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HETEROGENEOUS "CREDIT CHANNELS" AND OPTIMAL MONETARY POLICY IN A MONETARY UNION

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Abstract

The process of European monetary integration has prompted interest in the study of differences in financial systems and their consequences for monetary transmission mechanisms. This paper analyses the case of a monetary union composed of countries with heterogeneous "credit channels". In order better to insulate the economies from the asymmetric effects produced by differences in national financial systems, a money supply process based on the interest rate on bonds and its spread with respect to the bank lending rate is proposed. Using a two-country rational expectations model, this study highlights the properties of the optimal monetary instrument with respect to a wide range of stochastic disturbances.

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1. Introduction[1]

The process of European monetary integration has prompted interest in the study of differences in financial systems (both structural and institutional) among EU countries and their consequences for monetary transmission mechanisms. In the spirit of Bernanke and Blinder (1988), heterogeneity in the structure of financial intermediation and the degree and composition of firms’ and households’ indebtedness could imply differences in the effectiveness of the “credit channel” of monetary policy among EU countries. Empirical studies seem to confirm the importance of these asymmetries. For example, the presence of a “credit channel” has been found in the United Kingdom (Dale and Haldane, 1993a, 1993b, 1995) and Italy (Buttiglione and Ferri, 1994; Angeloni et al., 1995; Bagliano and Favero, 1995), but not in France (Bellando and Pollin, 1996) or, at least at the macro level, in Germany (Barran, Coudert and Mojon, 1995). Apart from their different conclusions, these econometric studies point out the high information content of the spread between bank lending rates and market rates in explaining loan market disturbances and their impact on the evolution of real output (see also Kashyap, Stein and Wilcox, 1992).

The aim of this paper is to analyse the optimal monetary policy in the case of a monetary union composed of countries with heterogeneous “credit channels”. In order better to insulate the economies from the asymmetric effects produced by differences in national financial systems, the

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classic money supply process proposed by Poole (1970) is modified to consider the spread between the interest rates on loans and bonds as an additional feedback variable. In fact, while the interest rate on bonds embodies information mainly on money market equilibrium, the spread also indicates the state of the credit and goods markets.

In the case of credit market and supply shocks, the optimal monetary policy is highly sensitive to the degree of asymmetry between national “credit channels” and is not univocally determined. Moreover, if the economies are hit by supply disturbances, the optimal rule with respect to inflation has opposite consequences on output variance. If the performances of the “pure” policies are compared (money targeting, interest rate on bonds pegging and spread pegging) the main result is that the country with a more effective “credit channel” always performs better (in terms of both prices and output stabilisation) independently of the selected policy rule.

The remainder of the paper is organised as follows. Section 2 presents the analytical framework, based on a two-country rational expectations model along the lines of Turnovsky and d’Orey (1989) and Monticelli (1993), which is then used to analyse the case of a monetary union (Section 3). After discussing the objective function of the area-wide monetary authority (Section 4), Section 5 investigates the properties of the money supply process as indicated by Poole (1970) and derives the analytical solution for the optimal monetary policy. The implications of an alternative money supply process based on the spread between interest rates on loans and on bonds are studied in Section 6. Section 7 analyses the optimal monetary rule based on a more complete money supply process that uses as feedback variables both the interest rate on bonds and the spread. The last section summarises the main conclusions.

2. The analytical framework

The analysis is based on a two-country rational expectations model in which both economies are subject to real and nominal disturbances. The specification is log-linear and, in order to simplify the analytical forms, all variables are expressed as deviations from their trend level.

The system of equations is summarised in Table 1 where asterisks are used to indicate variables pertaining to the foreign country. The model has three assets (money, bonds and loans) and a common traded good.

The first pair of equations indicates equilibrium on the money market. In particular, (1) represents a standard money demand function where, for simplicity, the income elasticity is set to one, while (2) specifies the money supply process as indicated by Poole (1970). In the presence of stochastic disturbances, movements in the interest rate embody information on the nature of current shocks, so that the optimal monetary instrument is defined by a feedback rule from interest rate changes to money stock. In terms of the classical *IS-LM* model, this means fixing the optimal slope of the *LM* function by making money supply interest sensitive. The policy instrument is represented by k, which optimises the stabilisation objectives of the authorities. In particular, imposing a non-negative *LM* slope ($\frac{1}{2a + k}$), this parameter will

List of variables:

a= elasticity of demand for money with respect to the interest rate on bonds,

b= elasticity of exports with respect to foreign real output (it also represents the degree of interrelation between the countries),

b^d= nominal bond demand by households (b^{dh}) and banks (b^{db}), expressed in logs,

b^s= nominal supply of bonds by firms, expressed in logs,

d= elasticity of aggregate demand with respect to the real exchange rate,

e= exchange rate, measured in terms of units of domestic currency per unit of foreign currency, expressed in logs,

E_t= expectation operator, conditional on information at time t,

f= measure of the overall (common to the capital and credit markets) effect on output of a change in the real interest rate on bonds (“money channel”),

g= measure of the “price surprise” effect on real output,

h= elasticity of demand for loans with respect to the spread,

i= nominal interest rate, expressed in units,

k, x= policy instruments,

K= capital stock,

l^d= nominal loan demand, expressed in logs,

l^s= nominal loan supply, expressed in logs,

m^s= nominal money supply, expressed in logs,

p= price of output expressed in logs,

q= elasticity of supply of loans with respect to the spread,

$\tilde{r}$ = spread between the interest rate on loans (r) and bonds (i), expressed in units,

r= bank reserves, expressed in logs,

u_{ld}= stochastic disturbance in the demand for loans,

u_{ls}= stochastic disturbance in the supply of loans,

u_{md}= stochastic disturbance in the demand for money,

u_p= stochastic disturbance in the purchasing power parity condition (“price-wedge”),

u_{yd}= stochastic disturbance in aggregate demand,

u_{ys}= stochastic disturbance in aggregate supply,

v= measure of the effect on real output of a change in the *spread* (“credit channel”),

w= elasticity of demand for loans with respect to real output,

W= nominal wealth, expressed in logs,

y= real output in logs, measured as a deviation from its natural rate level,

z= elasticity of supply of loans with respect to real output.

Table 1

THE ANALYTICAL FRAMEWORK

	Home country		Foreign country	
		Money market		
Money demand	$m_t^d - p_t = y_t - ai_t + u_{md}$		$m_t^{*d} - p_t^* = y_t^* - ai_t^* + u_{md}^*$	(1)
Money supply	$m_t^s = m_t + ki_t$	(a, k and $k^*>0$)	$m_t^{*s} = m_t^* + k^* i_t^*$	(2)
		Loan market		
Loan demand	$l_t^d - p_t = wy_t - h\tilde{r}_t + u_{ld}$		$l_t^{*d} - p_t^* = wy_t^* - h\tilde{r}_t^* + u_{ld}^*$	(3)
Loan supply	$l_t^s = zm_t + q\tilde{r}_t + u_{ls}$	(h, q, w and $z>0$)	$l_t^{*s} = zm_t^* + q\tilde{r}_t^* + u_{ls}^*$	(4)
		Bond market		
Households' and firms' budget constraint	$m_t + b_t^{dh} + K = l_t^d + b_t^s + W$		$m_t^* + b_t^{*dh} + K^* = l_t^{*d} + b_t^{*s} + W^*$	(5)
Banks' budget constraint	$l_t^s + b_t^{db} + r_t = m_t$	($W>0$)	$l_t^{*s} + b_t^{*db} + r_t^* = m_t^*$	(6)
		Output market		
Aggregate demand	$y_t^d = by_t^* - f(i_t - \underset{t}{E}\, p_{t+1} + p_t) + - d(p_t - p_t^* - e_t) - v\tilde{r} + u_{yd}$		$y_t^{*d} = by_t - f(i_t^* - \underset{t}{E}\, p_{t+1}^* + p_t^*) + - d(p_t^* - p_t + e_t) - v^*\tilde{r}^* + u_{yd}^*$	(7)
Aggregate supply	$y_t^s = g(p_t - \underset{t-1}{E}\, p_t) + u_{ys}$	($0<b<1$, $f>v>0$, $d>0$ and $g>0$)	$y_t^{*s} = g(p_t^* - \underset{t-1}{E}\, p_t^*) + u_{ys}^*$	(8)
		Arbitrage conditions		
Uncovered interest parity		$i_t = i_t^* + \underset{t}{E}\, e_{t+1} + e_t$		(9)
Purchasing power parity		$p_t = p_t^* + e_t + u_p$		(10)

lie in the interval $-2a<k<$¥ which includes the “pure” policies of money targeting ($k=0$) and interest rate pegging ($k=$¥).[2]

Ignoring currency in circulation, the equilibrium in the money market coincides with that in the deposit market. In fact, setting for simplicity the interest rate on deposits to zero, equation (1) also represents the demand for deposits, while the supply of money is equal to the supply of deposits by definition.

Following Bernanke and Blinder (1988), the loan market is characterised by imperfect substitutability between bonds and loans: borrowers (households and firms) and lenders (banks) choose, respectively, their liability and asset composition according to the spread between the interest rates on loans (r) and on bonds (i). The demand for credit (3) is negatively influenced by the spread and positively related to real output for transaction motive (working capital or liquidity considerations). The loan supply (4) depends positively on money (equal to deposits) and the spread (it is implicitly assumed that the rate of return on excess reserves is zero). The loan market clears by quantities ($l_d=l_s$) and there is no credit rationing.

Given total wealth, the bond market is conveniently suppressed by Walras’ law using the private sector budget constraints (5) and (6).

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[2] In the analysis of this paper the policy variable k is assumed to be controlled without error while no use is made of the proximate target concept. This approach has been followed, on the one hand, to simplify the analysis and, on the other, because the treatment of the money stock as a stochastic function of the monetary base would not have changed the

The equilibrium in the goods market is given by equations (7) and (8). The first relation represents output demand, which depends upon the other country's output (via exports) and the real exchange rate (measure of competitiveness). Moreover, output demand is influenced by the cost of financing for investment and consumption. The conditions of the capital market are captured by the real interest rate on bonds ("money channel"), while the spread ($\tilde{r} = r - i$) underlines the conditions of the credit market ("lending channel").

In fact, if borrowers (not only households but also "small" firms) have no access to the capital market they have to rely on bank credit for external funding. In this situation, bonds and loans are imperfect substitutes and every change in the composition of bank assets influences the spread and investment financing.[3]

The parameter v has a special role in the model and represents the only asymmetry in the economic structure of the two countries. It shows the strongest impact of monetary transmission on output due to the relative importance of

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results of the study.

[3] Some observations on the form used for output demand are worth making. Equation (7) is equivalent to that which considers separately the influence of the two real interest rates on bonds and loans. In fact, if the other variables are fixed and for simplicity the interest rates are expressed in nominal terms, the relation $Y = -f'i - v'r$ is equivalent to $Y = -fi - v(r - i)$ with $Y_i = -f' = -f + v < 0$ and $Y_r = -v' = -v < 0$. The only restriction to impose is that, given $Y_i < 0$, it must be that $f > v$. However, the interpretation of f' is different from that of f. The former explains only the effects on output of the interest rate condition in the bond market, while the latter ($f = f' + v'$) measures the overall effect of a change in i, which is common to the bond and credit markets, assigning v the task of capturing the effect caused by the "peculiarity" of the loan market with respect to the capital market.

intermediate versus direct financing. In summary, the value of this parameter is expected to be high if the private debt market is less developed, and/or firms' and households' indebtedness is significant and dependent on the banking sector. In order to emphasise the source of divergence of a heterogeneous financial system on the monetary transmission mechanism described by the model, the home country has been considered as being more dependent on the "credit channel" ($v>v^*$).

Equation (8) is a standard Lucas supply function where deviations in output from its natural rate (normalised to zero) are a positive function of unanticipated movements in the price level.

The two country blocks are linked by equations (9) and (10) which represent, respectively, the uncovered interest parity (*UIP*) condition and a stochastic version of the purchasing power parity (*PPP*) condition. The *UIP* postulates the perfect substitutability between the bonds of two countries. On the contrary, loans are considered imperfect substitutes, not only because bank credit depends on customer relationships that facilitate concentration in local markets,[4] but also on account of the lack of an efficient secondary market for credit, which prevents arbitrage. The *PPP* assumes perfect substitutability in the output market, except for the random disturbance, u_p, which incorporates market imperfections.[5] This wedge between the prices of the two

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[4] In some cases credit relationships are characterised by a "lock-in" effect that allows the bank to extract a monopolistic rent from the client (Sharpe, 1990; Rajan, 1992).

