the pre-adjustment periods, the third and fourth that from the adjustment period with adjustment taking place in a downturn and the last two that from adjustment taking place in a normal period.

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