[5] The simplifying assumption of an *exogenous* disturbance to the *PPP* condition, used as a short-cut to model the real exchange, will be relaxed in Section 3. In a framework in which the difference between the goods produced in the two countries is explicitly recognised, the real

countries could be determined by a difference in the monopolistic power of firms and trade unions (Minsky and Ferri, 1984). In particular, the capacity of firms to influence prices (mark-up) depends on institutional factors (monopoly legislation) and demand elasticities, while the ability of trade unions to influence wages (and so prices) depends on workers' participation and the level of unemployment (see Layard, Nickell and Jackman, 1991).

All the stochastic variables are assumed to be independently distributed with zero mean and finite variance (u ~id $(0, s_u)$).

3. The case of a monetary union

This section analyses the implications of a monetary union in the two-country model discussed above. The introduction of a single currency at the area-level means not only an irrevocably fixed exchange rate,[6] but also specifying a common monetary policy.

The irrevocable fixing of the exchange rate (for example to one, so that e_t=0) modifies the *UIC* and the *PPP*.

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exchange rate can be derived as a function of the shocks hitting the two economies (see Monticelli, 1993).

[6] This assumption, in line with the Delors Committee (Committee for the Study of Economic and Monetary Union, 1989), implies that an irrevocably fixed exchange rate would make national currencies perfect substitutes and therefore be equivalent to the creation of a common currency. For the institutional aspect of the difference between having separate national currencies with irrevocably fixed exchange rates and a common currency, see amongst others, De Grauwe (1994) and Gros and Thygesen (1992).

Table 2

THE CASE OF A MONETARY UNION

	Home country	Foreign country	
	Money market		
Money demand	$m_t^d - p_t = y_t - ai_t + u_{md}$ (*a* and *k*>0)	$m_t^{*d} - p_t^* = y_t^* - ai_t^* + u_{md}^*$	(1)
Money supply	$m' = (m_t + m_t^*)^s = m_t + m_t^* + ki_t$		(2′)
	Loan market		
Loan demand	$l_t^d - p_t = wy_t - h\tilde{r}_t + u_{ld}$	$l_t^{*d} - p_t^* = wy_t^* - h\tilde{r}_t^* + u_{ld}^*$	(3)
Loan supply	$l_t^s = z(m_t + m_t^*) + q\tilde{r}_t + u_{ls}$	$l_t^{*s} = z(m_t + m_t^*) + q\tilde{r}_t^* + u_{ls}^*$	(4′)
	(*h*, *q*, *w* and *z*>0)		
	Bond market		
Households’ and firms’ budget constraint	$m_t + b_t^{dh} + K = l_t^d + b_t^s + W$	$m_t^* + b_t^{*dh} + K^* = l_t^{*d} + b_t^{*s} + W^*$	(5)
Banks’ budget constraint	$l_t^s + b_t^{db} + r_t = m_t$	$l_t^{*s} + b_t^{*db} + r_t^* = m_t^*$	(6)
	(*W*>0)		
	Output market		
Aggregate demand	$y_t^d = by_t^* - f(i_t + p_t) + - d(p_t - p_t^*) - v\tilde{r} + u_{yd}$	$y_t^{*d} = by_t - f(i_t^* + p_t^*) + - d(p_t^* - p_t) - v^*\tilde{r}^* + u_{yd}^*$	(7′)
Aggregate supply	$y_t^s = g(p_t - {}_{t-1}E\, p_t) + u_{ys}$	$y_t^{*s} = g(p_t^* - {}_{t-1}E\, p_t^*) + u_{ys}^*$	(8)
	(0<*b*<1, *f*>*v*>0, *d*>0 and g>0)		
	Arbitrage conditions		
Uncovered interest parity	$i_t = i_t^*$		(9′)
Stochastic law of one price	$p_t = p_t^* + u_p$		(10′)

The first condition (see equation (9′) in Table 2) implies the uniqueness of the interest rate on bonds at the area-level (the same default risk is assumed between borrowers in the two countries), while the second condition shows that the difference in prices depends only on the disturbance u_p (see equation (10′)). In particular, assuming homogeneous "anti-trust" legislation, this wedge in prices should reflect differences between national labour markets (unemployment levels) that determine different wage pressures on price and demand.

The monetary policy is determined by equation (2′), which represents an extension at the area-level of the money supply process discussed in Section 2 (see Monticelli, 1993). It is worth noting that this rule determines the overall amount of money ($m' = (m_t + m_t^*)^S$), while its distribution between the two countries is endogenously determined.

Since the monetary market is perfectly integrated, the loan supply depends on the overall amount of money at the area-level (see equation 4′). The special case of credit markets′ segmentation (loan supply depends only on national deposits while interbank loans are negligible) will also be considered in the discussion.

Assuming that the optimal policy is constant over time, given that the model is in deviation form, all the expectations based on information at time t can be set to zero ($\underset{t}{E}\, p_{t+1} = \underset{t}{E}\, p^*_{t+1} = 0$). On the other hand, even if the rational expectations hypothesis assumes that agents know the model period by period, the parameters are not necessarily constant over time and may cause "price surprise" effects ($_{t-1}E\, p_t$ could be different from zero). Dealing with a rational expectations model, a classic "three-step" method has been used. The solutions for prices and real output, reported in Table 3, show the well-known neo-classic result: the only effect of monetary policy is on prices, while real incomes are a function only of the disturbances. The expressions suggest the following comments.

(i) The endogenous distribution of money between the two countries produces a symmetric effect ($A_0=B_0=A_1=B_1$). A different result is obtained if credit markets are segmented (loan supply depends only on national deposits). In this case, given the assumption $v>v^*$, it is possible to prove that $A_0=B_0>A_1=B_1$ and, therefore, price levels are lower if $m<m^*$.

(ii) Disturbances in loan supply and demand have an opposite effect on the endogenous variables. In particular, a positive supply shock, by generating a reduction in the cost of bank credit, increases real output and inflation, while a positive demand shock produces an increase in the spread, causing the opposite result. Moreover, given the financial integration between the two economies, internal and external shocks on the loan market enter the equations with the same sign, while the absolute value is directly proportional to the different effectiveness of the "credit channel" (given $v>v^*$, $A_2=B_2>A_3=B_3$).

(iii) Internal and external money demand shocks influence prices and income with the same intensity. This result depends on the symmetry of the parameter f and

Table 3

REDUCED FORM OF THE MODEL WITH A MONEY SUPPLY PROCESS BASED ON THE INTEREST RATE ON BONDS

(11) $p_t = A_0 m + A_1 m^* + A_2(u_{ls} - u_{ld}) + A_3(u_{ls^*} - u_{ld^*}) + A_4(u_{md} + u_{md^*}) + A_5(u_{yd} + u_{yd^*}) + A_6 u_{ys} + A_7 u_{ys^*} + A_8 u_p$

(12) $p_t^* = B_0 m + B_1 m^* + B_2(u_{ls} - u_{ld}) + B_3(u_{ls^*} - u_{ld^*}) + B_4(u_{md} + u_{md^*}) + B_5(u_{yd} + u_{yd^*}) + B_6 u_{ys} + B_7 u_{ys^*} + B_8 u_p$

(13) $y_t = C_2(u_{ls} - u_{ld}) + C_3(u_{ls^*} - u_{ld^*}) + C_4(u_{md} + u_{md^*}) + C_5(u_{yd} + u_{yd^*}) + C_6 u_{ys} + C_7 u_{ys^*} + C_8 u_p$

(14) $y_t^* = D_2(u_{ls} - u_{ld}) + D_3(u_{ls^*} - u_{ld^*}) + D_4(u_{md} + u_{md^*}) + D_5(u_{yd} + u_{yd^*}) + D_6 u_{ys} + D_7 u_{ys^*} + D_8 u_p$

where:

$A_0 = B_0 = A_1 = B_1 = \dfrac{2fJ_2 + (2a + k)(v + v^*)z}{4fJ_2 + (2a + k)(2fJ_2 + v + v^*)} > 0$

$A_2 = B_2 = [(2a + k)v] / \Delta > 0$

$A_3 = B_3 = [(2a + k)v^*] / \Delta > 0$

$A_4 = B_4 = (-2fJ_2) / \Delta < 0$

$A_5 = B_5 = (2a + k)J_2 / \Delta > 0$

$A_6 = B_6 = -[2fJ_2 + (J_1 J_2 + vw)(2a + k)] / \Delta < 0$

$A_7 = B_7 = -[2fJ_2 + (J_1 J_2 + v^* w)(2a + k)] / \Delta < 0$

$A_8 = \{(2a + k)[gJ_1 J_2 + fJ_2 + v^*(1 + gw)] + 2fJ_2(1 + g)\} / \Delta > 0$

$B_8 = -\{(2a + k)[gJ_1 J_2 + fJ_2 + v(1 + gw)] + 2fJ_2(1 + g)\} / \Delta < 0$

$\Delta = (2a + k)[2(f + gJ_1)J_2 + (v + v^*)(1 + gw)] + 4fJ_2(1 + g) > 0$

$0 < J_1 = 1 - b < 1$ and $J_2 = h + q > 0$

$C_0 = C_1 = D_0 = D_1 = 0$

$C_2 = D_2 = [g(2a + k)v] / \Delta > 0$

$C_3 = D_3 = [g(2a + k)v^*] / \Delta > 0$

$C_4 = D_4 = (-2fgJ_2) / \Delta < 0$

$C_5 = D_5 = g(2a + k)J_2 / \Delta > 0$

$C_6 = \{2fJ_2(2 + g) + [2fJ_2 + gJ_1 J_2 + v + v^*(1 + gw)](2a + k)\} / \Delta > 0$

$C_7 = -g[2fJ_2 + (J_1 J_2 + v^* w)(2a + k)] / \Delta < 0$

$C_8 = g\{2fJ_2(1 + g) + [fJ_2 + gJ_1 J_2 + v^*(1 + g)](2a + k)\} / \Delta > 0$

$D_6 = -g[2fJ_2 + (J_1 J_2 + vw)(2a + k)] / \Delta < 0$

$D_7 = \{2fJ_2(2 + g) + [2fJ_2 + gJ_1 J_2 + v^* + v(1 + gw)](2a + k)\} / \Delta > 0$

$D_8 = -g\{2fJ_2(1 + g) + [fJ_2 + gJ_1 J_2 + v(1 + g)](2a + k)\} / \Delta < 0$

$J_2 = h + q$ which represent, respectively, the relevance of the "money channel" and the sum of the elasticities of demand and supply of loans with respect to the spread.

(iv) Also shocks to output demand have the same effect on price levels and real incomes. In fact, as a result of the stochastic law of one price, the different competitiveness between the two countries has no influence on the sum of outputs and therefore the parameter d, denoting the elasticity of output demand in relation to the measure of competitiveness, does not enter the solutions.

(v) All financial and aggregate demand shocks affecting the two economies enter the equations in additive form and, therefore, the monetary union tends to reduce the effects of such disturbances only if they are negatively correlated.

(vi) Supply shocks have a different effect on the endogenous variables. On the one hand, a supply shock (internal or external) produces a variation in the price levels with an intensity that is directly related to the relative importance of the credit channel. Considering the assumption $v>v^*$, the variation in prices is relatively higher if the supply shock is related to the home country and vice versa. On the other hand, the effects on output are asymmetric. An internal positive supply shock produces a boom while an external positive supply shock causes a depression. Also in this case the intensity of a change in output depends on the relative weight of the "credit channel" ($C_6>D_7$ and $C_7>D_6$).

(vii) Also a shock in the "price-wedge" affects price levels and real incomes in opposite directions. The modelling of u_p as an exogenous stochastic variable is a simplifying assumption which gives tractability to the model. In general, solving the model without imposing condition (10'), it is possible to obtain the following analytical expression of the stochastic component of the law of one price, which is endogenously determined by the shocks hitting the economies:

$$
\begin{aligned}
u_p &= \Phi_1(\underline{d})(v - v^*)m' + \Phi_2(u_{ls} - u_{ld}) - \Phi_3(u_{ls^*} - u_{ld^*}) + \\
&\quad + \Phi_4(v - v^*)(u_{md} - u_{md^*}) + \Phi_5 u_{yd} - \Phi_6 u_{yd^*} - \Phi_7(\underline{d})u_{ys} + \Phi_8(\underline{d})u_{ys^*}
\end{aligned}
\tag{10''}
$$

where the coefficients $\Phi_i > 0$ (for i=1,...,8) are functions of the parameters of the model. In this case d, denoting the elasticity of output demand in relation to the measure of competitiveness, enters the coefficients of money supply and supply shocks. It is worth noting that the asymmetry $v>v^*$ determines a positive "price wedge" also in the absence of economic shocks. Moreover, given the relation $\Phi_2 > \Phi_3$, $\Phi_5 < \Phi_6$ and $\Phi_7 > \Phi_8$, u_p responds also to symmetric shocks hitting the two economies. Only when the two credit channels have the same effectiveness ($v=v^*$) does the price wedge have zero mean and is not influenced by money demand shocks. In this case, $\Phi_2 = \Phi_3$, $\Phi_5 = \Phi_6$, $\Phi_7 = \Phi_8$ and u_p responds only to asymmetric shocks, as already pointed out by Monticelli (1993).

(viii) Only when the sources of instability discussed in (vi) and (vii) are negligible does the stabilisation objective of each of the countries coincide with that of the monetary-area as a whole.

4. The objective function of the area-wide monetary authority

The monetary policy at the area-level is assumed to be unique and formulated by an independent council which represents the two countries. The primary objective of this authority is supposed to be the maintenance of price stability.[7] Therefore the monetary instrument *k* is chosen in order to minimise the following loss function:

$$(15) \qquad \min_{k} \; L^{p} = Var\,(p) + Var\,(p^{*}).$$

Since all the variables are expressed as deviations from their trend level, this objective function is equivalent to stabilising inflation.[8]

Following the basic idea in Rogoff (1985), the minimisation of the objective function (15) could have

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[7] These hypotheses are consistent with the Maastricht Treaty. The Governing Council, which formulates the monetary policy of the Community (intermediate monetary policy, interest rate pegging and supply of reserves in the system), is composed of six members constituting the Executive Board and the Governors of the national central banks. Each member of the Governing Council has one vote (see Art. 10 of the Maastricht Treaty). Moreover, Art. 2 of the Statute for the European System of Central Banks (ESCB) states that “the primary objective of the ESCB shall be to maintain price stability”.

[8] It is worth noting that, given A_8-B_8=1, the objective function (15) is equivalent to minimising the variance of the sum of the variables ($Var(p)+Var(p^*)=Var(p+p^*)$).

different welfare implications regarding output stabilisation. The latter can be measured by:

$$(16) \qquad L^{y} = Var\,(y) + Var\,(y^{*})$$

In this case, the consequences of a “too conservative” area-wide central bank should also encompass the possible asymmetric effects in the economies in terms of real output.

Using the definition of variance and the assumption of independently distributed shocks, the following equations are obtained:

$$(15') \qquad \begin{aligned} L^{p} &= Var(p_t) + Var(p_t^{*}) = 2A_2^2(s_{uls}^2 + s_{uld}^2) + 2A_3^2(s_{uls*}^2 + s_{uld*}^2) + \\ &+ 2A_4^2(s_{umd}^2 + s_{umd*}^2) + 2A_5^2(s_{uyd}^2 + s_{uyd*}^2) + 2A_6^2 s_{uys}^2 + 2A_7^2 s_{uys*}^2 + \\ &+ (A_8^2 + B_8^2)s_{up}^2 \end{aligned}$$

$$(16') \qquad \begin{aligned} L^{y} &= Var(y_t) + Var\,(y_t^{*}) = 2C_2^2(s_{uls}^2 + s_{uld}^2) + 2C_3^2(s_{uls*}^2 + s_{uld*}^2) + \\ &+ 2C_4^2(s_{umd}^2 + s_{umd*}^2) + 2C_5^2(s_{uyd}^2 + s_{uyd*}^2) + (C_6^2 + D_6^2)s_{uys}^2 + \\ &+ (C_7^2 + D_7^2)s_{uys*}^2 + (C_8^2 + D_8^2)s_{up}^2 \end{aligned}$$

with the expressions of the symbols identical to those reported in Table 3. The only source of asymmetry in price stabilisation is represented by the shock in the “price-wedge” ($A_8{}^{1}B_8$). On the other hand, internal ($C_6{}^{1}D_6$) and external ($C_7{}^{1}D_7$) supply shocks could also determine a different sensitivity of the results with respect to output stabilisation.

5. The optimal monetary policy à la Poole

The combined policy described by equation (2’) focuses on the parameter k and becomes a pure interest rate rule when k=¥ and a pure money stock supply when $k=0$. It is obvious that, except in special cases, the optimal monetary rule $-2a < k < \infty$ is superior to both of the “pure” policies.

The properties of the optimal monetary instrument k can be investigated using implicit differentiation of the objective function (15). The response of the optimal monetary rule to all kinds of shocks is reported in Table 4.

In the case of disturbances in the credit market or in output demand (wherever they occur), k is modified in order to increase the slope of the *LM* curve (in particular, if a is relatively low the optimal rule can be approximated using money targeting). On the other hand, the predominance of money demand shocks calls for interest rate pegging (the *LM* curve is horizontal).

Considering the Bernanke-Blinder model, where the *IS* line is replaced with the *CC* (Commodities and Credit) equilibrium, these results are similar to the classic Poole (1970) findings: in the case of a high variability of the *CC* curve (shocks in the loan and goods markets) a money stock supply rule is preferable, while interest rate pegging is superior in the presence of *LM* disturbances (shocks in money demand).

Table 4

THE PROPERTIES OF THE OPTIMAL MONETARY POLICY *K*

$$\text{sign}\,\frac{\P k}{\P s^2_{uls}} = \text{sign}\,\frac{\P k}{\P s^2_{uld}} = -\text{sign}[\frac{(2a + k)v}{\Delta}\ \frac{4f(1 + g)J_2 v}{\Delta^2}] < 0$$

$$\text{sign}\,\frac{\P k}{\P s^2_{uls*}} = \text{sign}\,\frac{\P k}{\P s^2_{uld*}} = -\text{sign}[\frac{(2a + k)v^*}{\Delta}\ \frac{4f(1 + g)J_2 v^*}{\Delta^2}] < 0$$

$$\text{sign}\,\frac{\P k}{\P s^2_{umd}} = \text{sign}\,\frac{\P k}{\P s^2_{umd*}} = -\text{sign}[-\frac{4f^2 J_2^2[2J_2(f + gJ_1) + (v + v^*)(1 + gw)]}{\Delta^3}] > 0$$

$$\text{sign}\,\frac{\P k}{\P s^2_{uyd}} = \text{sign}\,\frac{\P k}{\P s^2_{uyd*}} = -\text{sign}[\frac{(2a + k)J_2}{\Delta}\ \frac{4fJ_2^2(1 + gw)}{\Delta^2}] < 0$$

$$\text{sign}\,\frac{\P k}{\P s^2_{uys}} = -\text{sign}[\frac{-[2fJ_2 + (J_1 J_2 + vw)(2a + k)]}{\Delta}\quad \frac{2fJ_2\{2J_2(f - J_1) + +v[1 - (2 + g)w] + v^*(1 + gw)\}}{\Delta^2}]$$

$$\text{sign}\,\frac{\P k}{\P s^2_{uys*}} = -\text{sign}[\frac{-[2fJ_2 + (J_1 J_2 + v^* w)(2a + k)]}{\Delta}\quad \frac{2fJ_2\{2J_2(f - J_1) + +v^*[1 - (2 + g)w] + v(1 + gw)\}}{\Delta^2}]$$

$$\text{sign}\,\frac{\P k}{\P s^2_{up}} = -\text{sign}[\ \frac{2fJ_2(1 + g)(v^* - v)^2(1 + gw)^2(2a + k)}{\Delta^3}] < 0$$

where:

$$\Delta = (2a + k)[2(f + gJ_1)J_2 + (v + v^*)(1 + gw)] + 4fJ_2(1 + g) > 0$$

$0 < J_1 = 1 - b < 1$ and $J_2 = h + q > 0$

If the economy is hit by an internal supply shock, the sign of the derivative is highly sensitive to the elasticity of demand for loans with respect to real output (w) and to the degree of asymmetry between the "credit channels" ($v>v^*$). The result is also influenced by the sign of the difference between the "money channel" (f) and $J_1 = 1 - b$, where b represents the interrelation between the economies. The different cases are shown in Figure 1 where "*m*" (money targeting) represents $\frac{\P k}{\P s^2_{uys}} < 0$, while "*i*" (interest rate pegging) indicates $\frac{\P k}{\P s^2_{uys}} > 0$.

Sufficient conditions for the optimal monetary rule to move towards money targeting are given by:

$$(17) \qquad w > \frac{1}{2+g} \quad \text{and} \quad v > \frac{2J_2(f - J_1)}{(2+g)w - 1} + \frac{1+gw}{(2+g)w - 1} v^*,$$

which are more likely to be fulfilled in the case of an expansive credit cycle, an ineffective "money channel" and a low interrelation between the economies (see Figure 1). Vice versa, there is a call for interest rate pegging.

In the case of an external shock, given $v>v^*$, $\frac{\P k}{\P s^2_{uys*}}$ is more likely to be positive. In fact, the condition for the derivative to be less than zero becomes $v^* > \frac{2J_2(f - J_1)}{(2+g)w - 1} + \frac{1+gw}{(2+g)w - 1} v$, which is satisfied only if $f < J_1$(see Figure. 2). Moreover, in this case monetary policy is less reactive to the credit cycle. This outcome is due to

Figure 1

THE OPTIMAL MONETARY POLICY IN THE CASE OF AN INTERNAL SUPPLY SHOCK

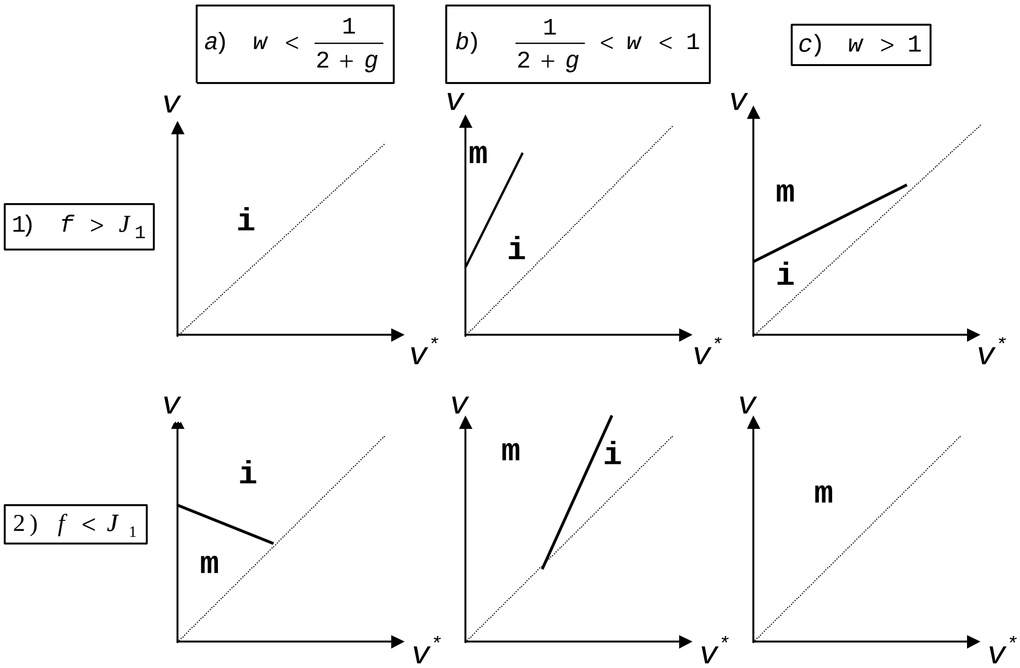
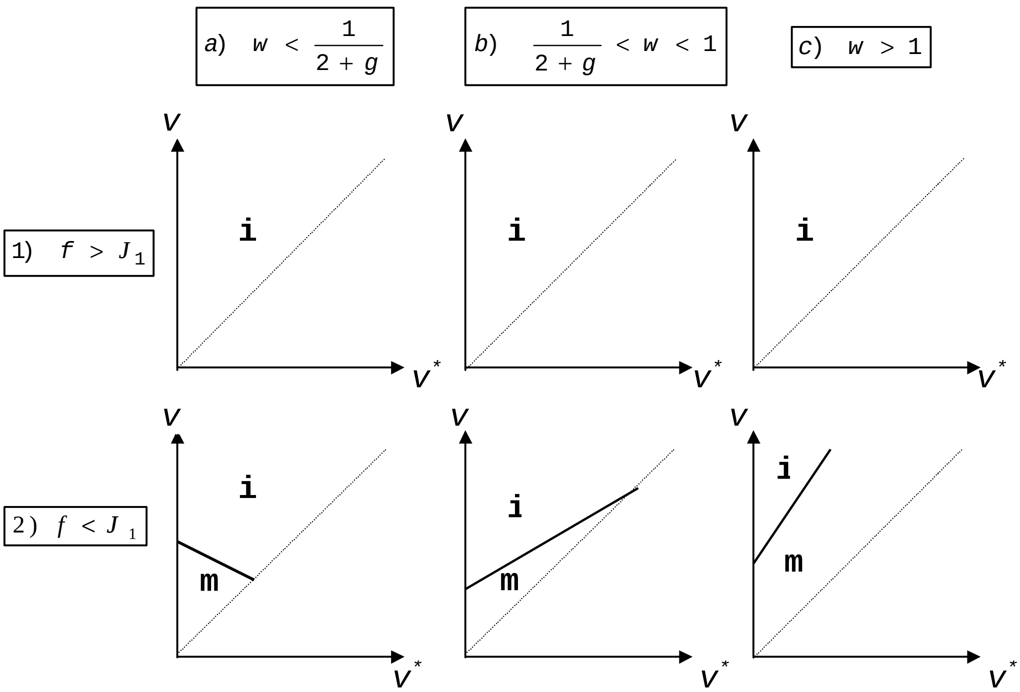


Figure 2

THE OPTIMAL MONETARY POLICY IN THE CASE OF AN EXTERNAL SUPPLY SHOCK



the fact that a less effective “credit channel” will have less direct influence on prices.

When the economies are hit by a “price-wedge” shock, the sign of $\frac{\P k}{\P s_{up}^{2}}$ is negative, implying that the optimal policy instrument moves towards money targeting. Given the assumption $v>v^{*}$, this kind of shock determines a greater loss for the foreign country (it is worth noting that in the cases analysed before there was no asymmetry in the effects on the countries). A comparison between the “pure” policies shows that interest rate pegging is superior to money targeting in terms of minimisation of this asymmetric effect.[9] Therefore, even if an “*m*” rule reduces the overall loss, interest rate pegging determines a smaller difference between national results.

To analyse the consequences of the optimal monetary rule *k* (referred to prices) for output stabilisation, the same procedure was applied to the variance of real income (see equation 16). The results, reported in Table 5, show that in the case of financial (credit and money markets), aggregate demand and “price-wedge” shocks, *k* is optimal also in terms of output.

---

[9] In particular, it is possible to prove that the differences in performance ($\Delta L^{p} = L^{p} - L^{*p}$) are given by the following expressions $\Delta L_{i}^{p} = L_{i}^{p} - L_{i}^{*p} = \frac{(v^{*} - v)(1 + gw)s_{up}^{2}}{2J_{2}(f + gJ_{1}) + (v + v^{*})(1 + gw)}$ and $\Delta L_{m}^{p} = L_{m}^{p} - L_{m}^{*p} = \frac{a(v^{*} - v)(1 + gw)s_{up}^{2}}{2J_{2}[f(1 + a) + g(f + aJ_{1})] + a(v + v^{*})(1 + gw)}$, with $\left|\Delta L_{i}^{p}\right| < \left|\Delta L_{m}^{p}\right|$.

In the case of supply shocks the derivatives prove that output variance minimisation leads to opposite results with respect to (15). For example, if the economy is hit by an internal supply shock, conditions (17) are sufficient for $\frac{\P k}{\P s^{2}_{uys}}$ to be positive (see Figure 3). In the case of external shocks, given $v>v^{*}$, $\frac{\P k}{\P s^{2}_{uys}}$ is more likely to be negative (see Figure 4).

Regarding the difference in performance between the economies, it is possible to prove that in the case of a symmetric supply shock ($s^{2}_{uys} = s^{2}_{uys^{*}}$) interest rate pegging is neutral, while money targeting determines a smaller loss in terms of output for the country which has a more effective "credit channel" (and therefore a steeper *AD* curve).[10]

Following the solution method described in Modigliani and Papademos (1990), it is possible to find the analytical expression for the optimal monetary rule (see Appendix):[11]

---

[10] In fact, the differences in performance are given, respectively, by $\Delta L^{y}_{i} = L^{y}_{i} - L^{*y}_{i} = 0$ and $\Delta L^{y}_{m} = L^{y}_{m} - L^{*y}_{m} = \frac{2agw(v^{*} - v)s^{2}_{uys}}{\{2J_{2}[f(1+a) + g(f + aJ_{1})] + a(v + v^{*})(1 + gw)\}^{2}}$.

[11] In synthesis this approach consists of: (i) calculating a number of partial equilibria equal to the number of policy variables to be controlled; (ii) using a linear combination of the equations obtained in (i) to define the combination policy; (iii) rewriting this combination policy in terms of the objective variables and disturbances; (iv) minimising this new expression with respect to the policy instruments and substituting the results in (ii) in order to obtain the analytical expression of the optimal policy rule.

Table 5

THE EFFECTS OF THE OPTIMAL MONETARY POLICY *K* ON OUTPUT STABILISATION

$$\text{sign}\,\frac{\P k}{\P s^2_{uls}} = \text{sign}\,\frac{\P k}{\P s^2_{uld}} = -\text{sign}\Big[\frac{(2a+k)gv}{\Delta}\ \frac{4fg(1+g)J_2v}{\Delta^2}\Big] < 0$$

$$\text{sign}\,\frac{\P k}{\P s^2_{uls*}} = \text{sign}\,\frac{\P k}{\P s^2_{uld*}} = -\text{sign}\Big[\frac{(2a+k)gv^*}{\Delta}\ \frac{4fg(1+g)J_2v^*}{\Delta^2}\Big] < 0$$

$$\text{sign}\,\frac{\P k}{\P s^2_{umd}} = \text{sign}\,\frac{\P k}{\P s^2_{umd*}} = -\text{sign}\Big[\frac{-4f^2g^2J_2^2[2J_2(f+gJ_1)+(v+v^*)(1+gw)]}{\Delta^3}\Big] > 0$$

$$\text{sign}\,\frac{\P k}{\P s^2_{uyd}} = \text{sign}\,\frac{\P k}{\P s^2_{uyd*}} = -\text{sign}\Big[\frac{(2a+k)gJ_2}{\Delta}\ \frac{4fgJ_2^2(1+gw)}{\Delta^2}\Big] < 0$$

$$\text{sign}\,\frac{\P k}{\P s^2_{uys}} = -\text{sign}\Big[\frac{2fgJ_2\{2J_2(f-J_1)+v[1-(2+g)w]+v^*(1+gw)\}\,\{2fJ_2(2+2a+k)+(2a+k)[v(1-gw)+v^*(1+gw)]\}}{\Delta^3}\Big]$$

$$\text{sign}\,\frac{\P k}{\P s^2_{uys*}} = -\text{sign}\Big[\frac{2fgJ_2\{2J_2(f-J_1)+v^*[1-(2+g)w]+v(1+gw)\}\,\{2fJ_2(2+2a+k)+(2a+k)[v^*(1-gw)+v(1+gw)]\}}{\Delta^3}\Big]$$

$$\text{sign}\,\frac{\P k}{\P s^2_{up}} = -\text{sign}\Big[\frac{2fg^2J_2(2a+k)(1+g)(v^*-v)^2(1+gw)^2}{\Delta^3}\Big] < 0$$

where:

$$\Delta = (2a+k)[2(f+gJ_1)J_2 + (v+v^*)(1+gw)] + 4fJ_2(1+g) > 0$$

$$0 < J_1 = 1-b < 1 \quad \text{and} \quad J_2 = h+q > 0$$

Figure 3

THE EFFECTS OF THE OPTIMAL MONETARY POLICY *K* ON OUTPUT STABILISATION:

THE CASE OF AN INTERNAL SUPPLY SHOCK

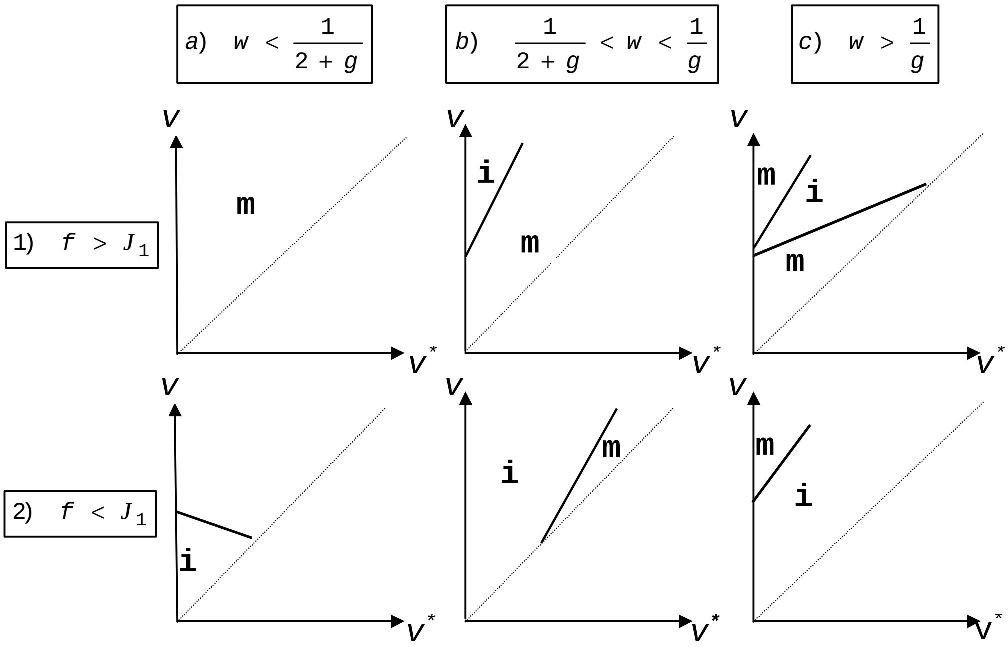
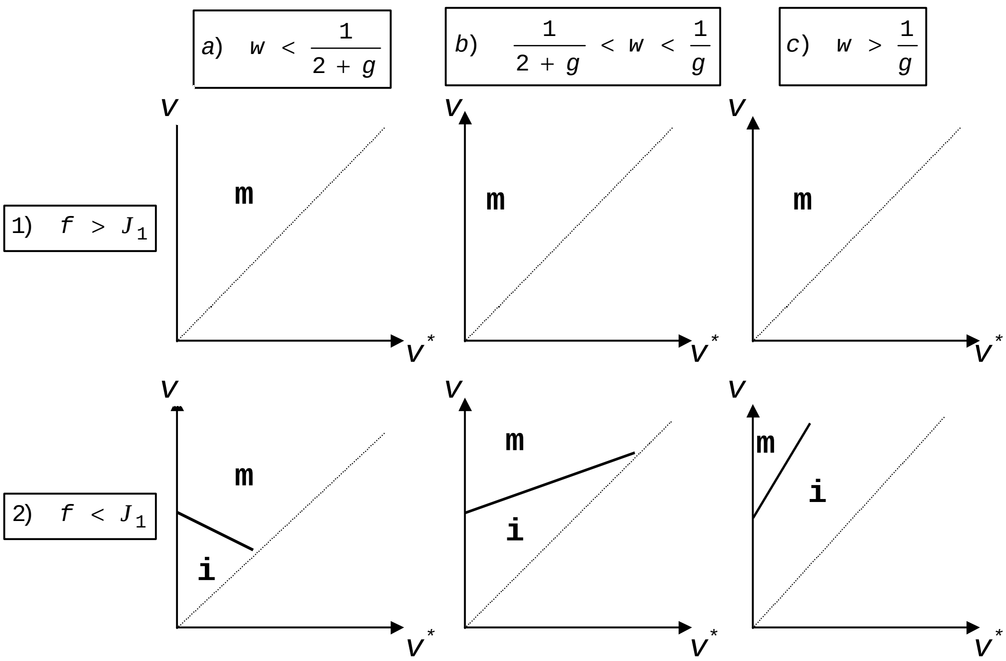


Figure 4

THE EFFECTS OF THE OPTIMAL MONETARY POLICY *K* ON OUTPUT STABILISATION:

THE CASE OF AN EXTERNAL SUPPLY SHOCK



$$
(18) \qquad \frac{m_t'}{i_t} = \frac{\begin{array}{l} 4fJ_2C_1(s^2_{umd} + s^2_{umd*} + s^2_{uys} + s^2_{uys*}) + 2aC_2C_3[4v^2(s^2_{uls} + s^2_{uld}) + \\ +4v^{*2}(s^2_{uls*} + s^2_{uld*}) + 4J_2^2(s^2_{uyd} + s^2_{uyd*}) + 4(J_1J_2 + vw)^2 s^2_{uys} + \\ +4(J_1J_2 + v^*w)^2 s^2_{uys*} + (1 + gw)^2(v^* - v)^2 s^2_{up}] \end{array}}{\begin{array}{l} 2z(v + v^*)C_1(s^2_{umd} + s^2_{umd*} + s^2_{uys} + s^2_{uys*}) + C_2C_3[4v^2(s^2_{uls} + s^2_{uld}) + \\ +4v^{*2}(s^2_{uls*} + s^2_{uld*}) + 4J_2^2(s^2_{uyd} + s^2_{uyd*}) + 4(J_1J_2 + vw)^2 s^2_{uys} + \\ +4(J_1J_2 + v^*w)^2 s^2_{uys*} + (1 + gw)^2(v^* - v)^2 s^2_{up}] \end{array}},
$$

where:

$$
\begin{aligned}
C_1 &= [2J_2(f + gJ_1) + (v + v^*)(1 + gw)]^2 \\
C_2 &= 2fJ_2 + v + v^* \\
C_3 &= (1 + g)^2.
\end{aligned}
$$

Expression (18) provides the following insights into the characteristics of the optimal feedback rule.

(i) The ratio $0<\frac{m_t'}{i_t}<\infty$ lies in an interval that includes the situations of vertical *LM* (i_t=¥), money targeting ($m_t' = 0$) and interest rate pegging ($i_t = 0$).[12]

(ii) In the case of a money demand shock the ratio becomes $\frac{m_t'}{i_t} = \frac{2fJ_2}{z\,(v + v^*)}$. The intuition behind this condition can be explained in the *CC-LM* and *AD-AS* framework referred to the whole union. In fact, the result depends on the slope of the *CC* (influenced by f, J_2 and the average credit

[12] Using the money supply process described by equation (2′), these three situations are reached by setting the value of k, respectively to $-2a$, 0, and ¥.

channel, $\bar{v} = \frac{v + v^*}{2}$) and on the sensitivity of the *CC* to a change in the money stock (z). The case in favour of interest rate pegging applies when f and J_2 are high and $\bar{v}$ is low (*CC* is almost horizontal), while z is negligible (the supply of credit does not react to a change in m' and *CC* is fixed).

(iii) If the economies are hit by a disturbance in the credit market or output demand (both shocks move the *CC* curve horizontally), the optimal rule depends on the slope of the *LM* curve. In fact, equation (18) collapses to $\frac{m_t'}{i_t} = 2a$ and the feedback rule depends on money demand elasticity with respect to the interest rate. For example, if the latter is low and *LM* becomes rigid, monetary targeting is preferable.

(iv) Also with “price-wedge” shocks, the optimal rule depends on the condition $\frac{m_t'}{i_t} = 2a$, and the same conclusion expressed in (iii) is valid. Nevertheless the nature of the result is different because the wedge in prices influences national aggregate demand also through the “money channel”. In particular, these effects move in opposite directions with the same intensity (f is common) and the overall result on output demand for the union does not change.

(v) In the case of an internal supply shock the optimal monetary rule becomes $\frac{m_t'}{i_t} = \frac{4fJ_2C_1 + 8aC_2C_3(J_1J_2 + vw)^2}{2z(v + v^*)C_1 + 4C_2C_3(J_1J_2 + vw)^2}$. Given the complexity of the formula, only differential calculus with respect to each parameter can explain the characteristic of the feedback rule. In particular, it is possible to prove that $\frac{m_t'}{i_t} = f(\underset{+}{a}, \underset{-}{z})$. As expected, a monetary targeting rule is preferable when the absolute value of the slopes of *LM* (influenced by *a*) and *CC* movements (influenced by *z*) are high. On the other hand, as already pointed out, the choice of the optimal monetary rule with respect to *CC* and *AS* slopes depends upon the effectiveness of money (f) and credit (v, v^*) channels. These conclusions also apply in the case of an external supply shock, where the optimal rule is $\frac{m_t'}{i_t} = \frac{4fJ_2C_1 + 8aC_2C_3(J_1J_2 + v^*w)^2}{2z(v + v^*)C_1 + 4C_2C_3(J_1J_2 + v^*w)^2}$. The difference between an internal and an external supply shock is represented by the conditions under which the optimal rule switches from money targeting to interest rate pegging. In particular, given $v>v^*$, the value of $\frac{m_t'}{i_t}$ is more likely to be low if the supply shock hits the country that has a relatively more effective “credit channel” (this can also be seen by comparing the critical areas in Figures 1 and 2).

6. An alternative money supply process based on the spread

The aim of this section is to analyse a different money supply process which uses the spread between the interest rates on loans and bonds as a feedback variable. The intuition behind this policy simulation is that the spread could embody more information about current shocks than the interest rate on bonds. From an analytical point of view, it is possible to hypothesise different cases, nested in the following equation based also on national rates on loans:

$$(2'') \quad m' = (m_t + m_t^*)^s = m_t + m_t^* - x_1 r_t - x_2 r_t^* + x_3\, i_t\,,$$

where x_1, x_2 and x_3 are policy instruments chosen jointly by the monetary authority to minimise its objective function.

Using the spread definition and considering, for simplicity, only one policy instrument ($x=2x_1=2x_2=x_3$) it is possible to obtain the following expression of the money supply process (2′′):

$$(19) \quad m' = (m_t + m_t^*)^s = m_t + m_t^* - x\,\frac{\tilde{r}_t + \tilde{r}_t^*}{2}$$

based on the average spread between the two countries.

Imposing (i) a non negative *LM* slope ($\frac{xw + 2J_2}{4aJ_2}$), (ii) a well behaved real balance effect (when prices increase *LM* intercept rises) and (iii) a positive effect of money on prices (quantitative theory), the value of the policy instrument x will lie in the following interval:

$$(20) \quad min[-\frac{2J_2}{w}, -2J_2, -\frac{2av}{f}] < x < ¥,$$

which includes the “pure” policies of money targeting (x=0) and average spread pegging (x=¥).[13] In particular, the average

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[13] The sign of the influence that the spread has on the money supply stock depends on the nature of the shock and cannot be established a priori. For example, an increase in the spread could be caused by an autonomous

spread pegging policy (hereafter the "*s*" rule) can be interpreted as a means to reduce, on the one hand, credit market imperfections (a spread reduction means that the cost of capital supplied by banks tends to be equal to the cost of financing in the bond market) and, on the other hand, differences between the credit systems of the two countries. In fact, given the existence of "monitoring" and "screening" costs, the spread cannot be negative and the average spread tends toward zero only if both national spreads converge toward zero.

Using equation (19) to solve the model, the reduced form reported in Table 6 can be obtained. This solution, given condition (20), has the same characteristics as that of the model with the money supply process based on the interest rate on bonds (see comments (i)-(viii) in Section 3).

The properties of the monetary instrument x are analysed using implicit differentiation of the objective function of the monetary authority (see Table 7).

An internal shock on the credit market reduces the value of x. Only if the difference between the "credit channels" is higher than a critical value $\left(v - v^{*} > \frac{2J_2[f(1 + a) + g(f + aJ_1)]}{a\,(1 + gw)}\right)$ are the derivatives $\frac{¶x}{¶s^2_{uls}}$ and $\frac{¶x}{¶s^2_{uld}}$ positive and spread control becomes necessary to

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reduction in loan supply (which implies the need for an "easy" money policy) or an increase in loan demand (which calls for a "tight" monetary policy).

Table 6

REDUCED FORM OF THE MODEL WITH THE MONEY SUPPLY PROCESS BASED ON THE SPREAD

(21) $p_t = A_0 m + A_1 m^* + A_2(u_{ls} - u_{ld}) + A_3(u_{ls^*} - u_{ld^*}) + A_4(u_{md} + u_{md^*}) + A_5(u_{yd} + u_{yd^*}) + A_6 u_{ys} + A_7 u_{ys^*} + A_8 u_p$

(22) $p_t^* = B_0 m + B_1 m^* + B_2(u_{ls} - u_{ld}) + B_3(u_{ls^*} - u_{ld^*}) + B_4(u_{md} + u_{md^*}) + B_5(u_{yd} + u_{yd^*}) + B_6 u_{ys} + B_7 u_{ys^*} + B_8 u_p$

(23) $y_t = C_2(u_{ls} - u_{ld}) + C_3(u_{ls^*} - u_{ld^*}) + C_4(u_{md} + u_{md^*}) + C_5(u_{yd} + u_{yd^*}) + C_6 u_{ys} + C_7 u_{ys^*} + C_8 u_p$

(24) $y_t^* = D_2(u_{ls} - u_{ld}) + D_3(u_{ls^*} - u_{ld^*}) + D_4(u_{md} + u_{md^*}) + D_5(u_{yd} + u_{yd^*}) + D_6 u_{ys} + D_7 u_{ys^*} + D_8 u_p$

where:

$A_0 = B_0 = A_1 = B_1 = \dfrac{f(J_2 + xz) + az(v + v^*)}{xf + 2fJ_2(1 + a) + a(v + v^*)}$ | $C_0 = C_1 = D_0 = D_1 = 0$

$A_2 = B_2 = (xf + 2av) / \Delta > 0$ | $C_2 = D_2 = g(xf + 2av) / \Delta > 0$

$A_3 = B_3 = (xf + 2av^*) / \Delta > 0$ | $C_3 = D_3 = g(xf + 2av^*) / \Delta > 0$

$A_4 = B_4 = (-2fJ_2) / \Delta < 0$ | $C_4 = D_4 = (-2fgJ_2) / \Delta < 0$

$A_5 = B_5 = 2aJ_2 / \Delta > 0$ | $C_5 = D_5 = 2agJ_2 / \Delta > 0$

$A_6 = B_6 = -[2J_2(f + aJ_1) + w(xf + 2av)] / \Delta < 0$ | $C_6 = \{xf(2 + gw) + 2J_2[2f(1 + a) + g(f + aJ_1)] + 2a[v + v^*(1 + gw)]\} / \Delta > 0$

$A_7 = B_7 = -[2J_2(f + aJ_1) + w(xf + 2av^*)] / \Delta < 0$ | $C_7 = -g[2J_2(f + aJ_1) + w(xf + 2av^*)] / \Delta < 0$

$A_8 = \{(1 + gw)(xf + 2av^*) + 2J_2[f(1 + a) + g(f + aJ_1)]\} / \Delta > 0$ | $C_8 = g\{(xf + 2av^*)(1 + gw) + 2J_2[f(1 + a) + g(f + aJ_1)]\} / \Delta > 0$

$B_8 = -\{(1 + gw)(xf + 2av) + 2J_2[f(1 + a) + g(f + aJ_1)]\} / \Delta < 0$ | $D_6 = -g[2J_2(f + aJ_1) + w(xf + 2av)] / \Delta < 0$

$\Delta = 2[xf + a(v + v^*)](1 + gw) + 4J_2[f(1 + a) + g(f + aJ_1)] > 0$ | $D_7 = \{xf(2 + gw) + 2J_2[2f(1 + a) + g(f + aJ_1)] + 2a[v(1 + gw) + v^*]\} / \Delta > 0$

$0 < J_1 = 1 - b < 1$ and $J_2 = h + q > 0$ | $D_8 = -g\{(xf + 2av)(1 + gw) + 2J_2[f(1 + a) + g(f + aJ_1)]\} / \Delta > 0$

Table 7

THE PROPERTIES OF THE MONETARY POLICY *X*

$$\text{sign}\,\frac{\P x}{\P s^2_{uls}} = \text{sign}\,\frac{\P x}{\P s^2_{uld}} = -\text{sign}\left[\frac{(xf + 2av)}{\Delta}\ \frac{2f\{2J_2[f(1 + a) + g(f + aJ_1) + a(v^* - v)(1 + gw)]\}}{\Delta^2}\right]$$

$$\text{sign}\,\frac{\P x}{\P s^2_{uls*}} = \text{sign}\,\frac{\P x}{\P s^2_{uld*}} = -\text{sign}\left[\frac{(xf + 2av^*)}{\Delta}\ \frac{2f\{2J_2[f(1 + a) + g(f + aJ_1) + a(v - v^*)(1 + gw)]\}}{\Delta^2}\right] < 0$$

$$\text{sign}\,\frac{\P x}{\P s^2_{umd}} = \text{sign}\,\frac{\P x}{\P s^2_{umd*}} = -\text{sign}\left[\frac{-2fJ_2}{\Delta}\ \frac{4f^2J_2(1 + gw)}{\Delta^2}\right] > 0$$

$$\text{sign}\,\frac{\P x}{\P s^2_{uyd}} = \text{sign}\,\frac{\P x}{\P s^2_{uyd*}} = -\text{sign}\left[-\ \frac{2aJ_2}{\Delta}\ \frac{4afJ_2(1 + gw)}{\Delta^2}\right] > 0$$

$$\text{sign}\,\frac{\P x}{\P s^2_{uys}} = -\text{sign}\left[\frac{-[2J_2(f + aJ_1) + w(xf + 2av)]}{\Delta}\ \frac{2f\{2J_2f[1 - w(1 + a)] + 2aJ_1J_2 + aw(v - v^*)(1 + gw)\}}{\Delta^2}\right]$$

$$\text{sign}\,\frac{\P x}{\P s^2_{uys*}} = -\text{sign}\left[\frac{-[2J_2(f + aJ_1) + w(xf + 2av^*)]}{\Delta}\ \frac{2f\{2J_2f[1 - w(1 + a)] + 2aJ_1J_2 + aw(v^* - v)(1 + gw)\}}{\Delta^2}\right]$$

$$\text{sign}\,\frac{\P x}{\P s^2_{up}} = -\text{sign}\left[-\ \frac{4a^2f(v - v^*)^2(1 + gw)^3}{\Delta^3}\right] > 0$$

where:

$$\Delta = 2[xf + a(v + v^*)](1 + gw) + 4J_2[f(1 + a) + g(f + aJ_1)] > 0$$

$0 < J_1 = 1 - b < 1$ and $J_2 = h + q > 0$

contain the effect on price variability. In the case of an external shock on the credit market, x always moves towards money targeting $\left(\frac{\partial x}{\partial \sigma^2_{uls^*}} = \frac{\partial x}{\partial \sigma^2_{uld^*}} < 0\right)$, because, given the flatter *CC*, the effect on the interest rate structure is limited.

As expected, if the economies are hit by disturbances in money and aggregate demand the derivatives are positive. In particular, a pure spread pegging rule perfectly insulates the economies from this kind of shocks. This result represents, on the one hand, an indication of the superiority of spread versus interest rate pegging (in fact, the latter insulates the economies only from money demand shocks) and, on the other hand, an extension to classic Poole findings (in the case of *IS* and *LM* shocks, spread pegging is also superior to money targeting).

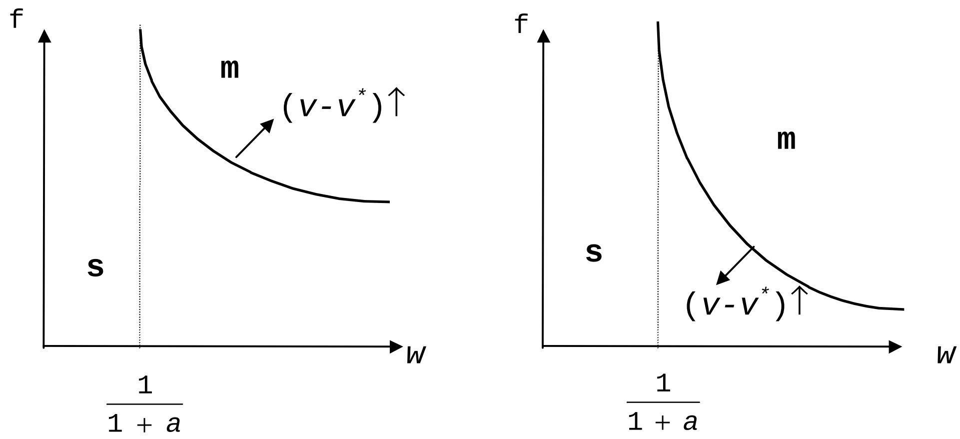
The sign of the derivative is also positive for “price-wedge” shocks, implying that x moves towards spread pegging. In this case, however, the setting of $x=\infty$ does not perfectly insulate the economies. In fact, by using the objective function (15) and the expressions in Table 6, it is possible to show that each country suffers a symmetric loss of $\frac{1}{2}\sigma^2_{up}$.

In the case of supply shocks, the sign of the derivatives depends on loan demand elasticity with respect to income (w) and on the difference between the two “credit channels” $(v - v^*)$. In fact, these parameters influence the slope of the *AD* curve and, hence, the price reaction to supply disturbances. Given $v>v^*$, in the case of an internal shock, the optimal monetary policy is more likely to move towards spread pegging, which is more effective in containing the indirect effect due to the banking sector. Vice versa, in the case of an external shock, there is less need to neutralise the consequences produced through the “credit channel” (compare the critical areas in Figures 5a and 5b). Moreover, with both an expansive credit cycle (*w* high) and an effective “money channel” (*f* high), money targeting is preferable wherever the supply disturbance occurs (see Figures 5a and 5b).[14]

Figure 5

THE OPTIMAL MONETARY POLICY *X* IN THE CASE OF SUPPLY SHOCKS

a)Internal b)External



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[14] When random disturbances affect the supply side of the home economy, a sufficient condition for $\frac{\partial x}{\partial s^2_{uys}}$ to be positive is $w < \frac{1}{1+a}$, while it would be negative if $w > \frac{1}{1+a}$ and $f > \frac{2aJ_1J_2 + aw(v - v^*)(1+gw)}{2aJ_2[w(1+a) - 1]}$. In the case of an external supply shock, given $v>v^*$, the derivative is more likely to be negative (see Figure 5b). In particular, if the difference between national “credit channels” is high ($v - v^* > \frac{2J_2f[1 - w(1+a)] + 2aJ_1J_2}{(1+gw)aw}$), money targeting is preferable.

Table 8

THE EFFECTS OF THE MONETARY POLICY *X* ON OUTPUT STABILISATION

$$\text{sign}\,\frac{\partial x}{\partial s^2_{uls}} = \text{sign}\,\frac{\partial x}{\partial s^2_{uld}} = -\text{sign}\Big[\frac{2fg^2\{2J_2[f(1+a)+g(f+aJ_1)+ +a(v^*-v)(1+gw)]\}(xf+2av)}{\Delta^3}\Big]$$

$$\text{sign}\,\frac{\partial x}{\partial s^2_{uls*}} = \text{sign}\,\frac{\partial x}{\partial s^2_{uld*}} = -\text{sign}\Big[\frac{2fg^2\{2J_2[f(1+a)+g(f+aJ_1)+ +a(v-v^*)(1+gw)]\}(xf+2av^*)}{\Delta^3}\Big] < 0$$

$$\text{sign}\,\frac{\partial x}{\partial s^2_{umd}} = \text{sign}\,\frac{\partial x}{\partial s^2_{umd*}} = -\text{sign}\Big[\frac{-2fgJ_2}{\Delta}\;\frac{4f^2gJ_2(1+gw)}{\Delta^2}\Big] > 0$$

$$\text{sign}\,\frac{\partial x}{\partial s^2_{uyd}} = \text{sign}\,\frac{\partial x}{\partial s^2_{uyd*}} = -\text{sign}\Big[\frac{-2agJ_2}{\Delta}\;\frac{4afgJ_2(1+gw)}{\Delta^2}\Big] > 0$$

$$\text{sign}\,\frac{\partial x}{\partial s^2_{uys}} = -\text{sign}\Big[\frac{4fg\{f[x+2J_2(1+a)]+av(1-gw)+av^*(1+gw)\}\{2J_2f[1-w(1+a)]+2aJ_1J_2+aw(v-v^*)(1+gw)\}}{\Delta^3}\Big]$$

$$\text{sign}\,\frac{\partial x}{\partial s^2_{uys*}} = -\text{sign}\Big[\frac{4fg\{f[x+2J_2(1+a)]+av^*(1-gw)+av(1+gw)\}\{2J_2f[1-w(1+a)]+2aJ_1J_2+aw(v^*-v)(1+gw)\}}{\Delta^3}\Big]$$

$$\text{sign}\,\frac{\partial x}{\partial s^2_{up}} = -\text{sign}\Big[-\frac{4a^2fg(v-v^*)^2(1+gw)^3}{\Delta^3}\Big] > 0$$

where:

$$\Delta = 2[xf + a(v+v^*)](1+gw) + 4J_2[f(1+a)+g(f+aJ_1)] > 0$$

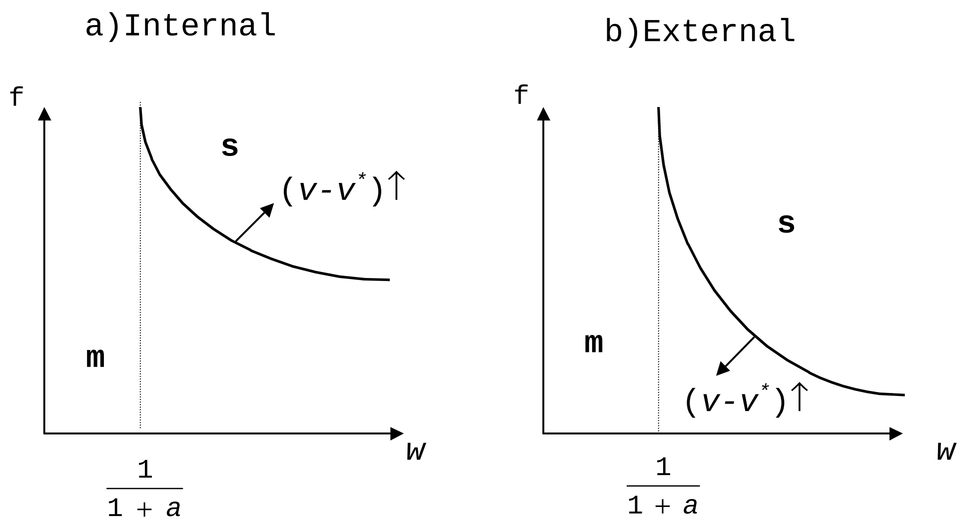
$0 < J_1 = 1 - b < 1$ and $J_2 = h + q > 0$

The consequences of the monetary rule *x* (designed to minimise price variation) for output stabilisation can be analysed from the results reported in Table 8. In the cases of financial (credit and money markets), aggregate demand and “price-wedge” shocks, *x* is also optimal in terms of output stabilisation. In the remaining case of supply disturbances, similarly to the outcomes obtained in Section 5, the optimal monetary rule *x* has opposite consequences on output variance (this can be seen comparing Figure 6 with Figure 5).

With reference to the effects between countries, the result in terms of both prices and output stabilisation is in favour of spread pegging, which always determines symmetric losses ($L_s^p - L_s^{*p} = L_s^y - L_s^{*y} = 0$).

Figure 6

THE EFFECTS OF THE OPTIMAL MONETARY POLICY *X* ON OUTPUT STABILISATION



7. What are the consequences for the optimal monetary rule?

If the economies are hit by a wide range of stochastic disturbances (not only *IS* and *LM* shocks as in Poole (1970), but also by credit market, *AS* and “price-wedge” disturbances), the optimal money supply process should consider both the interest rate on bonds and the spread as feedback variables. In fact, while the former embodies information mainly on money market disequilibria (*LM*), the latter also indicates the state of credit and goods markets (*CC*).

Using the following money supply process to solve the model:

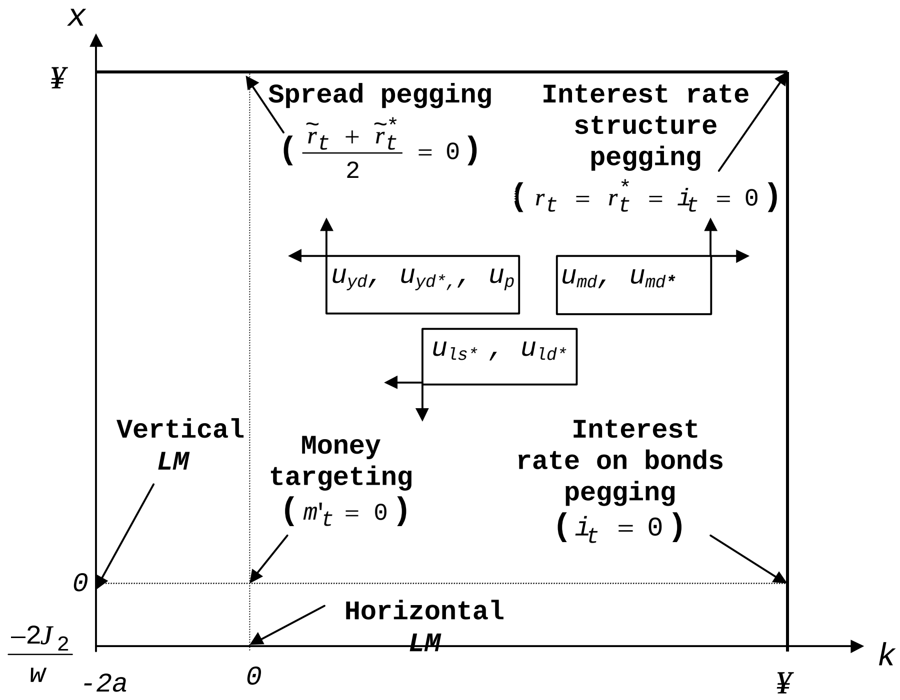
$$(25) \qquad m' = (m_t + m_t^*)^s = m_t + m_t^* + k i_t - x \frac{\tilde{r}_t + \tilde{r}_t^*}{2},$$

it is possible to obtain a reduced form that has the same characteristics as those reported in Tables 3 and 6 (the only difference is that each parameter is a function of both *k* and *x*). Moreover, the study of the optimal monetary rule (obtained using partial differentiation with respect to *k* and *x*) confirms the conclusions reported in Tables 4-5 and 7-8. In this case the monetary instrument (*k*, *x*) consists in choosing a point in the two-dimensional space (see Figure 7).

Each shock moves the optimal combination (*k, x*) in a different direction, with an intensity that is a function of the absolute value of the parameters of the model. Given the asymmetry between national credit channels, the law of motion of the optimal monetary instrument is univocally determined only for the disturbances represented in Figure 7.

Figure 7

THE LAW OF MOTION OF THE OPTIMAL MONETARY INSTRUMENT
WITH TWO FEEDBACK VARIABLES (*K, X*)

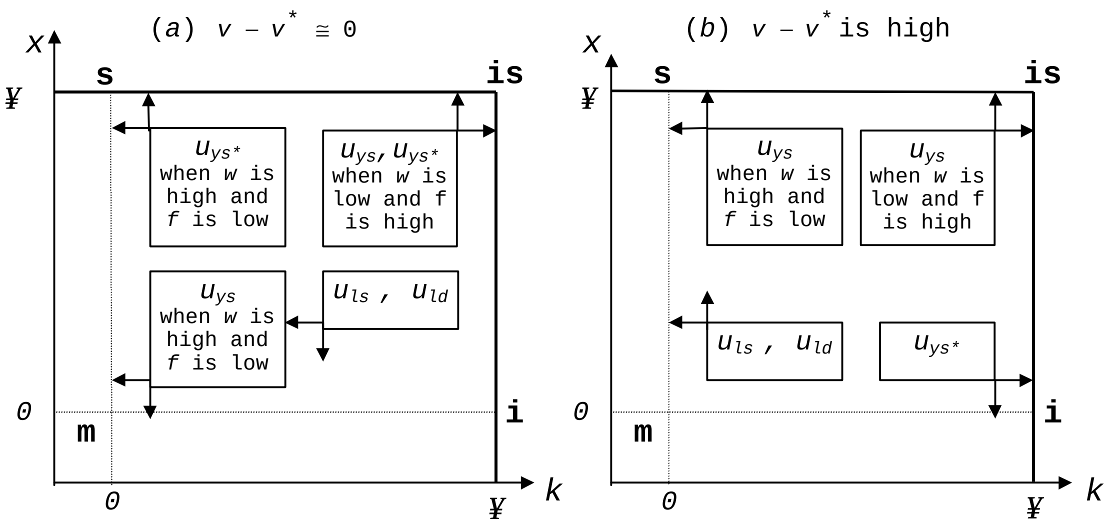


In the case of aggregate demand and "price-wedge" shocks the optimal monetary policy moves towards the North-West (this means that the three "pure" policies are in the following order of preference $s \succ m \succ i$), while if the economies are hit by money demand disturbances, complete control of the interest rate structure is preferable ($s, i \succ m$). An external credit market shock calls for money targeting ($m \succ s, i$).

As regards the remaining disturbances, their law of motion is strongly influenced by the effectiveness of money and credit channels and their degree of asymmetry. If the monetary union is characterised by approximately homogeneous "credit channels" ($v - v^* \cong 0$), the optimal policy is

Figure 8

THE LAW OF MOTION OF THE OPTIMAL MONETARY INSTRUMENT IN THE CASE OF (*A*) HOMOGENEOUS AND (*B*) HETEROGENEOUS CREDIT CHANNELS



represented by Figure 8a*,* while Figure 8b shows the case of heterogeneous "credit channels" (the difference $v - v^{*}$ is high)*.* If the economy is hit by internal credit market shocks, the optimal monetary rule coincides with that of an external disturbance only in the case of homogeneous "credit channels" (compare Figures 7 and 8a) and money targeting is preferable ($m \succ s, i$). On the other hand, if the national "credit channel" difference increases, home credit market shocks move the optimal monetary policy towards the North-West ($s \succ m \succ i$, see Figure 8b).With supply shocks, in the case of homogeneous "credit channels", monetary policy is highly sensitive to loan demand elasticity with respect to income (*w*) and the effectiveness of the money channel (*f*). Only when *w* is low and *f* is high (see Figure 8a) does the optimal rule coincide for internal and external shocks and a complete control of the interest rate structure is preferable ($s,i \succ m$). On the other hand, in the case of an expansive credit cycle combined with an ineffective money channel (w is high while f is low), an internal supply disturbance calls for money targeting ($m \succ s,i$), while an external supply shock moves towards spread pegging ($s \succ m \succ i$).

In the case of heterogeneous “credit channels”, the reaction to supply disturbances changes. In fact, an internal disturbance calls for spread pegging, except in the case of an effective money channel (f is high) and a low value of w which moves the optimal monetary policy towards the North-East ($s,i \succ m$, see Figure 8b). On the other hand, if the economy is hit by an external shock, interest rate pegging is preferable.

It is worth noting that if the credit channels are ineffective ($v = v^* \cong 0$) the optimal monetary rule only depends upon money channel effectiveness. In particular, a supply shock (wherever it occurs) moves the policy towards spread pegging if f is low and towards interest pegging if f is high.

As already pointed out, in the case of supply disturbances, the optimal monetary rule (in terms of price stabilisation) has asymmetric consequences for real output. These asymmetries depend not only on the absolute value of specific parameters of the model, but also on the proposed optimal monetary process. In fact, the latter, on the one hand improves the performance of the classic Poole’s rule but, on the other hand it seems to embody limited information with respect to supply-side shocks.

8. Conclusions and open questions

This paper has analysed the optimal monetary policy in a monetary union composed of countries with heterogeneous “credit channels”. In order to insulate better the economies from the asymmetric effects produced by differences in national financial systems, the classic money supply process proposed by Poole (1970) has been modified to consider the spread between the interest rates on loans and bonds as an additional feedback variable. Using a two-country rational expectations model, this study has highlighted the properties of the optimal monetary instrument with respect to a wide range of stochastic shocks. The main conclusions can be summarised as follows.

In the case of complete monetary integration between the economies, not only money and aggregate demand shocks, but also credit market disturbances influence prices and income in an additive form and, therefore, the monetary union tends to reduce the effects of such shocks only if they are negatively correlated. With regard to their intensity, given the assumptions of the model, credit market disturbances affect the endogenous variables in an idiosyncratic way (heterogeneous “credit channels”), while money and aggregate demand shocks (wherever they occur) influence prices and income with the same intensity (homogeneous “money channels” and stochastic law of one price). On the other hand, supply shocks and “price-wedge” disturbances have an asymmetric effect on national fundamentals (opposite in sign in the case of real output) and only when they are negligible does the stabilisation objective of each country coincide with that of the monetary area as a whole.

The optimal monetary policy has to consider as feedback variables both the interest rate on bonds and the average spread. While the former embodies information mainly about money market disequilibria, the latter also indicates the state of the credit market and aggregate demand.

Using the spread as a feedback variable in the money supply process determines, on the one hand, the reduction of asymmetric effects due to national credit market differences and, on the other, an insulation of the economies from money and aggregate demand disturbances. This result indicates the superiority of spread versus interest rate pegging (which insulates the economies only from money demand shocks) and represents an extension to classic Poole findings (in the case of *IS* and *LM* shocks, spread pegging is also superior to money targeting). In the face of credit market disturbances, money targeting is generally preferable, except in the case of an internal shock associated with high asymmetry between national "credit channels" which requires spread pegging to contain the effect on prices and output variability.

When random disturbances affect the supply side of the economies the signalling power of the proposed money process seems to be more limited. In fact, the optimal monetary policy becomes highly sensitive to the elasticity of demand for loans with respect to real output, the effectiveness of money and credit channels and their degree of asymmetry. Moreover, the selected rule with respect to inflation leads to opposite consequences on output variance.

The country with a more effective "credit channel" always performs better in terms of both prices and output stabilisation. This asymmetry tends to vanish if the monetary rule moves towards spread pegging.

The main message from this paper is that in the case of a monetary union among countries with different financial structures, monetary policy should respond by “leaning against the wind” with more intensity than if the countries were identical. Each kind of shock changes the optimal rule in a specific direction with an intensity that is a function of the parameters of the model. Moreover, if the difference between national “credit channels” reaches a critical value, with credit market and supply disturbances, the law of motion of the optimal rule switches, depending upon the country in which the shock has originated.

Further research could be directed towards tackling the following three issues. First, the general analytical framework of the model could be used to analyse the consequences for the monetary transmission process of structural and institutional differences between countries (for example, heterogeneous “money channels” or differences in the elasticity of some variables). Second, the results of the optimal monetary rule proposed in this study could be improved using price levels (or real outputs) as feedback variables to insulate better the economies from supply side shocks. Thirdly, an analysis of the monetary instrument problem should also take into account (autonomous) national fiscal policies. In this case, in fact, the optimal policy rule also depends on fiscal policy coordination at the area-level and should include as feedback variable also an indicator of their degree of asymmetry.

APPENDIX

Determination of the analytical expression of the optimal monetary rule

The analytical expression of the optimal monetary rule can be obtained from the procedure described in Modigliani and Papademos (1990). With two control variables (money and interest rates), this approach first requires the calculation of two partial equilibria: *CC-AS* and *LM-AS*.[15]

The *CC-AS* equilibrium can be obtained by the following system:

(A.1) $z(m_t + m_t^*) + q\tilde{r}_t + u_{ls} = p_t + wy_t - h\tilde{r}_t + u_{ld}$

(A.2) $z(m_t + m_t^*) + q\tilde{r}_t^* + u_{ls*} = p_t^* + wy_t^* - h\tilde{r}_t^* + u_{ld*}$

(A.3) $y_t + y_t^* = b(y_t + y_t^*) - f(2i_t + p_t + p_t^*) - v\tilde{r}_t + -v\tilde{r}_t^* + u_{yd} + u_{yd*}$

(A.4) $y_t = g(p_t - {}_{t-1}E\, p_t) + u_{ys}$

(A.5) $y_t^* = g(p_t^* - {}_{t-1}E\, p_t^*) + u_{ys}^*$

(A.6) $p_t = p_t^* + u_p$

that expressed in matrix form gives:

[15] Only the proposed combination of partial equilibria (*CC-AS* and *LM-AS*) is consistent with the rational expectations nature of the model. In fact, since only the aggregate supply includes price expectations, the classical combination *CC-LM* (that determines the *AD*) and *AS* is not applicable.

$$\begin{bmatrix} -1 & 0 & -w & 0 & J_2 & 0 \\ 0 & -1 & 0 & -w & 0 & J_2 \\ f & f & J_1 & J_1 & v & v^* \\ -g & 0 & 1 & 0 & 0 & 0 \\ 0 & -g & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p_t \\ p_t^* \\ y_t \\ y_t^* \\ \tilde{r}_t \\ \tilde{r}_t^* \end{bmatrix} = \begin{bmatrix} -z(m_t + m_t^*) + (u_{ld} - u_{ls}) \\ -z(m_t + m_t^*) - (u_{ld^*} + u_{ls^*}) \\ -2fi_t + (u_{yd} + u_{yd^*}) \\ u_{ys} - g \underset{t-1}{E} p_t \\ u_{ys} - g \underset{t-1}{E} p_t^* \\ u_p \end{bmatrix}$$

where the interest rate on bonds and price expectations have been treated as exogenous variables.

Applying the “three-step” method to eliminate rational expectations, the following reduced form for prices is obtained:

$$\begin{aligned} p_t = {} & A_0 m + A_1 m^* + A_2 i_t + A_3(u_{ls} - u_{ld}) + A_4(u_{ls^*} - u_{ld^*}) + \\ & + A_5(u_{yd} + u_{yd^*}) + A_6 u_{ys} + A_7 u_{ys^*} + A_8 u_p \end{aligned}$$

$$\begin{aligned} p_t = {} & B_0 m + B_1 m^* + B_2 i_t + B_3(u_{ls} - u_{ld}) + B_4(u_{ls^*} - u_{ld^*}) + \\ & + B_5(u_{yd} + u_{yd^*}) + B_6 u_{ys} + B_7 u_{ys^*} + B_8 u_p , \end{aligned}$$

where:

$$A_0 = B_0 = A_1 = B_1 = \frac{(v + v^*)z}{\Delta} > 0$$

$$A_2 = B_2 = -2fJ_2 / \Delta < 0$$

$$A_3 = B_3 = v / \Delta > 0$$

$$A_4 = B_4 = v^* / \Delta > 0$$

$$A_5 = B_5 = J_2 / \Delta > 0$$

$$A_6 = B_6 = -(J_1 J_2 + vw) / \Delta < 0$$

$$A_7 = B_7 = -(J_1 J_2 + v^* w) / \Delta < 0$$

$$A_8 = [J_2(f + gJ_1) + v^*(1 + gw)] / \Delta > 0$$

$$B_8 = -[J_2(f + gJ_1) + v(1 + gw)] / \Delta < 0$$

$$\Delta = 2J_2(f + gJ_1) + (v + v^*)(1 + gw) > 0$$

$$0 < J_1 = 1 - b < 1 \quad \text{and} \quad J_2 = h + q > 0.$$

The expression of the sum of prices is given by:

$$(p_t + p_t^*) = \frac{\begin{array}{c} 2v(u_{ls} - u_{ld}) + 2v^*(u_{ls^*} - u_{ld^*}) + 2J_2(u_{yd} - u_{yd^*}) + \\ -2(J_1J_2 + vw)u_{ys} + 2(J_1J_2 + v^*w)u_{ys^*} + (1 + gw)(v^* - v)u_p \end{array}}{2J_2(f + gJ_1) + (v + v^*)(1 + gw)} +$$

(A.7)
$$+ \frac{2z(v + v^*)(m_t + m_t^*)}{fJ_2 + v + v^*} - \frac{4fJ_2}{2fJ_2 + v + v^*} i_t.$$

The LM-AS equilibrium is obtained by the following system:

(A.4) $\quad y_t = g(p_t - {}_{t-1}E\, p_t) + u_{ys}$

(A.5) $\quad y_t^* = g(p_t^* - {}_{t-1}E\, p_t^*) + u_{ys}^*$

(A.8) $\quad m_t + m_t^* = p_t + p_t^* + y_t + y_t^* - 2ai_t + u_{md} + u_{md^*}$,

where equation (A.8) considers the interest rate as an exogenous variable (k=0).

Summing (A.4) and (A.5) it is possible to find an analytical expression for $y_t + y_t^*$. Substituting this expression in (A.8) it is easy to solve for $p_t + p_t^*$. Taking expectations and substituting, the following reduced form is obtained:

$$\text{(A.9)} \quad p_t + p_t^* = m_t + m_t^* - 2ai_t - \frac{u_{md} + u_{md^*} + u_{ys} + u_{ys^*}}{1 + g}.$$

The second step consists in using a linear combination of the parts of equations (A.7) and (A.9) which refer to the monetary instruments of the money supply process. The following expression, equivalent to equation (2’) in Table 2, is obtained:

$$\text{(A.10)} \quad k_0\{\frac{2z(v + v^*)m_t'}{2fJ_2 + v + v^*} - \frac{4fJ_2}{2fJ_2 + v + v^*}\, i_t\} + k_1\{m_t' - 2ai_t\} = 0,$$

where $k_0 + k_1 = 1$ and $m_t' = (m_t + m_t^*)^s$.

Using equations (A.7) and (A.9), expression (A.10) can be rewritten as:

$$k_0\{(p_t + p_t^*) - \frac{\begin{gathered}2v(u_{ls} - u_{ld}) + 2v^*(u_{ls^*} - u_{ld^*}) + \\ 2J_2(u_{yd} - u_{yd^*}) + -2(J_1J_2 + vw)u_{ys} + \\ +2(J_1J_2 + v^*w)u_{ys^*} + (1 + gw)(v^* - v)u_p\end{gathered}}{2J_2(f + gJ_1) + (v + v^*)(1 + gw)}\} +$$
$$+k_1\{(p_t + p_t^*) + \frac{u_{md} + u_{md^*} + u_{ys} + u_{ys^*}}{1 + g}\} = 0,$$

which, expressed as a function of $p_t + p_t^*$, is equivalent to:

$$p_t + p_t^* = (1 - k_1)\frac{\begin{gathered}2v(u_{ls} - u_{ld}) + 2v^*(u_{ls^*} - u_{ld^*}) + \\ +2J_2(u_{yd} - u_{yd^*}) + -2(J_1J_2 + vw)u_{ys} + \\ +2(J_1J_2 + v^*w)u_{ys^*} + (1 + gw)(v^* - v)u_p\end{gathered}}{2J_2(f + gJ_1) + (v + v^*)(1 + gw)} +$$
$$- k_1\frac{u_{md} + u_{md^*} + u_{ys} + u_{ys^*}}{1 + g}.$$

Using the relationship $Var\,(p_t) + Var\,(p_t^*) = Var\,(p_t + p_t^*)$ explained in footnote 8 of Section 4):

(A.11) $Var\,(p_t) + Var\,(p_t^*) = Var\,(p_t + p_t^*) = (1 - k_1)^2 A + k_1^2 B\,,$

where:

$$A = \frac{\begin{array}{l} 4v^2(s_{uls}^2 + s_{uld}^2) + 4v^{*2}(s_{uls^*}^2 + s_{uld^*}^2) + \\ +4J_2^2(s_{uyd}^2 + s_{uyd^*}^2) + 4(J_1J_2 + vw)^2 s_{uys}^2 + \\ +4(J_1J_2 + v^*w)^2 s_{uys^*}^2 + (1 + gw)^2(v^* - v)^2 s_{up}^2 \end{array}}{[2J_2(f + gJ_1) + (v + v^*)(1 + gw)]^2} > 0$$

$$B = \frac{s_{umd}^2 + s_{umd^*}^2 + s_{uys}^2 + s_{uys^*}^2}{(1 + g)^2} > 0\,.$$

Given the normalisation $k_0 = 1 - \ _1$, the first order condition to minimise (A.11) is represented by:

$$\frac{\P\, Var\, p_t \ + Var\, p}{\P k_1} = - \quad - k \ A + \ k_1 B =$$

$$k_1 = \frac{A}{A + B} \quad \text{and} \quad k \quad = \frac{B}{A + B}.^{16}$$

Substituting $_0$ and k_1

analytical expression of the optimal policy rule is obtained:

The sufficient for the of a minimum given by: $\dfrac{^2[Var(\quad Var(p^*)]}{\P \ _1^2} = 2A + B) \quad .$

(18)
$$\frac{m_t'}{i_t} = \frac{\begin{array}{l} 4fJ_2C_1(s^2_{umd} + s^2_{umd*} + s^2_{uys} + s^2_{uys*}) + 2aC_2C_3[4v^2(s^2_{uls} + s^2_{uld}) + \\ +4v^{*2}(s^2_{uls*} + s^2_{uld*}) + 4J_2^2(s^2_{uyd} + s^2_{uyd*}) + 4(J_1J_2 + vw)^2 s^2_{uys} + \\ +4(J_1J_2 + v^*w)^2 s^2_{uys*} + (1 + gw)^2(v^* - v)^2 s^2_{up}] \end{array}}{\begin{array}{l} 2z(v + v^*)C_1(s^2_{umd} + s^2_{umd*} + s^2_{uys} + s^2_{uys*}) + C_2C_3[4v^2(s^2_{uls} + \\ +s^2_{uld}) + 4v^{*2}(s^2_{uls*} + s^2_{uld*}) + 4J_2^2(s^2_{uyd} + s^2_{uyd*}) + 4(J_1J_2 + vw)^2 s^2_{uys} + \\ +4(J_1J_2 + v^*w)^2 s^2_{uys*} + (1 + gw)^2(v^* - v)^2 s^2_{up}] \end{array}}$$

where:

$$C_1 = [2J_2(f + gJ_1) + (v + v^*)(1 + gw)]^2$$
$$C_2 = 2fJ_2 + v + v^*$$
$$C_3 = (1 + g)^2.$$

References

Angeloni, I., L. Buttiglione, G. Ferri and E. Gaiotti (1995), "The Credit Channel of Monetary Policy across Heterogeneous Banks: The Case of Italy", Banca d'Italia, Temi di discussione, No. 256.

Bagliano, A. and C. Favero (1995), "The Credit Channel of Monetary Transmission: The Case of Italy", Milano, Università Cattolica del Sacro Cuore, mimeo.

Barran, F., V. Coudert and B. Mojon (1995), "Transmission de la politique monétaire et crédit bancaire: une application à trois pays de l'OCDE", *Revue Économique*, Vol. 46, pp. 393-413.

Bellando, R. and J. P. Pollin (1996), "Le canal du crédit en France depuis la déréglementation financière", *Revue Économique*, Vol. 47, pp. 731-43.

Bernanke, B. and A. S. Blinder (1988), "Credit, Money and Aggregate Demand", *American Economic Review*, Vol. 78, 435-39.

Buttiglione, L. and G. Ferri (1994), "Monetary Policy Transmission via Lending Rates in Italy: Any Lessons from Recent Experience?", Banca d'Italia, Temi di discussione, No. 224.

Committee for the Study of Economic and Monetary Union (Delors Committee) (1989*), Report on Economic and Monetary Union in the European Community*, Luxembourg, Office for Official Publications of the European Communities.

Dale, S. and A. Haldane (1993a), "Bank Behaviour and the Monetary Transmission Mechanism", *Bank of England Quarterly Bulletin*, No. 33.

______________________________ (1993b), "Interest Rate Control in a Model of Monetary Policy", Bank of England Working Paper Series, No. 17.

______________________________ (1995), "Interest Rate and the Channel of Monetary Transmission: Some Sectoral Estimates", *European Economic Review*, Vol. 39, pp. 1611-26.

De Grauwe, P.(1994), *The Economics of Monetary Integration: Second Revised Edition*, Oxford, Oxford University Press, Second Edition.

Gros, D. and N. Thygesen (1992), *European Monetary Integration*, London, Longman.

Kashyap, A. K., J. C. Stein and D. W. Wilcox (1992), “Monetary Policy and Bank Lending”, NBER Working Paper Series, No. 4015.

Layard, R., S. Nickell and R. Jackman (1991), *Unemployment: Macroeconomic Performance and the Labour Market*, Oxford, Oxford University Press.

Minsky, H. P. and P. Ferri (1984), “Prices, Employment and Profits”, *Journal of Post Keynesian Economics*, Vol. 6, pp. 489-99.

Modigliani, F. and L. Papademos (1990), “The Supply of Money”, in B. M. Friedman and H. Hahn, *Handbook in Monetary Economics*, Amsterdam, North Holland.

Monticelli, C. (1993), “Monetary Coordination under an Exchange Rate Agreement and the Optimal Monetary Instrument”, Banca d’Italia, Temi di discussione, No. 214.

Poole, W. (1970), “Optimal Choice of Monetary Instruments in a Single Stochastic Macro Model”, *Quarterly Journal of Economics*, Vol. 84, pp. 197-216.

Rajan, R. (1992), “Insiders and Outsiders: The Choice between Informed and Arm’s Length Debt”, *Journal of Finance*, Vol. 47, pp. 1367-400.

Rogoff, K. (1985), “The Optimal Degree of Commitment to an Intermediate Monetary Target”, *Quarterly Journal of Economics*, Vol. 100, pp. 1169-90.

Sharpe, S. (1990), “Asymmetric Information, Bank Lending and Implicit Contract: A Stylized Model of Customer Relationships”, *Journal of Finance*, Vol. 45, pp. 1069-87.

Turnovsky, S. J. and V. d'Orey (1989), "The Choice of Monetary Instrument in Two Interdependent Economies under Uncertainty", *Journal of Monetary Economics*, Vol. 23, pp. 121-33.

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